International Bridgelab Symposium
For Laser Acceleration Route toward Reality
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Gif sur Yvette

Bridgelab: Collider Consideration

Wim Leemans
Lawrence Berkeley National Lab
and
Toshiki Tajima
LMU,MPQ, Garching

Acknowledgments for Collaboration and advice: G. Mourou, C. Barty, C. Labaune, P. Chomaz, D. Payne, H. Videau, P. Martin, V. Malka, , F. Krausz, T. Esirkepov, S. Bulanov, M. Kando, W. Sandner, A. Suzuki, M. Teshima, X. Q. Yan, B. Cros, J. Chambaret, E. Esarey, R. Assmann, R. Heuer, A. Caldwell, S. Karsch, F. Gruener, M. Zepf, M. Somekh, E. Desurvire, D. Normand, J. Nilsson, W. Chou, F. Takasaki, M. Nozaki, K. Yokoya, J. Urakawa, C. Schroeder, S. Chattopadhay, S. Cheshkov, C. Chiu, M. Downer, P. Bolton, K. Ueda, K. Nakajima, Y. Kato, A. Sergeev, J. Koutchoouk, C. Siders, J. Rossbach, A. Chao, R. Ruth, P. Chen, J. Chen

Multistaged LWFA collider design

Collider physics consideration

beamstrahlung, disruption, emittance degradation, gamma emission (spectral degradation)

Driver development

independent and a huge challenge

Option for future collider (Raubenheimer-SLAC)

Examples of TeV Collider Parameters

	Laser	Plasma	CLIC	ILC
CMS Energy (GeV)	1000	1000	1000	1000
Luminosity (10^{34} $cm^{-2}s^{-1}$)	2.4	3.5	2.3	2.8
Luminosity in 1% of Ecms	~2	1.3	1.1	1.9
Bunch charge (10^{10})	3.80E-06	1	0.37	2
Bunches / train	193	125	312	2820
Repetition rate (Hz)	1.50E+07	100	50	4
Beam Power (MW)	11.6	20	9.2	36.2
Emittances $\varepsilon_{n,x}$ / $\varepsilon_{n,y}$ (mm-mrad)	1e-4 / 1e-4	2 / 0.05	0.7 / 0.02	10 / 0.04
IP Spot sizes sx/sy (nm)	1.0 / 1.0	140 / 3.2	140 / 2	554 / 3.5
IP bunch length sz (μm)	0.1 –> 300	10	30	300
Drive beam / Laser / RF Power (MW)	58	58	36.8	80
Gradient (MV/m)	400	25000	100	31.5
Two linac length (km)	~4	~6	14	47
Drive beam / Laser / RF generation eff.	60%	45%	49%	53.95%
Drive beam / Laser / RF coupling eff.	20%	35%	25%	49.01%
Overall efficiency	12%	15.70%	12.10%	17.90%
Site Power (MW)	~137	~170	~150	300

Raubenheimer(2008)

Keys issues of future colliders

Beam Acceleration

* Largest cost driver for a linear collider is the acceleration
  – ILC geometric gradient is ~20 MV/m → 50km for 1 TeV
* Size of facility is costly → higher acceleration gradients
  – High gradient acceleration requires high peak power and structures that can sustain high fields
    • Beams and lasers can be generated with high peak power
    • Dielectrics and plasmas can withstand high fields
* Many paths towards high gradient acceleration
  – High gradient microwave acceleration } ~100 MV/m
  – Acceleration with laser driven structures
  – Acceleration with beam driven structures } ~1 GV/m
  – Acceleration with laser driven plasmas
  – Acceleration with beam driven plasmas } ~10 GV/m

Issues for LWFA Collider

- Collider Physics issues (what is unique and challenging to LWFA) ----this lecture

  strong acceleration (compactness)

  small emittance (strong beam)

  strong transverse force/large betatron oscillations

  large quantum beamstrahlung effects

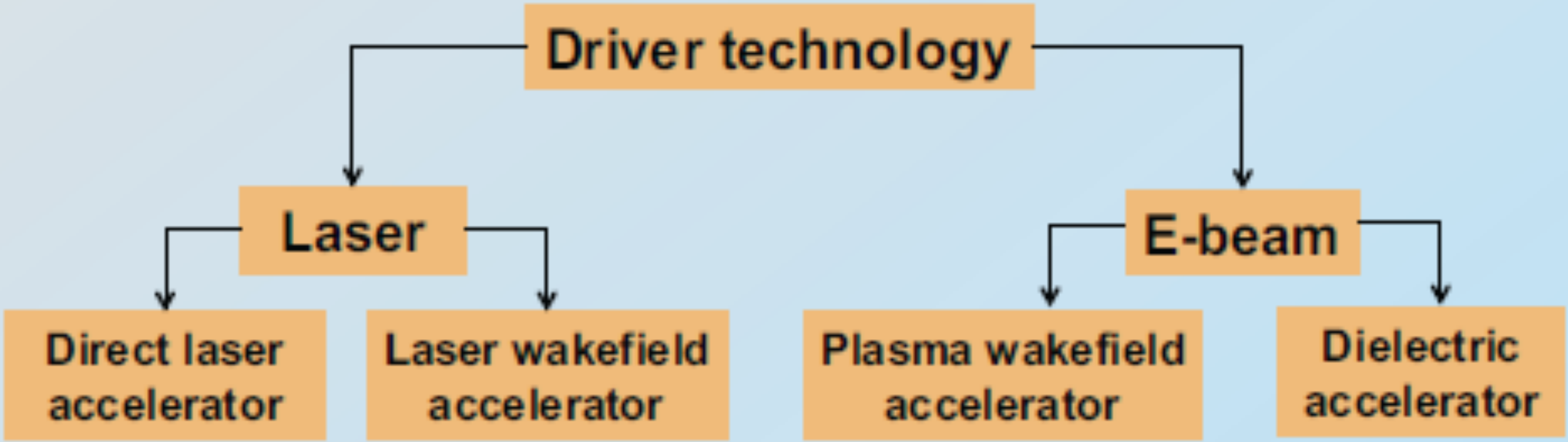
  miniature finesse issues

- Driver issues (high rep rate, high average power lasers) -----this afternoon

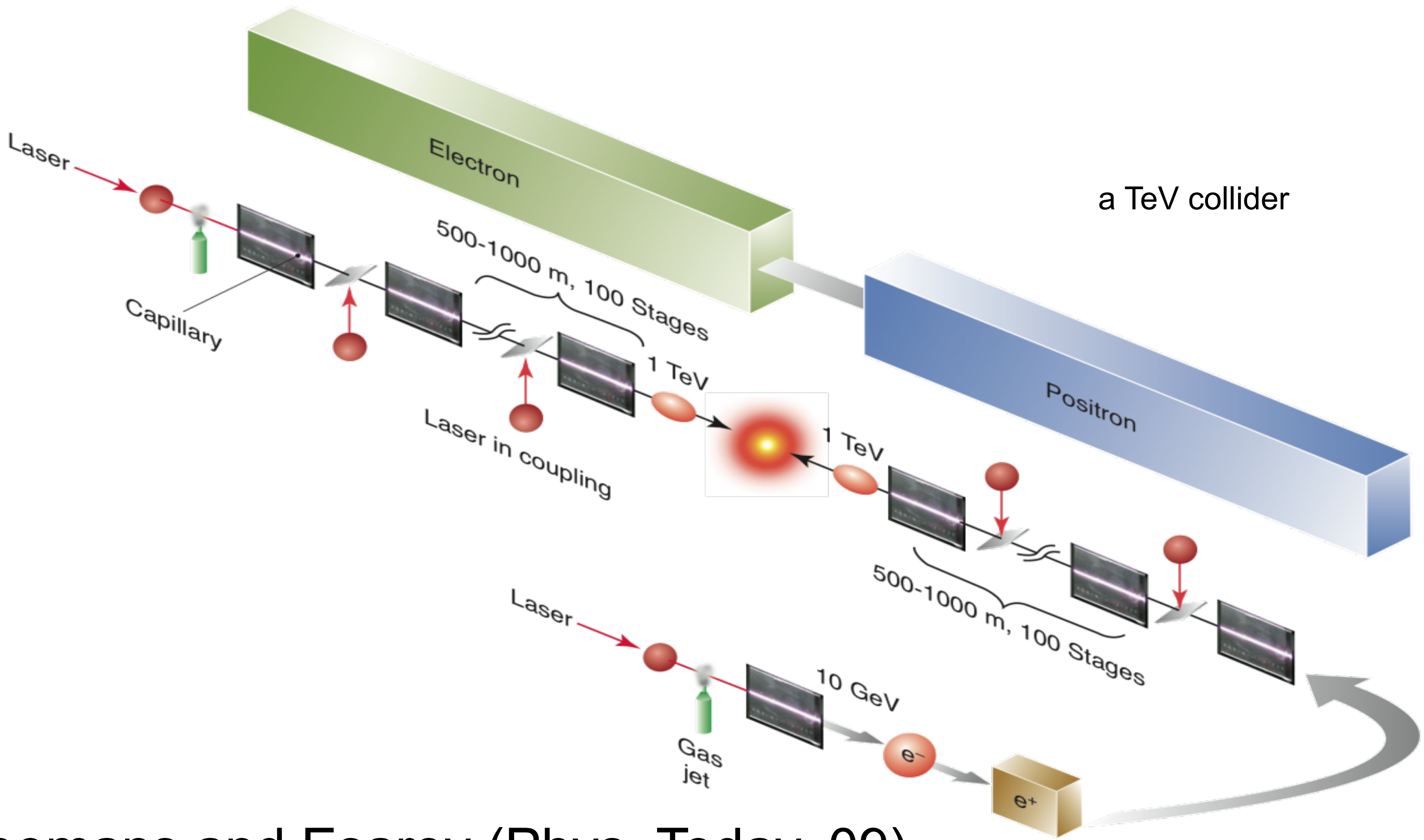
Motivation and overview

Leemans (AAC ,2010)

- Collider size set by maximum particle energy and maximum achievable gradient limited by breakdown
- Motivates R&D for ultra-high gradient technology



Laser driven collider concept



Leemans and Esarey (Phys. Today, 09)
ICFA-ICUIL Joint Task Force on Laser Acceleration(Darmstadt,10)

ICFA-ICUIL Joint Task Force
on laser acceleration (Darmstadt, 2010)

W. Leemans, Chair

Case	1 TeV	10 TeV (Scenario I)	10 TeV (Scenario II)
Energy per beam (TeV)	0.5	5	5
Luminosity (10^{34} cm^{-2}s^{-1})	1.2	71.4	71.4
Electrons per bunch ($\times 10^9$)	4	4	1.3
Bunch repetition rate (kHz)	13	17	170
Horizontal emittance $\gamma\varepsilon_x$ (nm-rad)	700	200	200
Vertical emittance $\gamma\varepsilon_y$ (nm-rad)	700	200	200
β^* (mm)	0.2	0.2	0.2
Horizontal beam size at IP σ^*_x (nm)	12	2	2
Vertical beam size at IP σ^*_y (nm)	12	2	2
Luminosity enhancement factor	1.04	1.35	1.2
Bunch length σ_z (μm)	1	1	1
Beamstrahlung parameter Υ	148	8980	2800
Beamstrahlung photons per electron n_γ	1.68	3.67	2.4
Beamstrahlung energy loss δ_E (%)	30.4	48	32
Accelerating gradient (GV/m)	10	10	10
Average beam power (MW)	4.2	54	170
Wall plug to beam efficiency (%)	10	10	10
One linac length (km)	0.1	1.0	0.3

Collider subgroup
List of parameters
(W. Chou)

Table 1
Collider parameters

JTF Report #2

Case	1 TeV	10 TeV (Scenario I)	10 TeV (Scenario II)
Wavelength (μm)	1	1	1
Pulse energy/stage (J)	32	32	1
Pulse length (fs)	56	56	18
Repetition rate (kHz)	13	17	170
Peak power (TW)	240	240	24
Average laser power/stage (MW)	0.42	0.54	0.17
Energy gain/stage (GeV)	10	10	1
Stage length [LPA + in-coupling] (m)	2	2	0.06
Number of stages (one linac)	50	500	5000
Total laser power (MW)	42	540	1700
Total wall power (MW)	84	1080	3400
Laser to beam efficiency (%) [laser to wake 50% + wake to beam 40%]	20	20	20
Wall plug to laser efficiency (%)	50	50	50
Laser spot rms radius (μm)	69	69	22
Laser intensity (W/cm^2)	3×10^{18}	3×10^{18}	3×10^{18}
Laser strength parameter a_0	1.5	1.5	1.5
Plasma density (cm^{-3}), with tapering	10^{17}	10^{17}	10^{18}
Plasma wavelength (μm)	105	105	33

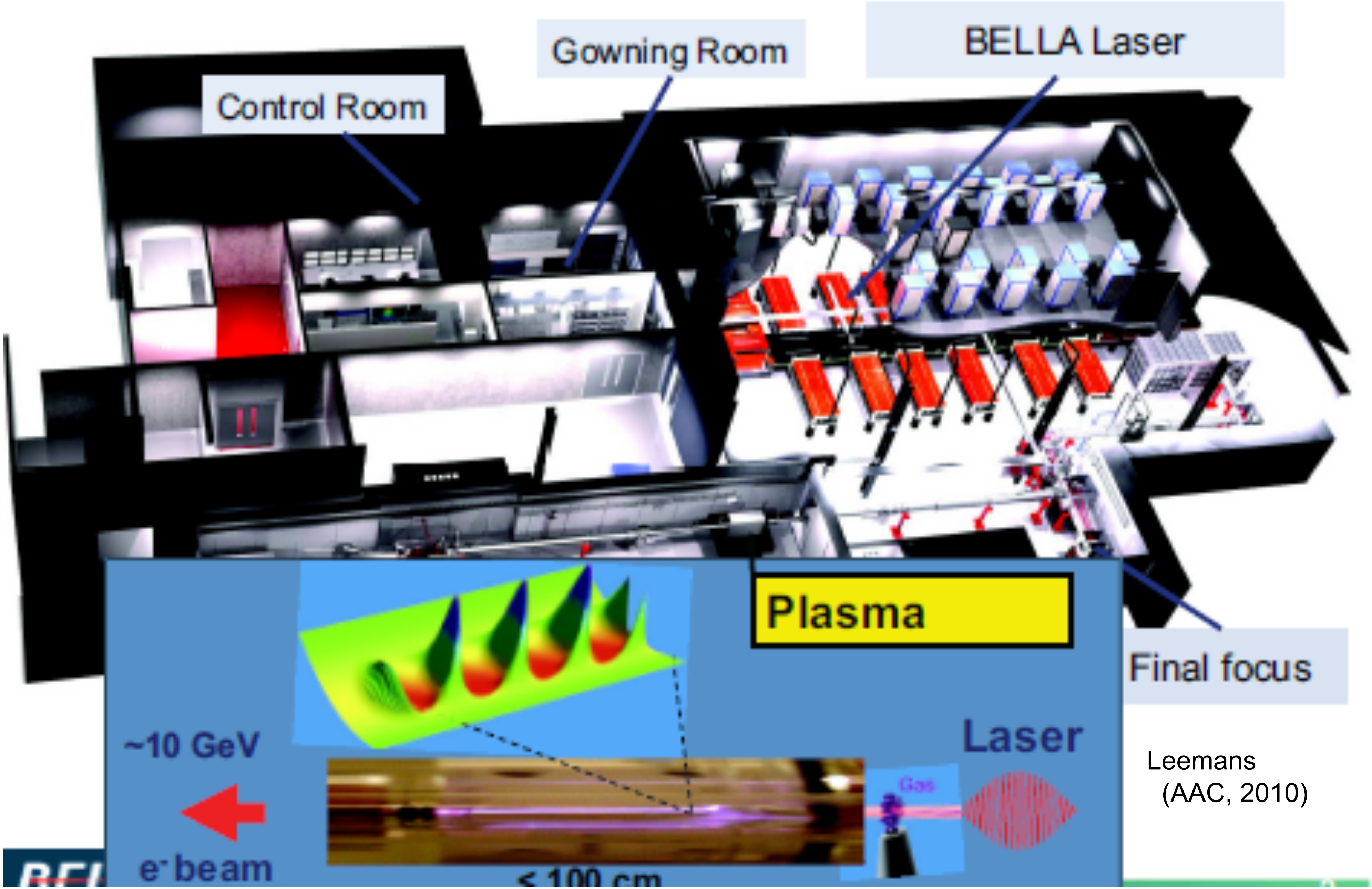
Collider subgroup
List of parameters
(W. Chou)

Table 2
Laser parameters

JTF Report #3: Comparison of Choices

Accelerator	Beam	Beam energy (GeV)	Beam power (MW)	Efficiency AC to beam	Note on AC power
PSI Cyclotron	H+	0.59	1.3	0.18	RF + magnets
SNS Linac	H^-	0.92	1.0	0.07	RF + cryo + cooling
TESLA (23.4 MV/m)	e^+/e^-	250 × 2	23	0.24	RF + cryo + cooling
ILC (31.5 MV/m)	e^+/e^-	250 × 2	21	0.16	RF + cryo + cooling
CLIC	e^+/e^-	1500 × 2	29.4	0.09	RF + cooling
LPA	e^+/e^-	500 × 2	8.4	0.10	Laser + plasma

BELLA Project: state-of-the-art PW-facility for laser accelerator science



Leemans
(AAC, 2010)

BELLA Laser: 40 J in 30 fs, 1 Hz

Leemans(AAC, 2010)

- Laser power and energy requirement:
  - Simple energy balance:
    - 400 pC at 10 GeV = 4 J
    - Assume laser -> beam efficiency = 10 %

    40 J
  - Point design
    - 40 J, 30 fs laser provides wide parameter range
      - Highly non-linear regime a>> 1
      - Mildly non-linear regime a ~ 1
- 1 Hz repetition rate:
  - Fast enough to do:
    - Rapid parameter scans: science with error bars
  - Slow enough that:
    - Thermal loading of optics remains low
    - Radiation yield remains < 5 mR/hr outside shielding

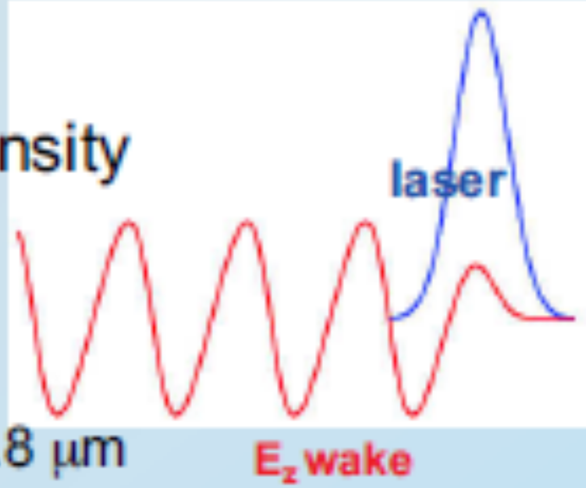
Basic design of a plasma accelerator: single-stage limited by laser energy

Leemasn(AAC, 2010)

- Laser pulse length and plasma density
  - $k_p \sigma_z \leq 1, \quad \sigma_z \sim \lambda_p \sim n^{-1/2}$
- Wakefield regime determined by laser intensity
  - Linear (a_0<1) or blowout (a_0>1)

$$a_0^2 \simeq 7.3 \times 10^{-19} [\lambda(\mu\mathrm{m})]^2 I_0(\mathrm{W/cm}^2),$$

  - Ex: $a_0 = 1$ for $I_0 = 2\mathrm{x}10^{18}$ W/cm^2 and $\lambda_0 = 0.8$ μm



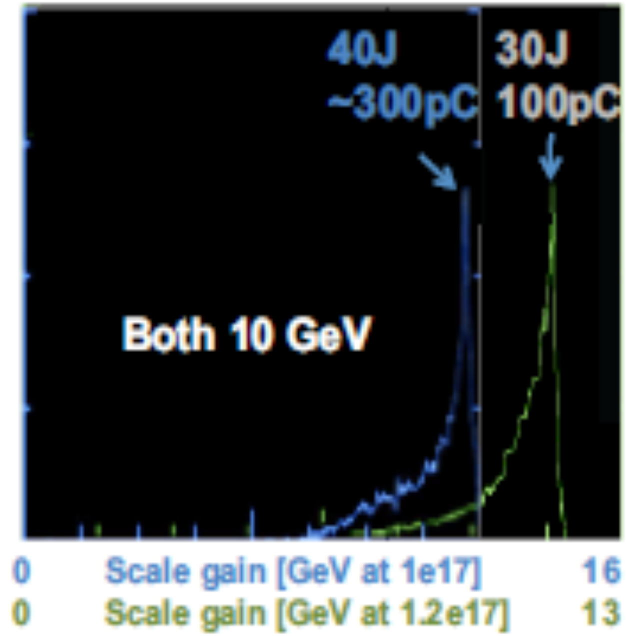
- Accelerating field determined by density and laser intensity
  - $E_z \sim (a_0^2/4)(1+a_0^2/2)^{-1/2}\, n^{1/2} \sim 10$ GV/m
- Energy gain determined by laser energy via depletion*
  - Laser: Present CPA technology 10's J/pulse
  - Bunch parameters also determined via beam loading

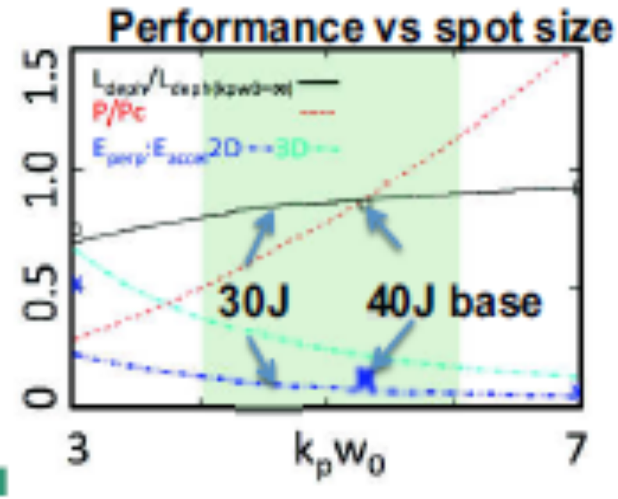
*Shadwick, Schroeder, Esarey, Phys. Plasmas (2009)

Leemans(AAC, 2010)

10 GeV reachable with BELLA laser at 30 – 40 J

- 10 GeV design scalable with density
  - $E_{bunch} \sim n^{-1}$, $E_{laser} \sim n^{-3/2}$, $Q \sim n^{-1/2}$
- Baseline laser performance: 40 J in 30 fs
  - At 10^{17}/cc → 10 GeV 300pC*,**
- For 30 J: increase density and lower charge
  - At 1.2×10^{17}/cc → 10 GeV for ~ 100pC
- Alternative: reduce spot size 15% at 10^{17}/cc
  - Performance vs. spot size characterized**
  - Should allow ~ 200pC at 10 GeV w/ 30J





*Cormier-Michel et al, AAC 2008, ** Geddes et al, PAC 2009. & SciDAC Review 2009

BELLA

First LWFA Collider Study (1997)

Studies of Laser-Driven 5 TeV e^+e^- Colliders in Strong Quantum Beamstrahlung Regime

M. Xie[1], T. Tajima[2], K. Yokoya[3] and S. Chattopadhyay[1]

[1] *Lawrence Berkeley National Laboratory, USA*
[2] *University of Texas at Austin, USA*
[3] *KEK, Japan*

Abstract.
We explore the multidimensional space of beam parameters, looking for preferred regions of operation for a e^+e^- linear collider at 5 TeV center of mass energy. Due to several major constraints such a collider is pushed into certain regime of high beamstrahlung parameter, Υ, where beamstrahlung can be suppressed by quantum effect. The collider performance at high Υ regime is examined with IP simulations using the code CAIN. Given the required beam parameters we then discuss the feasibility of laser-driven accelerations. In particular, we will discuss the capabilities of laser wakefield acceleration and comment on the difficulties and uncertainties associated with the approach. It is hoped that such an exercise will offer valuable guidelines for and insights into the current development of advanced accelerator technologies oriented towards future collider applications.

INTRODUCTION

It is believed that a linear collider at around 1 TeV center of mass energy can be built more or less with existing technologies. But it is practically impossible to go much beyond that energy without employing a new, yet largely unknown method of acceleration. However, apart from knowing the details of the future technologies, certain collider constraints on electron and positron beam parameters are considered to be quite general and have to be satisfied, e.g. available wall plug power and the constraints imposed by collision processes: beamstrahlung, disruption, backgrounds, etc. Therefore it is appropriate to explore and chart out the preferred region in parameter space based on these constraints, and with that hopefully to offer valuable guide-

With a plasma density of $10^{17}\mathrm{cm}^{-3}$, such a gradient can be produced in the linear regime with more or less existing T^3 laser, giving a plasma dephasing length of about 1 m [13]. If we assume a plasma channel tens of μm in width can be formed at a length equals to the dephasing length, we would have a 10 GeV acceleration module with an active length of 1 m. Of course, creating and maintaining a plasma channel of the required quality is no simple matter. To date, propagation in a plasma channel over a distance of up to 70 Rayleigh lengths (about 2.2 cm) of moderately intense pulse ($\sim 10^{15}\mathrm{W/cm}^2$) has been demonstrated [14]. New experiment aiming at propagating pulses with intensities on the order of $10^{18}\mathrm{W/cm}^2$ (required for a gradient of 10 GeV/m) is underway [13].

Table 1. Beam Parameters at Three Values of Beam Power

CASE	P_b(MW)	$N(10^8)$	f_c(kHz)	ϵ_y(nm)	$\beta_y(\mu\mathrm{m})$	σ_y(nm)	$\sigma_z(\mu\mathrm{m})$
I	2	0.5	50	2.2	22	0.1	0.32
II	20	1.6	156	25	62	0.56	1
III	200	6	416	310	188	3.5	2.8

Table 2. Results Given By the Formulas

CASE	Υ	D_y	F_{oide}	n_γ	δ_E	n_p	$\mathcal{L}_g(10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1})$
I	3485	0.93	0.89	0.72	0.2	0.19	1
II	631	0.29	0.89	0.72	0.2	0.12	1
III	138	0.081	0.91	0.72	0.2	0.072	1

Table 3. Results Given By CAIN Simulations

CASE	n_γ	δ_E	σ_e/E_0	n_p	$\mathcal{L}/\mathcal{L}_g(\mathrm{W_{cm}} \in 1\%)$	$\mathcal{L}/\mathcal{L}_g(\mathrm{W_{cm}} \in 10\%)$
I	1.9	0.38	0.42	0.28	0.83	1.1
II	0.97	0.26	0.36	0.12	0.65	0.80
III	0.84	0.21	0.32	0.06	0.62	0.75

Although a state-of-the-art T^3 laser, capable of generating sub-ps pulses with 10s of TW peak power and a few Js of energy per pulse [11], could almost serve the need for the required acceleration, the average power or the rep rate of a single unit is still quite low, and wall-plug efficiency inadequate. In addition, injection scheme and synchronization of laser and electron pulse from

Xie et al. (1997)

Collider Physics I

Basic parameters and scalings of LWFA Collider in Maximizing luminosity with constraintsof beamstrahlung , disruption, and γ emission

$$f_c = \left(\frac{P_b}{E_{cm}}\right)\left(\frac{1}{N}\right) \quad (1)$$

$$\sigma_y = \left(\frac{1}{\sqrt{4\pi}}\right)\left(\frac{1}{\sqrt{R}}\right)\left(\sqrt{\frac{P_b}{E_{cm}\mathcal{L}_g}}\right)(\sqrt{N})$$

$$\Upsilon = \left(\frac{5\sqrt{\pi}r_e^2}{6\alpha mc^2}\right)\left(\frac{\sqrt{R}}{1+R}\right)\left(\sqrt{\frac{E_{cm}^3\mathcal{L}_g}{P_b}}\right)\left(\frac{\sqrt{N}}{\sigma_z}\right)$$

$$D_y = (16\pi mc^2 r_e)\left(\frac{R}{1+R}\right)\left(\frac{\mathcal{L}_g}{P_b}\right)(\sigma_z)$$

$$n_\gamma = 2.54U_0(\Upsilon)F, \quad \delta_E = 1.24\Upsilon U_1(\Upsilon)F \quad (5)$$

$$F = \left(\frac{5\sqrt{\pi}r_e^2}{3\lambda_c}\right)\left(\frac{\sqrt{R}}{1+R}\right)\left(\sqrt{\frac{E_{cm}\mathcal{L}_g}{P_b}}\right)(\sqrt{N}). \quad (6)$$

$$f_c \sim 1/N, \quad \sigma_y \sim \sqrt{N}, \quad D_y \sim \sigma_z, \quad \Upsilon \sim \sqrt{N}/\sigma_z \quad (7)$$

$$n_\gamma \sim U_0(\Upsilon)\sqrt{N}, \quad \delta_E \sim \Upsilon U_1(\Upsilon)\sqrt{N}. \quad (8)$$

In the limit $\Upsilon \gg 1$, $U_0(\Upsilon) \to 1/\Upsilon^{1/3}$, $\Upsilon U_1(\Upsilon) \to 1/\Upsilon^{1/3}$. Eq.(8) becomes

$$n_\gamma \sim (N\sigma_z)^{1/3}, \quad \delta_E \sim (N\sigma_z)^{1/3}. \quad (9)$$

First paper on LWFA collider

Xie, M., Tajima, T., Yokoya, K. and Chattopadyay, S., *Studies of Laser-Driven 5TeV e+e- Colliders in Strong Quantum Beamstrahlung Regime*, (AIP Conference Proceedings, New York, 1997), **398**, p. 233-242.

Collider Physics II

Xie et al (1997)

Optimization for LWFA Collider at IP

Collision parameter dependence

Collision energy spectrum

First LWFA Collider design

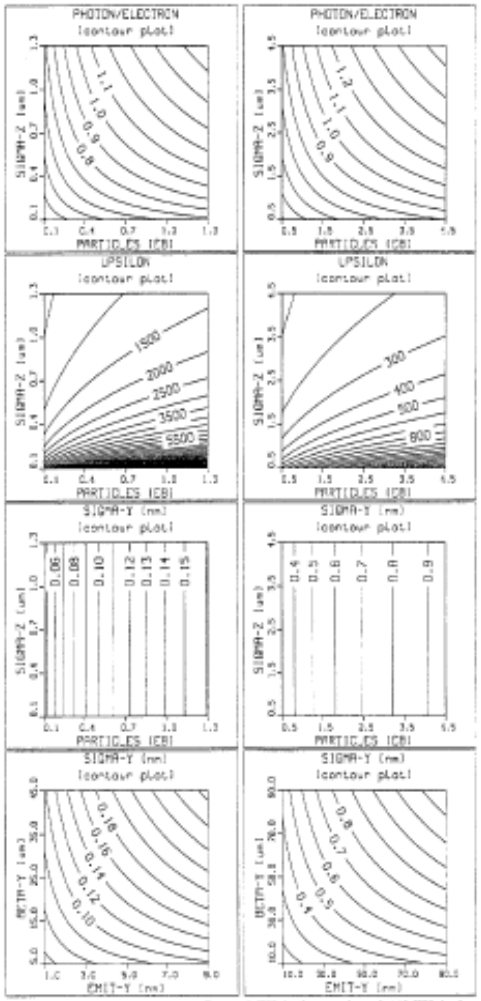
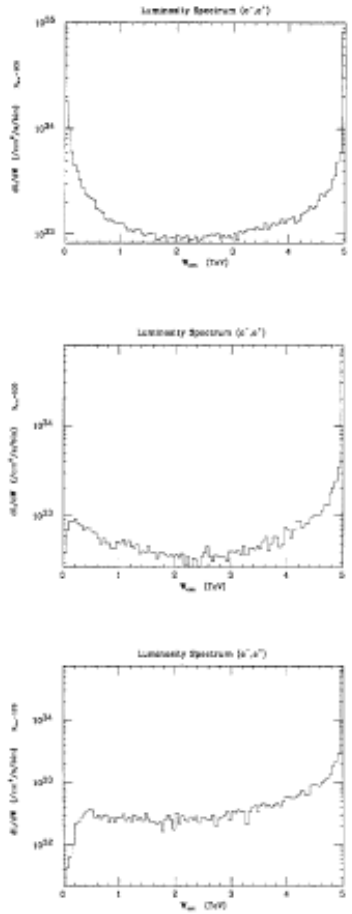


FIGURE 1. Parameter scans for P_b = 2MW (column 1) and 20MW (column 2).



e^+e^- luminosity spectrum for case I (top), II (middle), III (bottom)

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CASE	n_γ	δ_E	σ_ϵ/E_0	n_p	$\mathcal{L}/\mathcal{L}_g(W_{cm} \in 1\%)$	$\mathcal{L}/\mathcal{L}_g(W_{cm} \in 10\%)$
I	1.9	0.38	0.42	0.28	0.83	1.1
II	0.97	0.26	0.36	0.12	0.65	0.80
III	0.84	0.21	0.32	0.06	0.62	0.75

Physics under extremely strong **quantum beamstrahlung regime** Y >> 1 (the shorter the bunch, the smaller Y)

Collider Physics III

LWFA properties under multistage collider design

First multistage model for LWFA collider

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 3, 071301 (2000)

Particle dynamics in multistage wakefield collider

S. Cheshkov, T. Tajima,* and W. Horton
Department of Physics and Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712

K. Yokoya
KEK National Laboratory for High Energy Physics, Tsukuba-Shi, Japan

(Received 24 January 2000; published 27 July 2000)

The dynamics of particles in laser pulse-driven wakefields over multistages in a collider is studied. A map of phase space dynamics over a stage of wakefield acceleration induced by a laser pulse (or electron beam) is derived. The entire system of a collider is generated with a product of multiple maps of wakefields, drifts, magnets, etc. This systems map may include offsets of various elements of the accelerator, representing noise and errors arising from the operation of such a complex device. We find that an unmitigated strong focusing of the wakefield coupled with the alignment errors of the position (or laser beam aiming) of each wakefield stage and the unavoidable dispersion in individual particle betatron frequencies leads to a phase space mixing and causes a transverse emittance degradation. The rate of the emittance increase is proportional to the number of stages, the energy of the particles, the betatron frequency, the square of the misalignment amplitude, and the square of the betatron phase shift over a single stage. The accelerator with a weakened focus in a channel can, therefore, largely suppress the emittance degradation due to errors.

PACS numbers: 52.40.Nk, 52.65.Cc, 52.75.Di, 05.40.−a

I. INTRODUCTION

The use of plasma waves excited by laser beams for electron acceleration was proposed by Tajima and Dawson [1].

$$\mathcal{L} = \frac{f_c N^2}{4\pi\sigma_x\sigma_y} = \frac{\gamma f_c N^2}{4\pi\sqrt{\epsilon_x\beta_x^*}\sqrt{\epsilon_y\beta_y^*}}, \qquad (1)$$

where f_c is the collision frequency, N is the particle num-

Collider Physics IV

LWFA model for collider

Stage accelerator matrix

$$\frac{d\gamma}{dz} = k_p \Phi_0 \cos(\Psi), \quad (19)$$

$$\frac{d\Psi}{dz} = \frac{k_p}{2\gamma_p^2}. \quad (20)$$

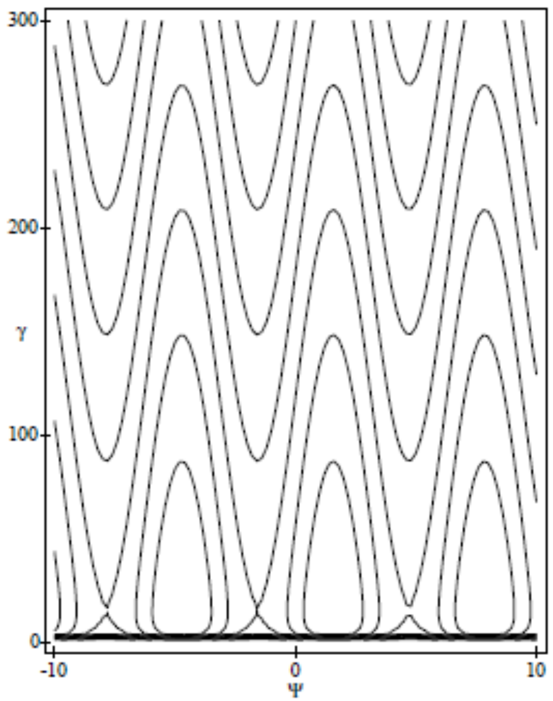


FIG. 1. The longitudinal phase space: electron Lorenz factor γ vs its phase with respect to the wakefield Ψ. The parameters used were $\gamma_p = 15$, $\Phi_0 = 0.2$.

Cheshkov et al (2000)

Longitudinal dynamics

The linearized equations of motion for the longitudinal degrees of freedom are

$$\delta\Psi_{n+1} = \delta\Psi_n, \quad (23)$$

$$\delta\gamma_{n+1} = 2\gamma_p^2\Phi_0[\cos(\Psi_s + \Delta) - \cos(\Psi_s)]\delta\Psi_n + \delta\gamma_n, \quad (24)$$

Transeverse dynamics

$$\ddot{u} + \left[\omega_\beta^2 \sin(\omega_s z + \Psi_s + \delta\Psi_n) - \frac{1}{2}\frac{\ddot{\gamma}}{\gamma} + \frac{1}{4}\frac{\dot{\gamma}^2}{\gamma^2}\right]\tilde{u} = 0, \quad (25)$$

where

$$\omega_s = \frac{k_p}{2\gamma_p^2}, \quad (26)$$

$$\omega_\beta = \left(\frac{4\Phi_0}{\gamma r_s^2}\right)^{1/2} \quad (27)$$

$$M = \begin{pmatrix} \cos(\frac{\omega}{\omega_s}\Delta) & \frac{1}{\omega}\sin(\frac{\omega}{\omega_s}\Delta) \\ -\omega\sin(\frac{\omega}{\omega_s}\Delta) & \cos(\frac{\omega}{\omega_s}\Delta) \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \quad (29)$$

where L is the drift distance between the wakefield stages and $1/\omega$ is the betatron length in the wakefield. The matrix (29) may be also written as

$$M = \begin{pmatrix} \cos(\frac{\omega}{\omega_s}\Delta) & \frac{1}{\omega}\sin(\frac{\omega}{\omega_s}\Delta) + L\cos(\frac{\omega}{\omega_s}\Delta) \\ -\omega\sin(\frac{\omega}{\omega_s}\Delta) & -L\omega\sin(\frac{\omega}{\omega_s}\Delta) + \cos(\frac{\omega}{\omega_s}\Delta) \end{pmatrix}. \quad (30)$$

The transverse map $\mathcal{M}$ for the whole accelerator system is

$$\mathcal{M} = M^N, \quad (31)$$

Collider Physics V

Cumulative effects over multistages

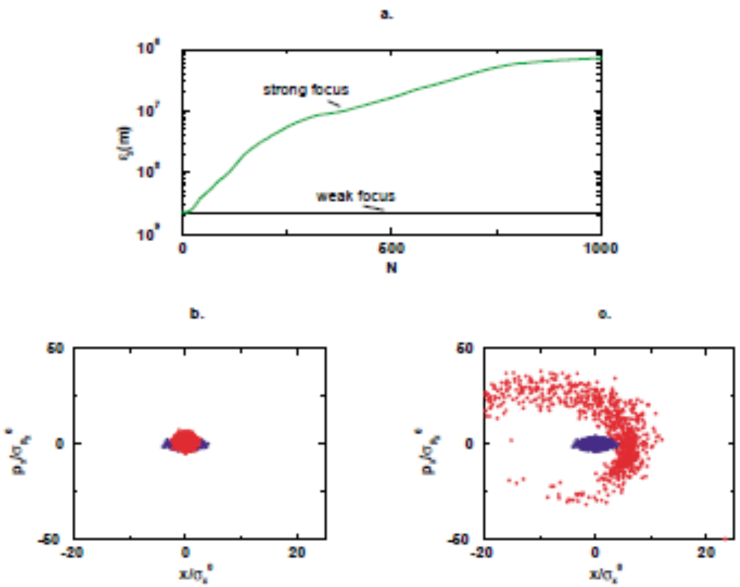
Strong LWFA betatron oscillations lead to emittance degradation

severe transverse emittance growth. Basically, what ...ens is that the particles rotate at different angular ve...ies in the transverse phase space and, if there is a ...position shift present, we get a characteristic banana-...ed distribution (see Fig. 3c) (it is banana shaped only ...e dislocation size is larger than the beam size, but in ...case the particle distribution gets diluted because of ...nisalignments). This process critically depends on the ...nitude of the betatron frequency spread. This means ...the typical strength of the focusing force is of great ...rtance. Of course, additional information can be ex-...ed from the other total phase space cross sections; see ...4. However, here we concentrate on the transverse ...tance as a figure of merit due to its importance to the ...luminosity of the collider. The effect of plasma noise ...ther noise, such as laser or the boundary) on the par-...dynamics over a stage may also be incorporated in a

$$\begin{pmatrix} \tilde{x}_{n+1} \\ \dot{\tilde{x}}_{n+1} \end{pmatrix} = M_n \begin{pmatrix} \tilde{x}_n - \tilde{\mathcal{D}}_n \\ \dot{\tilde{x}}_n \end{pmatrix} + \begin{pmatrix} \tilde{\mathcal{D}}_n \\ 0 \end{pmatrix}, \qquad (34)$$

where $\mathcal{D}_n$ is the stochastic misalignment ($\tilde{\mathcal{D}}_n = \sqrt{\gamma_n}\,\mathcal{D}_n$). The longitudinal degrees of freedom are not affected. For this map to describe realistically the electron motion, we assume that $\sigma_{\mathcal{D}} \ll r_s$. The total transverse map (in the presence of errors) can be written in the form

$$\begin{pmatrix} \tilde{x}_{n+1} \\ \dot{\tilde{x}}_{n+1} \end{pmatrix} = M_n M_{n-1} \cdots M_2 (1 - M_1) \begin{pmatrix} \tilde{\mathcal{D}}_1 \\ 0 \end{pmatrix} + \cdots (1 - M_n) \begin{pmatrix} \tilde{\mathcal{D}}_n \\ 0 \end{pmatrix} + M_n M_{n-1} \cdots M_1 \begin{pmatrix} \tilde{x}_1 \\ \dot{\tilde{x}}_1 \end{pmatrix}. \qquad (35)$$



$$\langle \mathcal{D} \rangle = 0, \qquad (38)$$

$$\langle \mathcal{D}(z_1)\mathcal{D}(z_2) \rangle = \sigma_{\mathcal{D}}^2 l\, \delta(z_1 - z_2). \qquad (39)$$

Applying the theory of random walk of a harmonic oscillator driven by a random force, we obtain

$$\langle \tilde{x} \rangle = 0, \qquad \langle \dot{\tilde{x}} \rangle = 0, \qquad \langle \tilde{x}\dot{\tilde{x}} \rangle = 0, \qquad (40)$$

$$\langle \tilde{x}^2 \rangle = Dz = DNl, \qquad \langle \dot{\tilde{x}}^2 \rangle = D\omega^2 z, \qquad (41)$$

where the diffusion coefficient D is given by

$$D = \frac{1}{2}\gamma\omega^2 l \sigma_{\mathcal{D}}^2. \qquad (42)$$

We are also assuming that the emittance growth is large (compared to the initial emittance). So, using (40) and (41), we obtain

$$\Delta\epsilon \approx \omega D\, z = \frac{1}{2}\gamma\omega(\omega l)^2 \sigma_{\mathcal{D}}^2 N. \qquad (43)$$

$$\Delta\epsilon \approx \frac{1}{2}\gamma\omega(\omega l)^2\sigma_{\mathcal{D}}^2 \left(\frac{\gamma}{\Delta\gamma}\right)^{1/2} \sqrt{N \ln\left(1 + \frac{\Delta\gamma N}{\gamma}\right)}, \qquad (44)$$

where γ is the initial particle energy. Typically, $\Delta\gamma \approx a_0^2 E_0 l$ and $\omega \propto \frac{a_0}{r_s}$, so we obtain

$$\Delta\epsilon \propto \frac{l^{3/2} a_0^2 \sigma_{\mathcal{D}}^2}{r_s^3 E_0^{1/2}} \sqrt{N \ln\left(1 + \frac{\Delta\gamma N}{\gamma}\right)}. \qquad (45)$$

Cheshkov et al (2000)

Collider Physics VI

Optimization for LWFA collider

Strategy of synchronous orbit operation

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High energy laser-wakefield collider with synchronous acceleration

C. Chiu,[1] S. Cheshkov,[1] and T. Tajima[1,2]
[1]*Department of Physics, University of Texas at Austin, Austin, Texas 78712*
[2]*Lawrence Livermore National Laboratory, Livermore, California 94550*
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A recent study on a high energy accelerator system which involves multistage laser wakefield acceleration shows that the system is very sensitive to jitters due to misalignment between the beam and the wakefield. In particular, the effect of jitters in the presence of a strong focusing wakefield and initial phase space spread of the beam leads to severe emittance degradation of the beam. One way to improve the emittance control is to mitigate the wakefield by working with a plasma channel. However, there are limitations in this approach. Our present investigation does not involve a plasma channel. Instead of averaging over the full phase range of the quarter-wave acceleration, we treat the phase range as a variable. We have found that, for a fixed final acceleration energy and a small phase slip, the final emittance is inversely proportional to the total number of stages. This leads us to consider an accelerator system which consists of superunits, where each superunit consists of closely spaced short tubes, or chips, with the wakefield of each chip being created by an independent laser pulse. There is a relatively large gap between adjacent superunits. With this arrangement the beam electrons are accelerated with a small phase slip; i.e., the phase of the beam is approximately synchronous with respect to the wakefield. This system is designed to have resilience against jitters. It has its practical limitations. We also consider a "horn model" with an exact synchronous acceleration based on a scheme suggested by Katsouleas. Computer simulation of both the chip model and the horn model confirms an expected $(\sin\psi)^{3/2}$ law for emittance degradation in the small phase angle region. Thus the choice of a small loading phase together with a small phase slip provides another important ingredient in controlling emittance degradation.

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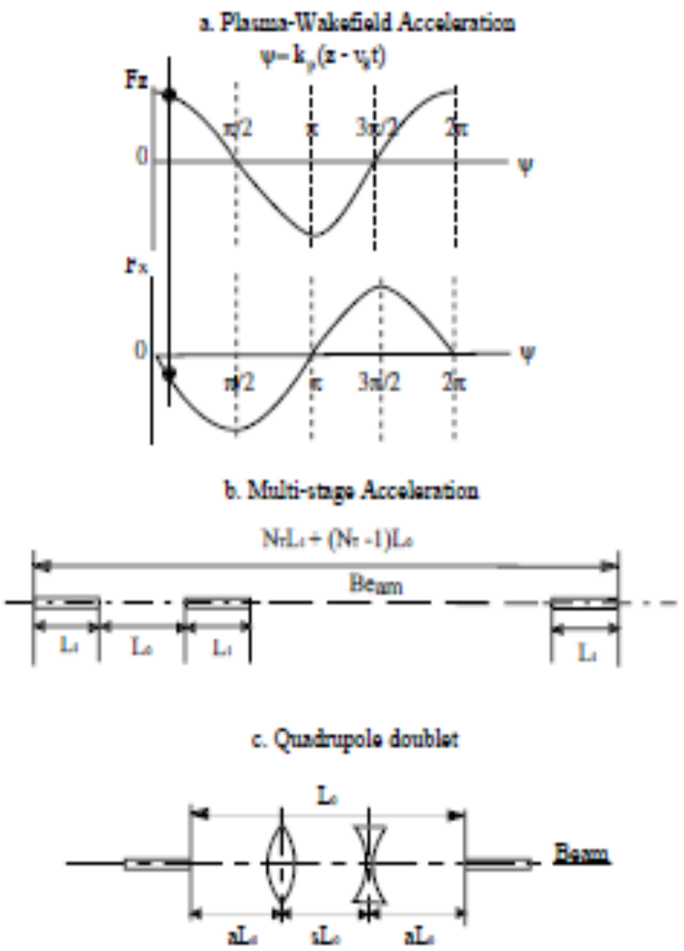
Collider Physics VII

optics of the LWFA collider stages



Chiu et al (2000)

orentz factor from the nth stage to the $n+1$th stage for typical particle[2] is given by

$$\gamma_{n+1} = \gamma_n + \Delta\gamma + \frac{\partial \Delta\gamma}{\partial \psi}\delta\psi, \qquad (7)$$

where the increase in the Lorentz factor over an acceleration stage is given by

$$\Delta\gamma = \Delta\gamma_{max}[\sin(\psi_s + \Delta) - \sin\psi_s],$$

with

$$\Delta\gamma_{max} = 2\gamma_p^2\Phi_0,$$

$$\frac{\partial \Delta\gamma}{\partial \psi} = \Delta\gamma_{max}[\cos(\psi_s + \Delta) - \cos\psi_s].$$

To the extent that one neglects the order of $\frac{1}{2\gamma_p^2}$, for a typical particle, the deviation of its longitudinal phase from the center of the beam in going from one stage to the next remains fixed; i.e.,

$$\delta\psi_{n+1} = \delta\psi_n = \delta\psi. \qquad (8)$$

C. Transverse iterative map

Transverse equation of motion. For the transverse motion of the beam particles in the x direction, we work with the two variables p_x and x. The equations of motion for these two variables are given by the Lorentz force equation and the definition of momentum,

$$\frac{dp_x}{dz} = \frac{dp_x}{cdt} = -\frac{eE_x}{c} \quad \text{and} \quad \frac{dx}{dz} = \frac{p_x}{m\gamma c}. \qquad (9)$$

It is shown in Ref. [5] that, in terms of the variable $u = \sqrt{\gamma}x$, the transverse force is approximately harmonic. The two equations of motion lead to

$$\frac{d^2u}{dz^2} \approx \frac{1}{mc\sqrt{\gamma}}\frac{dp_x}{dz} = -\frac{1}{mc\sqrt{\gamma}}\frac{eE_x}{c} = -\Omega^2 u, \qquad (10)$$

where

$$\Omega^2 = \frac{1}{mc\sqrt{\gamma}}\frac{4e}{c\sqrt{\gamma}r_s^2}\frac{\Phi_0 E_{bk}}{k_p}\sin\psi = \frac{\pi a_0^2}{r_s^2\gamma}\sin\psi. \qquad (11)$$

Jitters and the transverse map. So far the system is Hamiltonian and thus the emittance of the electron beam is preserved. Now consider jitters in the transverse directions, which, as mentioned earlier, may be due to the misalignment at each stage between the wakefield with respect to the beam line. We follow a procedure similar to those for the generation of random phase space variables. At each acceleration stage a random number χ is generated based on a normalized Gaussian distribution with a width unity. Denote the modified jitter displacement in the x direction by $D = \sqrt{\gamma}\sigma_D\chi$. This leads to a following recurrence relation in going from the nth stage to the $n+1$th stage:

$$\begin{pmatrix} u_{n+1} \\ u'_{n+1} \end{pmatrix} = M_{gap}M_{wk}\begin{pmatrix} u_n - D \\ u'_n \end{pmatrix} + \begin{pmatrix} D \\ 0 \end{pmatrix}. \qquad (12)$$

The wakefield acceleration matrix is given by

$$M_{wk} = \begin{bmatrix} \cos\theta & \frac{1}{\Omega}\sin\theta \\ -\Omega\sin\theta & \cos\theta \end{bmatrix}, \qquad \theta = \Omega L_1. \qquad (13)$$

Here L_1 is the spatial interval of acceleration, which is the tube length; see Fig. 1(b). From Eq. (6), $L_1 = 2\gamma_p^2\Delta_1/k_p$, where Δ_1 is the phase slip over the corresponding spatial interval. For a gap with a free space interval L_0, the corresponding transport matrix is given by

$$M_{gap} = S(L) = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}. \qquad (14)$$

Magnets. It is well known that the presence of magnets increases the stability of electron orbits. Figure 1(c) shows the layout with magnets. Within the gap there is a pair of quadrupoles separated by a distance sL_0, and the distance between each of the magnets to the corresponding end of the tube is given by aL_0. So $2a + s = 1$. With magnets, the matrix M_{gap} is to take on the following form:

$$M_{gap} \to S(aL_0)M(f)S(sL_0)M(-f)S(aL_0)$$

$$= \begin{bmatrix} 1 + \frac{s}{b} - \frac{as}{b^2} & [1 - \frac{a^2s}{b^2}]L_0 \\ -\frac{s}{b^2L_0} & 1 - \frac{s}{b} - \frac{as}{b^2} \end{bmatrix}, \qquad (15)$$

where $b = f/L_0$ and f is the magnitude of the focal length which is assumed to be the same for both the convergent and the divergent quadrupoles. The magnet matrix in the thin lens approximation, for focal length f, is given by

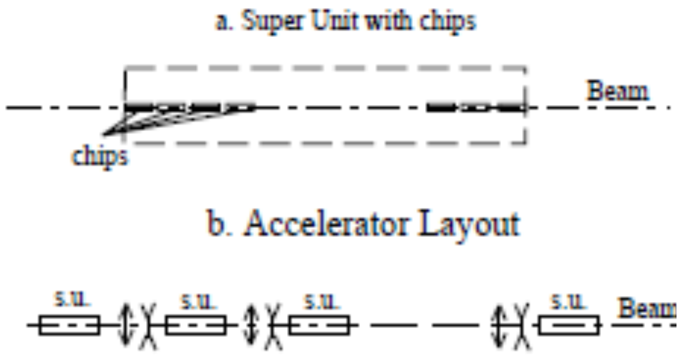
$$M(f) = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}. \qquad (16)$$

[2]Comments on a typical beam particle: Technically we could have introduced beam particle labels, i.e., $i = 1, 2, \ldots, N_0$. Then the ith particle would have a Lorentz factor of $\gamma_i = \gamma_0 + \delta\gamma_i$. Here γ_0 is the Lorentz factor at the "center" of the beam. To be precise, $\delta\gamma_i = \sigma_\gamma\chi_1(i)$ with $\chi_1(i)$ being a random number generated by a Gaussian distribution having a unit width. By the construction here, σ_γ is the Gaussian width, or simply the width, of the variable $\delta\gamma$. For brevity throughout the text we will suppress the beam particle label and refer to, for example, $\gamma = \gamma_0 + \delta\gamma$ as the Lorentz factor for a typical particle which has a width σ_γ. Similarly, the same typical particle will have a longitudinal phase ψ, with a width σ_ψ and a random variable χ_2 from $\{\chi_2(i)\}$. We will also apply the same convention to its transverse coordinates x and x'. They have their widths and the corresponding random variables from the set of $\{\chi_3(i)\}$ and $\{\chi_4(i)\}$.

Collider Physics VIII

Minimization strategy of emittance growth due to LWFA betatron effects

$$\epsilon \propto \langle \Omega^3 \rangle \propto (\sin\psi_m)^{3/2}, \qquad ($$



the local wakefield, as defined earlier, is ψ_s, then the electron phase relative to the laser pulse defined by the local plasma wave number k_p is

$$k_p s_1 = 2\pi N_{\rm load} - \psi_s,$$

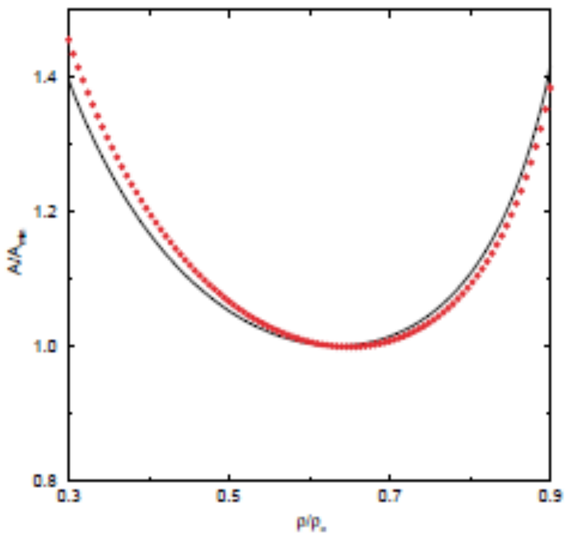


FIG. 8. (Color) Relationship between the normalized cross section and the normalized density function in a nozzle flow. The solid circles are for the monoatomic plasma and the curve is for the diatomic plasma.

$$\frac{1}{k_p}\frac{dk_p}{dz} = \frac{1}{2\pi N_{\rm load} - \psi_s}\frac{d\psi}{dz} = \frac{1}{2(2\pi N_{\rm load} - \psi_s)c}\frac{\omega_p^3}{\omega_0^2}. \qquad (35)$$

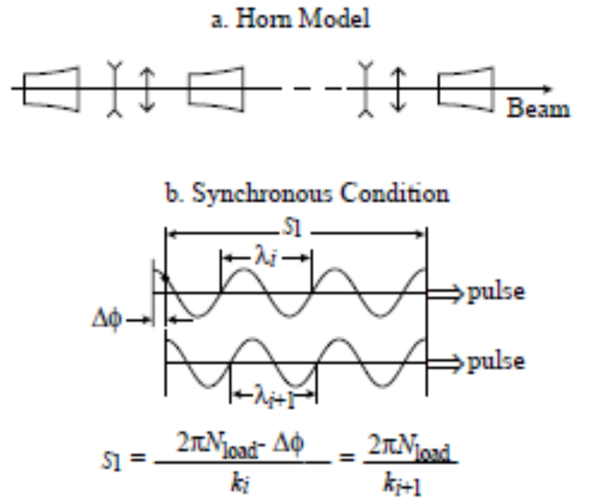


FIG. 9. The horn model. (a) Matching condition for synchronous acceleration for the case where $\psi_s = 0$. (b) A schematic layout of the horn model.

$$\Delta\epsilon = \epsilon_f - \epsilon_0 \propto \frac{(\sin\psi_m)^{3/2}\sigma_D^2}{N}.$$

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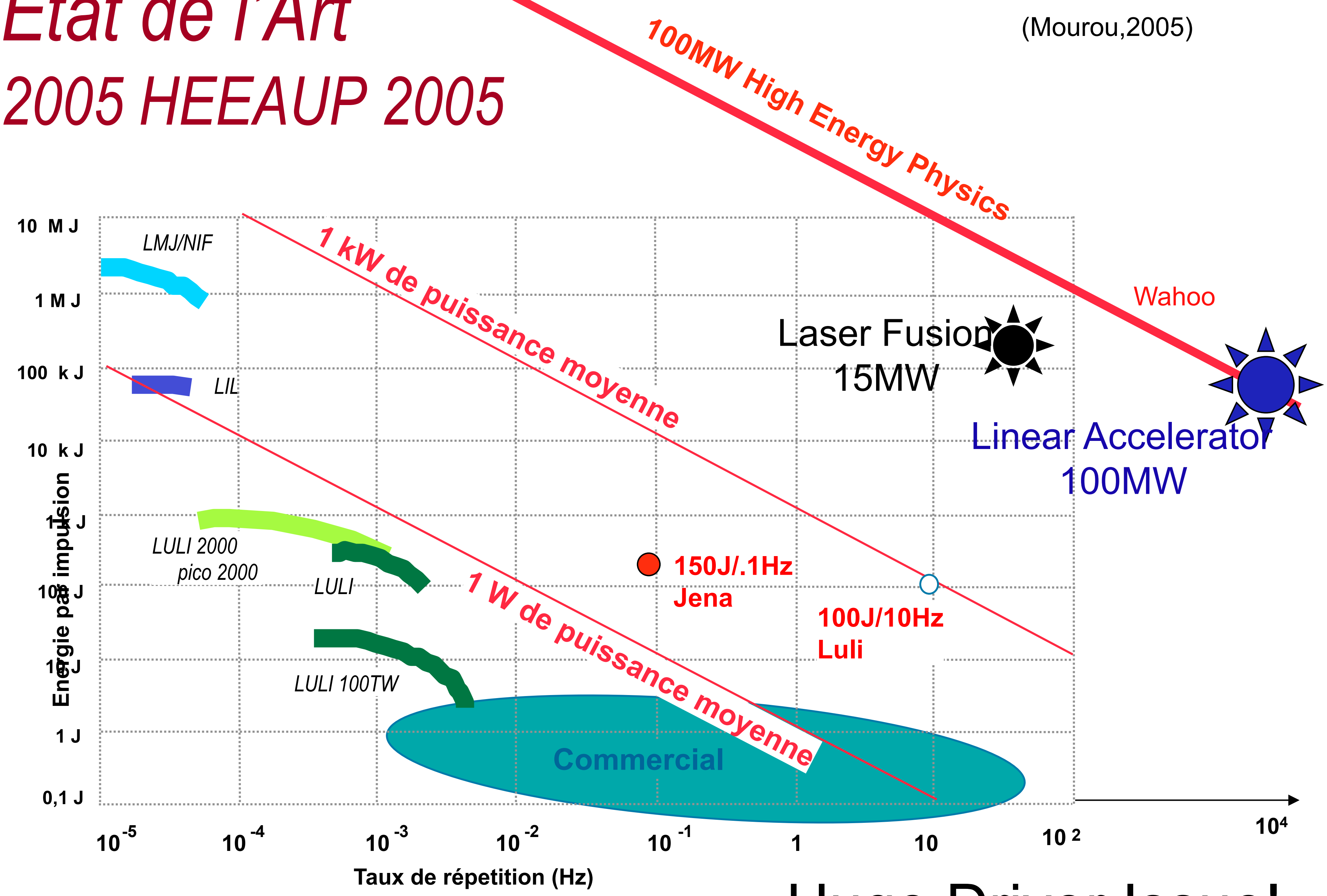
(Mourou,2005)



Huge Driver Issue!

conclusions

- LWFA provides unique tool to a future collider option
- More compact and thus less expensive collider framework
- Collider physics requirements: luminosity maximization, small beam, large betatron emittance preservation: tough challenge
- Driver laser for collider: a huge challenge, topic of discussion for the afternoon