Trying to improve on the jump method with Sequential Monte Carlo

Siminole Meeting, October 26th 2010

Rémi Bardenet

LAL, LRI, University Paris-Sud XI

26 octobre 2010

Outline

'Take home' message

A progressive scan of the FADC traces will allow better MCMC proposals for muon counting and more.

Summary

Main problem

The signal model

$\mathcal{P}$	$(y_{1:N}$	$\vert$	k_N	$\tau_{1:k_N}$	$u_{1:k_N}$	$\theta)$
	signal		muon number	arrivals	muon param.	tank param.

This likelihood has been finely parametrized (cf Balazs' work).

Target distribution

$$\pi_N(k_N, \tau_{1:k_N}, u_{1:k_N}, \theta) = \mathcal{P}(k_N, \tau_{1:k_N}, u_{1:k_N}, \theta | y_{1:N})$$

Explicitly deriving π_N

$$\pi_N(k_N, \tau_{1:k_N}, u_{1:k_N}) \propto \mathcal{P}(y_{1:N} | k_N, \tau_{1:k_N}, u_{1:k_N}) \times \mathcal{P}(k_N, \tau_{1:k_N}, u_{1:k_N})$$

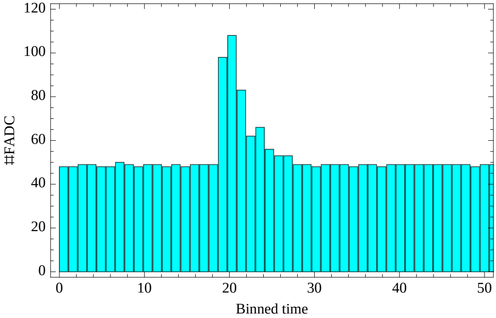
$$\mathcal{P}(k_N, \tau_{1:k_N}, u_{1:k_N}) = \left(\prod_{i=1}^{k_N} \mathcal{P}(u_i | \tau_i) \right) \times \mathcal{P}(\tau_{1:k_N}, k_N)$$

$$\mathcal{P}(\tau_{1:k_N}, k_N) = k_N! 1_{(0<\tau_1<\ldots<\tau_{k_N})} \prod_{i=1}^{k_N} \mathcal{P}(\tau_i) \times \mathcal{P}(k_N).$$

$$\mathcal{P}(k_N) = \mathcal{POI}(\overline{k_N} \times F_\tau(t_N))$$

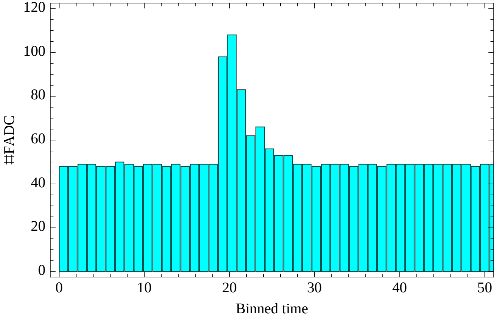
What prior should we take for $\overline{k_N}$? Note that it can depend on θ.

Reversible Jump MCMC



- Try to estimate the posterior of interest by directly trying several realizations of k, τ, u, θ.
- It is hard to find good proposals without looking at the data !

Key idea : add bins one at a time



- Run a SMC sweep, sequentially approximating

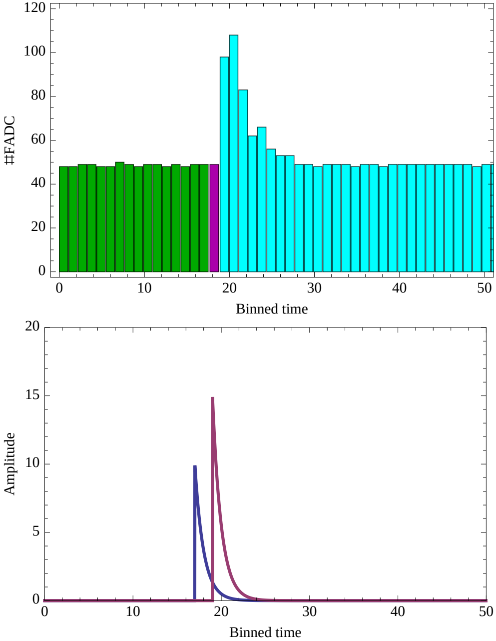
$$\pi_n = \mathcal{P}(k_n, \tau_{1:k_n}, u_{1:k_n} | \theta, y_{1:n}), n = 1..N$$

- Plug π_N into a higher-level MH algorithm, taking

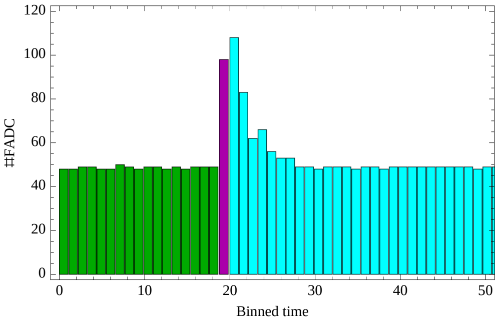
$$\pi_N(k', \tau', u') \otimes q(\theta' | \theta)$$

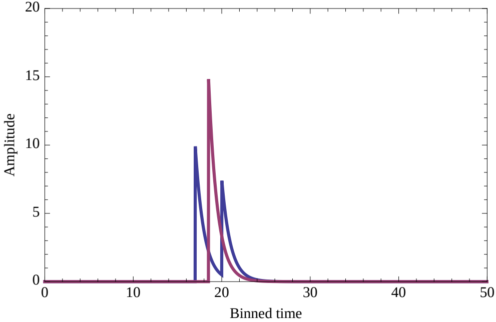
as a proposal (particle MCMC [AnDoHo10]).

Following a few SMC steps together

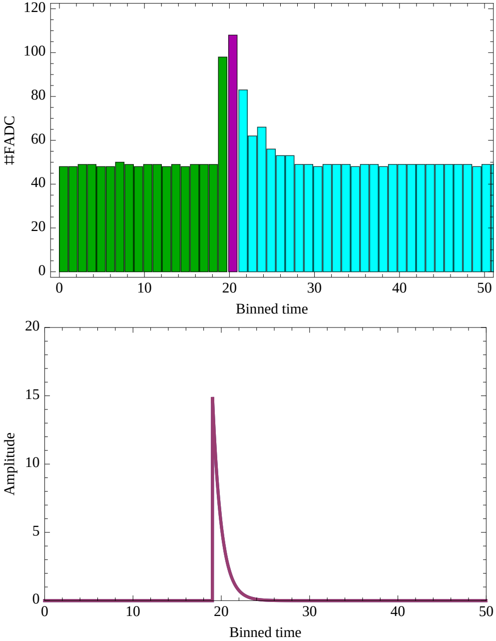


Following a few SMC steps together





Following a few SMC steps together



Summary

Difficulties

- Adding several muons at a time
  → draw a Poissonian number of muons to add, use F_τ and $\overline{k_N}$.
- Model the EM signal in a tractable fashion
  → use between-bin covariance through a shot noise process ?
- The spaces on which the π_n are defined are not of strictly increasing dimension
  → need for SMC samplers [DeDoJa06, DoMoJa06, WhJoGo10].

'Take home' message

A progressive scan of the FADC traces will allow better MCMC proposals for muon counting and more.

'To do' list

- Implement the model and the SMC procedure in C++/Root (currently Matlab),
- Assess it on simulated data,
- Treat the EM part,
- Try to use "foreseeing" to propose even better moves ?