Testing SU(3) flavour symmetry in non-leptonic B and D decays

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#solvingbeautifulpuzzles

Why study non-leptonic B decays?

- Only hadrons in the process, both initial and final state
- Very challenging to describe theoretically
- Tremendous amount of data for different final states and observables
- Important probes for CP violation
  - Large CP asymmetries with processes with tiny BRs and vice versa
  - Probes for new sources of CP?

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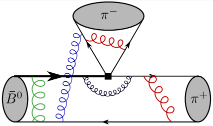
Challenge: Calculation of Hadronic matrix elements

How to handle nonleptonic B decays?



QCD Factorization

Beneke, Buchalla, Neubert, Sachrajda

- Disentangle perturbative (calculable) and non-perturbative dynamics using HQE
- Systematic expansion in α_s and $1/m_b$ (studied up to α_s^2) Bell, Beneke, Huber, Li

$$\langle \pi^+\pi^- | \mathcal{Q}_i | B \rangle = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^-} + T_i^{II} \otimes \Phi_{\pi^-} \otimes \Phi_{\pi^+} \otimes \Phi_B$$

- Non-perturbative form factors and LCDAs
  - from data, lattice or Light-Cone Sum Rules
- No systematic framework to compute power corrections (yet?)
- Strong phases suffer from large uncertainties
- Theoretical challenge: reliable computations of observables

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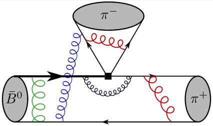

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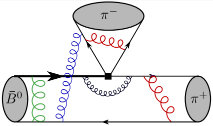

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Flavour symmetries (Isospin or $SU(3)$)

- In subsystems e.g. Fleischer, Jaarsma, KKV, Malami [2017,2018]
- Global $SU(3)$ fit to $B \to PP$ decays Huber, Tetlalmatzi-Xolocotzi [2111.06418], Berthiaume et al.[2311.18011] Burgos Marcos, Reboud, KKV [2504.05209]

Light-cone sumrules

- Calculation of soft-gluon contribution Piscopo, Rusov [2307.07594] (see also Khodjamirian, Mannel, Melic, Melcher [0509049])

Full $SU(3)_F$ analysis of $B \to PP$ decays

Based on M. Burgos Marcos, M. Reboud, KKV [2504.05209]

Topologies non-leptonic B decays

$$\lambda_i^{(p)} \equiv V_{ib} V_{ip}^*, \bar{H}_1^{12} = \delta_{pd}, \bar{H}_1^{13} = \delta_{ps}$$

$$\mathcal{A}(\bar{B}_q \to M_1 M_2) = i \frac{G_F}{\sqrt{2}} \left[\lambda_u^{(p)} A_p^{ut} + \lambda_c^{(p)} A_p^{ct} \right],$$

- A_p^{ut} are "tree" topologies
- A_p^{ct} are "penguin" topologies
- Typically: topological diagram or in irreducible $SU(3)_F$ representation amplitudes

$$\begin{aligned} A^{ut}_{p,\text{topo}} \equiv \quad & T \; B_i (M)^i_j \bar{H}^{jl}_k (M)^k_l + C \; B_i (M)^i_j \bar{H}^{lj}_k (M)^k_l + A \; B_i \bar{H}^{il}_j (M)^j_k (M)^k_l \\ & + E \; B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + T_{PA} \; B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + T_P \; B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l \end{aligned}$$

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & 0 \end{pmatrix} \qquad B_i = (B^+, B^0, B_s^0)$$

* excluding η modes

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- For penguins replace: $T \to P_T, C \to P_C, A \to P_{TA}, T_P \to P, E \to P_{TE}, T_{PA} \to P_A,$
- One-to-one correspondence with irreducible representation He,Wang[1803.04227]

  $\to$ 12 complex parameters: two redundant operators

$SU(3)_F$-limit analysis

- Full $SU(3)_F$ symmetry fit: $\chi^2 = 32.3$ for 15 degrees of freedom (34 constraints for 19 parameters)
- p-value $= 0.58\%$ below our threshold: fit is not satisfactory
- Similar conclusion found in Berthiaume et all. [2311.18011]

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Difference with previous works:

→ Include mixing-induced CP asymmetry

→ Include mixing effects in B_s^0 branching ratios

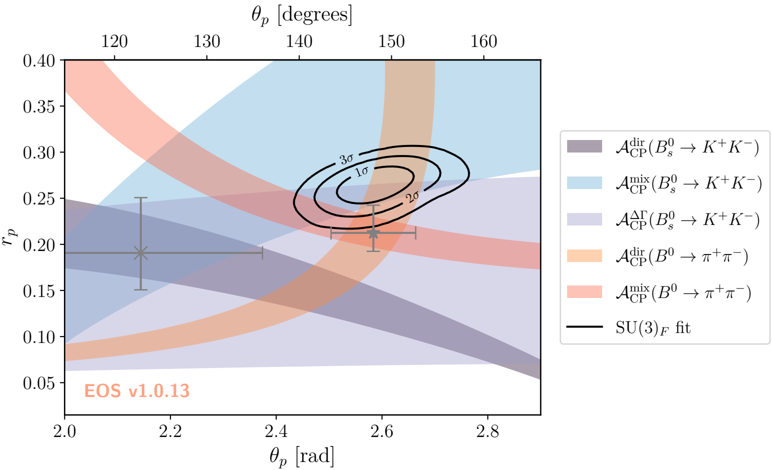
→ Include correlations by using ratios of branching ratios directly

→ Omit $\eta^{(\prime)}$ modes

$$\mathcal{A}_{\rm CP}(t) = \frac{\Gamma(B_q^0(t)\to f) - \Gamma(\bar{B}_q^0(t)\to f)}{\Gamma(B_q^0(t)\to f) + \Gamma(\bar{B}_q^0(t)\to f)} = \frac{\mathcal{A}_{\rm CP}^{\rm dir}\cos(\Delta M_q t) + \mathcal{A}_{\rm CP}^{\rm mix}\sin(\Delta M_q t)}{\cosh(\Delta\Gamma_q t/2) + \mathcal{A}_{\rm CP}^{\Delta\Gamma}\sinh(\Delta\Gamma_q t/2)},$$

$SU(3)_F$-limit analysis

M. Burgos Marcos, M. Reboud, KKV [2504.05209] $r_p e^{i\theta_p} = \frac{A^{ct}_{p,\text{topo}}}{A^{ut}_{p,\text{topo}}}$



- U-spin partners: $B^0_d \to \pi^+\pi^-$ and $B^0_s \to K^+K^-$
- CP asymmetries already show $SU(3)_F$ breaking at 2σ level

Factorizable $SU(3)_F$ breaking effects

Based on M. Burgos Marcos, M. Reboud, KKV [2504.05209]

Including $SU(3)_F$ breaking

M. Burgos Marcos, M. Reboud, KKV [2504.05209]

- Systematic approach to include linear $SU(3)_F$ breaking as perturbation in the Hamiltonian Gronou et al. [9504327]
- Here: Use $SU(3)_F$ breaking in decay constants and form factors
- Adopt parametrization used in QCDF:

$$\mathcal{A}(\bar{B}_q \to M_1 M_2) = i\frac{G_F}{\sqrt{2}} \sum_{r=u,c} A^{B_q}_{M_1 M_2} \Big\{ BM_1 \left(\alpha_1 \delta_{ru} \hat{U} + \alpha_4^r \hat{I} + \alpha^r_{4,EW} \hat{Q}\right) M_2 \Lambda_r$$
$$+ BM_1 \Lambda_r \operatorname{Tr}\left[\left(\alpha_2 \delta_{ru} \hat{U} + \alpha^r_{3,EW} \hat{Q}\right) M_2\right]$$
$$+ B\left(\beta_2 \delta_{ru} \hat{U} + \beta_3^r \hat{I} + \beta^r_{3,EW} \hat{Q}\right) M_1 M_2 \Lambda_r$$
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\end{aligned}$$

$$\Lambda_r = \begin{pmatrix} 0 \\ \lambda_r^{(d)} \\ \lambda_r^{(s)} \end{pmatrix}, \qquad \hat{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \hat{Q} = \frac{3}{2} Q = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix},$$

Factorizable $SU(3)_F$ breaking

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- Factorizable $SU(3)_F$ breaking: $A^{B_q}_{M_1 M_2} = M_B^2 \, F_0^{B\to M_1}(m^2_{M_2})\, f_{M_2}$
- Maximal in $B_d \to \pi^+\pi^-$ versus $B_s \to K^+K^-$: 20% effect
- β parameters are power suppressed annihilation operators in QCDF
  $A^{B_q}_{M_1 M_2} \beta_i^p \to B^{B_q}_{M_1 M_2}\, b_i^p$ where $B^{B_q}_{M_1 M_2} = f_{B_q} f_{M_1} f_{M_2}$

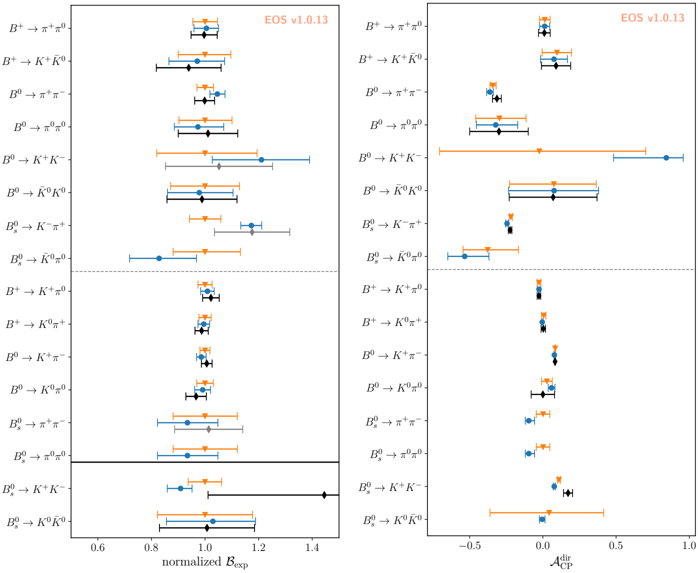
Factorizable $SU(3)_F$ breaking analysis

M. Burgos Marcos, M. Reboud, KKV [2504.05209]

- Fit all available $B \to PP$ modes; 23 real parameters
- Excellent description of the data with p-value of 32.2%
  - $\rightarrow$ $SU(3)_F$ breaking of $20 - 30\%$ gives perfect description of data
  - $\rightarrow$ Identify several decays were new data could improve our understanding
  - $\rightarrow$ No puzzling patterns in $B \to \pi K$ modes

Factorizable $SU(3)_F$ breaking

M. Burgos Marcos, M. Reboud, KKV [2504.05209]; black Data, Orange $SU(3)_F$ breaking analysis, Blue $SU(3)_F$ analysis



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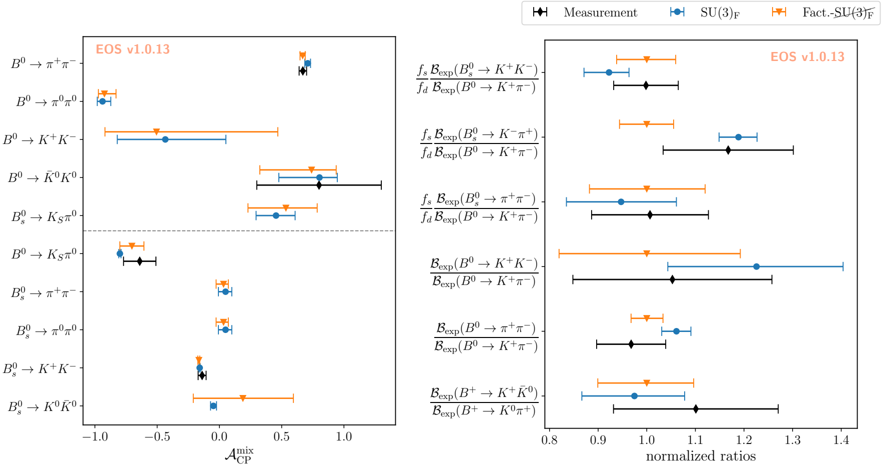

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Key modes to improve:

- Branching ratios of $B^0_{d,s} \to K^0 \bar{K}^0$ and $B^+ \to K^0 K^+$, $B^+ \to K^0 \pi^+$
- CP asymmetries in pure annihilation modes like: $B \to K^+ K^-$
- Updates of the – already precise – $B^0_s \to K^+ K^-$ decay

$B \to \pi K$ puzzle

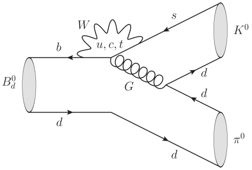
The $B \to K\pi$ Puzzle

e.g. Buras, Fleischer, Recksiegel, Schwab [2004, 2007];Fleischer, Jaeger, Pirjol, Zupan [2008]
Neubert, Rosner [1998]; Beaudry, Datta, London, Rashed, Roux [2018]; Fleischer, Jaarsma, KKV [2018]

(Longstanding) Puzzling patterns in $B \to \pi K$ data

$$\delta(\pi K) \equiv A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

- $\delta(\pi K)^{\rm exp} = (11.5 \pm 1.4)\%$
- 8σ from 0



- Color-suppressed tree (and EW penguins) contribute!
  - $\delta(\pi K)^{\rm QCDF} = (2.1^{+2.8}_{-4.6})\%$ [Bell, Beneke, Huber, Li]
  - or via $SU(3)$ [Fleischer, Jaarsma, Malami, KKV [2018]]
- Hint for NP in the EWP sector?
- Our fit: $\left|\frac{\alpha^c_{4,\rm EW}}{\tilde{\alpha}^c_4}\right| \sim 1 \to$ large EW effects!

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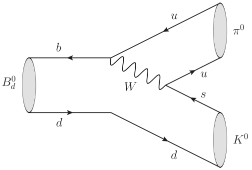

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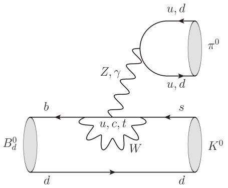

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Isospin sumrule

$$\Delta_{\rm SR} = -\mathcal{A}_{\rm CP}^{\rm dir}(B^+ \to \pi^0 K^+)\frac{2\mathcal{B}(B^+ \to \pi^0 K^+)}{\mathcal{B}(B_d^0 \to \pi^- K^+)}\frac{\tau_{B^0}}{\tau_{B^+}} - \mathcal{A}_{\rm CP}^{\rm dir}(B_d^0 \to \pi^0 K^0)\frac{2\mathcal{B}(B_d^0 \to \pi^0 K^0)}{\mathcal{B}(B_d^0 \to \pi^- K^+)}$$
$$+ \mathcal{A}_{\rm CP}^{\rm dir}(B_d^0 \to \pi^- K^+) + \mathcal{A}_{\rm CP}^{\rm dir}(B^+ \to \pi^+ K^0)\frac{\mathcal{B}(B^+ \to \pi^+ K^0)}{\mathcal{B}(B_d^0 \to \pi^- K^+)}\frac{\tau_{B^0}}{\tau_{B^+}} \ ,$$

- Sensitive to new physics effects: $\Delta_{\rm SR}^{\rm QCDF} \sim 1\%$ [Bell, Beneke, Huber, Li]
- $\Delta_{\rm SR}^{\rm exp.} = 0.12 \pm 0.08$
- Our analysis: $\Delta_{\rm SR}^{\rm Fact.SU(3)_F} = 0.097 \pm 0.050$
- Sizable uncertainty, but in agreement with theoretical expectations

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Comparing with QCDF

Comparison with QCDF

M. Burgos Marcos, M. Reboud, KKV [2504.05209]

- Several redefinitions required
- Only sensitive to combinations of parameters

  $\rightarrow$ $\tilde{\alpha}_1 + \tilde{\alpha}_4^u = 0.584 \pm 0.024$

  $\rightarrow$ $\frac{3}{2}\alpha_{4,\mathrm{EW}}^c + \tilde{\alpha}_4^c = -\big(0.102 \pm 0.001\big) + \big(0.044 \pm 0.002\big)\, i,$

$$\left|\frac{\alpha_{4,EW}^c}{\tilde{\alpha}_1}\right| \sim \left|\frac{\tilde{\alpha}_4^c}{\tilde{\alpha}_4^u}\right| \sim \left|\frac{\alpha_{3,EW}^c}{\tilde{\alpha}_2}\right| \in [10^{-4}, 10^{-2}]\,,$$

- Challenging to compare due to mixture with end-point divergent terms

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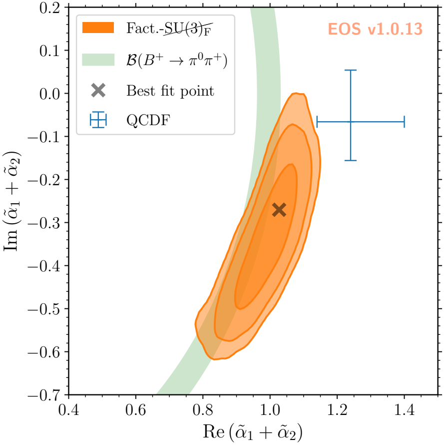

Conclusion

- With Factorizable $SU(3)_F$ breaking we get a perfect description of the data
- 20-30% $SU(3)_F$ breaking sufficient!

- Challenging to compare to QCDF as only *combinations* are constrained

  $\rightarrow$ Identify several modes where new data would help

- In progress:

  $\rightarrow$ Include $B \rightarrow VP$ modes
  $\rightarrow$ Linear $SU(3)_F$-breaking analysis in $B \rightarrow PP$
  $\rightarrow$ Detailed comparison with QCDF

What about η modes?

Comment on η modes

Bolognani, Nierste, Schacht, KKV [2410.08138]

$|\eta_8\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle}{\sqrt{6}}$, $|\eta_0\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle}{\sqrt{3}}$,

- Within $SU(3)_F$ a *single, universal* mixing angle between η and η' states cannot be used!
- η' decouples *completely* from η modes

$$\begin{aligned} \langle P\eta|H|D\rangle &= \cos\theta\langle P\eta_8|H|D\rangle - \sin\theta\langle P\eta_0|H|D\rangle \\ \langle P\eta'|H|D\rangle &= \sin\theta\langle P\eta_8|H|D\rangle' + \cos\theta\langle P\eta_0|H|D\rangle' , \end{aligned}$$

and most importantly

$$\begin{aligned} \langle P\eta_8|H|D\rangle' &\neq \langle P\eta_8|H|D\rangle , \\ \langle P\eta_0|H|D\rangle' &\neq \langle P\eta_0|H|D\rangle . \end{aligned}$$

- Linear $SU(3)_F$ breaking can be included using the powercounting: $\theta = \mathcal{O}(\varepsilon)$
- Allows to relate $\langle P\eta_8|H|D\rangle$ and $\langle PP'|H|D\rangle$

Comment on η modes

M. Burgos Marcos, M. Reboud, KKV [2504.05209]

- Modes with η not included
- Postdict modes in full $SU(3)_F$

Channel	Branching Ratios in units of 10^{-6} Experimental value	$\mathbf{SU(3)_F}$
$B^+ \to \eta\pi^+$	4.02 ± 0.27	$3.1^{+0.9}_{-1.6}$
$B^0 \to \eta\pi^0$	$0.41^{+0.17+0.05}_{-0.15-0.07}$[81]	0.44 ± 0.11
$B_s^0 \to \eta\bar{K}^0$	Not available	$0.376^{+0.063}_{-0.050}$
$B^0 \to \eta\eta$	$0.5 \pm 0.3 \pm 0.1$ [82]	$0.35^{+0.13}_{-0.11}$
$B^+ \to \eta K^+$	2.4 ± 0.4	$3.84^{+0.18}_{-0.17}$
$B^0 \to \eta K^0$	$1.23^{+0.27}_{-0.23}$	3.4 ± 0.10
$B_s^0 \to \eta\pi^0$	$< 10^3$ [83]	$11.4^{+14.4}_{-9.9}$
$B_s^0 \to \eta\eta$	$100 \pm 105 \pm 23$[84]	$10.6^{+5.9}_{-9.0}$

- Requires closer look at form factors and decay constants

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What about Charm?

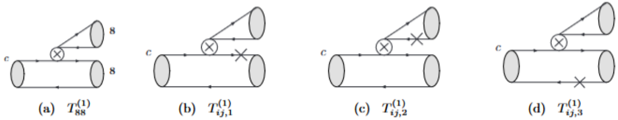
Testing linear $SU(3)_F$ breaking

Bolognani, Nierste, Schacht, KKV [2410.08138]
See: Gronau [1995]; Mueller, Nierste, Schacht [1503.06759]

$$H_{SU(3)_F-\mathrm{break}} = (m_s - m_d)\bar{s}s$$

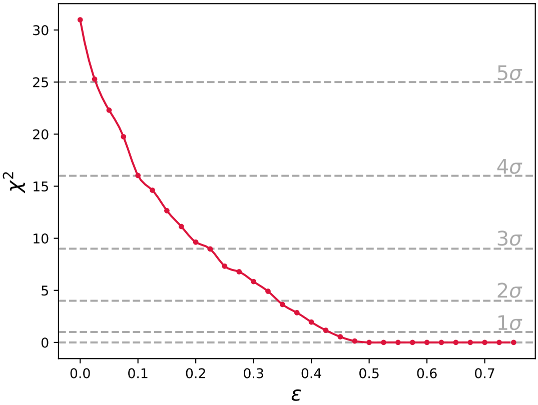
- Feynman rule is denoted by a cross on the s-quark line
- Allows to write the SU(3)$_F$-breaking decomposition of the eight $D \to P\eta'$
  - Completely decouple from η and P modes*
- After redefinition: 17 parameters
- Include constraint on $SU(3)_F$ breaking through:

$$\left|\frac{\mathcal{A}_{SU(3)-\mathrm{break}}(D\to P\eta')}{\mathcal{A}_{\mathrm{SU(3)-lim}}(D\to P\eta')}\right| \leq \varepsilon$$



Testing linear $SU(3)_F$ breaking

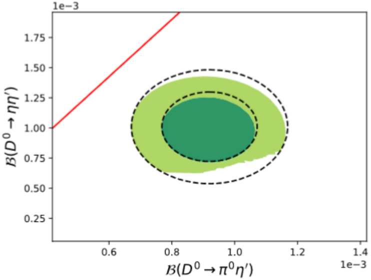
Bolognani, Nierste, Schacht, KKV [2410.08138]

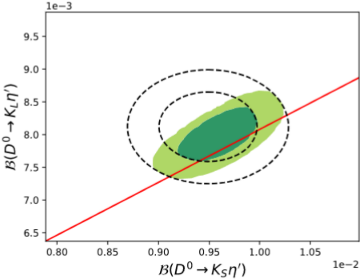


- Nominal fit $\varepsilon = 30\%$ gives slight 2.5σ tension with data

Testing linear $SU(3)_F$ breaking

Bolognani, Nierste, Schacht, KKV [2410.08138]





- Improved measurements of $D^0 \to \pi^0\eta'$ and $D^0 \to \eta\eta'$ can further test $SU(3)_F$ in charm
- Comparison with study of all $D \to PP$ decays in progress

Comparison with QCDF?

- Factorization not expected to work, maybe for tree?
- Could use large N_c arguments?

Decay ampl. $\mathcal{A}(d)$	$\hat{T}_{18}$	$\hat{T}^{(1)}_{18,1}$	$\hat{T}^{(1)}_{18,2}$	$\hat{T}^{(1)}_{88}$	$\hat{C}^{(1)}_{18,1}$	$\hat{C}_{81}$	$\hat{C}^{(1)}_{81,1}$	$\hat{C}^{(1)}_{81,2}$	$\hat{C}^{(1)}_{88}$
				SCS					
$\mathcal{A}(D^0 \to \pi^0\eta')$	0	0	0	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	$-\frac{1}{\sqrt{3}}$
$\mathcal{A}(D^0 \to \eta\eta')$	0	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	-1
$\mathcal{A}(D^+ \to \pi^+\eta')$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
$\mathcal{A}(D_s^+ \to K^+\eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
				CF					
$\mathcal{A}(D^0 \to \overline{K}^0\eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D_s^+ \to \pi^+\eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$-\sqrt{\frac{2}{3}}$	0	0	0	0	0
				DCS					
$\mathcal{A}(D^0 \to K^0\eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D^+ \to K^+\eta')$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	0	0	0	0	0

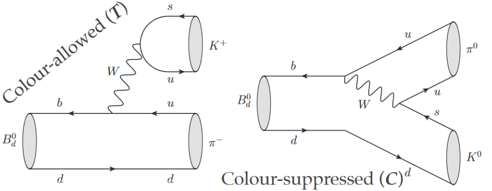
- Many redefinitions, do not allow to obtain T alone

Conclusion

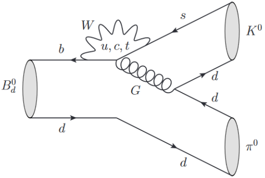
- Entering an era in B decays where we can test $SU(3)_F$
- Further tests of $SU(3)_F$ in charm of interest
- Could help to improve QCD-based description of the data
  - Insights into weak annihilation?
- In progress:
  - $\rightarrow$ Include $B \rightarrow VP$ modes
  - $\rightarrow$ Linear $SU(3)_F$-breaking analysis in $B \rightarrow PP$
  - $\rightarrow$ Detailed comparison with QCDF
  - $\rightarrow$ Updated analysis of $D \rightarrow PP$

Backup-Slides

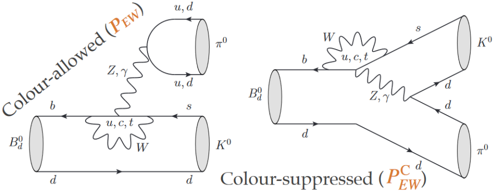
Tree topologies



QCD penguin (*P*)



Electroweak (EW) penguins



- Many decay topologies
- Different CKM dependencies
- Different dominant contributions
- Some more challenging to compute

Parametrizations

- In QCDF $\alpha(M_1 M_2)$ depend on final states (tiny effect)
- One-to-one correspondence with amplitudes

$$
\begin{aligned}
T &= A^{B_q}_{M_1 M_2}\,\tilde{\alpha}_1, & C &= A^{B_q}_{M_1 M_2}\,\tilde{\alpha}_2, & T_P &= A^{B_q}_{M_1 M_2}\,\tilde{\alpha}^u_4, \\
A &= B^{B_q}_{M_1 M_2}\,\tilde{b}_2, & E &= B^{B_q}_{M_1 M_2}\,\tilde{b}_1, & T_{PA} &= B^{B_q}_{M_1 M_2}\,\tilde{b}^u_4,
\end{aligned}
$$

- if $A^{B_q}_{M_1 M_2}$ is universal

$$
A^{B_q}_{M_1 M_2} = M_B^2\, F_0^{B\to M_1}(m^2_{M_2})\, f_{M_2},
$$

Redefinitions

$$
\begin{aligned}
\tilde{\alpha}_1 &\equiv \alpha_1 + \frac{3}{2}\alpha^{u}_{4,\mathrm{EW}}, & \tilde{\alpha}_2 &\equiv \alpha_2 + \frac{3}{2}\alpha^{u}_{3,\mathrm{EW}}, \\
\tilde{\beta}_1 &\equiv \beta_1 + \frac{3}{2}\beta^{u}_{4,\mathrm{EW}}, & \tilde{\beta}_2 &\equiv \beta_2 + \frac{3}{2}\beta^{u}_{3,\mathrm{EW}}, \\
\tilde{\alpha}^{r}_4 &\equiv \alpha^{r}_4 + \beta^{r}_3 - \frac{1}{2}\left(\alpha^{r}_{4,\mathrm{EW}} + \beta^{r}_{3,\mathrm{EW}}\right), & \tilde{\beta}^{r}_4 &\equiv \beta^{r}_4 - \frac{1}{2}\beta^{r}_{4,\mathrm{EW}},
\end{aligned}
$$

Weak annihilation

- Strong constraints on several combinations from $B_d^0 \to K^+K^-$ decay and its U-spin partner $B_s^0 \to \pi^+\pi^-$
- $\tilde{b}_1 + 2\tilde{b}_4^u = -\left(3.7^{+9.6}_{-8.2}\right) + \left(13.7^{+6.9}_{-11.9}\right) i$, and
$\frac{3}{2} b_{4,EW}^c + 2\tilde{b}_4^c = -\left(1.5^{+1.4}_{-1.5}\right) + \left(11.0^{+4.6}_{-5.2}\right) i$.
- $\left|\frac{\tilde{b}_2}{\tilde{b}_1}\right| = 0.99 \pm 0.05$

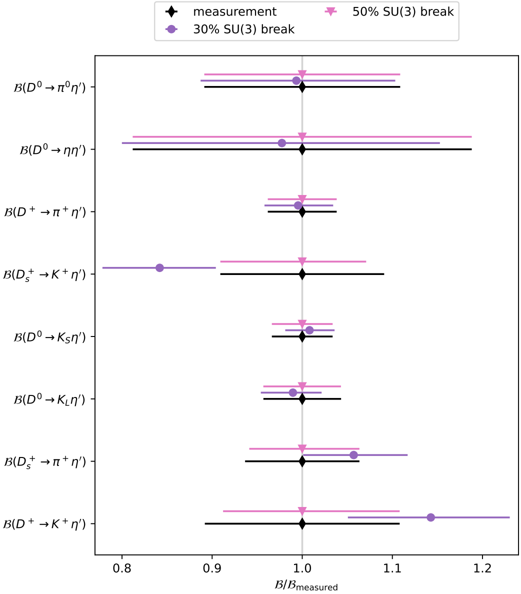
Less constrained parameters have:

$$\left|\frac{b_{4,EW}^c}{\tilde{b}_1}\right| \sim \left|\frac{b_{3,EW}^c}{\tilde{b}_2}\right| \sim \left|\frac{\tilde{b}_4^c}{\tilde{b}_4^u}\right| \in [10^{-4}, 10^{-2}] \,,$$

indicating that the electroweak parameters are small and that $\tilde{b}_4^c$ is much smaller than $\tilde{b}_4^u$.

Testing linear $SU(3)_F$ breaking

Bolognani, Nierste, Schacht, KKV [2410.08138]



Testing linear $SU(3)_F$ breaking

Bolognani, Nierste, Schacht, KKV [2410.08138]

Observable	Global theory fit	Experiment
$\mathcal{B}(D^0 \to \pi^0\eta')$	$(9.15^{+1.00}_{-0.99}) \cdot 10^{-4}$	$(9.2 \pm 1.0) \cdot 10^{-4}$
$\mathcal{B}(D^0 \to \eta\eta')$	$(0.98 \pm 0.18) \cdot 10^{-3}$	$(1.01 \pm 0.19) \cdot 10^{-3}$
$\mathcal{B}(D^+ \to \pi^+\eta')$	$(4.95 \pm 0.19) \cdot 10^{-3}$	$(4.97 \pm 0.19) \cdot 10^{-3}$
$\mathcal{B}(D_s^+ \to K^+\eta')$	$(2.22 \pm 0.17) \cdot 10^{-3}$	$(2.64 \pm 0.24) \cdot 10^{-3}$
$\mathcal{B}(D^0 \to K_S\eta')$	$(9.56^{+0.27}_{-0.25}) \cdot 10^{-3}$	$(9.49 \pm 0.32) \cdot 10^{-3}$
$\mathcal{B}(D^0 \to K_L\eta')$	$(8.04^{+0.26}_{-0.29}) \cdot 10^{-3}$	$(8.12 \pm 0.35) \cdot 10^{-3}$
$\mathcal{B}(D_s^+ \to \pi^+\eta')$	$(4.17 \pm 0.23) \cdot 10^{-2}$	$(3.94 \pm 0.25) \cdot 10^{-2}$
$\mathcal{B}(D^+ \to K^+\eta')$	$(2.11 \pm 0.17) \cdot 10^{-4}$	$(1.85 \pm 0.20) \cdot 10^{-4}$

Redefinitions

Bolognani, Nierste, Schacht, KKV [2410.08138]

$$
\begin{aligned}
\hat{T}^{(1)}_{18,1} &= T^{(1)}_{18,1} + T^{(1)}_{18,3} + A^{(1)}_{18,1} + A^{(1)}_{81,1} + 3A^{(1)}_{H,1} + 3\sqrt{2}A^{(1)}_{88} + 3\sqrt{2}E^{(1)}_{88}\,, \\
\hat{T}^{(1)}_{18,2} &= T^{(1)}_{18,2} + A^{(1)}_{18,2} + A^{(1)}_{81,2} + A^{(1)}_{81,3} + 3A^{(1)}_{H,2} - \frac{3}{\sqrt{2}}A^{(1)}_{88} - \frac{3}{\sqrt{2}}E^{(1)}_{88}\,, \\
\hat{T}^{(1)}_{88} &= T^{(1)}_{88} + 2A^{(1)}_{88} + 3E^{(1)}_{88}\,, \\
\hat{C}^{(1)}_{18,1} &= C^{(1)}_{18,1} + C^{(1)}_{18,2} + P^{(1)}_{18,\mathrm{break}} + P^{(1)}_{81,\mathrm{break}}\,, \\
\hat{C}^{(1)}_{81,1} &= C^{(1)}_{81,1} + E^{(1)}_{18,1} + E^{(1)}_{18,3} + E^{(1)}_{81,1} + 3E^{(1)}_{H,1}\,, \\
\hat{C}^{(1)}_{81,2} &= C^{(1)}_{81,2} + E^{(1)}_{18,2} + E^{(1)}_{81,2} + E^{(1)}_{81,3} + 3E^{(1)}_{H,2}\,, \\
\hat{C}^{(1)}_{88} &= C^{(1)}_{88} - E^{(1)}_{88}\,.
\end{aligned}
$$