



MICHIGAN STATE
UNIVERSITY

Optical potentials: Why do we care? How do we construct them? What are the current frontiers?

Chloë Hebborn

Why did I choose this topic?

Optical potentials are ubiquitous in nuclear physics and important for analysis of experiments!

Optical potentials for the rare-isotope beam era

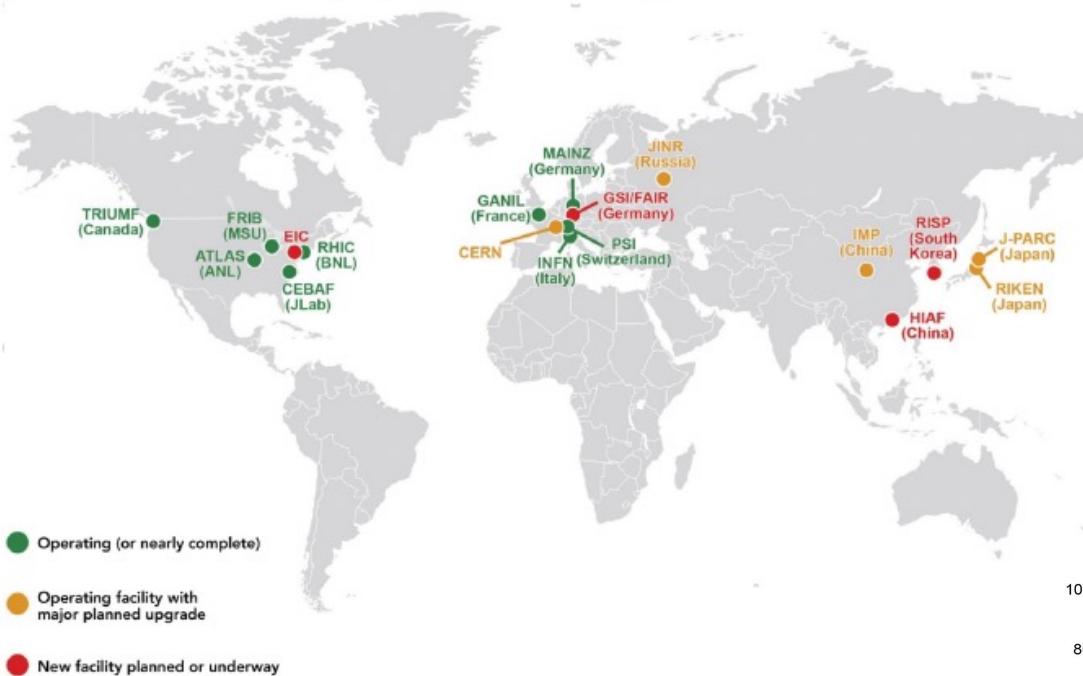
- Many recent developments
(whitepaper JPG 50, 060501 (2023))

Various authors from the Paris's region

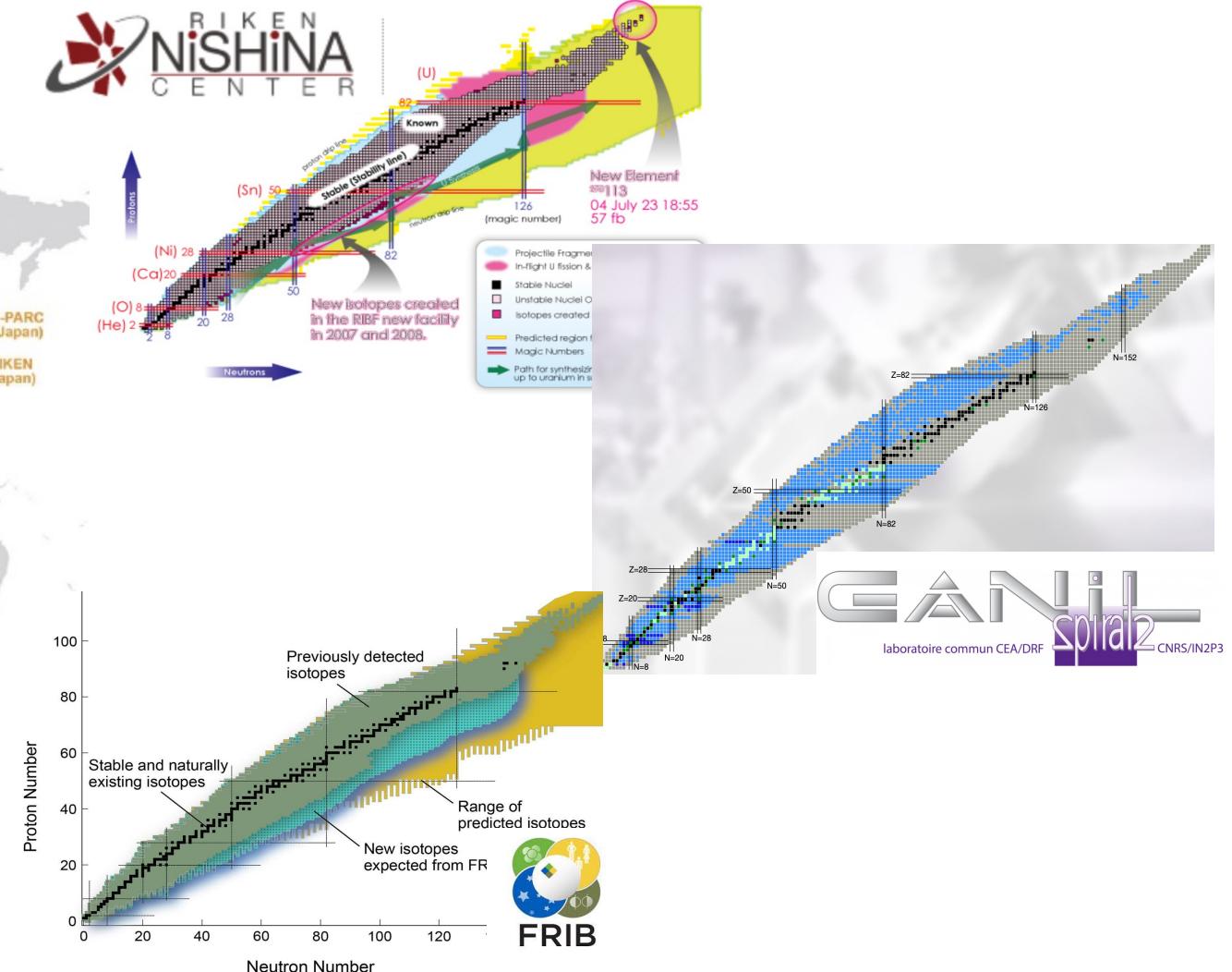
- + many developments since then
(also by **Soma**)!

C Hebborn^{1,2} , F M Nunes^{1,3} , G Potel² , W H Dickhoff⁴ ,
J W Holt⁵ , M C Atkinson^{2,6} , R B Baker⁷ , C Barbieri^{8,9} ,
G Blanchon^{10,11} , M Burrows¹² , R Capote¹³ ,
P Danielewicz^{1,3} , M Dupuis^{10,11} , Ch Elster⁷ ,
J E Escher² , L Hlophe² , A Idini¹⁴ , H Jayatissa¹⁵ ,
B P Kay¹⁵ , K Kravvaris² , J J Manfredi¹⁶ , A Mercenne¹⁷ ,
B Morillon^{10,11} , G Perdikakis¹⁸ , C D Pruitt² ,
G H Sargsyan² , I J Thompson² , M Vorabbi^{19,20}  and
T R Whitehead¹ 

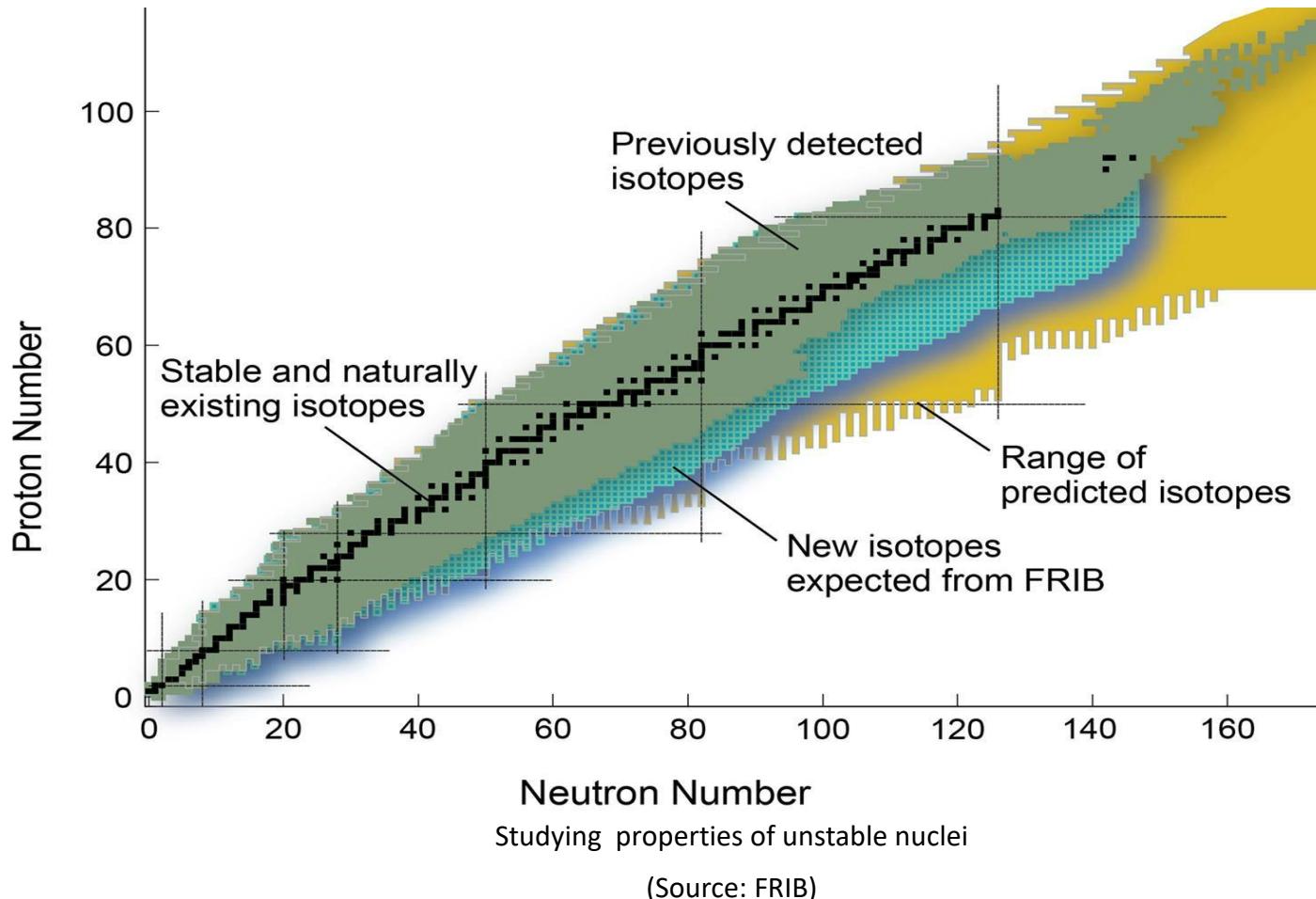
Exciting time to be a nuclear physicists as RIB facilities enable the study of many unstable nuclei!



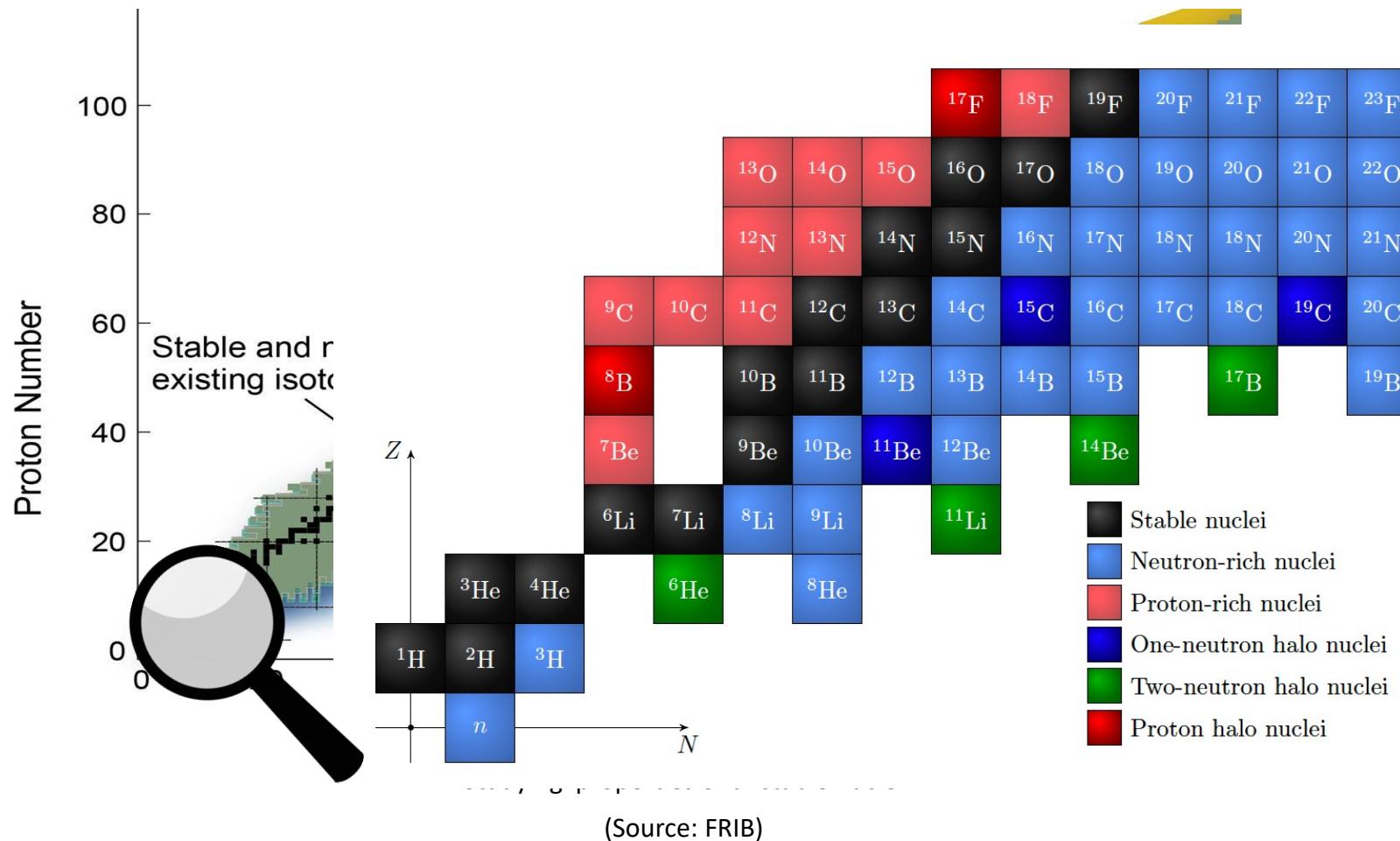
Non-exhaustive map of RIB facilities



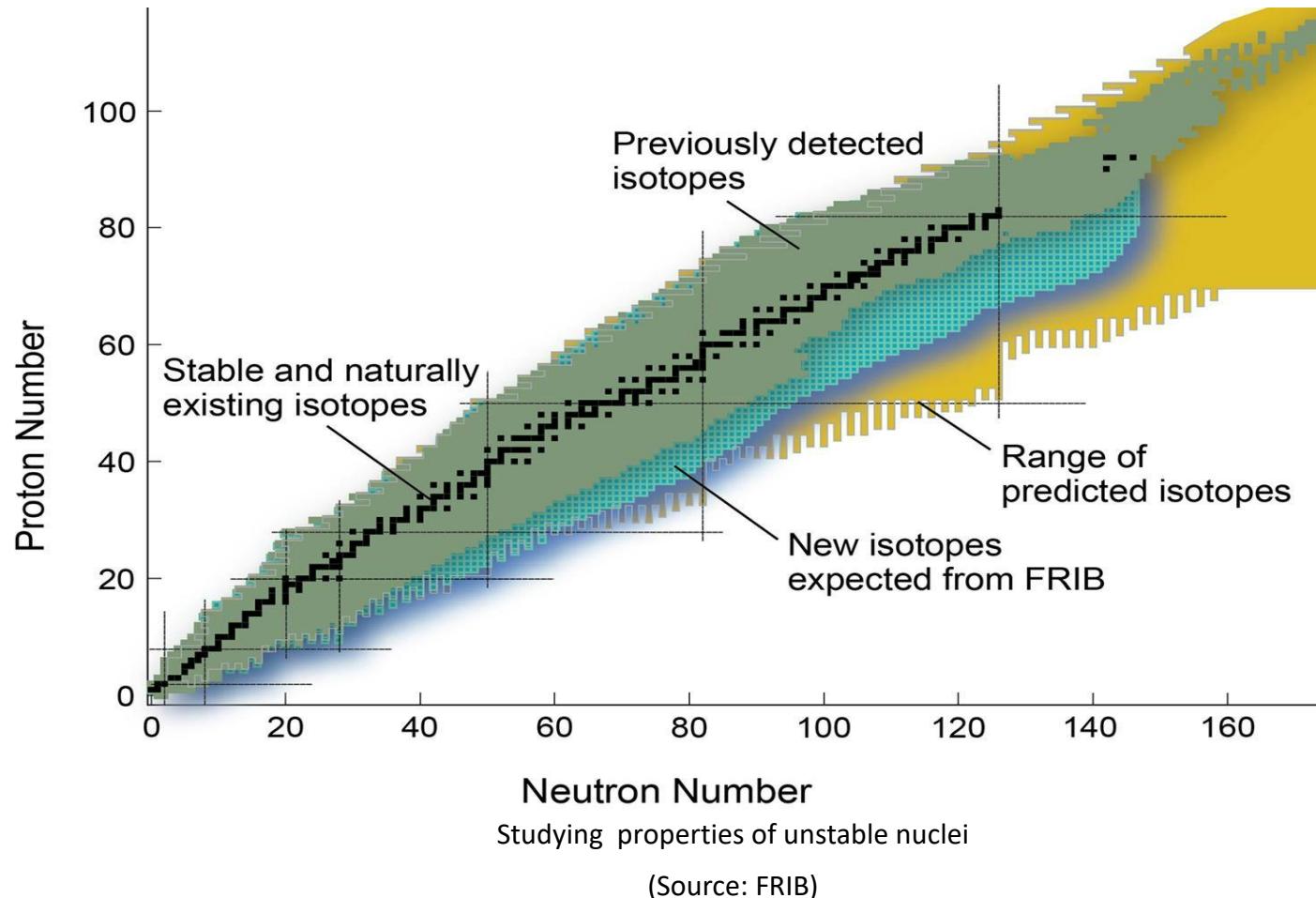
This will allow to explore the structure of nuclei away from stability, and potentially new exotic phenomena



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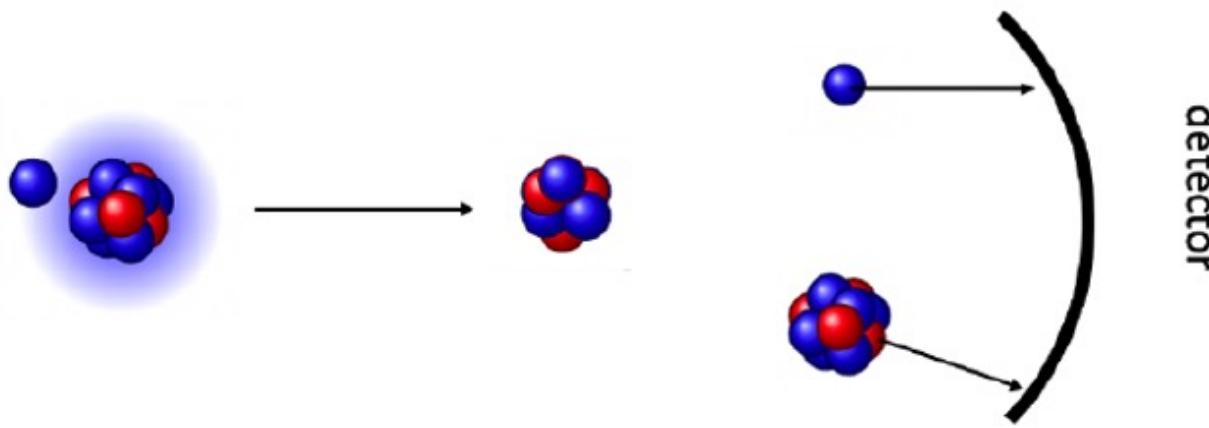


This will allow to explore the structure of nuclei away from stability, and potentially new exotic phenomena

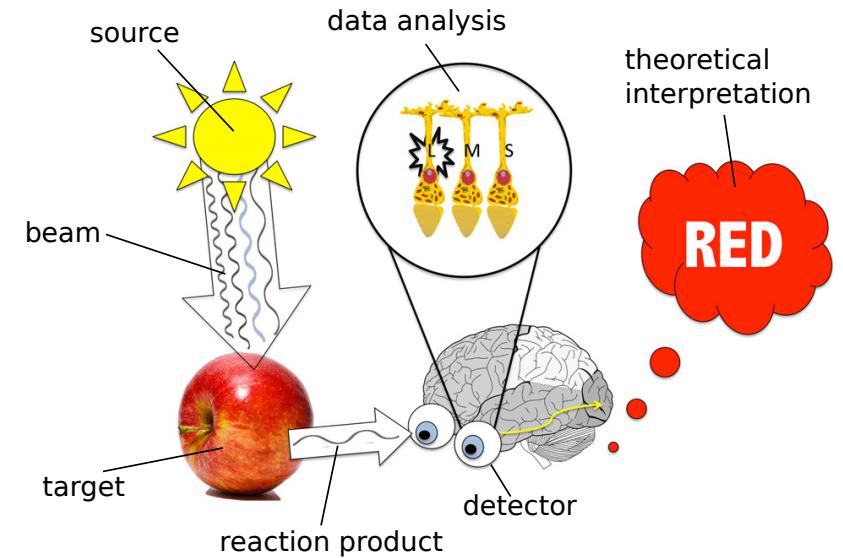


Will we find more exotic systems, in the mid and heavy mass regions?

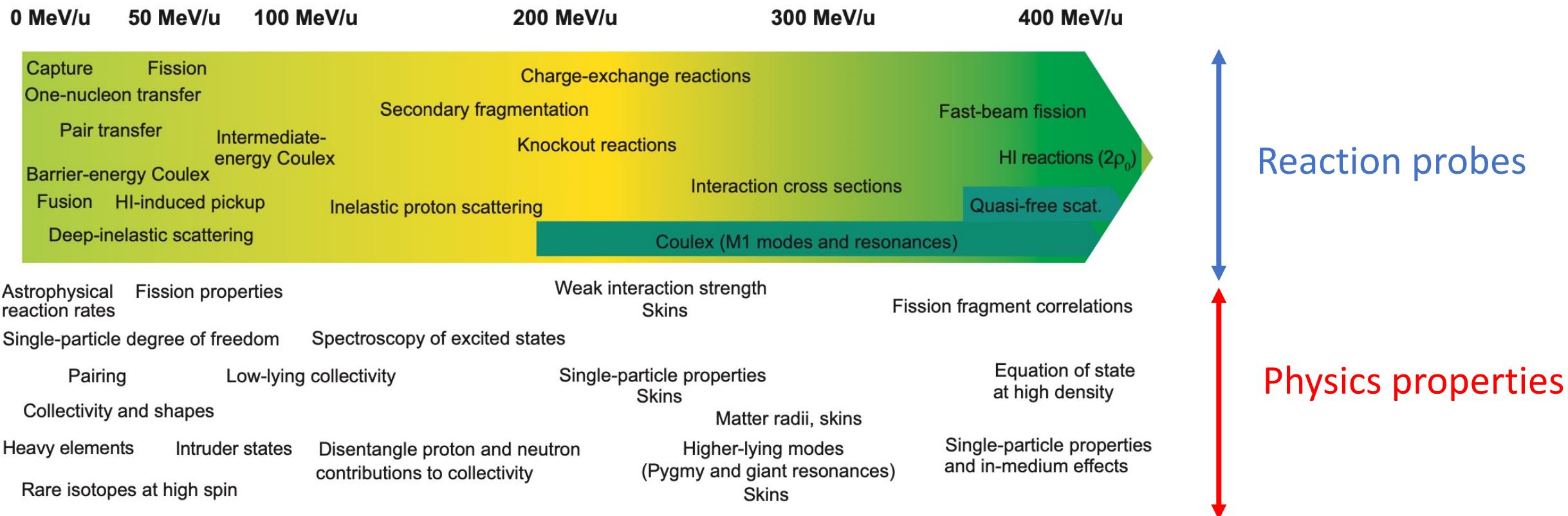
Being too short lived, unstable nuclei are often produced and studied through reactions



Often at energy $\sim 5A$ MeV to 400 MeV



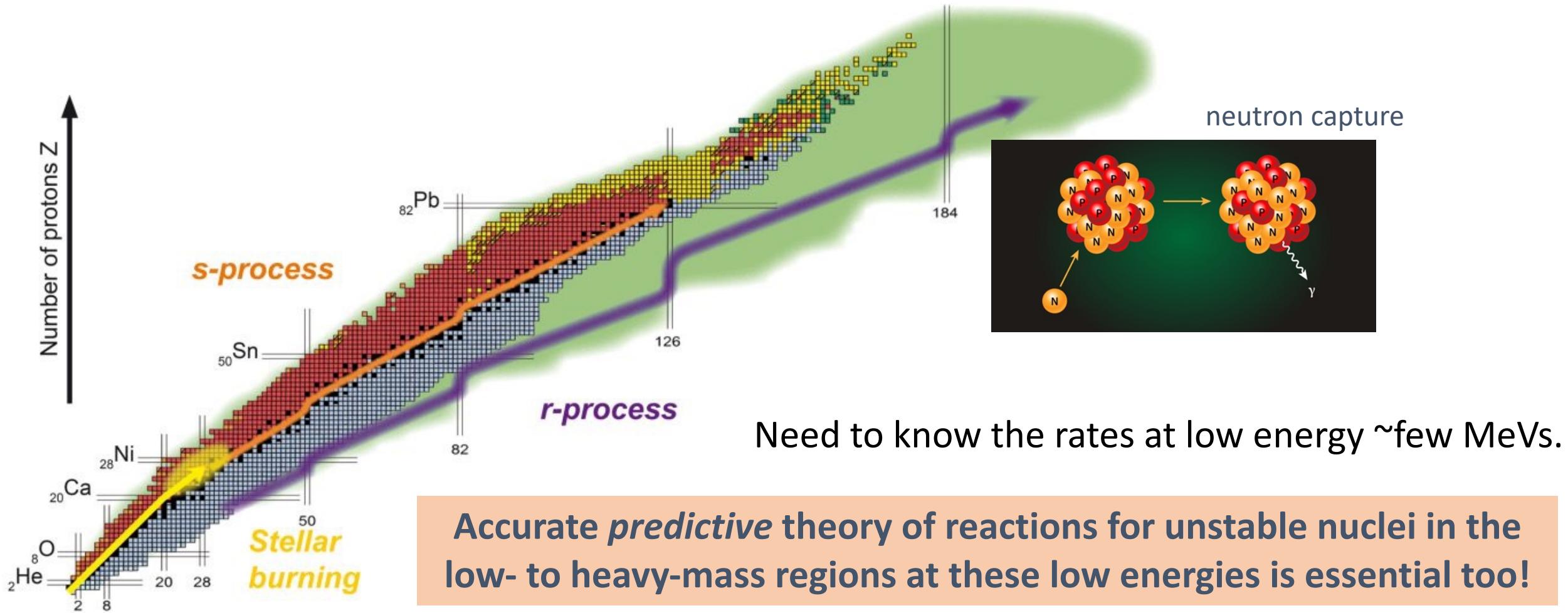
Many different reactions to probe different properties



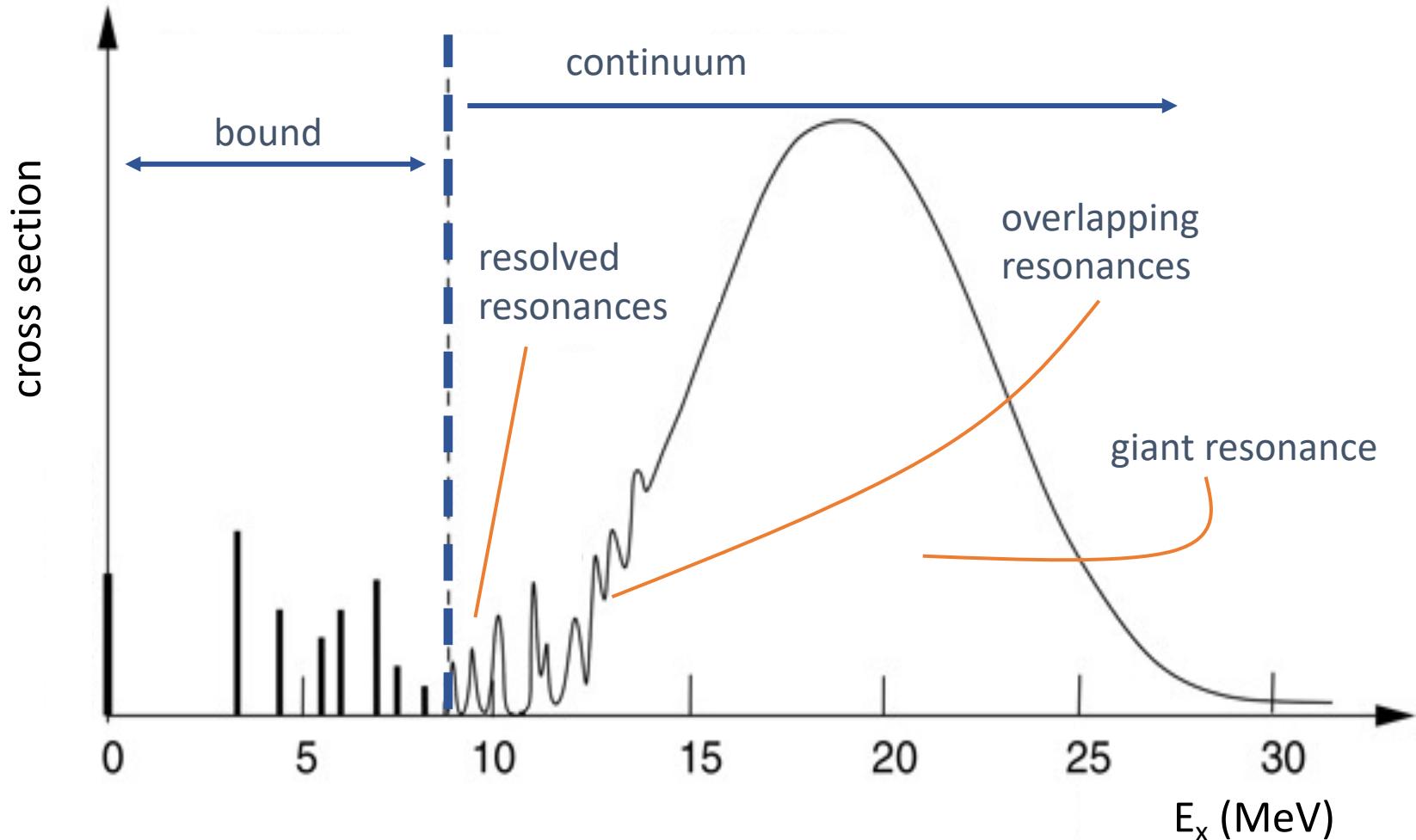
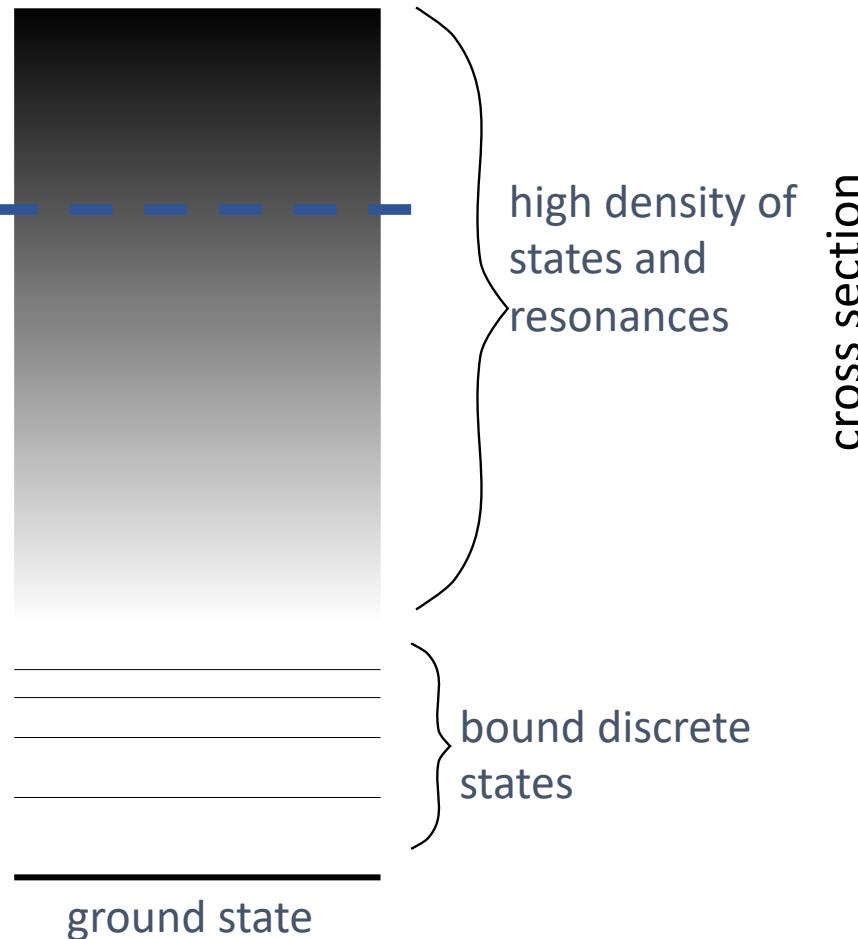
[FRIB 400 whitepaper]

Accurate theory of reactions for unstable nuclei in the low- to heavy-mass regions at *all these energies* are crucial!

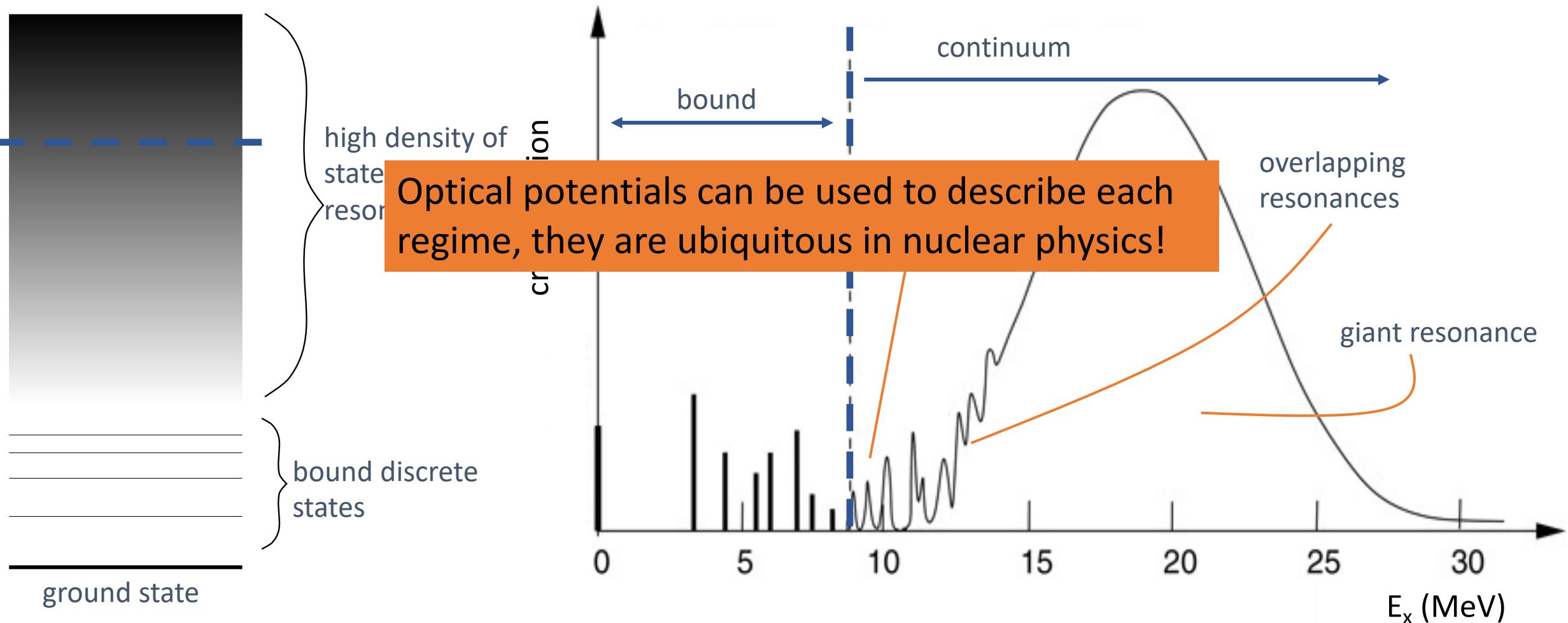
Studying the properties of unstable systems will also help us constrain reactions of astrophysical interest



Need to predict accurately a wide range of reactions using a unified model... an impossible challenge!



Need to predict accurately a wide range of reactions using a unified model... an impossible challenge!



Outline of this talk

- **What is an optical potential?
& why it is difficult to build an *accurate* one using ab initio theory?**
- Status of optical potentials:
 - Main techniques to construct them
 - How do they perform for various probes?
 - What are their uncertainties?
- What tools exist for reaction calculations and what can we do to reduce the uncertainties associated with optical potentials?

Every reaction is formally solution to a many body problem

- Many-body Hamiltonian

$$H \Psi(\vec{r}, \vec{r}_1, \dots, \vec{r}_A) = E_{tot} \Psi(\vec{r}, \vec{r}_1, \dots, \vec{r}_A)$$

$$\text{with } H = H_A(\vec{r}_1, \dots, \vec{r}_A) + h_0 + V(\vec{r}, \vec{r}_1, \dots, \vec{r}_A)$$



- One can rewrite this equation by projecting on the elastic channel ϕ_0

$$(h_0 + V_{opt})\phi_0 = E\phi_0$$

[Feshbach RMP **36** 1076 (1964)]

by defining the optical potential

$$V_{opt} = V_{00} + \sum_{j,k \neq 0} V_{0j} \frac{1}{E_{jk}^+ - H} V_{k0}$$

Many body propagator

$$G_{jk}^+ = \frac{1}{E_{jk}^+ - H}$$

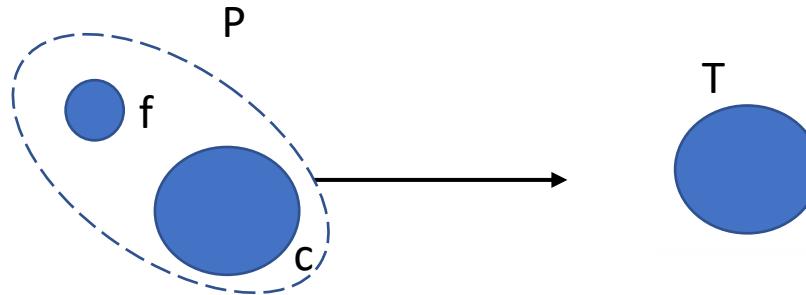
→ Need to know the many body dynamics, i.e., properties of all states up to $\sim E$

- This projection has a cost: optical potentials are **E dependent, complex and non local** ☺

Often we are using these optical potentials in three body models to study the structure of nuclei

Goal: study the projectile

(its spectroscopic factors, ANCs, binding energy,etc)



Projectile seen as core+fragment

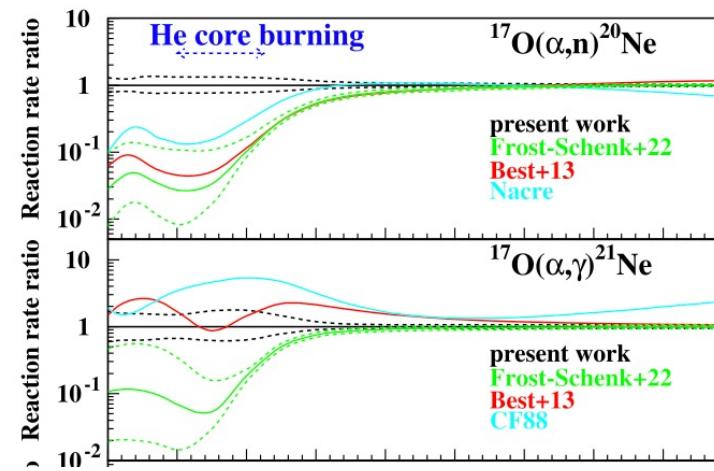
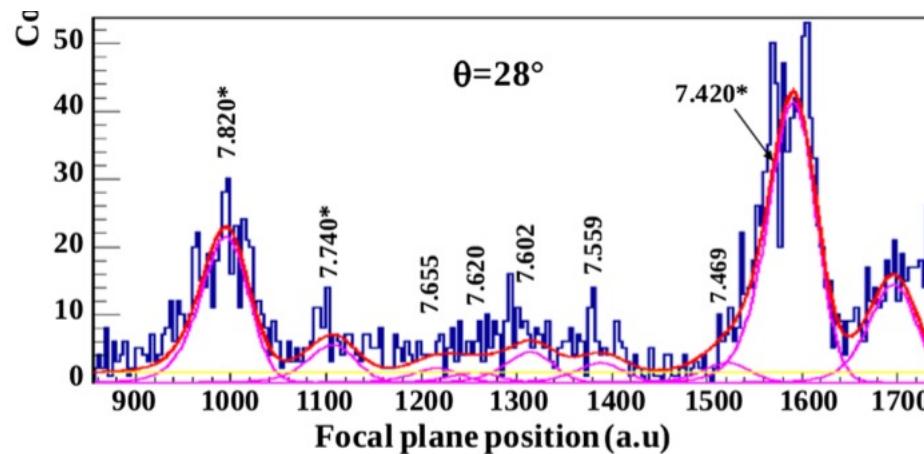
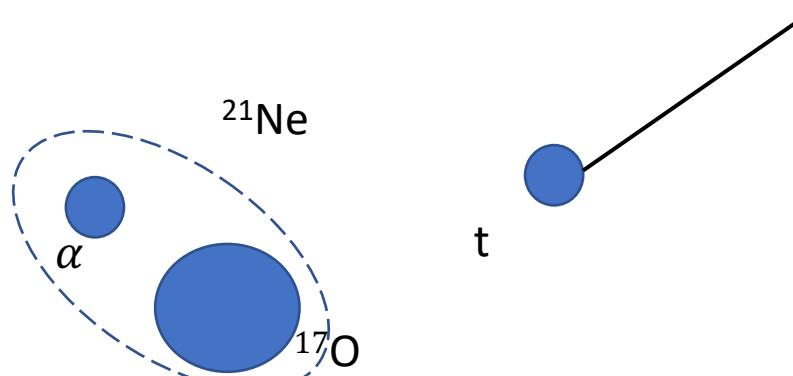
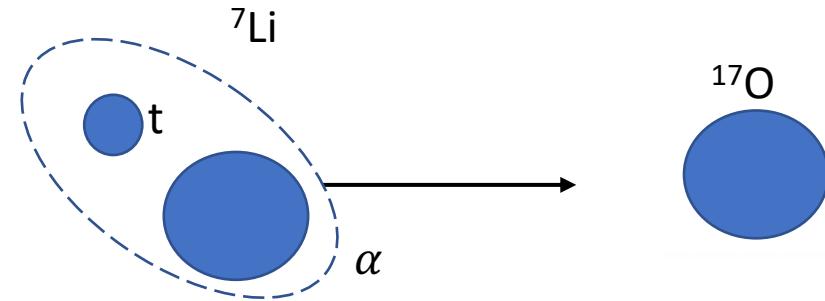
Solve the 3-body problem with 3-b Hamiltonian:

$$H^{3b} \approx T_{cT} + T_{fT} + V_{opt}^{cT} + V_{opt}^{fT} + h_P$$

(+3-b force almost always neglected)

Example of experimental study

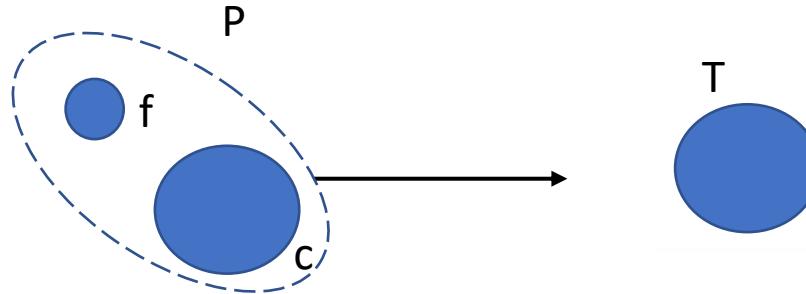
Use of $^{17}\text{O}(^{7}\text{Li}, \text{t})^{21}\text{Ne}$ reaction to constrain $^{17}\text{O}(\alpha, \text{n})^{20}\text{Ne}$ and $^{17}\text{O}(\alpha, \gamma)^{21}\text{Ne}$ astrophysical rates [Hammache *et al.* PRL **132** 182701 (2024)]



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(its spectroscopic factors, ANC_s, binding energy,etc)



Projectile seen as core+fragment

$$H^{3b} \approx T_{cT} + T_{fT} + V_{opt}^{cT} + V_{opt}^{fT} + h_p \quad (+3\text{-}b \text{ force almost always neglected})$$

→ To study accurately the **projectile**, one needs to know accurately the **optical potentials**
(and have an accurate three-body solver)

→ Note that the Hamiltonian is defined up to a unitary transformation..

→ need consistency between **optical potentials** and **the projectile description**

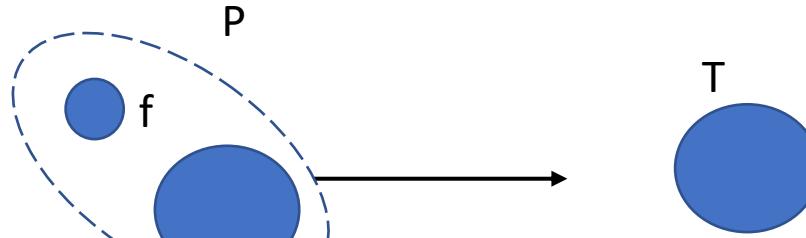
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Goal: study the projectile

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Projectile seen as core

$$H^{3b} \approx T_{cT} + T_{fT} + V_{opt}^c + V_{opt}^f + V_P \quad (V_P \text{ is often almost always neglected})$$



Can we construct these optical potentials starting from their exact formulation and microscopic theories, e. g. ab initio methods?

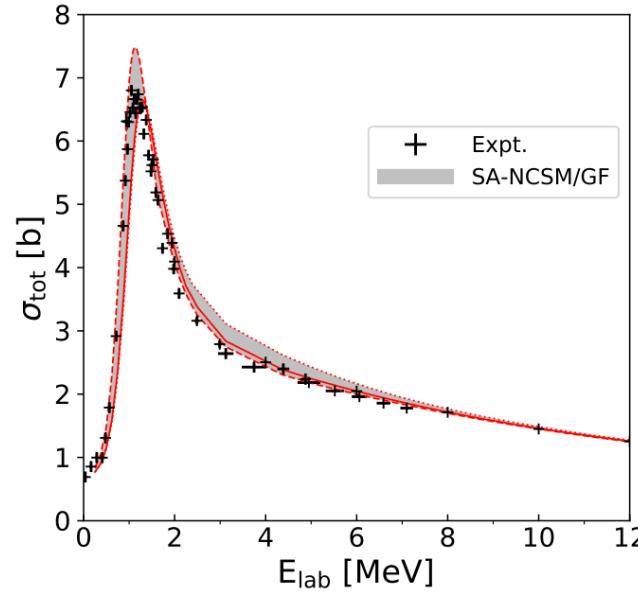
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Two examples of accurate ab initio predictions of reactions computing many-body propagators

From no-core-shell model
 $^4\text{He}(\text{n},\text{n})^4\text{He}$ at low E



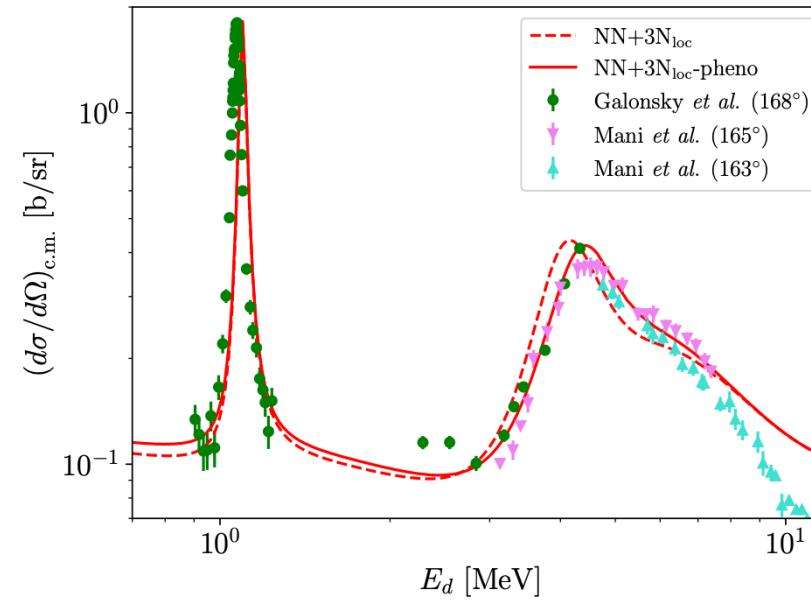
[Burrows *et al.* PRC **109** 014616 (2024)]

Two examples of accurate ab initio predictions of reactions computing many-body propagators

From no-core-shell model

$^4\text{He}(n,n)^4\text{He}$ at low E

From no-core shell model
with continuum



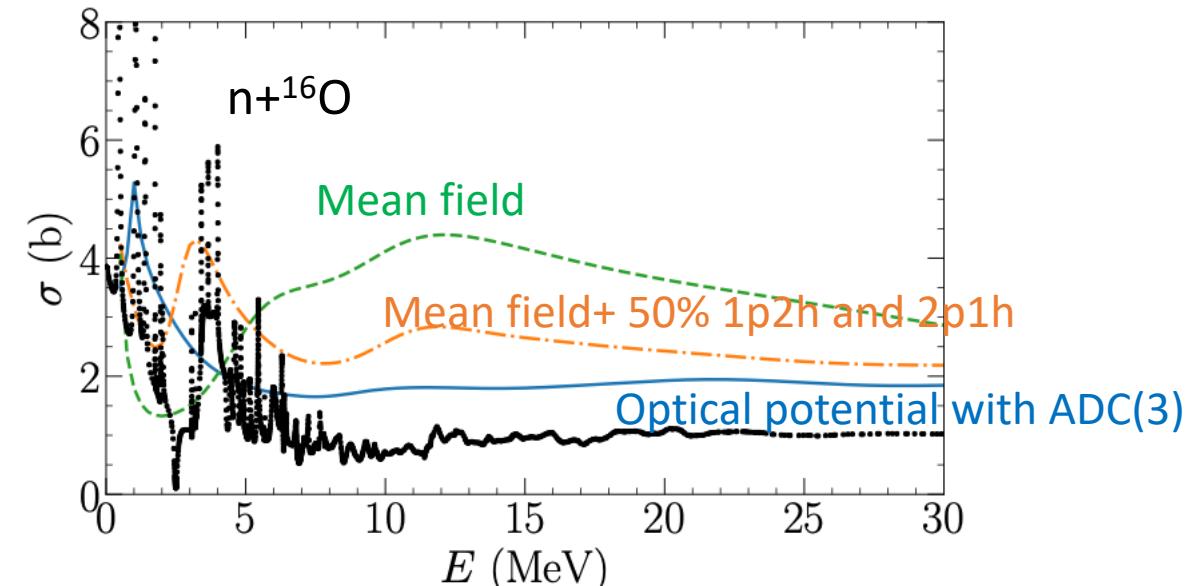
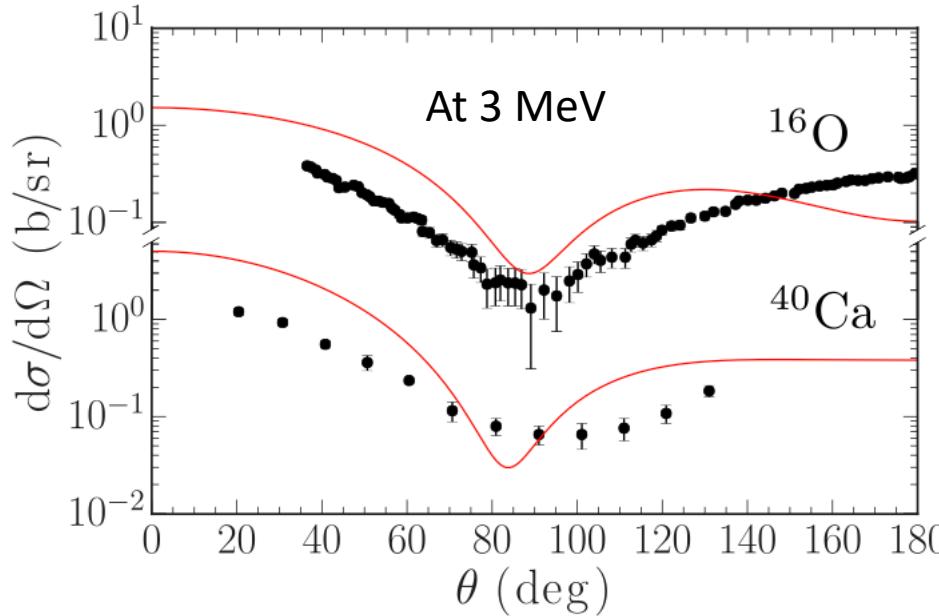
$^4\text{He}(d, d)^4\text{He}$

[Hebborn *et al.* PRL **129** 042503
(2022)]

This work well, why are we not doing this for all reactions?

What happens for heavier systems?

Ab initio potentials from SCGF [Idini *et al.* PRL **123** 092501 (2019)]



Limited accuracy → lack of absorption due to missing correlations

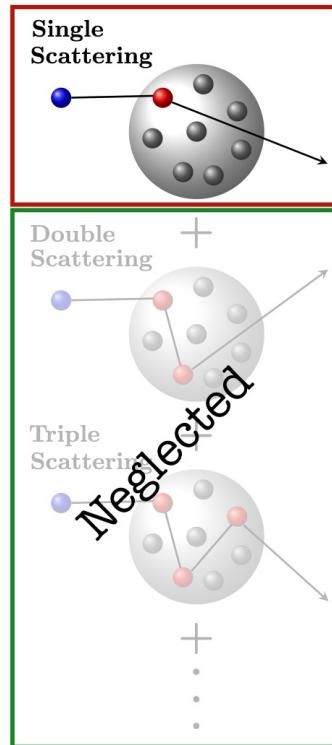
What do we do in the meantime? *Like everything that is too hard to solve from first principles, we rely on approximations & phenomenology ☺*

Use of the impulse approximation to develop the Watson multiple scattering theory

Work from the Lippman-Schwinger equation $T = V + VG - T$

With G many body propagator and V projectile-target interaction

Spectator expansion:



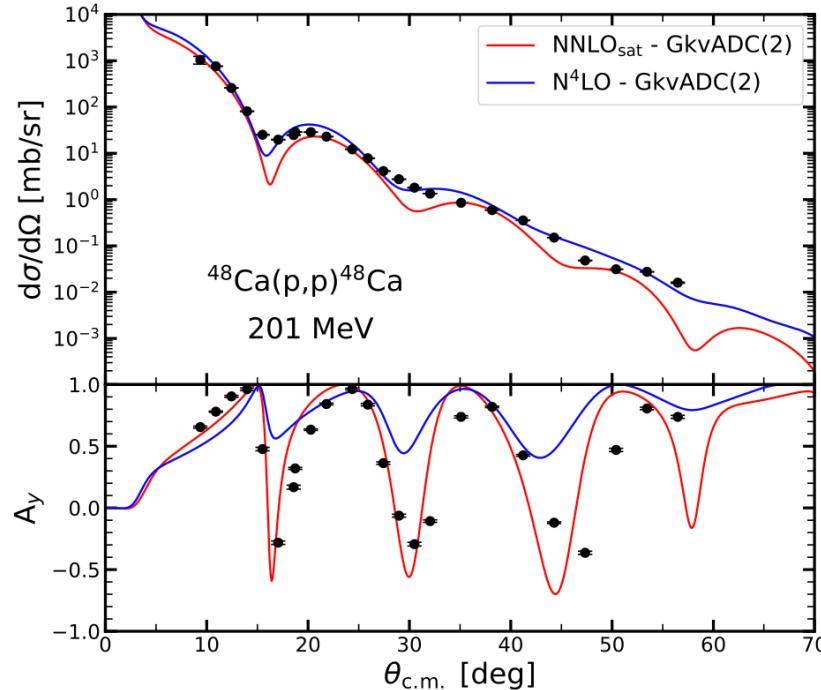
In practice:

- truncate at first order: folding density and t_{NN}
- recently: using **ab initio density** and **chiral-eft interaction**

Vorabbi (Surrey), Elster (Ohio University),
Durant/Capel (Mainz), etc

Picture from Vorabbi's slides

Ab initio Watson multiple scattering theory: recent results



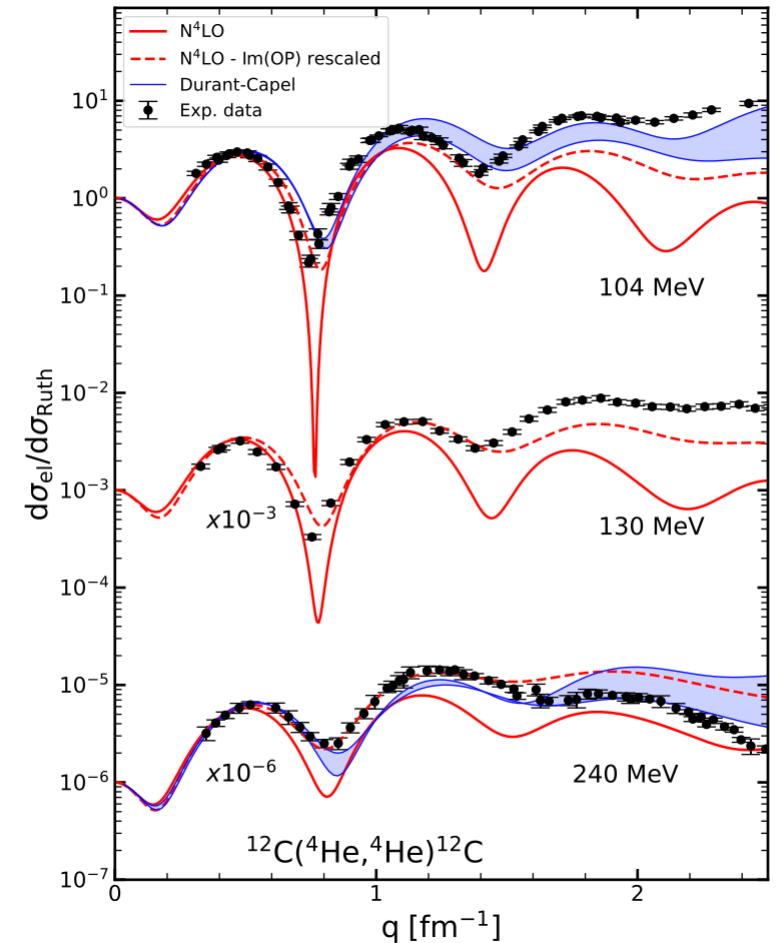
Vorabbi, ..., Soma, *et al.*
PRC **109** 034613 (2024)

Works well at high energy
for nucleon-nucleus
(accuracy of chiral EFT?)

Difficult to go to
higher order...

Some works on the way
to do nucleus-nucleus
potential...

→ some challenges
& developments ahead

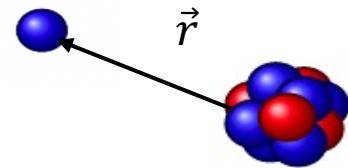


Vorabbi *et al.* PRL **135** 172501 (2025)

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Three main approaches to construct optical potentials



Ab initio & microscopic theories



Phenomenological approach

Interactions directly obtained from many-body Hamiltonian

A priori should be accurate to extrapolate away from stability (if ab initio theory accurate)

Difficult to have all many-body correlations -> lack absorption at energies (where MST not valid)

Phenomenological approach

- Local parametrization

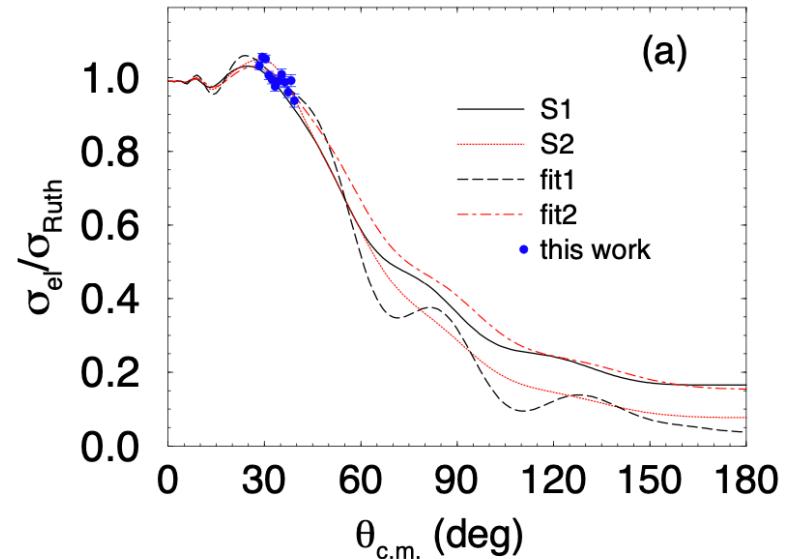
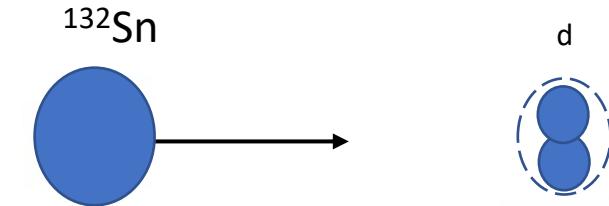
Typically local Wood-Saxon parametrization,

$$U_{opt} = V_R f_{WS}(r_R, a_R) + i W_I f_{WS}(r_I, a_I)$$

fit for one energy and one target

→ often use when elastic data are available!

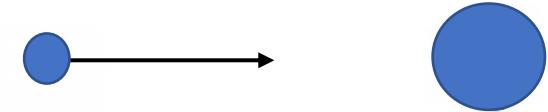
→ Angular range often limited by inverse kinematics



Fit of $d(^{132}\text{Sn}, ^{132}\text{Sn})d$ cross sections to analyze
 $d(^{132}\text{Sn}, ^{133}\text{Sn})p$ Jones, Nunes, et al. PRC **84**, 034601 (2011)

Comes with **uncertainties** and **lack predictive power**... and for most unstable nuclei, no elastic data available...

Phenomenological approach: global parametrizations



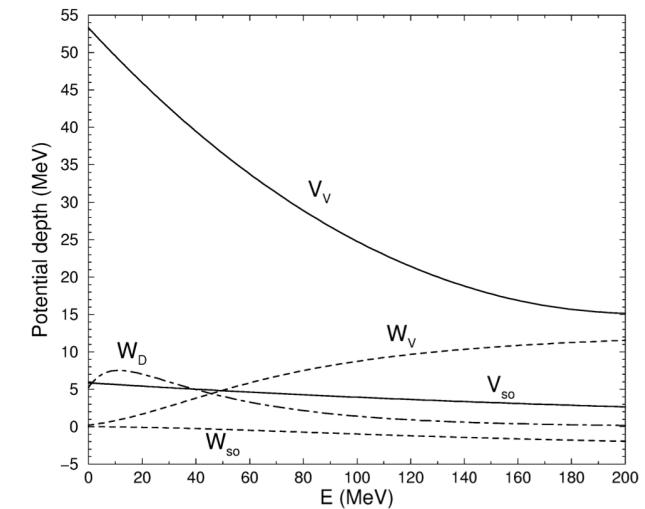
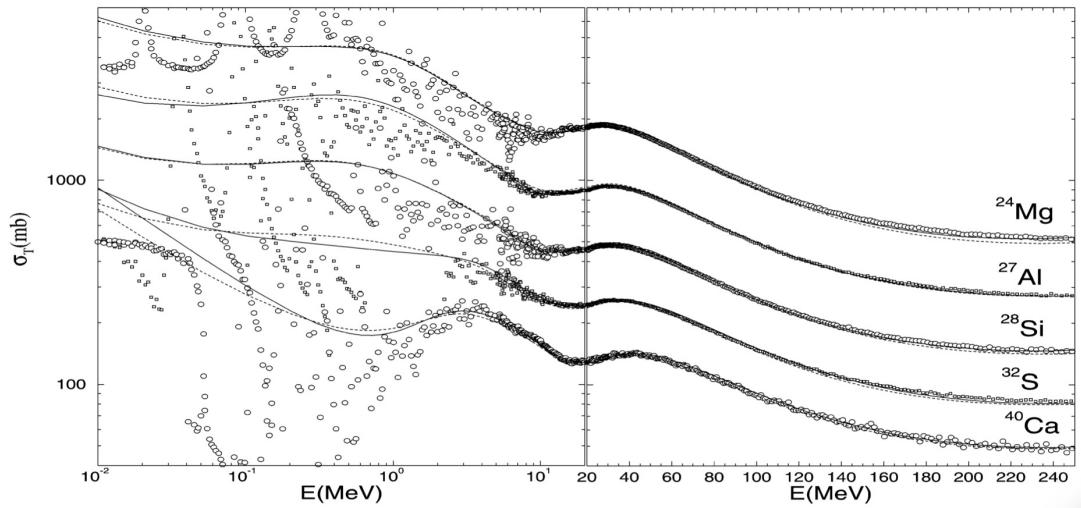
- Fit to elastic, reaction cross section, polarization data, etc

- Global parametrization
 Typically local,
 spherical,
 L-independent,
 isospin dependent,
 but strongly E-dependent

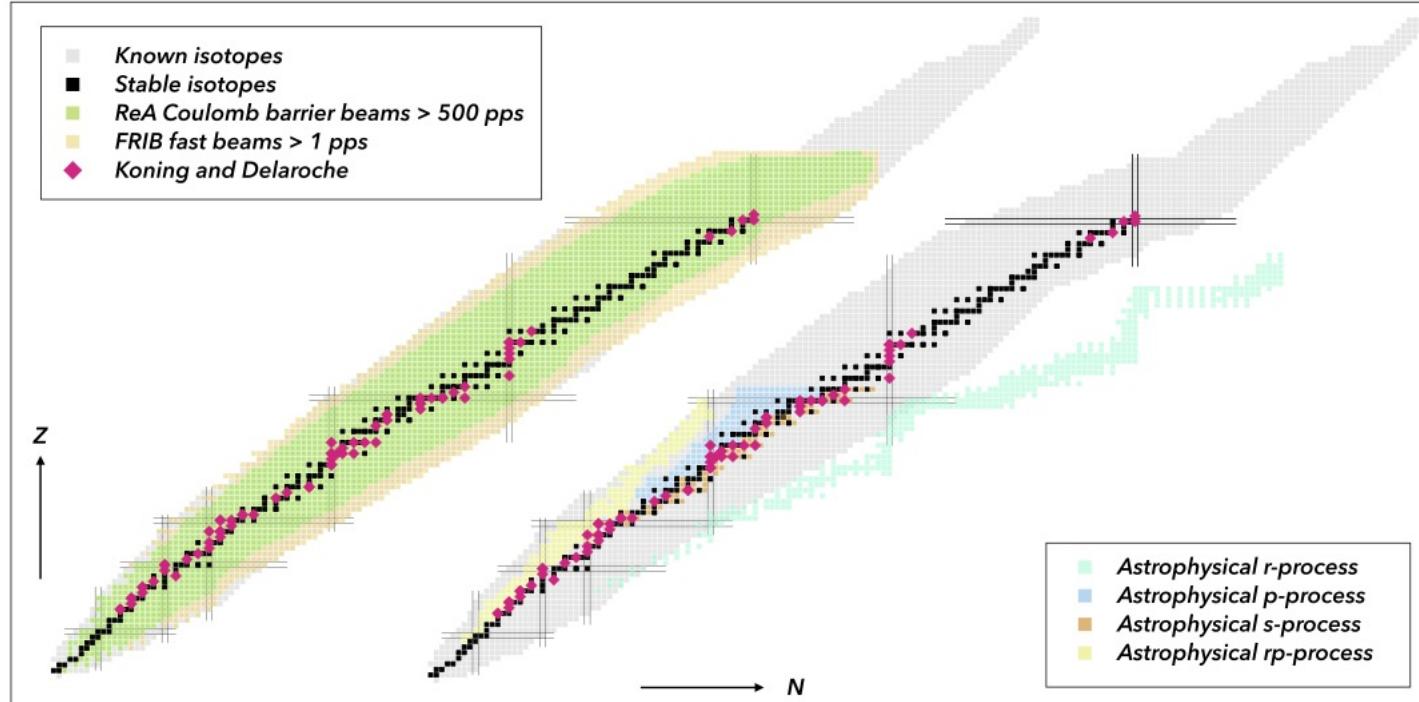
$$\begin{aligned}
 \mathcal{V}_V(r, E) &= V_V(E) f(r, R_V, a_V), \\
 \mathcal{W}_V(r, E) &= W_V(E) f(r, R_V, a_V), \\
 \mathcal{W}_D(r, E) &= -4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D), \\
 \mathcal{V}_{SO}(r, E) &= V_{SO}(E) \left(\frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r, R_{SO}, a_{SO}), \\
 \mathcal{W}_{SO}(r, E) &= W_{SO}(E) \left(\frac{\hbar}{m_\pi c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r, R_{SO}, a_{SO}).
 \end{aligned}$$

As usual, the form factor $f(r, R_i, a_i)$ is a Woods-Saxon shape

$$f(r, R_i, a_i) = (1 + \exp[(r - R_i)/a_i])^{-1},$$



Phenomenological approach: global parametrizations, fitted to stable nuclei...

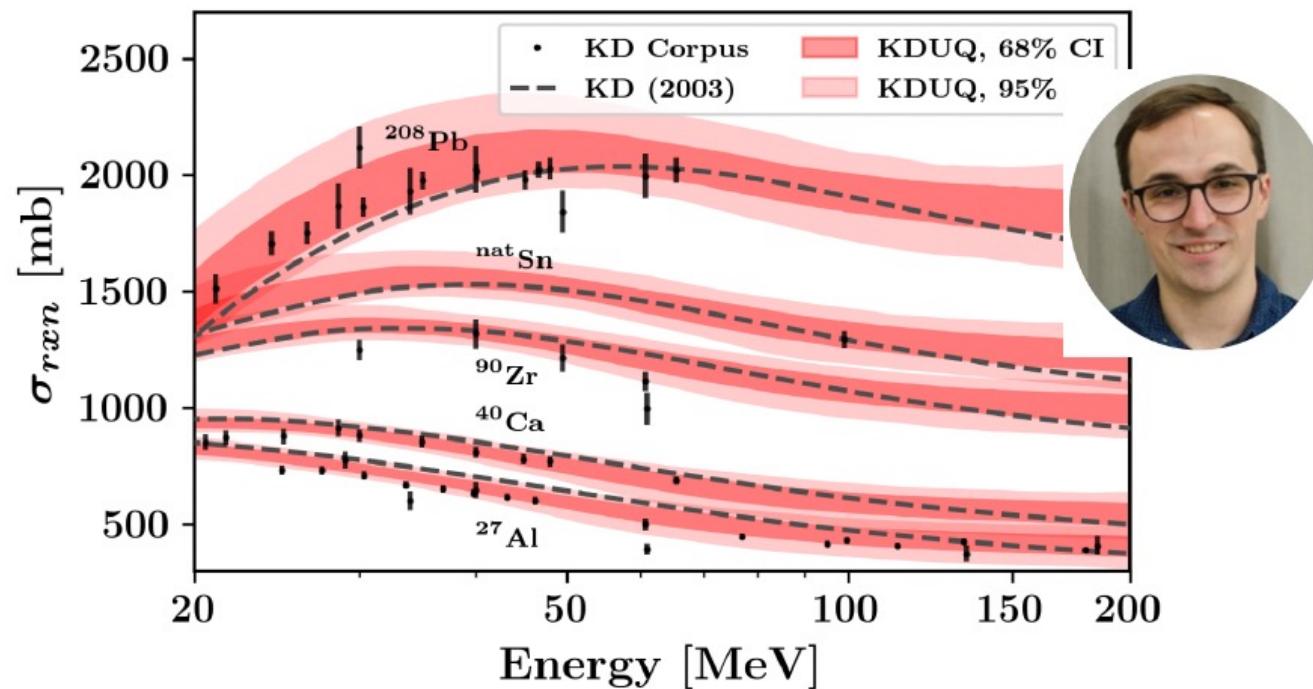


Whitepaper : JPG 50 060501 (2023)

What are their uncertainties? Does it extrapolate well away from stability?

Phenomenological approach: global parametrizations, recent effort from LLNL

KDUQ : Global fit using Bayesian statistics by Cole Pruitt



[Pruitt *et al.* PRC 107, 014602 (2023) including python scripts !]

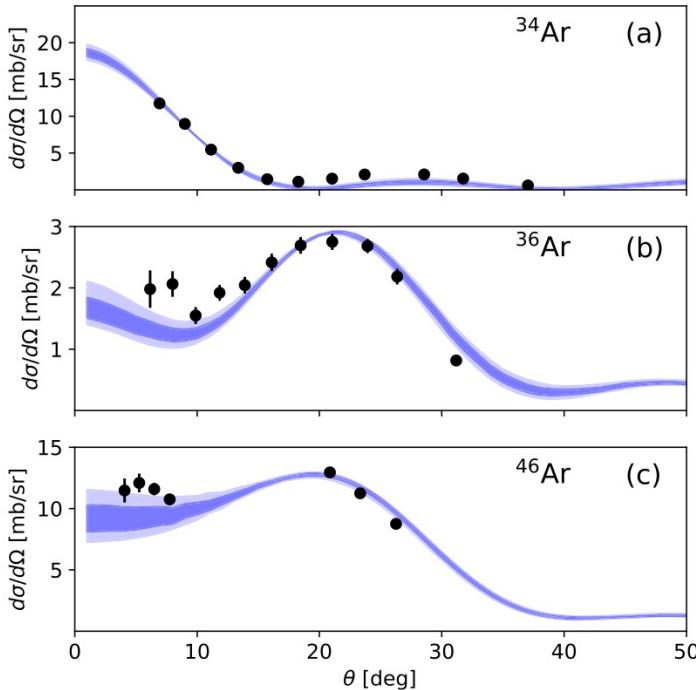
During these last years: quantification of optical potential uncertainties in different reaction channels

Motivation: support experimental efforts and assign realistic uncertainties on properties inferred from reaction measurements

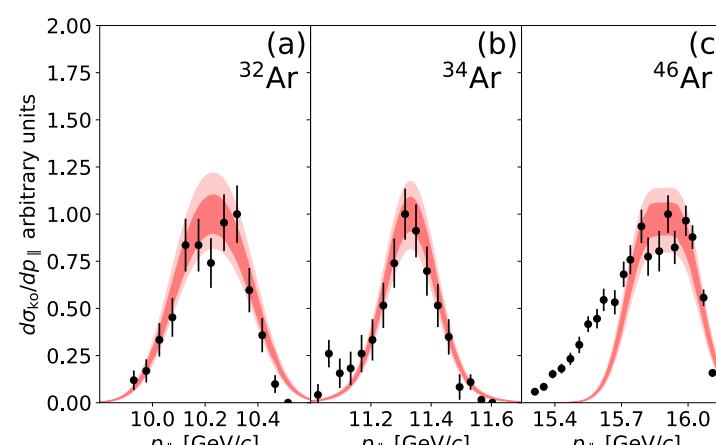
some selected examples...

During these last years: quantification of optical potential uncertainties in different reaction channels

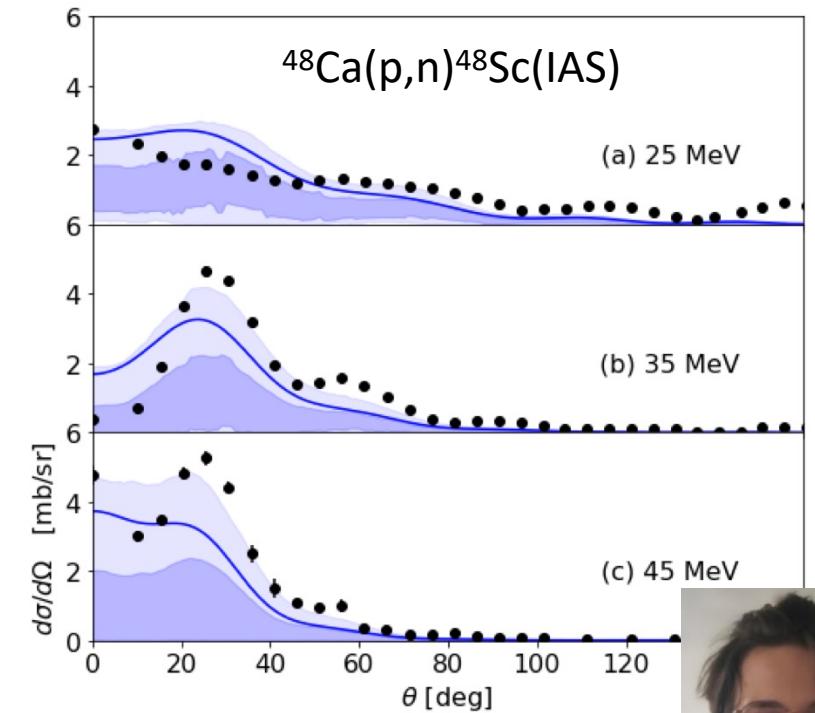
Transfer (d,p) & (p,d):
~5-10%



Heavy-ion Knockout :
~15-20%



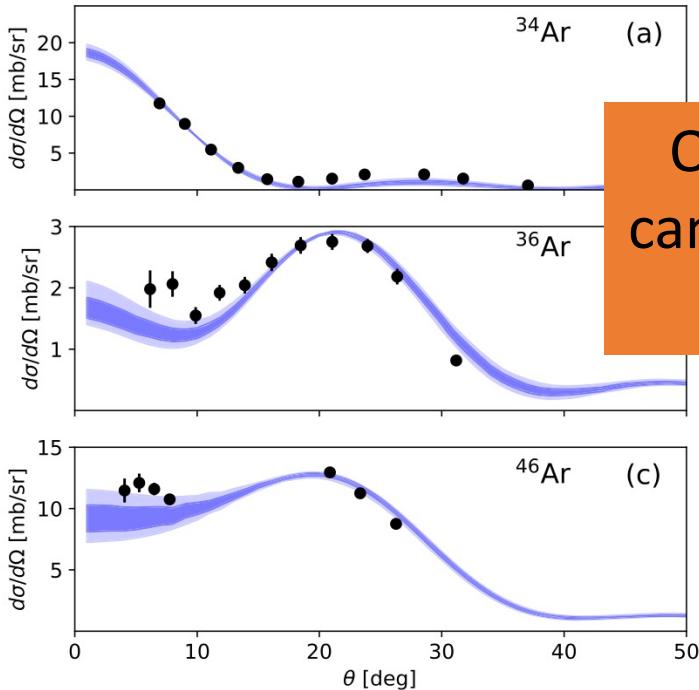
Charge-exchange (Lane formalism):
60-100%



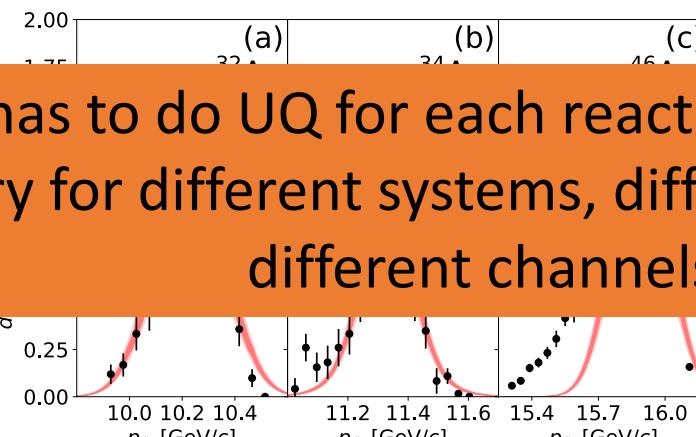
Hebborn *et al.* PRL **131**, 212503 (2023) & Frontiers in physics (2025).
Smith *et al.* PRC **110**, 034602 (2024).

During these last years: quantification of optical potential uncertainties in different reaction channels

Transfer (d,p) & (p,d):
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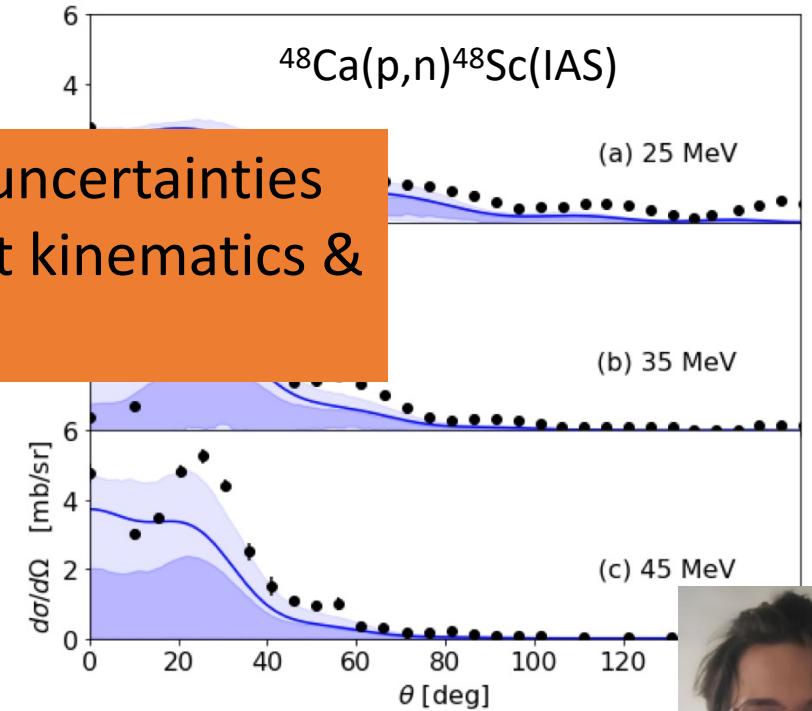


Heavy-ion Knockout :
~15-20%



One has to do UQ for each reaction: uncertainties can vary for different systems, different kinematics & different channels...

Charge-exchange (Lane formalism):
60-100%



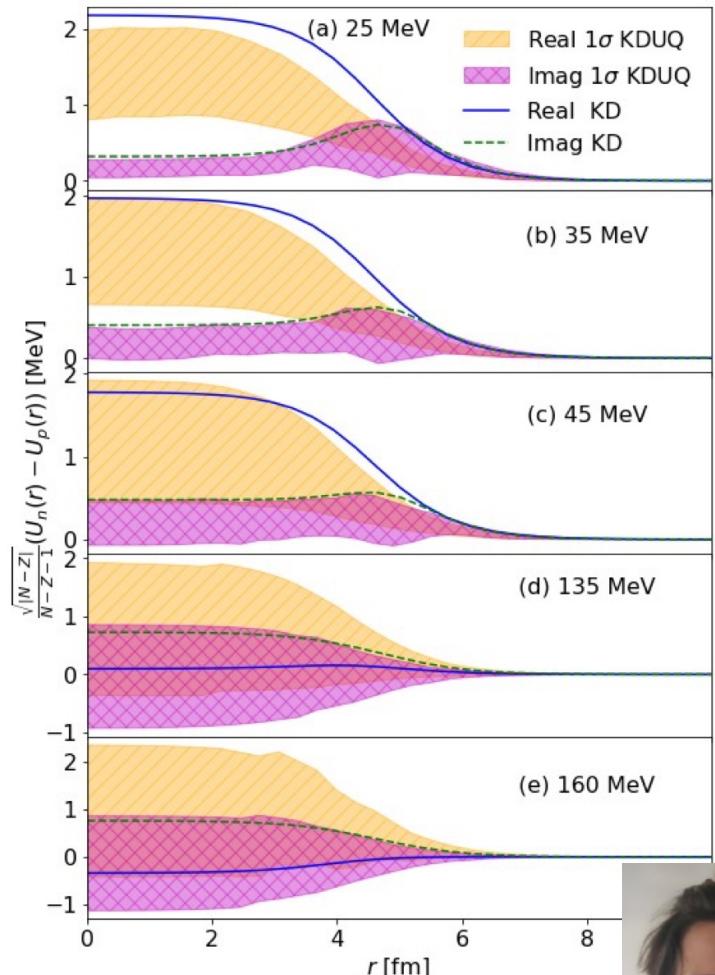
Hebborn *et al.* PRL **131**, 212503 (2023) & Frontiers in physics (2025).
Smith *et al.* PRC **110**, 034602 (2024).

Why charge-exchange uncertainties are so large?

For (p,n) reactions, the chex to the IAS can be obtained as (Lane formalism)

$$\sigma_{(p,n)} \propto | \langle \Psi_n | U_n - U_p | \psi_p \rangle |^2$$

→ operator depends on the **isospin dependence** of the optical potentials... **poorly constrained..**



Smith *et al.* PRC 110, 034602 (2024).

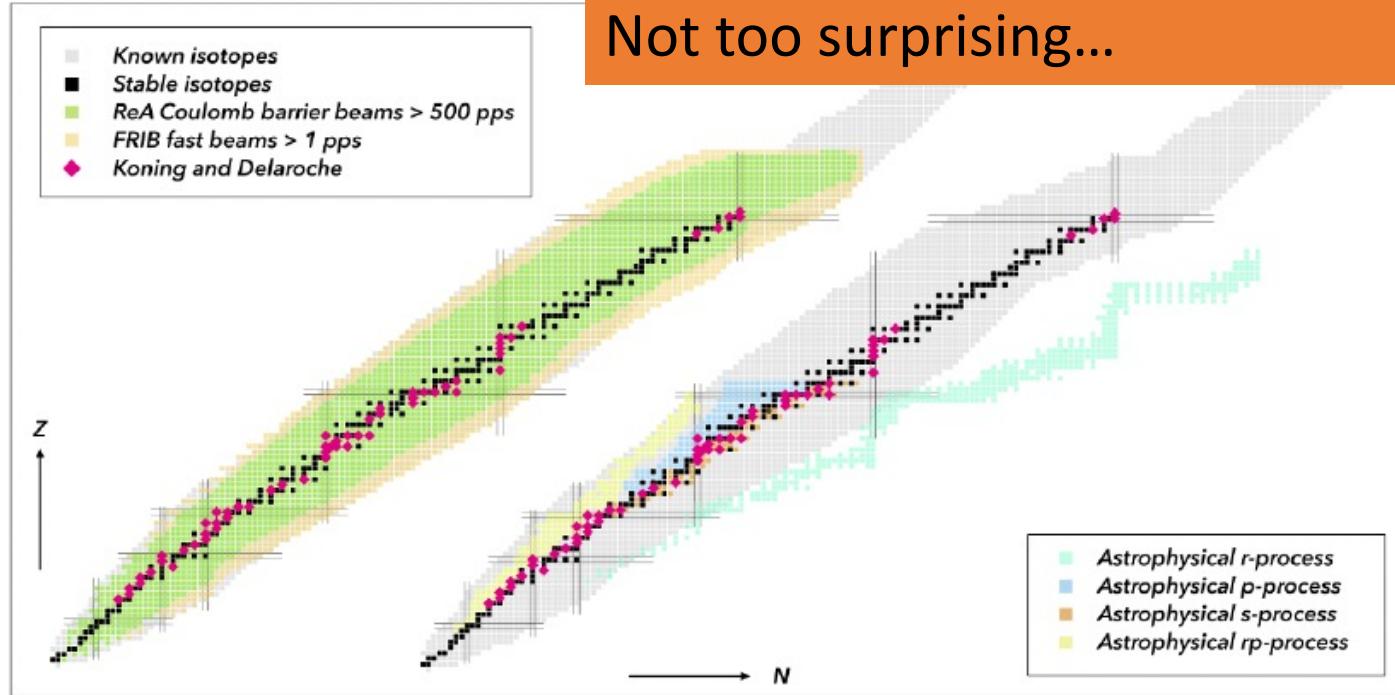


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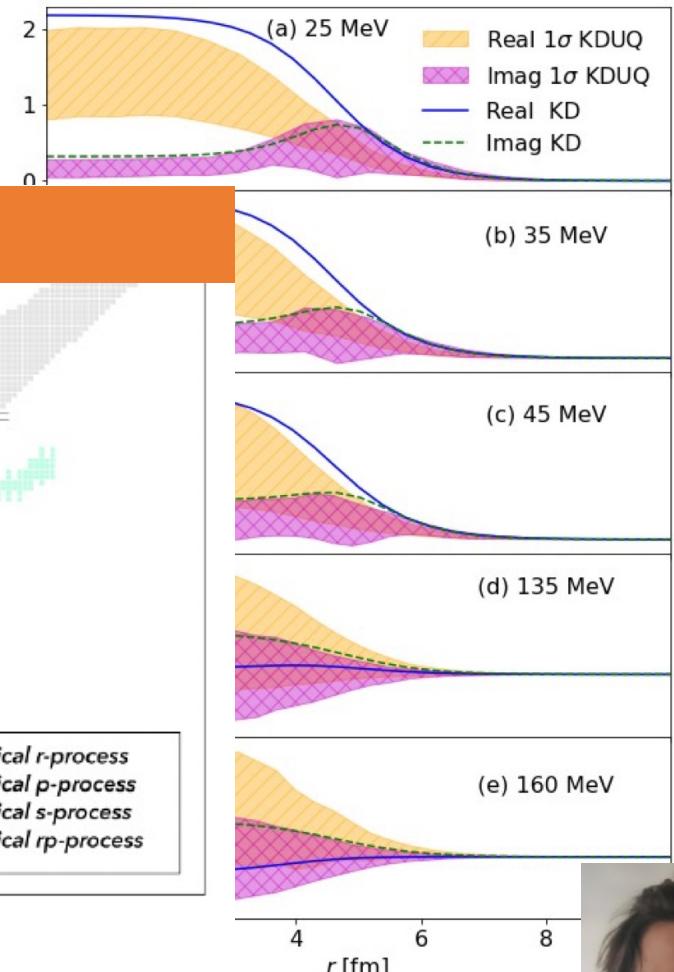
$$\sigma_{(p,n)} \propto | \Psi |^2$$

→ operator dependence of the optical potential



Whitepaper : JPG 50 060501 (2023)

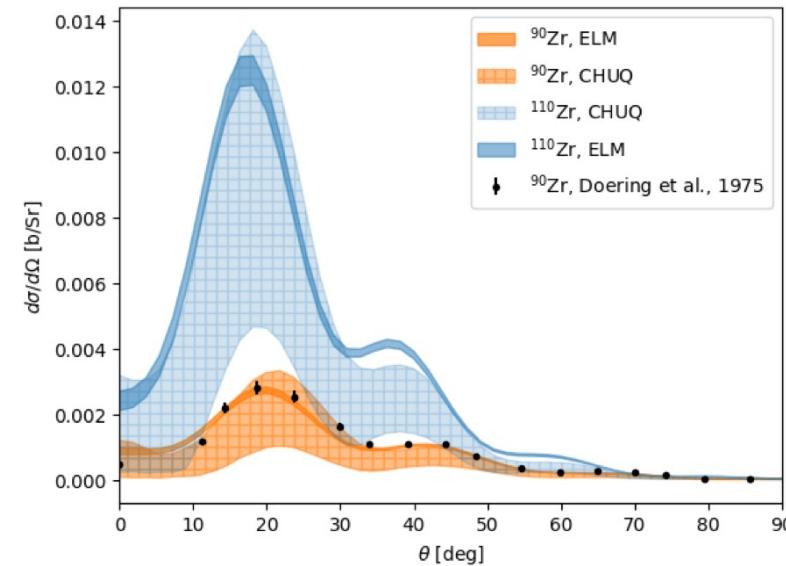
Journal of Physics: Conference Series, 034602 (2024).



How to reduce the uncertainties of purely phenomenological approaches?

Different possibilities (and different groups pursue different paths):

- Add different reaction channels in the fit...

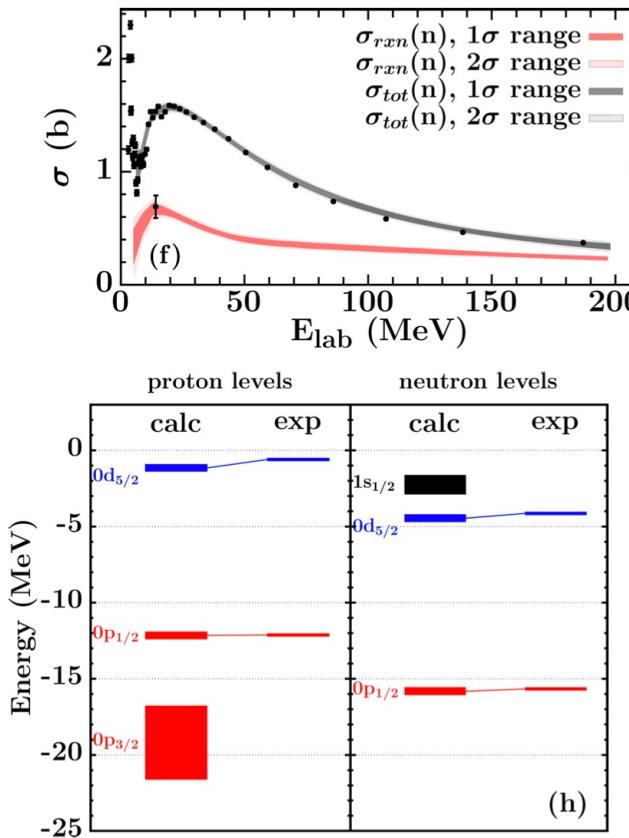


[Kyle Beyer \(postdoc at MSU\): talk at IJClab](#)

How to reduce the uncertainties of purely phenomenological approaches?

Different possibilities (and different groups pursue different paths):

- Add different reaction channels in the fit...
- Add physics constraints
dispersion relation (to impose causality)
inclusion bound and scattering data



Dispersive
optical model
for the oxygen
chain

[Pruitt *et al.* PRC
102 034601 (2020)]

How to reduce the uncertainties of purely phenomenological approaches?

Different possibilities (and different groups pursue different paths):

- Add different reaction channels in the fit...
- Add physics constraints
dispersion relation (to impose causality)
inclusion bound and scattering data
- Choose more physics-informed
parametrization (what is the shape of non-locality?)

PHYSICAL REVIEW C 109, 064609 (2024)

Universal separable structure of the optical potential

H. F. Arellano^{1,2} and G. Blanchon^{2,3}

[Seminar by Guillaume at IJClab Oct 1](#)

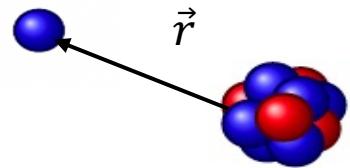
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 - dispersion relation (to impose causality)
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- Choose more physics-informed
 - parametrization (what is the shape of non-locality?)
- ...

Stay tuned!

Three main approaches to construct optical potentials



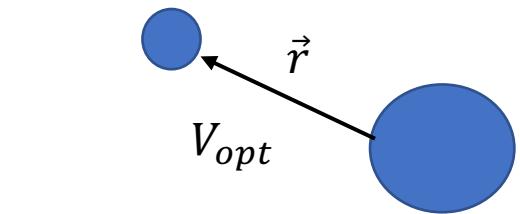
Ab initio & microscopic theories

Interactions directly obtained from many-body Hamiltonian

A priori should be accurate to extrapolate away from stability (if ab initio theory accurate)

Difficult to have all many-body correlations -> lack absorption at energies (where MST not valid)

Semi-microscopic approach



Phenomenological approach

Fit to existing data

Ad hoc choice of phenomenological forms

unclear how it extrapolates but first studies show large uncertainties...

Semiclassical approach often relies on folding densities :
nucleon-nucleus scattering (example JLM-B)

Folding approach for nucleon nucleus

$$U_p(\mathbf{r}, E) = \int \mathcal{V}_p(|\mathbf{r} - \mathbf{r}'|, E, \rho^{(0)}, \alpha^{(0)}) \rho^{(0)}(\mathbf{r}') d\mathbf{r}'.$$

Microscopic densities

JLM-B effective interaction:

$$\begin{aligned} V_p(\rho, \alpha, E) \\ = \lambda_V(E) [V_0(\rho, E) \pm \alpha \lambda_{V_1}(E) V_1(\rho, E)] \\ + i \lambda_W(E) [W_0(\rho, E) \pm \alpha \lambda_{W_1}(E) W_1(\rho, E)]. \end{aligned}$$

Fitted to data to better reproduce experiments

Semiclassical approach often relies on folding densities :
nucleon-nucleus scattering (example JLM-B)

Folding approach for nucleon nucleus

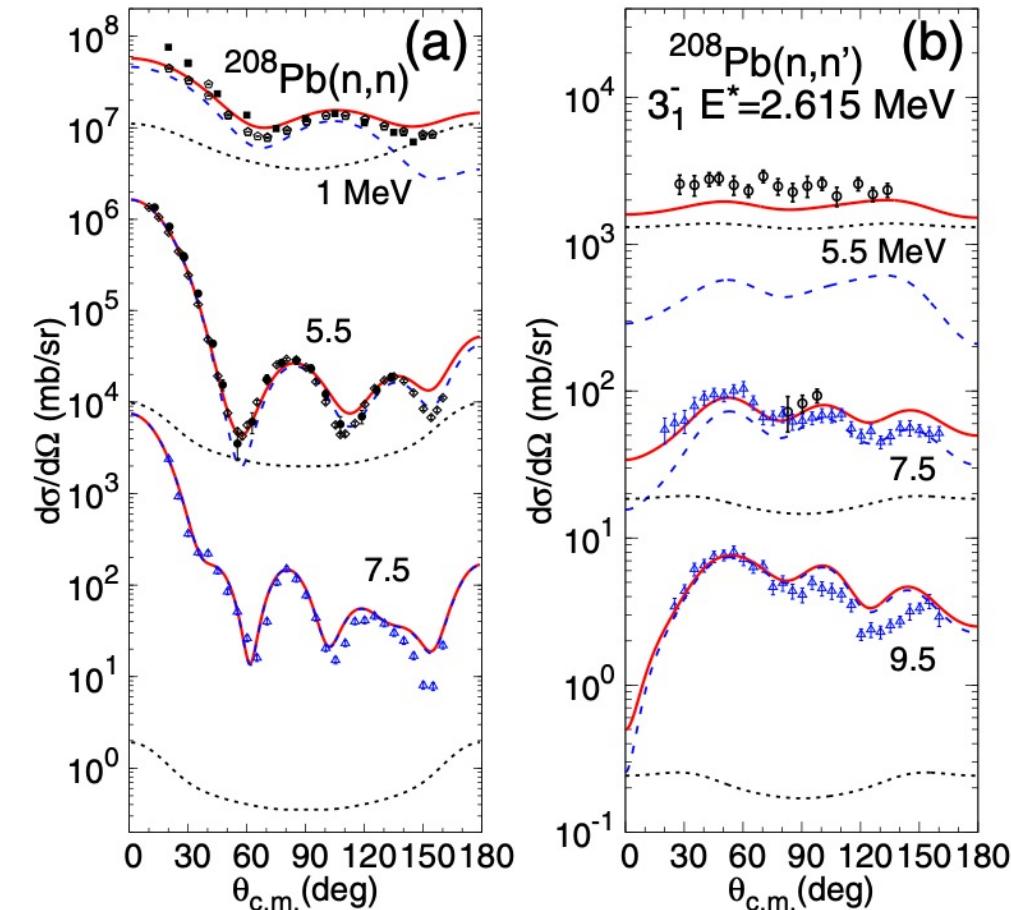
$$U_n(\mathbf{r}, E) = \int \mathcal{V}_n(|\mathbf{r} - \mathbf{r}'|, E, \rho^{(0)}, \alpha^{(0)}) \rho^{(0)}(\mathbf{r}') d\mathbf{r}'.$$

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Works well 😊 Uncertainties not quantified (yet?)



[Dupuis *et al.*. PRC 100 044607 (2019)]

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Mic

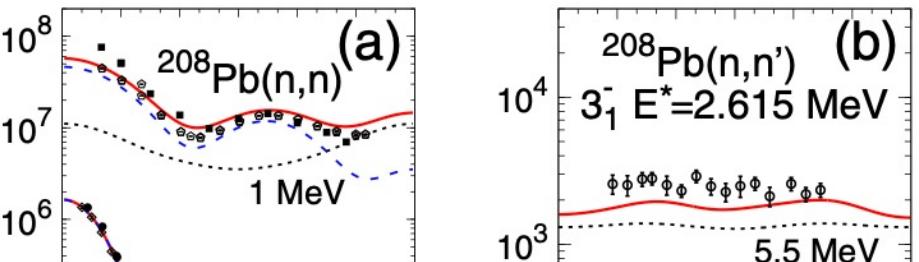
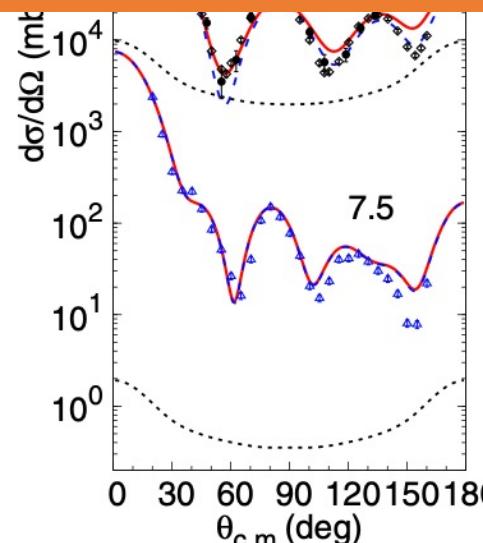
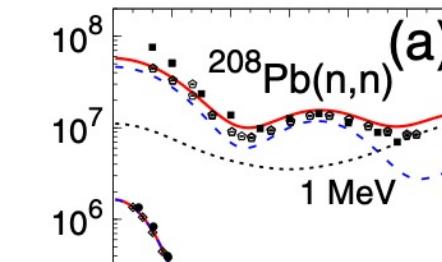
JLM-B effective int Come see Marc's seminar on Feb 5 at 2pm !

$$V_n(\rho, \alpha, E)$$

$$= \lambda_V(E) [V_0(\rho, E) \pm \alpha \lambda_{V_1}(E) V_1(\rho, E)] \\ + i \lambda_W(E) [W_0(\rho, E) \pm \alpha \lambda_{W_1}(E) W_1(\rho, E)].$$

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Semiclassical approach often relies on folding densities :
nucleus-nucleus scattering using double folding (1 example)

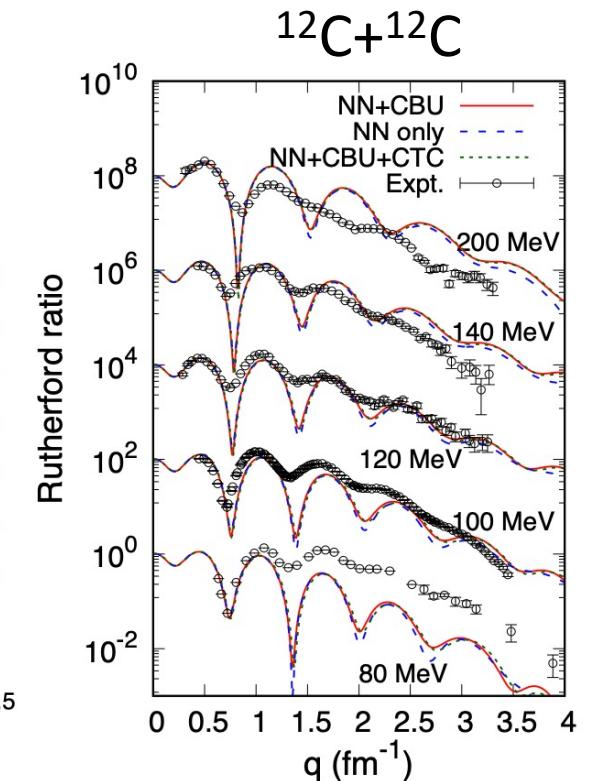
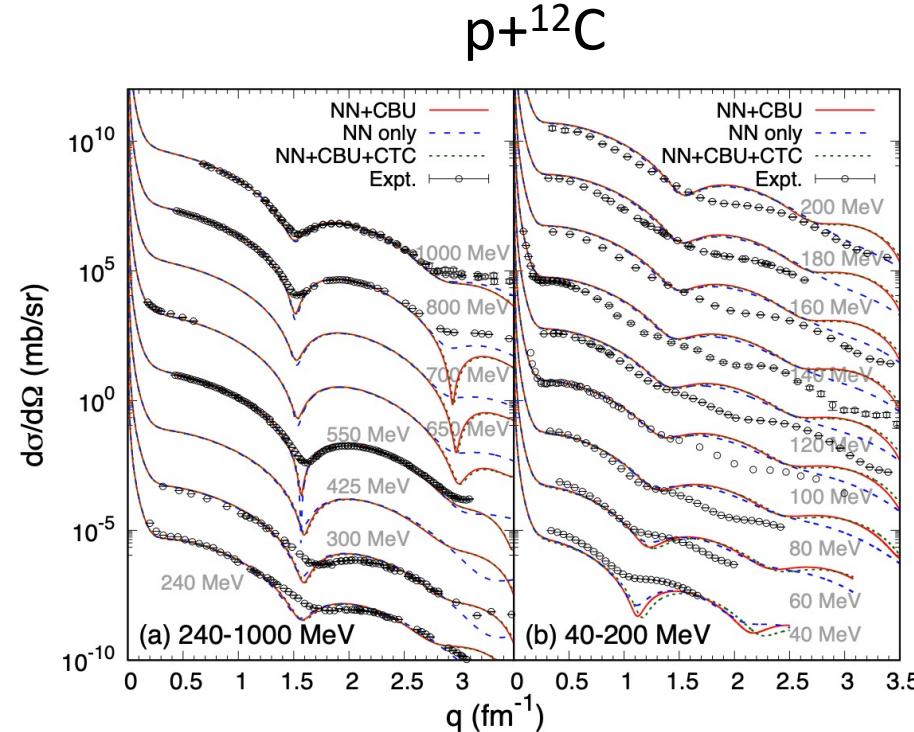
Double folding $U_{\text{opt}} = \int \int \rho_{\text{projectile}} V_{\text{NN}}^{\text{eff}} \rho_{\text{target}}$

Glauber analysis:

Valid at high energy !

[Glauber, Lectures in
Theoretical Physics (1959)]

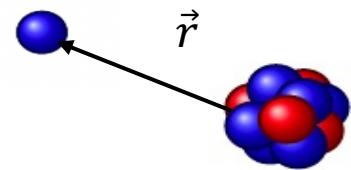
[Horiuchi *et al.* arXiv:2512.20100]



Uncertainties not quantified yet → Andy is working on it ☺

Three main approaches to construct optical potentials

Ab initio & microscopic theories



Typically obtained from microscopic densities and folded

Main tool for nucleus nucleus potentials!

Radial dependence fixed but energy dependence fitted

Phenomenological approach

A diagram showing two blue spheres representing nuclei. A vector arrow labeled \vec{r} points from the left sphere to the right sphere, and another vector arrow labeled V_{opt} points from the right sphere back to the left sphere, representing a central potential.

Interactions directly obtained from many-body Hamiltonian

A priori should be accurate to extrapolate away from stability (if ab initio theory accurate)

Difficult to have all many-body correlations -> lack absorption at energies where MST not valid

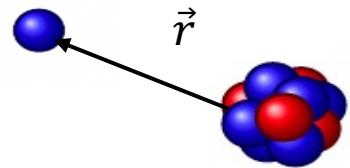
Semi-microscopic approach

Fit to existing data

Ad hoc choice of phenomenological forms

unclear how it extrapolates but first studies show large uncertainties...

Three main approaches to construct optical potentials



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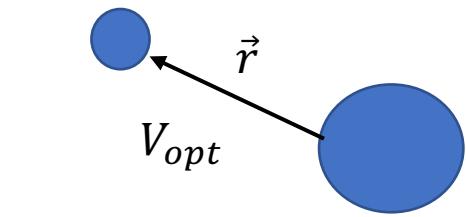
More extensive comparisons of different approaches done in JPG 50, 060501 (2023)

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Outline of this talk

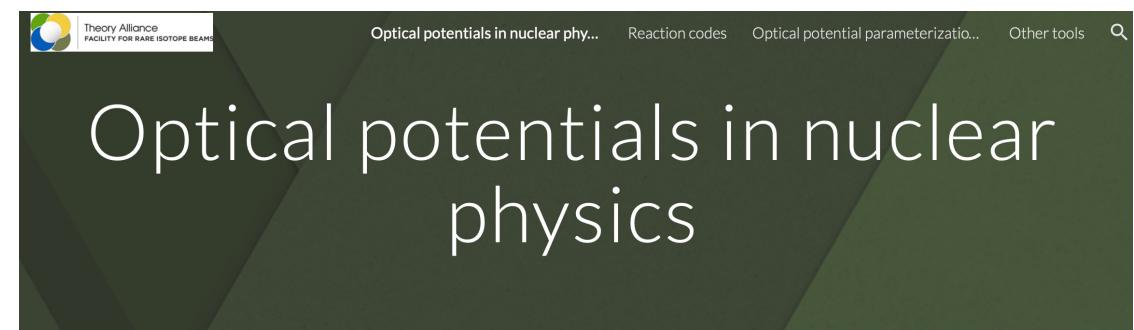
- What is an optical potential?
& why it is difficult to build an *accurate* one using ab initio theory?
- Status of optical potentials:
 - Main techniques to construct them
 - How do they perform for various probes?
 - What are their uncertainties?
- **What tools exist for reaction calculations and what can we do to reduce the uncertainties associated with optical potentials?**

What tools exist for reactions calculations?

I. Available [tools](#) to fit optical potentials
(Bayesian wrapper, emulator, etc)



II. Available global parametrizations
in RIPL 3 library & in [here](#)



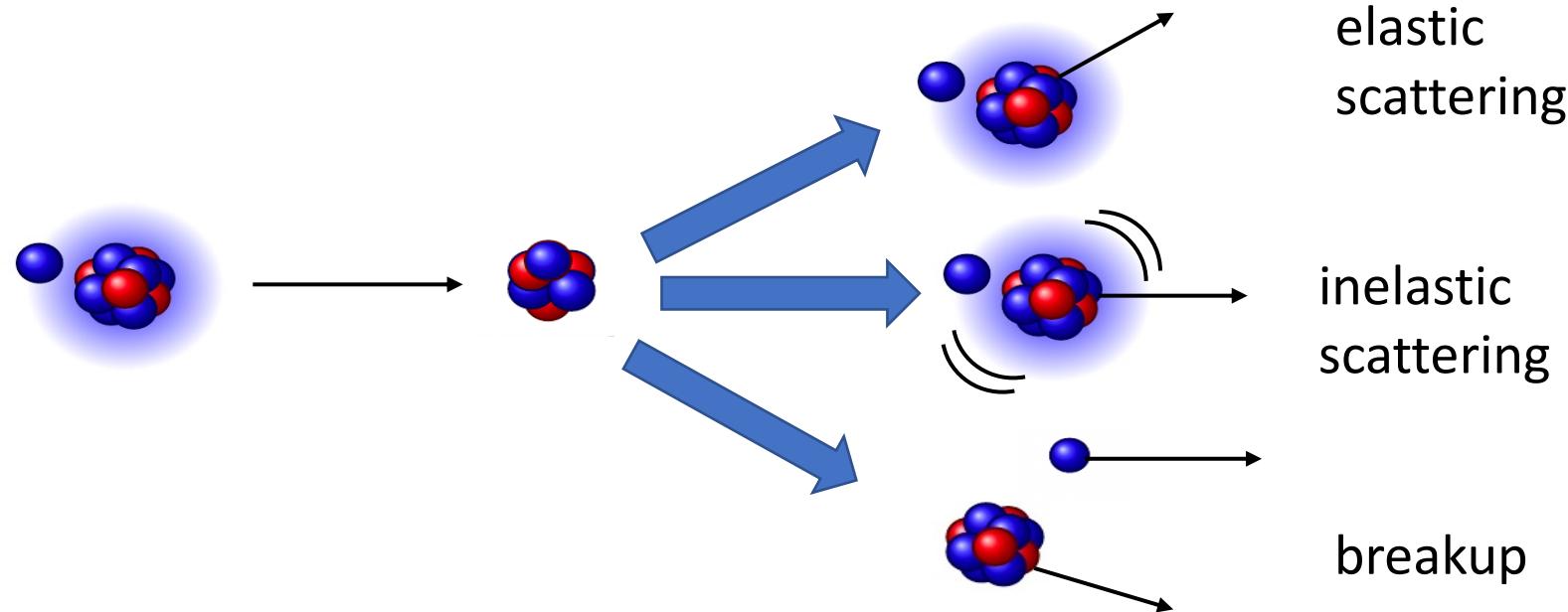
III. Folding codes in [here](#)

IV. Reaction codes in [here](#) (two-nucleon transfer, quasifree scattering, etc)

→ Particularly for (N,d) and (d,N), straightforward with TWOFRN

What can be done to reduce the optical potentials uncertainties in the analysis of reaction data?

Measure various channels and identify ratio of observables that cancel uncertainties



Capel et al. PRC **88** 044602 (2013):

$$R_{quasi} = \frac{\left(\frac{d\sigma_{bu}}{d\Omega dE} \right)_{E=0.1 \text{ MeV}}}{\frac{d\sigma_{el}}{d\Omega} + \frac{d\sigma_{inel}}{d\Omega}}$$

Conclusions and prospects

- Optical potentials are central to the analysis of reactions
→ Need to quantify their uncertainties in reaction analysis
- Many tools already exist: if you want to use them, ask me questions ☺
- Phenomenological optical potentials carry large uncertainties...
→ Need to make *more developments*
in particular: isospin dependence, nucleus-nucleus potentials
→ The not-too-far future (a personal view):
improving semi-microscopic methods to integrate easily microscopic inputs and performing UQ
- **Ongoing** : uncertainties in other reaction probes
nucleus-nucleus reaction cross sections, (p, pN), finding ratios to reduce uncertainties
(work lead by Andrew Smith, discussion with Ogata, collaboration with Capel & Nunes)

Thank you for your attention ...

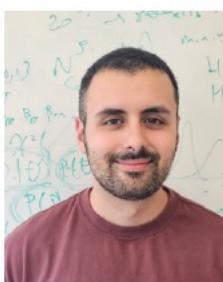
& thanks the few-body reaction group at MSU



Filomena Nunes



Chloë Hebborn



Ibrahim
Abdurrahman



Kyle Beyer



Patrick McGlynn



Cate Beckman



Manuel
Catacora Rios



Andy Smith



Zetian Ma



Daniel Shiu



Pablo Giuliani



Grigor Sargsyan

Backup slides

From the Dyson equation, similar derivation

Optical potentials is identified as the self energy Σ^*

$$G = G_0 + G_0 \Sigma^* G$$

Where $G_0 = \frac{1}{E - H_A - h_0}$ free propagator.

Need to know overlap function and energy of all states in the A+1 continuum and in the A-1 bound regime... difficult...

And one can obtain

$$\Sigma^* = (G_0)^{-1} - G^{-1}$$

In practice G_0 mean field propagator and one can write (Lehmann representation)

$$G_{\ell j}(r, r'; E) = \sum_m \frac{\langle \Psi_0^A | a_{r\ell j} | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_{r'\ell j}^\dagger | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} + \sum_n \frac{\langle \Psi_0^A | a_{r'\ell j}^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_{r\ell j} | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta},$$

How do these different approach compare?

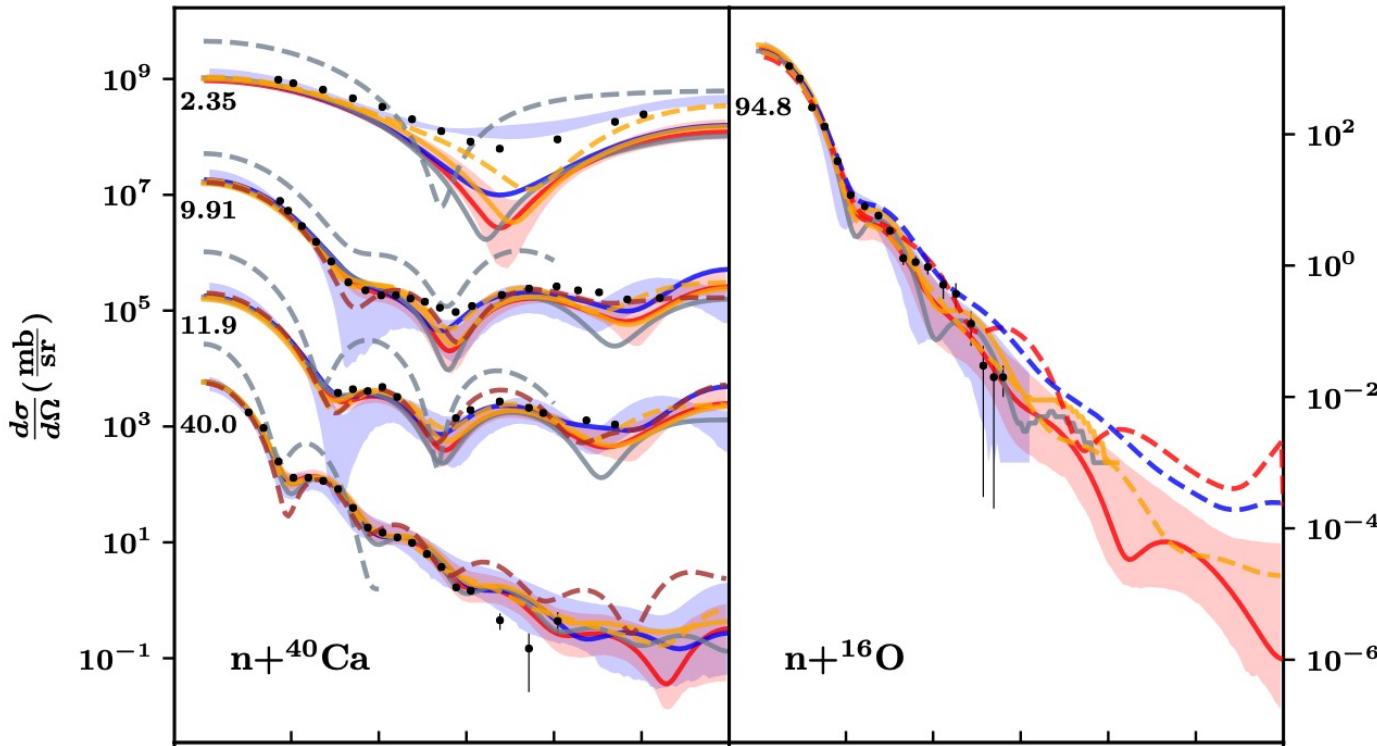
Comparison done in a program at FRIB in 2022 [whitepaper: JPG 50, 060501 (2023)]

	Mass	Energy	D.	Mic.	UQ
KD	$24 \leq A \leq 209$	$1 \text{ keV} \leq E \leq 200 \text{ MeV}$	✗	✗	✗
KDUQ	$24 \leq A \leq 209$	$1 \text{ keV} \leq E \leq 200 \text{ MeV}$	✗	✗	✓
DOM (STL)	C, O, Ca, Ni, Sn, Pb isotopes	$-\infty < E < 200 \text{ MeV}$	✓	✗	✓
MR	$12 < Z < 83$	$E < 200 \text{ MeV}$	✓	✗	✗
MBR	$12 < Z < 83$	$E < 200 \text{ MeV}$	✓	✗	✗
NSM	$^{40}\text{Ca}, ^{48}\text{Ca}, ^{208}\text{Pb}$	$E < 40 \text{ MeV}$	✓	✓	✗
SCGF	O, Ca, Ni isotopes	$E < 100 \text{ MeV}$	✓	✓	✗
MST-B	$A \leq 20$	$E \gtrsim 70 \text{ MeV}$	✗	✓	✗
MST-V	$4 \leq A \leq 16$	$E \gtrsim 60 \text{ MeV}$	✗	✓	✗
WLH	$12 \leq A \leq 242$	$0 \leq E \leq 150 \text{ MeV}$	✗	✓	✓
JLMB	$A > 30$	$1 \text{ keV} < E < 340 \text{ MeV}$	✗	✓	✗

Legend:

- Upward arrow: phenomenological
- Downward arrow: microscopic
- Vertical double-headed arrow: Semi-phenomenological

How do these different approach compare?



• experiment	— MBR	— JLMB
— KD	— MST-B	— NSM
— DOM (STL)	— MST-V	— KDUQ
— MR (2007)	— SCGF	— WLH

