Related Technologies :
1) Radiofrequency
and
2) cryogenics and superconductivity

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Part 1: Radiofrequency Technologies for Particle Accelerators

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Outline

- Introduction: from DC to RF accelerator (15 min)
- Fundamental of RF (15 min)
- A simple example of an RF cavity (15 min)
- Break (10 min)
- RF around cavities (10 min)
- Wake field (10 min)
- Various shapes of RF cavities (10 min)
- Conclusion

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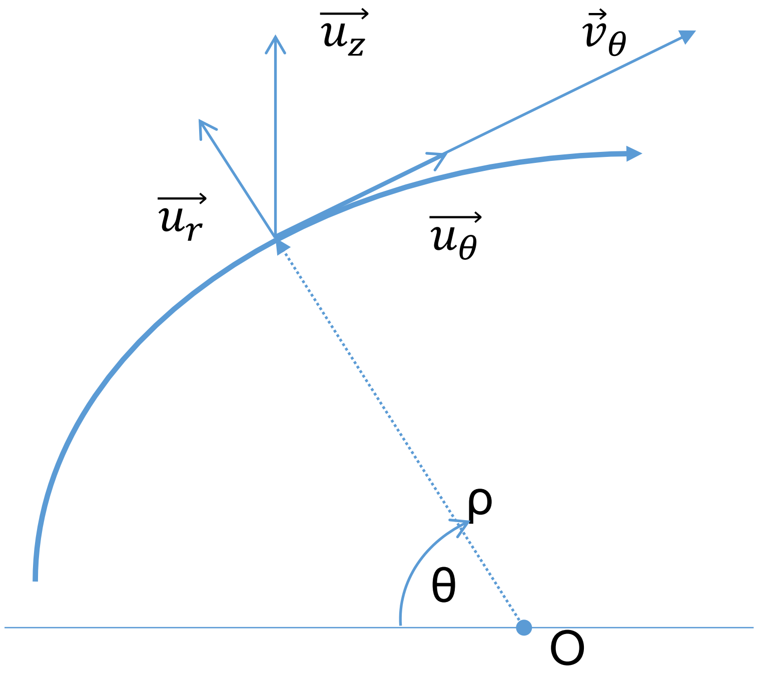
Lorentz force



Assuming $\overrightarrow{E} \rightarrow E_\theta$ and $\overrightarrow{B} \rightarrow B_z$

Lorentz Force: $\frac{d\vec{p}}{dt} = e\vec{E} + e\vec{v} \times \vec{B}$

By developing in the coordinate system:

$$\frac{d(mv_\theta)}{dt}.\overrightarrow{u_\theta} - m\frac{v_\theta^2}{\rho}.\overrightarrow{u_r} = eE_\theta.\overrightarrow{u_\theta} + ev_\theta B_z.\overrightarrow{u_r}$$

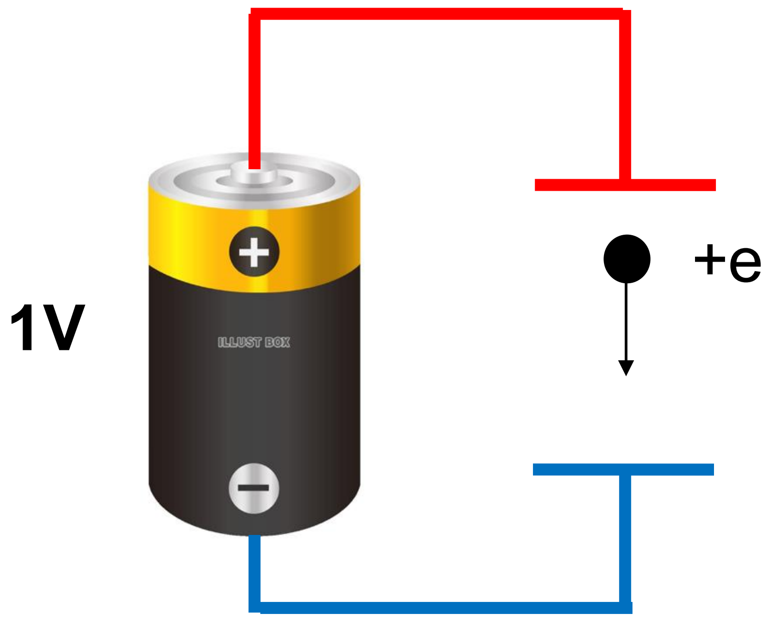
Then, by identification on unit vectors:

$$\frac{dp_\theta}{dt} = eE_\theta$$

$$\frac{p_\theta}{e} = \rho B_z$$

In conventional particle accelerators, we need

1. Magnetic fields to bend the trajectory (magnets)
2. Electric fields to accelerate and decelerate the beam (cavities)



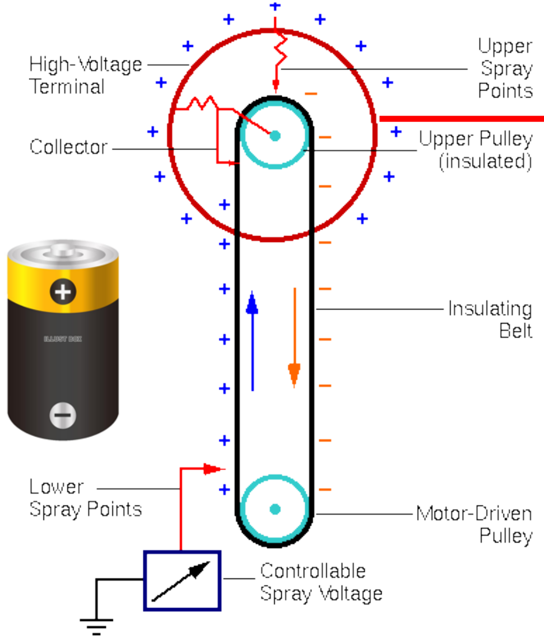
Electron's rest mass
in the natural unit
$m(c^2) = 511$ keV

Kinetic energy of a charge +e $(1.6 \times 10^{-19}\text{C})$ accelerated by 1 V

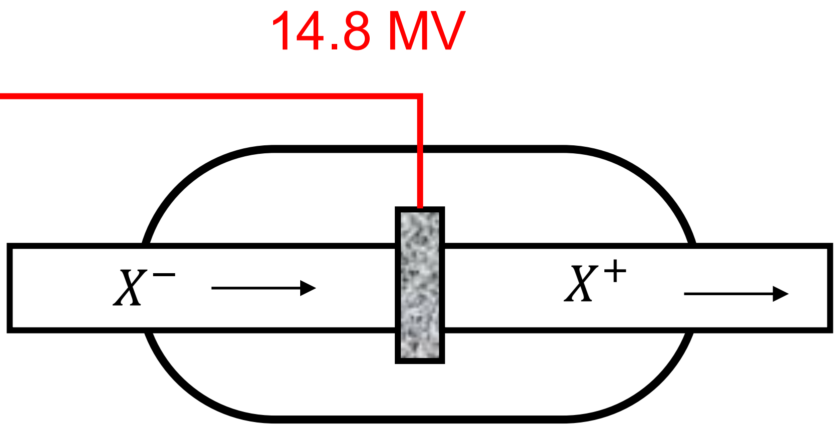
$E = 1$ **eV**

Modern science ≫ MeV (Neutrons>**1GeV**, hard X-rays>**10GeV**, Higgs boson>**125+90 GeV**)

Example of DC accelerator: ALTO Tandem



https://www.eag.com/app-note/rutherford-backscattering-spectrometry-rbs-tutorial/

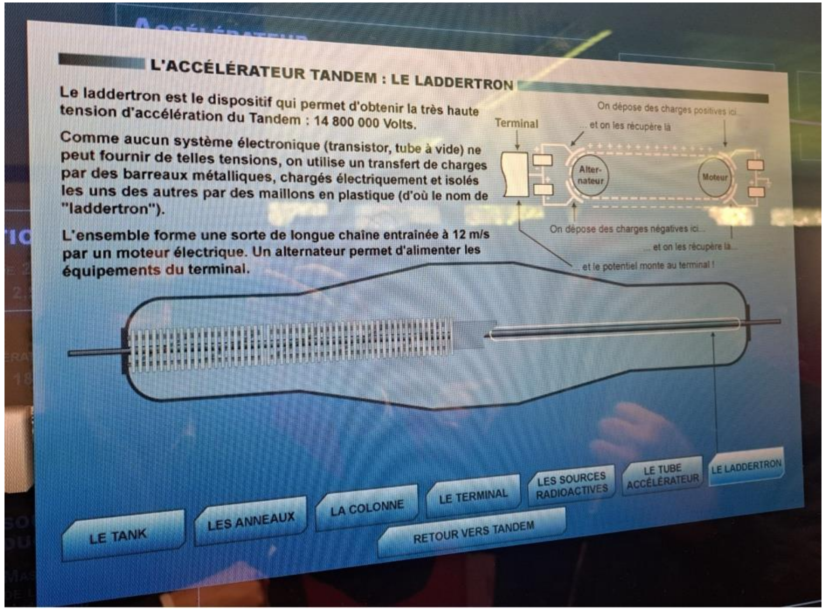


https://alto.ijclab.in2p3.fr/en/facility/alto-heb-en/tandem-2/

- Rubber belt transport charges → very high voltage
- 1st accelerate negative ions → charge stripped → re-accelerate position ions
- One can reach > 10 MV charged particles

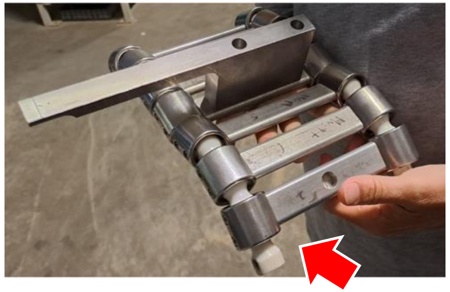
ALTO Tandem: reality

Schematic



Old belt

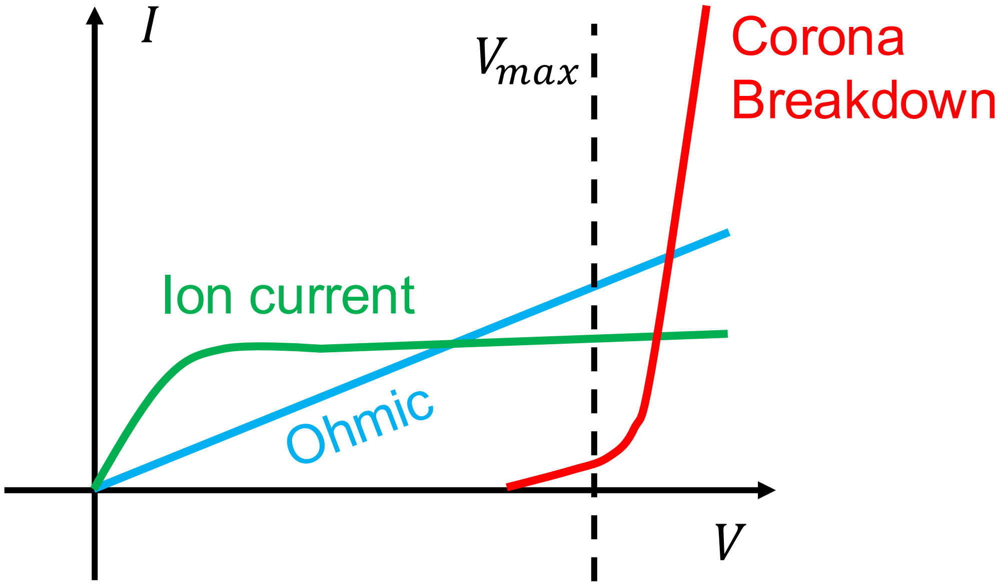
New belt



ALTO Tandem has been operated since 1973 for physics with heavy ions → excellent user facility at Orsay

(cf. A friend of mine in University of Tokyo wrote PhD thesis out of the nuclear physics result of the ALTO Tandem 12 years ago)

Limitation of DC accelerator



- HV is limited to O(10 MV) → break down even in insulating gas (SF_6)
- One cannot study GeV physics with a DC accelerator

→ Breakthrough is necessary at this point

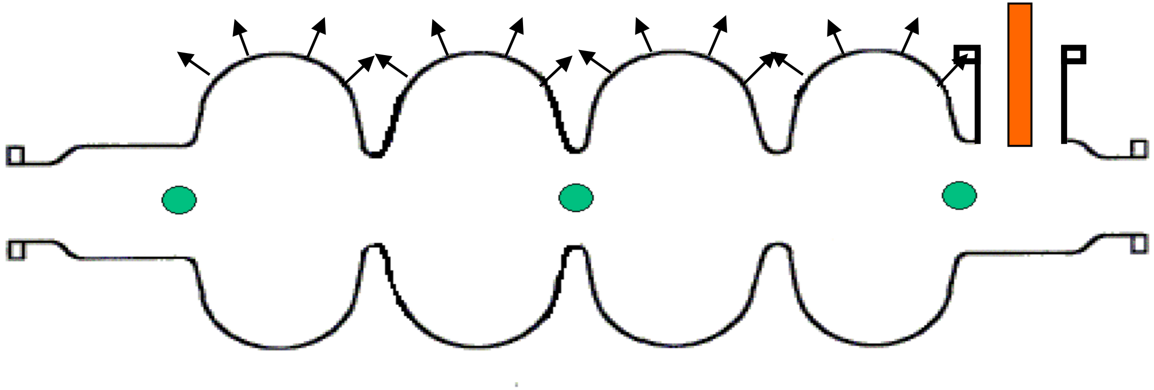
Maxwell equation

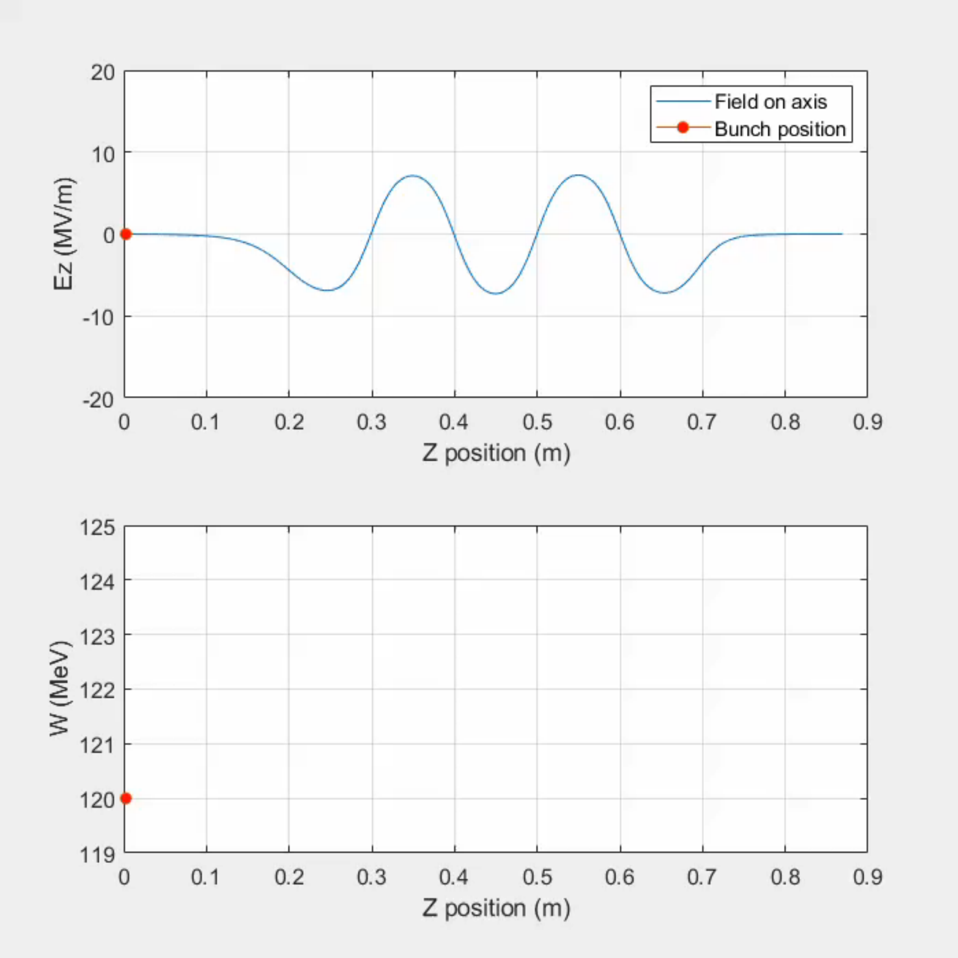
$$\begin{cases} \nabla \cdot \boldsymbol{E} = 0 \\ \nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

Boundary condition

$$\begin{cases} \boldsymbol{n} \times \boldsymbol{E} = 0 \\ \boldsymbol{n} \cdot \boldsymbol{B} = 0 \end{cases}$$

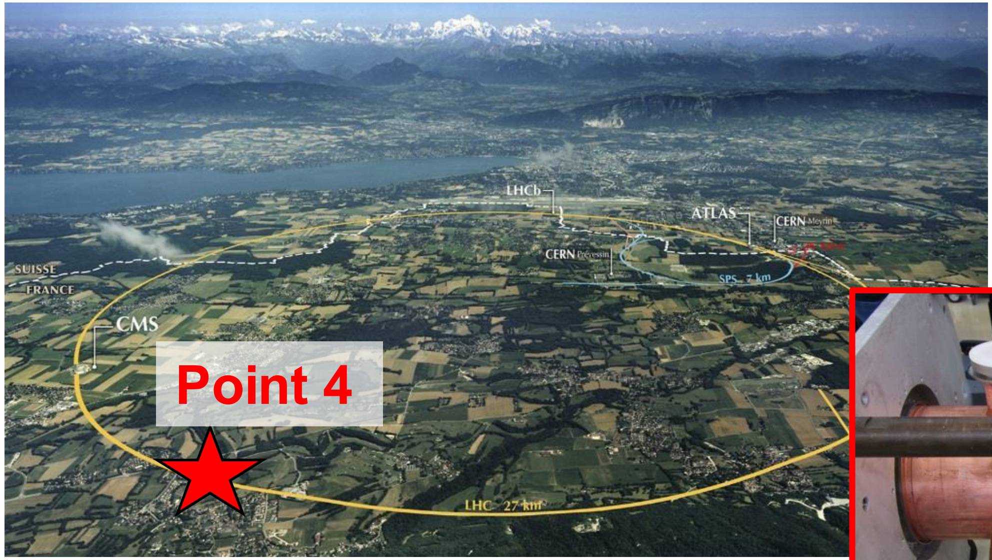
Courtesy: F. Bouly

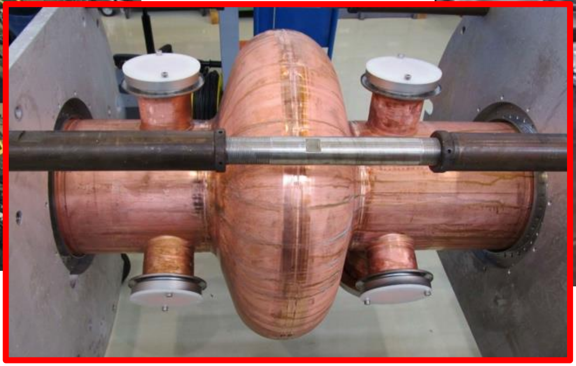


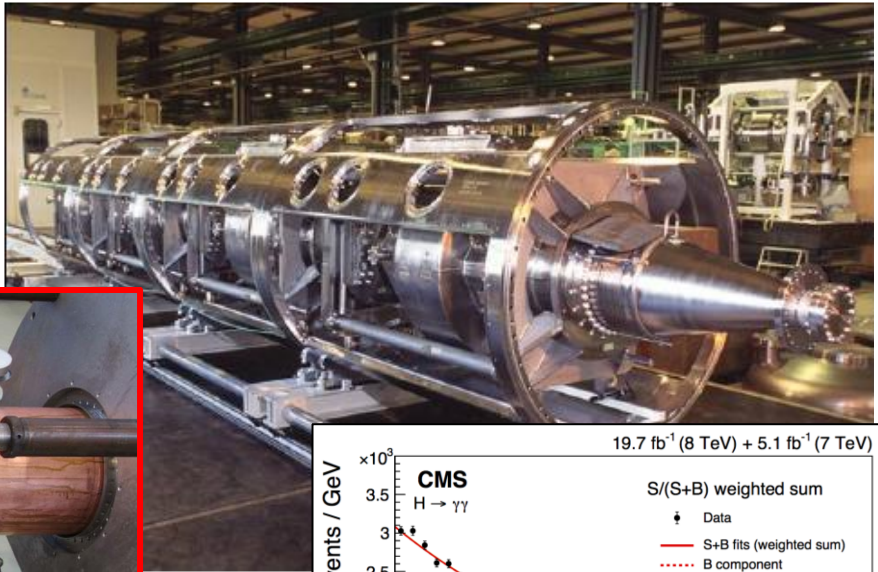


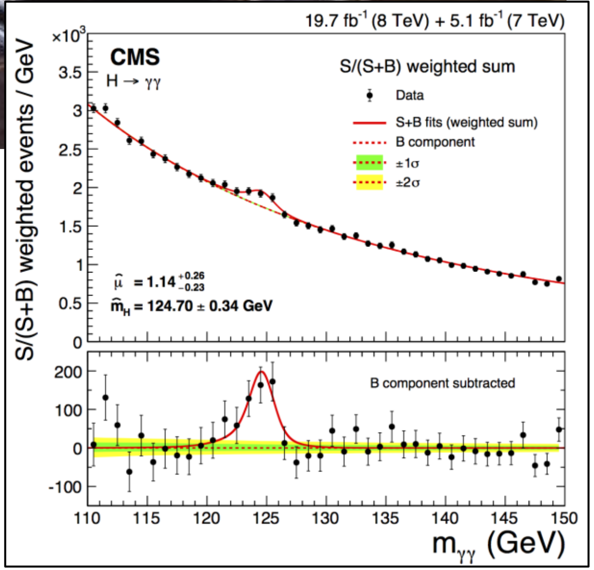
Cascade of field → >> GeV in total

Example of RF accelerators: LHC at CERN









- 8 superconducting RF cavities for each ring (in total16 cavities)
- 2 MV per each → +16 MV per turn for protons
- 11245 lap/s → 0.18 TeV/s → 7 TeV within 20 minutes
- Center mass energy 14 TeV → accessible to TeV physics
  - ggH→γγ at 125 GeV << 14 TeV due to parton distribution function

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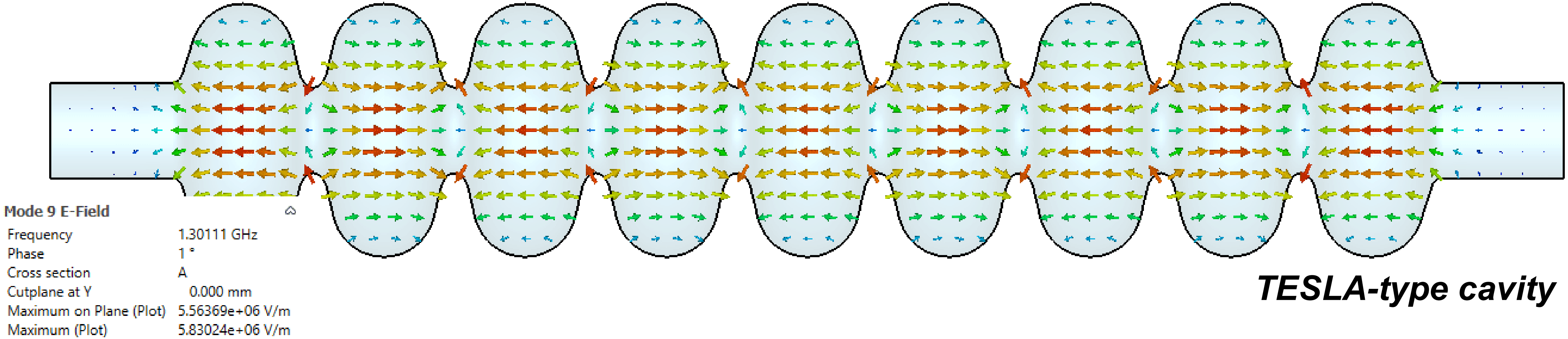
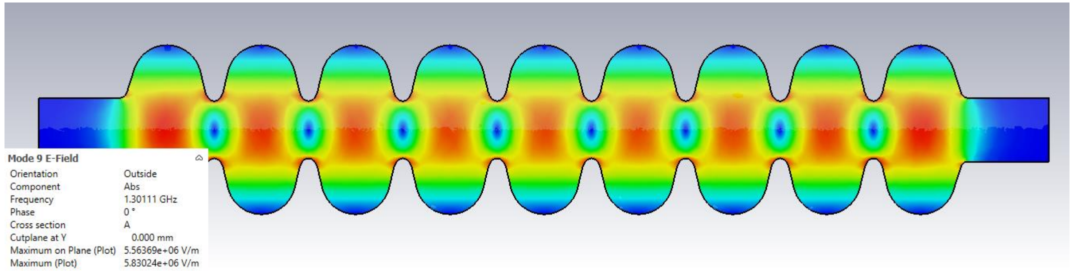
E-field vector

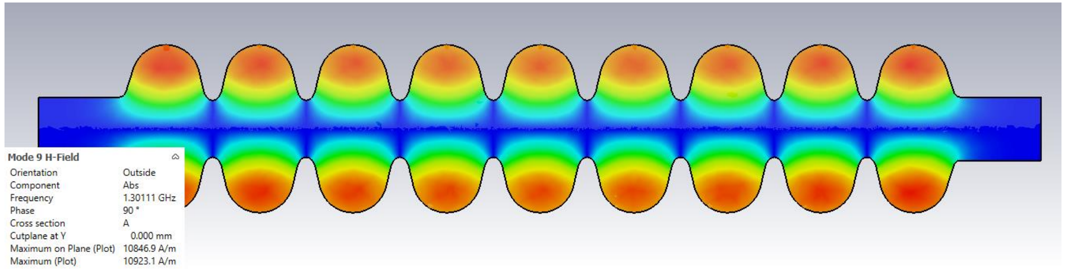


TESLA-type cavity

E-field contour



B-field contour



→ For better understandings, a bit of mathematics is helpful ☺

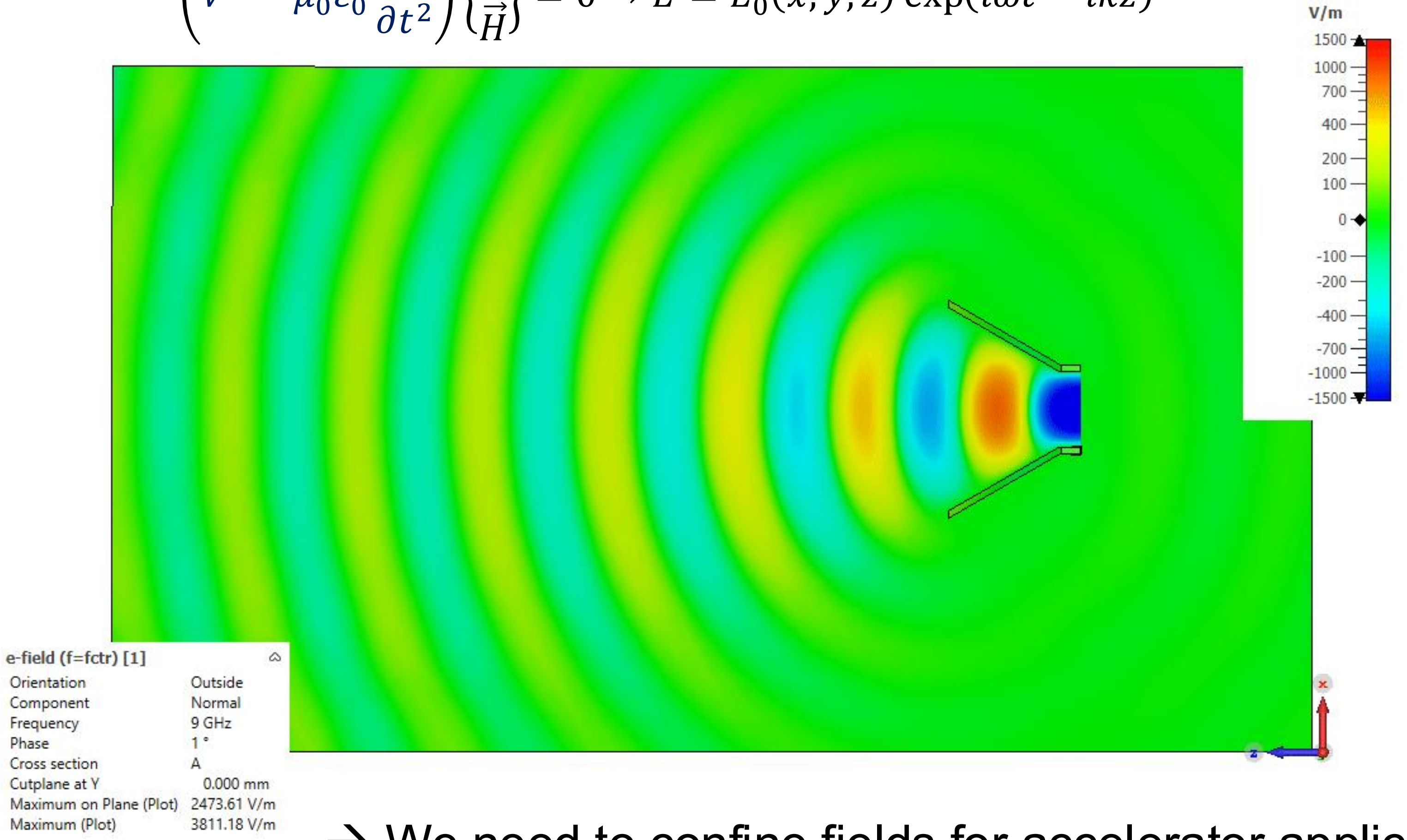
Starting point: Maxwell equations

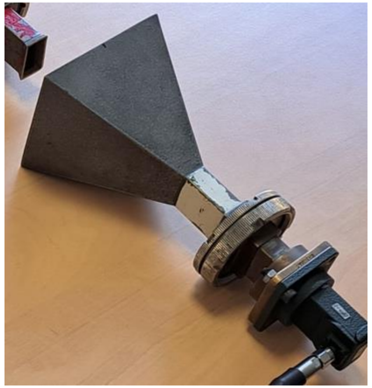
	With charges and current	In vacuum
Gauss Law	$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$	$\vec{\nabla}.\vec{E} = 0$
No monopole	$\vec{\nabla}.\vec{B} = 0$	$\vec{\nabla}.\vec{B} = 0$
Ampere law	$\vec{\nabla} \times \vec{B} = \mu_0.\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$	$\vec{\nabla} \times \vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$
Faraday law	$\vec{\nabla} \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$

→ **Wave equation** $$\left(\nabla^2 - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$ $$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

➢ The boundary condition is necessary to solve these partial differential equations

$$\left(\nabla^2 - \mu_0\varepsilon_0 \frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0 \rightarrow \vec{E} = \vec{E}_0(x,y,z)\exp(i\omega t - ikz)$$

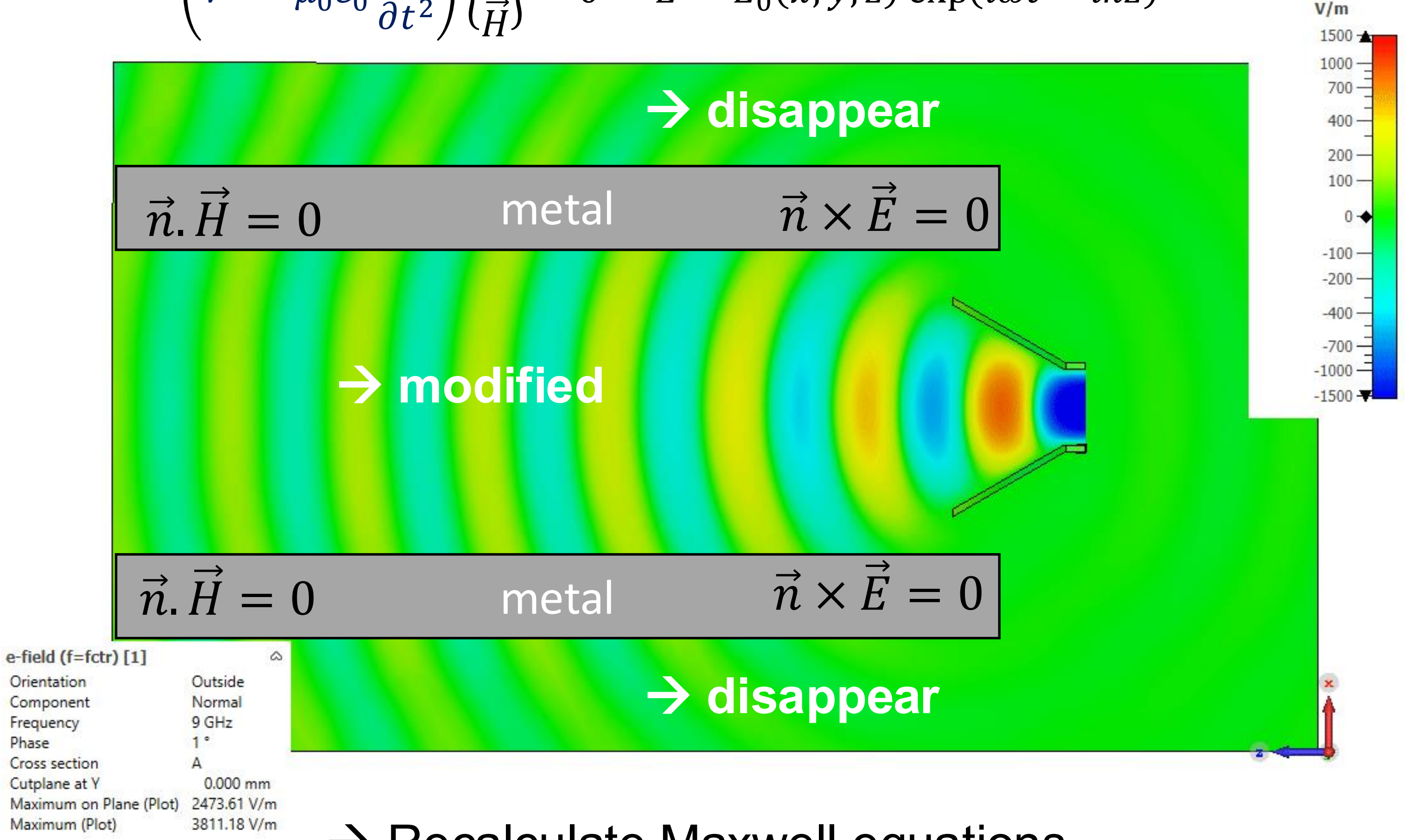


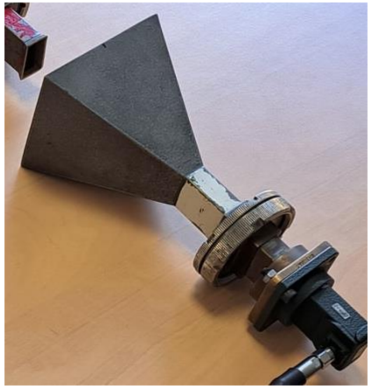


→ We need to confine fields for accelerator applications

A solution in free space

$$\left(\nabla^2 - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0 \rightarrow \vec{E} = \vec{E}_0(x,y,z)\exp(i\omega t - ikz)$$





→ Recalculate Maxwell equations

Propagation mode in a waveguide

Difference from free space: propagation constant β

$$\vec{E}(\vec{x},t) = \vec{E}(\rho)e^{i\omega t - i\beta z}$$

$$\vec{H}(\vec{x},t) = \vec{H}(\rho)e^{i\omega t - i\beta z}$$

$$\left(\nabla_t^2 + \frac{\omega^2}{c^2} - \beta^2\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$

Cut-off wave number

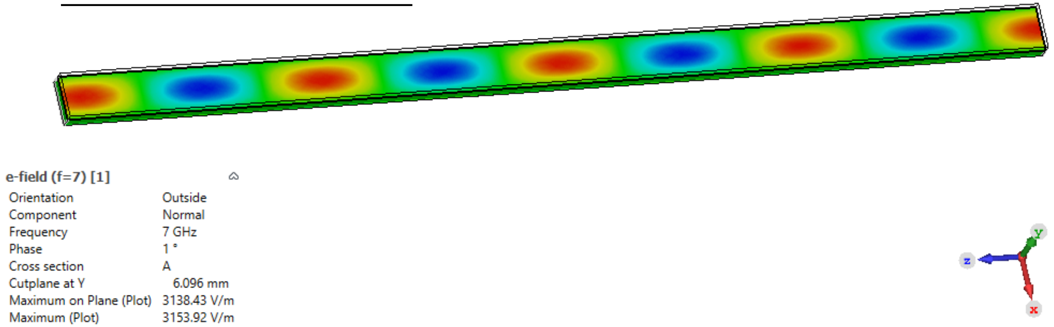
$$k_c^2 = \frac{\omega^2}{c^2} - \beta^2 \geq 0$$

Relation in transverse fields

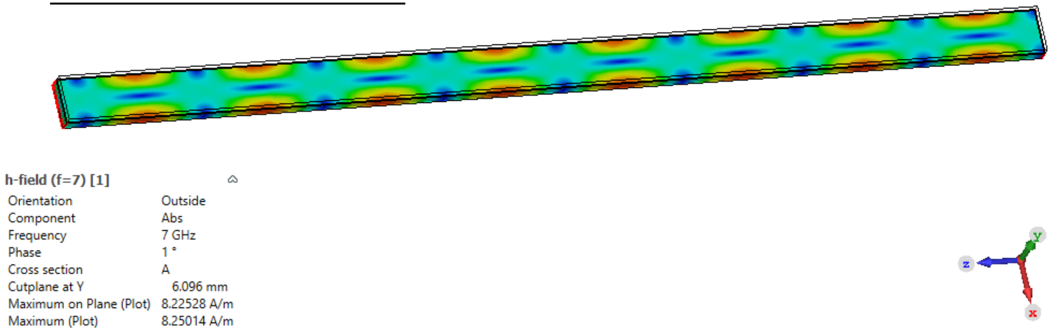
$$\overrightarrow{H_t} = \frac{\vec{z} \times \overrightarrow{E_t}}{Z}$$

Wave impedance Z

E-field of TE10



H-field of TE10

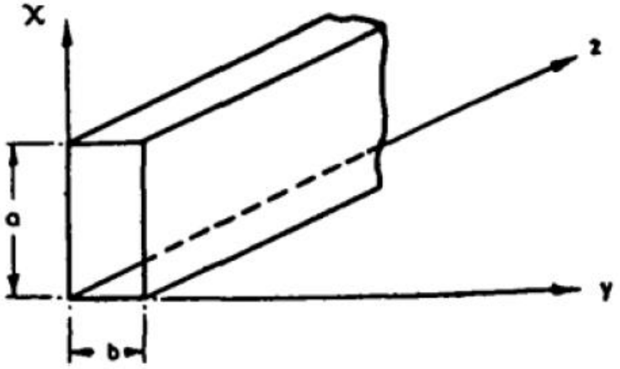


2 sets of modes

- *transverse magnetic (TM)* modes
- *transverse electric (TE)* modes

$$Z_{TM} = \frac{\beta}{\varepsilon\omega} \qquad Z_{TE} = \frac{\mu\omega}{\beta}$$

Analytical solutions of rectangular waveguides



- The propagating constant is

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

- Each mode has a cutoff frequency

$$f_{c_{m,n}} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

with $k_x = \frac{m\pi}{a}$; $k_y = \frac{n\pi}{b}$

- Boundaries conditions

TE : $\begin{cases} e_x(x,y) = 0 & y = 0, b \\ e_y(x,y) = 0 & x = 0, a \end{cases}$ TM : $\begin{cases} e_z(x,y) = 0 & y = 0, b \\ e_z(x,y) = 0 & x = 0, a \end{cases}$

- Cut-off wave number : $k_c^2 = k_x^2 + k_y^2$
- Solution of the form ($e^{-i\beta z}$ disregarded)

TE : $H_z = A_{mn} \cos k_x x \sin k_y y$

TM : $E_z = B_{mn} \cos k_x x \sin k_y y$

$$E_x = \frac{i\omega\mu n\pi}{k_c^2 b} A_{mn} \cos k_x x \sin k_y y \qquad H_x = \frac{i\omega\varepsilon n\pi}{k_c^2 b} B_{mn} \sin k_x x \cos k_y y$$

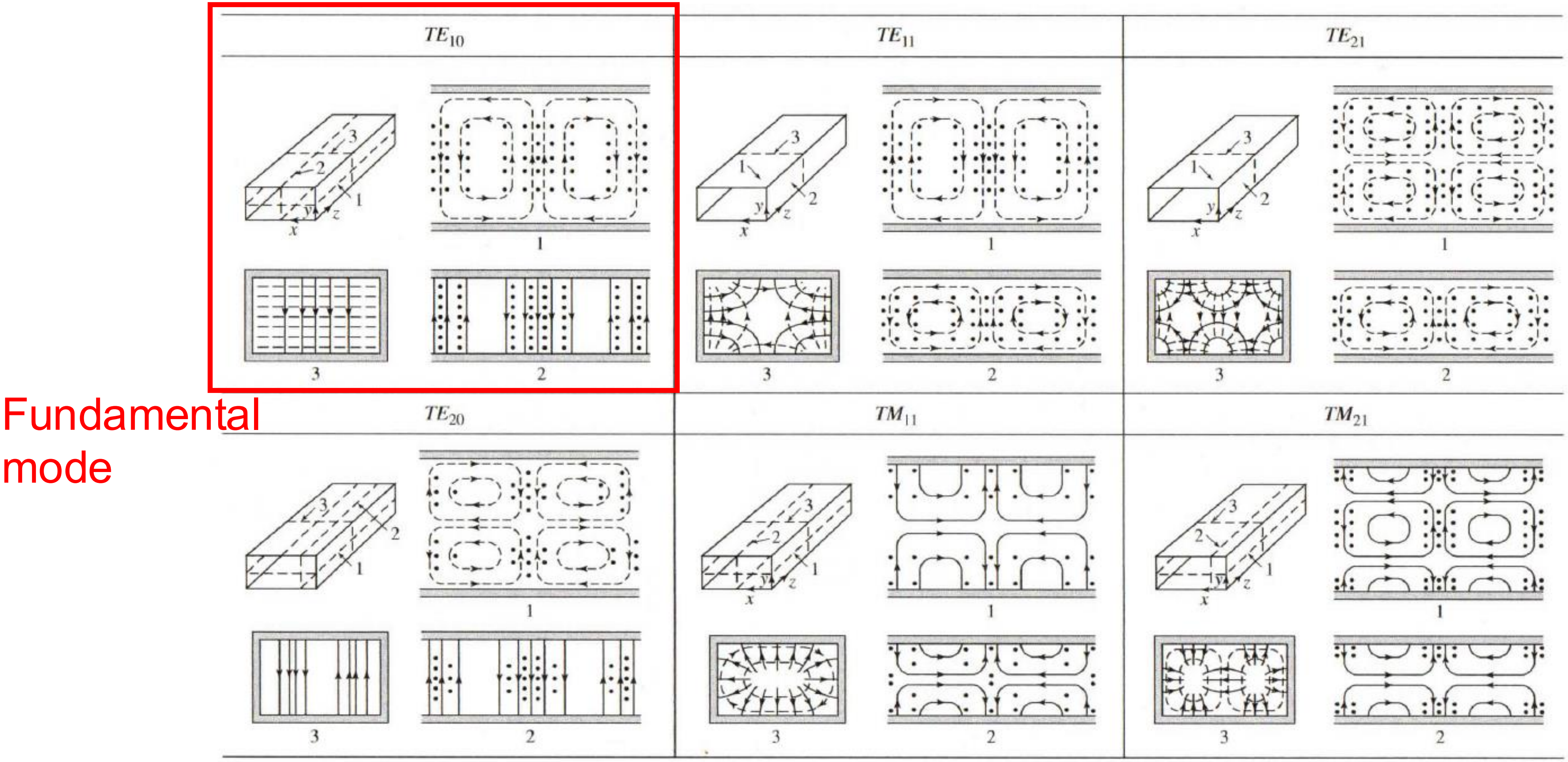
$$E_y = \frac{-i\omega\mu m\pi}{k_c^2 a} A_{mn} \sin k_x x \cos k_y y \qquad H_y = \frac{-i\omega\varepsilon m\pi}{k_c^2 a} B_{mn} \cos k_x x \sin k_y y$$

$$H_x = \frac{i\beta m\pi}{k_c^2 a} A_{mn} \sin k_x x \cos k_y y \qquad E_x = \frac{-i\beta m\pi}{k_c^2 a} B_{mn} \cos k_x x \sin k_y y$$

$$H_y = \frac{i\beta n\pi}{k_c^2 b} A_{mn} \cos k_x x \sin k_y y \qquad E_y = \frac{-i\beta n\pi}{k_c^2 b} B_{mn} \sin k_x x \cos k_y y$$

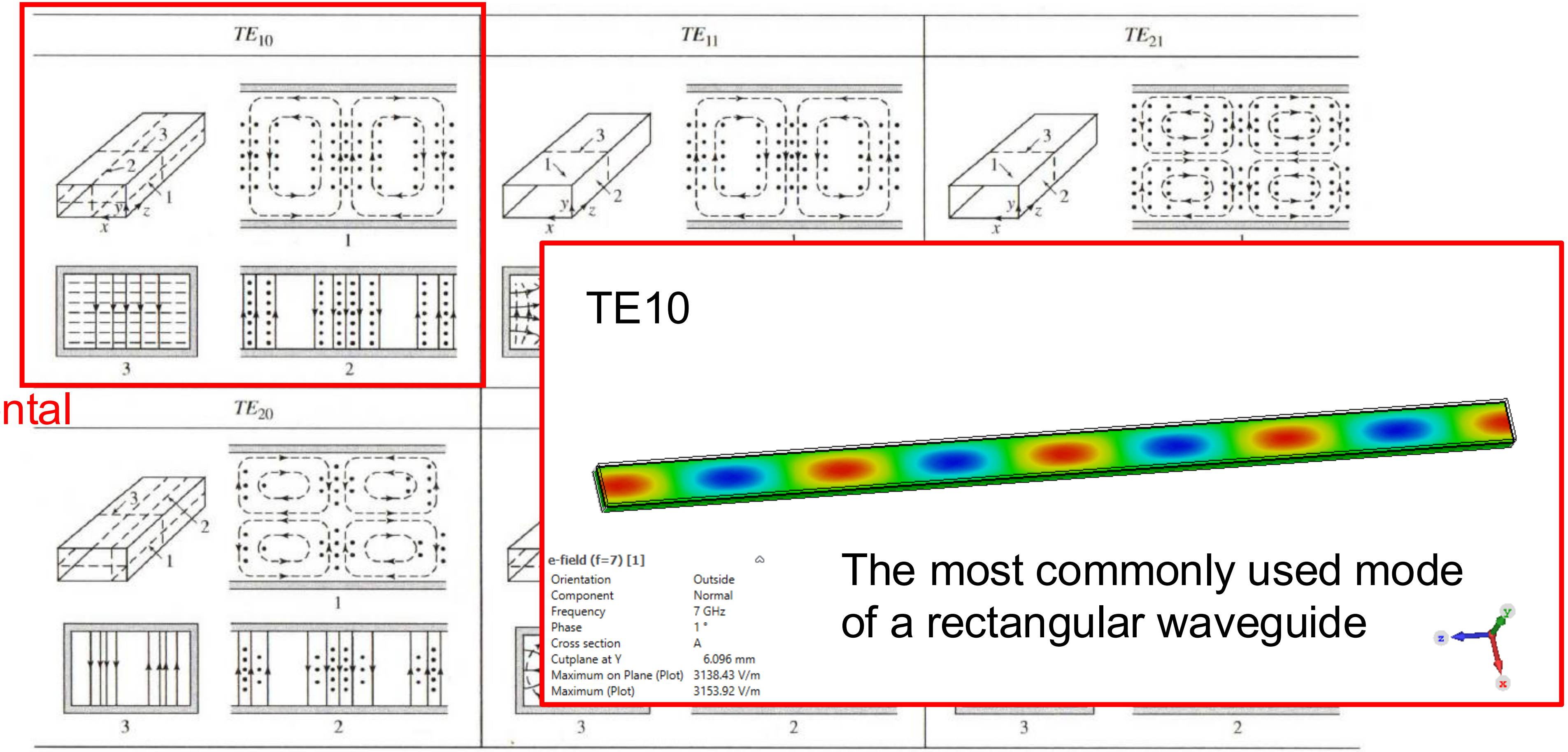
No TE_{00}, TM_{00}, TM_{01} and TM_{10}

The lowest modes are TE_{10} TM_{11}



Field lines for some of the lower order modes of a rectangular waveguide (from D.M. Pozar)

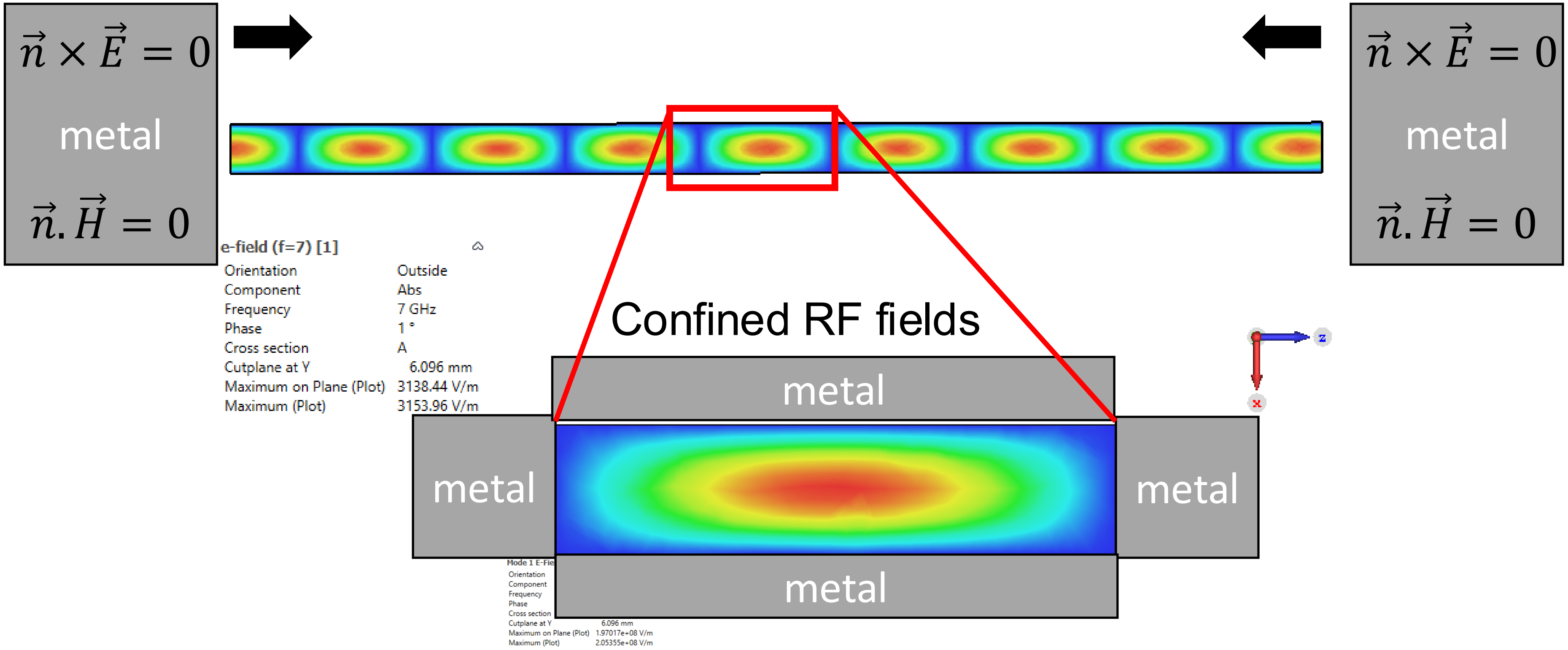
Visualized modes



Fundamental mode

The most commonly used mode of a rectangular waveguide

Field lines for some of the lower order modes of a rectangular waveguide (from D.M. Pozar)



However, TE10 in rectangular cavity cannot accelerate charged particles

→ One need a strong electric field at the center of the resonator

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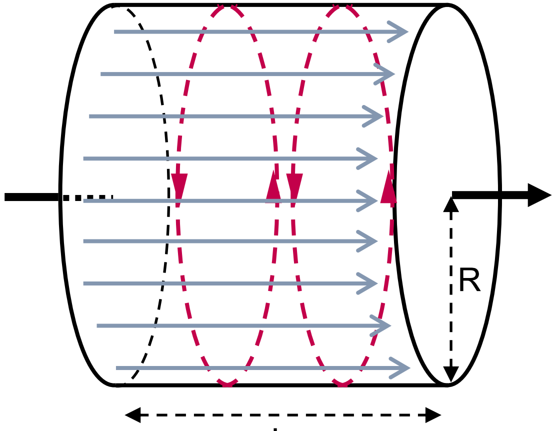
Pill-box cavity TM010

The simplest cavity is cylindrical geometry with radius R and length L



Transverse magnetic modes with longitudinal electric field are used for acceleration ($TM_{0,n,p}$)

Boundaries conditions:

- On cavity wall => $\vec{n}.\vec{H} = 0$
- Optionally at $z = 0$ and $z = L$ => $\vec{n}.\vec{H} = 0$ and $H = 0$

Applying Maxwell's equation and boundaries conditions yield to Helmholtz equation:

$$(\nabla^2 + \omega^2 \mu \epsilon)\mathrm{H} = 0$$

- There is infinity of TM_{0XX} solutions (modes)
- All modes are defined by frequency ω_n and the EM fields:

$$H_n(r,z) = [0, H_{\theta,n}(r,z), 0]$$
$$E_n(r,z) = [E_{r,n}(r,z), 0, E_{z,n}(r,z)]$$

Good cavity: higher accelerating voltage V_{acc} with lower power loss P_c on the cavity wall
→ Shunt impedance R_{sh} to be maximized

$$R_{sh} \equiv \frac{V_{acc}^2}{P_c}$$

The field level is related to stored energy of the useful mode (TM010) inside the cavity

$$U \equiv 2\mu \int_V \frac{H^2}{4} dV = 2\varepsilon \int_V \frac{E^2}{4} \mathrm{dV} \ (= \kappa V_{acc}^2)$$ κ (geometrical factor)

The loss is given by surface integral of RF magnetic field on the cavity wall with surface resistance R_s

$$P_c = \frac{R_S}{2} \int H^2 dS$$

→ Unloaded quality factor is defined

$$Q_0 \equiv \frac{\omega U}{P_c} = \frac{1}{R_s} \boxed{\omega\mu \frac{\int_V H^2 dV}{\int H^2 dS}}$$ $\equiv G$ (geometrical factor)

Performance of cavities 2/2

The shunt impedance can be written

$$R_{sh} \equiv \frac{V_{acc}^2}{P_c} = \frac{U}{\kappa P_c} = \frac{Q_0}{\kappa\omega} = \frac{1}{\omega}\boxed{\frac{G}{\kappa}}\boxed{\frac{1}{R_s}}$$

→ Surface resistance given by material

→ geometry

(R_{sh}/Q is also geometrical)

The cavities can be optimized by better geometry and material (normal conducting vs superconducting)

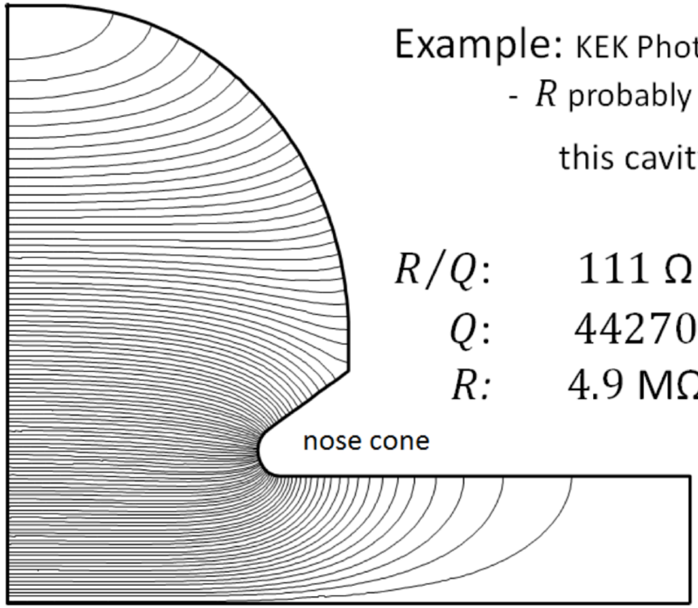
Other figures of merit (geometrical):

$\frac{E_{pk}}{E_{acc}}$: peak electric field is on the cavity wall NOT the accelerating field → breakdown

$\frac{B_{pk}}{E_{acc}}$: peak magnetic field is also crucial in superconducting cavities → quench

Optimization of the cavity shape: different philosophy

Maximize R_{sh}/Q

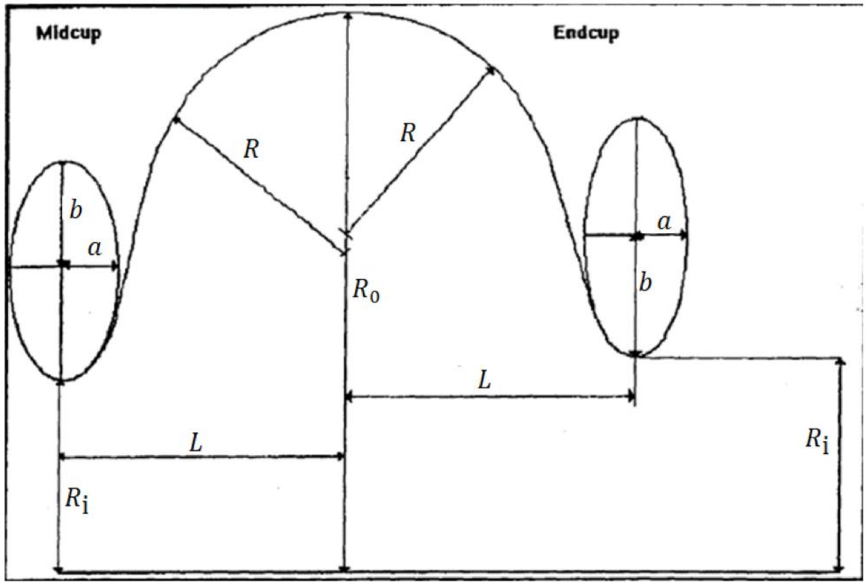


Example: KEK Photon Factory 500 MHz

- R probably *as good as it gets* -

	this cavity	optimized pillbox
R/Q:	111 Ω	107.5 Ω
Q:	44270	41630
R:	4.9 MΩ	4.47 MΩ

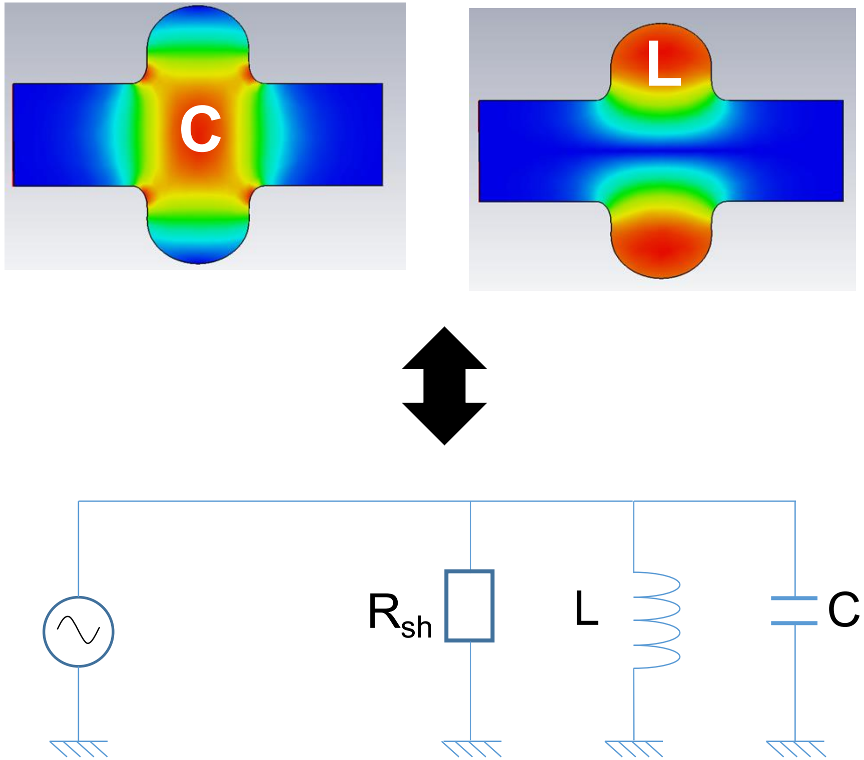
So so R_{sh}/Q but reduce B_{pk}/E_{acc}



D. Proch, http://accelconf.web.cern.ch/accelconf/SRF93/papers/srf93g01.pdf

- Depending on the project demand, frequency, priority, etc, the best structure is different
- A lot of phenomena (ultimate performance, risk of discharge, quench etc) → compromise
- Lessons learned: over-optimization in one figure of merit may overlook a showstopper!

Equivalent circuit approach



A resonating cavity can be modeled by a lumped capacitance C, resistance R and inductance L

The input impedance is: $$Z_{in} = \left(\frac{1}{R_{sh}} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

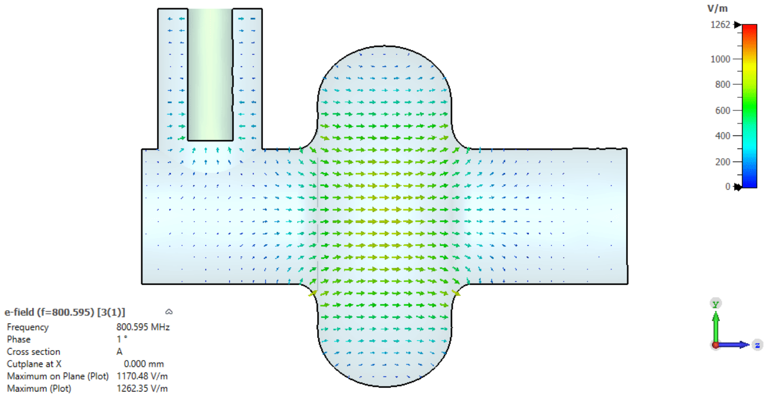
Assuming $\omega = \omega_0 + \Delta\omega$ $(\Delta\omega \ll \omega_0)$ and $\omega_0 = {}^{1}/_{\sqrt{LC}}$

$$Z_{in} = \frac{R_{sh}}{1 + 2Q\frac{\Delta\omega}{\omega_0}}$$ with $$Q = \frac{\omega_0}{2\Delta\omega} = \omega_0 R_{sh} C = \frac{R_{sh}}{\omega_0 L}$$

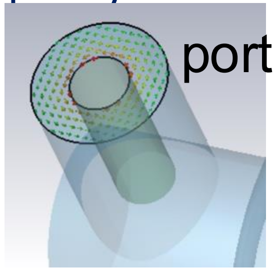
Remark 1: $R_{sh} \neq R_s$

Remark 2: R_{sh} conventionally used in general circuit theory differs by factor 2

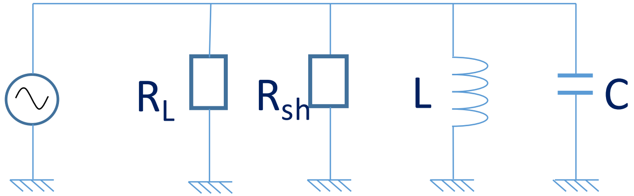
From the knowledge of the ω_0, Q and R/Q of the RF structure, one get the equivalent resonant circuit



Resonant cavity is fed by coupling port in order to provide EM wave. As intrinsic quality factor, one can define the external quality factor:



$$Q_{ext} = \frac{\omega_0 U}{P_e} = \frac{\omega_0 U}{\frac{1}{2}\int_{S_{port}} \vec{E} \times \vec{H} dS}$$



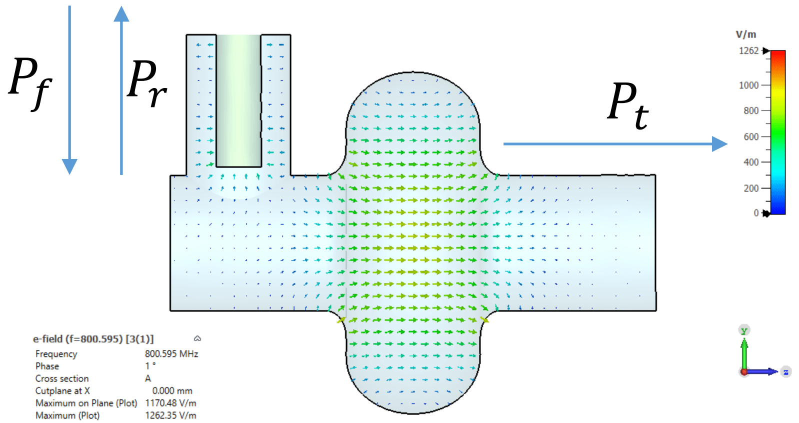
From RLC equivalent circuit, the external coupling yields at additional losses to an external load R_L in parallel

$$Q_L = \frac{R_{tot}}{\omega_0 L} \quad \text{with} \quad R_{tot} = \frac{R_{sh} R_L}{R_{sh} + R_L}$$

Then, external coupling is: $Q_{ext} = \frac{R_L}{\omega_0 L} \Rightarrow \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$

Generalization of this concept leads at: $\frac{1}{Q_L} = \frac{1}{Q_0} + \sum_i \frac{1}{Q_i}$

Coupling factor and pulse response

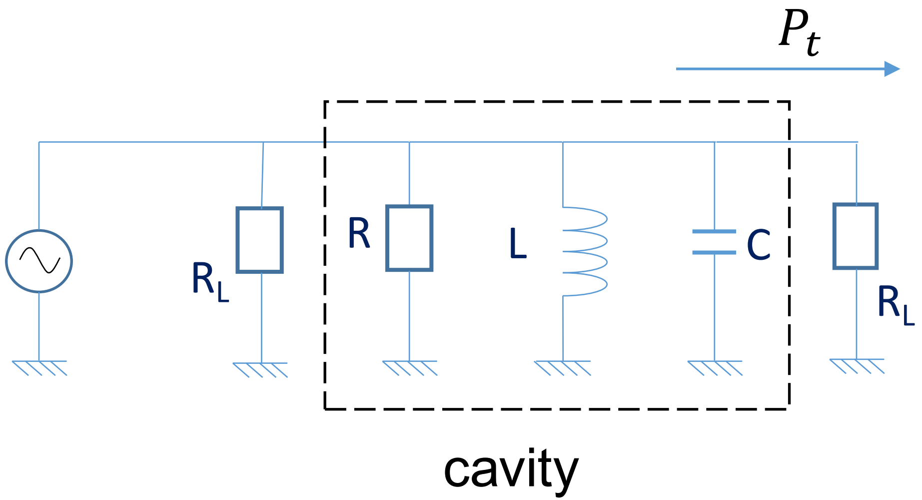


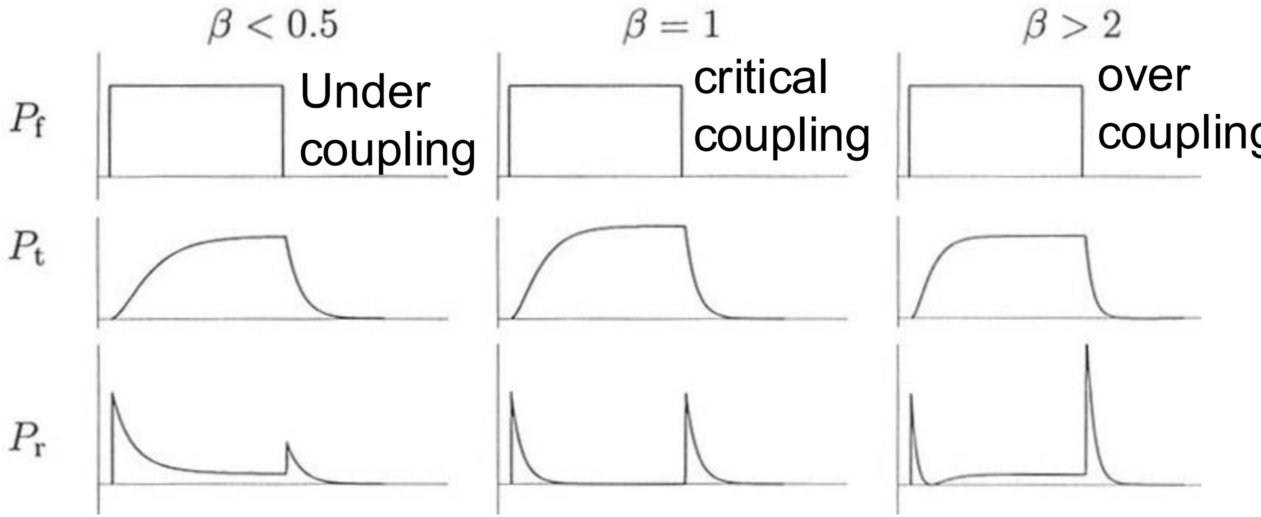
Without beam, the intrinsic quality factor is

$$Q_0 = Q_L(1+\beta_i+\beta_t) \text{ with } \beta_i = \frac{Q_0}{Q_i} \text{ and } \beta_t = \frac{Q_0}{Q_t}$$

When the RF generator is turned off, the EM field is solution of the equation:

$$E(t) + \tau.\frac{dE(t)}{dt} + \frac{1}{\omega_0^2}.\frac{d^2E(t)}{dt^2} = 0 \text{ with } \tau = \frac{Q_L}{\omega_0} = \frac{1}{\Delta\omega}$$



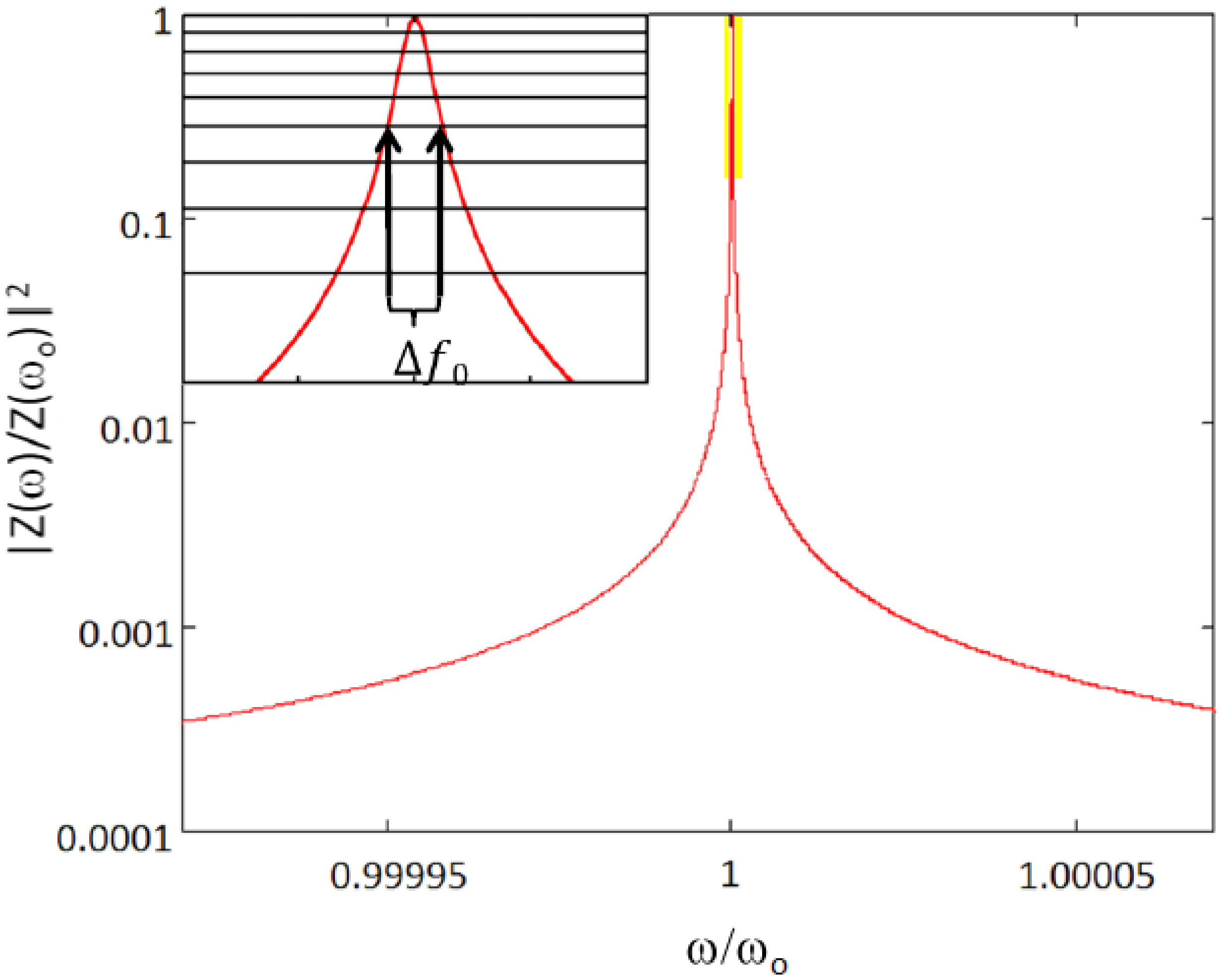


maximum of $Z(\omega)$

$$\omega_0 \equiv {}^{1}/_{\sqrt{LC}}$$

Sharpness of resonance

$$Q_L = \frac{f}{\Delta f} = \frac{U}{\Delta U}$$



Outline

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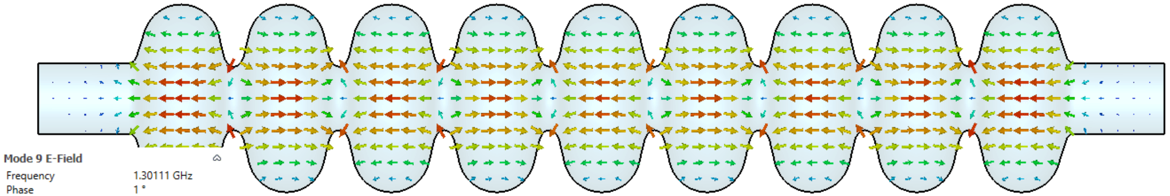
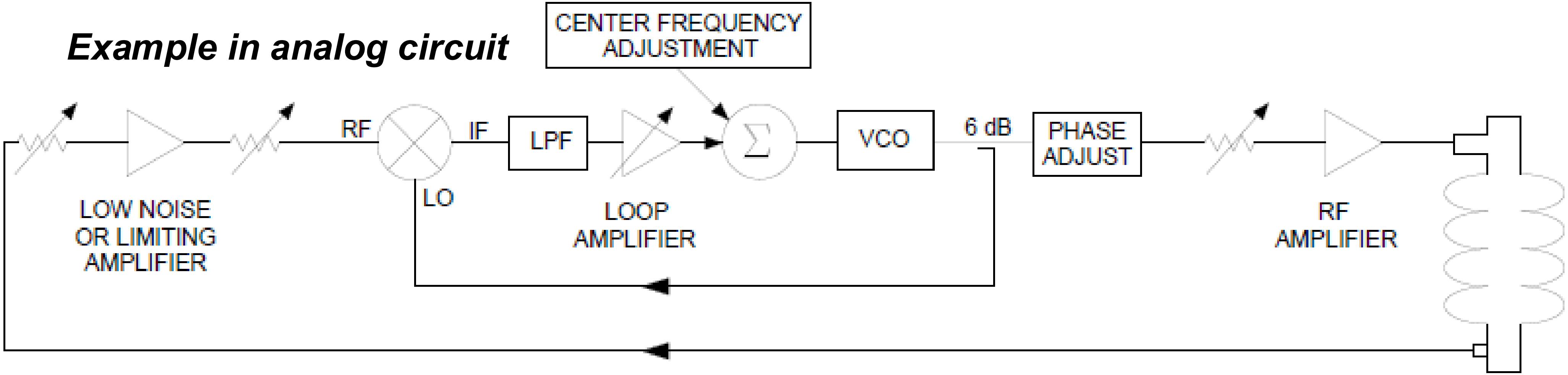
We need RF inside

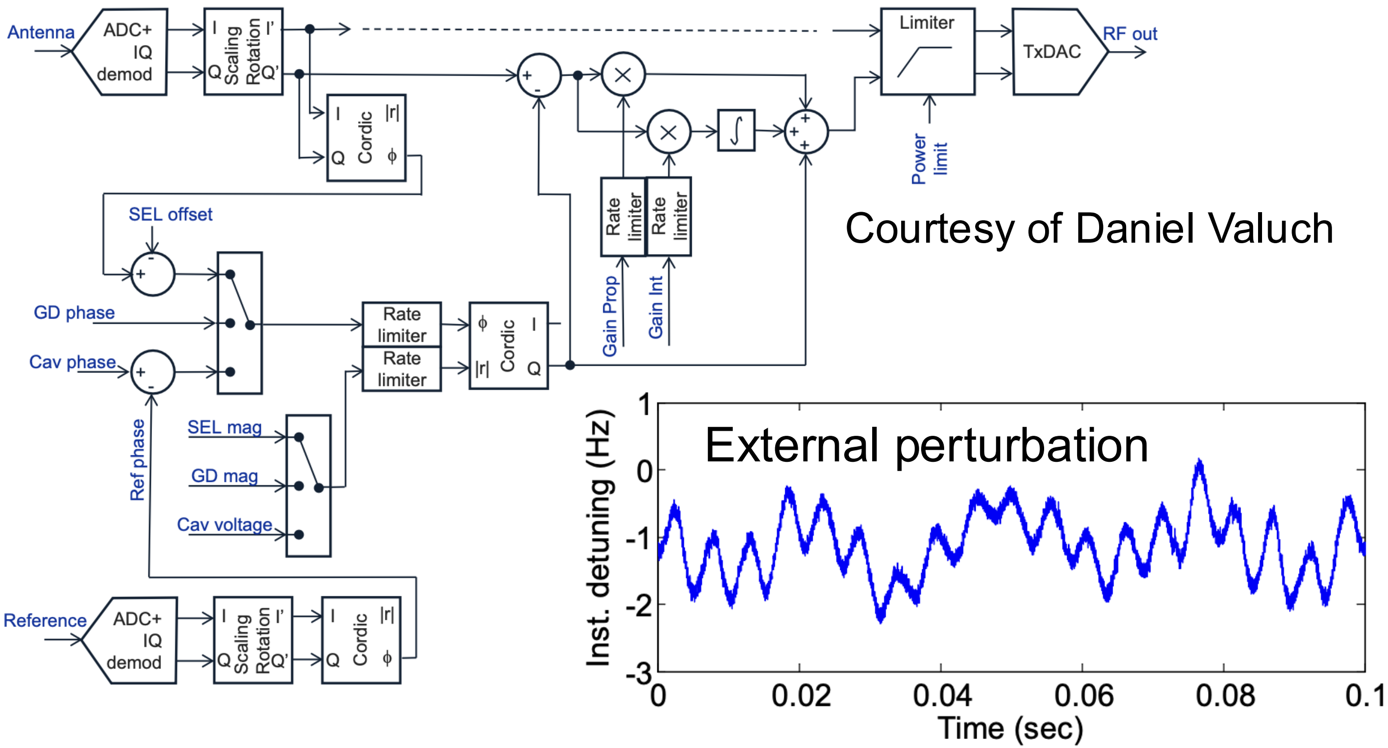


Example in analog circuit

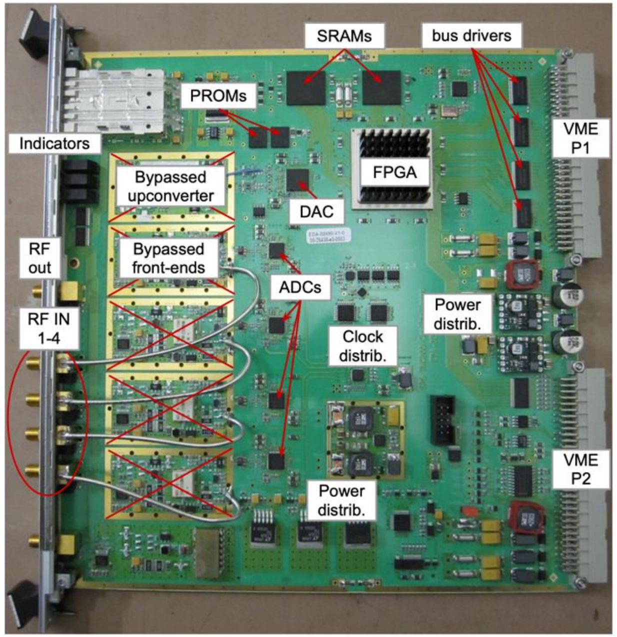


- A low-power RF circuit locks frequency, phase, and amplitude of the superconducting cavity
- An RF amplifier generates useful power level
- A power coupler feeds RF to the cavity
- Tuner controls resonant frequency of the cavity

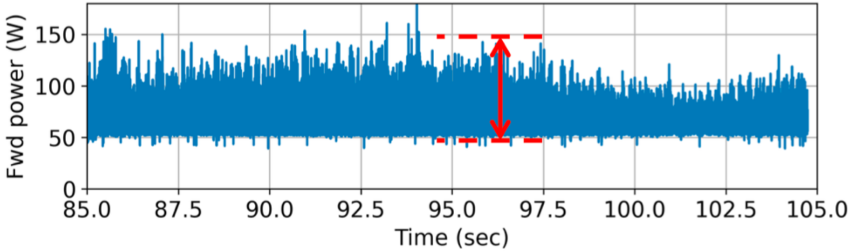
LLRF: digital system and control algorithm



M. Elias master thesis



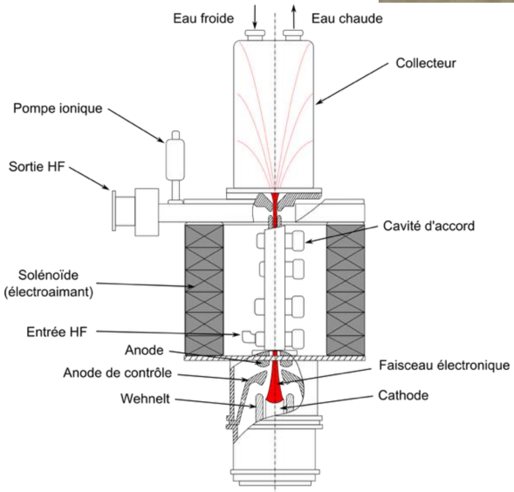
FB to keep cavity field constant



- Design analog and digital circuit to cope with various phenomena to keep cavity field stable
- Control theory and implementation in FPGA
- Directly handled power: 1 mW → needs amplifier

Vacuum tubes / klystrons

Amplification via the RF & DC beam interaction

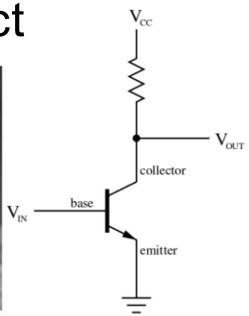


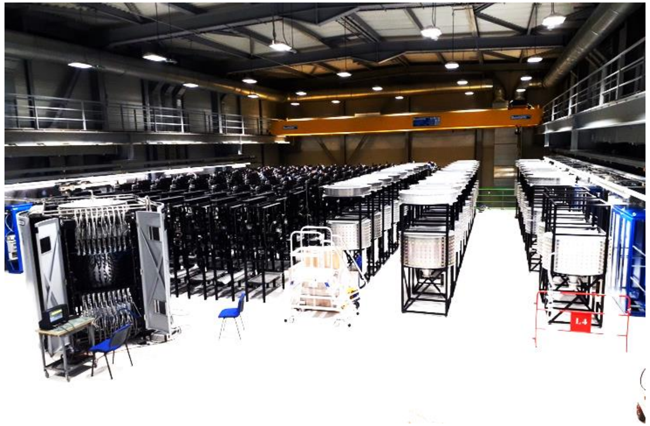
Par Julien Hillairet — Travail personnel, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=3165202

Transistor-based solid-state amplifiers

Amplification via the tunneling effect

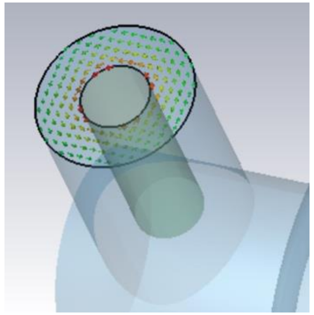
Shockley Bardeen Brattain

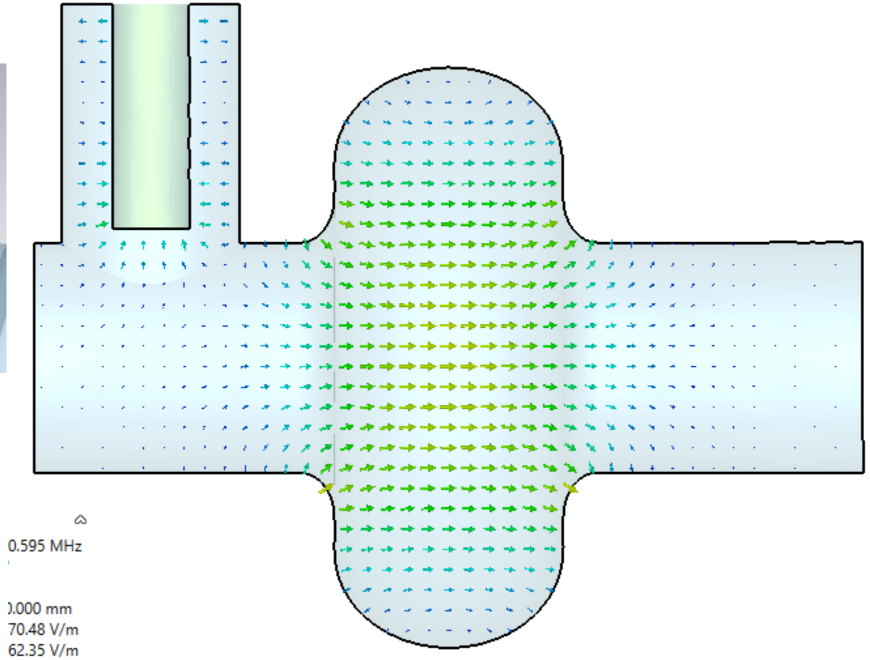


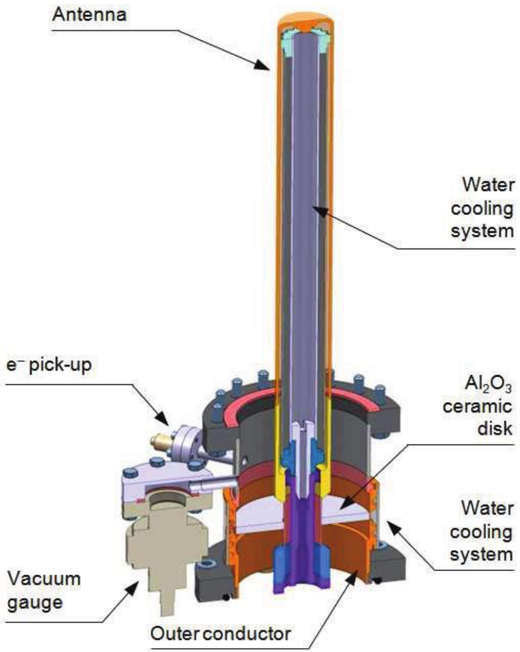


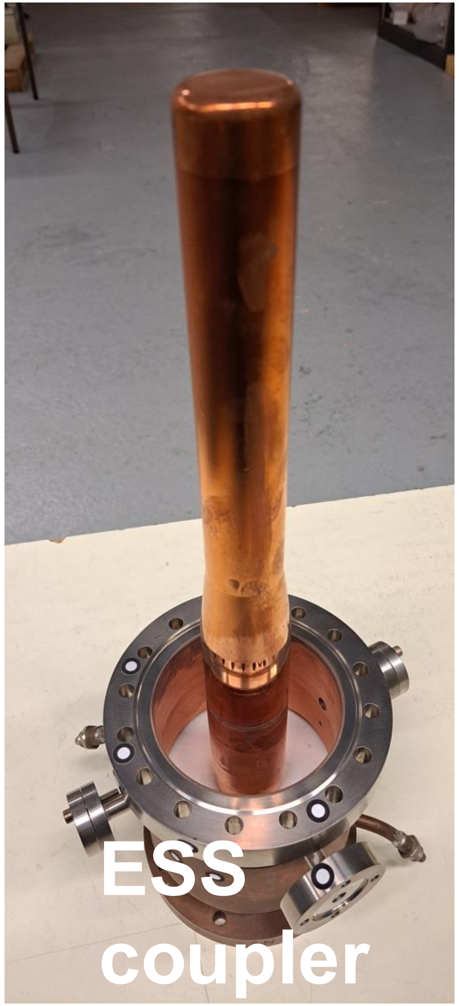
Courtesy: Eric Montesinos

port









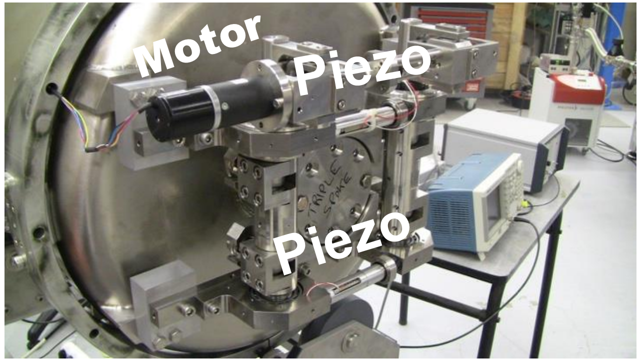
Power flow (Poynting vector) through the port gives coupler Q

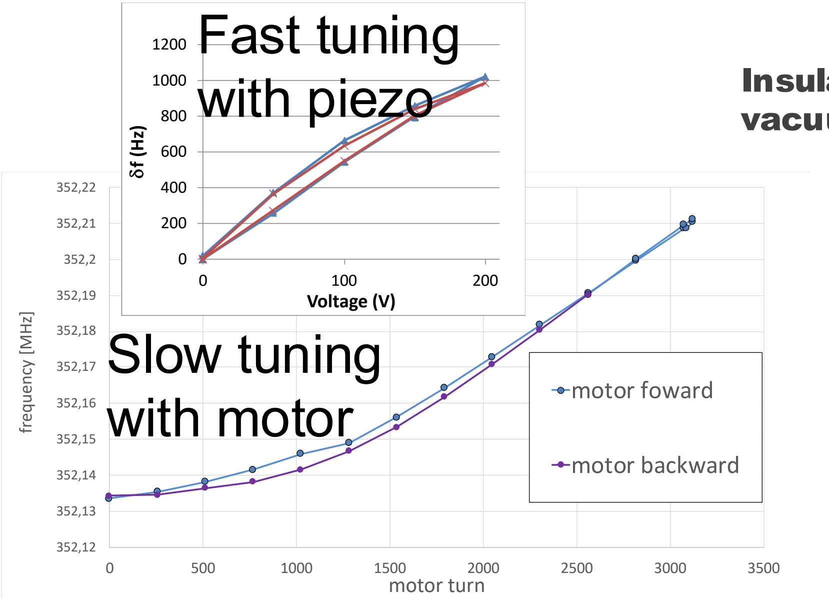
$$Q_{ext} = \frac{\omega_0 U}{P_e} = \frac{\omega_0 U}{\frac{1}{2}\int_{S_{port}} \vec{E} \times \vec{H} dS}$$

Total Q of the cavity is thus shifted from unloaded Q_0

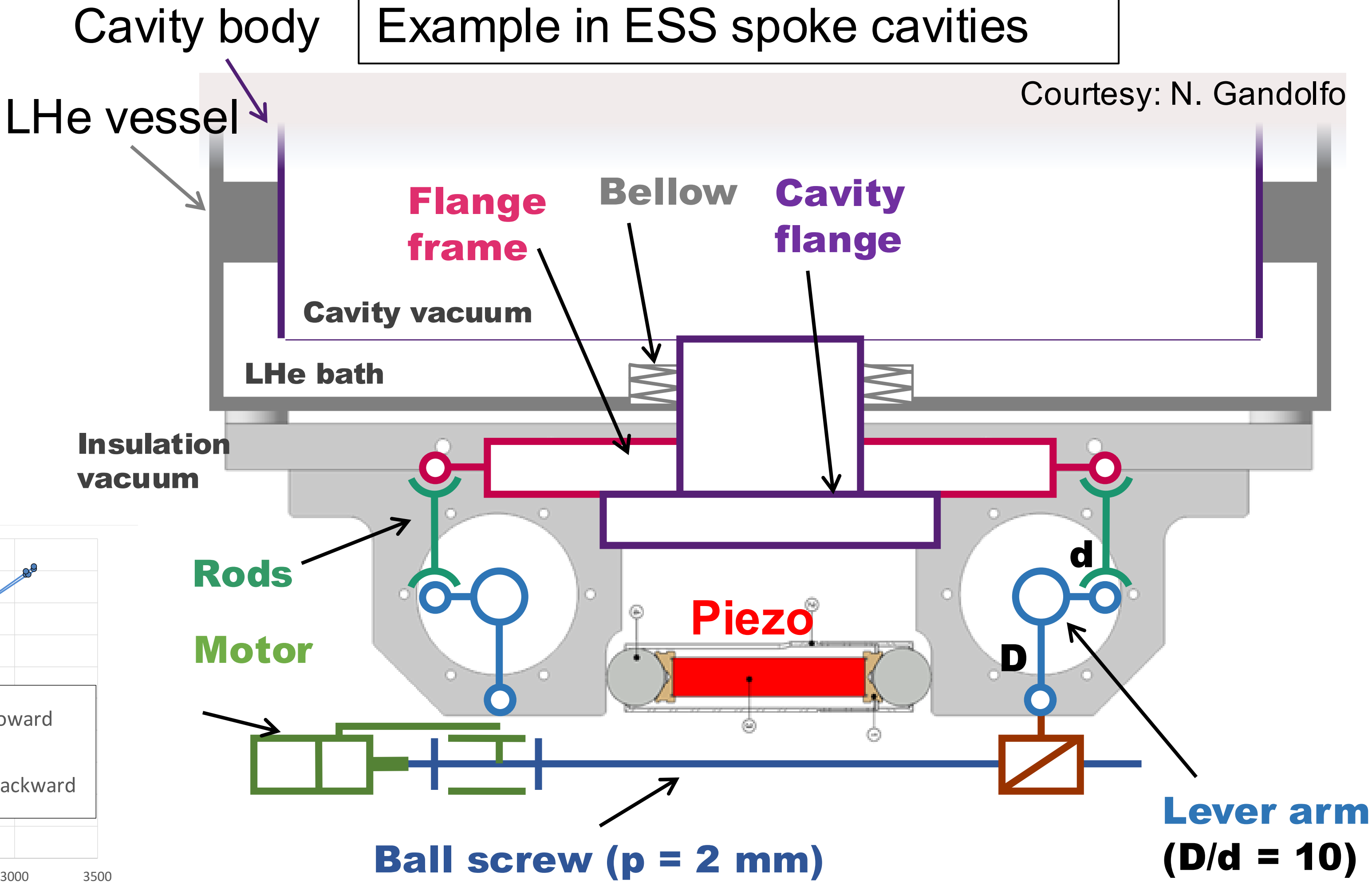
$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

Cavity tuner

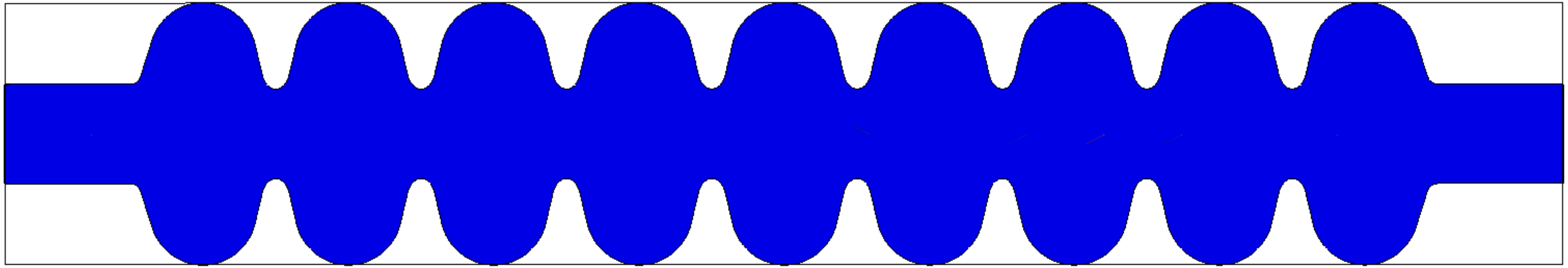




Example in ESS spoke cavities



Outline



$$\begin{cases} \nabla \cdot \boldsymbol{E} = \rho(t, \boldsymbol{r}) \\ \nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}(t, \boldsymbol{r}) + \dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

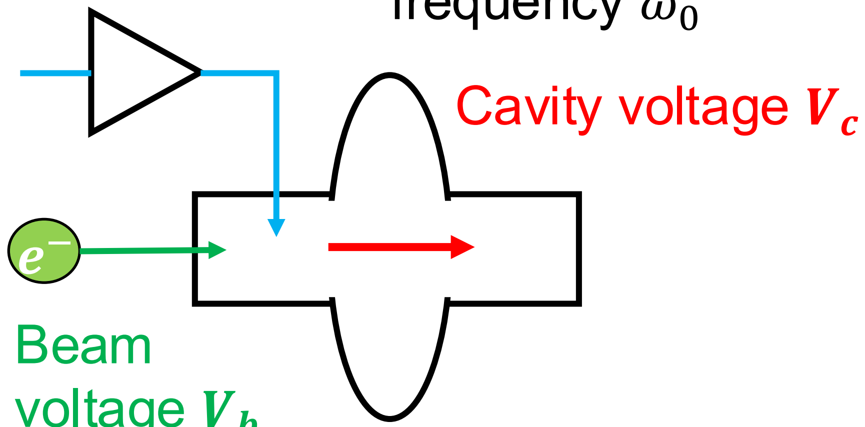
Trajectory of charged particles → Induce RF fields

- Influence to accelerating mode (beam loading)
  → Compensation with amplifier
- Influence to other modes
  → kick following bunches
  → Needs to be damped (Higher Order Mode damper)

Beam loading

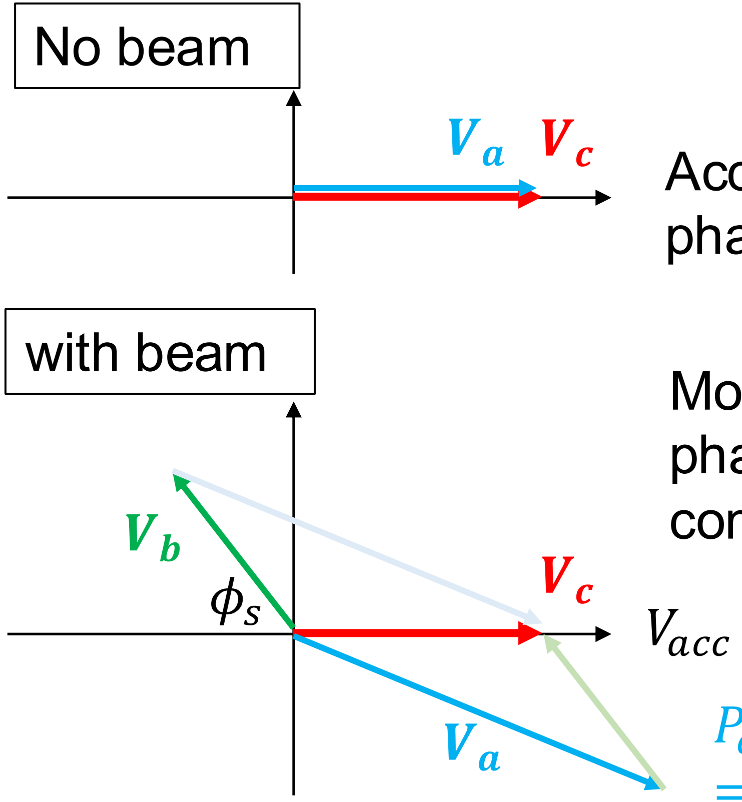
amplifier voltage $\boldsymbol{V_a}$

Cavity resonant frequency ω_0



Beam voltage $\boldsymbol{V_b}$

All at RF frequency ω_{RF}



Accelerating phase V_{acc} $\boldsymbol{V_c} = \boldsymbol{V_a}$

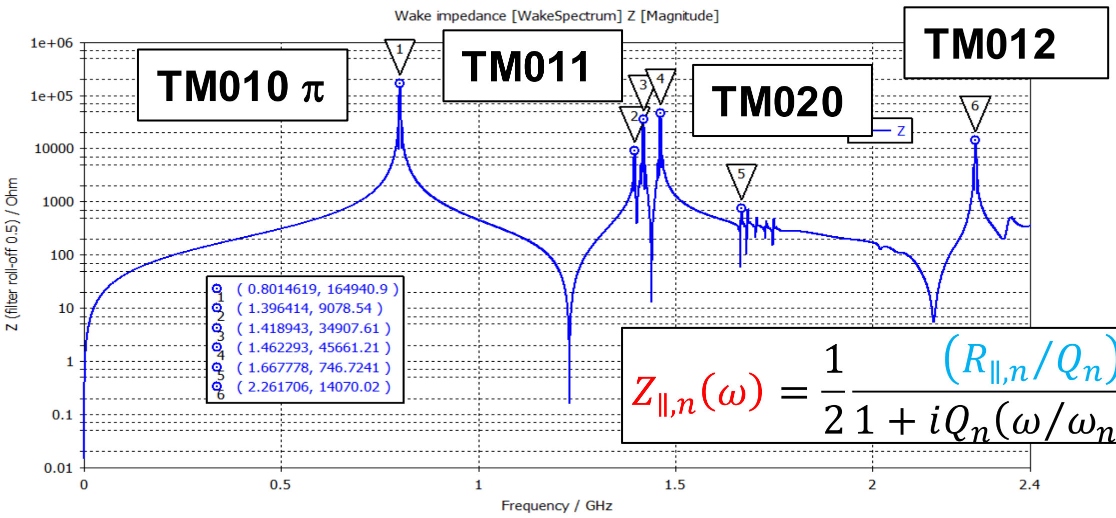
Modified amplitude and phase of amplifier to compensate V_b

$$\boldsymbol{V_c} = \boldsymbol{V_b} + \boldsymbol{V_a}$$

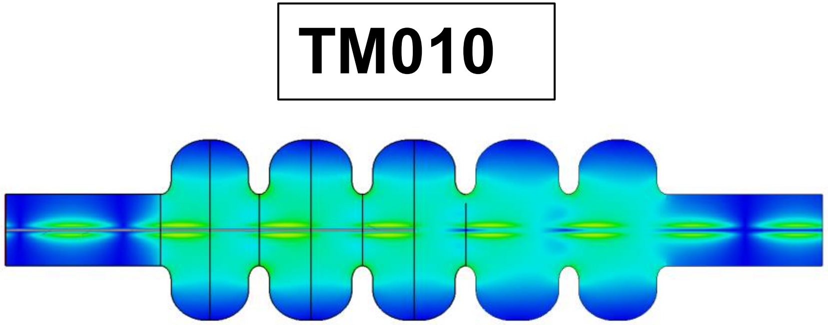
$$P_a \propto |\boldsymbol{V_a}|^2 = V_b^2 + V_c^2 + 2V_bV_c \cos\phi_s$$

- One needs power higher than without beam to compensate the beam effect
- Further optimization: cavity detuning to save power (beyond the level of this lecture)

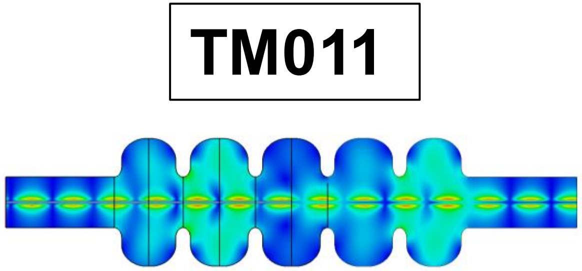
Higher Order Modes (HOM) waked by beam

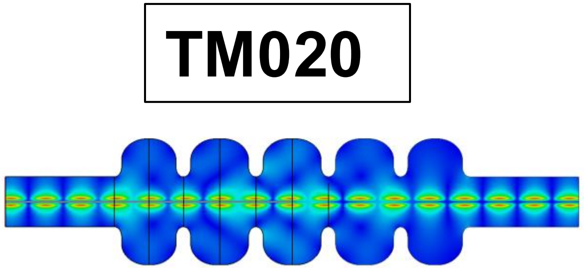


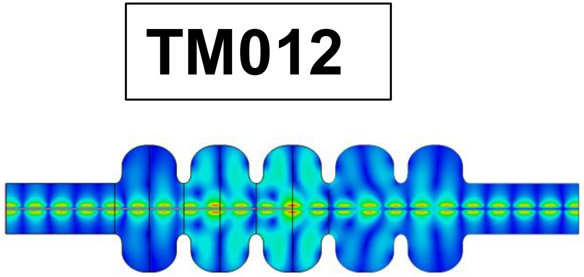
$$Z_{\parallel,n}(\omega) = \frac{1}{2}\frac{\left(R_{\parallel,n}/Q_n\right)\cdot Q_n}{1 + iQ_n(\omega/\omega_n - \omega_n/\omega)}$$



Accelerating mode in another phase → beam loading





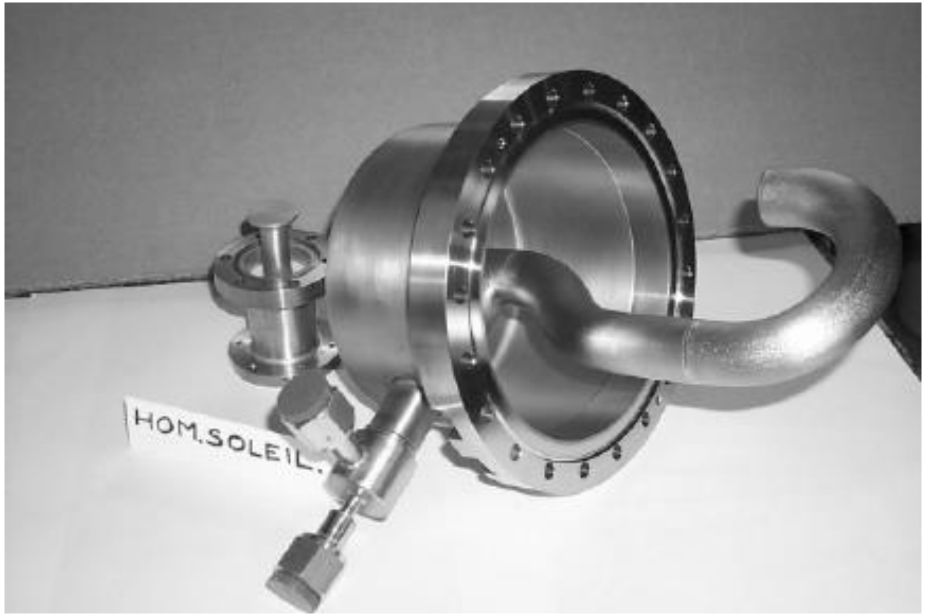


→ Beam charge and current generate additional RF

→ Some modes disturb beam → Beam instability

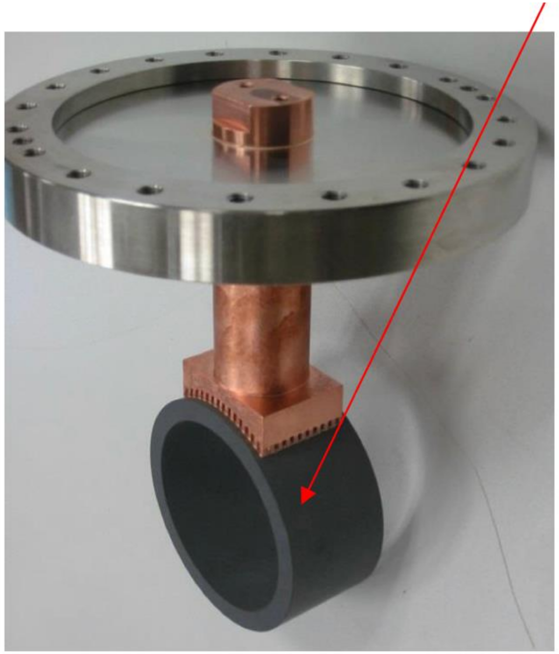
Eliminate HOMs: HOM coupler and absorbers

HOM couplers



Extract HOMs outside the cavities

HOM absorbers



Absorbs HOMs in the beam pipe

Key technology in high-current accelerators
(LHC, FCC, SuperKEKB, PERLE, EIC, etc)

https://indico.fnal.gov/event/10102/contributions/813/attachments/436/508/Solyak_HOM_Absorber.pdf

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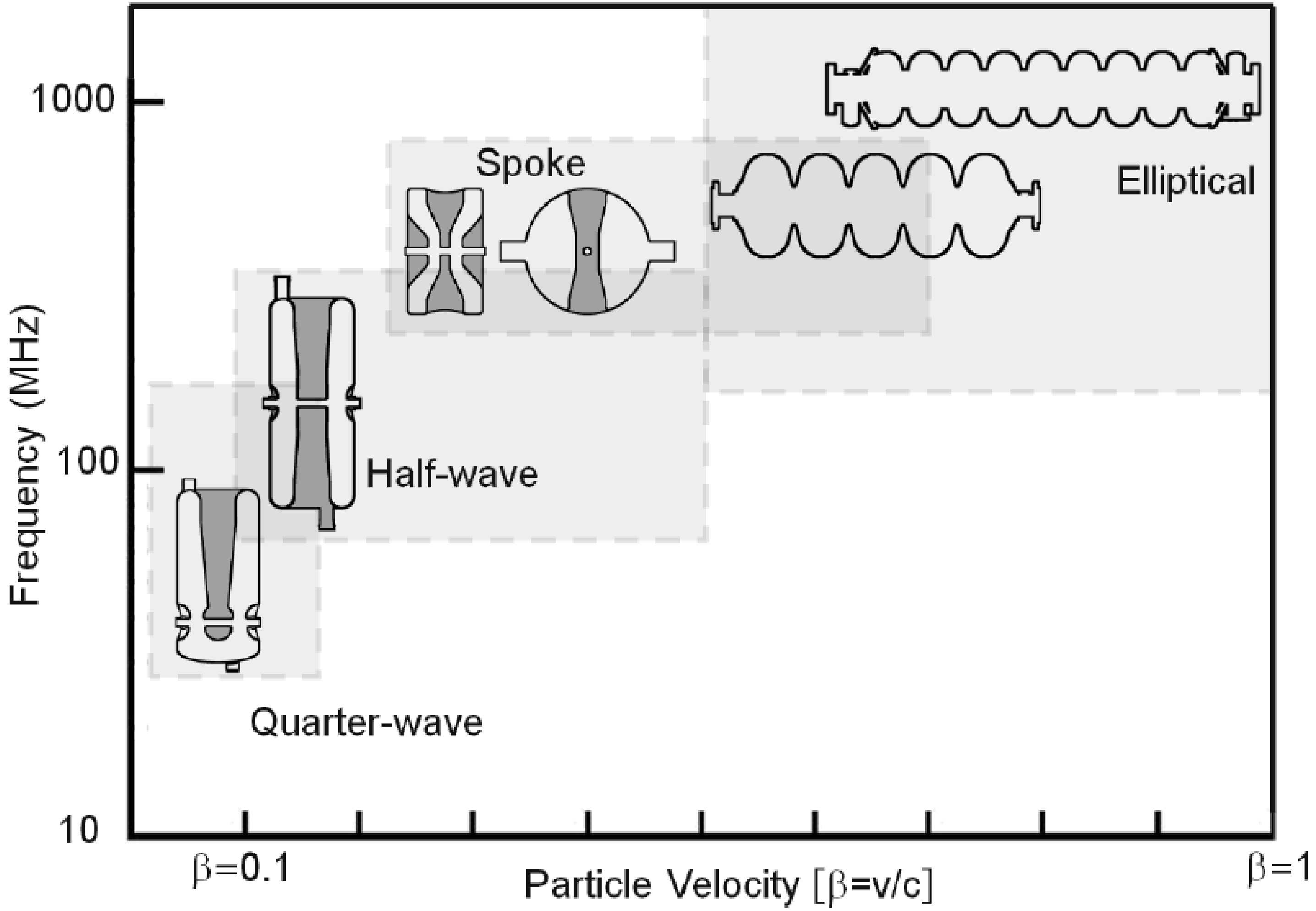
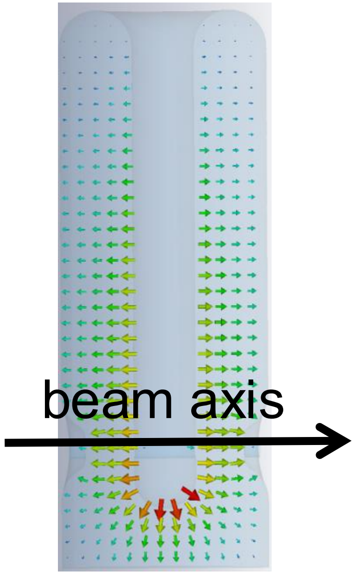
Various shapes for acceleration

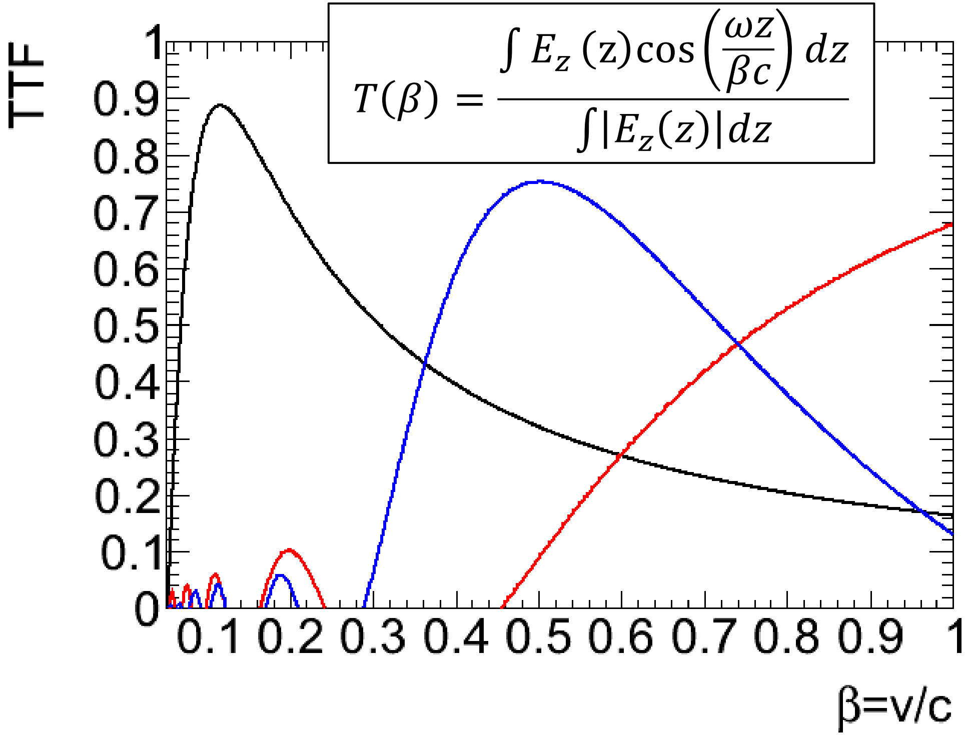


Why different shapes? Example in cavities

TEM_{00} modes in a quarter-wave or half-wave cavity

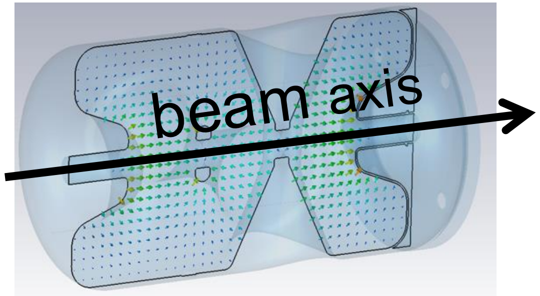


- p+ upstream (<1GeV)
- Heavy ion

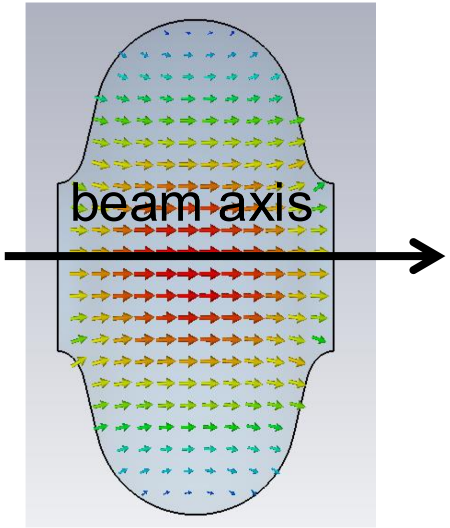


TEM modes in a spoke cavity

- p+ (<1GeV)

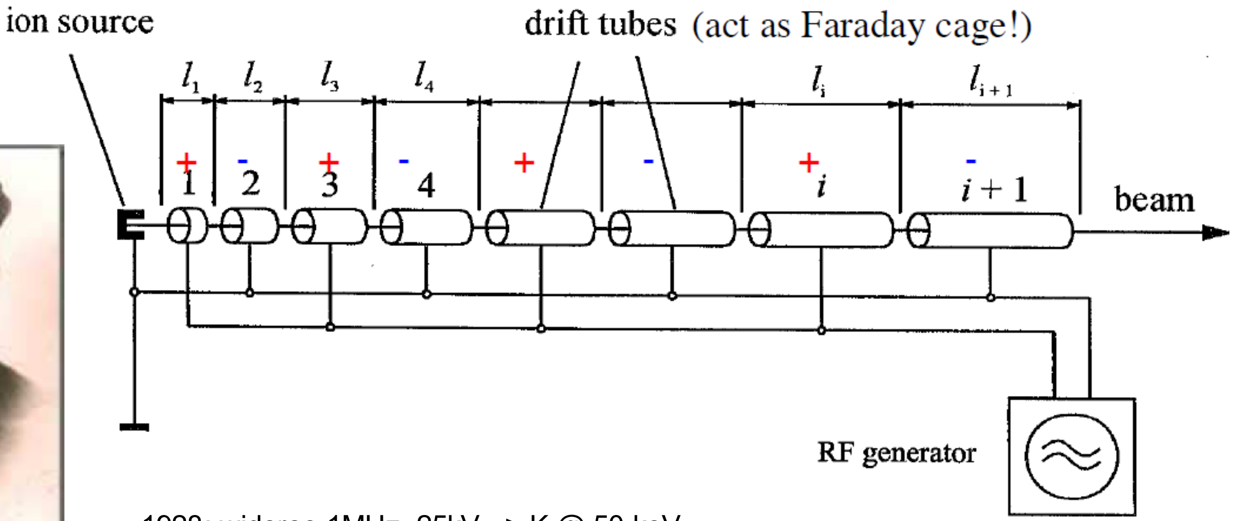


TM_{010} modes in an elliptical cavity



- p+ downstream (>1GeV)
- e-, e+ (>0.5MeV)

Wideroe structure:



1928: wideroe 1MHz, 25kV => K @ 50 keV
1931: Lawrence 7MHz => Hg @ 1.26 MeV

$$V(t) = V_0 \sin(\omega t)$$

For non-relativistic particles => $K_n = nqV_0 \sin(\varphi) = \frac{1}{2} m v_n{}^2$

For constant φ in all gaps, the distance between each at the n^th drift tube length:

$$l_n = v_n \frac{T}{2} = \frac{k}{2} \beta_n \lambda = \frac{\sqrt{n}}{f} \sqrt{\frac{qV_0 \sin(\varphi)}{2m}}$$

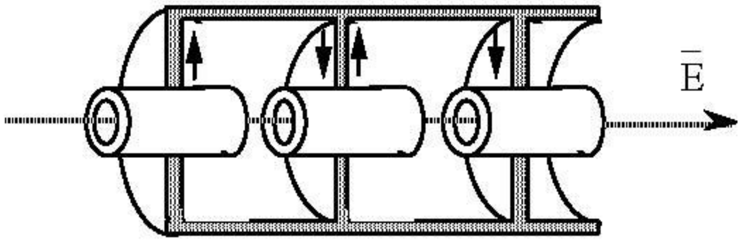
Not efficient (π mode)

Drift tube inside a cavity

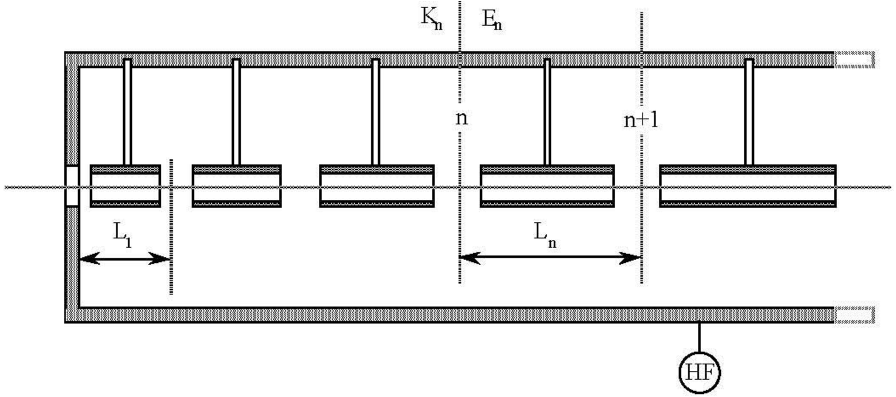
Alvarez structure:

Working on 2π mode, $\beta = 0.1 - 0.35, f = 100 - 500MHz$

Fermilab DTL, 425 MHz



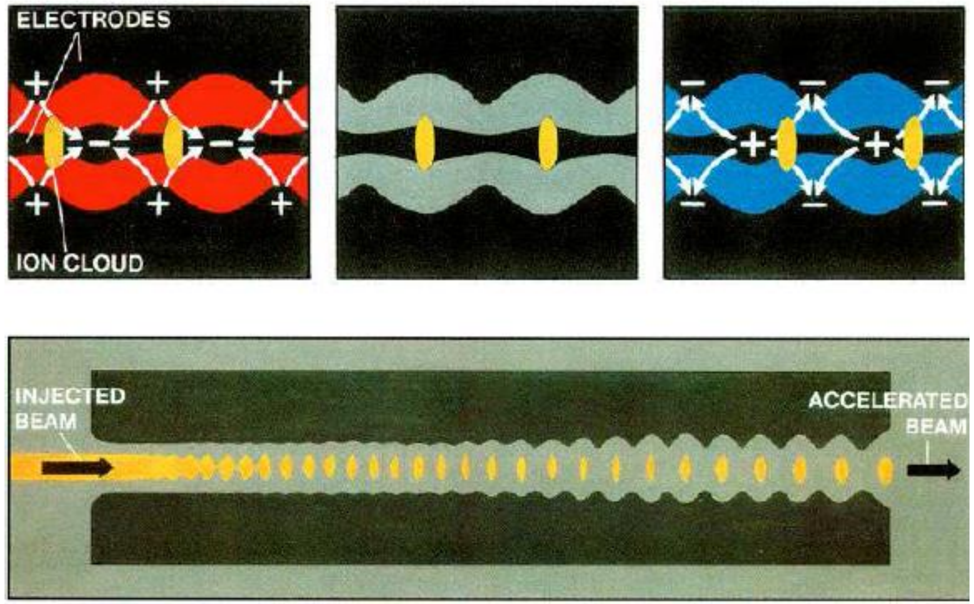
Surface current allows removing cavity wall

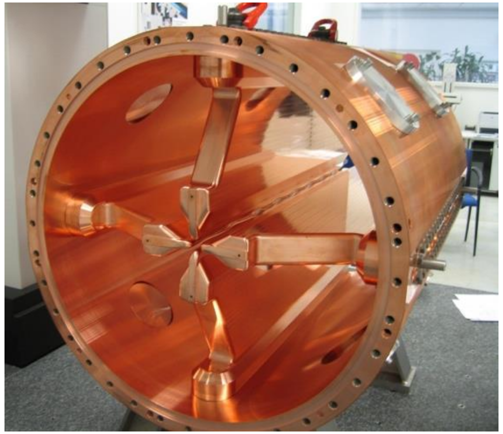


RFQ

Efficient for very low beta particles and light ions acceleration.

Best for $\beta = 0.005 - 0.03, \mathrm{f} = 10{\sim}400\ MHz$





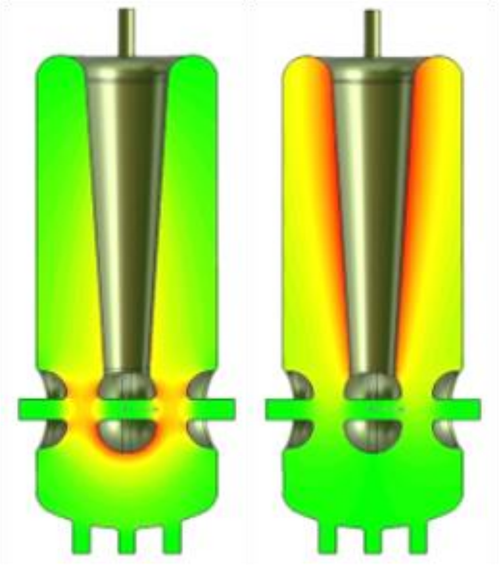
Spiral2 RFQ section, 88 MHz, 240kW
20 kev/u => 0.75 MeV/u, 5 mA

2 types: 4-rod (low f), 4-vane (high f)
High acceptance, low operational cost

Quarter-wave resonators (IJCLab's speciality)

QWR structures:

Low frequency, large accelerating gaps, suitable for heavy ions v <0.15c



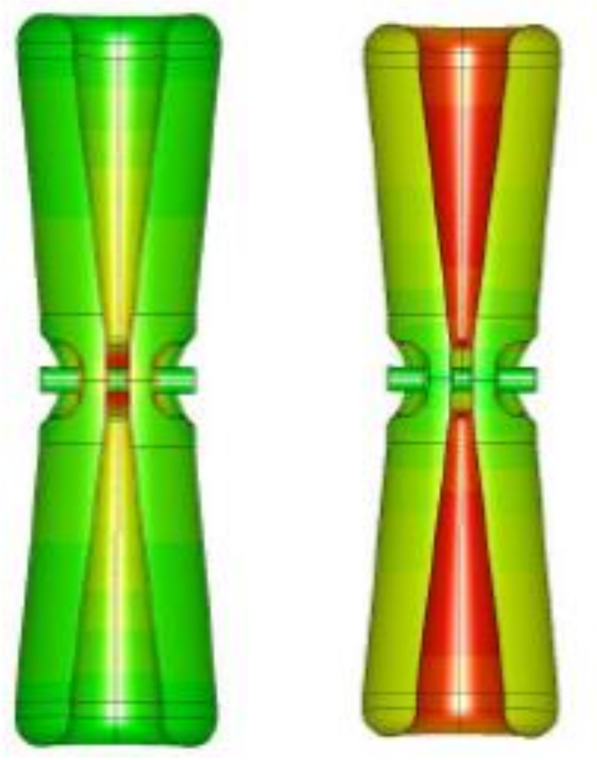
E field H field

Why not high beta QWR?

- Physically large
- Strong magnetic steering on beam axis

Half-wave resonators

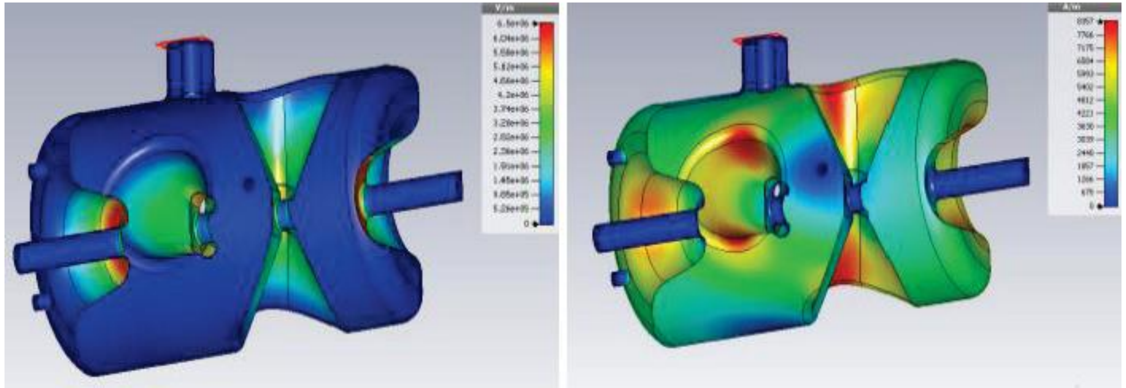
Low frequency, large accelerating gaps, suitable for heavy ions v<0.5c



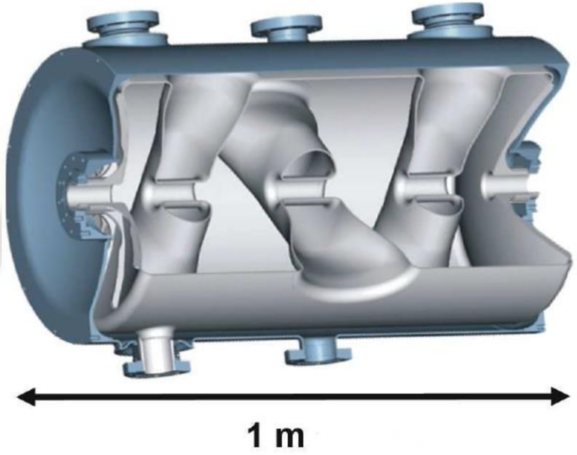
E field *H field*

Spoke cavities (IJCLab's speciality)

Suitable of proton with 0.15c < v < 0.7c



ESS 2-spoke cavity, β=0.5, f=352 MHz

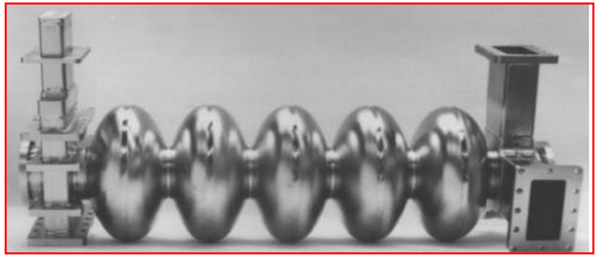


RIA 3-spoke cavity, β=0.63, f=340 MHz

Elliptical cavities

For relativistic particles (f>1GHz for electrons, f≈700 MHz for proton).

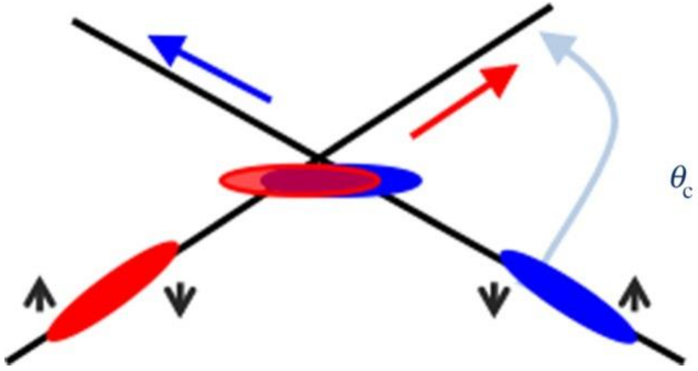
XFEL 9-cell cavity, f=1.3 GHz

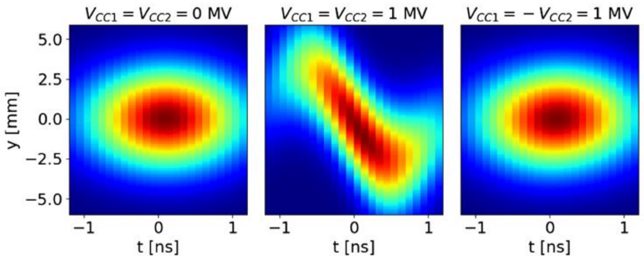


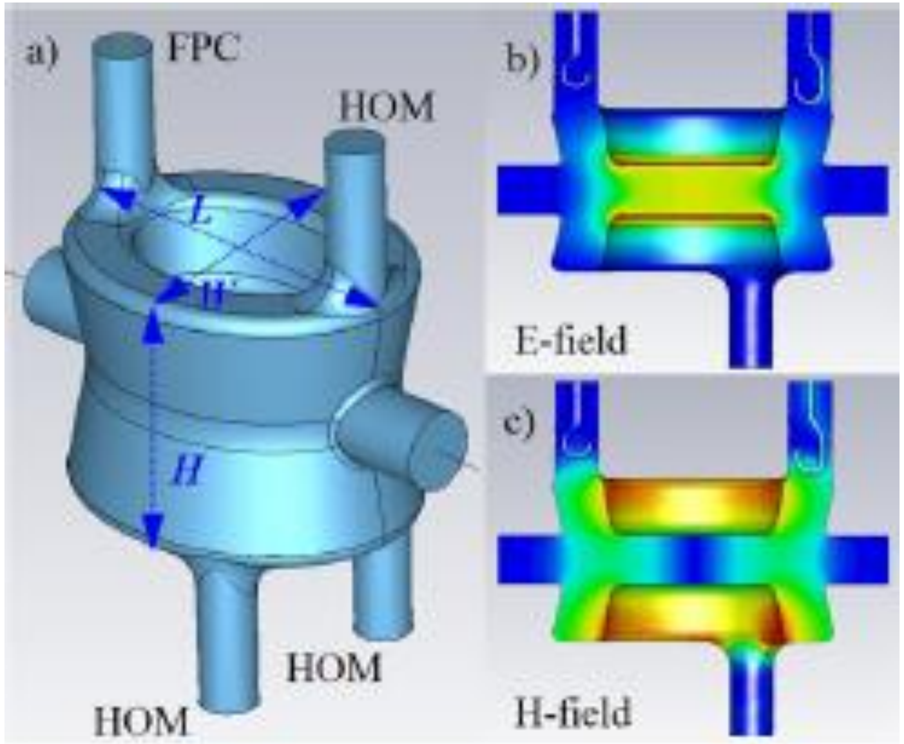
CEBAF 5-cell cavity, f=1.5 GHz

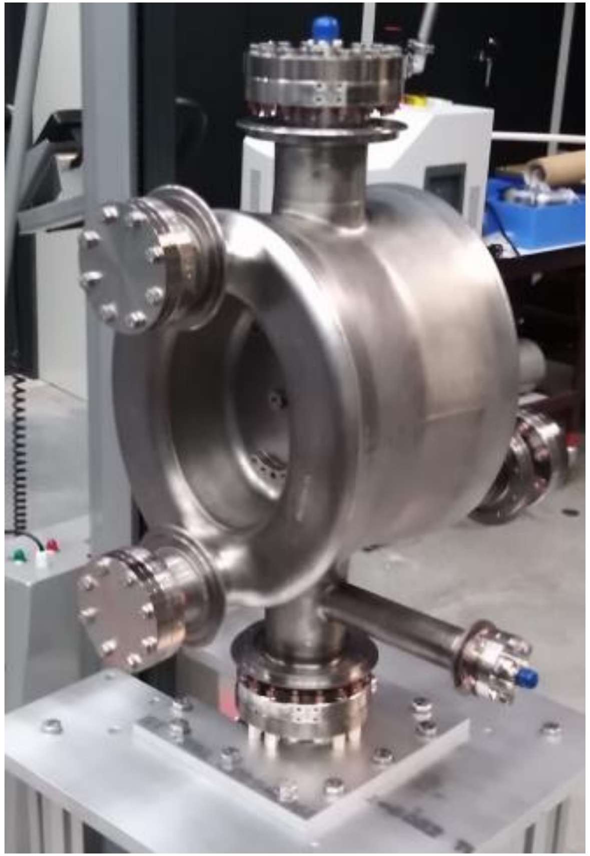
ILC Ichiro shape 9-cell cavity, f=1.3 GHz

ESS 5-cell cavity, β=0.86, f=704 MHz









Phys. Rev. Accel. Beams 24, 062001 2021

For better luminosity

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Conclusion: *Radiofrequency Technologies for Particle Accelerators*

- DC accelerator is excellent for low energy heavy ions but with limited energy level
  - RF is a way to go beyond GeV
- RF is in short just a solution of Maxwell equations but contains rich physics and technical challenges behind it
  - Free space → waveguide → resonant cavities
  - Pill box cavity, optimized cavities, figures of merits
  - equivalent circuit, quality factor
- RF cavities are operated with ancillaries
  - control system, amplifier, coupler, tuner
- RF cavities suffer from back-action of beam: wakefield
  - Beam loading, higher order modes, handed by HOM couplers & absorbers
- Various RF cavity shape for different purpose
  - TEM type: QWR, HWR, spoke for slow particles
  - TM type: elliptical cavities for fast particles (electrons, protons in down stream)
  - Deflecting cavities: crab cavity for crabbing
- Excellent interplay of physics and engineering to enable particle acceleration!

Part 2: Superconductivity and Cryogenics for Particle Accelerators

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Outline

- Introduction: thermodynamics and benefit of cooling (10 min)
- Basics of cryogenics (15 min)
- BCS superconductivity (15 min)
- Non-BCS superconductivity (5 min)
- Break (10 min)
- Superconducting magnet (10 min)
- Superconducting RF cavities (20 min)
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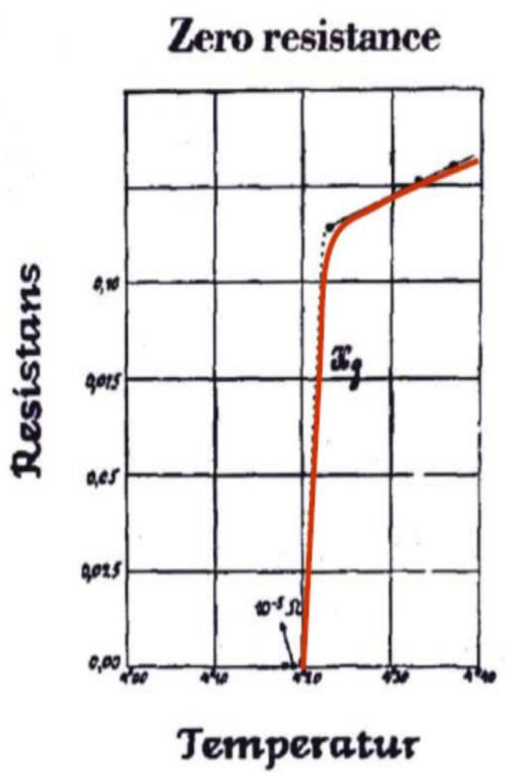
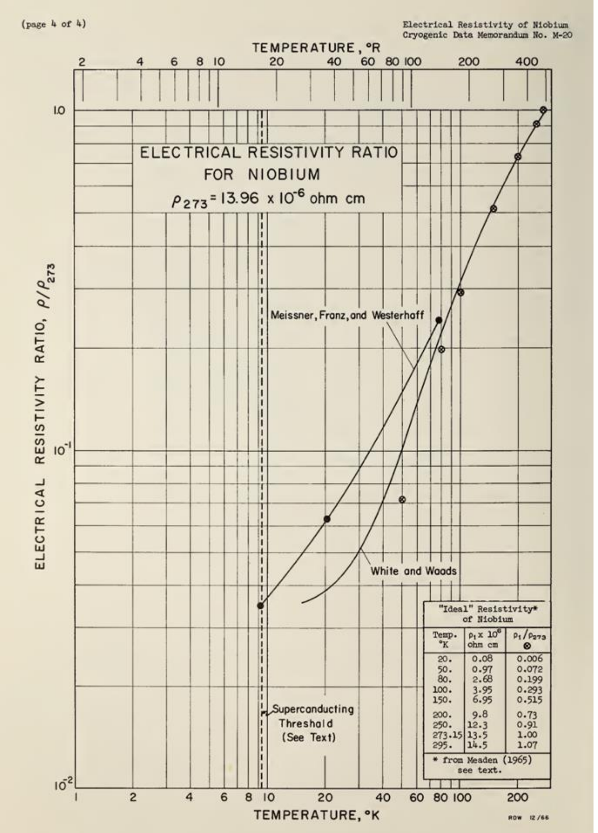
Cooling down to reduce electric loss

Superconducting (Hg)



Niobium



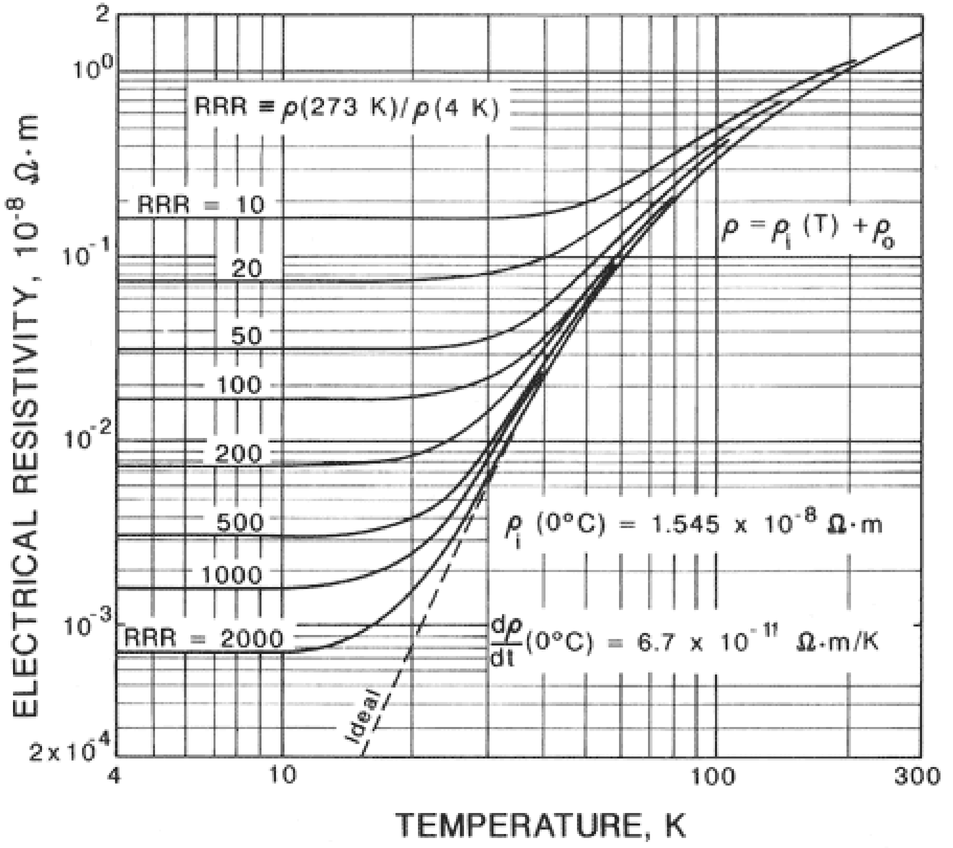
"Ideal" Resistivity* of Niobium

Temp. °K	ρ_i x 10^6 ohm cm	ρ_i/ρ_{273} ⊗
20.	0.08	0.006
50.	0.97	0.072
80.	2.68	0.199
100.	3.95	0.293
150.	6.95	0.515
200.	9.8	0.73
250.	12.3	0.91
273.15	13.5	1.00
295.	14.5	1.07

* from Meaden (1965) see text.

https://nvlpubs.nist.gov/nistpubs/Legacy/TN/nbstechnicalnote365.pdf

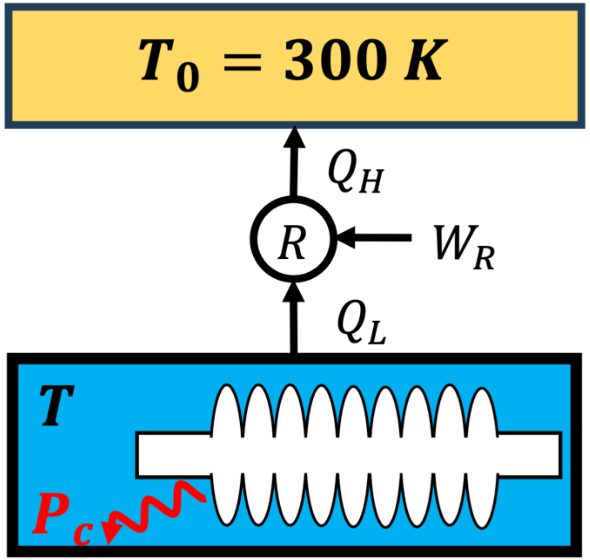
Copper



https://www.copper.org/resources/properties/cryogenic/

Cost of ideal cryogenics

Cooling power 1 W depends on temperature



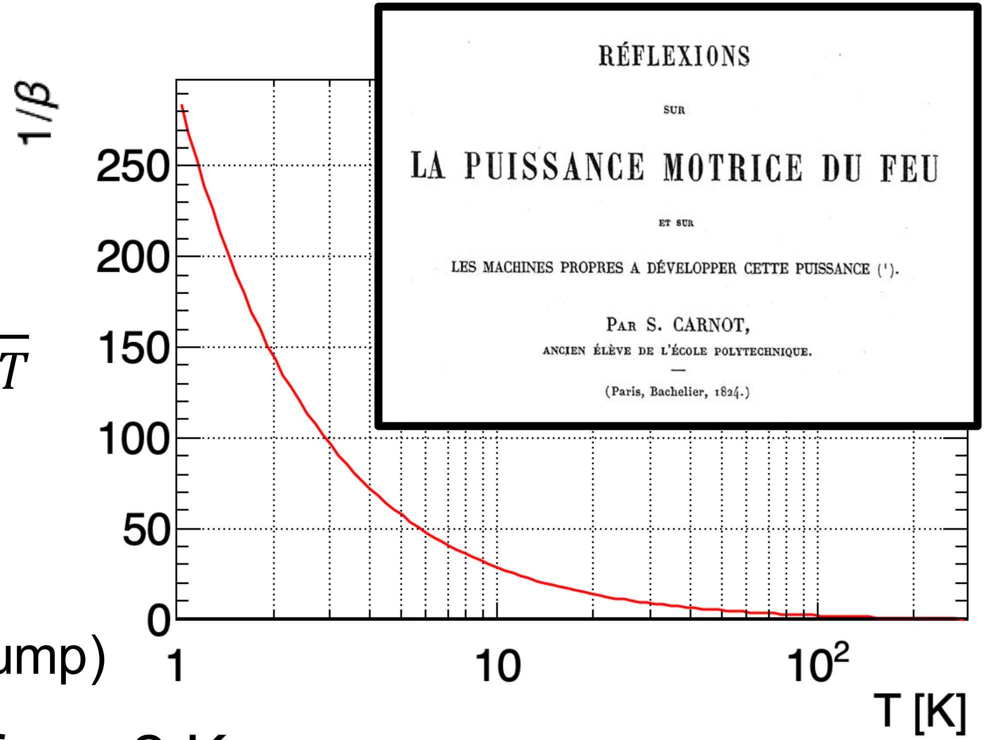
Carnot's theorem

$$\beta = \frac{Q_L}{W_R} = \frac{Q_L}{Q_H - Q_L} = \frac{T}{T_0 - T}$$

Required power

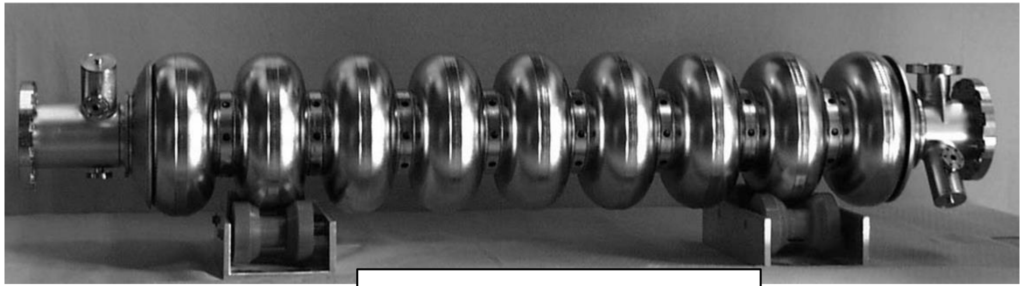
$$P_{cryo} > \frac{P_c}{\beta}$$

(be careful about logical jump)

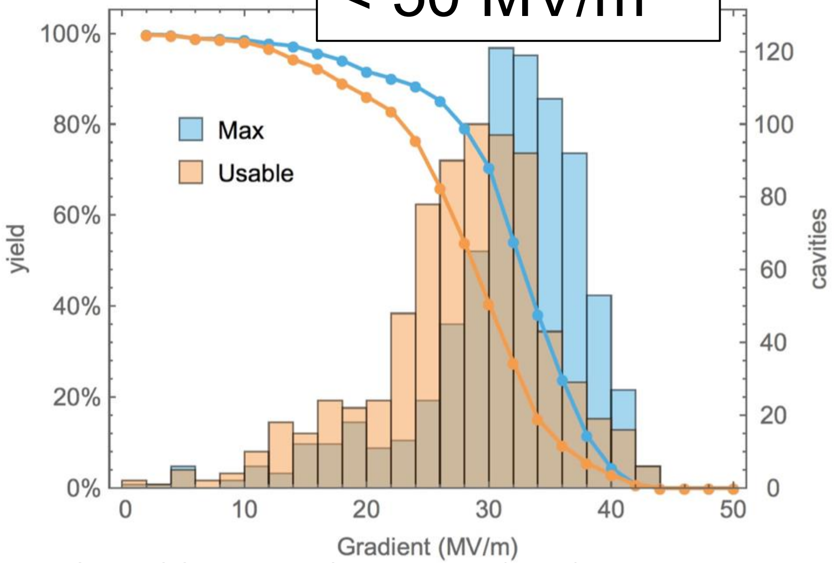


- We may need >150 W to evacuate 1 W from 2 K
- Cryogenics is an option if the resistance is improved better than this factor
    - Or some applications (eg superconductors) are not feasible at warm

Superconducting niobium cavities (TESLA)

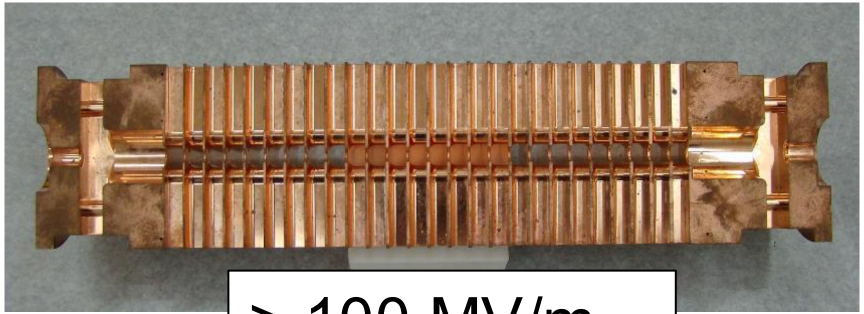


< 50 MV/m



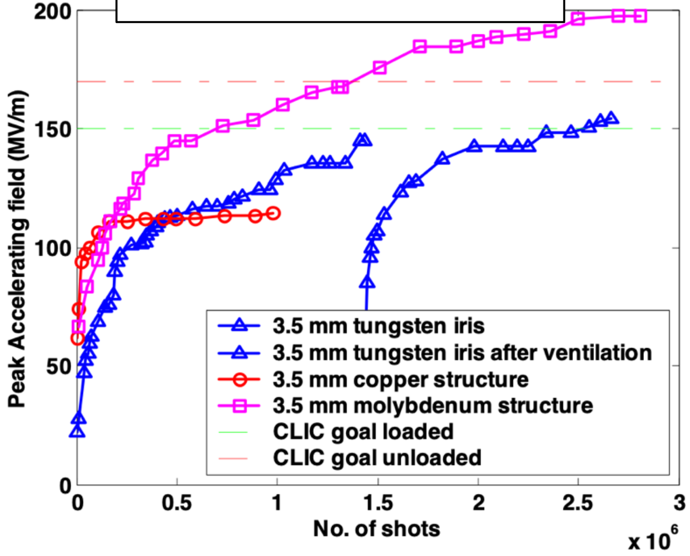
PHY REV ST - ACCEL BEAMS, **3**, 092001 (2000)
PHY REV ACCEL BEAMS 20, 042004 (2017)

Normal conducting copper cavities



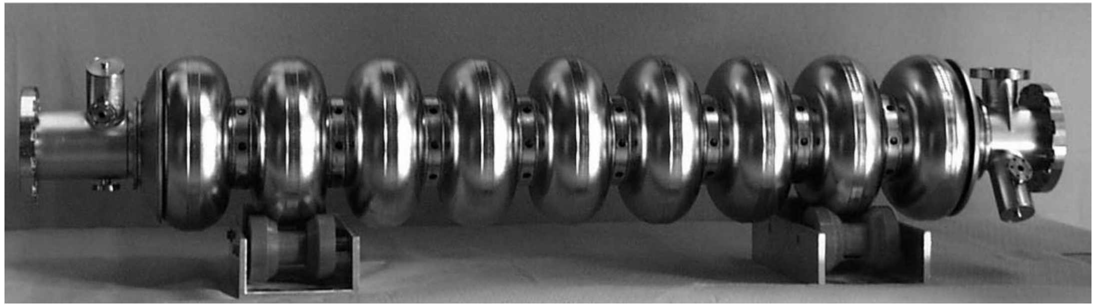
> 100 MV/m

$> \times 2$

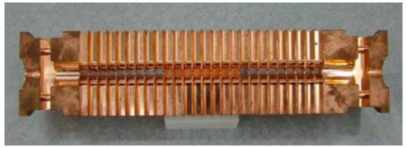


Courtesy: Walter Wuensch

Aperture

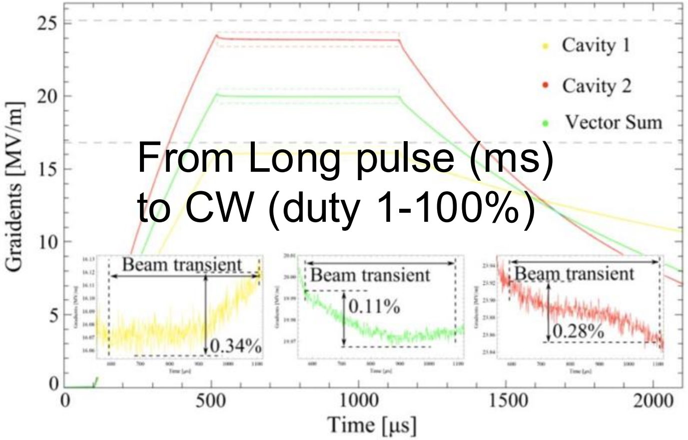


Superconducting cavities have high gradient at low frequency
→ large aperture (ILC: ϕ70 mm)



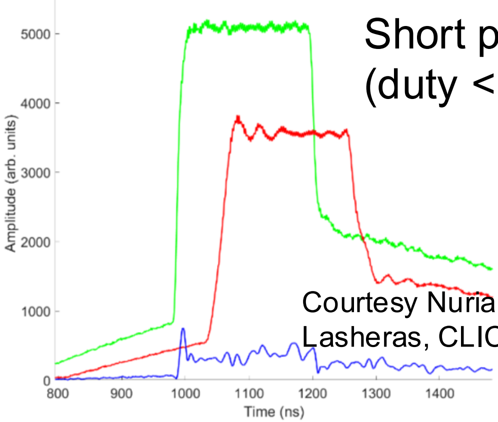
Normal conducting cavities are efficient at high frequency
→ small aperture (CLIC X-band: around ϕ3 mm)

Pulse length and duty cycle



From Long pulse (ms)
to CW (duty 1-100%)

M. Omet
SRF2021
lecture

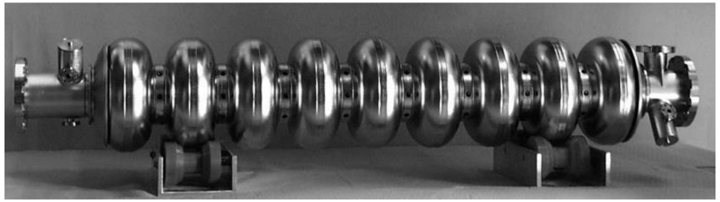


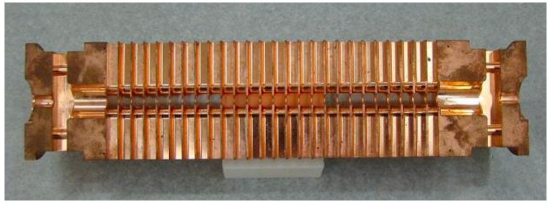
Short pulse (µs)
(duty <<1%)

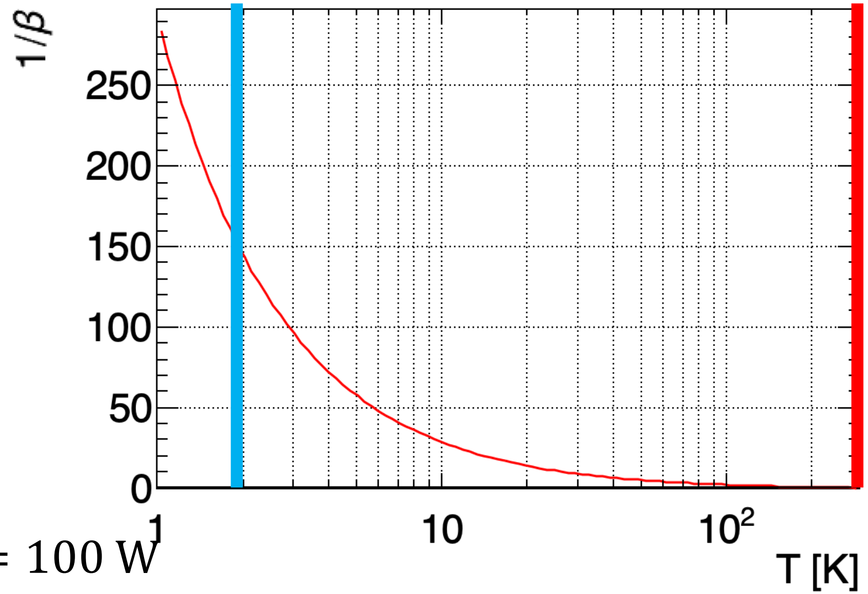
Courtesy Nuria Catalan Lasheras, CLIC

SC cavities' quality factor
$\times 10^{6}$
than copper cavities
→ power dissipation
$\times 10^{-6}$
but in ***cryogenics***!

SC vs NC cavities and cooling







<u>SC cavities</u>

$P_c = 100$ W (CW)
Duty cycle 10^{-2}
$T = 2$ K

$P_{NET} \sim 100 \text{ W} \times 1\% \times 150 = 150 \text{ W}$

<u>NC cavities</u>

$P_c = 10$ MW (CW)
Duty cycle 10^{-5}
Water cooling

$P_{NET} \sim 10 \text{ MW} \times 10^{-5} \times 1 = 100 \text{ W}$

- Short pulsed NC cavities and long pulsed SC cavities are similar in power consumption
  - If we need very long pulse or CW beam (LEP, FCCee, LCLSII) → SC cavities
  - If we need very short pulse beam (LCLS) → NC cavities
  - If we need CW but with low energy (many storage rings) → NC cavities
- Linear colliders are on the boarder: both SC (ILC) and NC (CLIC) may be OK

Carnot cycle is unrealistic at all

Thermodynamics does not include characteristic time constant
→ Carnot cycle gives maximum efficiency in quasi-static process ($\Delta t \to \infty$)
→ Power (work per time) is in trade off with the efficiency

PRL117 190601 (2016)

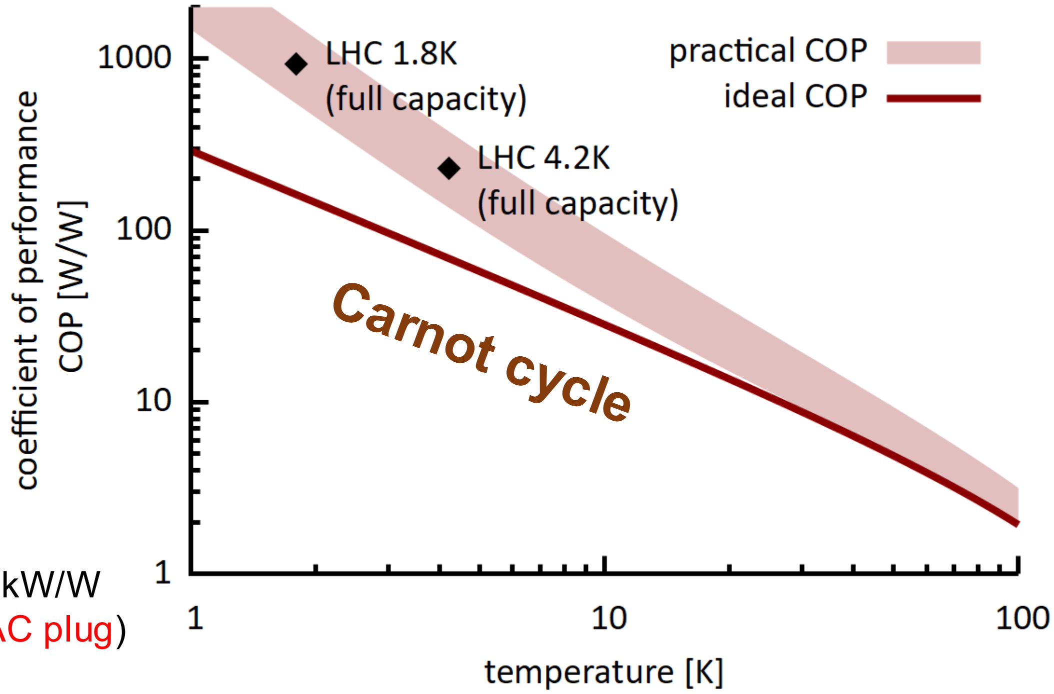
$$P_c \leq \bar{\Theta}\beta_L \eta(\eta_C - \eta)$$

We lose useful power if efficiency η is too good approaching to Carnot η_C

In addition, more practical limitations further degrade the efficiency

→ Around 1 kW is necessary to evacuate 1 W from 2 K

(typically 5 kW/W @ 2 K for AC plug)



→ If the power loss of the device improves by $>10^3$ by cooling, 2 K operation is beneficial

Outline

- Introduction: thermodynamics and benefit of cooling (10 min)
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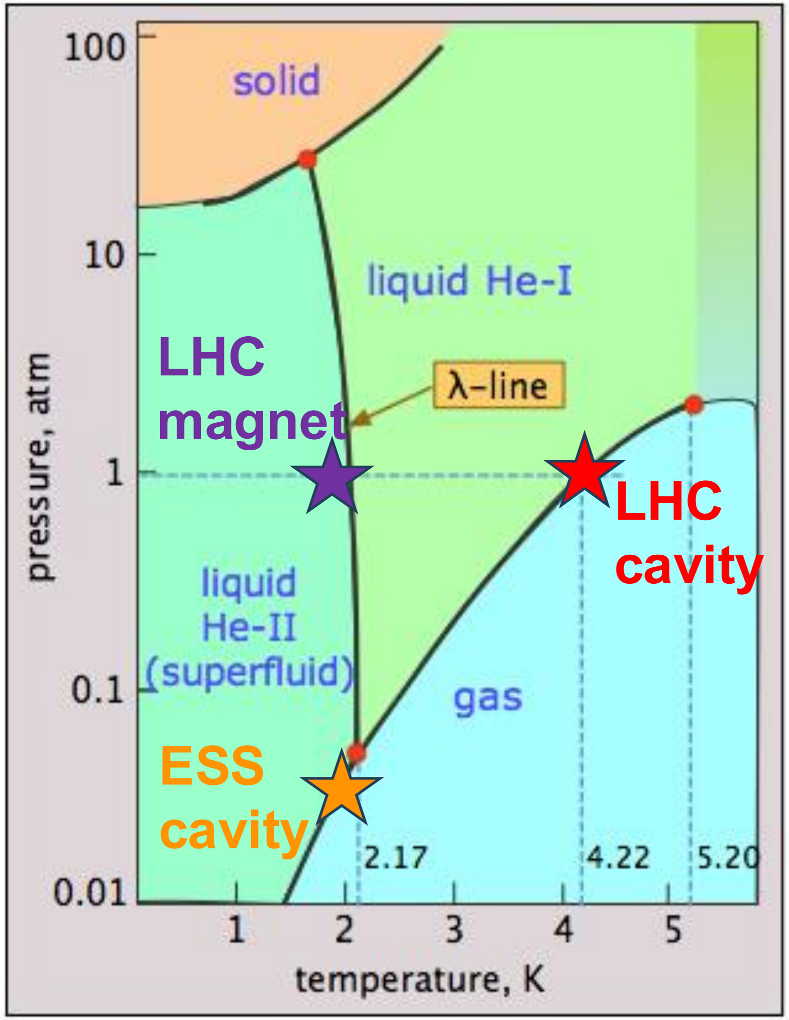
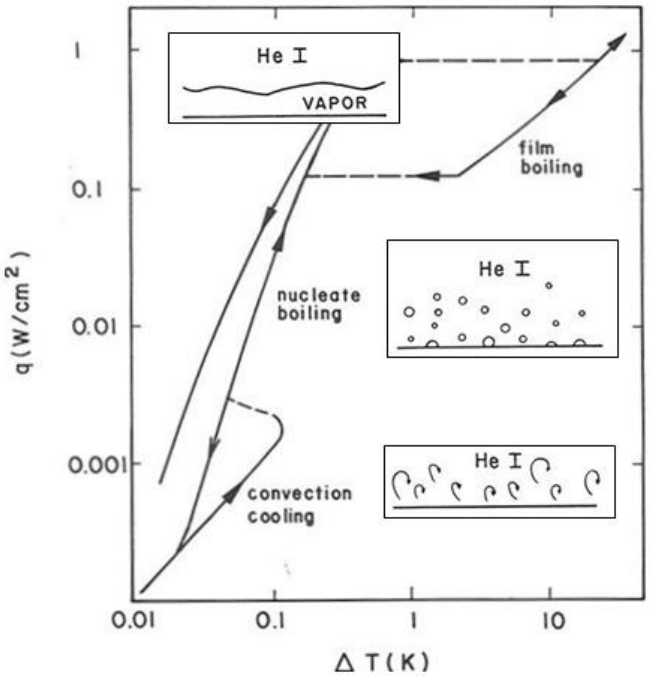
- Superconducting RF cavities (20 min)
- Conclusion

Cavities and magnets are in liquid helium

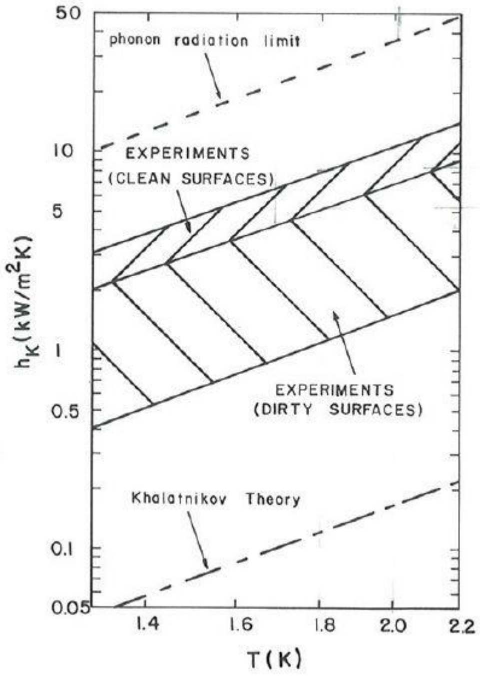


Normal helium (He-I)



~100 W/m²/K

Superfluid helium (He-II)



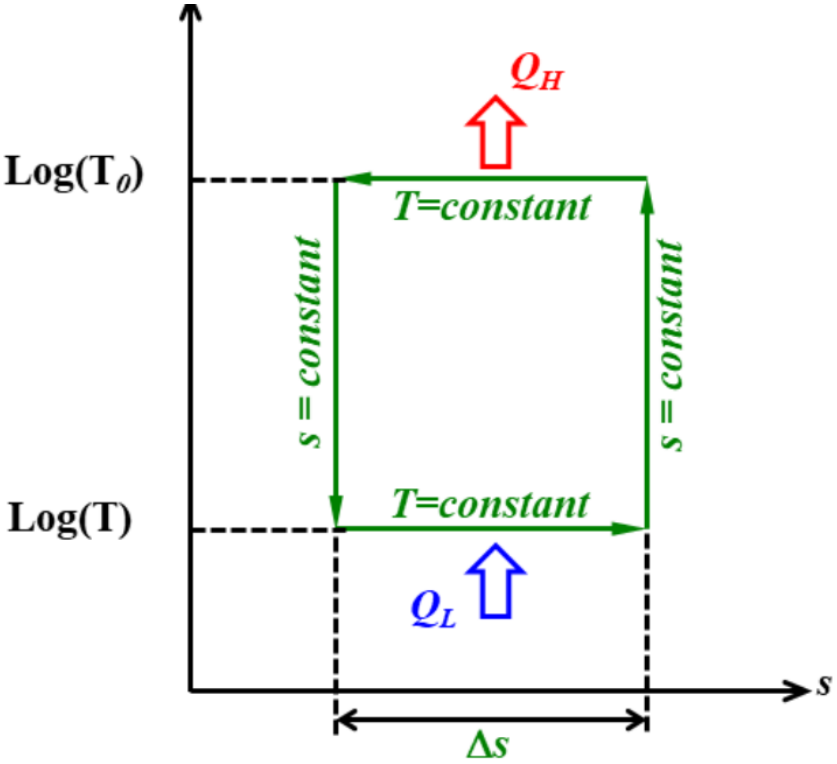
~ 5000 W/m²/K

We need to liquify gas helium!

S. W. Van Sciver, "Helium cryogenics", Plenum press, 1986

Ideal cooling cycle: Carnot cycle

Courtesy: Nusair M. Hasan

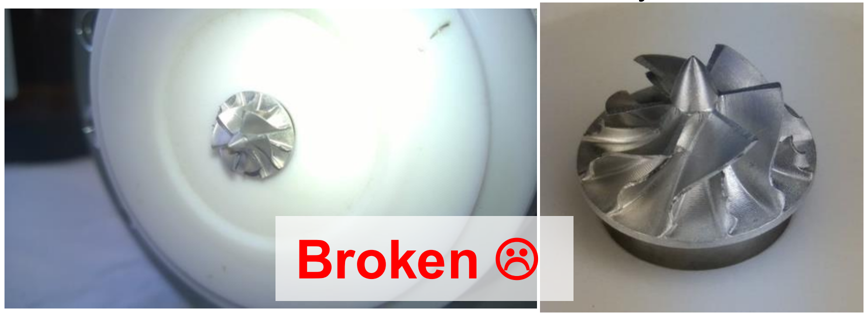


- Combination of ientropic (reversible adiabatic) and isothermal compression & expansions
- Expander & compressor are the key components
- A Carnot cycle could theoretically generate LHe but requires unrealistically high pressure

Isentropic expansion and isothermal + isobar compression

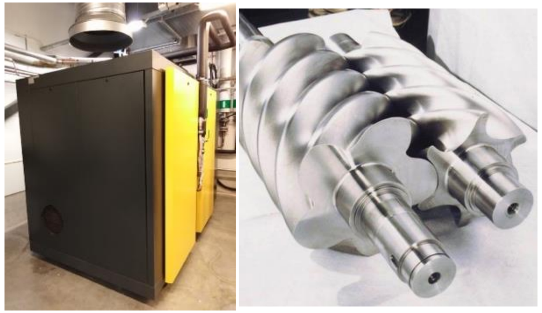
Turboexpander (expansion turbine)

Courtesy: M. Pierens



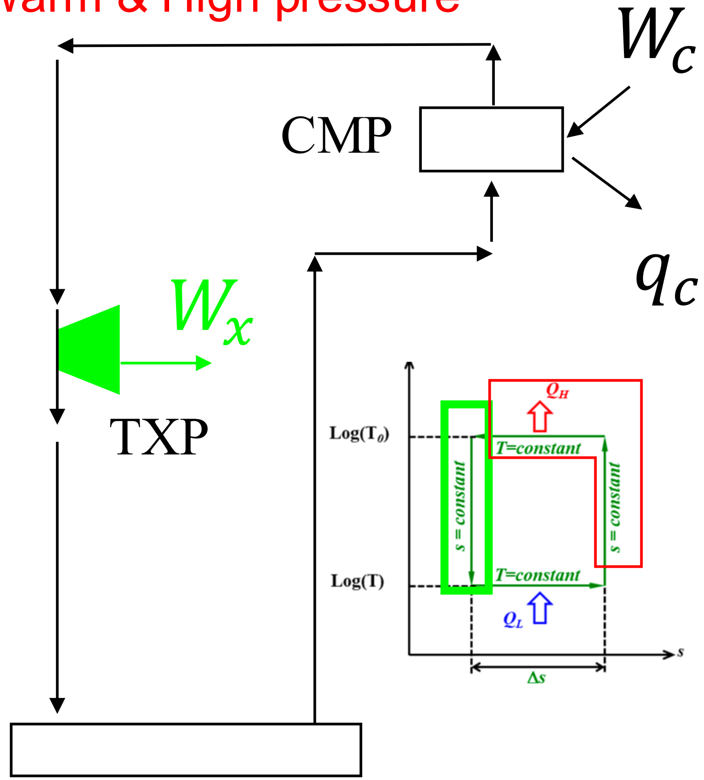
isentropic (=reversible adiabatic) expansion

Compressor



Reversed-modified Joule (Brayton) cycle

Warm & High pressure



Cold & low pressure

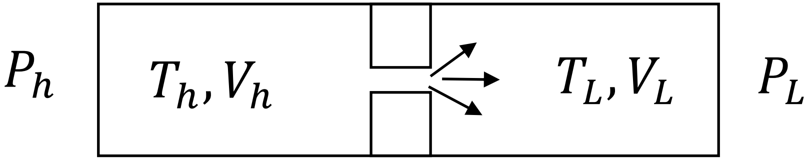
<u>Remarks</u>

- Isobar process → Joule cycle ≠ Carnot cycle
- Weak points (and expensive!) of a cryogenic facility
  - If your accelerator is down, maybe one of them are out of order
- Not used for liquefaction of helium alone practically
  - Turbine does not like LHe

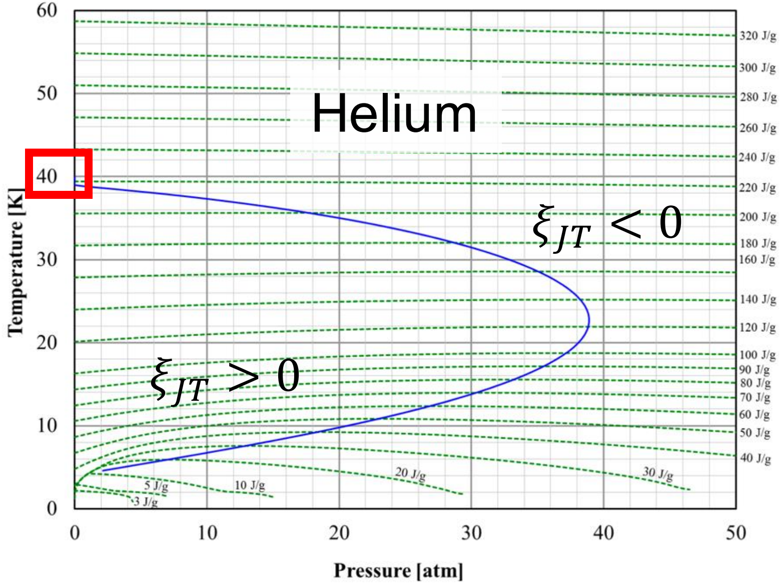
→ Another process needs to be combined

Joule-Thomson (JT) process is isenthalpic (irreversible adiabatic)

Adiabatic ≠ isentropy ($ds \geq d'Q/T$)



$$H = U_h + P_h V_h = U_L + P_L V_L$$



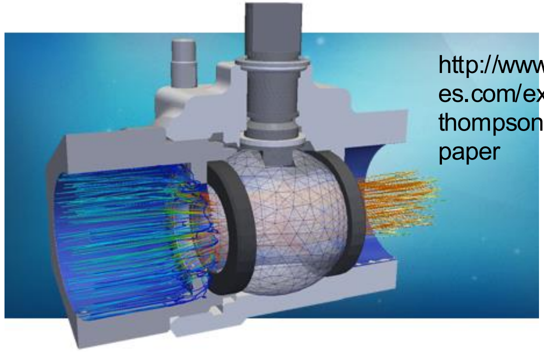
JT coefficient

$$\xi_{JT} = \left(\frac{\partial T}{\partial P}\right)_H$$

$\xi_{JT} < 0$ (→ $T_h < T_L$): heating

$\xi_{JT} > 0$ (→ $T_h > T_L$): cooling

$\xi_{JT} = 0$ at reverse temperature



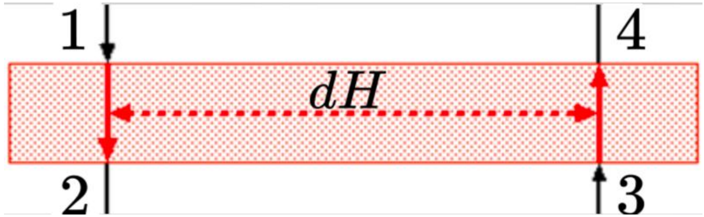
http://www.pretechnologies.com/experience/joule-thompson-cooling-white-paper

He Liquefier is made of i) compressor ii) turbine iii) JT valve

Heat exchanger for pre-cooling

Heat exchanger profits from cold return gas to enhance the liquefaction yield

$$H_4 = H_3 + dH$$



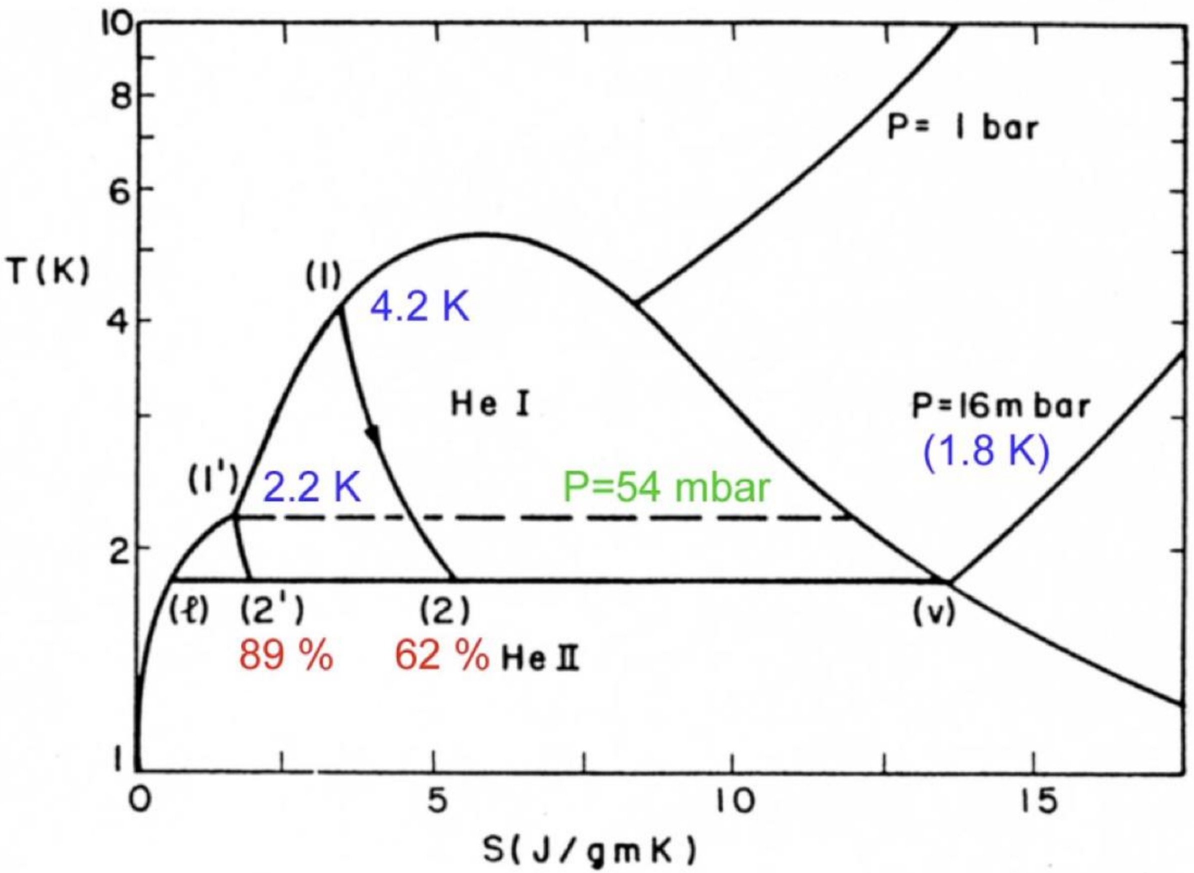
$$H_2 = H_1 - dH$$

If temperature before the JT valves is

- 4.2K → 62%
- 2.2K → 89%

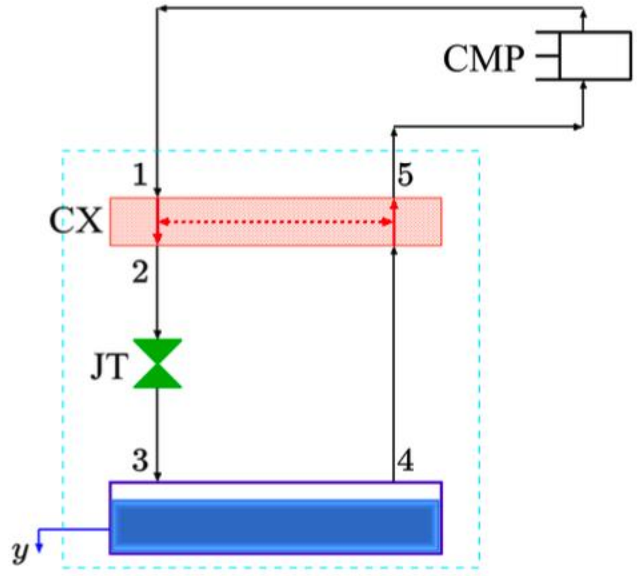
Superfluid is generated by pumping down to 30 mbar

Courtesy: Elias Waagaard



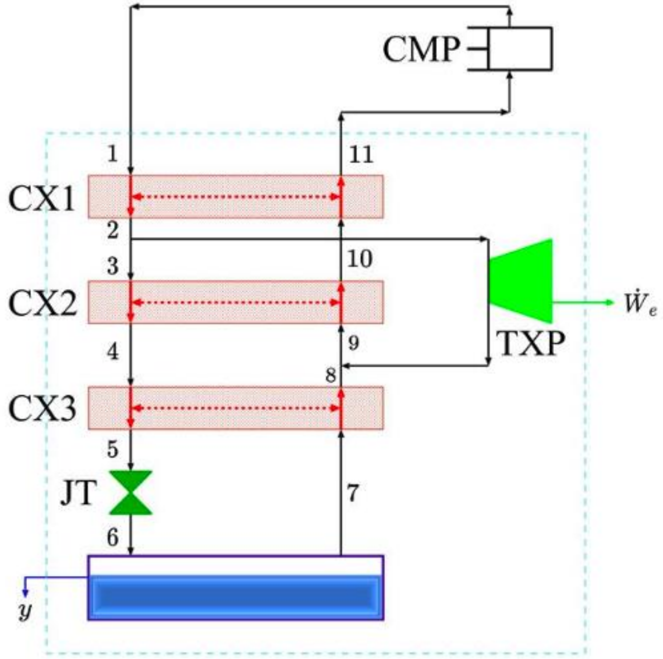
Van Sciver S. W. “Helium Cryogenics”, Plenum Press (1986)

Linde cycle



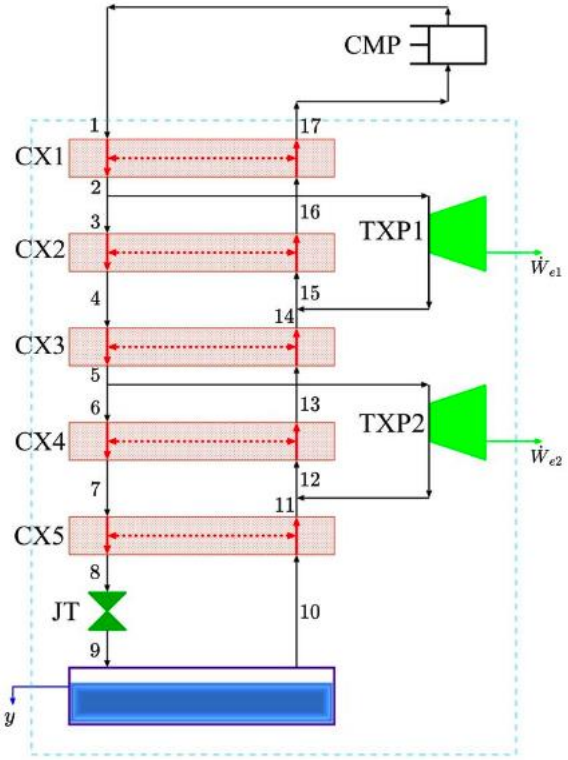
Joule-Thomson liquefication

Claude cycle



Joule cycle refrigeration +
Joule-Thomson liquefication

Collins cycle

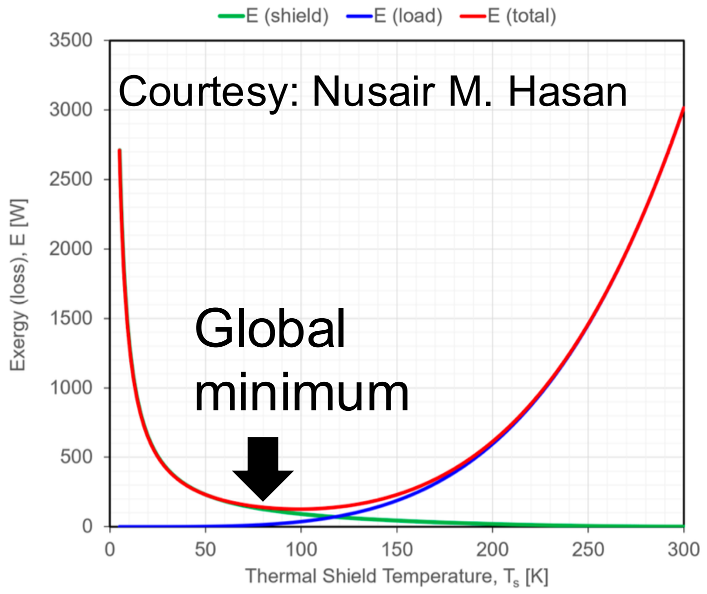


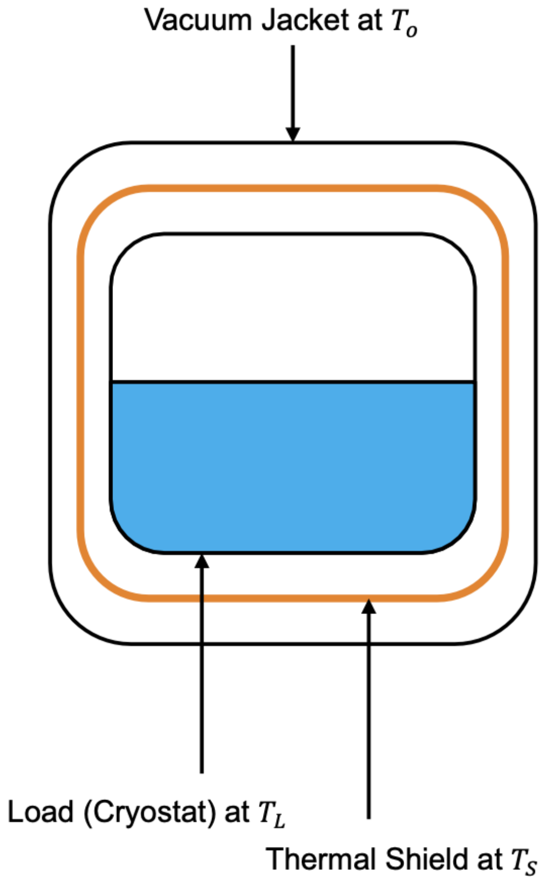
Courtesy: Elias Waagaard

Insulation vacuum and radiation shield

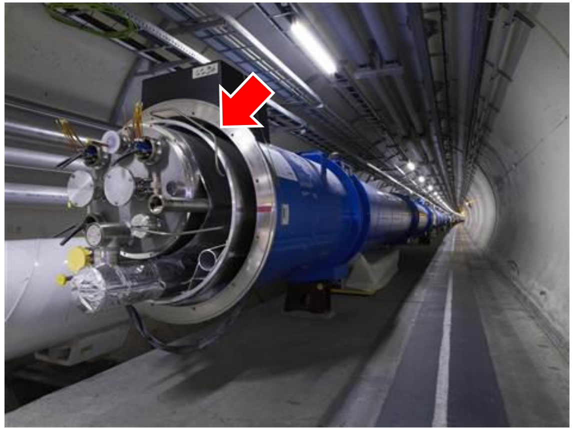
Stephan Boltzmann: $q = \epsilon A \sigma T^4$

- Intersect the thermal radiation by intermediate temperature
- Easier to evacuate heat at higher temperature





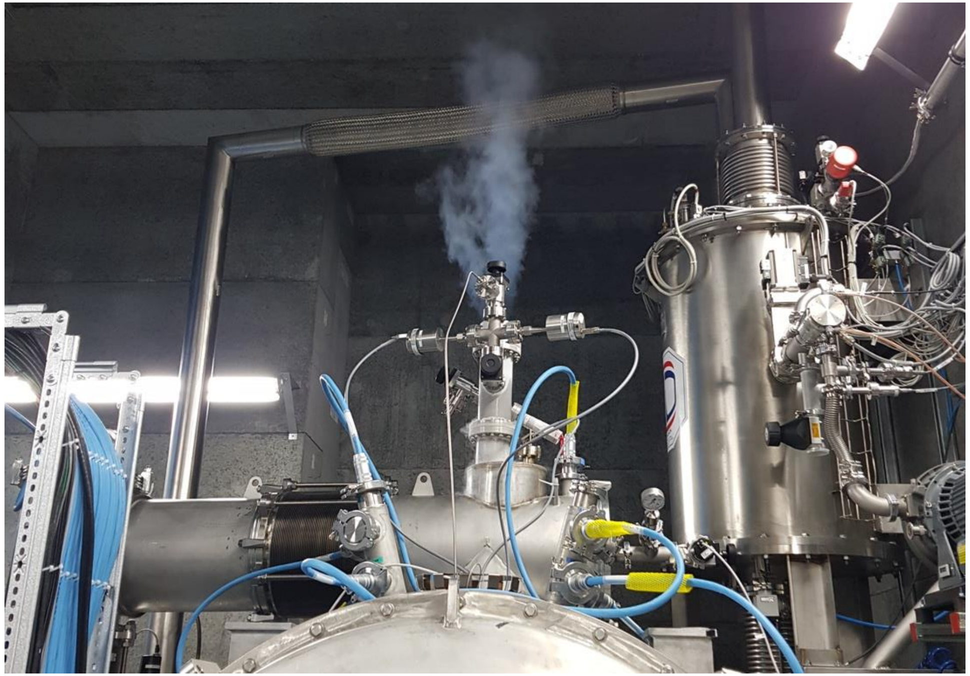
Multilayer Insulation (MLI)



LHC dipole

High power + cryogenics → Lesson learned from incidents ☹

3 months after I got a staff position



High power RF → SC cavity quenched by multipacting → wrong interlock setup → Overpressure in LHe → Rupture disk burst → no mechanical damage

3 months after I started PhD in particle physics

Defect in soldering joint → resistive (magnet quench itself was NOT the trigger)→ Electrical arc → He goes into isolation vacuum → helium boiled off → mechanical damage

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Ohm's law

Applied DC electric field E

DC Current J

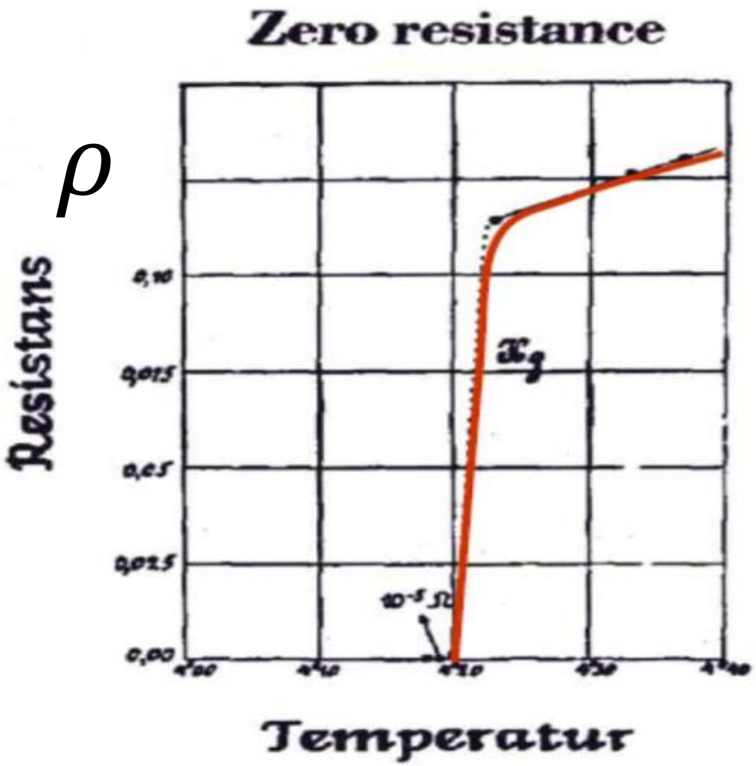
DC resistivity ρ

$$\rho \equiv \frac{E}{J}$$

DC conductivity σ

$$\sigma = \frac{1}{\rho} \equiv \frac{J}{E}$$

Cool down the resistor…



Heike Kamerlingh Onnes

Nobel prize in 1913

$\boldsymbol{\rho = 0}$ below transition temperature T_c

Challengers for microscopic theory of superconductors

J. Schmalian, arxiv:1008.0447

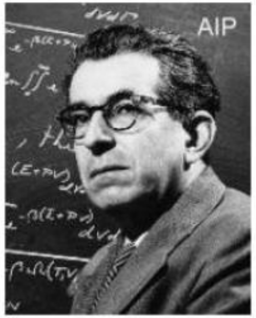

Albert Einstein
(1879-1955)


Niels Bohr
(1885-1962)


Ralph Kronig
(1905-1995)

John Bardeen
(1908-1991)


Werner Heisenberg
(1901-1976)



Fritz London
(1900-1954)


Lev D. Landau
(1908-1968)


Felix Bloch
(1905-1983)

Léon Brillouin
(1889 -1969)

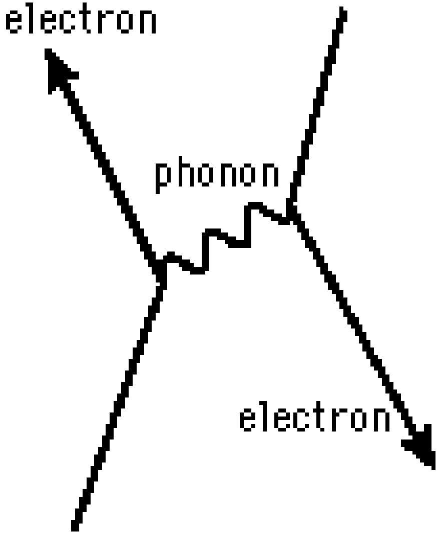

Max Born
(1882-1970)

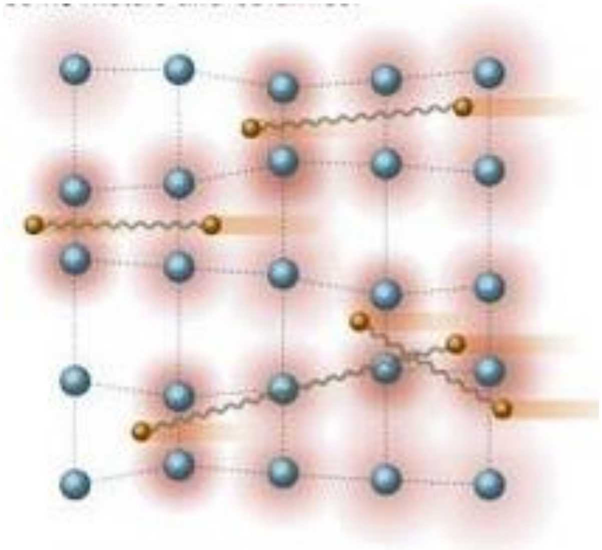

Herbert Fröhlich
(1905-1991)


Richard Feynman
(1918-1988)

A lot of models…all failed ☹
Development of quantum field theory in many body problems was necessary…

Theory of superconductor in equilibrium





John Bardeen

Leon Cooper

John Robert Schrieffer

<u>Cooper pair: Composite boson</u>

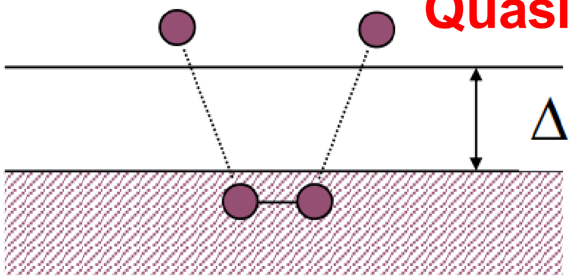
Two electrons are bounded by something (phonon) → effective Hamiltonian $\mathcal{H}_{BCS}$

Mean field approximation + Variational method (+other approximations…)

$$\mathcal{H}_{BCS}|\Phi_0\rangle = E|\Phi_0\rangle$$

Solution: superconducting gap

Quasi-particles



$\Delta_o \propto \Psi_o$

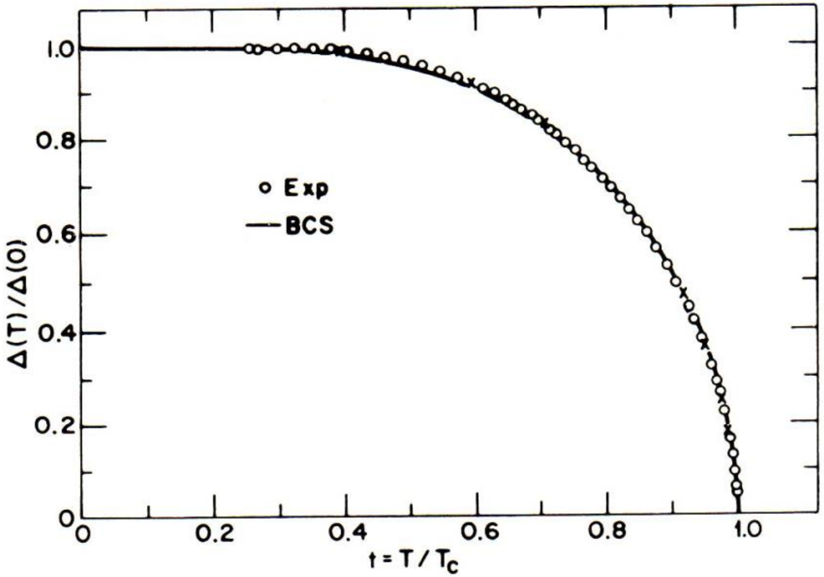
Scaler wave function

- The Cooper pair needs certain amount of energy to be broken
- The cause of Ohmic loss, stochastic scattering of one single electron by phonon or impurity **cannot break the pair**

  →No DC loss

Self-consistent gap equation

$$\Delta = N(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2 + \Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$

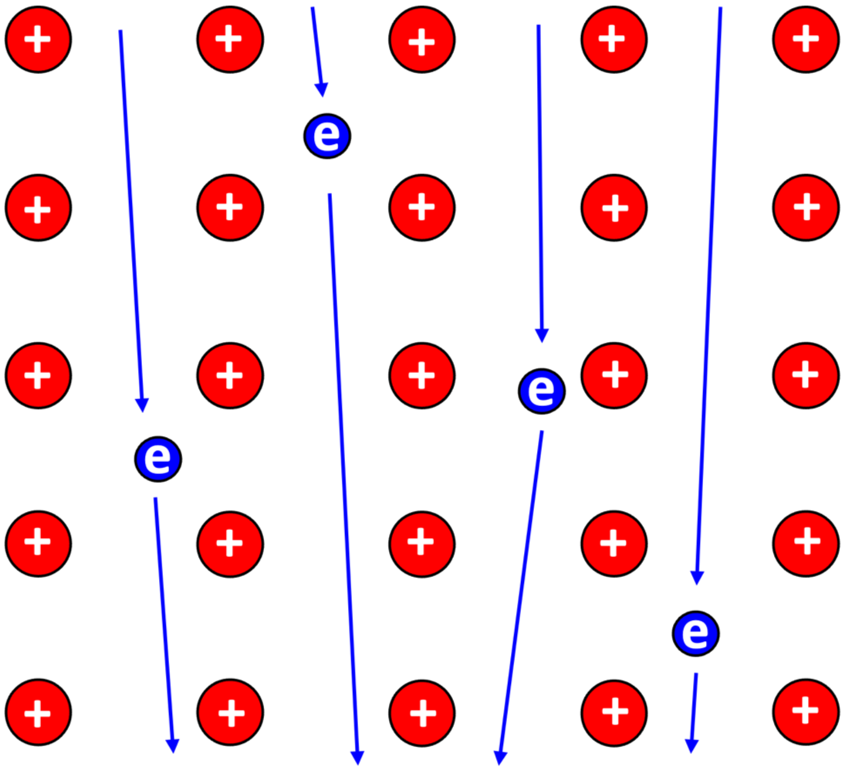


The Equilibrium state of conventional superconductor was understood !

→ In this lecture, we try to obtain qualitative insight of the phenomenon

Electrons in a perfect metal are free (independent)

Perfectly periodic potential by ions does ***NOT*** scatter electrons (Bloch's theorem)



These electrons are ***NOT*** our favorite elementary particle of

$$m = 511\ \mathrm{keV}$$

These electrons are ***dressed*** by complicated electromagnetic property of metals to have an effective mass m^* given by a band structure

→ ***Quasi-particles***

In reality, imperfection causes quasi-particle scattering

Electrons in a real metals show Ohmic loss

<u>Imperfections causes ***local*** scattering</u>

1. Impurity, defects (scattering time τ_{def})
2. Lattice vibration, phonon (τ_{ph})

Total scattering time

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{def}}} + \frac{1}{\tau_{ph}}$$

<u>Macroscopic phenomenology (Drude model)</u>

An electron accelerated by an electric field

$$m^* \frac{dv}{dt} = -eE$$

is scattered by imperfections per τ, and its velocity relaxes to a mean velocity

$$\langle v \rangle = -\frac{e}{m^*} E\tau$$

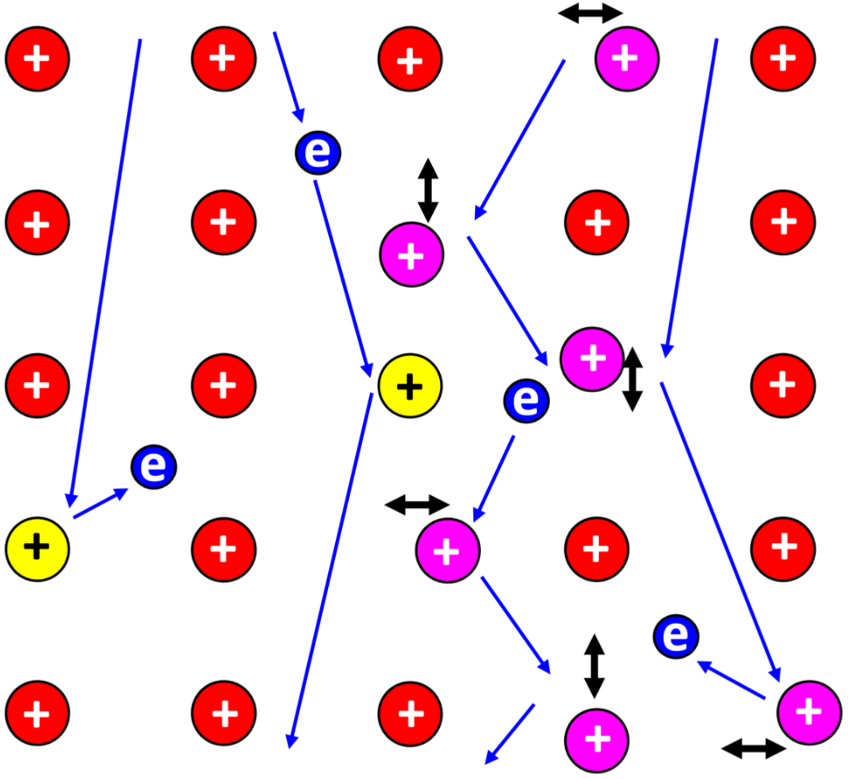
Electric current is a collective flow of n electrons

$$j = -en\langle v \rangle = \boxed{\frac{e^2 n\tau}{m^*}} E$$

Ohm's law

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

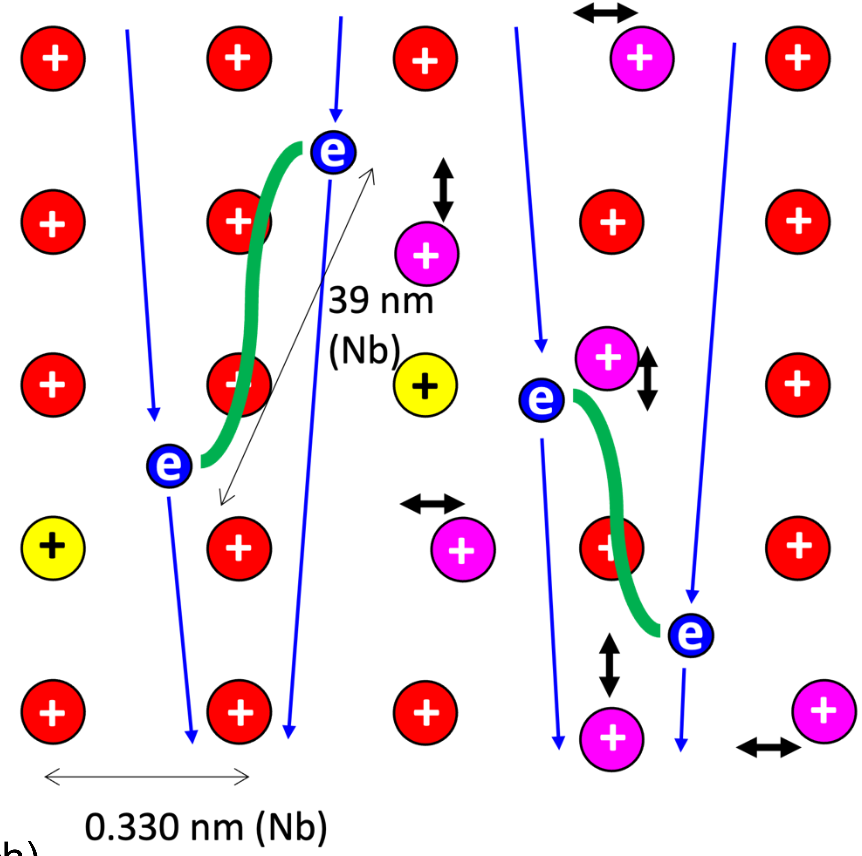
Electrical conductivity σ



Paired electrons can avoid Ohmic loss

If electrons ***in a distance*** (>39 nm) are bounded, ***local*** (< 0.5 nm) scattering can be avoided

Any small attractive interaction V between electrons can lead to a ***Cooper pair*** coupled with an energy 2Δ, below critical temperature T_c

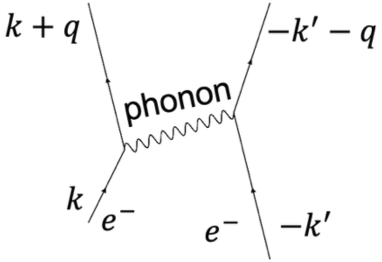


BCS gap equation (1957)

Non-perturbative!

$$\Delta = n(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$

Classical superconductors' attractive potential is from ***longitudinal mode of lattice vibration***



If energy transfer $|\epsilon_{k+q} - \epsilon_k|$ is smaller than phonon energy the interaction is attractive (Flöhlich)
→ Eliashberg's strong coupling superconductor (1960)

Implication of no scattering?

No scattering

$$m^* \frac{\partial \langle v \rangle}{\partial t} = -eE$$

generates super-current

$$j_s = -en_S \langle v \rangle$$

$$\rightarrow \frac{\partial j_s}{\partial t} - \frac{n_S e^2}{m^*} E = 0$$

Apply $\nabla \times$ from the left

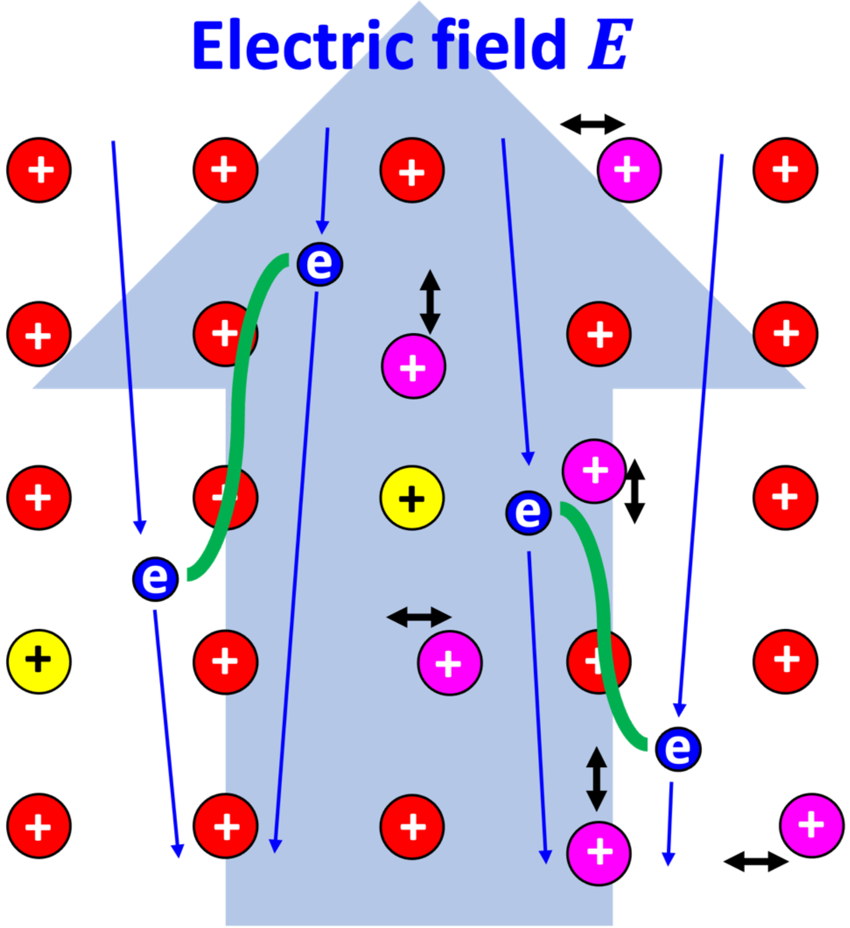
$$\frac{\partial}{\partial t}(\nabla \times j_s) - \frac{n_S e^2}{m^*} \nabla \times E = 0$$

$\sim \nabla \times \boldsymbol{B}/\mu_0$; $-\frac{\partial \boldsymbol{B}}{\partial t}$

leads to

$$\frac{\partial}{\partial t}\left[\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2} \boldsymbol{B}\right] = 0$$

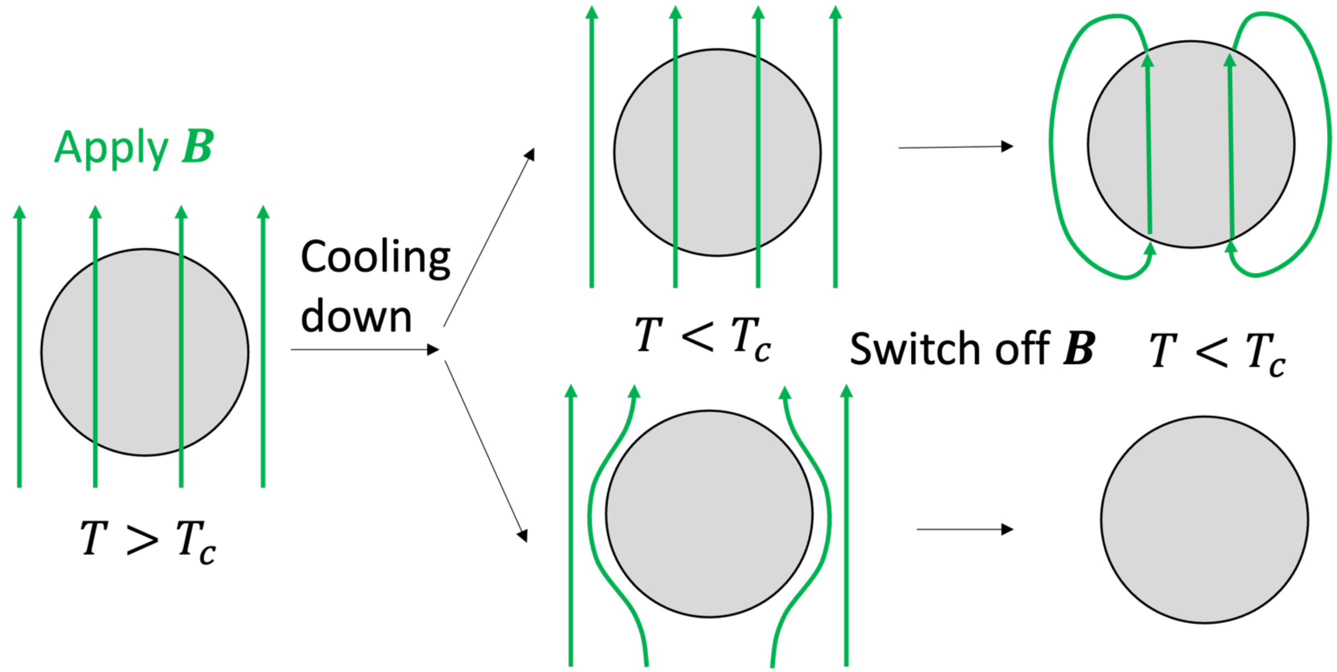
$$\lambda_L^2 \equiv \frac{m^*}{n_s e^2 \mu_0}$$



Constant of time

→ Initial condition before phase transition $T > T_c$ must be preserved

Meissner effect **differentiates them**



Perfect Electric Conductor

$$\nabla^2\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) - \frac{1}{\lambda_L^2}\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) = 0$$

Preserve initial condition!

Superconductor

Zero field!

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

London equation

Additional constraint (broken Gauge symmetry)

Superconductivity is a thermodynamical state which expels magnetic fields and cannot be explained by classical electrodynamics → beyond the scope of this lecture ☹

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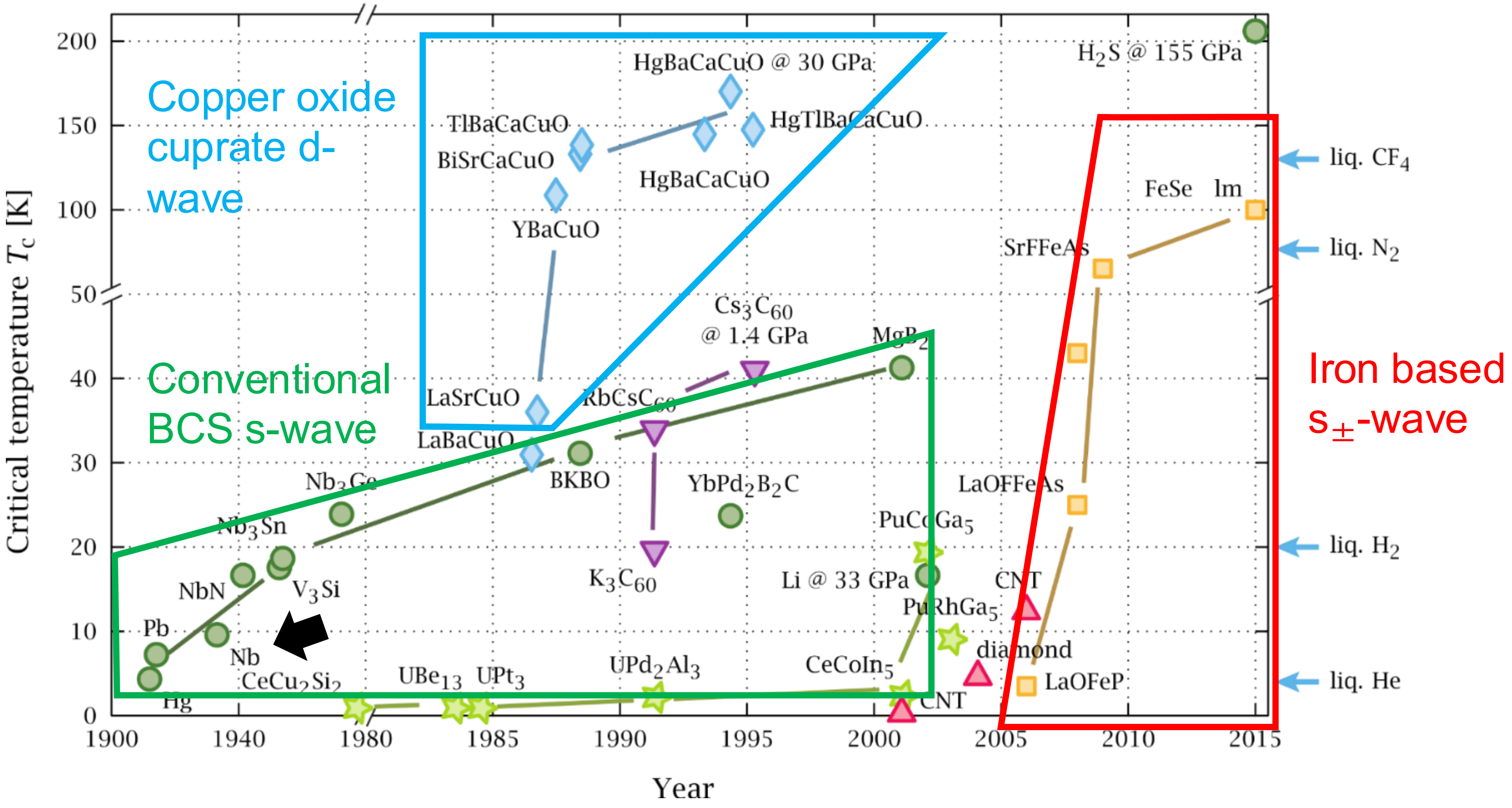
BCS and non BCS superconductors



State of the art of BCS/non-BCS superconductors

- BCS superconductors (1911)
  - Example: Hg, Al, Sn, Pb, Nb, NbN, NbTi, Nb_3Sn, …, MgB_2 (?)
  - Phonon mediated Cooper pairs
  - Application: superconducting magnet, cavities, detectors, qubit
- Cooper oxide superconductors (discovered in 1986)
  - Example: YBCO, …
  - Fundamental theory unknown (Coulomb repulsive force generates correlation ?)
  - Application: magnet, current lead, beam screen, dark matter detector, etc
- Iron based superconductors (discovered in 2008)
  - Example: Ba-122, FeSeTe, …
  - Fundamental theory unknown (Spin internation generates correlation ?)
  - Application: magnet, current lead, dark matter detector, cavities (?), etc
- More fancy superconductors…

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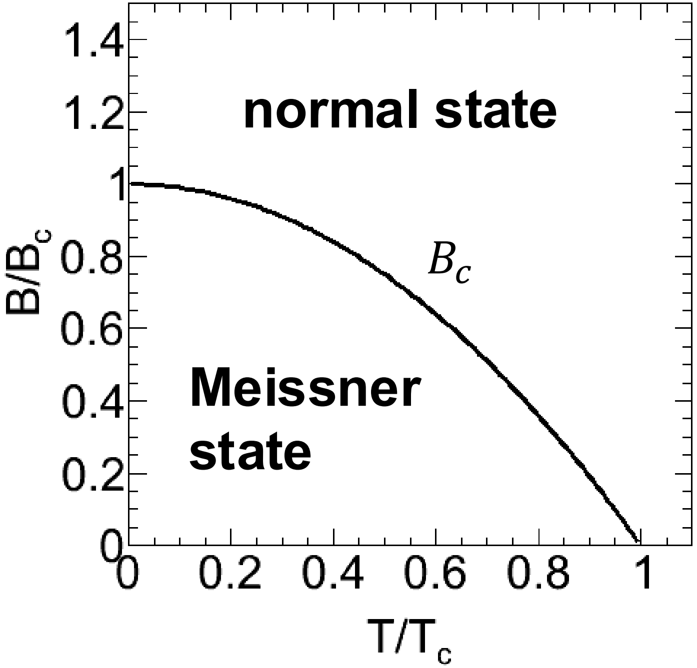
Strong but *static* magnetic field: Type-I vs Type-II

Type-I

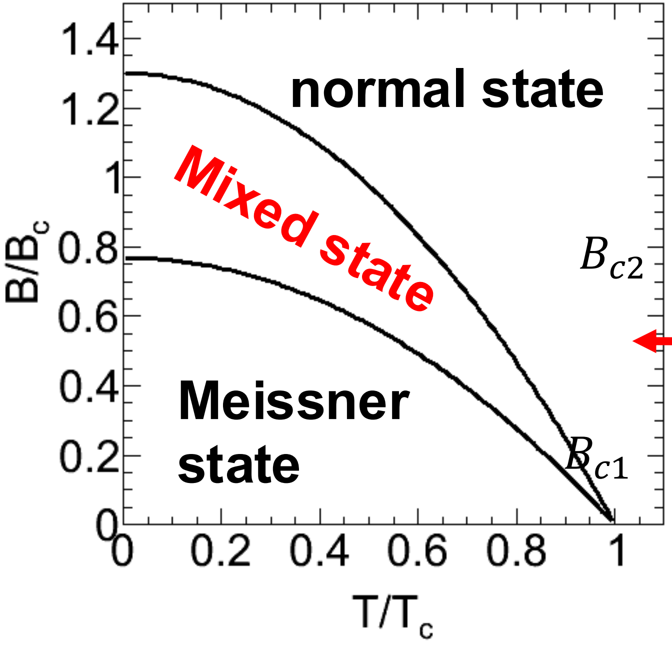
$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Pb} \sim \frac{28\text{ nm}}{71\text{ nm}} \sim 0.40$$

Type-2

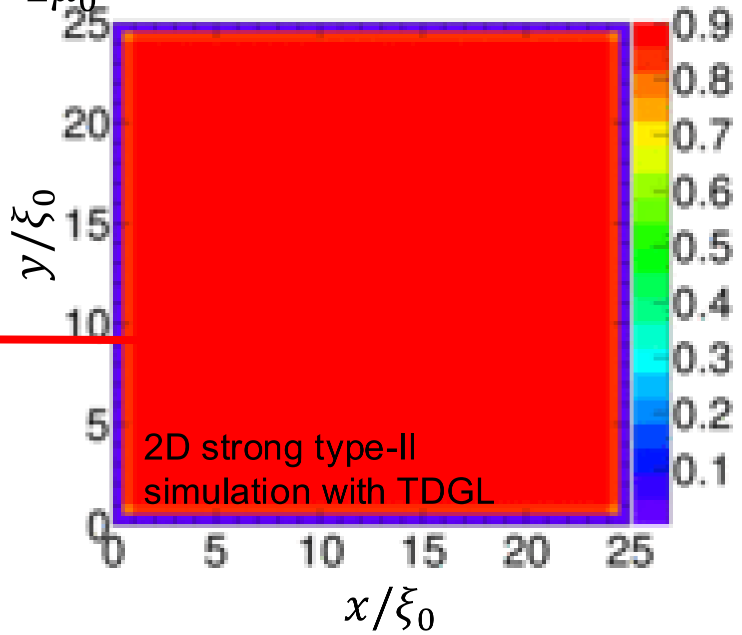
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Nb} \sim \frac{36\text{ nm}}{39\text{ nm}} \sim 0.92$$

Stabilized by NC/SC boundary energy

$$\frac{1}{2\mu_0}(\xi_0 B_c^2 - \lambda_L B^2) < 0 \text{ for } B > B_{c1}$$



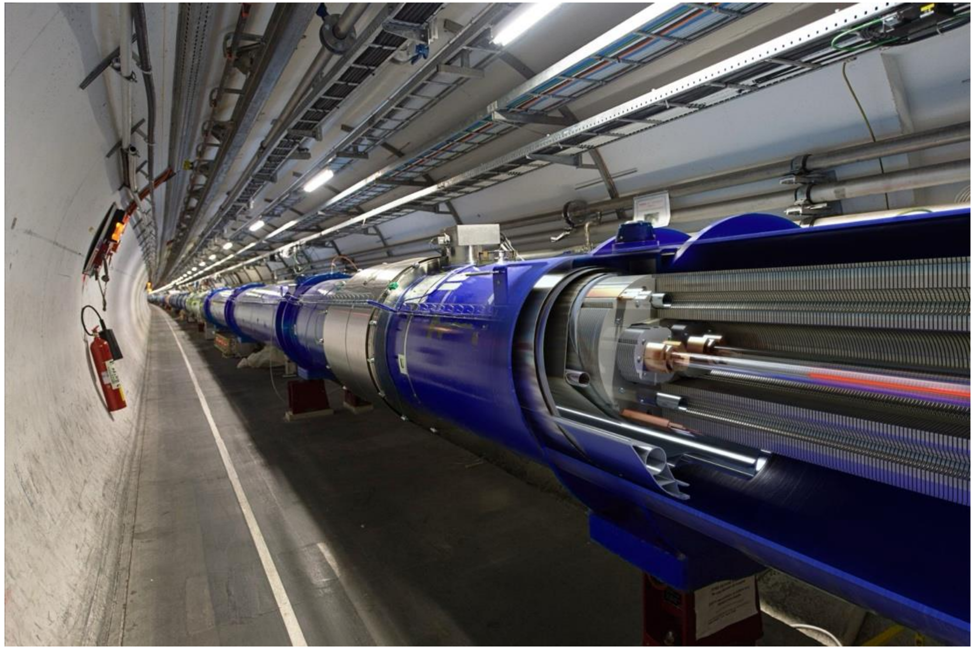
Without pinning centers, type-II ***traps*** magnetic flux if $B > B_{c1}$

Quantized flux $\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}$ Wb

SC magnet and cavities in different states

DC Magnetic fields in magnets to bend the trajectory (Mixed state)

RF Electric fields in cavities to accelerate the beam (Meissner state)

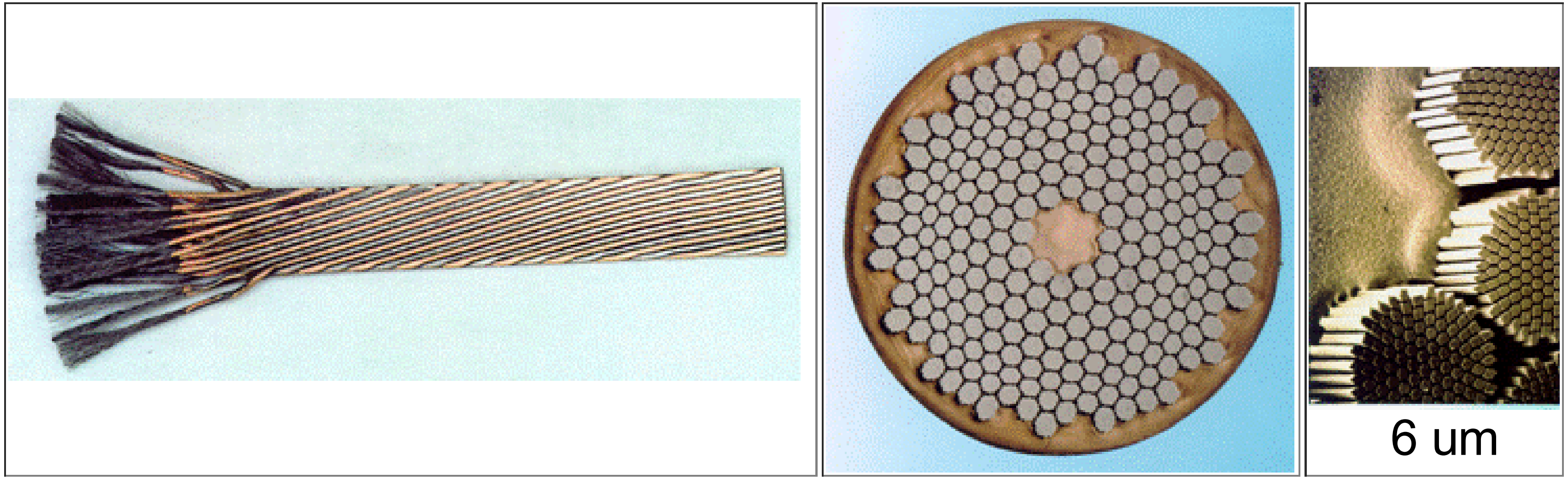


DC vs RF is qualitatively different in superconducting devices

SC magnet cables: made of filaments

Rutherford cable

NbTi filaments embedded in copper matrix for heat and current → LHC magnets



Cable | Strand | Filaments

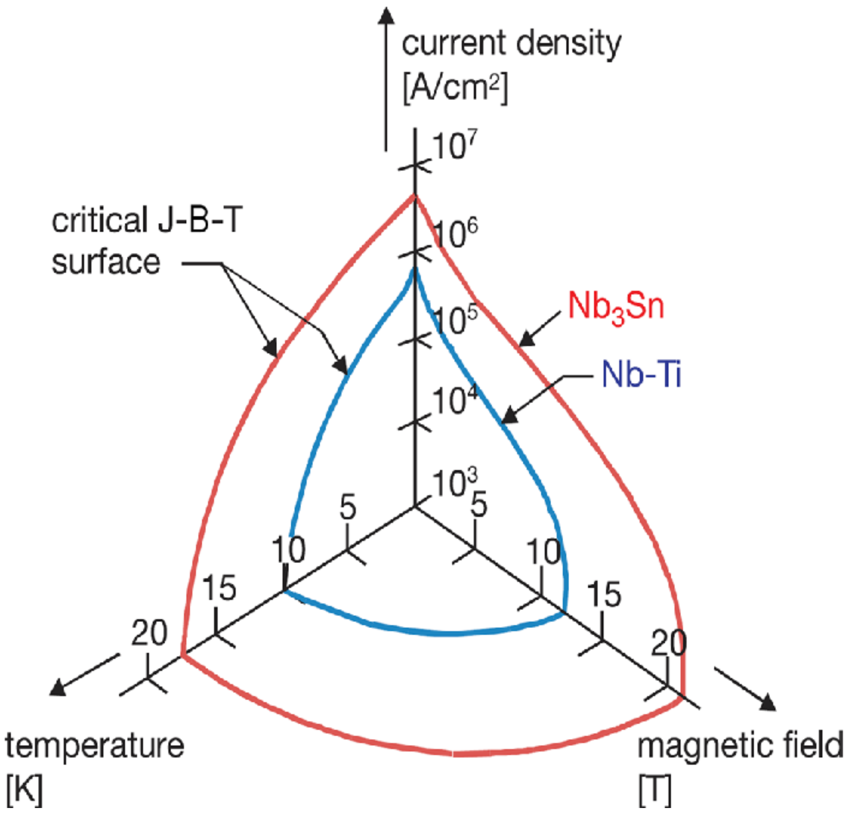
0.825 mm

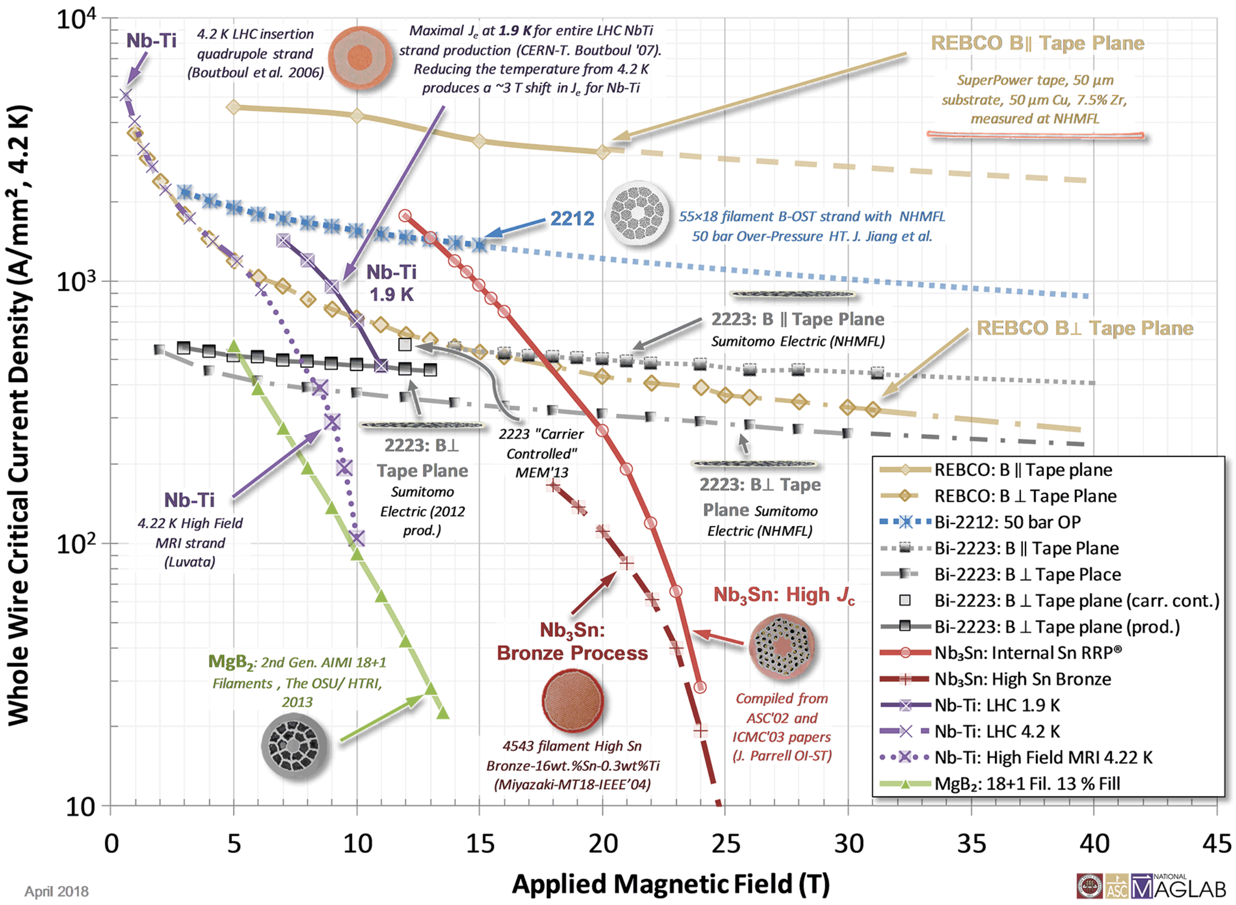
In LHC :
1200 magnets
7600 km of cable
36 strands per cable
6400 filaments per strands

→ Total length of filaments 10 times the distance between earth and sun !!!

The NbTi materials is exposed to 8.3 T >> Hc1 at 1.9 K → Mixed state

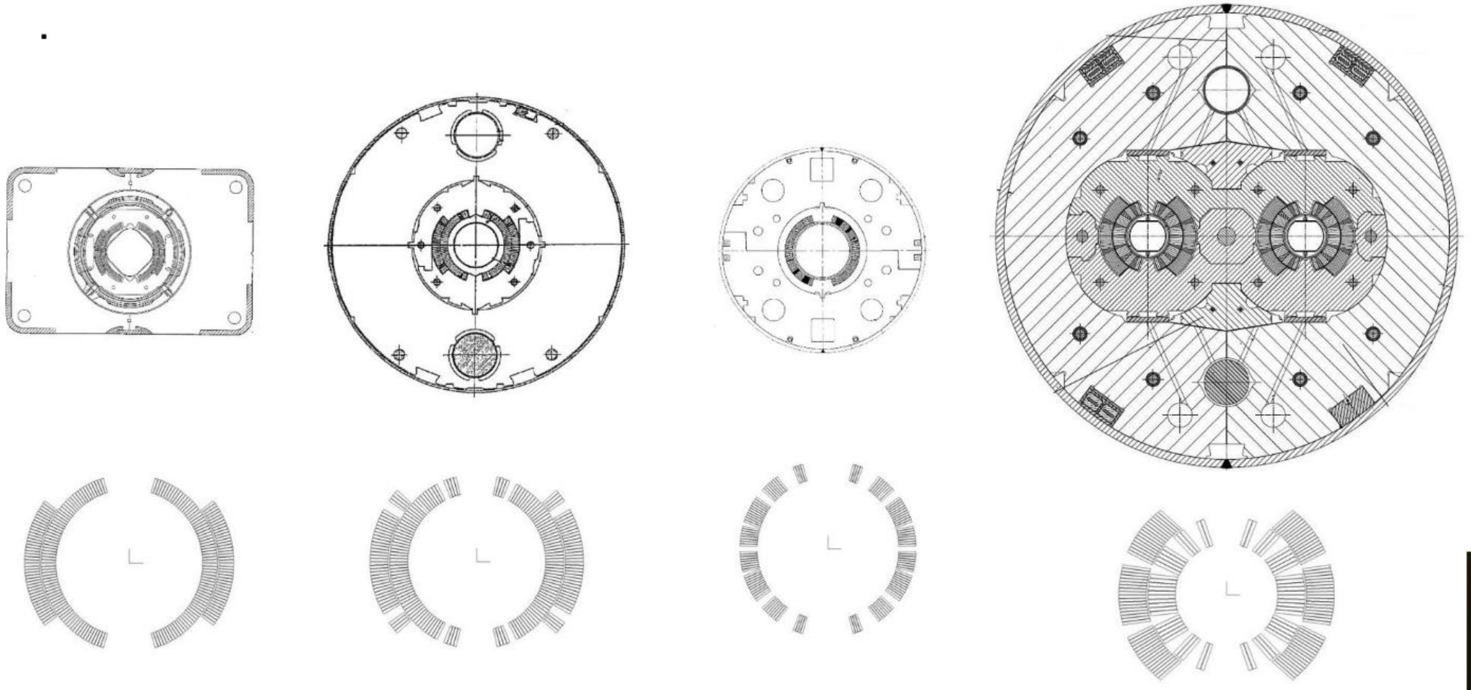
Critical current density





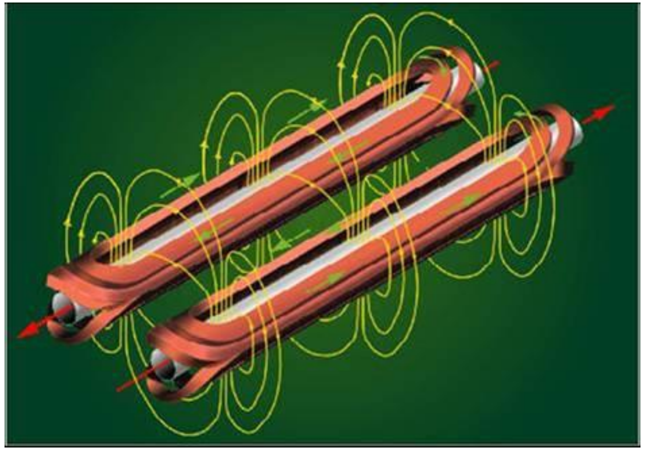
M. E. Ladd et al https://doi.org/10.1007/s10334-023-01085-z

SC dipole: history of NbTi

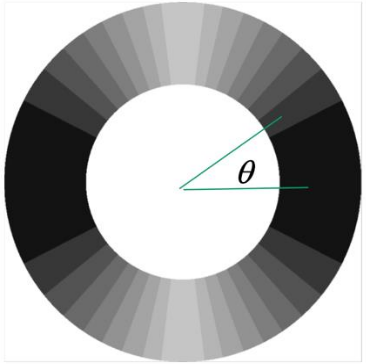


Tevatron	HERA	RHIC	LHC
76 mm bore	75 mm bore	80 mm bore	56 mm bore
B = 4.4 T	B = 5.0 T	B = 3.5 T	B = 8.34 T
T = 4.2 K	T = 4.5 K	T = 4.3-4.6 K	T = 1.9 K
first beam 1983	first beam 1991	first beam 2000	first beam 2008

Courtesy G. d. Rijk and P. Ferracin, CERN



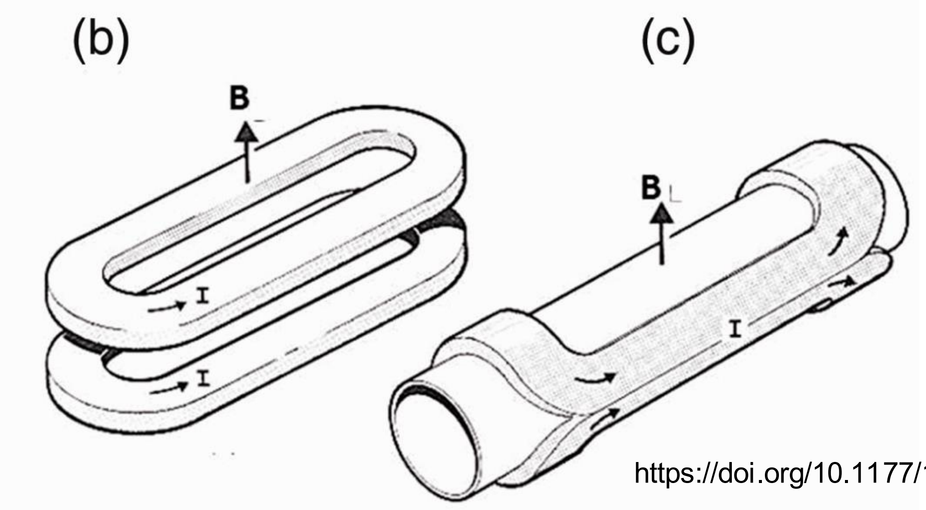
$$J = J_0 \cos\theta$$



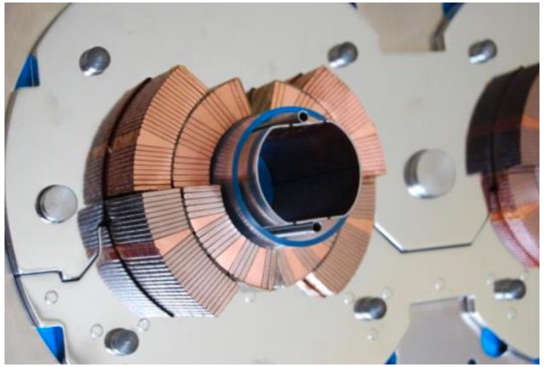
$\cos\theta$ current distribution gives pure dipole field

Different dipoles

Baseline: $\cos\theta$ (racetrack or saddle)



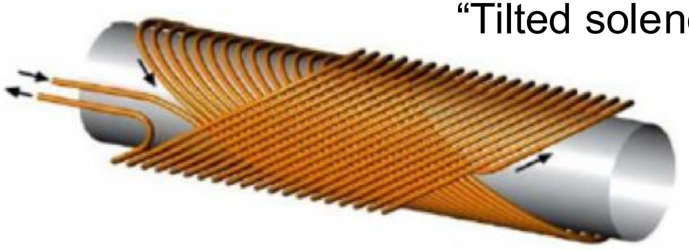
https://doi.org/10.1177/15280837166612



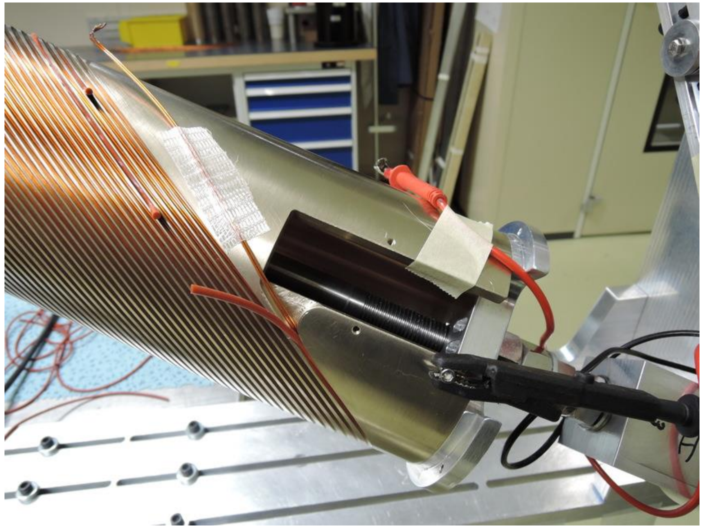
Courtesy G. d. Rij, M. Wilson,

Canted $\text{Cos}\,\theta$ (CCT)

“Tilted solenoid”



Proposed in 1960s realized recently



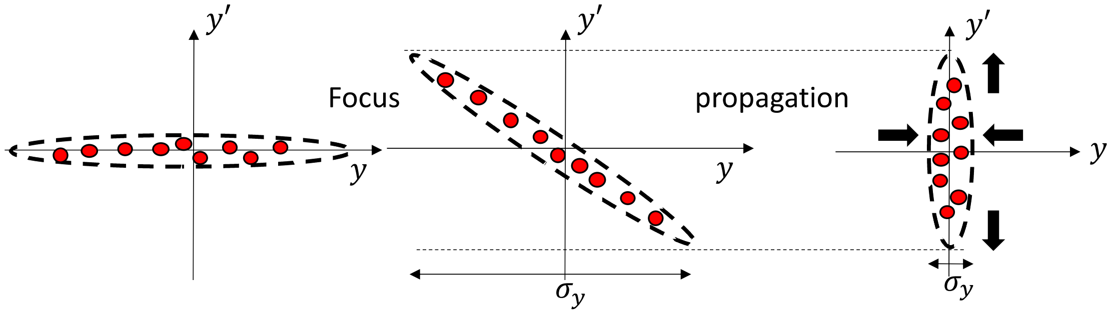
https://home.cern/news/news/engineering/new-life-old-technology-canted-cosine-theta-magnets

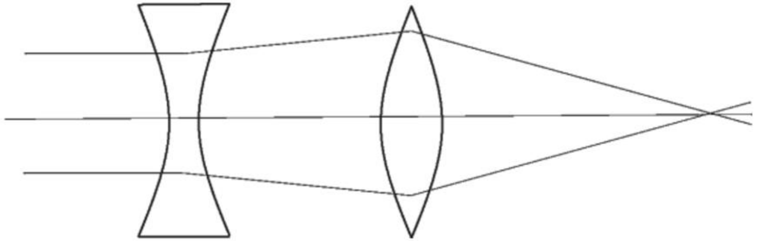
SC quadrupoles

Lorentz force is not only for bending trajectory

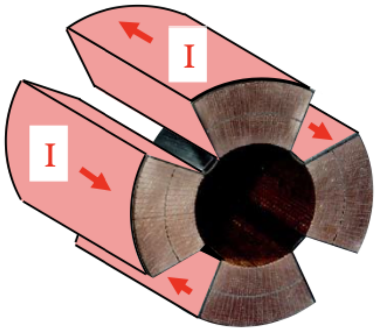
$$\frac{d(mv_\theta)}{dt}.\overrightarrow{u_\theta} - m\frac{v_\theta^2}{\rho}.\overrightarrow{u_r} = eE_\theta.\overrightarrow{u_\theta} + ev_\theta B_z.\overrightarrow{u_r}$$

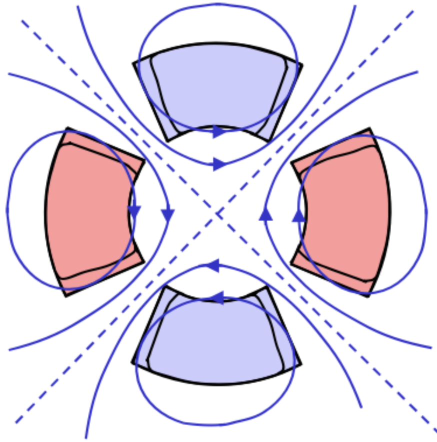
Focusing and defocusing of a group of charged particles (bunch) are crucial for beam transport and luminosity at collision

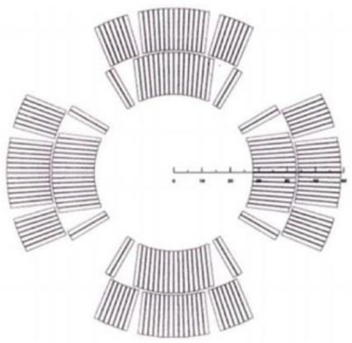




Implemented by quadrupole magnets







Courtesy G. d. Rij

SC vs NC magnets: typical and exception

$$p[\text{GeV}/c] = 0.3 \times \rho[\text{m}] \times B[\text{T}]$$

SC magnet

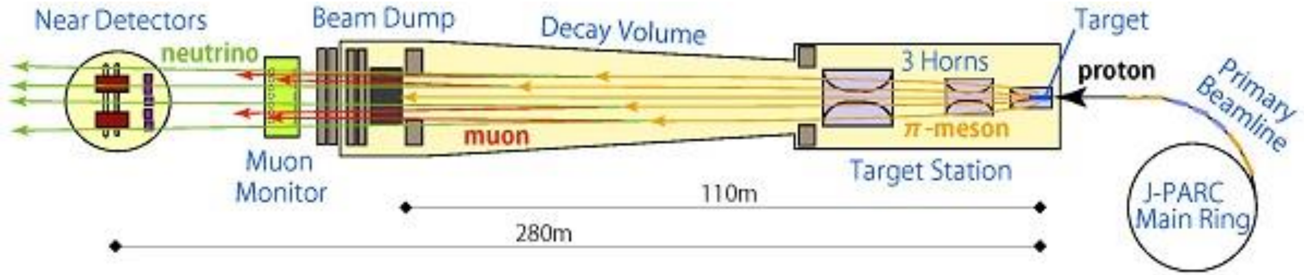
- Strong field (>0.2 T) steady state
- Typically proton circular machine
- Ex) 7 TeV & 8.3 T at LHC

NC magnet

- Weak field (<0.2 T) steady state
- Typically electron circular machine
- Ex) 90 GeV & 0.13 T at LEP

Short pulsed NC magnet

https://j-parc.jp/Neutrino/en/nu-facility.html



- Separate π^+ and π^- for neutrino and anti-neutrino
  - Too high radiation → SC magnet may not survive
  - Coincidence time window with neutrino detector
- Strong field (2 T) for short pulse 5 μs (every 2-3 seconds)

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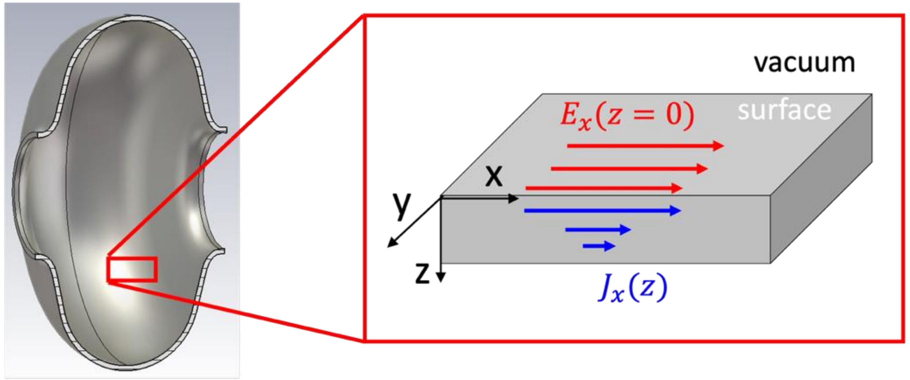
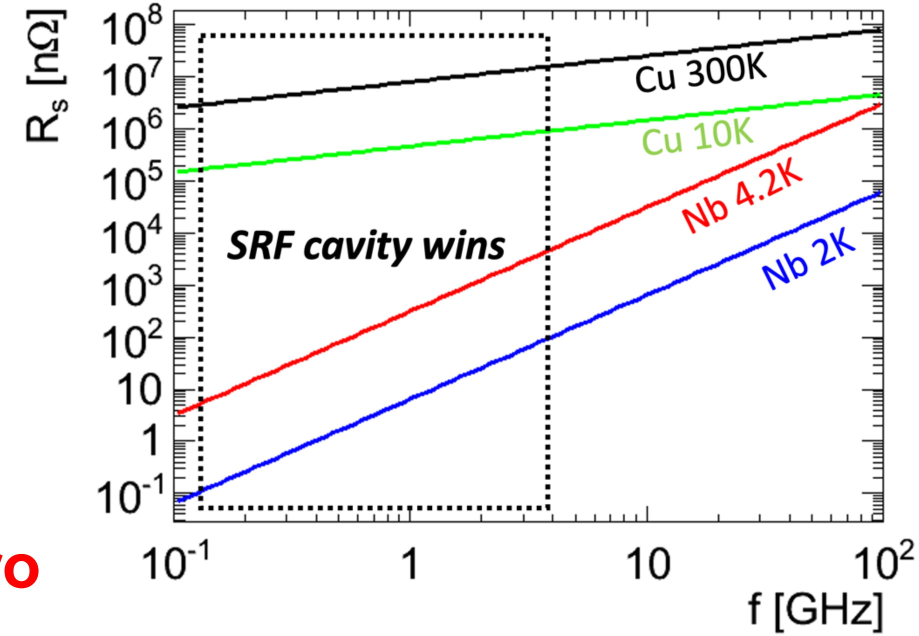


Local surface resistance

$$R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right)$$

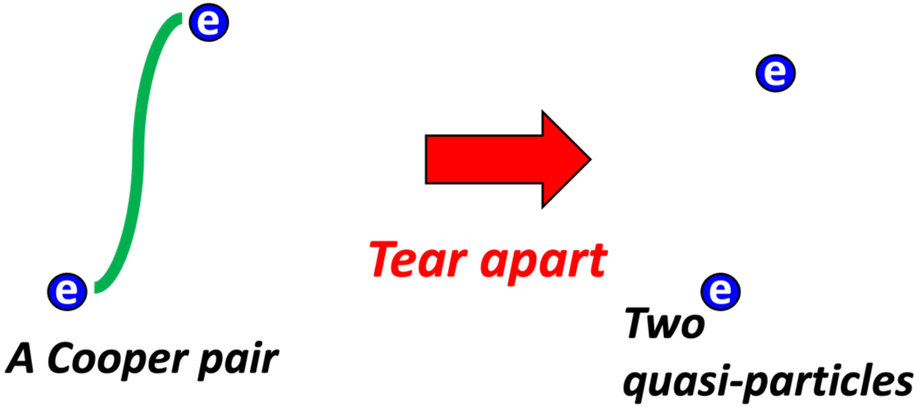
Normal conducting (Cu) $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} \propto f^{1/2}$

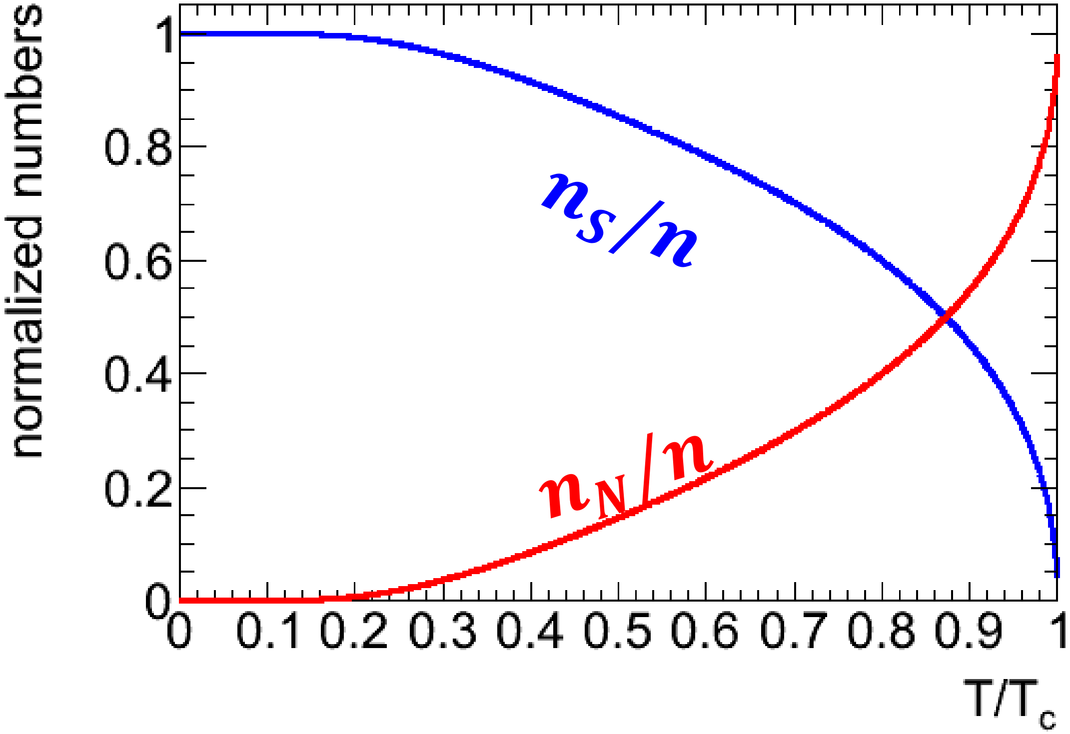
Super- conducting (Nb) $R_s = \frac{Af^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) \propto f^2$



Superconducting R_s is small but non zero

Thermal excitation of quasi-particles





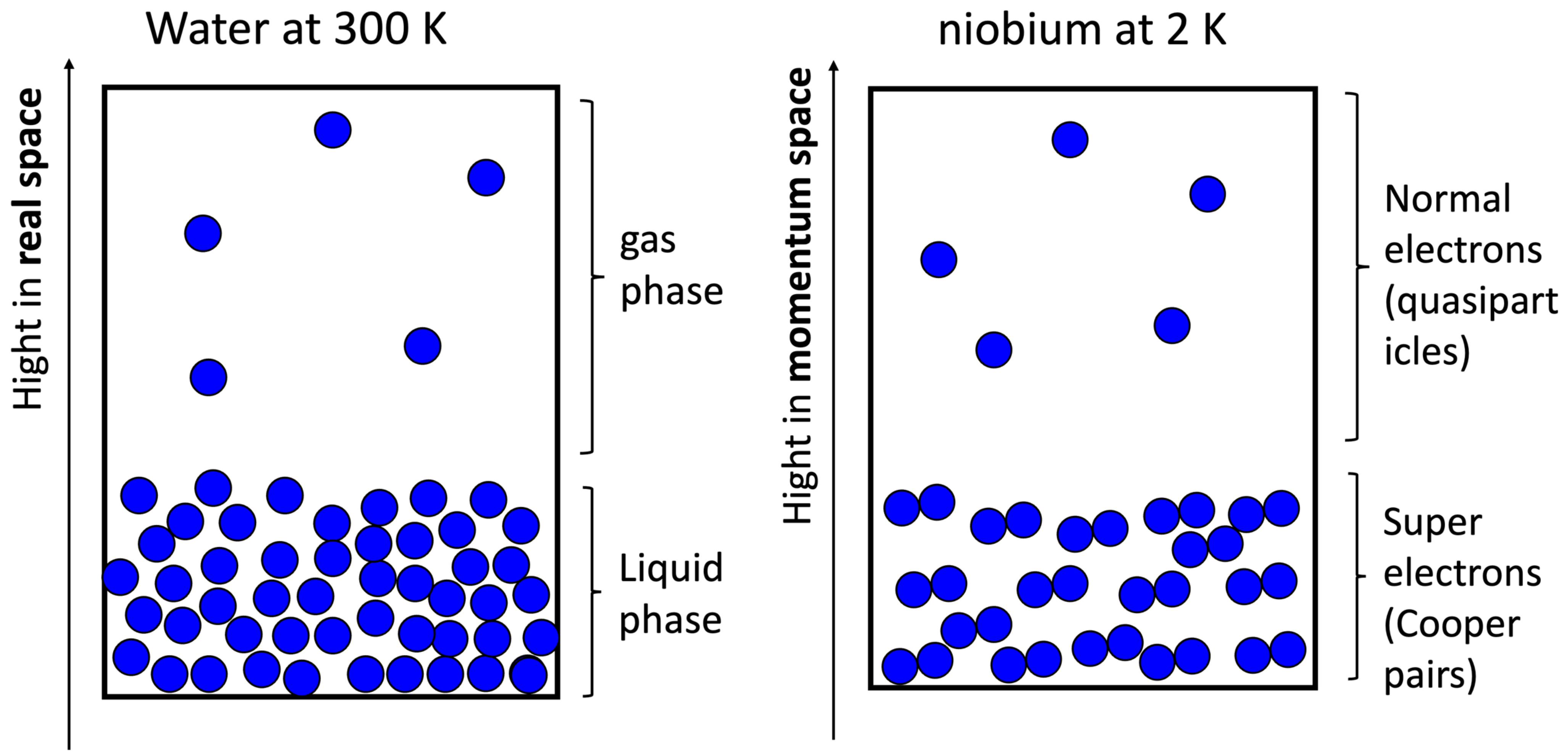
At finite temperature $0 < T < T_c$, these two states are ***in thermal equilibrium***

# of quasiparticles: $n_N \sim \exp\left(-\frac{\Delta}{k_B T}\right)$

# of electrons in Cooper pairs: $n_S \sim n - n_N$

Quasi-particles (~normal conducting electrons) still exist if $T > 0$

Coexistence of Cooper pairs and quasi-particles



Supercurrent

$$\frac{\partial j_s}{\partial t} - \frac{n_S e^2}{m^*} E = 0$$

$$j_s = j_0 \exp(i\omega t)$$

$$j_s = -i \frac{n_S e^2}{m^* \omega} E$$

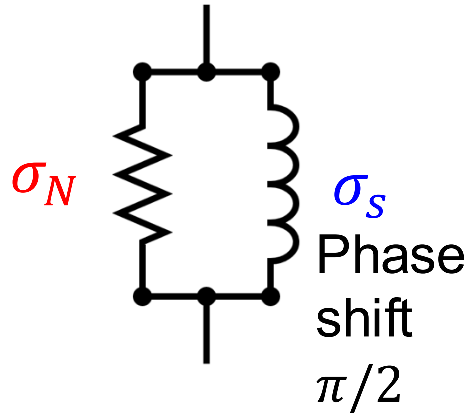
$\equiv \sigma_S$

Normal current

Ohm's law →

$$j_N = \frac{n_N e^2 \tau}{m^*} E$$

$\equiv \sigma_N$

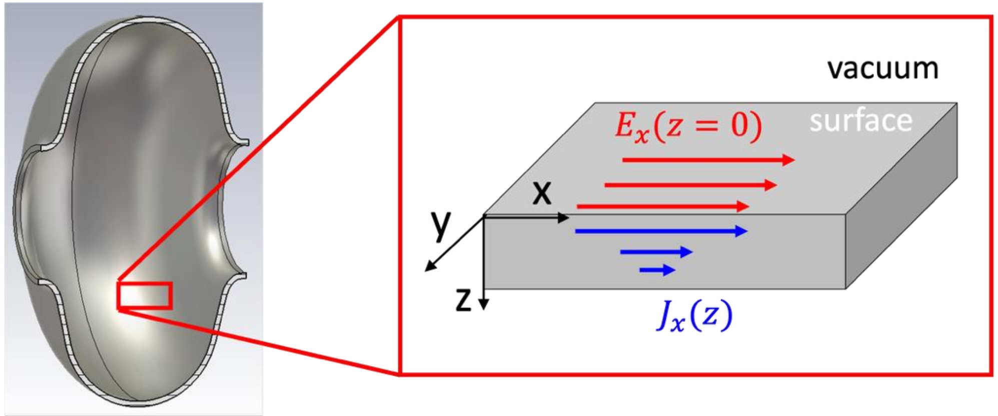


Total current induced by RF

$$j = j_S + j_N \quad \rightarrow \quad j = (\sigma_N - i\sigma_S)E$$

Dissipation by quasi-particles →resistive

Inertia of Cooper pairs → inductive



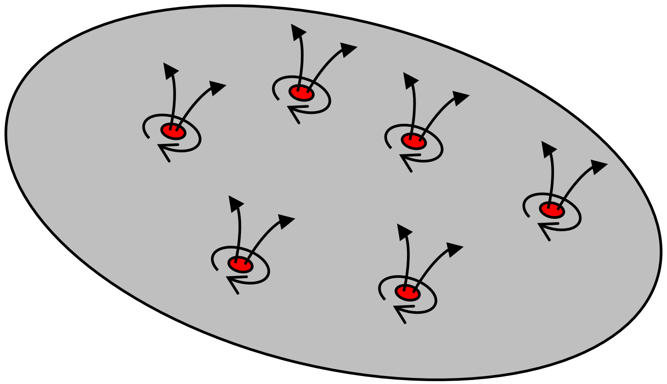
$$\sigma_N = \frac{e^2 n_N \tau}{m^*} \propto n_N \propto \exp\left(-\frac{\Delta}{k_B \boldsymbol{T}}\right)$$

$$\begin{cases} j_x = (\sigma_N - i\sigma_S)E_x \\ E_x(z) = E_0 \exp(-z/\lambda_L) \end{cases} \rightarrow R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right) \sim \frac{1}{2}\frac{\sigma_N}{\sigma_S}\sqrt{\frac{\omega\mu_0}{\sigma_S}} = \frac{\mu_0^2}{2}\lambda_L^3\sigma_N\omega^2 > 0$$

<u>Lessons</u>

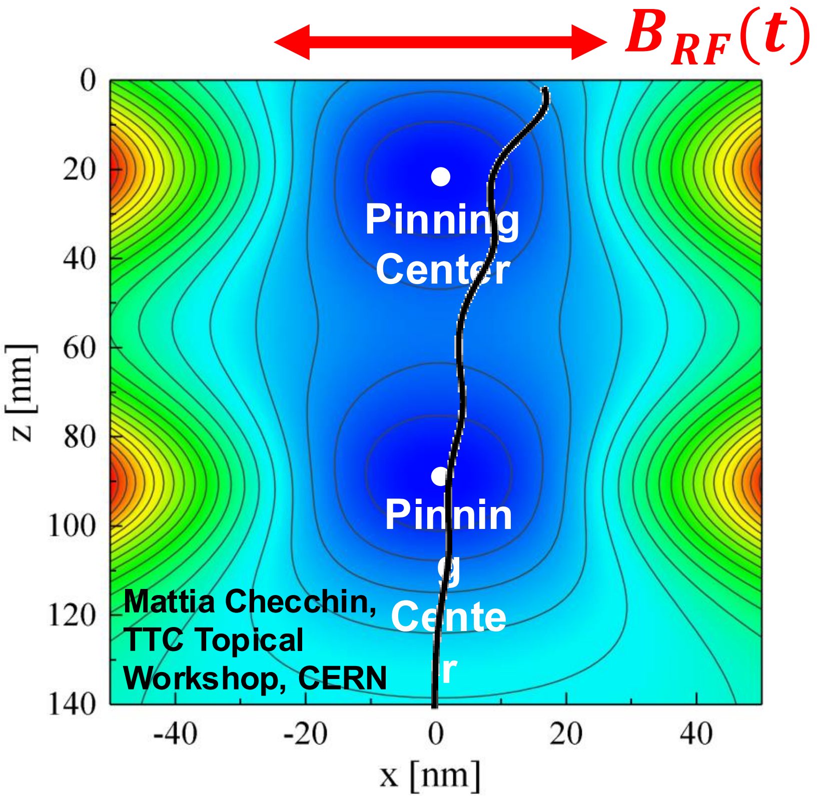
- One origin of the finite R_s of superconductors is quasi-particles
- Quasi-particles are thermally activated from Cooper pairs at $0 < T < T_c$
- R_s exponentially decreases by lower T because quasi-particles are frozen out
- Higher RF frequency increases $R_s \sim \omega^2$
- Order of magnitude is 10 nΩ

Static trapped magnetic field in SC cavities



$$R_{\mathrm{mag}} \sim N \times \pi\xi_0^2 \times R_{\mathrm{n}} \sim \frac{B_{\mathrm{ext}}}{2B_{c2}} R_{\mathrm{n}}$$

- N: Flux number density
- $\pi\xi_0^2$: Normal conducting area
- R_{n}: Normal conducting



Earth field B_{ext}=50uT
B_{c2}~400 mT (Nb)
R_n~1.3 mΩ at 1.3 GHz (Nb)

$$R_{\mathrm{mag}} \sim 80\mathrm{n}\Omega > R_{BCS}(2K) \sim 10\mathrm{n}\Omega$$

An excellent magnetic shield is mandatory for SC cavities

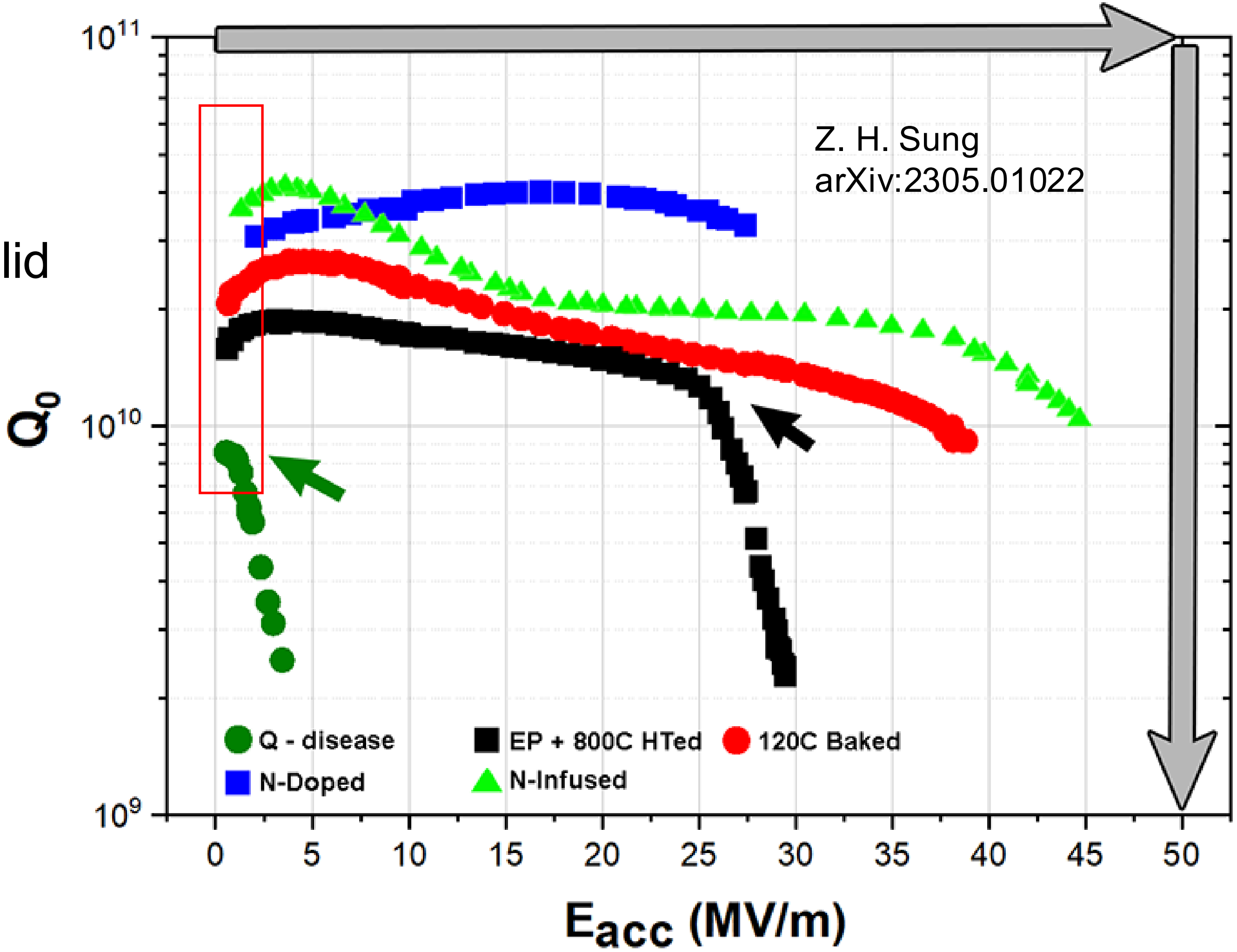
Quality factor vs accelerating gradient

$$Q_0 \equiv \frac{\omega U}{P_c} = \frac{1}{R_s} \boxed{\omega\mu \frac{\int_V H^2 dV}{\int H^2 dS}}$$

geometrical

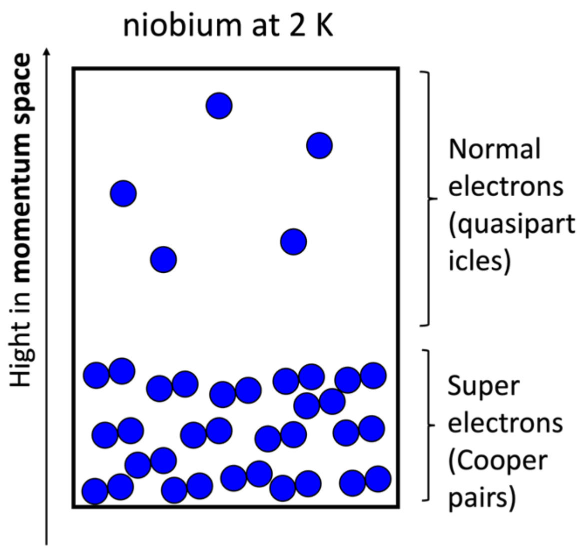
- The established theory in only valid for low external RF field
- The simplest nonequilibrium problem is solved in the linear response theory
- Very tiny difference in surface impacts the results (<10 nΩ physics!)
- We do not have complete theory that explains the nonlinear phenomena in Q vs E

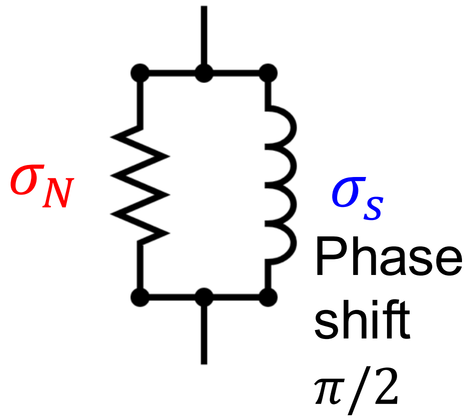
→ Open research field!



Comparison: magnet vs cavity

- Magnets are operated in strong DC fields
  - The two-fluid circuit is short → no loss
  - Finite loss during ramping up and down the field
  - Magnetic fields are trapped in Mixed state
    - Strong pinning force is manipulated to avoid flux flow (otherwise, loss)
- Cavities are operated in RF fields
  - The phase shift due to Cooper pairs' inertia causes finite loss in quasiparticles
  - Oscillating trapped flux kills the cavity performance
  - Operation in as clean as the Meissner state
    - RF field itself does not penetrate due to superheating effect and surface barrier





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Conclusion: ***Superconductivity and Cryogenics for Particle Accelerators***

- Cooling down improves loss but becomes less efficient
    - Superconducting vs normal conducting under the ideal Carnot cycle
    - Carnot cycle is NOT realistic at all: trade off in efficiency and power evacuation
- Superconducting devices are typically operated inside liquid helium
    - Cryogenics is enabled by physics and engineering of thermodynamics
- Superconductivity is macroscopic quantum phenomenon
    - Meissner effect is the most characteristic phenomenon (spontaneously broken Gauge symmetry in the ground stage)
    - Zero DC loss is caused by Cooper pairs
    - Classical electrodynamics can give us some insight with limitation
- Several different superconducting mechanisms are known
    - Only BCS-type SCs are well understood (phonon-mediated correlation) and Copper oxide & iron-based SC are still under devate
- SC magnet are operated in the Mixed state of type-II superconductors
- SC cavities are operated in the Meissner state of type-II superconductors
    - Non-equibrilium phenomenon is the remaining challenge of BCS-type SCs
    - Two-fluid model gives us some insight with limitations

Conclusion: *rich physics in the research field of accelerators*