EXCALIBUR : Relativistic raytracing for weak & strong lensing analysis

Laurent Magri-Stella[1,2], Narei Lorenzo Martinez[2,3], Vincent Reverdy[2,3]

[1] Université Savoie Mont Blanc
[2] Laboratoire d'Annecy de Physique des Particules (LAPP), CNRS
9 Chemin de Bellevue, 74940 Annecy-le-Vieux
[3] Centre National de la Recherche Scientifique (CNRS)

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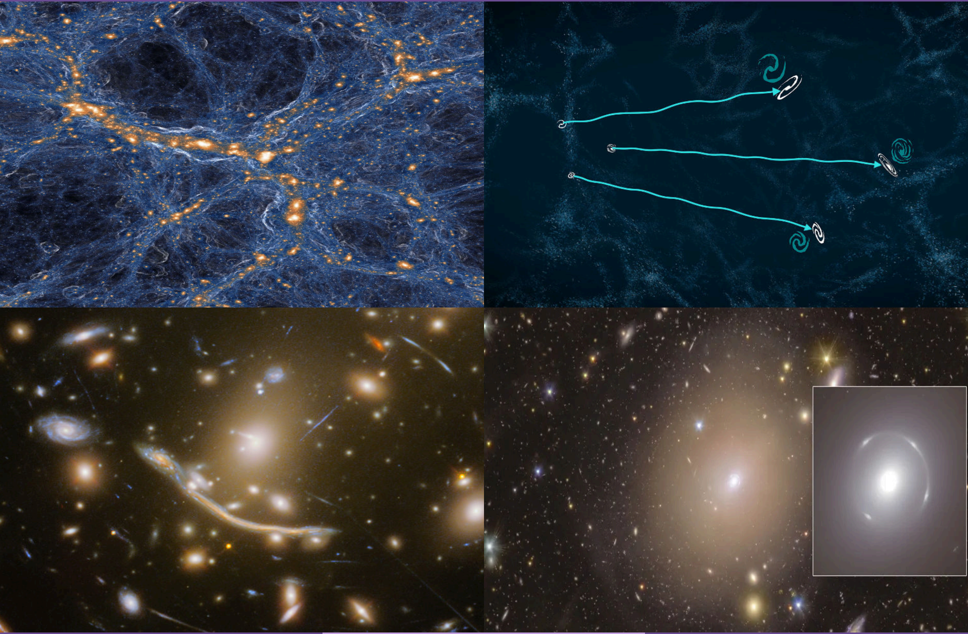
Galaxy clusters as a key cosmological probe



The upcoming surveys

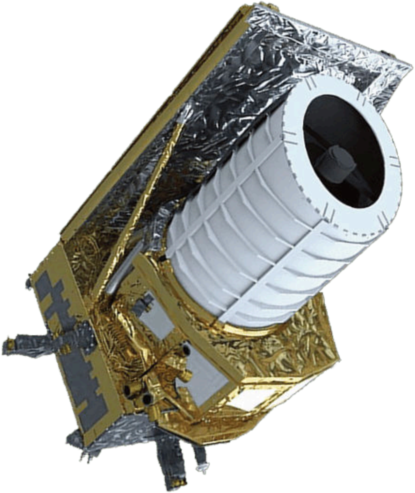
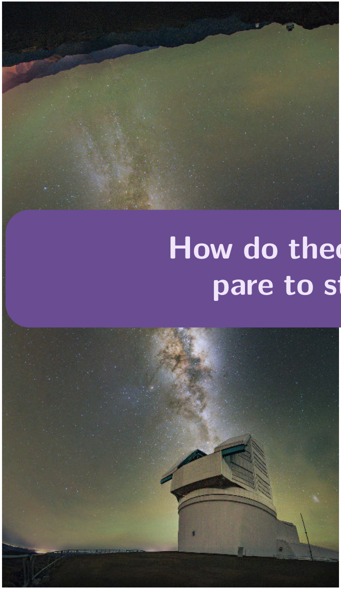


Figure 1: Vera C. Rubin Observatory (LSST) and Euclid satellite.

An important question



How do theoretical uncertainties compare to statistical uncertainties?

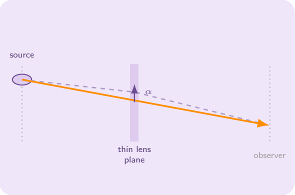
Figure 2: Vera C. Rubin Observatory (LSST) and Euclid satellite.

Towards an answer ?

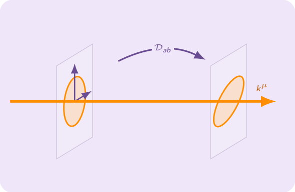
1 **Weak lensing**

2 **Sachs formalism**

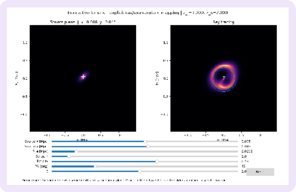
3 **EXCALIBUR**



Current weak-lensing practice and its main approximations.



Relativistic beam propagation and distortions from curvature.



Relativistic raytracing for weak & strong lensing in FLRW space-time.

Weak lensing: a brief overview

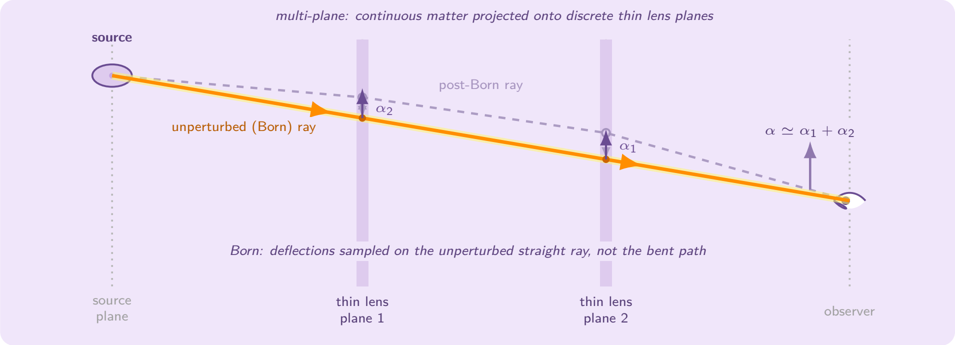


Figure 3: Current approximations in weak lensing frameworks: 1/ Born approximation 2/ thin lens planes. The dashed ray represents the post Born approach, which starts to be more commonly used in simulations, has yet to be implemented in analysis pipelines.

Lens equation and magnification matrix

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta}) \qquad \frac{\partial \beta_i}{\partial \theta_j} \approx \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = A_{ij}$$

Weak lensing: convergence, shear, and flexion

First order approximation of the magnification matrix

$$A = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \gamma \begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix}$$

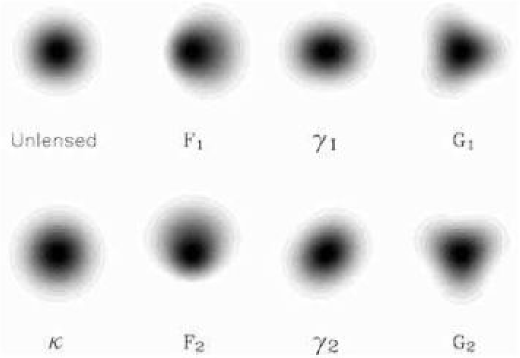


Figure 4: Effects of the different lensing fields on a Gaussian galaxy of radius 1 arcsec. 10% convergence/shear and 0.28 arcsec^{-1} flexion (which is a very high value for this quantity, chosen only to visualize) are applied. From Bacon et al. 2006

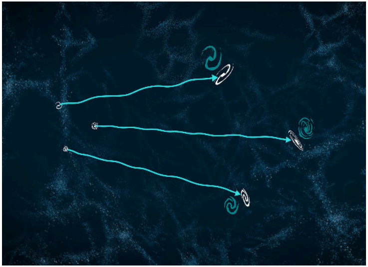
A relativistic description of light propagation

Goal

Describe light propagation directly in space-time, beyond the standard weak-lensing approximations.

Three ingredients

1. a background metric describing the geometry;
2. the geodesic equation for photon trajectories;
3. the Sachs formalism for beam deformations.



Why this drops the approximations

- Born → geodesic integrated along the true light path.
- Thin lens → continuous fields sampled in 4D.

1/ A background metric describing the geometry

The Friedmann-Lemaitre-Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & \dfrac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2\sin^2\theta \end{pmatrix}$$

where:

- $a(t)$: scale factor
- k: curvature parameter $(k = 0, +1, -1)$

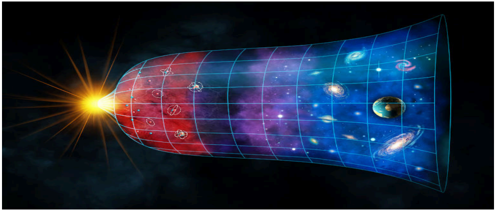


Figure 5: The story of the Universe: from the Big Bang to the present day.
Credit: Natalie Mayer

1/ A perturbed background metric describing the geometry

The perturbed Friedmann-Lemaitre-Robertson-Walker metric

$$g_{\mu\nu} = \begin{pmatrix} -c^2(1+2\Phi) & 0 & 0 & 0 \\ 0 & a^2(t)(1-2\Psi) & 0 & 0 \\ 0 & 0 & a^2(t)(1-2\Psi) & 0 \\ 0 & 0 & 0 & a^2(t)(1-2\Psi) \end{pmatrix}$$

in

- cartesian coordinates;
- flat space ($k = 0$).

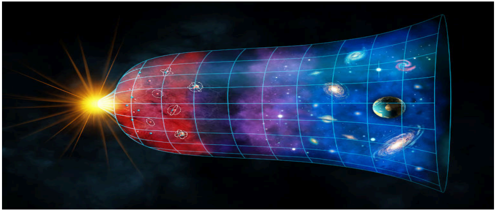
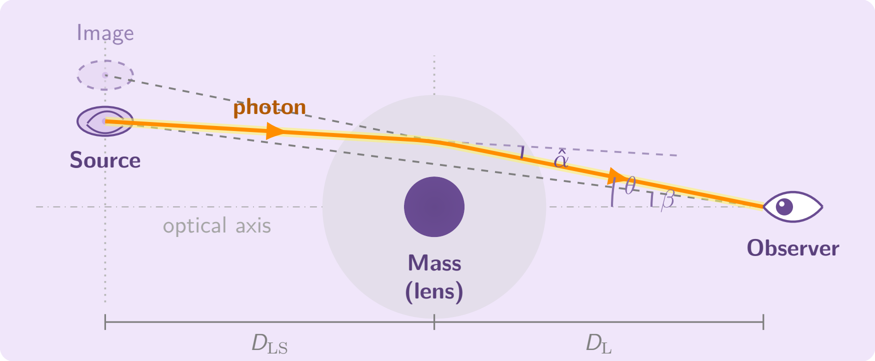


Figure 6: The story of the Universe: from the Big Bang to the present day.
Credit: Natalie Mayer

2/ The geodesic equation for photon trajectories

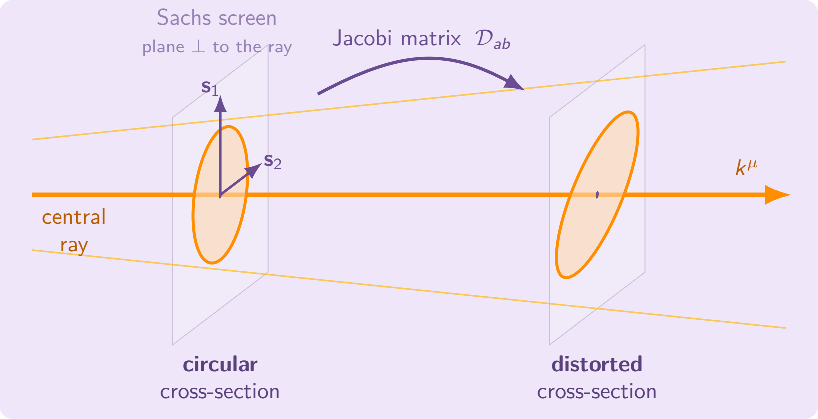


Geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} = 0 \qquad g_{\mu\nu} k^\mu k^\nu = 0$$

The metric fixes the Christoffel symbols, which determine how photon trajectories curve through inhomogeneous matter distributions.

3/ The Sachs formalism for beam deformations



Optical tidal matrix

$$\mathcal{R}_{AB} = R_{\mu\nu\rho\sigma} k^\mu s_A^\nu s_B^\rho k^\sigma$$

$$\mathcal{R}_{AB} = -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu \delta_{AB} + C_{\mu\nu\rho\sigma} s_A^\mu k^\nu k^\rho s_B^\sigma$$

Focusing equation and A_{ij}

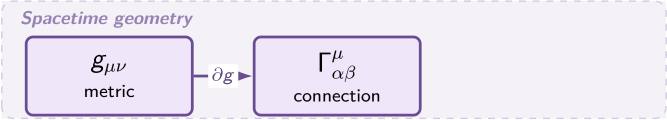
$$\frac{\mathrm{d}^2 \mathcal{D}_{AB}}{\mathrm{d}v^2} = \mathcal{R}_{AC}\, \mathcal{D}_{CB}$$

$$\mathcal{D} = A\bar{\mathcal{D}} + \dots$$

The tensor chain: from the metric to the Jacobi matrix

The **metric alone** fixes the whole chain: each tensor follows from derivatives of the previous one.

The tensor chain: from the metric to the Jacobi matrix



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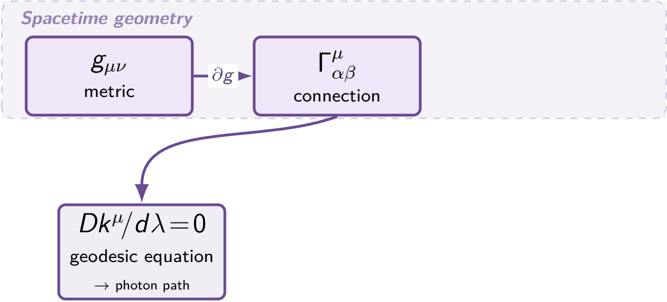

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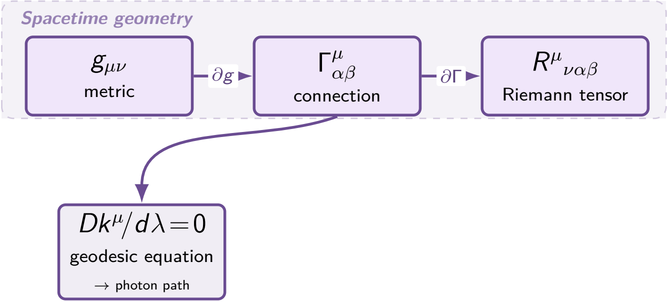

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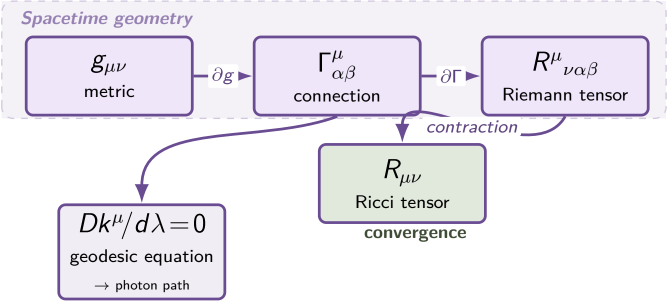

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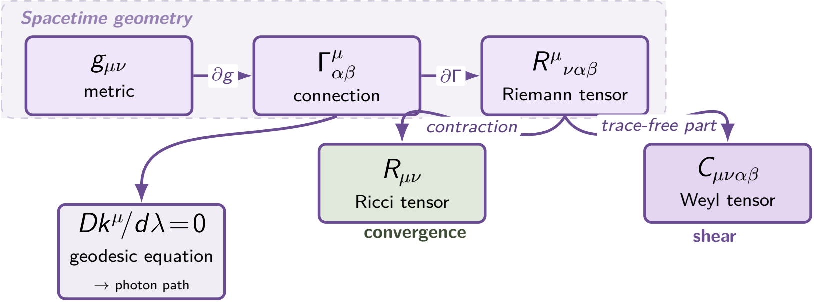

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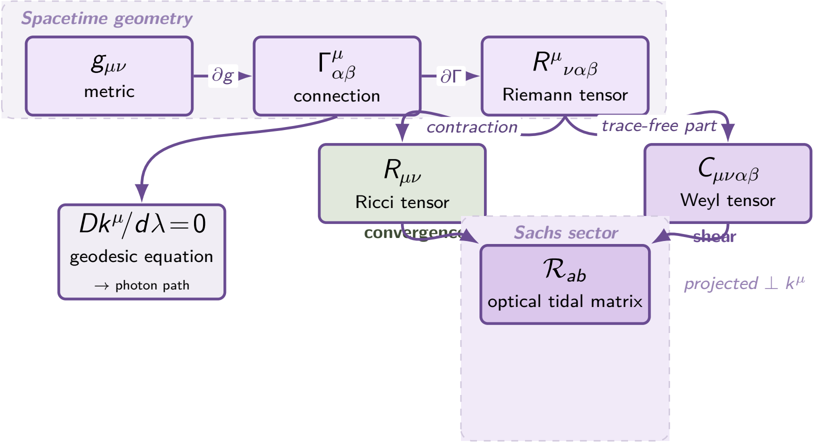

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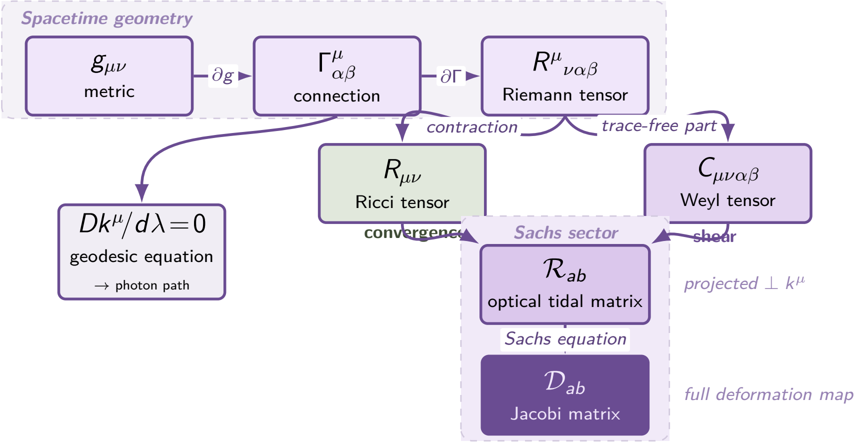

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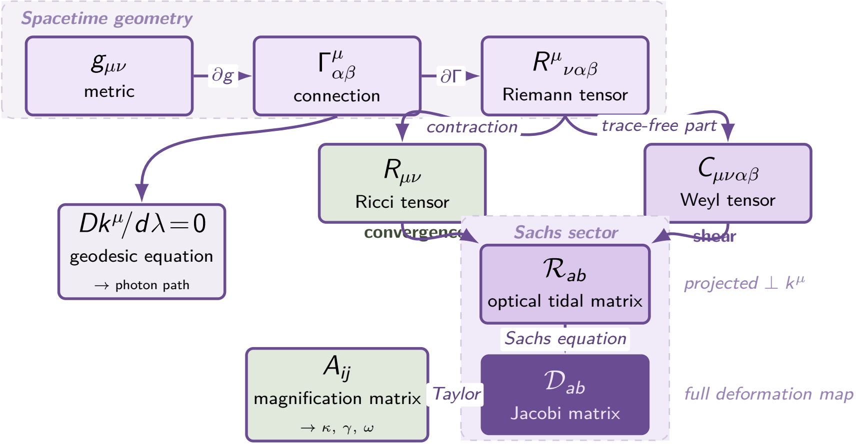

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Introducing EXCALIBUR

E X C A L I B U R

EX	**EX**act
CA	**CA**lculation (of)
LI	**LI**ght
B	**B**eams
U	**U**sing
R	**R**elativity

EXCALIBUR aims to be a modular and performant tool to compute light propagation in curved space-times.

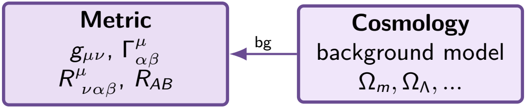
EXCALIBUR: numerical pipeline

Metric

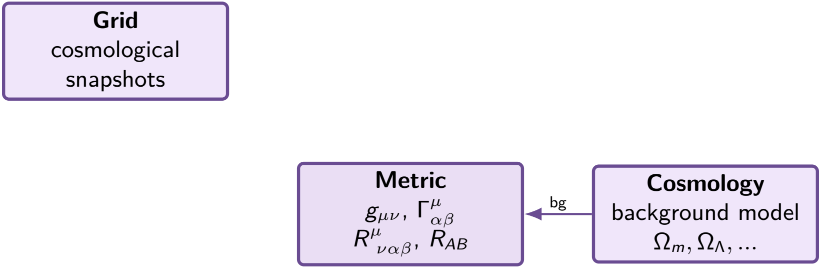
$g_{\mu\nu}$, $\Gamma^{\mu}_{\alpha\beta}$

$R^{\mu}{}_{\nu\alpha\beta}$, R_{AB}

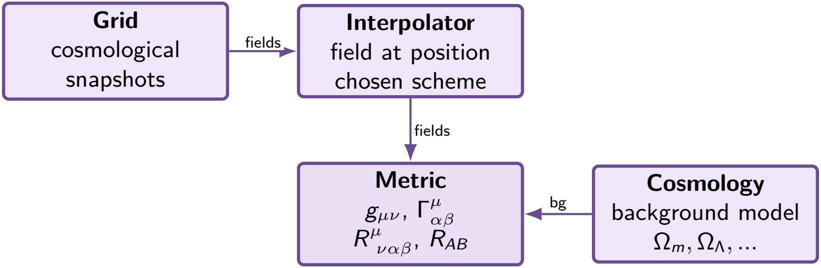
EXCALIBUR: numerical pipeline



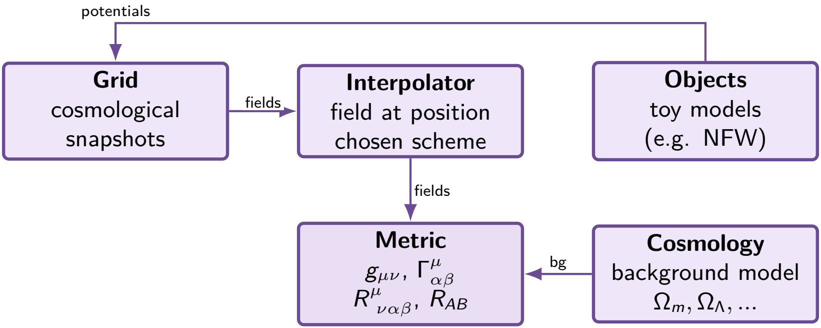
EXCALIBUR: numerical pipeline



EXCALIBUR: numerical pipeline



EXCALIBUR: numerical pipeline



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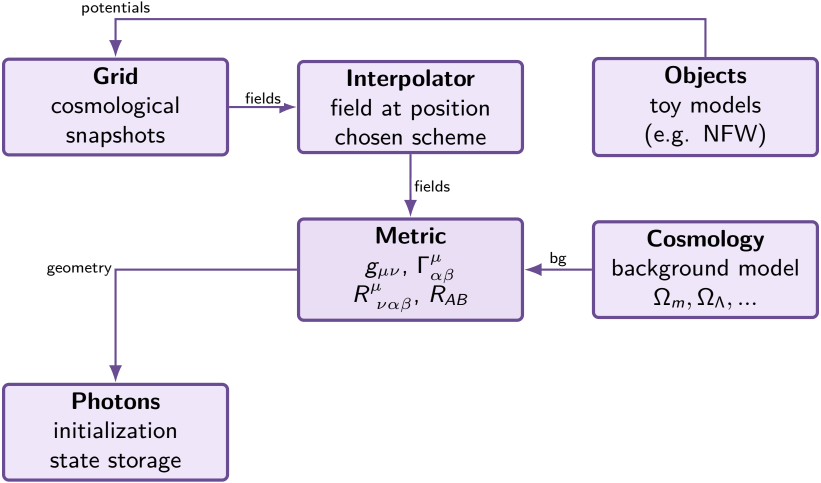

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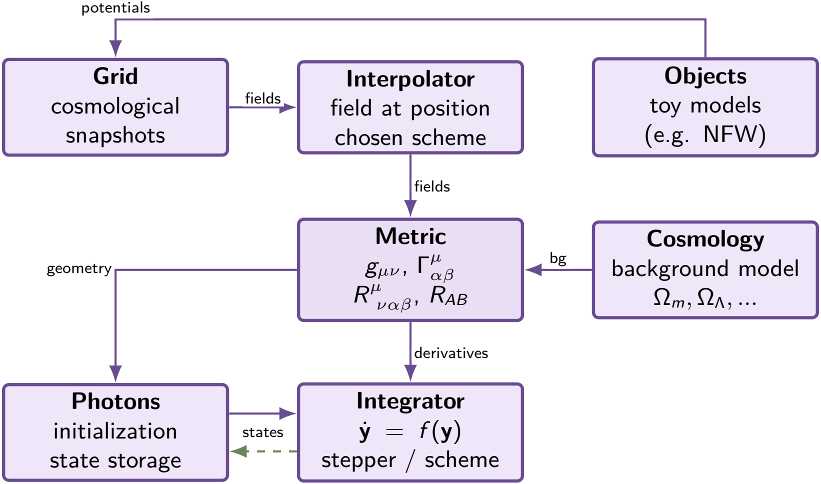

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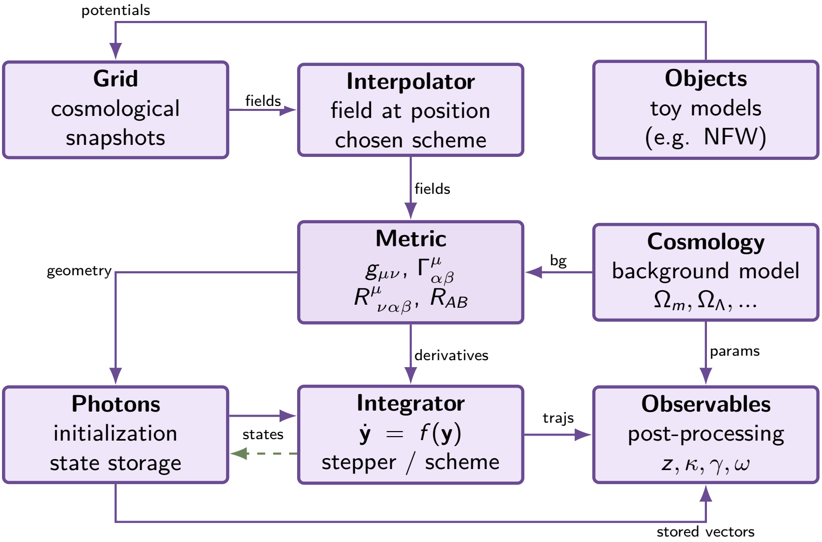
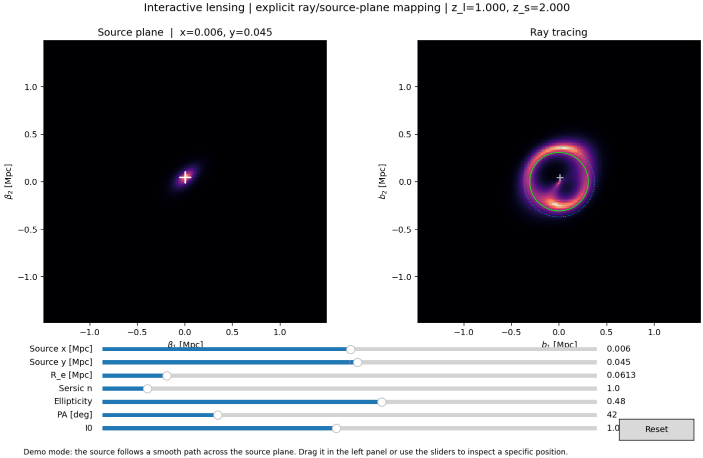


What EXCALIBUR enables



`lmagristella.github.io/random-science/`

Code validation

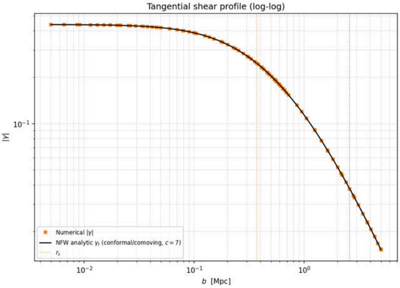


Figure 7: Shear profile of an NFW halo: numerical vs analytical.

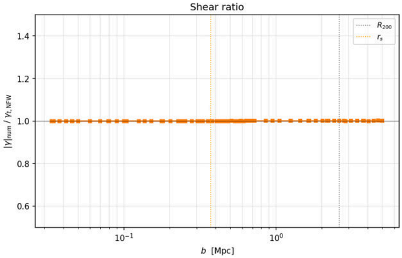
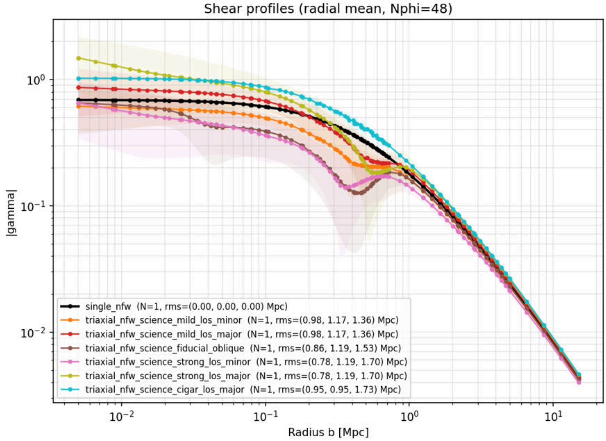
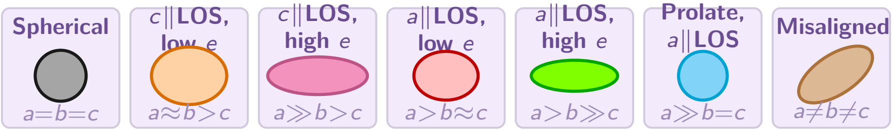


Figure 8: Ratio of numerical to analytical shear profile for an NFW halo.

Validation of lensing quantities

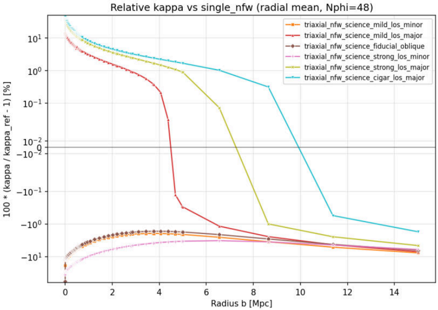
Compare simulated κ and γ profiles of an NFW halo to the analytical NFW predictions.

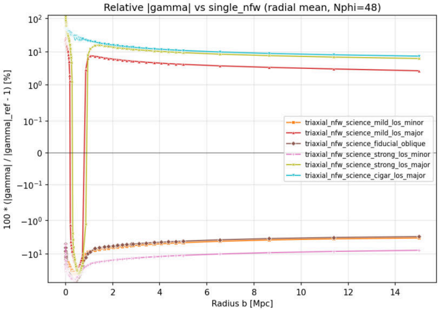


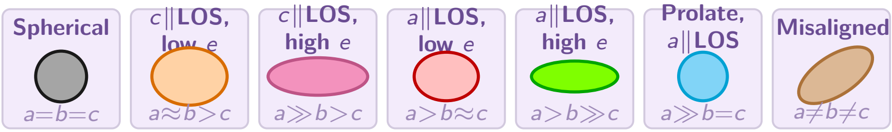


Sky-plane projections ($\perp$ LOS)

Current work: model errors on mass and shape

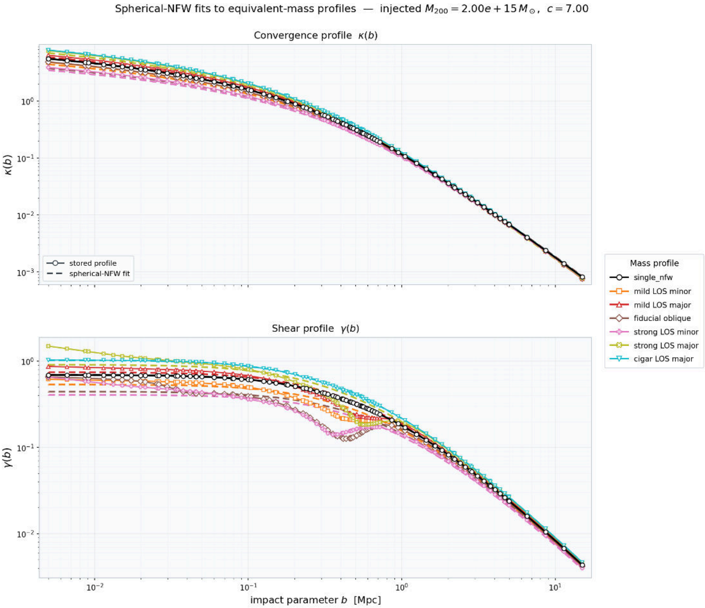




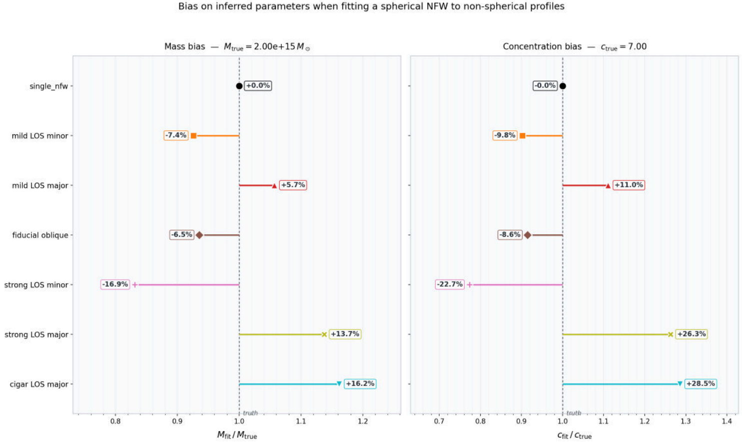


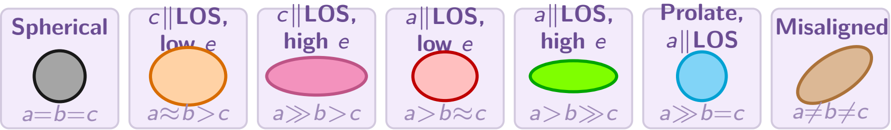
Sky-plane projections ($\perp$ LOS)

Fitting a spherical NFW model ?



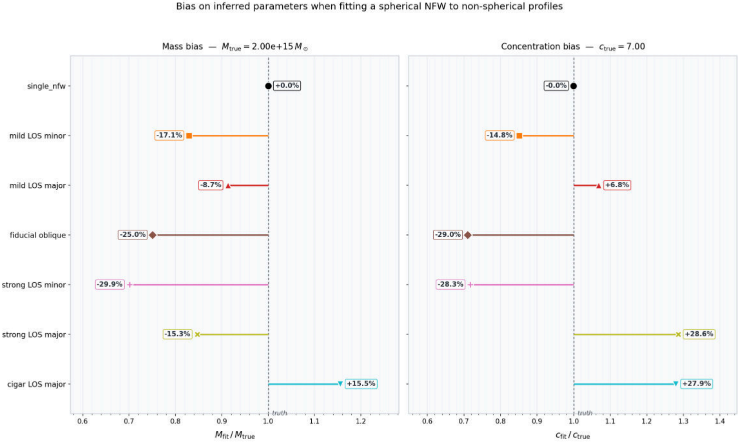
κ fit biases

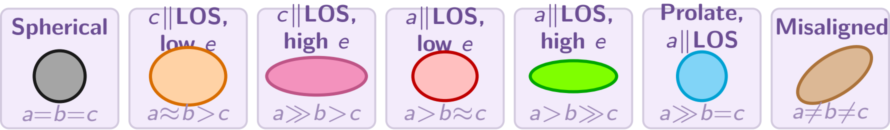




Sky-plane projections ($\perp$ LOS)

γ fit biases





Sky-plane projections ($\perp$ LOS)

Discussion

1. **Bias on the cluster mass** — 5–20%

   worst for large ellipticities, $c \parallel LOS$, and prolate halos

2. **Bias on the concentration** — 5–30%

   same dependence on the halo geometry

3. **Large impact on cosmology**

   amplified by the exponential cutoff of the halo mass function dN/dM at high mass

Consistent with Corless & King (2007)

Same setup

Spherical NFW fits to **triaxial** halos with the odd axis along the LOS

Truth: $M_{200} = 10^{15}\, M_\odot$, $C = 4$

Same trend

- Prolate $(Q > 1) \Rightarrow$ **over**estimated M and C
- Oblate $(Q < 1) \Rightarrow$ **under**estimated M and C

Independent confirmation via a purely statistical approach to triaxial halo modeling.

6 *V. L. Corless & L. J. King*

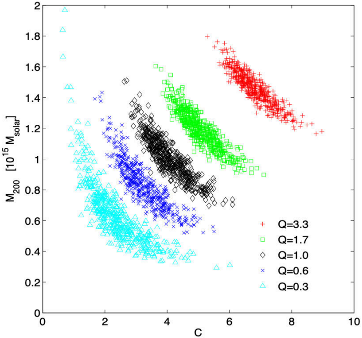


Figure 3. Best-fit concentration C and virial mass M_{200} for a simple NFW model fit to five triaxial lenses with $C = 4$, $M_{200} = 10^{15}\ \mathrm{M}_\odot$, circularly symmetric on the sky with odd axis oriented along the line of sight. Halos with axis ratio Q (ratio between odd axis and similar axes) less than one are oblate ($Q = 0.3 \rightarrow a = 0.3$), while those with Q greater than one are prolate ($Q = 3.3 \rightarrow a = b = 0.3$).

Conclusion and outlook

Some (yet to determine precisely) perspectives

Lens geometry effects
NFW potentials in an FLRW background

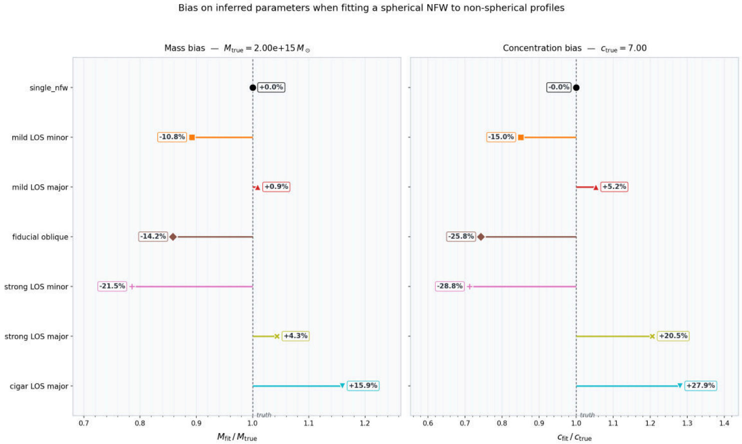
Other metrics?
Beyond the cosmological principle (LTB, Szekeres, . . .)

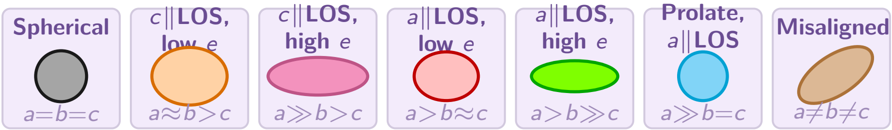
Model inversion?
Raytracing-based lens and cosmology inference

Take home message

excalibur: a modular relativistic raytracing code to quantify lensing effects and systematics. Lens geometry can bias shear profiles by $\approx 20\%$

Appendix: Joint κ, γ fit biases





Sky-plane projections $(\perp \mathrm{LOS})$