Percolation and (Super)Conduction: 2d superconductors and correlated disorder

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TIDES
Two-dimensional & Disordered Superconductors

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Swiss National Science Foundation
DQMP Department of Quantum Matter Physics
UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Percolation and (Super)conduction:
2d superconductors and correlated disorder

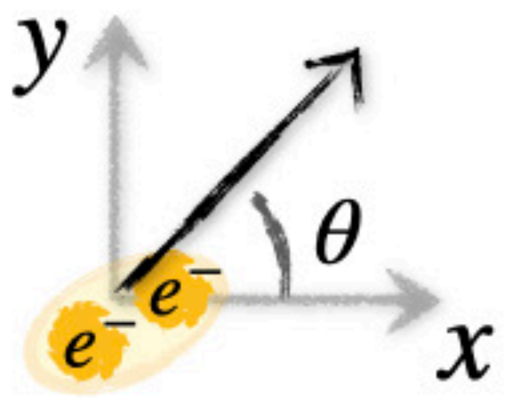
Overview

- Intro to Superconductivity in 2D: BKT vs inhomogeneities
- The Random Impedance Network model
- Twist angle inhomogeneities in multilayer graphene
- Outlooks


Superconductivity in 2D

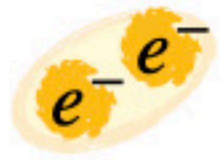
Superconductivity

Superconducting order parameter: spontaneous symmetry breaking of $\mathbf{U(1)}$ **symmetry**

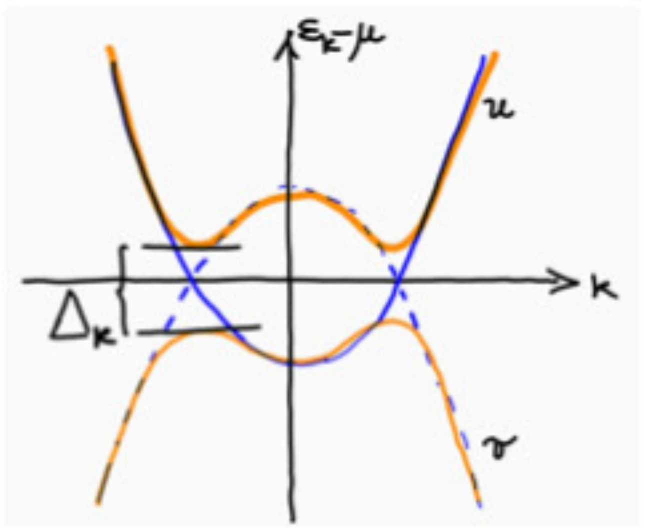


$$\Delta_{\mathbf{k}} = \underbrace{\langle c^\dagger_{\mathbf{k}\sigma} c^\dagger_{-\mathbf{k}\sigma'} \rangle}_{\text{Cooper pairs}} = |\Delta_{\mathbf{k}}| e^{i\theta}$$

Global phase rigidity

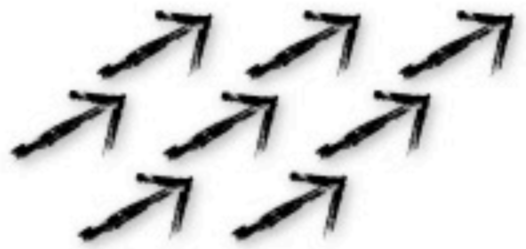


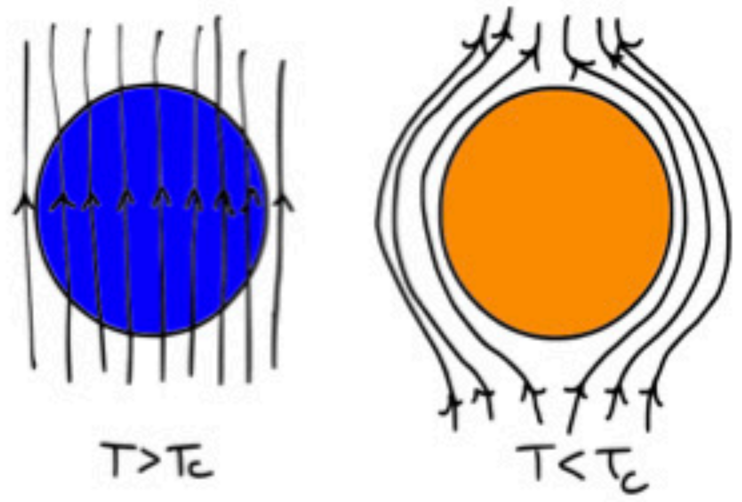
Energy gap Δ



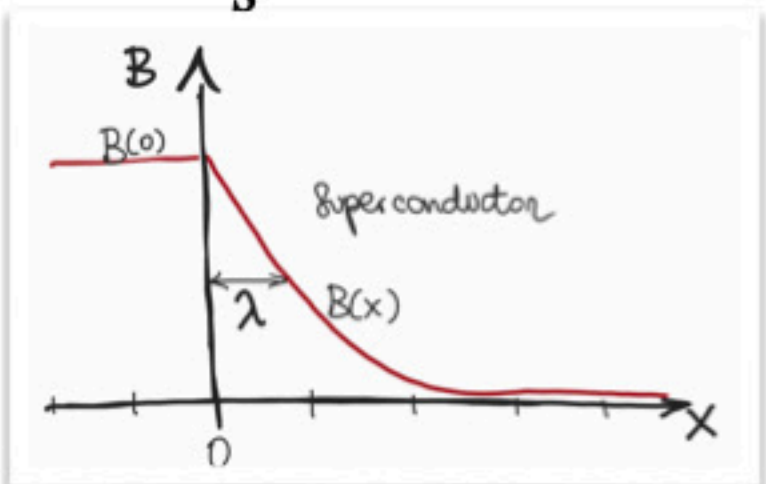
Superfluid stiffness J_s

energetic cost to twist θ





$J_s \propto \lambda^{-2}$



Superfluid density: n_s

density of coherent carriers

Usually $\Delta \ll J_s$

2d & strongly correlated $J_s \ll \Delta$

$J_s \propto n_s$ ☑ Homogeneous systems ☒ Inhomogeneous systems

Superconductivity
in two dimensions

Raychaudhuri, Dutta, J. Phys.: Condens. Matter ***34*** *083001 (2022)*

No ext fields nor currents

$$H \propto J_S \int |\nabla \theta(\mathbf{r})|^2 d\mathbf{r}$$

$$\Updownarrow$$

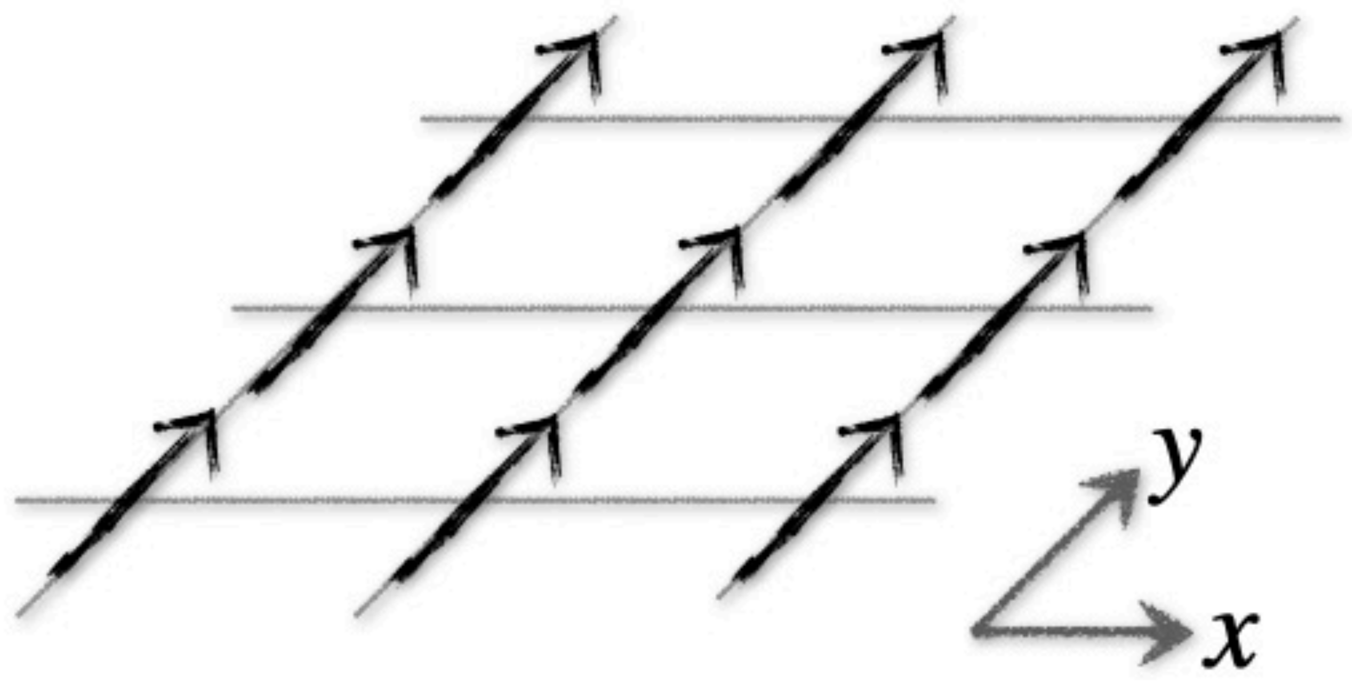
(Discrete) XY model

$$H = -J \sum_{\langle ij \rangle} \mathbf{s}_i \cdot \mathbf{s}_j$$

Mermin-Wagner theorem: no SSB in d<3

In 2d phase fluctuations destroys

$$\langle \Delta e^{i\theta_i} \rangle = 0$$



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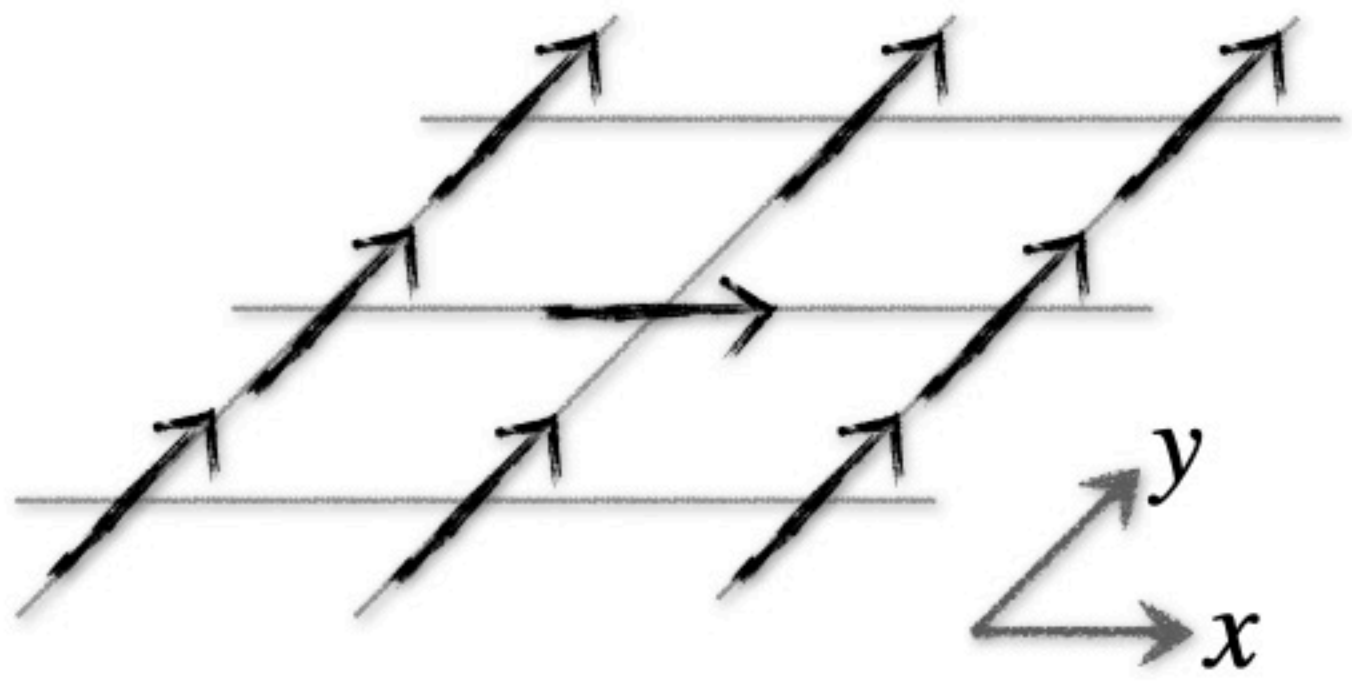

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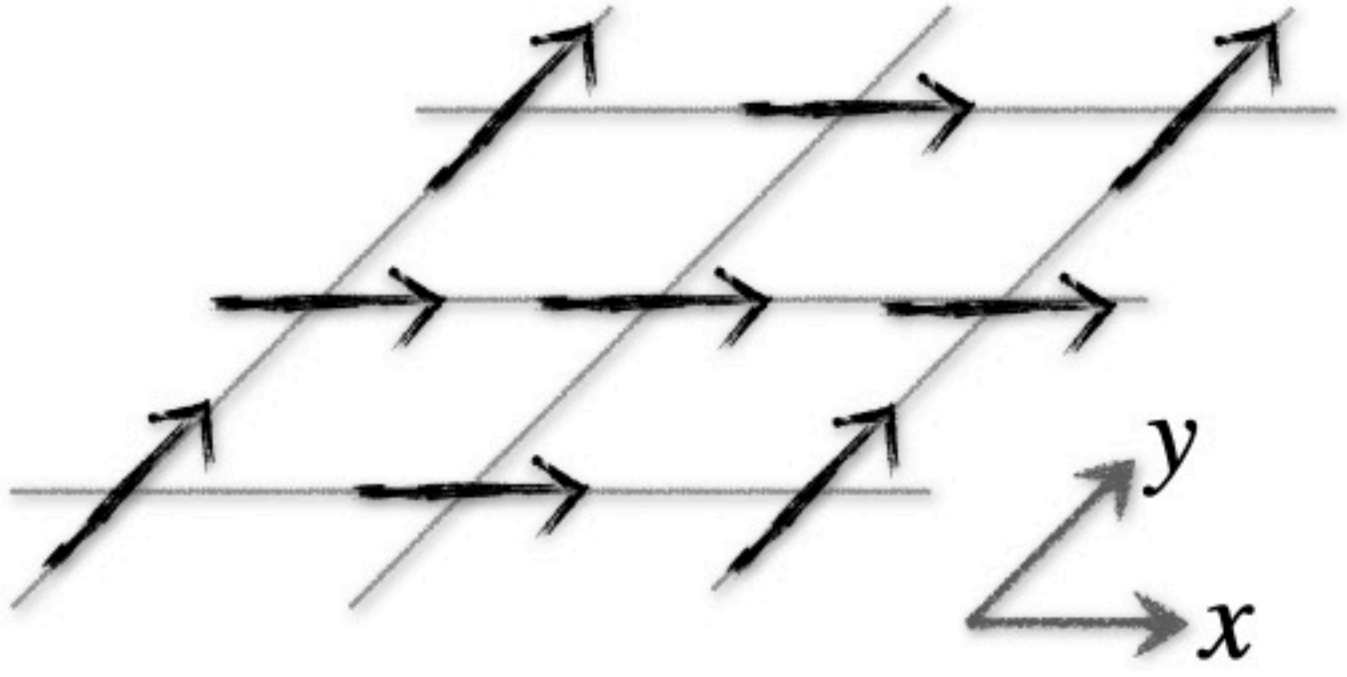

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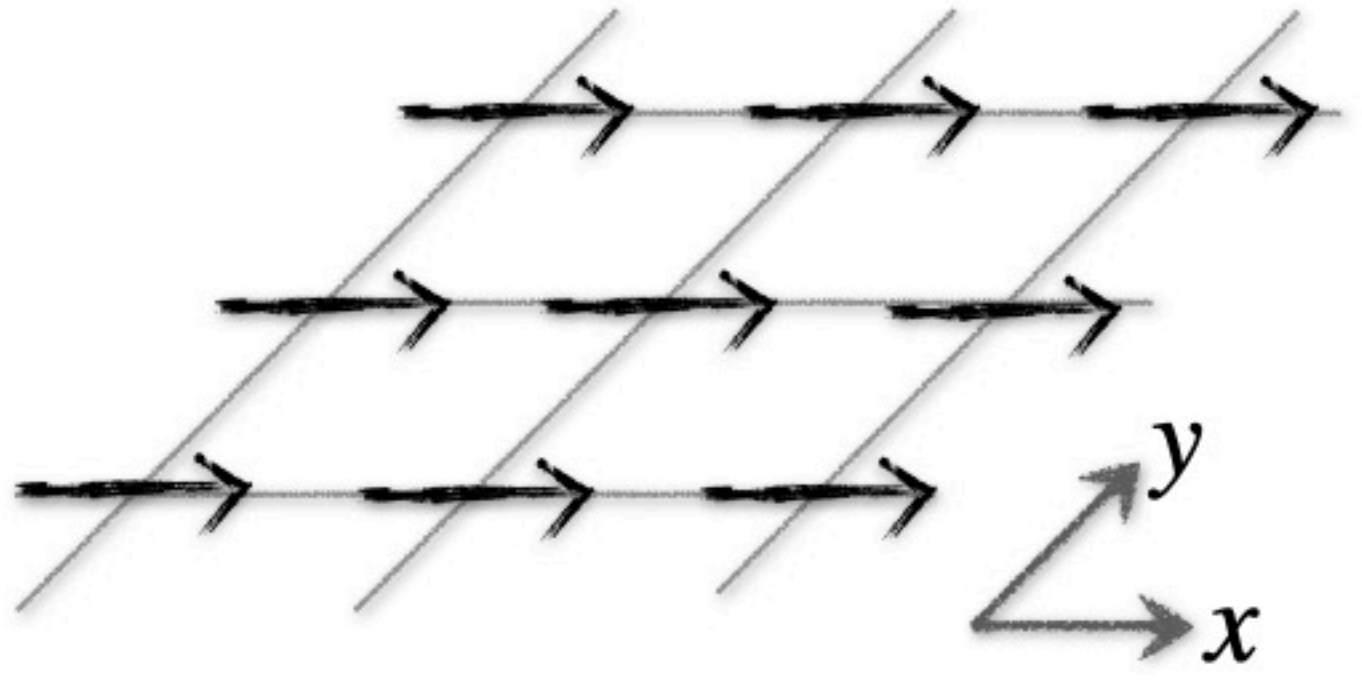

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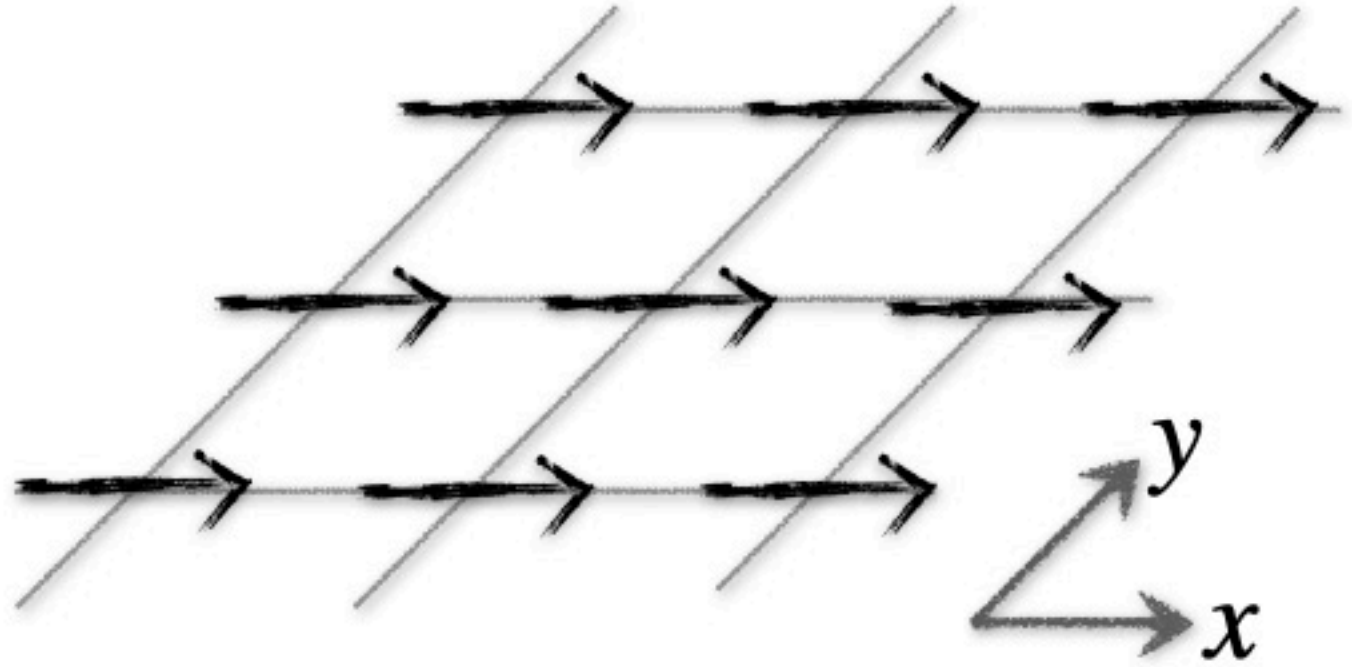


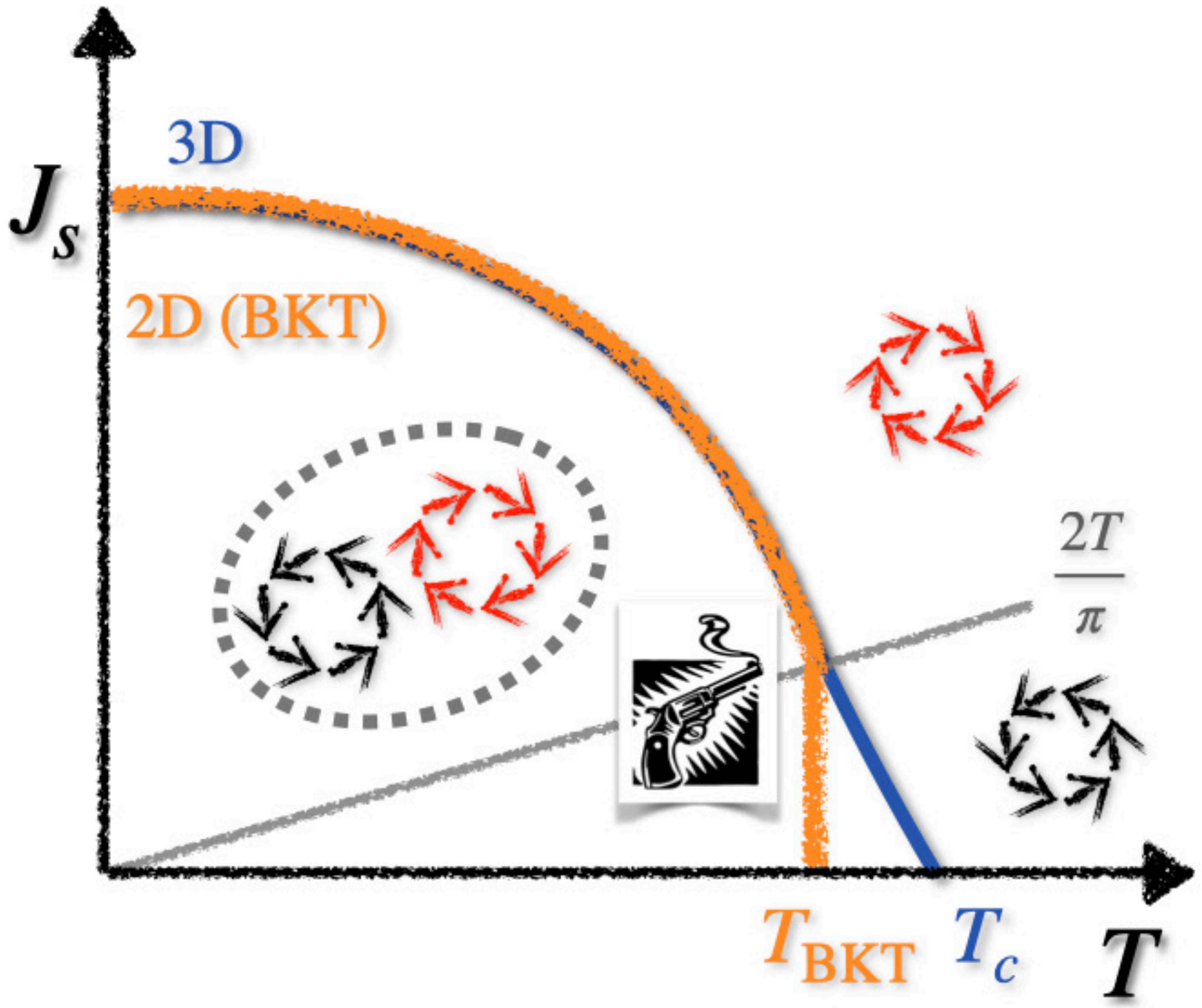
(Quasi-long-range order)

Berezinskii-Kosterlitz-Thouless (BKT)
topological phase transition

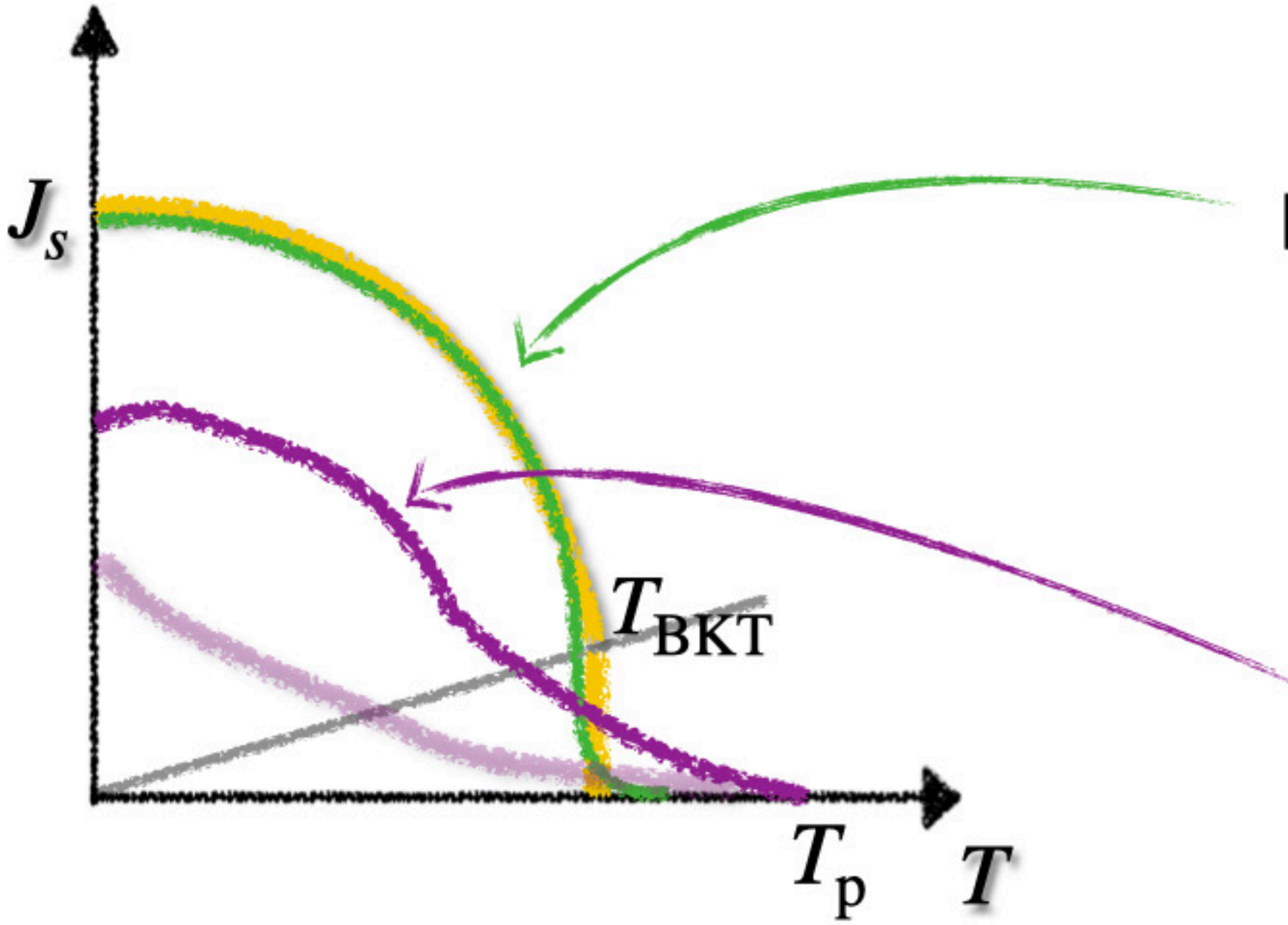
$\mathbf{U(1)}$ symmetry class in $d < 3$

$$J_s \propto \langle \Delta^2 e^{i\theta_i} e^{i\theta_j} \rangle \neq 0$$

$$J_s(T_{\mathrm{BKT}}) = 2T_{\mathrm{BKT}}/\pi$$

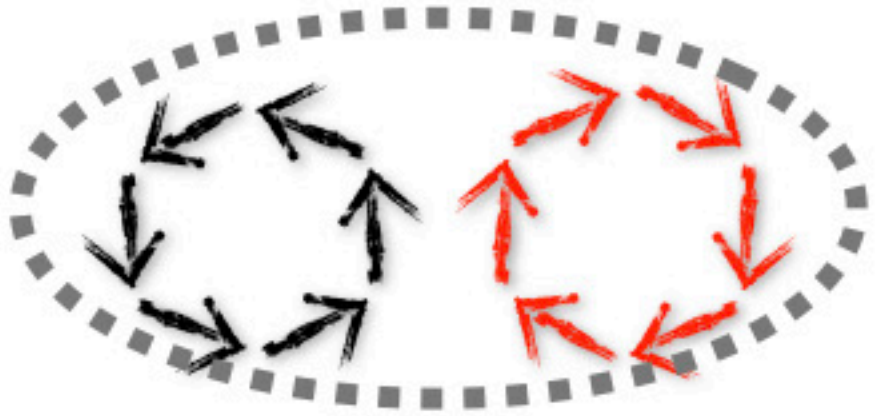


Superconductivity
in two dimensions



BKT physics: smeared out w uncorrelated disorder

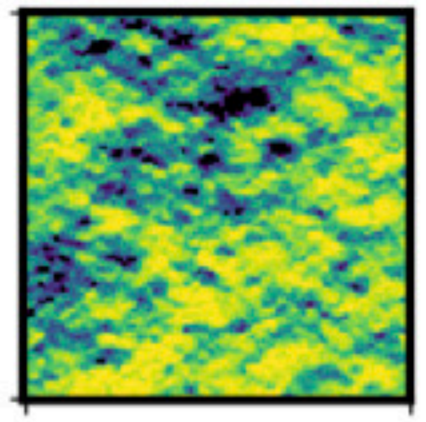
$$J_s(T_{\mathrm{BKT}}) = 2T_{\mathrm{BKT}}/\pi$$

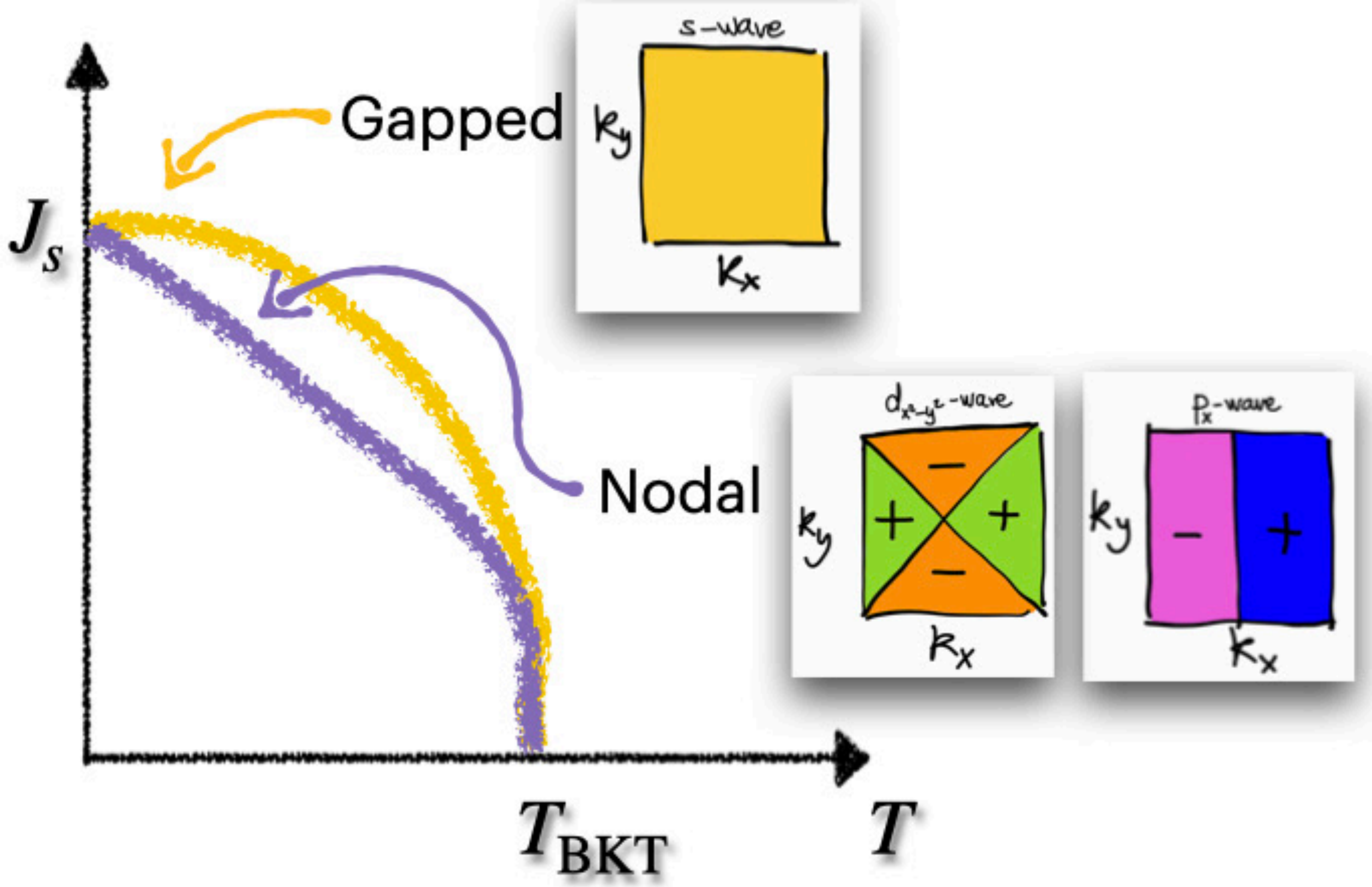


Percolation: Correlated disorder/large scale inhomogeneities

Critical temperature

$$T_{\mathrm{c}} \equiv T_p$$





Intrinsic/extrinsic disorder can give apparent exotic features!

⟶ Disentangle disorder from quantum effects!

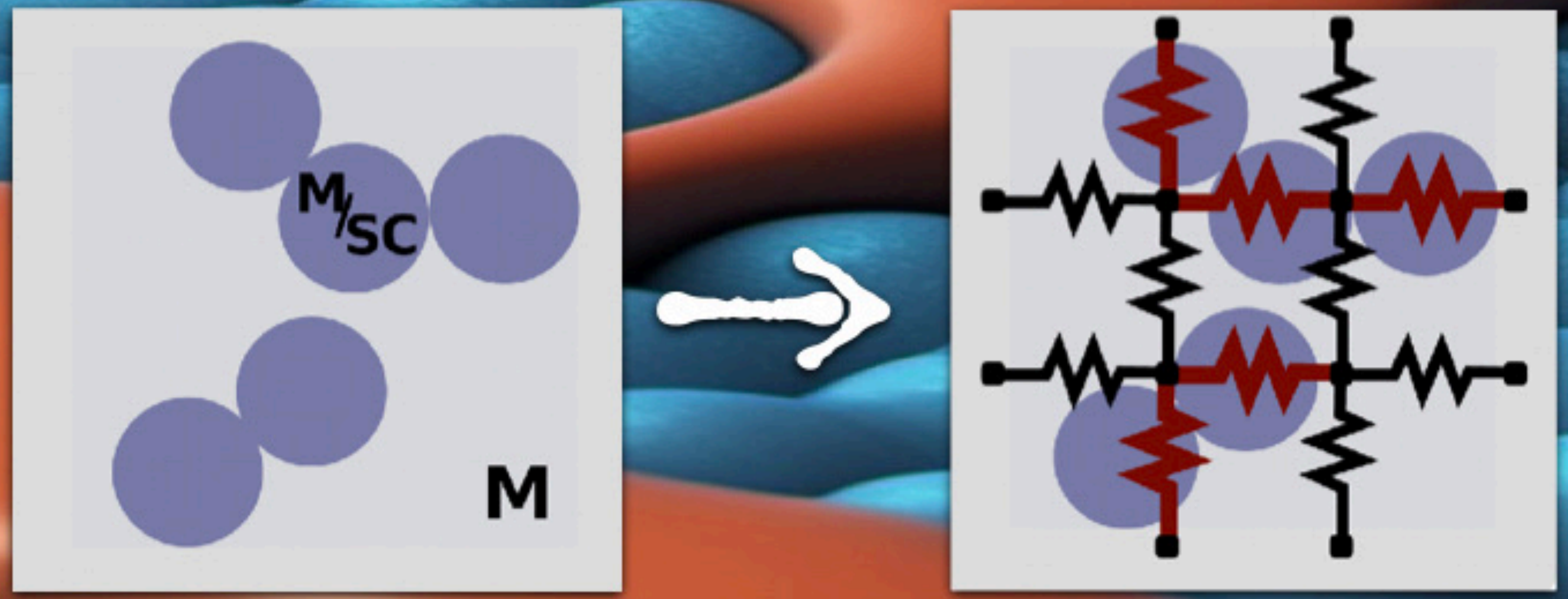
Random Impedance Network

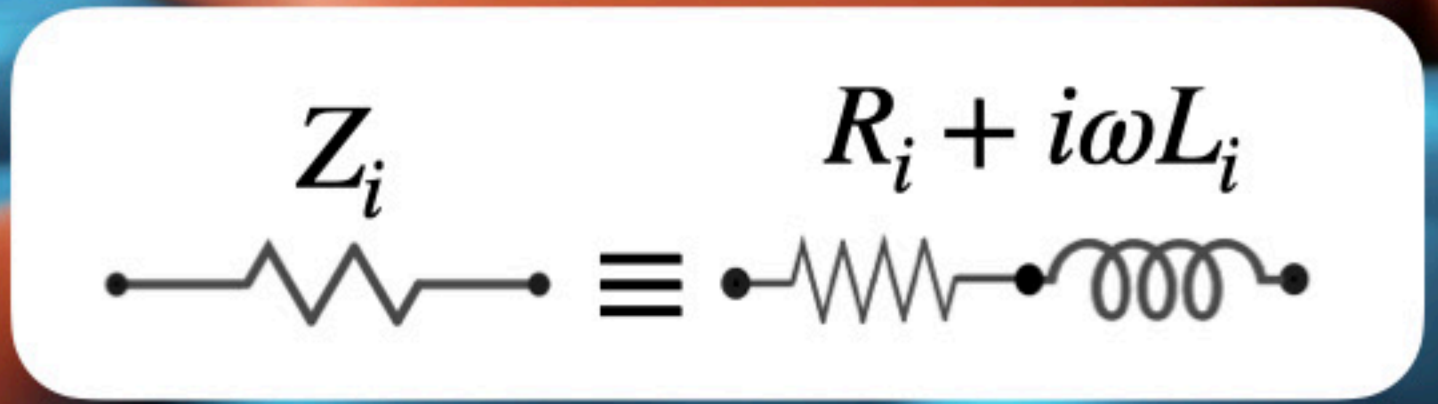
REVIEWS OF MODERN PHYSICS 1973

Percolation and Conduction

Scott Kirkpatrick

Random Resistor Network





Random Impedance Network (RIN)

DC transport: Random Resistor Network (RRN)

REVIEWS OF MODERN PHYSICS 1973

Percolation and Conduction

Scott Kirkpatrick

* No phase fluctuations
* Only inhomogeneities
* Percolating transition
* (Local phase coherence)

Effective Medium (EM) solution:
uniformly distributed mesoscopic inhomogeneities $\sum_{i=m,s} w_i \frac{Z_{\text{em}} - Z_i}{Z_{\text{em}} + Z_i} = 0$

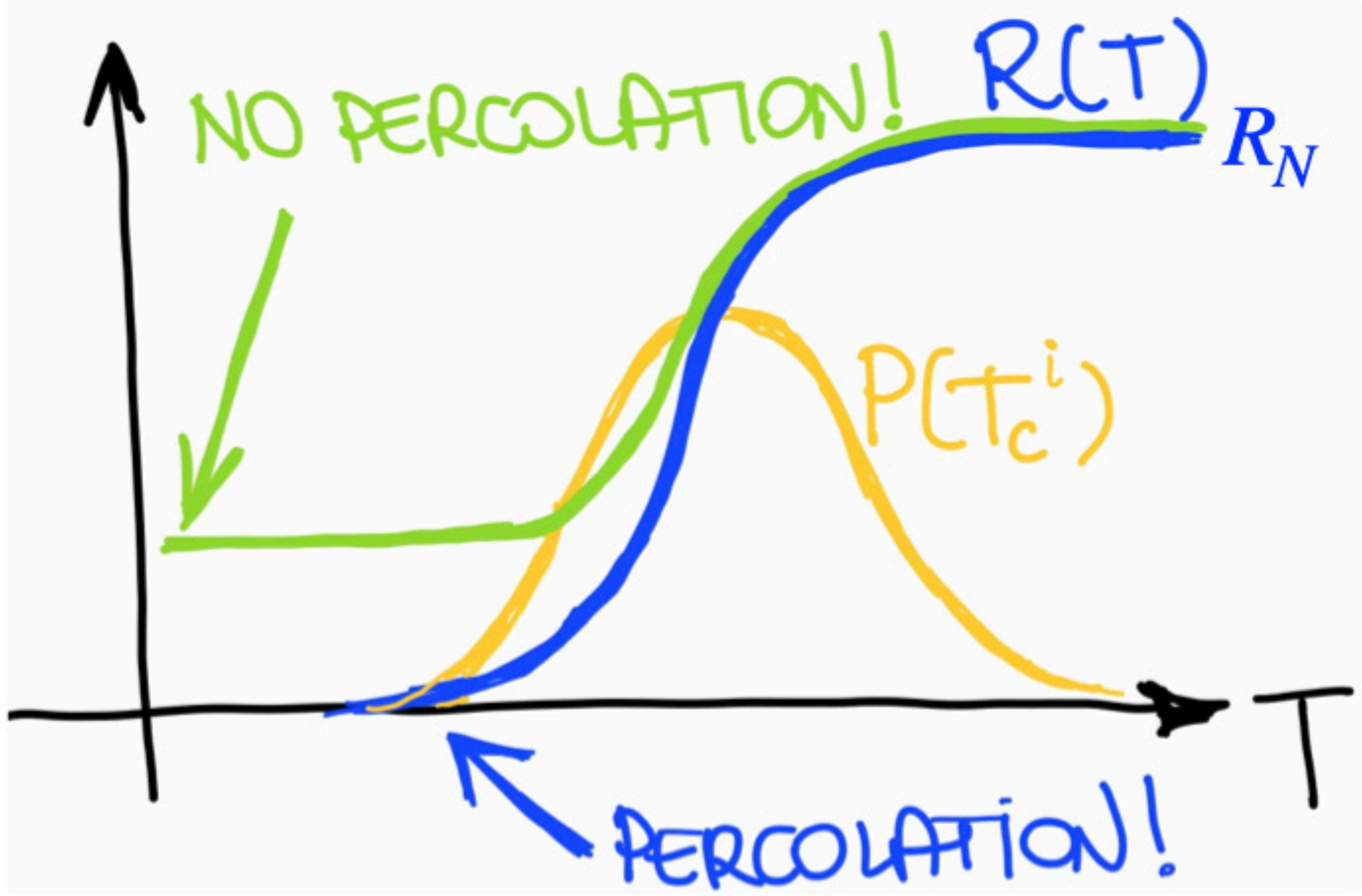
Exact solution:
solve Kirchoff & Ohm equations

w_s superconducting bonds

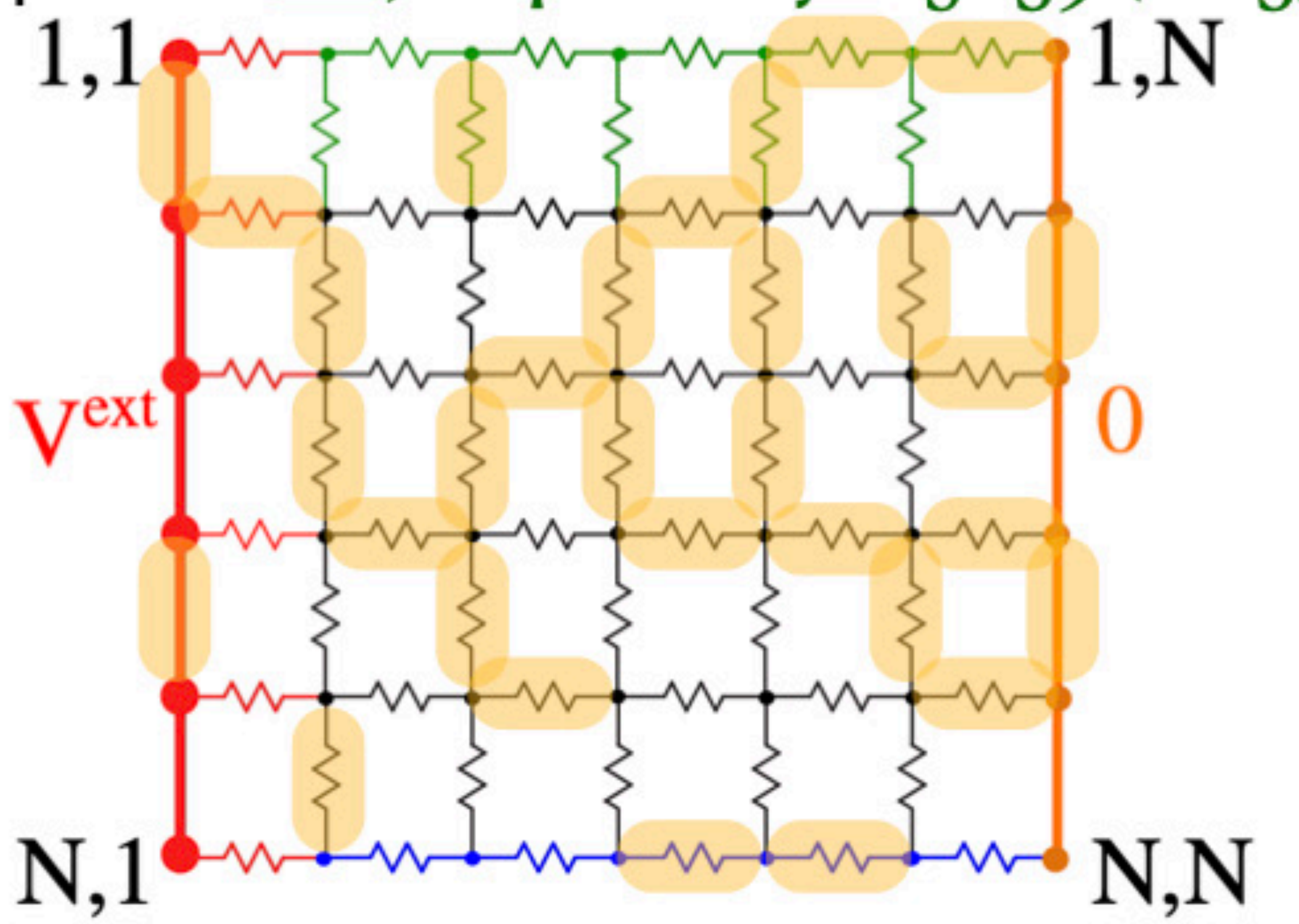
$$R_i = \begin{cases} R_N, \text{ if } T > T_c^i \\ 0, \text{ if } T \le T_c^i \end{cases} \qquad w_s P(T_c^i)$$

$w_m = 1 - w_s$ metallic bonds

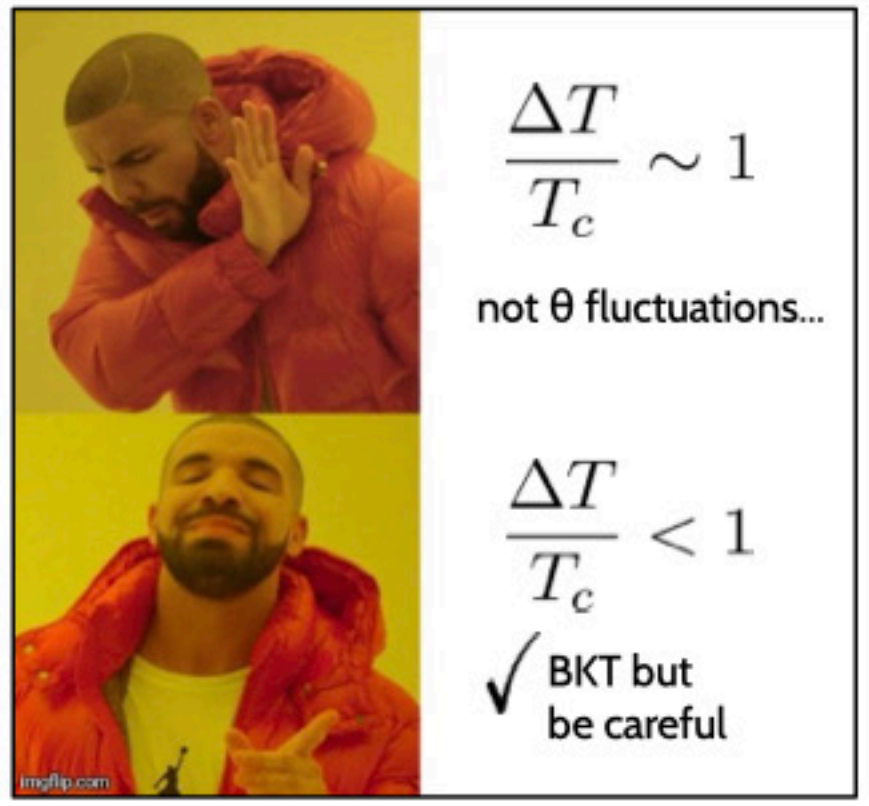
$$R_i = R_N$$



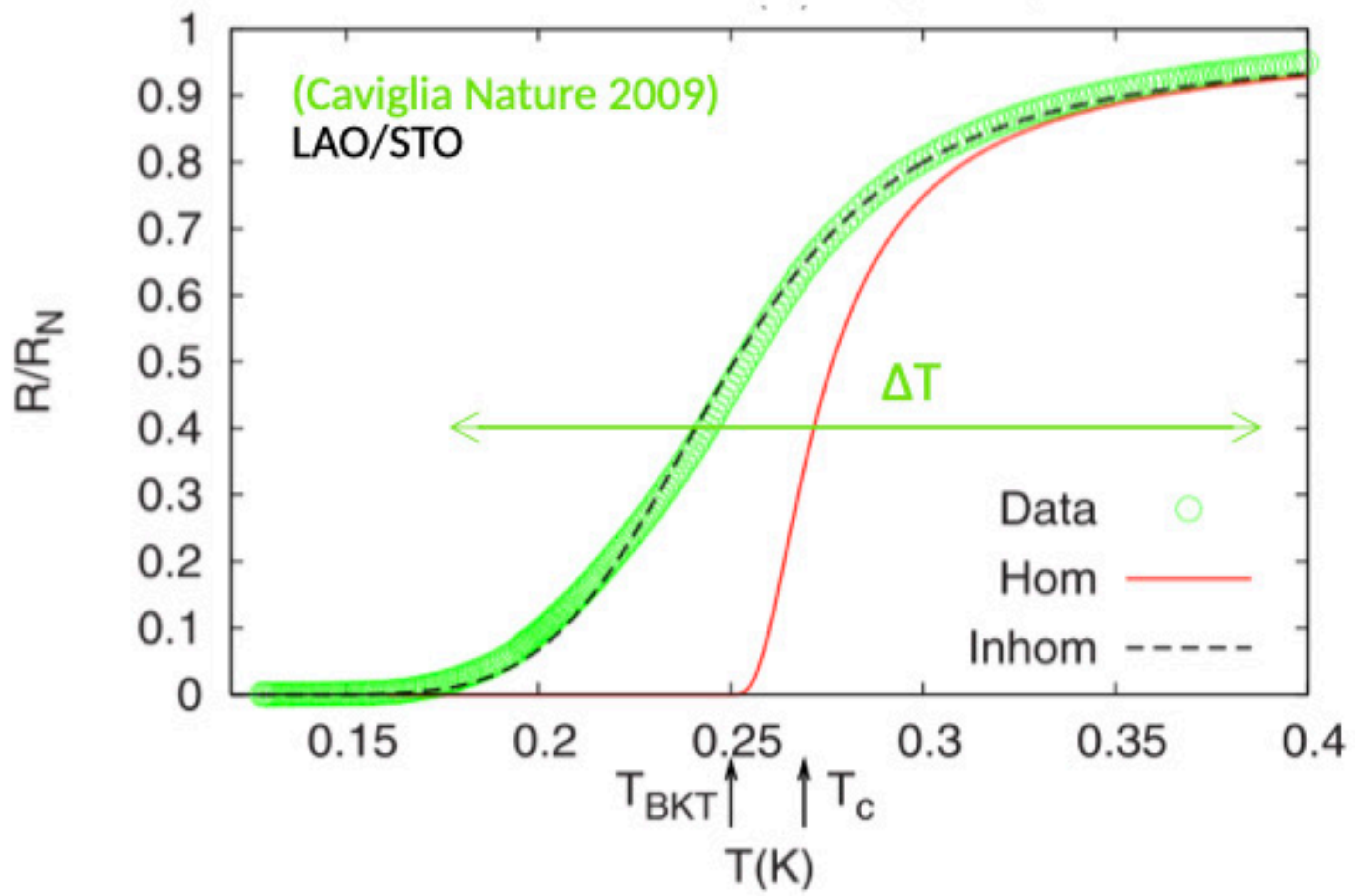
App.A1 in **GV**, Scipost Phys. 15 239 (2023)



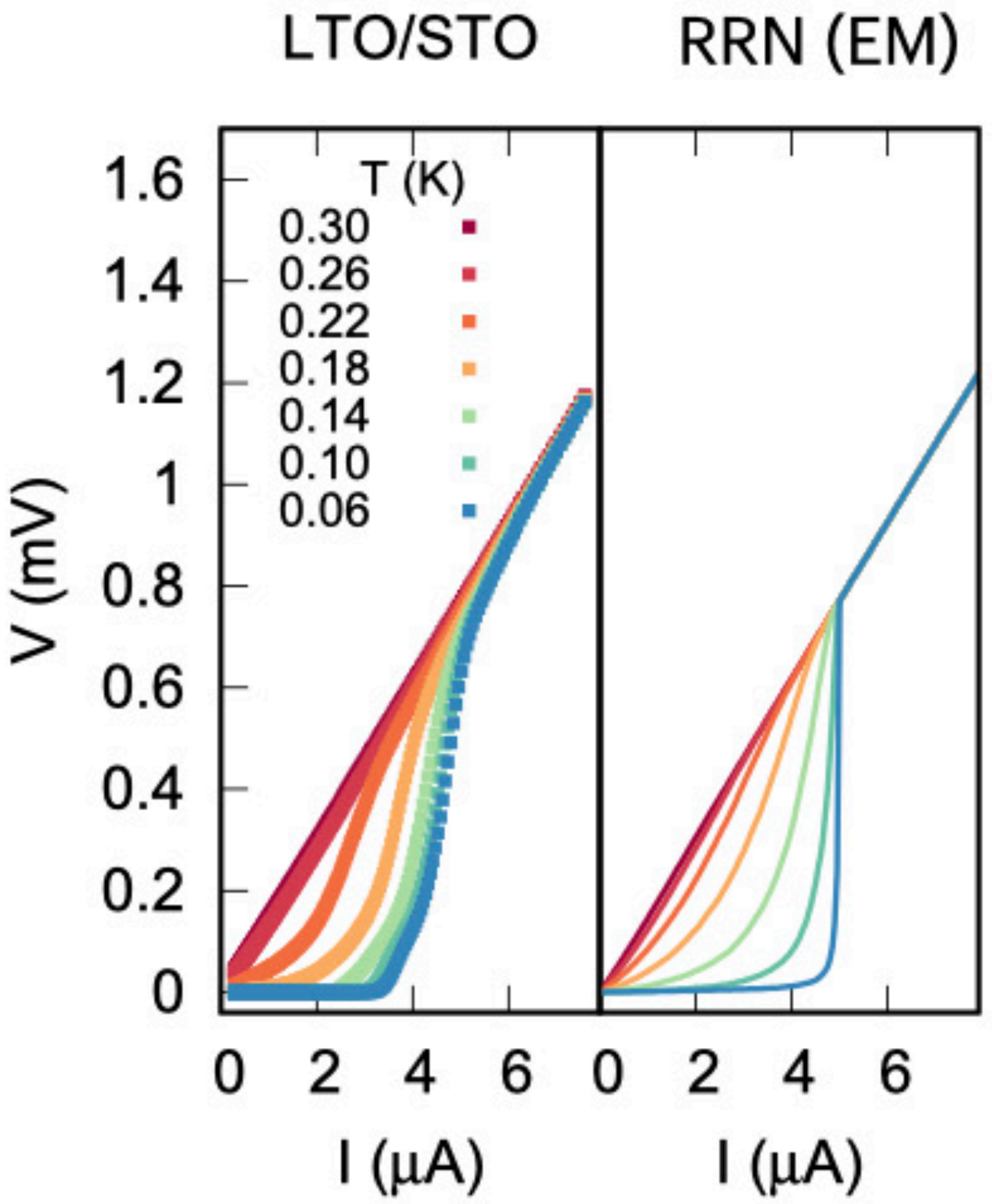
DC transport



Benfatto et al, PRB (2009)

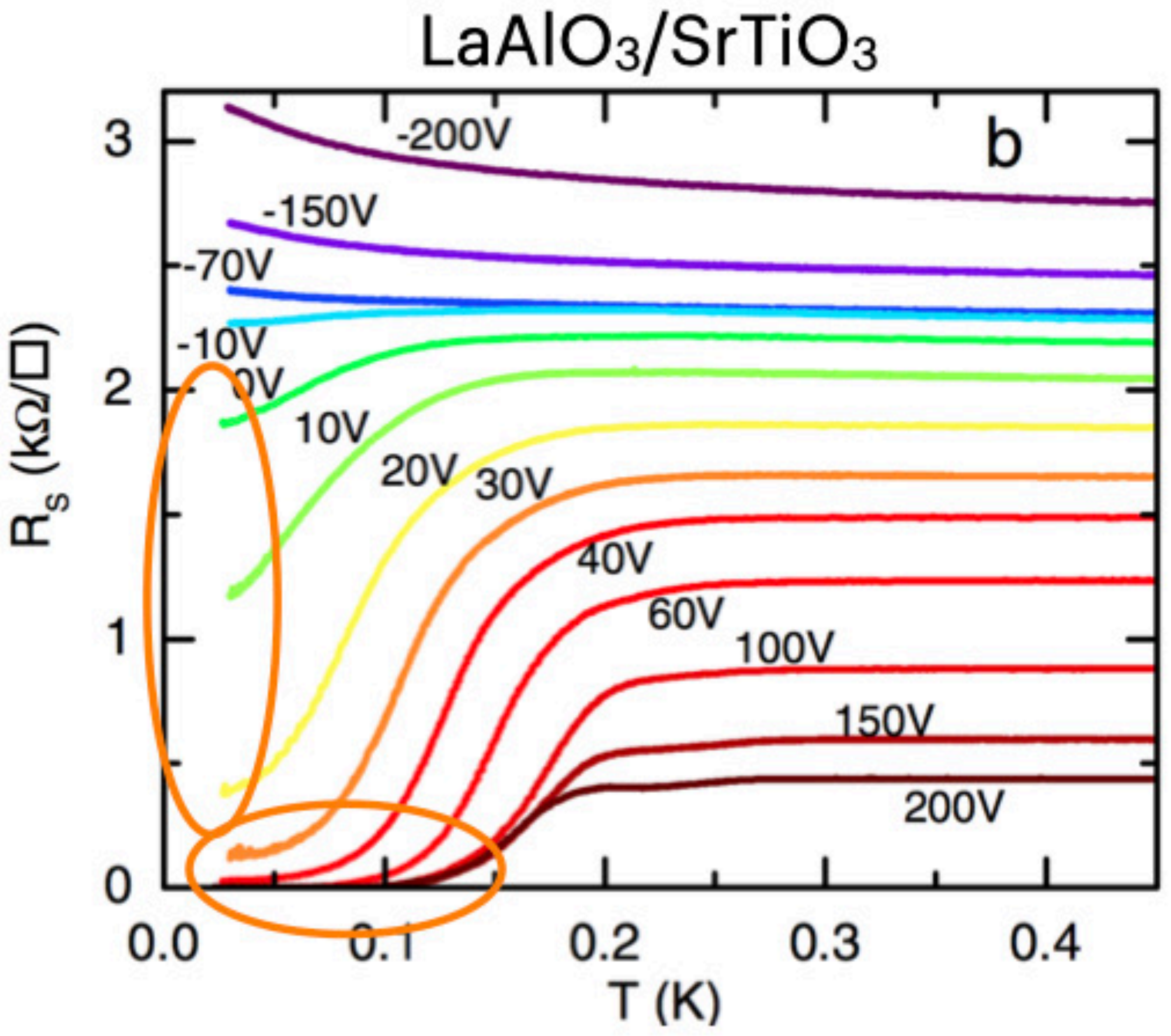


Non-linear IV characteristics

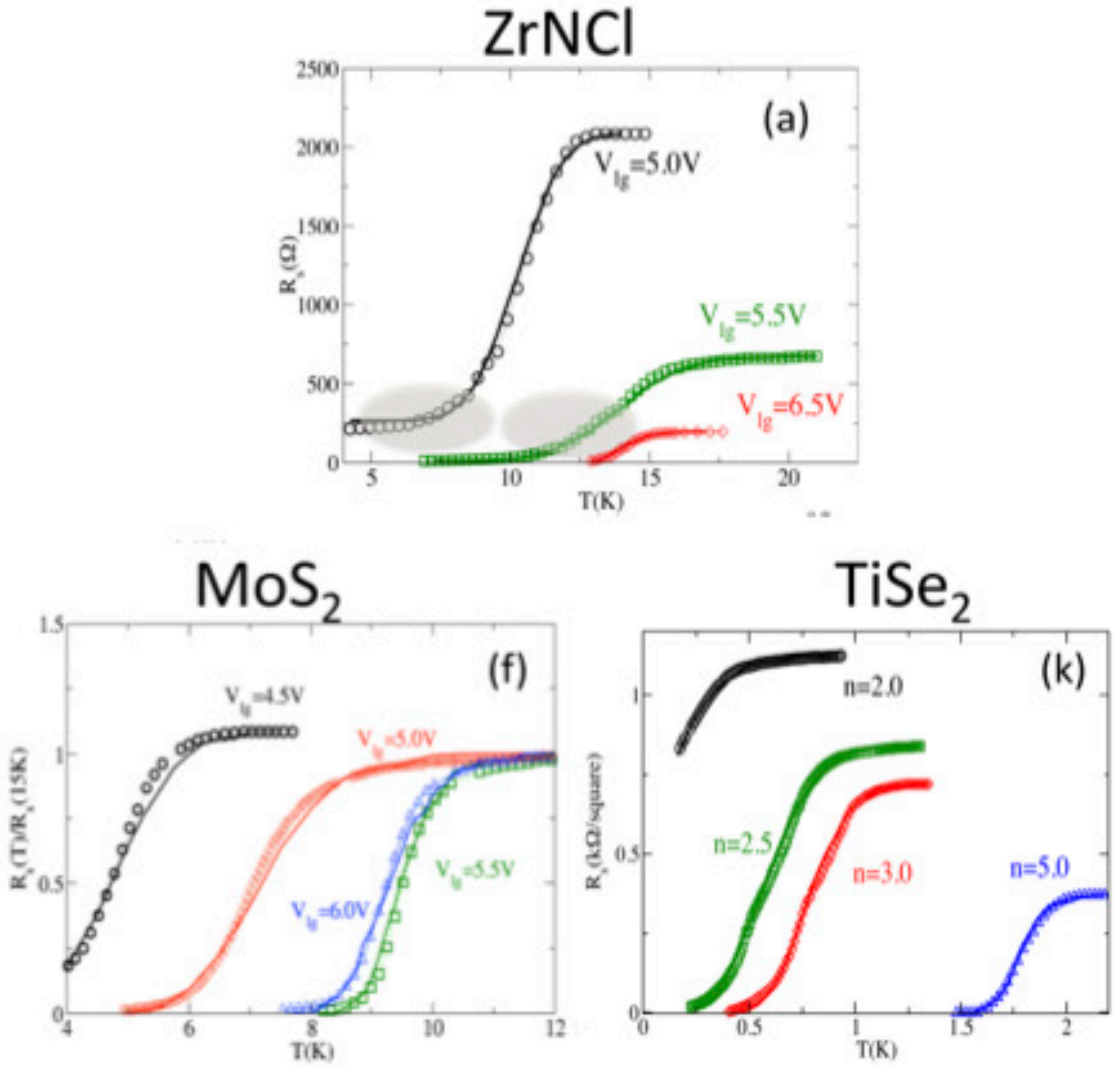


***GV** et al, PRB **100**, 064506 (2019)*

Large M-SC transitions, long tails, anomalous metal

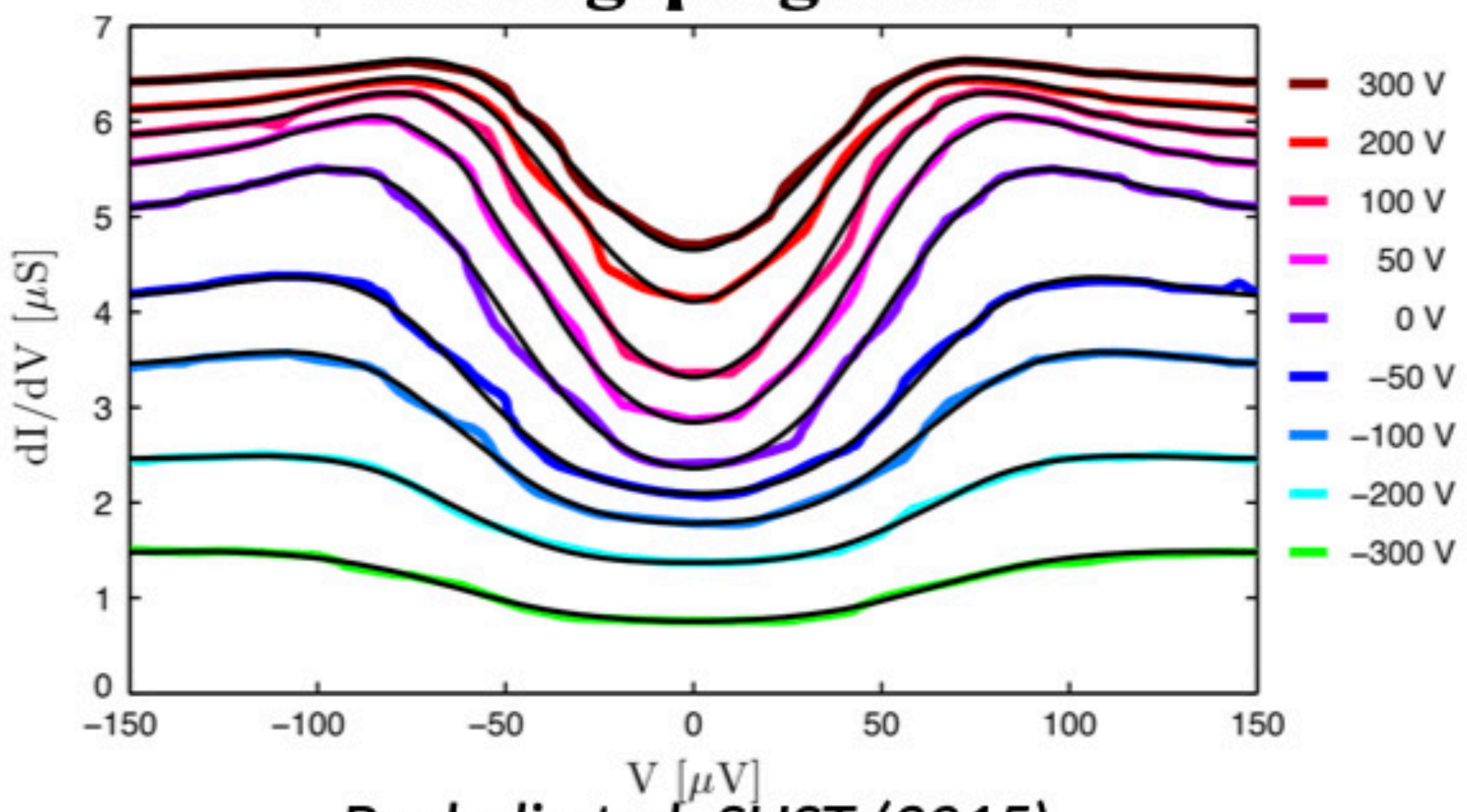


Biscaras et al, PRL (2012)



ZrNCl, MoGe$_2$, TiSe$_2$ in Dezi et al, PRB (2009)

Pseudo-gap signatures



Bucheli et al, SUST (2015)

AC (superfluid) response

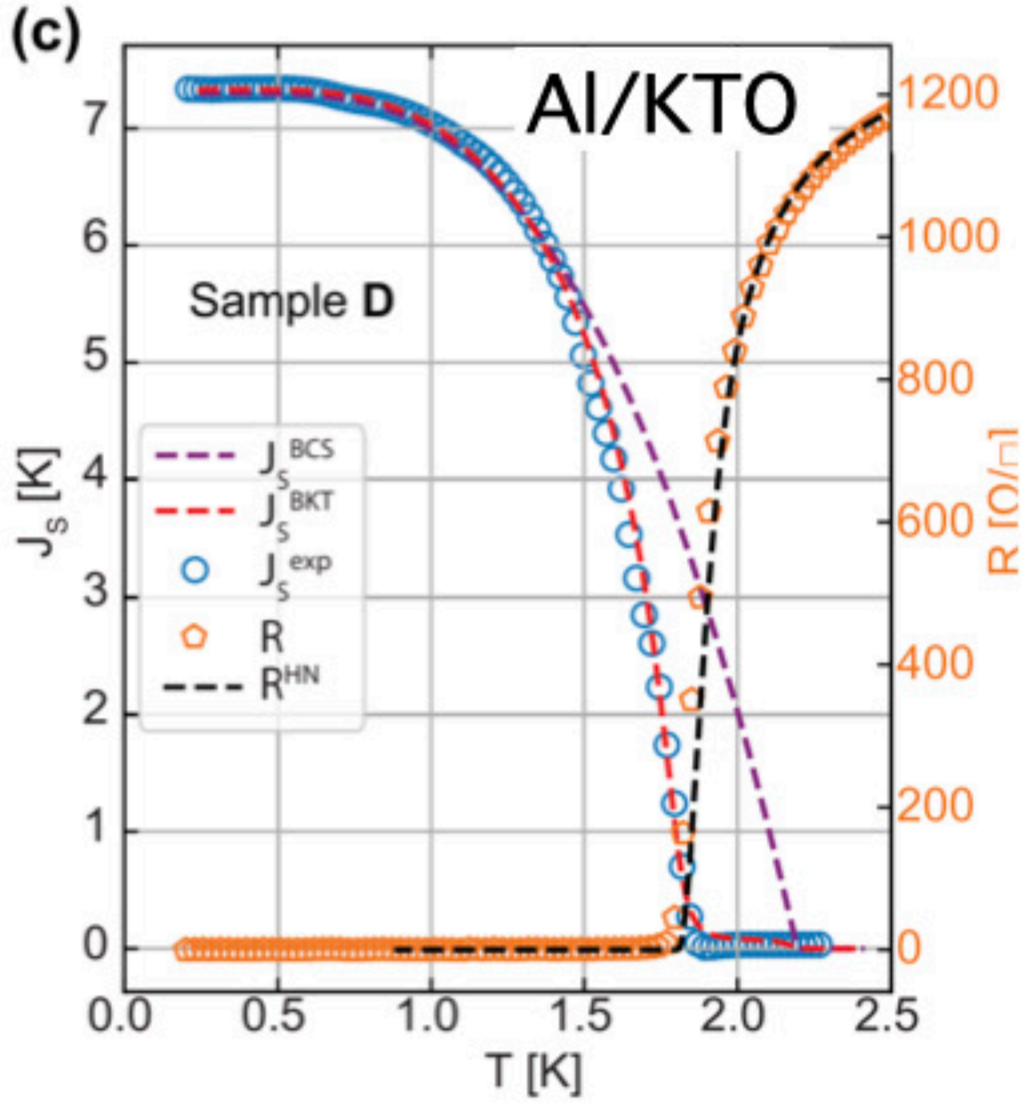
$$\sigma(\omega_0, T) = \underbrace{\sigma_1(\omega_0, T)}_{\text{unpaired e}^-} - i\,\overbrace{\sigma_2(\omega_0, T)}^{= \frac{1}{\omega_0 L_K} \propto J_s(T)}$$

Resonant microwave transport

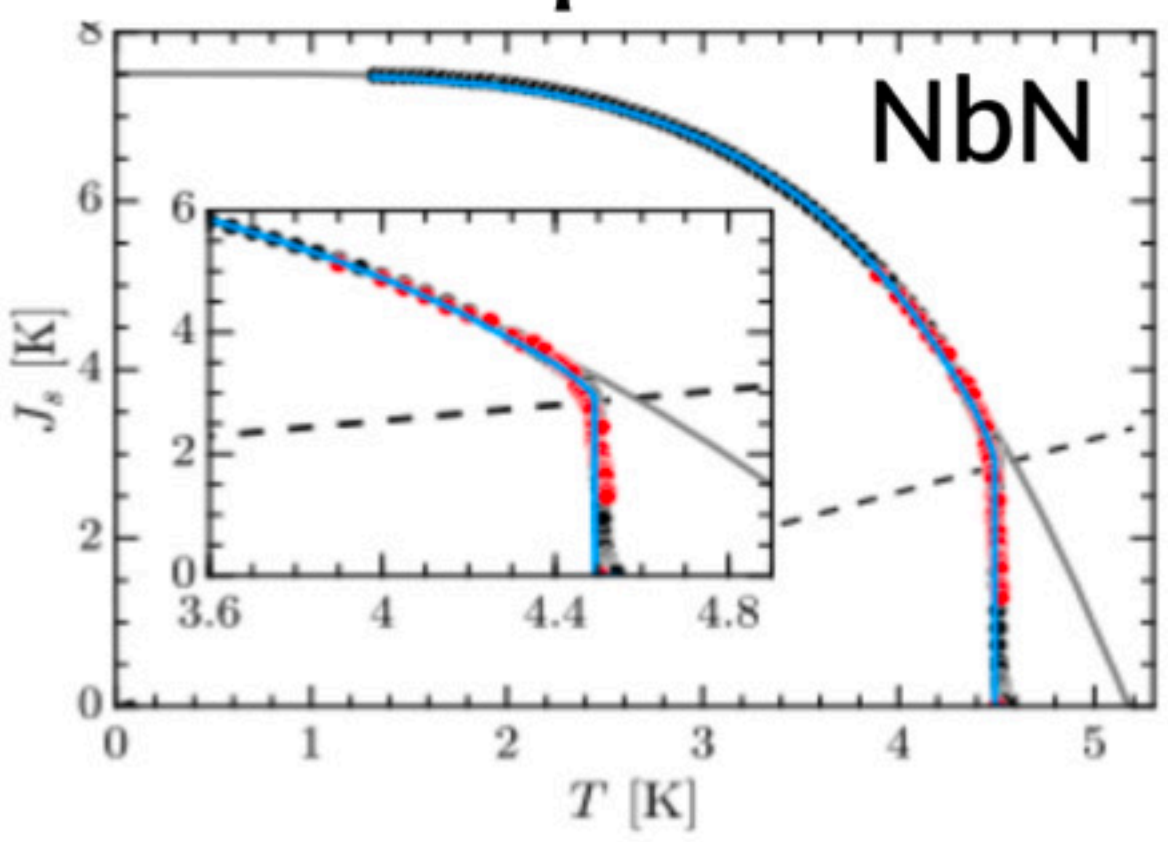
Resonance frequency $\omega_0 = 2\,\mathrm{GHz}$

Nicolas Bergeal's group (ESPCI, Paris)

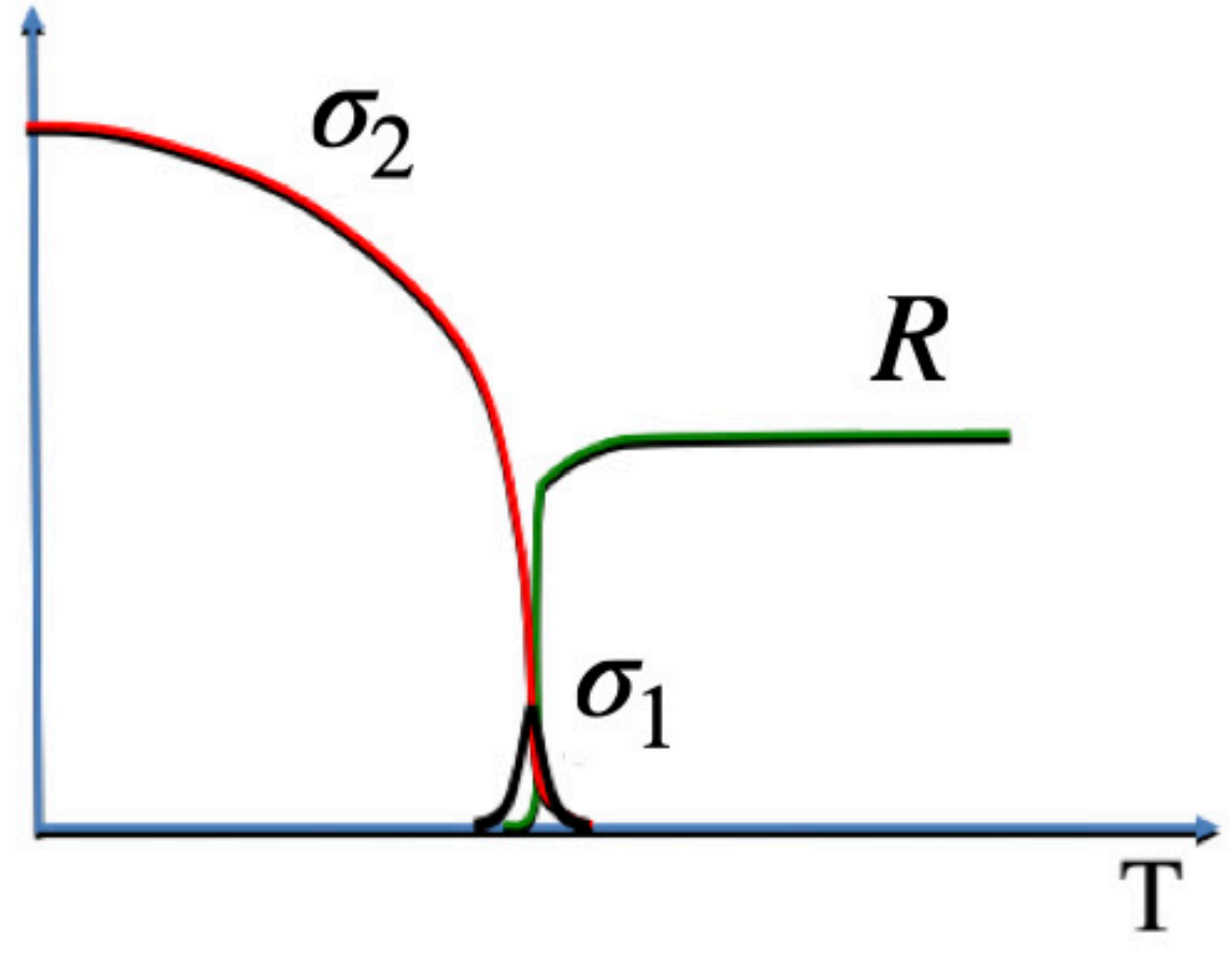
Mallik et al., Nat. Comm. (2022)



Christoph Strunk's talk



Weitzel et al., PRL (2023)



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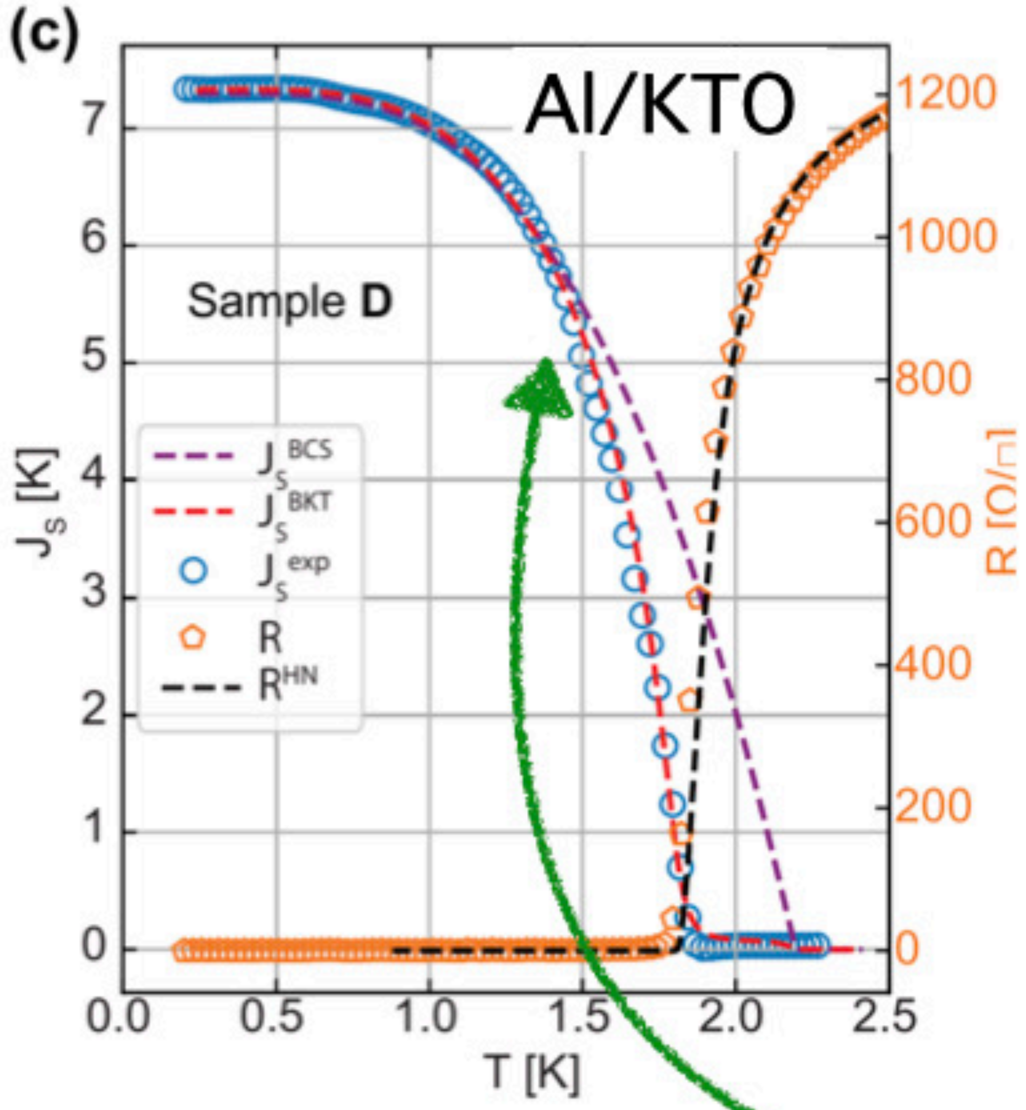

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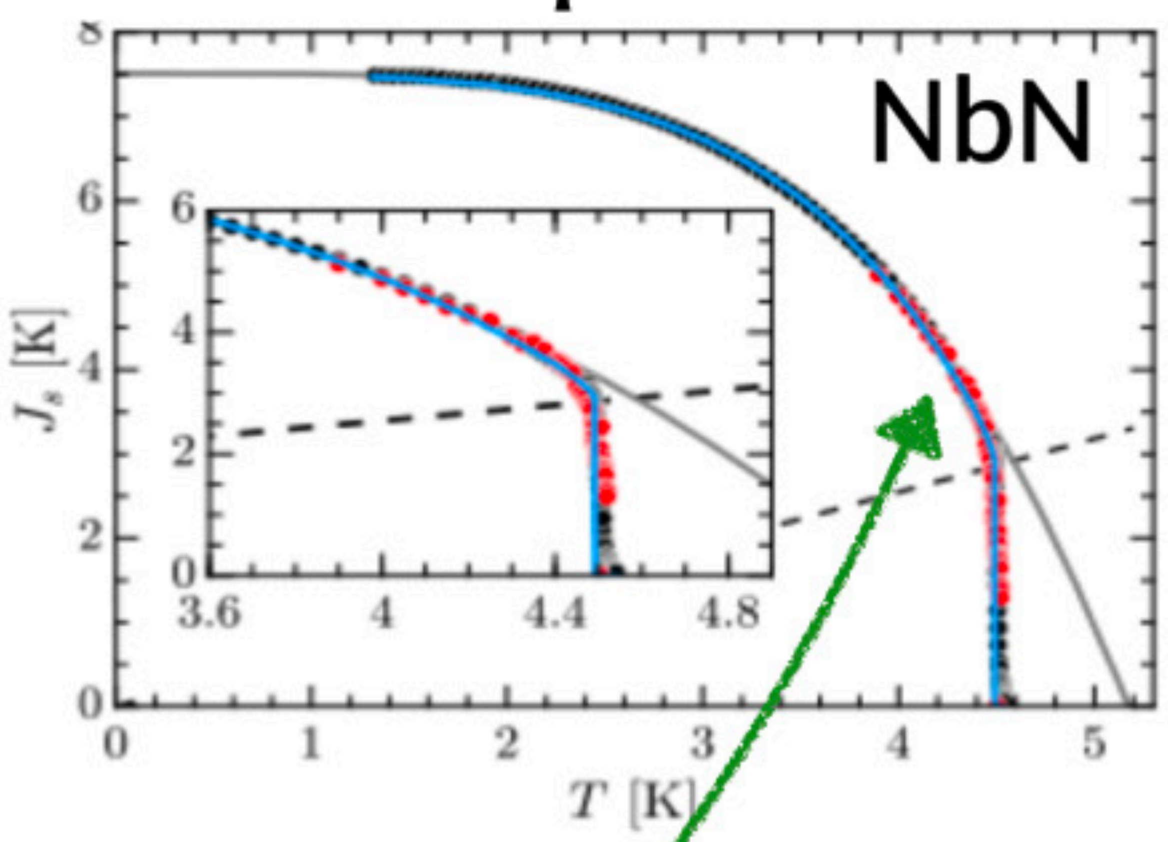


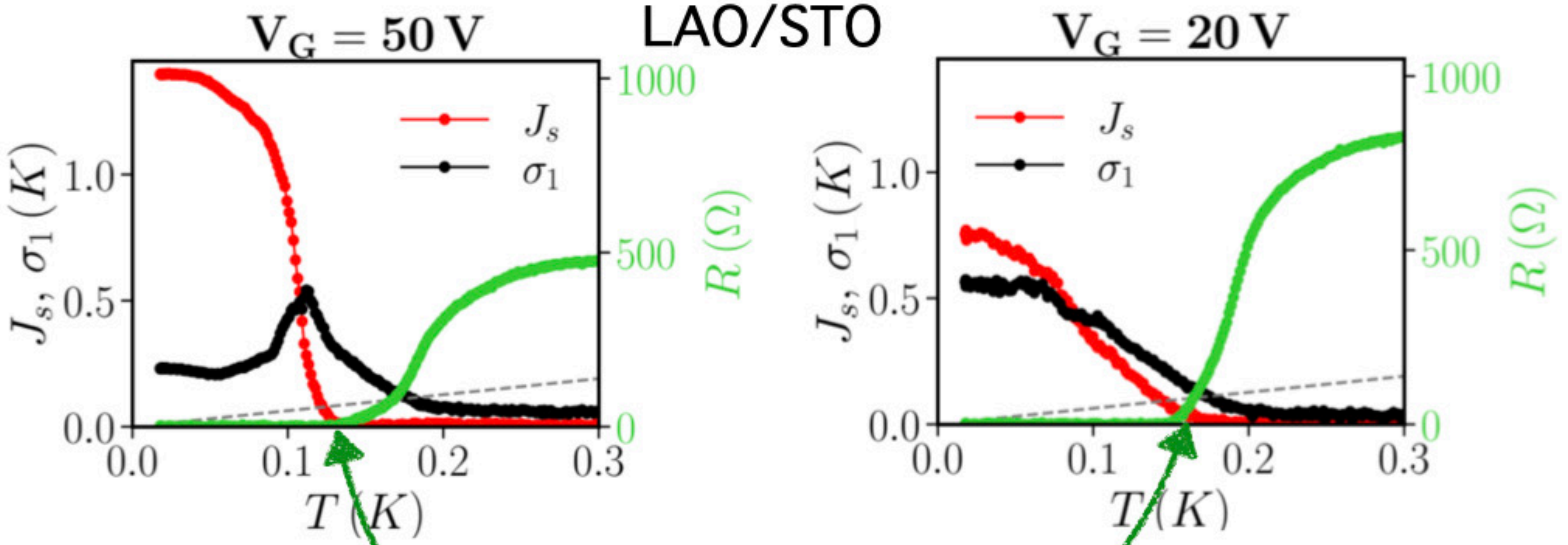
Weitzel et al., PRL (2023)

BKT fit!

GV, Scipost Phys. 15 239 (2023)

Nicolas Bergeal's group (ESPCI, Paris)

LAO/STO



No jump ⇸ no BKT

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$$\sigma(\omega_0, T) = \underbrace{\sigma_1(\omega_0, T)}_{\text{unpaired e}^-} - i\,\overbrace{\sigma_2(\omega_0, T)}^{= \frac{1}{\omega_0 L_K} \propto J_s(T)}$$

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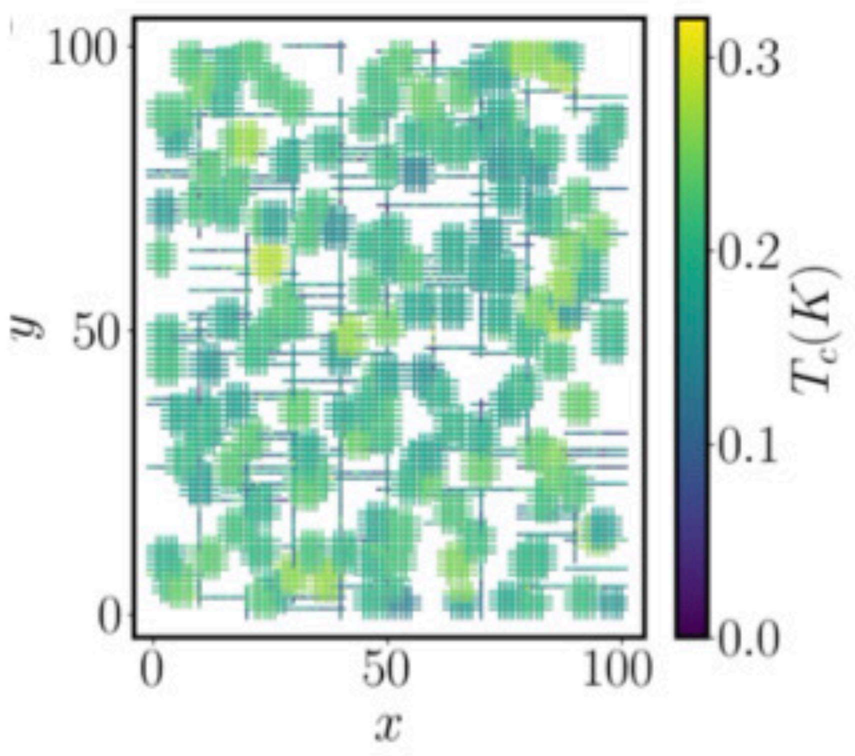
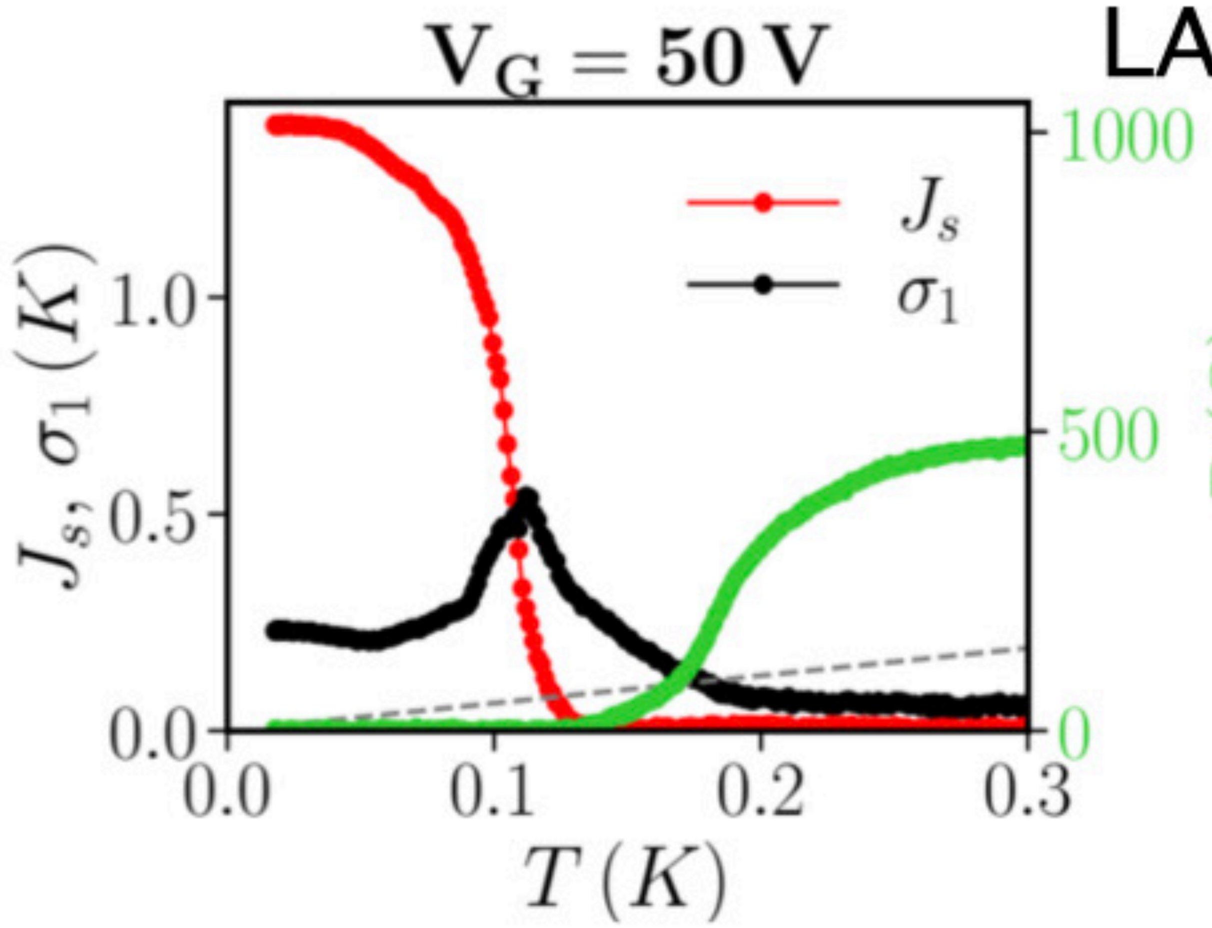
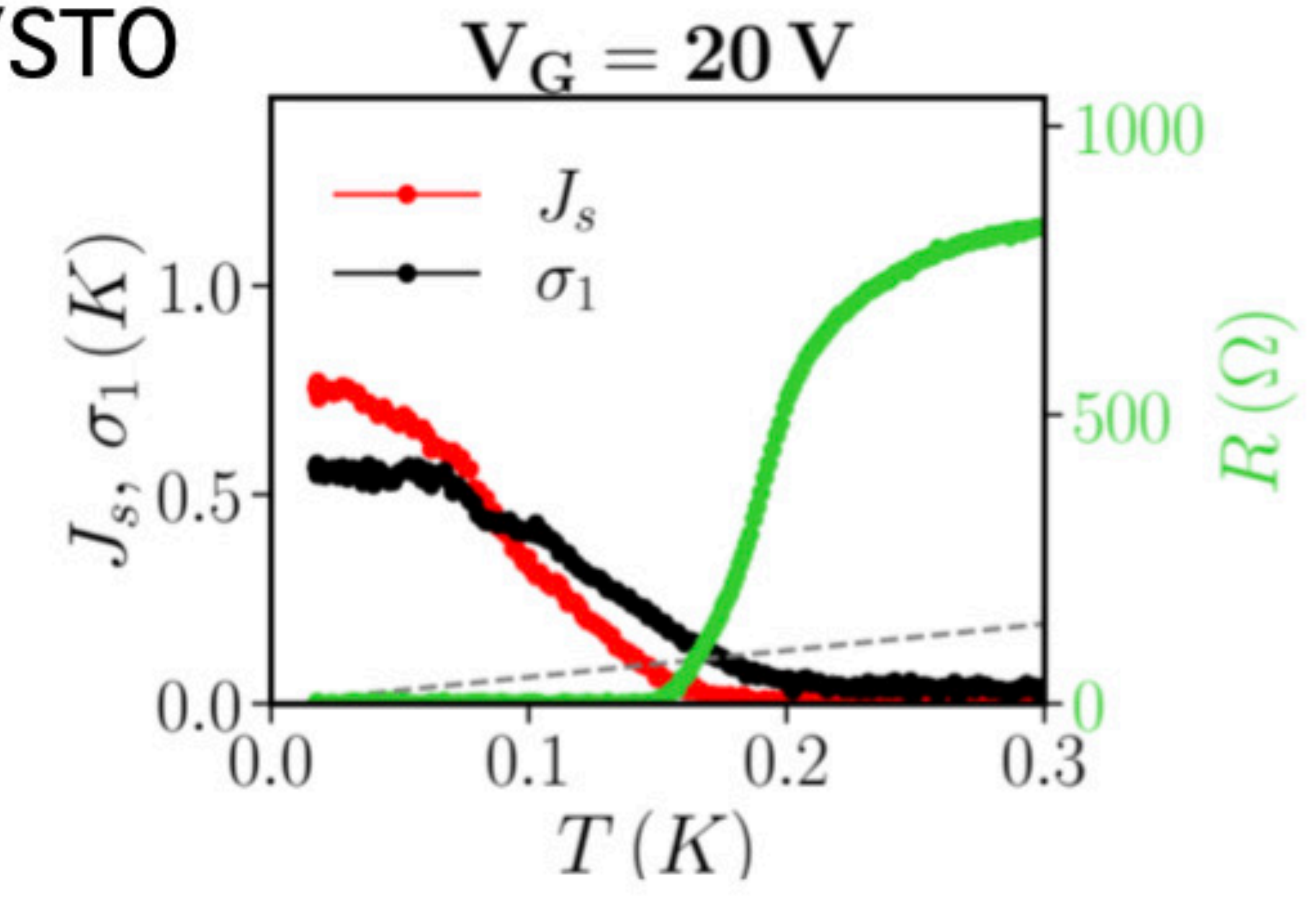
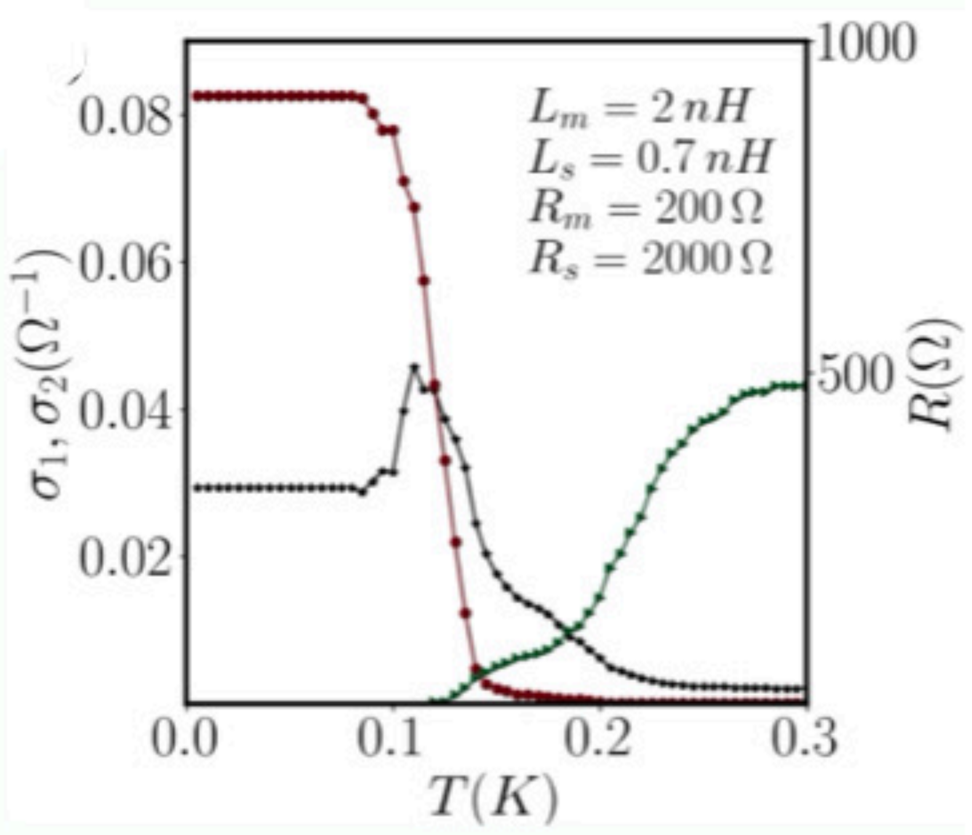
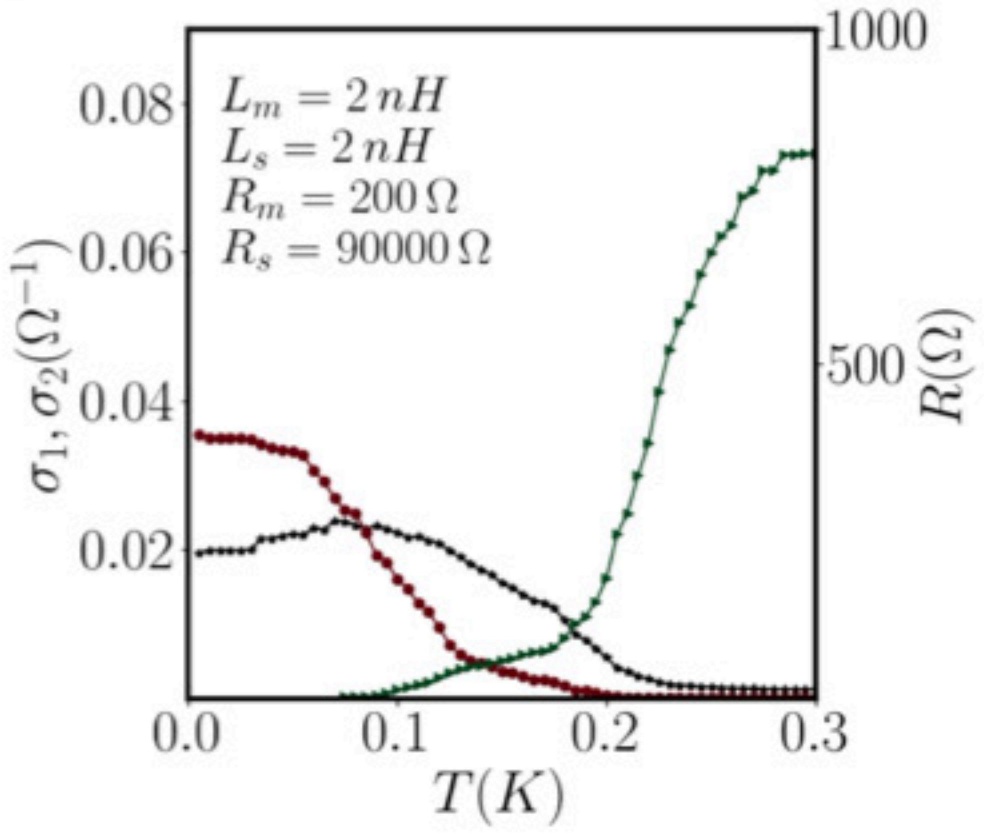










RIN results:

- same w_{sc} different superfluid response $J_s \neq n_s$
- Huge residual σ_1 revealed different responses of metal and superconducting clusters:
  - $R_m \neq R_s$ and $L_m \neq L_s$

Take home messages so far

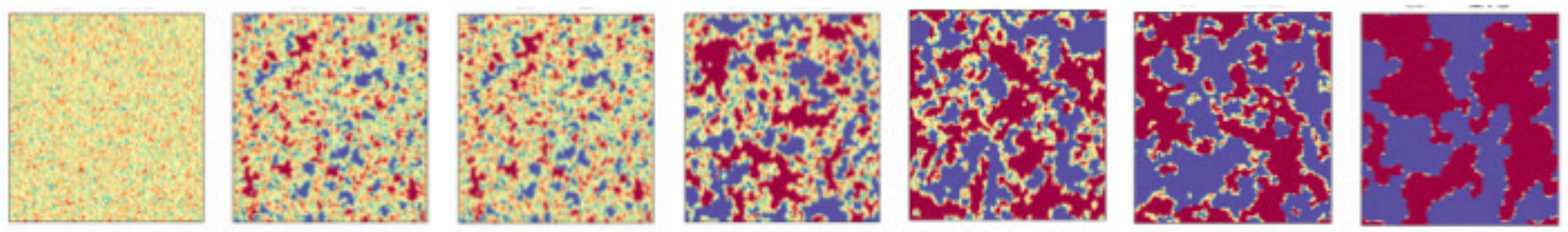
RIN EM solution:
<u>GV</u> et al, Condens. Mat. **5(2)** (2020)
<u>GV</u> et al, Nanomat. **11(8)** (2021)
RIN exact solution:
<u>GV</u> et al, Scipost Phys. **15** 239 (2023)

- **RRN** and **RIN** well captures the **general phenomenology**

- They can give important **intuitions on the microscopic details**

- Completely **agnostic** wrt **microscopic origin** of inhomogeneities

    - Cuprates: impurities + phase competition → emerging filamentarity with loss of BKT transition



<u>GV</u> et al, ScipostPhys. **15** 230 (2023)

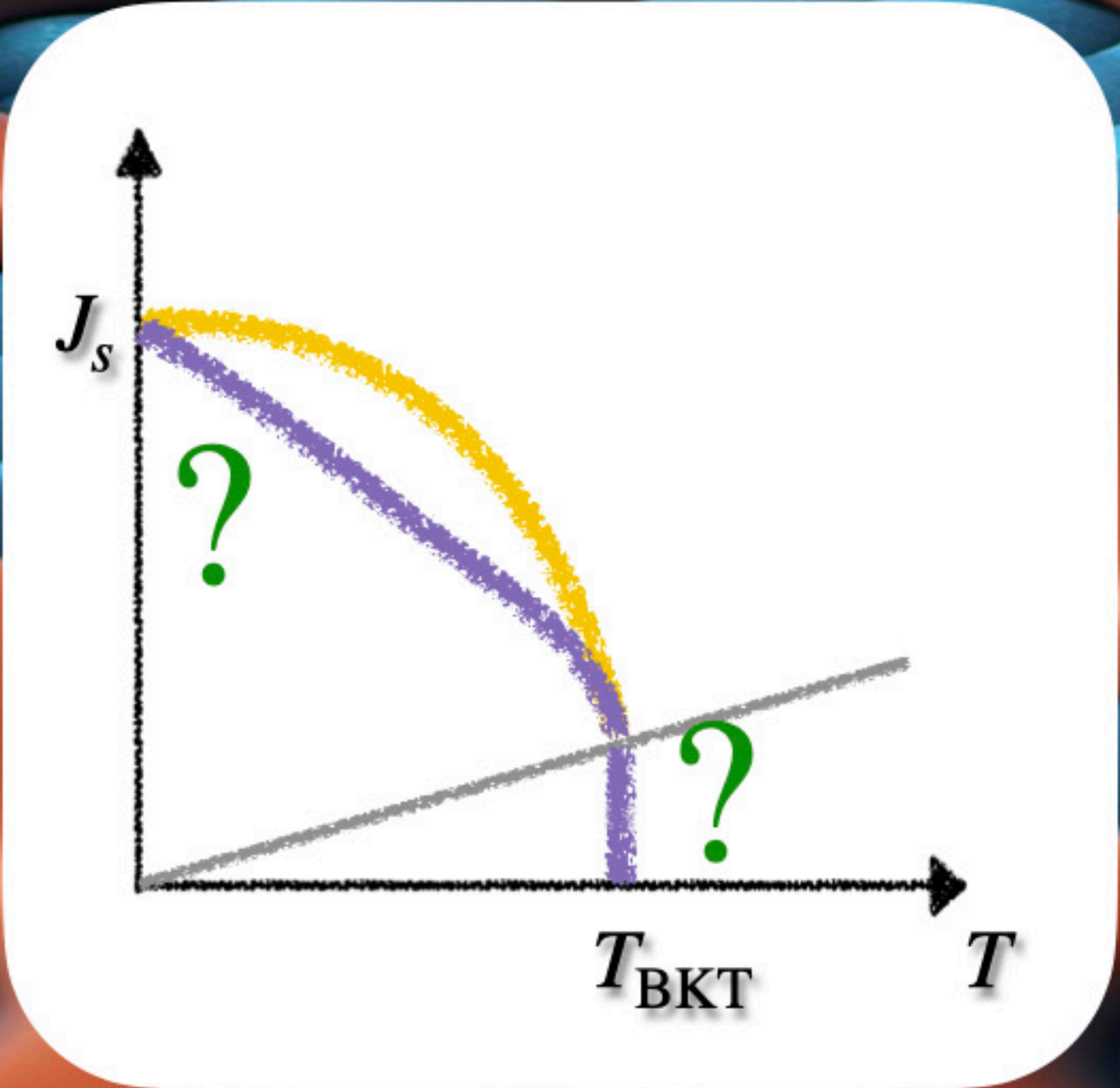
    - **Multilayer graphene: twist angle inhomogeneity**

Uri Nature (2020), Turkel Science (2022), Dolleman PRB (2024), Mukherjee NatMat (2025),...

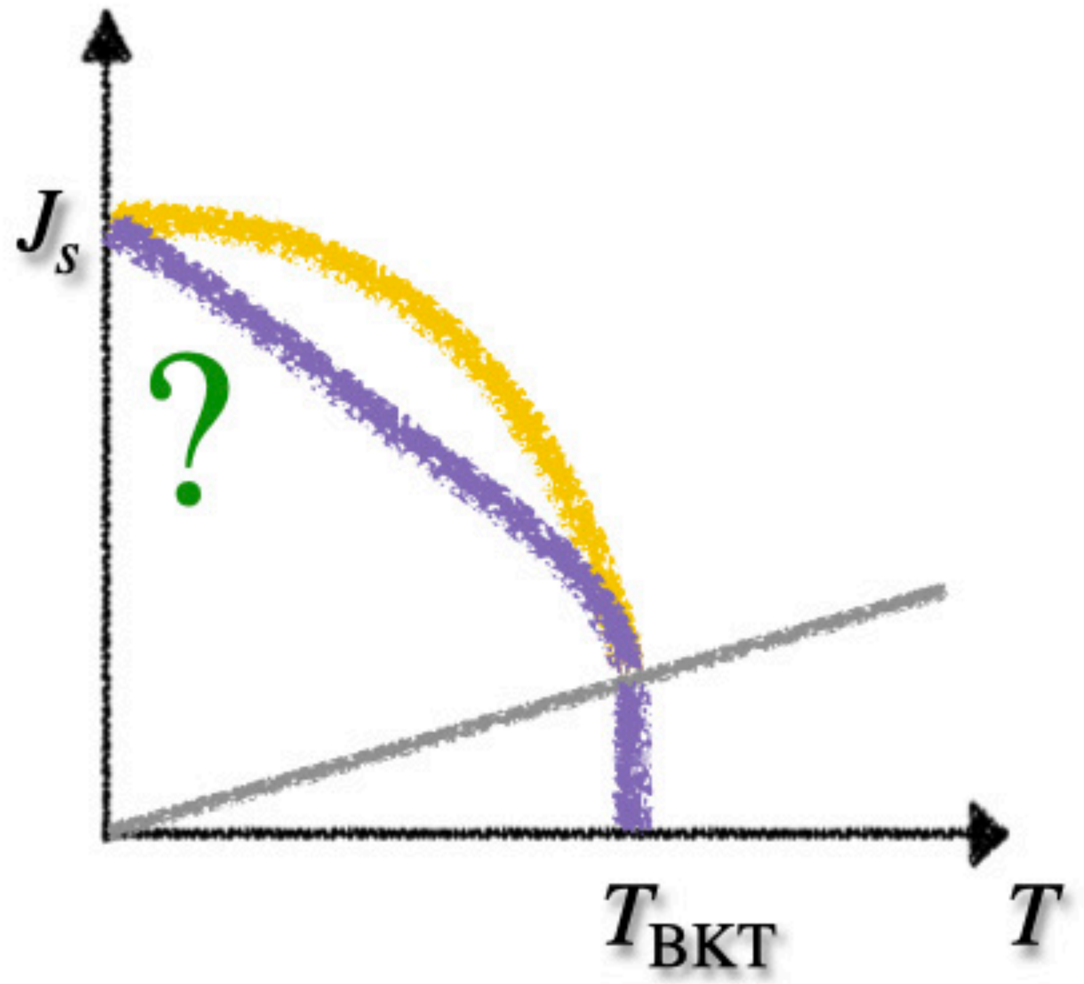
Maccari, Rademaker, and <u>GV</u>, arXiv:2606.17191 (2026)

Twisted multilayer graphene

- What form of disorder? BKT transition?
- Superconducting order parameter ?



"Unconventional" superconductivity ?

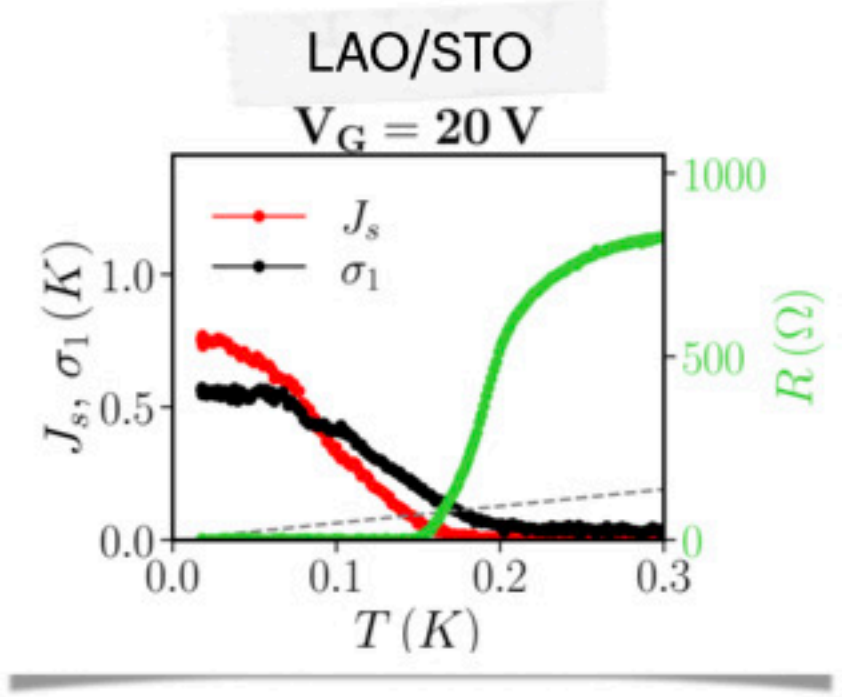


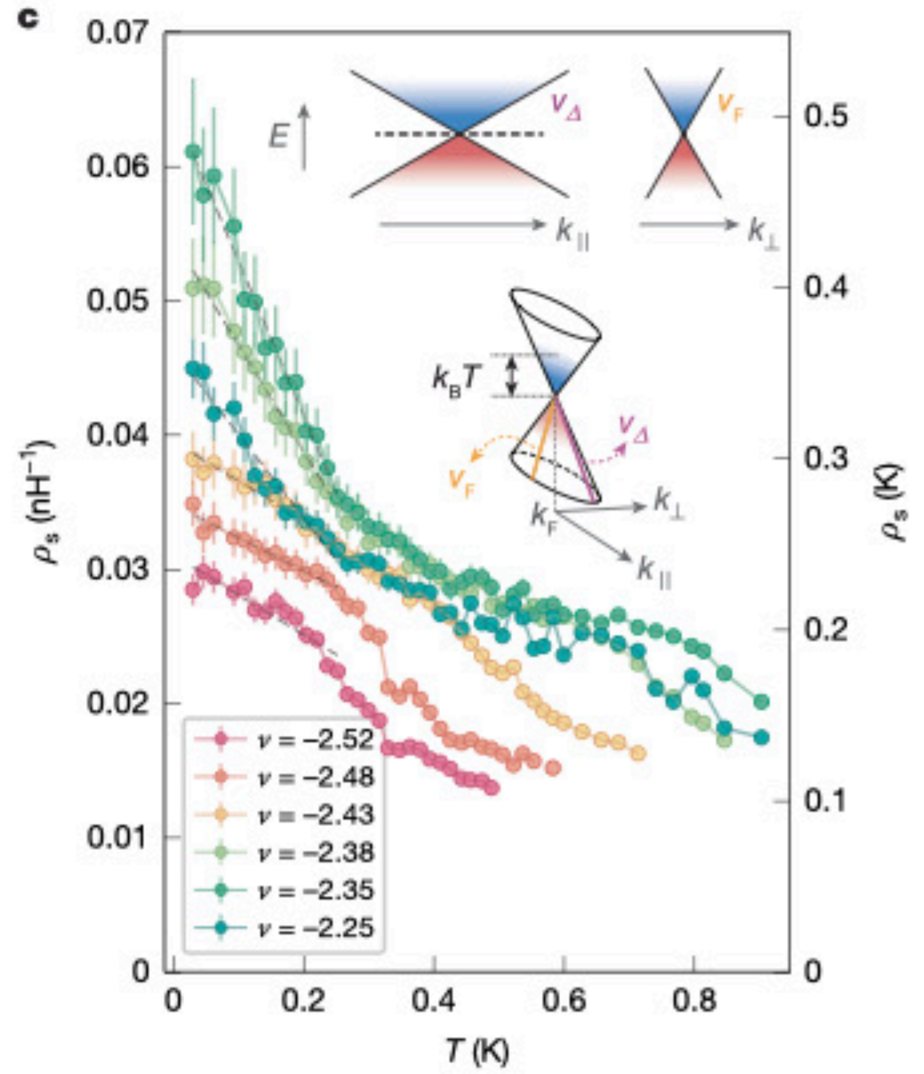
Order parameter symmetry?

Nodal?

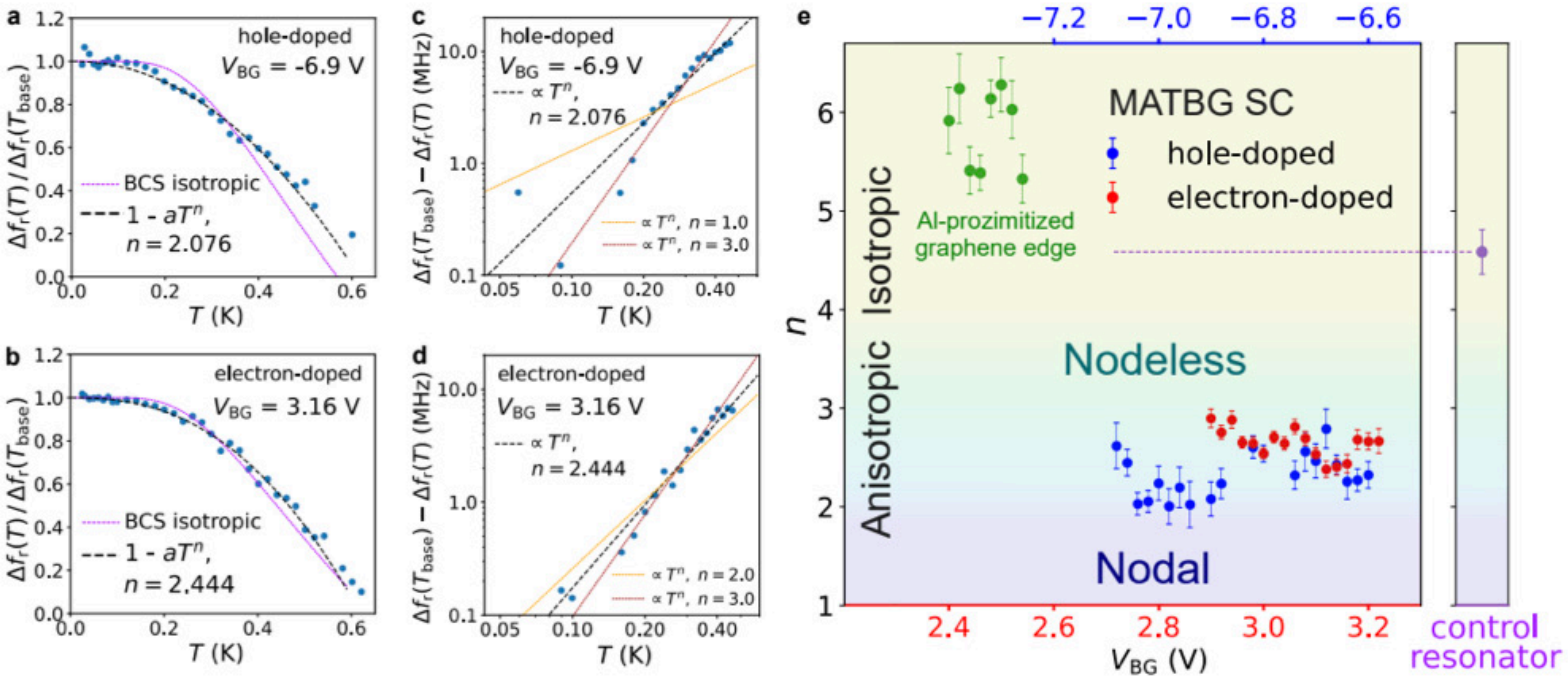
BKT analysis in transport suggest presence of inhomogeneities

Negative electronic compressibility: *Dolleman et al, PRB (2024)*





TTG: *Banerjee et al, Nature (2025),*
Mukherjee et al, Nature Mat. (2025)
Park et al, Science (2025)



TBG: *Tanaka et al, Nature (2025)*

Twisted multilayer graphene:
Reproducibility issues and twist angle inhomogeneities

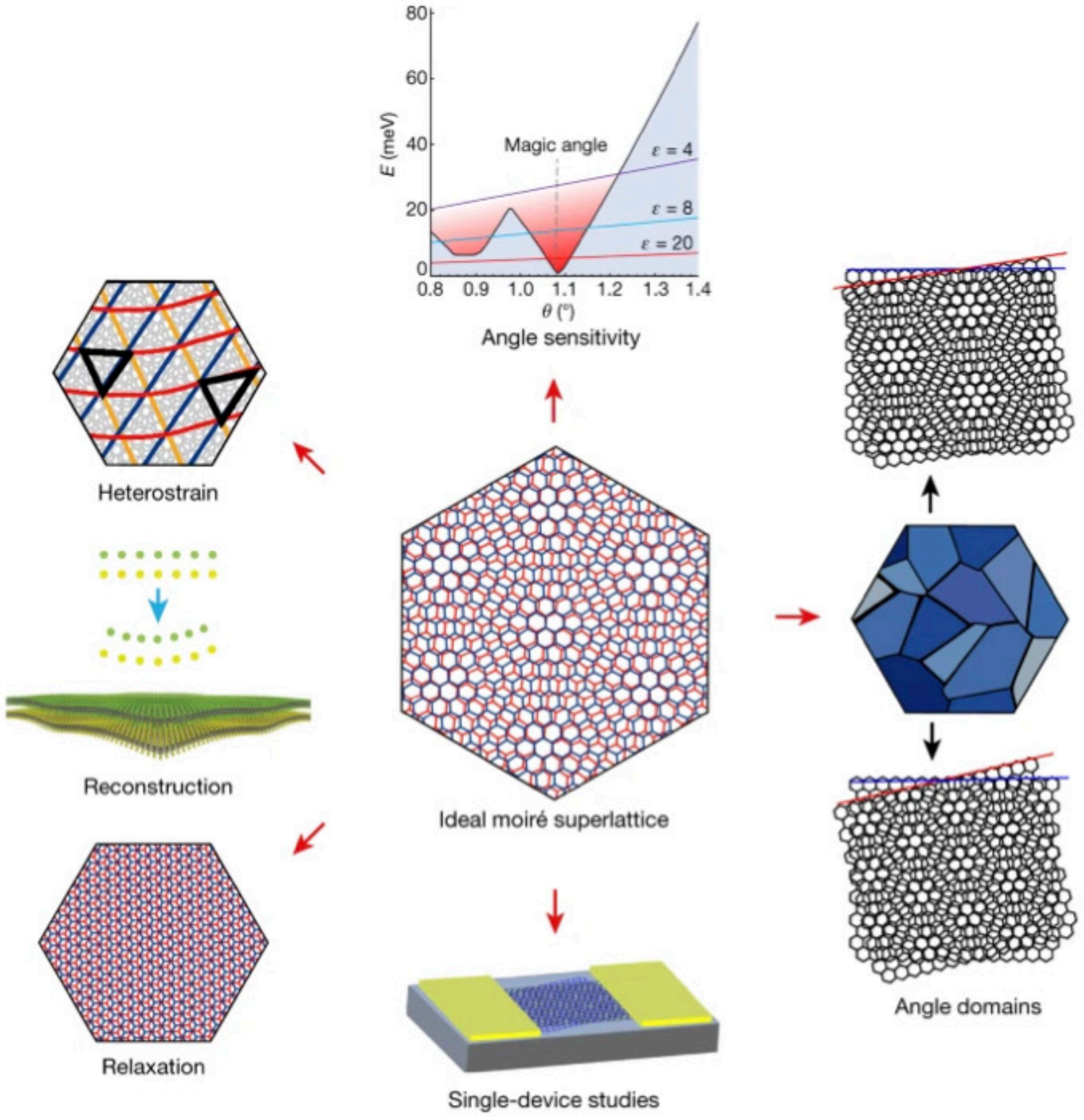
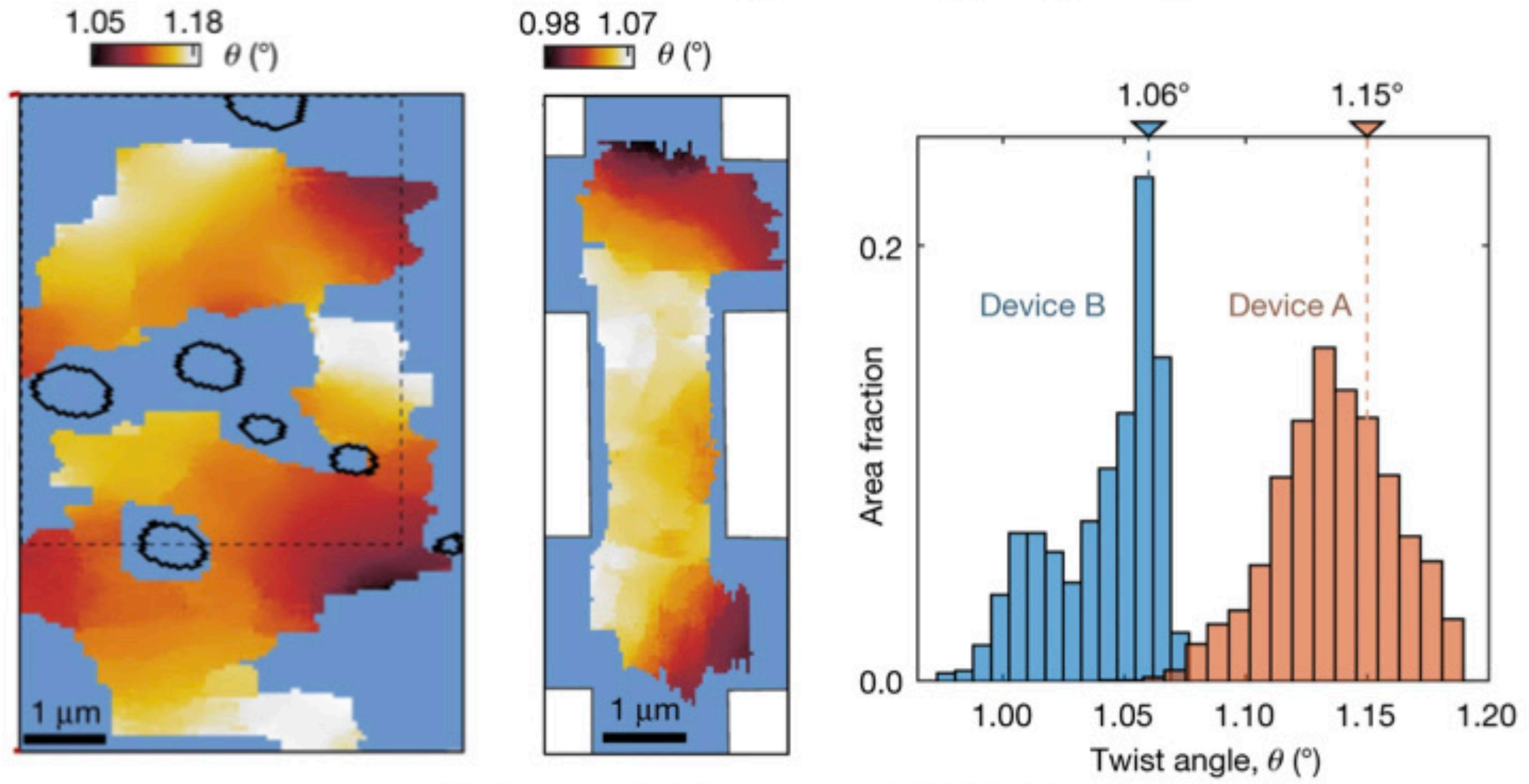


Fig. 1: Various factors and phenomena affecting the reproducibility of moiré materials studies.

Chun Ning Lau et al, Nature (2022)

Mapping the twist-angle disorder and Landau levels in magic-angle graphene



Uri et al, Nature (2020)

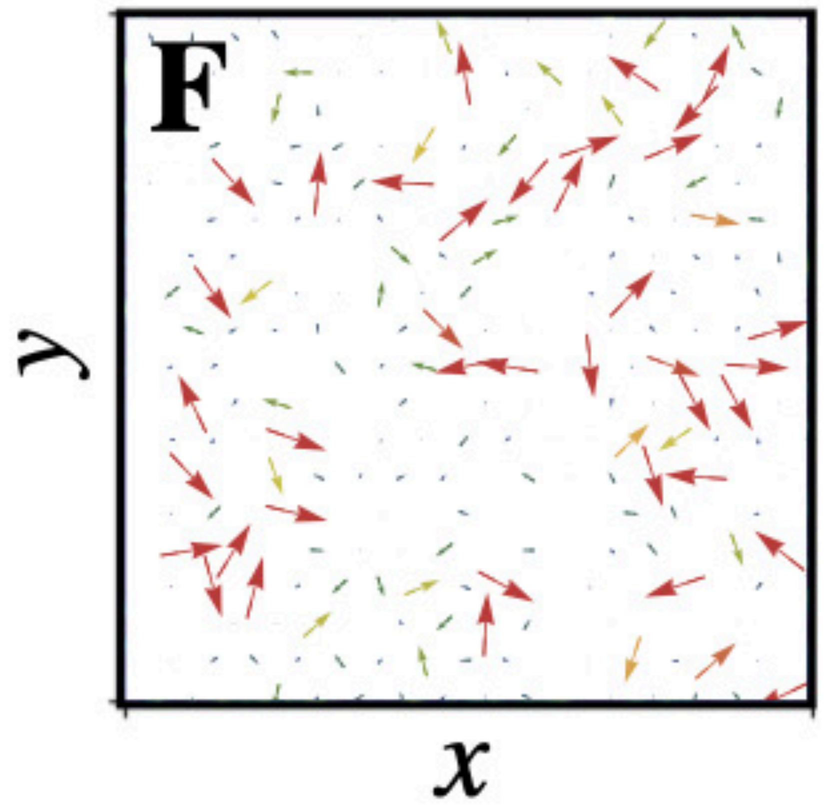
periods. We find a correlation between the degree of θ disorder and the quality of the MATBG transport characteristics and show that even state-of-the-art devices—which exhibit correlated states, Landau fans and superconductivity—display considerable local variation in θ of up to 0.1 degrees, exhibiting substantial gradients and networks of jumps, and may contain areas with no local MATBG behaviour. We observe that the

Beechem et al, ACS Nano (2014)
Turkel et al, Science (2022)

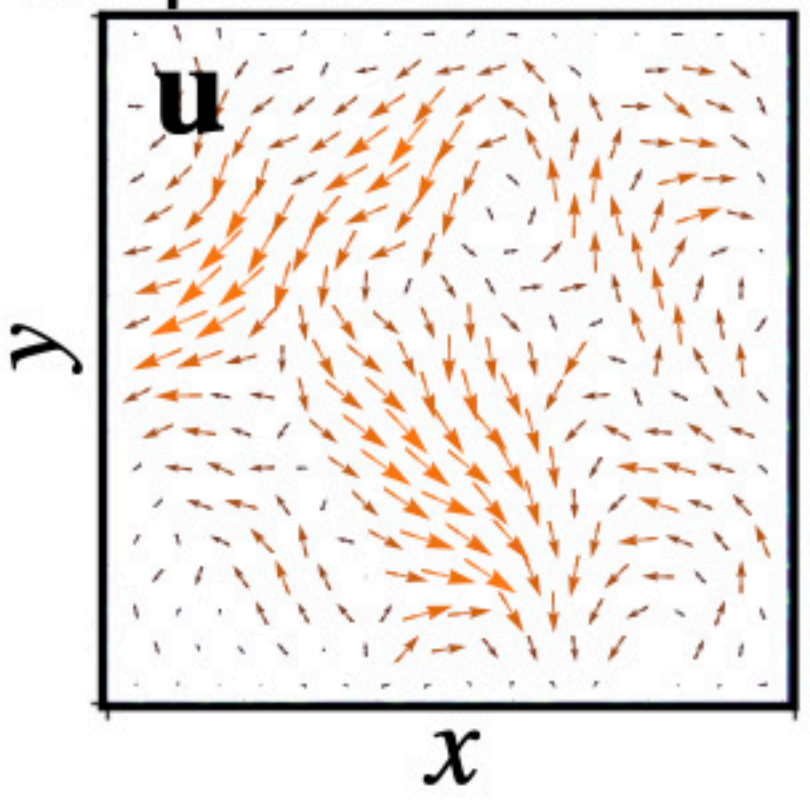
Long-range correlations from elasticity of the solid

Elastic solid: $\Delta \mathbf{u}(\mathbf{x}) + \frac{1}{1-2\nu}\nabla(\nabla\cdot\mathbf{u}(\mathbf{x})) = \mathbf{F}$

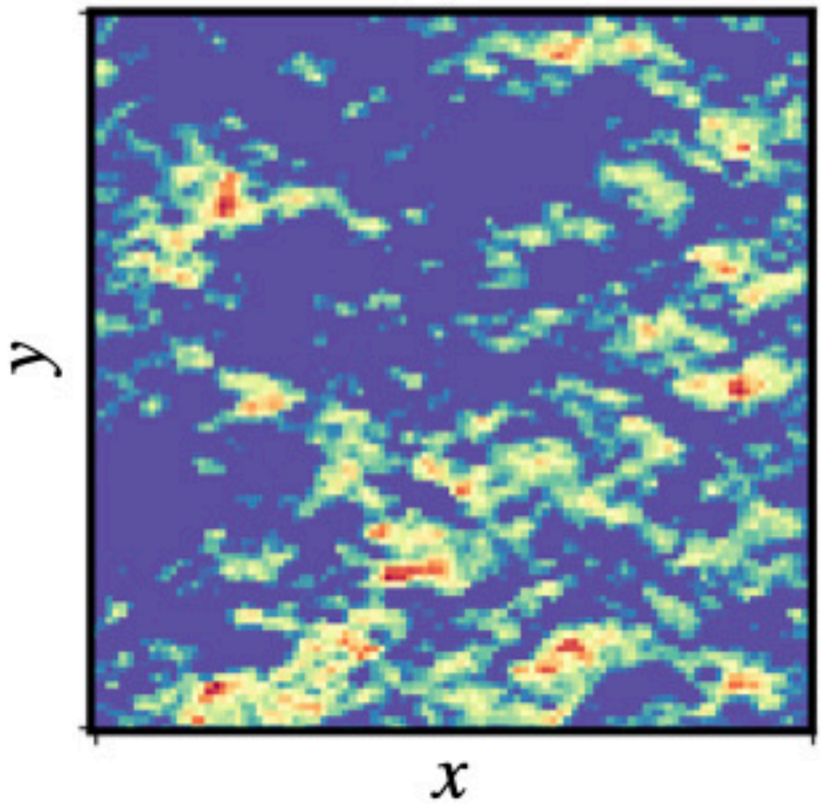
Local uncorrelated forces $\mathbf{F}$

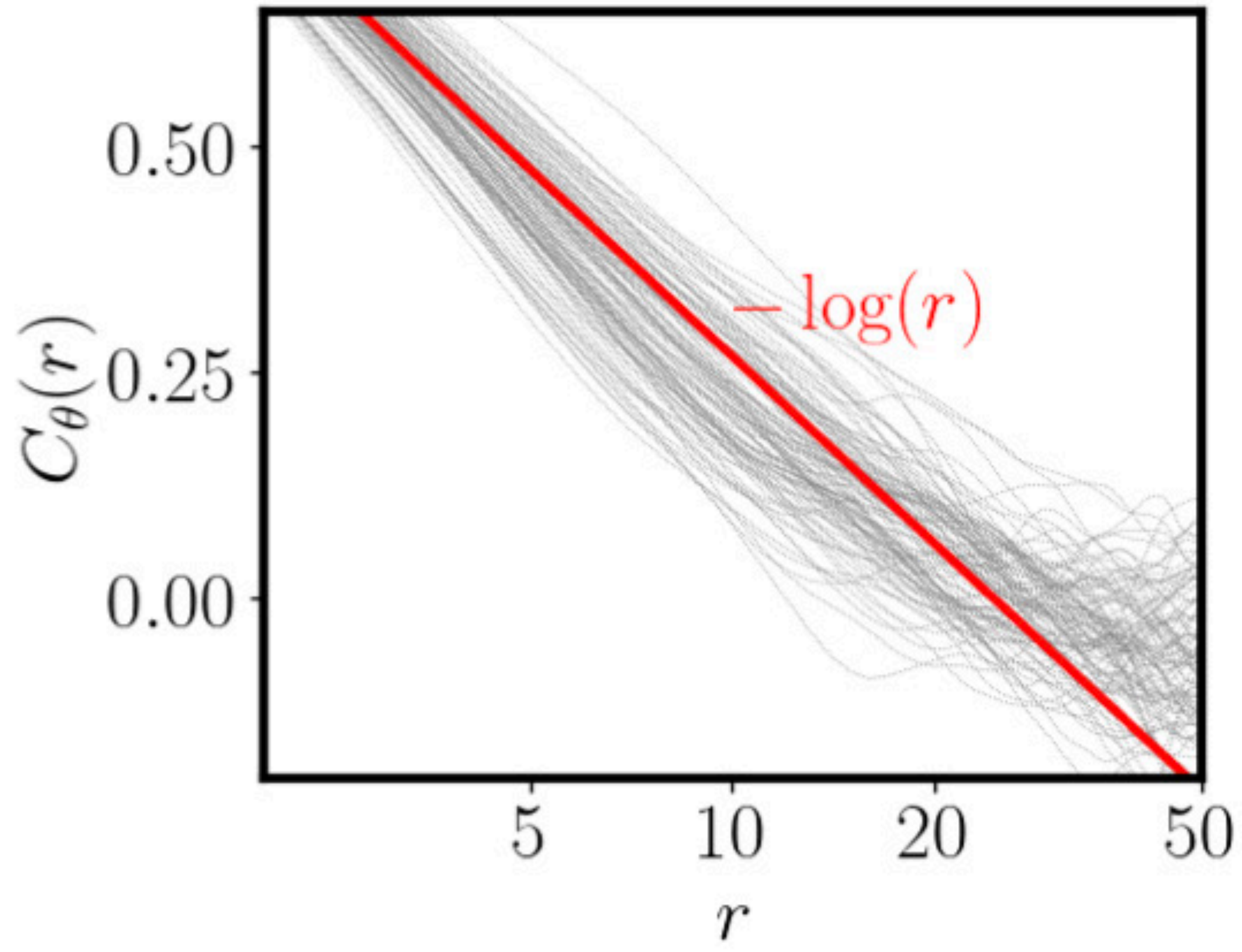


Generates spatial displacements $\mathbf{u}$



$\delta\theta = \frac{1}{2}\nabla\times\mathbf{u}$





Random uncorrelated impurities

$\langle \mathbf{F}(\mathbf{x})\rangle = 0\ ,\ \langle F_a(\mathbf{x}_1)F_b(\mathbf{x}_2)\rangle = F^2\delta_{ab}\delta(\mathbf{x}_1-\mathbf{x}_2)$

$$C_\theta(\mathbf{r}_1-\mathbf{r}_2) = \langle\delta\theta(\mathbf{r}_1)\delta\theta(\mathbf{r}_2)\rangle \sim -\log(r)$$

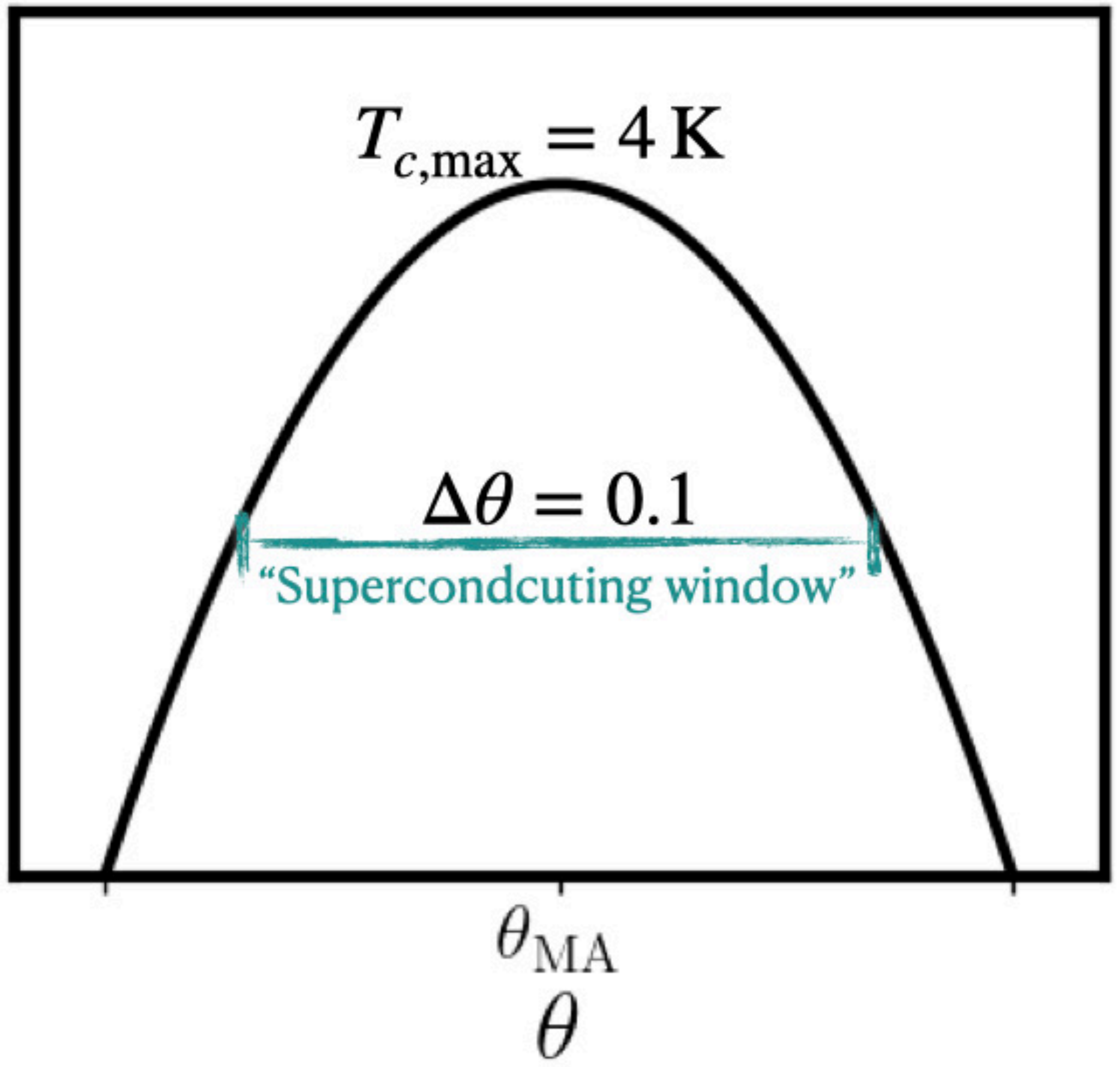
Long-range correlations of twist angles even for very few impurities

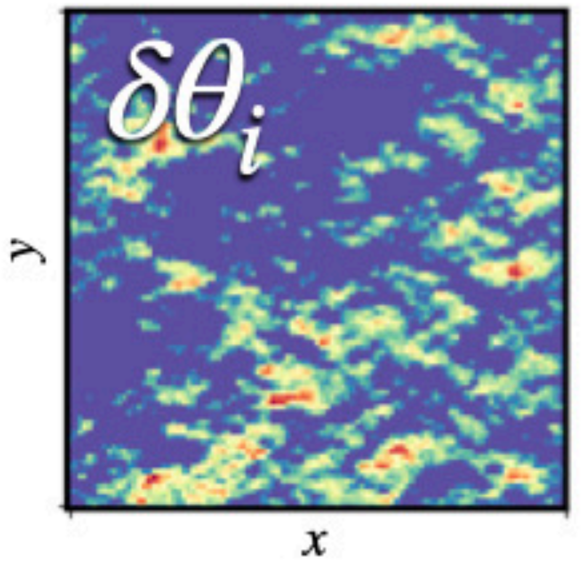
Maccari, Rademaker, and GV, arXiv: 2606.17191 (2026)

From twist maps to temperature maps

$$\theta_i = \theta_0 + r_{\text{maps}} \delta\theta_i$$

$$T_c(\theta_i) = T_{c,\max}\left(1 - \frac{(\theta_i - \theta_{\text{MA}})^2}{\Delta\theta^2}\right)$$





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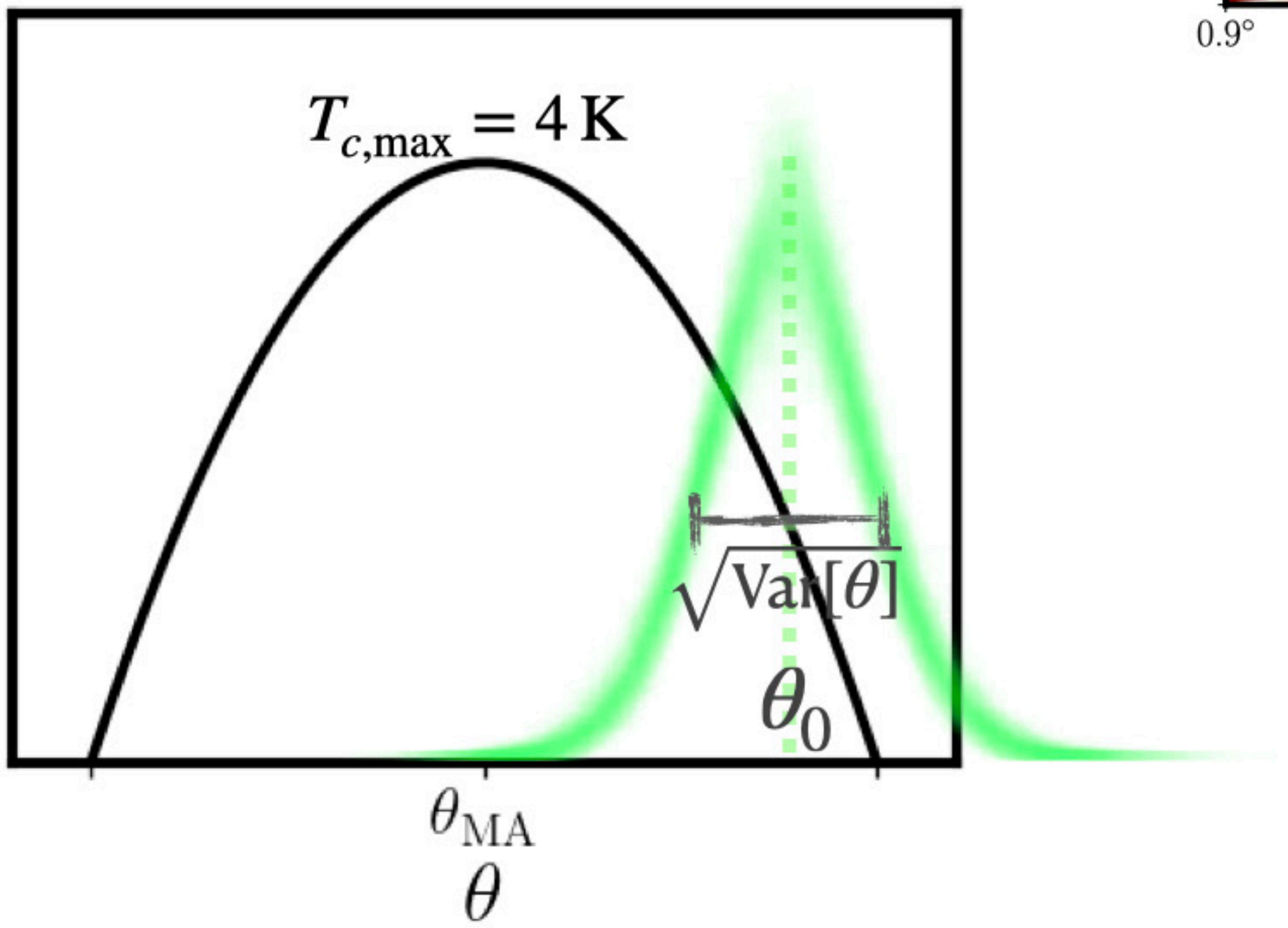
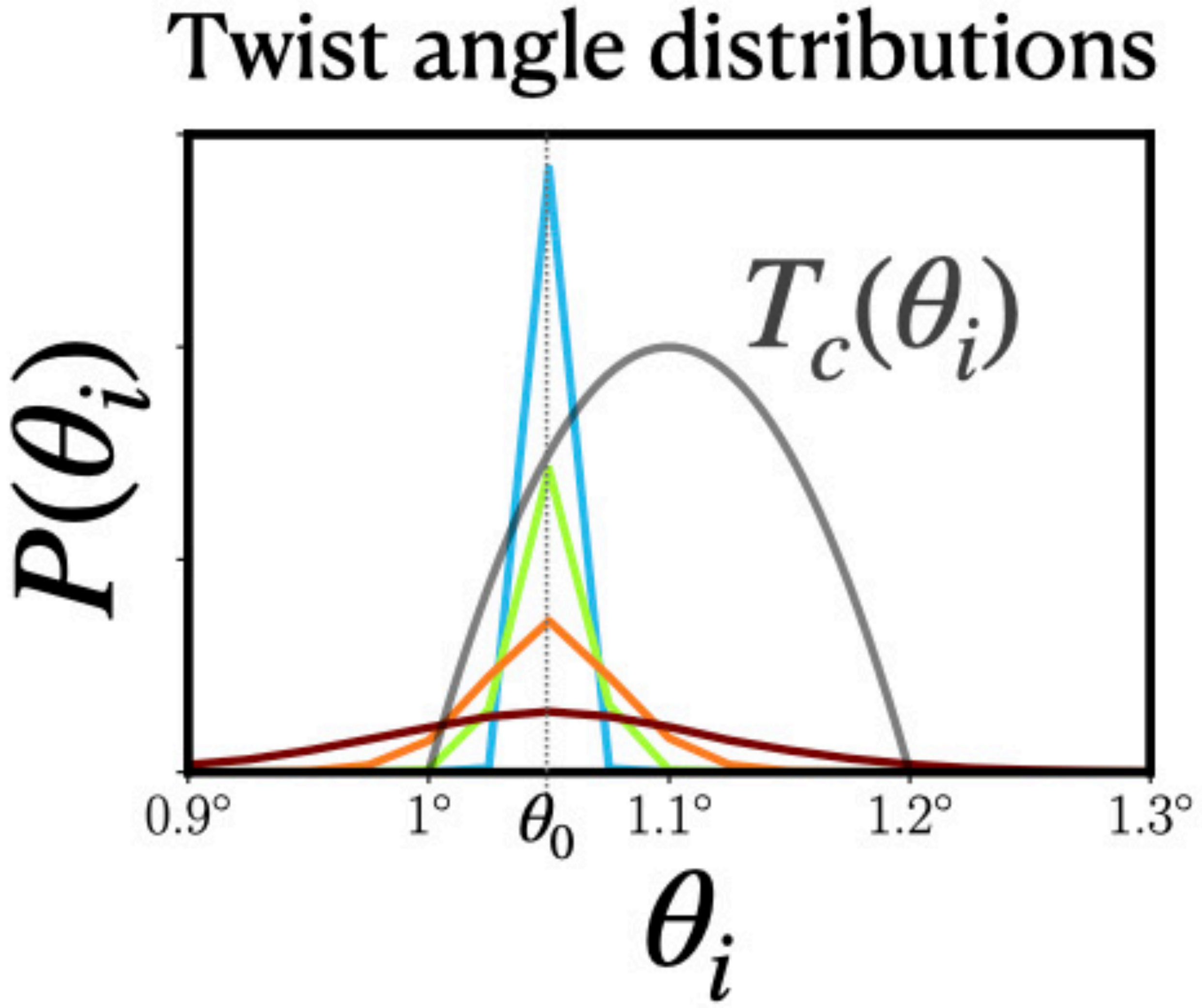
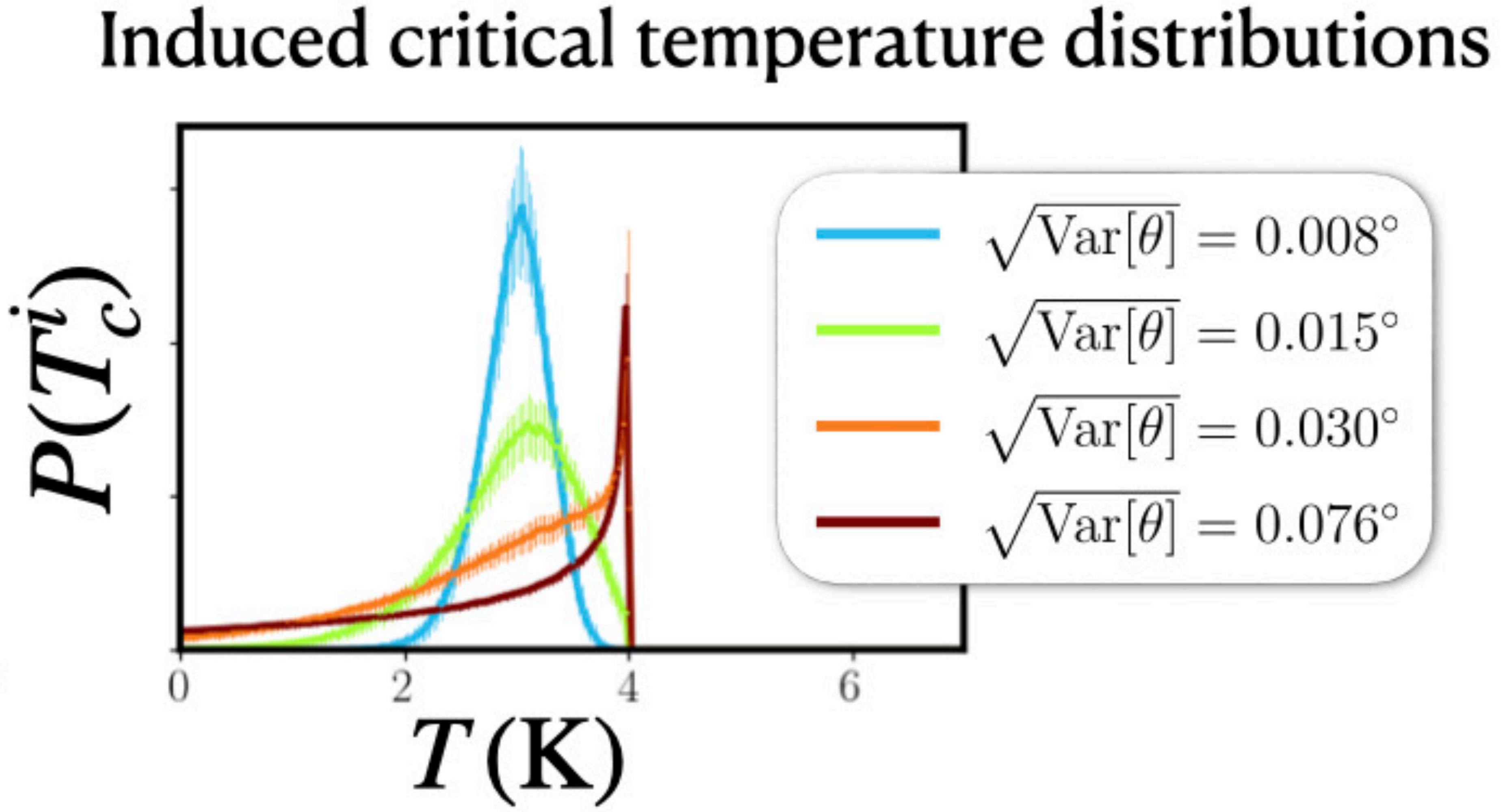
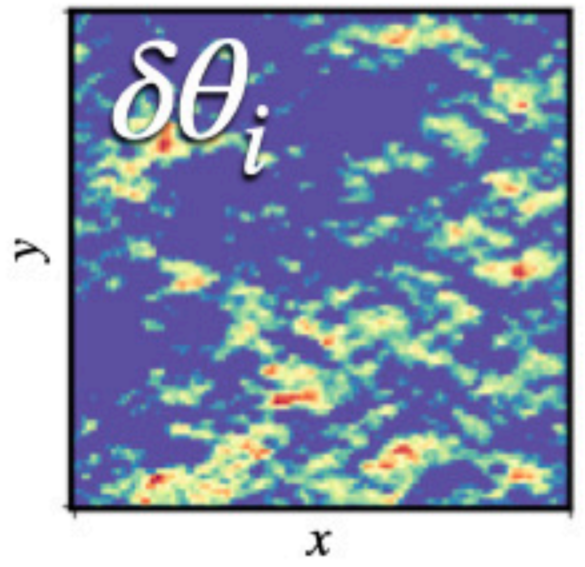
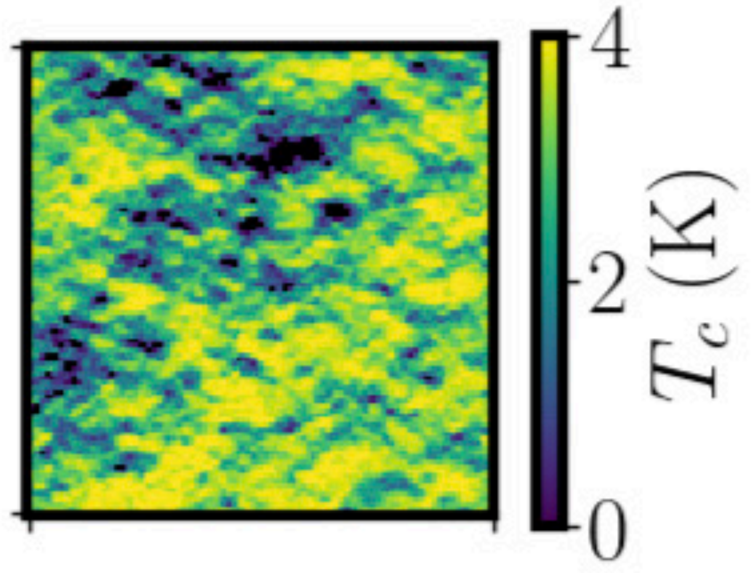
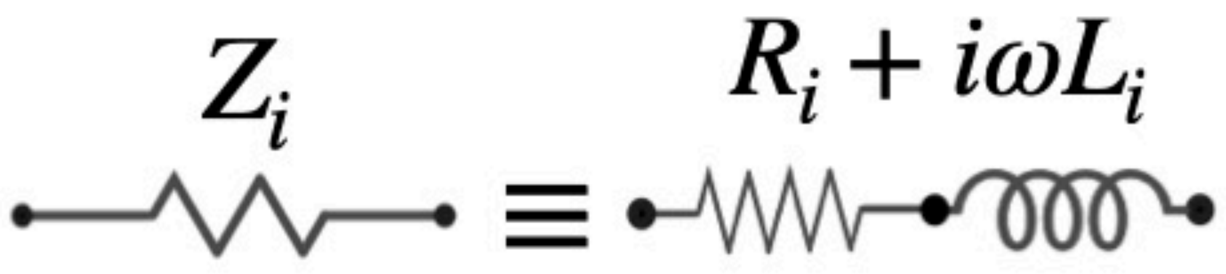












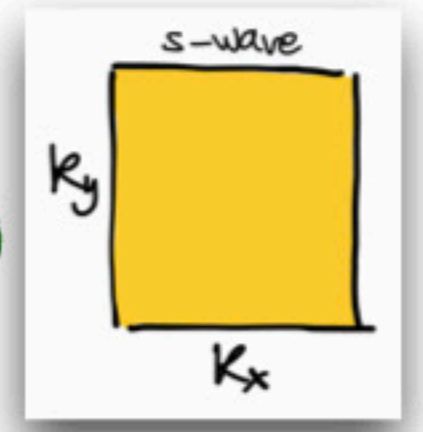
$$R_i = \begin{cases} R, T > T_c^i \\ 0, T \leq T_c^i \end{cases}$$

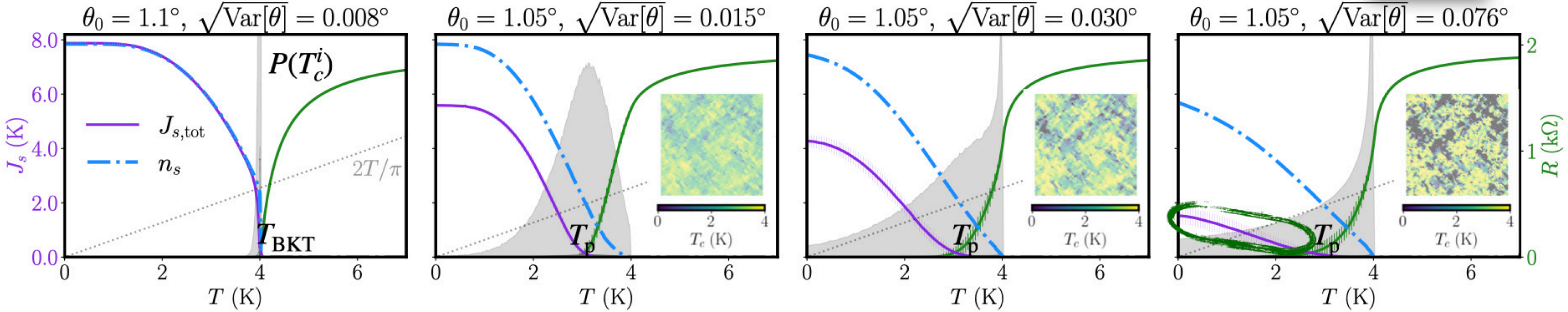
$R_i = R_N = 2\,\text{k}\Omega,\, L_i = L_0 \sim \text{nH}\,,\, \omega_0 = 2\,\text{GHz}$

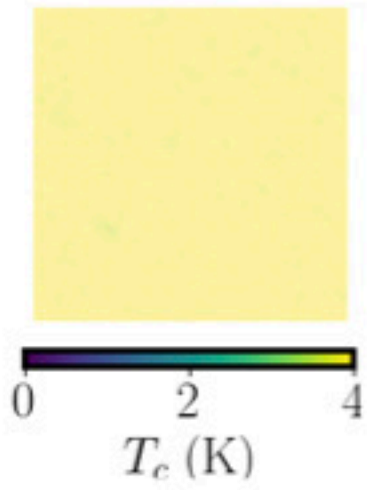
RIN results

Random Impedance Network: Results

Superfluid stiffness: $J_s(T,\omega_0) = \dfrac{\hbar^2\omega_0}{4e^2}\sigma_2(T,\omega_0)$ with local $J_{s,i}(T) \propto f(T,T_c^i)\begin{cases}\text{gapped}\\ \text{nodal}\end{cases}$







Exceptionally clean system with $\theta_0 \equiv \theta_{\mathrm{MA}}$ then BKT jump and SC order parameter can be observed

Inhomogeneities and correlations drive the system into a percolating transition

⟶ Superfluid density $n_s \propto \int_T^\infty P(T_c^i) f(T,T_c^i)\, dT_c^i$ and $J_s(T)$ are not proportional in inhomogeneous systems

⟶ Assuming s-wave, disorder can induce apparent "nodal-like" features

⟶ Key diagnostic observable: $\sigma_1(T,\omega_0)$!

Random Impedance Network: Results

$$\sigma(\omega_0, T) = \sigma_1(\omega_0, T) - i\sigma_2(\omega_0, T)$$

key diagnostic observable

it gives info on unpaired e^-

$\longrightarrow \sigma_1(\omega_0, T)$ "quantifies" $J_s \neq n_s$

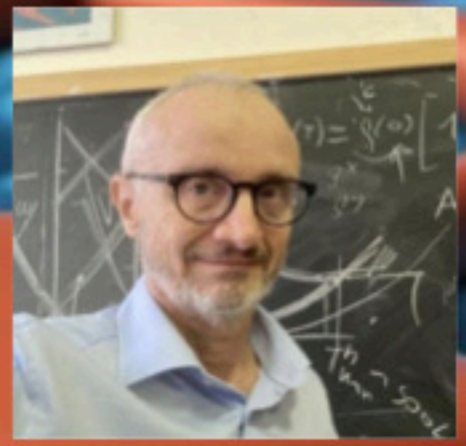
Effective medium solution $\neq$ correlated disorder $\longrightarrow \sigma_1(\omega_0, T)$ indirect probe for correlations

$\sigma_{1,\mathrm{em}}$ peaked at $T_{p,\mathrm{em}} : w_s(T_{p,\mathrm{em}}) = 0.5$

Acknowledgements

Theory:

Sergio Caprara
(Sapienza)



Marco Grilli
(Sapienza)

Ilaria Maccari
(ETH Zurich)

Lara Benfatto
(Sapienza)

Louk Rademaker
(Unige&Leiden)

RIN EM solution:
GV et al, PRB **100** 064506 (2019)
GV et al, Condens. Mat. **5(2)** (2020)
GV et al, Nanomat. **11(8)** (2021)
RIN exact solution:
GV et al, Scipost Phys. **15** 239 (2023)
CO+SC competition:
GV et al, Scipost Phys. **15** 230 (2023)
Twist angle inhomogeneities:
Maccari, Rademaker, and GV,
arXiv: 2606.17191 (2026)

Experiments:

ESPCI PARIS

Ramesh C. Budhani — MORGAN STATE UNIVERSITY
Gyanendra Singh — ICMAB INSTITUT DE CIÈNCIA DE MATERIALS DE BARCELONA
Alexis Jouan
Cheryl Feuillet-Palma
Jérôme Lesueur
Nicolas Bergeal

Swiss National Science Foundation

DQMP Department of Quantum Matter Physics

UNIVERSITÉ DE GENÈVE
FACULTÉ DES SCIENCES

Conclusions

- **RIN**: powerful tool to understand inhomogeneos systems
- **Twisted multilayer graphene:**
  - **Elastic solid:** long-range correlations
  - Crucial in macroscopic observables
- Superfluid **stiffness** $\not\equiv$ superfluid **density**
- Key observable is $\sigma_1(T)$!
- Compare transport and local probes

RIN EM solution:
GV et al, PRB **100** 064506 (2019)
GV et al, Condens. Mat. **5(2)** (2020)
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GV et al, Cond. Matt. **8(3)** 54 (2023)
GV et al, Scipost Phys. **15** 239 (2023)

Outlooks

⟶ Response in frequency?
⟶ **Non-linear effects** in the **RIN**?
⟶ Study **SIT** with RIN ?

Thank you!

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Back up

$$\frac{R}{R_N} \sim \left(\frac{\xi_0}{\xi}\right)^2 \sim e^{-2b/\sqrt{t}}$$

Works only at $T \to T^+_{BKT}$

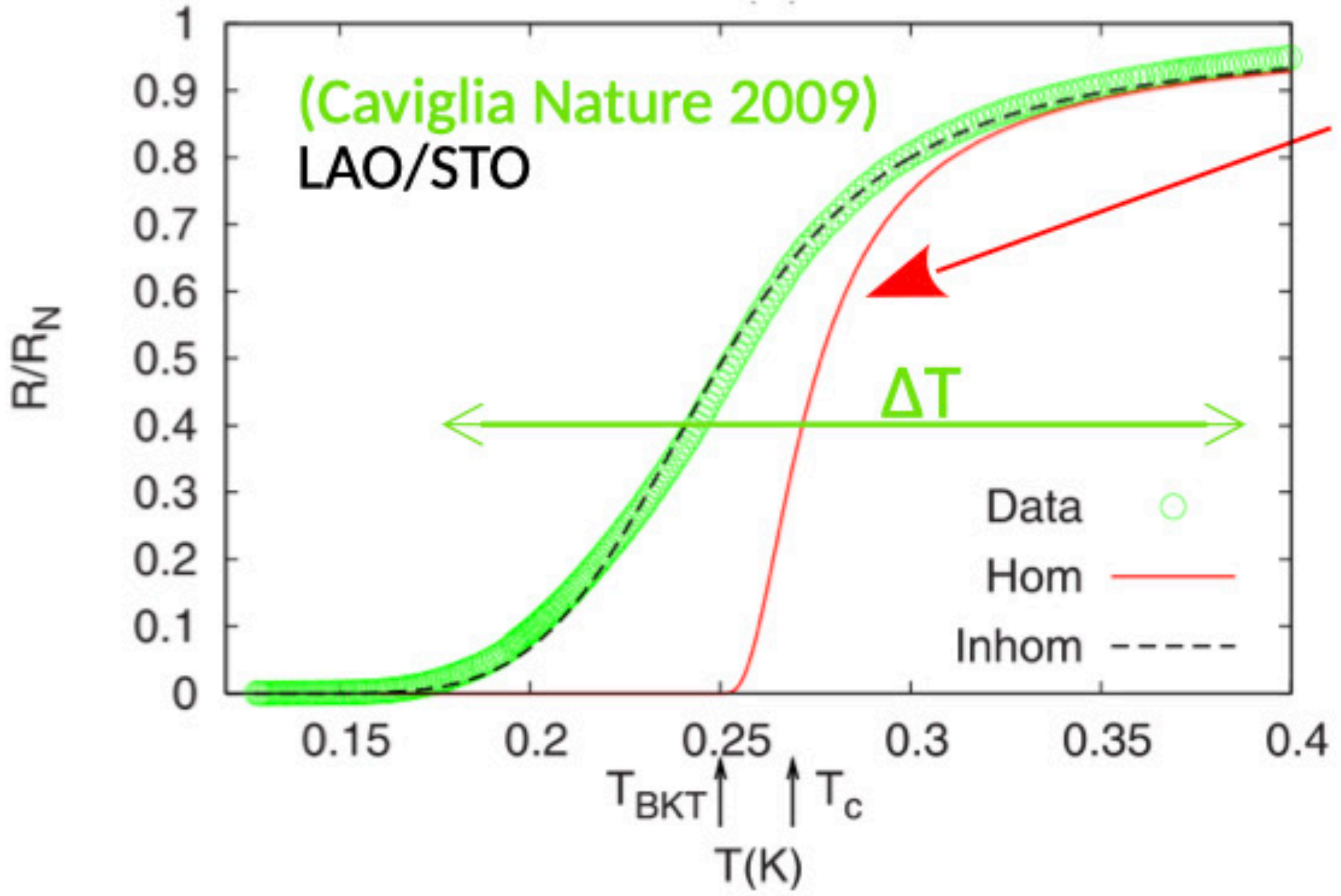
PHYSICAL REVIEW B **80**, 214506 (2009)

Broadening of the Berezinskii-Kosterlitz-Thouless superconducting transition by inhomogeneity and finite-size effects

L. Benfatto,[1,2] C. Castellani,[2] and T. Giamarchi[3]

(almost) Full R(T) range

$$\frac{R}{R_N} = \frac{1}{1+(\xi/\xi_0)^2}$$



BKT fluctuations

$$\frac{R}{R_N} = \frac{1}{1+\left(k\sinh\left(\frac{b}{\sqrt{t}}\right)\right)^2}$$

$$t = \frac{T - T_{BKT}}{T_{BKT}},$$

$$\boxed{b^2 k^2 = R_N/R_c}$$

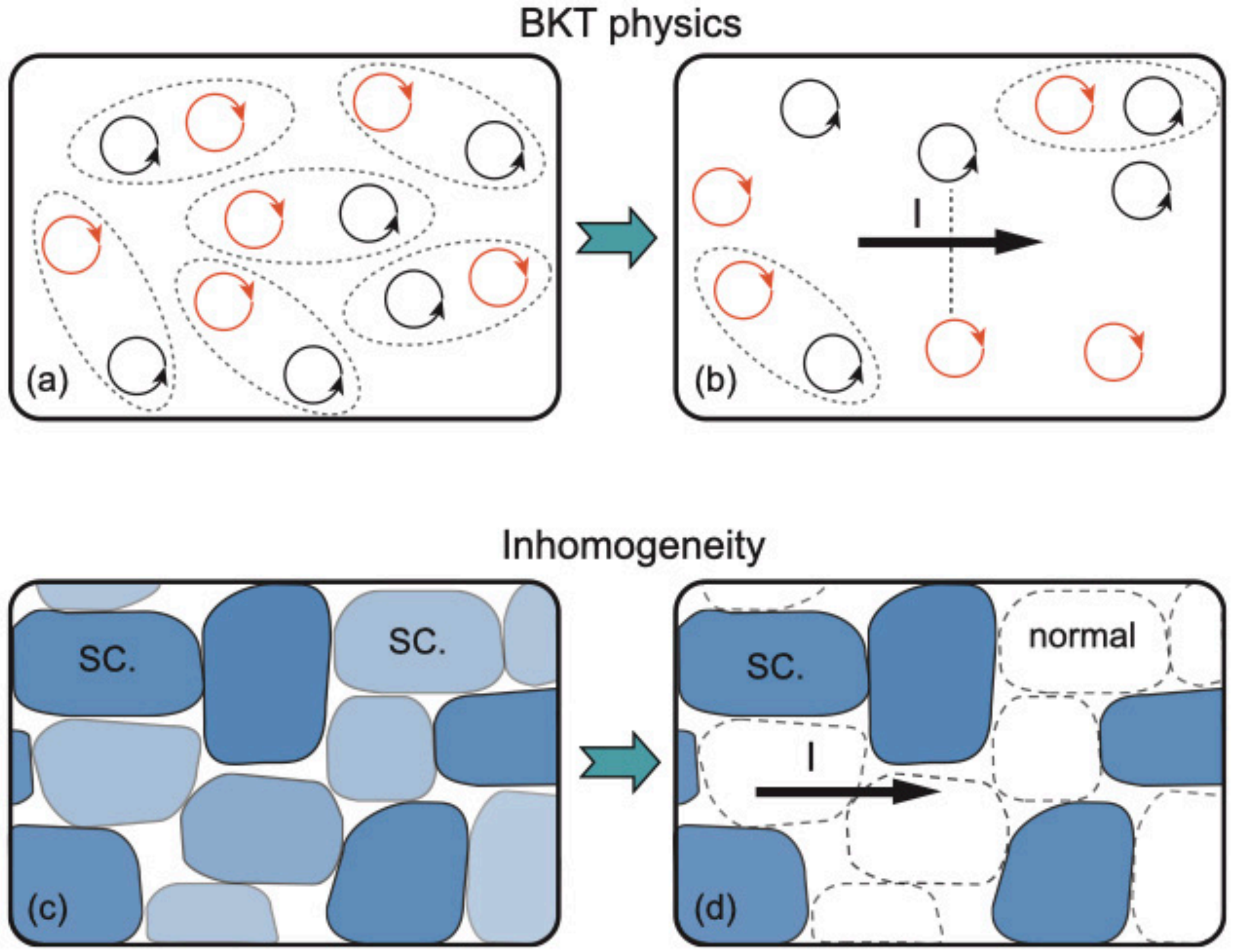
$$\underline{t_c \simeq 2.72 R_N/R_c \sim 0.01 \div 0.1}$$

Cooper pair fluct

$$R_c = 16\hbar^2/e^2 = 65.6\,\mathrm{k}\Omega$$

$$b \sim \frac{\mu}{J_S}\sqrt{t_c}$$

$$\underline{k \sim 1,\ b \sim 0.15}$$



Rule of thumb:

$$\frac{\Delta T}{T_c} \sim 1$$

not θ fluctuations...

$$\frac{\Delta T}{T_c} < 1$$

✓ but be careful

Finite size effects and **inhomogeneities**, **correlated disorder** can prevent the observation of BKT signatures

Landau-Lifshitz: Elasticity theory

Σ_{ik} stress tensor

u_{ik} strain tensor

Elastic solids

$$F_{ik} \equiv 0 = \frac{\partial \Sigma_{ik}}{\partial x_{ik}}$$ (no external forces)

With ext. forces

$$\frac{\partial \Sigma_{ik}}{\partial x_k} + \rho f_i = 0$$

ν: Poisson's ratio $-1 \leq \nu \leq \frac{1}{2}$

E: Young modulus

$$\Sigma_{ik} = \frac{E}{1+\nu}\left(u_{ik} + \frac{\nu}{1-2\nu} u_{ll}\delta_{ik}\right)$$

$\vec{u} = (u_x, u_y)$ displacement:

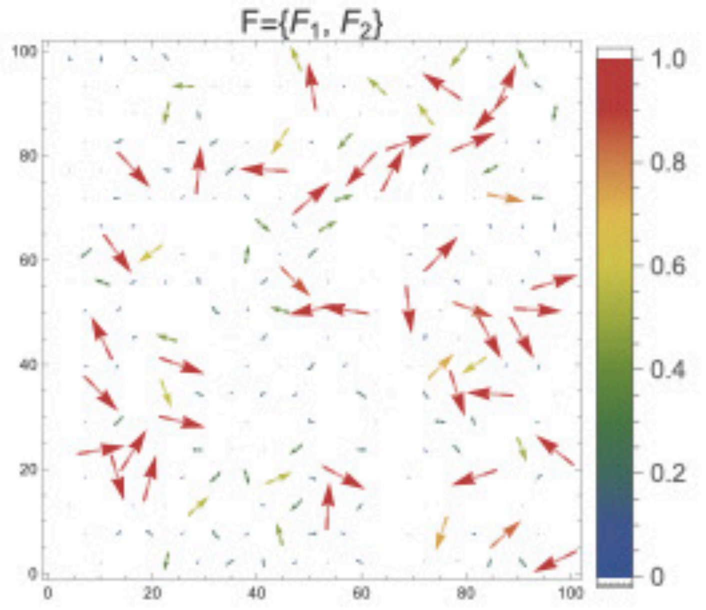
$$u_{ik} = \frac{1}{2}\left(\partial_k u_i + \partial_i u_k\right) = \frac{1}{2}\left(\partial_y u_x + \partial_x u_y\right)$$

$$\ldots\ \nabla^2 \vec{u} + \frac{1}{1-2\nu}\vec{\nabla}(\nabla \cdot \vec{u}) = -\frac{2(1+\nu)}{E}\rho \vec{f}$$

Landau-Lifshitz: Elasticity theory

$$\frac{\partial \Sigma_{ik}}{\partial x_k} + \rho f_i = 0$$

Stress tensor Σ , local forces $\mathbf{f}$ (impurities) in a medium of density ρ

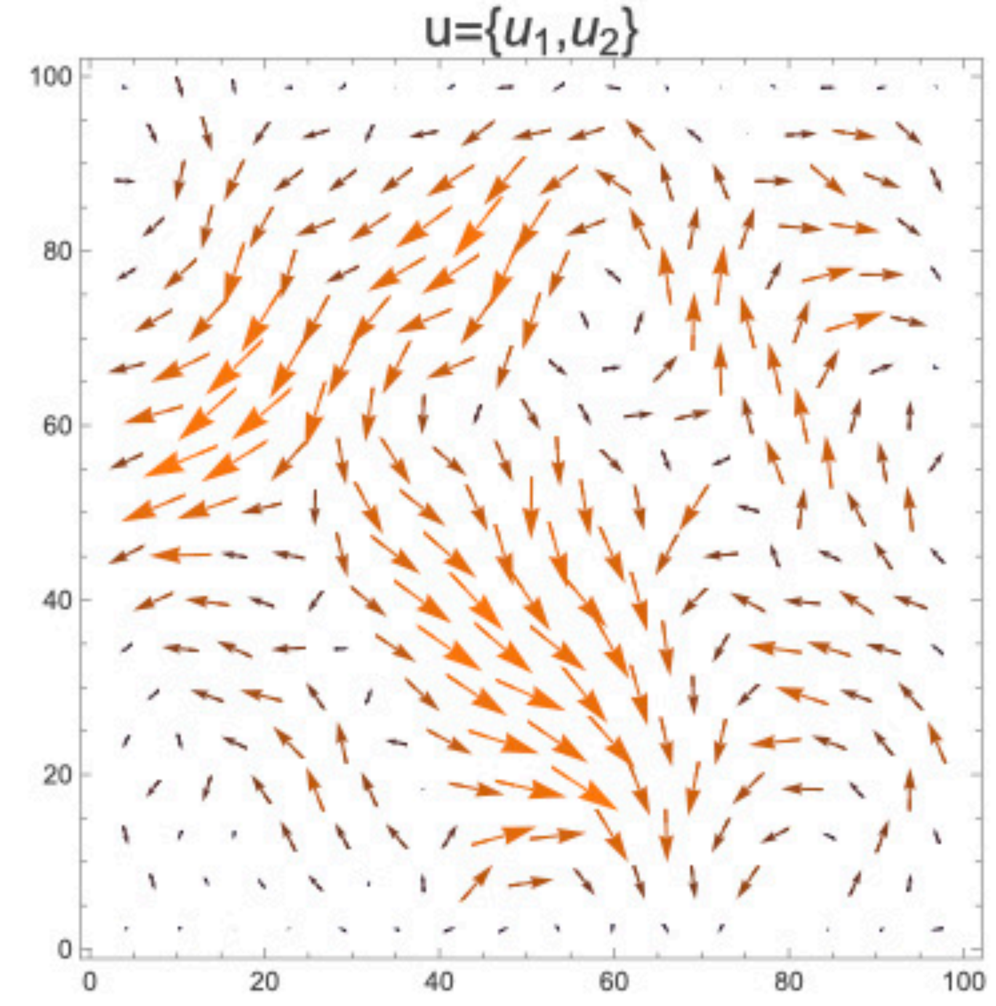


$$\Delta \mathbf{u} + \frac{1}{1-2\nu} \nabla(\nabla \cdot \mathbf{u}) = -\rho \mathbf{f} \frac{2(1+\nu)}{E}$$

Multilayer graphene:

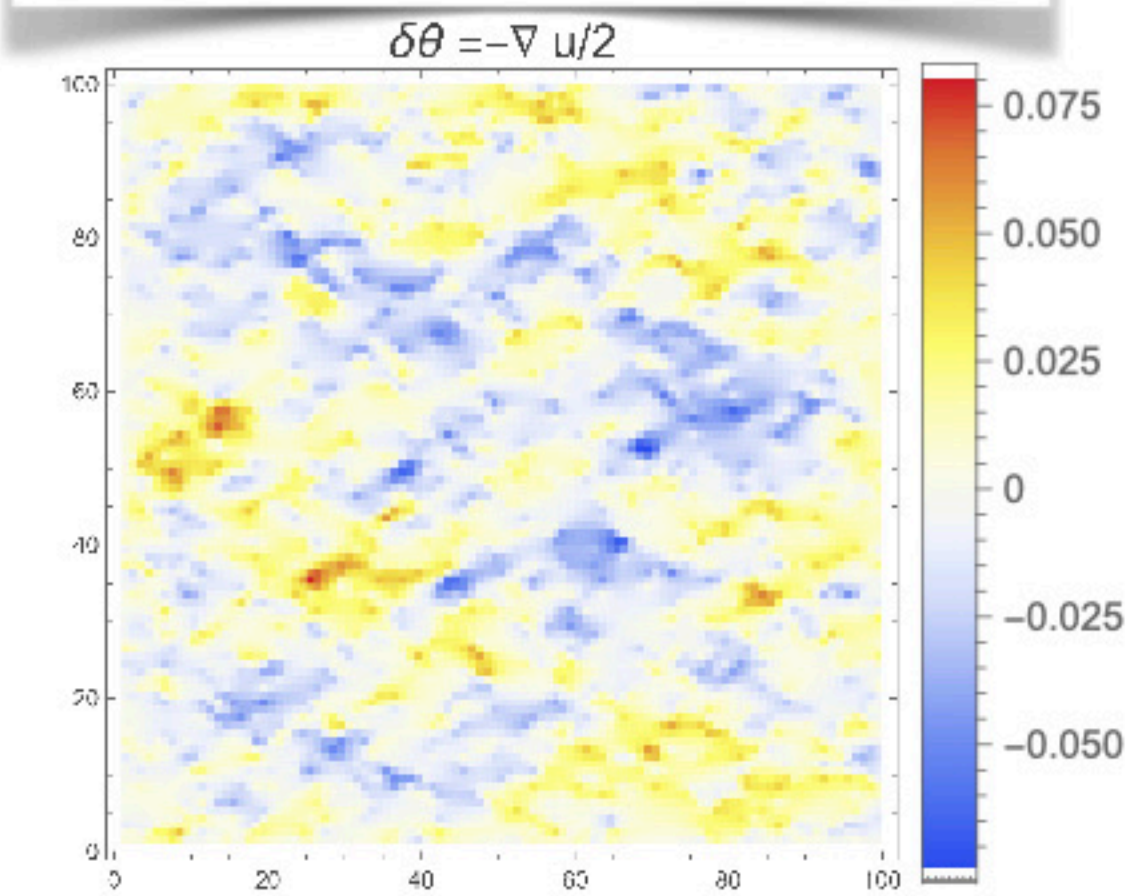
$\nu \approx 0.16 \div 0.2$ (Poisson's ratio)

$E \sim 0.5 \div 1$ TPa (Young modulus)



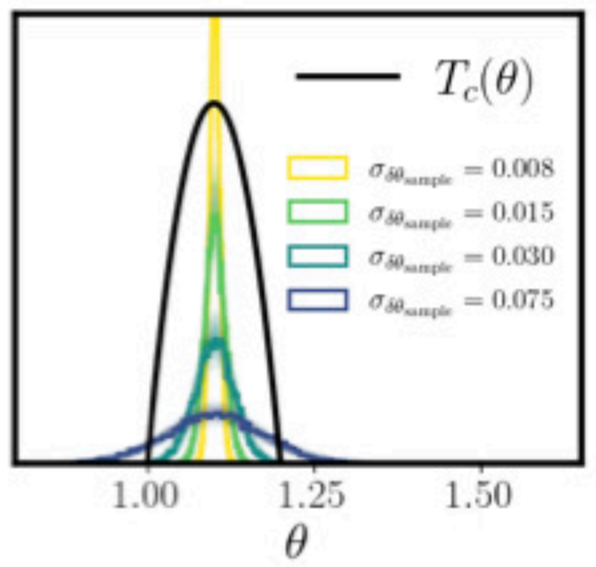
$\mathbf{u} = (u_x, u_y)$ displacement, ν Poisson's ratio, E Young modulus

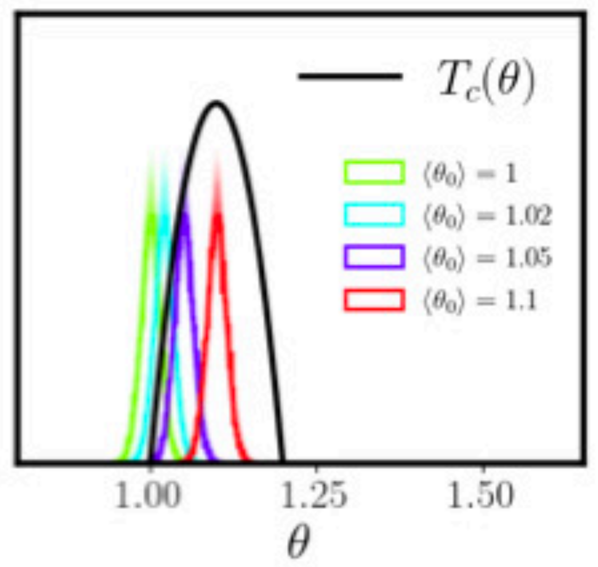
$$\theta(\mathbf{r}) \equiv -\frac{1}{2} \hat{z} \cdot (\nabla \times \mathbf{u})$$



$$C_\theta(\mathbf{r_1} - \mathbf{r_2}) = \langle \theta(\mathbf{r_1}) \theta(\mathbf{r_2}) \rangle$$

Map into $T_c(\theta) \rightarrow$ Transport through RIN



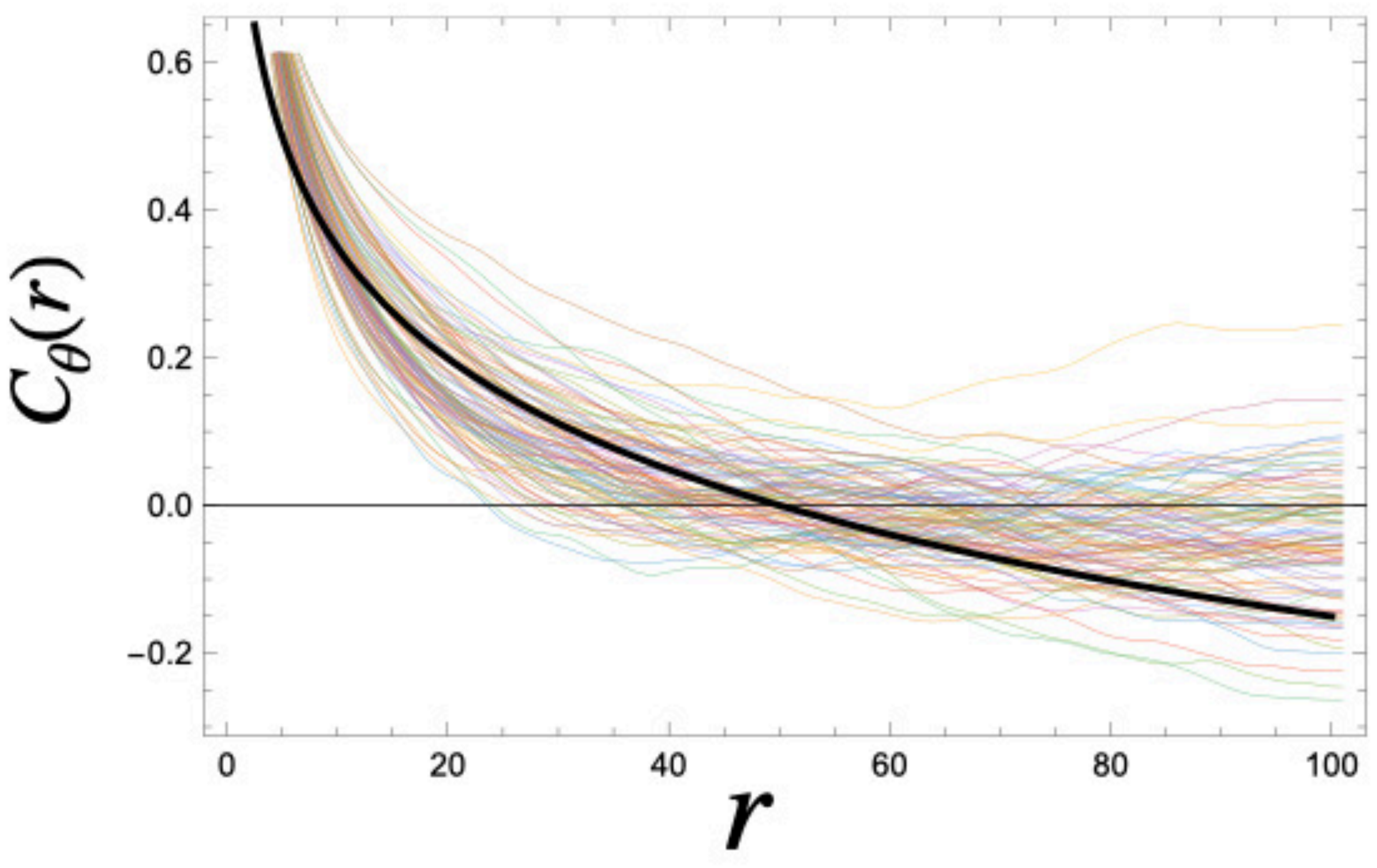


Twist angle correlations

$$\frac{\partial \Sigma_{ik}}{\partial x_k} + \rho f_i = 0$$

$$\Delta \mathbf{u} + \frac{1}{1-2\nu}\nabla(\nabla \cdot \mathbf{u}) = -\rho \mathbf{f}\frac{2(1+\nu)}{E}$$

$\mathbf{u} = (u_x, u_y)$ displacement, ν Poisson's ratio, E Young modulus



Continuum model

$$\hat{G}^{-1}(\mathbf{r})\mathbf{u}(\mathbf{r}) = \mathbf{F}(\mathbf{r}) \xrightarrow{\text{FT}} \mathbf{u}(\mathbf{k}) = \hat{G}(\mathbf{k})\mathbf{F}(\mathbf{k})$$

Compute $\hat{G}(\mathbf{r}) = \begin{pmatrix} G_{xx}(\mathbf{r}) & G_{xy}(\mathbf{r}) \\ G_{yx}(\mathbf{r}) & G_{yy}(\mathbf{r}) \end{pmatrix}$ and $\mathbf{u}(\mathbf{r})$

$$\theta(\mathbf{r}) \equiv -\frac{1}{2}\hat{z}\cdot(\nabla \times \mathbf{u})$$

$$C_\theta(\mathbf{r_1} - \mathbf{r_2}) = \langle \theta(\mathbf{r_1})\theta(\mathbf{r_2})\rangle$$

In the case of ***uncorrelated random forces***

$$\langle F_c(\vec{x}_1')F_d(\vec{x}_2')\rangle = F^2\delta_{cd}\delta(\vec{x}_1' - \vec{x}_2')$$

$$C_\theta(\mathbf{r_1} - \mathbf{r_2}) \sim -\log|\mathbf{r_1} - \mathbf{r_2}|$$