Gauge anomalies on shell and collinear factorization

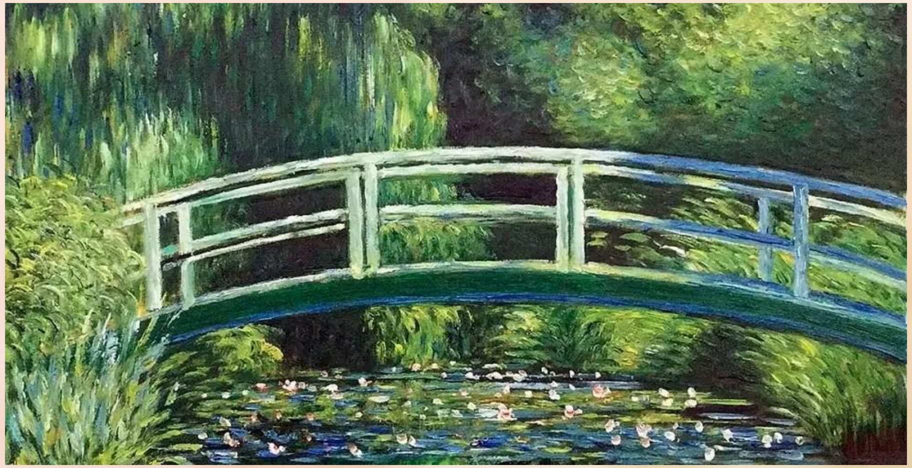
Edoardo Alviani, IJCLab – Université Paris-Saclay

alviani@ijclab.in2p3.fr

Based on work 2509.03368 in collaboration with Adam Falkowski

Soon!

A river can be crossed with a bridge or a boat.



A storm calls for

Resistance to wind fluctuations

Resistance to water waves fluctuations

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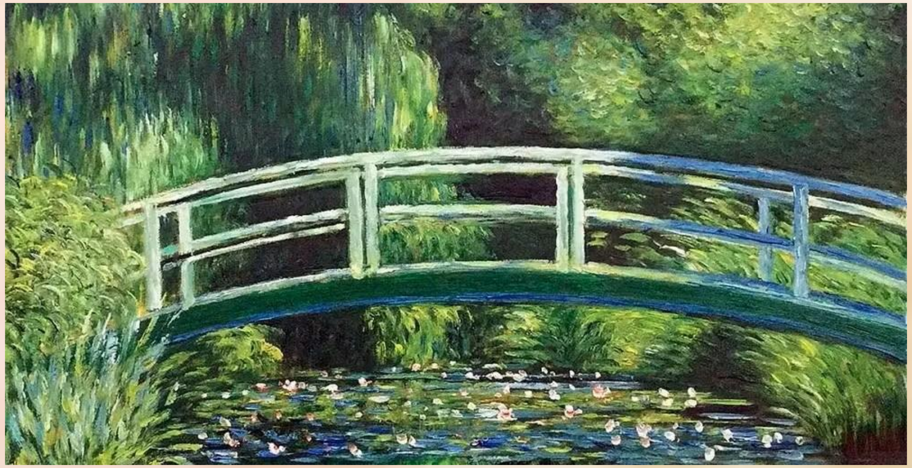

A storm calls for

Resistance to wind fluctuations

Resistance to water waves fluctuations

In the same way any theory has to be **resistant to quantum fluctuations** regardless of the formalism we use to navigate it.

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization
4. Gauge anomalies on shell

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Why???

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Fermion masses and mixing angles from gauge symmetries [Ibanez, Ross]

A Resolution of the Flavor Problem of Two Higgs Doublet Models with an Extra U(1)_H Symmetry for Higgs Flavor

[Ko, Omura, Yu ’12]

Anomaly Holography, the Wess-Zumino-Witten Term, and Electroweak Symmetry Breaking

[Gripaios ’08]

Why???

- Gauge anomalies must cancel when gaugeing flavour symmetries
- Anomaly cancellation constrains viable Higgs sectors

1. Introducing the question: quantum fields

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Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states, gives photon correct number of physical polarizations

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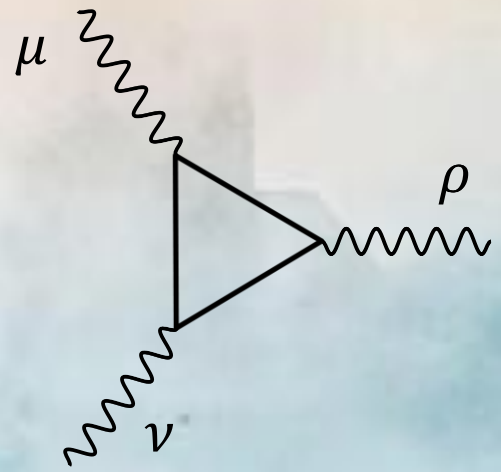

Anomaly: breaking of **a** symmetry upon computing loop correlators.

Not problematic in principle.

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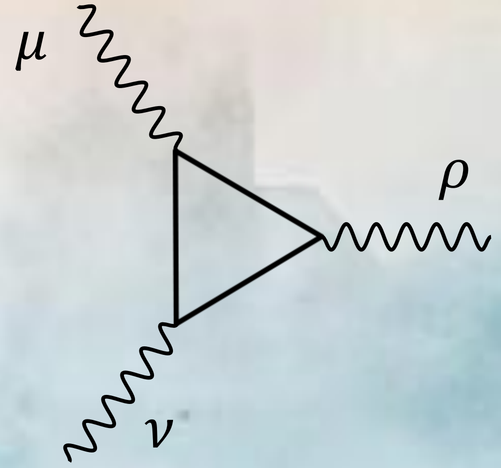
∂_μ  $\neq 0$

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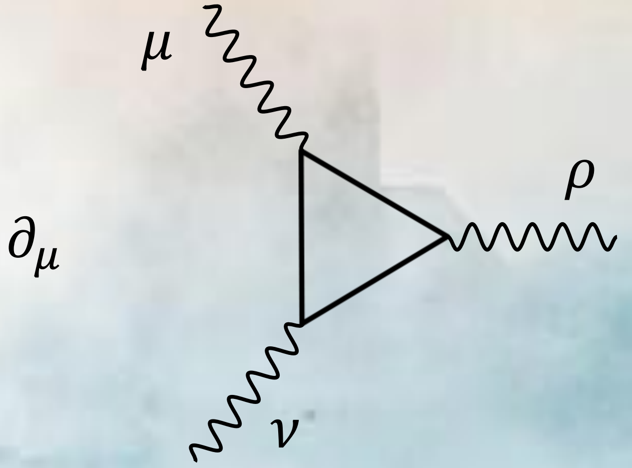

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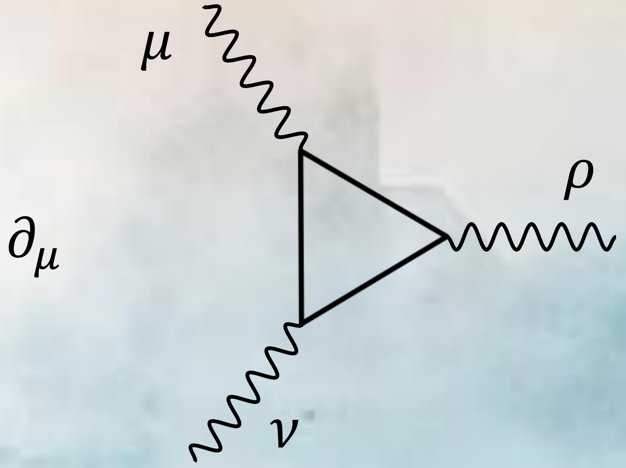

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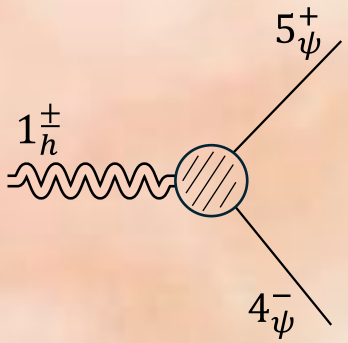
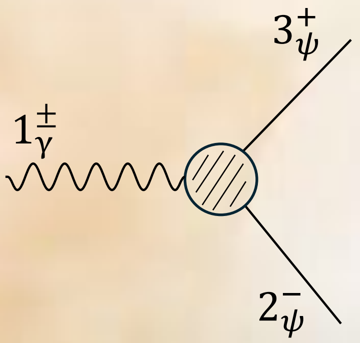
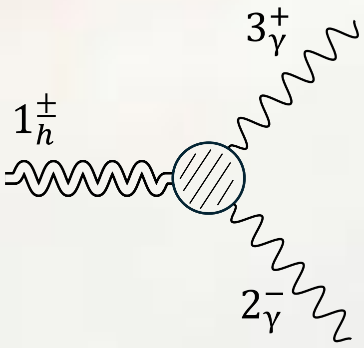
Physical constraints on the spectrum of the theory

1. Introducing the question: on-shell methods

We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.







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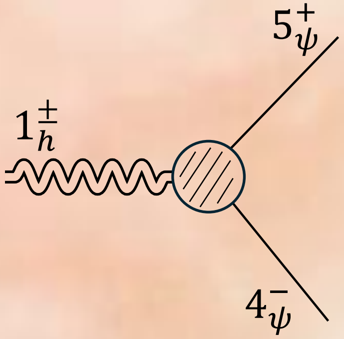
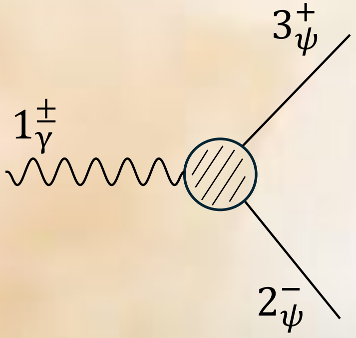
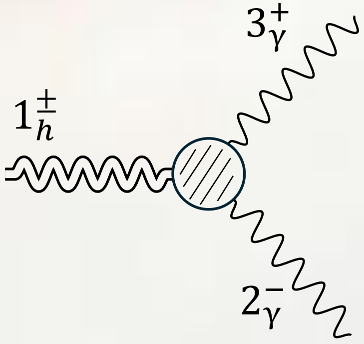
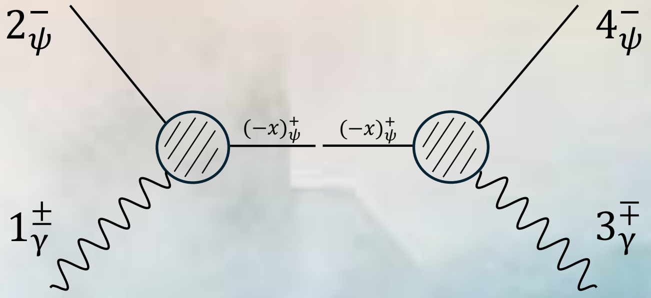
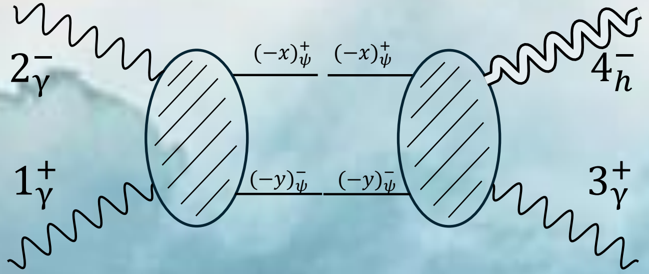






Higher-point and loop-amplitudes can be constructed from a set of three-point ones by unitarity





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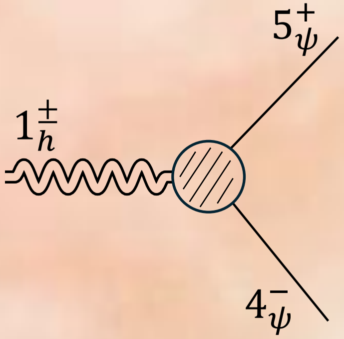

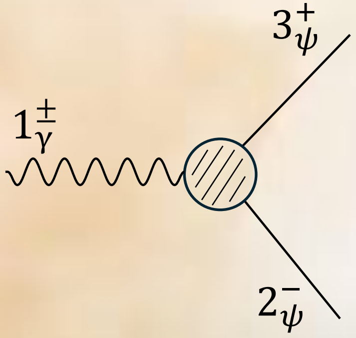

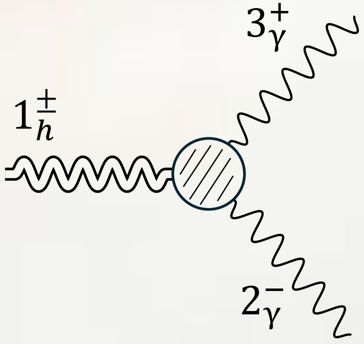

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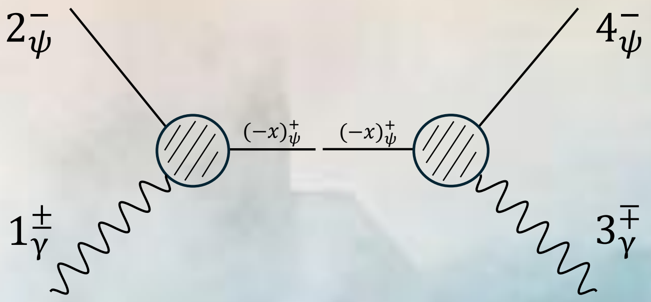

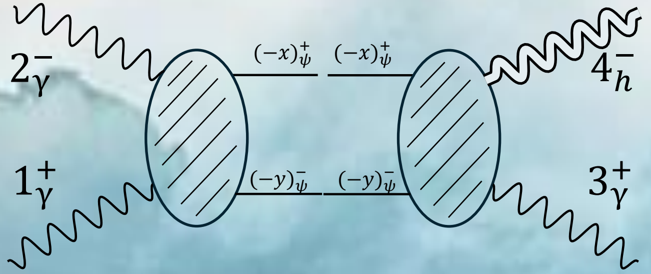

Gauge invariance is bypassed from the get-go

Edoardo Alviani

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum.

On-shell formulation of QFT bypasses gauge invariance.

Where are such constraints from in the on-shell formulation?

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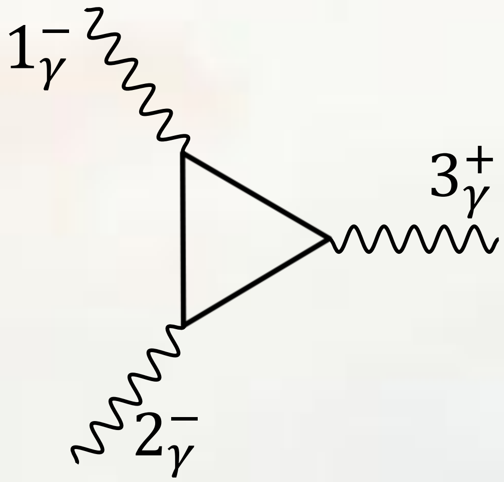
Where are such constraints from in the on-shell formulation?

Answer: **anomalies lead to a breakdown of collinear factorization**

2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.



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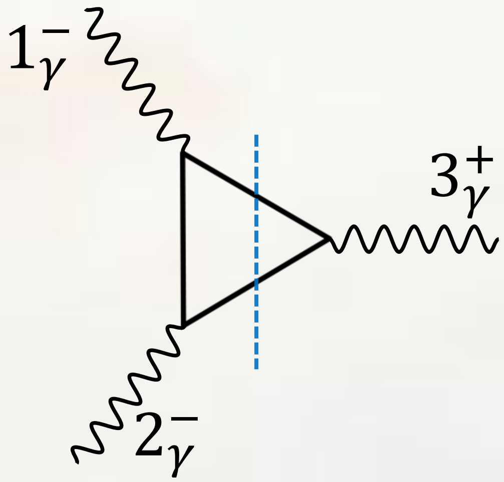

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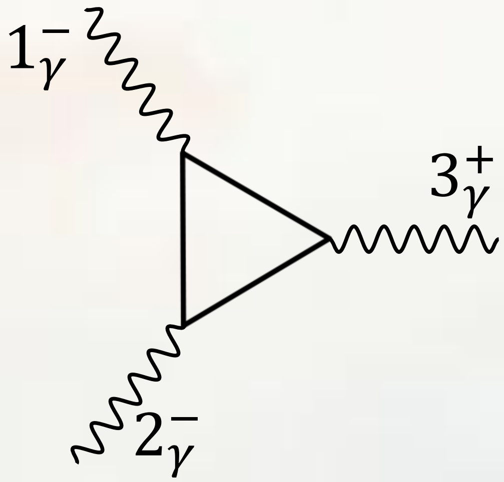

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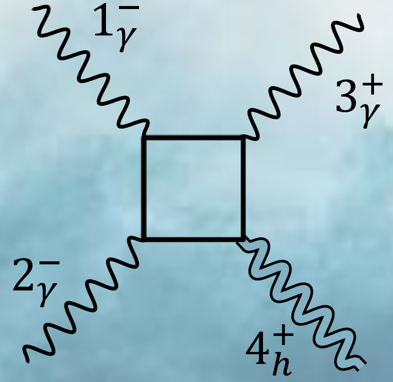

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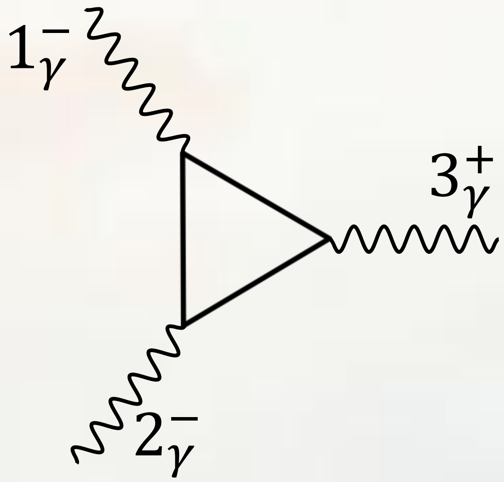

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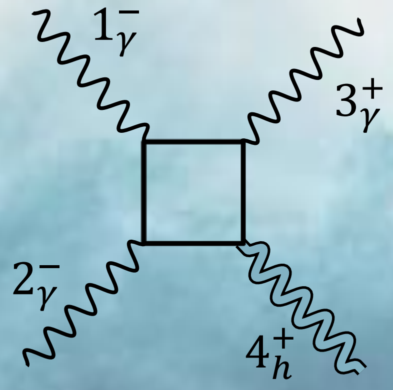
$$\mathcal{M}^{(1)}[1^-_\gamma 2^-_\gamma 3^+_\gamma 4^+_h] = M_{3\gamma h} + R_{3\gamma h},$$

$$M_{3\gamma\mathrm{h}} \equiv -\left[\sum_{\mathrm{n}=1}^{\mathrm{N}} s_n Q_n^3\right] \frac{\sqrt{2}e^3}{16\pi^2 M_{\mathrm{Pl}}} \langle 12\rangle^2 [34]^2 [4|p_1 p_2|4] F_{3\gamma\mathrm{h}}(s,t),$$

$$F_{3\gamma h}(s,t) \equiv \frac{1}{s^3}\left\{4\log\left(\frac{t}{u}\right) + \frac{t-u}{s}\left[\log\left(\frac{t}{u}\right)^2 + \pi^2\right]\right\}\Big|_{u=-s-t}.$$



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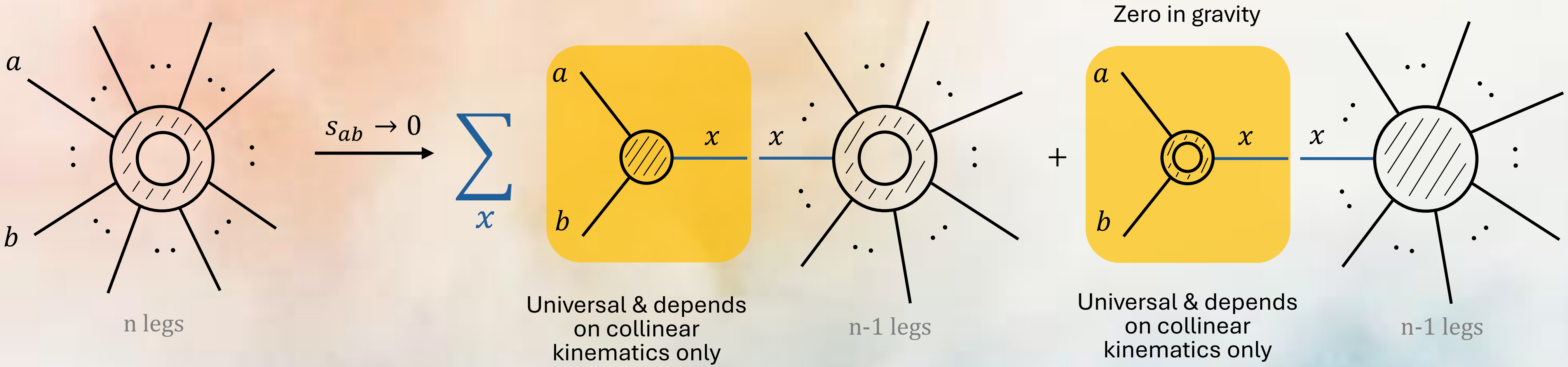
Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as [Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]



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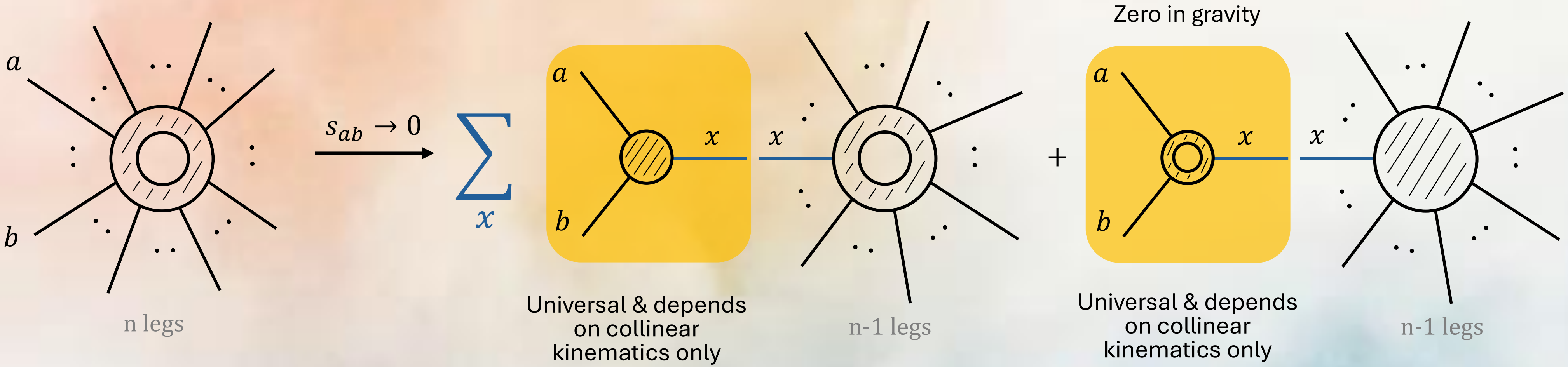

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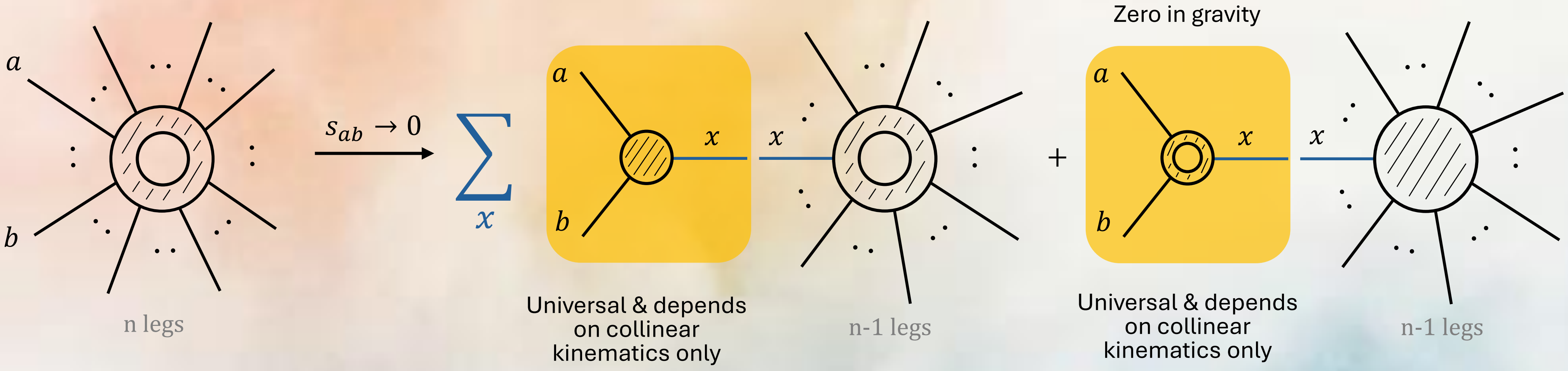


$\sim \frac{1}{\sqrt{s_{ab}}}$ for x photon

$\sim s_{ab}^0$ for x graviton

Crucially, this factorization holds for $n \geq 5$

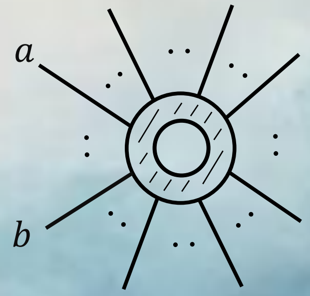
Any deviation from this pattern leads to an inconsistency.

4. Gauge anomalies on-shell

We constructed the following algorithm to detect anomalous theories fully on-shell

Compute $M_n^{(1)}$ using unitarity

Pick suspicious $M_n^{(1)}$: the ones proportional to cancellation conditions (e.g. $\sum_n Q_n^3$)



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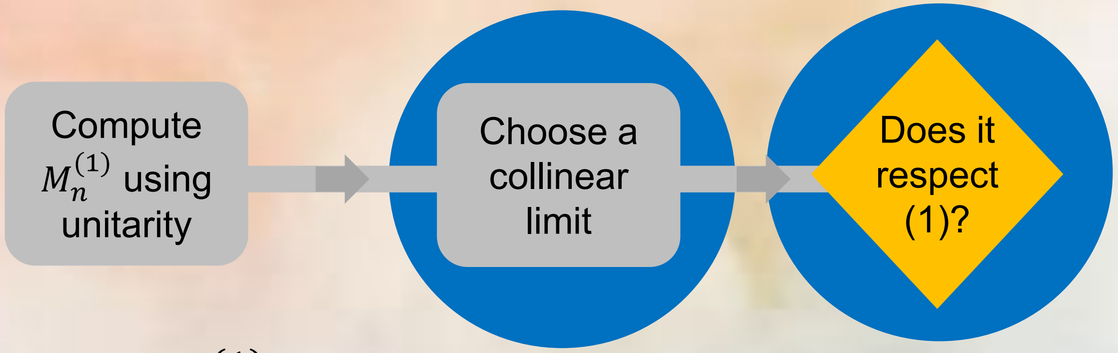

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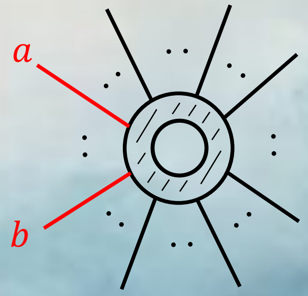

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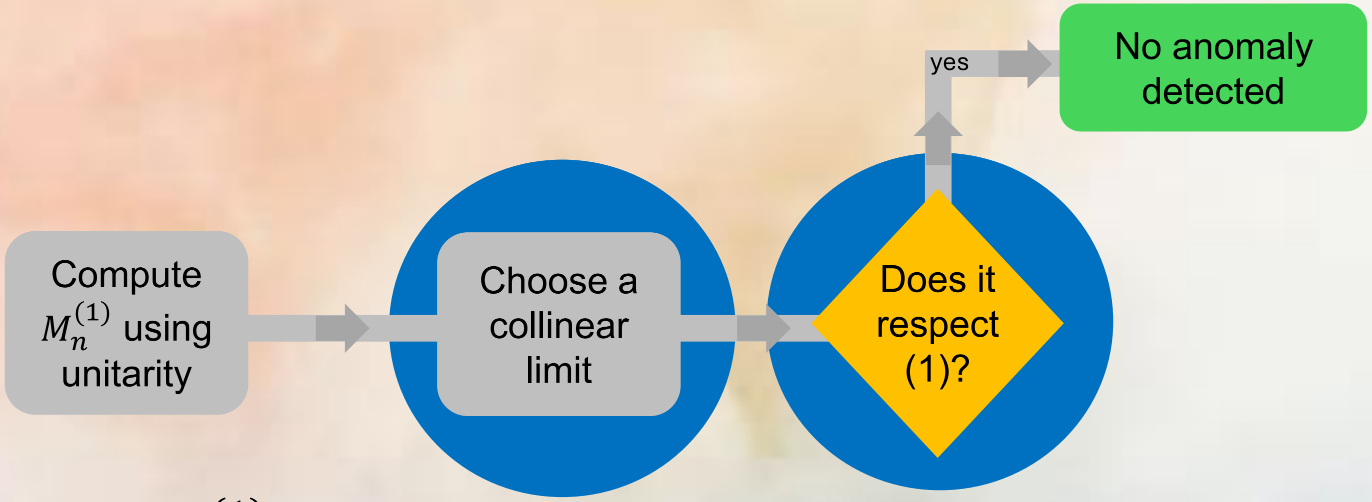

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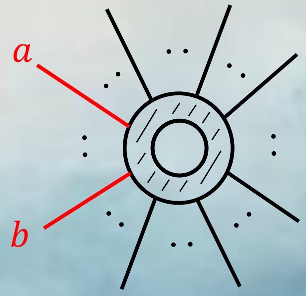

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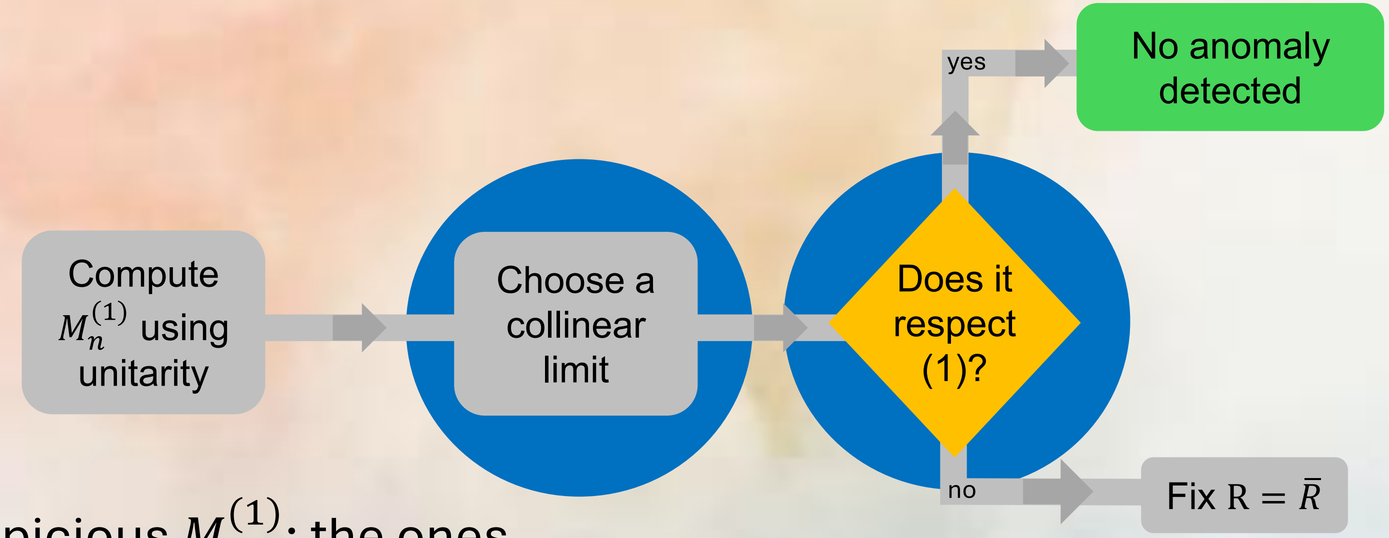

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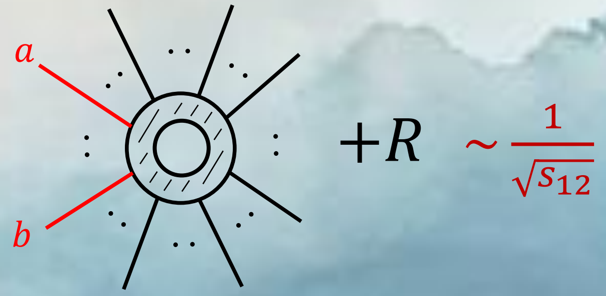

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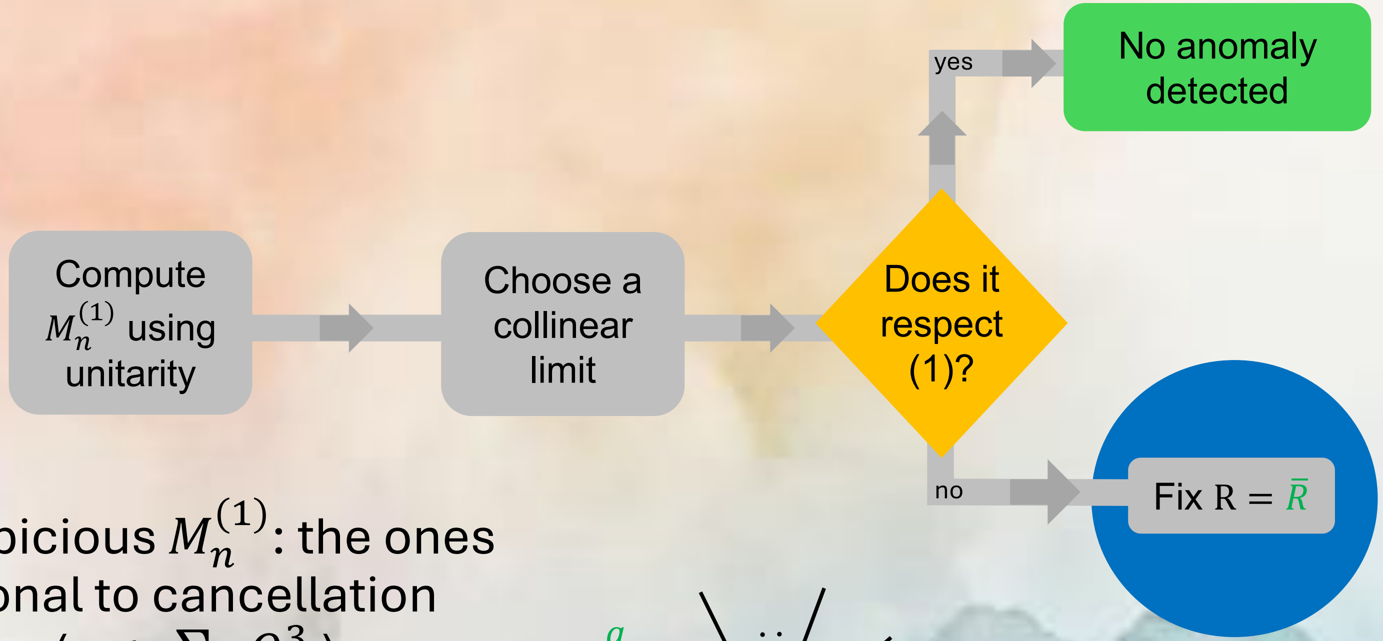

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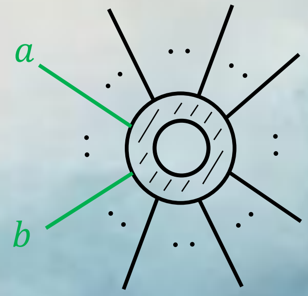

$+\bar{R} \sim O(\sqrt{s_{12}})$

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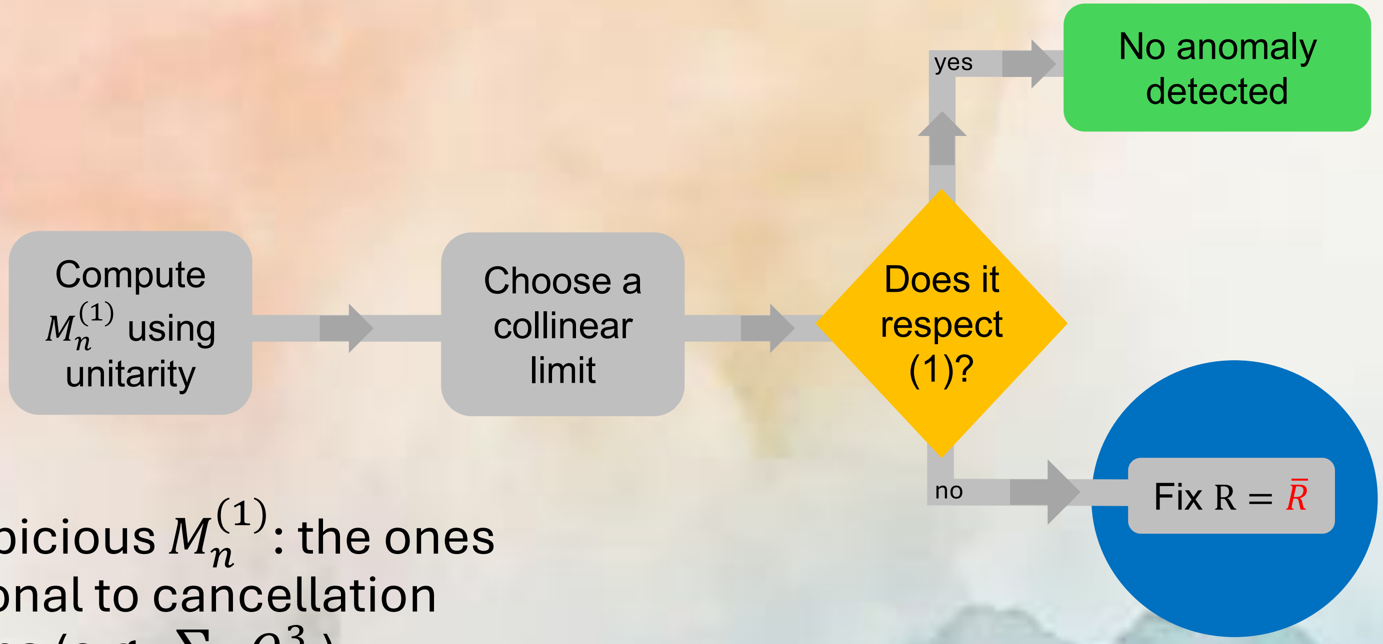

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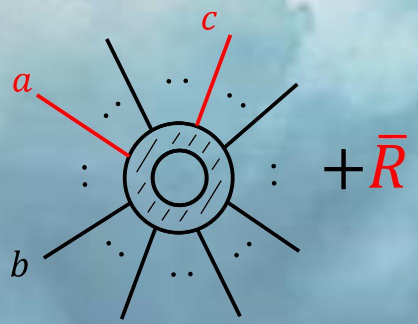

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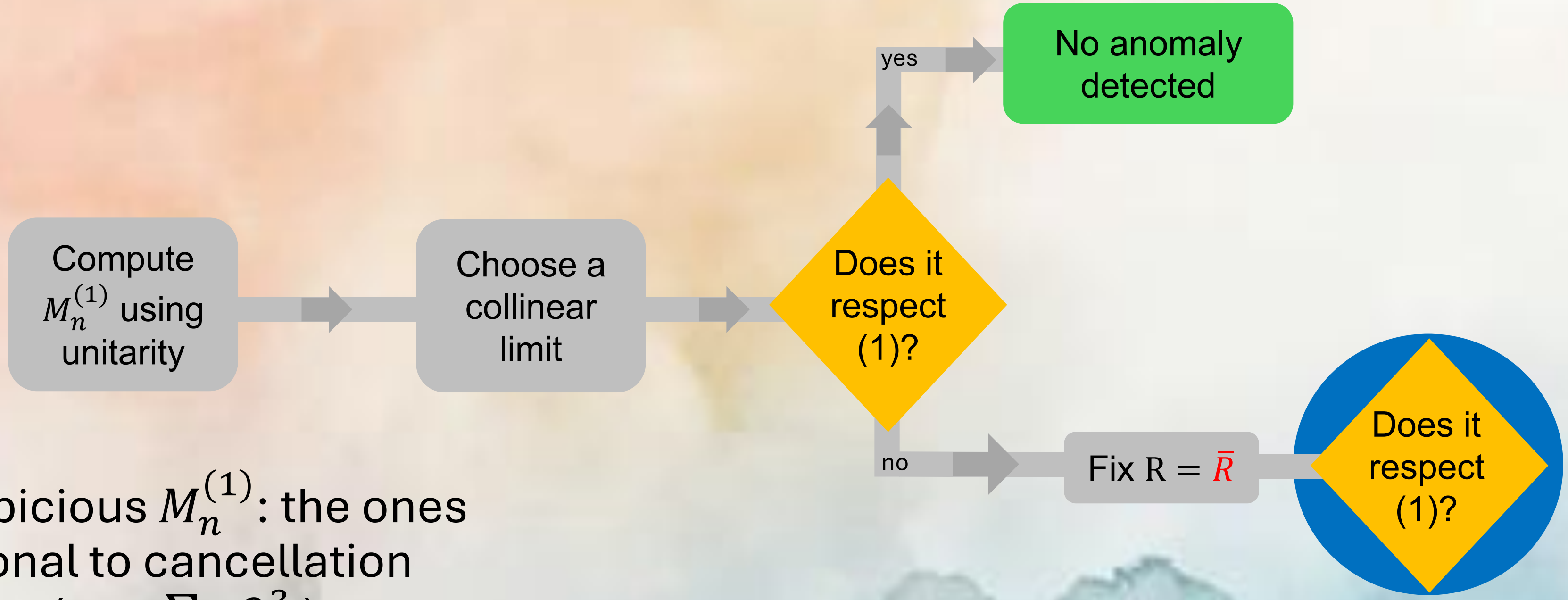

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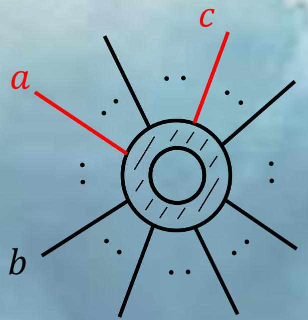

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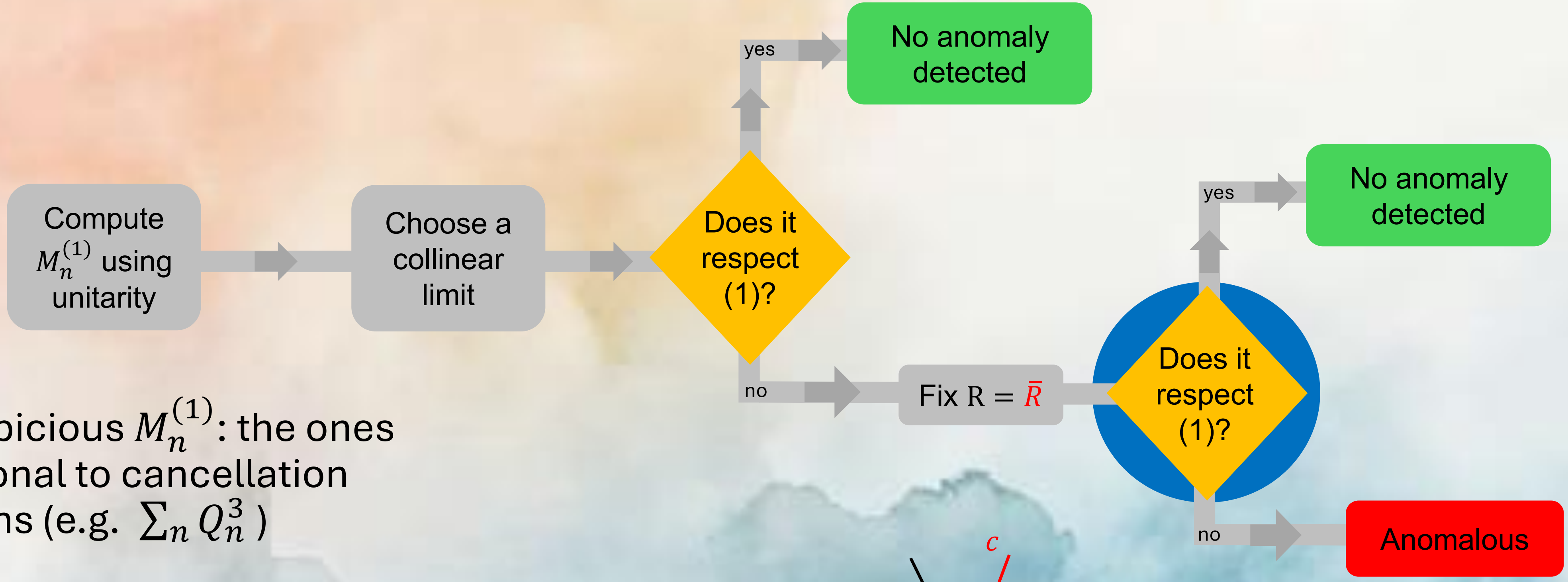

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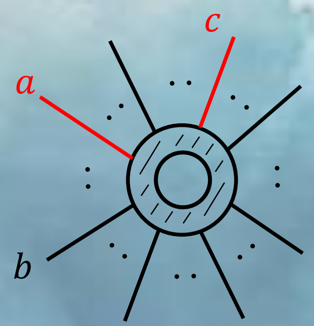

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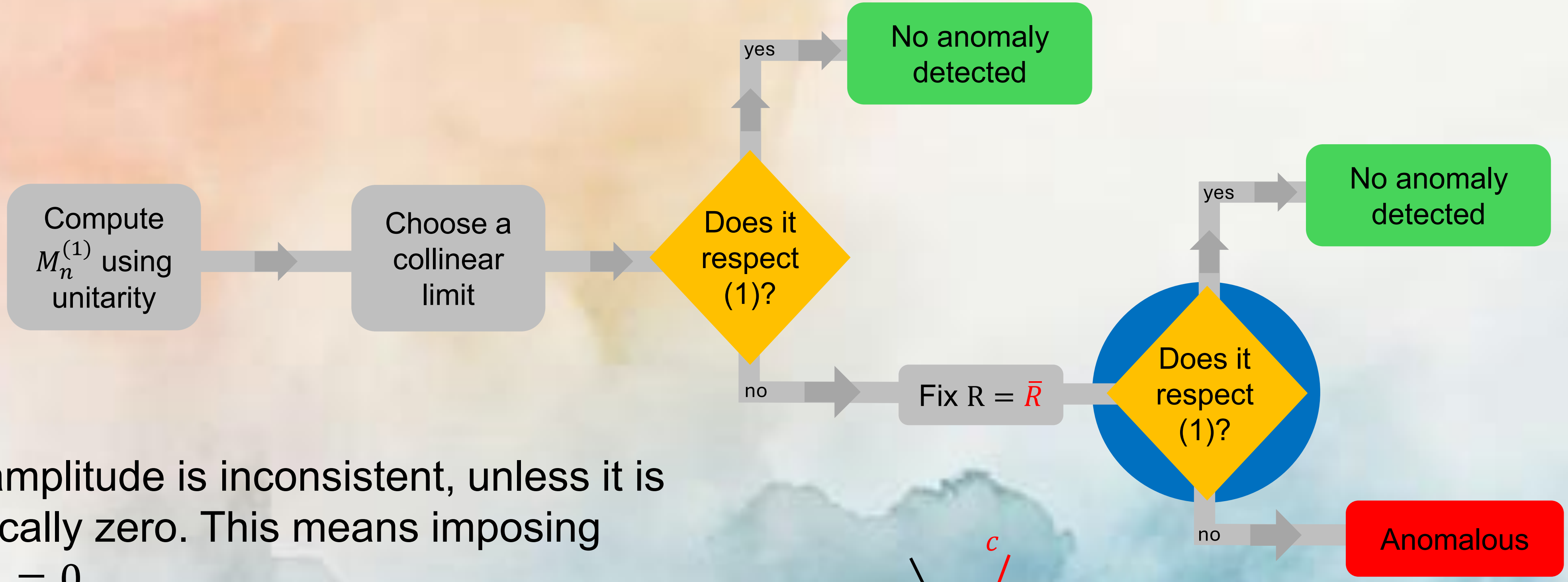

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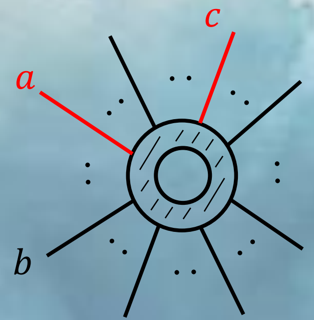

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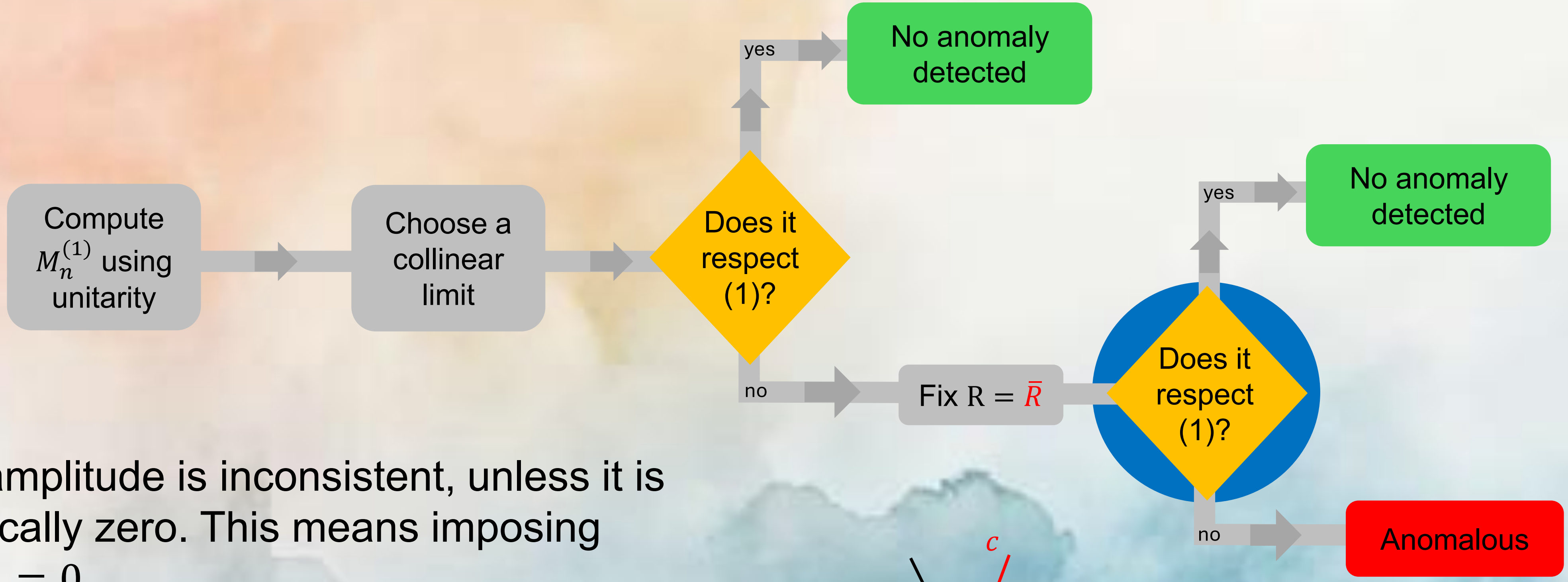

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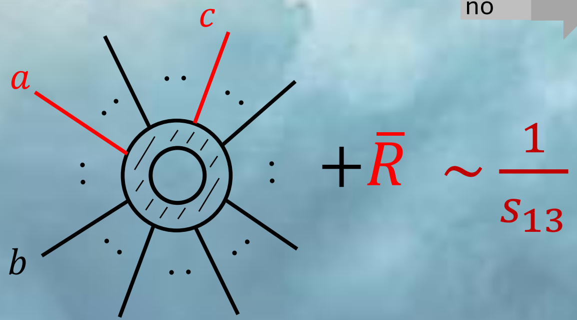
Anomaly cancellation conditions follow from restoring collinear factorization.



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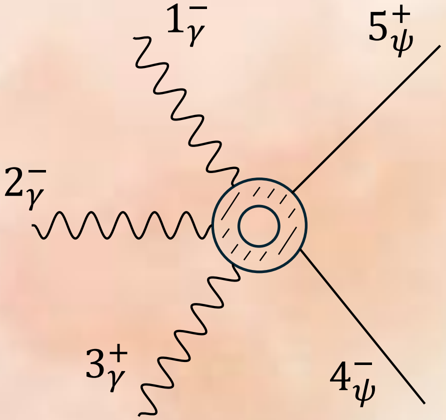

U(1)^3

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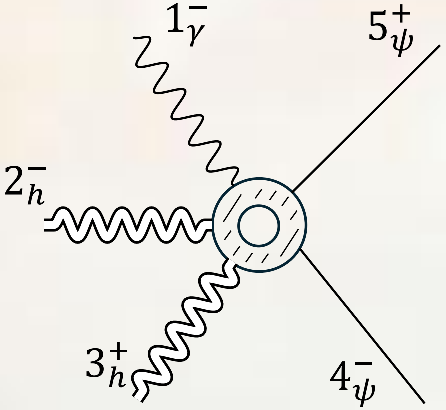

U(1)-gravitational

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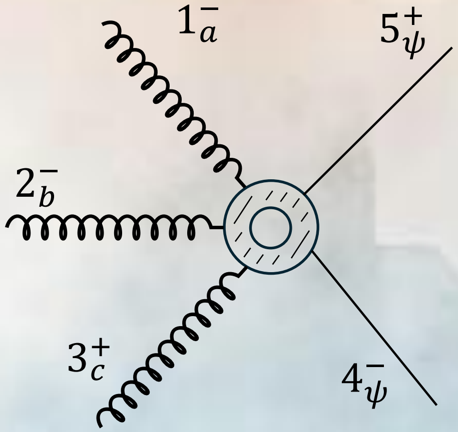

SU(3)^3

$$\sum_{\mathcal{R}} n_{\mathcal{R}} d_{\mathcal{R}}^{abc} = 0$$

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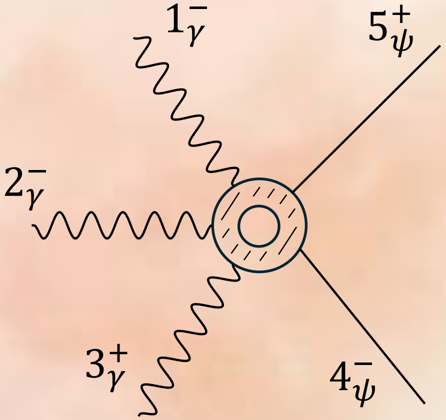

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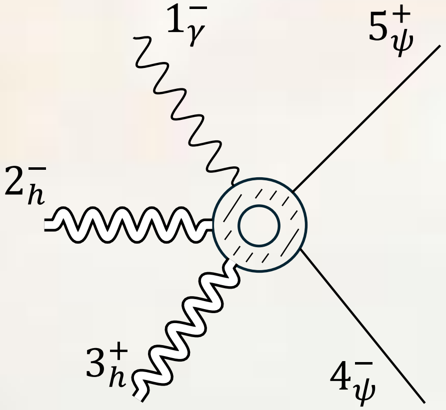
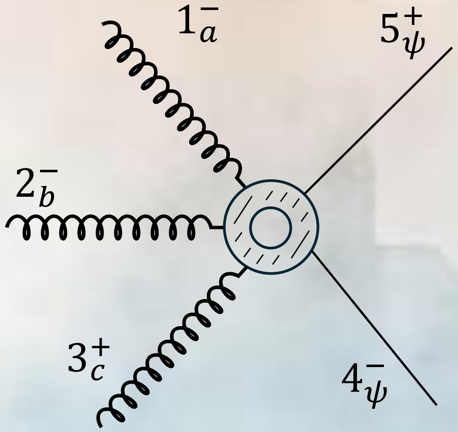
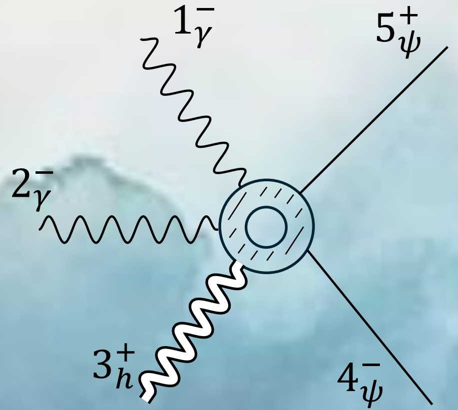


$$\sum_n Q_n = 0$$

$\mathrm{SU}(3)^3$



$$\sum_{\mathcal{R}} n_{\mathcal{R}} d^{abc}_{\mathcal{R}} = 0$$



$$\sum_n Q_n^2 \neq 0$$

5. Outlook

In this work we showed how:

- Gauge anomalies appear on shell as breakdown of collinear factorization
- Restoring the factorization yields gauge anomaly cancellation conditions

For the future:

- Include Green-Schwarz formalism: how new DOF restore factorization
- Study global anomalies via a gravity probe
- Possible on-shell proof of the Adler-Bardeen theorem

5. Outlook

In this work we showed how:

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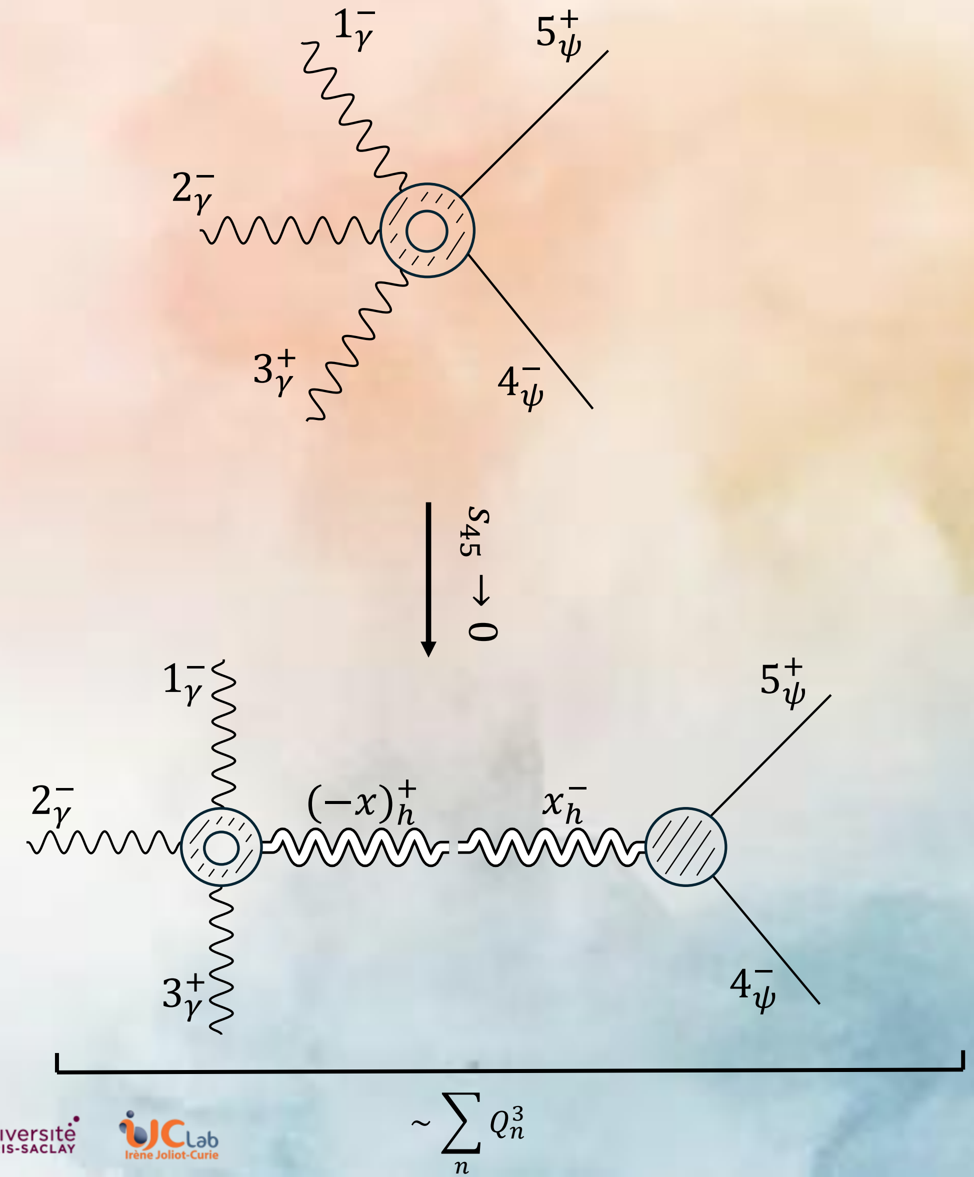
And I thank you!

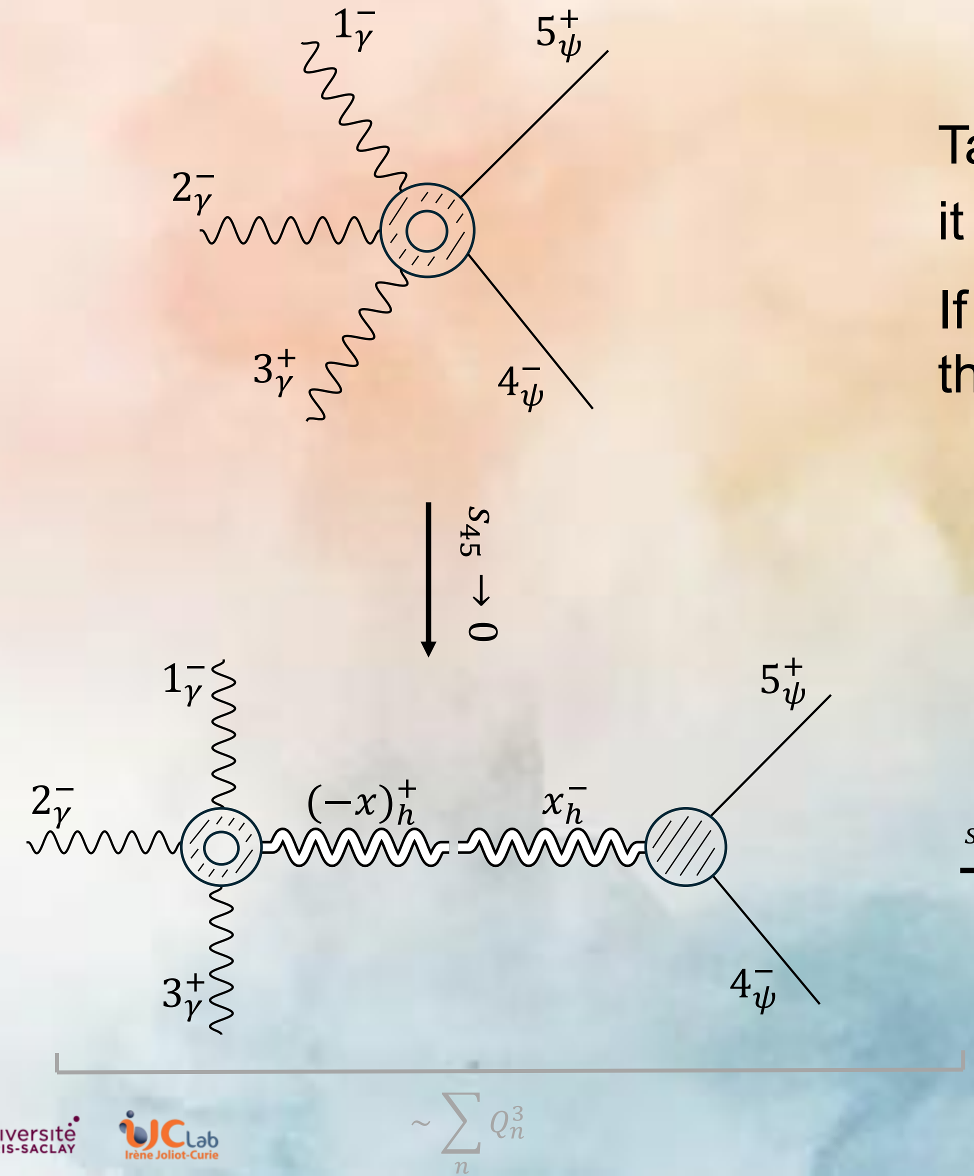
Backup slides

Gauge anomalies on-shell: U(1)^3



Start from a five point amplitude and take a collinear limit to expose the structure of interest.

Gauge anomalies on-shell: $U(1)^3$



Take a second collinear limit to test the pole structure. Does it scale as $\frac{1}{\sqrt{s_{12}}}$ or s_{12}^0?

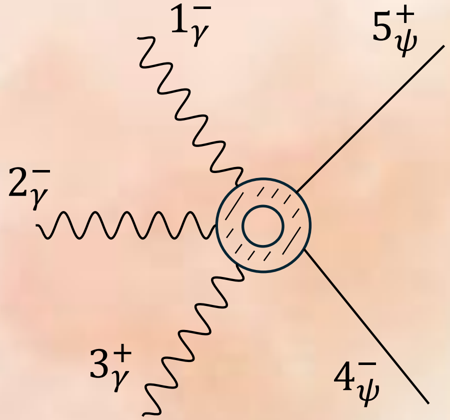
If yes, can we **interpret this as a collinear factorization** of the 5-point amplitude?

$$\xrightarrow{s_{12} \to 0} \underbrace{A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} \right.}_{\sim \frac{1}{\sqrt{s_{12}}}} \left. + O(s_{12}{}^0, s_{45}{}^1) \right) + R$$

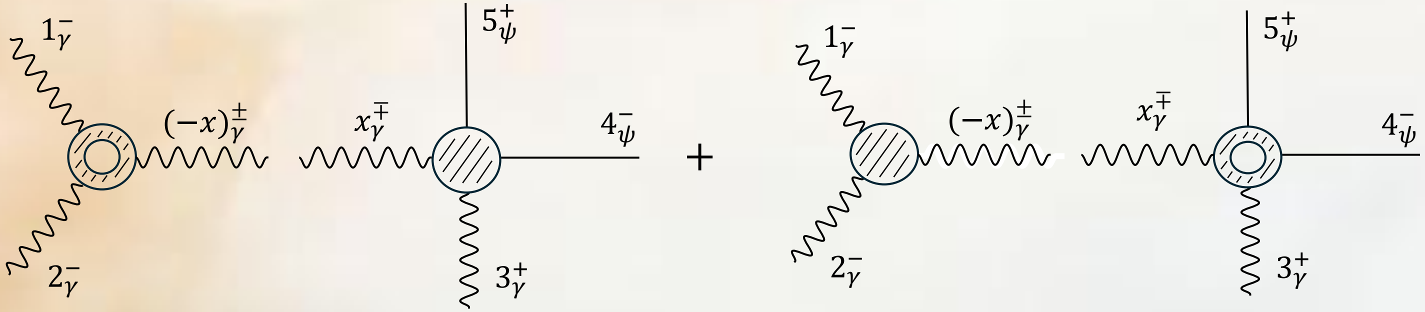
$$S_{QED} = \frac{< 12 >^2 [35]^3 < 3|p_2\sigma|4] < 45 >^2}{s_{12} s_{45}} \sim \sqrt{s_{12}} + O(s_{12})$$

$$A = \frac{-\sqrt{2} e^3}{16\pi^2 M_{Pl}^2}$$

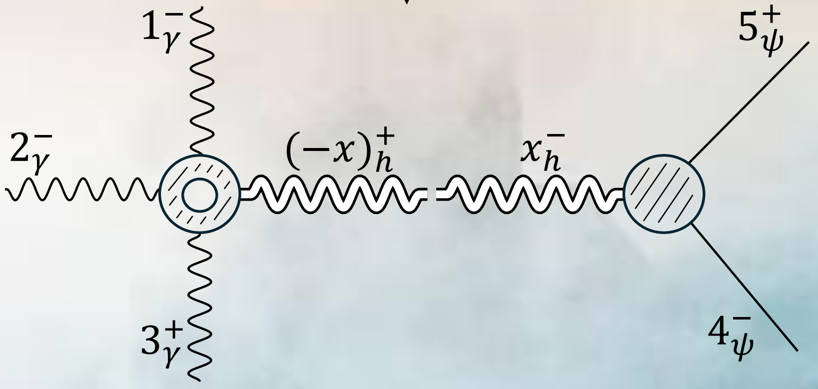
Gauge anomalies on-shell: $U(1)^3$



$\xrightarrow{s_{12} \to 0}$



$\big\downarrow s_{45} \to 0$

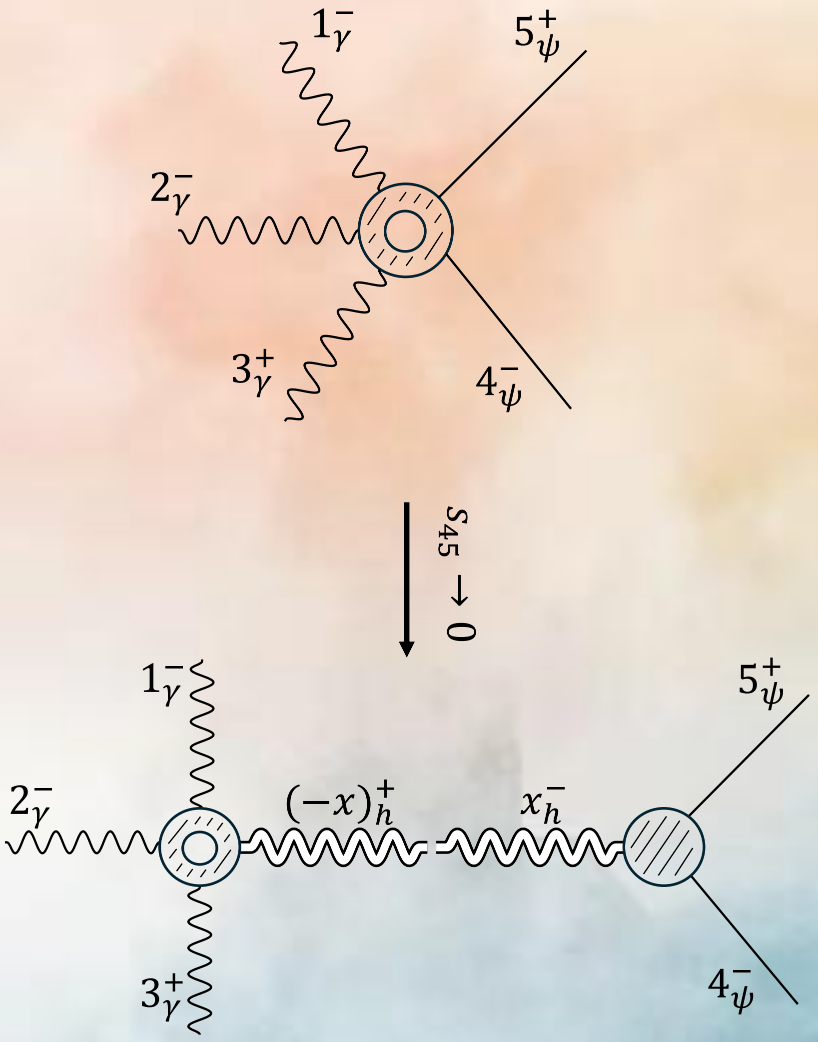


$\sim \sum_n Q_n^3$

$$\xrightarrow{s_{12} \to 0} \underbrace{\boxed{A\, S_{QED}}}_{}\!\!\overset{\sim \frac{1}{\sqrt{s_{12}}}}{} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O\left(s_{12}{}^0, s_{45}{}^1\right) \right) + R$$

The **collinear interpretation fails**: it leads to non universal splitting functions.
We can still save the amplitude by **adjusting** R.

Gauge anomalies on-shell: U(1)^3



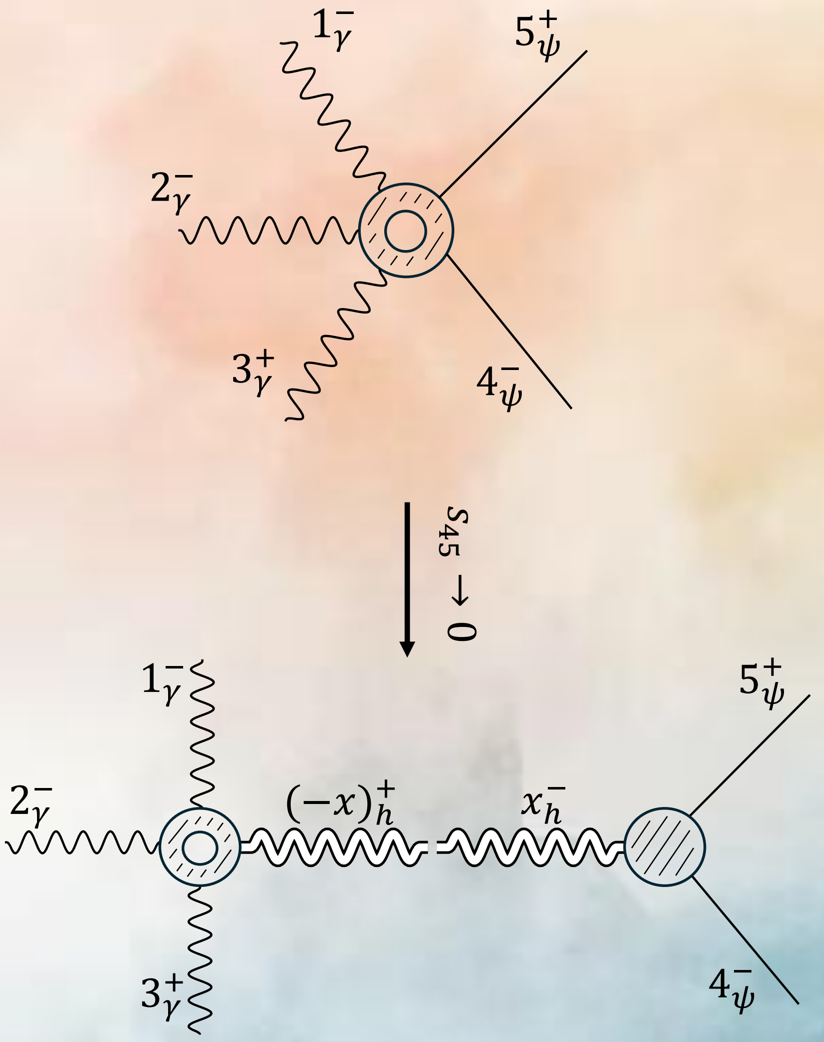
$\sim \sum_n Q_n^3$

R **cancels** the s_{12} singularity

$$R = A \sum_n Q_n^3 \; S_{QED} \frac{1}{s_{12} s_{13}}$$

$$\xrightarrow{s_{12} \to 0} \quad A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \sim O(\sqrt{s_{12}})$$

Gauge anomalies on-shell: U(1)^3



$\sim \sum_n Q_n^3$

R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

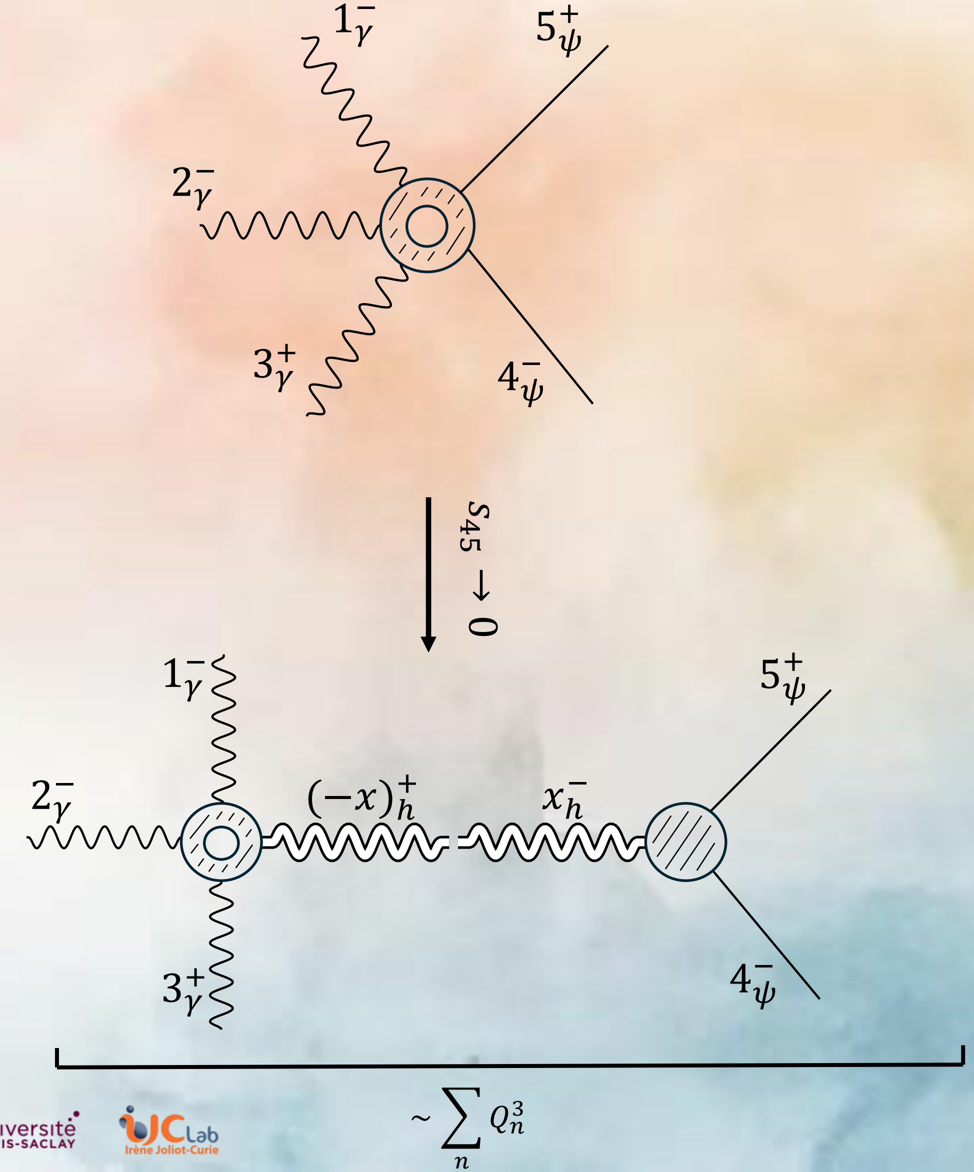
$$\xrightarrow{s_{13} \to 0} \quad R = A \sum_n Q_n^3 \boxed{S_{QED} \frac{1}{s_{12} s_{13}}} \sim \frac{1}{s_{13}}$$

$$\xrightarrow{s_{12} \to 0} \quad A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \sim O(\sqrt{s_{12}})$$

$$S_{QED} = \frac{<12>^2\, [35]^3 <3|p_2 \sigma|4] <45>^2}{s_{12} s_{45}} \sim \sqrt{s_{12}} + O(s_{12})$$

$$A = \frac{-\sqrt{2} e^3}{16 \pi^2 M_{Pl}^2}$$

Gauge anomalies on-shell: $U(1)^3$



R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

The amplitude is inconsistent, unless this piece is identically zero. This means imposing

$$\sum_n Q_n^3 = 0$$

Anomaly cancellation conditions follow from restoring collinear factorization

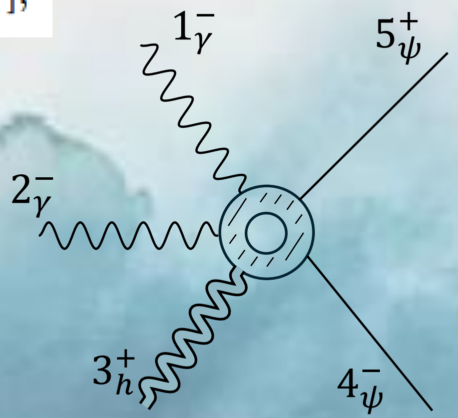
Crosscheck of collinear factorization theorem

$$\mathcal{M}^{(1)}[1_\gamma^- 2_\gamma^- 3_h^+ 4_h^+ 5_h^-] \supset -\frac{\mathcal{M}^{(1)}\left[1_\gamma^- 2_\gamma^- 3_h^+ K_h^+\right]\mathcal{M}^{(0)}[(-K)_h^- 4_h^+ 5_h^-]}{s_{45}} = M_{2\gamma 3h} + R'_{2\gamma 3h},$$

$$M_{2\gamma 3h} = \left[\sum_n Q_n^2\right] \frac{c^2}{16\pi^2 M_{\text{Pl}}^3} \frac{\langle 45\rangle}{[45]} \frac{[34]^5}{[12]^3[35]^2} \frac{[13][24]}{s_{13}} F_{2\gamma 2h}(s_{12}, s_{13}),$$

$$R'_{2\gamma 3h} = \frac{R_{2\gamma 2h}}{s_{45}} \frac{\langle K5\rangle^6}{M_{\text{Pl}}\langle K4\rangle^2\langle 45\rangle^2},$$

$$R'_{2\gamma 3h} \xrightarrow{1\|3} \text{Split}^{(1)}_{\gamma^-}\left[1_\gamma^- 3_h^+\right]\mathcal{M}^{(0)}[P_\gamma^+ 2_\gamma^- 4_h^+ 5_h^-] + \text{Split}^{(0)}_{\gamma^+}\left[1_\gamma^- 3_h^+\right]\mathcal{M}^{(1)}[P_\gamma^- 2_\gamma^- 4_h^+ 5_h^-],$$



$\Rightarrow \sum_n Q_n^2 \neq 0$