The magnetic dipole moments of the charged leptons

A stroll through the standard model to find out what may lie beyond it...

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OUTLINE

- Introduction: What are we talking about?

- Experimental aspects

- Theory

- QED contributions
- Weak contributions
- Strong interactions

- Conclusions - Perspectives for the future

The charged leptons of the three-family standard model

$q = \pm 1$ (charge) $\quad s = \frac{1}{2}$ (spin)

- $e^{\pm}$

- $\mu^{\pm}$

- $\tau^{\pm}$

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- $\tau^{\pm}$ $\quad m_\tau = 1776.93(09)\ \mathrm{MeV}$

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$$m_e = 0.510\,998\,950\,00(15)\ \mathrm{MeV}$$

$$\tau_e \gtrsim 10^{28}\ \mathrm{yrs} \quad (e \to \nu_e \gamma)$$

- $\mu^{\pm}$

$$m_\mu = 105.6583755(23)\ \mathrm{MeV}$$

$$\tau_\mu = 2.1969811(22) \cdot 10^{-6} s \quad (\mu^- \to e^- \bar{\nu}_e \nu_\mu)$$

- $\tau^{\pm}$

$$m_\tau = 1776.93(09)\ \mathrm{MeV}$$

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Response of a charged particle to an external static electromagnetic field

static electric field $\longrightarrow$ total charge, electric dipole moment,...

static magnetic field $\longrightarrow$ magnetic dipole moment,...

Natural starting point:
quantum mechanics of a charged particle (m_ℓ, q_ℓ) in an electromagnetic field

$\longrightarrow$ Schrödinger equation with the minimal coupling precription

$$i\hbar\,\frac{\partial\varphi}{\partial t} \;=\; \left[\frac{(-i\hbar\boldsymbol{\nabla} - (q_\ell/c)\boldsymbol{\mathcal{A}})^2}{2m_\ell} + \,q_\ell\mathcal{A}_0\right]\varphi$$

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$\longrightarrow$ Schrödinger-Pauli equation

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with $\boldsymbol{\mu_\ell}\;=\;g_\ell\left(\frac{q_\ell}{2m_\ell c}\right)\mathbf{S},\;\mathbf{S}=\hbar\,\frac{\boldsymbol{\sigma}}{2},$ i.e. $g_\ell^{\text{Pauli}}=2$ [g_ℓ = gyromagnetic factor]

Relativistic effects?

$\longrightarrow$ Dirac equation with the minimal coupling precription

$$i\hbar\,\frac{\partial\psi}{\partial t} \;=\; \left[c\boldsymbol{\alpha}\cdot\left(-i\hbar\boldsymbol{\nabla}-\frac{q_\ell}{c}\boldsymbol{\mathcal{A}}\right)+\beta m_\ell c^2+q_\ell\mathcal{A}_0\right]\psi$$

In the non relativistic limit, this reduces to the Pauli equation for the two-component spinor φ describing the large components of the Dirac spinor ψ,

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with $\boldsymbol{\mu_\ell} = g_\ell\left(\frac{q_\ell}{2m_\ell c}\right)\mathbf{S},\ \mathbf{S} = \hbar\,\frac{\boldsymbol{\sigma}}{2},$ i.e. $g_\ell^{\rm Dirac} = 2$

Misses an important aspect: ceaseless emission and absorption of virtual particles (SM or not!)

$\longrightarrow$ requires QFT

One wants to probe the response of a charged lepton to an external, static, and weak electromagnetic field $\longrightarrow$ linear response: $-q_\ell A^\rho J_\rho$

$$\langle \ell; p'|J_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p', p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\Big[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\Big]\mathrm{u}(p)$$

uses only the conservation of the electromagnetic current $J_\rho \equiv \bar{\psi}_\ell\gamma_\rho\psi_\ell$, $k_\mu \equiv p'_\mu - p_\mu$

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/q_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\ G_M(k^2) = F_1(k^2) + F_2(k^2)$$

$$G_E(0) = 1 \quad G_M(0) = 1 + F_2(0) = g_\ell/2$$

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At tree level in the SM

$$F_1^{\text{tree}} = 1 \quad \text{and} \quad F_2^{\text{tree}} = F_3^{\text{tree}} = F_4^{\text{tree}} = 0$$

Tree-level contributions to $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ would require operators of dimensions > 4 $\longrightarrow$ no counterterms available in the SM!

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops

$\longrightarrow$ calculable! (i.e. UV finite)

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The *anomalous* magnetic moment is induced at loop level:

$$a_\ell \equiv \frac{g_\ell - g_\ell^{\text{Dirac}}}{g_\ell^{\text{Dirac}}} \; (\equiv F_2(0))$$

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... provided one can measure it and *predict it with enough precision !*

Why the muon?

The anomalous magnetic moment of a lepton ℓ is a dimensionless quantity

New-Physics scale Λ_{NP} enters through

$$a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda_{\mathrm{NP}}^2}$$

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$\longrightarrow$ caveat: assumes that NP is decoupling and couples in a LFU way!

Experimental aspects

• The electron case

1947: hf splitting in H and D (0.2% discrepancy with the value $g_e^{\mathrm{Dirac}} = 2$)

[J. Nafe, E. B. Nelson, I. I. Rabi, Phys. Rev. 71, 914 (1947)]

1958: first direct measurement of g_e for *free* electrons

[H. G. Dehmelt, Phys. Rev. 109, 381 (1958)]

1968 $\longrightarrow$ 1987: Penning trap type experiments $\longrightarrow$ single trapped electron (geonium)

$$\begin{aligned} a_{e^-}^{exp} &= 1\,159\,652\,188.4(4.3)\cdot 10^{-12} \quad [3.7\,\mathrm{ppb}] \\ a_{e^+}^{exp} &= 1\,159\,652\,187.9(4.3)\cdot 10^{-12} \quad [3.7\,\mathrm{ppb}] \end{aligned}$$

[R.S. van Dyck Jr. et al., PRL 59, 26 (1987)]

New series of high precision measurements at Harward and at NW Univ.

$$a_e^{exp} = 1\,159\,652\,180.85(76) \cdot 10^{-12} \quad [0.66\,\text{ppb}]$$

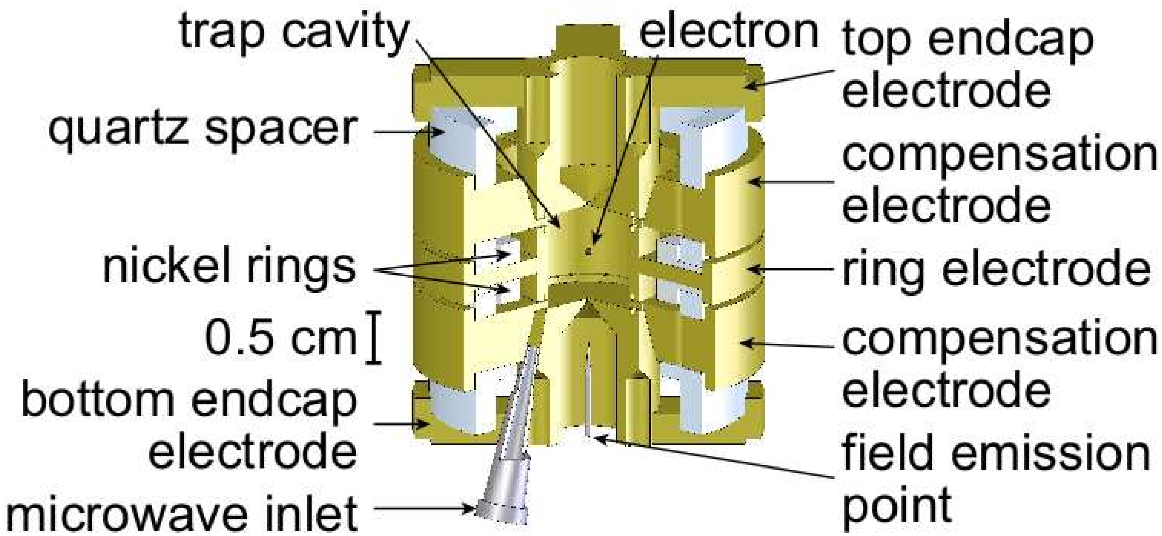
[Odom et al., PRL 97, 030801 (2006)]

$$a_e^{exp} = 1\,159\,652\,180.73(28) \cdot 10^{-12} \quad [0.24\,\text{ppb}]$$

[D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)]

$$a_e^{exp} = 1\,159\,652\,180.59(13) \cdot 10^{-12} \quad [0.11\,\text{ppb}]$$

[X. Fan, T. G. Myers, B. A. D. Sukra, G. Gabrielse, PRL 130, 071801 (2023)]



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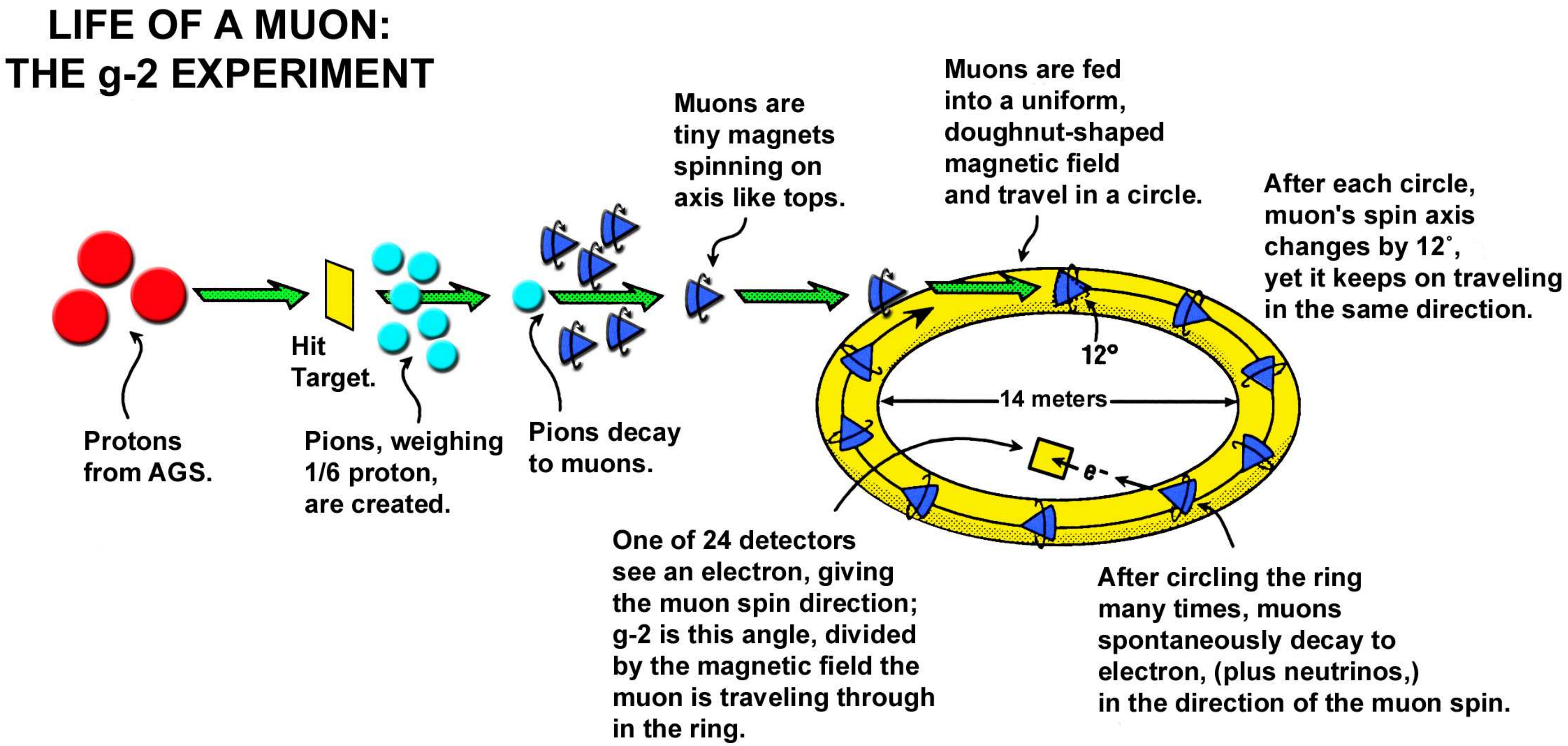

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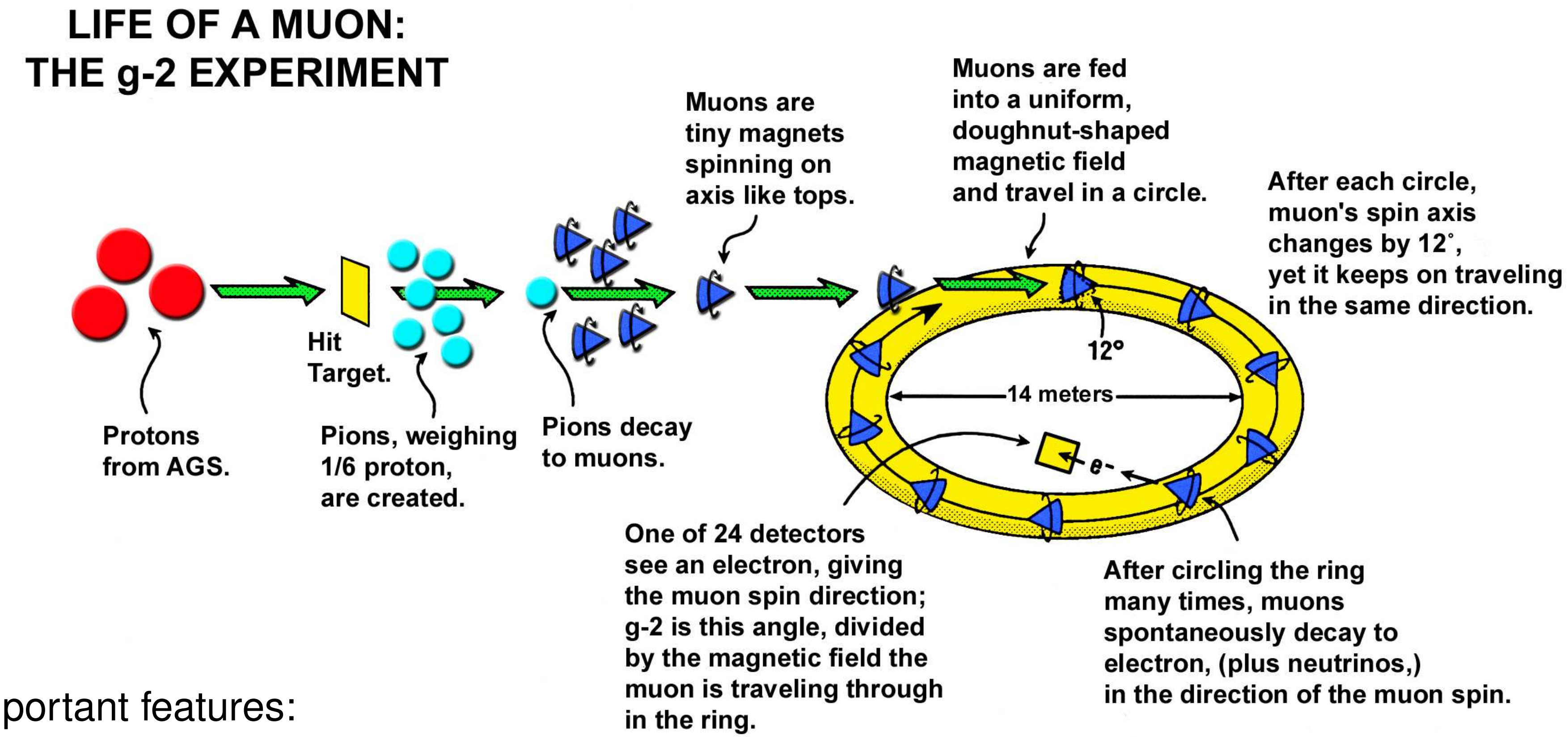

Two important features:

- the most energetic muons emitted in the decay of the pions are forward polarized

- the most energetic positrons are emitted in the direction of the spin of the decaying μ^+

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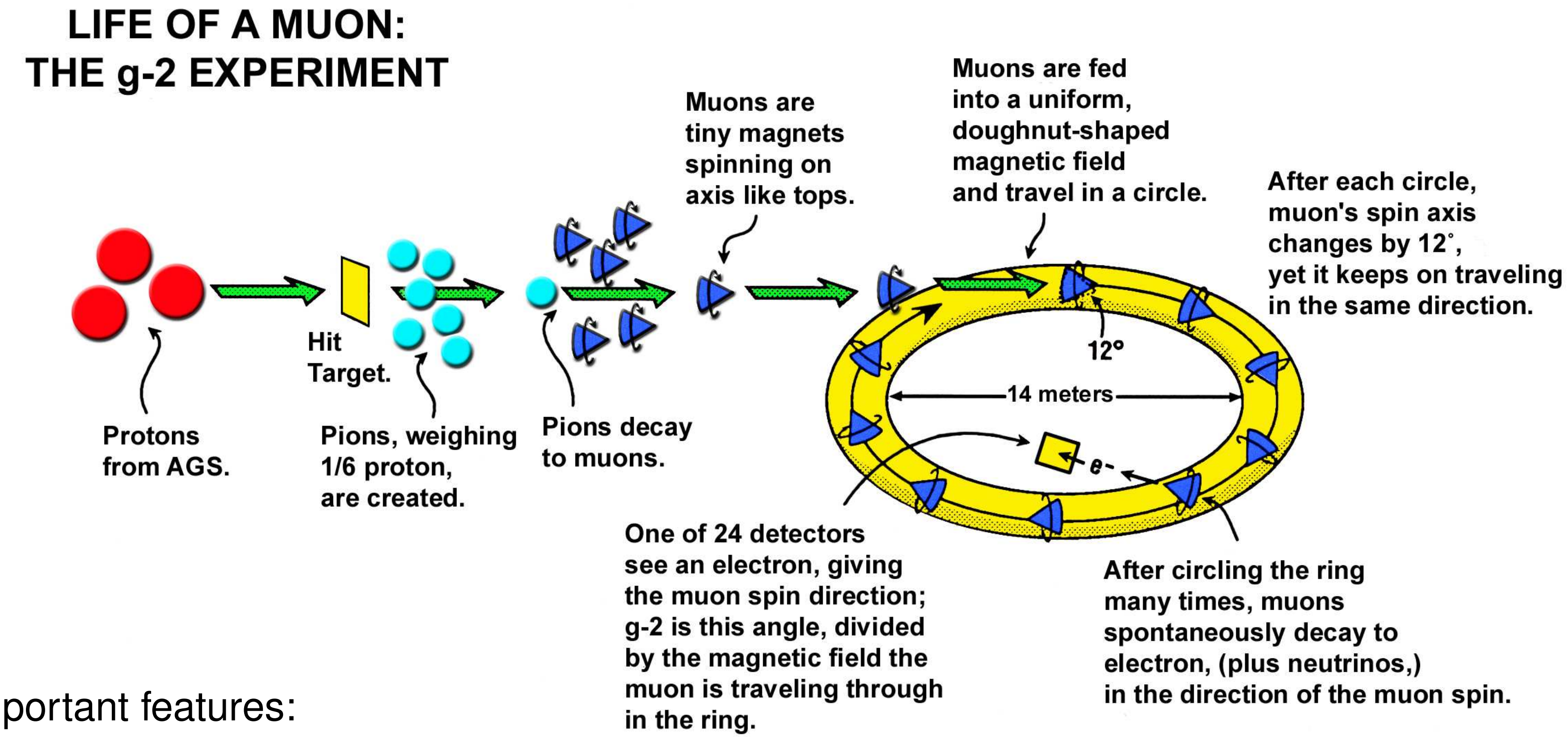

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- the most energetic positrons are emitted in the direction of the spin of the decaying μ^+

These are experimental facts, no need to assume SM is valid!

- muon storage ring experiment $[\vec{\beta}\cdot\vec{E}=\vec{\beta}\cdot\vec{B}=0]$

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2-1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B}+\vec{E}]$$

- $\gamma \sim 29.3$ [electrostatic focusing will not affect the spin]

Muon g-2 Coll., H. N. Brown et al., Phys. Rev. Lett. 86, 2227 (2001)

- $|d_\mu| < 1.9\cdot 10^{-19}$ [95% CL] Muon g-2 Coll., G. W. Bennett et al., Phys. Rev. D 80 (2009)

- muon storage ring experiment $[\vec{\beta}\cdot\vec{E}=\vec{\beta}\cdot\vec{B}=0]$

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- Still need to measure the magnetic field $\longrightarrow$ NMR probes $\longrightarrow \omega_{\rm p} = -g_{\rm p}\frac{e}{2m_{\rm p}}B$

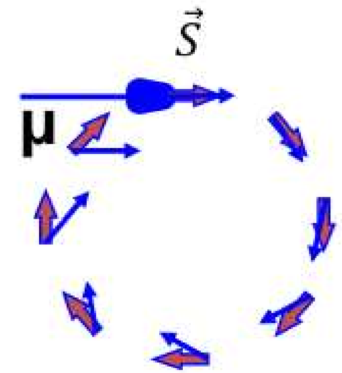
$$a_\mu = \frac{g_{\rm p}}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_{\rm p}} = \frac{g_e}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_e}\frac{\mu_{\rm p}}{\mu_e}$$

$$\frac{\Delta g_e}{g_e} = 0.26\text{ppt},\quad \frac{\Delta(m_\mu/m_e)}{m_\mu/m_e} = 22\text{ppb},\quad \frac{\Delta\mu_{\rm p}/\mu_e}{\mu_{\rm p}/\mu_e} = 3\text{ppb},\quad \frac{\Delta\omega_{\rm p}}{\omega_{\rm p}} = 70\text{ppb}$$

Key ingredients

1) Polarized muons

~95% polarized for forward decay



2) Precession proportional to (g-2)

$$\omega_a = \omega_{spin} - \omega_{cyclotron} = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

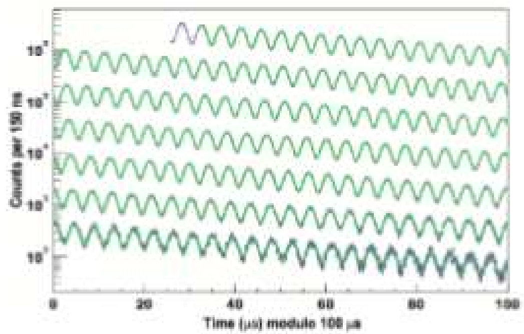
$a_\mu = (g-2)/2$

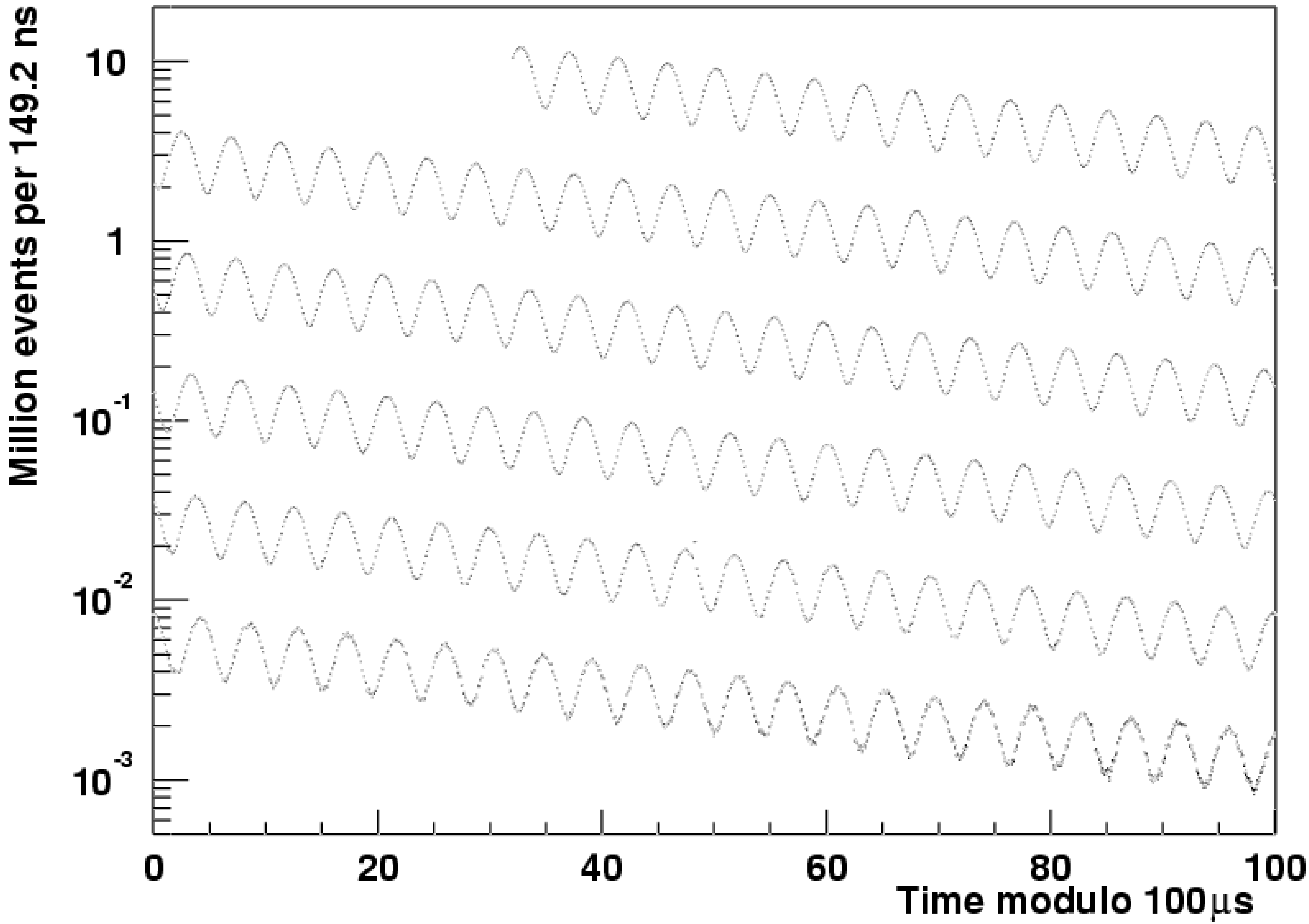
3) P_μ magic momentum = 3.09 GeV/c

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

4) Decay e^+ emitted preferably in spin direction of the muon

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$





Million events per 149.2 ns
10
1
10^-1
10^-2
10^-3
0
20
40
60
80
100
Time modulo 100μs

BNL-E821

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72
FNAL E989	2021	μ^+	11 659 204.0(5.4)	0.46
FNAL E989	2023	μ^+	11 659 205.7(2.5)	0.21
FNAL E989	2025	μ^+	11 659 207.10(1.62)	0.139

$$a_\mu^{\mathrm{exp}}(\mathrm{FNAL\ E989}) = 116\,592\,0705(148) \cdot 10^{-12}\ [127\ \mathrm{ppb}]$$

$$a_\mu^{\mathrm{exp}}(\mathrm{WA}) = 116\,592\,0715(145) \cdot 10^{-12}\ [124\ \mathrm{ppb}]$$

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- $e^+e^- \to \tau^+\tau^-\gamma$

$-0.052 < a_\tau^{exp} < +0.058$ (L3, 1998, 95% CL) Phys. Lett. B 434, 169 (1998)

$-0.068 < a_\tau^{exp} < +0.065$ (OPAL, 1998, 95% CL) Phys. Lett. B 431, 188 (1998)

- $e^+e^- \to e^+e^-\tau^+\tau^-$

$-0.052 < a_\tau^{exp} < +0.013$ (DELPHI, 2004, 95% CL) $a_\tau^{exp} = -0.018(17)$

Eur. Phys. J. C 35, 159 (2004)

- Reanalysis of experiments $\longrightarrow$ $-0.007 < a_\tau^{exp} < +0.005$

G. A. Gonzalez-Sprinberg, A. Santamaria, J. Vidal, Nucl. Phys. B 582, 3 (2000)

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$\longrightarrow$ Various proposals to improve the situation exist: arXiv:0707.2496, arXiv:0807.2366 arXiv:1601.07987, arXiv:1803.00501, arXiv:1810.06699, arXiv:1908.05180, arXiv:2002.05503, arXiv:2111.10378, arXiv:2506.19557

Standard Model Prediction

Standard Model Prediction

Disclaimer:

only a very compact overview of a very rich and elaborate subject

many interesting topics will only be briefly discussed

plenty of details available in the *White Paper* and its recent update

T. Aoyama et al., Phys. Rep. 887, 1 (2020), arXiv:2006.04822 [hep-ph]

R. Aliberti et al., arXiv:2505.21476 [hep-ph]

SM prediction

Considering SM contributions only, one has, by order of importance

$$a_\ell = a_\ell^{\text{QED}} + a_\ell^{\text{had}} + a_\ell^{\text{weak}}$$

a_ℓ^{QED}: loops with only photons and leptons

a_ℓ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons

a_ℓ^{weak}: loops with also contributions from the electroweak sector

SM prediction

Considering SM contributions only, one has, by order of importance

$$a_\ell = a_\ell^{\text{QED}} + a_\ell^{\text{had}} + a_\ell^{\text{weak}}$$

a_ℓ^{QED}: loops with only photons and leptons $\longleftarrow$ fully perturbative

a_ℓ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons $\longleftarrow$ fully non-perturbative

a_ℓ^{weak}: loops with also contributions from the electroweak sector
$\longleftarrow$ perturbative with (small) non-perturbative pieces

Standard Model Prediction

I QED

QED contribution :

$\longrightarrow$ loops with only photons and leptons

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$$(\alpha/\pi)^4 = 2.91 \ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76 \ldots \cdot 10^{-14}$$

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$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

QED contribution :

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

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$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

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mass-dependent (non-universal) contributions (multi-flavour QED)

– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

– Massive fermions with $m_{\ell'} \gg m_\ell$ contribute to a_ℓ through powers of $m_\ell^2/m_{\ell'}^2$, times logarithms

– Light degrees of freedom with $m_{\ell'} \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m_{\ell'}^2)$

QED contribution :

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

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$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

For the electron, $A_1^{(2n)}$ matter most, whereas $A_2^{(2n)}$ and $A_3^{(2n)}$ are suppressed by powers of m_e^2/m_μ^2 and m_e^2/m_τ^2 (times logarithms)

QED contribution :

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

For the muon, $A_1^{(2n)}$ are negligible, whereas $A_2^{(2n)}(m_\mu/m_e)$ are enhanced by powers of $\ln(m_\mu/m_e)$ $\quad\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

Even more true for the τ...

$$a_\ell{}^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$4.952(55)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$$a_\ell^{\mathrm{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

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$\Delta C_e^{(10)} \cdot (\alpha/\pi)^5 \sim 0.2 \cdot 10^{-13}$ $\qquad \Delta a_e^{\mathrm{exp}} = 1.3 \cdot 10^{-13}$

$$a_\ell^{\mathsf{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

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$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.6 \cdot 10^{-13} \qquad \Delta a_\mu^{\mathsf{exp}} = 1.48 \cdot 10^{-10}$$

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

QED prediction ?

$\longrightarrow$ requires an input for the fine structure constant α that matches the experimental accuracy on a_e

$$\frac{\Delta a_e}{a_e} = 0.11\mathrm{ppb} \;\rightarrow\; \frac{\Delta\alpha}{\alpha} \sim 0.11\mathrm{ppb} \;\rightarrow\; \Delta\alpha \lesssim 1\cdot 10^{-12}$$

- quantum Hall effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \qquad [19.7\mathrm{ppb}]$$

[P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)]

- atomic recoil velocity through photon absorption

$$\alpha^2 = \frac{2R_\infty}{c}\cdot\frac{M_{\mathrm{atom}}}{m_e}\cdot\frac{h}{M_{\mathrm{atom}}}$$

$$\Delta R_\infty = 7\cdot 10^{-12} \qquad \Delta\left(\frac{M_{\mathrm{Rb}}}{m_e}\right) = 4.4\cdot 10^{-10}$$

$$\alpha^{-1}[Cs\,02] = 137.036\,0001(11) \qquad [7.7\ \mathrm{ppb}]$$

A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)

$$\alpha^{-1}[Rb\,06] = 137.035\,998\,84(91) \qquad [6.7\ \mathrm{ppb}]$$

P. Cladé et al, Phys. Rev. A 74, 052109 (2006)

$$\alpha^{-1}[Rb\,08] = 137.035\,999\,45(62) \qquad [4.6\ \mathrm{ppb}]$$

M. Cadoret et al, Phys. Rev. Lett. 101, 230801 (2008)

$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\ \mathrm{ppb}]$$

R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

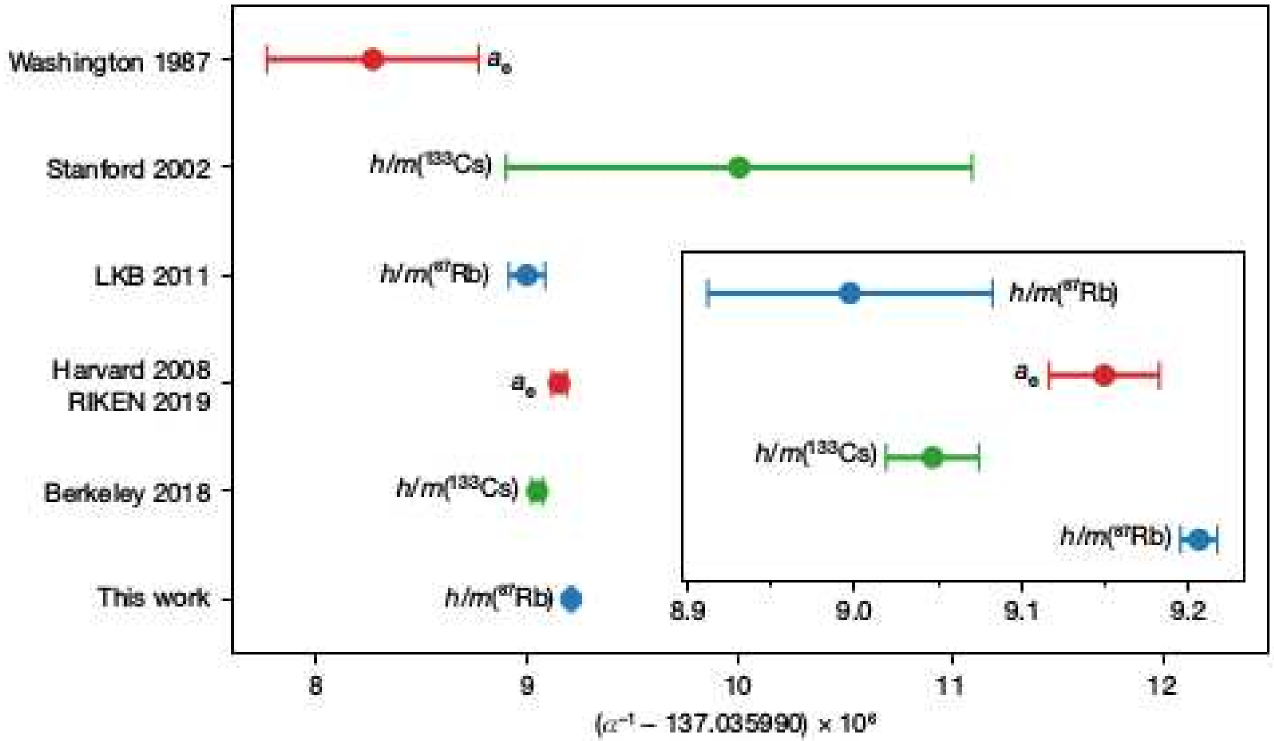
$$\alpha^{-1}[Rb\,11] = 137.035\,998\,995(85) \qquad [0.62\ \mathrm{ppb}]$$ [shift in R_∞]

$$\alpha^{-1}[Cs\,18] = 137.035\,999\,046(27) \qquad [0.20\ \mathrm{ppb}]$$

R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)

$$\alpha^{-1}[Rb\,20] = 137.035\,999\,206(11) \qquad [81\ \mathrm{ppt}]$$

L. Morel, Z. Yao, P. Cladé, S. Ghelladi-Khélifa, Nature 588, 61 (2020)



$\longrightarrow$ need to understand discrepancy between $\alpha(Cs\,18)$ and $\alpha(Rb\,20)$, but also between $\alpha(Rb\,11)$ and $\alpha(Rb\,20)$

$$\alpha^{-1}(Cs\,18) - \alpha^{-1}(Rb\,20) = -0.160(29) \cdot 10^{-6} \quad [5.5\sigma]$$

$\longrightarrow$ particularly important in view of the possiblity to improve the accuracy on $a_e^{\rm exp}$ by an order of magnitude!

G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

$$a_\mu^{\mathrm{QED}}(Cs) = 116\,584\,718.932(23)_\alpha(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[104] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{QED}}(a_e) = 116\,584\,718.833(13)_\alpha(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[103] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{QED}}(Rb) = 116\,584\,718.795(8)_\alpha(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[102] \cdot 10^{-11}$$

$\longrightarrow$

$$a_\mu^{\mathrm{QED}} = 116\,584\,718.8(2) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{exp}}(\mathrm{WA}) = 116\,592\,071.5(14.5) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{exp}}(\mathrm{WA}) - a_\mu^{\mathrm{QED}} = 735.3(1.5) \cdot 10^{-10}$$

$$a_\mu^{\rm QED}(Cs) = 116\,584\,718.932(23)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[104]\cdot 10^{-11}$$

$$a_\mu^{\rm QED}(a_e) = 116\,584\,718.833(13)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[103]\cdot 10^{-11}$$

$$a_\mu^{\rm QED}(Rb) = 116\,584\,718.795(8)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[102]\cdot 10^{-11}$$

$\longrightarrow$

$$a_\mu^{\rm QED} = 116\,584\,718.8(2)\cdot 10^{-11}$$

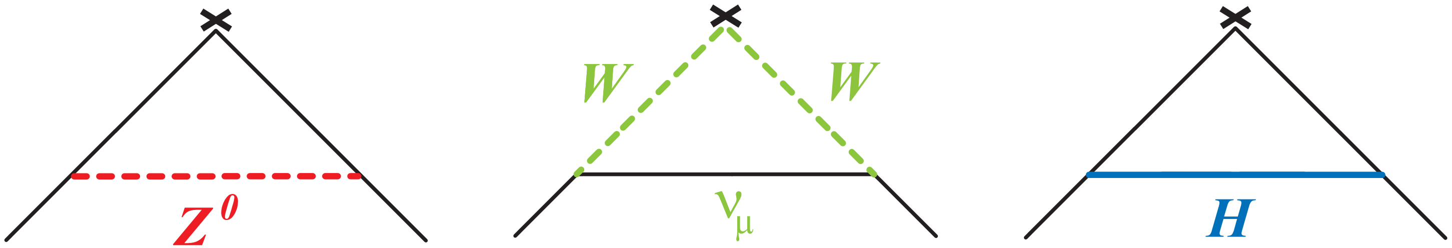
$$a_\mu^{\rm exp}({\rm WA}) = 116\,592\,071.5(14.5)\cdot 10^{-11}$$

$$a_\mu^{\rm exp}({\rm WA}) - a_\mu^{\rm QED} = 735.3(1.5)\cdot 10^{-10}$$

- QED provides more than 99.99% of the total experimental value
- The missing part has to be provided by weak and strong interactions (or else, new physics...)

Theory II: Weak interactions

Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$
$$= 19.479(1)\times 10^{-10}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

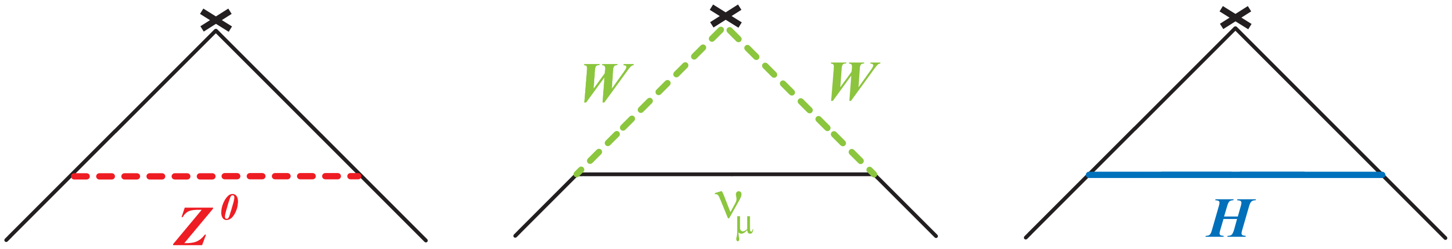
M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Including higher-loop corrections reduces the value by $\sim 25\%$

$$a_\mu^{\text{weak}} = 15.44(4)\cdot 10^{-10}$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} = 719.86(1.50)\cdot 10^{-10}$$

Weak contributions : W, Z,... loops



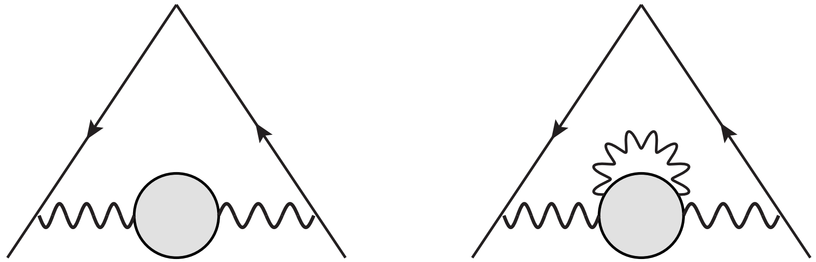
$$a_e^{\mathrm{weak}} = 0.030\,53(23) \cdot 10^{-12} \qquad [\Delta a_e^{\mathrm{exp}} = 0.13 \cdot 10^{-12}]$$

Standard Model Prediction

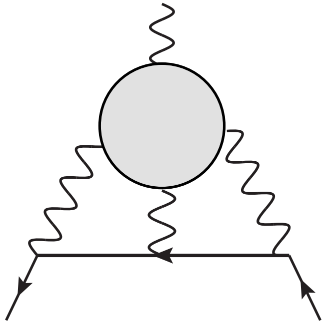
III Strong interactions

Contributions from strong interactions

- hadronic vacuum polarization (HVP)



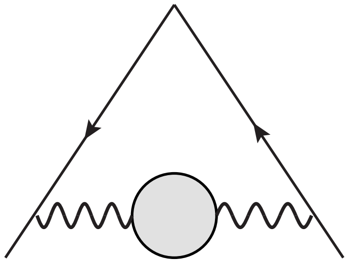
- (virtual) hadronic light-by-light (HLxL) scattering

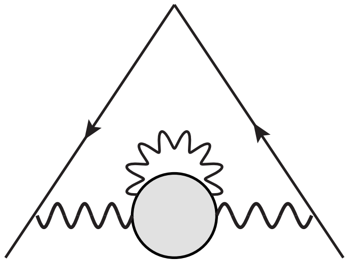


$$a_\ell^{\text{had}} = a_\ell^{\text{HVP-LO}} + a_\ell^{\text{HVP-NLO}} + a_\ell^{\text{HVP-NNLO}} + a_\ell^{\text{HLxL}}$$

Contributions from strong interactions

- hadronic vacuum polarization (HVP)





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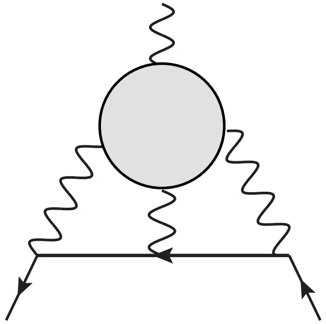
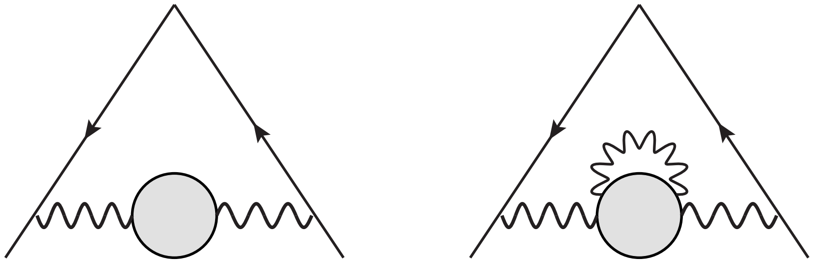


$\longrightarrow$ non-perturbative regime of QCD

Contributions from strong interactions

- hadronic vacuum polarization (HVP)



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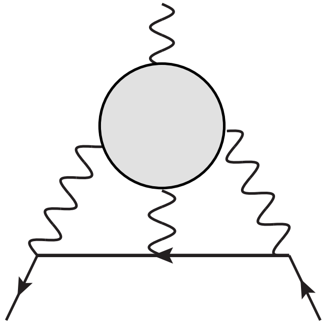
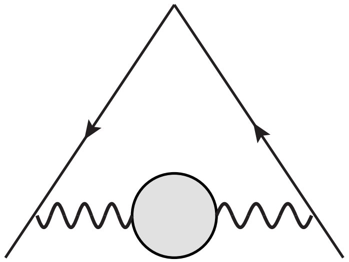


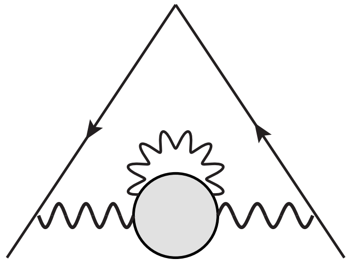
$\longrightarrow$ non-perturbative regime of QCD

- dispersive methods
- lattice QCD

Contributions from strong interactions

- hadronic vacuum polarization (HVP)





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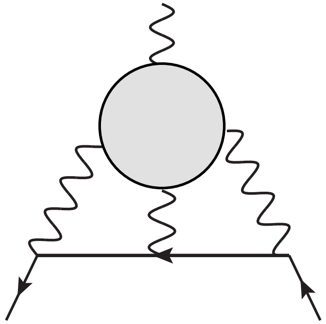

$\longrightarrow$ non-perturbative regime of QCD

- dispersive methods $\longrightarrow$ requires high-precision data for $e^+e^- \to \mathrm{hadrons}(+\gamma)$

- lattice QCD $\longrightarrow$ requires control over systematic effects

HVP: Dispersive evaluation

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t)$$

$$K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{t}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the low-energy region dominates

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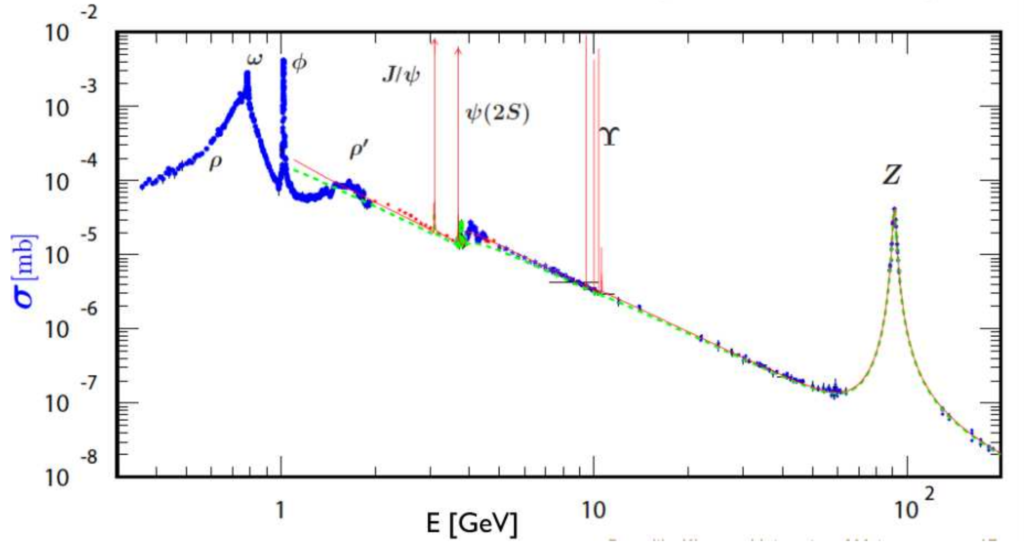
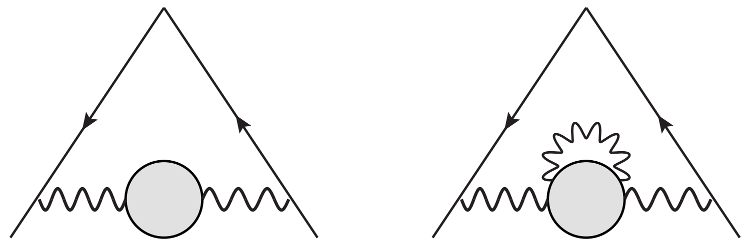
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the low-energy region dominates
- $a_\ell^{\text{HVP-LO}}$ is related to an experimental quantity



$\longrightarrow$



HVP: Dispersive evaluation

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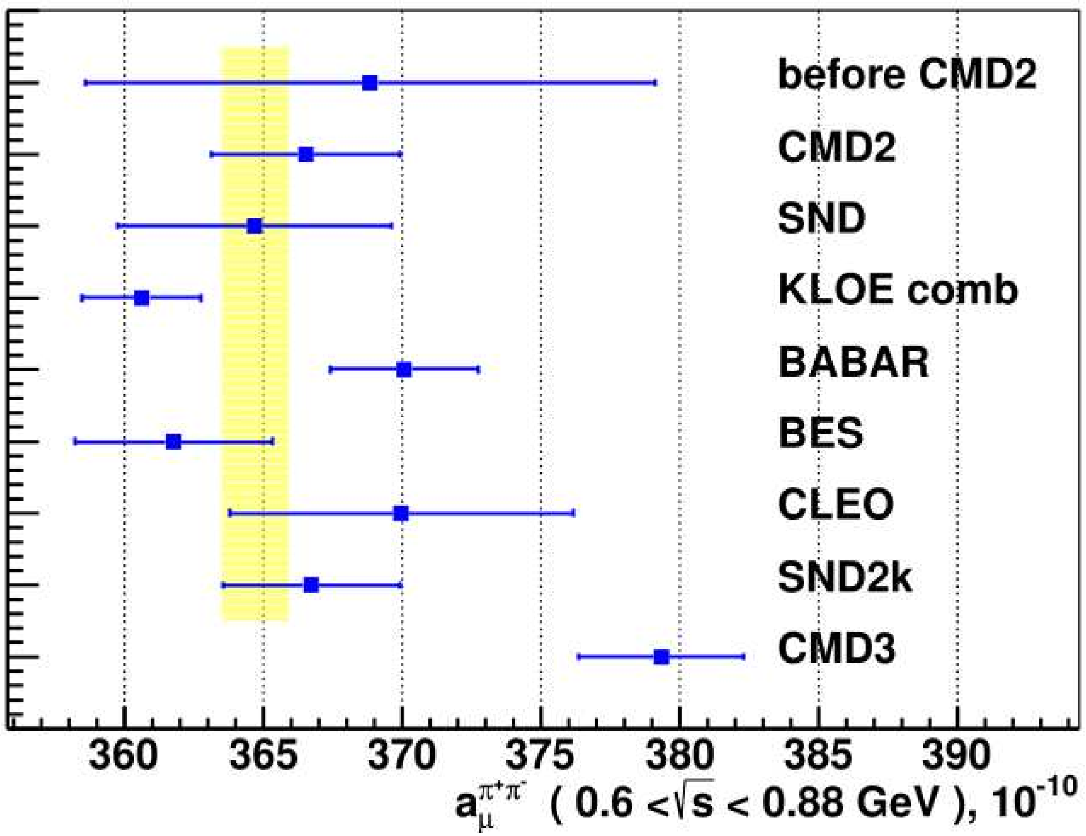
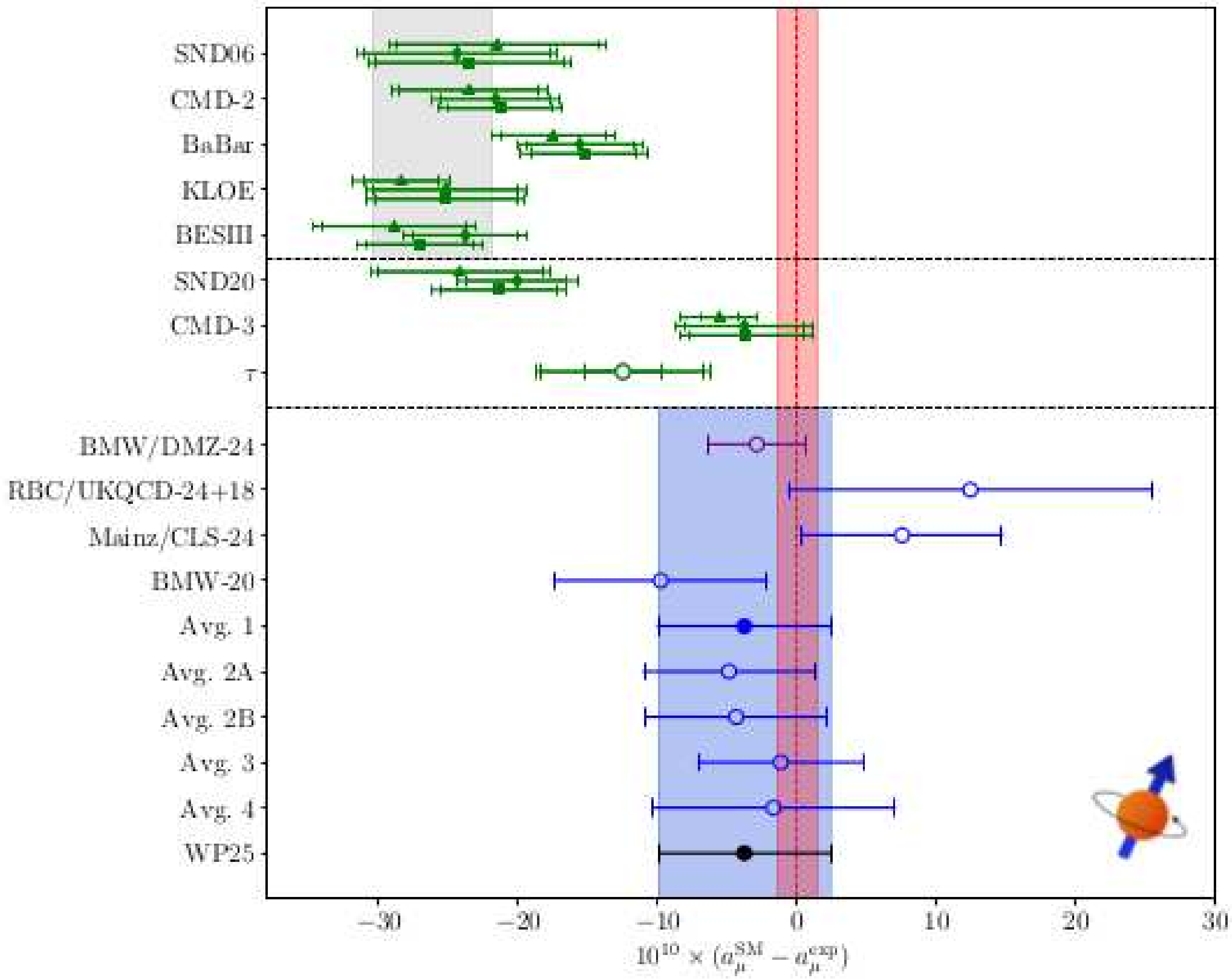
$$K(t) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{t}{m_\ell^2}}$$

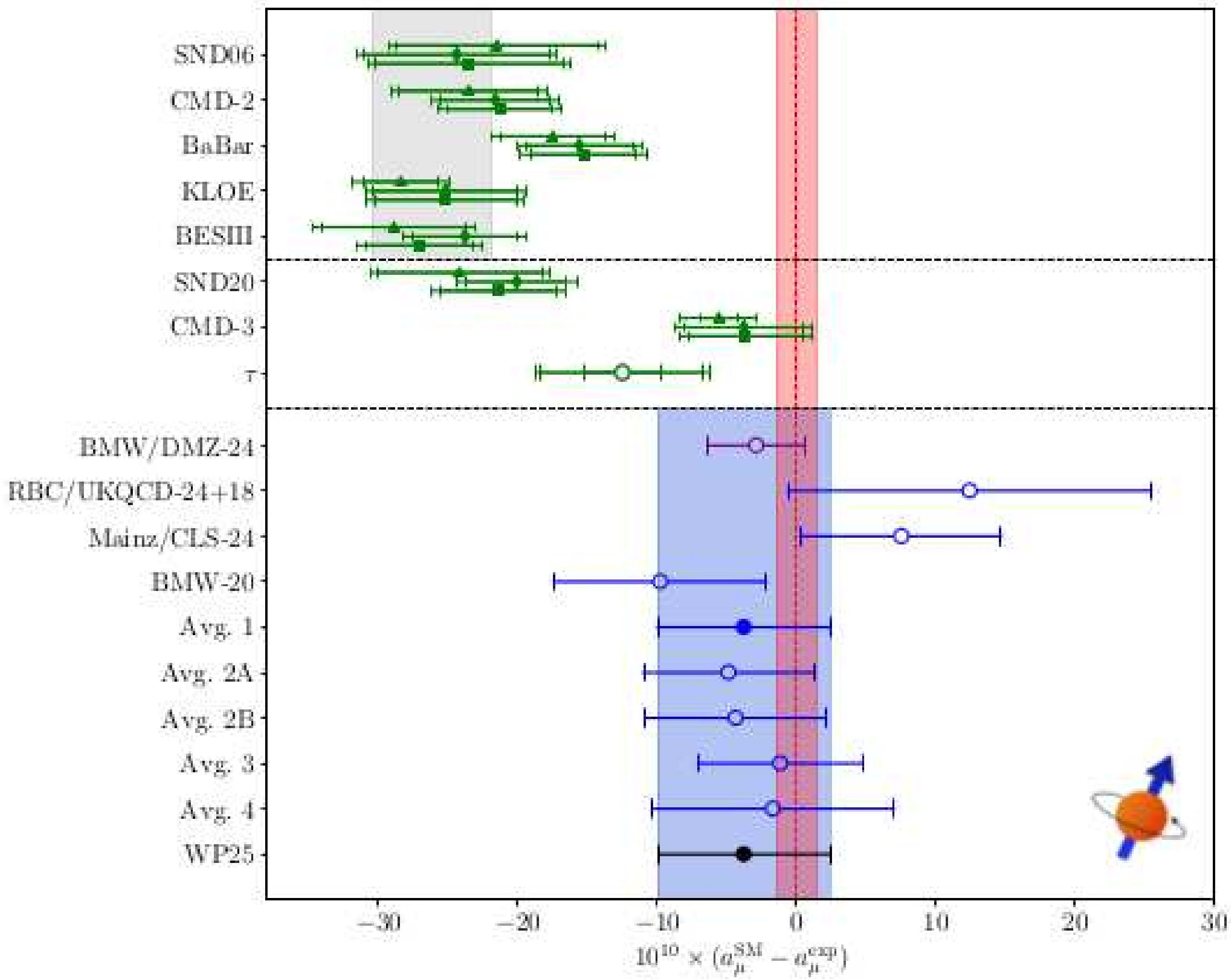


HVP: Lattice QCD



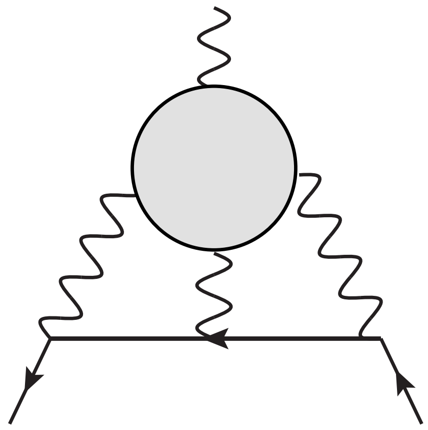
HVP: Lattice QCD

$$a_\mu^{\mathrm{HVP}}(\mathrm{WP25}) = 7045(61) \cdot 10^{-11}$$

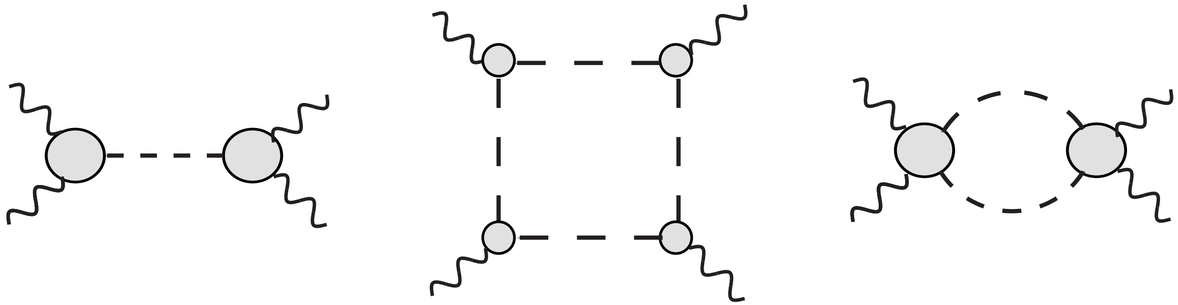


HLxL: Dispersive evaluation

- Occurs at order $\mathcal{O}(\alpha^3)$
- Not related, as a whole, to an experimental observable...

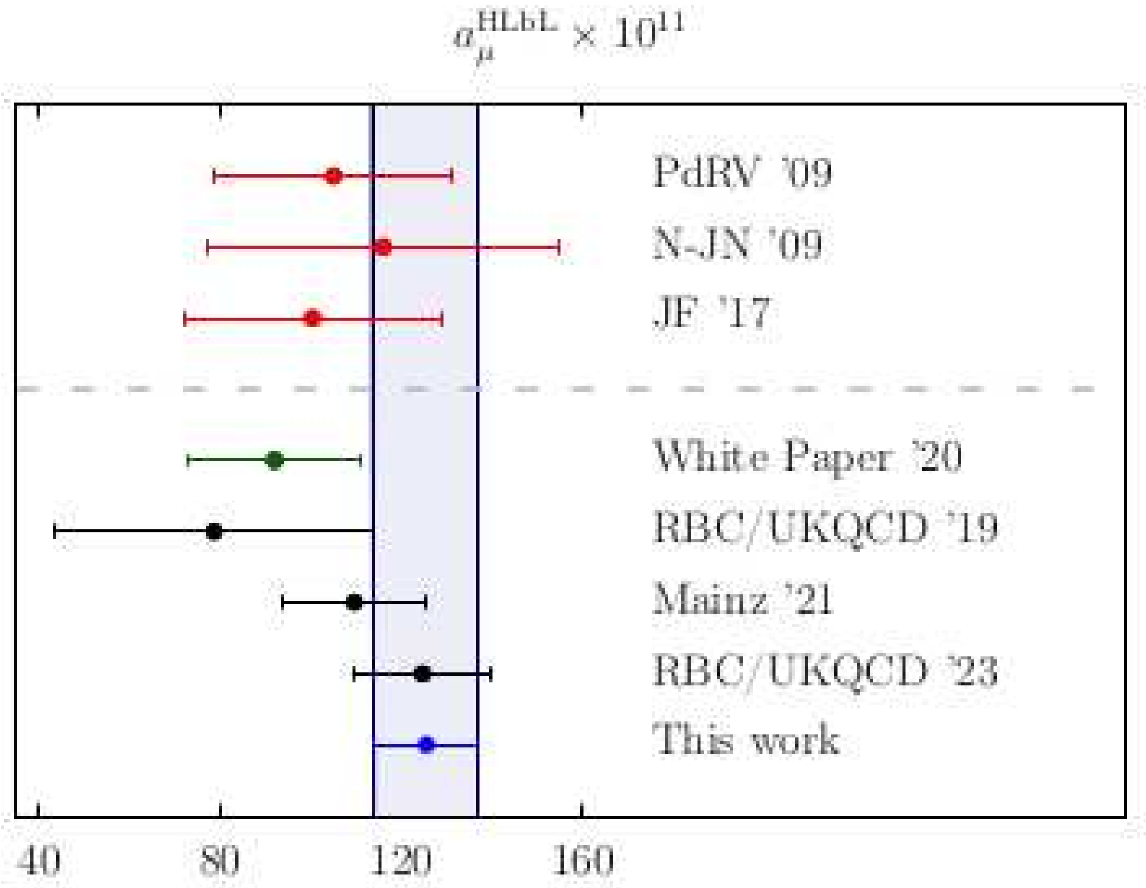
? $\longrightarrow$ 

- Identify specific contributions through the singularities they produce



- Use input from data (when available) for various form factors + QCD short-distance constraints

HLxL: Lattice QCD



$$a_\mu^{\mathrm{HLxL-LO}}(\mathrm{WP;\ analytic}) = 103.3(8.8) \cdot 10^{-11} \qquad a_\mu^{\mathrm{HLxL-LO}}(\mathrm{WP;\ LQCD}) = 122.5(9.2) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{HLxL}}(\mathrm{WP}) = 115.5(9.9) \cdot 10^{-11}$$

Summary

The anomalous magnetic moments of the electron and the muon are among the most precisely measured observables of the SM

$$a_\mu^{\rm exp} = 116\,592\,071.5(14.5) \cdot 10^{-11}\ [124\ \rm ppb]$$

$$a_e^{\rm exp} = 1\,159\,652\,180.59(13) \cdot 10^{-12}\ [110\ \rm ppt]$$

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Bringing the uncertainties down at a level where one can hope to become sensitive to NP is a real challenge, both from the experimental and theoretical point of view

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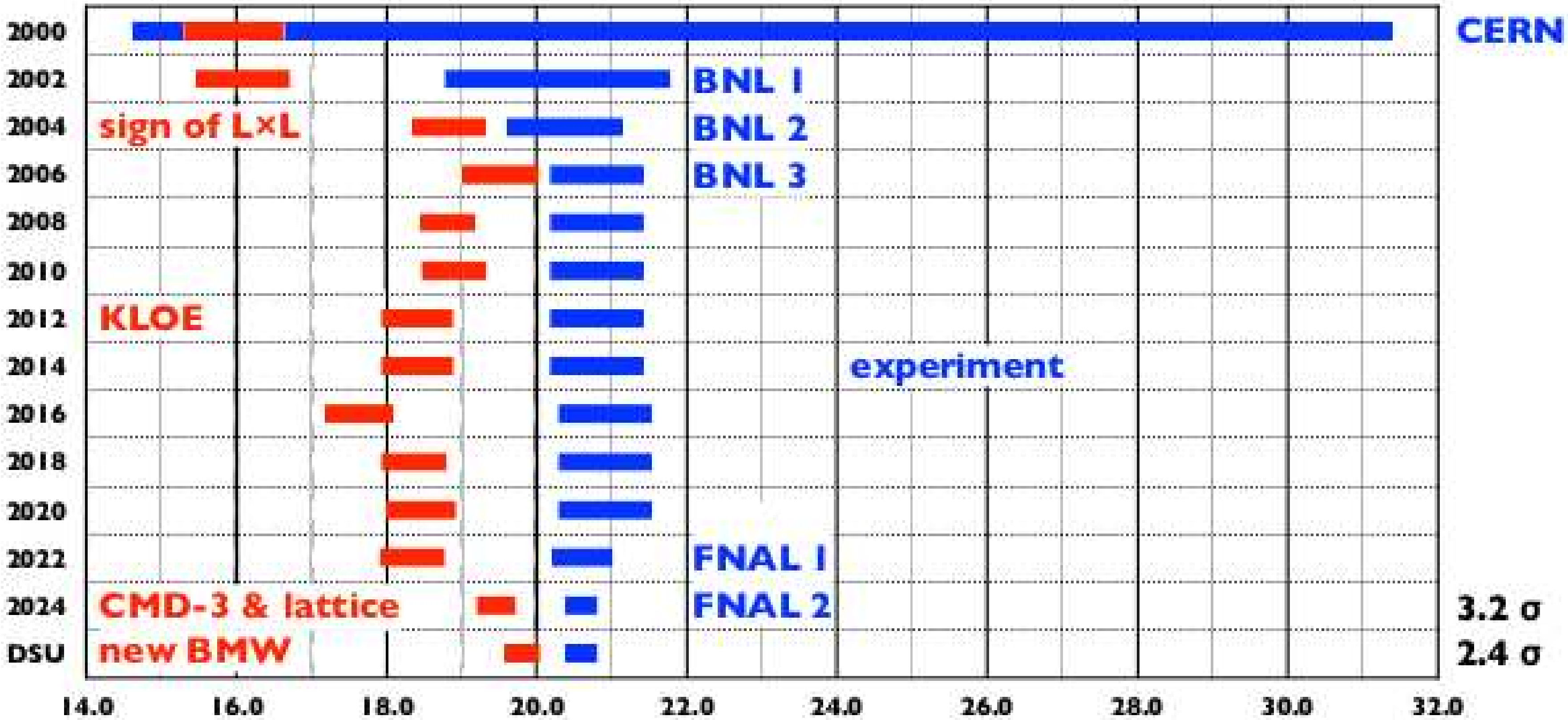
In the case of the muon,

$$a_\mu^{\rm SM} = 116\,592\,033(62) \cdot 10^{-11}\ [532\ \rm ppb] \qquad a_\mu^{\rm exp} - a_\mu^{\rm SM} = 38(63) \cdot 10^{-11}$$

the apparent agreement between theory and experiment hides important tensions between different experimental measurements of the $e^+e^- \to \rm hadrons(+\gamma)$ cross section. Situation remains to be clarified and understood



$$10^9 a_\mu - 1165900$$

J. Erler arXiv:2505.03457 [hep-ph]

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In the case of the electron, the interpretation of the measurement is limited by conflicting results for the determination of the fine-structure constants at the required level of precision

$$a_e^{\rm SM}[\alpha({\rm Cs})] = 1\,159\,652\,181.59(23)(0)(3) \cdot 10^{-12}\ [200\ {\rm ppt}] \qquad a_e^{\rm exp}({\rm NW22}) - a_e^{\rm SM}[\alpha({\rm Cs})] = -1.00(26) \cdot 10^{-12}$$

$$a_e^{\rm SM}[\alpha({\rm Rb})] = 1\,159\,652\,180.238(82)(4)(30) \cdot 10^{-12}\ [75\ {\rm ppt}] \qquad a_e^{\rm exp}({\rm NW22}) - a_e^{\rm SM}[\alpha({\rm Rb})] = +0.35(16) \cdot 10^{-12}$$

The anomalous magnetic moments of the electron and the muon are among the most precisely measured observables of the SM

$$a_\mu^{\rm exp} = 116\,592\,071.5(14.5) \cdot 10^{-11} \; [124 \; \rm ppb]$$

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Bringing the uncertainties down at a level where one can hope to become sensitive to NP is a real challenge, both from the experimental and theoretical point of view

In the case of the tau, an accurate experimental measurement of a_τ is lacking

Thanks for your attention!