

Tutorial

Superconductor-Insulator transition

Christoph Strunk

University of Regensburg

Outline

- Localization: suppression of diffusion by disorder

Abrahams et al. 1979: „There are no metals in two dimensions!“

- Superconductivity
- Suppression of zero-resistance state by phase fluctuations
- Suppression of Cooper-pairing by disorder
- Direct superconductor-insulator transition
- ‚Failed‘ superconductors and insulators:

Kravchenko et al. 1994: „There are ~~no~~ metals in two dimensions!“

Localization

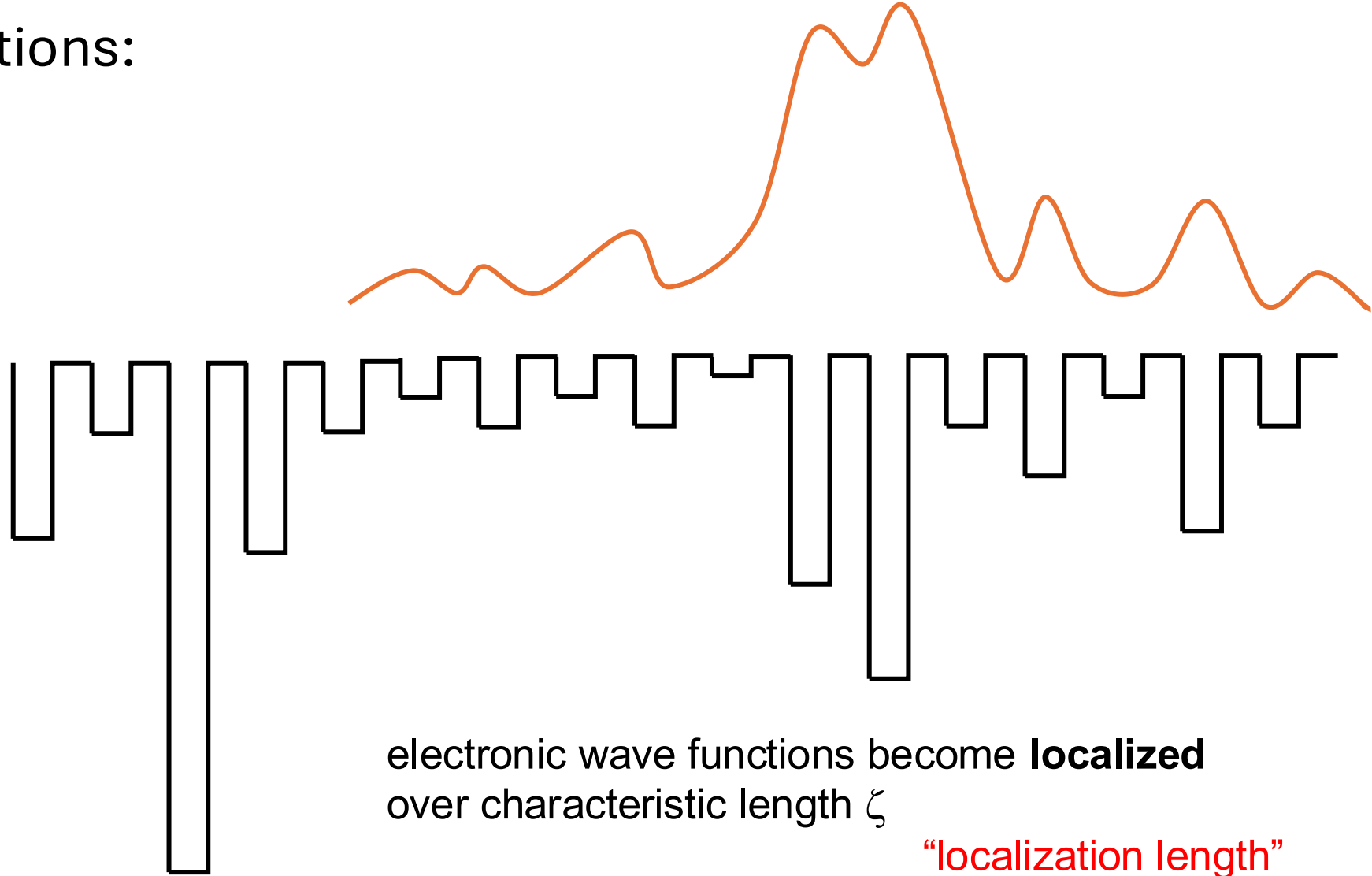
Numerical simulations show that diffusion is suppressed in presence of sufficiently strong on-site disorder

- Two seminal predictions:

P. W. Anderson,

**On the absence of localization
in certain random lattices**

Phys. Rev. **109**, 1492 (1958).



Localization

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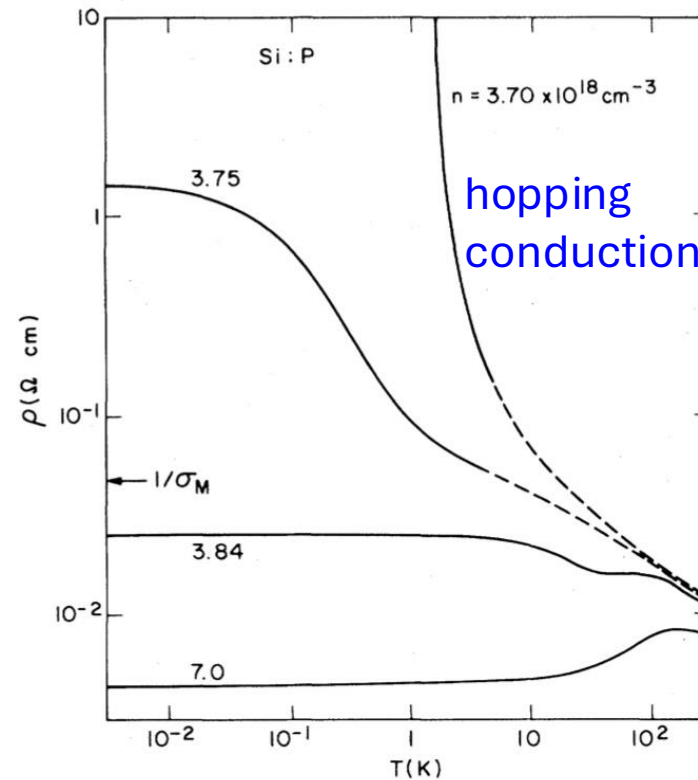
Phys. Rev. **109**, 1492 (1958).

T. F. Rosenbaum, R. F. Milligan, M. A. Paalanen,
G. A. Thomas, R. N. Bhatt, W. Lin

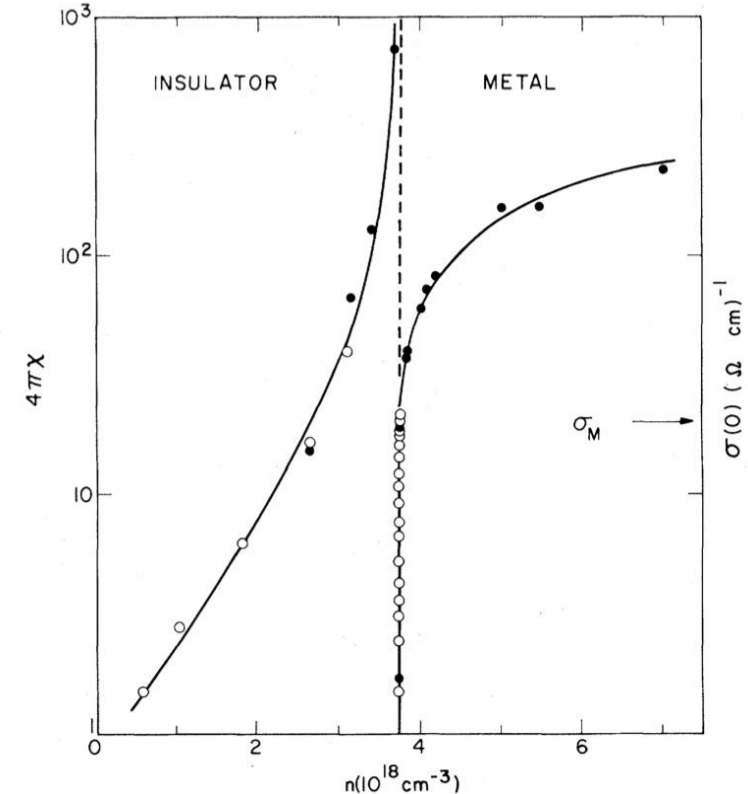
**Metal-insulator transition (MIT) in a doped
semiconductor**

Phys. Rev. B **27**, 7509 (1983)

Experimental realization:
doped semiconductors, e.g., Si:P



resistivity $\rho(0)$ diverges below
and saturates above MIT
in the limit $T \rightarrow 0$



both, $\rho(0)$ and $\chi(0)$
diverge at MIT

Localization

- Two seminal predictions:

E. Abrahams, P. W. Anderson,
D. C. Licciardello, T. V. Ramakrishnan,

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

Phys. Rev. Lett. **42**, 673 (1979)

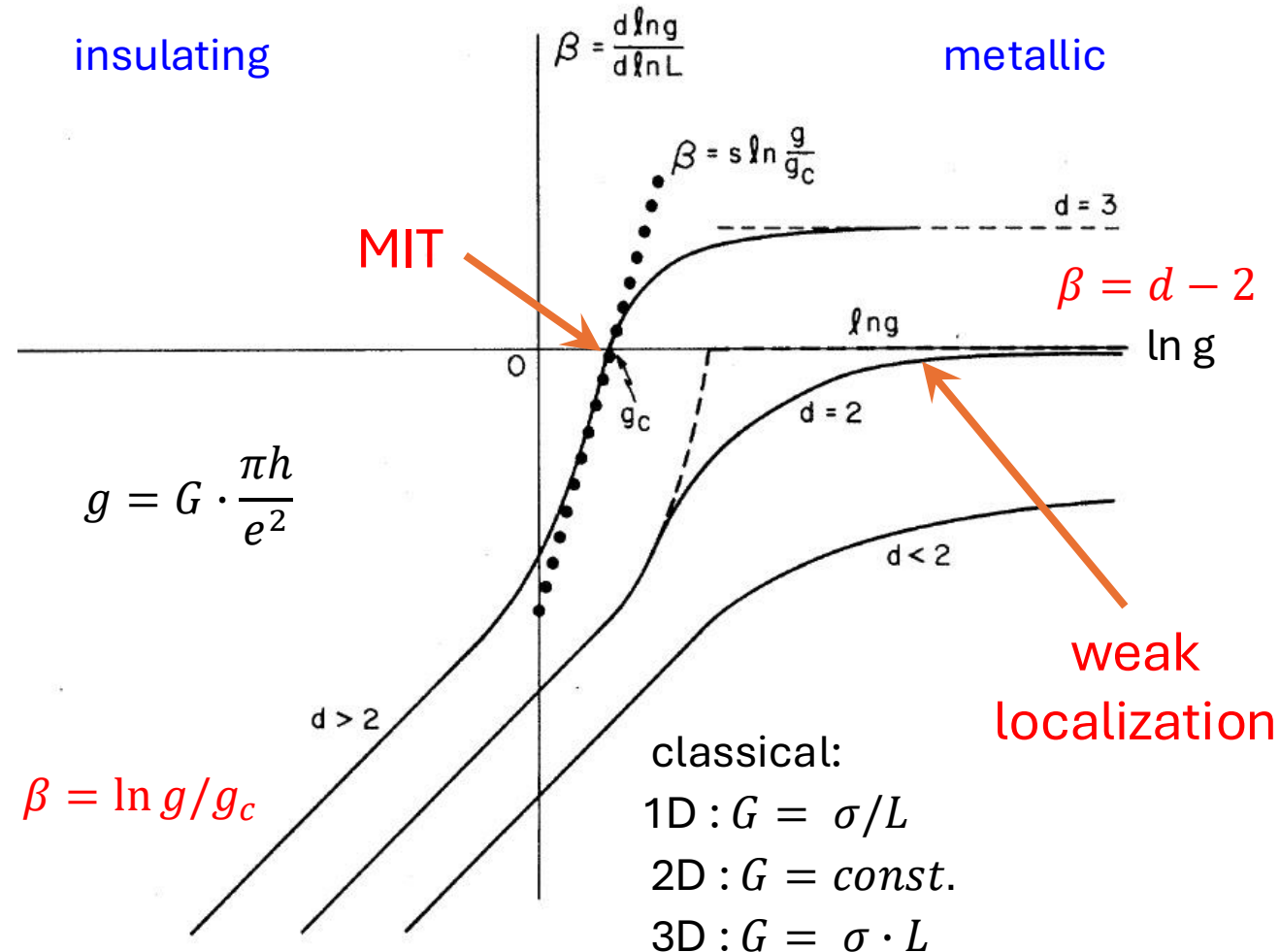
Conductance G of a hypercube becomes a
function of size L (in units of ζ):

classical regime: $G(L) = \sigma \cdot L^{d-2}$

localized regime: $G(L) \propto \exp(-L)$

strong localization

Scaling arguments show that in 2D and
in absence of interactions this occurs at
arbitrarily weak disorder



Weak Localization

If the film is larger than the phase-coherence length L_ϕ , then L has to be replaced by $L_\phi(T)$.

Hence, $R(T)$ was predicted to show T -dependent *quantum corrections*:

Altshuler, B. L., and A. G. Aronov, JETP Lett. **30**, 514 (1979).

Altshuler, B. L., and A. G. Aronov, JETP **50**, 968 (1979).

Altshuler, B.L., and A. G. Aronov, JETP Lett. **33**, 499 (1981).

SPOILER ALERT

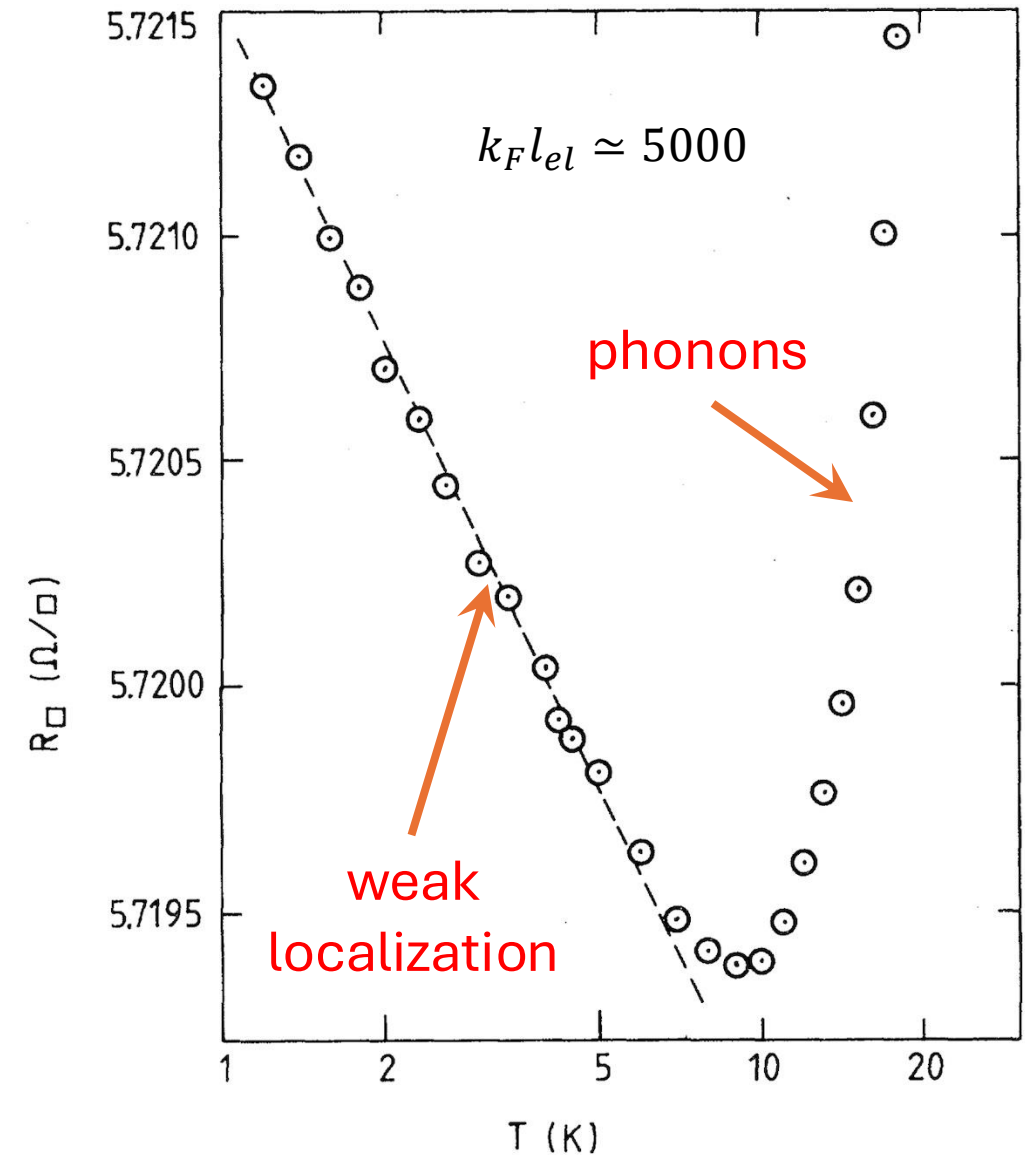
In order to see the predicted crossover to insulating behavior for systems with $k_F\ell \gg 1$, one would have to measure the conductance at exponentially low temperatures,

$$T < T^* \sim E_F \exp[-\pi k_F \ell]. \quad (1)$$

A. Kapitulnik, S. V. Kivelson, B. Spivak

Colloquium: Anomalous metals: Failed superconductors

Rev. Mod. Phys. **91**, 011002 (2019)



L. Van den Dries, C. Van Haesendonck, Y. Bruynseraede, G. Deutscher

Two-Dimensional Localization in Thin Copper Films

Phys. Rev. Lett. **46**, 565 (1981)

Localization

For a comprehensive review:

Disordered electronic systems

Patrick A. Lee

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

T. V. Ramakrishnan

Department of Physics, Banaras Hindu University, Varanasi-221005 (U.P.), India

Rev. Mod. Phys. **57**, 287 (1985)

Localization

Experimental difficulty:

Q: How to make homogeneously disordered thin films?

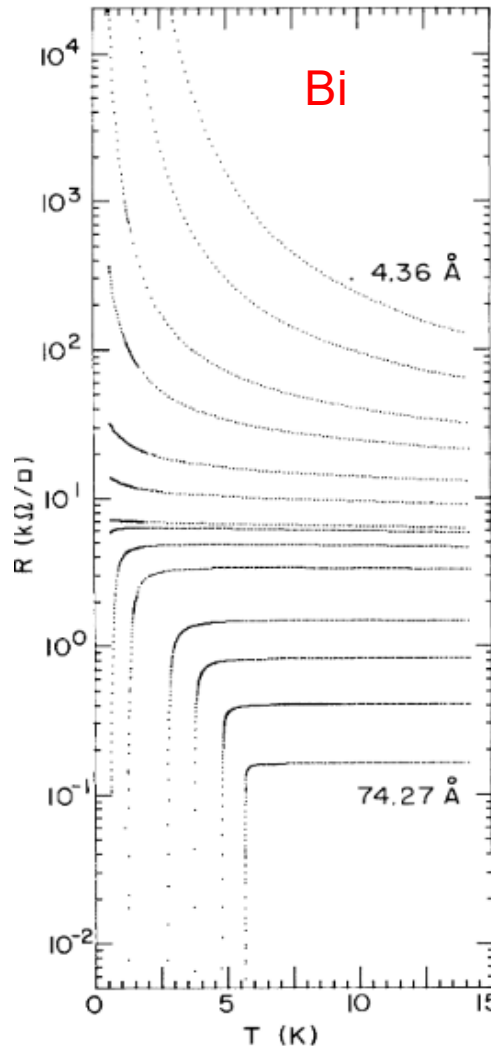
A: Use quench-condensed amorphous films (e.g.: Bi)

- Two seminal predictions:

E. Abrahams, P. W. Anderson,
D. C. Licciardello, T. V. Ramakrishnan,

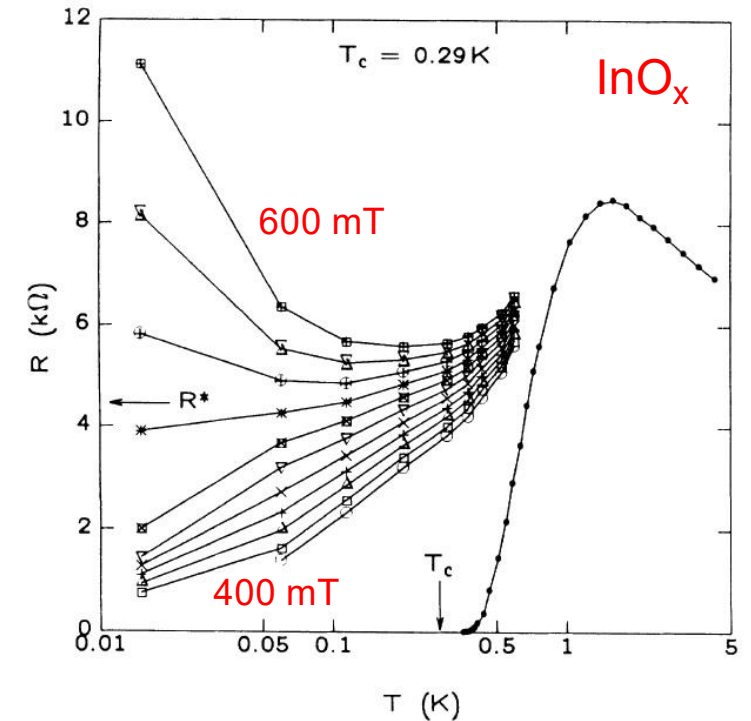
**Scaling Theory of Localization:
Absence of Quantum Diffusion
in Two Dimensions**

Phys. Rev. Lett. **42**, 673 (1979)



D. Haviland, Y. Liu, A. Goldman,
**Onset of superconductivity in
the two-dimensional limit**

Phys. Rev. Lett. **62**, 2180 (1989)



A. F. Hebard, M. A. Paalanen
**Magnetic-Field-Tuned Superconductor-
Insulator Transition in Two-Dimensional
Films**

Phys. Rev. Lett. **65**, 927 (1990)

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- Localization: suppression of diffusion by disorder

Abrahams et al. 1979: „There are no metals in two dimensions!“

- **Superconductivity**

Superconductivity

- Repulsive (Coulomb) part of electron-electron is strongly screened
- Arbitrarily small attractive interactions can lead to Cooper-instability
- Electrons condense pairwise into new ground state: **superfluid condensate**
- Excitation spectrum of quasiparticles displays energy gap Δ

In presence of disorder, superflow is significantly suppressed: $\vec{j}_s \propto 1/R_N$

P.W. Anderson:

- Nonmagnetic impurities should not affect Cooper pairing.

This is valid only for relatively weak disorder:

- Slowing down of electron diffusion reduces screening and T_c

Supercurrent

Normal electric currents are driven by electric potential (ϕ_{el}) differences :

$$\vec{j}_{\text{N}} = -\sigma_{\text{N}} \text{grad } \phi_{\text{el}} \quad \text{but, in superconductors:} \quad \text{grad } \phi_{\text{el}} \equiv 0 \quad (\text{in the stationary case})$$

So - what's drives a supercurrent?

$$\vec{j}_{\text{S}} = -2e |\psi|^2 \frac{\hbar}{m^*} \left(\boxed{\text{grad } \varphi} + \frac{2e}{\hbar} \vec{A} \right), \quad \text{where } \psi(\vec{r}) = |\psi(\vec{r})|^2 \exp i\varphi(\vec{r}) \quad \begin{array}{l} \text{complex} \\ \text{order} \\ \text{parameter} \end{array}$$

current-phase-relation (2nd Ginzburg-Landau equation/ 1st Josephson equation)

Ginzburg & Landau: supercurrents are driven by (gauge-invariant) phase differences!

electric potential differences generate a time evolution of the phase :

$$\boxed{\frac{\partial}{\partial t} \varphi(\vec{r}, t) = \frac{2e}{\hbar} \phi_{\text{el}}(\vec{r}, t)}$$

2nd Josephson equation



acceleration of superfluid
condensate, if $\text{grad } \phi_{\text{el}} \neq 0$

London equations

A) Take curl of current-phase-relation:

$$\text{rot } \vec{j}_s = -2e |\psi|^2 \frac{\hbar}{m^*} \left(\cancel{\text{rot grad } \varphi} + \frac{2e}{\hbar} \text{rot } \vec{A} \right) = - \underbrace{\frac{4e^2 |\psi|^2}{m^*}}_{\Lambda_L^{-1}} \vec{B}$$

Meissner – Effect !

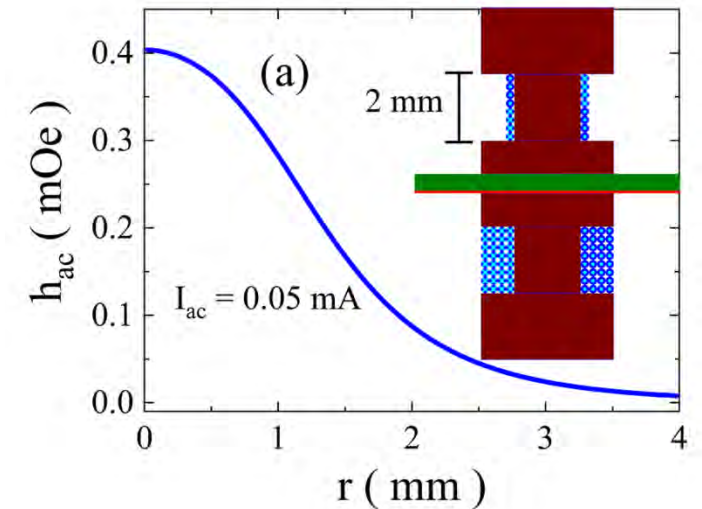
$$\Lambda_L^{-1} = \frac{4e^2 |\psi(T)|^2}{m^*} = \frac{1}{\mu_0 \lambda^2(T)}$$

London parameter, equivalent to
superfluid density
penetration depth
superfluid weight

$$J_s(T) = \frac{\hbar^2}{4e^2 k_B} \cdot \frac{d}{\Lambda_L}$$

superfluid phase stiffness

2nd London equation



see, e.g., S. Mandal et al.,
Supercond. Sci. Tech. **36**, 014004 (2023)

Two-coil method allows to measure Λ_L^{-1} in thin films with thickness $d \lesssim \lambda(T)$

London equations

B) Take time derivative of current-phase-relation:

$$\frac{\partial \vec{j}_S}{\partial t} = -2e |\psi|^2 \frac{\hbar}{m^*} \left(\text{grad} \frac{\partial \varphi}{\partial t} + \frac{2e}{\hbar} \frac{\partial \vec{A}}{\partial t} \right) = \Lambda_L^{-1} \vec{E} \quad \text{1st London equation}$$

ideal conductor : $\sigma = \infty$

$$\frac{\partial I_S}{\partial t} = \underbrace{\frac{wd}{\ell} \Lambda_L^{-1}}_{L_{\text{kin}}^{-1}} \cdot V \quad \text{the 1st London equation is an induction law !}$$

total inductance :
$$L = L_{\text{geo}} + L_{\text{kin}}$$
$$= \mu_0 \frac{w}{\ell} \left(d + \frac{\lambda^2}{d} \right) \quad L_{\text{kin}} \text{ important, if film thickness: } d \lesssim \lambda(T)$$

Why “superfluid density” is a seductive, but sometimes misleading concept

- it suggest a link to electron density $n_S = n_e/2$ and that at low temperature all electrons are paired up into Cooper pairs
- **wrong**: only electrons within Δ around Fermi surface pair up, the others are invisible because of Pauli principle
- in experiments, Λ_L^{-1} *strongly depends on mean free path*, while a carrier density should not: Cooper pairs propagate dissipationless, but still need diffusion times L^2/D_{electron} to travel a distance L in the superconductor
- Only the **ratio** $\Lambda_L^{-1} = \frac{4e^2 n_S}{m^*} = \frac{1}{\mu_0 \lambda^2(T)}$ is generally relevant, but not n_S and m^* separately
- supercurrent is simply given by: $\vec{j}_s = \frac{\hbar}{2e\Lambda_L} \left(\text{grad } \varphi + \frac{2e}{\hbar} \vec{A} \right)$
- dirty limit BCS-theory results in $\Lambda_L^{-1} = \frac{\Delta(0)}{\hbar R_N d} = \frac{\Delta(0)}{\hbar \rho_N} \neq \frac{1}{\rho_N \tau}$ with $\tau = \infty$

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Abrahams et al. 1979: „There are no metals in two dimensions!“

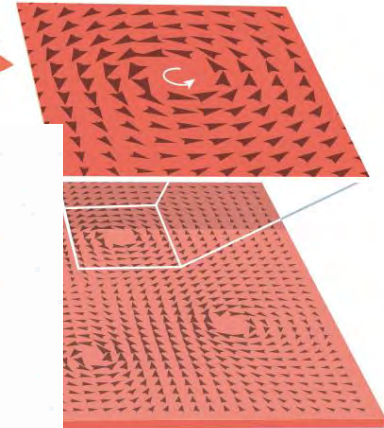
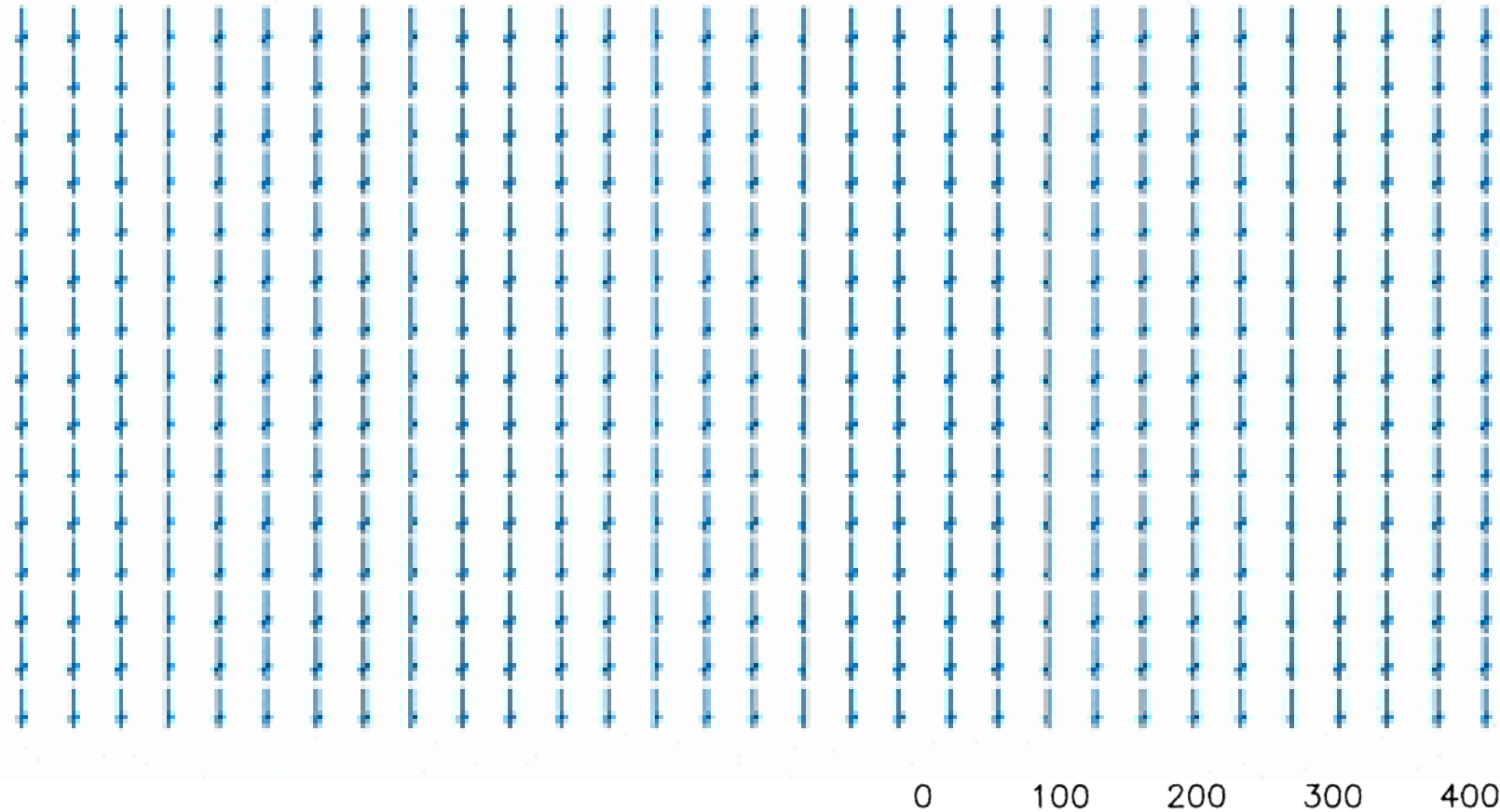
- Superconductivity
- **Suppression of zero-resistance state by phase fluctuations**

Berezinski-Kosterlitz-Thouless transition

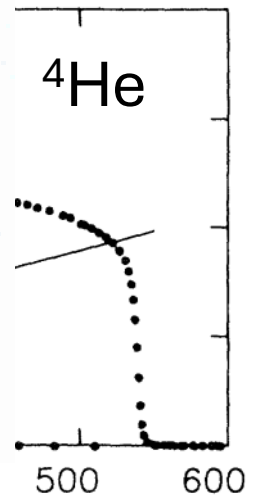
Superfluid stiffness J_s : energy scale for twisting the phase of the order parameter Ψ

Topological phase

- Unbinding of vortices at critical temperature
- Sudden drop in superfluid stiffness at universal value $J_s(T_{\text{BKT}}) = \frac{2\pi k_B T_{\text{BKT}}}{\hbar^2}$
- Clear experimental observation in superfluid ^4He



WATER TEMPERATURE



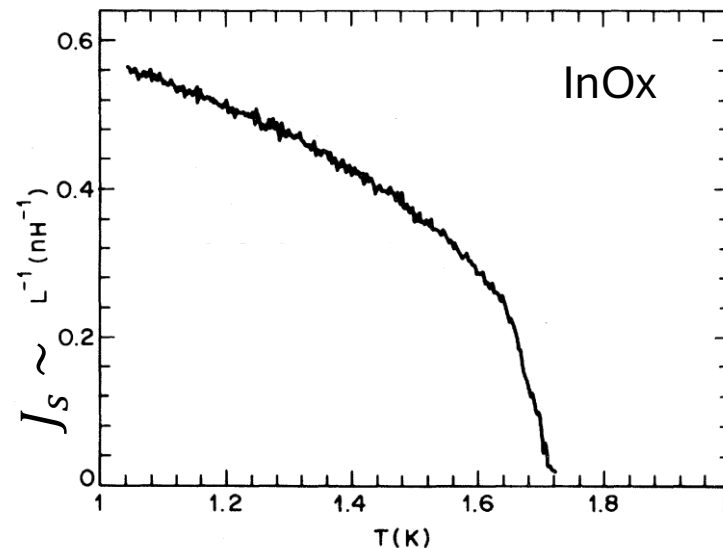
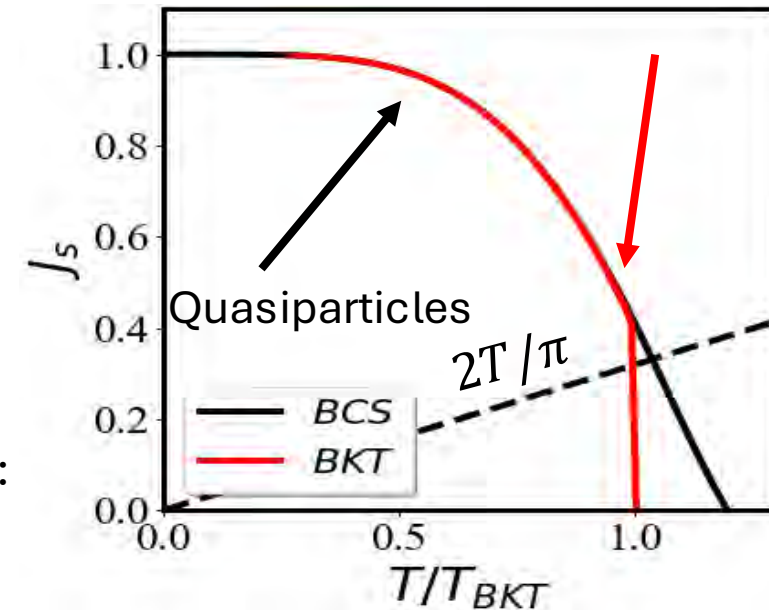
BKT transition in superconductors

In 2D superconductors situation much less clear:

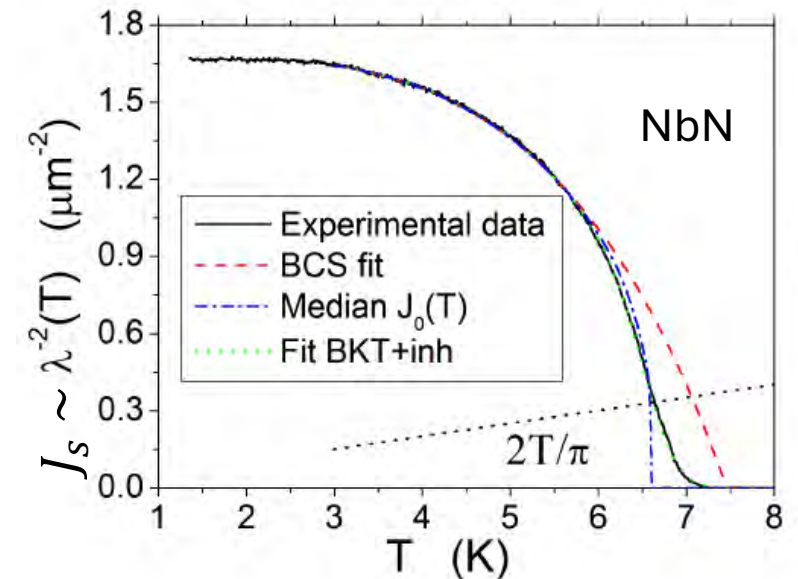
- Few comparisons of superfluid stiffness signatures and transport signatures on the same device
- BKT transition in superconducting films often displays strong broadening:
- Cannot be explained by finite size effects
(see poster L. Pfaffinger)
- Most probably due to material inhomogeneity

more recent experiments:

Cartoon:



Fiory et al., Phys. Rev. B **28**, 5075 (1983)

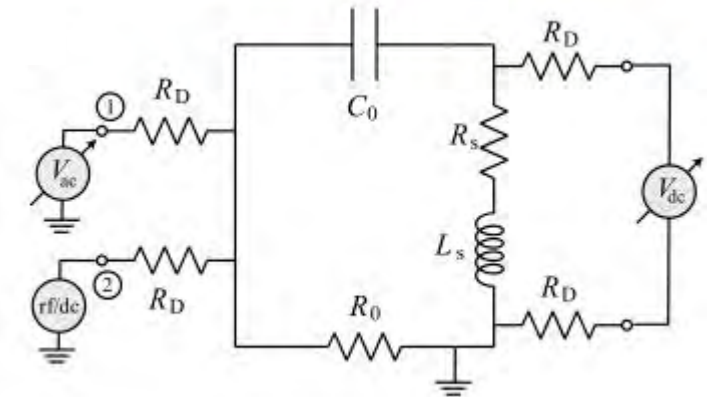
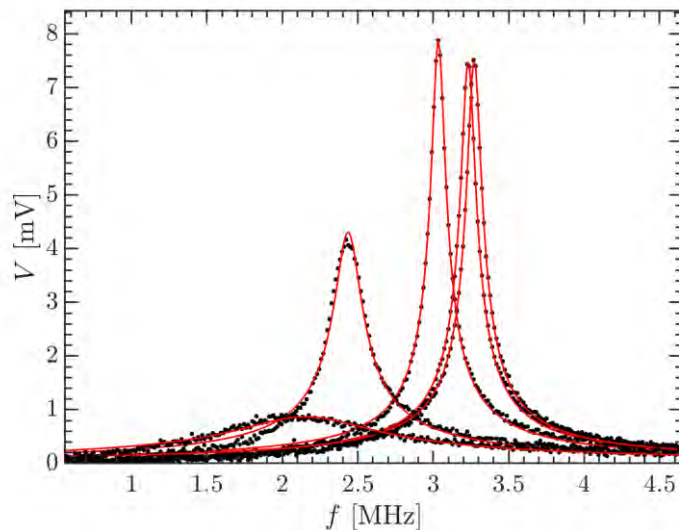


Yong et al., PRB **87**, 184505 (2013)

J_s from kinetic inductance measurements

Resonator method: put superconducting strip of sample in series with Cu-coil

- decrease in resonance frequency f_0 corresponds to increase in kinetic inductance of sample
- decrease in internal Q-factor Q_i corresponds to increase in sample resistance
- J_s is very sensitive to magnetic field



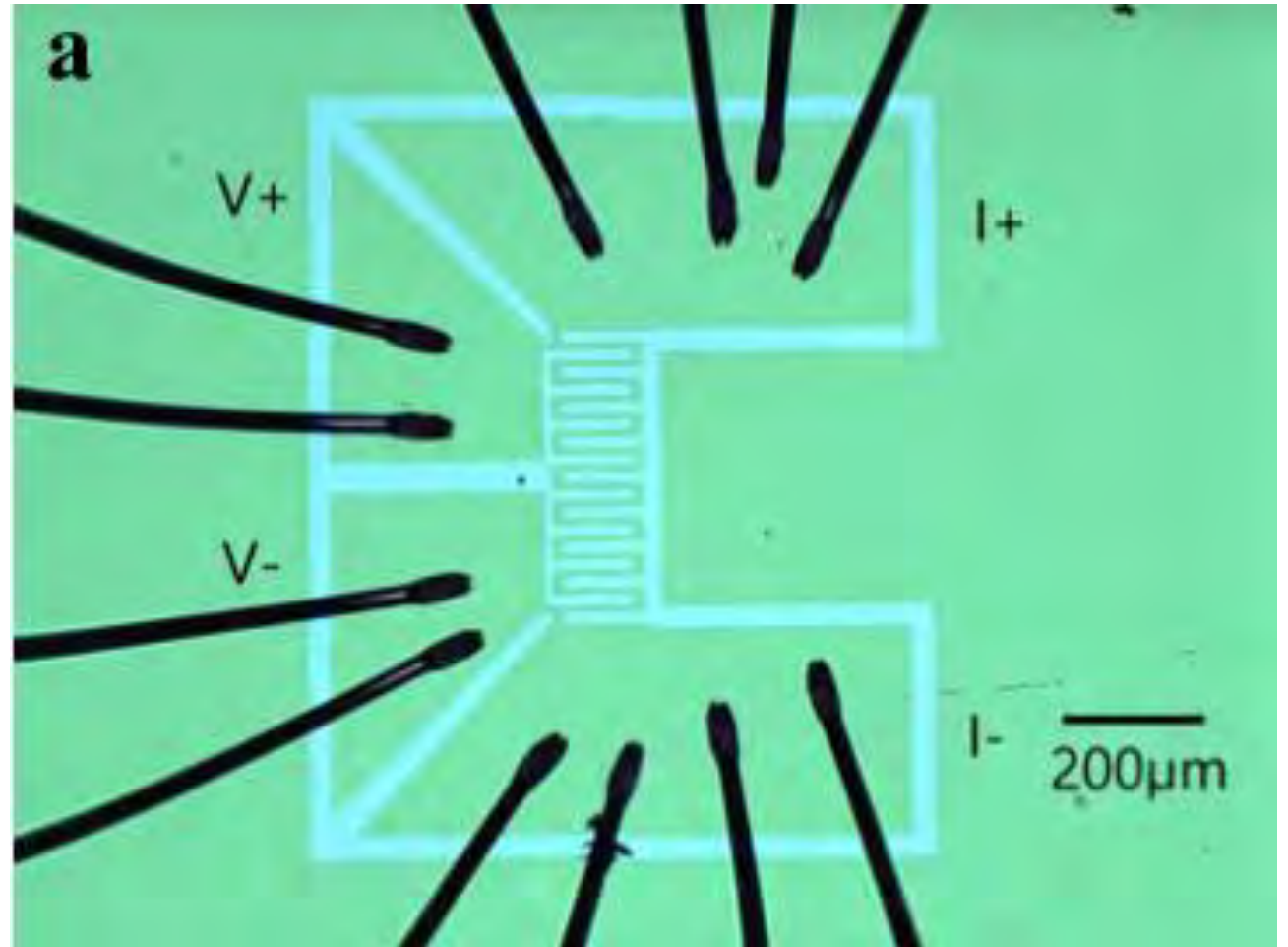
measure superfluid stiffness via kinetic inductance $L_{\blacksquare}$:

$$J_s = \frac{\Phi_0^2}{16\pi^2 k_B L_{\blacksquare}} = \frac{\hbar^2}{4e^2 k_B} \cdot \frac{d}{\mu_0 \lambda^2(T)}$$

$$\text{Dirty limit BCS: } J_s(T=0) = \frac{\Phi_0^2 \Delta(0)}{4\pi k_B \hbar R_N}$$

Our samples

- 3 nm thin films NbN films on Si/SiO₂ substrate
- Preparation by atomic layer deposition at IPHT Jena
- Patterned into long meanders (40 – 200 squares) for high resolution in resistance and inductance
- Vary widths (10, 50, 200 ... μm) and disorder levels ($R_{\square}(300\text{ K}) = 1.5 - 3.3\text{ k}\Omega$, controlled by N₂ during deposition or aging)



Our samples: linear resistance

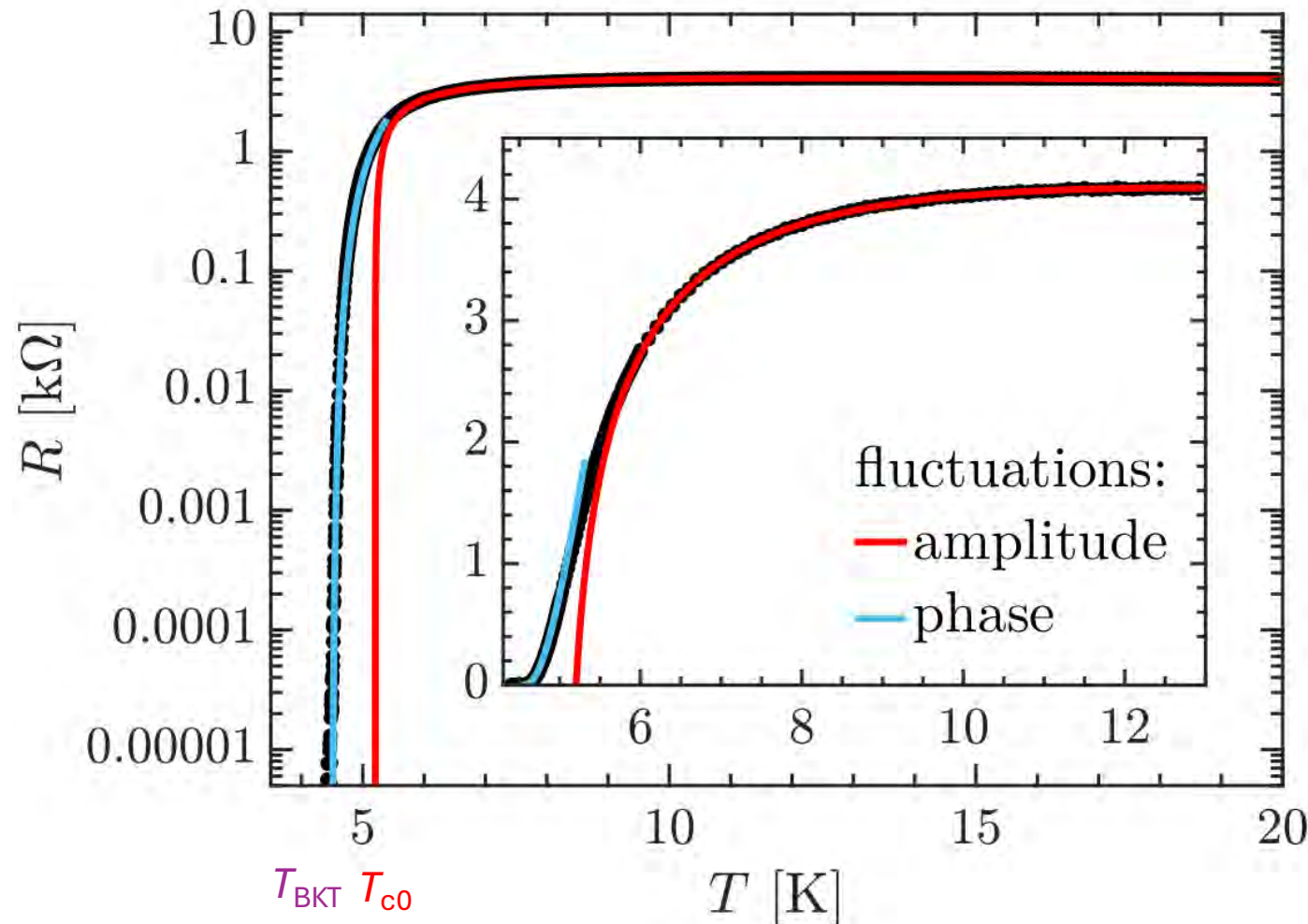
Resistance curves well described by fluctuation conductivity:

above T_{c0} :
amplitude fluctuations of the order parameter

Between T_{BKT} and T_{c0} :
phase fluctuations of the order parameter

Onset of resistance following a „square root cusp“ in temperature:

$$R \sim \exp\left(\frac{-b}{\sqrt{\frac{T}{T_{BKT}} - 1}}\right)$$



Expectations for DC IV -characteristics

non-linear transport characteristics :

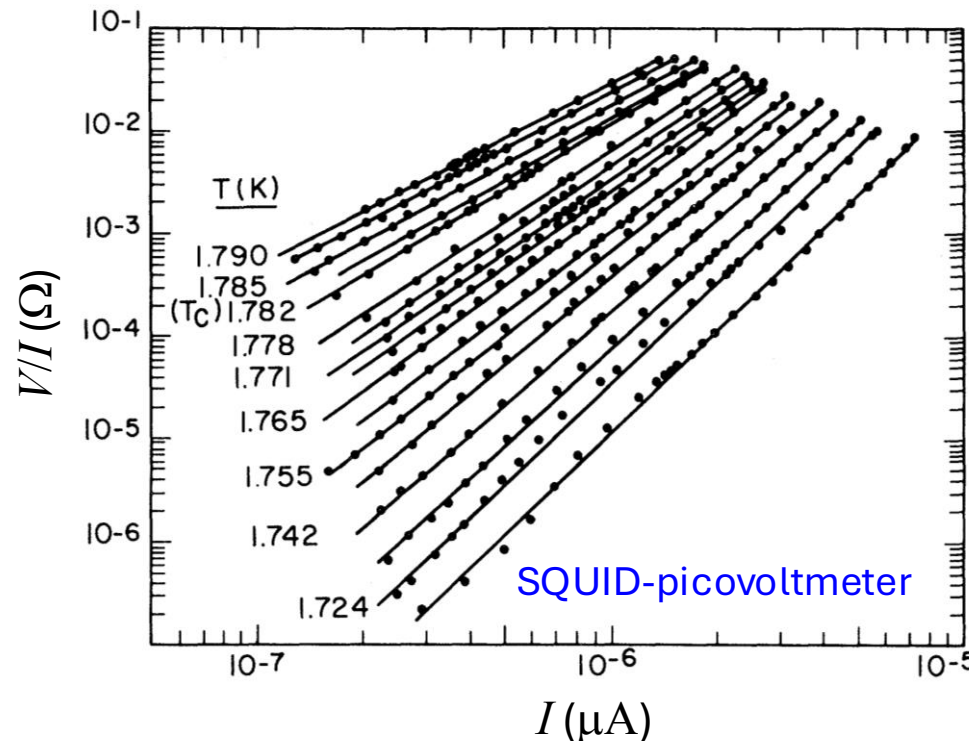
- onset of linear resistance at $T = T_{BKT} < T_{c0}$
- power law behavior of IVs: $V \sim I^\alpha$
- power law exponent jumps from $\alpha = 1$ to 3 at $T = T_{BKT}$
- Direct relation to superfluid stiffness:

$$\alpha(T) = 1 + \frac{\pi J_s(T)}{T}$$

so far very few direct comparisons between $V(I)$ and L_K

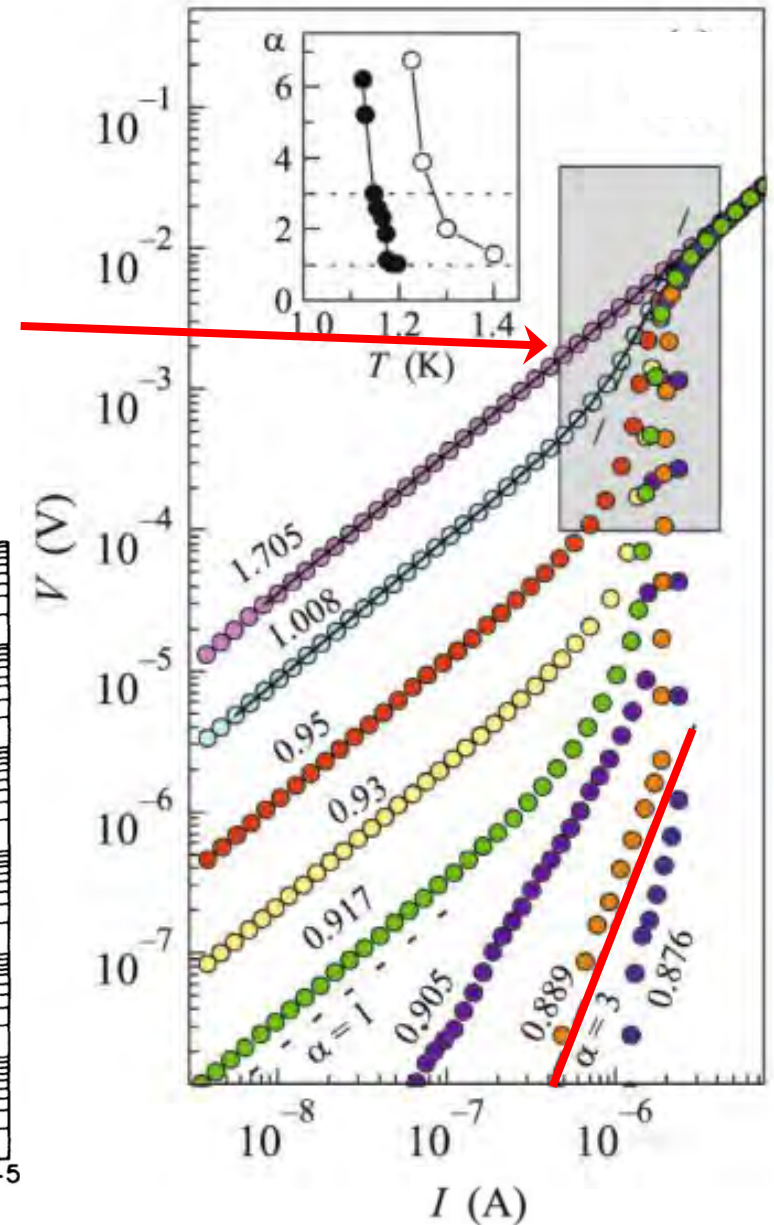
Theory:

Ambegaokar, Halperin, Siggia, Nelson
PRL **40**, 783 (1978), PRB **21**, 1806 (1980)



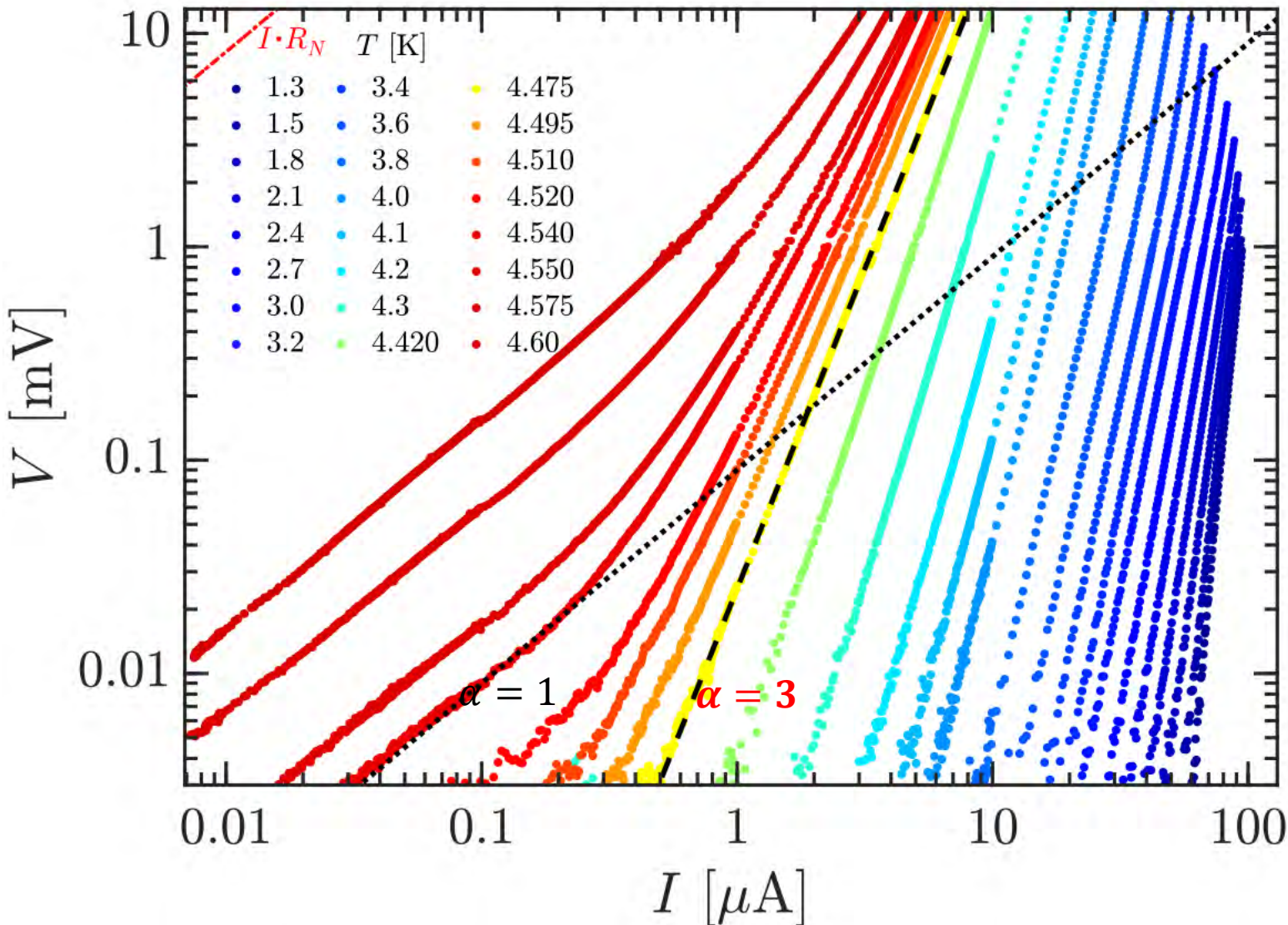
Fiory et al., Phys. Rev. B **28**, 5075 (1983)

electron
heating
regime!



Postolova et. al, JETP Lett., **100** (2014)

J_S and T_{BKT} from DC IV -characteristics



- Fast measurements (~ 2 s) of IV -characteristics to minimize heating
- Power-law behavior over 2-3 orders of magnitude in V

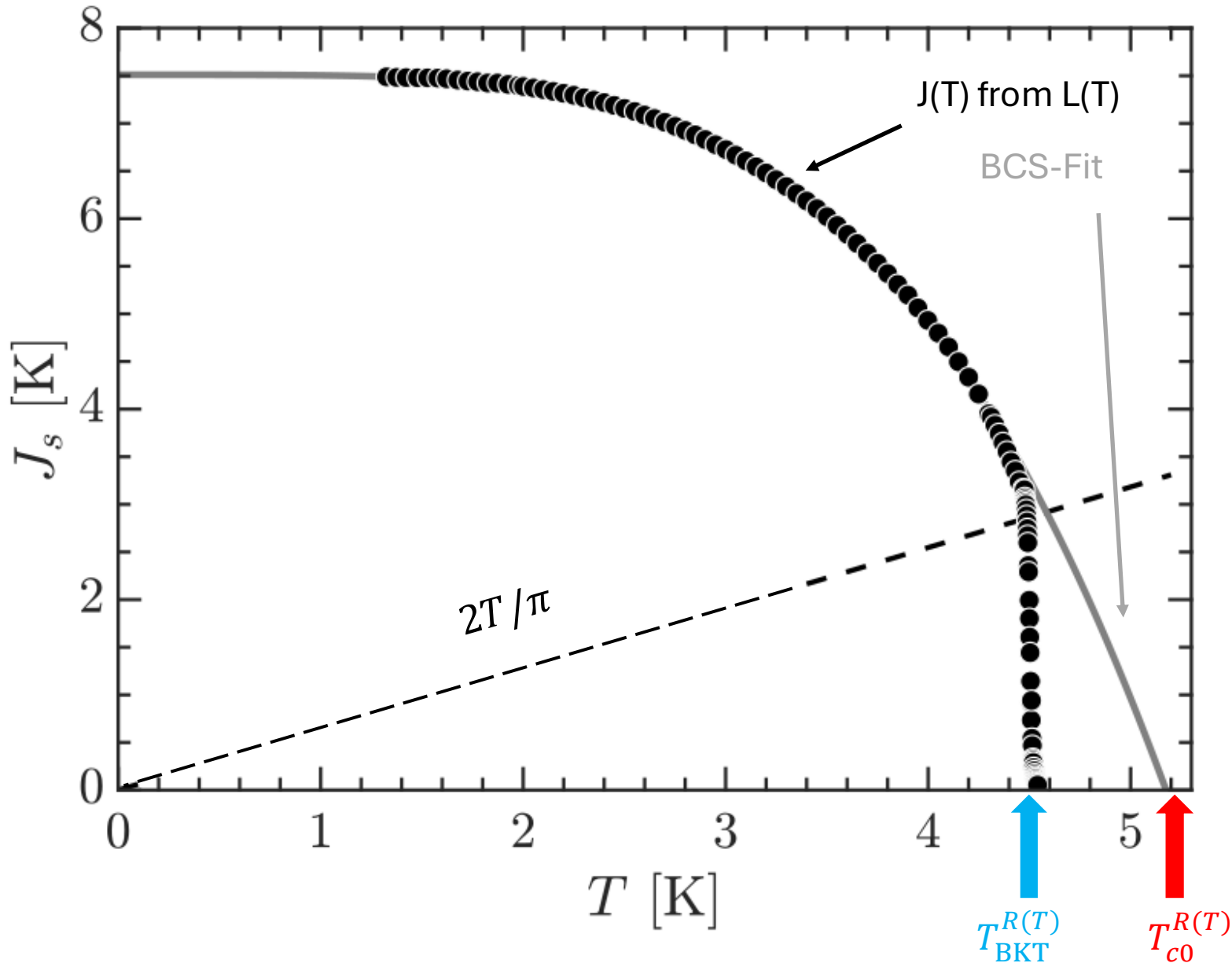
- J_S from power law exponent $\alpha(T)$:

$$\alpha(T) = 1 + \frac{\pi J_S(T)}{T}$$

- Comparison between $R(T)$ and $I(V)$:

- $T_{\text{BKT}}^{R(T)} = 4.471$ K,
- $T(\alpha = 3) = 4.475$ K

Temperature dependence of J_s



$$BKT: J_s(T_{BKT}) = \frac{2}{\pi} T_{BKT}$$

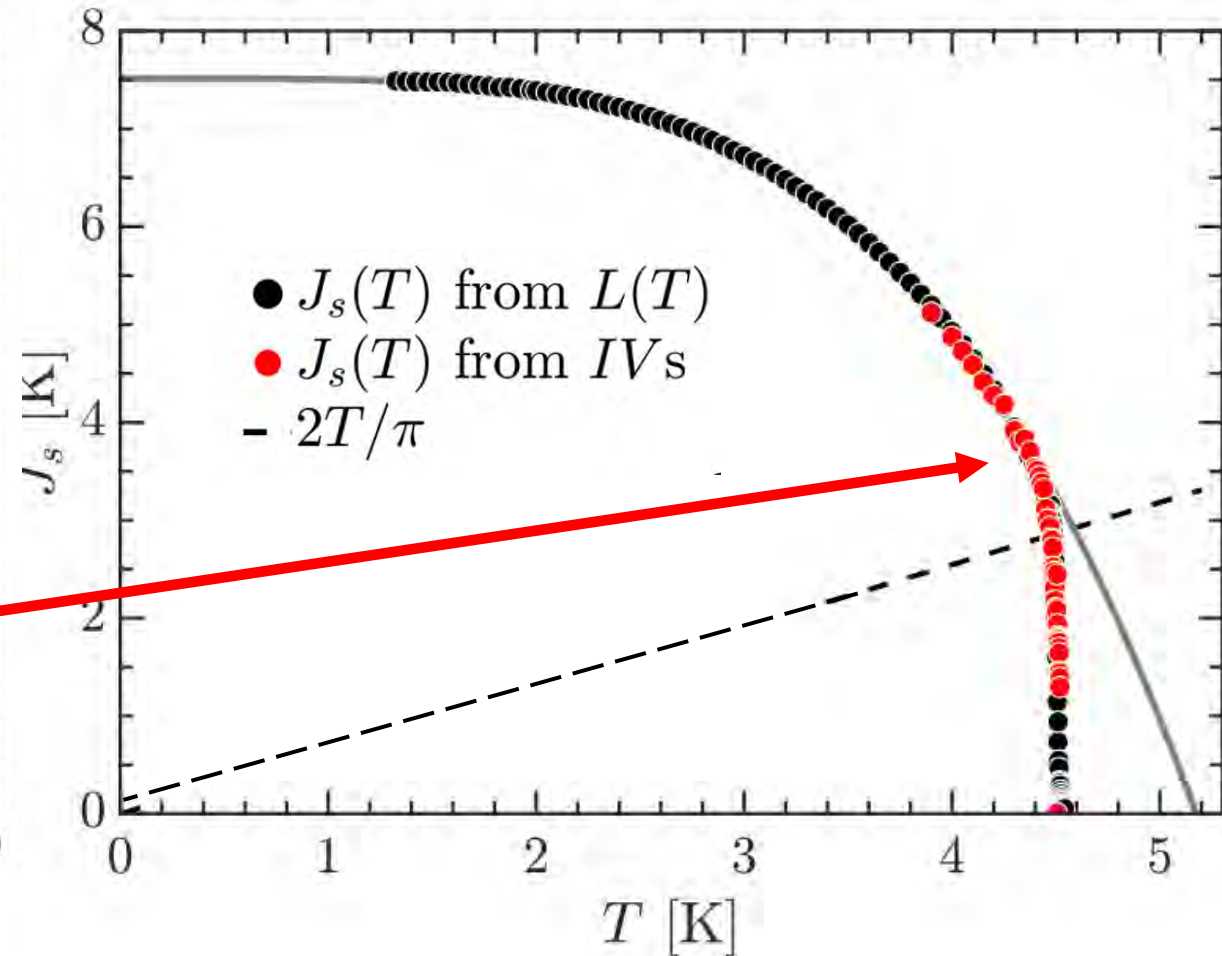
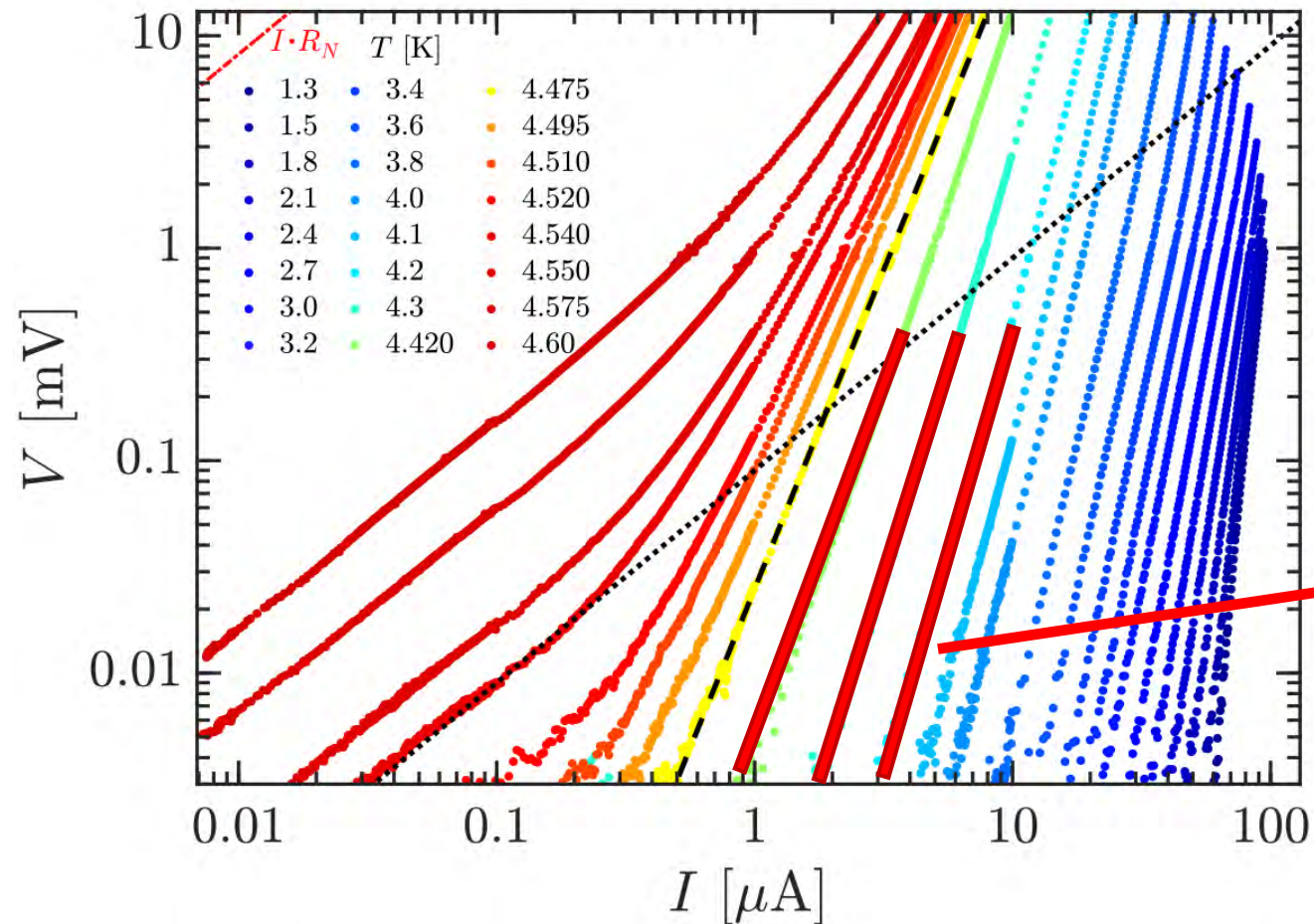
$$BCS: J_s(T) = J_0 \frac{\Delta(T)}{\Delta(0)} \tanh\left(\frac{\Delta(T)}{2k_B T}\right)$$

- sharp transition in $J_s(T)$
- excellent agreement between resistive and inductive transition temperatures

Comparison between $L(T)$ and $I(V)$

$$V(I) = A \cdot I^{\alpha(T)}$$

$$\alpha(T) = 1 + \frac{\pi J_s(T)}{T}$$



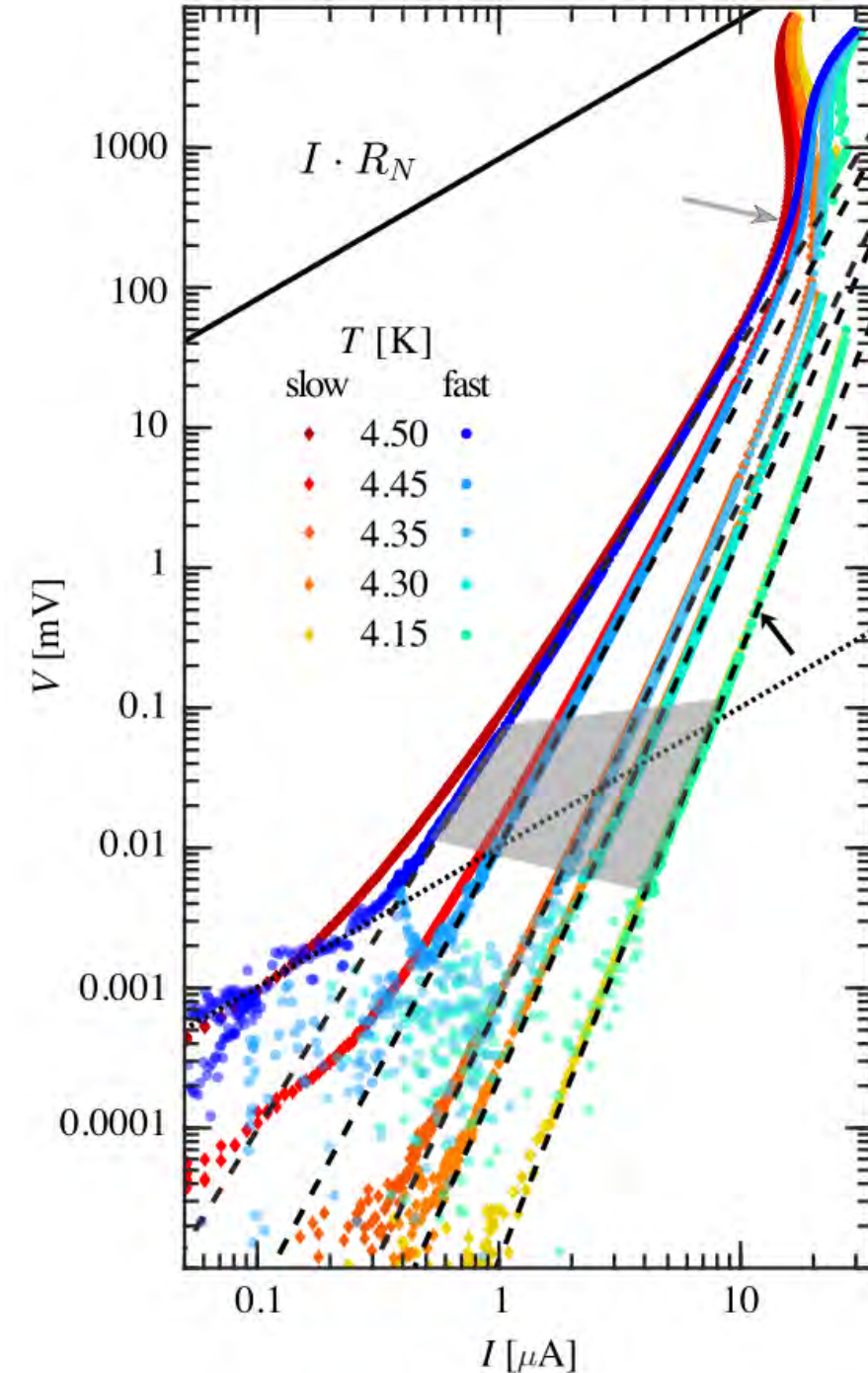
Excellent agreement between $J_s(T)$ from $L(T)$ and $I(V)$

A. Weitzel, PRL **131**, 186002 (2023)

Subtleties of IV characteristics

Comparison between fast (~ 2 s) and slow (~ 10 min):

- „Large“ area of agreement
- At few picowatts:
electron overheating (black arrow)
- At microwatts: stage heating (grey arrow)
- At low voltage: Ohmic tails



Questions?

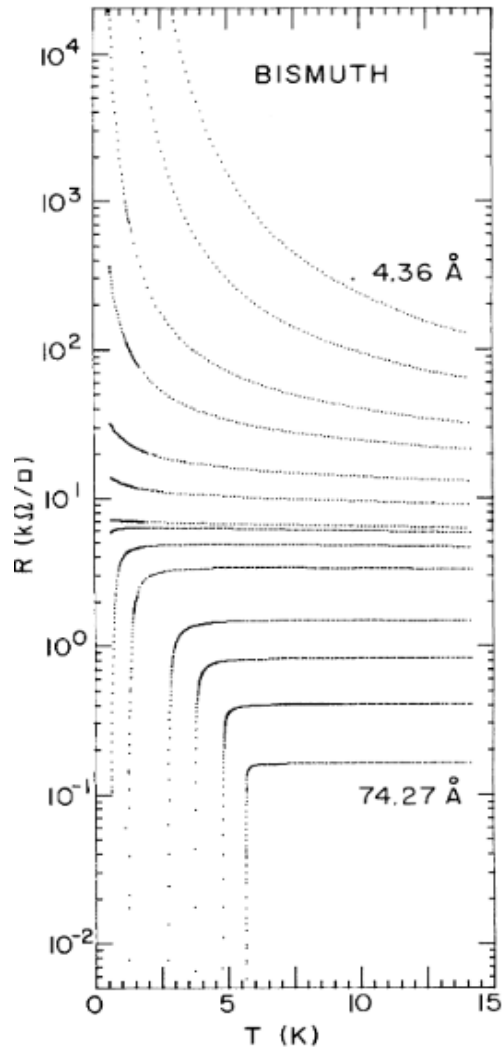
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Abrahams et al. 1979: „There are no metals in two dimensions!“

- Superconductivity
- Suppression of zero-resistance state by phase fluctuations
- **Suppression of Cooper-pairing by disorder**

From superconductor to insulator, a continuous quantum phase transition ?



Haviland et al., PRL **62** 2180, 1989

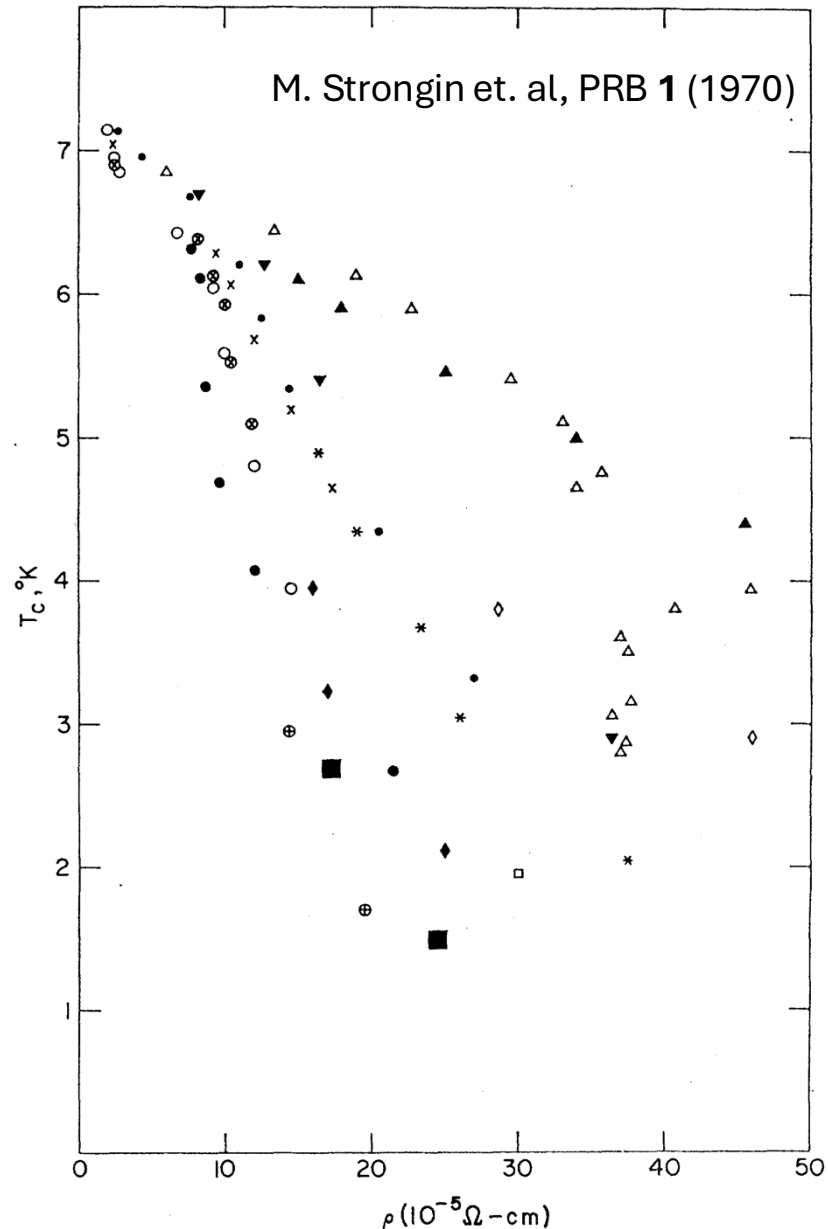
Effect of *strong* disorder on the superconducting order parameter

$$\Psi(\mathbf{r}) = \Delta(\mathbf{r})e^{i\varphi(\mathbf{r})}$$

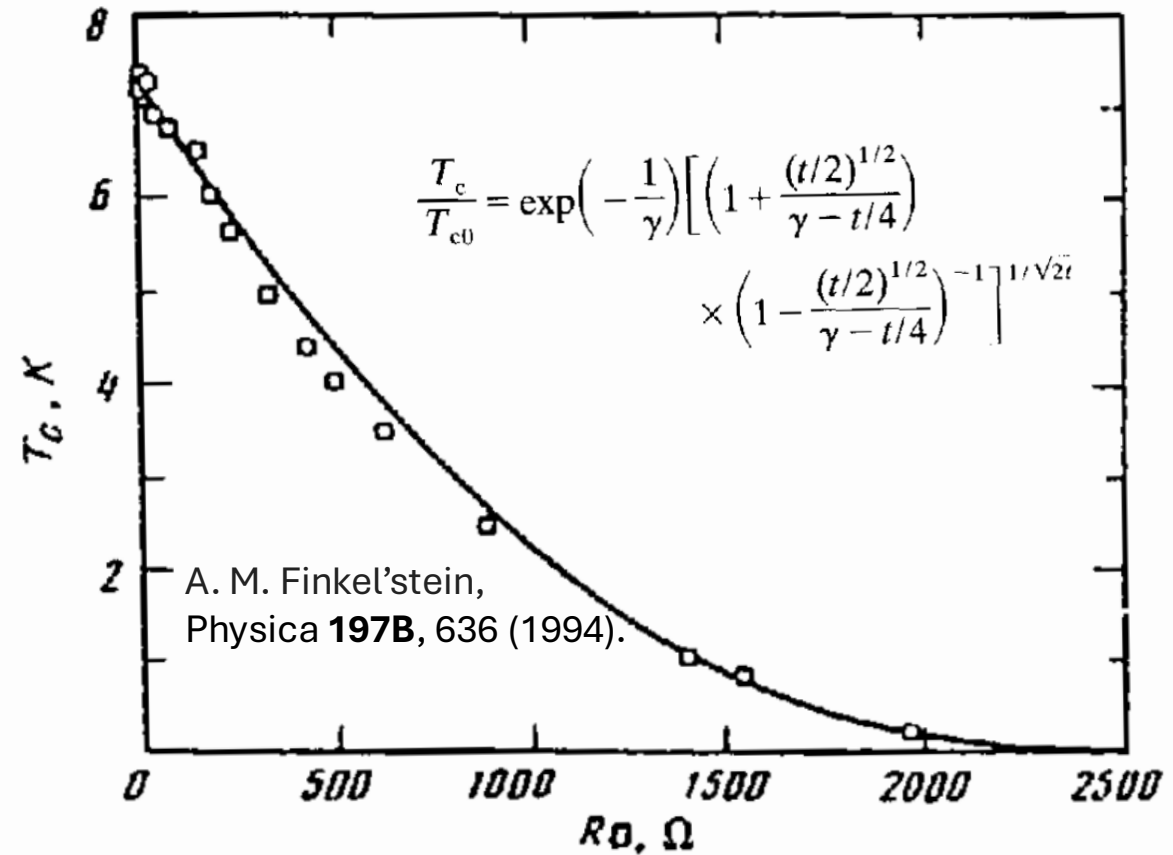
Fermionic route:
suppression of
amplitude of
superconducting
order parameter

Bosonic route:
amplitude of order parameter
remains finite,
but phase fluctuations destroy
superfluid *phase stiffness*

Fermionic approach



Enhancement of Coulomb interaction suppresses electron screening and thus pairing amplitude $\Delta(T)$ and mean field T_c



A. M. Finkel'stein, Physica **197B**, 636 (1994)

parameters t and γ are related to Drude scattering time and electron density

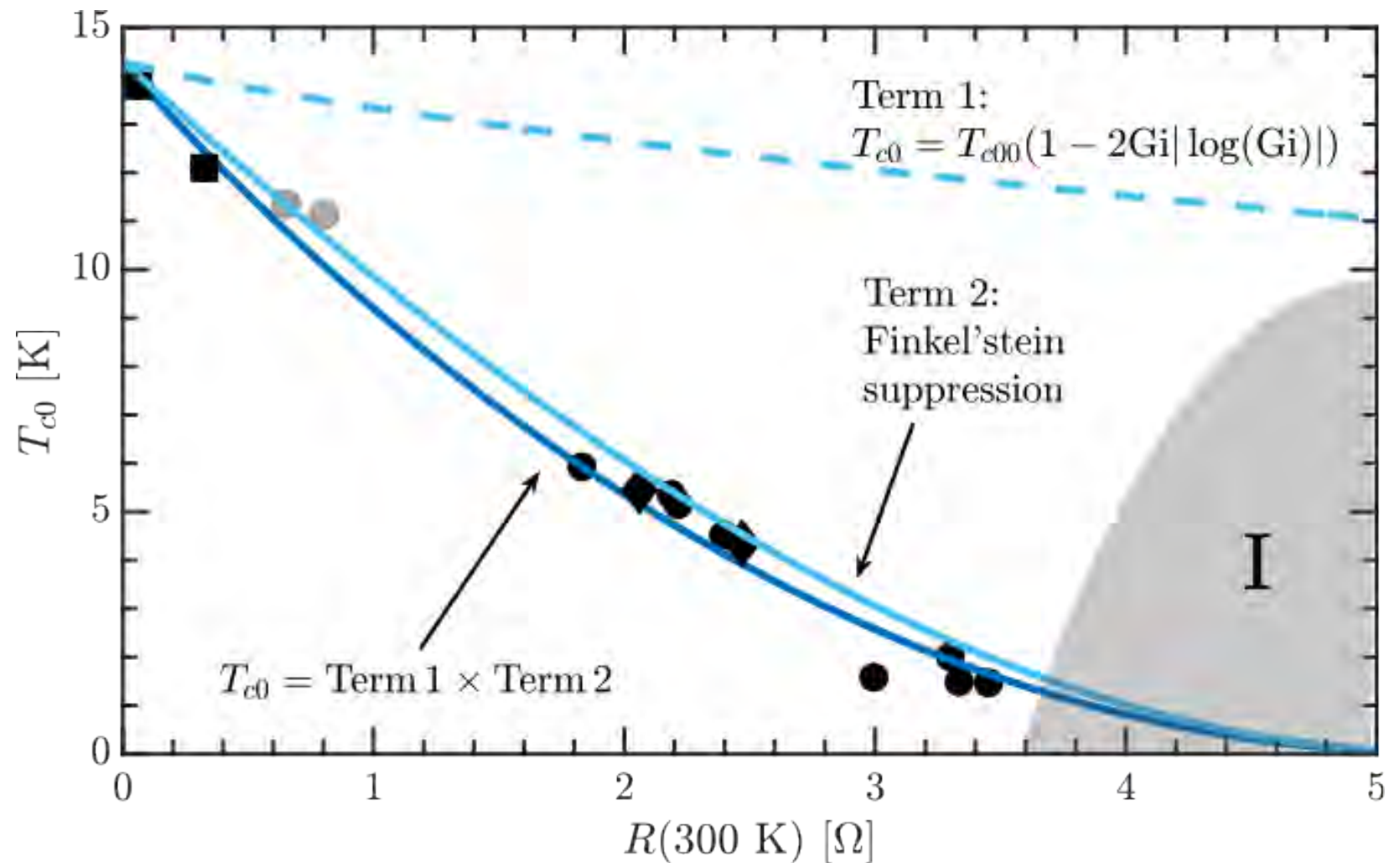
Fermionic approach

Enhancement of Coulomb interaction
suppresses pairing amplitude $\Delta(T)$ and T_{c0}

Monotonic suppression
of T_{c0} , but no SIT expected
from this theory

Data NbN:

(A. Weitzel, UR)



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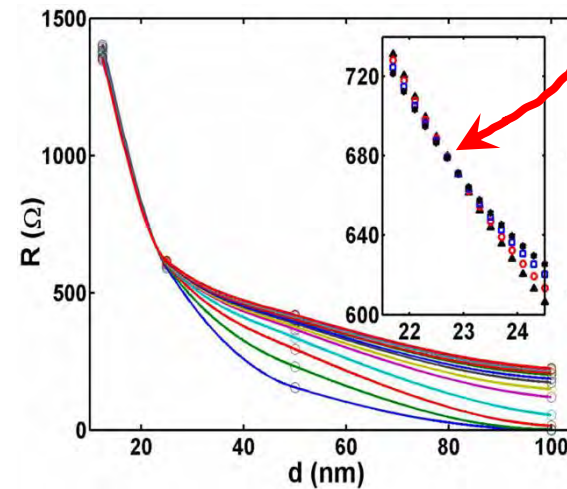
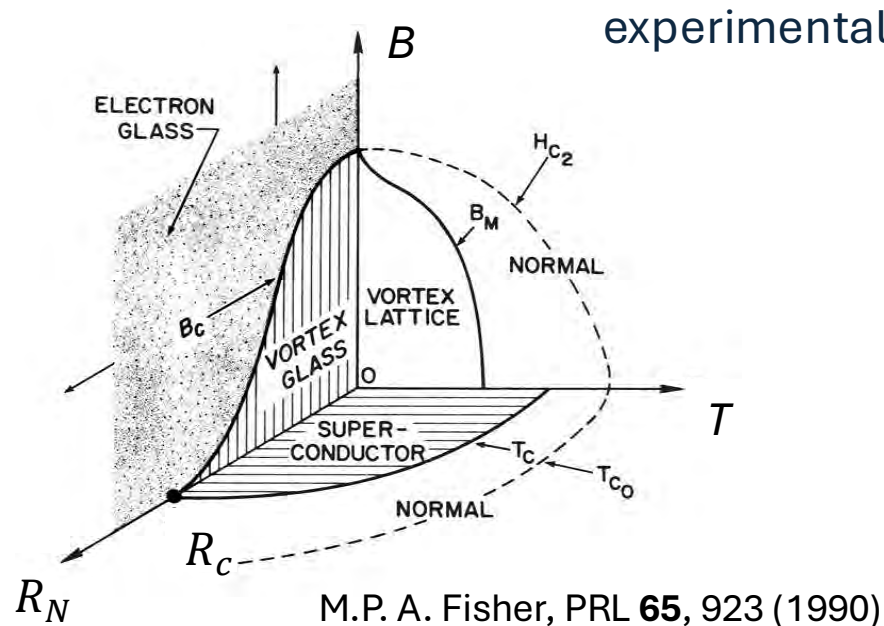
Abrahams et al. 1979: „There are no metals in two dimensions!“

- Superconductivity
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- Suppression of Cooper-pairing by disorder
- **Direct superconductor-insulator transition**

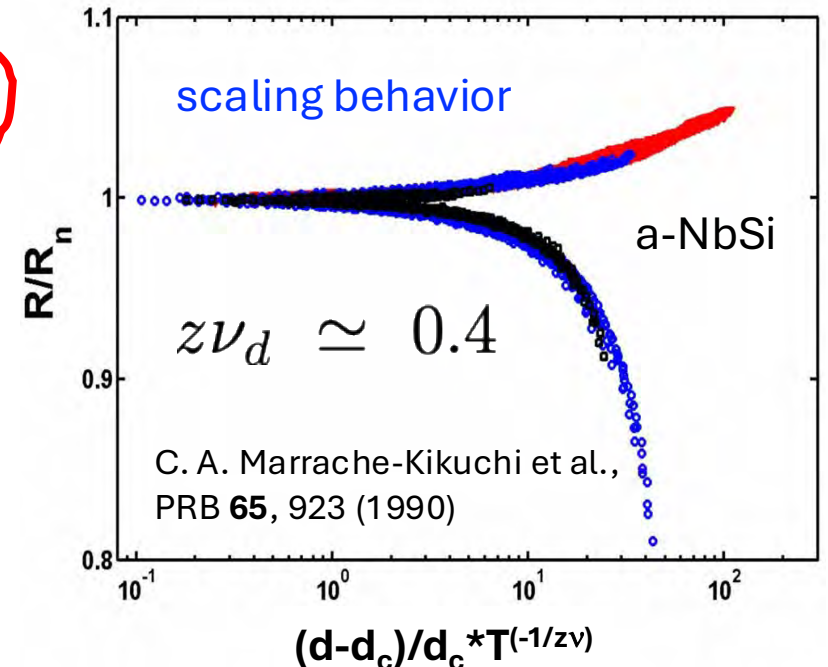
Bosonic approach

Continuous quantum phase transition between a **vortex glass phase** (superconducting) and a **Cooper-pair glass phase** (insulating), characterized by:

- diverging coherence length: $\xi \propto |R_c - R_N|^{-\nu}$
- vanishing characteristic energy: $T_{\text{BKT}} \propto \xi^{-z} \propto |R_c - R|^{z\nu}$
- phase transition at $T = 0$ in magnetic field: $B_c \propto |R_c - R|^{2\nu}$
- universal scaling behavior in magnetic field: $R_{\square} = \frac{h}{4e^2} f(|B - B_c|/T^{1/z_B\nu_B})$



in resistance isotherms

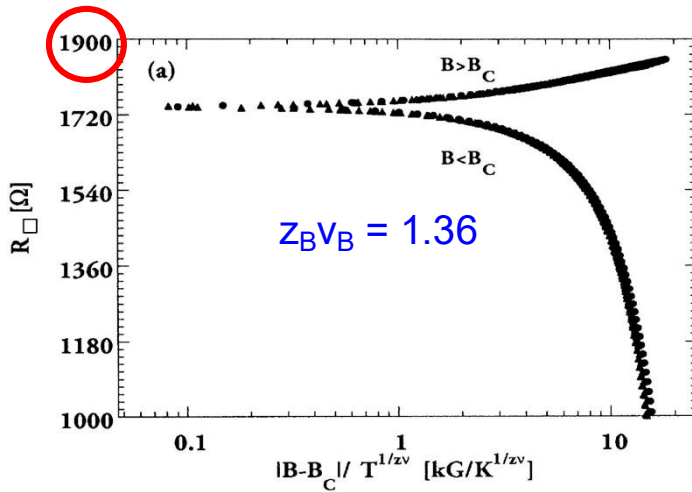
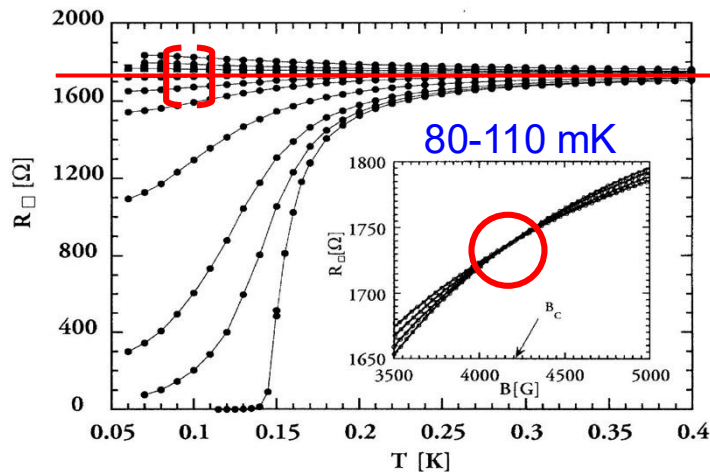


Bosonic approach

scaling behavior in magnetic field

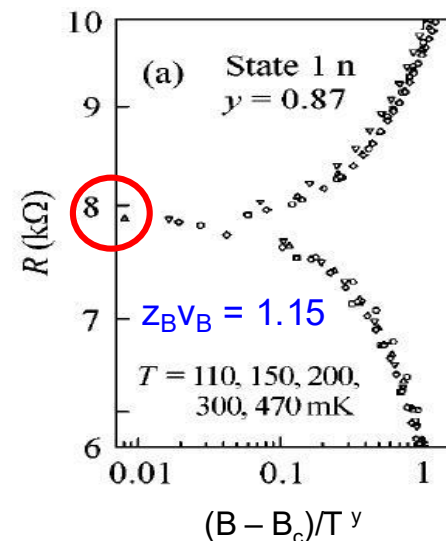
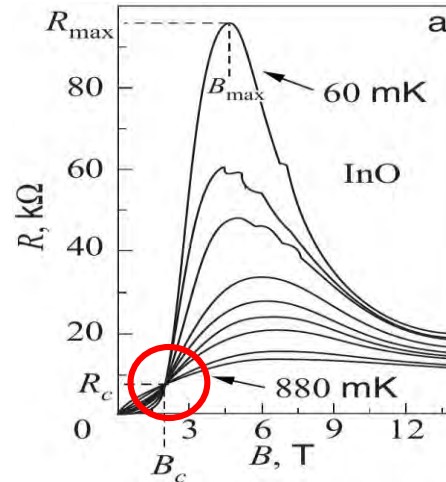
Amorphous MoGe

A. Yazdani and A. Kapitulnik
Phys. Rev. Lett. **74**, 3037 (1995)



Amorphous InO_x

V.F. Gantmakher et al.,
JETP **71**, 473 (2000)

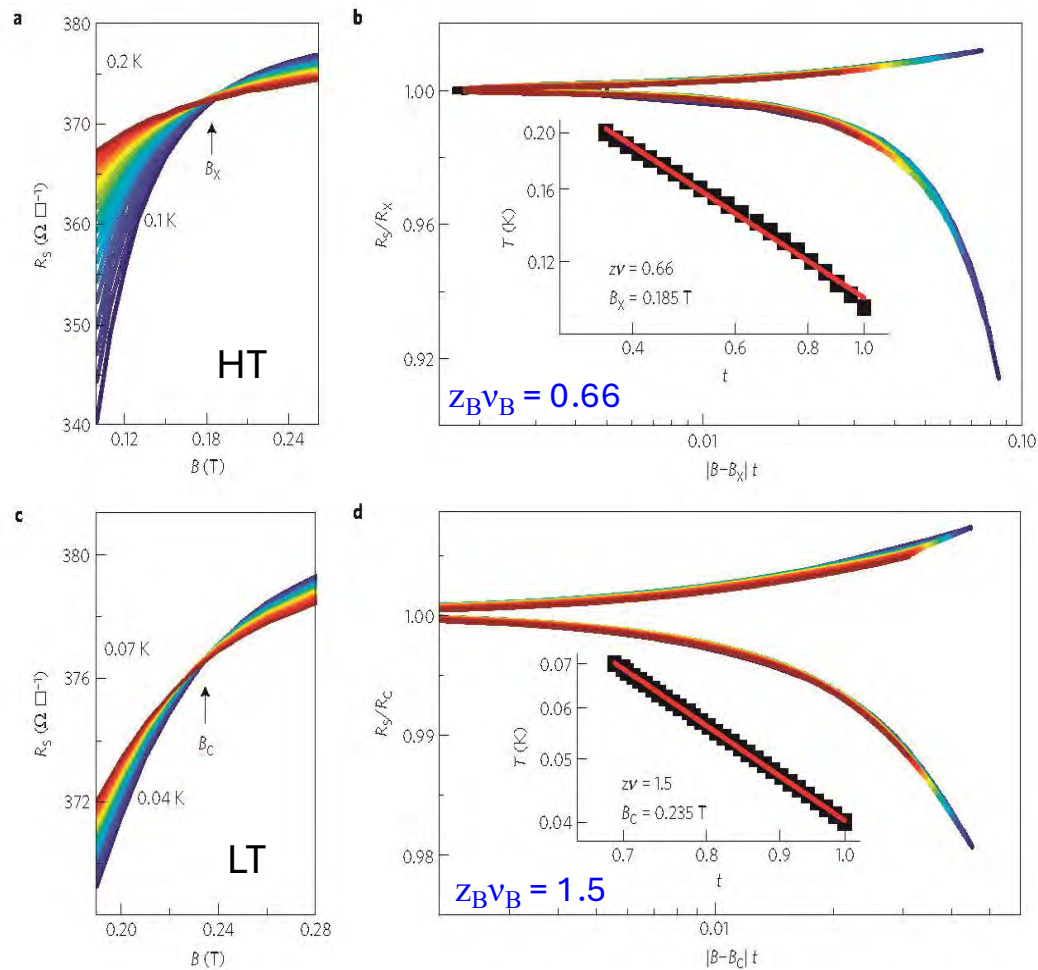


Questions:

- what is an insulator?
- what about about highly insulating states?
- how universal is the scaling?
- how significant is it?

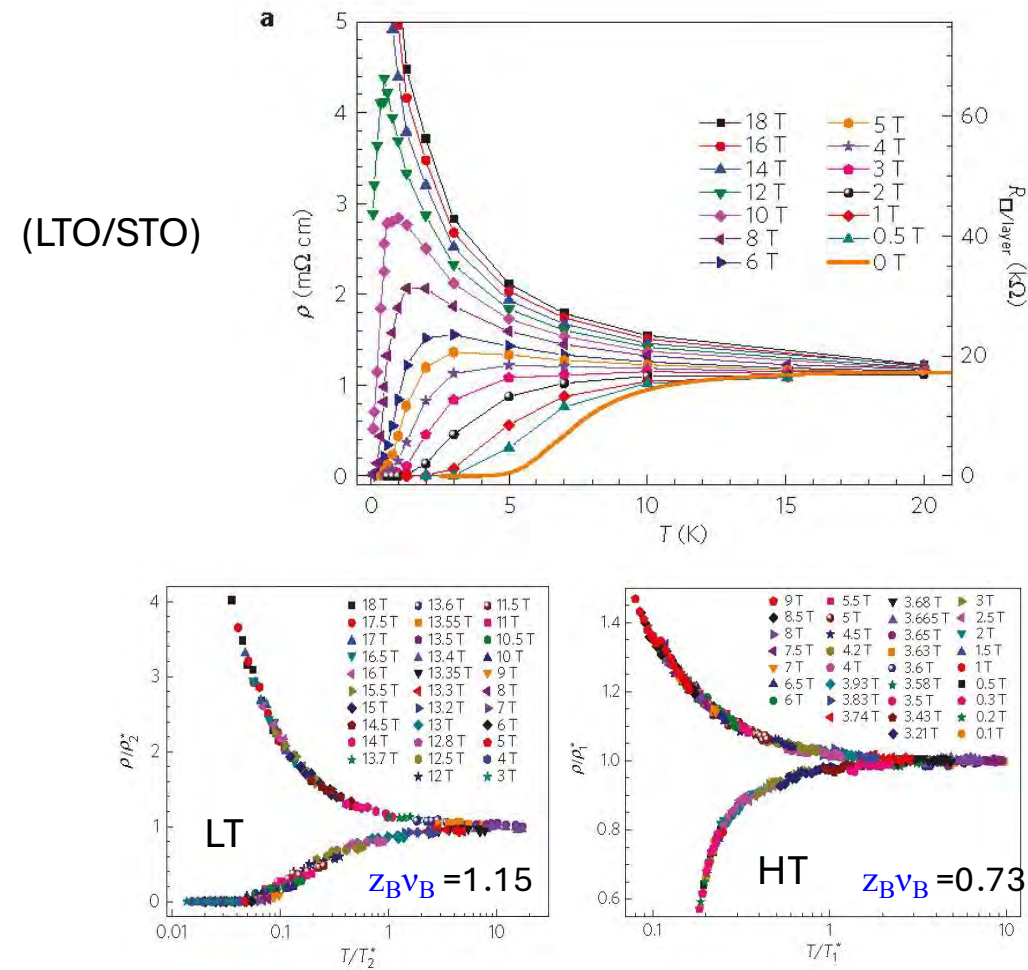
Multiple quantum criticality in a two-dimensional superconductor

J. Biscaras¹, N. Bergeal¹, S. Hurand¹, C. Feuillet-Palma¹, A. Rastogi², R. C. Budhani^{2,3}, M. Grilli⁴, S. Caprara⁴ and J. Lesueur^{1*}



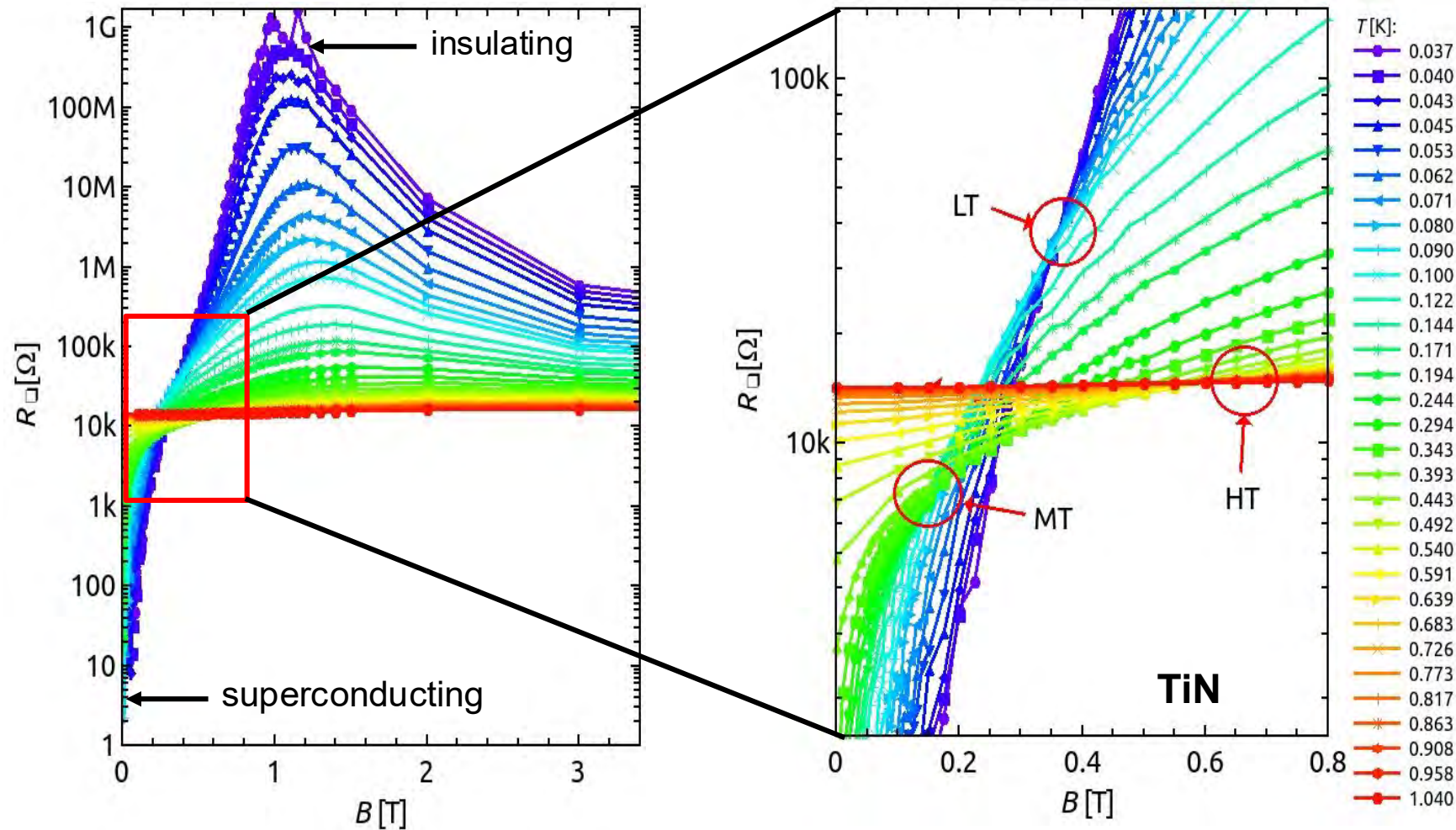
Two-stage magnetic-field-tuned superconductor-insulator transition in underdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

Xiaoyan Shi^{1†}, Ping V. Lin¹, T. Sasagawa², V. Dobrosavljević¹ and Dragana Popović^{1*}



Bosonic route

scaling behavior in magnetic field



three crossing points, where $R(B)$ s intersect!

Questions:

- what is an insulator?
- what about about highly insulating states?
- how universal is the scaling?
- how significant is it?

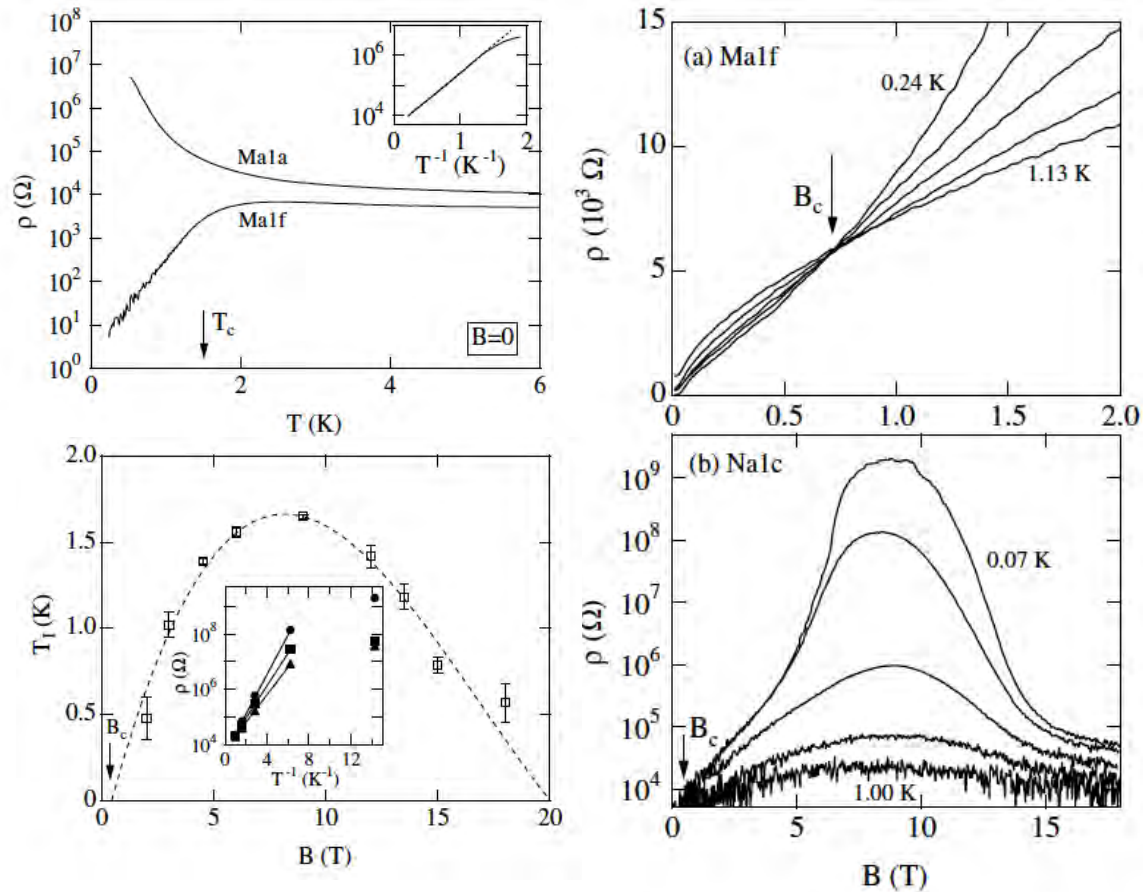
Direct superconductor/insulator transition with Arrhenius-like conductance on the insulating side

G. Sambandamurthy, D. Shahar, et al.

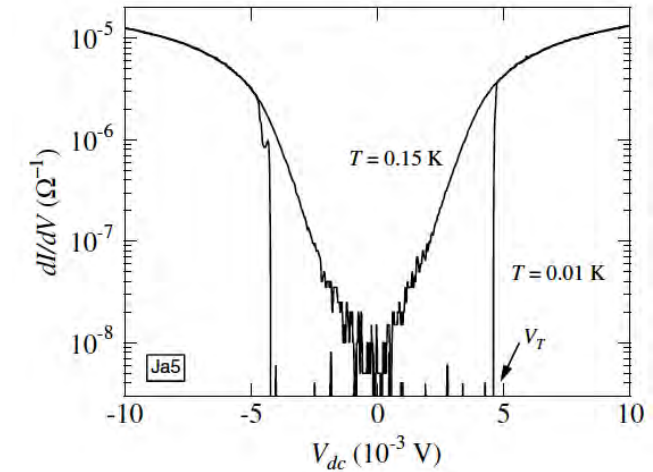
Superconductivity-Related Insulating Behavior

Phys. Rev. Lett. **92**, 107005 (2004)

InO_x

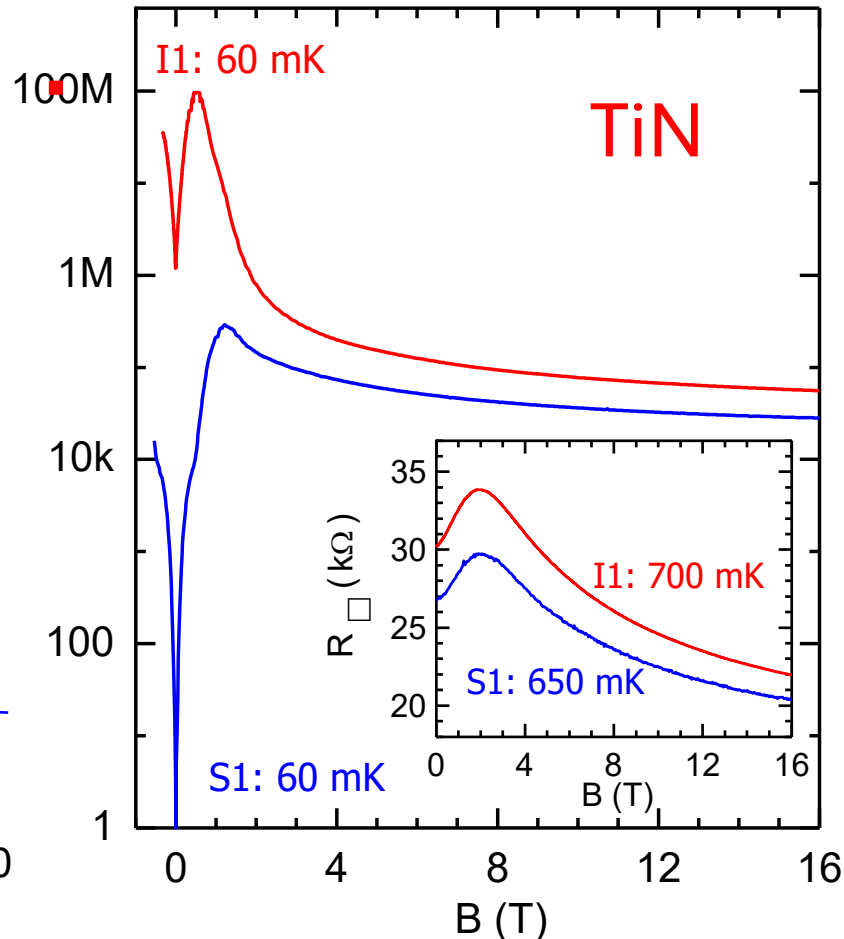
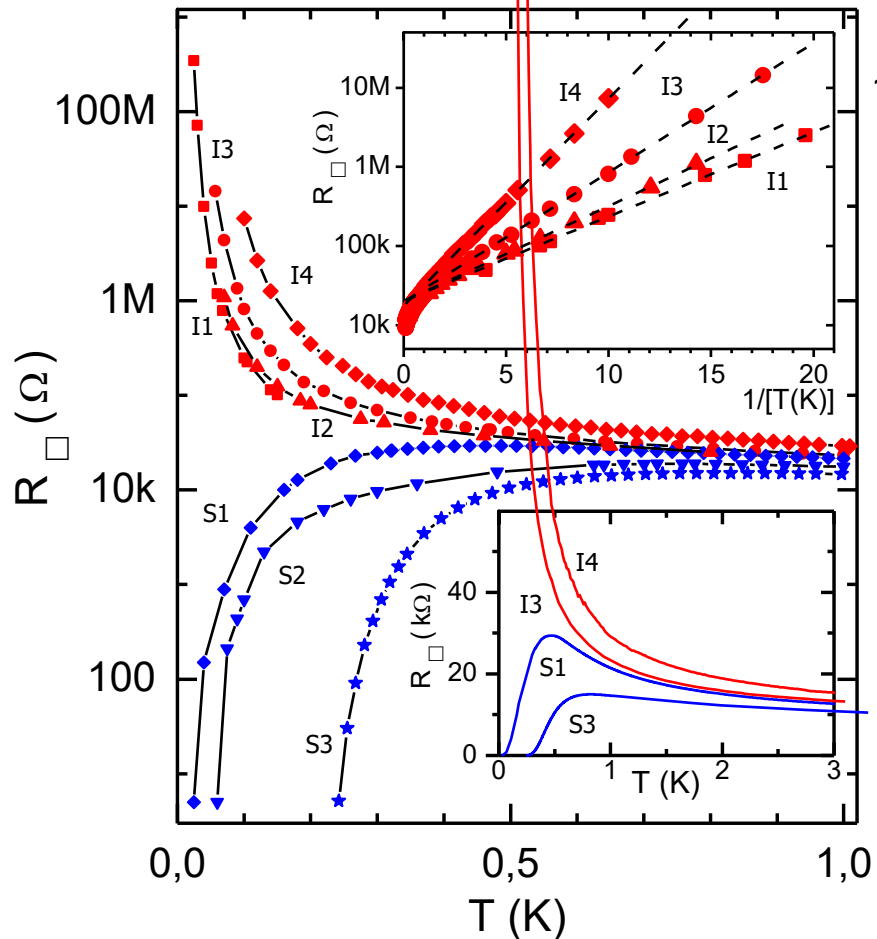


G. Sambandamurthy, D. Shahar, et al.,
**Experimental Evidence for a Collective Insulating
State in Two-Dimensional Superconductors**
Phys. Rev. Lett. **94**, 017003 (2005)



Direct superconductor/insulator transition with Arrhenius-like conductance on the insulating side

homogeneous **TiN**-films of 4-18 nm thickness show a disorder-induced SIT



Qualitatively similar behavior
for insulating and
superconducting films

Exponential decay of
magneto-resistance at high
fields:
gradual suppression of
superconducting OP

Very similar behavior of
insulating and
superconducting films
for $T > 600$ mK

Superconductivity important
for insulating behavior ?!

**quasi-metallic phase
at high B**

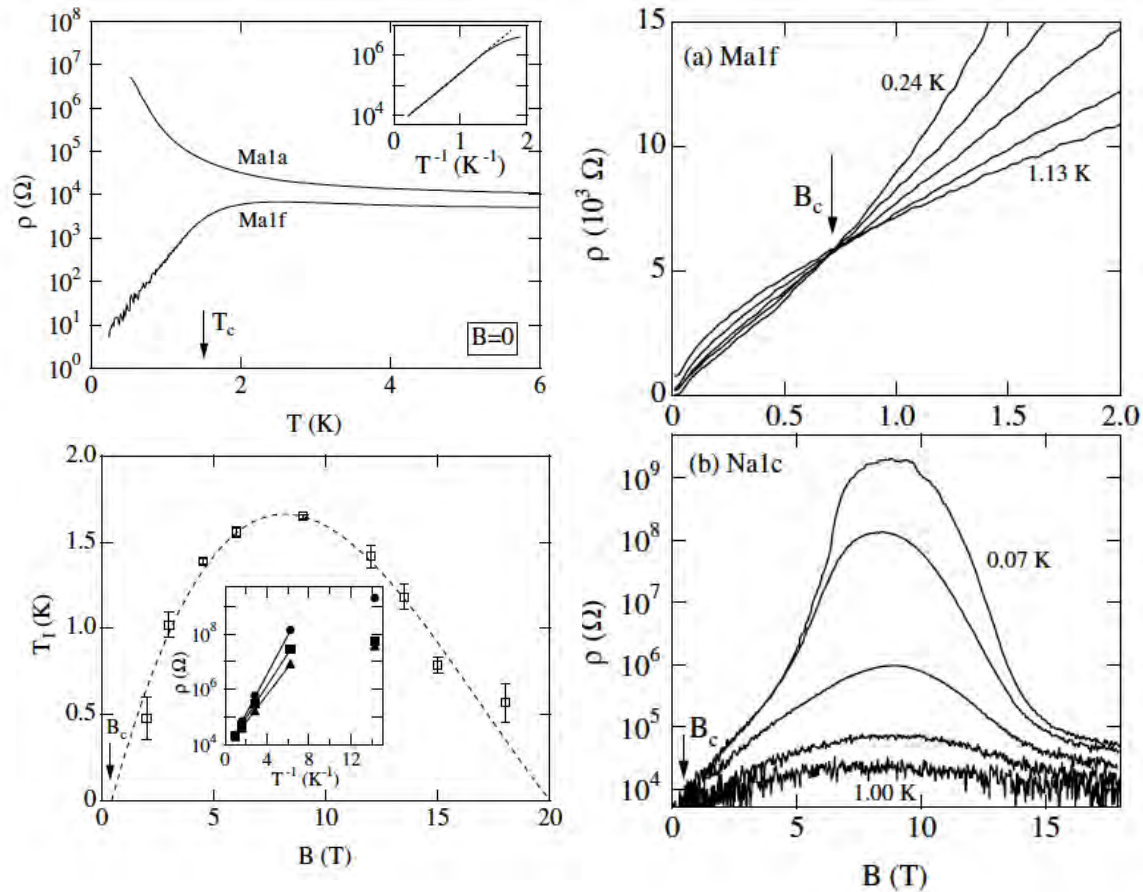
**low B :
,Cooper-pair' - insulator !**

Direct superconductor/insulator transition with Arrhenius-like conductance on the insulating side

G. Sambandamurthy, D. Shahar, et al.

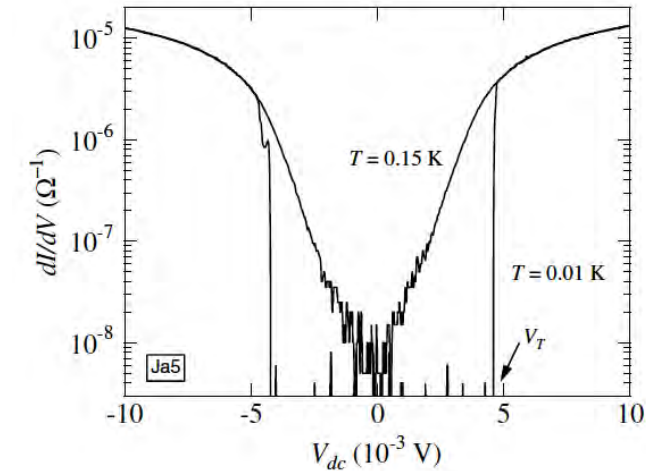
Superconductivity-Related Insulating Behavior

Phys. Rev. Lett. **92**, 107005 (2004)



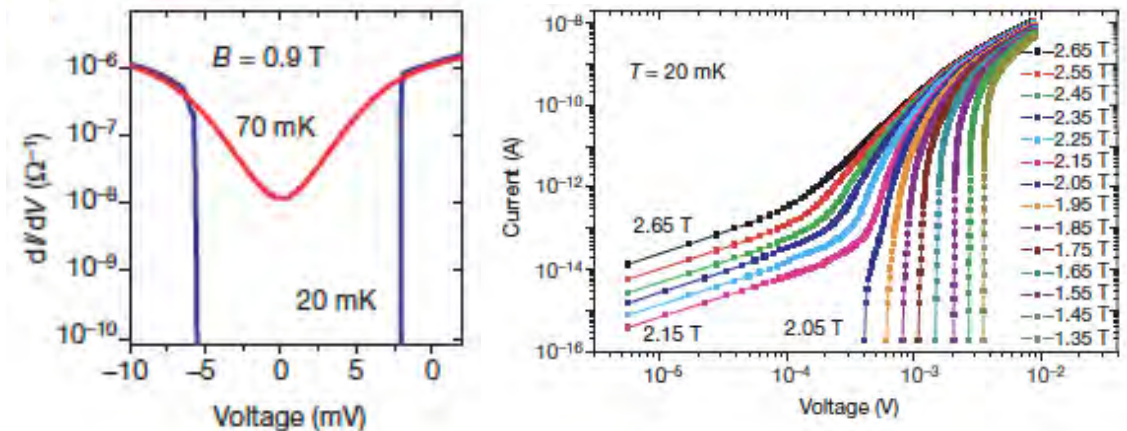
G. Sambandamurthy, D. Shahar, et al.,
**Experimental Evidence for a Collective Insulating
State in Two-Dimensional Superconductors**
Phys. Rev. Lett. **94**, 017003 (2005)

InO_x

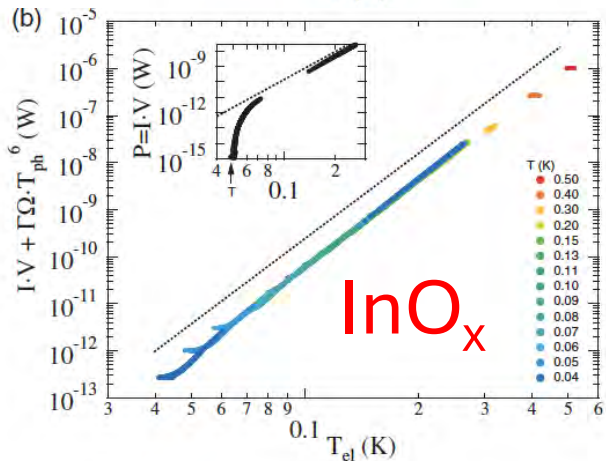
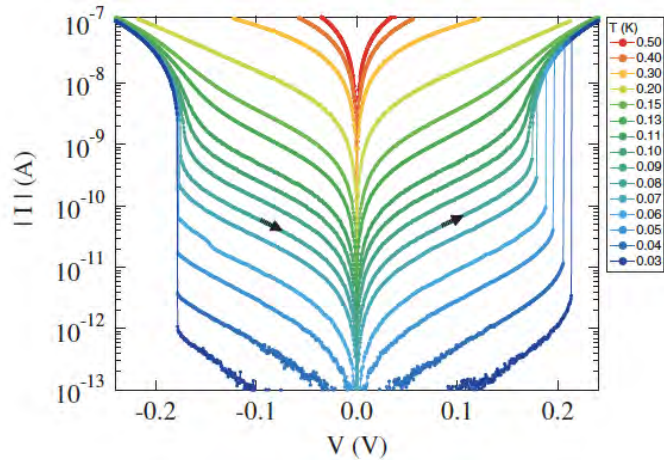


V. M. Vinokur, T. I. Baturina, C. Strunk, et al.
Superinsulator and quantum synchronization
Nature **452**, 613 (2008)

TiN



Alternative theory for jumps in $I(V)$ and the nature of the insulating state



M. Ovadia, B. Sacépé, D. Shahar
Electron-Phonon Decoupling in Disordered Insulators
 Phys. Rev. Lett. **102**, 176802 (2009)

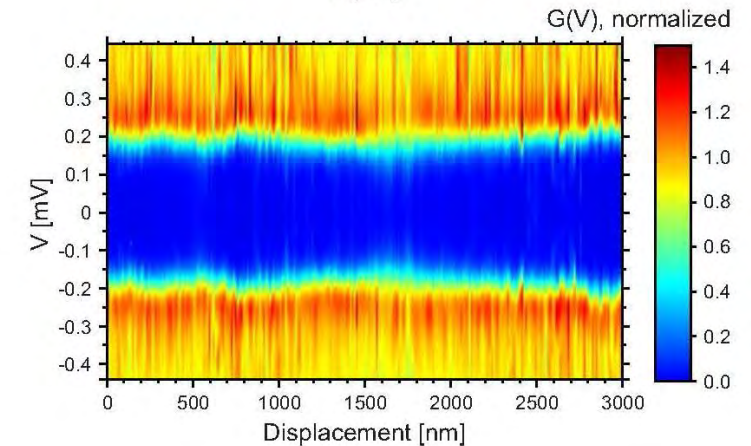
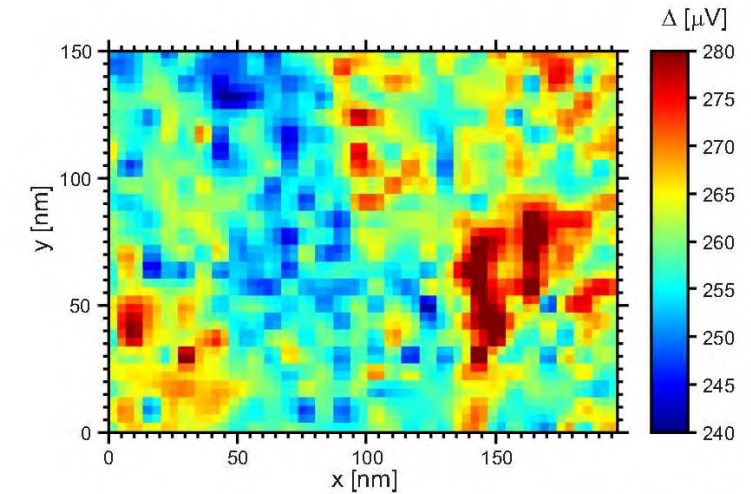
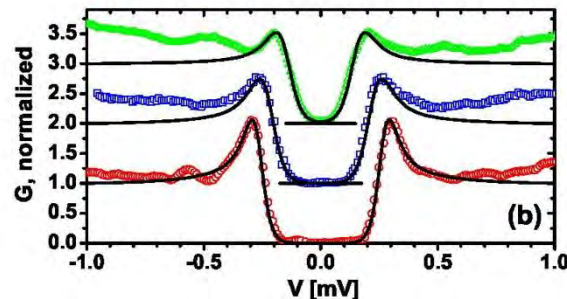
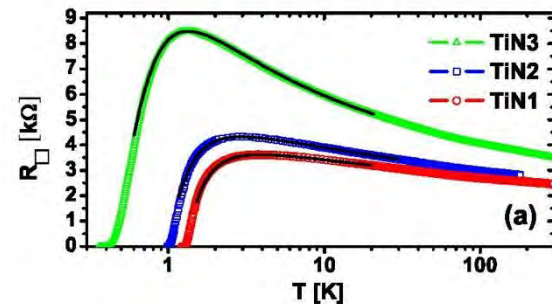
B. Altshuler, V. Kravtsov, I. Lerner, I. Aleiner,
Jumps in Current-Voltage Characteristics in Disordered Films
 Phys. Rev. Lett. **102**, 176803 (2009).

Assumptions:

- $I(V)$ is intrinsically linear
- T_{el} decouples from T_{ph} and is determined from heat balance equation:

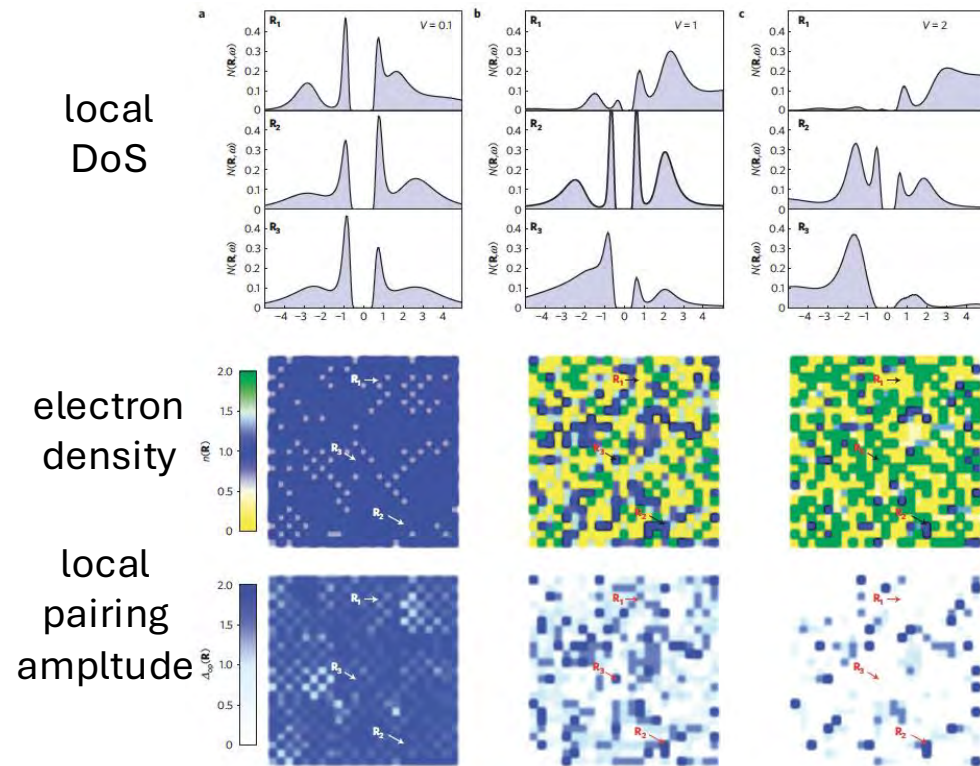
$$\frac{V^2}{R(T_{el})} = \Gamma \Omega (T_{el}^\beta - T_{ph}^\beta)$$

- $R(T)$ varies fast enough



B. Sacepe, C. Chapelier, T. I. Baturina,
 V.M. Vinokur, M.R. Baklanov, M. Sanquer
Disorder-Induced Inhomogeneities of the Superconducting State Close to the Superconductor-Insulator Transition
 Phys. Rev. Lett. **101**, 157006 (2008)

Emergent granularity of the SC order parameter



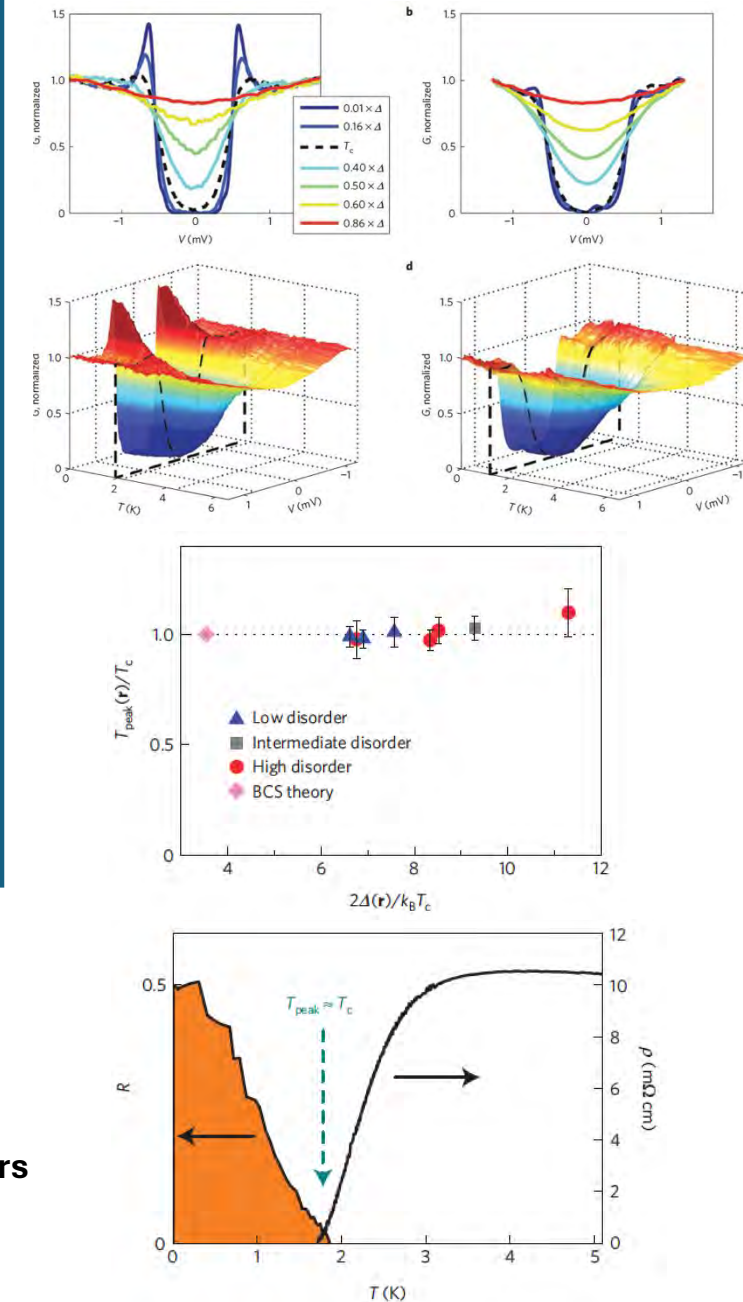
See: A. Ghosal, M. Randeria, N. Trivedi,
Inhomogeneous pairing in highly disordered s-wave superconductors
 PRL **81**, 3940 (1998), PRB **65**, 014501 (2001).

K. Bouadim, Y. L. Loh, M. Randeria, N. Trivedi,
Single- and two-particle energy gaps across the disorder-driven superconductor-insulator transition
 Nat. Phys. **7**, 884 (2011)

- Theory predicts correlation between onset of coherence peaks in spectra and zero resistance state:
- SIT is driven by the increase of incoherent pairs at the expense of the condensate.
- Close to the transition, Δ strongly fluctuates, indicating a spontaneously formed inhomogeneity.
- On the insulating side, the transport is dominated by incoherent Cooper pair hopping.

B. Sacépé, T. Dubouchet, Chapelier, M. Sanquer, M. Ovadia, D. Shahr, M. Feigel'man, L. Ioffe
Localization of preformed Cooper pairs in disordered superconductors
 Nat. Phys. **7**, 239 (2011)

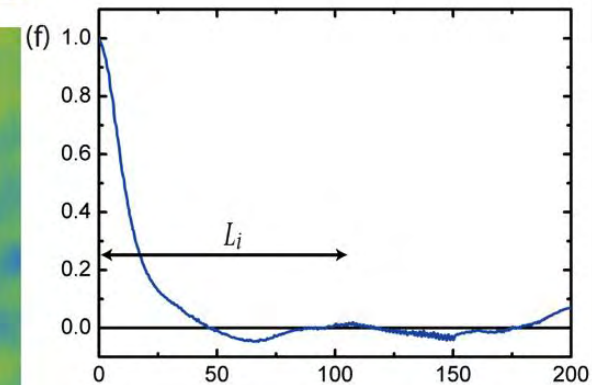
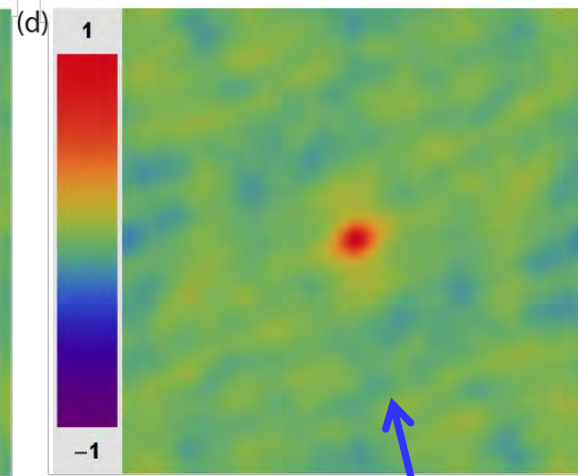
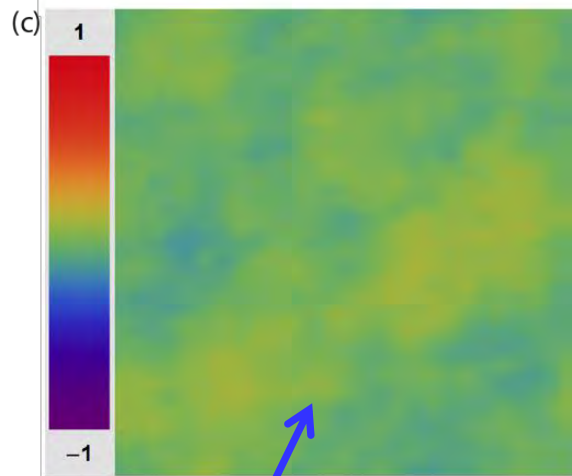
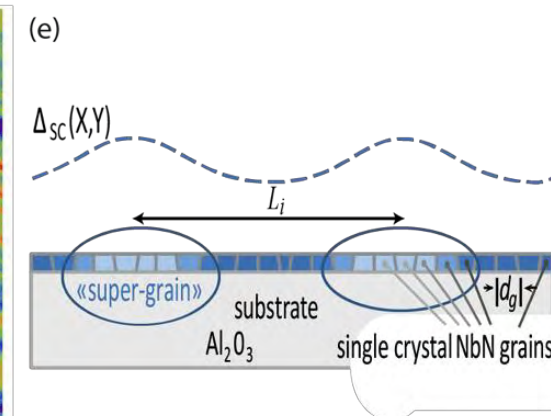
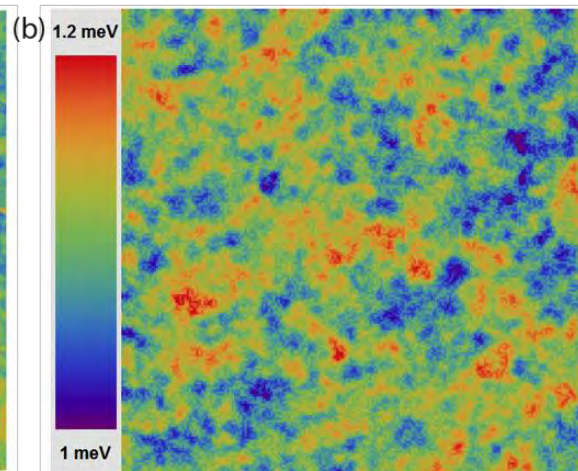
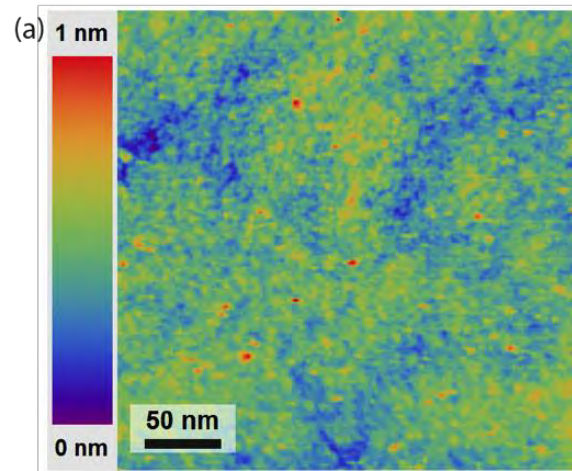
Experiment:



Emergent granularity in NbN thin films

topography

density of states



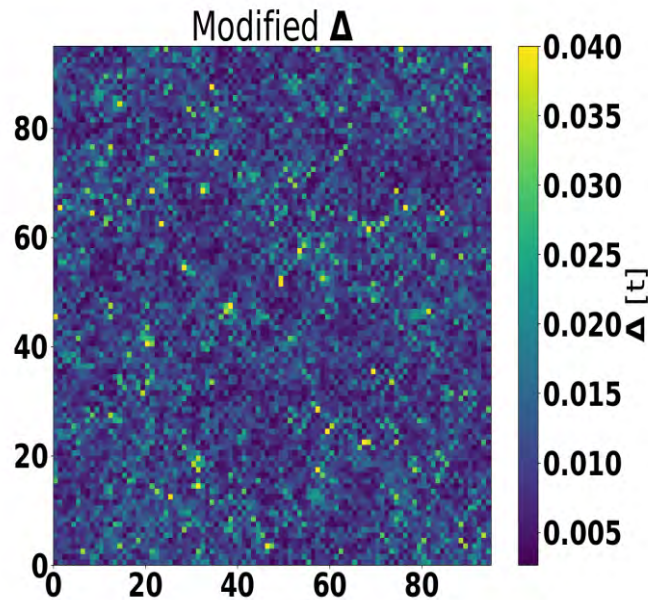
C. Carbillet et al., PRB **93**, 144509 (2016)

Roditchev group,
INSP Paris

no cross-correlation between topography and electronic granularity

auto-correlation of electronic granularity

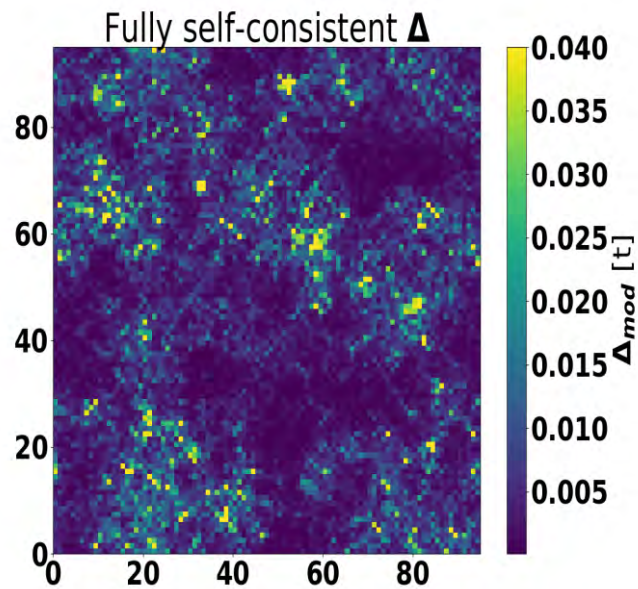
Numerical calculation of $\Delta(r)$ in presence of strong disorder



Pairing amplitude in real space the for a specific disorder configuration on a 96x96 lattice:

simplified self-consistency scheme

following Feigel'man, Ioffe, Kravtsov, Cuevas, Ann. Phys. 325, 1368 (2010)



fully self-consistent scheme

(see also Bouadim et al., Nat. Phys. 2011)

(same disorder and parameter configuration, phase fluctuations neglected)

calculations by M. Stosiek, F. Evers

Direct superconductor/insulator transition with Arrhenius-like conductance on the insulating side

G. Kopnov, O. Cohen, M. Ovadia, K. H. Lee, C. C. Wong, D. Shahar

Little-Parks Oscillations in an Insulator

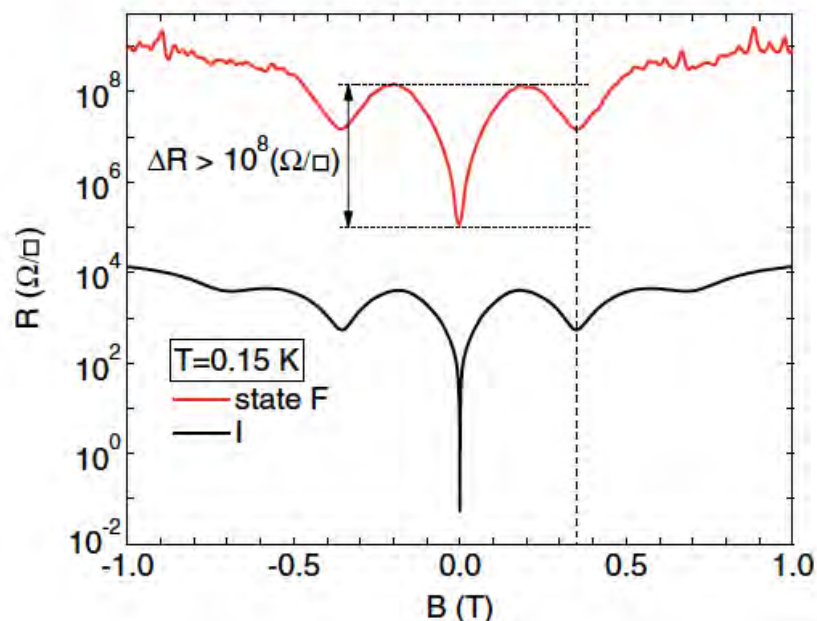
Phys. Rev. Lett. **109**, 167002 (2012)



42 nm 80 nm

nano-perforated InO_x - film

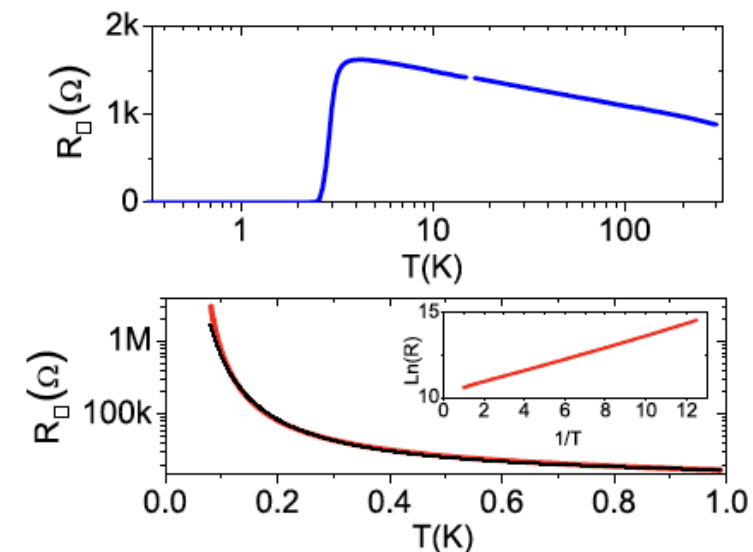
Evidence for
fluxoid quantization
in the insulating state



compare planar
tunnel junctions to
superconducting
and insulating
 InO_x - films

D. Sherman, G. Kopnov, D. Shahar, A. Frydman
**Measurement of a Superconducting Energy Gap
in a Homogeneously Amorphous Insulator**

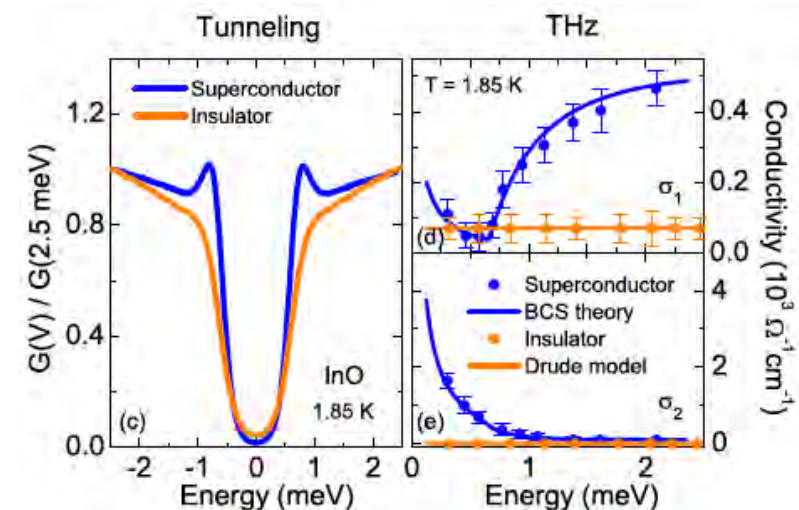
Phys. Rev. Lett. **108**, 177006 (2012)



Superconducting
gap persists on
insulating side, also
in THz-spectroscopy

D. Sherman, B. Gorshunov, S. Poran, N. Trivedi, E. Farber, M. Dressel, A. Frydman
**Effect of Coulomb interactions on the
disorder-driven superconductor-
insulator transition**

Phys. Rev. B **89**, 035149 (2014)



How to model the Cooper-pair insulator:

- thermally activated conductance
expt.: Palaanen & Hebard, Ovadyahu, and many others
theory: Fistul, Baturina, Vinokur, PRL **100** (2008)
Feigel'man, Ioffe, Kravtsov, Cuevas, Ann. Physics 325, 1368 (2010);
Feigelman, Ioffe, Mezard, PRB **82**, 184534 (2010)
- strongly non-monotonic magnetoresistance
expt.: Gantmakher et al., Shahar et al., Baturina et al. and others
Theory: Dubi, Meir, Avishai, Nature **449** 876 (2007)
- non-linearities in IV-characteristics
expt. Baturina et al.; Ovadia et al.,
theory: Vinokur et al., Altshuler et al.

Motivated by emergent inhomogeneity in superconducting condensate —

→ exploit knowledge about Josephson junction array (JJAs) and assume that superfluid condensate fractionalizes into small droplets with a random Josephson coupling

Josephson arrays in the charging regime

J. Mooij, B.J. van Wees, L.J. Geerlings, M. Peters, R. Fazio and G. Schön,
**Unbinding of Charge-Anti-Charge Pairs in Two-Dimensional arrays
of small Tunnel Junctions**
Phys. Rev. Lett. **65**, 645 (1990)

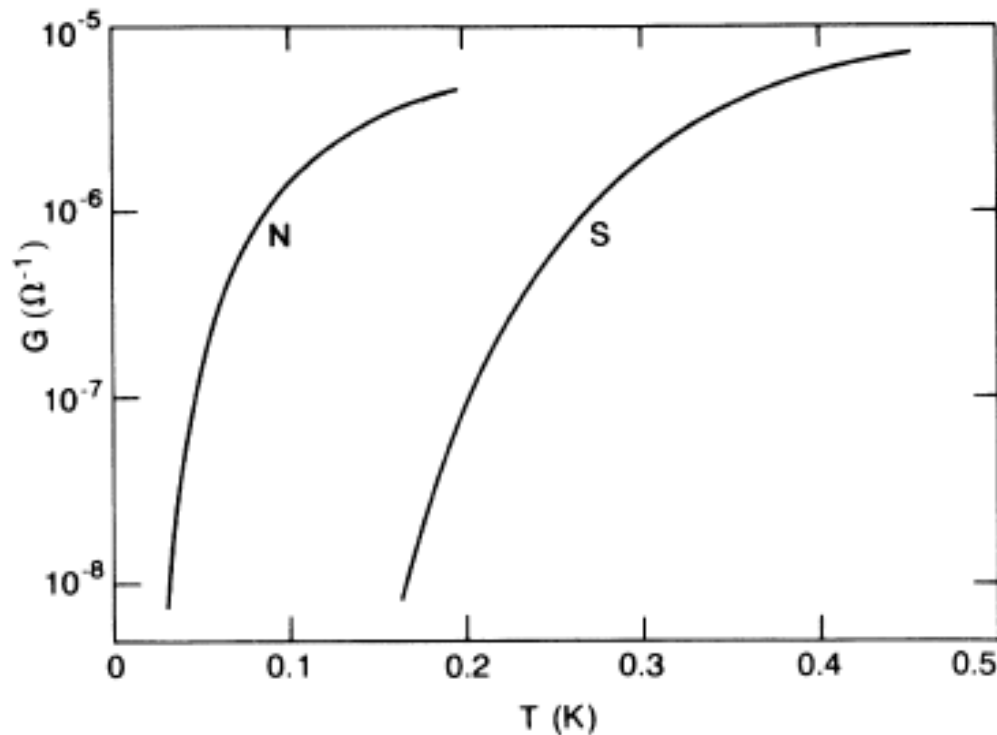


FIG. 3. Measured conductance of an array of $(100 \text{ nm})^2$ aluminum tunnel junctions, 190 cells long and 60 cells wide. *N* is in the normal state (magnetic field of 3T applied); *S* in the superconducting state.

consider JJ-arrays in the limit $E_J < E_C$

superconducting array turns insulating
at much higher temperatures !

H. v.d.Zant, R. Fazio

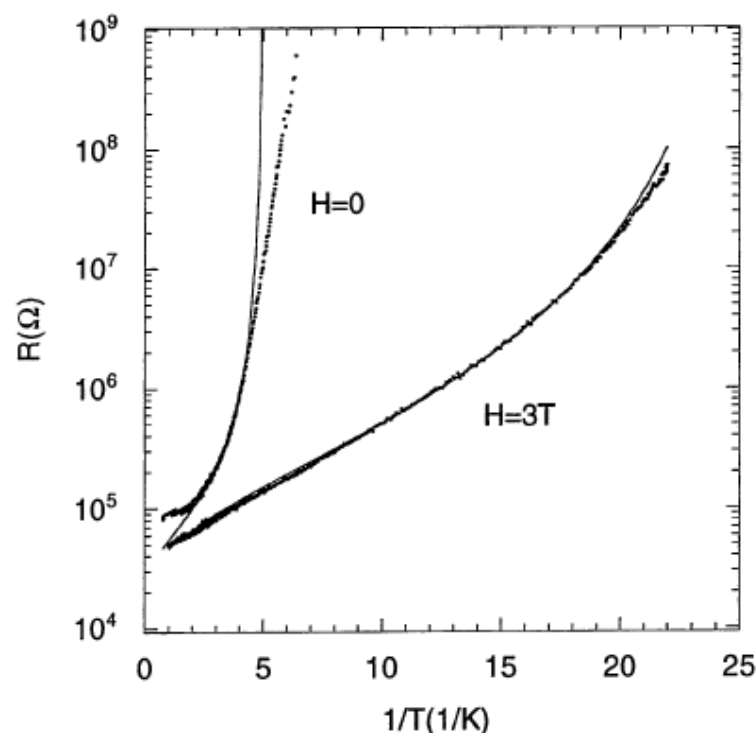
Phys. Repts. **355**, 235 (2001)

Super-exponential behavior in JJ-arrays

Journal of the Physical Society of Japan
Vol. 64, No. 1, January, 1995, pp. 19-21

Precursor of Charge KTB Transition in Normal and Superconducting Tunnel Junction Array

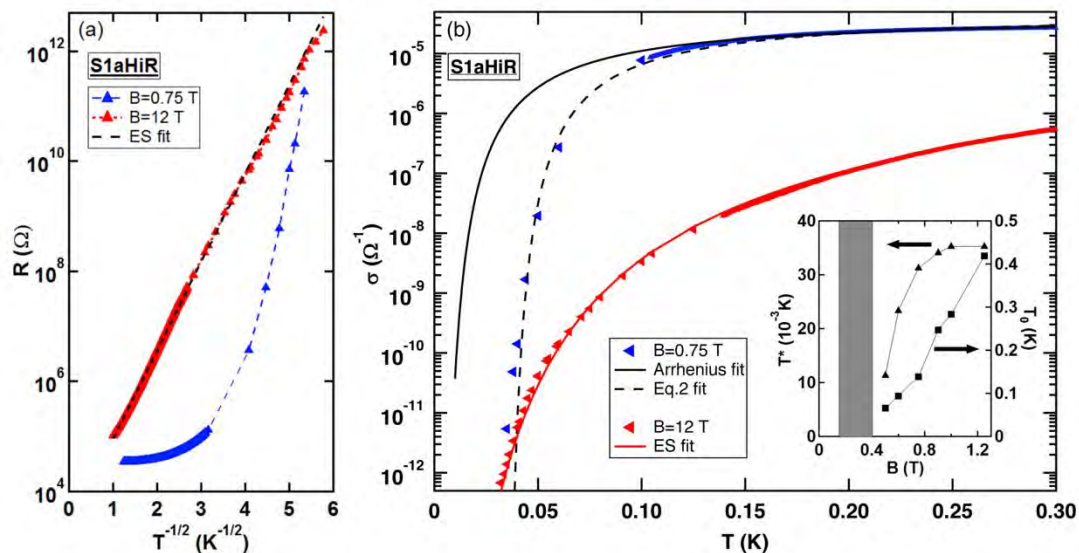
Akinobu KANDA and Shun-ichi KOBAYASHI



The array was 380 junctions in length and 331 junctions in width. Each junction had an area of $0.0072 (\mu\text{m})^2$, normal-state tunneling resistance $R_N=32 \text{ k}\Omega$ and the capacitance $C=1.1 \times 10^{-15} \text{ F}$. The self-capacitance of the island electrode was $5.1 \times 10^{-17} \text{ F}$. The method to estimate these values is described in ref. 7. Consequently, the charging energy $E_C=e^2/2C=0.81 \text{ K}$, the Josephson coupling energy $E_J=0.31 \text{ K}$ and $A=5.7$ junctions.

Resistance at low T grows faster than expected from an Arrhenius law!

Super-exponential resistance in homogeneously disordered films



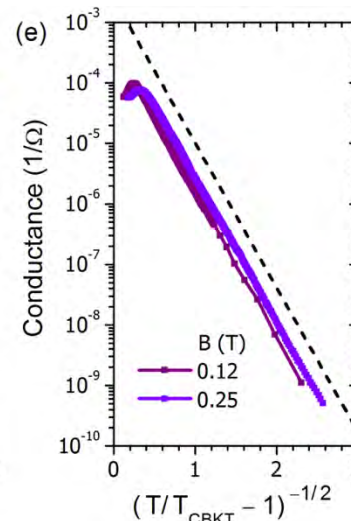
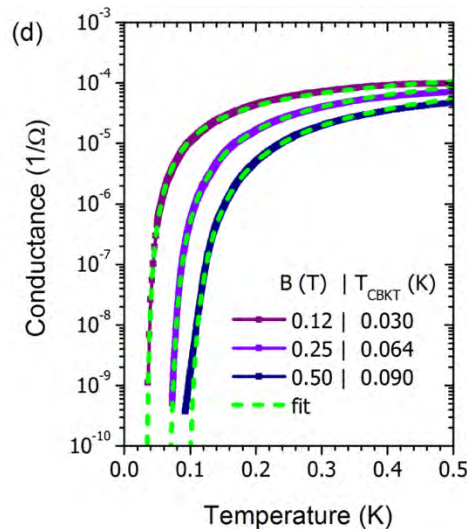
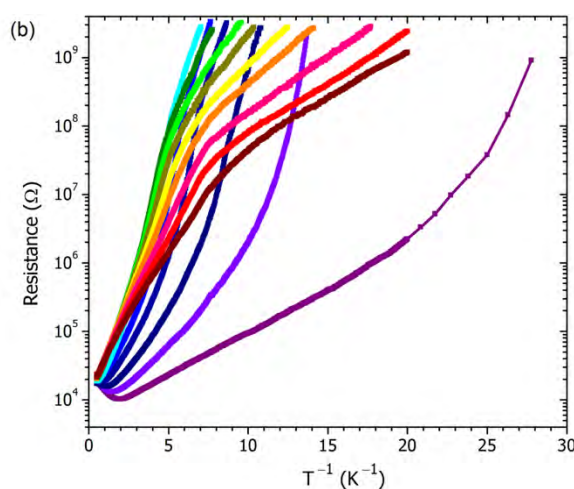
Evidence for a Finite-Temperature Insulator

M. Ovadia^{1,†}, D. Kalok², I. Tamir³, S. Mitra³, B. Sacépé^{1,2,3} & D. Shahar¹

$$R(T) \propto \exp\left(\frac{a}{1 - T/T^*}\right)$$

Sci. Rept. **5**, 13503 (2015)

InO_x



Charge Berezinskii-Kosterlitz-Thouless transition in superconducting NbTiN films

A. Yu. Mironov, T. F. Rosenbaum, V. V. Vinokur, T. I. Baturina, et. al.

NbTiN_x

$$R(T) \propto \exp\left(\frac{b}{(1 - T/T^*)^{1/2}}\right)$$

Sci. Rept. **8**, 4082 (2018)

Questions:

- What is the mechanism of destruction of superconductivity – fermionic or bosonic or both?
- Do Josephson coupled droplets with a fully developed order parameter exist at the SIT that can be Josephson coupled
- What is the role of Coulomb interactions within the bosonic approach?
- What is the transport mechanism of preformed pairs?
- When does hopping of Cooper pairs transition to the hopping of single electrons?

Outline

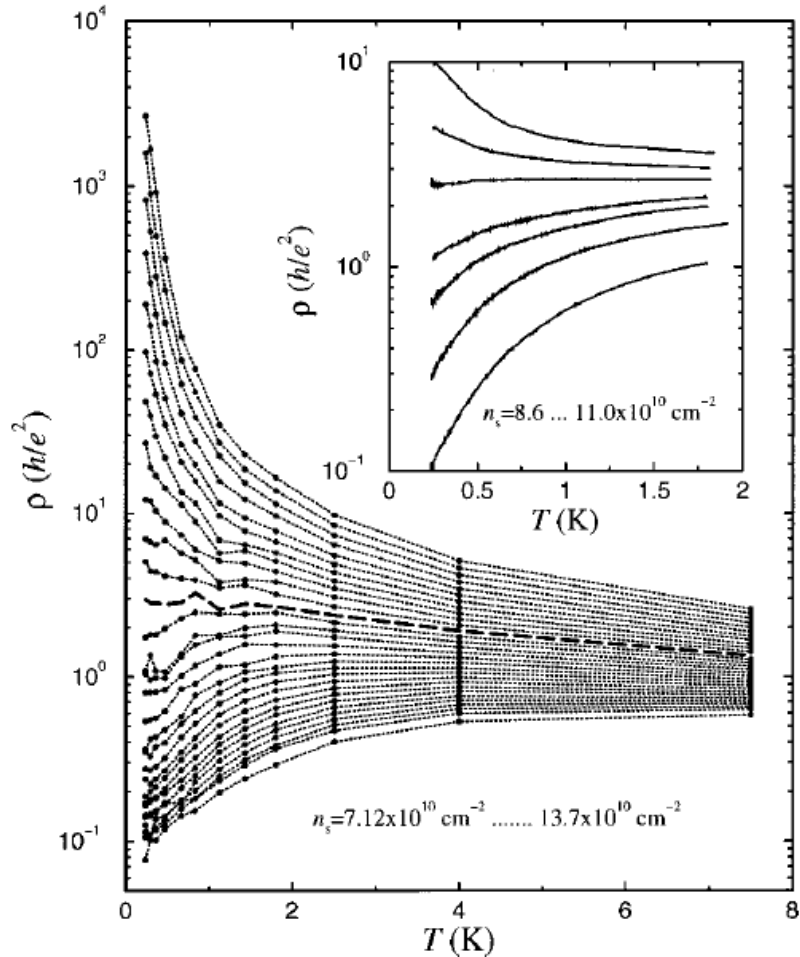
- Localization: suppression of diffusion by disorder

Abrahams et al. 1979: „There are no metals in two dimensions!“

- Superconductivity
- Suppression of zero-resistance state by phase fluctuations
- Suppression of Cooper-pairing by disorder
- Direct superconductor-insulator transition
- **„Failed‘ superconductors and insulators:**

Kravchenko et al. 1994: „There are ~~no~~ metals in two dimensions!“

Normal metals in two dimensions?



S. V. Kravchenko, D. Simonian, M. P. Sarachik
W. Mason, J. E. Furneaux

Electric Field Scaling at a $B = 0$ Metal-Insulator Transition in Two Dimensions

Phys. Rev. Lett. **77**, 4938 (1996).

- Electron-electron interactions matter!
- Already early theory suggested that MIT may occur in clean systems [Finkelstein (1984), Castellani, et al. (1984)]
- Clear metal insulator transition observed in the 2D electron systems in high mobility MOSFETs at very low density!
- Critical resistance $R_c \simeq \frac{h}{e^2} \simeq 25.8 \text{ k}\Omega$

REVIEWS OF MODERN PHYSICS, VOLUME 73, APRIL 2001

Colloquium: Metallic behavior and related phenomena in two dimensions

Elihu Abrahams

Center for Materials Theory, Physics Department, Rutgers University, Piscataway,
New Jersey 08854

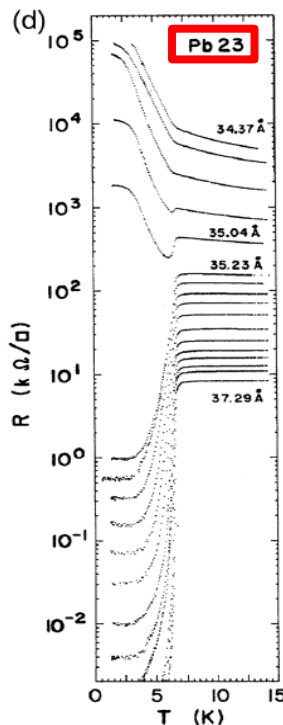
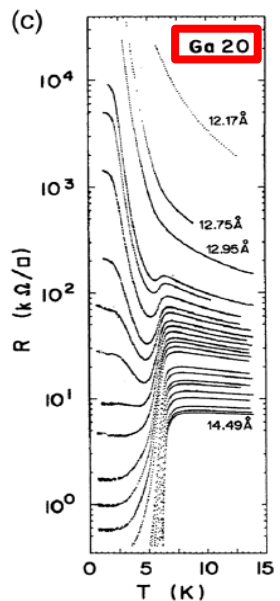
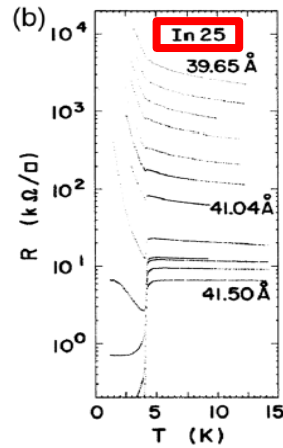
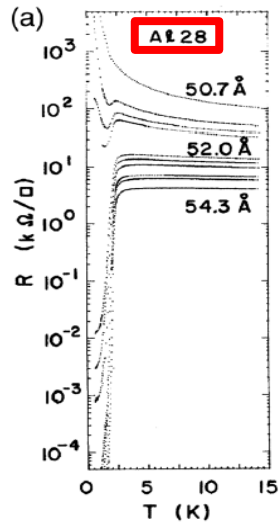
Sergey V. Kravchenko

Physics Department, Northeastern University, Boston, Massachusetts 02115

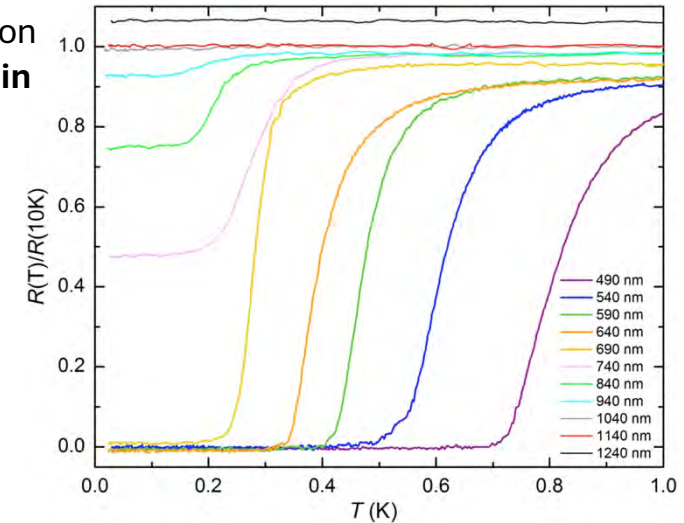
Myriam P. Sarachik

Physics Department, City College of the City University of New York, New York,
New York 10031

Granular superconductors and Josephson junction arrays



S. Eley, S. Gopalakrishnan, P. M. Goldbart, N. Mason
Approaching zero-temperature metallic states in mesoscopic superconductor-normal-superconductor arrays
 Nat. Phys. 8, 59 (2012).

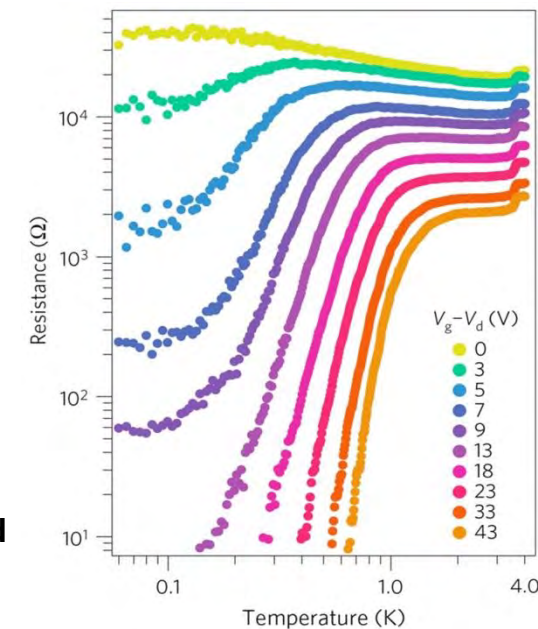


Low-T saturation observed for:

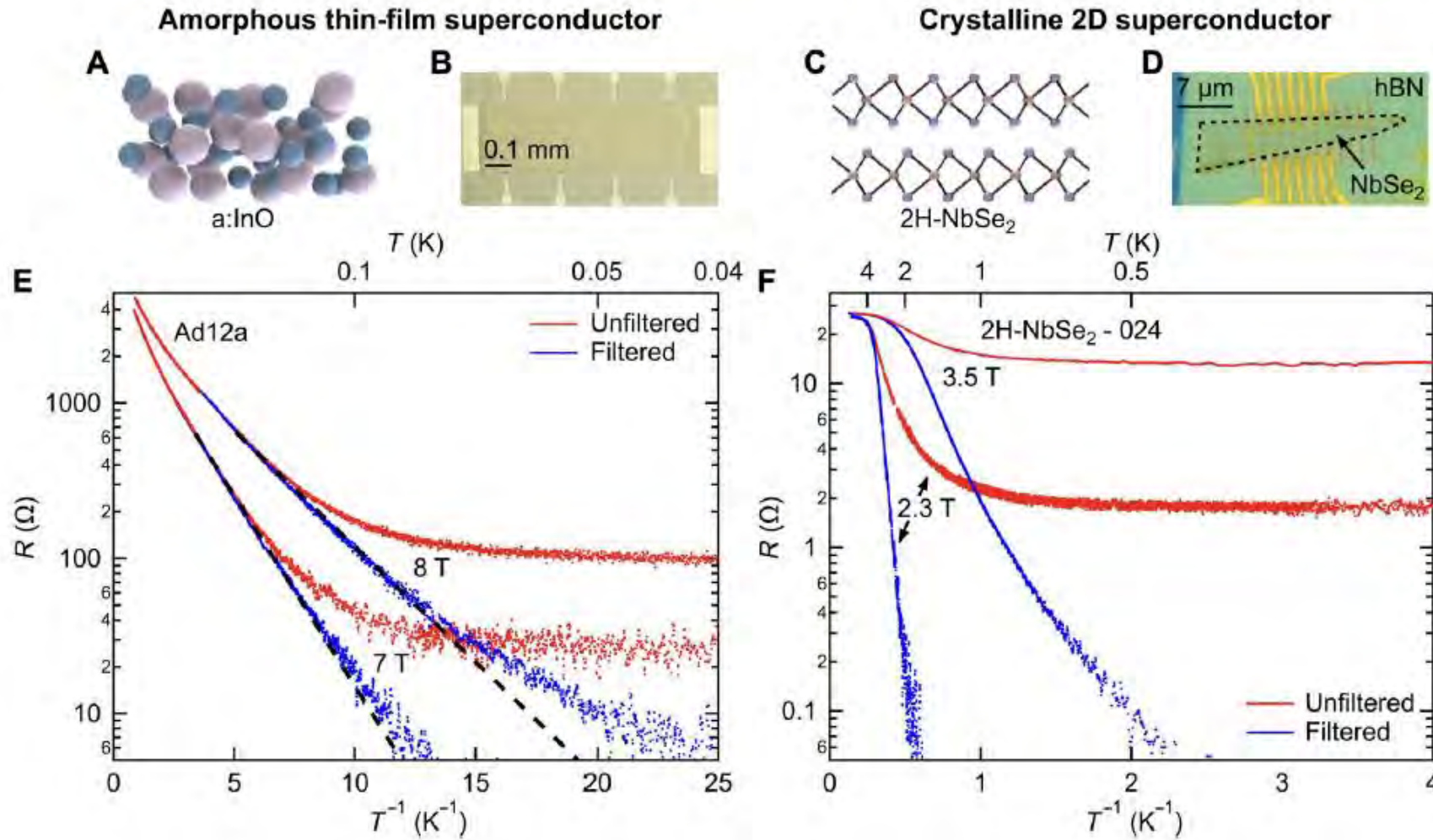
- granular films (random JJ-arrays)
- arrays of metallic SNS-junctions
- Sn droplets on graphene

H. M. Jaeger, D. B. Haviland, B. G. Orr, A. M. Goldman
Onset of superconductivity in ultrathin granular metal films
 Phys. Rev. B **40**, 182 (1989).

Z. Han, et al., B. Sacepe, and V. Bouchiat,
Collapse of superconductivity in a hybrid tin-graphene Josephson junction array
 Nat. Phys. **10**, 380 (2014).



Possible artifacts from insufficient rf-filtering of cryogenic wiring?



I. Tamir et al., C. Dean, D. Shahar

Sensitivity of the superconducting state in thin films

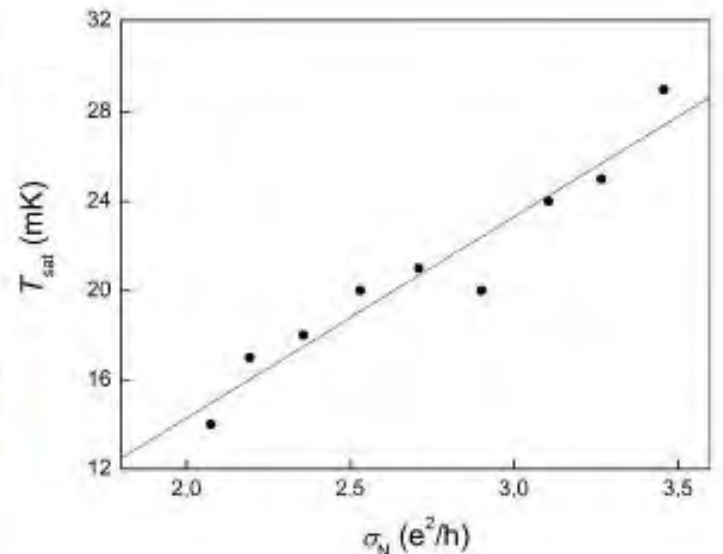
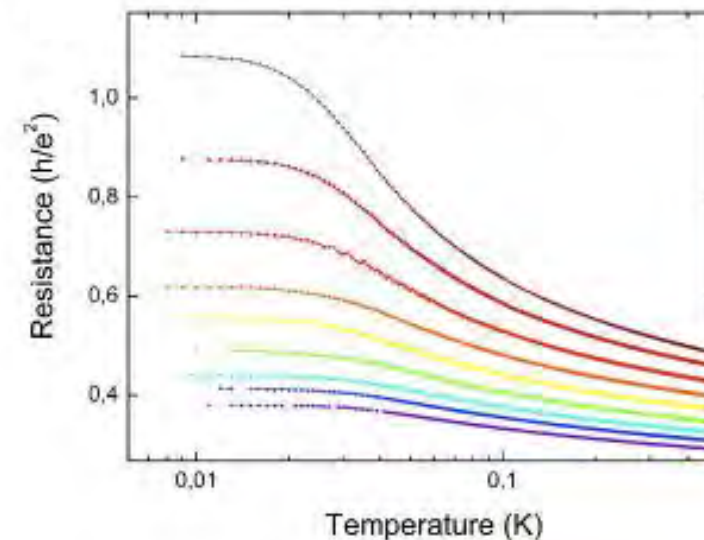
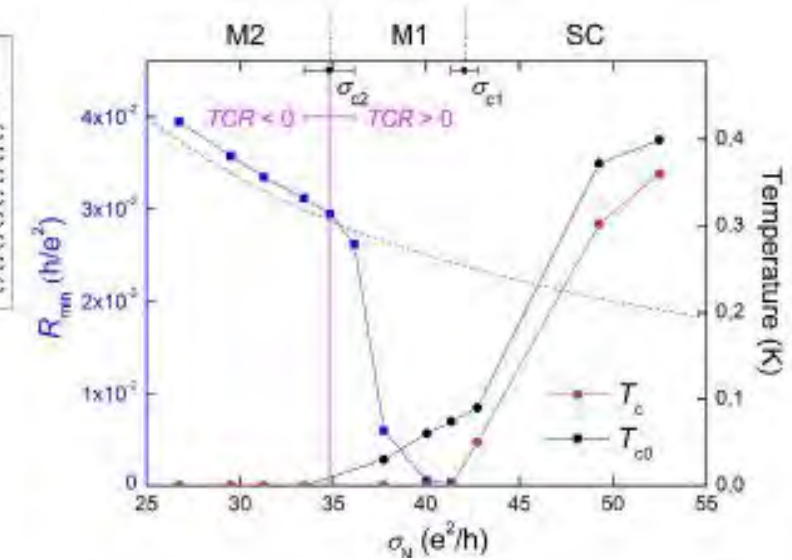
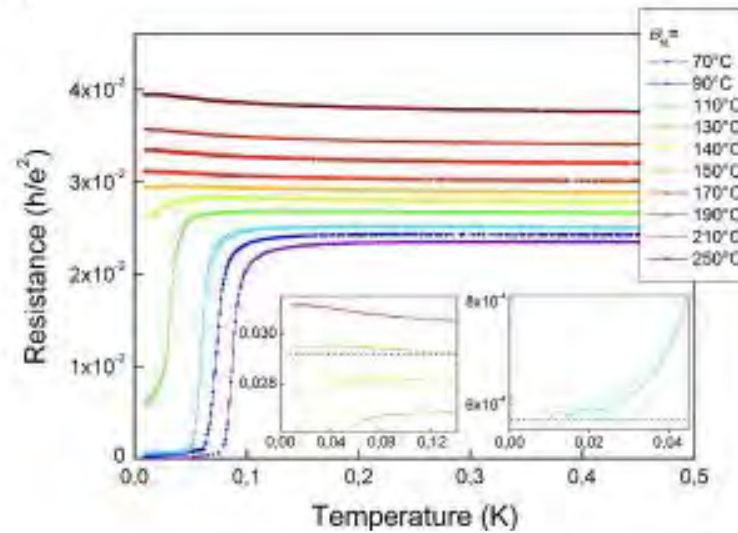
Sci. Adv. **5** eaau3826 (2019).

„Failed“ superconductors and insulators appear also with state-of-the-art filtering...

Tune sheet resistance of $\text{Nb}_{13.5}\text{Si}_{86.5}$ films by annealing:

- saturation for both, *superconducting* and *insulating* films
- the **Bose-metal** remains an enigma...

F. Couëdo, O. Crauste, A. A. Drillien, V. Humbert
L. Bergé, C. A. Marrache-Kikuchi, L. Dumoulin
**Dissipative phases across the
superconductor-to-insulator transition**
Sci. Rept. **6**:35834 (2016).



Overview

- Localization: suppression of diffusion by disorder

Abrahams et al. 1979: „There are no metals in two dimensions!“

- Superconductivity
- Suppression of zero-resistance state by phase fluctuations
- Suppression of Cooper-pairing by disorder
- Direct superconductor-insulator transition
- ‚Failed‘ superconductors and insulators:

Kravchenko et al. 1994: „There are ~~no~~ metals in two dimensions!“