Microwave Response of Superconductors with Paramagnetic Impurities

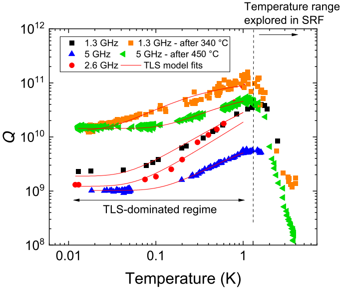
Mehdi Zarea and James A. Sauls

Hearne Institute of Iheoretical Physics, Louisiana State University

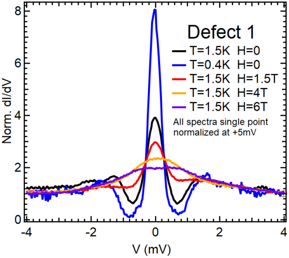
arXiv-2512.20565v1

HITP
Hearne Institute of Theoretical Physics
Louisiana State University

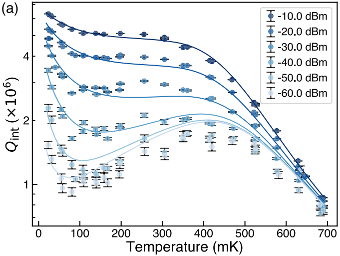
DE-AC02-07CH11359



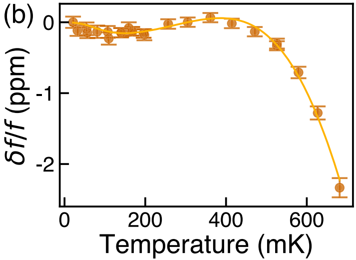
A.Romanenko *et al*, P.R.A, 13:034032,2020



[J.Makita *et al* ACS Nano 20, 8726, 2026]



K.D Crowley *et al*,Phys.Rev.X, 13:041005, 2023



Dissipation by quasiparticle excitations

Hot Nonequilibrium Quasiparticles in Transmon Qubits

K. Serniak,[1,*] M. Hays,[1] G. de Lange,[1,2] S. Diamond,[1] S. Shankar,[1] L. D. Burkhart,[1]
L. Frunzio,[1] M. Houzet,[3] and M. H. Devoret[1,†]
[1]*Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA*
[2]*QuTech and Kavli Institute of Nanoscience, Delft University of Technology, 2600 GA Delft, Netherlands*
[3]*Univ. Grenoble Alpes, CEA, INAC-Pheliqs, F-38000 Grenoble, France*

(Received 2 April 2018; revised manuscript received 27 July 2018; published 10 October 2018)

Nonequilibrium quasiparticle excitations degrade the performance of a variety of superconducting circuits. Understanding the energy distribution of these quasiparticles will yield insight into their generation mechanisms, the limitations they impose on superconducting devices, and how to efficiently mitigate quasiparticle-induced qubit decoherence. To probe this energy distribution, we systematically correlate qubit relaxation and excitation with charge-parity switches in an offset-charge-sensitive transmon qubit, and find that quasiparticle-induced excitation events are the dominant mechanism behind the residual excited-state population in our samples. By itself, the observed quasiparticle distribution would limit T_1 to $\approx 200\ \mu s$, which indicates that quasiparticle loss in our devices is on equal footing with all other loss mechanisms. Furthermore, the measured rate of quasiparticle-induced excitation events is greater than that of relaxation events, which signifies that the quasiparticles are more energetic than would be predicted from a thermal distribution describing their apparent density.

DOI: 10.1103/PhysRevLett.121.157701

The Message

- Even ***low concentration*** (1 ppm) of magnetic impurities in ***strong coupling*** regime causes significant dissipation in SRF cavities
- **Sub-gap states** near Fermi energy contribute to:
  - ***residual resistance*** at low temperatures
  - ***anomalies in quality factor behavior*** at low temperatures
  - ***anomalies in frequency shift*** near $T = 0$ and T_c
  - ***coherence peak in conductivity*** near T_c
- Measure the position and extent of the subgap states

Methodology

$$\widehat{g}(\hat{p},\mathrm{r};\varepsilon_n,t) \equiv \begin{pmatrix} g(\hat{p},\mathrm{r};\varepsilon_n)\hat{1} & f(\hat{p},\mathrm{r};\varepsilon_n)i\sigma_y \\ \bar{f}(\hat{p},\mathrm{r};\varepsilon_n)i\sigma_y & \bar{g}(\hat{p},\mathrm{r};\varepsilon_n)\hat{1} \end{pmatrix}$$

where $\hat{p}$ is the direction momentum on the Fermi surface

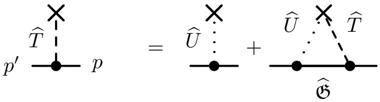
- Eilenberger equation: $\widehat{g}^2 = -\pi^2 \widehat{1}$

$$\left[i\varepsilon_n \widehat{\tau}_3 - \widehat{\Delta} - \widehat{\Sigma}_{imp}, \widehat{g} \right] + i\hbar \mathrm{v}_f \cdot \nabla \widehat{g} = 0 \qquad \widehat{\Delta} = \Delta \widehat{\tau}_1 i\sigma_y,$$

$\Delta = 2\pi\lambda \sum_n \int d\hat{p} f(\hat{p};\varepsilon_n),$ λ : e-phonon coupling constant

- Find $\widehat{\Sigma}_{imp}(\widehat{g})$ self-consistently using T-matrix

The impurity self-energy term



$$\widehat{\Sigma}_{\mathrm{imp}}(\mathrm{p};\varepsilon_n) = n_s\,\widehat{T}(\mathrm{p},\mathrm{p};\varepsilon_n)\,, \tag{1}$$

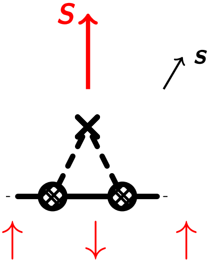
Non-magnetic impurities: $\widehat{U}(\mathrm{p}',\mathrm{p}) = v_{\mathrm{N}}\widehat{1}, \rightarrow \widehat{\Sigma}_{\mathrm{N}} = \frac{1}{2\tau_{\mathrm{N}}} N_f\,\widehat{g}$

scattering rate $\frac{1}{\tau_N}$ contributes to Drude conductivity

Non-magnetic impurities do not effect T_c and Δ: [Anderson-1959]

But they do contribute to conductivity and London penetration depth

A.A. Abrikosov and L.P. GorkovJETP, 12:1243, 1961



uncorrelated, classical spin

$H' = J\vec{S}\cdot\vec{s}$,

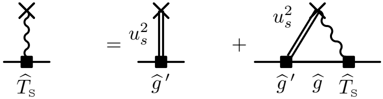
$\vec{S}$ impurity spin

$\vec{s} = (\hbar/2)\vec{\sigma}$ electron spin

Pair breaking rate: $1/\tau_s$

$\alpha_s \equiv 1/2\pi T_{c_0}\tau_s$ pair-breaking rate/pair-formation rate

L.Yu, Acta Physica Sinica, 21(1):75, 1965]
H.Shiba, PTP, 40(3):435–451, 1968]
A.I.Rusinov, JETP Lett., 9:146, 1969]



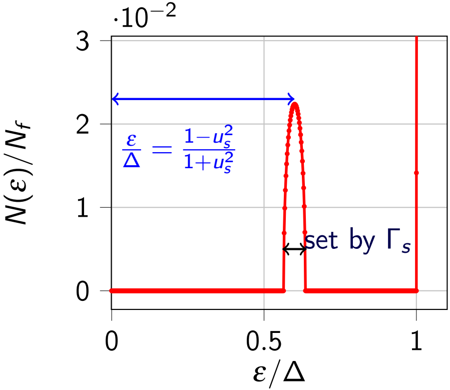
- impurity **concentration**

$\Gamma_s = \frac{c_s}{2\pi N_f} \qquad \frac{\Gamma_s}{T_{c0}} \approx \frac{N_s}{N_{at}} \frac{T_f}{T_{c0}}$

- **coupling strength**

$\bar{u}_s^2 = (\pi N_f J)^2 S(S+1)$

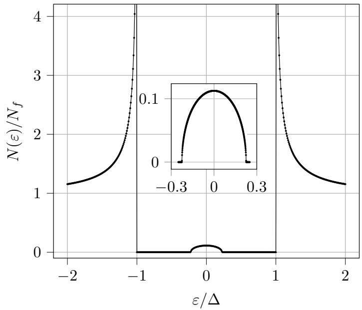
$\frac{1}{2\tau_s} = \Gamma_s \frac{\bar{u}_s^2}{(1+\bar{u}_s^2)^2 - 4f(\varepsilon)^2}$

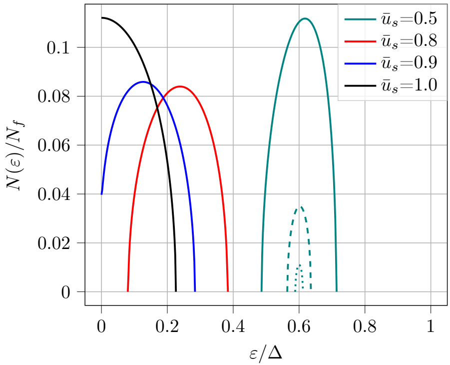


Contribution to Drude σ_n:

$\delta(\frac{1}{\tau_n}) = \Gamma_s \frac{2\bar{u}_s^4}{(1+\bar{u}_s^2)^2 - 4f^2}$

Impurity states inside the gap





$T/T_c = 0.02$

Solid lines: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-3}$

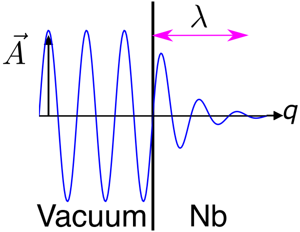
Dashed line: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-4}$ Dotted line: $\Gamma_S/2\pi T_{c_0} = 3.5 \times 10^{-5}$

The Current Linear Response Function

$$\widehat{\Sigma}_{EM}(\mathrm{r}, t) = -\frac{e}{c}\mathrm{v}_f \cdot \mathrm{A}(\mathrm{r}, t)\widehat{\tau}_3$$

$$\mathrm{J}^{R}(\mathrm{q}, \omega) = -\frac{c}{4\pi} K(\omega)\,\mathrm{A}(\mathrm{q}, \omega)\,.$$

$$\begin{aligned} K(\omega, T) &= \frac{1}{\lambda^2(\omega, T)} - i\frac{4\pi\omega}{c^2}\sigma_1(\omega, T) \\ &= \frac{4\pi e^2}{mc^2} n_s - i\frac{4\pi\omega}{c^2}\sigma_1(\omega, T) \end{aligned}$$



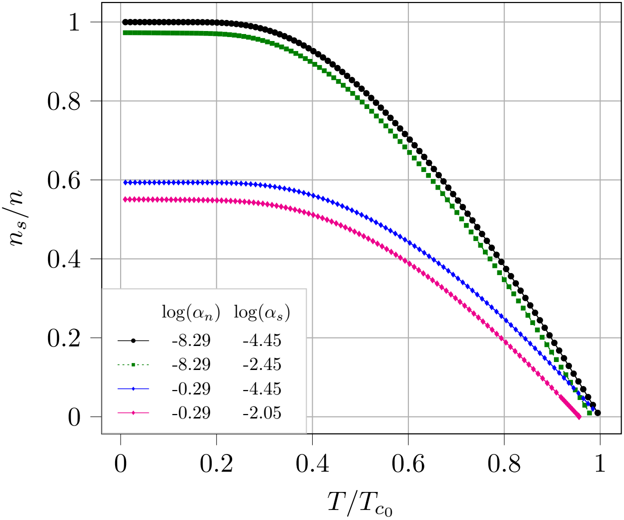
$\lambda(\omega, T)$: penetration depth

n_s superfluid fraction

σ_1: the dissipation of the EM field $\longrightarrow \sigma_n$

$\hbar\omega/\Delta \ll 1$

Superfluid fraction

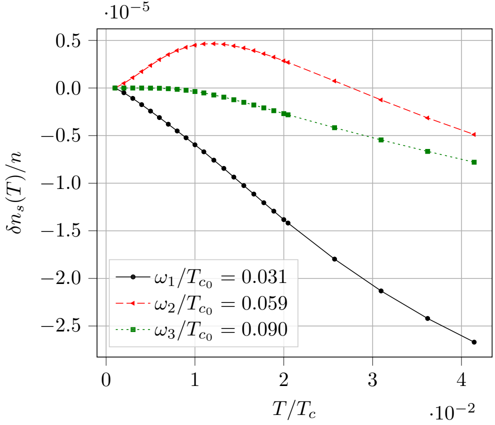


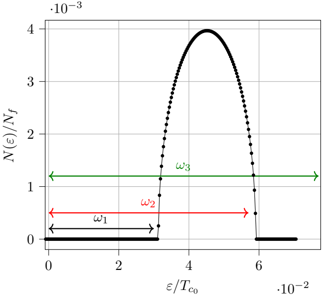
$\omega = 0.005\, T_{c_0}$

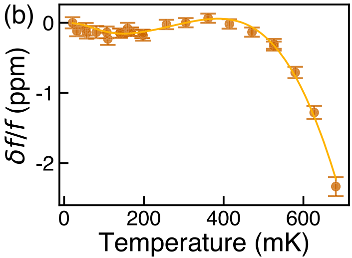
$(f \approx 1GHz)$

$\alpha_n = \Gamma_n/\pi T_{c_0}$ $\qquad \alpha_s = \Gamma_s/2\pi T_{c_0}$ $\qquad \bar{u}_S = 1.$

Superfluid fraction near zero temperature





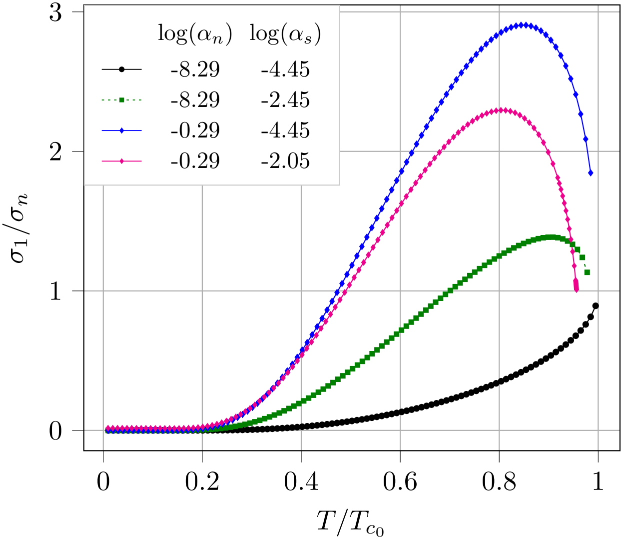


$\bar{u}_s = 0.975$ and $\Gamma_s/2\pi T_{c_0} = 8.9 \times 10^{-6}$

$$\frac{f - f_0}{f_0} = -\frac{1}{R}\mathrm{Re}\left(\frac{1}{\sqrt{K}}\right) \approx \frac{\lambda}{R}$$

[K.D Crowley *et al*,Phys.Rev.X, 13:041005,

Dissipative conductivity - near T_c

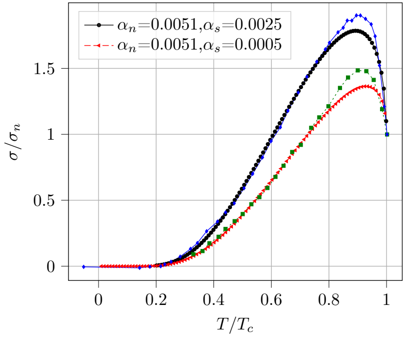


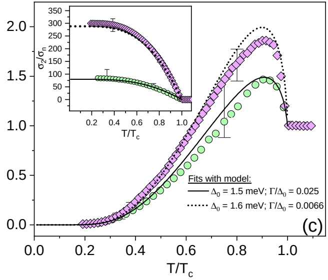
$\bar{u}_s = 1$

$\hbar\omega/k_B T_{c_0} \; = \; 0.005$

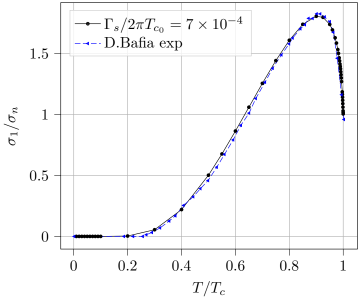
$(f \approx 1GHz)$.

$\sigma_n = \frac{ne^2\tau/m}{1+(\omega\tau)^2}$



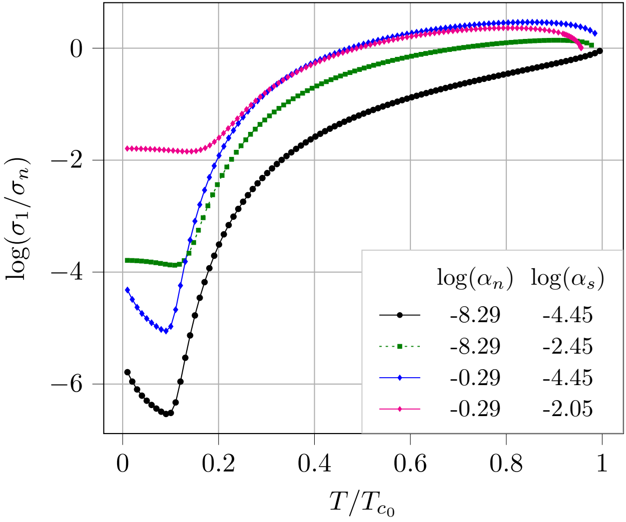


[Bafia *et al*, Phys. Rev. Appl., 23:054052, 2025]

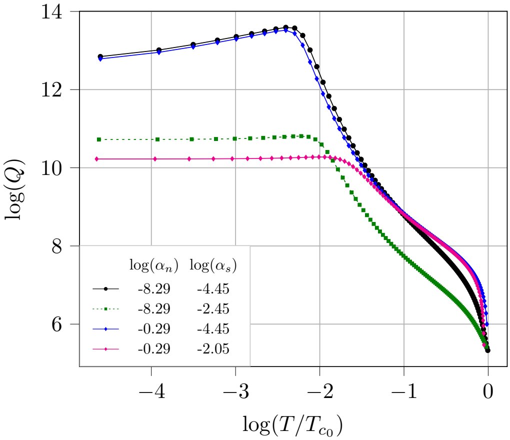


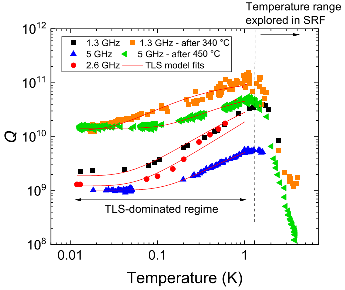
Problem: $\alpha_n = -0.013 < 0$

Conductivity at low temperatures



Quality factor





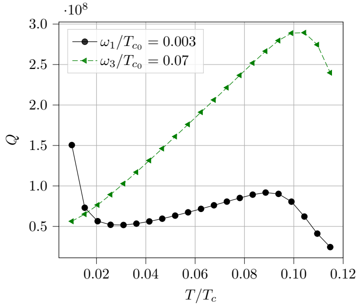
[A. Romanenko *et al*, Phys.Rev.Applied, 13:034032, 2020]

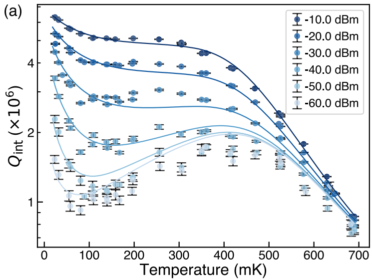
[J. He *et al*,Prog.Theor.Exp.Phys, 2025:063I01, 2025]

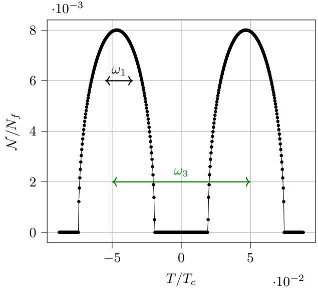
- dissipation by sub-gap quasi-particles,
- provides mechanism for residual resistivity,

$$\frac{1}{Q} = \frac{2}{R} Im\left(\frac{1}{\sqrt{K}}\right)$$

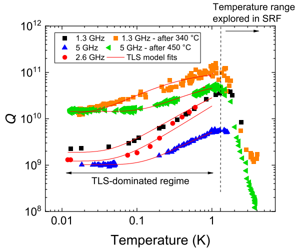
Quality factor, inferring sub-gap position







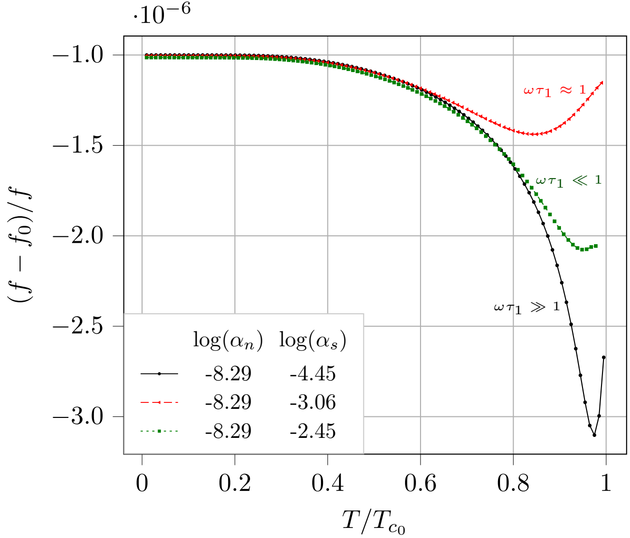
[K.D Crowley *et al*,Phys.Rev.X, 13:041005, 2023]



Intra-band: suppressed as $T \to 0$

Inter-band: present at $T = 0$

The anomaly in resonance frequency shift



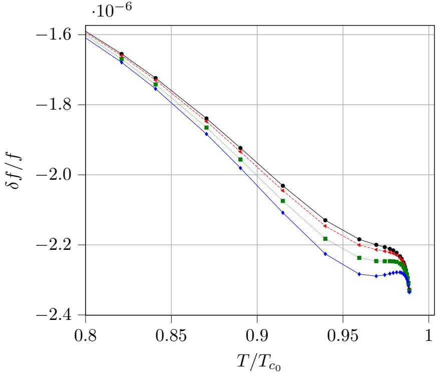
[R. Ghanbari *et al*,doi:10.18429/JACoW-SRF2023-MOPMB02]

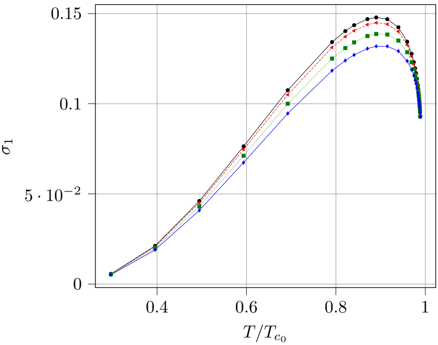
[N. Raut *et al*, arXiv:2308.09859]

Frequency shift and Conductivity

Frequency shift and Conductivity near T_c are sensitive to pair-breaking

- Same pair-breaking rate $1/\tau_s$
- Same normal state scattering rate
- Density and strength of magnetic impurities varies





Conclusion

The sub-gap spectrum of quasiparticles, generated by dilute concentrations of paramagnetic impurities,

- limits $Q(T)$ of SRF cavities operating at microwave frequencies at low temperatures.
- provides a theory for the zero-temperature residual resistance
- create anomalies in the resonant frequency

Thank you!

arXiv: 2512.20565