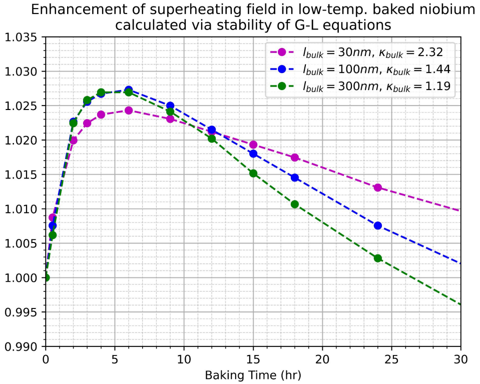
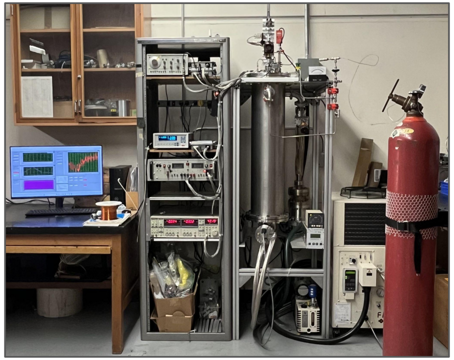
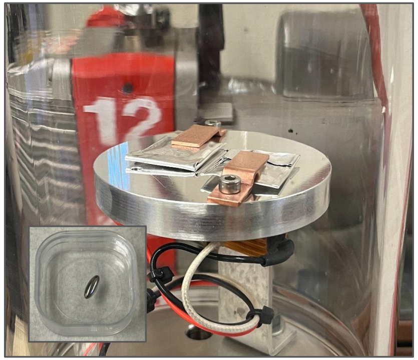
Field of first vortex penetration of inhomogeneous materials:
Predictions via Ginzburg-Landau theory and comparison to magnetometry measurements

TTC 2026 - CEA-CNRS-Université Paris Saclay
Lucas Wallace - University of Victoria, Canada







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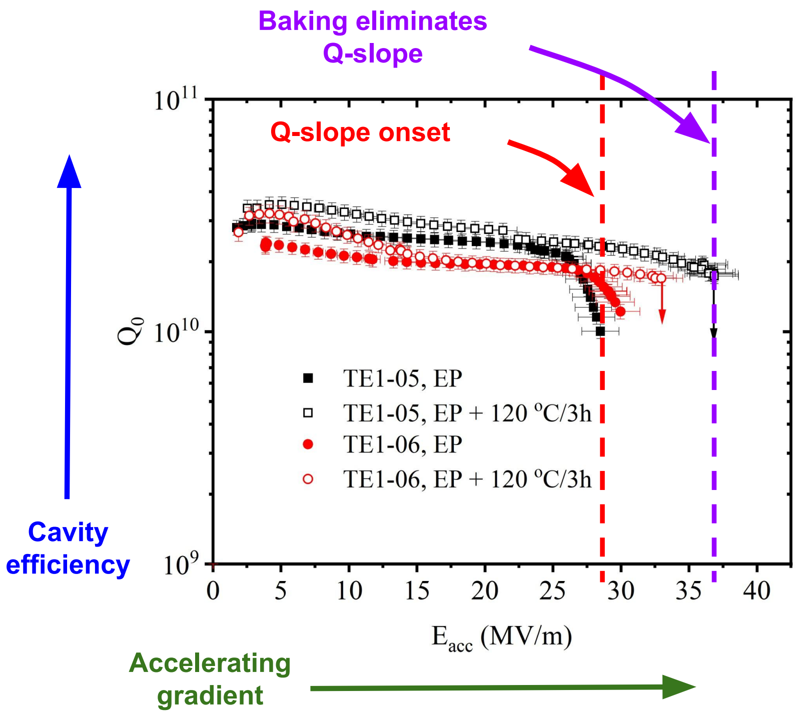
Experiment overview

Low-temperature baking has a long history of use in superconducting radio frequency cavities [1].

Used to eliminate high field Q-slope [1].

Recipes vary! 48hrs/120C is often used, but 3hrs/120C also works [2].

Why do different recipes work?



(1) G. Ciovati, “Effect of low-temperature baking on the radio-frequency properties of niobium superconducting cavities for particle accelerators,” doi: 10.1063/1.1767295
(2) B. D. Khanal and P. Dhakal, “Insight to the Duration of 120 °C Baking on the Performance of SRF Niobium Cavities,” doi: 10.1109/TASC.2023.3235311.

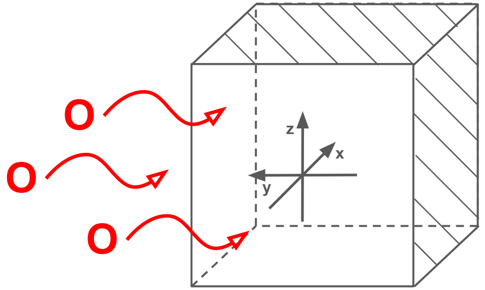
Experiment overview

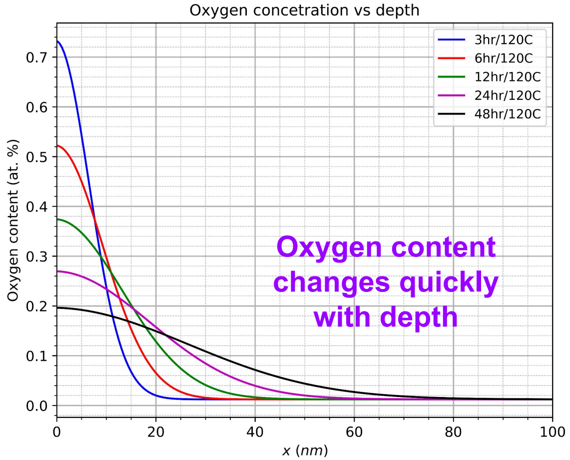
Low-temperature baking **dissolves near-surface oxygen**.

Oxygen concentration inhomogeneous near surface -> Difficult to predict behaviour.

Competing theories as to why this improves performance:

1. **Enhanced surface barrier?**
2. **Reduction in supercurrent density**?
3. **Reduces Hydride formation?**





Oxygen concentration calculated via diffusion sim by Ryan McFadden (TRIUMF)

Experiment overview

University of Victoria

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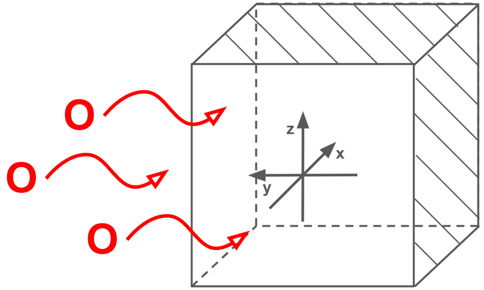
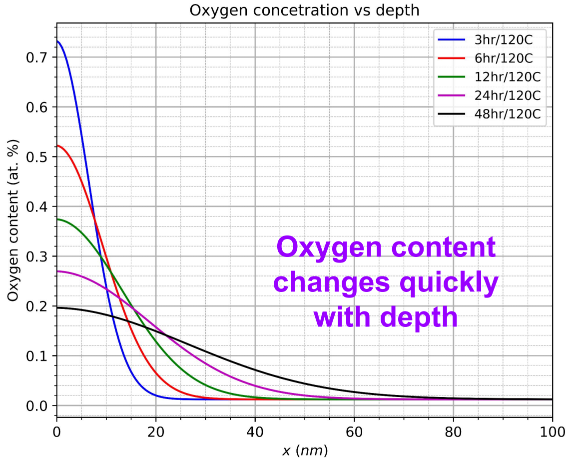
This study investigates the surface barrier





Oxygen concentration calculated via diffusion sim by Ryan McFadden (TRIUMF)

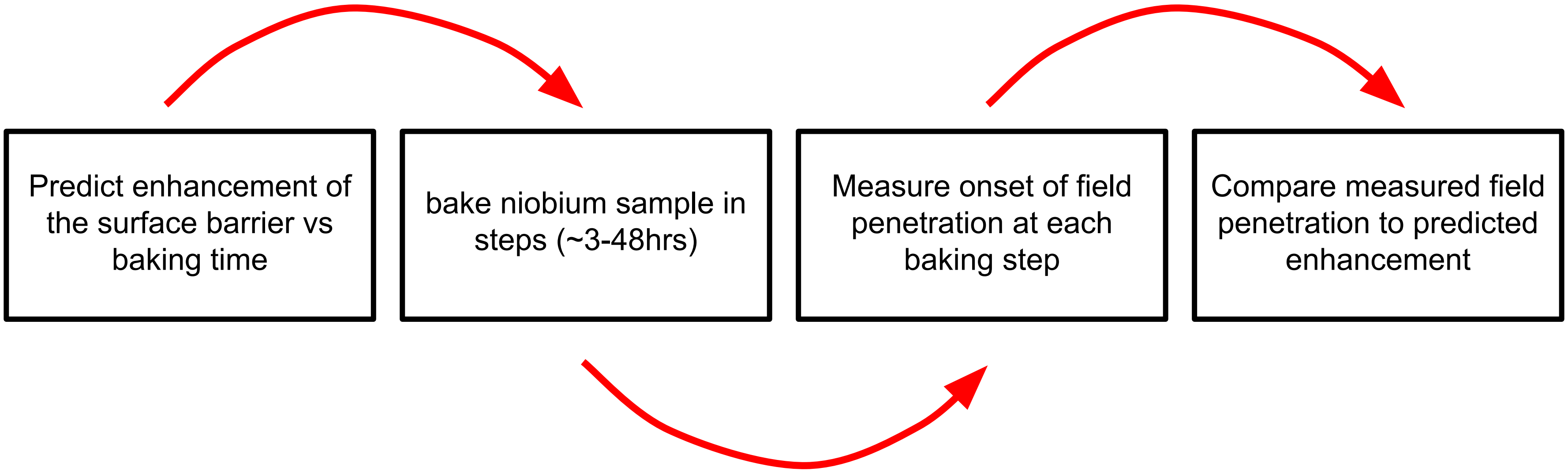
Experiment overview

The aim of this research is to answer the question:

Is it possible to measure an enhanced surface barrier in low-temperature baked niobium?

Experiment overview

To answer this question, we will:



Predicting performance in inhomogeneous materials

University of Victoria

To predict performance of baked Niobium, one must:

1. **Solve the inhomogeneous Ginzburg-Landau (G-L) equations.**

$$f - f^3 + \vec{\nabla}\left(\xi(\vec{x},T)^2\right)\cdot\vec{\nabla} f + \xi(\vec{x},T)^2\vec{\nabla}^2 f - \frac{\lambda(\vec{x},T)^2}{f^3}\left(\vec{\nabla}\times\vec{h}\right)^2 = 0$$

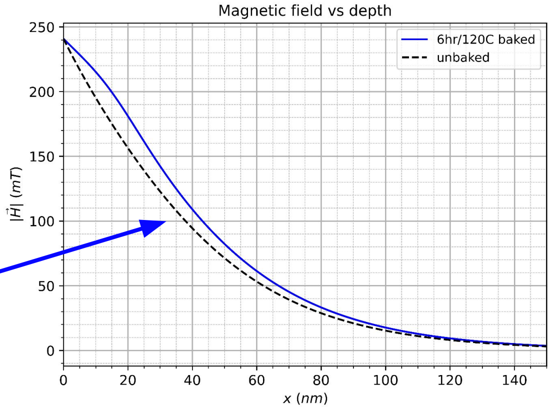
$$\vec{h} - \vec{\nabla}\times\left(\frac{2\lambda(\vec{x},T)^2}{f^2}\left(\vec{\nabla}\times\vec{h}\right)\right) = 0$$

Material parameters λ(x), ξ(x) depend on gradient of oxygen content, With dependence given by:

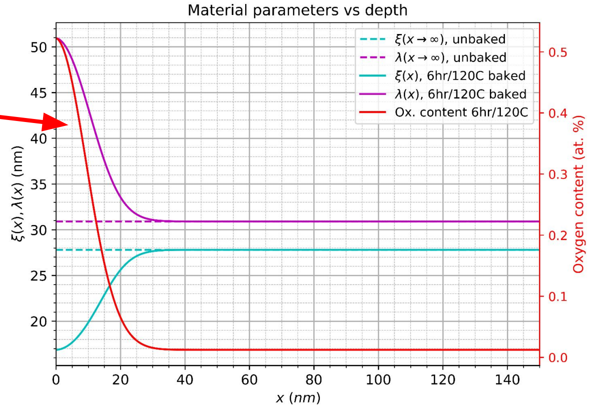
$$\xi(\vec{x},T) = 0.74\,\frac{\xi_0}{r}\left[1 + 0.88\,\frac{\xi_0}{l(\vec{x})}\right]^{-1/2}\left(1 - \frac{T}{T_c}\right)^{-1/2}$$

$$\lambda(\vec{x},T) = \frac{r\,\lambda_0}{\sqrt{2}}\left[1 + 0.88\,\frac{\xi_0}{l(\vec{x})}\right]^{1/2}\left(1 - \frac{T}{T_c}\right)^{-1/2}$$

H(x)



Ox. content

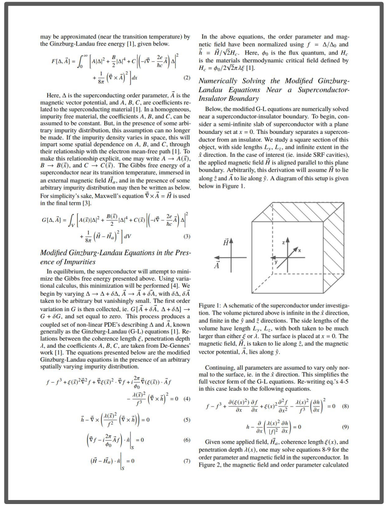


Predicting performance in inhomogeneous materials

University of Victoria

To predict performance of baked Niobium, one must:

2. **Determine the stability of these G-L solutions**



may be approximated (near the transition temperature) by the Ginzburg-Landau free energy [1], given below.

$$F[\Delta,\vec{A}] = \int_0^\infty \left[A|\Delta|^2 + \frac{B}{2}|\Delta|^4 + C\left|\left(-i\vec{\nabla} - \frac{2e}{\hbar c}\vec{A}\right)\Delta\right|^2 + \frac{1}{8\pi}\left(\vec{\nabla}\times\vec{A}\right)^2 \right] dx \quad (2)$$

Here, Δ is the superconducting order parameter, $\vec{A}$ is the magnetic vector potential, and A, B, C, are coefficients related to the superconducting material [1]. In a homogeneous, impurity free material, the coefficients A, B, and C, can be assumed to be constant. But, in the presence of some arbitrary impurity distribution, this assumption can no longer be made. If the impurity density varies in space, this will impart some spatial dependence on A, B, and C, through their relationship with the electron mean-free path [1]. To make this relationship explicit, one may write $A \to A(\vec{x})$, $B \to B(\vec{x})$, and $C \to C(\vec{x})$. The Gibbs free energy of a superconductor near its transition temperature, immersed in an external magnetic field $\vec{H}_a$, and in the presence of some arbitrary impurity distribution may then be written as below. For simplicity's sake, Maxwell's equation $\vec{\nabla}\times\vec{A} = \vec{H}$ is used in the final term [3].

$$G[\Delta,\vec{A}] = \int_V \left[A(\vec{x})|\Delta|^2 + \frac{B(\vec{x})}{2}|\Delta|^4 + C(\vec{x})\left|\left(-i\vec{\nabla} - \frac{2e}{\hbar c}\vec{A}\right)\Delta\right|^2 + \frac{1}{8\pi}\left(\vec{H} - \vec{H}_a\right)^2 \right] dV \quad (3)$$

Modified Ginzburg-Landau Equations in the Presence of Impurities

In equilibrium, the superconductor will attempt to minimize the Gibbs free energy presented above. Using variational calculus, this minimization will be performed [4]. We begin by varying $\Delta \to \Delta + \delta\Delta$, $\vec{A} \to \vec{A} + \delta\vec{A}$, with $\delta\Delta$, $\delta\vec{A}$ taken to be arbitrary but vanishingly small. The first order variation in G is then collected, ie. $G[\vec{A}+\delta\vec{A},\ \Delta+\delta\Delta] \to G + \delta G$, and set equal to zero. This process produces a coupled set of non-linear PDE's describing Δ and $\vec{A}$, known generally as the Ginzburg-Landau (G-L) equations [1]. Relations between the coherence length ξ, penetration depth λ, and the coefficients A, B, C, are taken from De-Gennes' work [1]. The equations presented below are the modified Ginzburg-Landau equations in the presence of an arbitrary spatially varying impurity distribution.

$$f - f^3 + \xi(\vec{x})^2\vec{\nabla}^2 f + \vec{\nabla}\xi(\vec{x})^2\cdot\vec{\nabla}f + i\frac{2\pi}{\phi_0}\vec{\nabla}(\xi(\vec{x}))\cdot\vec{A}f - \frac{\lambda(\vec{x})^2}{f^3}\left(\vec{\nabla}\times\vec{h}\right)^2 = 0 \quad (4)$$

$$\vec{h} - \vec{\nabla}\times\left(\frac{\lambda(\vec{x})^2}{f^2}\left(\vec{\nabla}\times\vec{h}\right)\right) = 0 \quad (5)$$

$$\left(\vec{\nabla}f - i\frac{2\pi}{\phi_0}\vec{A}f\right)\cdot\hat{n}\Big|_S = 0 \quad (6)$$

$$(\vec{H} - \vec{H}_a)\cdot\hat{n}\Big|_S = 0 \quad (7)$$

In the above equations, the order parameter and magnetic field have been normalized using $f = \Delta/\Delta_0$ and $\vec{h} = \vec{H}/\sqrt{2}H_c$. Here, ϕ_0 is the flux quantum, and H_c is the materials thermodynamic critical field defined by $H_c = \phi_0/2\sqrt{2}\pi\lambda\xi$ [1].

Numerically Solving the Modified Ginzburg-Landau Equations Near a Superconductor-Insulator Boundary

Below, the modified G-L equations are numerically solved near a superconductor-insulator boundary. To begin, consider a semi-infinite slab of superconductor with a plane boundary set at $x = 0$. This boundary separates a superconductor from an insulator. We study a square section of this object, with side lengths L_y, L_z, and infinite extent in the $\hat{x}$ direction. In the case of interest (ie. inside SRF cavities), the applied magnetic field $\vec{H}$ is aligned parallel to this plane boundary. Arbitrarily, this derivation will assume $\vec{H}$ to lie along $\hat{z}$ and $\vec{A}$ to lie along $\hat{y}$. A diagram of this setup is given below in Figure 1.

Figure 1: A schematic of the superconductor under investigation. The volume pictured above is infinite in the $\hat{x}$ direction, and finite in the $\hat{y}$ and $\hat{z}$ directions. The side lengths of the volume have length L_y, L_z, with both taken to be much larger than either ξ or λ. The surface is placed at $x = 0$. The magnetic field, $\vec{H}$, is taken to lie along $\hat{z}$, and the magnetic vector potential, $\vec{A}$, lies along $\hat{y}$.

Continuing, all parameters are assumed to vary only normal to the surface, ie. in the $\hat{x}$ direction. This simplifies the full vector form of the G-L equations. Re-writing eq.'s 4-5 in this case leads to the following equations.

$$f - f^3 + \frac{\partial(\xi(x)^2)}{\partial x}\frac{\partial f}{\partial x} + \xi(x)^2\frac{\partial^2 f}{\partial x^2} - \frac{\lambda(x)^2}{f^3}\left(\frac{\partial h}{\partial x}\right)^2 = 0 \quad (8)$$

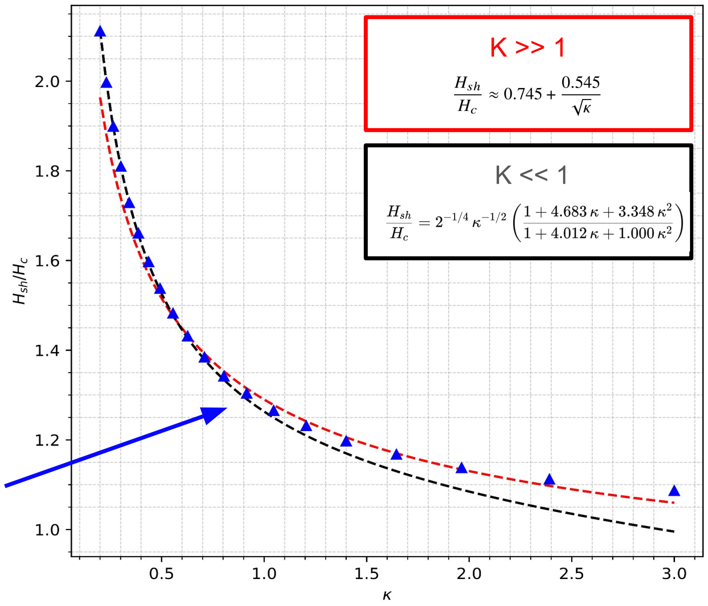
$$h - \frac{\partial}{\partial x}\left(\frac{\lambda(x)^2}{|f|^2}\frac{\partial h}{\partial x}\right) = 0 \quad (9)$$

Given some applied field, $\vec{H}_a$, coherence length $\xi(x)$, and penetration depth $\lambda(x)$, one may solve equations 8-9 for the order parameter and magnetic field in the superconductor. In Figure 2, the magnetic field and order parameter calculated

Stability determined via method in previous conference submission (SRF 2025, 10.18429/JACoW-SRF2025-TUP15)

Numerical stability of 1D G-L solver approximates superconductor stability quite well (for κ~1)

Superheating field of Nb vs K



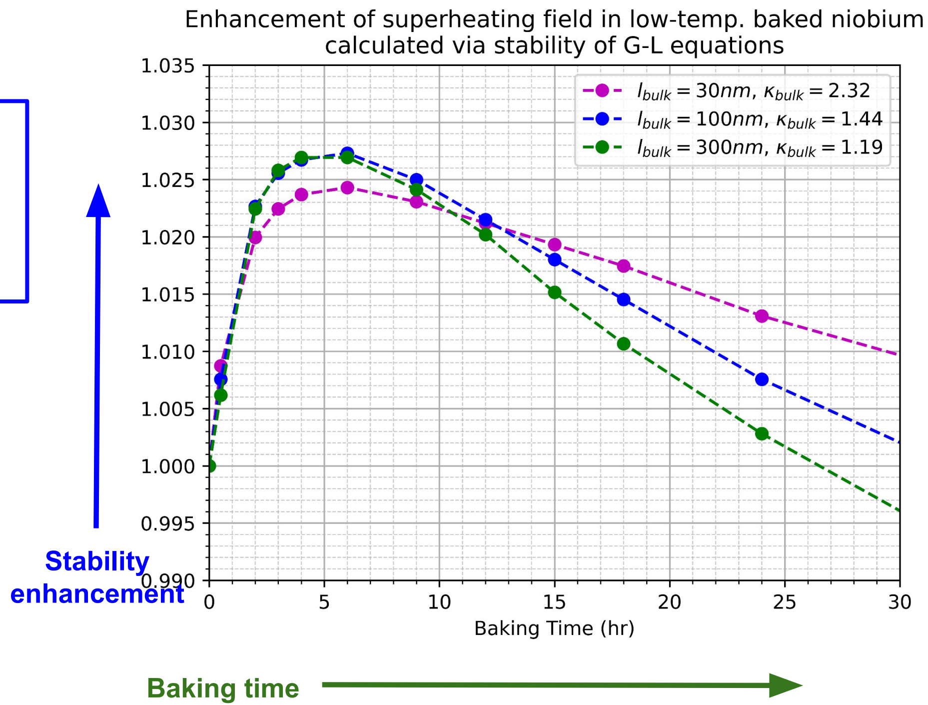
Predicting performance in inhomogeneous materials

Using simulated oxygen profiles -> **predict stability enhancement**

$$\text{Stability enhancement} = \frac{\text{Baked superheating field}}{\text{Unbaked superheating field}}$$

Max stability enhancement **~3%**

Max stability enhancement predicted at **~6hrs/120C**



Niobium sample preparation: Surface processing

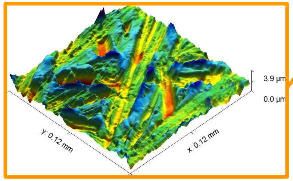
University of Victoria

Difficult to measure small % changes in field penetration. We must:

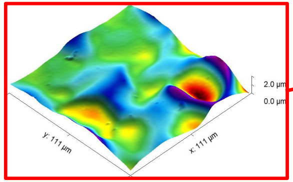
- Electropolish (EP) sample to a high degree.
- Heat treat sample (1400C/3hr) to reduce bulk pinning.
- Design highly repeatable measurement system.

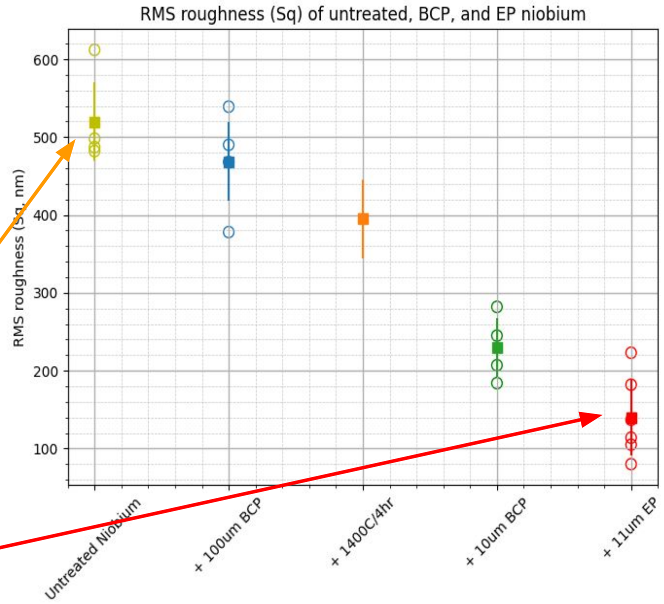
Niobium Ellipsoid

Untreated surface



Electropolished Surface



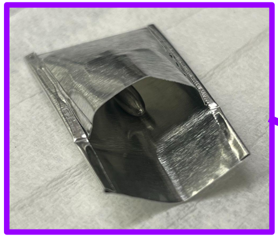


Sample treatment:
100umBCP + 1400C/3hr + 10umBCP + 11um EP
(Treatments courtesy of Philipp Kolb, James Keir, TRIUMF)

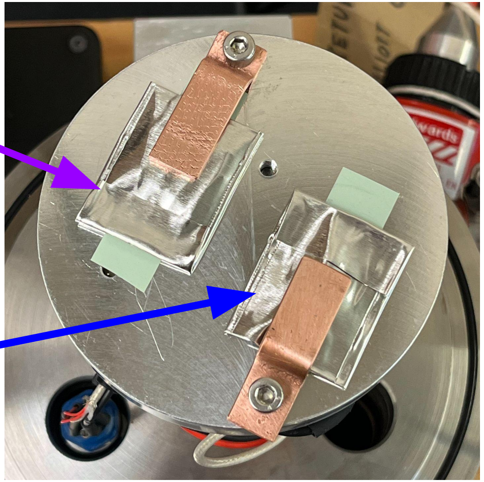
Niobium sample preparation: low-temp. baking

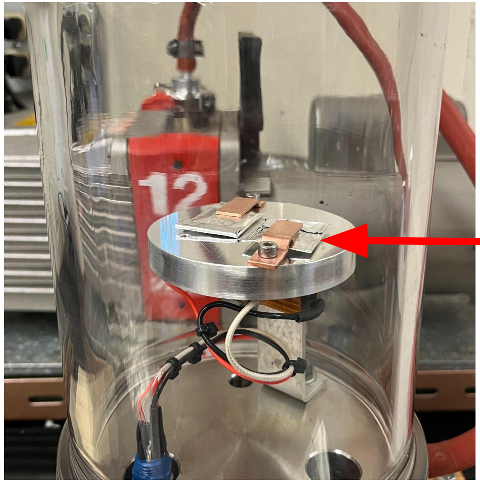
After processing, niobium sample was placed in foil bag, and **baked in a vacuum furnace** (designed as part of PhD work) at 120C.

Niobium ellipsoid



Witness samples





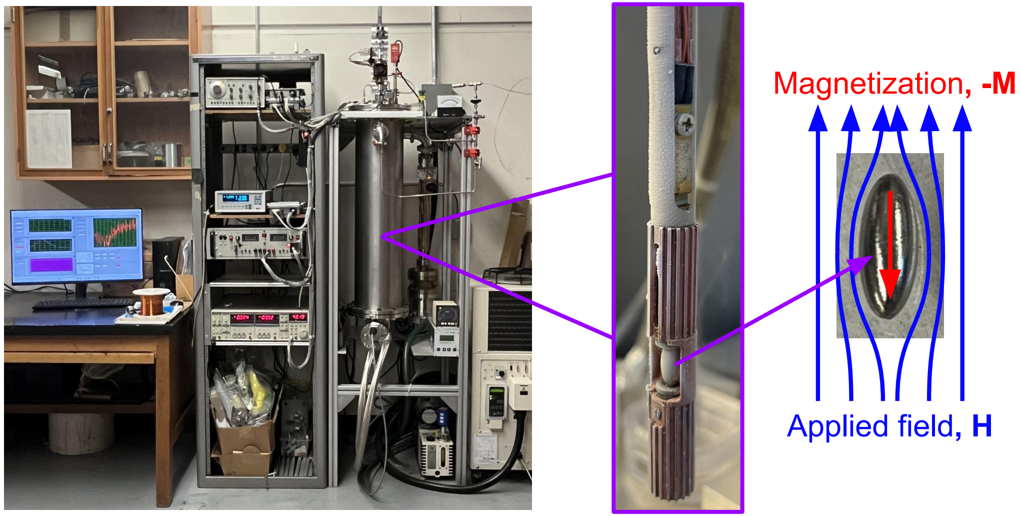
120C hot plate

Vacuum furnace

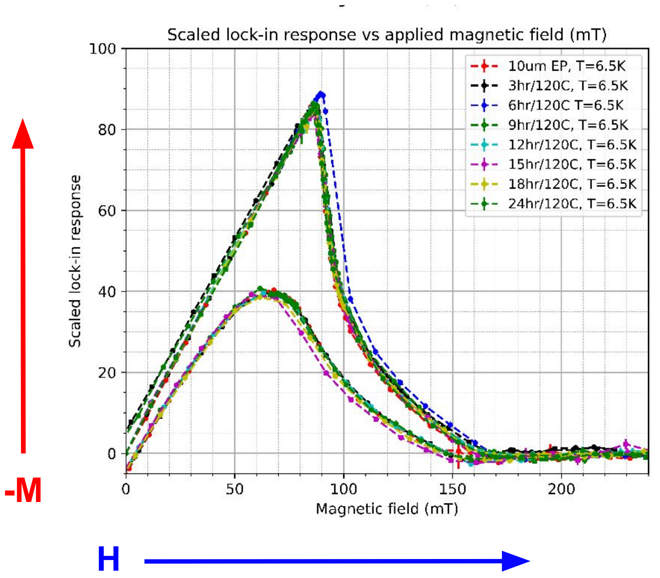
Magnetization measurements and onset of field penetration

University of Victoria

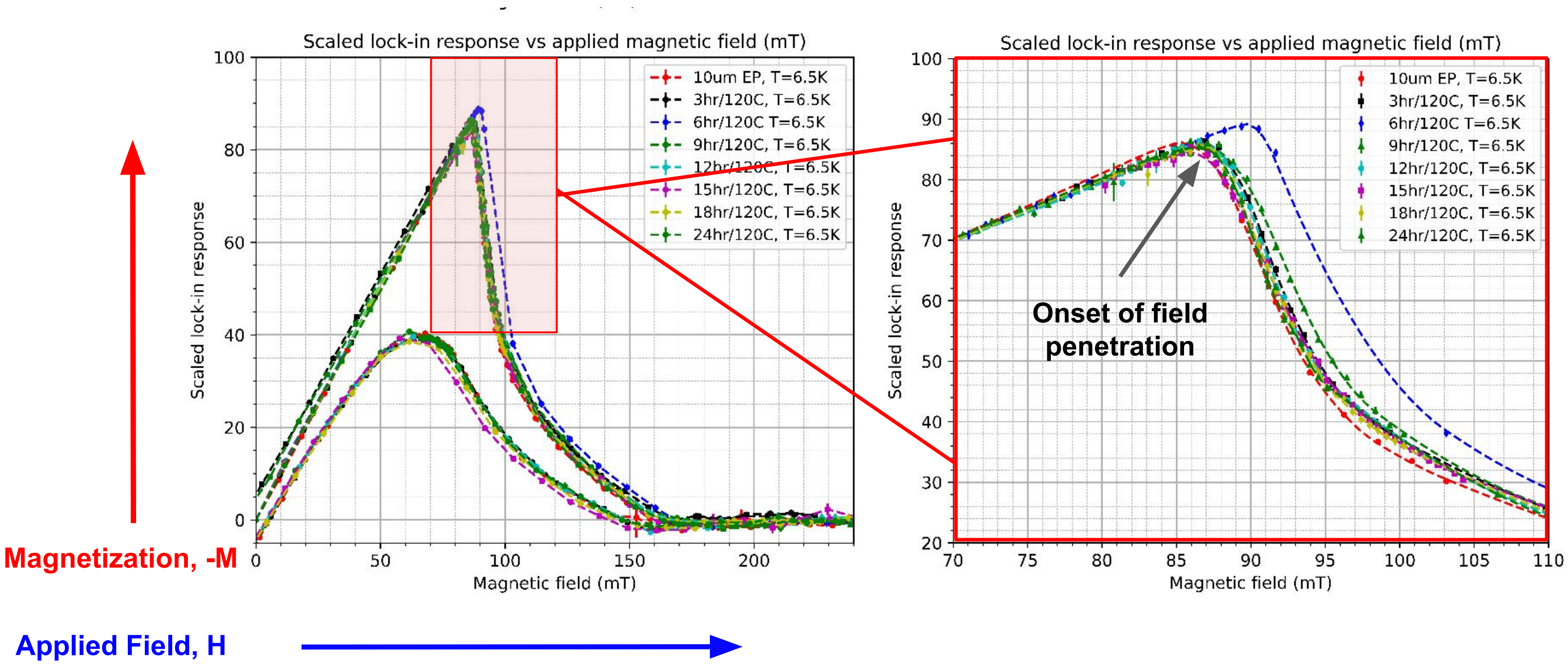
Magnetization of baked niobium samples measured in **vibrating sample magnetometer** (designed as part of PhD work) at the University of Victoria.



Vibrating Sample Magnetometer and control system

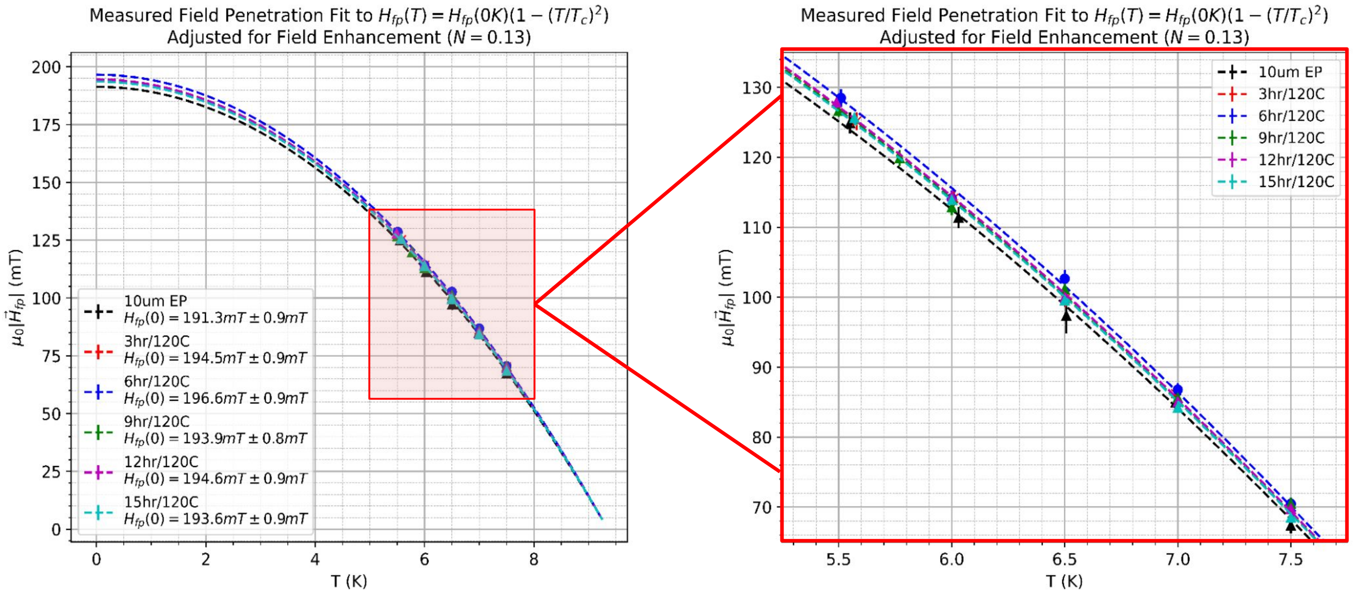


Magnetization measurements and onset of field penetration



Magnetization measurements and onset of field penetration

Onset of field penetration fit to Hfp(T)~Hfp(0)*(1-(T/Tc)^2)

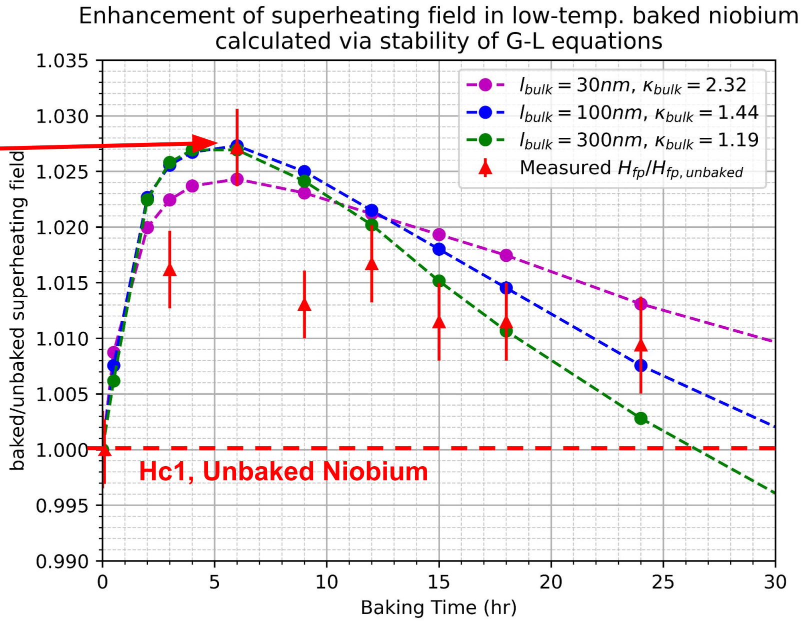


Comparison to predicted onset of field penetration

Maximum field penetration at **~6hr/120C bake**

Max stability enhancement **~3%**

Enhancement decays past ~6hr/120C baking



Comparison to predicted onset of field penetration

Returning to our original question:

Is it possible to measure an enhanced surface barrier in low-temperature baked niobium?

These measurements point to -> **Yes!**

Concluding remarks

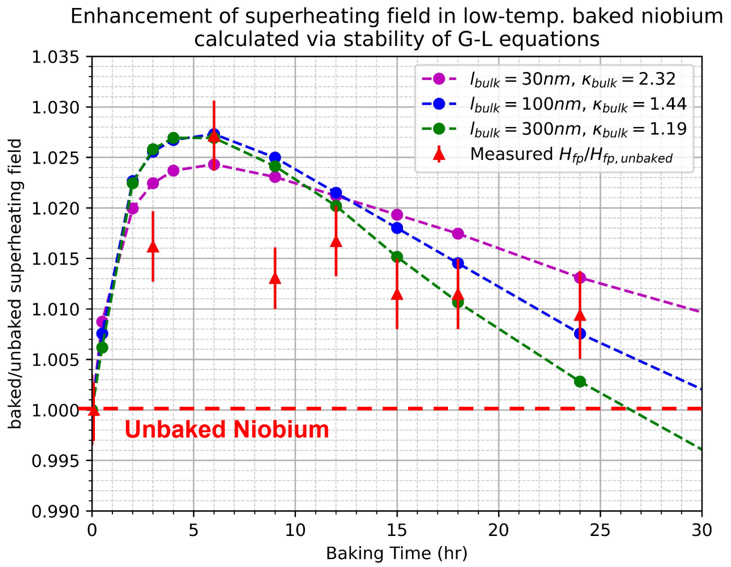
What else can we infer from these results?

It is possible to measure surface barrier effects in DC magnetometry experiments! Opens up small+cheap studies of low-temp. treatments.

The measured enhancement reasonably follows the predicted value -> use model for future predictions.

More oxygen at surface + stronger Ox. gradient -> Larger surface barrier?

Lower bulk kappa + low temp. bake -> larger surface barrier. Very clean baked niobium may perform better?



Concluding remarks

Thank you for listening!