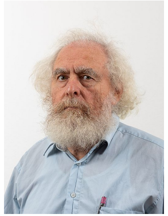
Bulk Localized Collective Excitations in Strongly Disordered Superconductors

The 17th of July, 2026

Anton Khvalyuk,

with: Thibault Charpentier, Mikhail Feigel'man, Nicolas Roch, and Benjamin Sacépé

Contents of this talk

01

Unusual phenomenology
Experimental motivation

02

Model Hamiltonian
Network model of electrodynamics

03

Temperature and frequency dependence of electromagnetic response

04

Phase diagram near SIT
Conclusions and Outlook

A bulk superconductor* = ?

*in equilibrium at given T

… for a physicist:

- Zero DC resistance
- Meissner effect
- Flux quantization

M. Tinkham, *Introduction to Superconductivity*

A bulk superconductor* = London equations

*in equilibrium at given T

… for a physicist:

- Zero DC resistance
- Meissner effect
- Flux quantization

… in equations:

Vector potential

$$\boldsymbol{j} = \frac{c}{4\pi\lambda_L^2}\left(\frac{\hbar c}{2e}\nabla\varphi - \boldsymbol{A}\right)$$

Current density

Superconducting phase

London penetration depth

M. Tinkham, *Introduction to Superconductivity*

A bulk superconductor* = London equations

*in equilibrium at given T

… for a physicist:

- Zero DC resistance
- Meissner effect
- Flux quantization

- **Spectral gap**
- Exponential dependence on temperature
- Anderson theorem
- …

… in equations:

$$\boldsymbol{j} = \frac{c}{4\pi\lambda_L^2}\left(\frac{\hbar c}{2e}\nabla\varphi - \boldsymbol{A}\right)$$

Semiclassical (BCS) theory

M. Tinkham, *Introduction to Superconductivity*

A bulk superconductor* = London equations

*in equilibrium at given T

$$\boldsymbol{j} = \frac{c}{4\pi\lambda_L^2}\left(\frac{\hbar c}{2e}\nabla\varphi - \boldsymbol{A}\right)$$

applies to *all* bulk superconductors

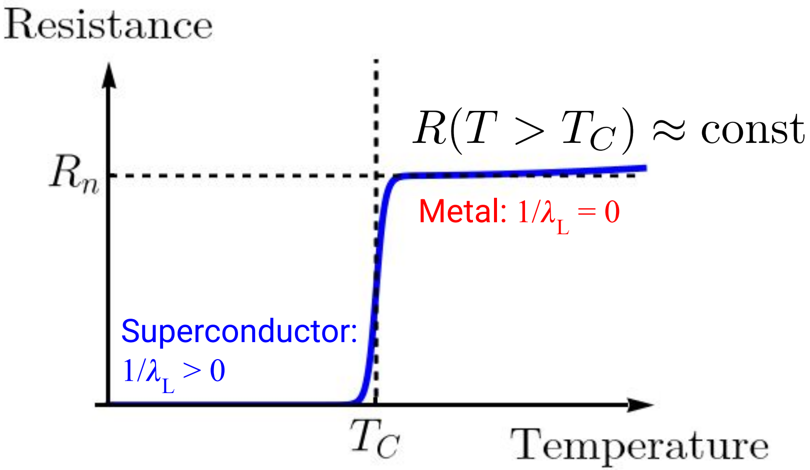
Semiclassical (BCS) theory

applies only to *moderately* disordered superconductors

M. Tinkham, *Introduction to Superconductivity*

Semiclassical theory of superconductors*

*s-wave



A.A. Abrikosov , L.P. Gor'kov, I.E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*
A. V. Svidzinsky, *Spatially Non-Uniform Problems in the Theory of Superconductivity*
P.-G. De Gennes, *Superconductivity of metals and alloys*

Semiclassical theory of superconductors*

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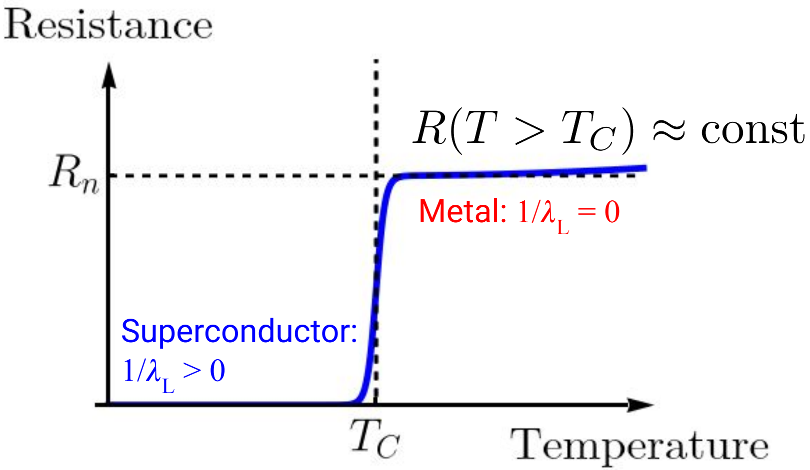

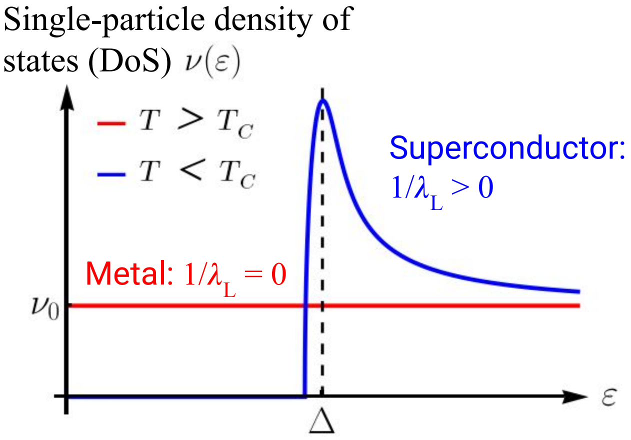

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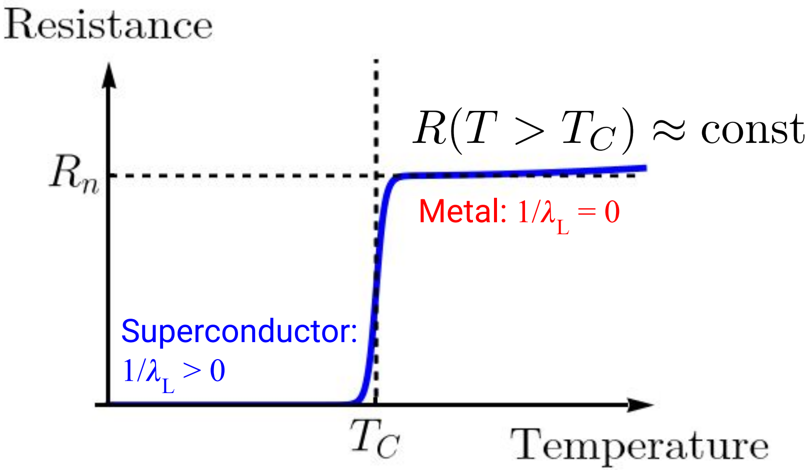

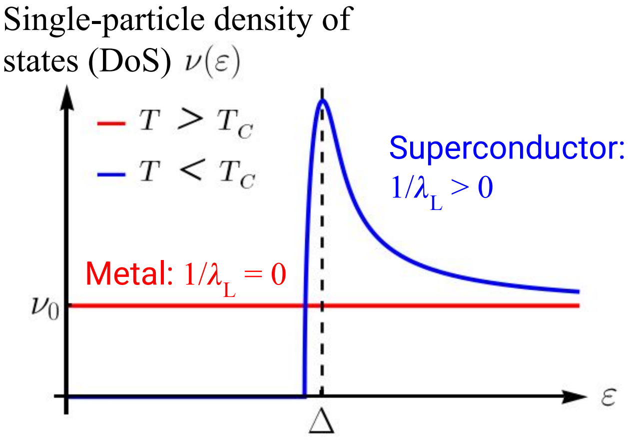

Exponential temperature dependencies:

$$4\pi\lambda_L^2/d = \frac{\hbar c^2}{\pi\Delta} R_\square \coth\frac{\Delta}{2T} \propto 1 + 2\exp\left\{-\frac{\Delta}{T}\right\} \quad (T \ll \Delta)$$

Normal state sheet resistance

A.A. Abrikosov , L.P. Gor'kov, I.E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*
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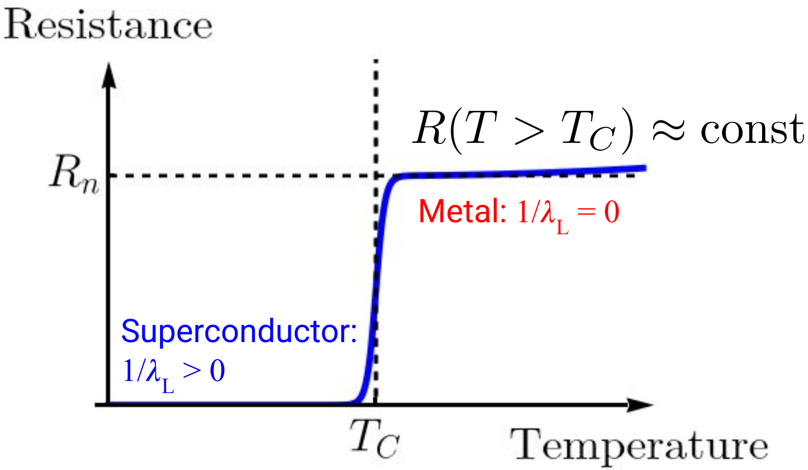
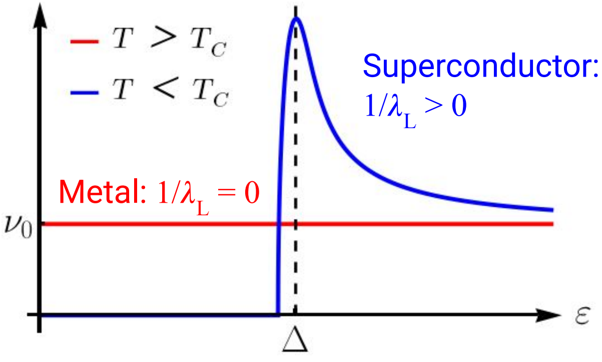


Single-particle density of states (DoS) $\nu(\varepsilon)$



Fermi wave number

Applicable if $k_F l \gg 1,\ k_F \xi_0 \gg 1 \quad \left(\xi_0 \sim \sqrt{\hbar D/\Delta}\right)$

mean free path

coherence length

Exponential temperature dependencies:

$$4\pi\lambda_L^2/d = \frac{\hbar c^2}{\pi\Delta} R_\square \coth\frac{\Delta}{2T} \propto 1 + 2\exp\left\{-\frac{\Delta}{T}\right\} \quad (T \ll \Delta)$$

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$\cancel{k_F l \gg 1}$

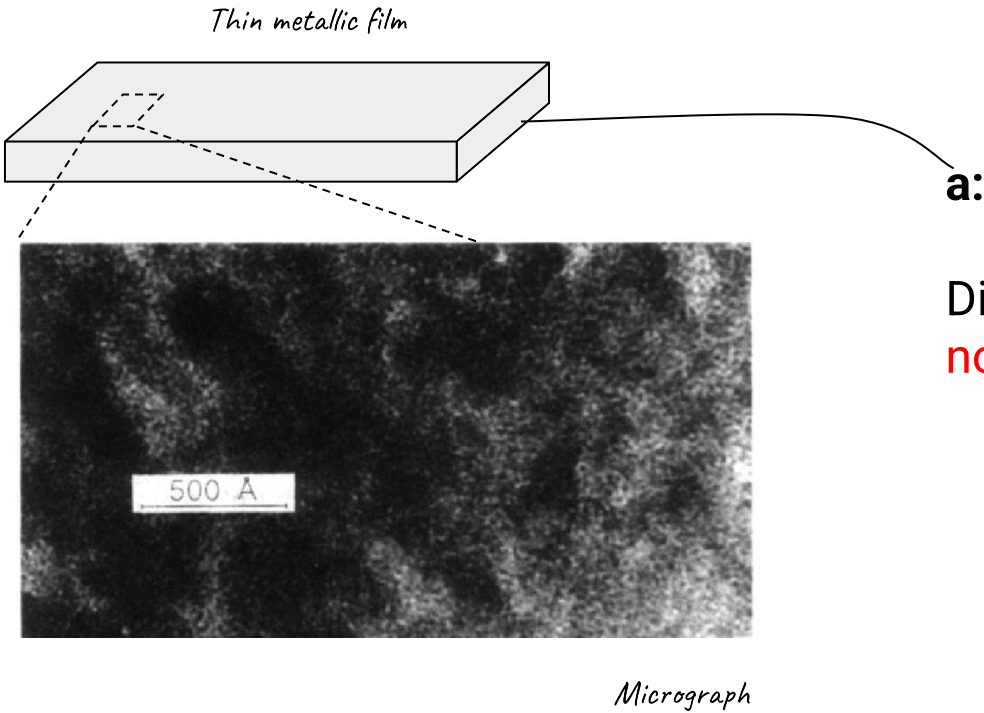
What happens to λ_{L} at strong disorder?

Grows with disorder

???

$$4\pi\lambda_L^2/d = \frac{\hbar c^2}{\pi\Delta} R_{\Box} \coth\frac{\Delta}{2T} \propto 1 + 2\exp\left\{-\frac{\Delta}{T}\right\} \quad (T \ll \Delta)$$

Films of amorphous Indium oxide (a:InO)



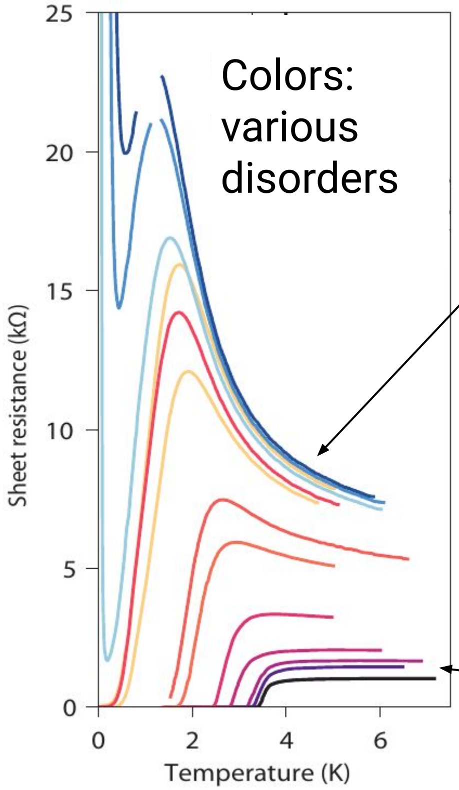
a:InO

Disorder: atomically homogeneous, no grains!

(oxygen vacancies)

D. Shahar and Z. Ovadyahu, PRB 1992; Z. Ovadyahu et al, J. Electrochem. Soc. 1983

Sheet resistance of a:InO films

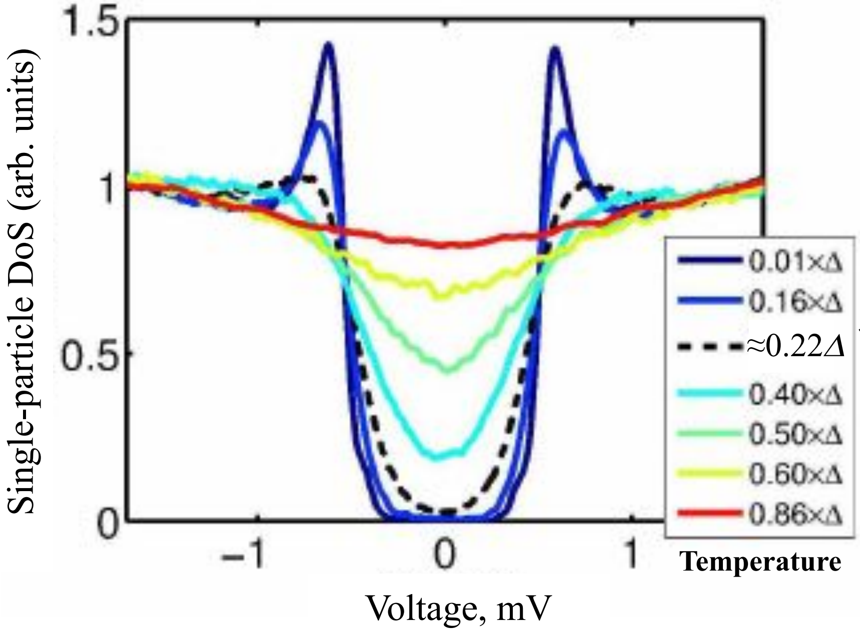


Strong disorder:
Not a good metal at $T > T_C$
Semiclassics not applicable

Weak disorder:
Metallic at $T > T_C$

T. Charpentier. PhD thesis, Université Grenoble Alpes, 2023.

Single-particle DoS for various *T*



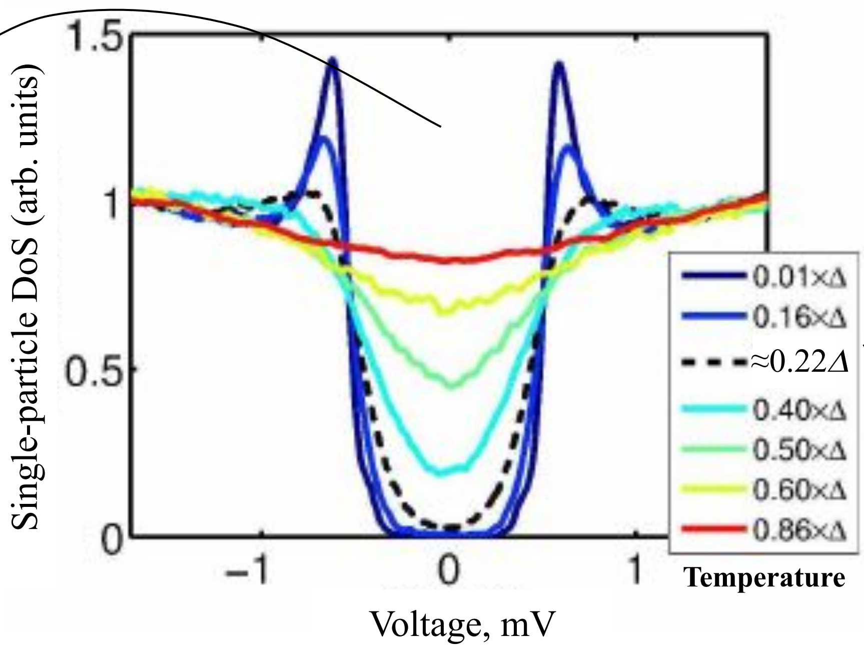
T_C, determined from resistance

B. Sacépé et al, Nat. Phys. 2011

Single-particle DoS for various T

Hard gap
at and above T_C

Cooper pairs form above T_C
Gap is not an order parameter



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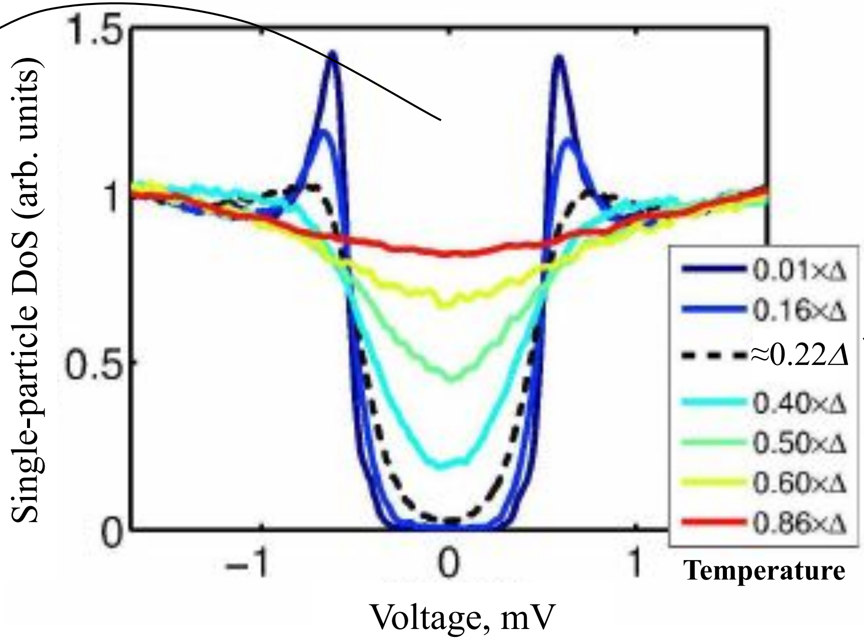

T_C, determined from resistance

$$\Delta/T_C \sim 4.5$$

Pair-breaking excitations are irrelevant at low T

B. Sacépé et al, Nat. Phys. 2011

Single-particle DoS for various T

Hard gap
at and above T_C

Cooper pairs form above T_C
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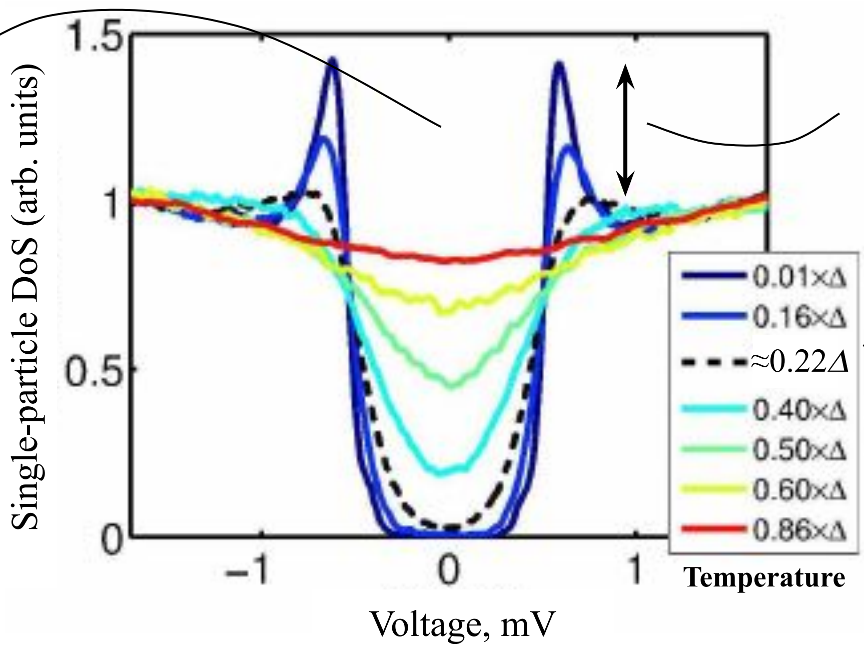


Coherence peak height
appears only below T_C

~Local order parameter

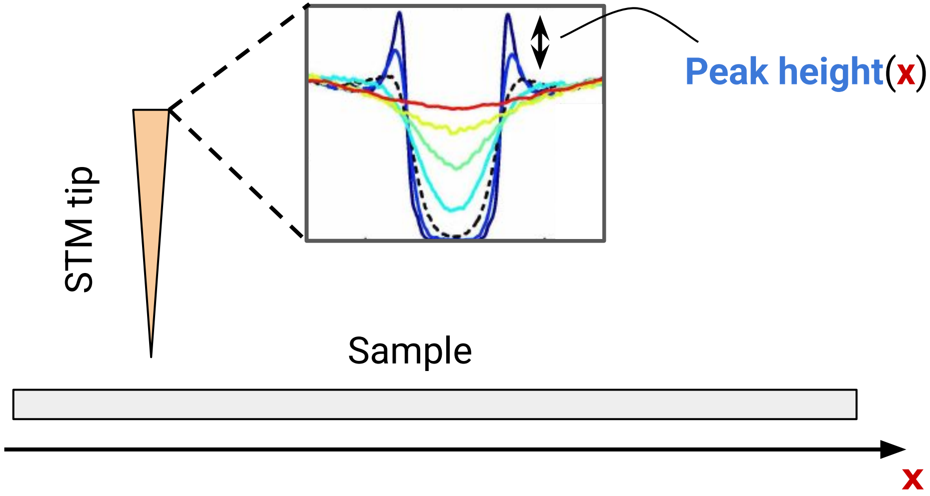
T_C, determined from resistance

$$\Delta/T_C \sim 4.5$$

Pair-breaking excitations are irrelevant at low T

B. Sacépé et al, Nat. Phys. 2011

Statistics of spatially resolved DoS features



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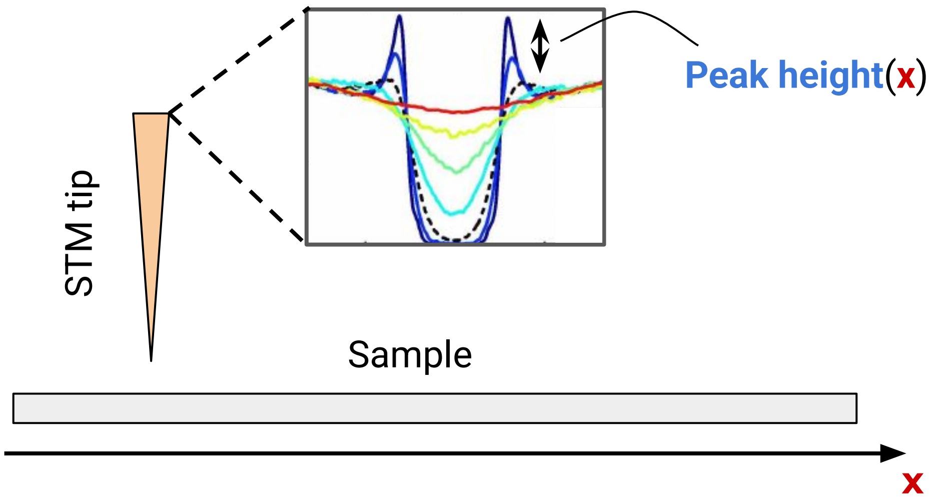
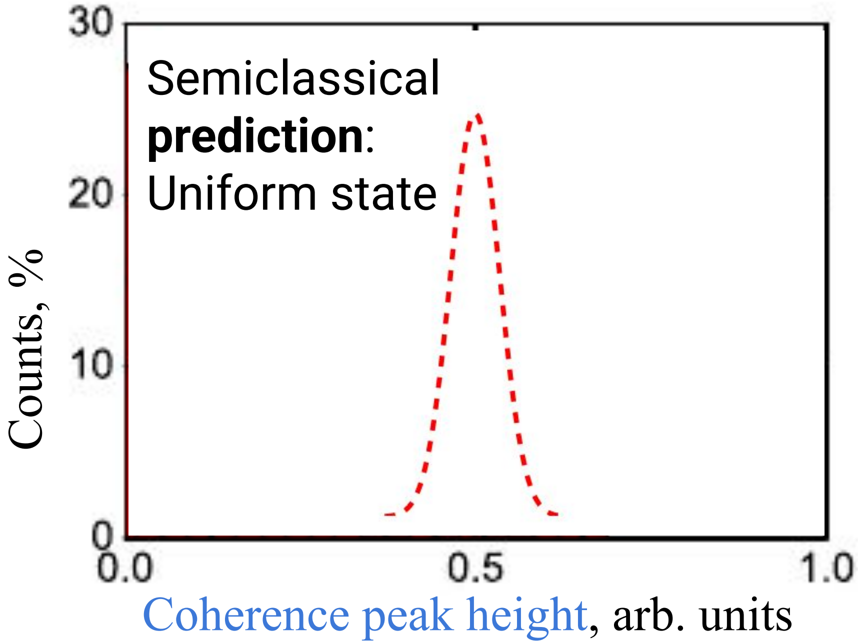
Statistics of spatially resolved DoS features





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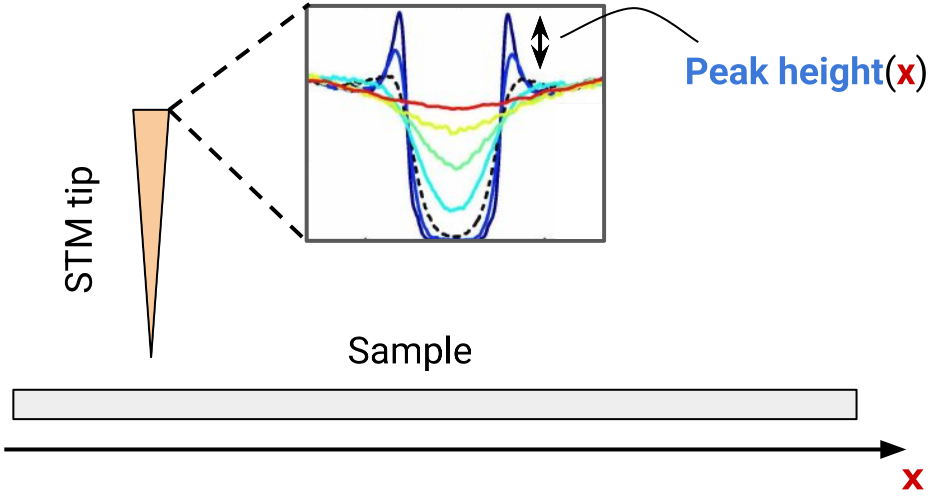
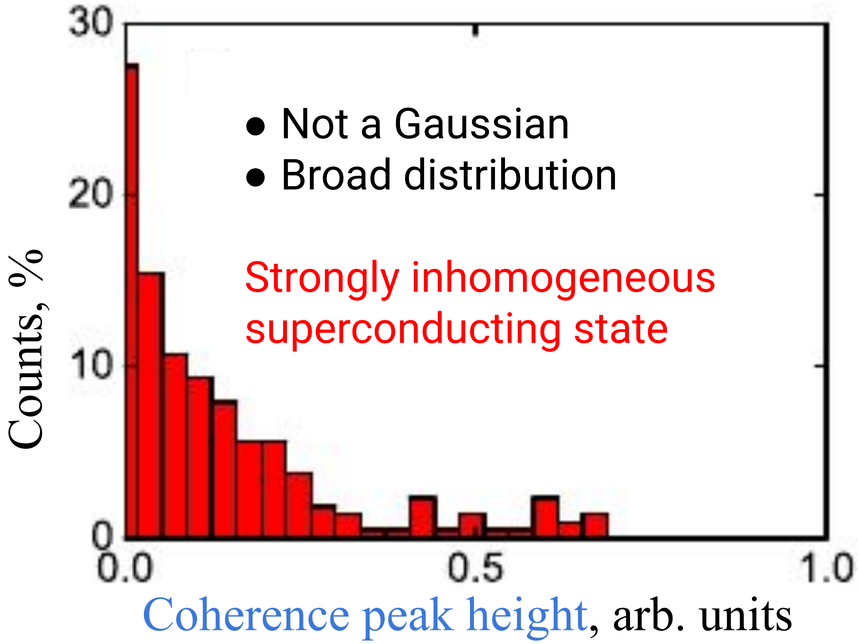




- Not a Gaussian
- Broad distribution

Strongly inhomogeneous superconducting state

B. Sacépé et al, Nat. Phys. 2011

a:InO needs a different microscopic theory

- Zero DC resistance
- Meissner effect
- Flux quantization

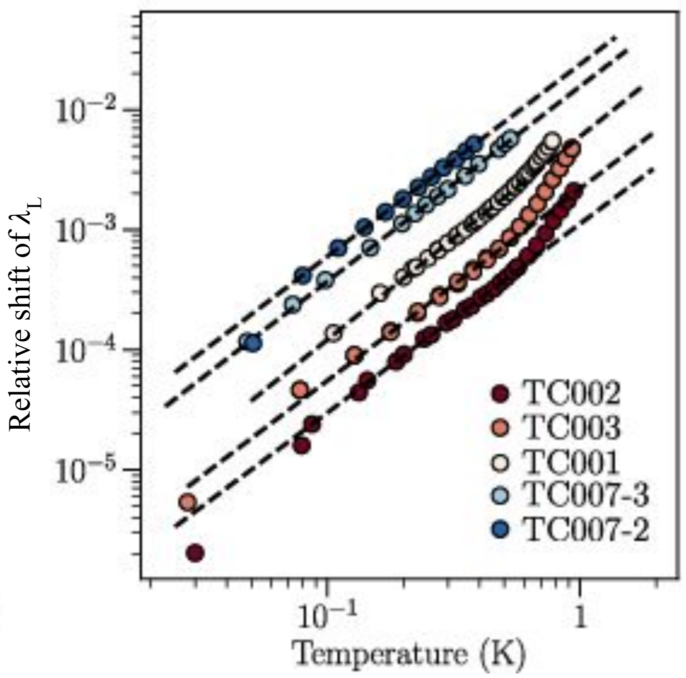
$$\boldsymbol{j} = \frac{c}{4\pi\lambda_L^2}\left(\frac{\hbar c}{2e}\nabla\varphi - \boldsymbol{A}\right)$$

The only reliable signature of superconductivity.
Need to measure that!

- ~~Spectral gap~~
- ~~Exponential dependence on temperature~~
- Anderson theorem
- …

~~**Semiclassical (BCS) theory**~~

T-dependence of λ_L



Dashed lines: **power law**

$$\frac{\lambda_L^2(T)}{\lambda_L^2(0)} - 1 = \left(\frac{T}{T_0}\right)^b$$

The effect is **pronounced**:

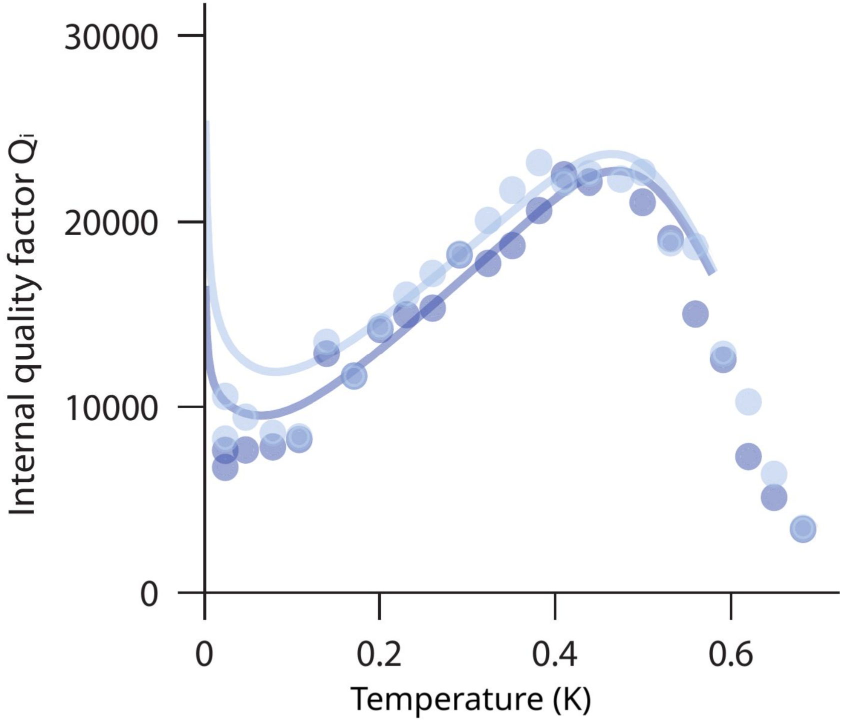
$$b \sim 1.6, \quad T_0 \sim 10^1 - 10^2\,\mathrm{K}$$

$$\cancel{\exp\left\{-\frac{\Delta}{T}\right\} \quad (T \ll \Delta)}$$

Pair breaking (BCS), collective modes,
surface contaminants, etc.
cannot explain this

T. Charpentier et al, Nat. Phys. 2025; AVK et al, PRB 2024

T-dependence of dissipation



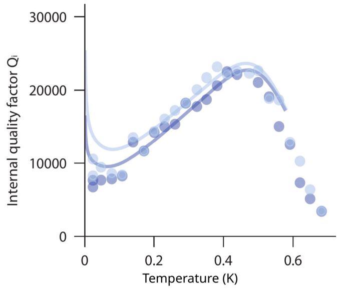
$Q \propto 1/\sigma_1(T)$ **grows** with *T*

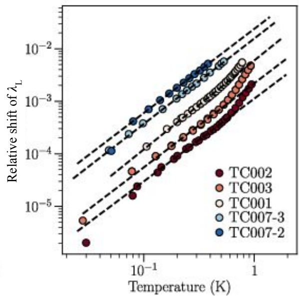
$\cancel{\exp\left\{-\frac{\Delta}{T}\right\} \quad (T \ll \Delta)}$

Pair breaking (BCS), collective modes, surface contaminants, etc. cannot explain this

T. Charpentier et al, Nat. Phys. 2025

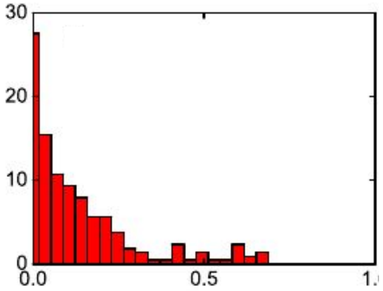
Q: What physical mechanism causes this?





Low-energy theory

A: inhomogeneity of the superconducting state.



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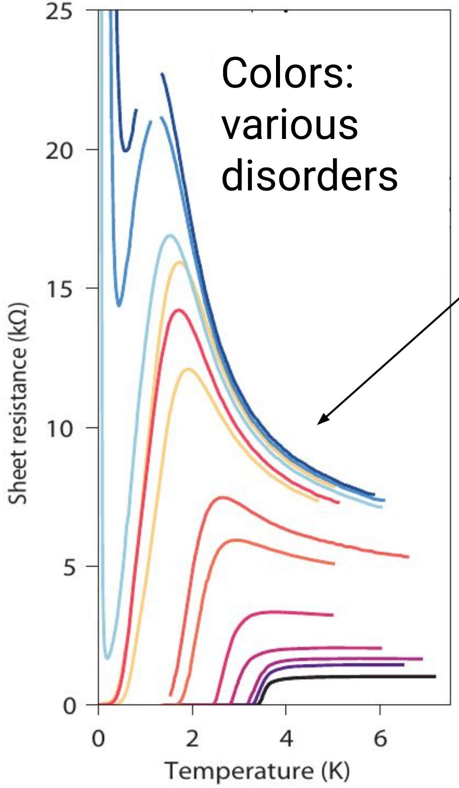
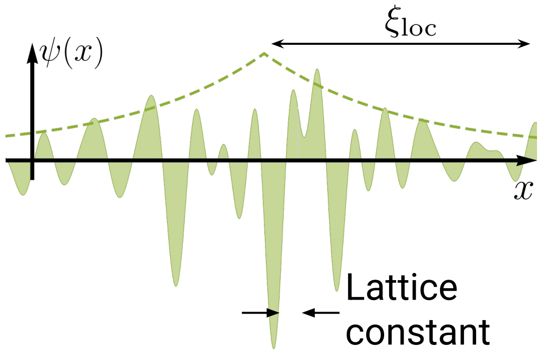
Localized superconductors

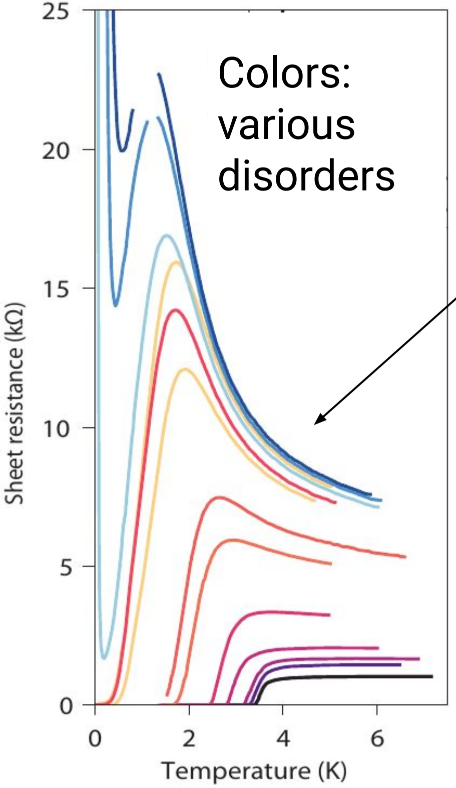


Large disorder:
Anderson insulator with large ξ_{loc}

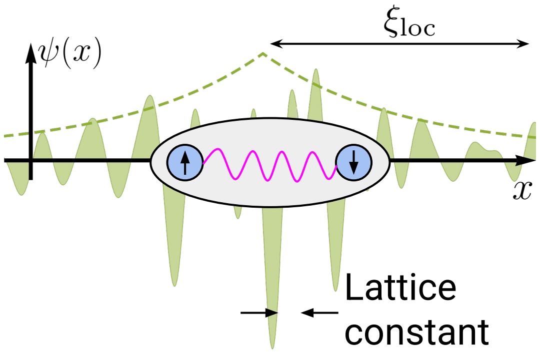


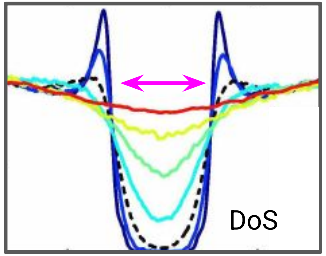
M.V. Feigel'man et al, Ann. Phys. 2010; B. Sacépé et al, Nat. Phys. 2020

Localized superconductors



Large disorder:
Anderson insulator with large ξ_{loc}
and preformed Cooper pairs





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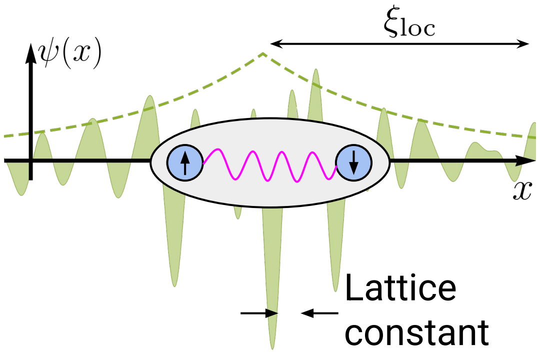
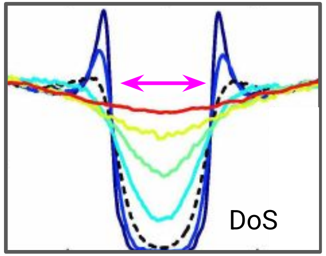
Localized superconductors

$$H = \textbf{?}$$





Localized Cooper pairs

$$H = ?$$

No quasiparticles:

$$\sum_{\sigma=\uparrow,\downarrow} a^{\dagger}_{i\sigma} a_{i\sigma} \, |\text{state}\rangle = \{0 \text{ or } 2\} \, |\text{state}\rangle$$

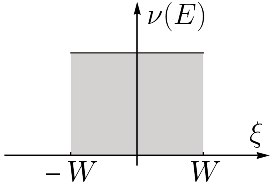
Ma and Lee, PRB 1985; M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel’man and L.B. Ioffe, PRL 2018

Localized Cooper pairs

$$H = - \sum_{i,\sigma=\downarrow,\uparrow} \xi_i a^\dagger_{i\sigma} a_{i\sigma}$$

Index of localized
single-particle state

Random energies,
DoS $\nu(\xi)$, scale *W*



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Hopping of localized Cooper pairs

$$H = -\sum_{i,\sigma=\downarrow,\uparrow} \xi_i a^\dagger_{i\sigma} a_{i\sigma} - \sum_{i\neq j} \left(D_{ij}\, a^\dagger_{i\downarrow} a^\dagger_{i\uparrow} a_{j\uparrow} a_{j\downarrow} + \text{h.c.} \right)$$

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Structure of the matrix elements?

No quasiparticles:

$$\sum_{\sigma=\uparrow,\downarrow} a^\dagger_{i\sigma} a_{i\sigma}\ |\mathrm{state}\rangle = \{0 \text{ or } 2\}\ |\mathrm{state}\rangle$$

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Energy dependence at the scale of ω_D

Wave function overlap

$$D_{ij} \approx \underbrace{\mathcal{D}(\xi_i - \xi_j; 0)}_{\text{Phonon-induced attraction}} \int d^3\boldsymbol{r}\, \bar{\psi}_i^2(\boldsymbol{r}) \psi_j^2(\boldsymbol{r})$$

$$T_c \ll \left(\nu_0 \xi_{\text{loc}}^3\right)^{-1} \ll \omega_D$$

Large but finite localization volume

No quasiparticles:

$$\sum_{\sigma=\uparrow,\downarrow} a^\dagger_{i\sigma} a_{i\sigma} \,|\text{state}\rangle = \{0 \text{ or } 2\}\, |\text{state}\rangle$$

M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel’man and L.B. Ioffe, PRL 2018

Matrix elements in an Anderson insulator

$$D_{ij} \approx \mathcal{D}(\xi_i - \xi_j; 0) \int d^3\boldsymbol{r}\, \bar{\psi}_i^2(\boldsymbol{r})\psi_j^2(\boldsymbol{r})$$

M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel’man and L.B. Ioffe, PRL 2018; B.Sacépé et al, Nat. Phys. 2020

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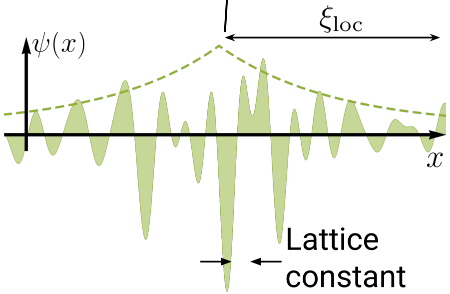
$$D_{ij} \approx \mathcal{D}(\xi_i - \xi_j; 0) \int d^3\boldsymbol{r}\, \bar{\psi}_i^2(\boldsymbol{r}) \psi_j^2(\boldsymbol{r})$$

- Decay with distance
- Fluctuate strongly



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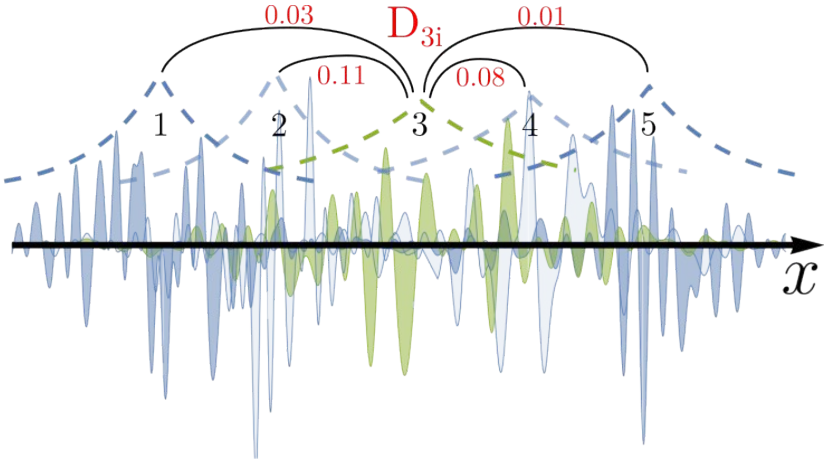


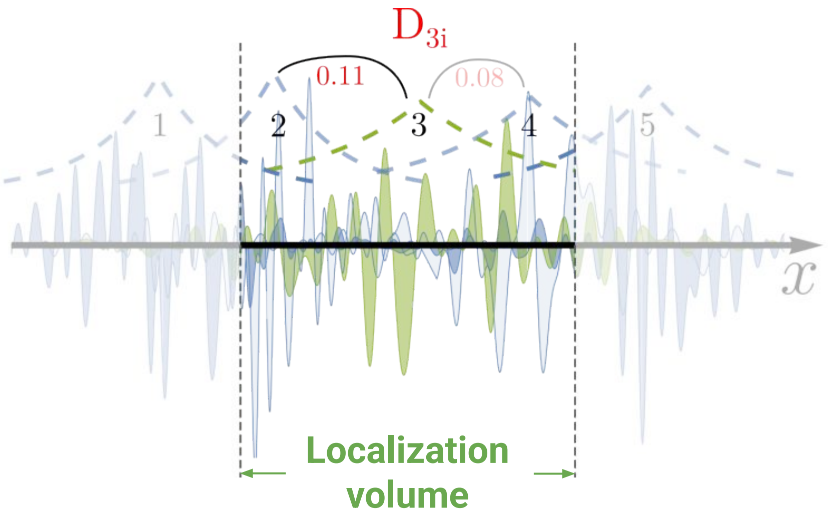
Single-particle wave functions

$$D_{ij} \approx \mathcal{D}(\xi_i - \xi_j; 0) \int d^3\boldsymbol{r}\, \bar{\psi}_i^2(\boldsymbol{r})\psi_j^2(\boldsymbol{r})$$

- Decay with distance (between *i, j*)
- Fluctuate strongly

M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel’man and L.B. Ioffe, PRL 2018; B.Sacépé et al, Nat. Phys. 2020

Matrix elements inside localization volume with binary distribution



$$D_{ij} = \begin{cases} E_F \frac{\lambda}{K}, & \text{"strong"}, \\ 0, & \text{"weak"} \end{cases}$$

Dimensionless Cooper coupling constant

- Neighbors chosen randomly in the localization volume
- *Constant* number of neighbors *K + 1*

Do not spoil the physics

M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel’man and L.B. Ioffe, PRL 2018

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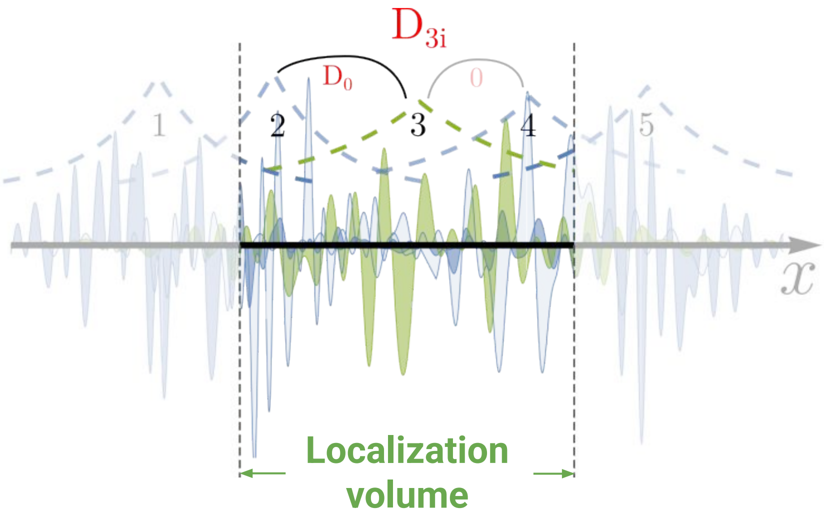
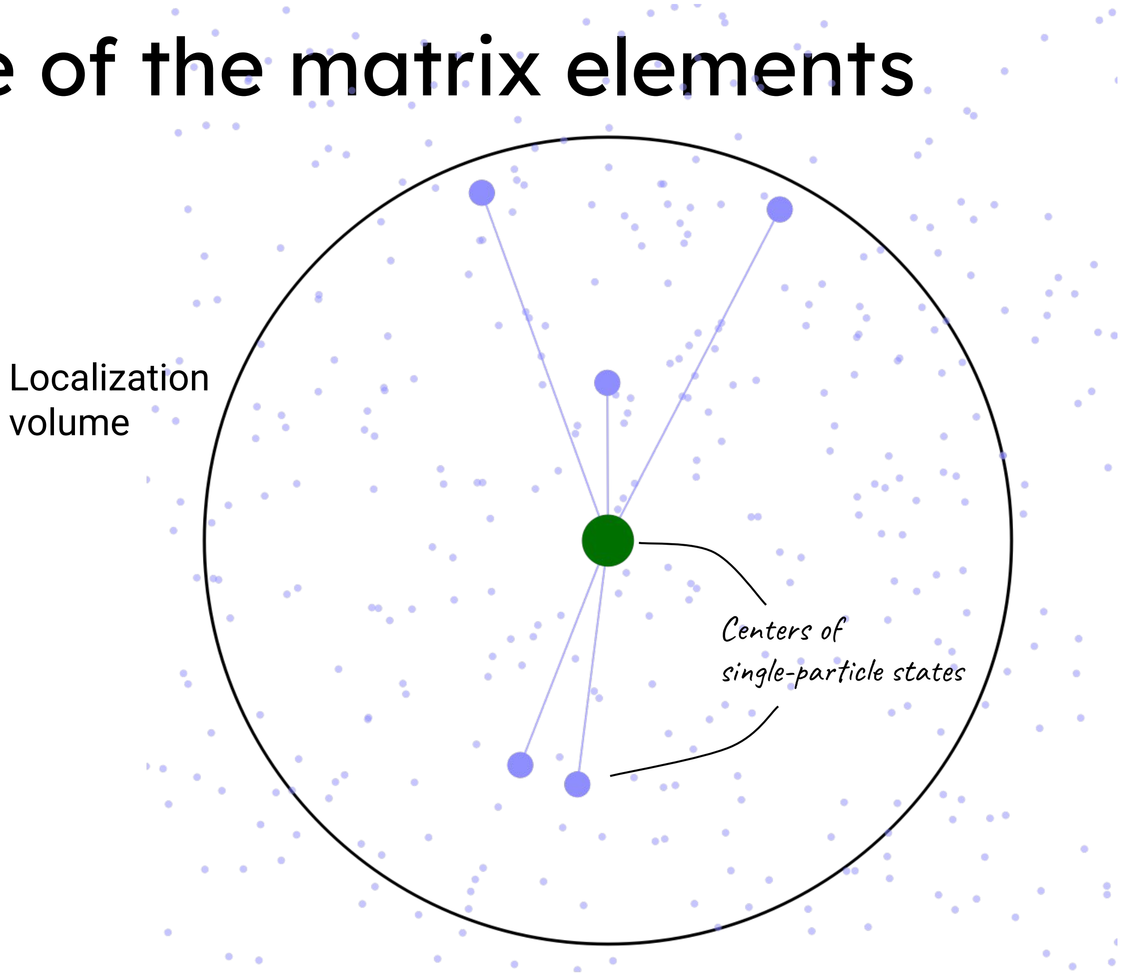


$$D_{ij} = \begin{cases} E_F \frac{\lambda}{K}, & \text{"strong"}, \\ 0, & \text{"weak"} \end{cases}$$

✓ notion of a **graph** embedded in real space
✓ **locally tree-like** structure of D_{ij}

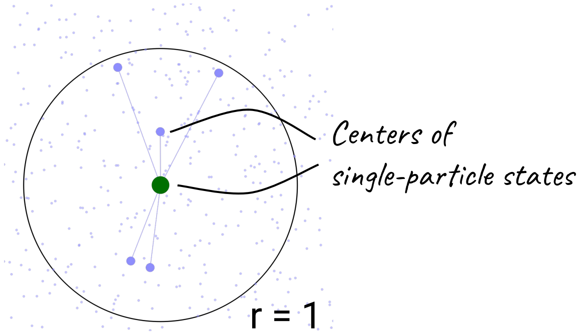
M.V. Feigel'man et al, Annals of Phys. 2010; M.V. Feigel'man and L.B. Ioffe, PRL 2018

Structure of the matrix elements

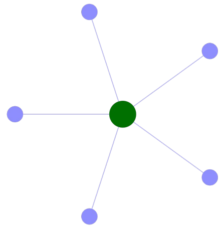


Structure of the matrix elements

Real space:



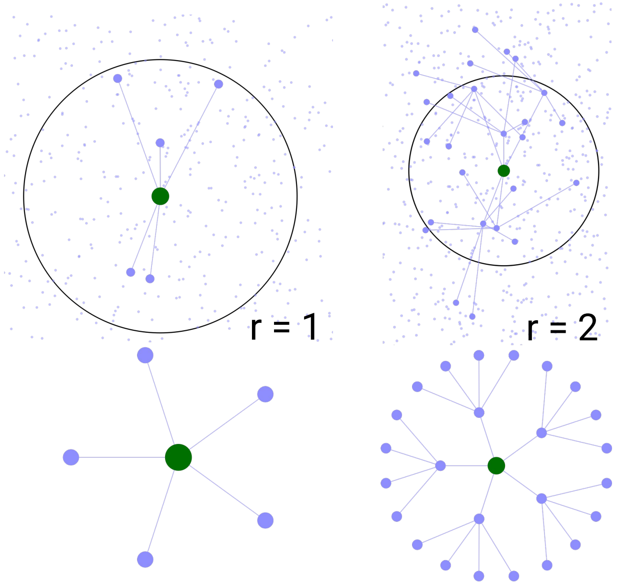
Topology:



Tree-like structure of the matrix elements

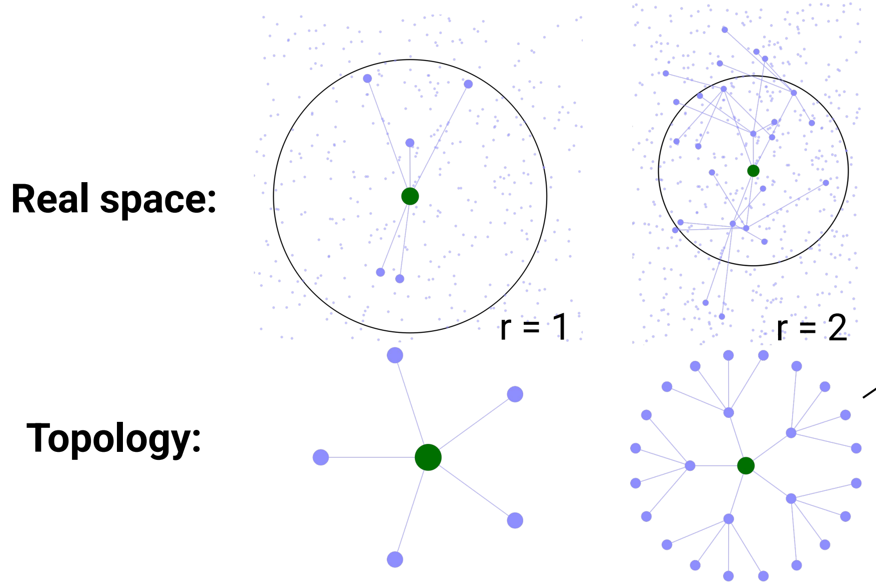
Real space:

Topology:



M.V. Feigel'man et al, Ann. Phys. 2010; B. Sacépé et al, Nat. Phys. 2020

Tree-like structure of the matrix elements



This structure allows for powerful theoretical methods (e.g., Belief Propagation)

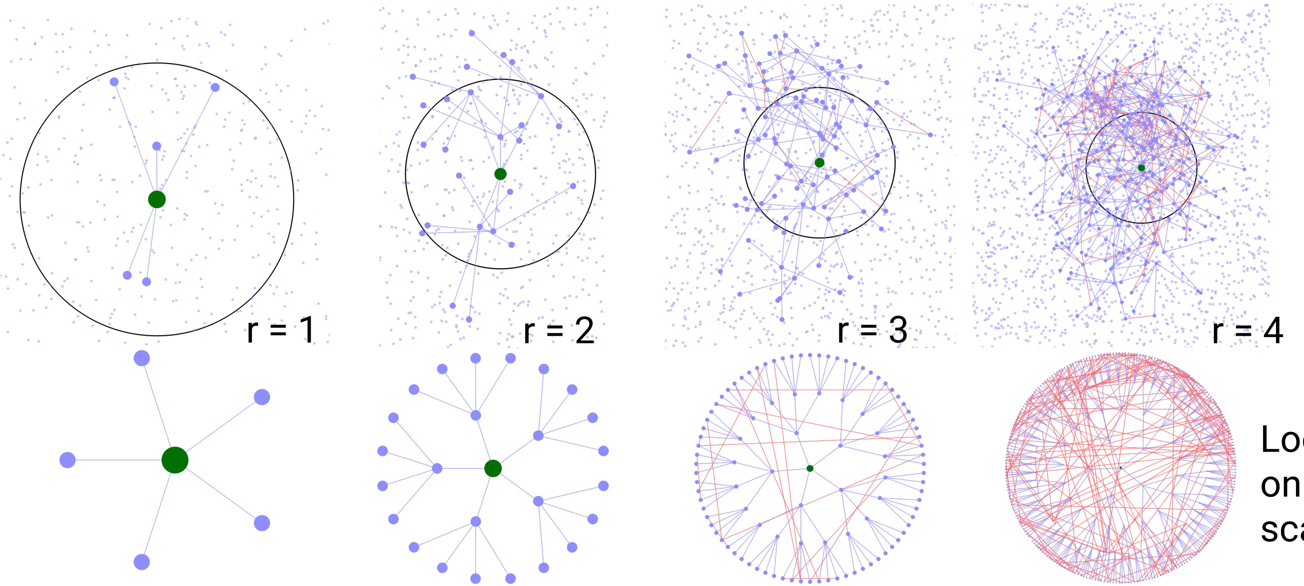
… but this is only at small scales!

M.V. Feigel'man et al, Ann. Phys. 2010; B. Sacépé et al, Nat. Phys. 2020

Locally tree-like structure of the matrix elements

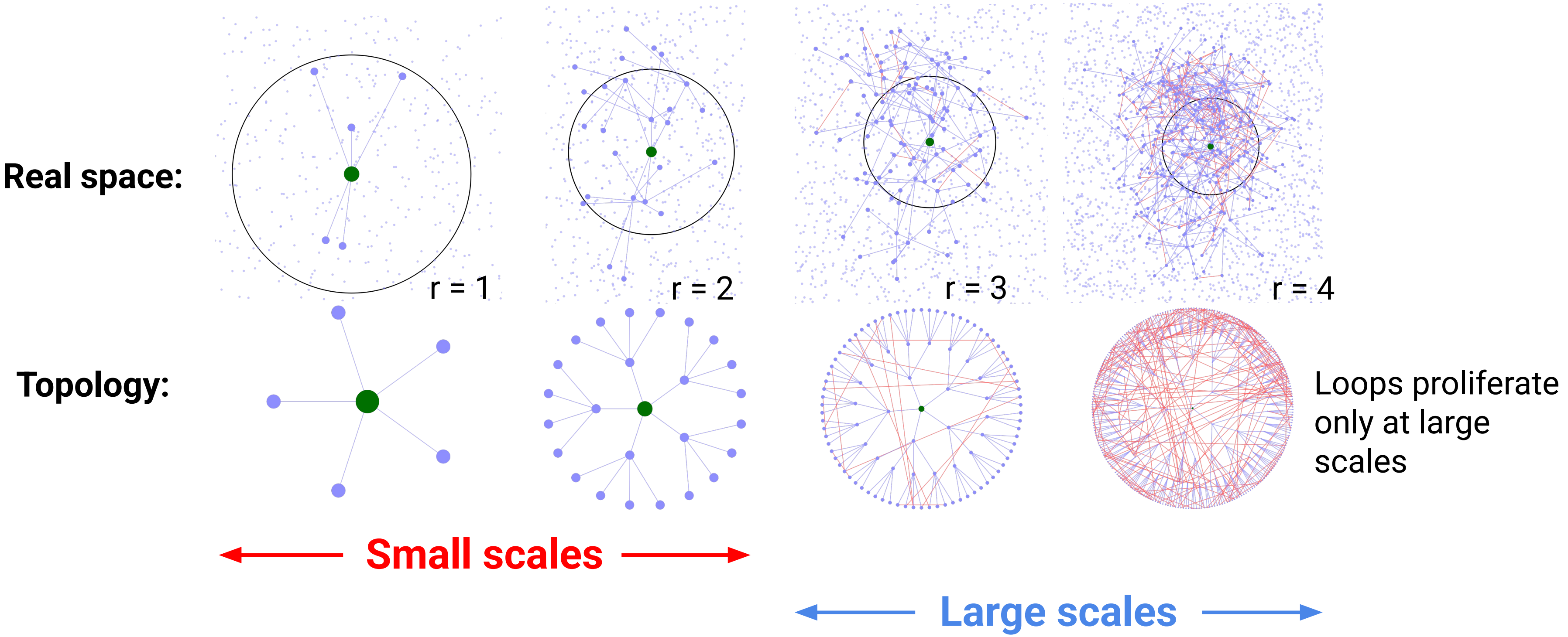
Real space:

Topology:



Loops proliferate only at large scales

Locally tree-like structure of the matrix elements



The problem of macroscopic response

Small scales:
Pseudospin
Hamiltonian

???

Large scales:
London
equations

Contents of this talk

01

Unusual phenomenology
Experimental motivation

02

Model Hamiltonian
Network model of electrodynamics

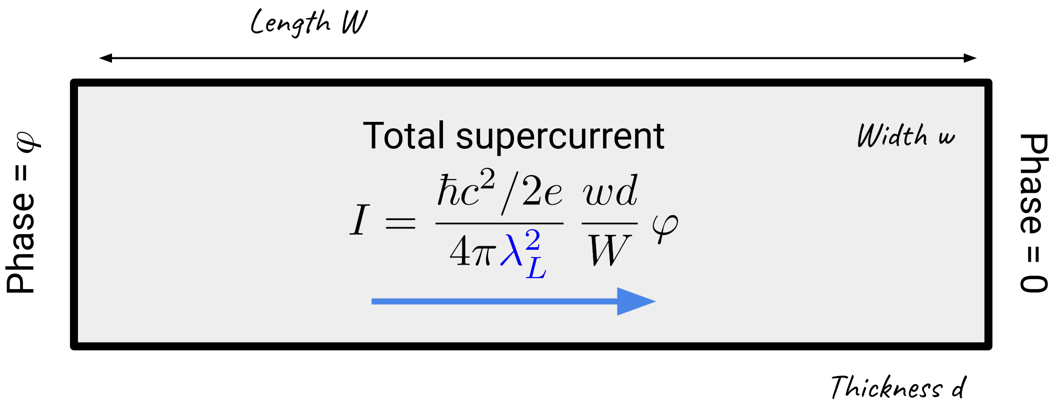
03

Temperature and frequency dependence of electromagnetic response

04

Phase diagram near SIT
Conclusions and Outlook

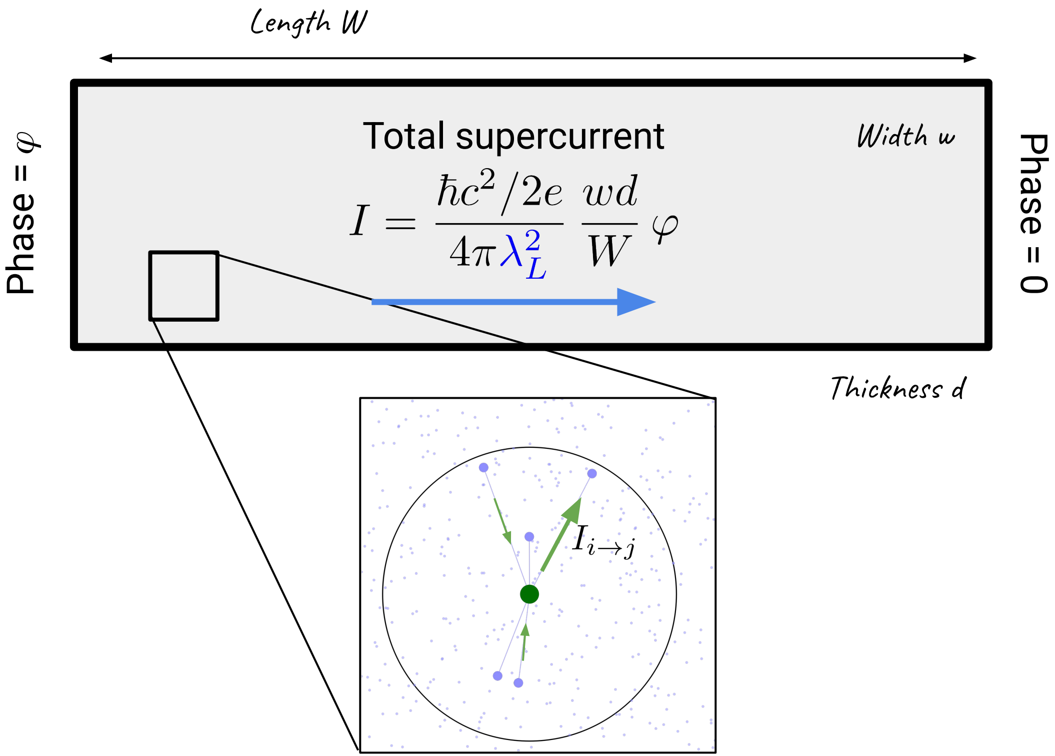
Network model at ω = 0



Network model at ω = 0

$$\sum_j I_{i\to j} = 0$$

$$\sum_{\text{boundary}} I_{i\to j} = I$$



Network model at ω = 0

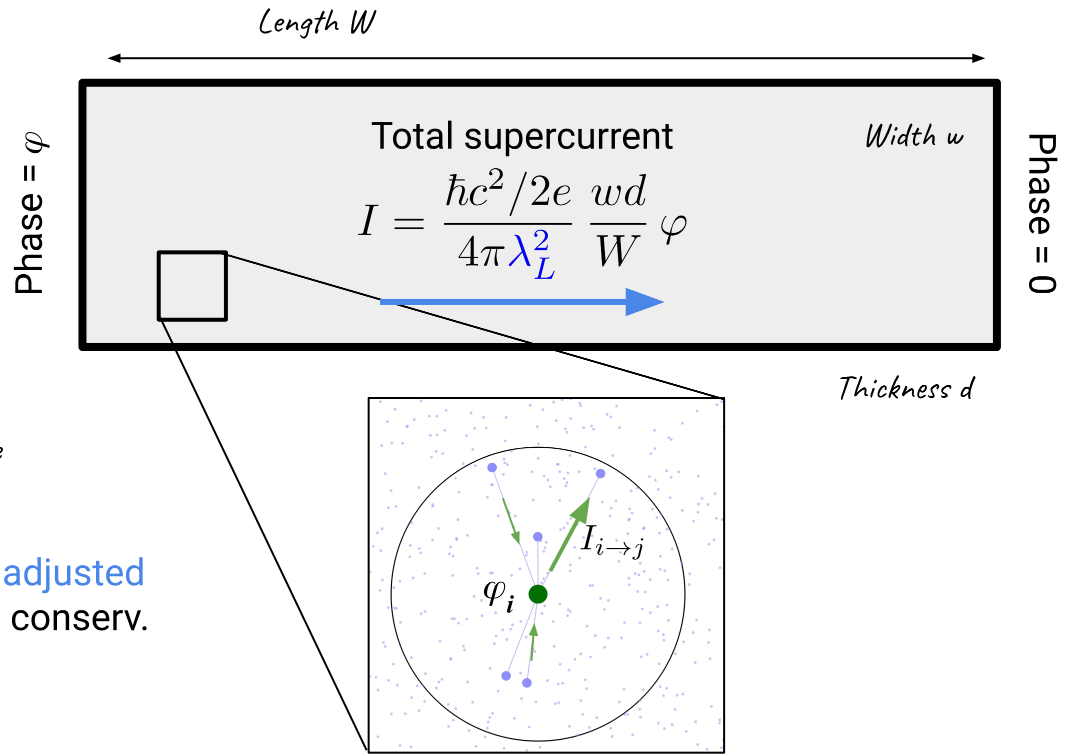
$$\sum_j I_{i\to j} = 0$$

$$\sum_{\text{boundary}} I_{i\to j} = I$$

$$I_{i\to j} = -R_{ij}\,\frac{\hbar}{2e}\,(\varphi_j - \varphi_i)$$ *Linear response*

Random, strongly disordered, determined from microscopics

Phases φ_i adjusted for charge conserv.



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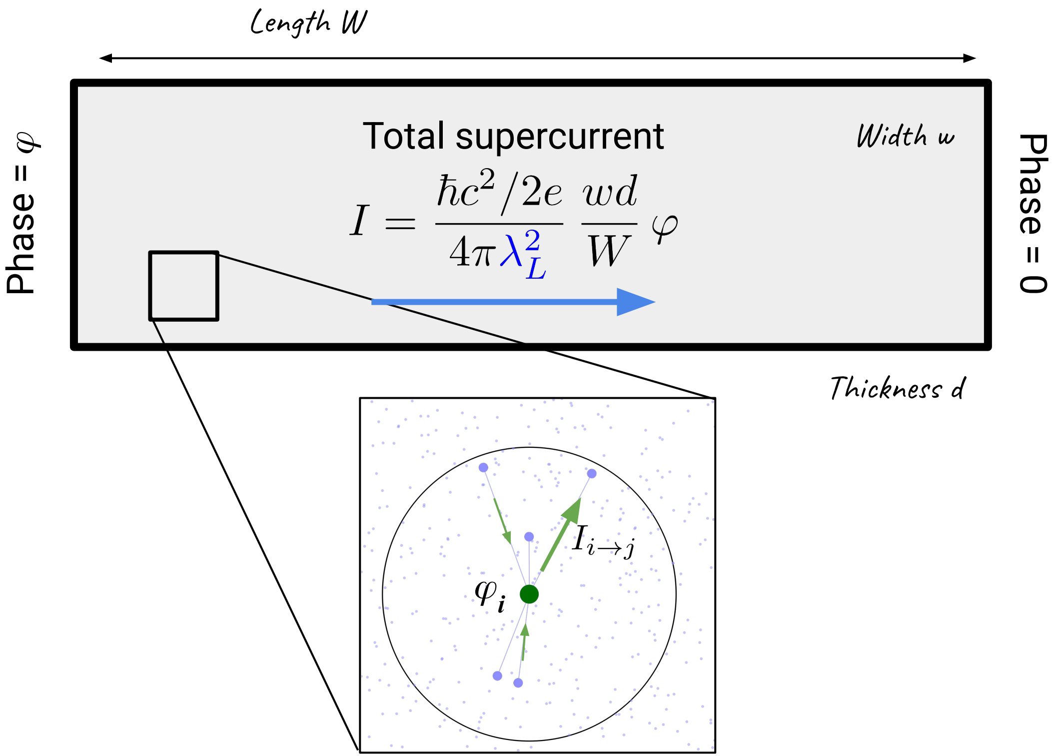


Applies only at ω = 0.
Derivation: NOT JJ model,
but microscopics (PRB 2024)

The problem of macroscopic response

Microscopic theory

Large scales:
London
equations

λ_L

Small scales:
Pseudospin
Hamiltonian

$\lambda,\ \kappa$

Disorder average?

Network model

semiclassics?

$$\frac{c^2}{4\pi\lambda_L^2}\delta^{\alpha\beta} = \int d^3\boldsymbol{r}'\,\overline{R^{\alpha\beta}(\omega;\boldsymbol{r},\boldsymbol{r}')}$$

A.A. Abrikosov , L.P. Gor’kov, I.E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*

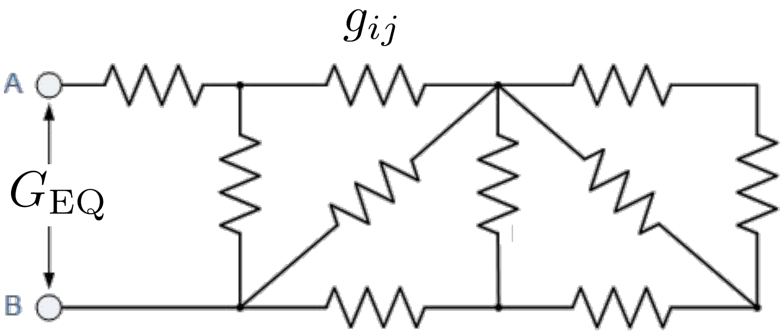
Mathematical equivalence:
network of conductances

$$\sum_j I_{i\to j} = 0$$ *Kirchoff's law*

$$\sum_{\text{boundary}} I_{i\to j} = I$$ *Charge conservation*

$$I_{i\to j} = -R_{ij}\,\frac{\hbar}{2e}\left(\varphi_j - \varphi_i\right)$$ *Ohm's law (linear response)*

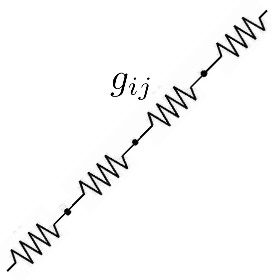
$$I = \frac{\hbar c^2/2e}{4\pi\lambda_L^2}\,\frac{wd}{W}\,\varphi$$ *Global Ohm's law*



$$G_{\mathrm{EQ}} = \frac{\text{cross section area}}{\text{length}}\,\sigma_{\mathrm{EQ}}$$

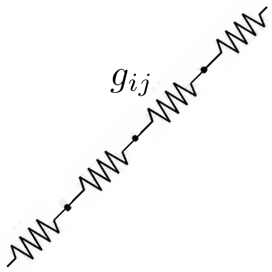
Image adopted from: https://www.electronics-tutorials.ws/resistor/res_5.html

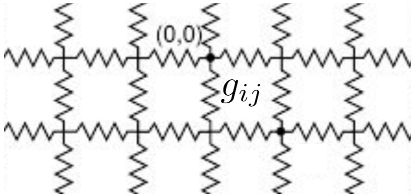
Conductivity in thermodynamic limit



$$\sigma_{\mathrm{EQ}} = \frac{1}{n_{1D}} \frac{1}{\overline{1/g_{ij}}}$$

Conductivity in thermodynamic limit





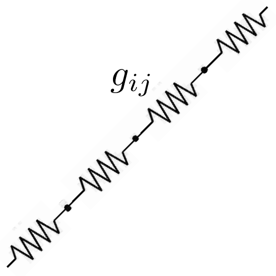
symmetric distrib. of $\ln g_{ij}$

$$\sigma_{\mathrm{EQ}} = \exp\left\{\overline{\ln g_{ij}}\right\} \, n_{2D}\overline{(\boldsymbol{r}_i - \boldsymbol{r}_j)^2}$$

$$\sigma_{\mathrm{EQ}} = \frac{1}{n_{1D}} \frac{1}{\overline{1/g_{ij}}}$$

A.M. Dykhne, JETP 1971

Conductivity in thermodynamic limit



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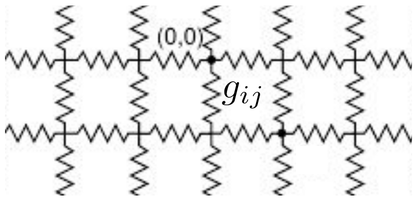

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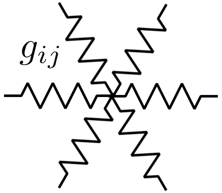
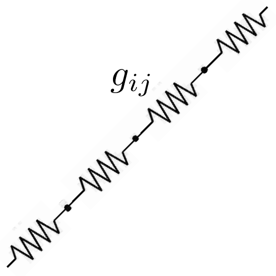
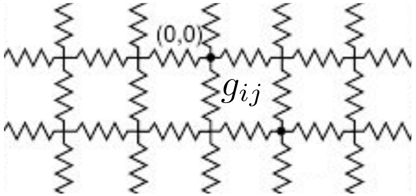


Large coordination number Z

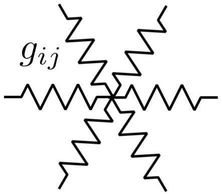
$$\sigma_{\mathrm{EQ}} = \frac{\overline{Z}}{2}\, \overline{g_{ij}} \times n\overline{(\boldsymbol{r}_i - \boldsymbol{r}_j)^2}$$

A.M. Dykhne, JETP 1971; S. Kirkpatrick et al, Rev. Mod. Phys. 1973





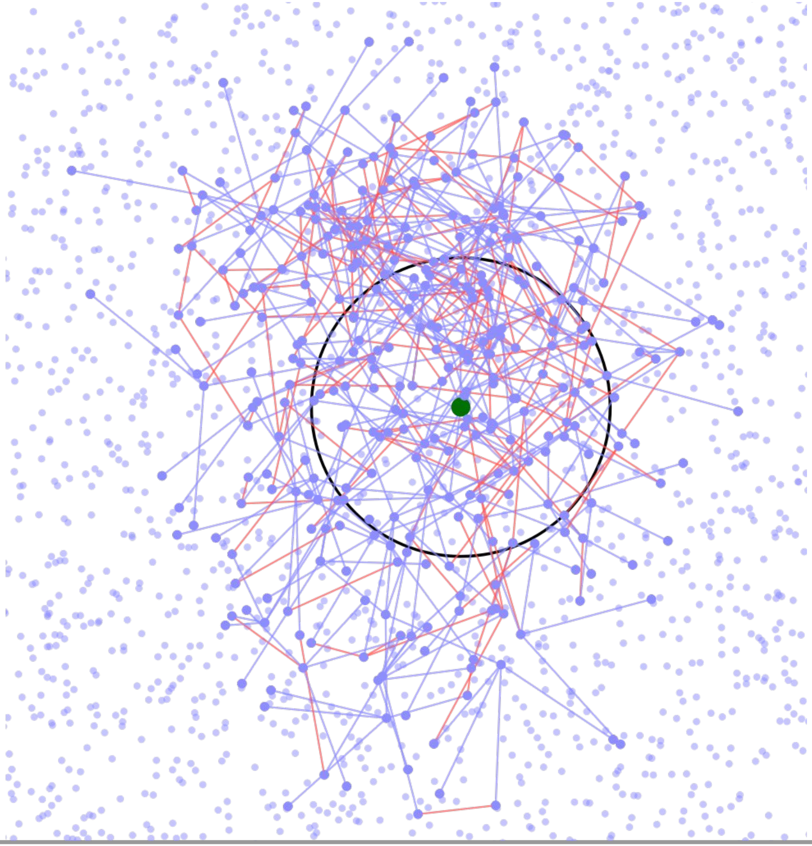
symmetric distrib. of $\ln g_{ij}$



Large coordination number Z

Conductivity = graph structure + distribution!

Network model:



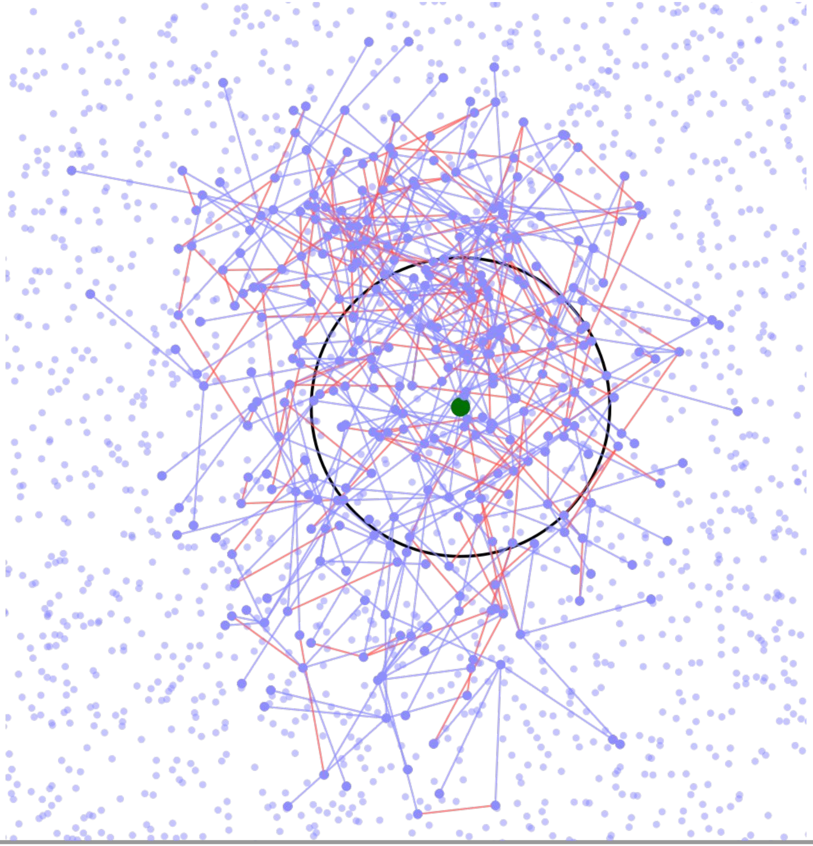
$$\frac{1}{n_{1D}} \frac{1}{\overline{\overline{1/g_{ij}}}} \quad ?$$

$$\frac{\overline{Z}}{2} \overline{g_{ij}} \times n\overline{(\boldsymbol{r}_i - \boldsymbol{r}_j)^2} \quad ?$$

$$\sigma_{\text{EQ}} = \quad \exp\left\{\overline{\ln g_{ij}}\right\} n_{2D}\overline{(\boldsymbol{r}_i - \boldsymbol{r}_j)^2} \quad ?$$

Another weird expression ?

Network model: Hard Problem



in semiclassics:

$$\frac{c^2}{4\pi\lambda_L^2}\delta^{\alpha\beta} = \int d^3\boldsymbol{r}'\,\overline{R^{\alpha\beta}(\omega;\boldsymbol{r},\boldsymbol{r}')}$$

$$\sigma_{\mathrm{EQ}} = \frac{\overline{Z}}{2}\,\overline{g_{ij}} \times \overline{n(\boldsymbol{r}_i - \boldsymbol{r}_j)^2}$$ (weak disorder)

Hard problem! (strong disorder)

A.A. Abrikosov , L.P. Gor'kov, I.E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*

The problem of macroscopic response

Microscopic theory

Large scales:
London
equations
λ_L

Small scales:
Pseudospin
Hamiltonian
$\lambda, \ \kappa$

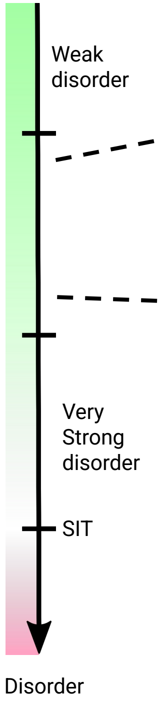
Network model
for λ_L

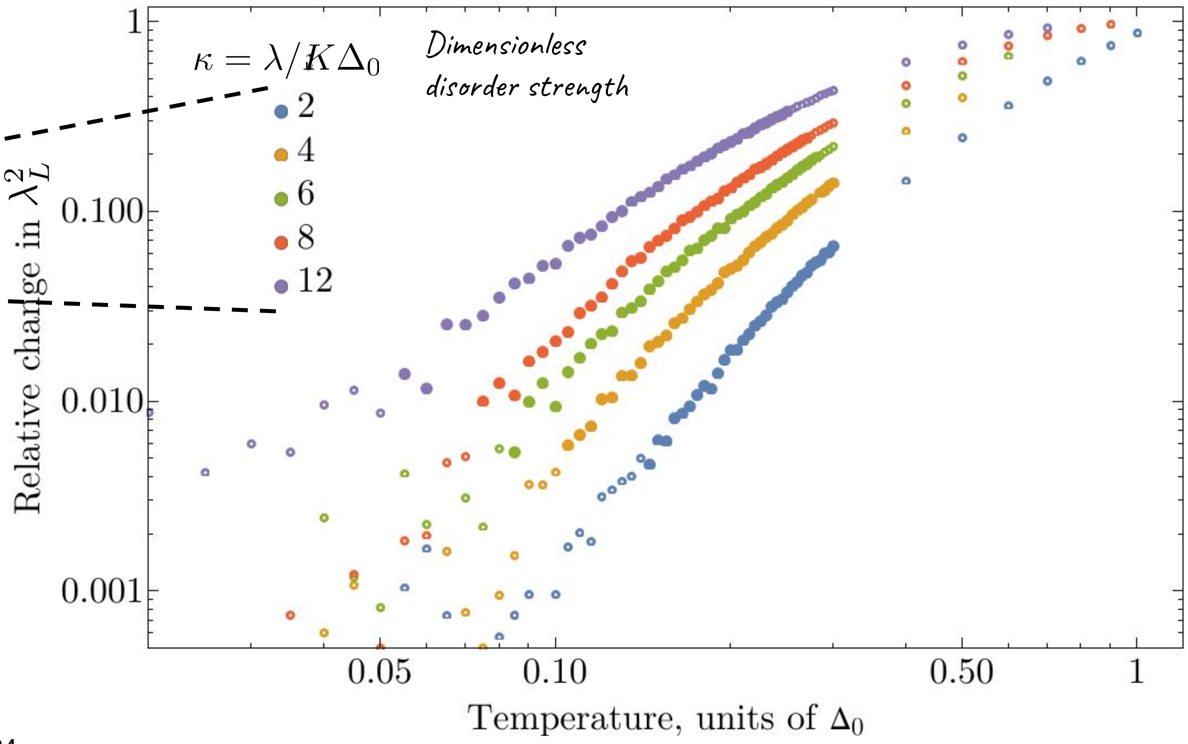
- Numerical analysis
- Percolation mapping
- Belief Propagation
- Analytic theory (WIP)

…

Contents of this talk

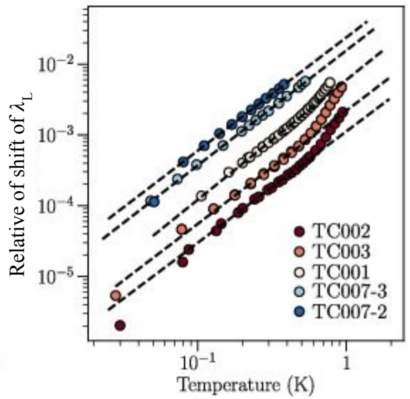
Results for strong disorder



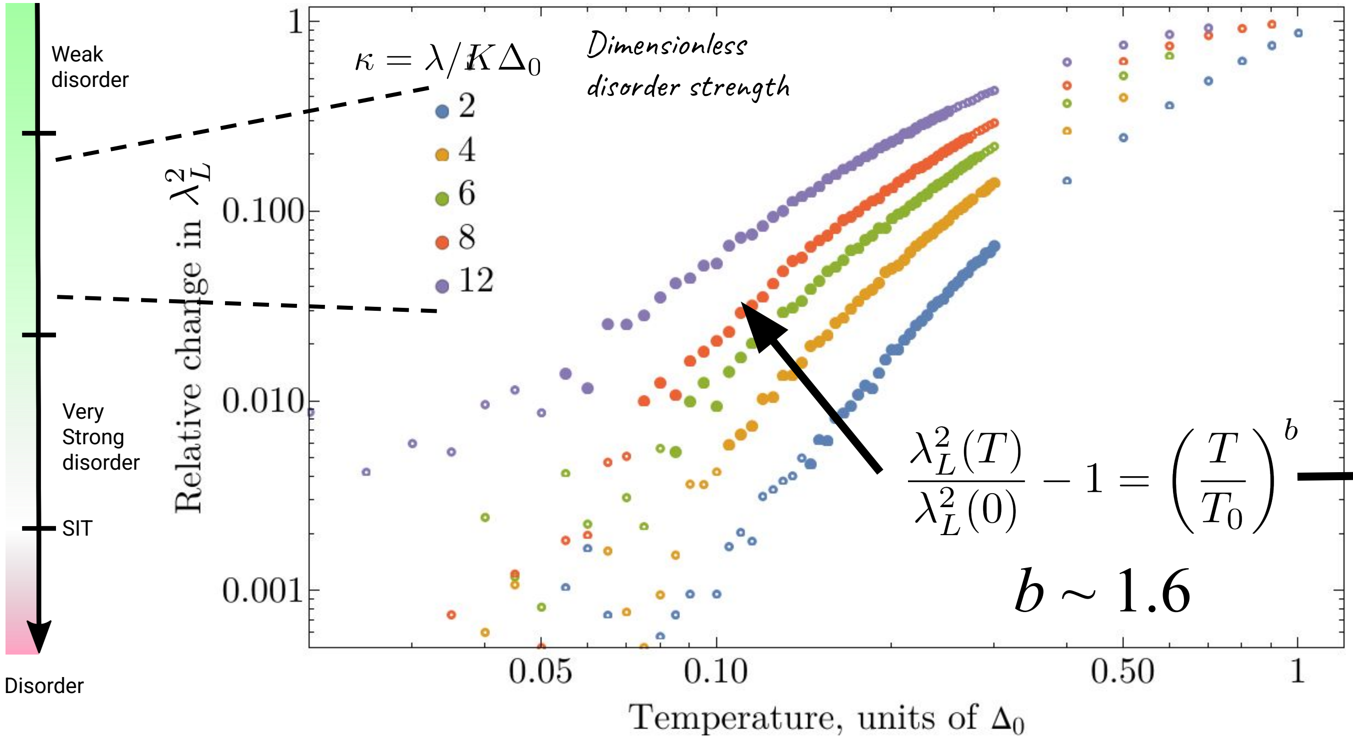


AVK et al, PRB 2024

Looks like power law, similar to experiment



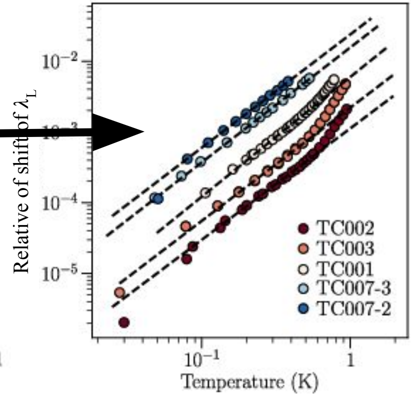
Results for strong disorder



$$\frac{\lambda_L^2(T)}{\lambda_L^2(0)} - 1 = \left(\frac{T}{T_0}\right)^b$$

$b \sim 1.6$

Looks like power law, similar to experiment



AVK et al, PRB 2024

Results for strong disorder ($T \ll \Delta_0$)

Disorder-dependent coefficient

Modified Bessel function

$$\frac{\lambda_L^2(T)}{\lambda_L^2} - 1 \propto -T^2 \frac{\partial}{\partial T} \int_0^\infty dh\, P_T(h) \int_{2h/T}^{+\infty} dx\, K_0(x)$$

Selects low-value tail
$\Delta < T$

Order parameter distribution
(T-dependent)

AVK et al, PRB 2024

Results for strong disorder ($\omega/2$, T << Δ_0)

Disorder-dependent coefficient

Modified Bessel function

$$\frac{\lambda_L^2(T)}{\lambda_L^2} - 1 \propto -T^2 \frac{\partial}{\partial T} \int_0^\infty dh\, P_T(h) \int_{2h/T}^{+\infty} dx\, K_0(x)$$

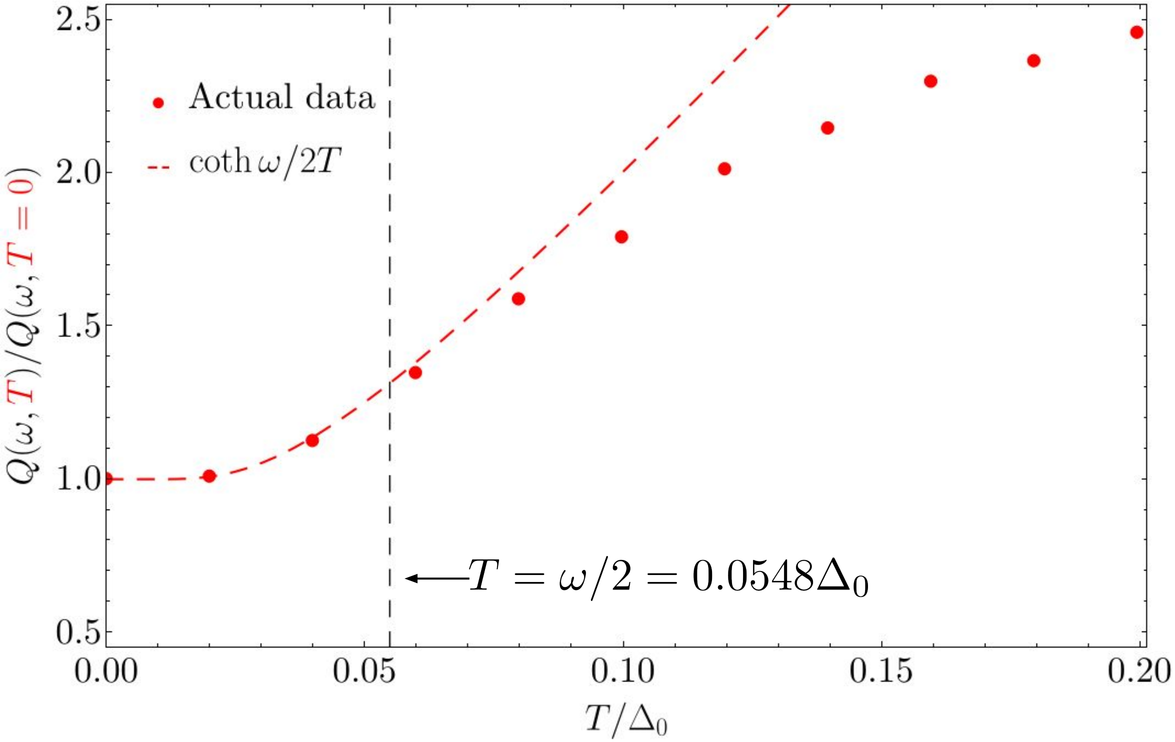
$$\frac{1}{Q(\omega, T)} \propto \frac{\omega}{2\Delta_0} \tanh \frac{\omega}{2T} \int_0^{\omega/2} dh\, P_T(h) \arccos \frac{2h}{\omega}$$

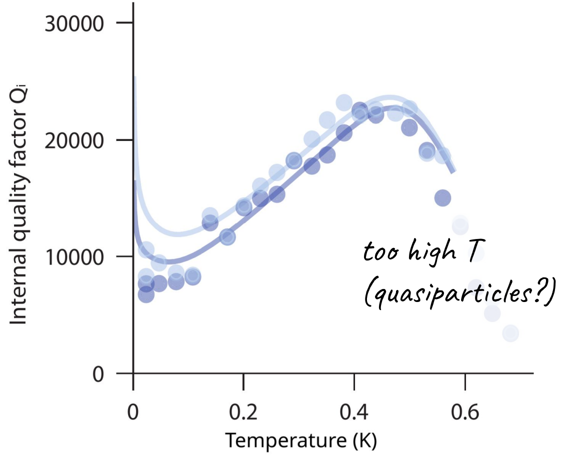
Selects low-value tail
Δ < T or ω/2

Order parameter distribution
(T-dependent)

AVK et al, PRB 2024; AVK and M.V. Feigel'man, PRL 2026

Q(T): temperature-dependent TLS

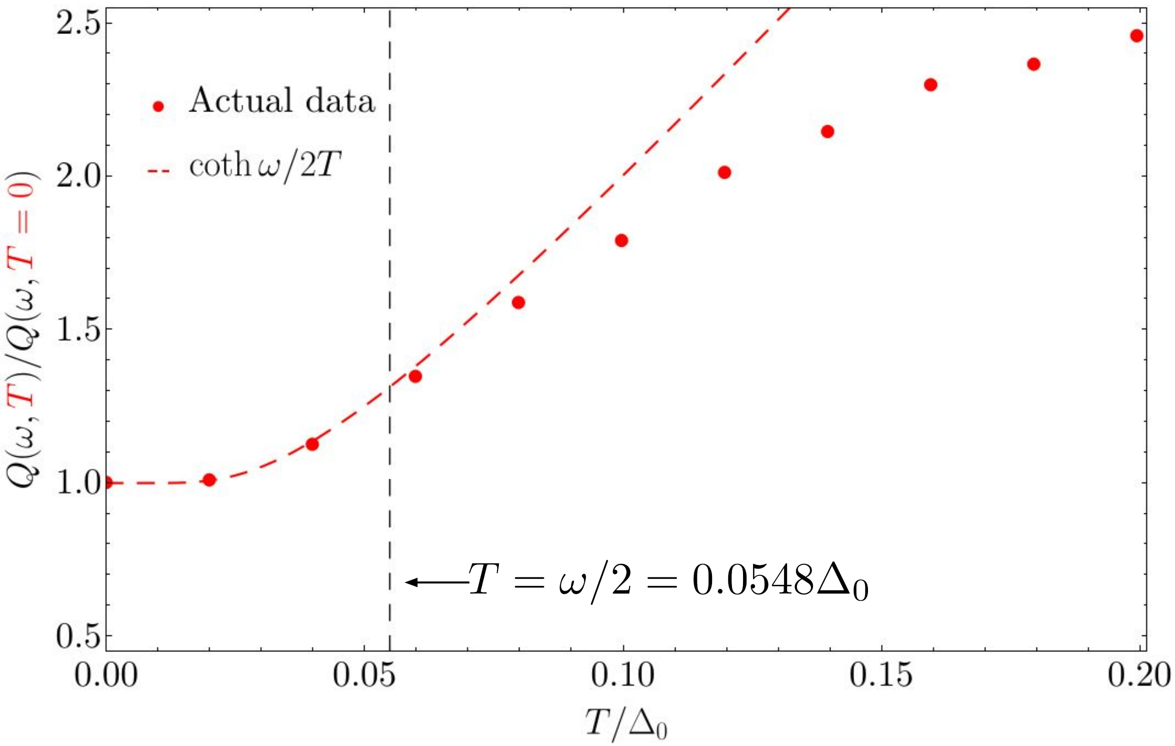


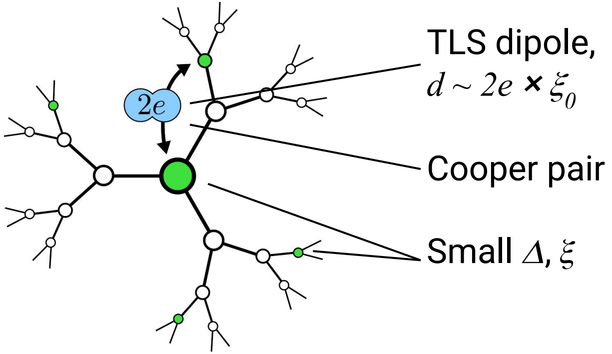


$K = 10$,
$\kappa = 10$,
$\lambda = 0.1373$

AVK and M.V. Feigel'man, PRL 2026

Q(T): temperature-dependent TLS





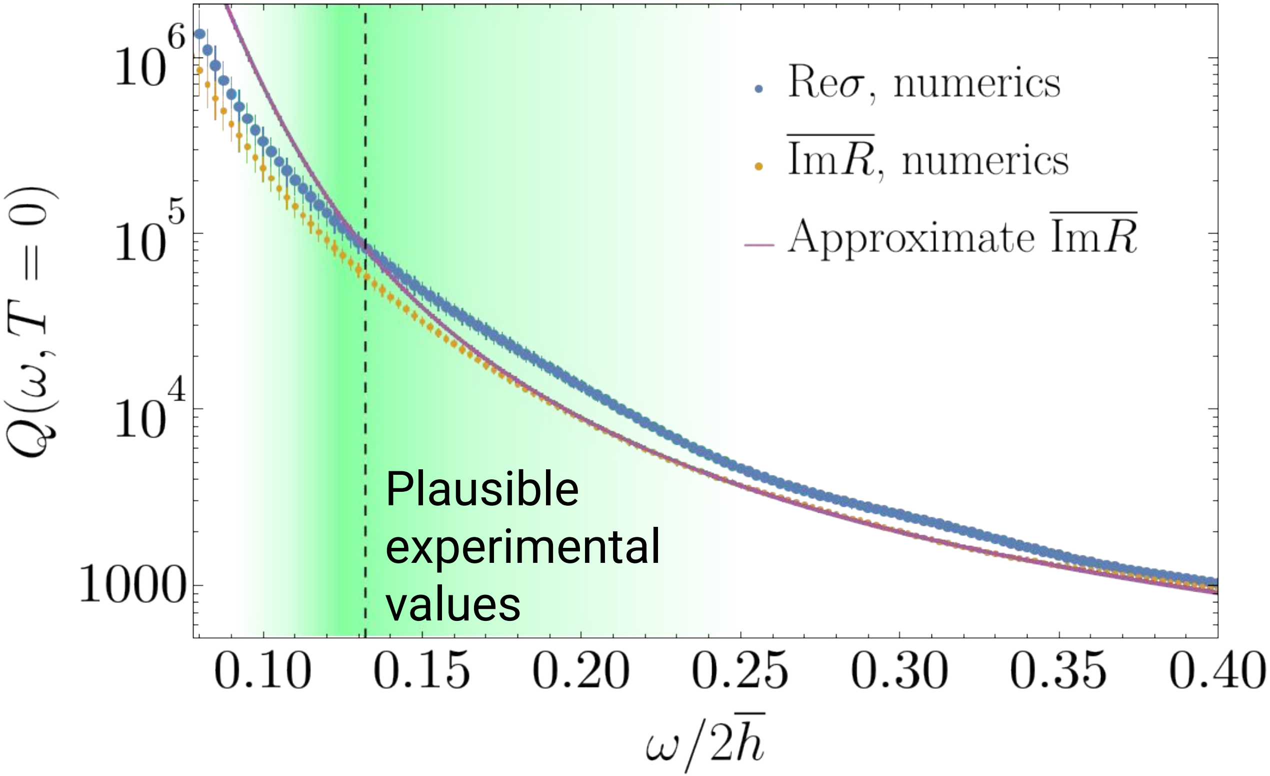
TLS-like **interpretation**:

1. *TLS saturation at low temperature*
2. *Higher TLS spectral density at higher T*

$K = 10$,
$\kappa = 10$,
$\lambda = 0.1373$

AVK and M.V. Feigel'man, PRL 2026

Strong frequency dependence of Q



$K = 10,$
$\kappa = 10,$
$\lambda = 0.1373$

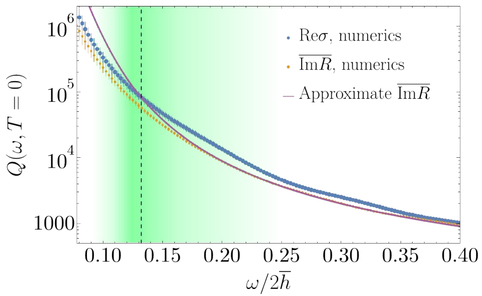
AVK and M.V. Feigel'man, PRL 2026

Probing the tails of Δ distribution

$$\frac{1}{Q(\omega, T)} \propto \frac{\omega}{2\Delta_0} \tanh \frac{\omega}{2T} \int\limits_0^{\omega/2} dh\, P_T(h) \arccos \frac{2h}{\omega}$$

$$P\left(h \lesssim \langle h \rangle\right) \propto \frac{d}{dh} \int\limits_0^{2h} d\omega\, \frac{h}{\sqrt{h^2 - (\omega/2)^2}} \frac{d}{d\omega} \frac{\coth \omega/2T}{\omega\, Q(\omega)}$$

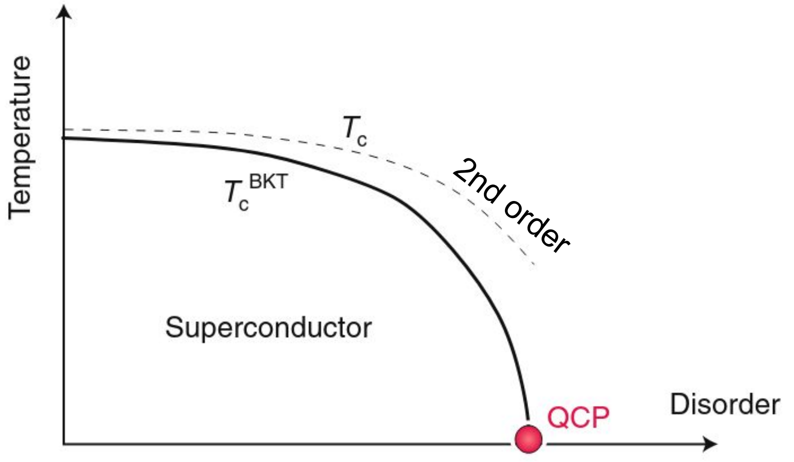
Only measurable quantities!



AVK and M.V. Feigel'man, PRL 2026

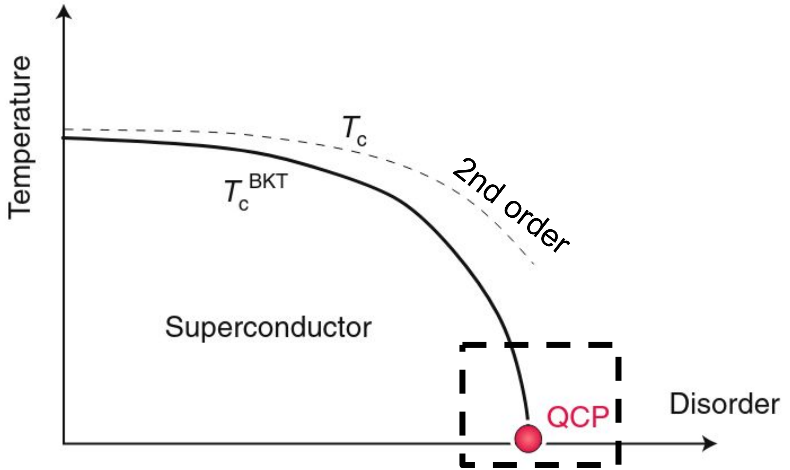
Contents of this talk

Superconductor-insulator phase diagram



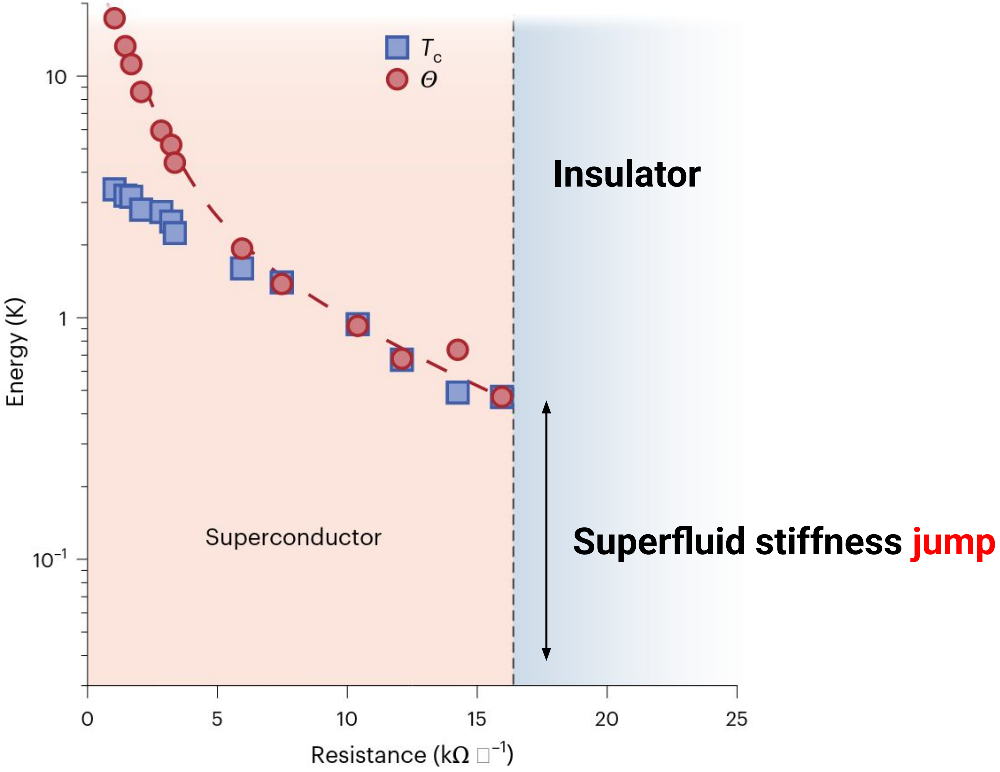
B. Sacépé et al, Nat. Phys. 2020.

Superconductor-insulator phase diagram



B. Sacépé et al, Nat. Phys. 2020.

First-order quantum breakdown of supercond.



T. Charpentier et al, Nat. Phys. 2025

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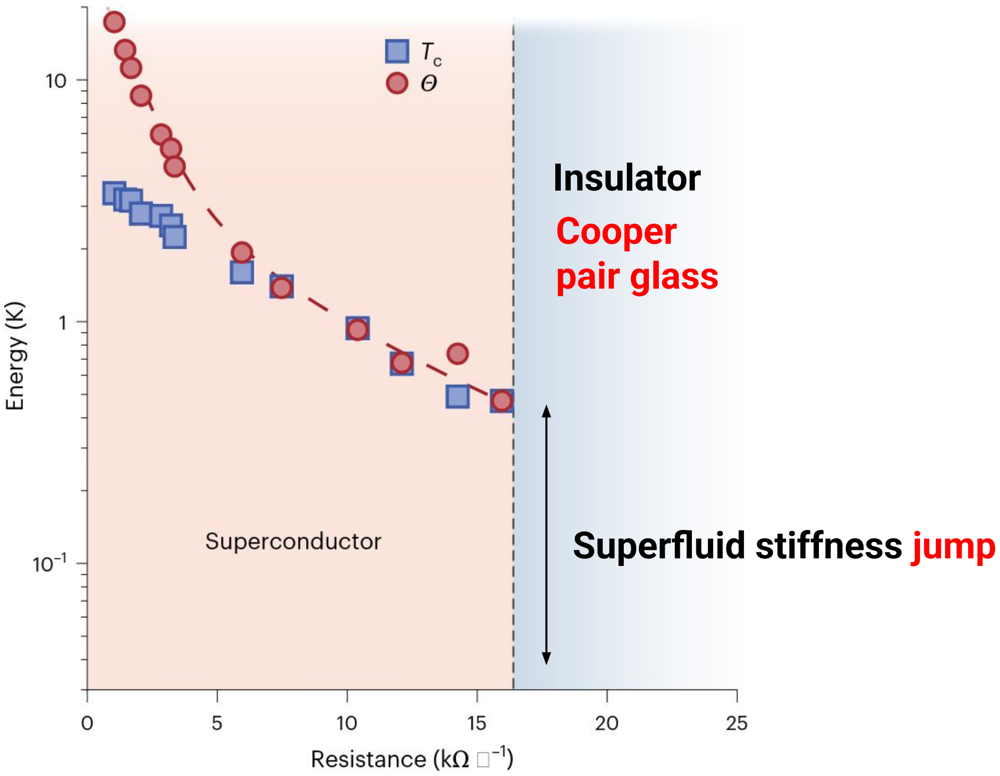
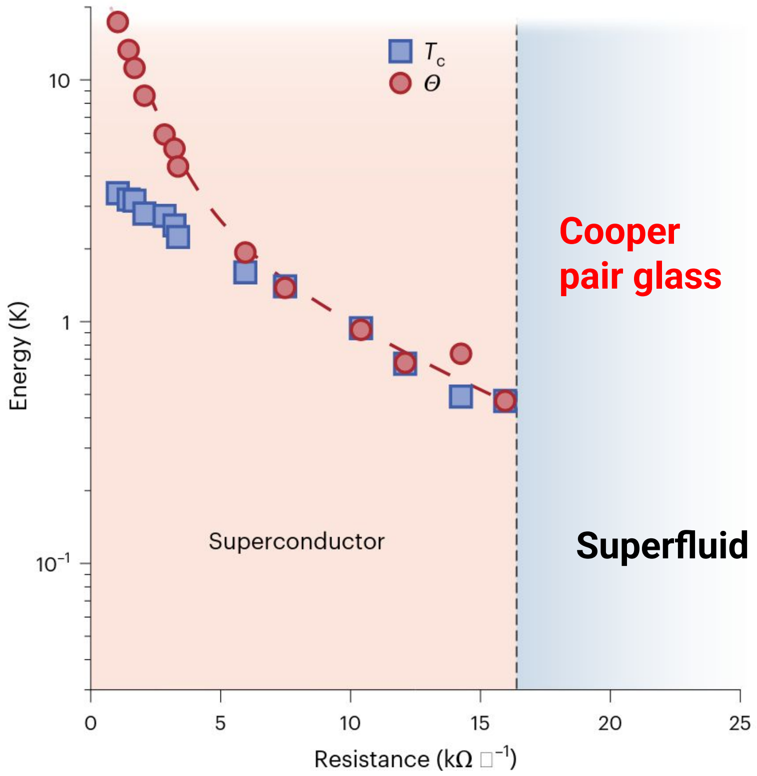


T. Charpentier et al, Nat. Phys. 2025

4/4

First order: supercond. to Cooper pair glass



Incompatible orders

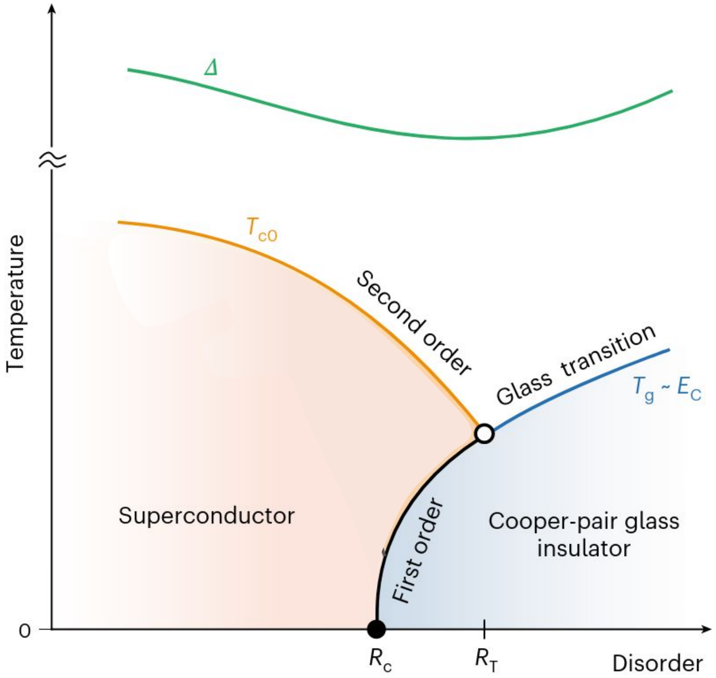
1st order phase transition

Superfluid stiffness jump

T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

Unconventional shape of the transition line

$$F_{\mathrm{SC}}(\text{disorder},T) = F_{\text{Cooper pair glass}}(\text{disorder},T)$$

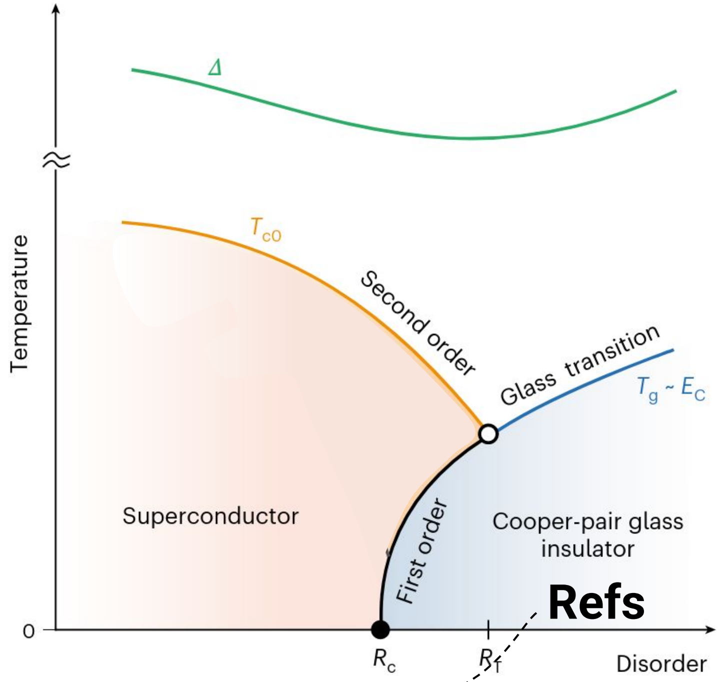


T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

Unconventional shape of the transition line

$$F_{\mathrm{SC}}(\text{disorder},T) = F_{\text{Cooper pair glass}}(\text{disorder},T)$$

$$F_{\mathrm{CPG}}(T) - F_{\mathrm{CPG}}(0) \sim -E_C\,(T/E_C)^4$$



Refs

T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

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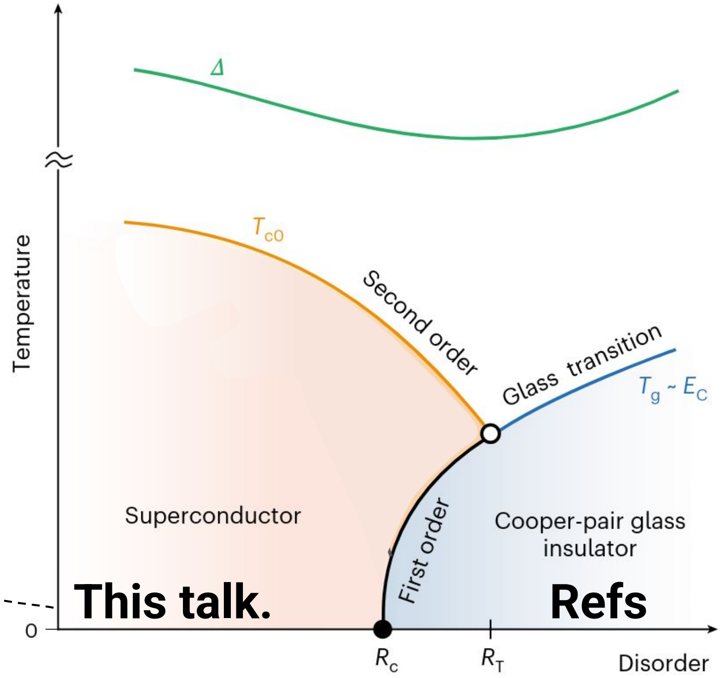
$$\cancel{F_{\mathrm{SC}}(T) - F_C(0) \propto \exp\{-\Delta/T\} \approx 0}$$

$$F_{\mathrm{SC}} \sim -T_0\,(T/T_0)^{\beta+1}\,,\ \ \beta \sim 1.6 < 3$$



T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

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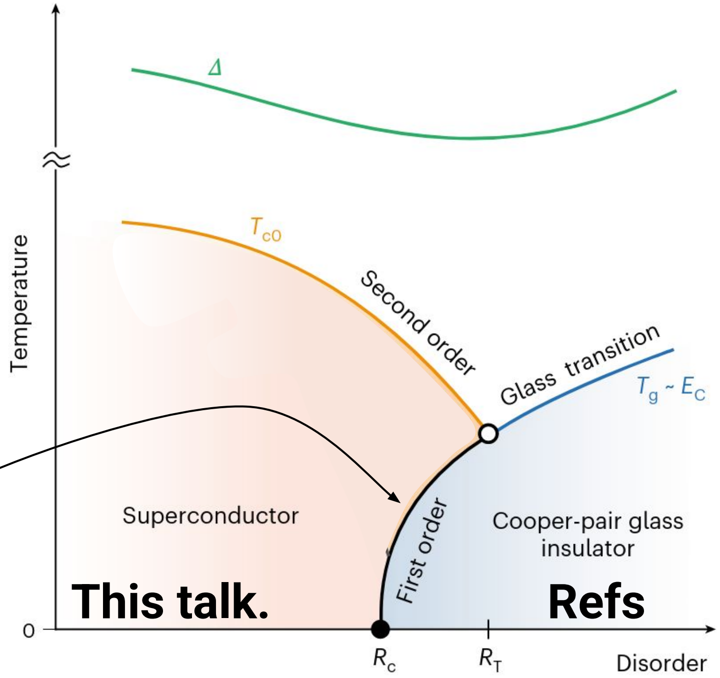
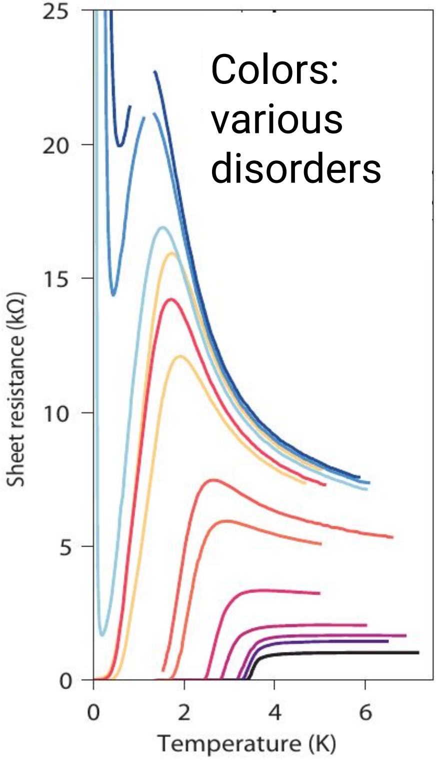
$$F_{\mathrm{SC}} \sim -T_0\,(T/T_0)^{\beta+1}, \quad \beta \sim 1.6 < 3$$

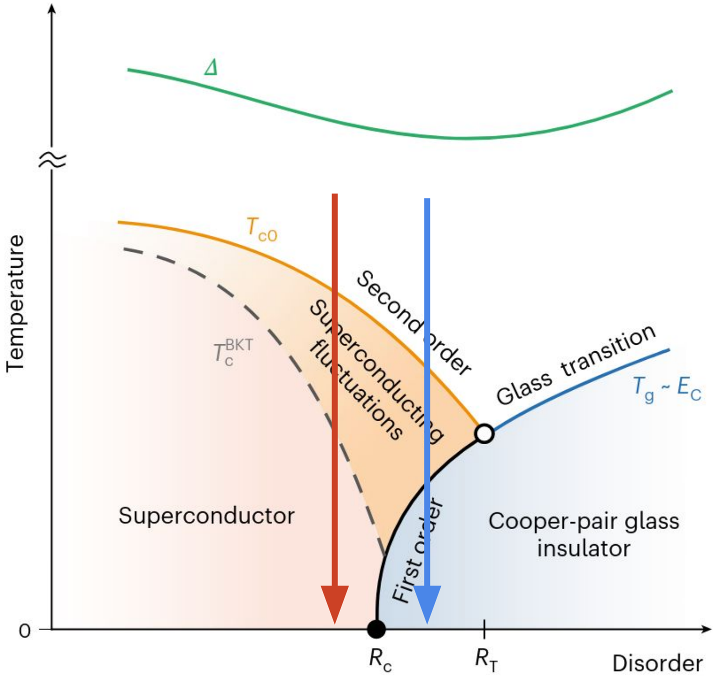
$$\frac{dT_c}{d\text{disorder}} = -\frac{\partial_{\mathrm{dis}}\,(F_{\mathrm{CPG}} - F_{\mathrm{SC}})}{\partial_T\,(F_{\mathrm{CPG}} - F_{\mathrm{SC}})} > 0$$



T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

Re-entrant resistance curves

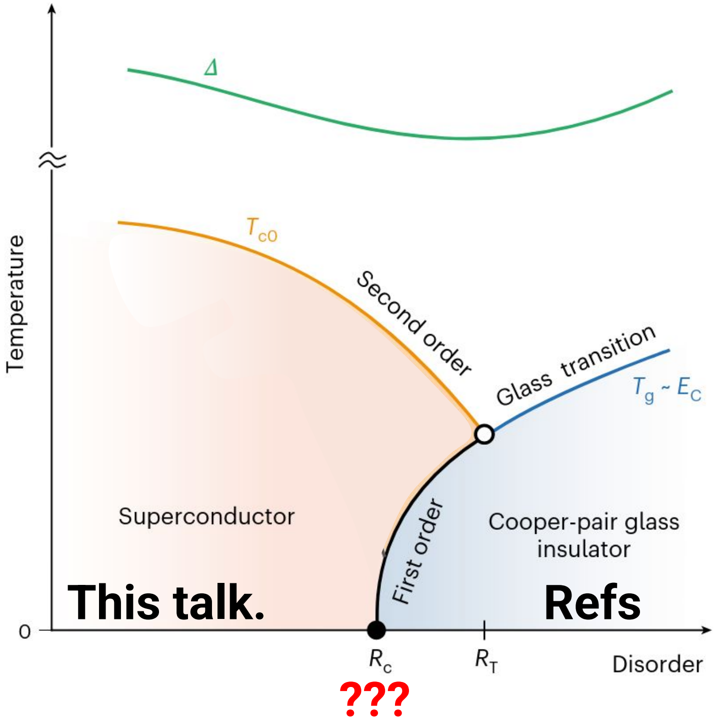




T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

Unconventional shape of the transition line

No good theory!



T. Charpentier et al, Nat. Phys. 2025; I. Poboiko et al., SciPost 2024

Contents of this talk

A novel physical mechanism:

1. The inhomogeneity of the superconducting state creates
bulk collective TLS-like excitations,

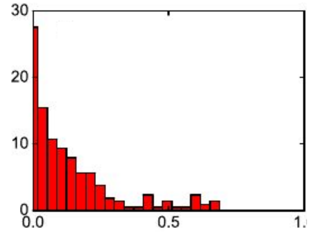
which:

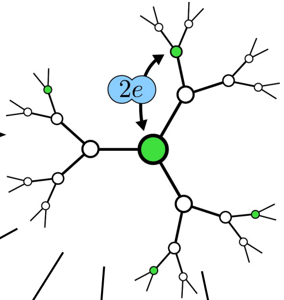
2. cause power-law increase of λ_L with T

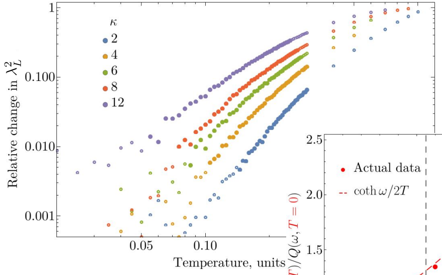
3. contribute to dissipation at finite ω

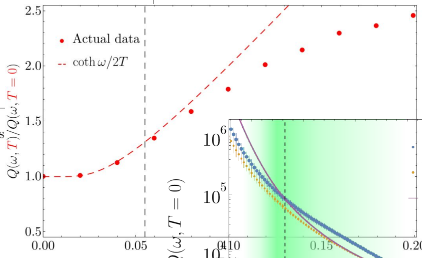
4. enable a probe of the extreme value statistics of the order parameter

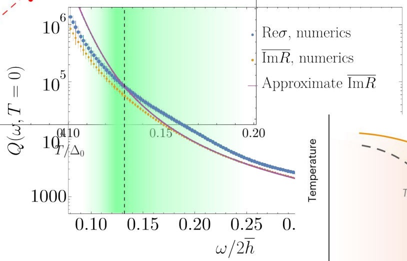
5. conspire with competing spin-glass order to form an unconventional phase diagram



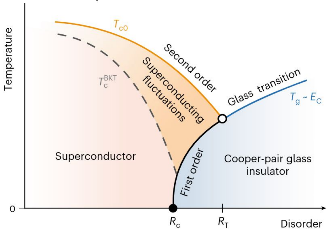






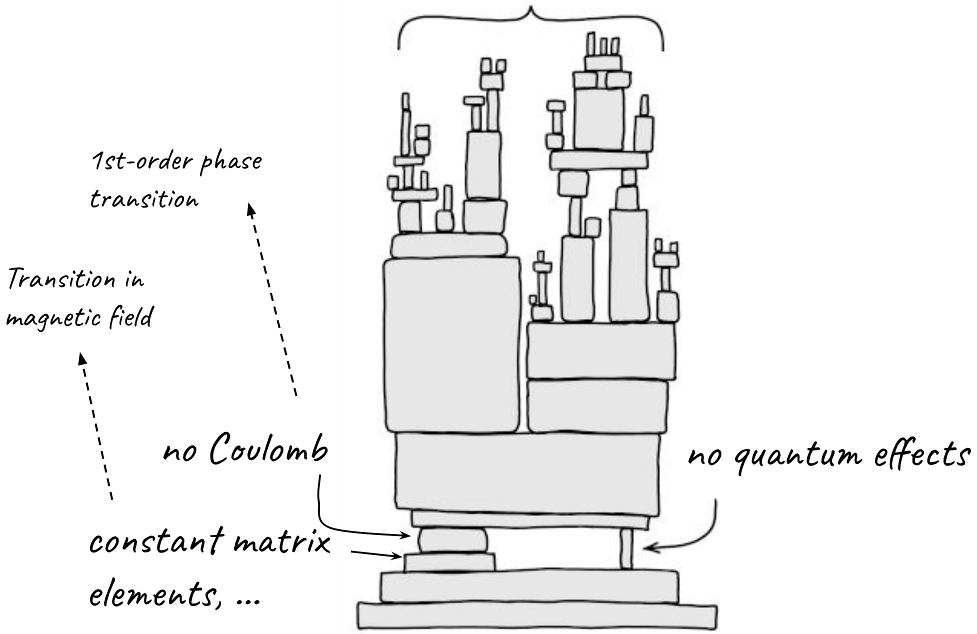


with Cooper pair glass

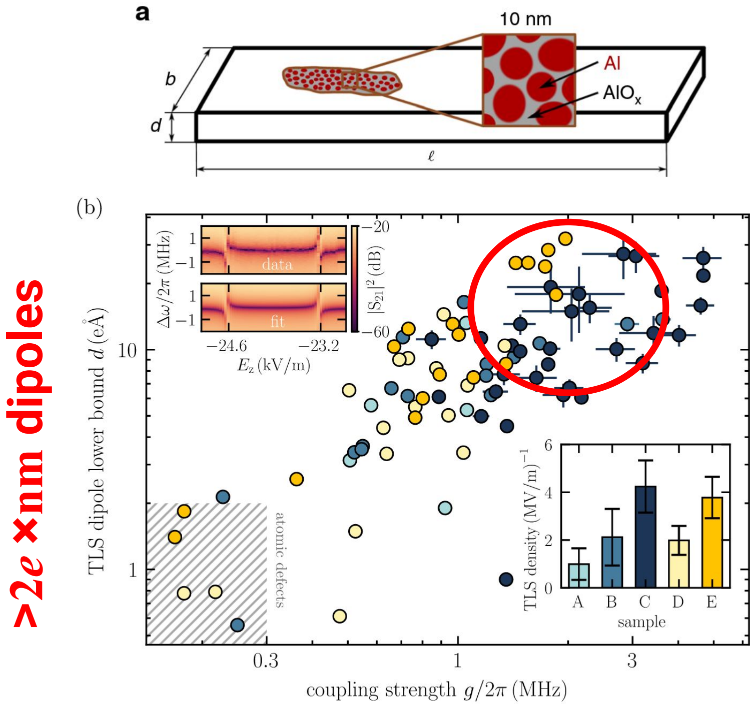


Outlook

This talk: Low-energy theory of strongly disordered superconductors



Granular Aluminum films: similar phenomenology



N. Maleeva et al, Nat. Comm. 2018; M. Kristen et al, PRL 2024