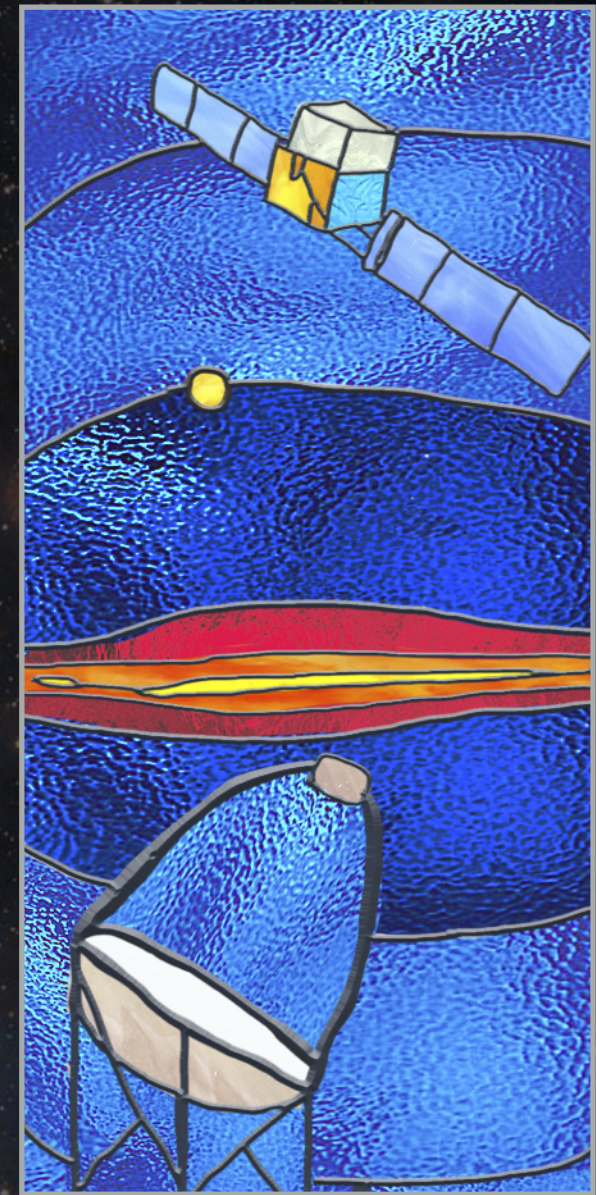
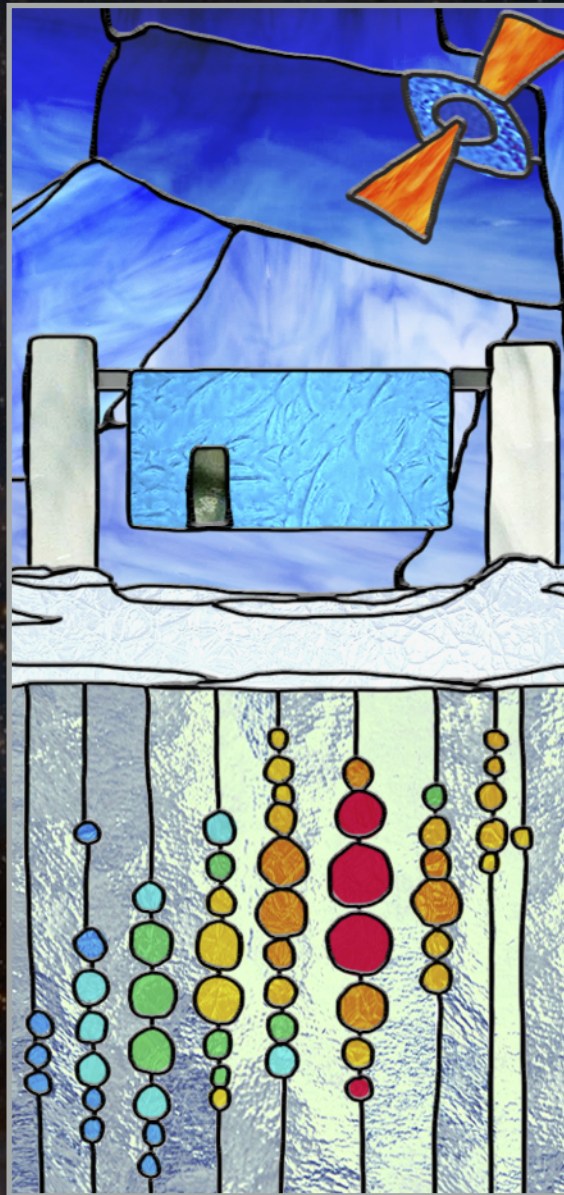




Multi-messenger Astrophysics - Lecture 1

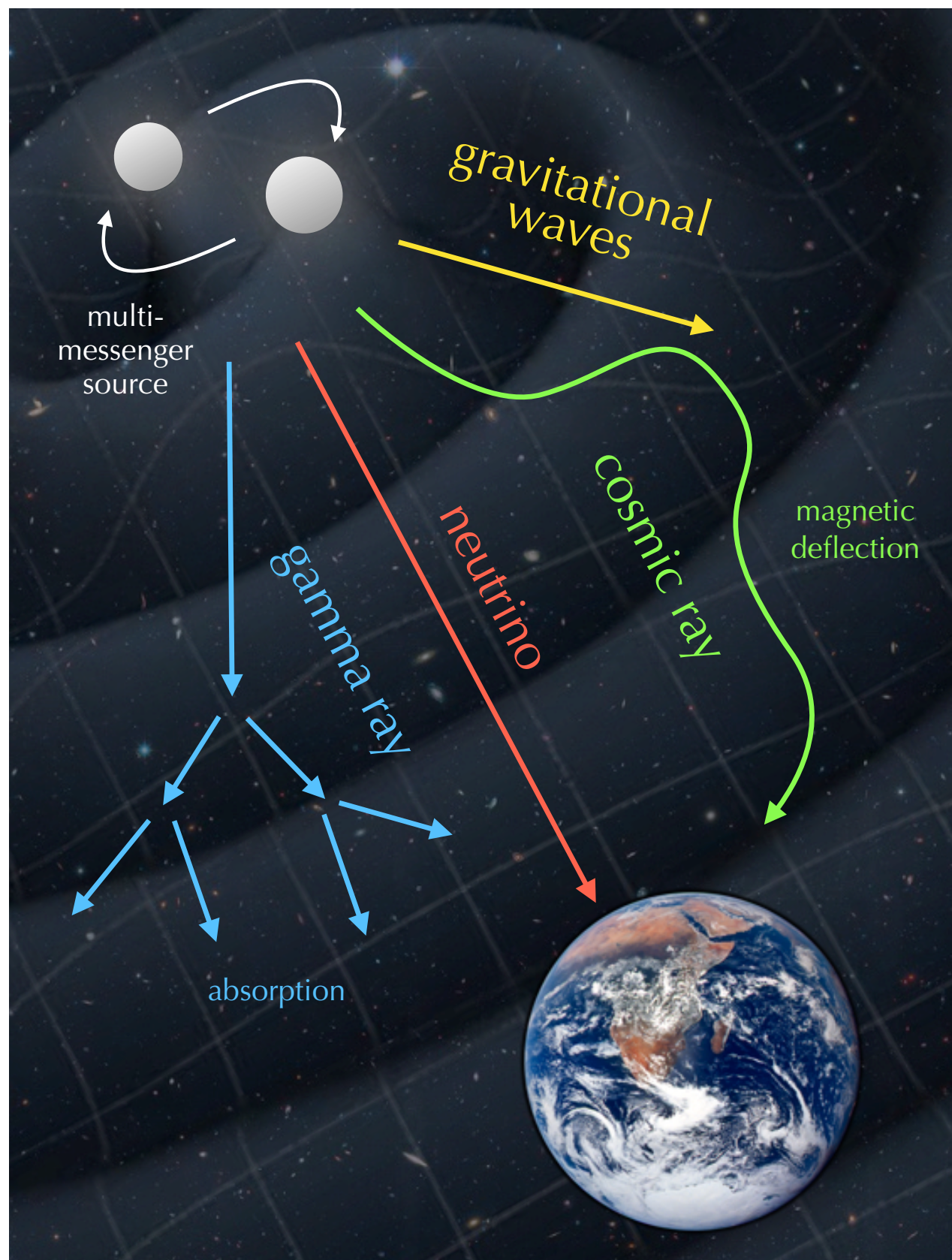
CARLSBERG
FOUNDATION

Markus Ahlers
Niels Bohr Institute
ISAPP Paris-Saclay 2026

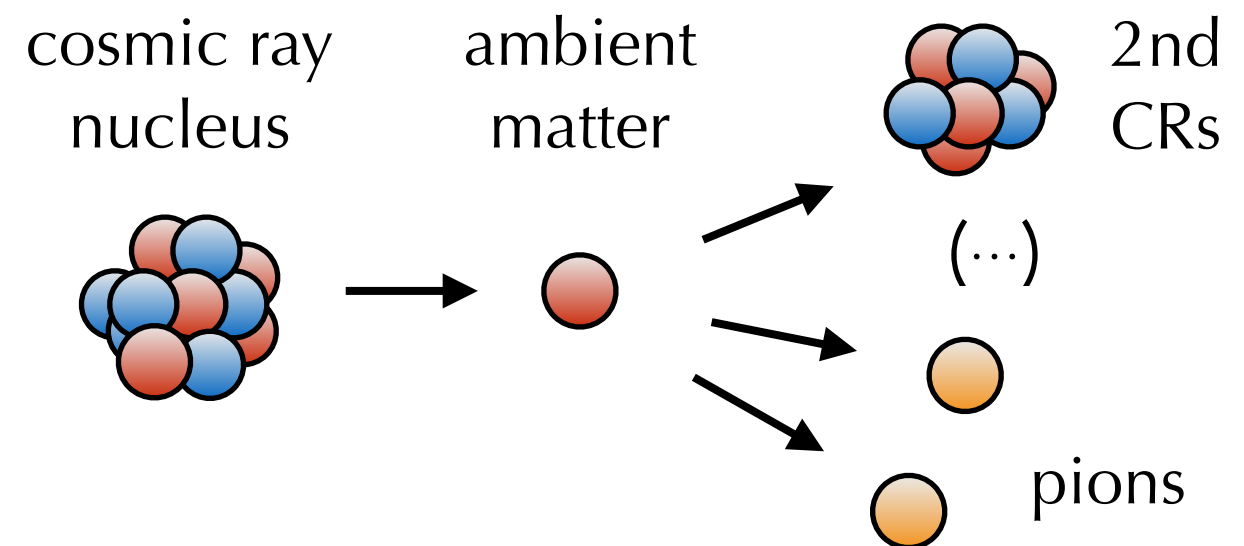
KØBENHAVNS
UNIVERSITET



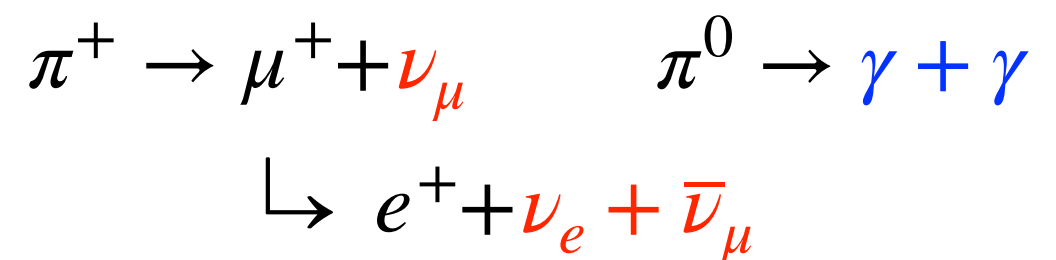
Multi-Messenger Astronomy



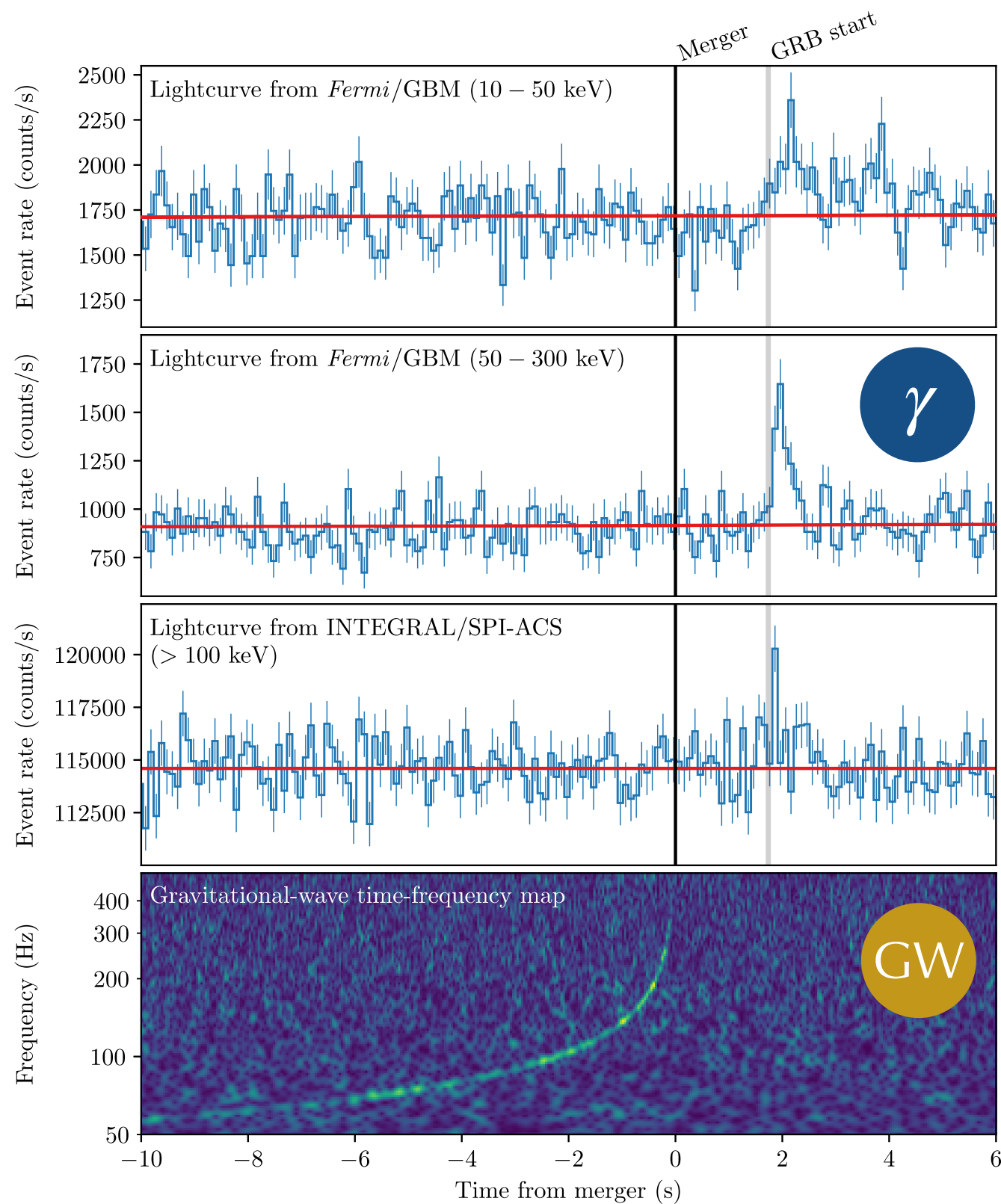
Acceleration of **cosmic rays (CRs)** - especially in the aftermath of cataclysmic events, sometimes visible in **gravitational waves**.



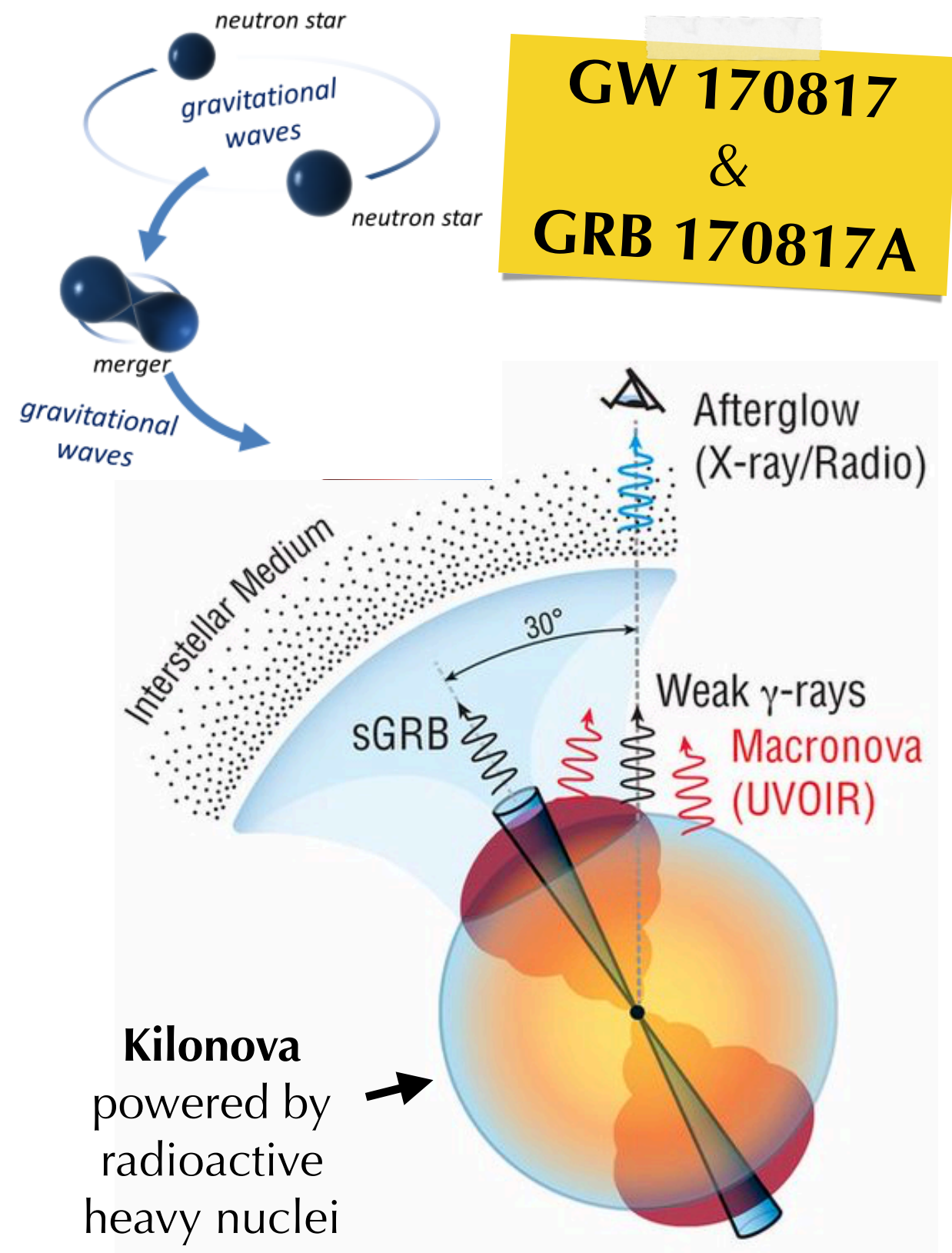
Secondary **neutrinos** and **gamma-rays** from pion decays:



Example: GW 170817

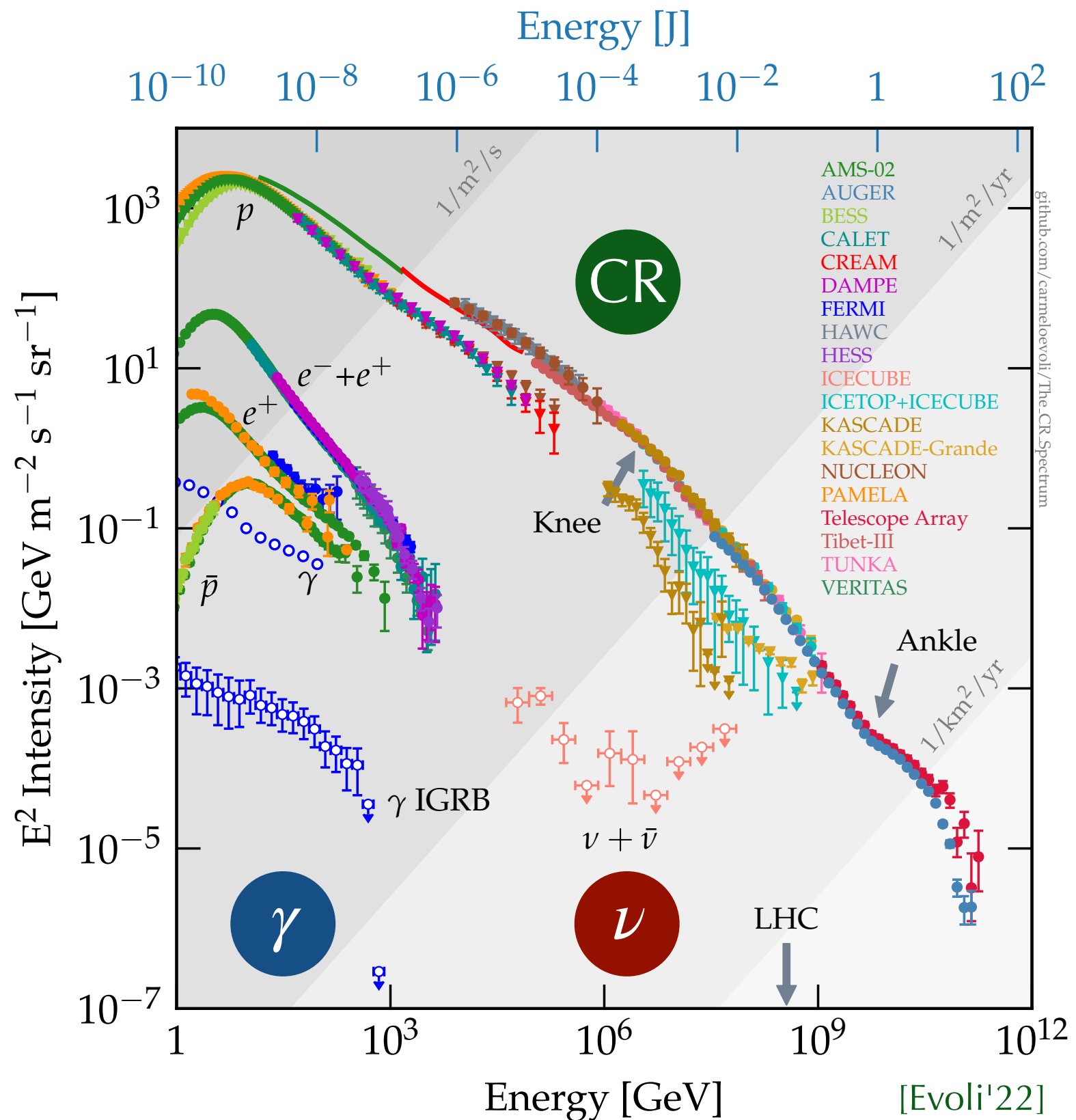


[LVD, *Fermi* & INTEGRAL, **ApJ** 848 (2017) no.2, L13]

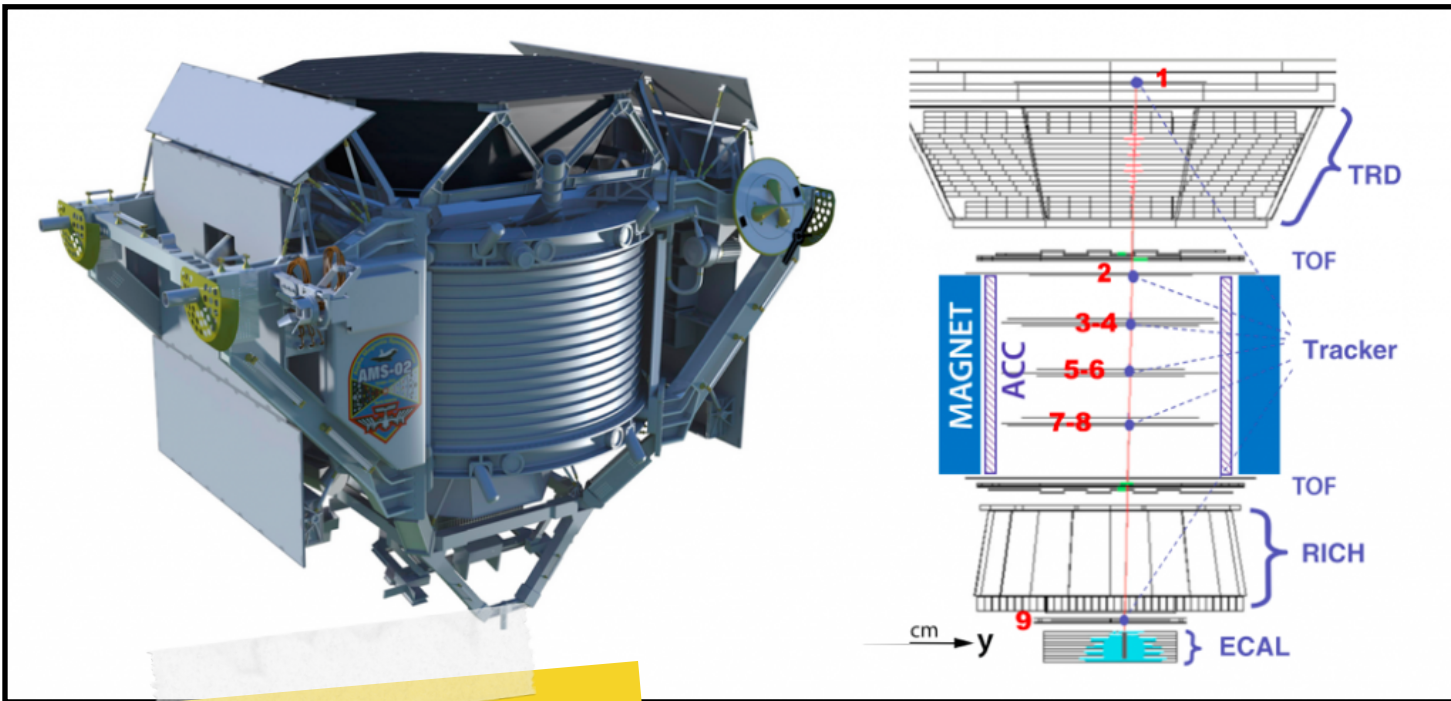


Cosmic Rays

- Cosmic rays (CRs) are **energetic nuclei** and (at a lower level) leptons.
- Spectrum follows a **power-law** over many orders of magnitude, indicating a **non-thermal origin**.
- **Direct observation** with satellite and balloon-borne experiments up to TeV
- **Indirect observation** as air showers above 10-100 TeV



Space-Based Detectors



AMS-02
onboard the
International Space
Station (ISS)

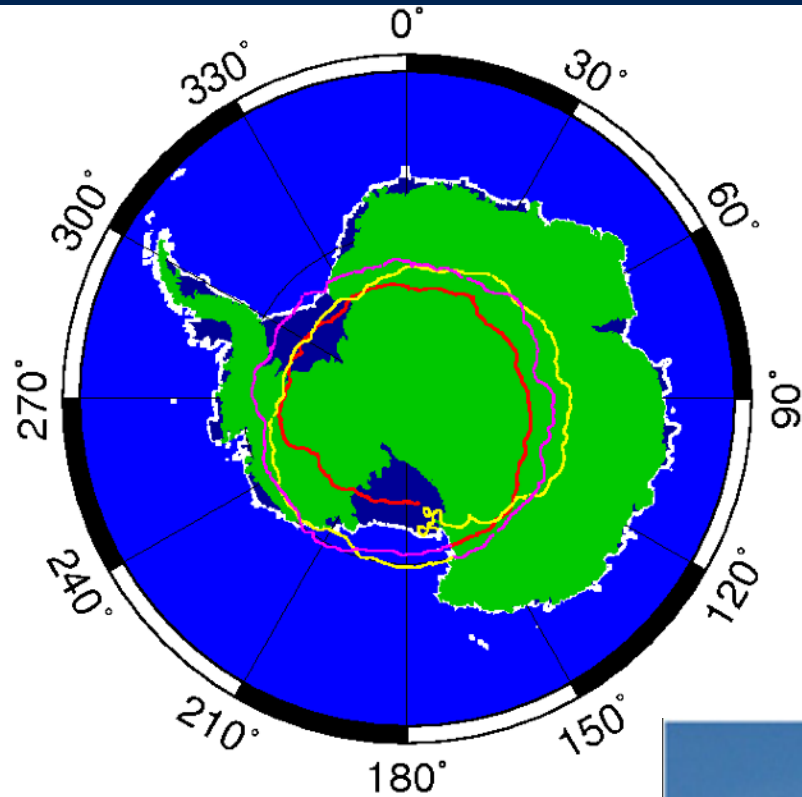


- Space-based cosmic ray observation allows for good particle identification due to magnetic spectrometers.
- Expensive and limited in size!

PAMELA detector
attached to Russian
commercial satellite



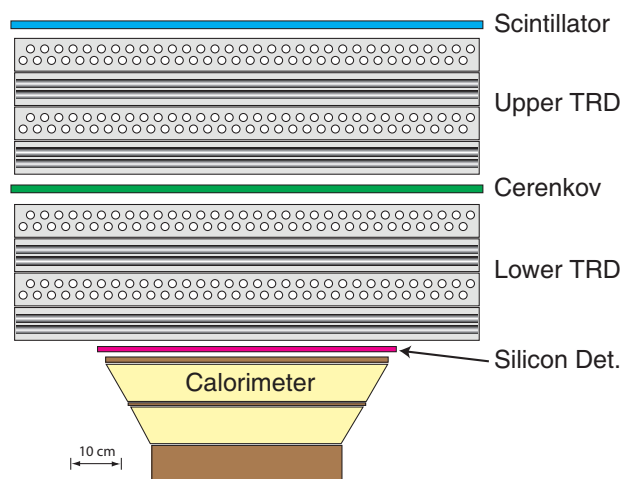
Balloon-Borne Experiments



flight path of
CREAM-V
December 1, 2009 -
January 8, 2010

- Stratospheric balloon-borne experiments are more cost efficient and allows for somewhat larger payloads.
- Limited in observation time of weeks per flight.

detector
schematics

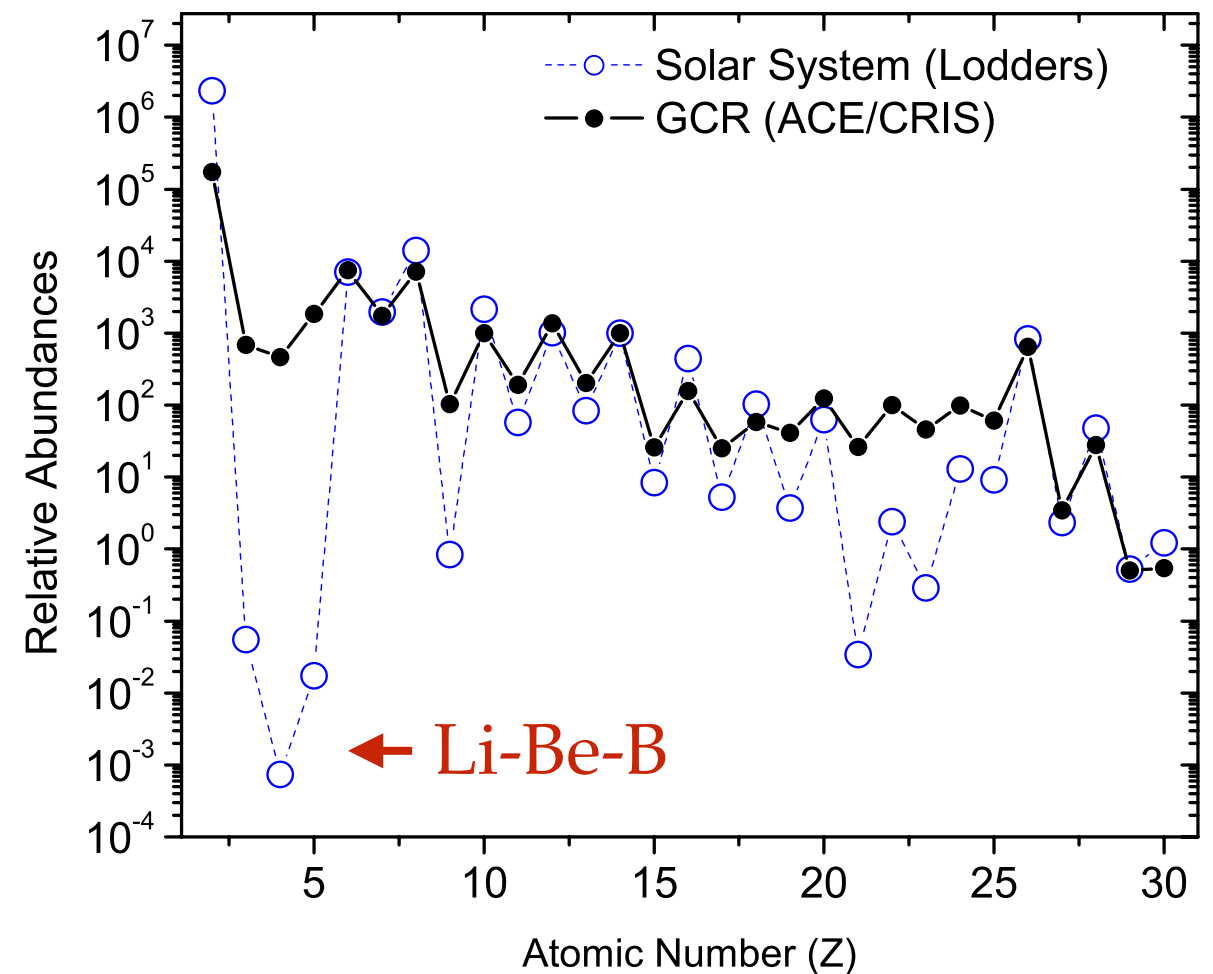
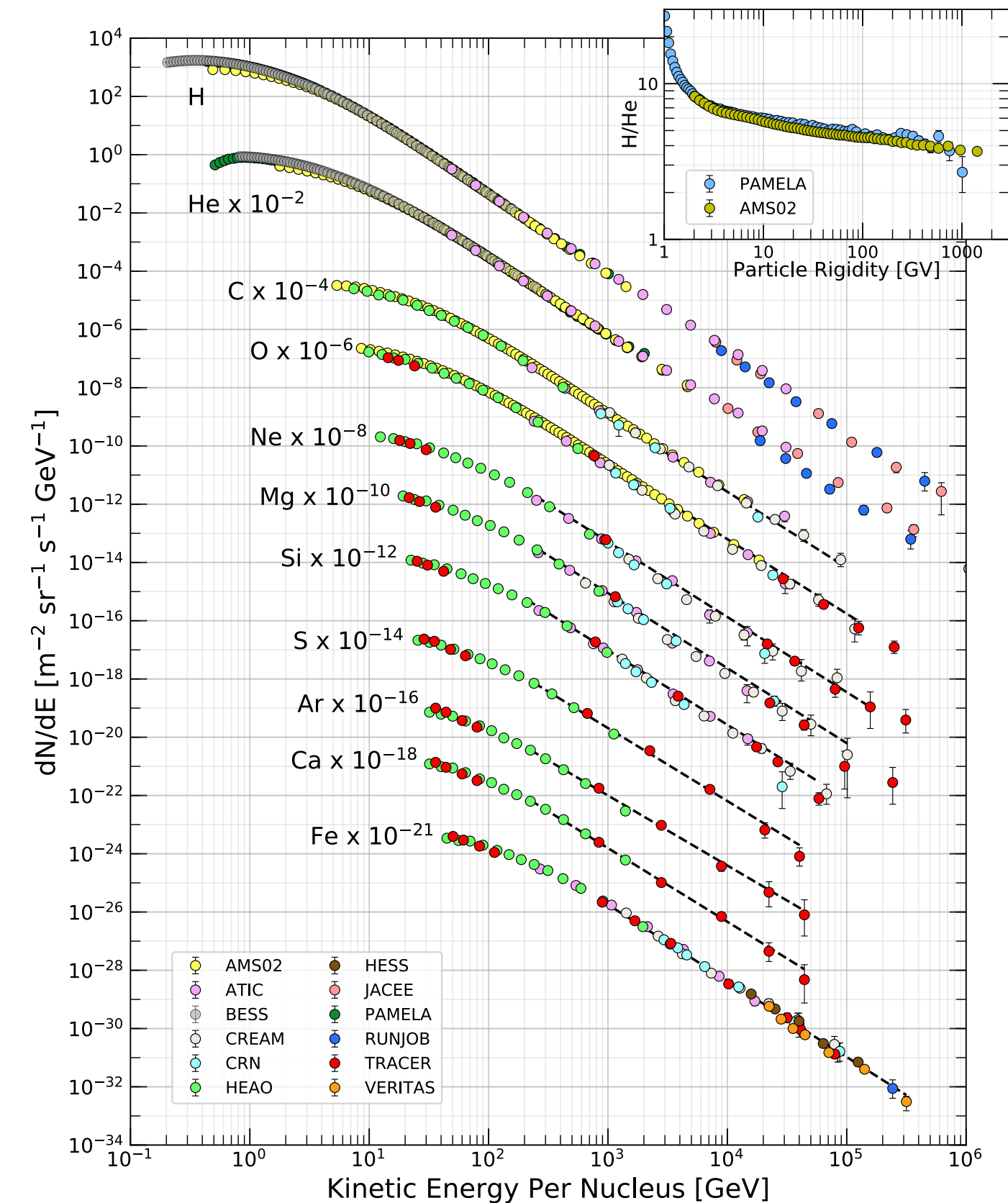


launch of **CREAM**
balloon-borne
experiment

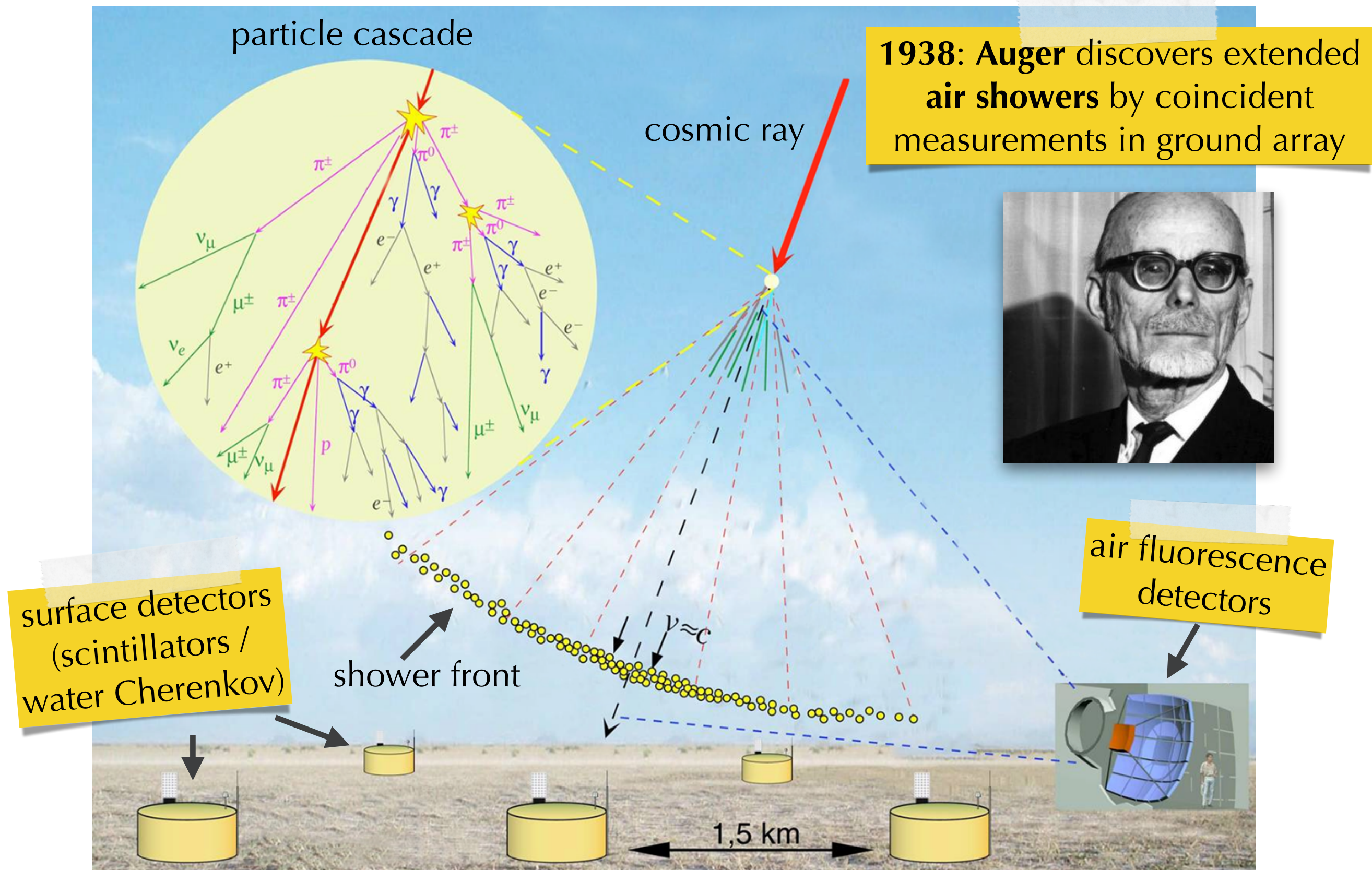


Cosmic Ray Composition

- Individual isotopes follow similar power-law spectra.
- Relative abundance differs from solar system.
- **CR spallation** on gas significant source of Li, Be & B.



Cosmic Ray Air Showers

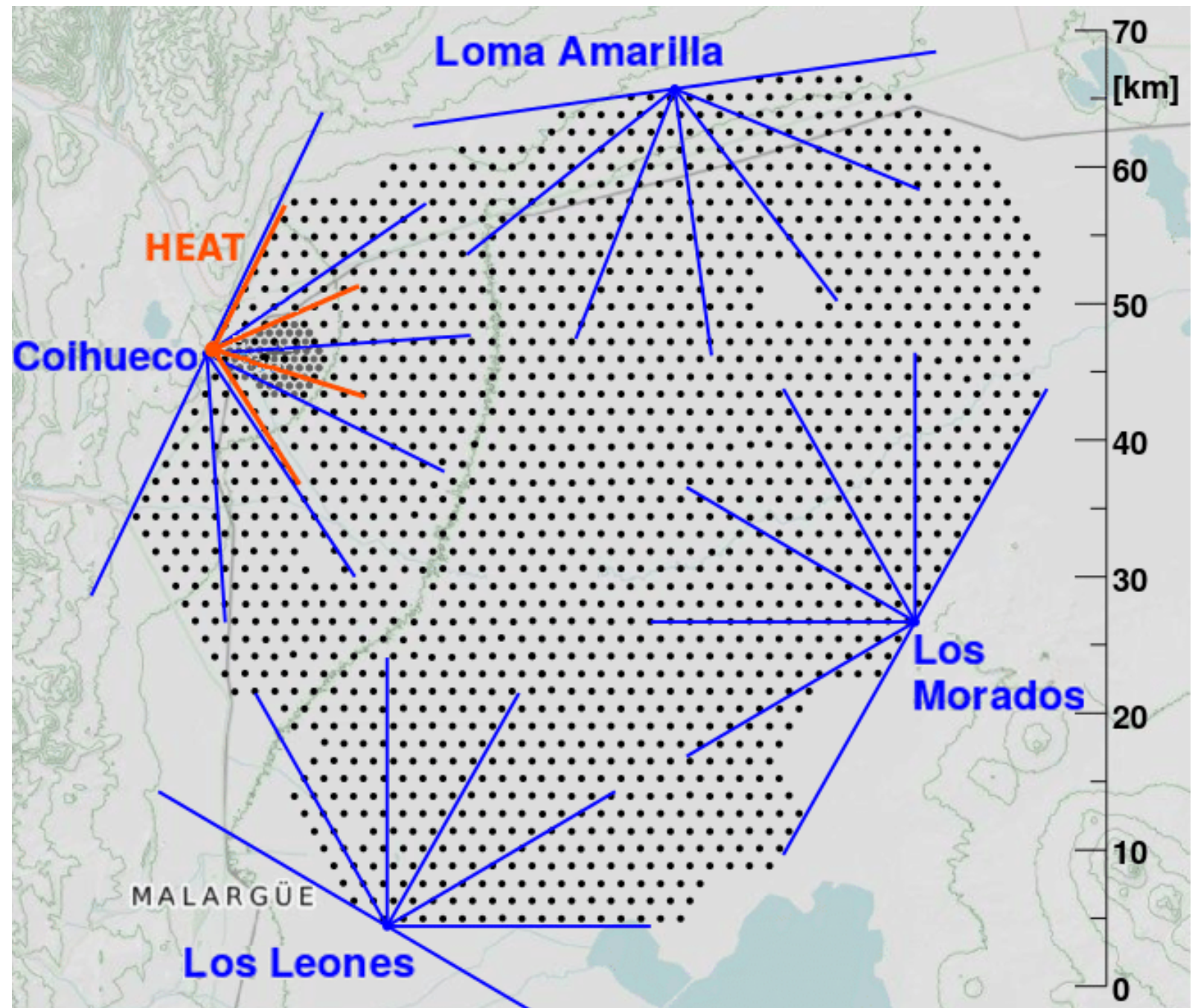


Example : Auger Observatory

fluorescence detector

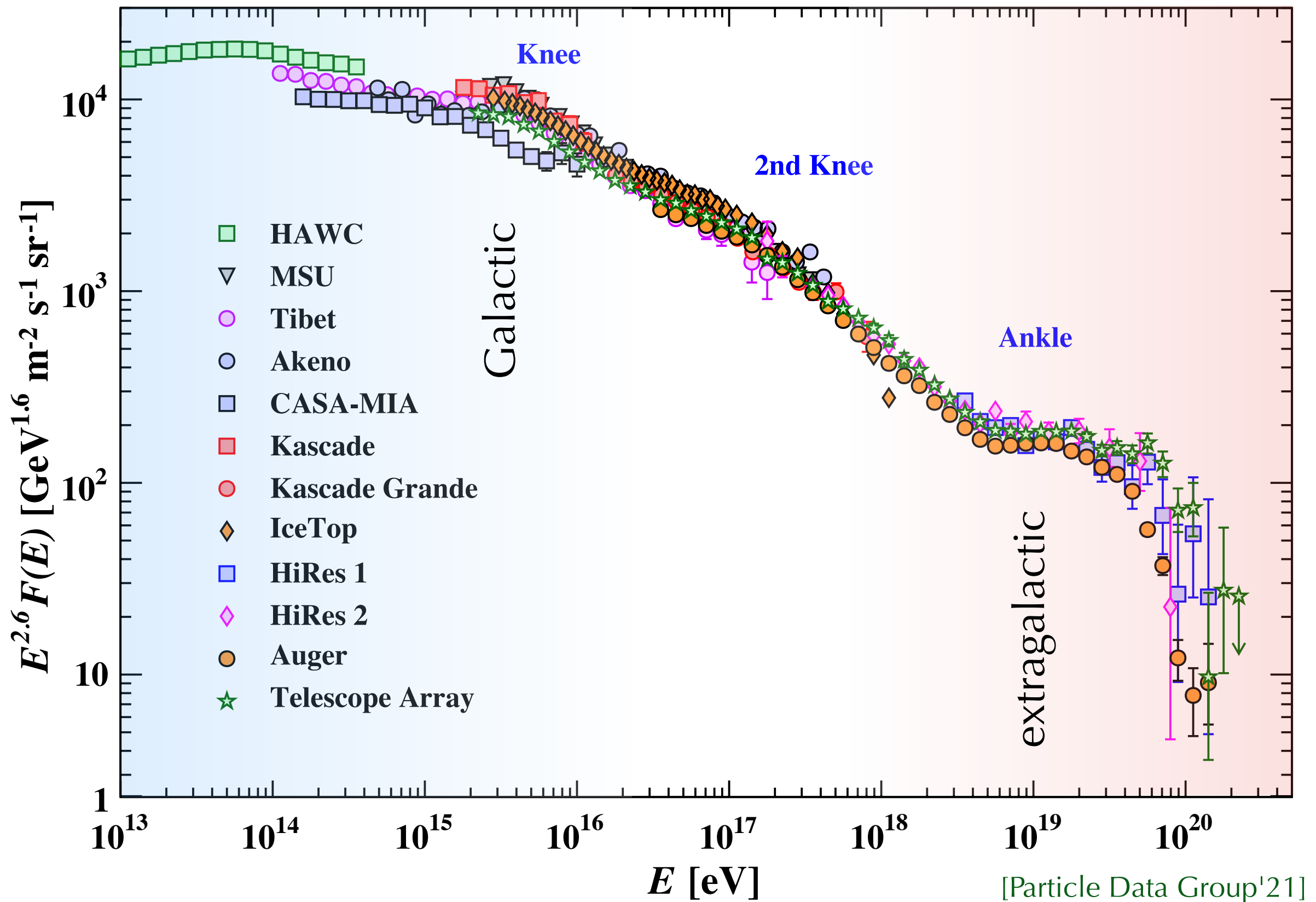


water Cherenkov detector



Auger Observatory covering 3000 km² with 1660 surface detectors and 4 fluorescence detectors

What is the Origin of Cosmic Rays?



Cosmic Ray Gyration

- CR momentum and energy:

$$\mathbf{p} = \gamma m \beta c \quad p_0 = \gamma m c$$

- CR motion under **Lorentz force**:

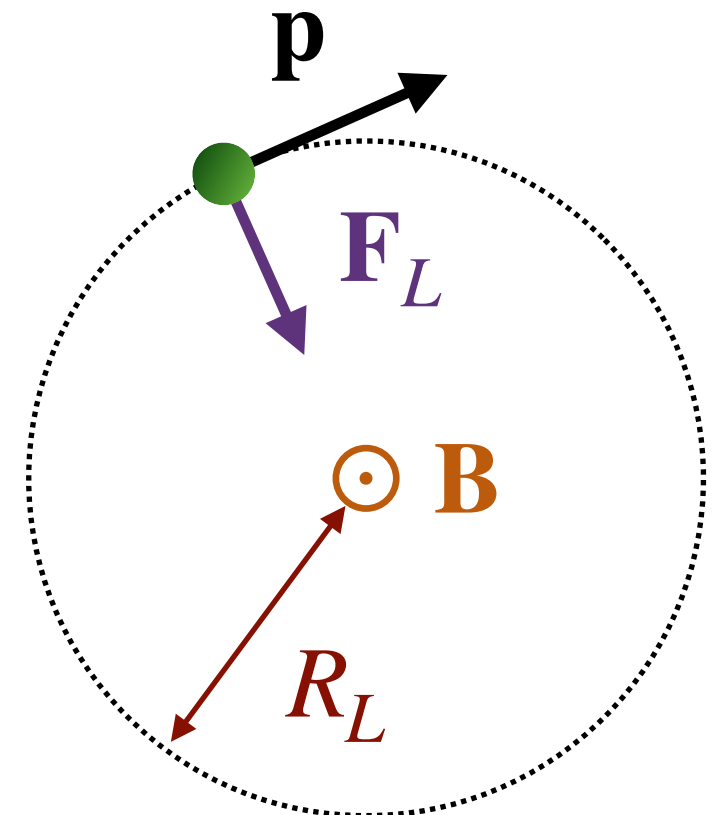
$$\dot{\mathbf{p}} = q\boldsymbol{\beta} \times \mathbf{B} = \mathbf{p} \times \left(\frac{q\mathbf{B}}{\gamma m c} \right) = \mathbf{p} \times \boldsymbol{\Omega}$$

- **Larmor frequency**:

$$\Omega_B = \frac{qB}{\gamma m c} = \frac{2\pi}{T}$$

- **Larmor radius**:

$$R_L = \frac{\beta c}{\Omega_B} = \frac{|\mathbf{p}| c}{qB} = \frac{\mathcal{R}}{B}$$



Note: Maxwell's Equations

- We will work in **Gaussian units** (*EM fields have same units*).
- **Maxwell's equations:**

$$\underbrace{\nabla \cdot \mathbf{B} = 0}_{\text{Gauss's law}}$$

$$\underbrace{\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B}}_{\text{Faraday's law}}$$

$$\underbrace{\nabla \cdot \mathbf{E} = 4\pi\rho}_{\text{Gauss's law}}$$

$$\underbrace{\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial}{\partial t} \mathbf{E}}_{\text{Ampere's law}}$$

- **Lorentz force:** $\mathbf{F} = q \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)$

- **Elementary charge unit:** $\frac{e^2}{\hbar c} = \alpha \simeq \frac{1}{137}$

Conversion between Metric Systems

- **Gaussian (G)** to **SI units** (recall relation $\epsilon_0\mu_0 = c^{-2}$):

$$\rho_{\text{SI}} = \sqrt{4\pi\epsilon_0} \rho_{\text{G}} \quad \mathbf{j}_{\text{SI}} = \sqrt{4\pi\epsilon_0} \mathbf{j}_{\text{G}} \quad \mathbf{E}_{\text{SI}} = \frac{\mathbf{E}_{\text{G}}}{\sqrt{4\pi\epsilon_0}} \quad \mathbf{B}_{\text{SI}} = \sqrt{\frac{\mu_0}{4\pi}} \mathbf{B}_{\text{G}}$$

- **Gaussian** to **Heaviside-Lorentz (HL) units** (common in particle physics):

$$\rho_{\text{HL}} = \sqrt{4\pi} \rho_{\text{G}} \quad \mathbf{j}_{\text{HL}} = \sqrt{4\pi} \mathbf{j}_{\text{G}} \quad \mathbf{E}_{\text{HL}} = \frac{\mathbf{E}_{\text{G}}}{\sqrt{4\pi}} \quad \mathbf{B}_{\text{HL}} = \frac{\mathbf{B}_{\text{G}}}{\sqrt{4\pi}}$$

- **Elementary charge unit:**

$$e_{\text{G}}^2 = \frac{e_{\text{SI}}^2}{4\pi\epsilon_0} = \frac{e_{\text{HL}}^2}{4\pi} \simeq \frac{\hbar c}{137}$$

- In addition, **natural HL units** further use $\hbar = c = 1$.

Natural Units

- In **natural units**, the speed of light c and Planck's reduced constant $\hbar$ are:

$$c = \hbar = 1$$

- Time, length, mass and momentum are now expressed in **units of energy**:

$$\tilde{x} \equiv \frac{x}{\hbar c} \quad \tilde{t} \equiv \frac{t}{\hbar} \quad \tilde{p} \equiv pc \quad \tilde{m} \equiv mc^2$$

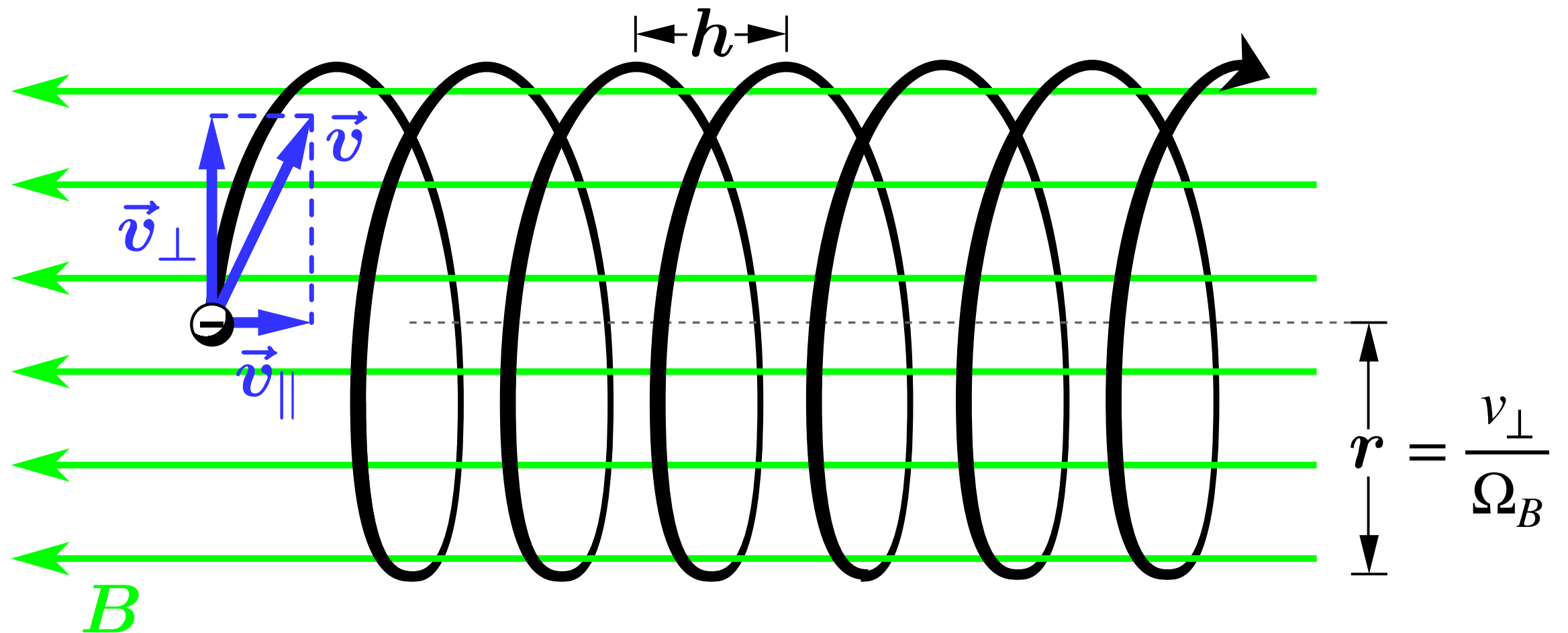
- Conversion back to standard (SI) units can be done via the identities:

$$c \simeq 3 \times 10^8 \text{ m/s} \quad \hbar c \simeq 200 \text{ MeV fm}$$

- It is also convenient to set **Boltzmann's constant** $k_B = 1$ with conversion:

$$\frac{k_B T}{300 \text{ K}} \simeq \frac{1}{40} \text{ eV}$$

Cosmic Ray Gyration

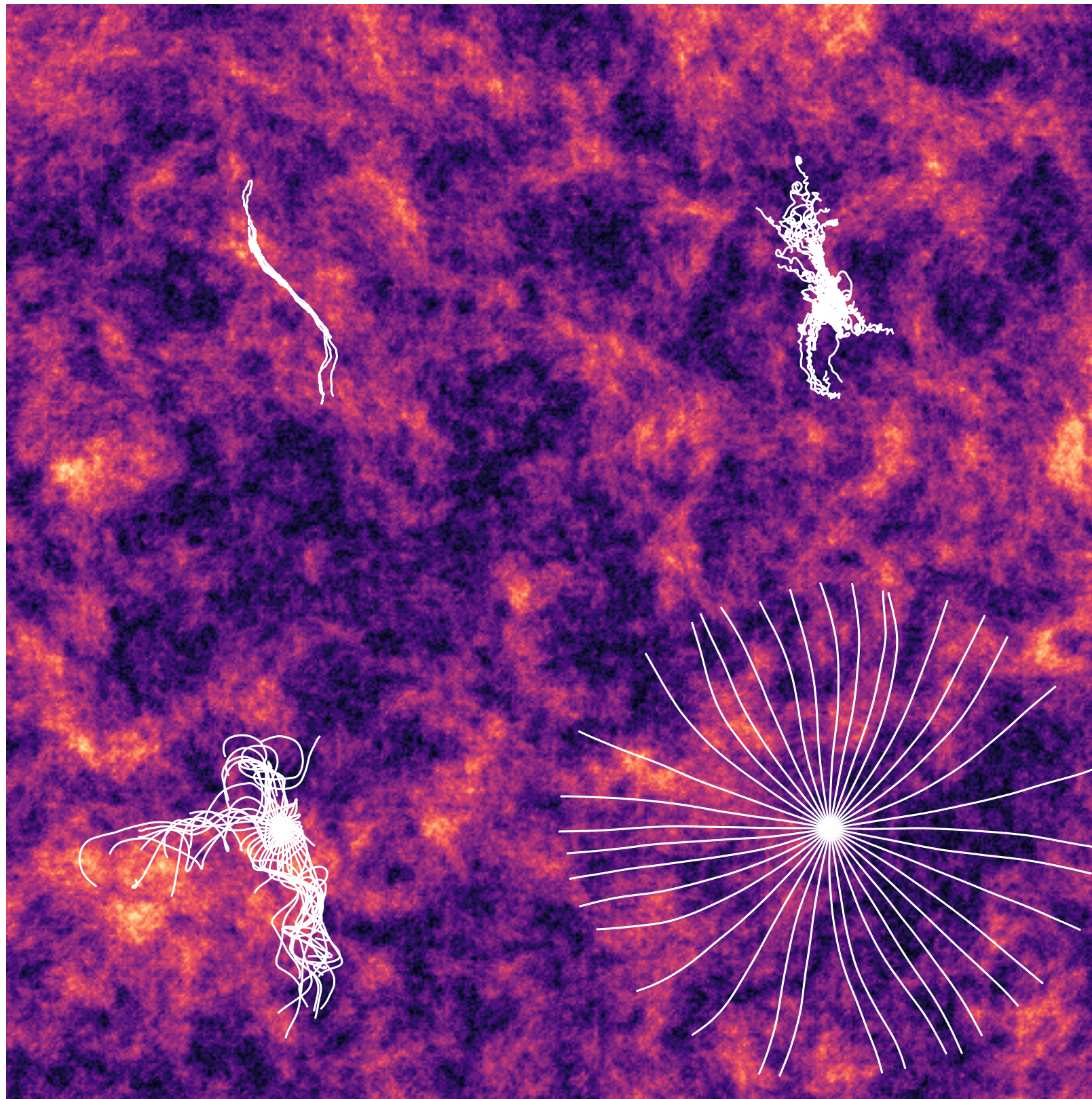


Pitch angle α between $\mathbf{v}(t)$ and $\mathbf{B}_0$ remains constant in time.

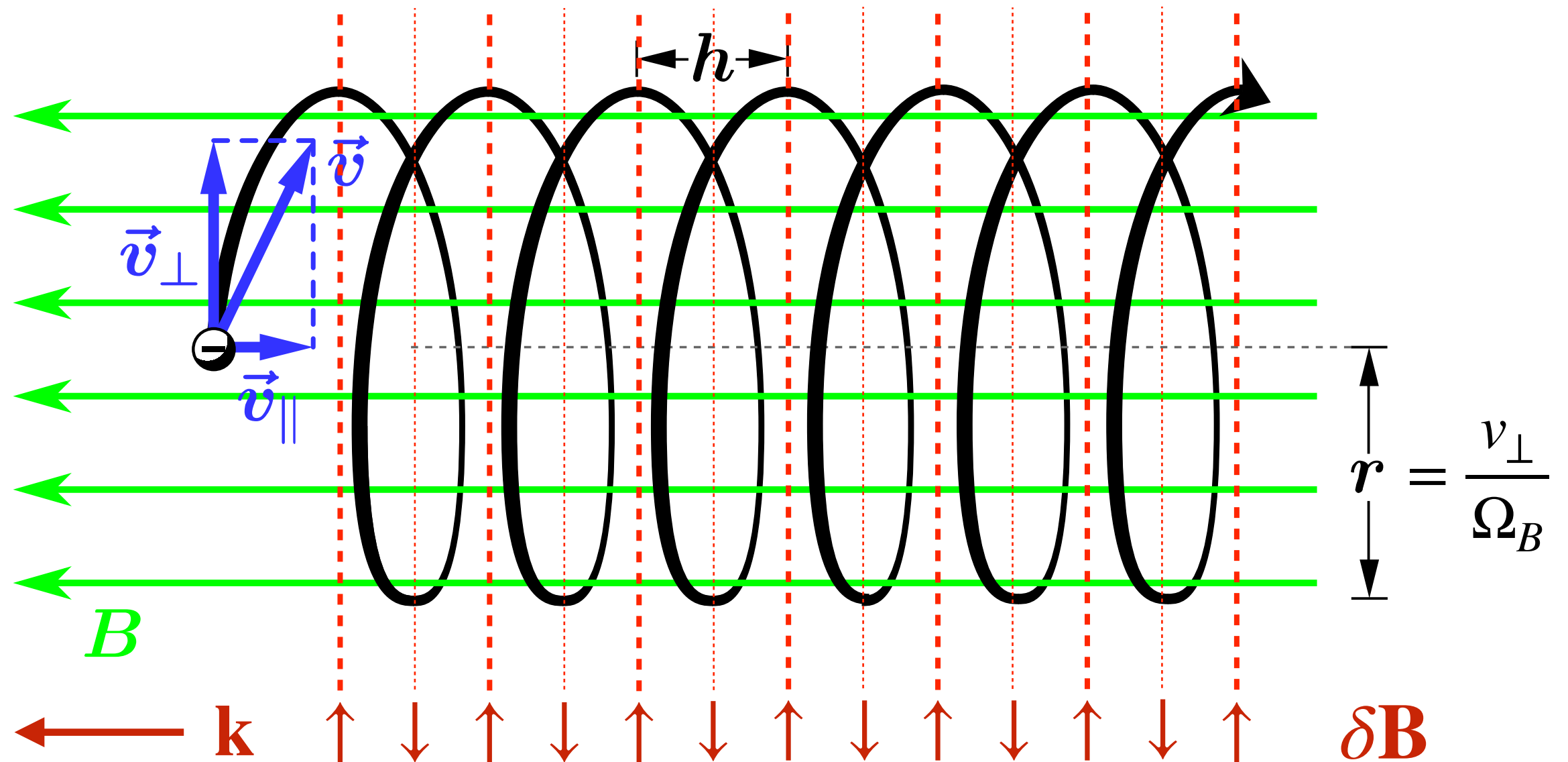
Path is a superposition of circular motion in the plane perpendicular to $\mathbf{B}_0$ and linear motion along $\mathbf{B}_0$ with velocity:

$$v_{\parallel} = \cos \alpha v \equiv \mu v.$$

Magnetic Turbulence



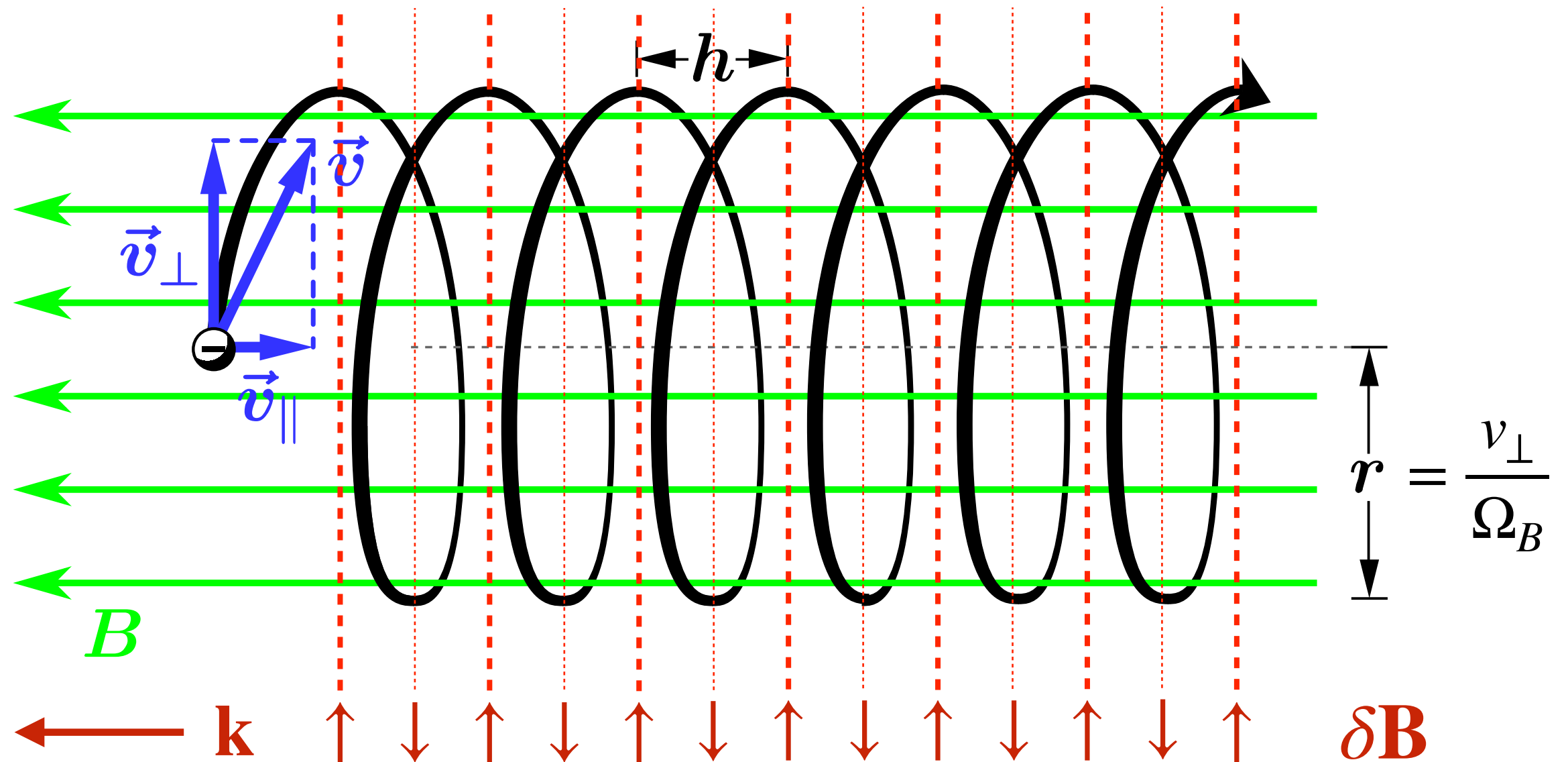
Resonant Scattering



Consider now a **magnetic perturbation** in form of a plane wave:

$$\delta \mathbf{B} = \delta B \mathbf{e}_x \cos(kz + \alpha)$$

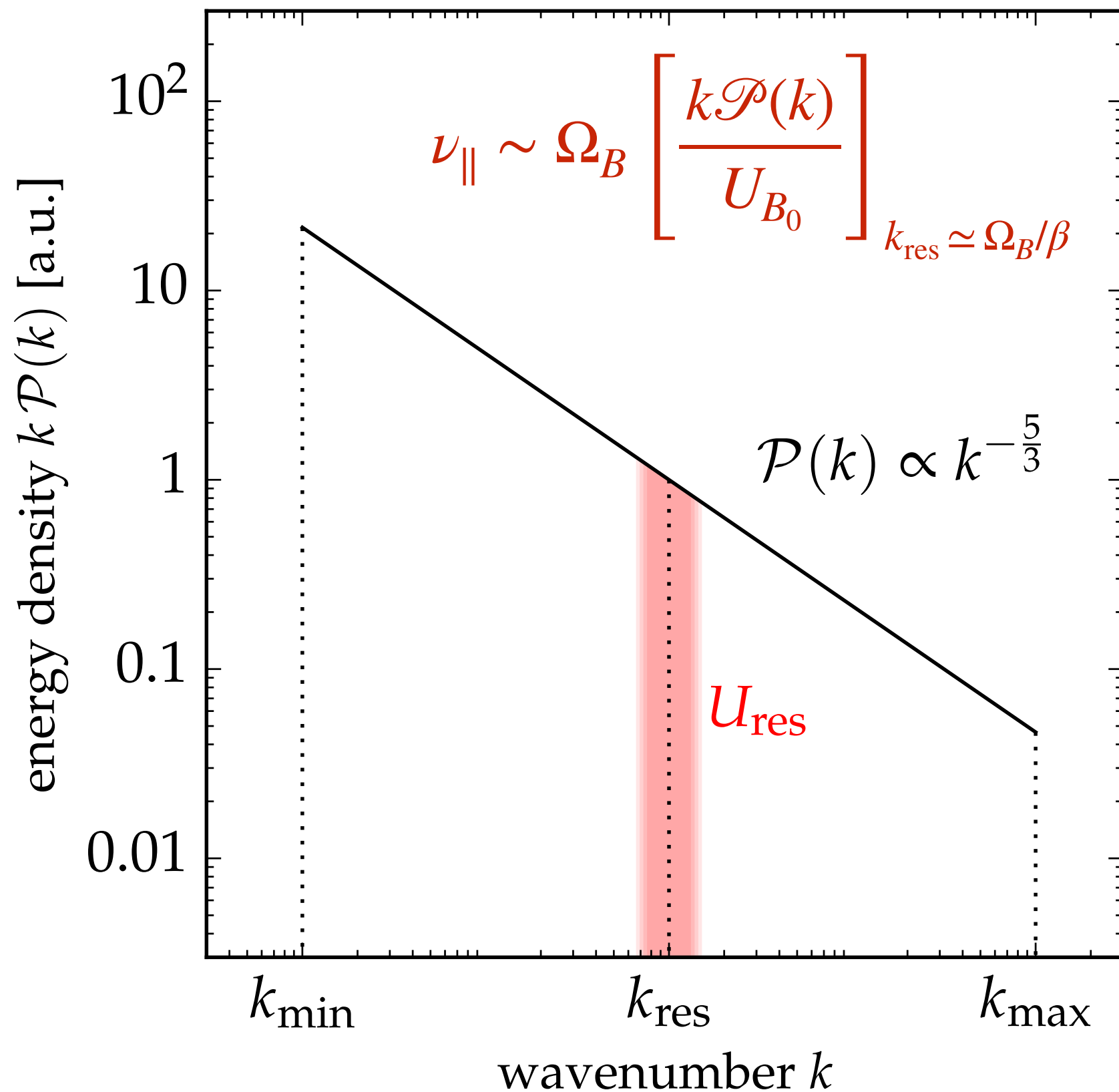
Resonant Scattering



The time-averaged Lorentz force $\delta \mathbf{F}_L = q \boldsymbol{\beta} \times \delta \mathbf{B}$ along the path has the strongest contribution at the **resonance**:

$$kv_\parallel = \pm \Omega_B$$

Resonant Scattering



Diffusion Approximation

- Treating the effect of magnetic turbulence as a collision term with rate $\nu_{\parallel}$ that isotropizes the PSD, we arrive at the **diffusion equation**:

$$\partial_t n - \partial_i \left(K_{ij} \partial_j n \right) = 0$$

- For background field $B_0 \mathbf{e}_z$ the **diffusion tensor** can be approximated:

$$\mathbf{K} \simeq \frac{\beta^2}{3} \begin{pmatrix} \nu_{\perp}^{-1} & \nu_A^{-1} & 0 \\ -\nu_A^{-1} & \nu_{\perp}^{-1} & 0 \\ 0 & 0 & \nu_{\parallel}^{-1} \end{pmatrix} \quad \begin{aligned} \nu_{\perp} &\simeq \nu_{\parallel} + \Omega_B^2 / \nu_{\parallel} \\ \nu_A &\simeq \Omega_B + \nu_{\parallel}^2 / \Omega_B \end{aligned}$$

- For instance, parallel diffusion in **Kolmogorov turbulence**:

$$\nu_{\parallel} \sim \Omega_B \left[\frac{k \mathcal{P}(k)}{U_{B_0}} \right]_{k_{\text{res}} \simeq \Omega/\beta} \sim \Omega_B^{2-\frac{5}{3}} \sim R_L^{-\frac{1}{3}}$$

Cosmic Ray Transport

$$\begin{aligned}
 \frac{\partial n_i}{\partial t} = & \frac{\partial}{\partial r_a} \left[K_{ab} \frac{\partial}{\partial r_b} n_i \right] && \text{(spatial diffusion)} \\
 & + \frac{\partial}{\partial p} \left[p^2 \tilde{K} \frac{\partial}{\partial p} \left(\frac{n_i}{p^2} \right) \right] && \text{(momentum diffusion)} \\
 & - \frac{\partial}{\partial r_a} (u_a n_i) && \text{(convection)} \\
 & - \frac{\partial}{\partial p} \left[\dot{p} n_i - \frac{p}{3} \left(\frac{\partial u_a}{\partial r_a} \right) n_i \right] && \text{(continuous energy loss)} \\
 & - \Gamma_i^{\text{dec}} n_i && \text{(CR decay)} \\
 & - c \rho_{\text{bgr}} \sigma_i n_i && \text{(loss from CR collisions)} \\
 & + c \rho_{\text{bgr}} \sum_j \int dE_j \frac{d\sigma_{j \rightarrow i}}{dE_i} n_j && \text{(gain from CR collisions)} \\
 & + Q_i && \text{(source term)}
 \end{aligned}$$

Galactic Cosmic Rays

- *Standard paradigm:*
Galactic CRs accelerated
in supernova remnants

[Baade & Zwicky'34]
[Ginzburg & Sirovatskii'64]

- diffusive shock
acceleration:

$$n_{\text{CR}} \propto E^{-\Gamma}$$

- rigidity-dependent escape
from Galaxy:

$$n_{\text{CR}} \propto E^{-\Gamma-\delta}$$

- ★ Cosmic-ray arrival
directions are randomized.

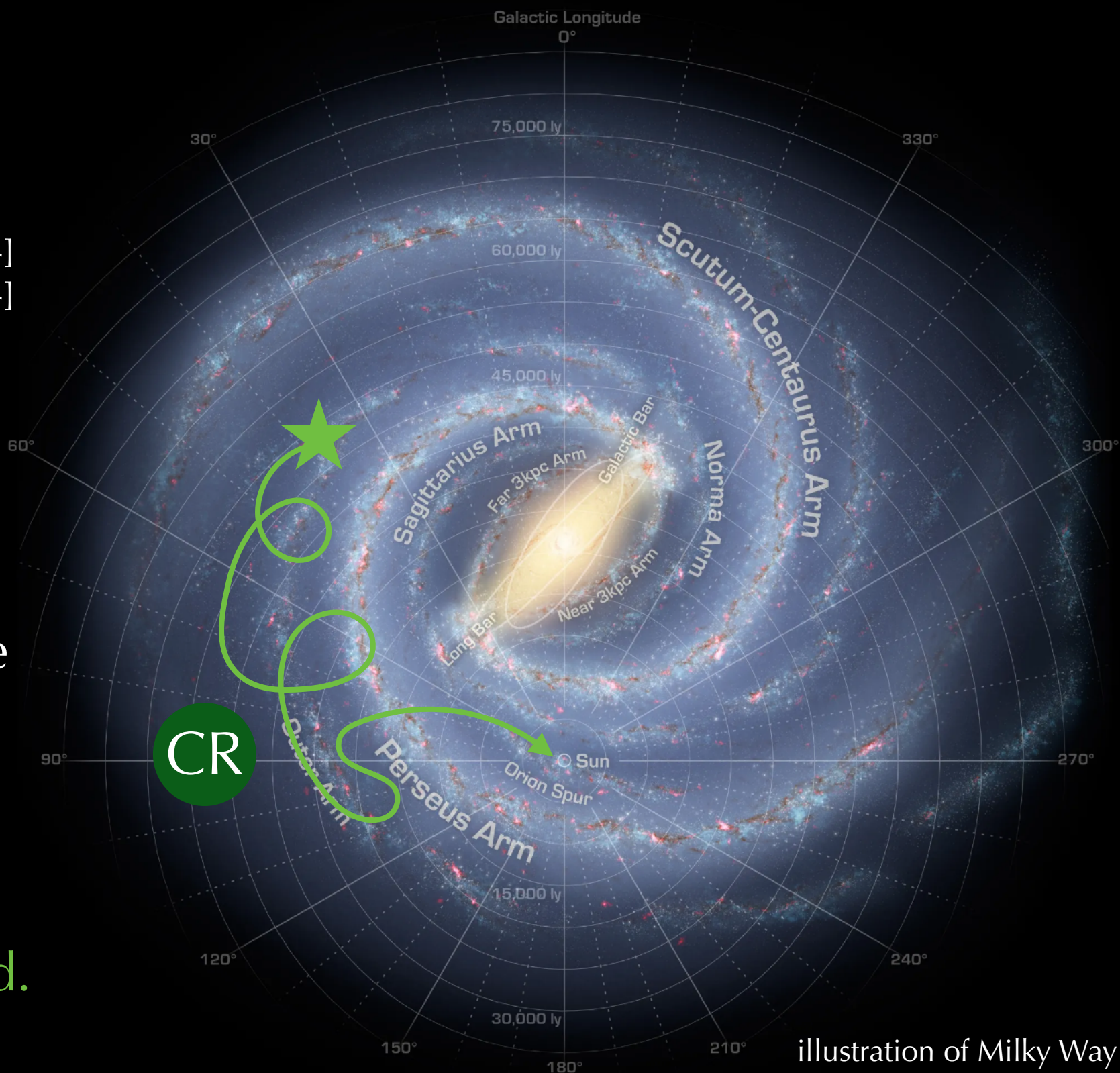
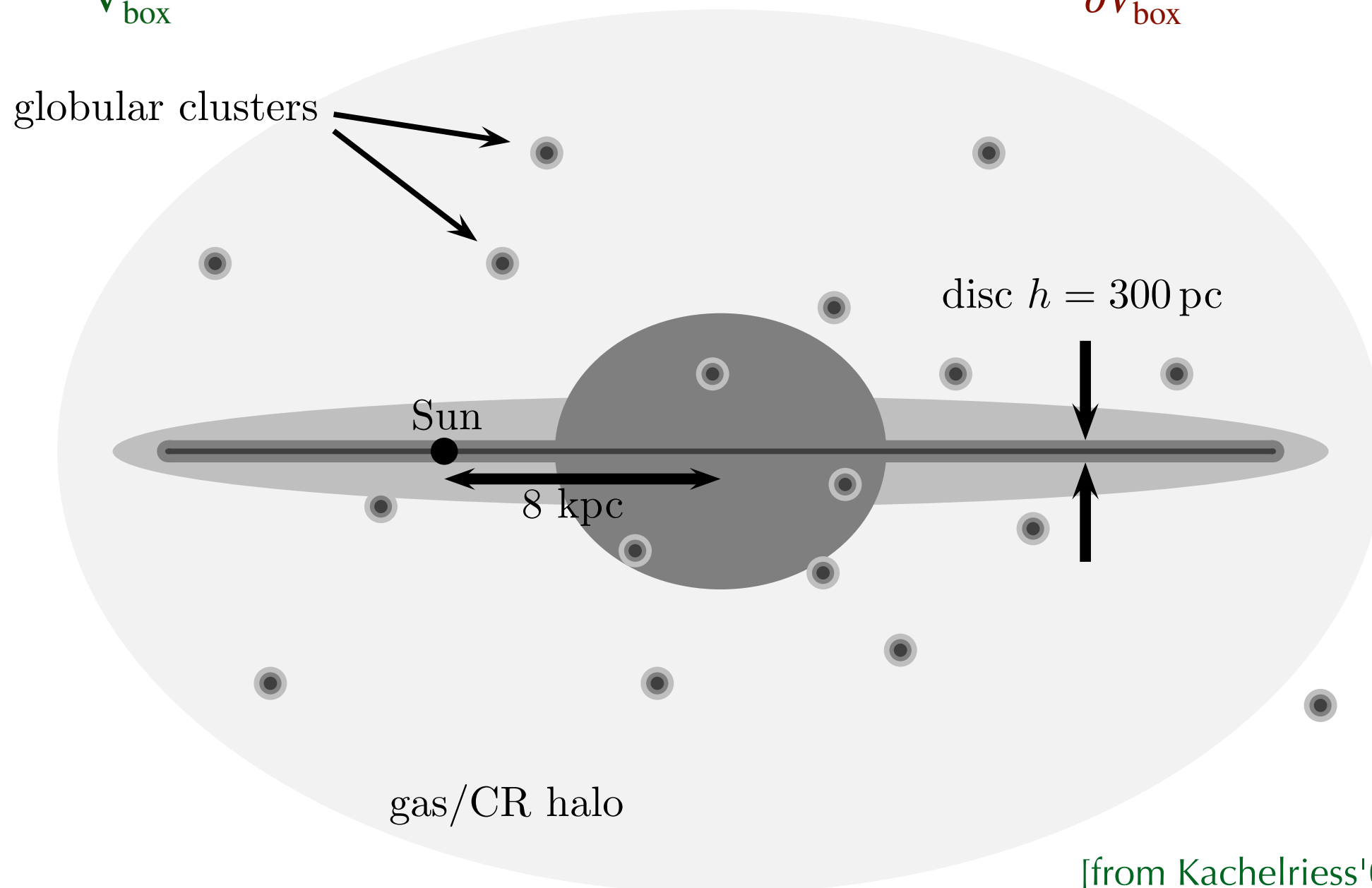


illustration of Milky Way
[Credit: NASA]

Leaky-Box Model

$$N_{\text{CR}} \equiv \int_{V_{\text{box}}} d^3r n_{\text{CR}}$$

$$\dot{N}_{\text{CR}} \simeq -\frac{N_{\text{CR}}}{\tau_{\text{loss}}} \equiv -\int_{\partial V_{\text{box}}} d\mathbf{A} \cdot \mathbf{K} \cdot \nabla n_{\text{CR}}$$



[from Kachelriess'08]

Secondary-To-Primary Ratio

- Abundance of Galactic CRs in the Li-Be-B group is larger than observed from solar system measurement.
- Phenomenon is related to production of secondary CRs (N_s) in primary CR (N_p) collisions in background molecular gas:

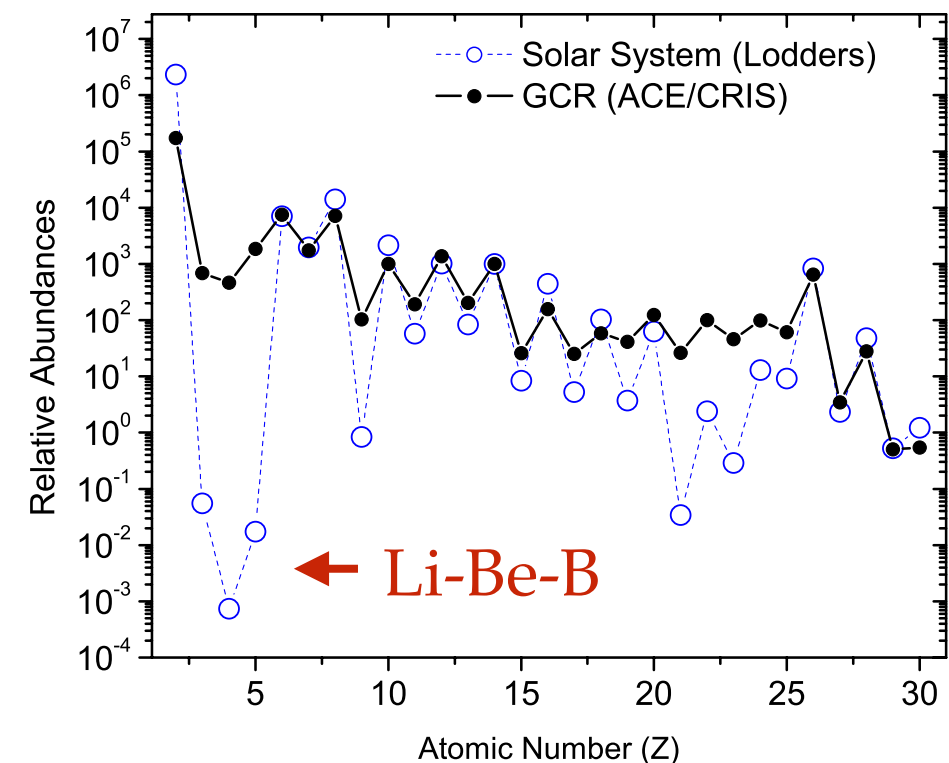
$$\partial_t N_s = -\frac{N_s}{\tau_{\text{loss}}} + c\rho\sigma_{p\rightarrow s}N_p$$

- Steady-state solution ($\partial_t N_s = \partial_t N_p = 0$):

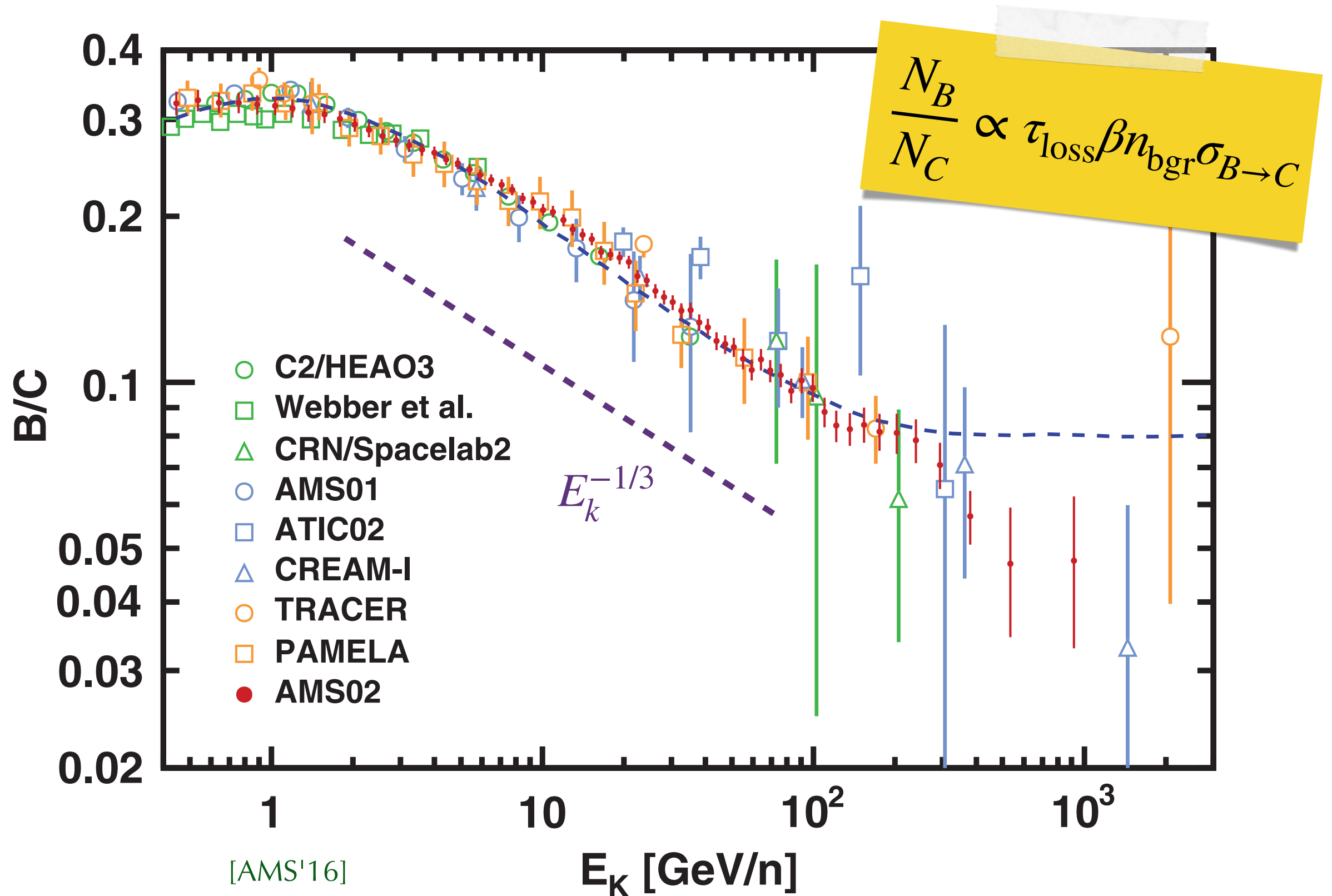
$$N_s = \tau_{\text{loss}} c\rho\sigma_{p\rightarrow s} N_p$$

- **Secondary-to-primary ratio:**

$$\frac{N_s}{N_p} = \tau_{\text{loss}}(E) c\rho\sigma_{p\rightarrow s} \propto E^{-\delta_{\text{diff}}}$$



Borton-To-Carbon Ratio



Radioactive Isotopes

- Secondary isotopes produced in CR interactions can be unstable, e.g. ^{10}Be with a lifetime $\tau_{\text{decay}} \simeq 1.5 \times 10^6$ years:

$$\partial_t N_s = -\frac{N_s}{\tau_{\text{loss}}} - \frac{N_s}{\tau_{\text{decay}}} + c\rho\sigma_{p \rightarrow s} N_p$$

- Steady-state solution is now:

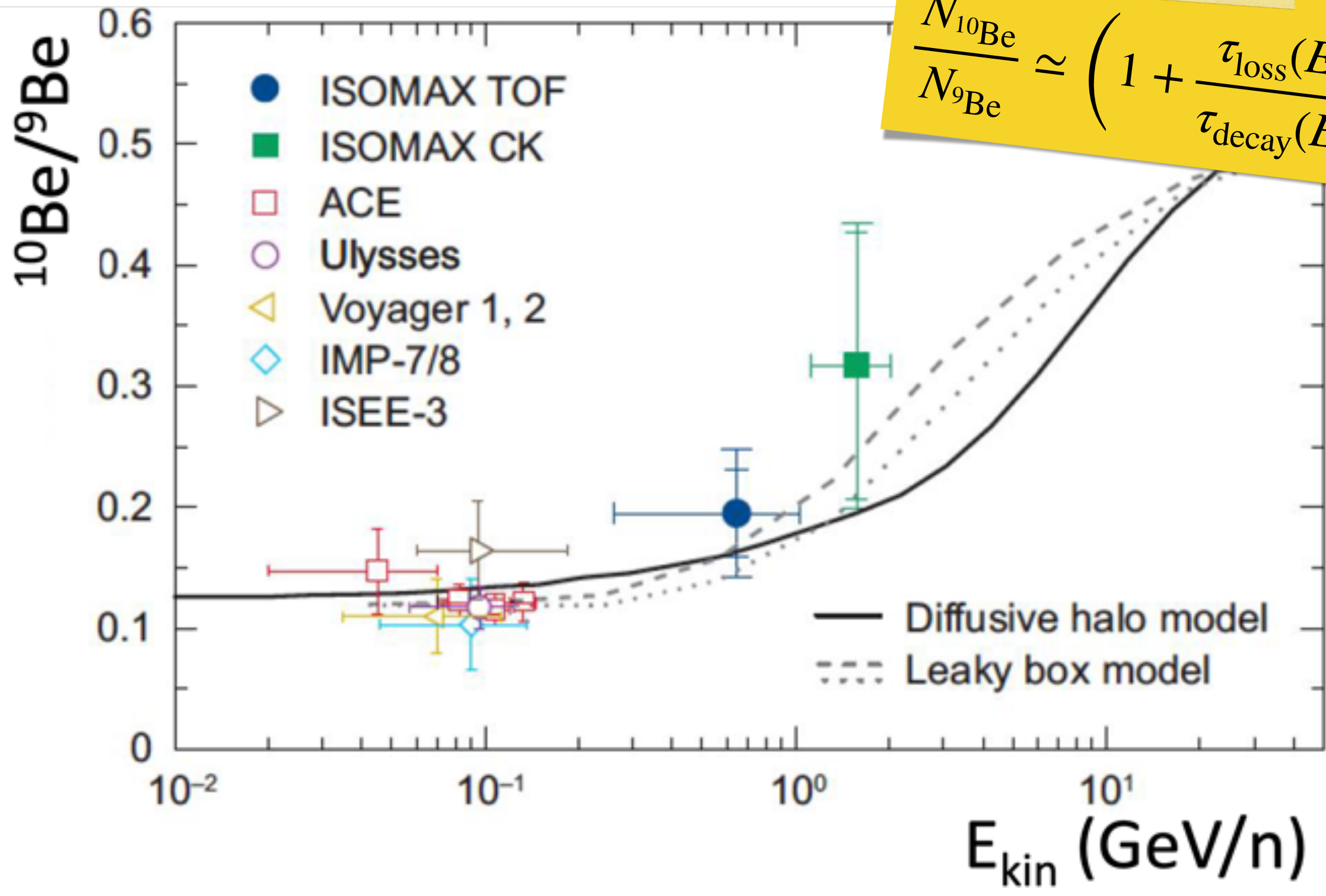
$$N_s = \frac{c\rho\sigma_{p \rightarrow s}}{\tau_{\text{loss}}^{-1} + \tau_{\text{decay}}^{-1}} N_p$$

- Beryllium is dominated by secondary CRs, so ratio of isotopes is:

$$\frac{N_{^{10}\text{Be}}}{N_{^9\text{Be}}} \simeq \left(1 + \frac{\tau_{\text{loss}}}{\tau_{\text{decay}}}\right)^{-1} \frac{\sigma_{p \rightarrow ^{10}\text{Be}}}{\sigma_{p \rightarrow ^9\text{Be}}}$$

- For $E \simeq 100$ MeV we get $\tau_{\text{loss}} \simeq 15 \times 10^6$ years.

Be¹⁰-to-Be⁹ Ratio



$$\frac{N_{^{10}\text{Be}}}{N_{^9\text{Be}}} \approx \left(1 + \frac{\tau_{\text{loss}}(E)}{\tau_{\text{decay}}(E)} \right)^{-1}$$

Energy Budget of Galactic CRs

- **Energy density** of observed CRs:

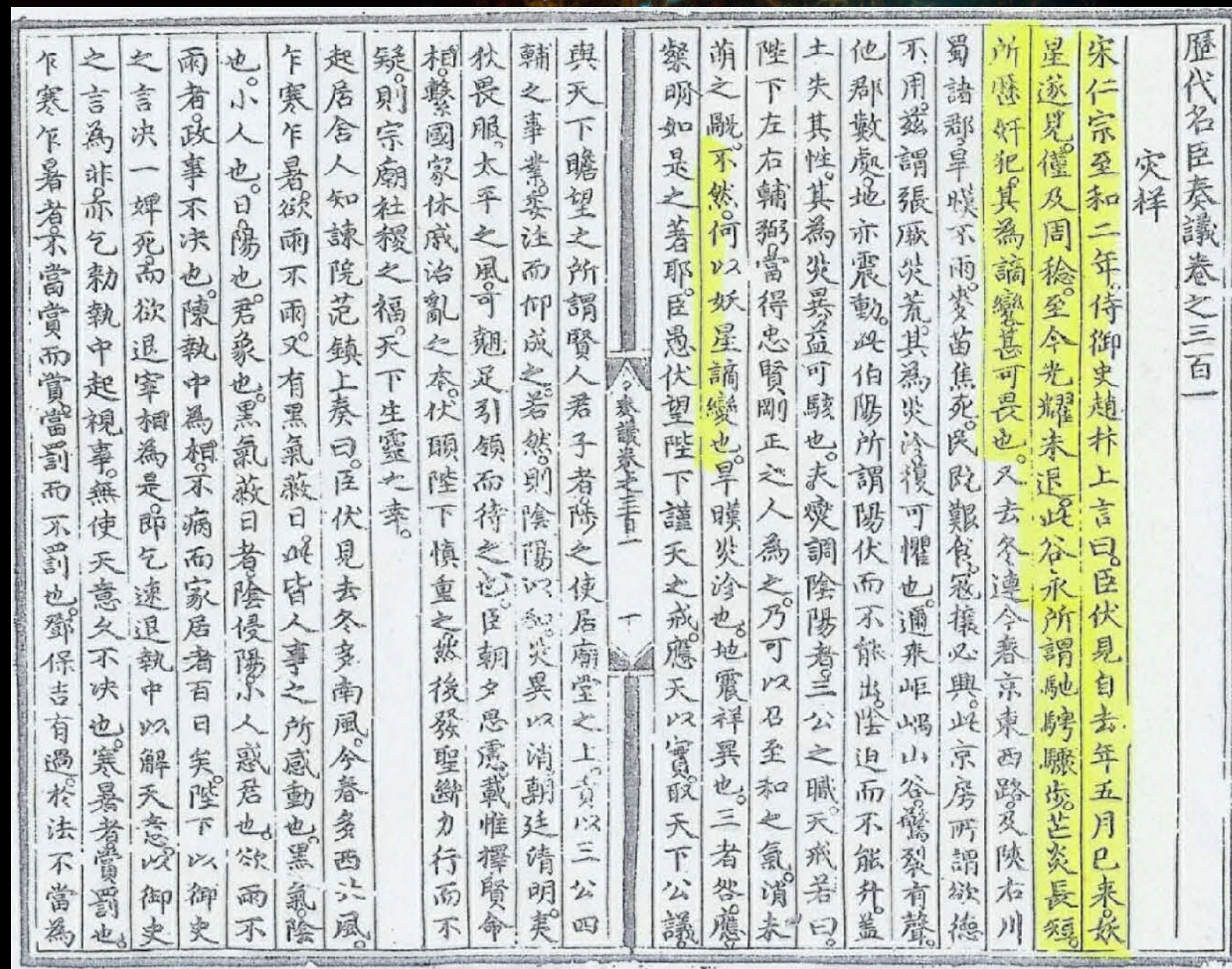
$$w_{\text{CR}} = \int dE E n_{\text{CR}}(E) \simeq 1 \frac{\text{eV}}{\text{cm}^3}$$

- Diffusion volume in leaky-box: $V \simeq 10\text{kpc} \times \pi(15\text{kpc})^2 \simeq 7000\text{kpc}^3$
- With $\tau_{\text{loss}} \simeq 15 \times 10^6$ years we can estimate the **required luminosity of Galactic CR sources**:

$$\mathcal{L}_{\text{CR}} = \frac{V \omega_{\text{CR}}}{\tau_{\text{loss}}} \simeq 5 \times 10^{53} \frac{\text{eV}}{\text{s}} \simeq 7 \times 10^{41} \frac{\text{erg}}{\text{s}}$$

- **Galactic core-collapse supernovae** release gravitational binding energy at the level of $W_{\text{kin}} \simeq 3 \times 10^{51}$ erg.
- **Can account for $\mathcal{L}_{\text{CR}}$ if $\mathcal{O}(10)\%$ of W_{kin} is converted to CRs.**

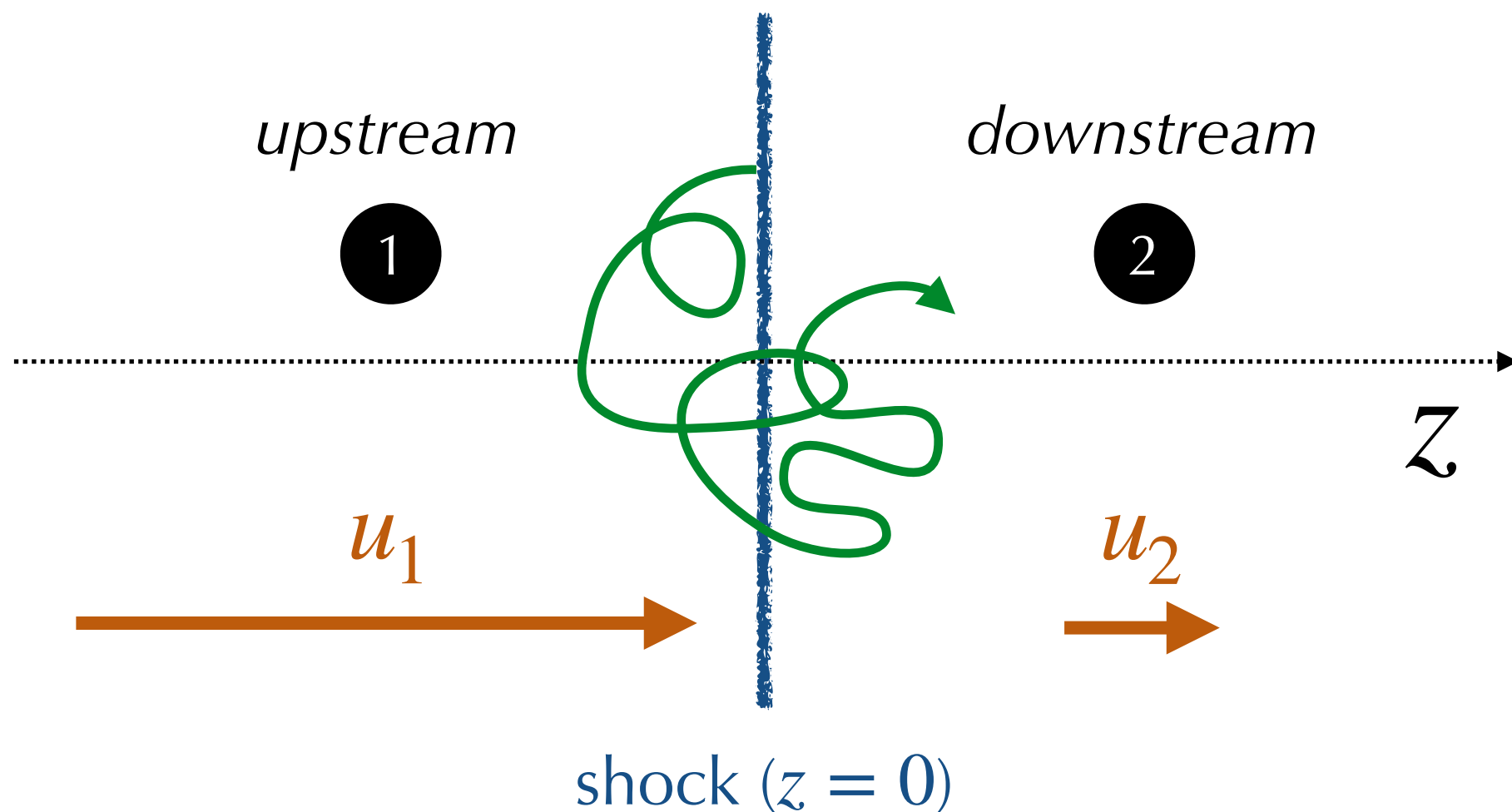
Supernovae



Crab Nebula (SN 1054)

Diffusive Shock Acceleration

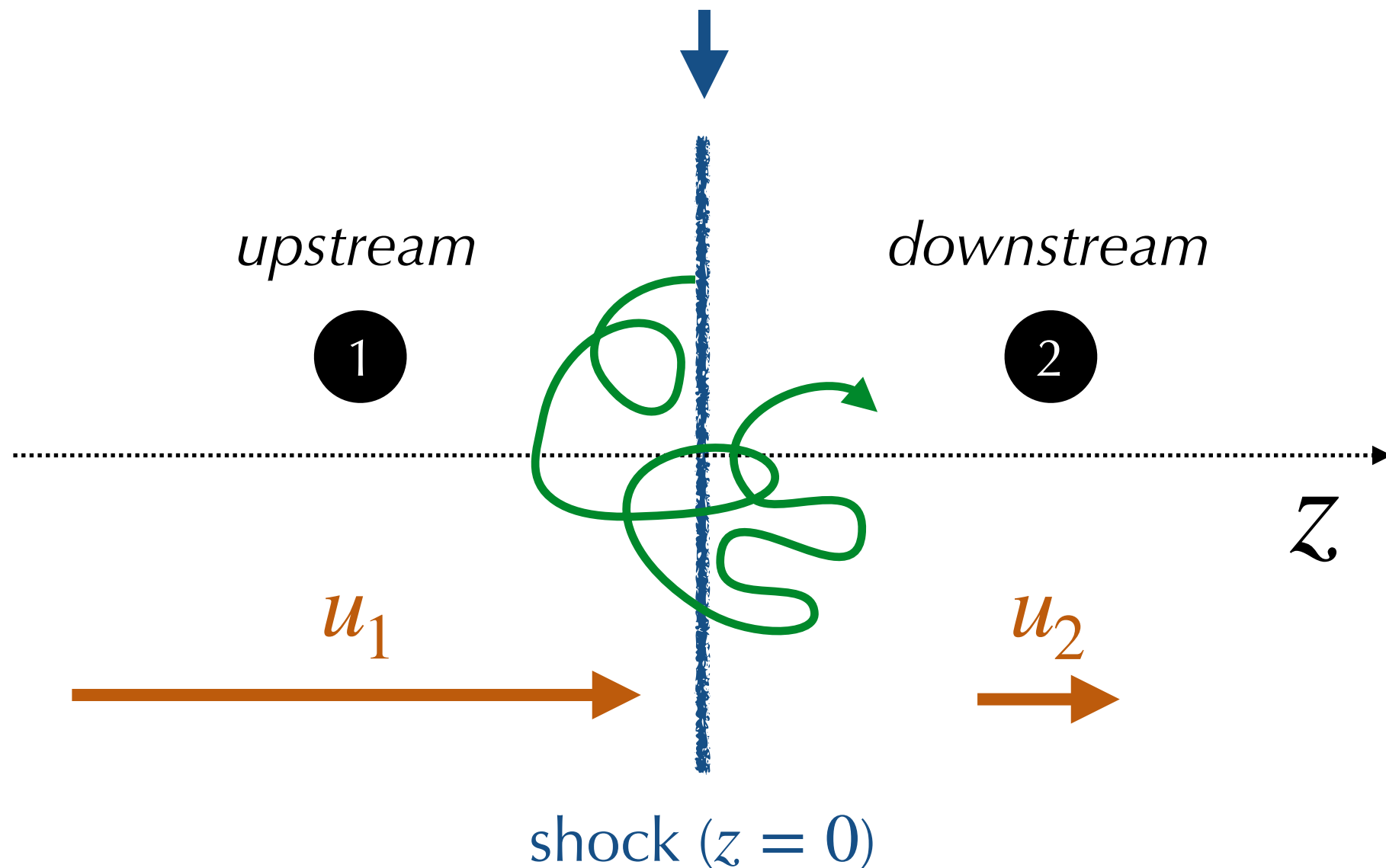
$$\underbrace{u \partial_z f}_{\text{convection}} - \underbrace{\partial_z \left(K_{\parallel} \partial_z f \right)}_{\text{diffusion}} - \underbrace{\frac{1}{3} \frac{du}{dz} p \partial_p f}_{\text{compression}} = Q$$



Diffusive Shock Acceleration

CR injection at shock ($z = 0$):

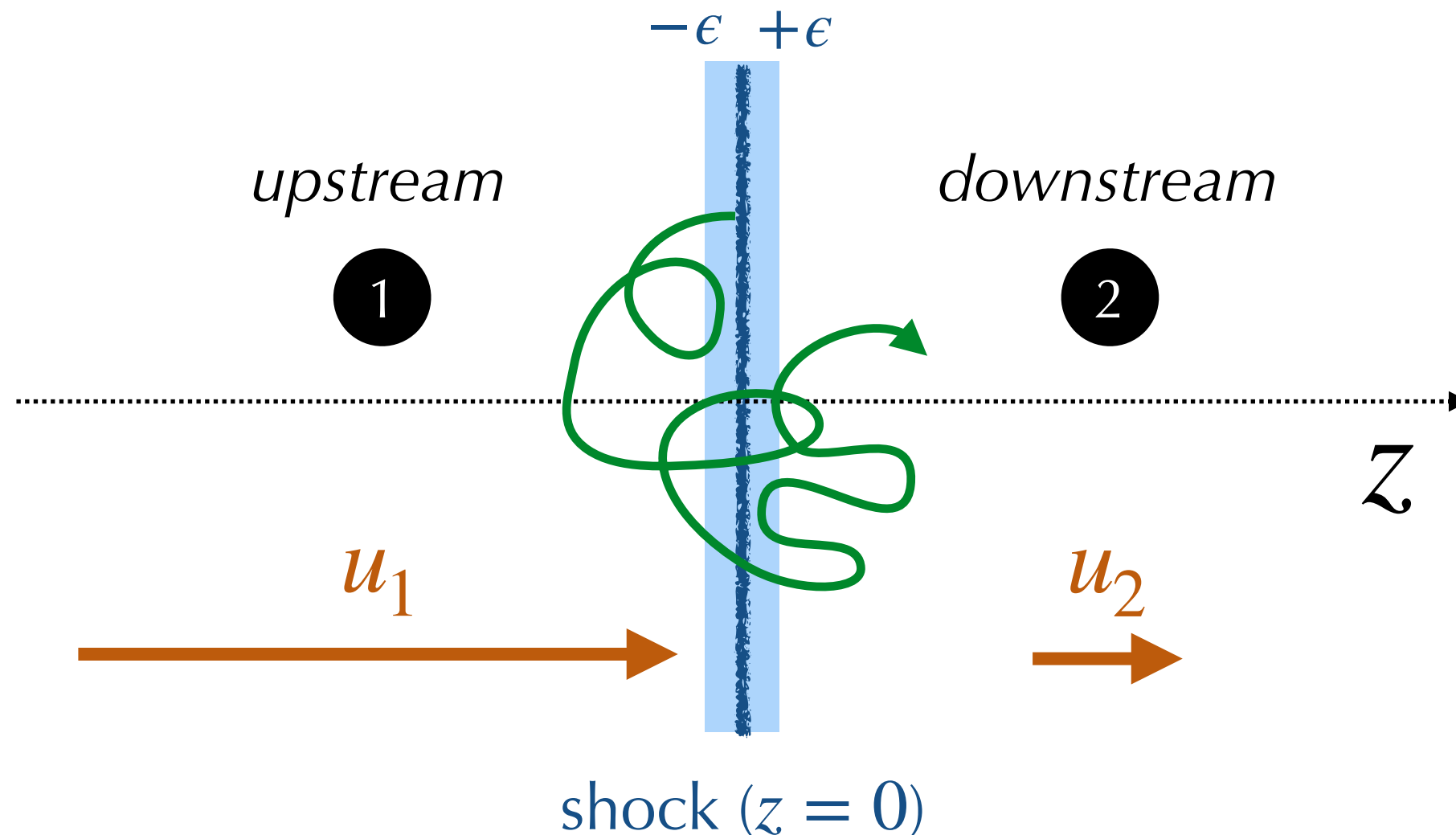
$$Q(p, z) = \mathcal{N} u_1 \delta(p - p_{\text{inj}}) \delta(z)$$



Diffusive Shock Acceleration

Integration of diffusion equation (DE) in the **shock vicinity**:

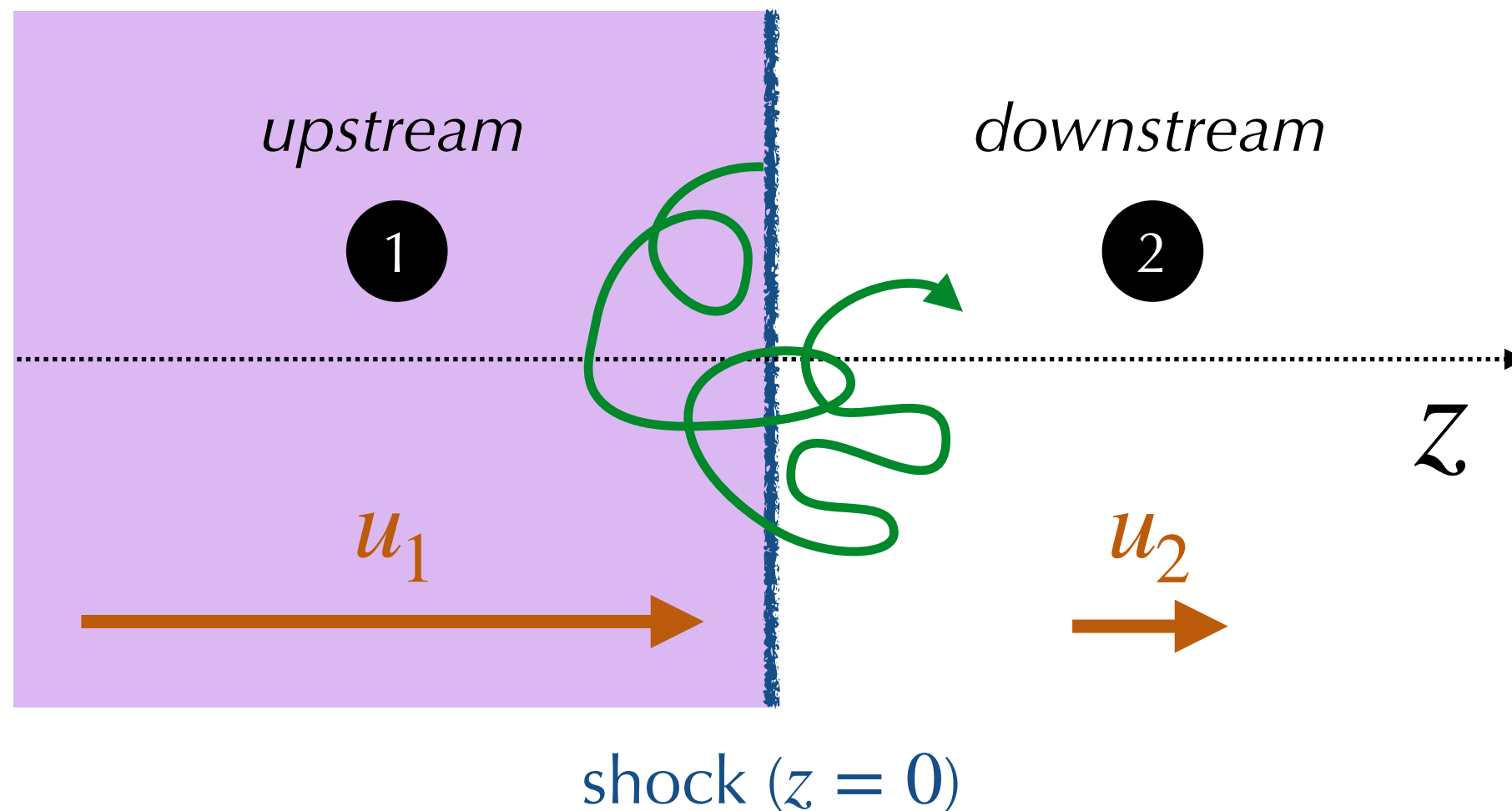
$$[K\partial_z f]_2 - [K\partial_z f]_1 + \frac{1}{3}(u_2 - u_1)p\partial_p f_0 + \mathcal{N}u_1\delta(p - p_{\text{inj}}) = 0$$



Diffusive Shock Acceleration

In **upstream region** $f \rightarrow 0$ for $z \rightarrow -\infty$, so integration of DE yields:

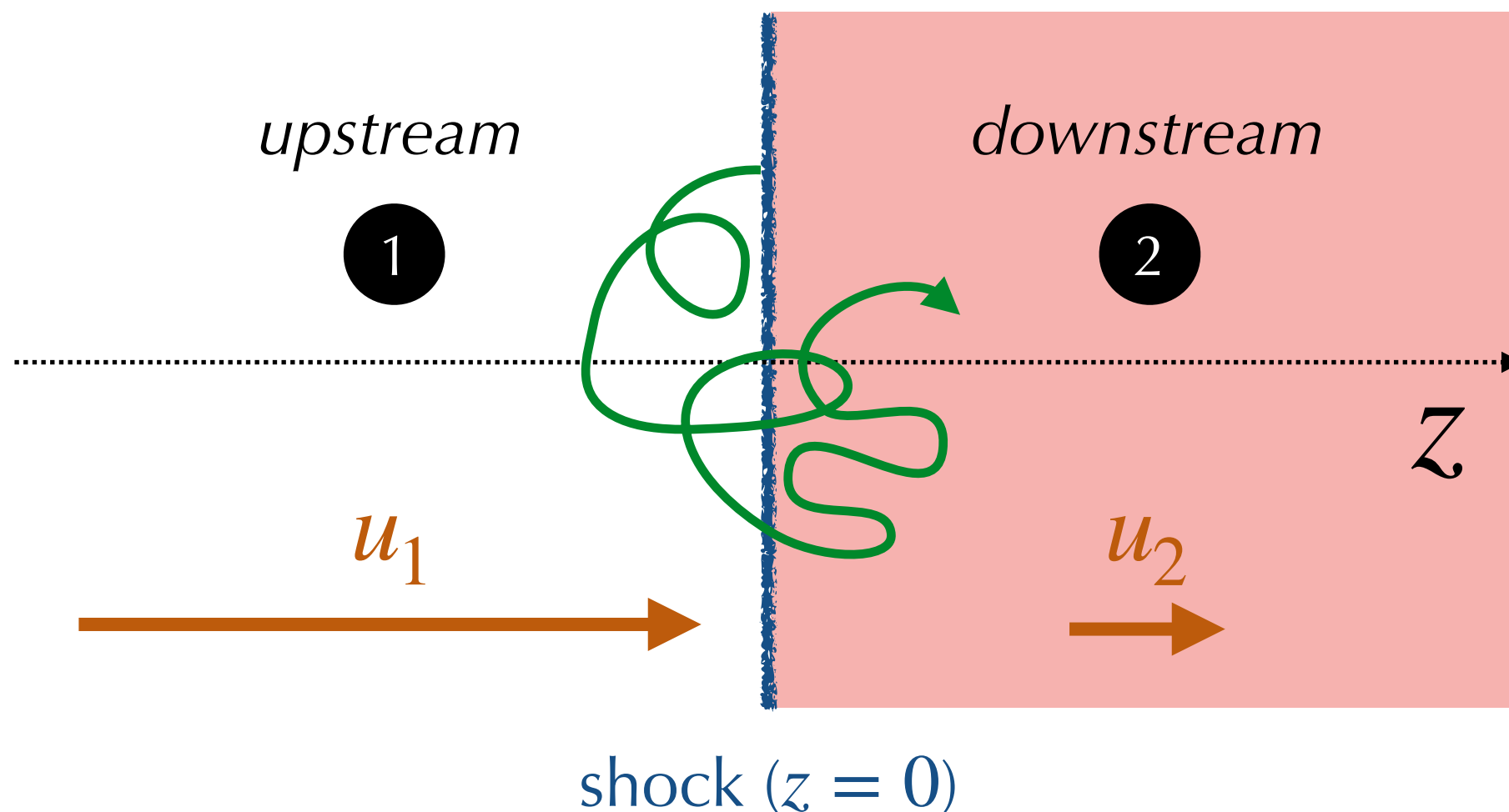
$$\partial_z [uf - K\partial_z f] = 0 \quad \rightarrow \quad [K\partial_z f]_1 = u_1 f_0$$



Diffusive Shock Acceleration

In **downstream region** $\partial_z f = 0$, so DE in **shock vicinity** becomes:

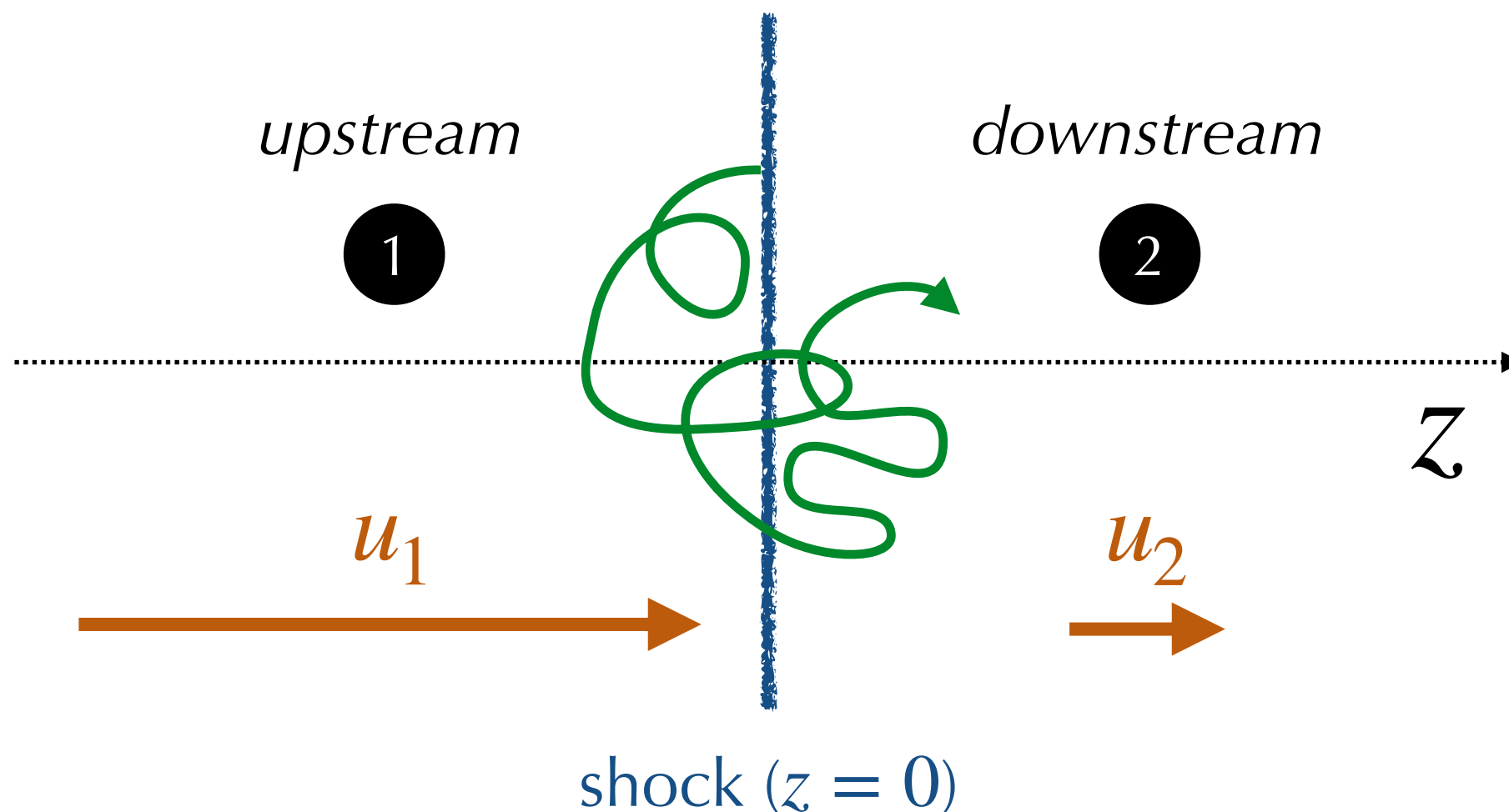
$$f_0 = \frac{u_2 - u_1}{3u_1} p \partial_p f_0 + \mathcal{N} \delta(p - p_{\text{inj}})$$



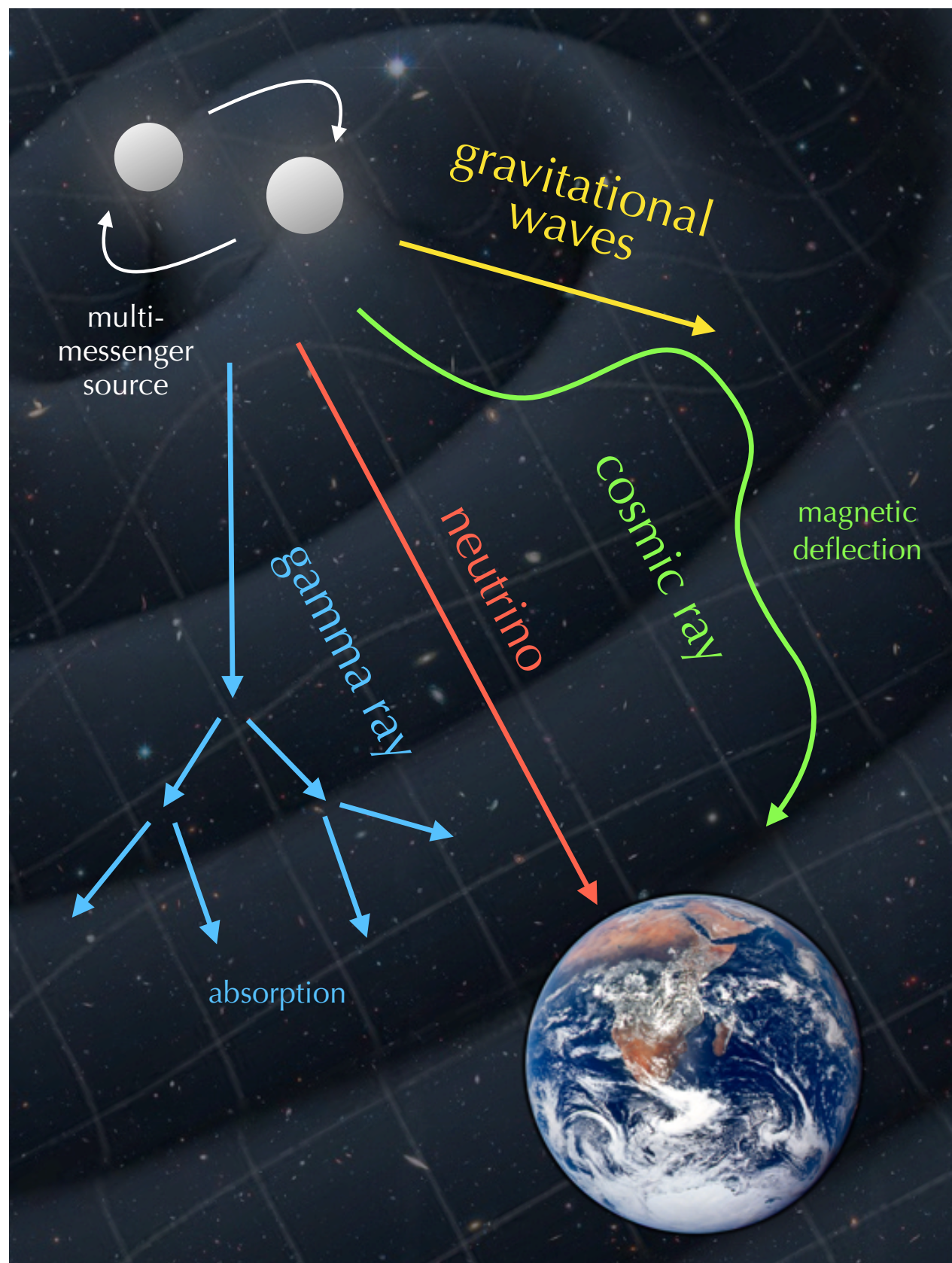
Diffusive Shock Acceleration

Solution is **power-law in momentum**:

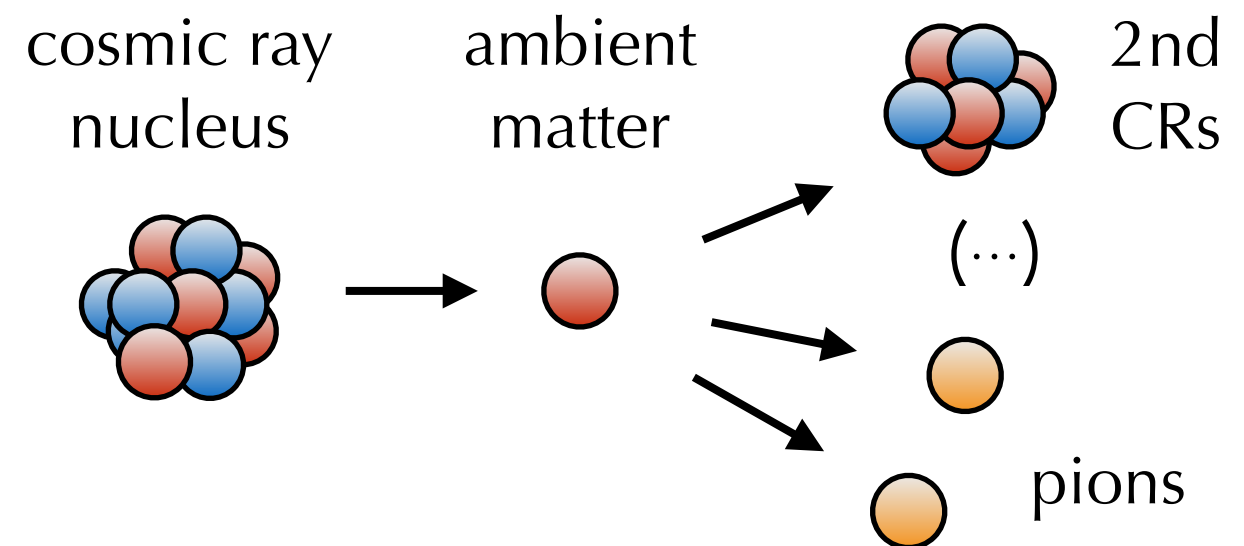
$$f_0(p) = \theta(p - p_{\text{inj}}) \frac{\gamma \mathcal{N}}{p_{\text{inj}}} \left(\frac{p}{p_{\text{inj}}} \right)^{-\gamma} \quad \gamma \equiv \frac{3}{1 - u_2/u_1}$$



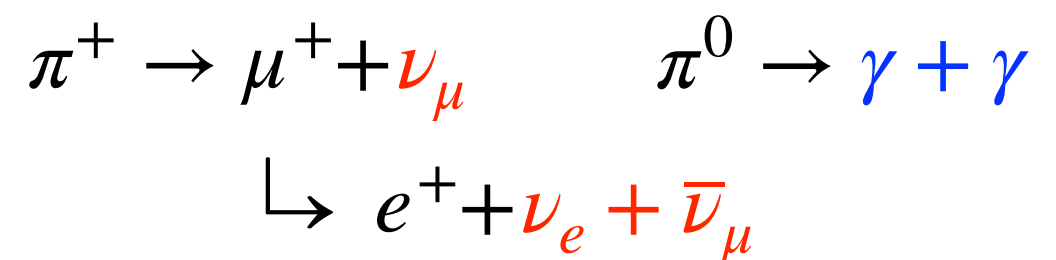
Hadronic Emission



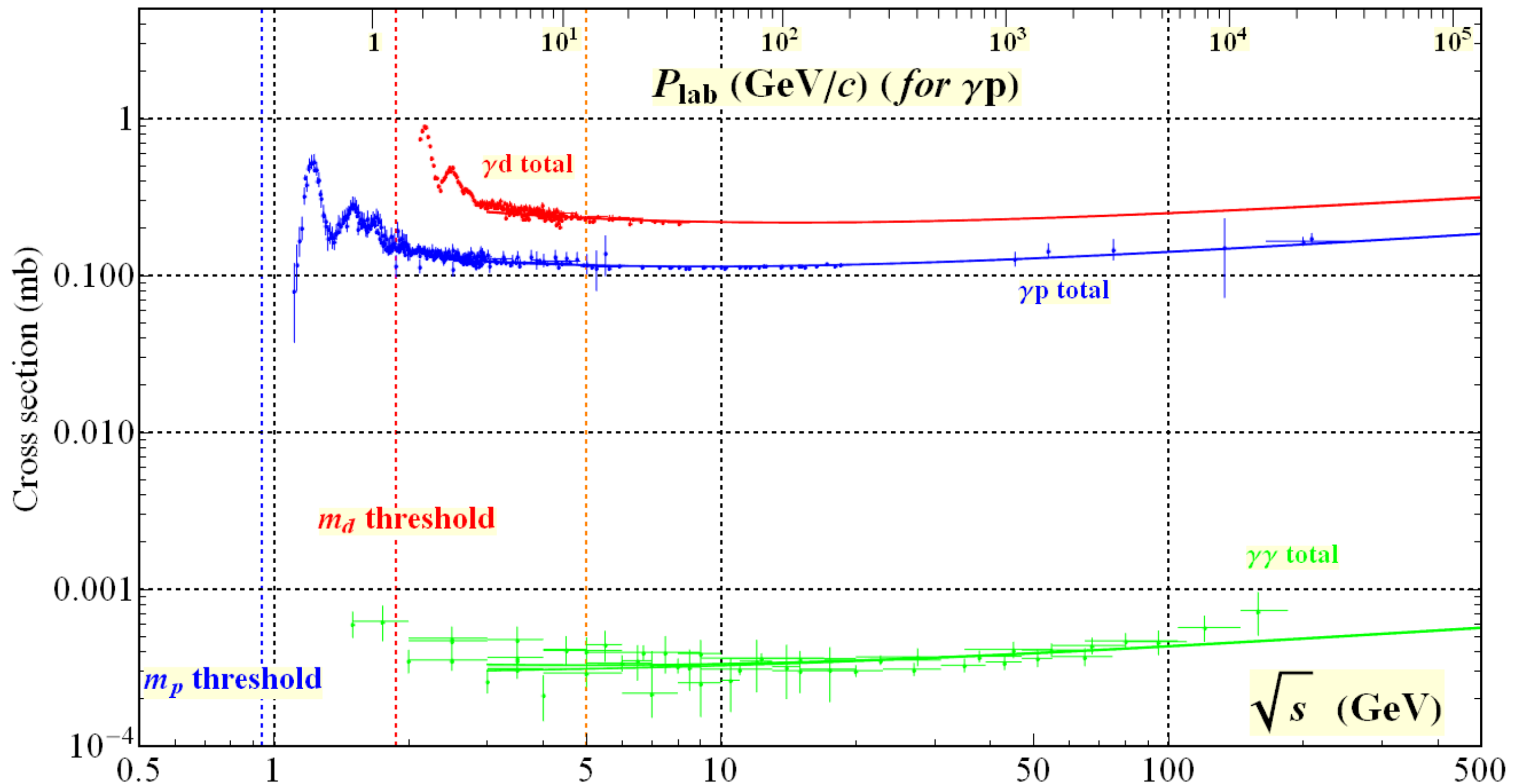
Acceleration of **cosmic rays (CRs)** - especially in the aftermath of cataclysmic events, sometimes visible in **gravitational waves (GW)**.



Secondary **neutrinos** and **gamma-rays** from pion decays:

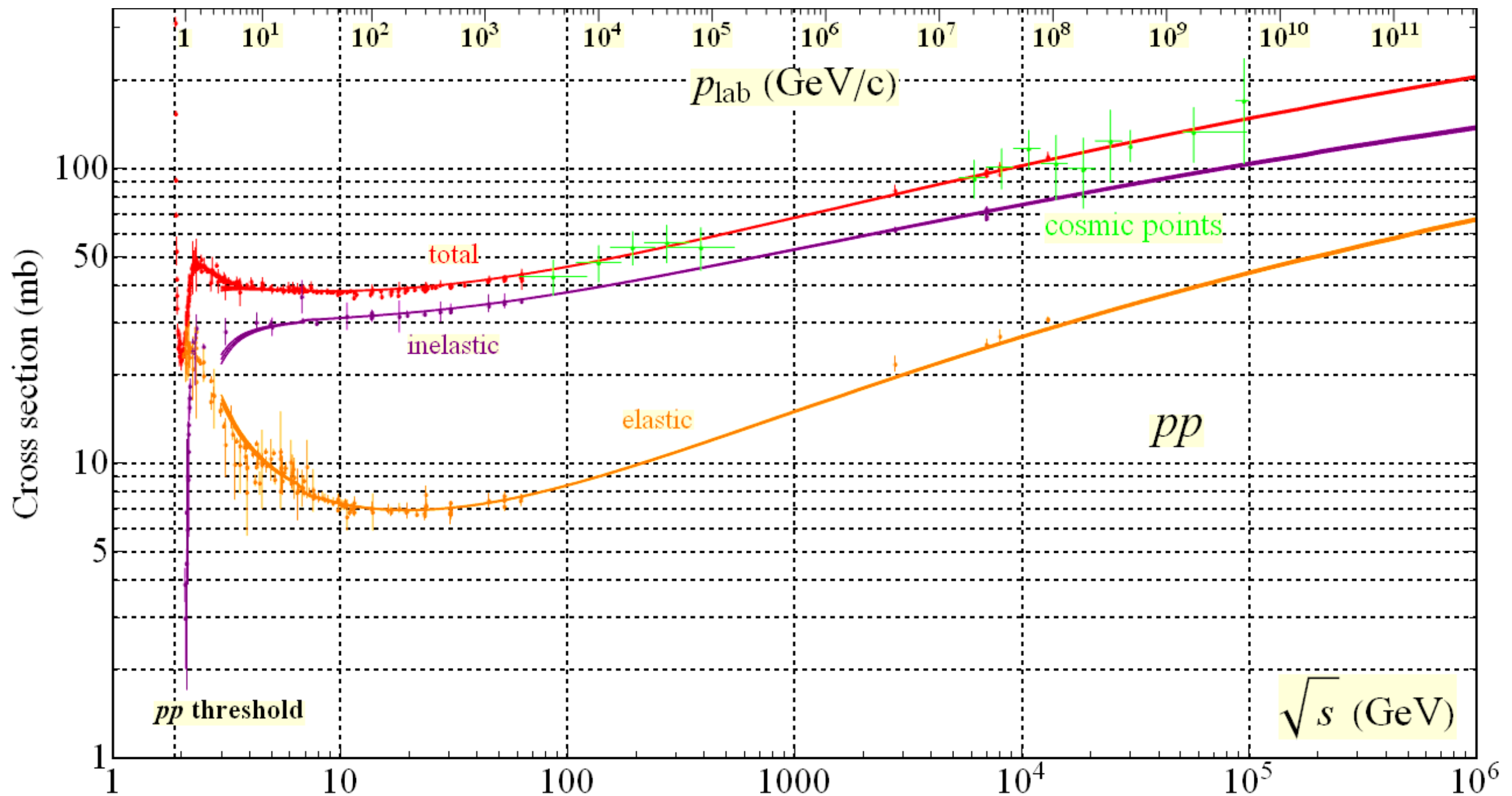


Proton-Photon ($p\gamma$) Interactions



[Particle Data Group; pdg.lbl.gov]

Proton-Proton (pp) Interactions



[Particle Data Group; pdg.lbl.gov]

Pion Production Efficiency

- Pion production depend on **target opacity** $\tau = \ell \sigma n$
- Bolometric **pion production efficiency** (with inelasticity κ):

$$f_{\pi} = 1 - e^{-\kappa \tau}$$

- Inelasticity per pion: $\kappa_{\pi} = \kappa / \langle N_{\pi} \rangle \simeq 0.17 - 0.2$
- Bolometric relation of the production rates Q :

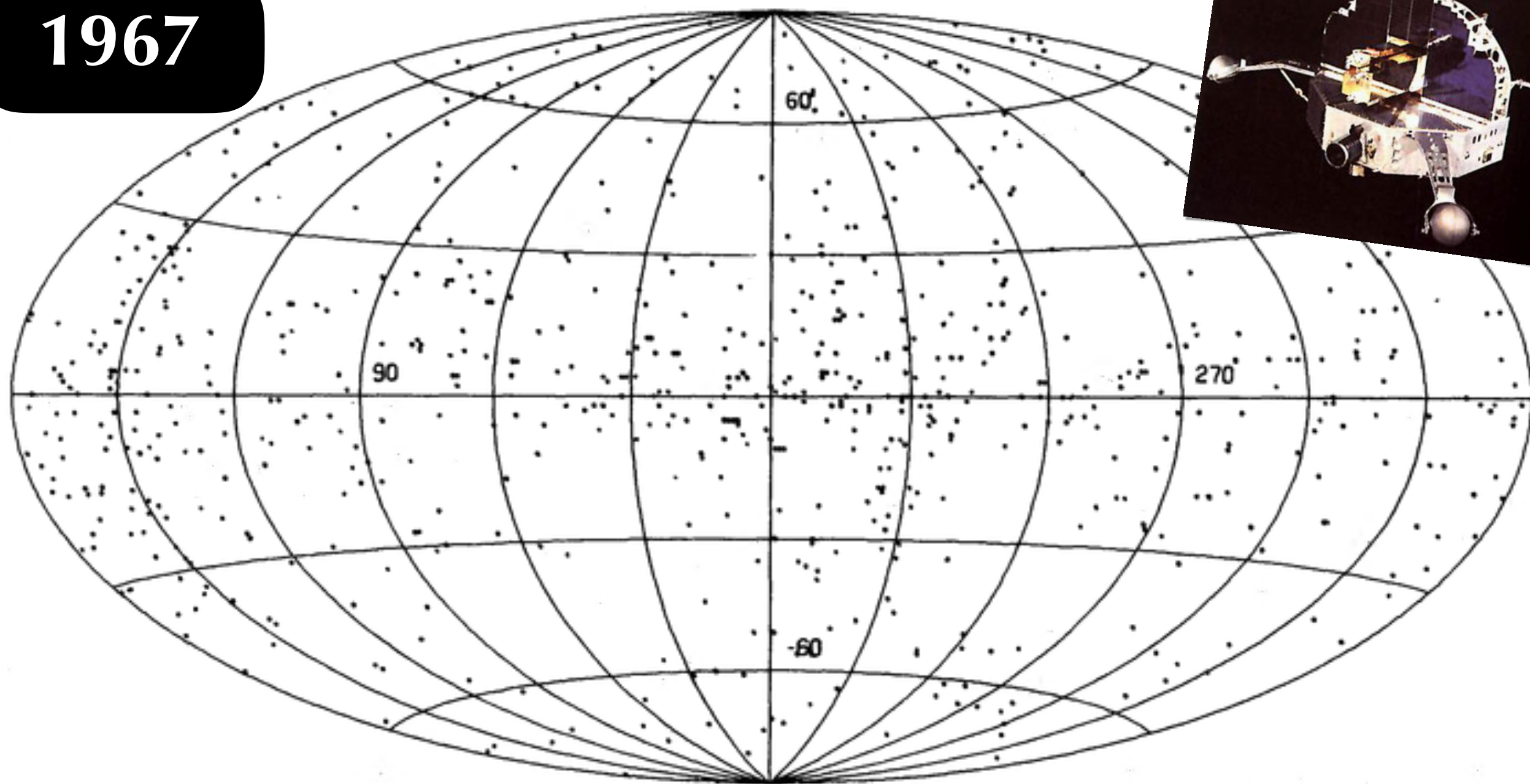
$$E_{\pi}^2 Q_{\pi^{\pm}} \simeq \frac{\langle N_{\pi^{\pm}} \rangle}{\langle N_{\pi^0} \rangle + \langle N_{\pi^{\pm}} \rangle} \left[f_{\pi} E_N^2 Q_N(E_N) \right]_{E_N = E_{\pi} / \kappa_{\pi}}$$

- **Charged-to-neutral pion ratio** K_{π} :

$$E_{\pi}^2 Q_{\pi^{\pm}} \simeq \frac{K_{\pi}}{1 + K_{\pi}} \left[f_{\pi} E_N^2 Q_N(E_N) \right]_{E_N = E_{\pi} / \kappa_{\pi}} \quad K_{\pi} = \frac{\langle N_{\pi^{\pm}} \rangle}{\langle N_{\pi^0} \rangle} = \begin{cases} 2 & \text{pp} \\ 1 & \text{p}\gamma \end{cases}$$

Gamma-Ray Astronomy

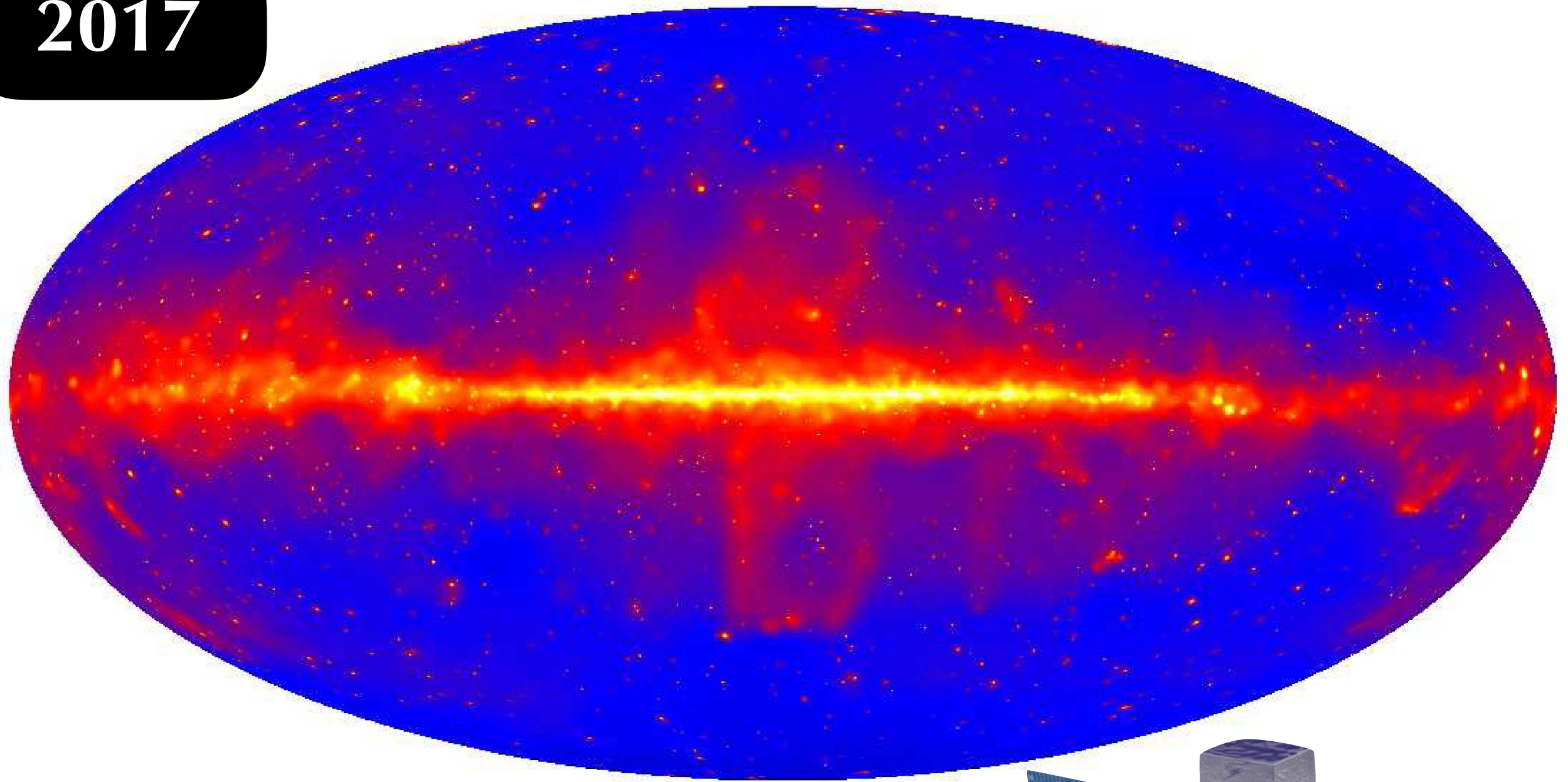
1967



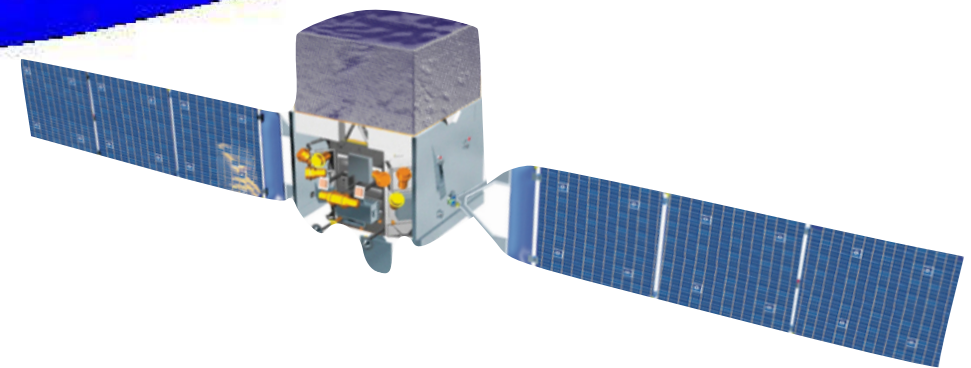
Orbiting Solar Observatory (OSO-3) (Clark & Kraushaar'67)

Gamma-Ray Astronomy

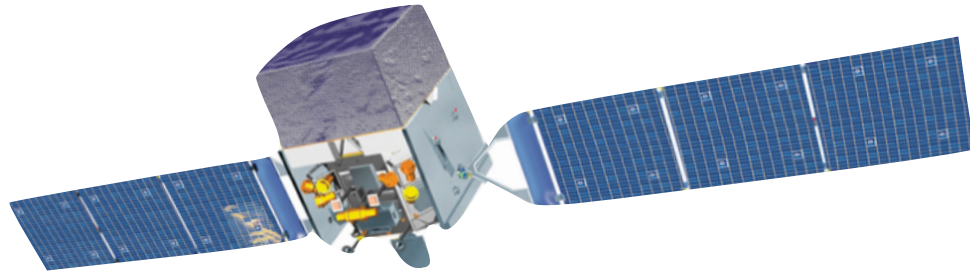
2017



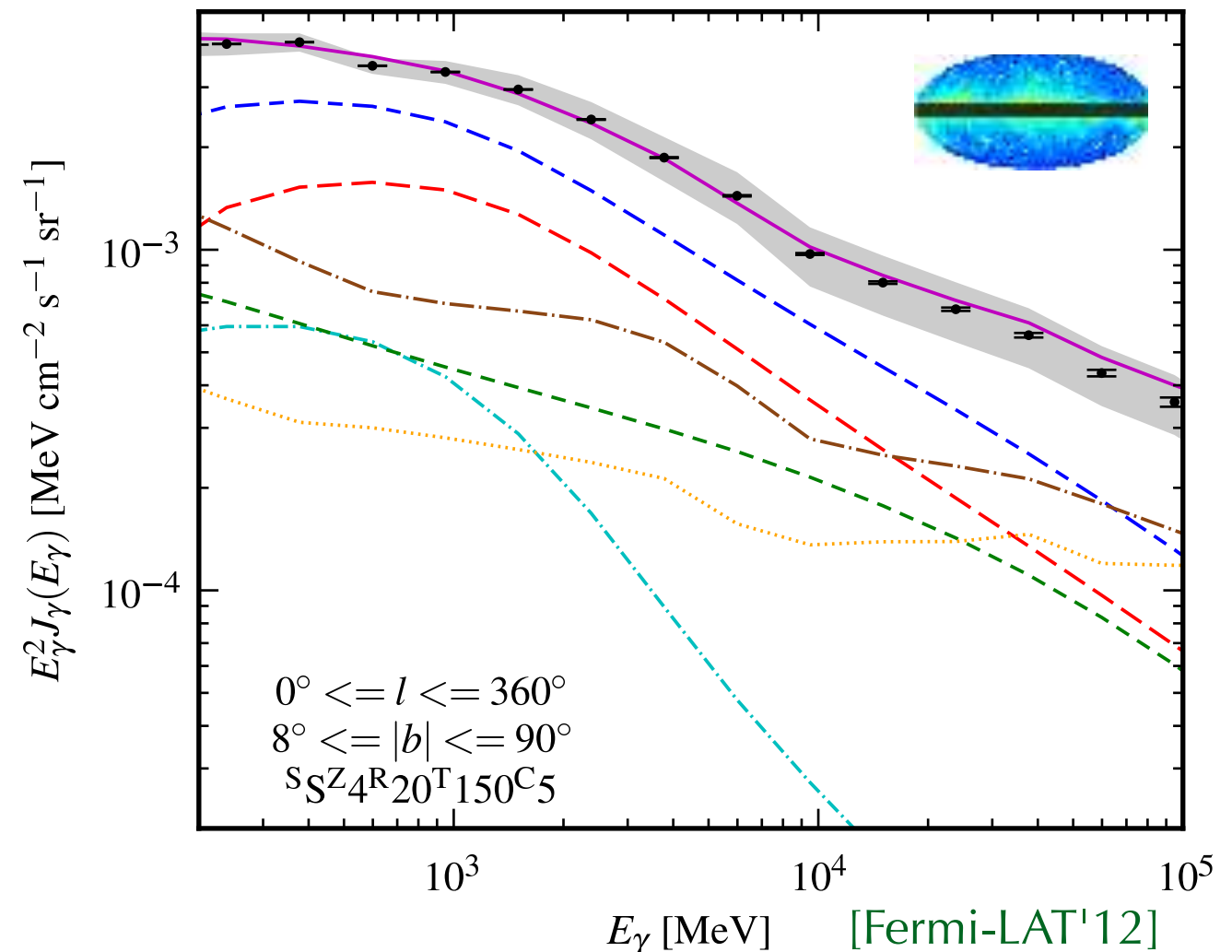
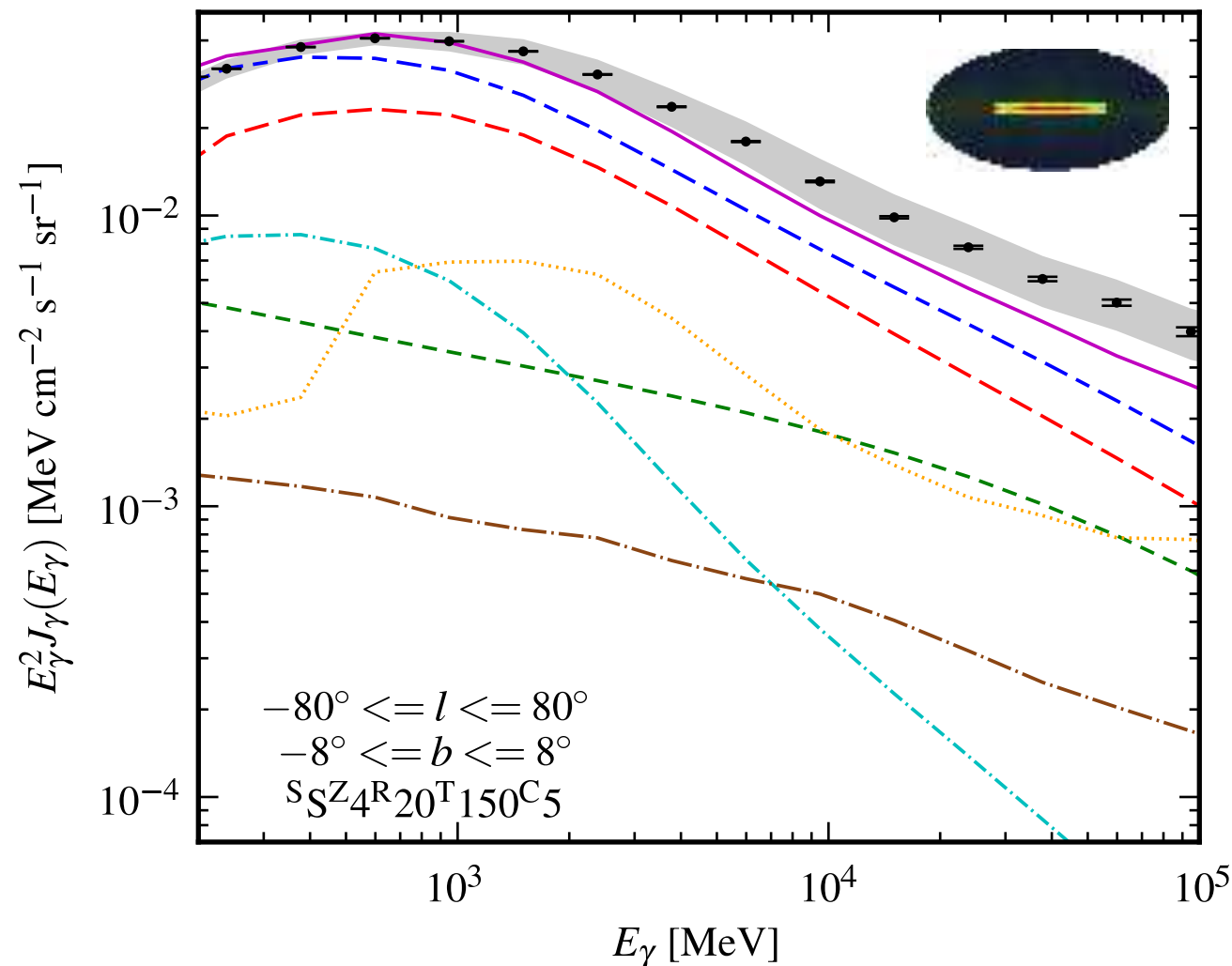
Fermi-LAT gamma-ray count map



Diffuse Galactic Gamma-Rays

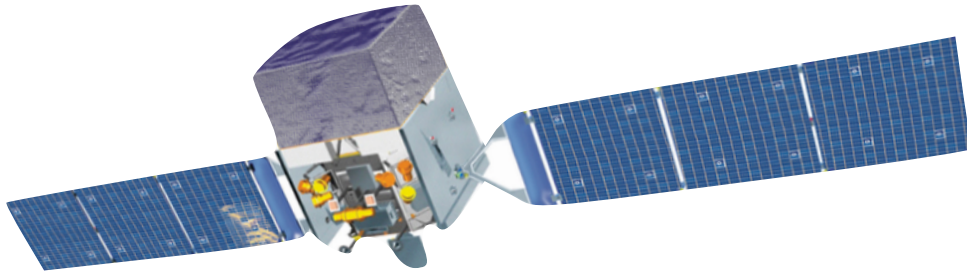


diffuse gamma-rays from
pion production

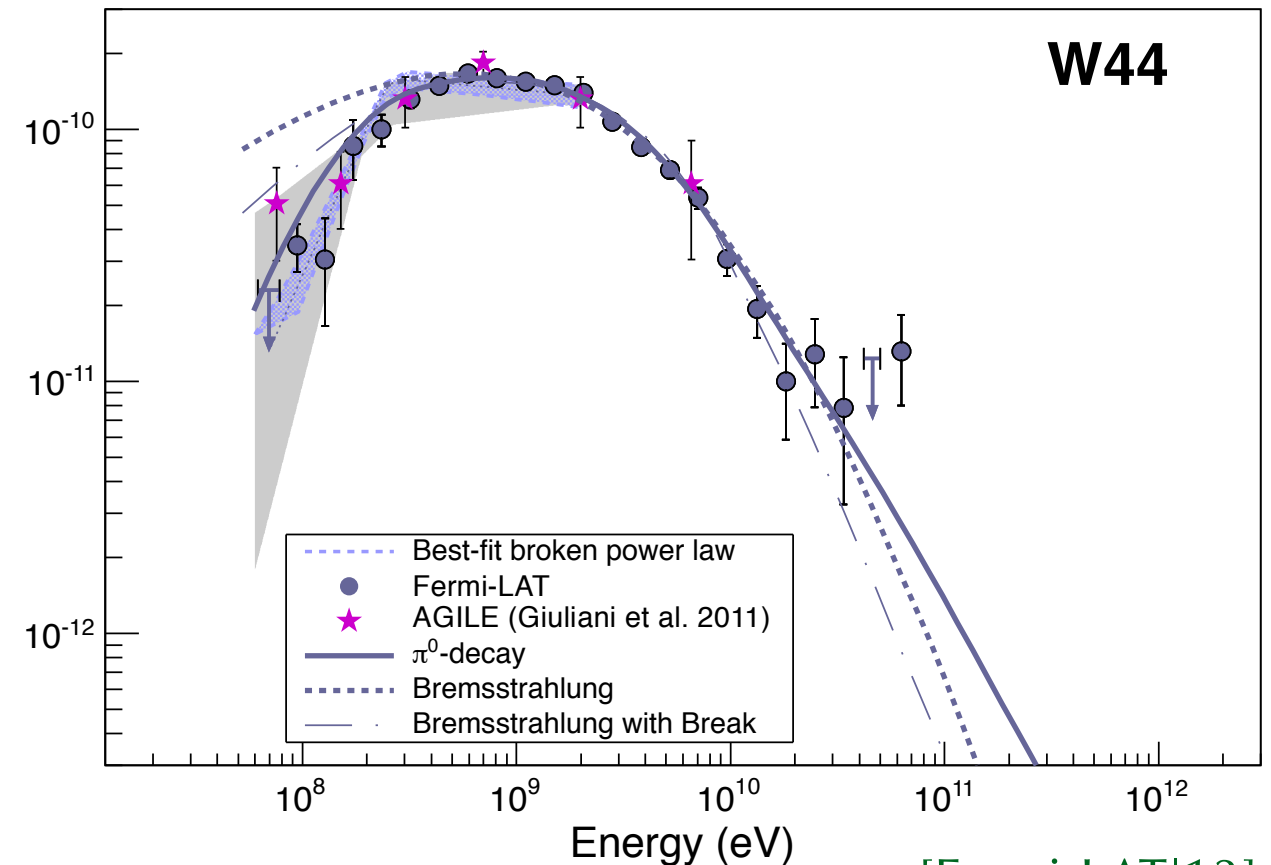
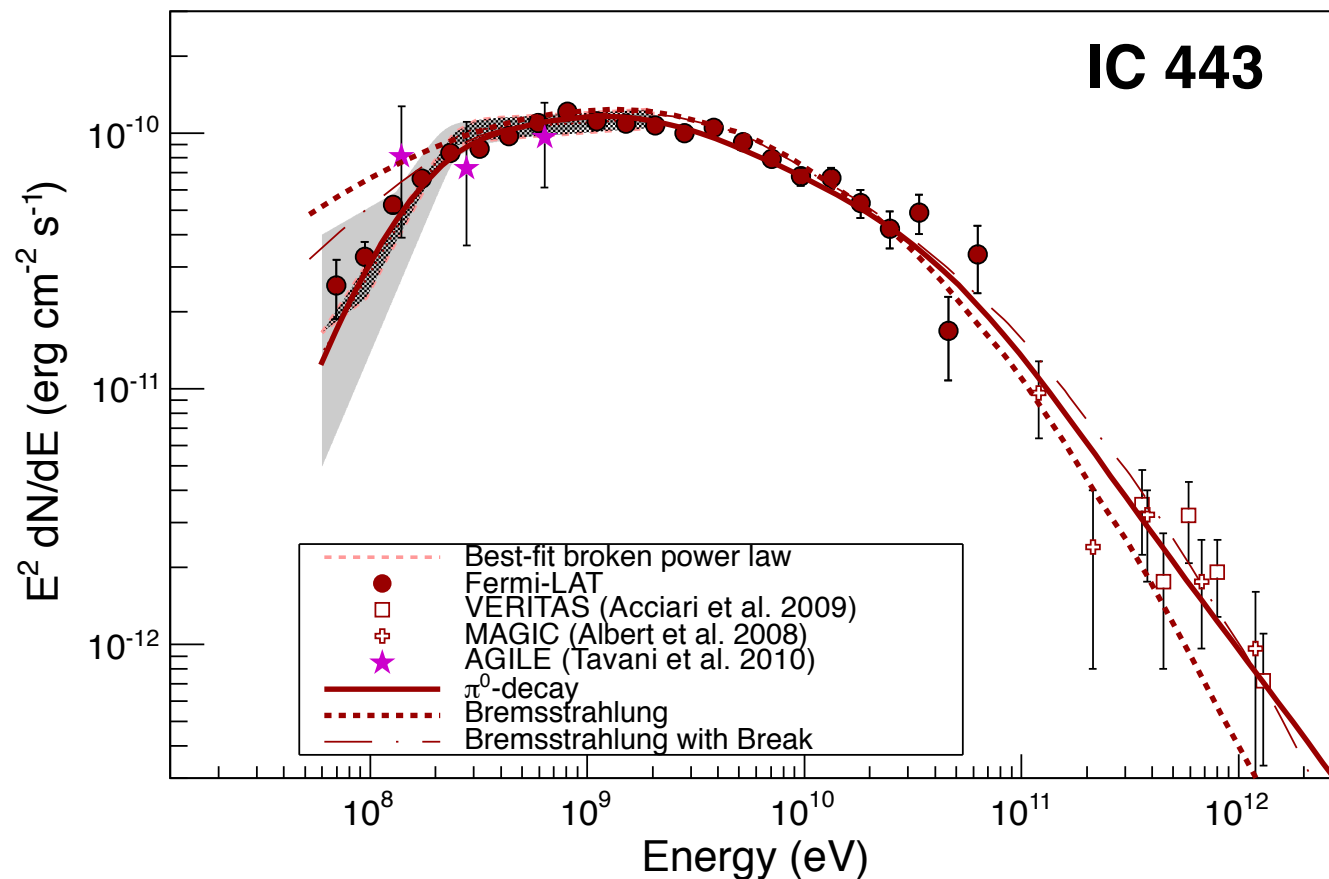


isotropic background (brown, long-dash-dotted), detected sources (orange, dotted),
 π^0 -decay (red, long-dashed), inverse-Compton (green, dashed)
 & bremsstrahlung (cyan, dash-dotted)

Pion "Bump" in SNR Spectra



pion-bump feature in γ -ray spectra of Galactic supernova remnants



[Fermi-LAT'13]

π^0 production in CR collisions with gas: $p + p \rightarrow \pi^0 + X$.

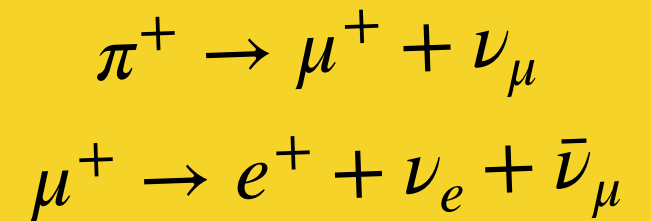
γ -ray in the rest-frame of the pion takes $E_\gamma^* = m_\pi/2 \simeq 67.5$ MeV.

Kinematics of the interaction produces a break at $E_\gamma \simeq 200$ MeV.

Gamma-Ray vs. Neutrinos

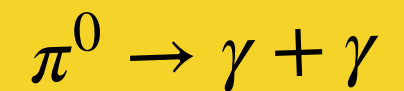
- **Neutrino emission** from charged pion decay:

$$\frac{1}{3} \sum_{\alpha} E_{\nu} Q_{\nu_{\alpha}}(E_{\nu}) \simeq [E_{\pi} Q_{\pi^{\pm}}(E_{\pi})]_{E_{\pi} \simeq 4E_{\nu}}$$



- **Gamma-ray emission** from neutral pion decay:

$$\frac{1}{2} E_{\gamma} Q_{\gamma}(E_{\gamma}) \simeq [E_{\pi} Q_{\pi^0}(E_{\pi})]_{E_{\pi} \simeq 2E_{\gamma}}$$



- **Multi-messenger relation** between neutrino and γ -ray emission:

$$\frac{1}{3} \sum_{\alpha} E_{\nu}^2 Q_{\nu_{\alpha}}(E_{\nu}) \simeq \frac{1}{4} K_{\pi} [E_{\gamma}^2 Q_{\gamma}(E_{\gamma})]_{E_{\gamma} \simeq 2E_{\nu}}$$

- **Note:** Observable γ -ray emission is attenuated in sources and, in particular, in extragalactic background radiation.