Phenomenological approach about muon component in hadronic showers

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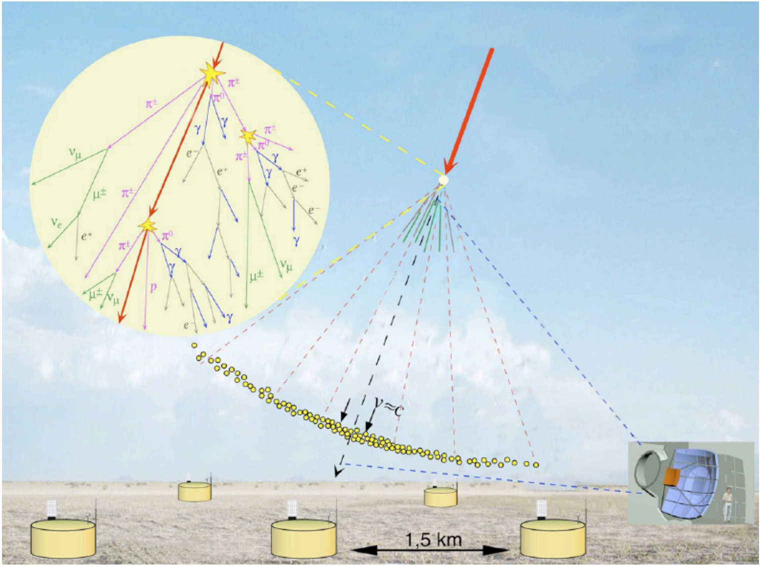
A hadronic shower in the Pierre Auger Observatory



Outline

1. **Tracking of charged pions component**
   - Few words about the toy model
   - Different predictions from the toy model
2. **Muon production from charged pions**
3. **Muon propagation from its birth to ground**
   - Phenomena taken into account for muon propagation
   - Muon energy spectrum at ground
   - Muon arrival time to the ground
4. **Conclusions**

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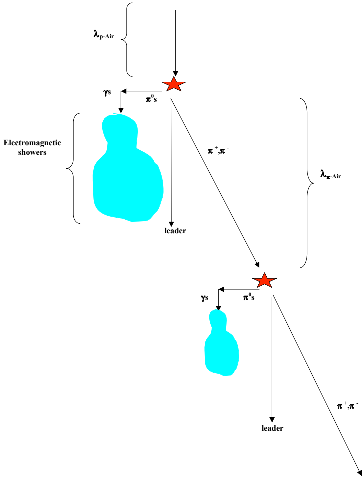
Motivation

Simple analytical description of the muon birth through average values

$\Longrightarrow$ predictions of the altitude, the energy and the number of muons... at birth.

Toy model for development of hadronic showers



Heitler model

$1/3 \Rightarrow \pi^0$: EM component,
$2/3 \Rightarrow \pi^\pm$: hadronic component.

Requirements in input

1. mean free paths p-air and π-air,
2. multiplicities p-air and π-air,

Competition between 2 phenomena

Interaction with atmosphere: $\pi^{\pm} + Air \rightarrow \sum \pi^{\pm} + \sum \pi^{0} + ...$

- $L_I[m] \simeq 10^3 exp\left(\frac{h[km]}{8}\right)$.

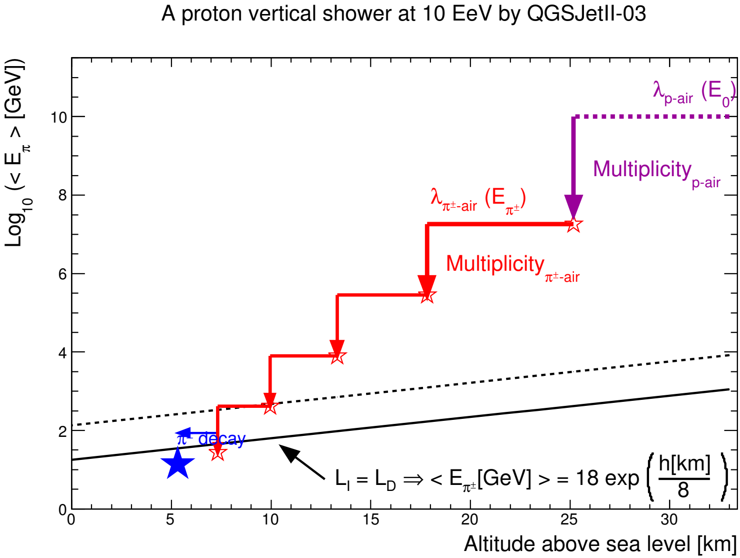
Decay: $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$

- $L_D[m] = \beta\gamma c\tau = 56 \times E_{\pi^{\pm}}[GeV]$.

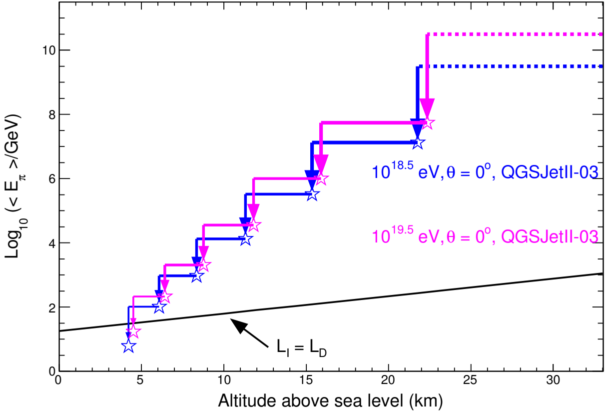
The algorithm

- $L_I < L_D$: pions interact with atmosphere (first phase),
- $L_I = L_D$: pions decay in muons (second phase).

Evolution of the hadronic component

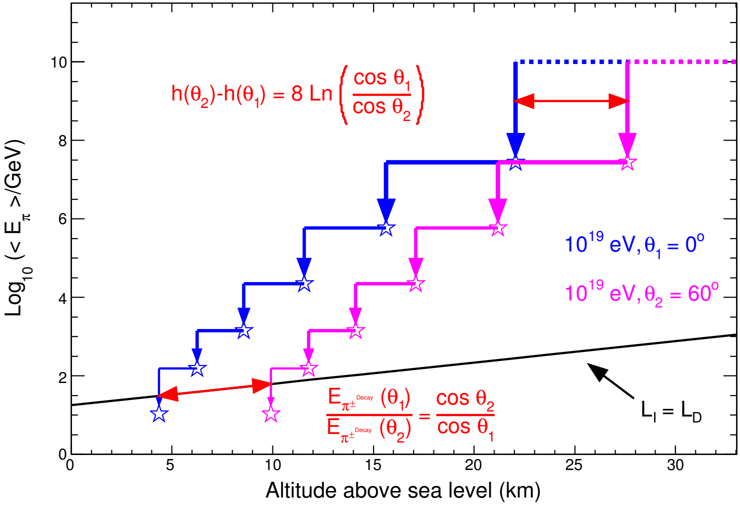


Influence of energy E_0 on muon production altitude

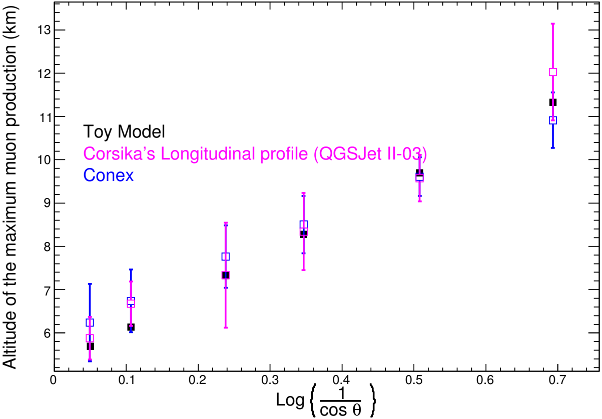


⟹ Altitude of the maximum muon production quite independent of the primary energy

Muon's energy and production altitude wrt zenith angle



Predictions wrt zenithal angle (proton, 10^{19}eV)



$\Longrightarrow$ Good agreement for toy model with longitudinal profiles from Conex and Corsika

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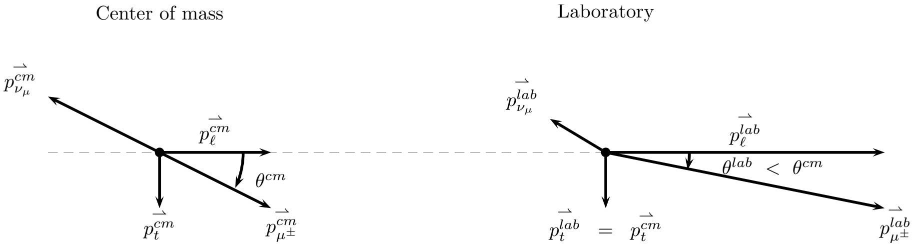
Kinematics of the charged meson decay

$$\pi^{\pm} \longrightarrow \mu^{\pm} + \nu_\mu \quad \text{or} \quad \mathsf{K}^{\pm} \longrightarrow \mu^{\pm} + \nu_\mu$$



Muon energy in the laboratory frame

$$E_\mu^{\mathsf{lab}} = \gamma(E_\mu^{\mathsf{cm}} + \beta\, p_\mu^{\mathsf{cm}} \cos\theta^{cm}) \simeq p_{\mu,\parallel}^{\mathsf{lab}}\, c$$

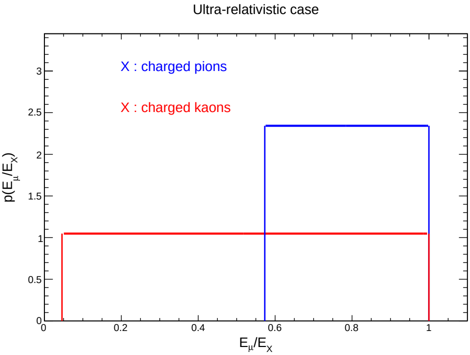
Transerve momentum in the laboratory frame

$$p_{\mu,\perp}^{\mathsf{lab}} = p_{\mu,\perp}^{\mathsf{cm}} = p_\mu^{\mathsf{cm}} \sin\theta^{cm}$$

$p(E_\mu|E_\pi)$: muon energy, knowing E_π (its mother)

$$\frac{1+r}{2} - \beta\frac{1-r}{2} \leq \frac{E_\mu}{E_\pi} \leq \frac{1+r}{2} + \beta\frac{1-r}{2} \,, \quad \text{with} \quad \mathrm{r} = \left(\frac{\mathrm{m}_\mu}{\mathrm{m}_\pi}\right)^2$$

Ultra-relativistic case: $\beta = 1$ ($v = c$)

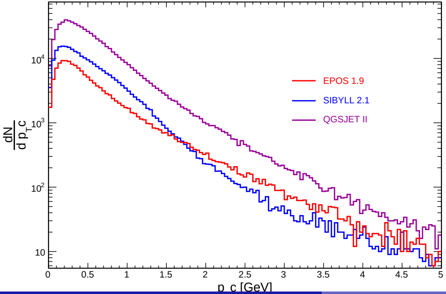


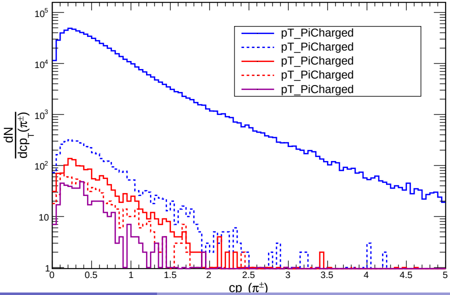
$$\begin{cases} 0.57 \;\leq\; E_\mu^{\text{lab}}/E_\pi \;\leq\; 1.00 \\ \Longleftrightarrow\; < E_\mu^{\text{lab}} >= 0.79\; E_\pi \\ 0.04 \;\leq\; E_\mu^{\text{lab}}/E_{\mathsf{K}} \;\leq\; 1.00 \\ \Longleftrightarrow\; < E_\mu^{\text{lab}} >= 0.52\; E_{\mathsf{K}} \end{cases}$$

$p(\alpha|E_\pi)$: muon emission angle, knowing E_π (its mother)

- α is the angle between the charged pion direction and the shower axis,
- the muon produced by pion decay is assumed colinear.

$$\alpha = \tan^{-1}\left(\frac{p_T}{p_L}\right) = \tan^{-1}\left[\frac{p_T}{\sqrt{E_\pi^2 - p_T^2 - m_\pi^2}}\right]$$





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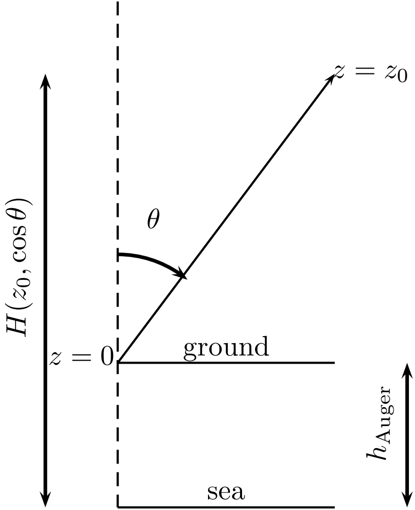
Motivation

Understand muon propagation from its production to its detection in the water tanks

$\Longrightarrow$ a way to obtain different hadronic characteristics in the shower development from muon detected at ground.

Muon decay



Mean free path for muon decay

$$L_{\mathsf{D}}[\mathsf{m}] = \sqrt{{\gamma_\mu}^2 - 1}\, c\tau_\mu$$
$$= 658\sqrt{\left(\frac{T_\mu}{m_\mu c^2} + 1\right)^2 - 1}$$

Evolution of muon number

$$N(z+dz) = N(z) + N(z) \times dz / L_{\mathsf{D},\mu}$$
$$\Rightarrow \frac{dN}{dz} = \frac{N}{L_{\mathsf{D},\mu}}$$

Ionization losses

- muon energy not constant during propagation: $T_\mu(z = z_0) \to T_\mu(z)$
- range in energy around minimum ionization

Average rate of muon energy loss

$$\left\langle -\frac{dT_\mu}{dz} \right\rangle = a \simeq 2\,\text{MeV g}^{-1}\text{cm}^2 \simeq 0.24\,\text{exp}\left(-\frac{H(z,\cos\theta)[\text{m}]}{8000}\right)\text{MeV m}^{-1}$$

(while using US standard model for atmosphere)

Kinetics muon energy at z

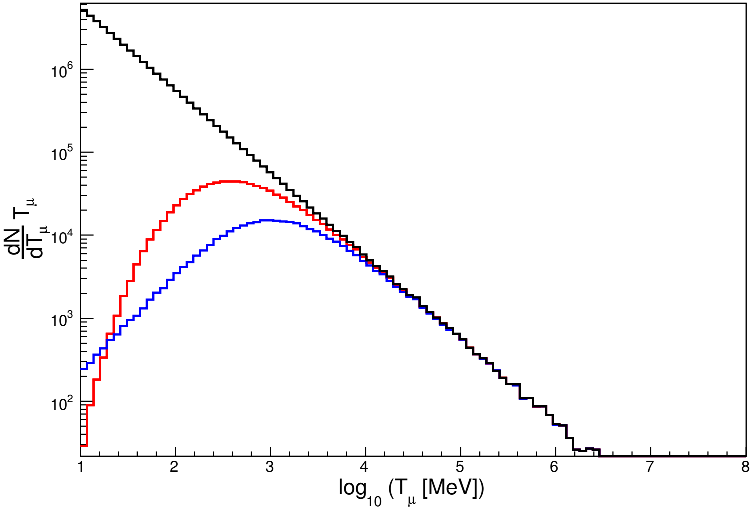
$$T_\mu(z) = T_\mu(z_0) - \int_{z_0}^{z} 0.24\,\text{exp}\left(-\frac{H(z,\cos\theta)[\text{m}]}{8000}\right)\text{dz}$$

$$T_\mu(z) = T_\mu(z_0) - \frac{1920}{\cos\theta}\text{exp}\left(-\frac{h_{\text{Auger}}}{8000}\right)\Bigg[\text{exp}\left(-\frac{z\cos\theta}{8000}\right) - \text{exp}\Bigg(-\frac{z_0\cos\theta}{8000}$$

1 – Muon propagation *by* a Toy Monte Carlo

```
Algorithm Toy Monte Carlo
for $i \leftarrow 1$ to $n$
    do $T_\mu \leftarrow$ {muon energy spectrum at birth (here $E^{-2}$)}
        while $z \geq 0$ and $T_\mu(z, z_0) \geq 0$
            do $T_\mu(z, z_0) \rightarrow T_\mu(z - d, z_0)$, with $d$ the step in distance
                $L_{\mathsf{D},\mu}(z - d, z_0)$
                if $\frac{d}{L_{\mathsf{D},\mu}(z-d, z_0)} \geq Rnd$, with $Rnd$ a float from an
                uniform distribution $]0, 1]$
                    then break the while loop : the muon decays
                        $\rightarrow Decay = +1$
                    else next iteration in the while loop
        if $Decay \neq +1$
            then add one event at the kinetic energy $T_\mu(z = 0, z_0)$ in
                the histogram
return final muon energy spectrum
```

1 – Toy MC : results for $H(z_0, \cos\theta = 1) = 5500$ meters



2 – Muon propagation *by* an analytical formula: $p_{\text{arrived}}(T_\mu, z_0, \cos\theta)$

- muon propagation from an altitude $H(z_0, \cos\theta)$ to the ground,
- formula without any approximation.

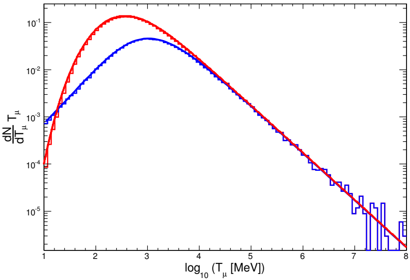
$$N_{\text{ground}} = N(z=0) = N(z_0) \left(\frac{1 - \frac{2A}{\gamma_\mu(z_0) + \sqrt{\gamma_\mu(z_0)^2 - 1}}}{1 - \frac{2A}{\gamma_\mu(0) + \sqrt{\gamma_\mu(0)^2 - 1}}} \right)^{\frac{8000}{658 A(z_0, \cos\theta) \cos\theta}}$$

with

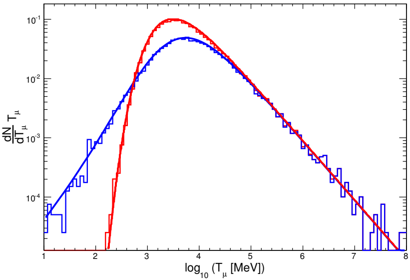
$$A(z_0, \cos\theta) = \frac{T_\mu(z_0) + m_\mu c^2}{m_\mu c^2} + \frac{1920}{\cos\theta} \frac{1}{m_\mu c^2} \exp\left(-\frac{H(z_0, \cos\theta)}{8000} \right).$$

2 – Muon propagation *by* an analytical formula: $p_{\text{arrived}}(T_\mu, z_0, \cos\theta)$

$$H(z_0, \cos\theta = 1) = 5\,500$$



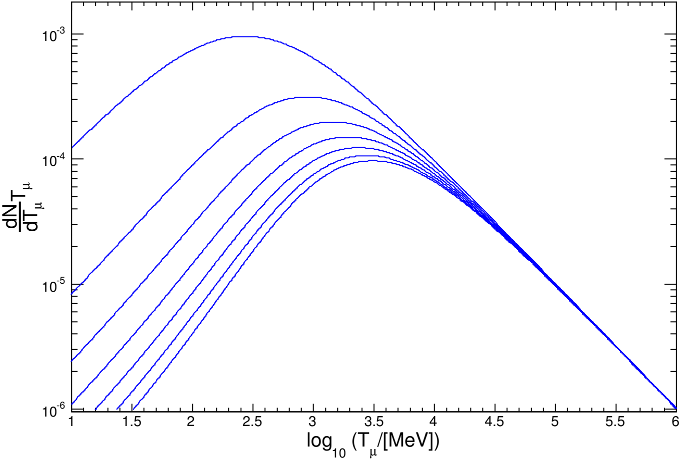
$$H(z_0, \cos\theta = 0.5) = 12\,000$$



Muon energy at ground

$$T_\mu(z = 0 | z_0, \cos\theta) = T_\mu(z_0) - \frac{1920}{\cos\theta} \exp\left(-\frac{h_{\text{Auger}}}{8000}\right)\left[1 - \exp\left(-\frac{z_0 \cos\theta}{8000}\right)\right]$$

Muon energy spectrum evolution wrt. $H(z_0, \cos\theta = 1)$



$$H(z_0, \cos\theta = 1) = 2, 4, 6, 8, 10, 12, 14 \; km$$

Muon arrival time to the ground: $t_\mu(T_\mu, z_0, \cos\theta)$

Muon velocity

- muon velocity changes during its travel

$$v_\mu = c \times \beta_\mu(z) = c \times \frac{p_\mu c}{E_\mu} = c \times \frac{\left[T_\mu(z) \times (T_\mu(z) + 2m_\mu c^2)\right]^2}{T_\mu(z) + m_\mu c^2}$$

Muon arrival time at $z = 0$

$$t_\mu(z=0) = \int_{z_0}^{z=0} \frac{1}{c} \frac{T_\mu(z) + m_\mu c^2}{\left[T_\mu(z) \times (T_\mu(z) + 2m_\mu c^2)\right]^2} \mathrm{dz}$$

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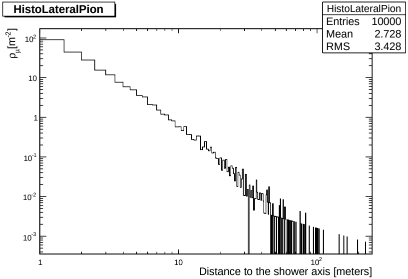
Summary

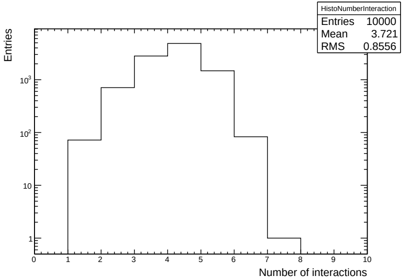
- the charged pions component can be understood easily by a model based on averaged values,
- the altitude of maximum muon production has been localized in the shower,
- physical quantities linked to the muon component are given.

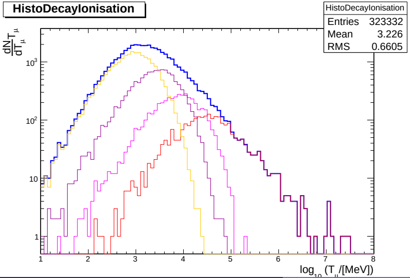
Do we have to extend our pdf's to the charged pion component ?

- the muon component give information about the last charged pion generation,
- the muon lateral extension comes from the charged pions $\Longrightarrow$ maybe, we need to parametrize *also* the charged pions component

A 1st attempt, *just to give an idea*







- we assume a p_T distribution for pions independ of the energy,
- pdf's of hadronic parameters come from CONEX.