

Light meson photoproduction with Regge theory: advancing hybrid meson searches at GlueX

Glòria Montaña

IJCLab PHE Seminar

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UNIVERSITAT DE
BARCELONA



EXCELENCIA
MARÍA
DE MAEZTU
04/2025-03/2031

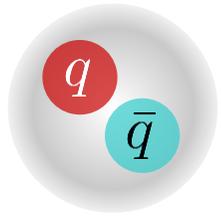
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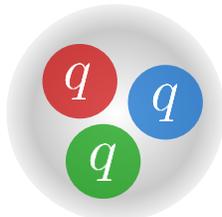
EXO HAD
EXOTIC HADRONS TOPICAL COLLABORATION

Exotic hadrons

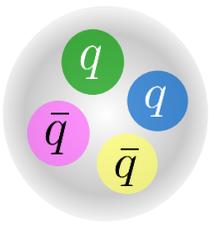
Mesons and baryons aren't the only states allowed by QCD



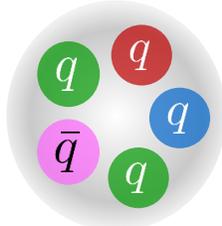
meson



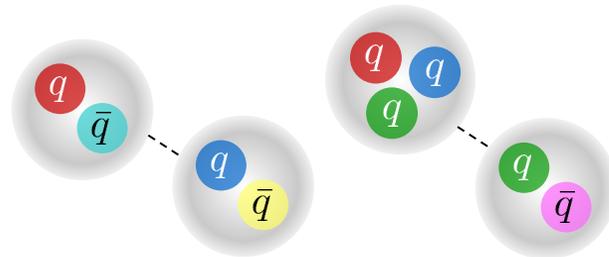
baryon



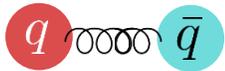
tetraquark



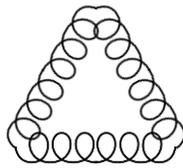
pentaquark



hadronic molecules



hybrid meson



glueball

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

... Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. ...

Phys.Lett. 8, 214 (1964)

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

G. Zweig *)

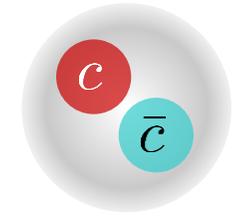
CERN - Geneva

In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAA$, etc., where \bar{A} denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA , that is, "deuces and treys".

CERN-TH-401 (1964)

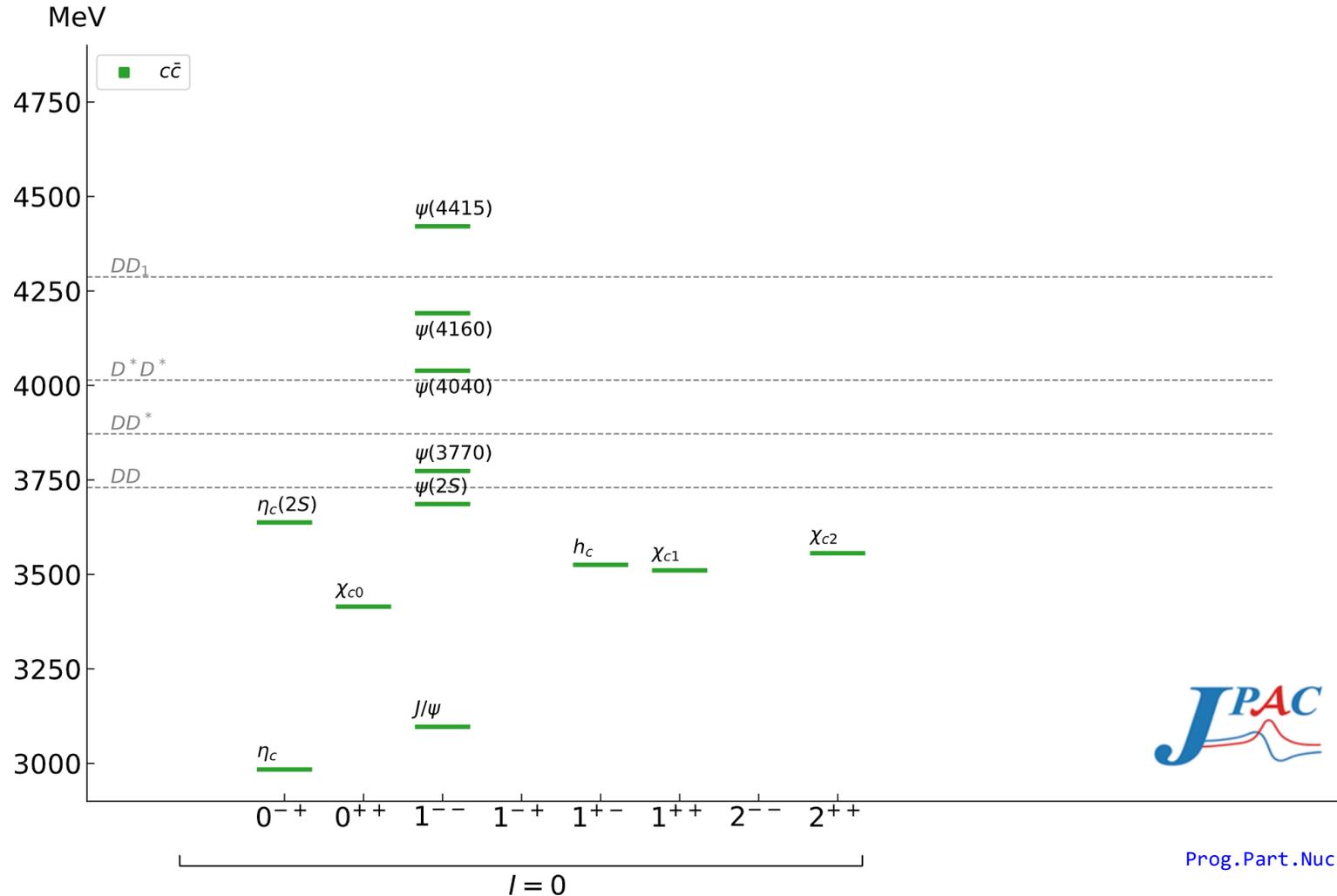
Hybrid mesons (with valence gluons) → insight into the gluon dynamics in the nonperturbative regime

Heavy hadron spectroscopy: $XYZP_c$



charmonia states

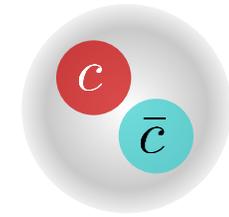
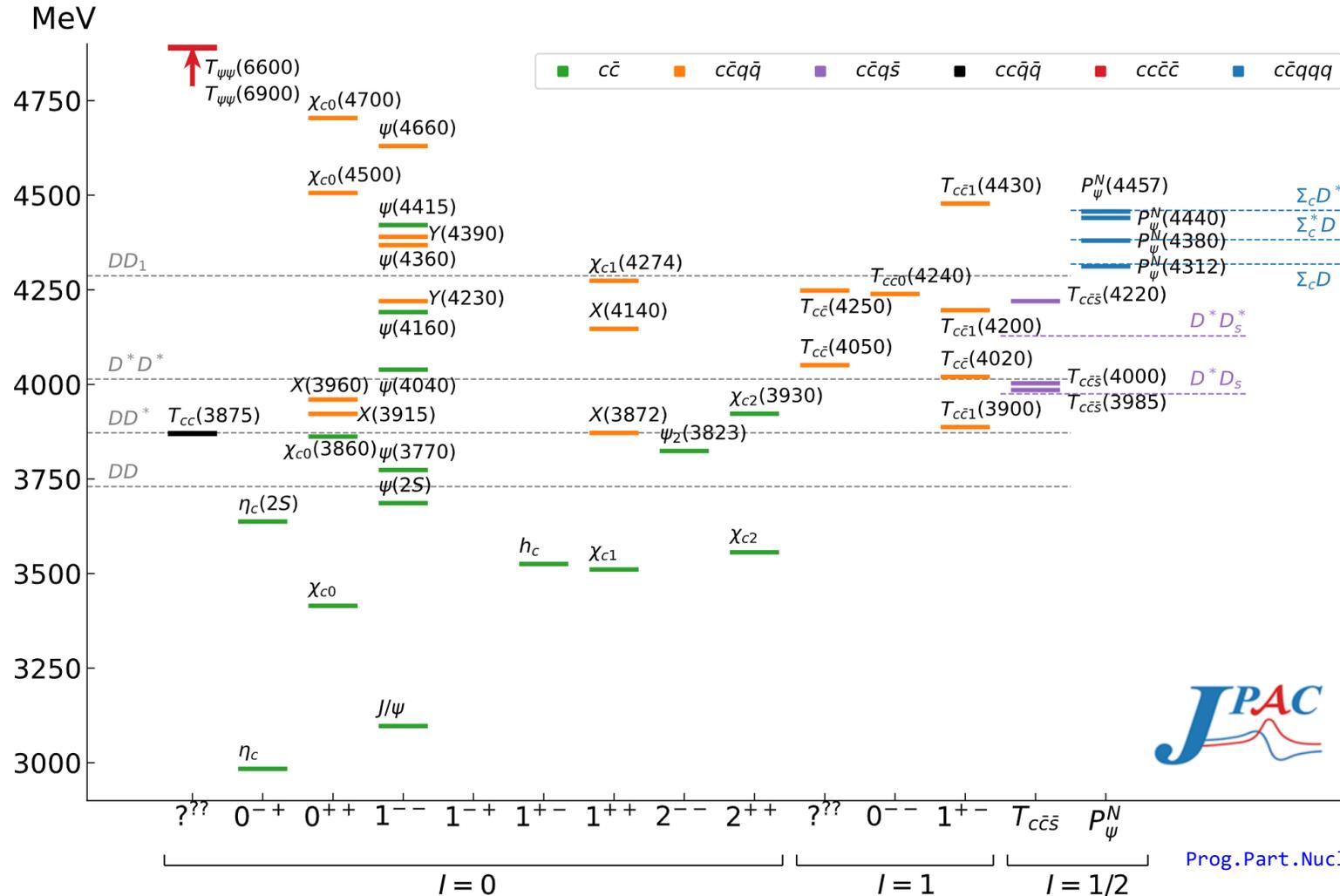
Before 2003 ...



Prog.Part.Nucl.Phys. 127, 103981 (2022)

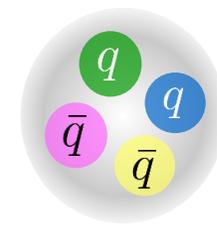
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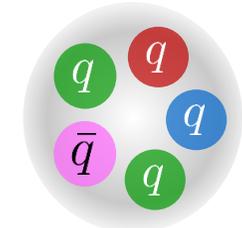


charmonia states

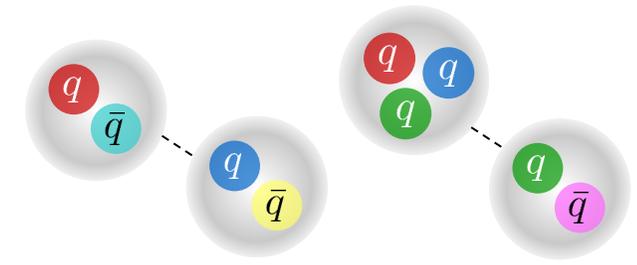
Exotic hadrons



tetraquark



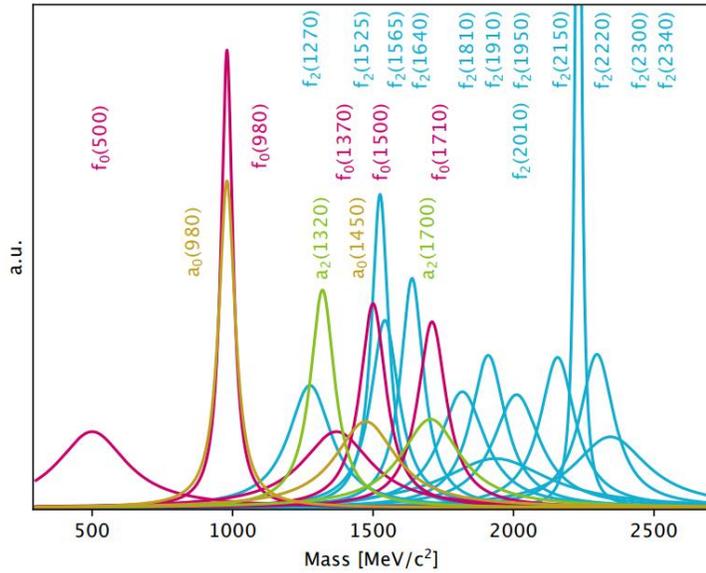
pentaquark



hadronic molecules



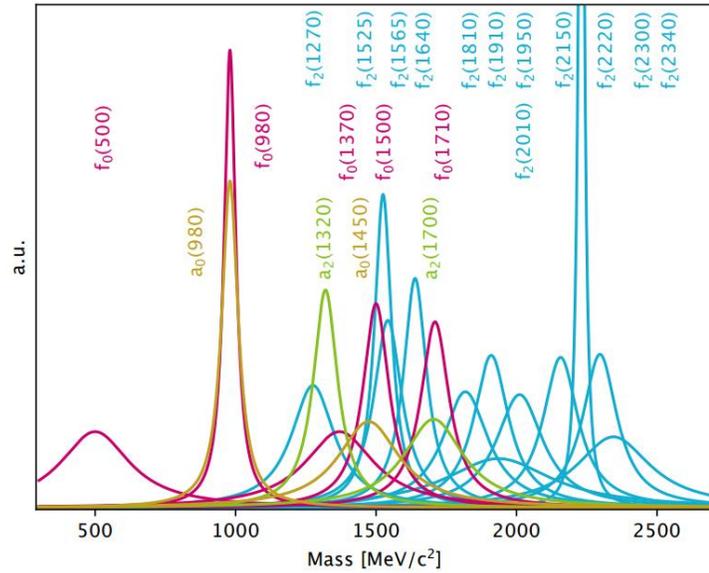
Light meson spectroscopy



Experimental challenges:

- * Many wide and overlapping states
- * Need complicated partial wave analysis (PWA) techniques

Light meson spectroscopy

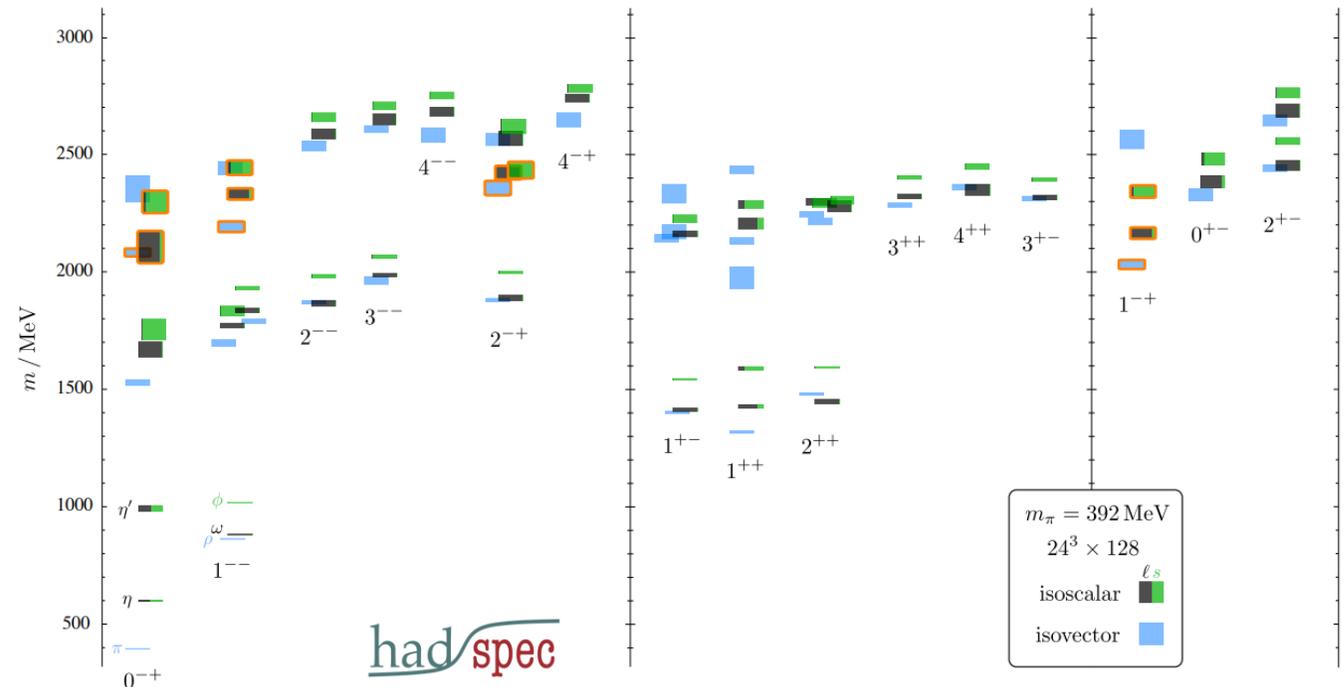


Lattice QCD predictions:

J.Dudek et al., Phys.Rev.D 88, 094505 (2013)

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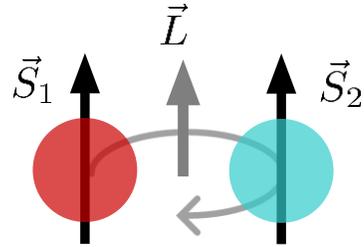
Exotic quantum numbers

Quantum numbers:

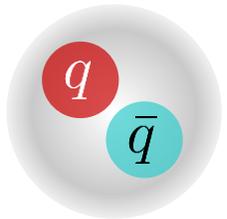
$$\vec{J} = \vec{L} \otimes \vec{S} \quad , \quad \vec{S} = \vec{S}_1 \otimes \vec{S}_2$$

$$P = -(-1)^L$$

$$C = (-1)^{L+S}$$



Conventional $q\bar{q}$ mesons:



J^{PC}	{	0^{++}	0^{-+}	
		1^{--}	1^{++}	1^{+-}
		2^{--}	2^{++}	2^{-+}
		3^{--}	3^{++}	3^{+-}
		\vdots	\vdots	\vdots

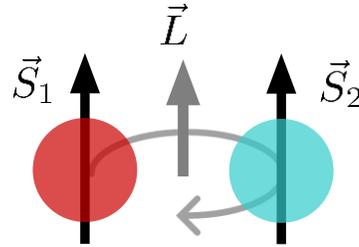
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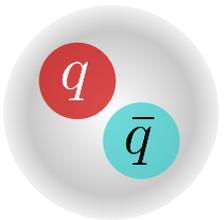
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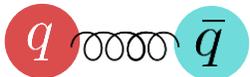
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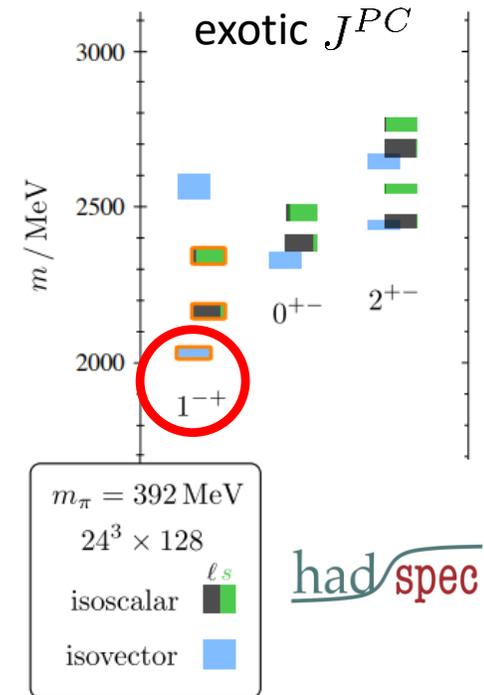
J^{PC}

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Hybrid mesons



* Predictions from lattice QCD:



J.Dudek et al., Phys.Rev.D 88, 094505 (2013)

* Gluonic fields in hybrid mesons give rise to states with **“exotic” quantum numbers** (not allowed for $q\bar{q}$)

→ “Smoking gun” to find exotic mesons!

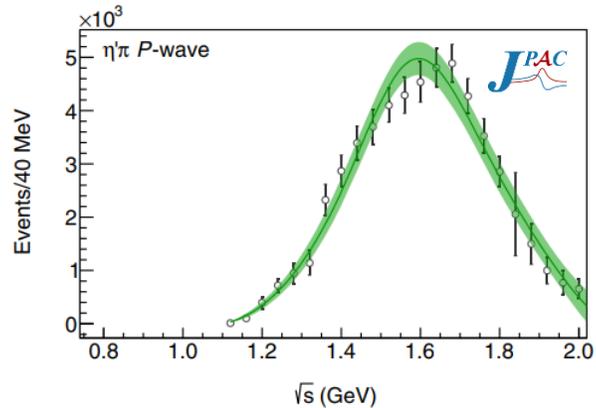
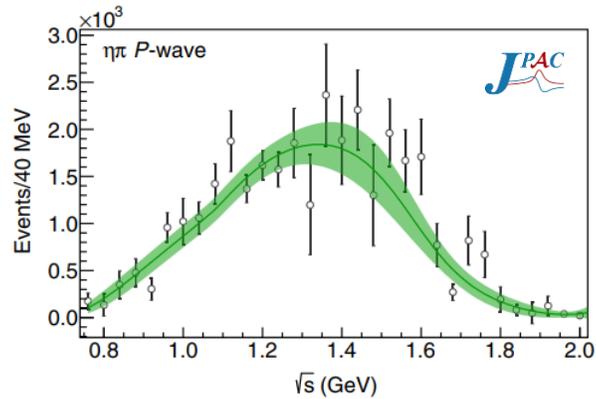
The $\pi_1(1600)$

Best experimental evidence from COMPASS with a pion beam

Historically, two 1^{-+} isovector candidates: $\pi_1(1400)$ and $\pi_1(1600)$



Data:
Phys.Lett.B 740, 303 (2015)



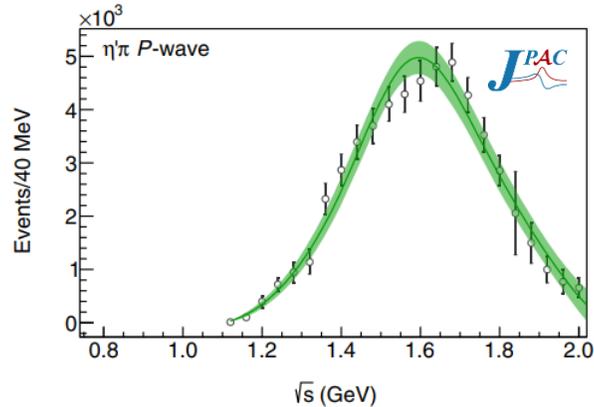
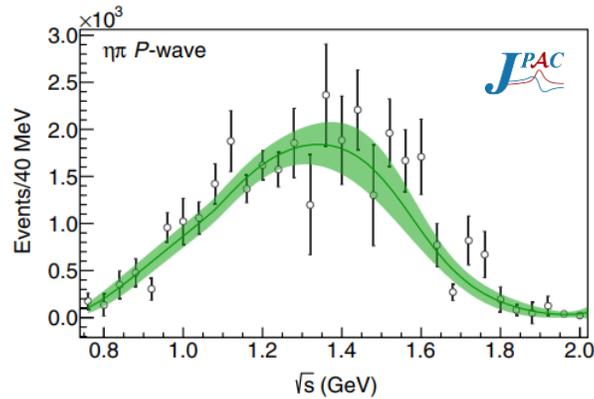
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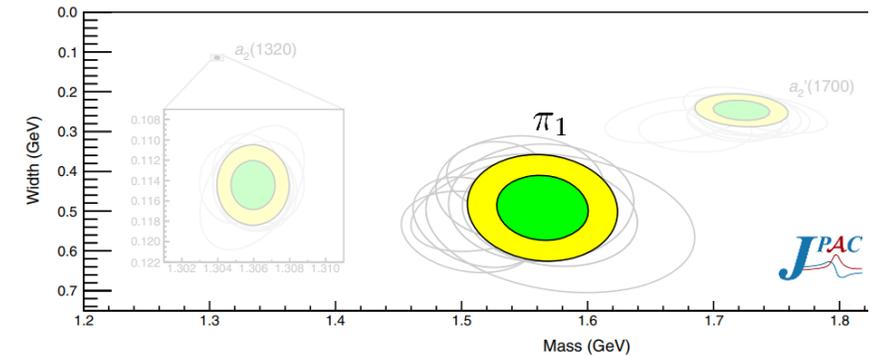
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* Coupled-channel analysis by JPAC:

→ Data is consistent with a **single resonance pole**

A.Rodas et al., Phys.Rev.Lett. 122, 042002 (2019)



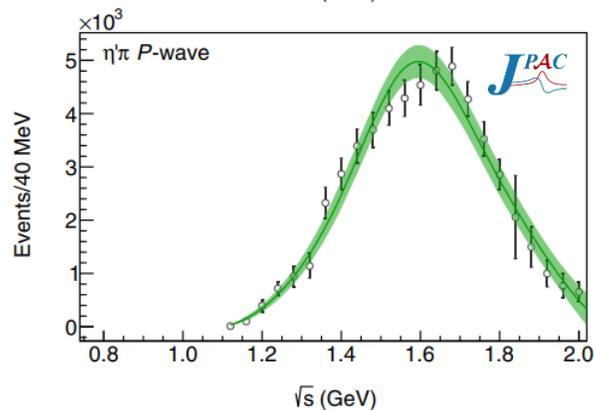
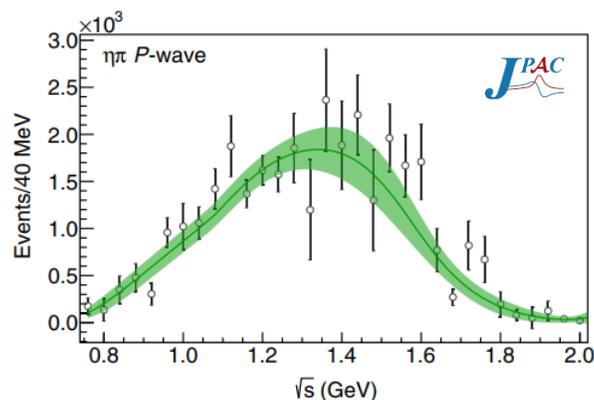
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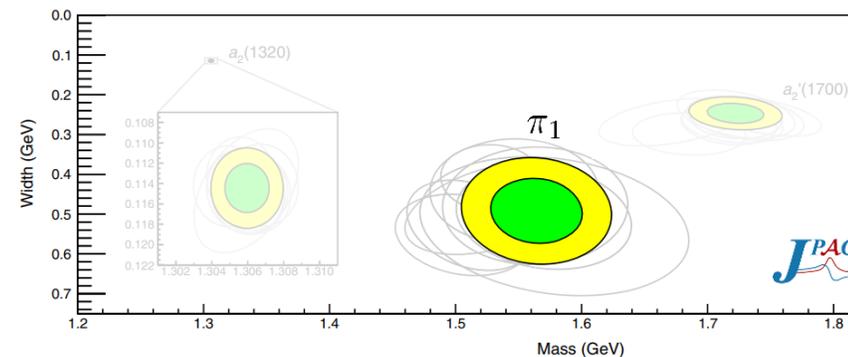


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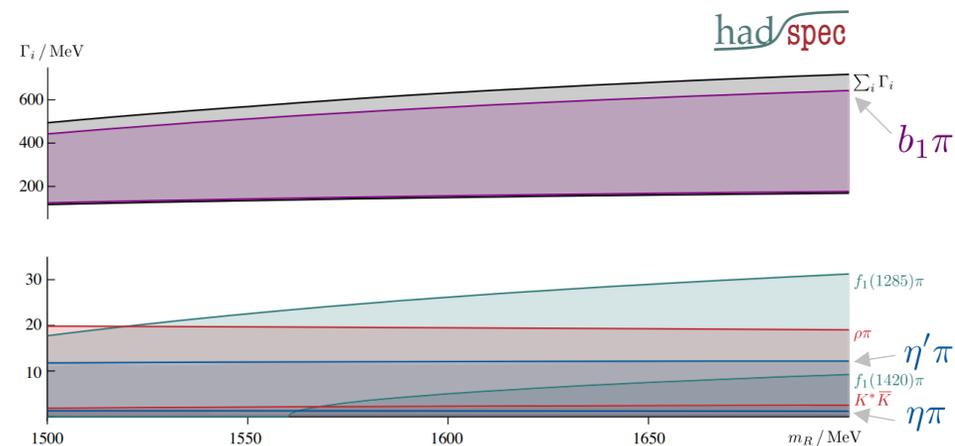
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* Confirmed by HadSpec:

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A.Woss et al., Phys.Rev.D 103 5, 054502 (2021)



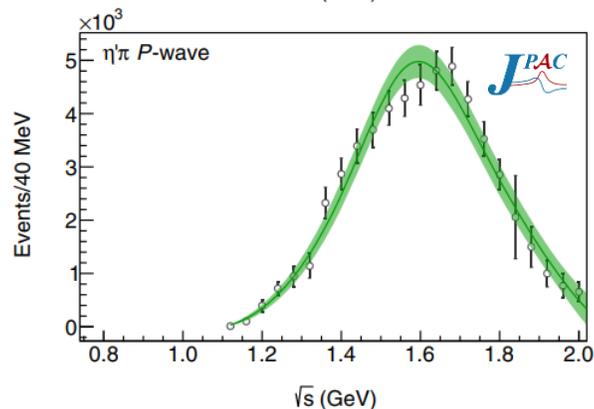
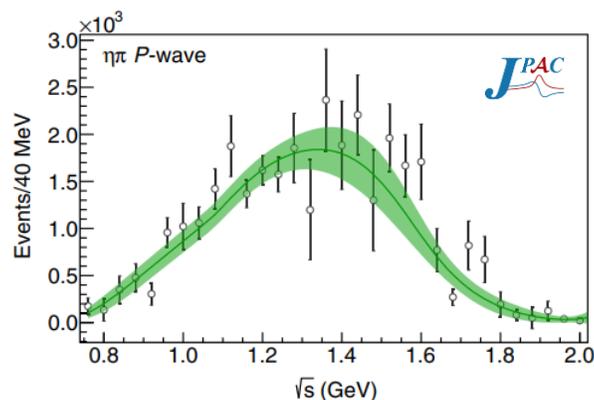
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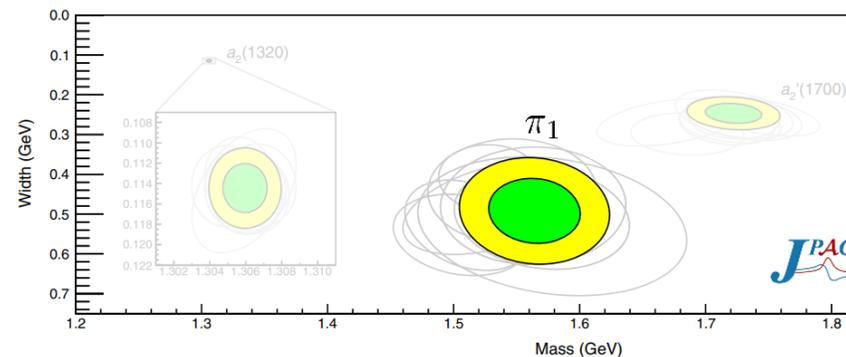


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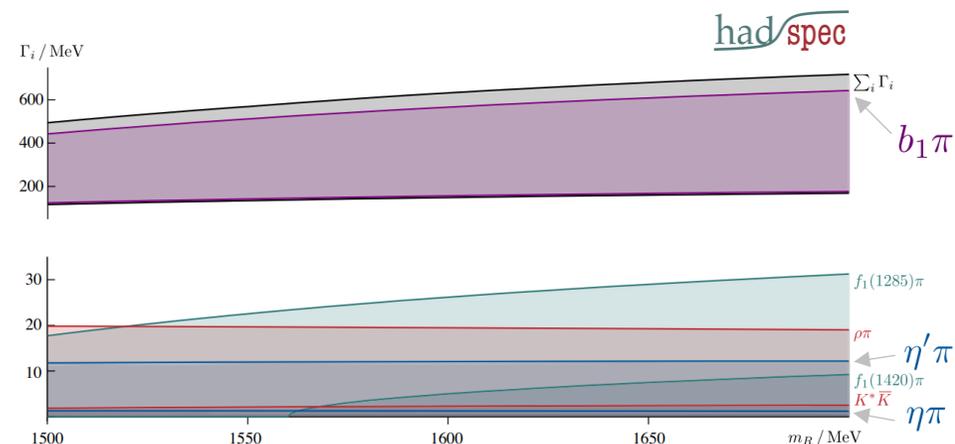
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* Experimental confirmation with a photon beam is the main goal of GlueX

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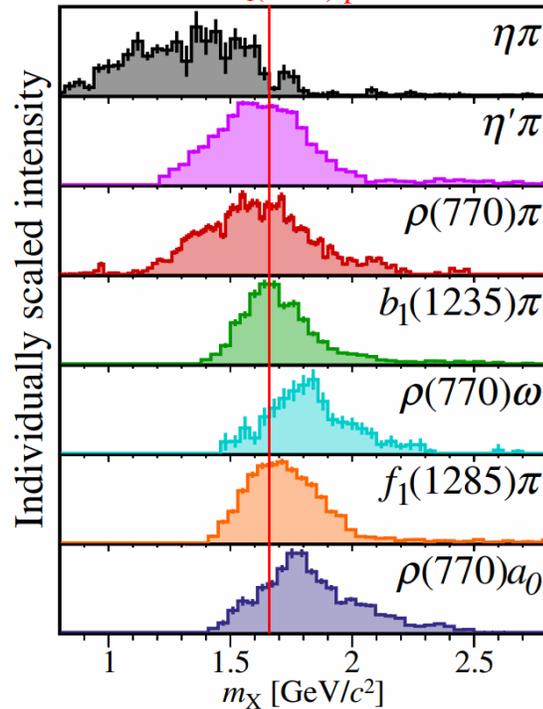
More recently...

- * Multiple decay channels with high statistics data from COMPASS

Talk by D.Spielbeck at HADRON2025 (2508.18908)

Spin-exotic $J^{PC}=1^{-+}$ waves at COMPASS
preliminary

Nominal $\pi_1(1600)$ position



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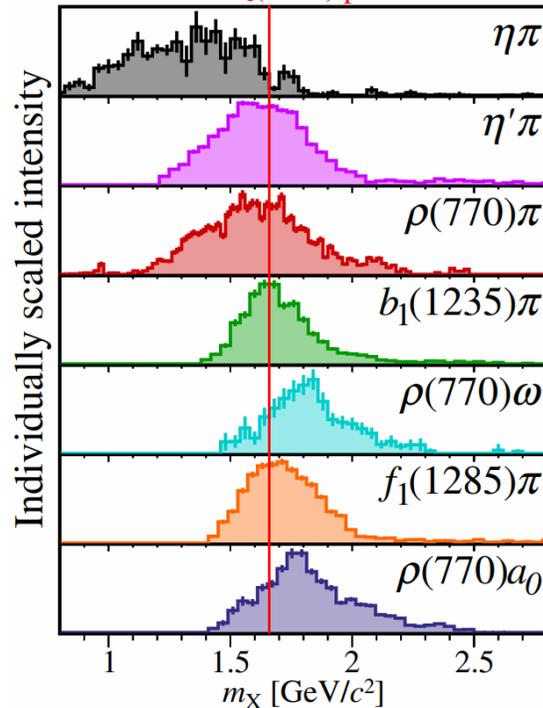
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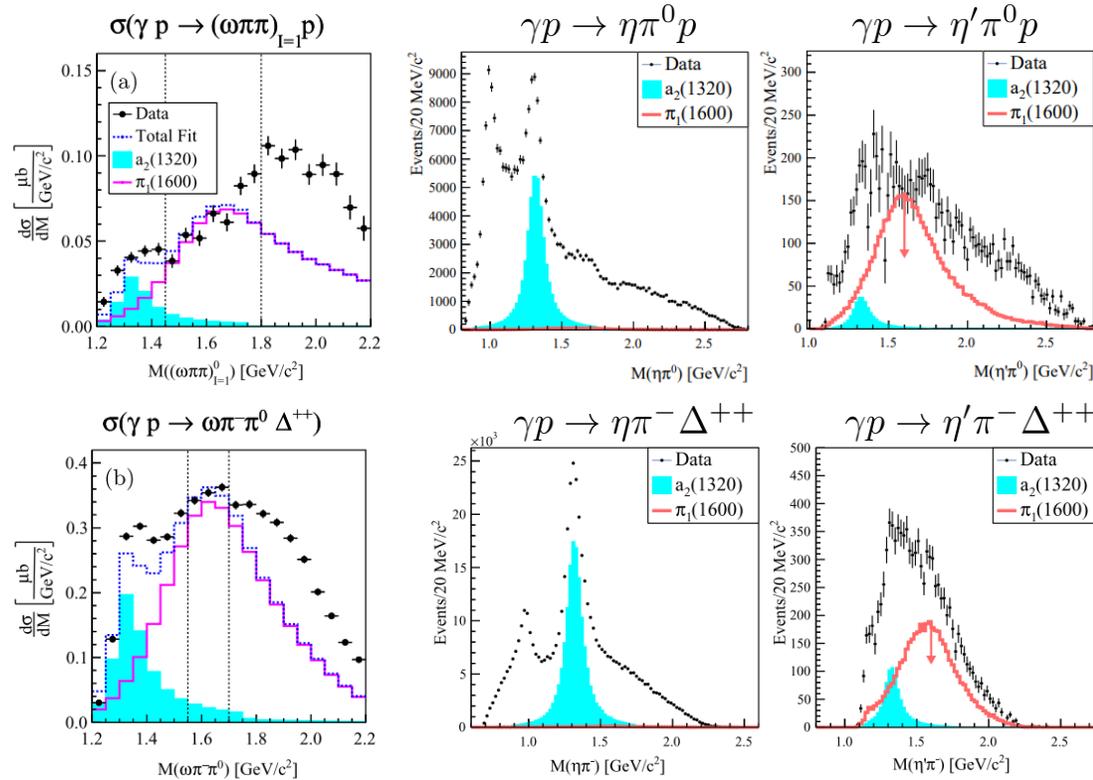
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Phys.Rev.Lett. 133 261903 (2024)



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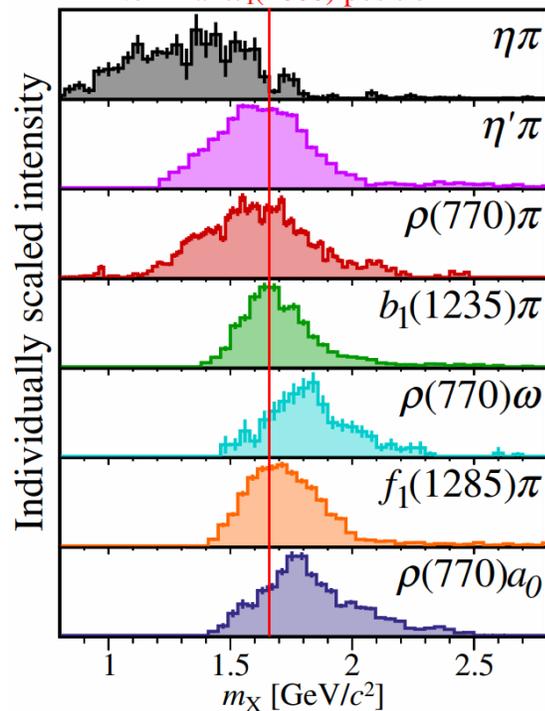
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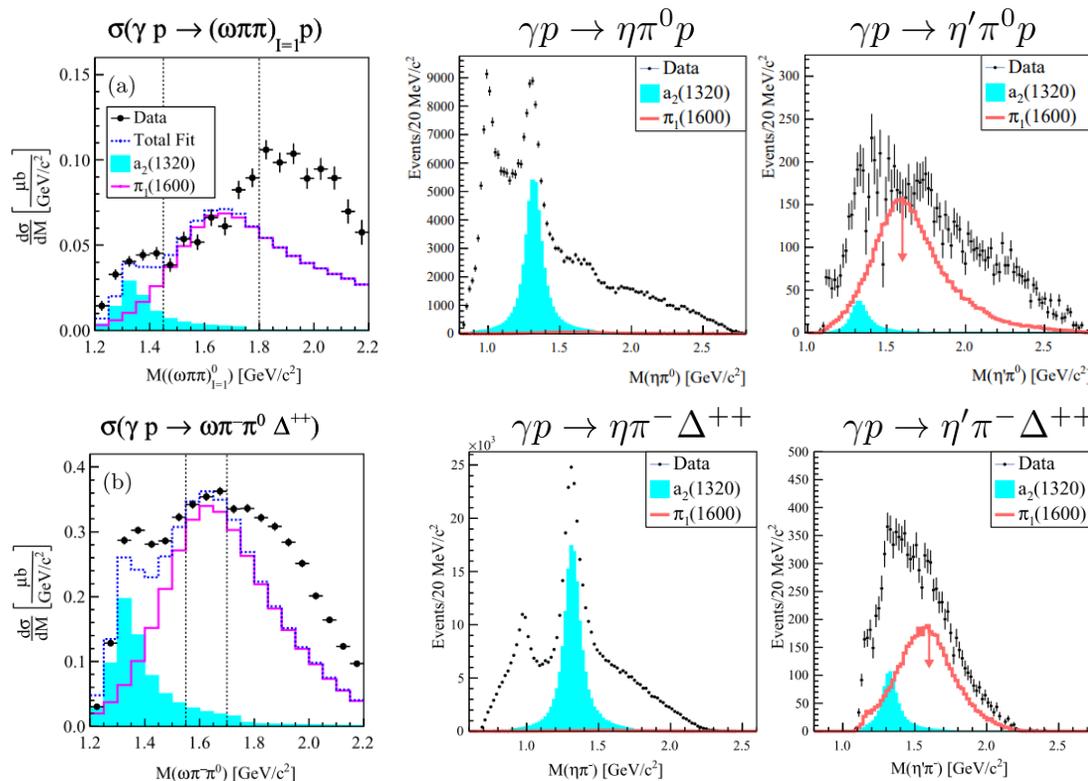
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Phys.Rev.Lett. 133 261903 (2024)



- * Observation of isospin partner $\eta_1(1855)$ by BES-III Phys.Rev.Lett. 129, 192002 (2022)

00 Motivation

01 Remarks on Scattering and Regge Theory

02 Pion exchange in pion photoproduction

03 Photoproduction of $\eta^{(\prime)}\pi$ in the double-Regge region

04 Other studies

05 Conclusions

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Principles of Scattering Theory

Probability conservation

Unitarity $SS^\dagger = 1$

if $S = 1 + iA$ then $-i(A - A^\dagger) = 2\text{Im} A = AA^\dagger$

Causality

Analyticity

$A(s)$ has singularities (poles and branch cuts) in the complex s plane.

Existence of antiparticles

Crossing symmetry $A_{ab \rightarrow cd}(s, t, u) = A_{a\bar{c} \rightarrow \bar{b}d}(t, s, u)$

Physical regions (in the case of equal mass particles):

s - channel: $a + b \rightarrow c + d$ $s \geq 4m^2, t \leq 0, u \leq 0$

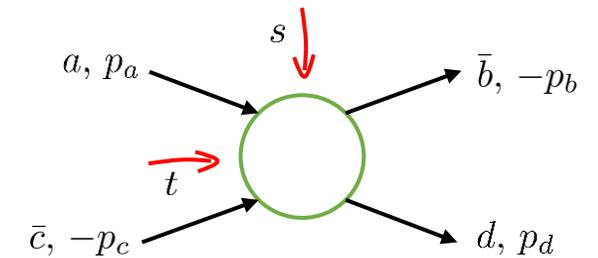
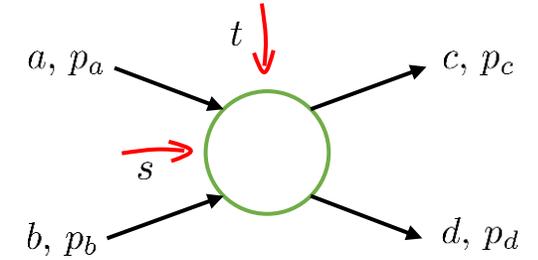
t - channel: $a + \bar{c} \rightarrow \bar{b} + d$ $t \geq 4m^2, s \leq 0, u \leq 0$

Conservation of orbital angular momentum

Partial wave expansion

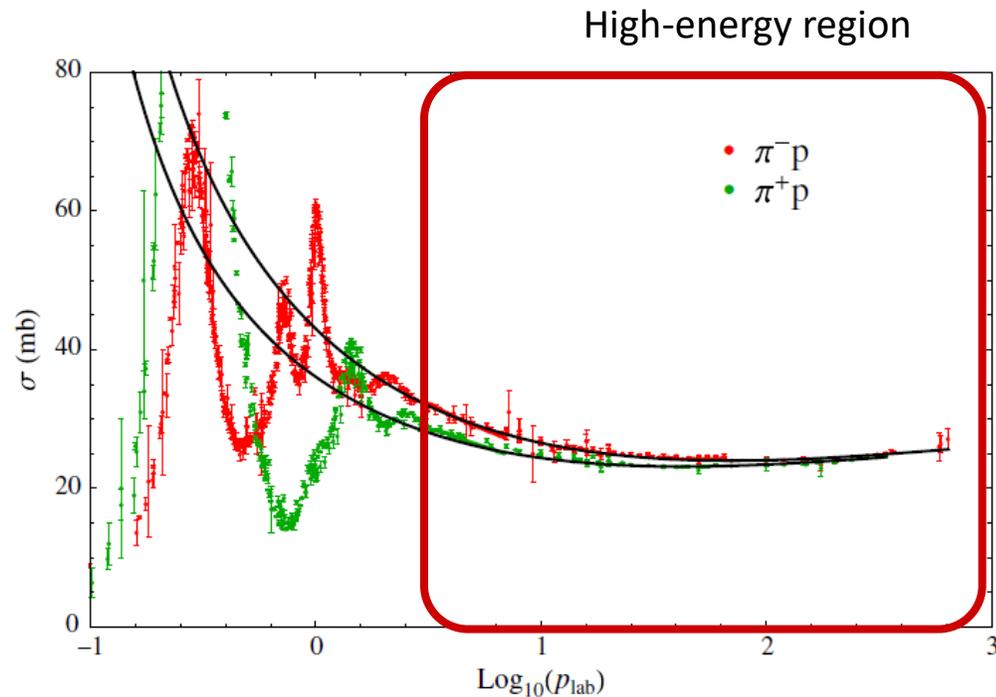
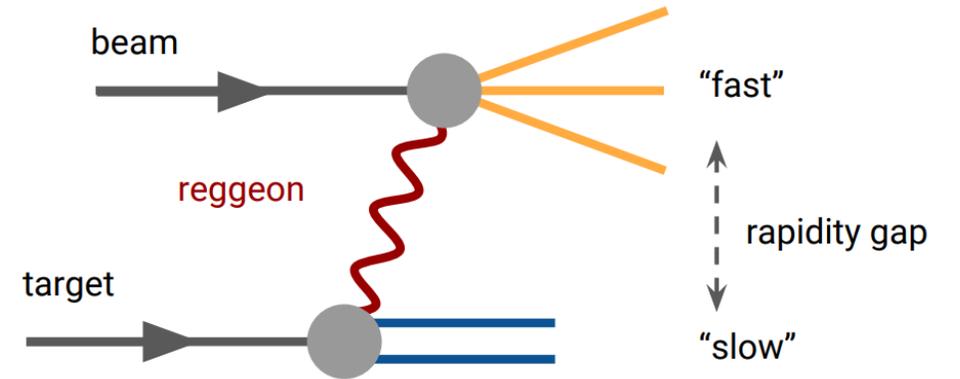
$$A(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_{\ell}(s) P_{\ell}(z)$$

$f_{\ell}(s)$: s - channel partial wave amplitudes
 $P_{\ell}(z)$: Legendre polynomials



Meson photoproduction at high energies

At high energies, meson photoproduction is dominated by the **exchange of Regge trajectories** in the t-channel



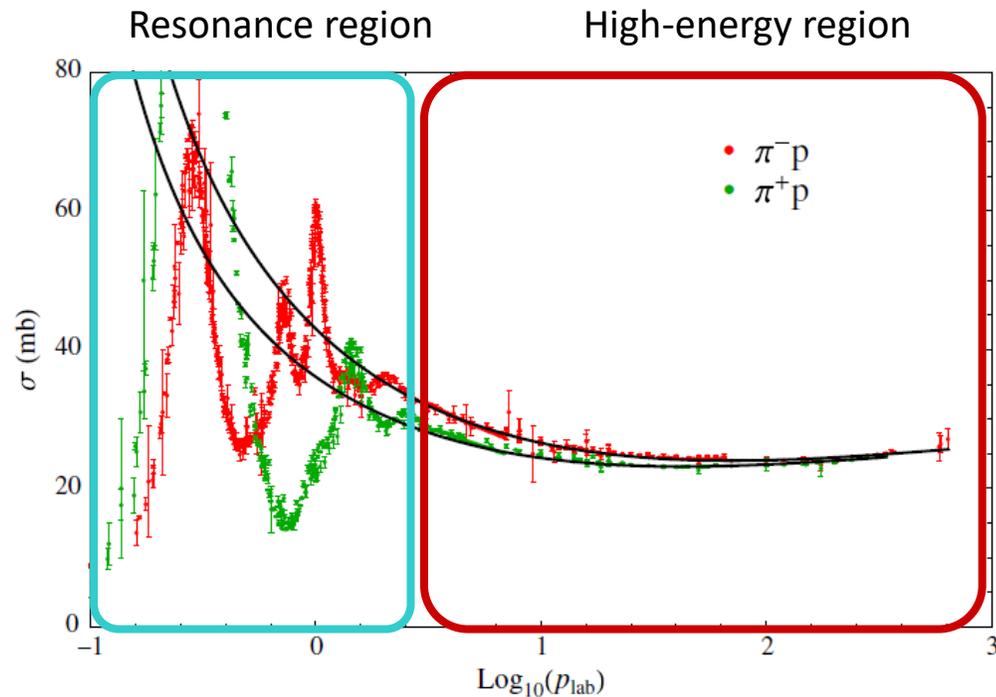
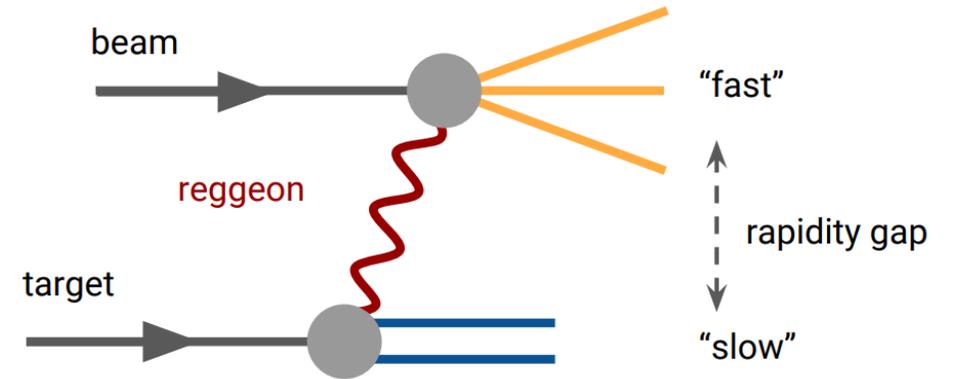
V.Mathieu et al., Phys.Rev.D 92, 074004 (2015)

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* **Duality** between resonance poles and Regge exchanges

Dolen, Horn, Schmid, Phys.Rev. 166, 1768 (1968), Veneziano, Nuovo Cim.A 57, 198 (1968)



V.Mathieu et al., Phys.Rev.D 92, 074004 (2015)

Low-energy: resonances

$$A(s, t) \sim \sum_r \frac{g_r}{s - s_r}$$

High energy: Regge exchanges

$$A(s, t) \sim \sum_i \frac{g_i}{t - t_i}$$

Analytically connected

Introduction to Regge Theory

T. Regge, *Nuovo Cim.* 18, 947 (1960)

At high energy, the scattering amplitude in the physical region of the s-channel is related to t-channel exchanges

$$A(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_{\ell}(t) P_{\ell}(z_t)$$

t-channel partial wave amplitudes

Regge limit
 $s \gg -t, m^2$

$$A(s, t) \sim s^{\ell_{\text{eff}}}$$
$$z_t = \cos \theta_t = 1 + \frac{2s}{t - 4m^2}$$
$$\lim_{z \rightarrow \infty} P_{\ell}(z) \sim z^{\ell}$$

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The concept of partial-wave amplitude can be extended to complex values of angular momentum

$$\{f_{\ell}(t)\} \rightarrow f(\ell, t) \text{ with } f(\ell, t) \rightarrow f_{\ell}(t), \ell \in \{0, 1, 2, \dots\}$$

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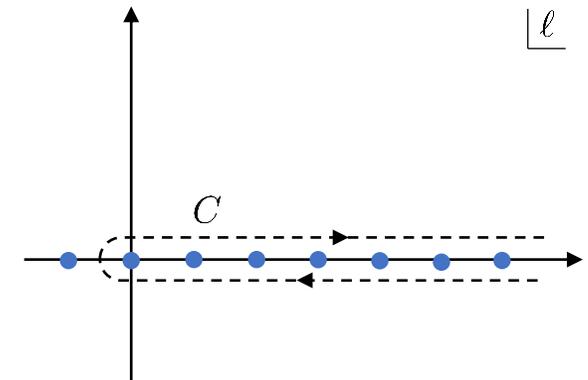
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Do the sum: Sommerfeld-Watson transform

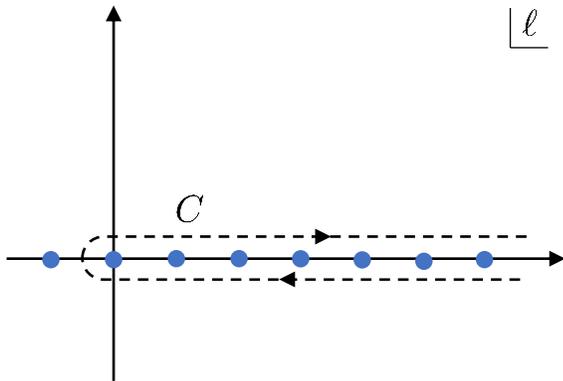
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Introduction to Regge Theory

Sommerfeld-Watson transform

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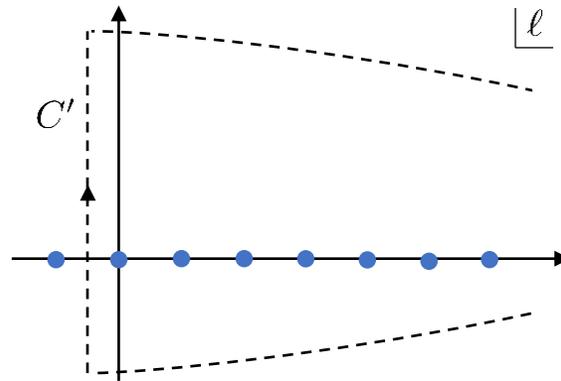
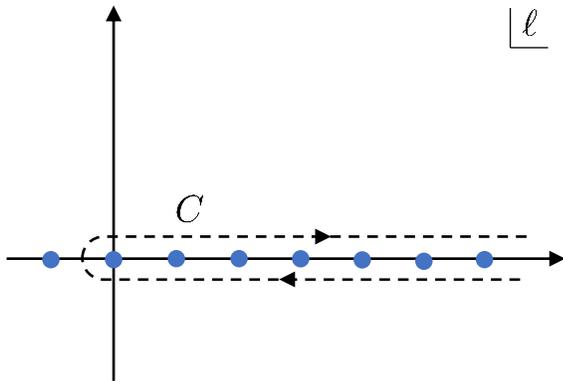


Introduction to Regge Theory

Sommerfeld-Watson transform

$$A(s, t) = -\frac{1}{2i} \int_C d\ell \frac{(2\ell + 1) P_\ell(-z_t) f(\ell, t)}{\sin \pi \ell}$$

* Deform the contour



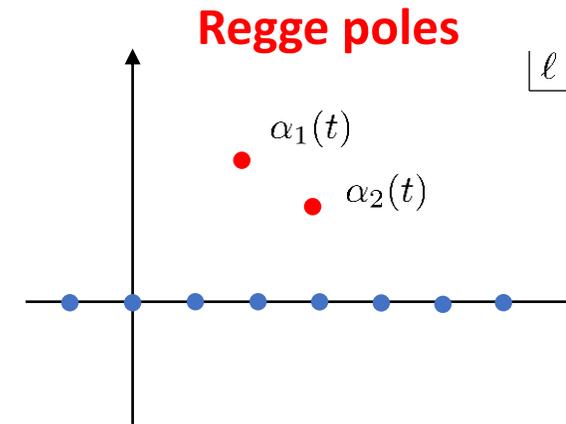
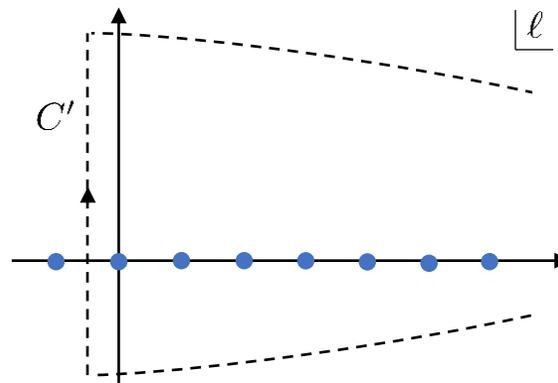
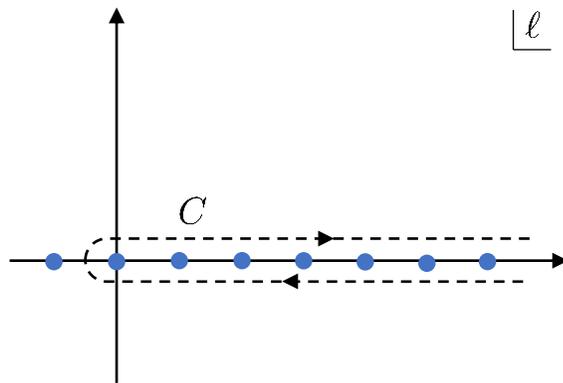
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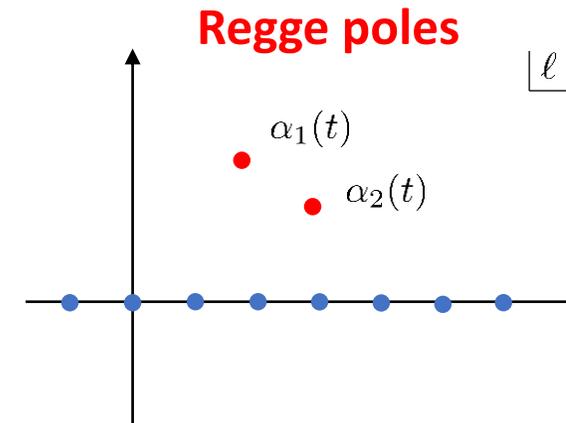
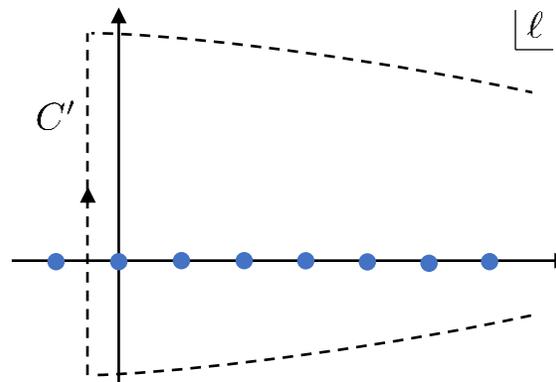
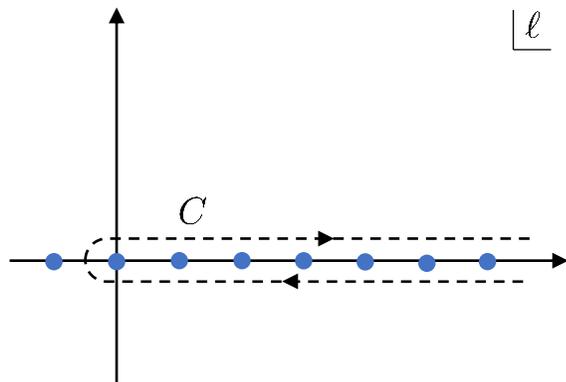
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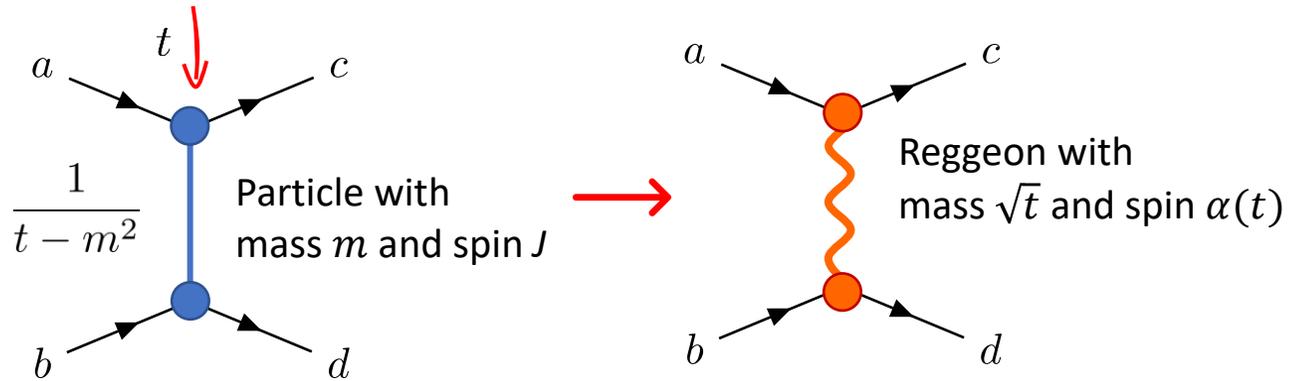
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$$A(s, t) = \underbrace{-\frac{1}{2i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} d\ell \dots}_{\text{background} \sim s^{-1/2}} - \underbrace{\sum_i \frac{\pi(2\alpha_i(t) + 1)\beta_i(t)}{\sin(\pi\alpha_i^\pm(t))} \frac{1}{2} P_{\alpha_i}(-z_t)}_{\text{pole contributions} \sim s^{\alpha(t)}}$$



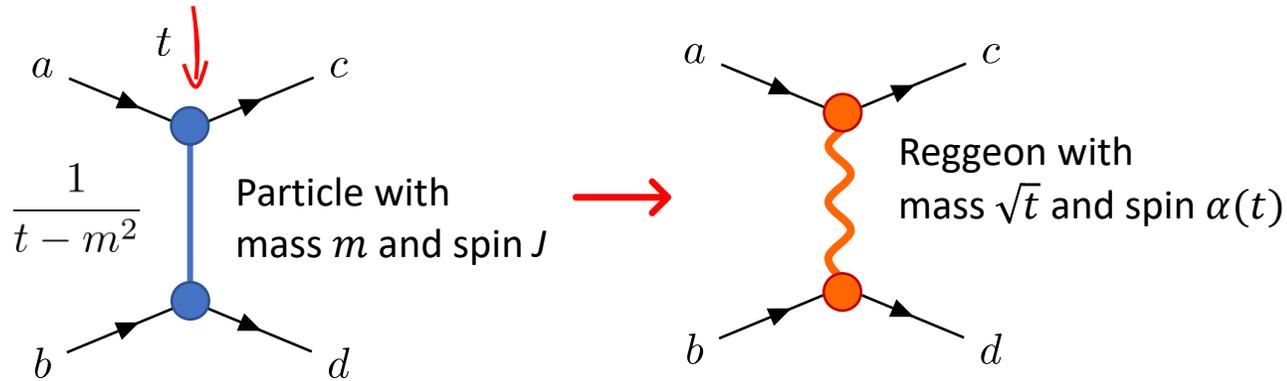
Introduction to Regge Theory



$$\mathcal{P}_{\text{Regge}} = \frac{\pi\alpha'}{\sin(\pi\alpha(t))} \frac{1 + \eta e^{-i\pi\alpha(t)}}{2} \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{1}{\Gamma(1 + \alpha(t))}$$

poles for integer $\alpha(t)$ signature factor $\eta = (-1)^J$ asymptotic behavior cancel non-physical poles

Introduction to Regge Theory

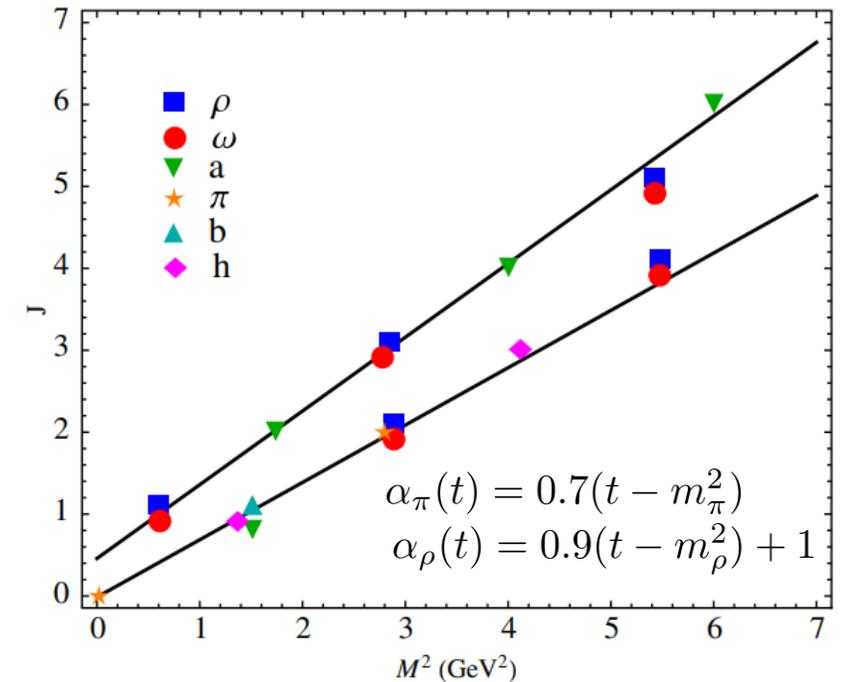


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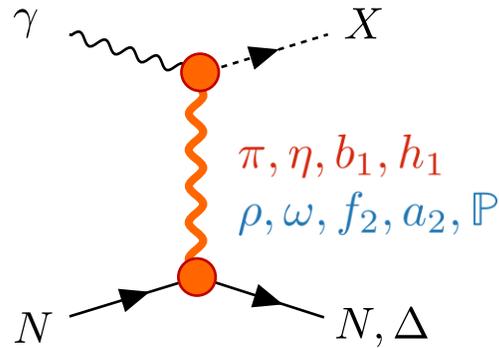
Resonances appear simultaneously as poles in energy and spin!

- * **Regge trajectories:** families with same quantum numbers but different spin
- * Almost straight lines (Chew-Frautschi plot)
- * In standard Regge theory parameterized by: $\alpha(t) = \alpha' t + \alpha_0$



Model for meson photoproduction

We “only” need to know the (dominant) crossed-channel exchanges

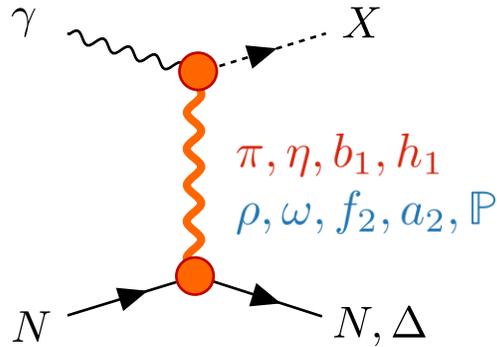


→ **Unnatural** parity ($P(-1)^J = -1$): $0^-, 1^+, 2^-, 3^+, \dots$

→ **Natural** parity ($P(-1)^J = +1$): $0^+, 1^-, 2^+, 3^-, \dots$

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→ **Natural** parity ($P(-1)^J = +1$): $0^+, 1^-, 2^+, 3^-, \dots$

* Neutral-exchange reactions:

$$\gamma p \rightarrow \pi_1^0 p$$

$$\gamma p \rightarrow b_1^0 p, \gamma p \rightarrow a_0^0 p, \gamma p \rightarrow a_2^0 p,$$

$$\gamma p \rightarrow \eta^{(\prime)} \pi^0 p \dots$$

→ Natural parity exchanges dominate

* Charge-exchange reactions:

$$\gamma p \rightarrow \pi_1^- \Delta^{++}$$

$$\gamma p \rightarrow b_1^- \Delta^{++}, \gamma p \rightarrow \pi^- \Delta^{++}, \gamma p \rightarrow a_0^- \Delta^{++}, \gamma \Delta^{++} \rightarrow a_2^- p,$$

$$\gamma p \rightarrow \eta^{(\prime)} \pi^- \Delta^{++} \dots$$

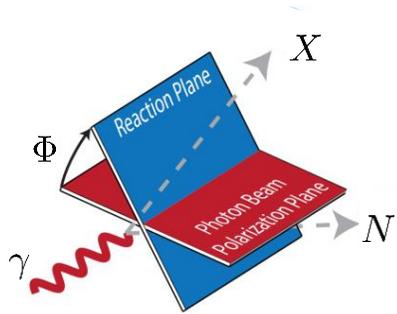
→ Small -t: unnatural (pion) exchanges favored

→ Larger -t: natural exchanges favored

Model for meson photoproduction

Linearly polarized photon beam at GlueX helps disentangle the production mechanisms

* Beam asymmetry



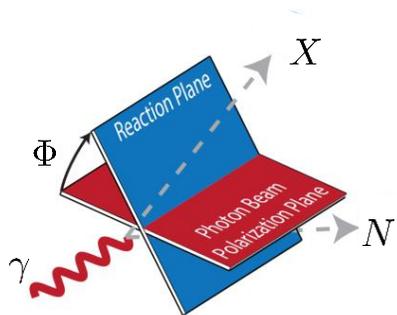
$$\Sigma = \frac{d\sigma_{\perp} - d\sigma_{\parallel}}{d\sigma_{\perp} + d\sigma_{\parallel}} \approx \frac{d\sigma_N - d\sigma_U}{d\sigma_N + d\sigma_U}$$

V.Mathieu et al., Phys.Rev.D 92, 074013 (2015)

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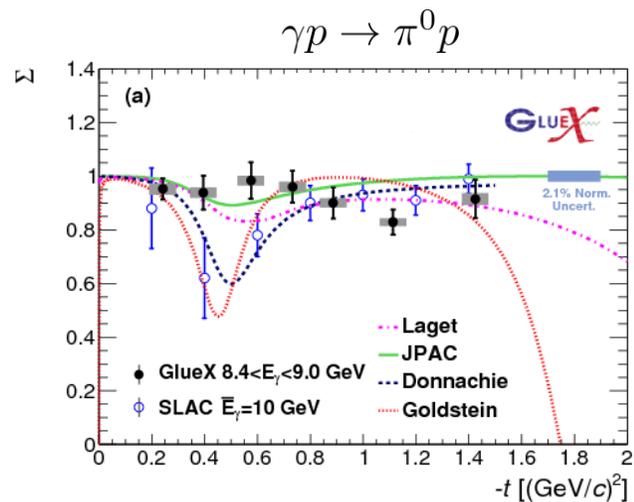
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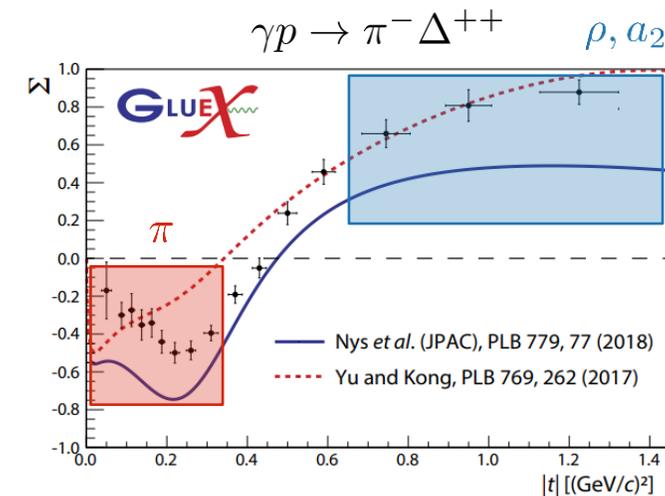
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→ Neutral- vs charge-exchange reactions:



GlueX Collaboration, Phys.Rev.C 95, 042201 (2017)



GlueX Collaboration, Phys.Rev.C 103, L022201 (2021)

00 Motivation

01 Remarks on Scattering and Regge Theory

02 Pion exchange in pion photoproduction

03 Photoproduction of $\eta^{(\prime)}\pi$ in the double-Regge region

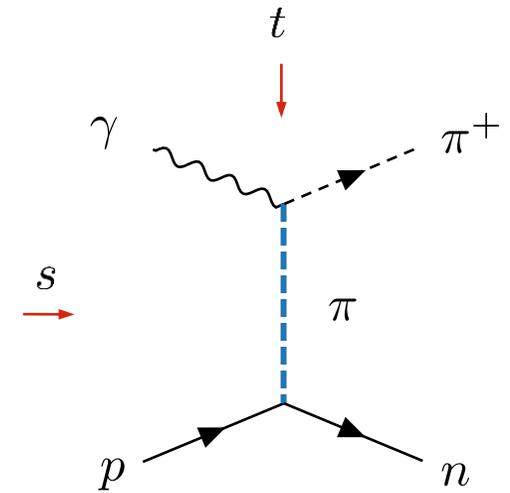
04 Other studies

05 Conclusions

Understanding pion exchange

What do we know?

- Dominates in charge exchange reactions at small momentum transfer
- Low energies: constrained by effective Lagrangians of QCD
- High energies: Regge theory



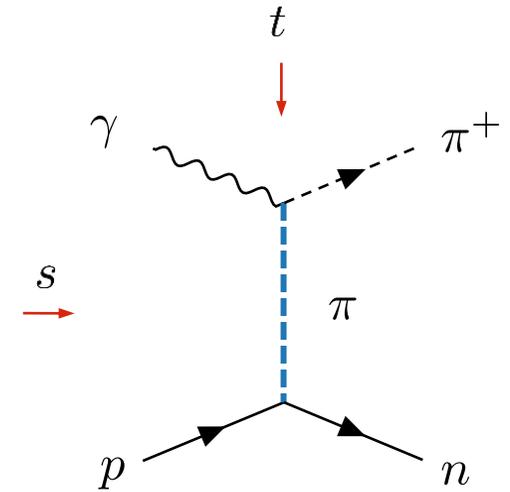
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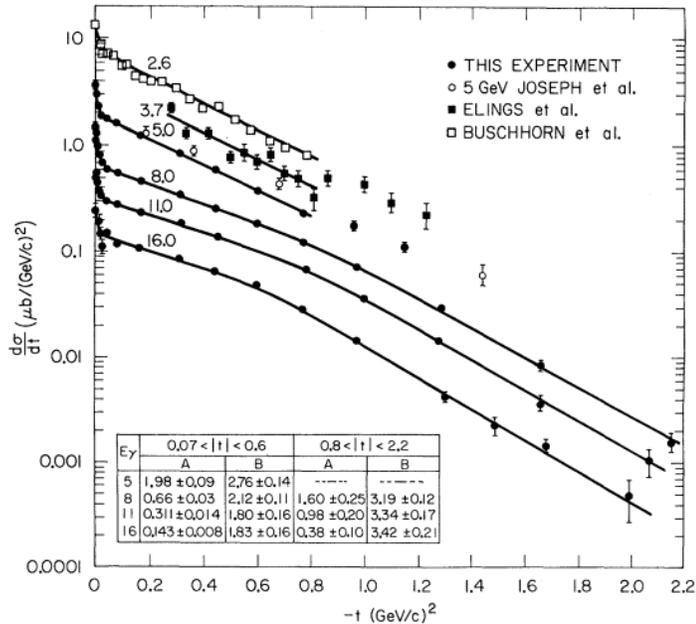
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Boyarski et al., *Phys.Rev.Lett.* 20, 300 (1968)

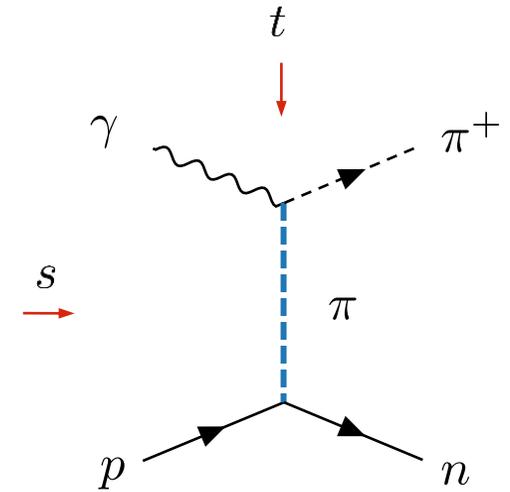


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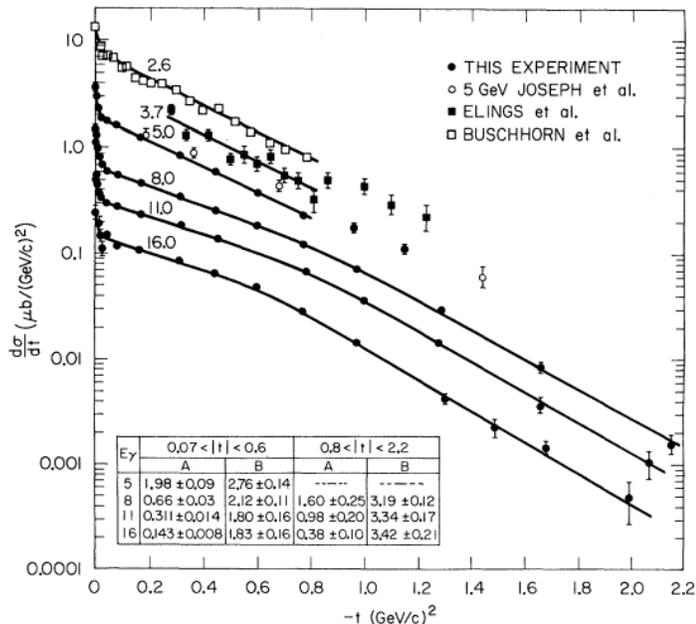
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Known issues:

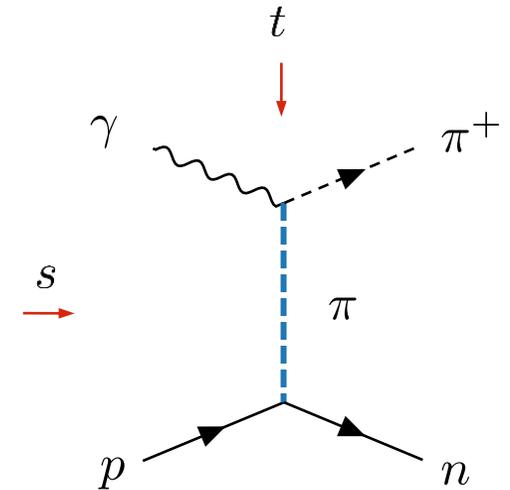
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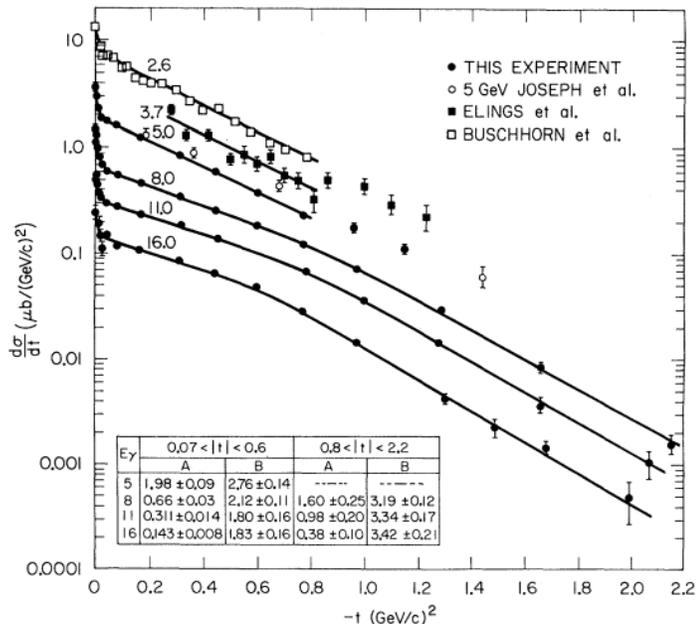
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Early solutions:

Parity-doublet conspirator of the pion

Ball, Frazer and Jacob, *Phys.Rev.Lett.* 20, 518 (1968)

Regge cuts and absorption (final state interactions)

Henyey, Kane, Pumplin, *Phys.Rev.* 182, 1579 (1969)

Nucleon Born terms

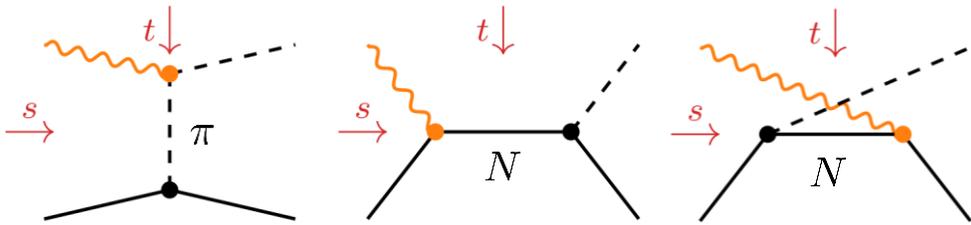
Jones, *Rev.Mod.Phys.* 52, 545 (1980) 545

Born amplitudes

In terms of Born diagrams, **current conservation** relates t-channel diagram (pion exchange) and s-, u-channel diagrams (nucleon exchanges)

$$A_{\mu\gamma\mu_i\mu_f} = \epsilon_{\mu\gamma}(k) \cdot J_{\mu_i\mu_f}$$

$$J_{\mu_i\lambda_f}^\mu = J_{\mu_i\mu_f,t}^\mu + J_{\mu_i\mu_f,s}^\mu + J_{\mu_i\mu_f,u}^\mu$$



$$J_{\mu_i\mu_f,t}^\mu = -\sqrt{2}e_\pi g_{\pi NN} \frac{q_t^\mu - p_\pi^\mu}{t - \mu^2} \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

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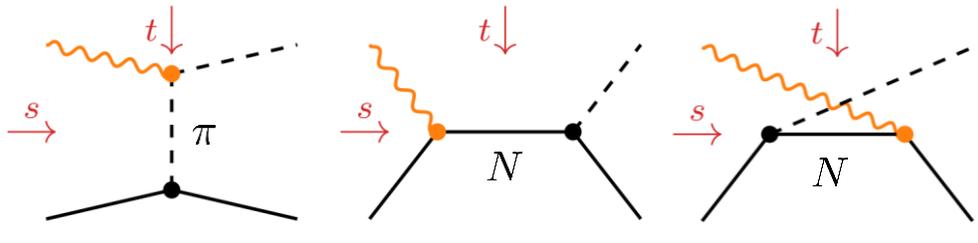
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Separate electric and magnetic contributions (gauge invariant alone)

$$A_{\mu\gamma\mu_i\mu_f} = A_{\mu\gamma\lambda_i\mu_f}^e + A_{\mu\gamma\mu_i\mu_f}^m, \quad A_{\mu\gamma\mu_i\mu_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \frac{(\epsilon_{\mu\gamma} \cdot p_\pi)}{t - \mu^2} + e_{N_i} \frac{(\epsilon_{\mu\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\mu\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

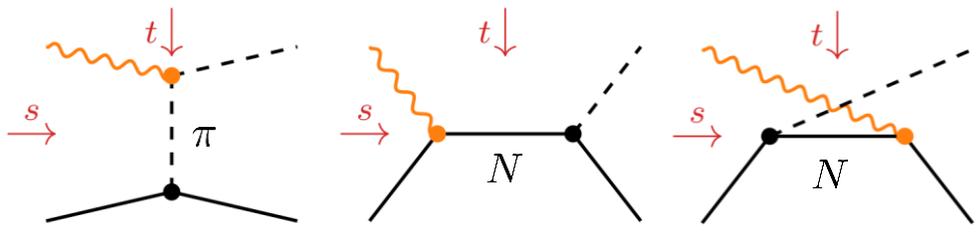
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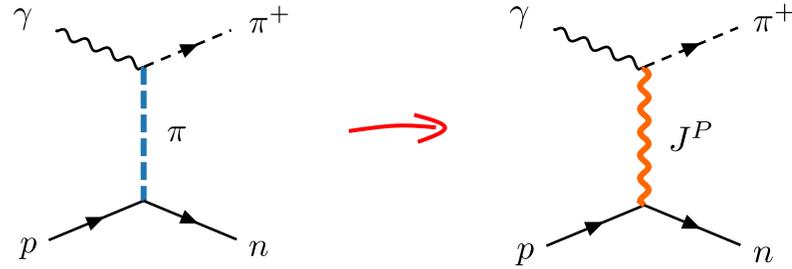
vanishes in t-channel CM

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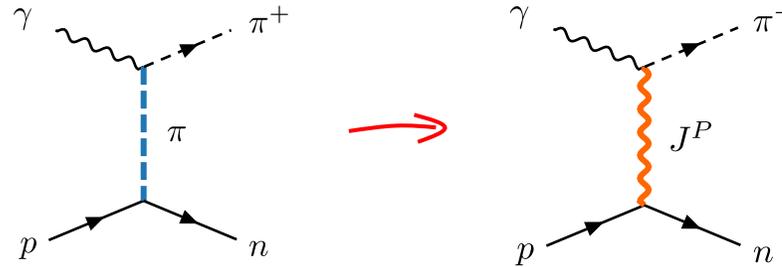
Reggeization of pion exchange

In the Regge-pole approximation:



Reggeization of pion exchange

In the Regge-pole approximation:



* Empirical prescription by Vanderhaegen, Guidal and Laget

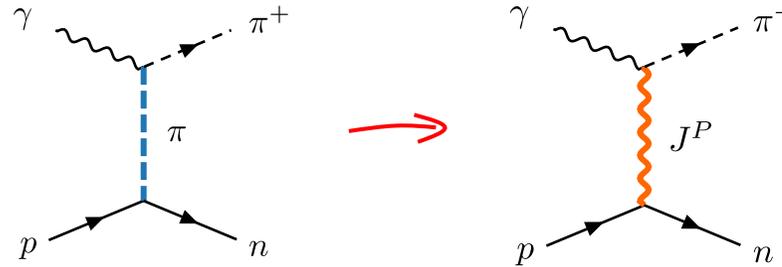
- Preserves gauge invariance and describes forward peak)

$$A_{\lambda_\gamma \lambda_i \lambda_f}^{\text{VGL}}(s, t) = A_{\lambda_\gamma \lambda_i \lambda_f}^{\text{Born}}(s, t) (t - m_\pi^2) \alpha' \frac{1 + e^{-i\pi\alpha(t)}}{2} \Gamma(-\alpha(t)) \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

Nucl.Phys.A 627, 645 (1997)

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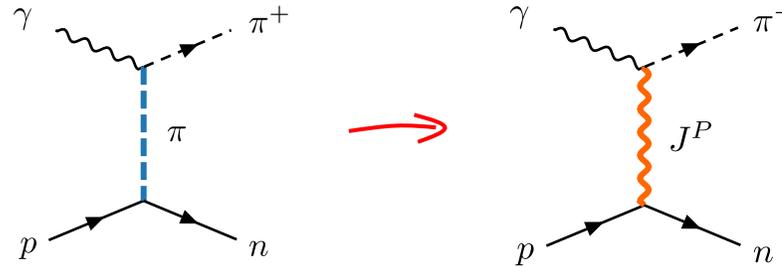
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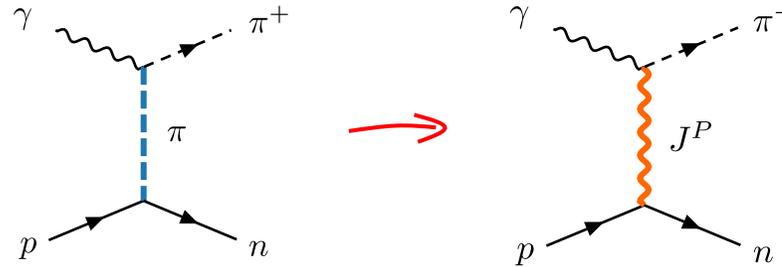
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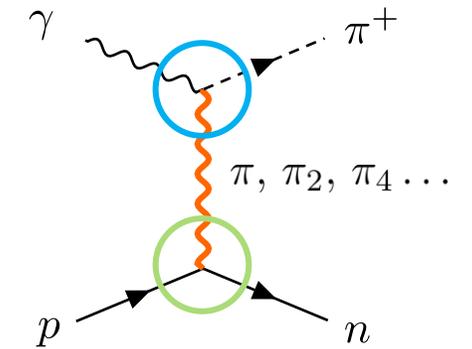
No pion pole?

Reggeization of pion exchange

GM, D.Winney, et al. (JPAC), Phys.Rev.D 110, 114012 (2024)

1. Build an amplitude for the exchange of a particle of arbitrary spin $J > 0$ (gauge invariant by construction)

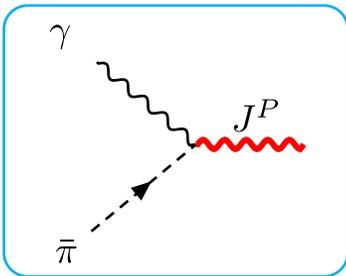
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Reggeization of pion exchange

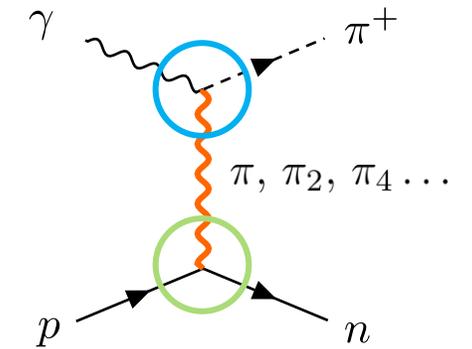
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$$1^- \otimes 0^- = 1^+ \left\{ \begin{array}{ll} L = 1 & J = 0 \\ L = \{J - 1, J + 1\} & J \geq 2 \end{array} \right\} \text{ one } L \text{ vs two } L\text{'s}$$

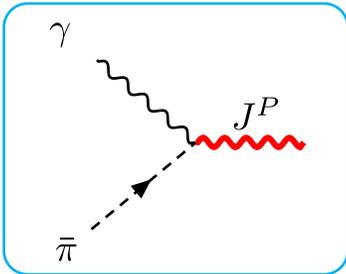
$$V_{\lambda_\gamma}^J(\sigma_J) = 2\sqrt{2} g_{\gamma\pi} \epsilon_{\nu_1 \dots \nu_J}^*(\sigma_J) \epsilon_\mu(k, \lambda_\gamma) (k^{\nu_1} \dots k^{\nu_{J-1}}) \left[k^{\nu_J} p_\pi^\mu - g^{\nu_J \mu} (k \cdot p_\pi) \right]$$



Reggeization of pion exchange

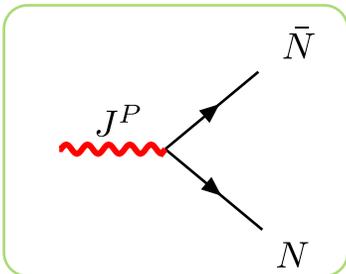
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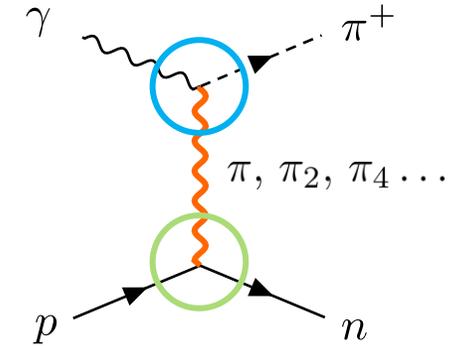
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$$\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^- \rightarrow L = J$$

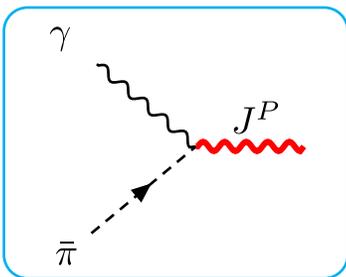
$$V_{\lambda_i \lambda_f}^J(\sigma_J) = g_{N\bar{N}} (P^{\nu_1} \dots P^{\nu_J}) \epsilon_{\nu_1 \dots \nu_J}(\sigma_J) \bar{u}(p_f, \lambda_f) \gamma_5 v(-p_i, \lambda_i) \quad (P^\nu = p_i^\nu + p_f^\nu)$$



Reggeization of pion exchange

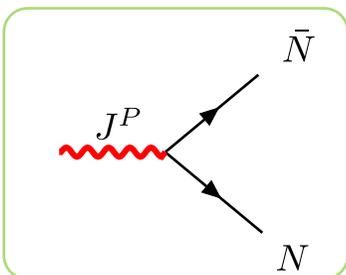
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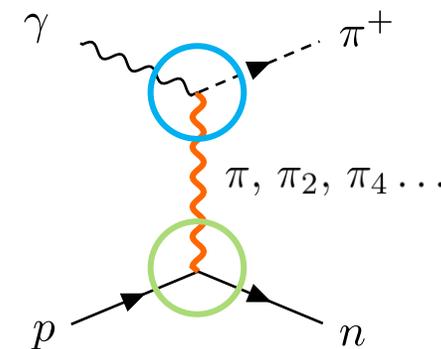
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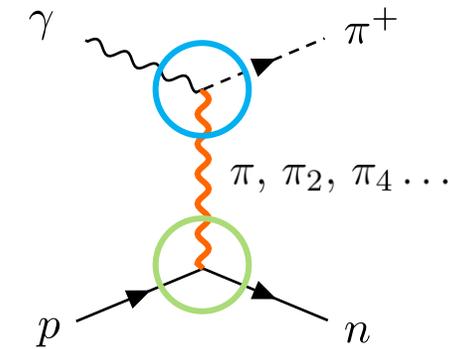
→ spin- J amplitude:
$$a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) \equiv \frac{2e_\pi g_J t}{J - \alpha_\pi(t)} (2\lambda_i \delta_{\lambda_i \lambda_f}) c_J^2 \sqrt{\frac{J+1}{J}} (-2p_t k_t)^J \quad \text{for } J > 0$$

Reggeization of pion exchange

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$$d_{\lambda_\gamma, 0}^J(\theta_t) = \sqrt{\frac{J+1}{2J}} d_{\lambda_\gamma, 0}^1(\theta_t) P_{J-1}^{11}(z_t)$$

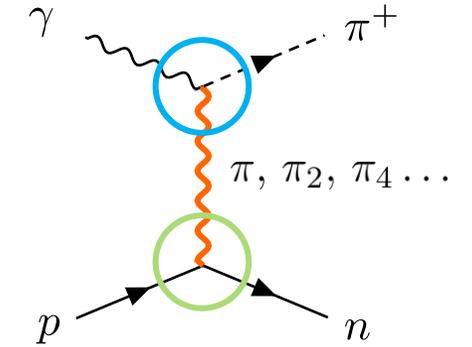


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$$P_{J-1}^{11}(z_t) = J {}_2\tilde{F}_1 \left(1 - J, J + 2; 2; \frac{1 - z_t}{2} \right)$$

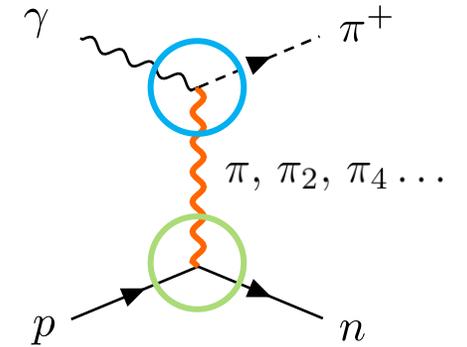
$$A_{\lambda_\gamma \lambda_i \lambda_f}^{J \rightarrow 0}(s, t) = \frac{2 e_\pi g_0 t}{-\alpha(t)} (2\lambda_i \lambda_\gamma \delta_{\lambda_i \lambda_f}) \frac{z_t}{\sqrt{1 - z_t^2}} \approx -i \frac{2 e_\pi g_{\pi NN} t}{m_\pi^2 - t} (2\lambda_i \lambda_\gamma \delta_{\lambda_i \lambda_f})$$

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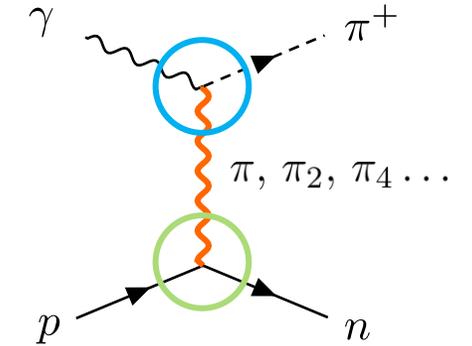
The $J \rightarrow 0$ amplitude is finite!

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The $J \rightarrow 0$ amplitude is finite!

3. Sum the tower of exchanges (e.g. Sommerfeld-Watson, or generating function for Jacobi polynomials)

$$A_{\lambda_\gamma \lambda_i \lambda_f}(s, t) = A_{\lambda_\gamma \lambda_i \lambda_f}^{J \rightarrow 0}(s, t) + \sum_{J=1} (2J+1) a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) d_{\lambda_\gamma \lambda_i - \lambda_f}^J(\theta_t)$$

$$\propto \alpha' \tau \frac{2}{\sqrt{\pi}} \Gamma\left(\alpha(t) + \frac{3}{2}\right) \Gamma(-\alpha(t)) (sR^2)^{\alpha(t)}$$

$$\text{vs } \mathcal{P}_\pi^{\text{Regge}} \propto \alpha' \tau \Gamma(-\alpha(t)) \left(\frac{s}{s_0}\right)^{\alpha(t)}$$

Interpretation in terms of Born diagrams

* In the s-channel CM frame

$$A_{\mu\gamma\mu_i\mu_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \frac{(\epsilon_{\mu\gamma} \cdot p_\pi)}{t - \mu^2} + \underbrace{e_{N_i} \frac{(\cancel{\epsilon_{\mu\gamma} \cdot p_i})}{s - M^2} + e_{N_f} \frac{(\epsilon_{\mu\gamma} \cdot p_f)}{u - M^2}}_{1/s} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

$\sim \sqrt{t}$ ↙

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$$A_{\lambda\gamma\lambda_i\lambda_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \frac{(\cancel{\epsilon_{\lambda\gamma} \cdot p_\pi})}{t - \mu^2} + e_{N_i} \frac{(\epsilon_{\lambda\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\lambda\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(-p_i)$$

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Re-write as 3 gauge-invariant terms proportional to the mass of the exchanged particle

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vanishes in t-channel CM ↙

↘ only contribution s-channel CM, subleading in the t-channel CM

$$\left. + \frac{1}{2} e_{N_i} \left(\frac{\epsilon \cdot p_\pi}{s - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{s - m_N^2} \right) - \frac{1}{2} e_{N_f} \left(\frac{\epsilon \cdot p_\pi}{u - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{u - m_N^2} \right) \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

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⏟
1/s

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vanishes in t-channel CM ↙

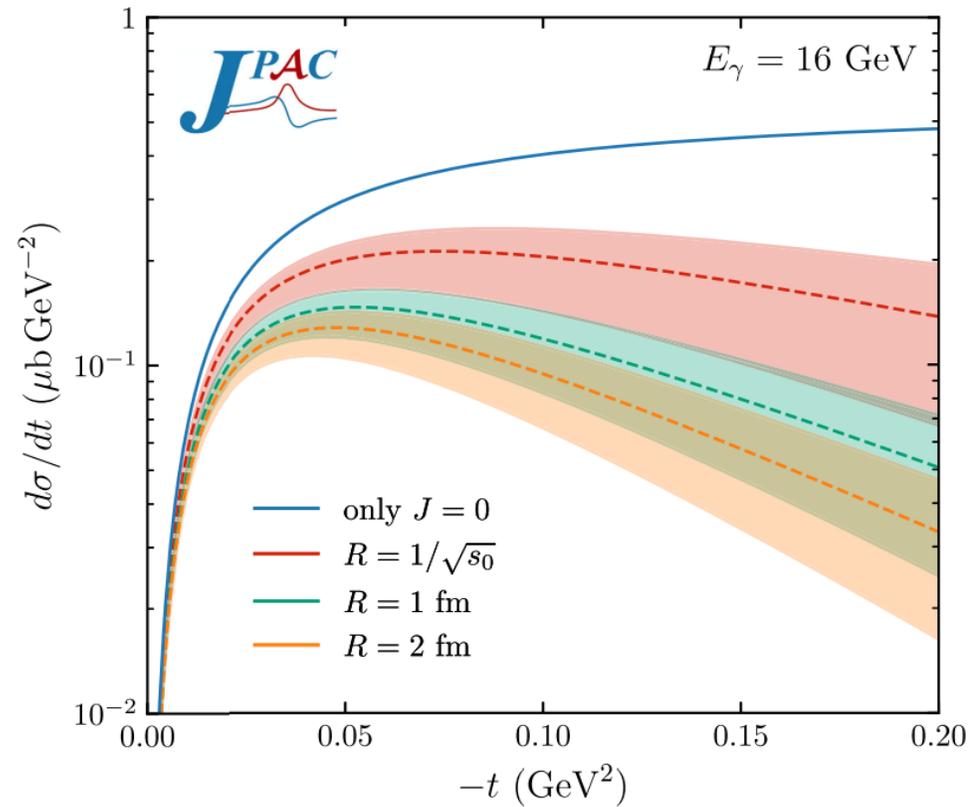
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$$+ \frac{1}{2} e_{N_i} \left(\frac{\epsilon \cdot p_\pi}{s - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{s - m_N^2} \right) - \frac{1}{2} e_{N_f} \left(\frac{\epsilon \cdot p_\pi}{u - m_N^2} + \frac{\epsilon \cdot P}{s - u} \frac{t - m_\pi^2 - k^2}{u - m_N^2} \right) \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

Results for pion photoproduction

* Reggeized pion exchange

$$A_{\lambda_\gamma \lambda_i \lambda_f}^{\text{Regge}}(s, t) \propto t(sR^2)^{\alpha(t)}$$

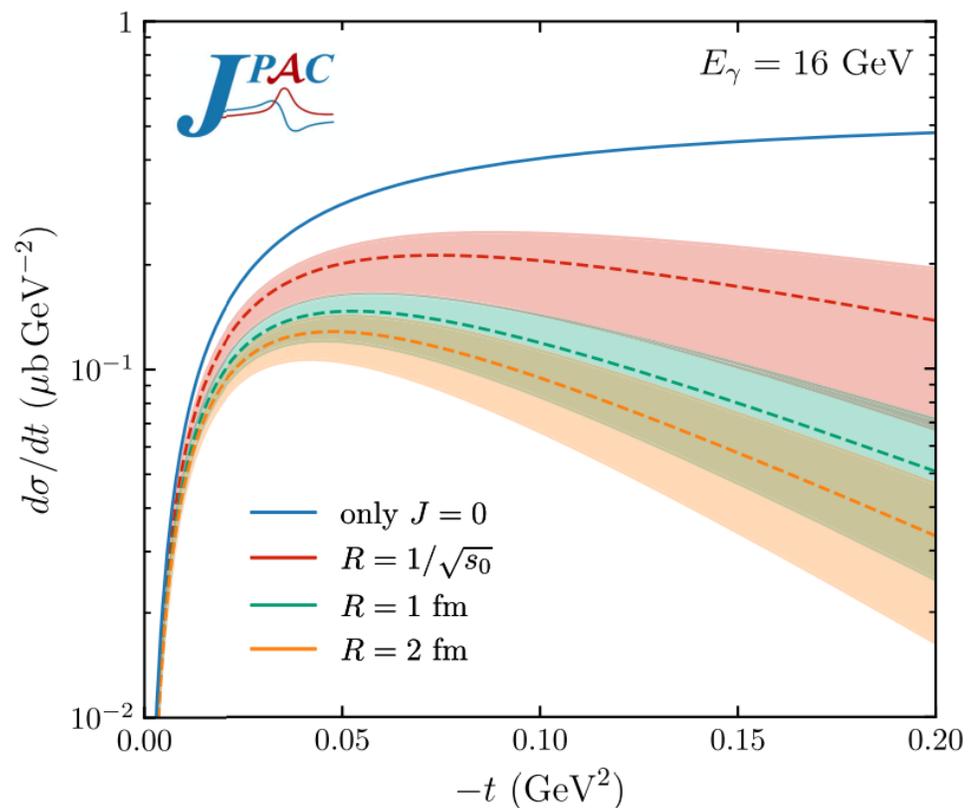


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GM, D.Winney, et al. (JPAC), Phys.Rev.D 110, 114012 (2024)

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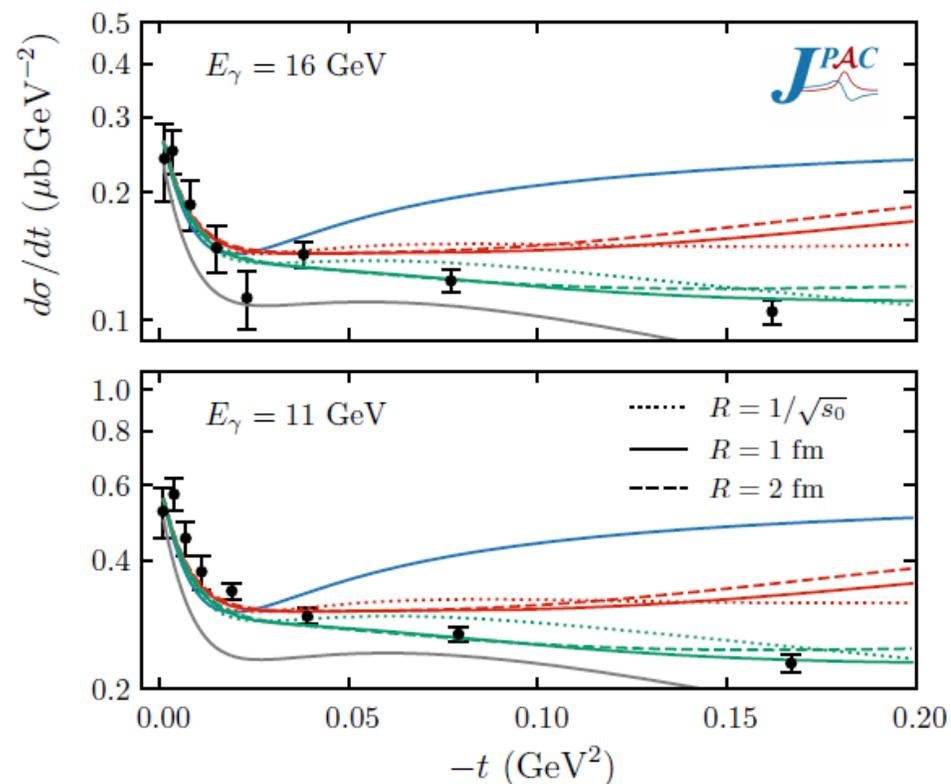
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* Adding the nucleon magnetic term

- With a form factor (missing Regge exchanges)

Comparison with VGL approach (grey line)



00 Motivation

01 Remarks on Scattering and Regge Theory

02 Pion exchange in pion photoproduction

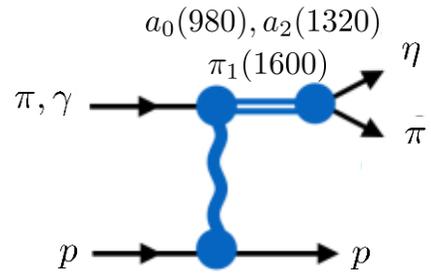
03 Photoproduction of $\eta^{(\prime)}\pi$ in the double-Regge region

04 Other studies

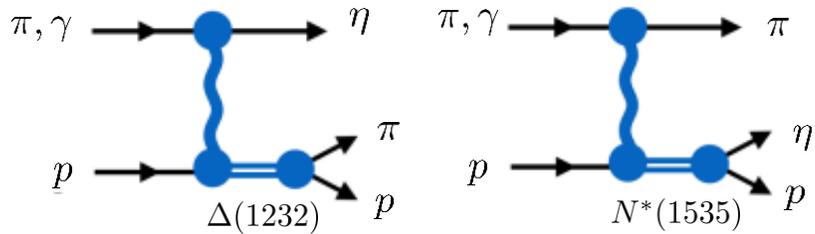
05 Conclusions

$\eta^{(\prime)}\pi$ final state

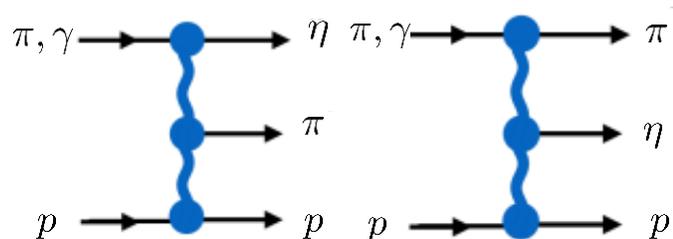
* Meson resonances



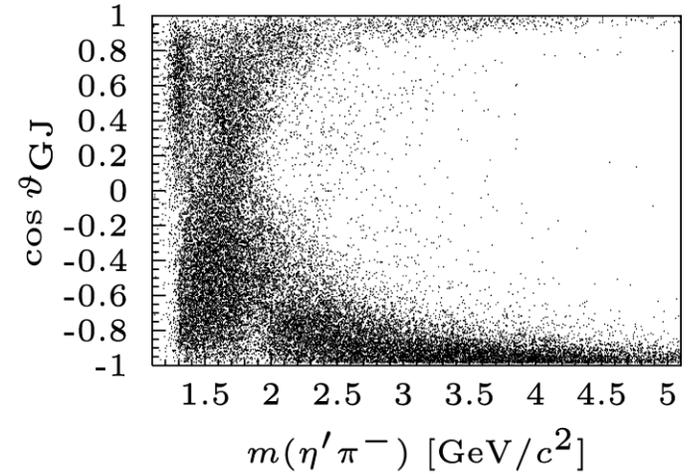
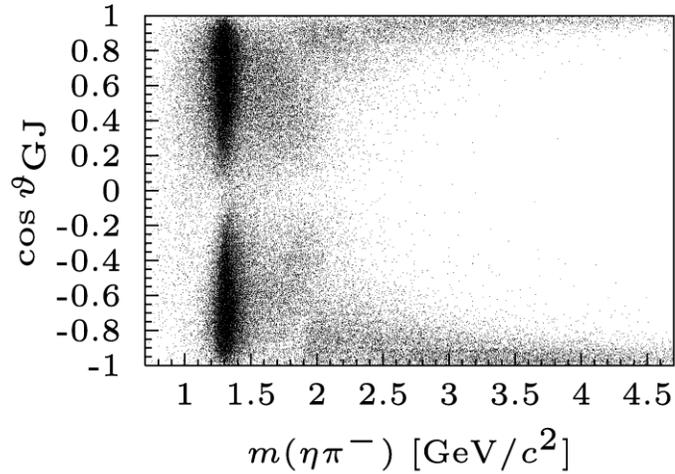
* Baryon resonances



* Double Regge production



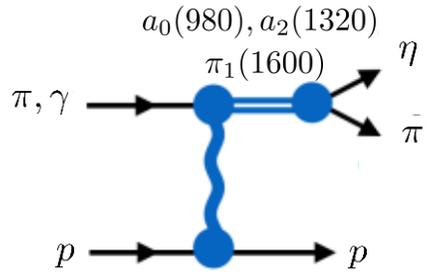
Pion beam



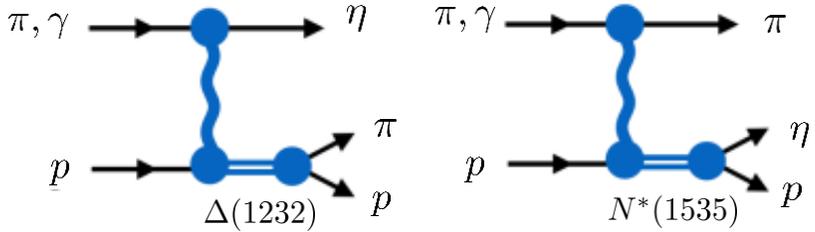
COMPASS, *Phys.Lett.B* 740, 303 (2015)

$\eta^{(\prime)}\pi$ final state

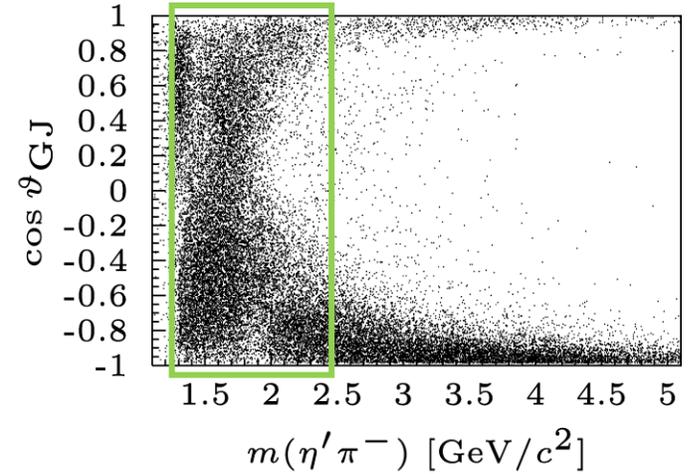
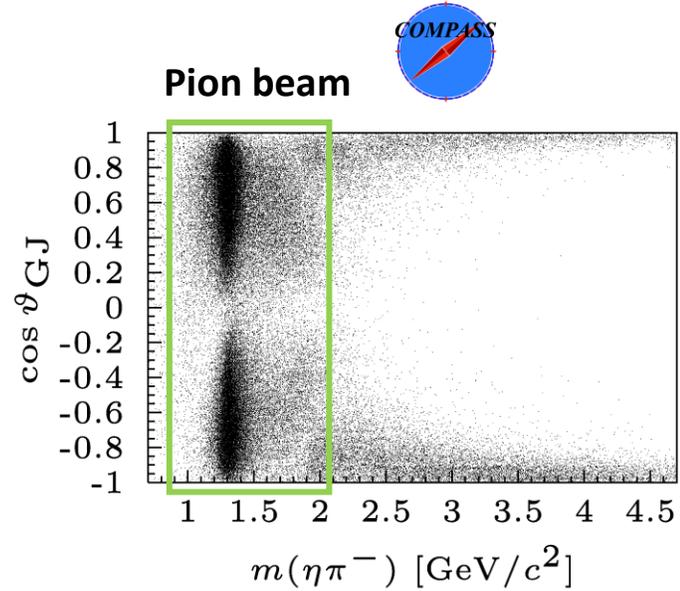
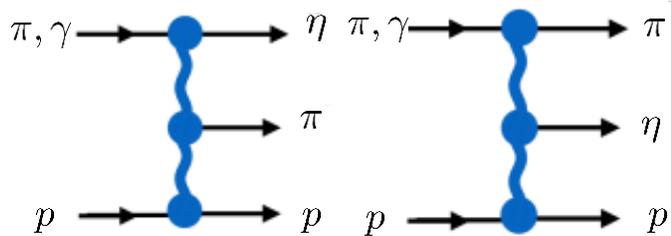
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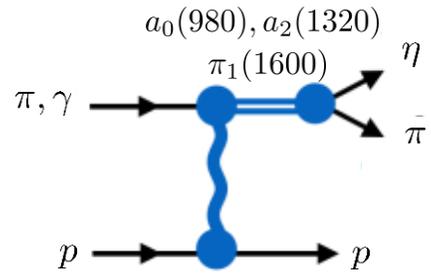
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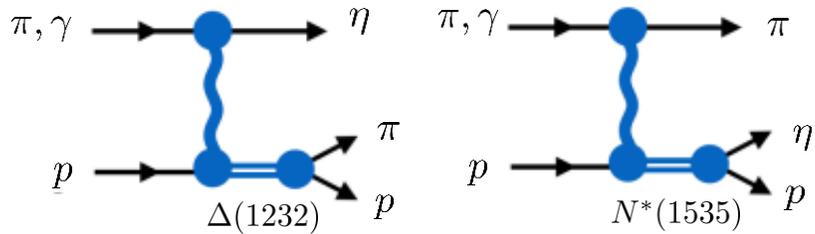
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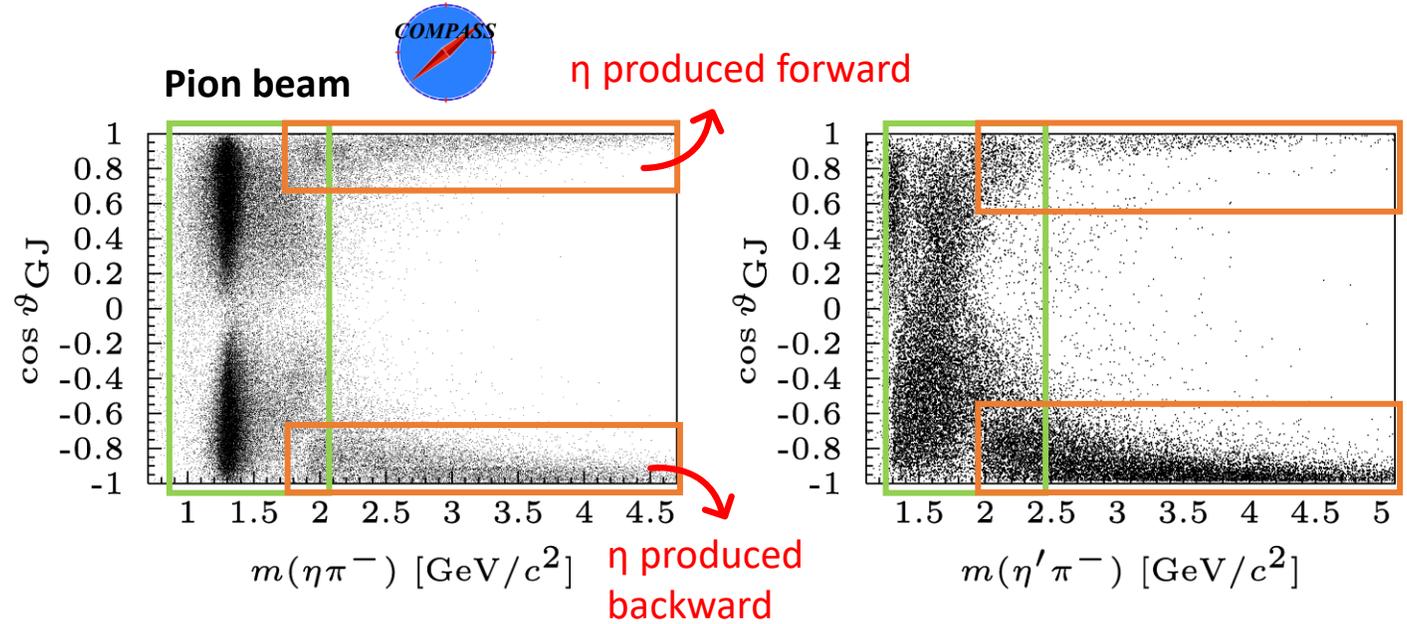
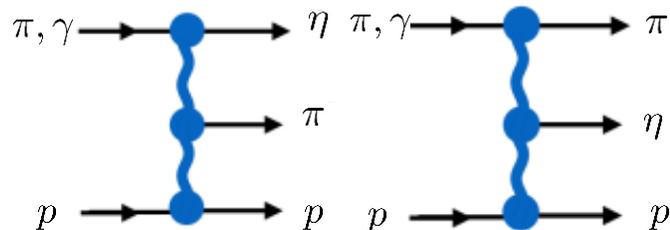
* Meson resonances



* Baryon resonances



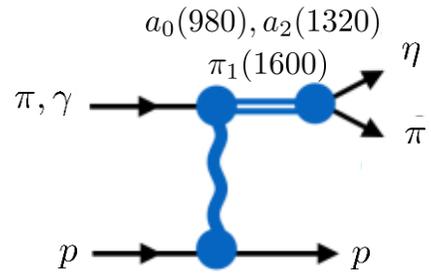
* Double Regge production



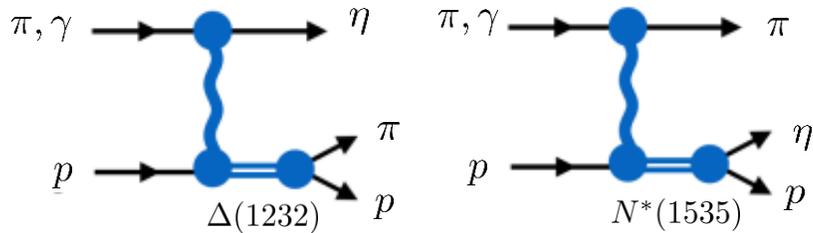
COMPASS, *Phys.Lett.B* 740, 303 (2015)

$\eta^{(\prime)}\pi$ final state

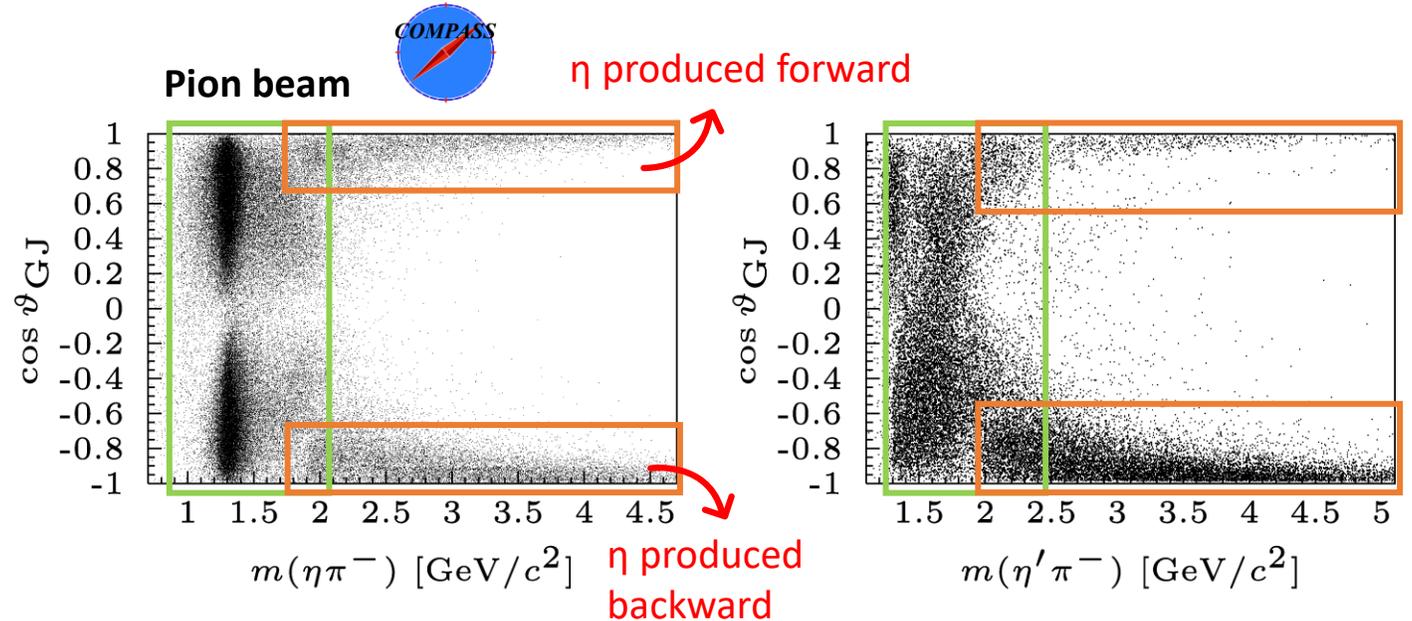
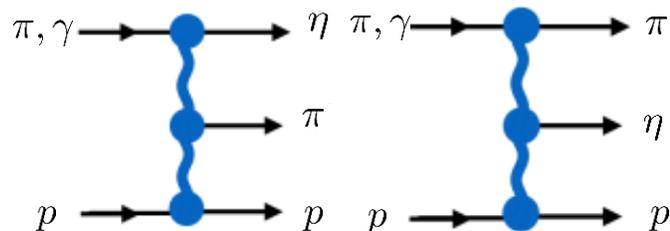
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* Forward-backward asymmetry:

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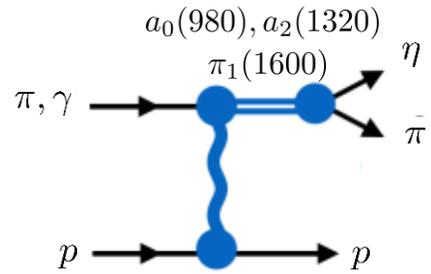
Interference between even and odd partial waves

→ Odd partial waves are exotic (e.g. 1^{-+})

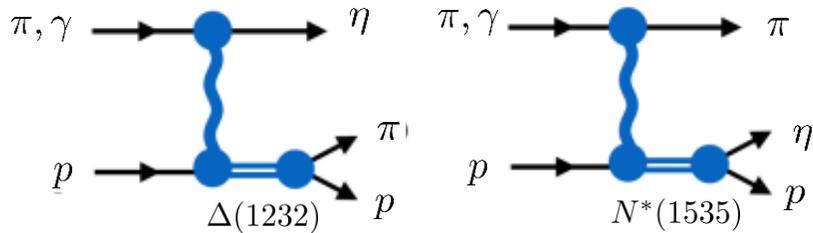
Presence of exotic partial waves (also at low energies)

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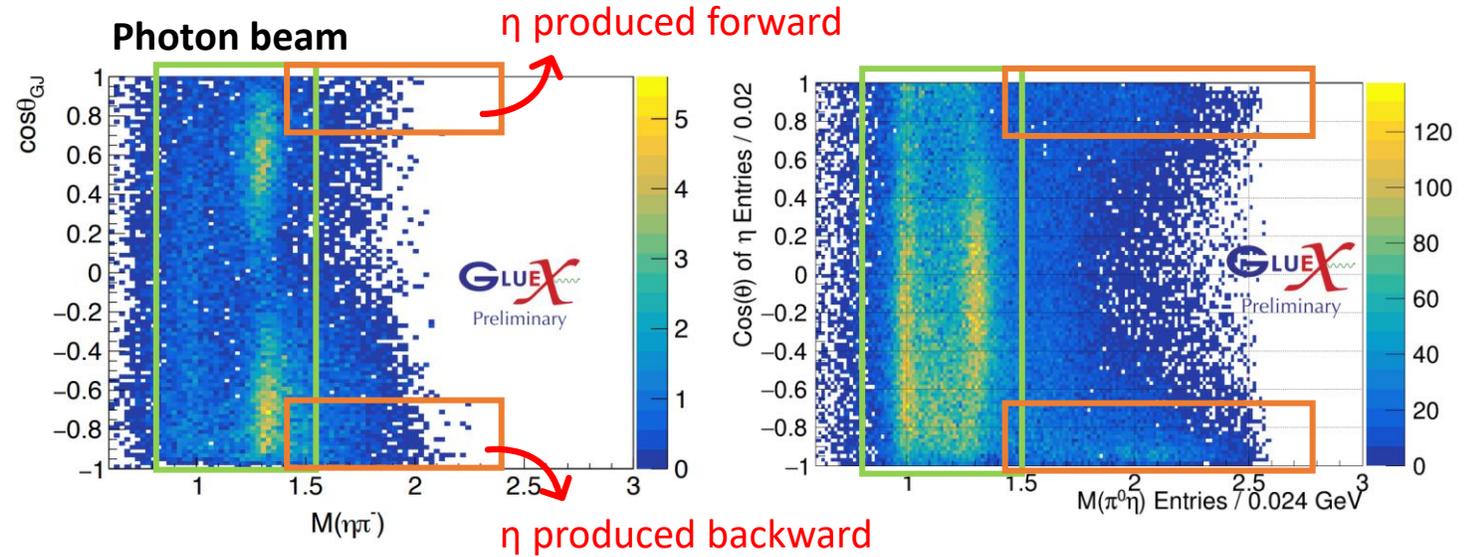
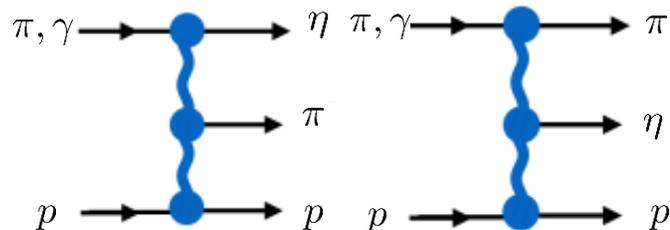
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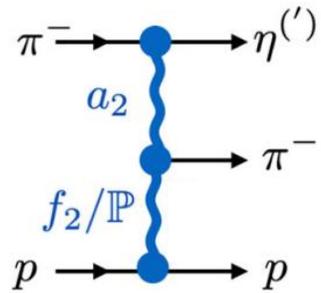
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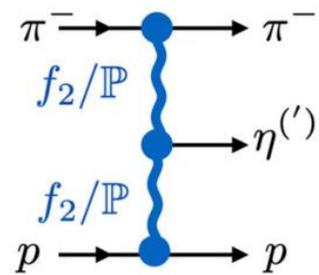
Double Regge production of $\eta^{(\prime)}\pi$ at COMPASS

* Dominant exchanges:

Fast η $\cos \theta \approx +1$



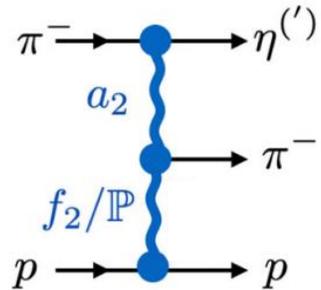
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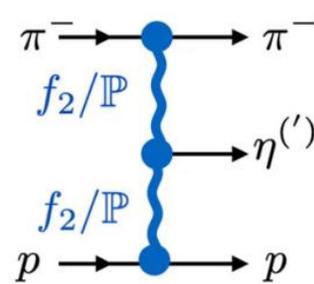
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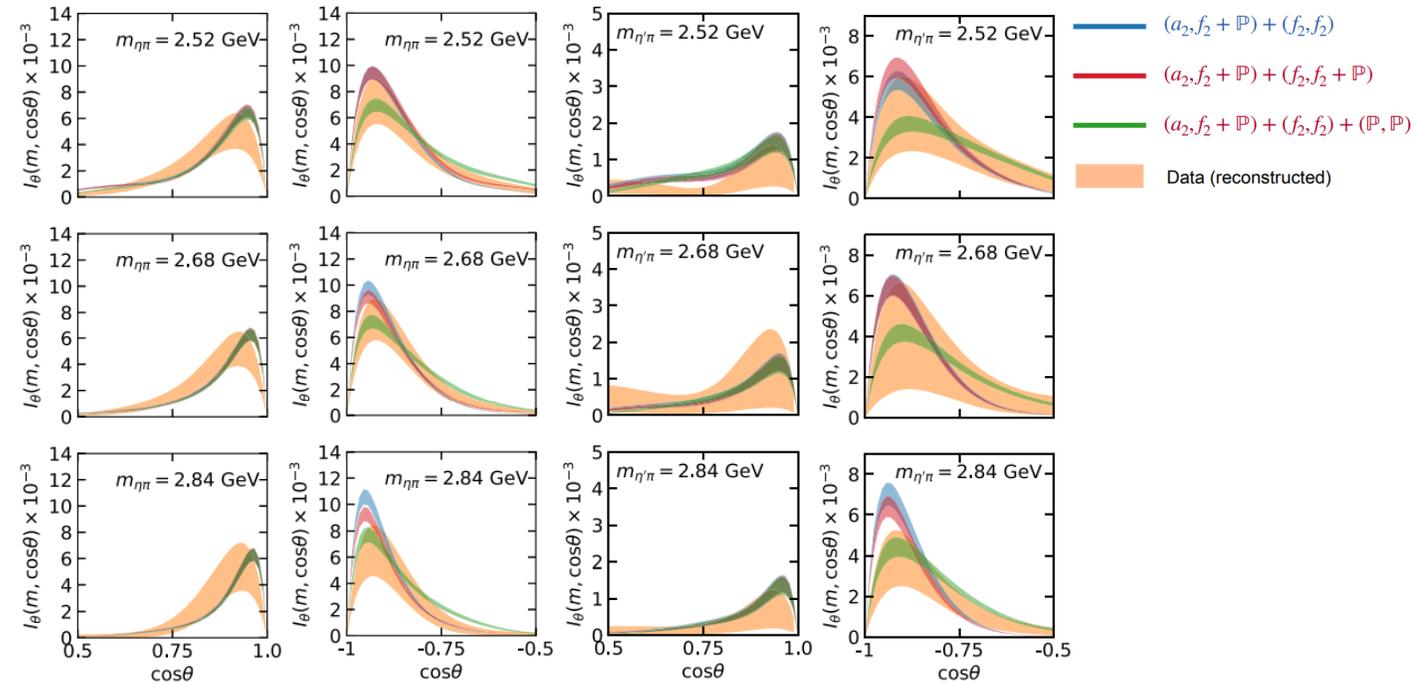
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* Pomeron exchange needed to describe the data

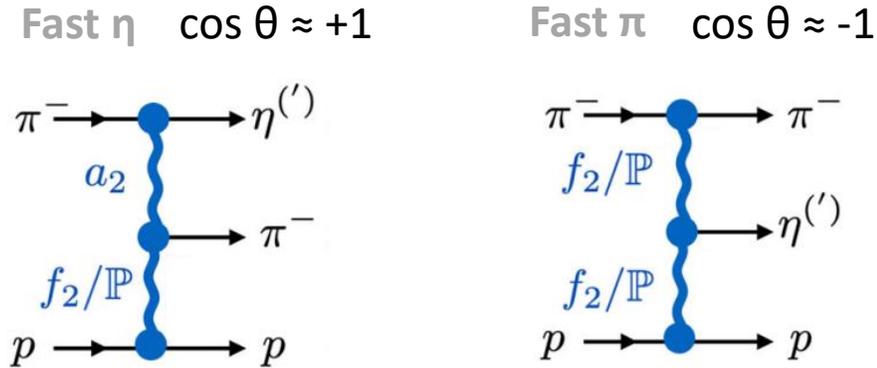
* (\mathbb{P}, \mathbb{P}) more important for $\eta'\pi$

L.Bibrzycki, et al. (JPAC), Eur.Phys.J.C 81, 647 (2021)



Double Regge production of $\eta^{(\prime)}\pi$ at COMPASS

* Dominant exchanges:



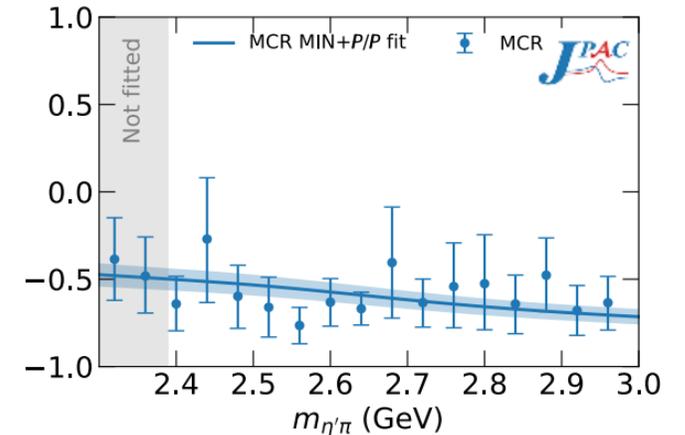
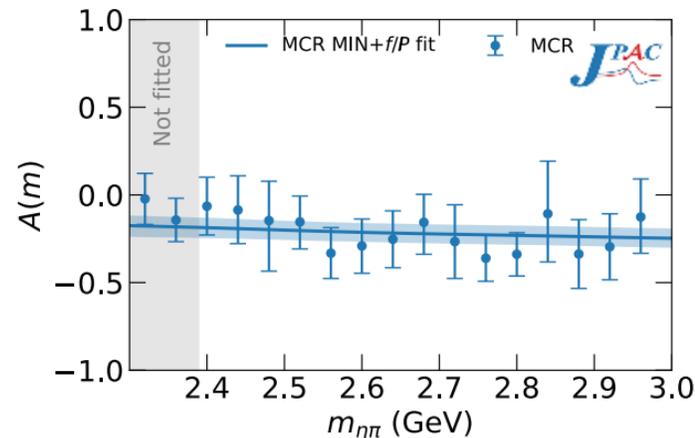
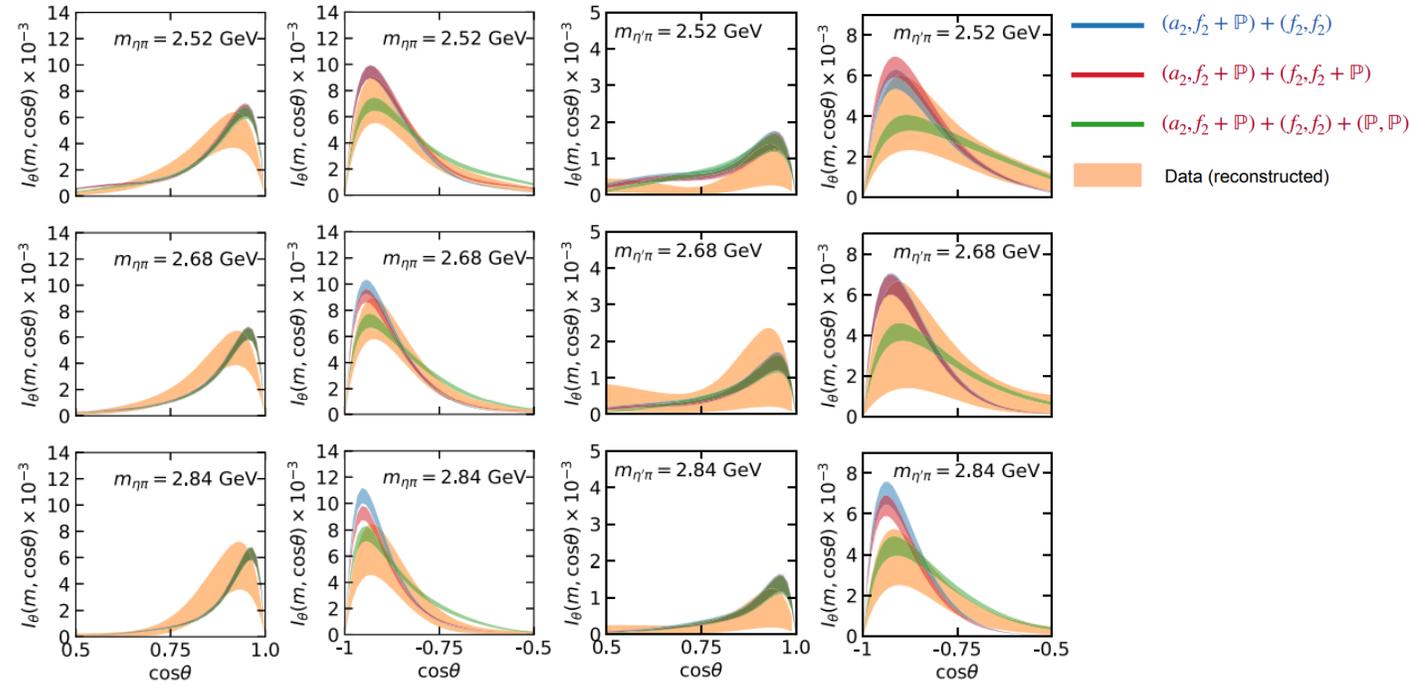
* Pomeron exchange needed to describe the data

* (\mathbb{P}, \mathbb{P}) more important for $\eta'\pi$

* Asymmetry originating mainly from $(a_2, f_2 + \mathbb{P}) \neq (f_2, f_2 + \mathbb{P})$, and also (\mathbb{P}, \mathbb{P}) , specially for $\eta'\pi$

* Pomeron couples more strongly to η'

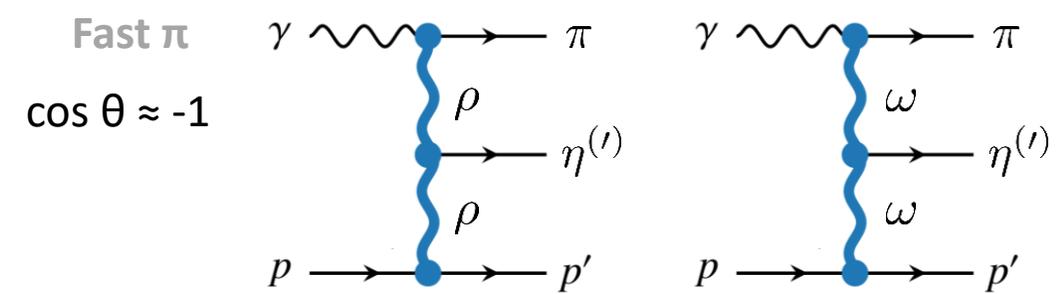
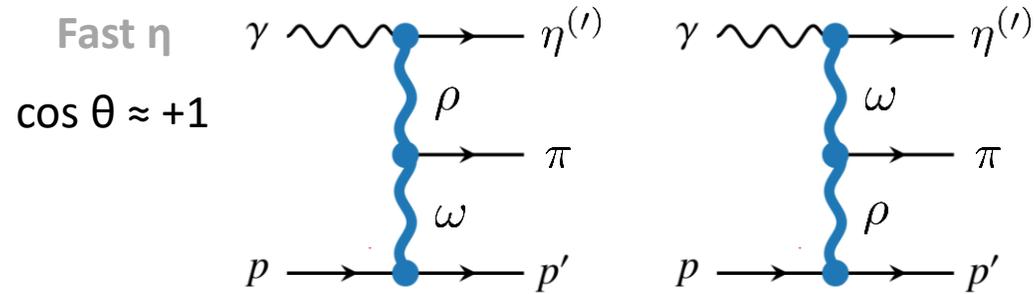
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Double Regge photoproduction of $\eta^{(\prime)}\pi$

GM, V.Mathieu, et al. (JPAC), Phys.Lett.B 872, 140101 (2026)

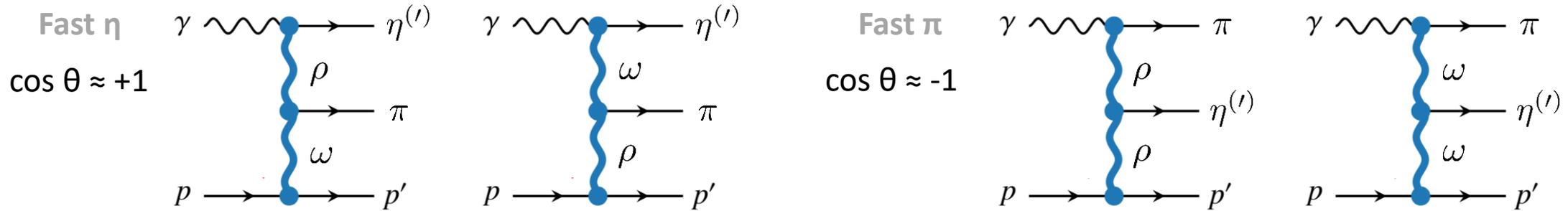
* Vector (natural parity) exchanges are dominant



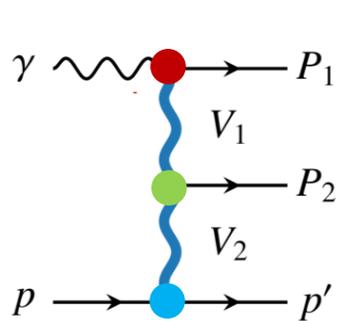
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* Vector (natural parity) exchanges are dominant



* Lorentz structure of the vertices:



Top vertex: $\langle V_1 | \gamma P_1 \rangle = \frac{g_{\gamma V_1 P_1}}{m_0} i \epsilon_{\alpha\beta\mu\nu} \epsilon_{\gamma}^{\alpha*} q_{\gamma}^{\beta} q_{V_1}^{\mu} \epsilon_{V_1}^{\nu}$

Middle vertex: $\langle V_2 | V_1 P_2 \rangle = \frac{g_{V_1 V_2 P_2}}{m_0} i \epsilon_{\alpha\beta\mu\nu} \epsilon_{V_1}^{\alpha*} q_{V_1}^{\beta} q_{V_2}^{\mu} \epsilon_{V_2}^{\nu}$

Bottom vertex: $\langle N' | V_2 N \rangle = \bar{u}(\vec{q}_{p'}, \mu') \left[(g_1^{V_2} + g_2^{V_2}) \gamma_{\nu} - g_2^{V_2} \frac{(q_p + q_{p'})_{\nu}}{2m_p} \right] u(\vec{q}_p, \mu)$

All couplings are known

Double Regge photoproduction of $\eta^{(\prime)}\pi$

GM, V.Mathieu, et al. (JPAC), Phys.Lett.B 872, 140101 (2026)

* Identify the kinematical factor

$$K_{\lambda_\gamma \lambda \lambda'} = 2m_p t_1 |\vec{p}_1| |\vec{p}_2| \lambda_\gamma \sum_{\xi=-1}^1 (-\xi) e^{-i\xi\omega} d_{\lambda_\gamma \xi}^1(\theta_1) d_{\xi \Lambda}^1(\theta_2)$$

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* **Reggeization** \rightarrow sum over spins J_1 and J_2

$$K_{\lambda_\gamma\lambda\lambda'} \rightarrow K_{\lambda_\gamma\lambda\lambda'} \times R(\alpha_1(t_1), \alpha_2(t_2), s_1, s_2, \kappa)$$

with $\kappa^{-1} \equiv \frac{s}{\alpha' s_1 s_2}$

Double Regge photoproduction of $\eta^{(\prime)}\pi$

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* **Reggeization** \rightarrow sum over spins J_1 and J_2

$$K_{\lambda_\gamma \lambda \lambda'} \rightarrow K_{\lambda_\gamma \lambda \lambda'} \times R(\alpha_1(t_1), \alpha_2(t_2), s_1, s_2, \kappa) \quad \text{with} \quad \kappa^{-1} \equiv \frac{s}{\alpha' s_1 s_2}$$

based on the model of Shimada, Martin, and Irving

Shimada, Martin, Irving, Nucl. Phys. B 142 (1978)

poles for integer $\alpha_i(t)$

$$R(\alpha_1, \alpha_2, s_1, s_2, \kappa) = [(\alpha_1 - 1)\Gamma(1 - \alpha_1)] [(\alpha_2 - 1)\Gamma(1 - \alpha_2)] \times [\xi_1 \xi_{21} \kappa^{1-\alpha_1} V(\alpha_1, \alpha_2, \kappa) + \xi_2 \xi_{12} \kappa^{1-\alpha_2} V(\alpha_2, \alpha_1, \kappa)] \left(\frac{s_1}{s_0}\right)^{\alpha_1-1} \left(\frac{s_2}{s_0}\right)^{\alpha_2-1}$$

\downarrow signature factors
 \downarrow vertex function
 $\underbrace{\hspace{10em}}$ asymptotic behavior

$$V(\alpha_1, \alpha_2, \kappa) = \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_2)} {}_1F_1(1 - \alpha_1, 1 - \alpha_1 + \alpha_2, -\kappa)$$

Results for $\eta^{(\prime)}\pi$ photoproduction

GM, V.Mathieu, et al. (JPAC), Phys.Lett.B 872, 140101 (2026)

Use linear Regge trajectories: $\alpha_i(t_i) = 1 + \alpha'(t_i - m_{V_i}^2)$, with $\alpha' = 0.9 \text{ GeV}^{-2}$

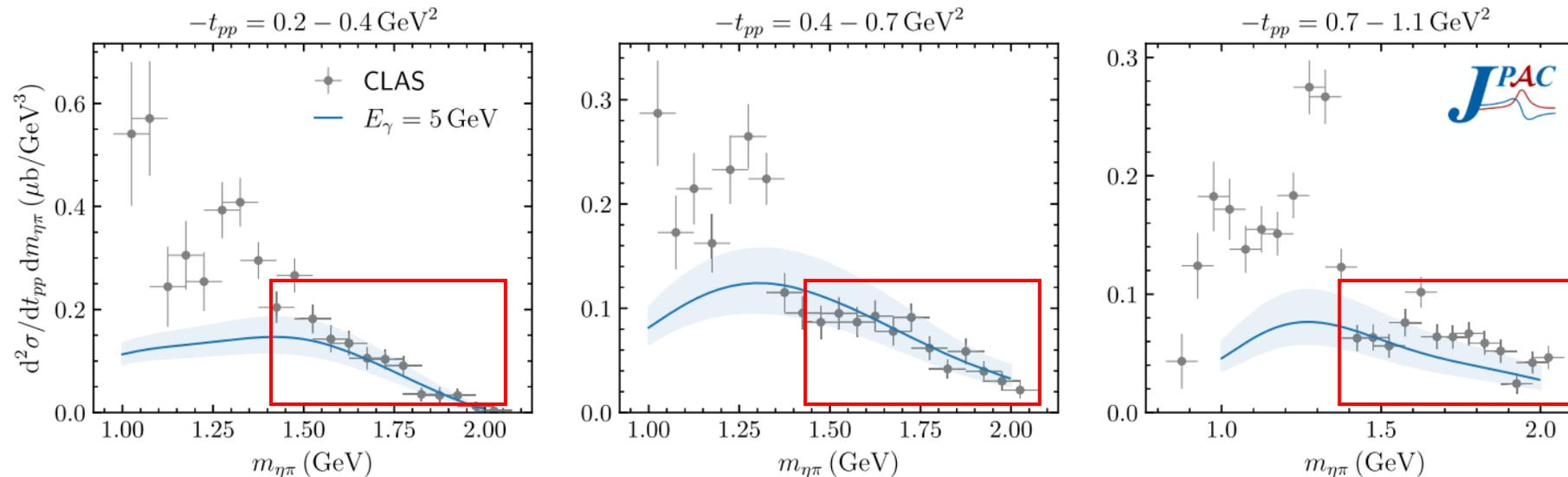
All parameters of the model completely fixed ($s_0 = 1 \text{ GeV}^2$)

* Comparison with CLAS data (not a fit):

[Phys.Rev.C 102, 032201 \(2020\)](#)

Shaded bands:

$0.95 \text{ GeV}^2 < s_0 < 1.05 \text{ GeV}^2$

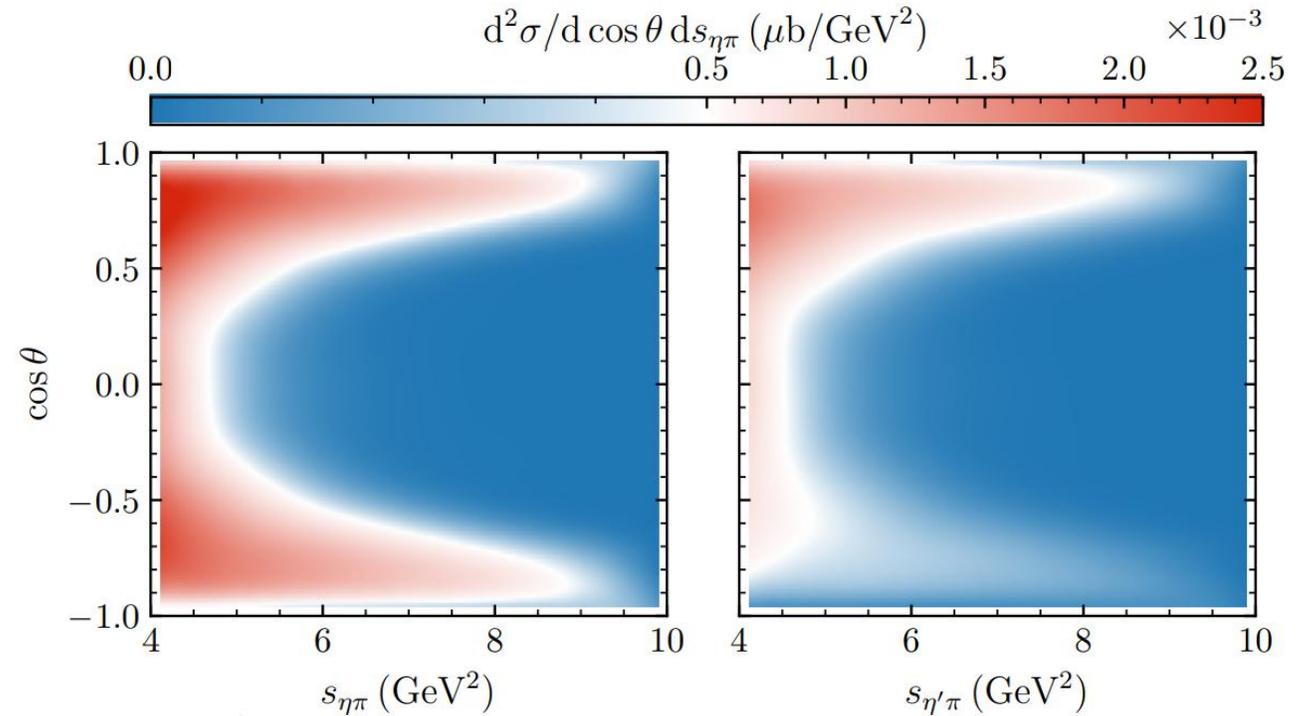
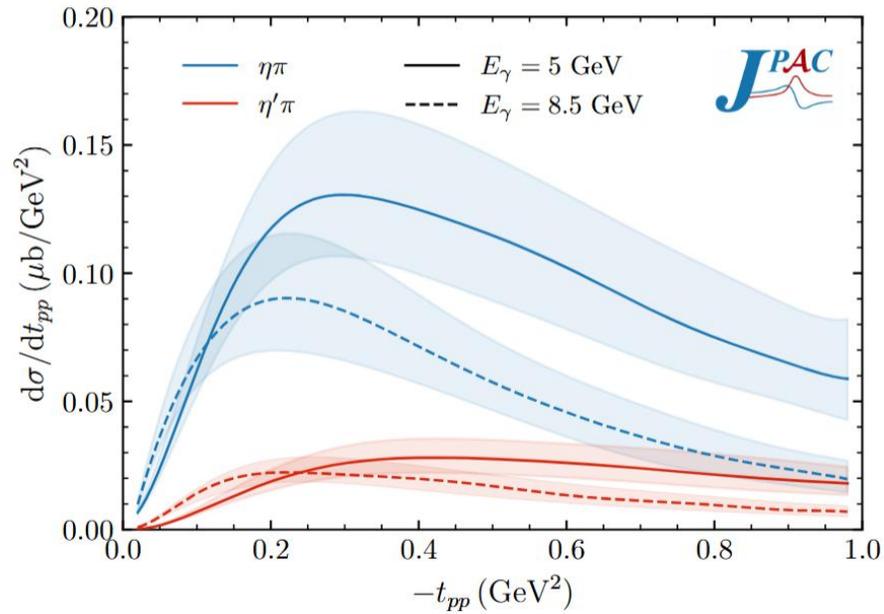


→ Good description of the data above the resonance region

Results for $\eta^{(\prime)}\pi$ photoproduction

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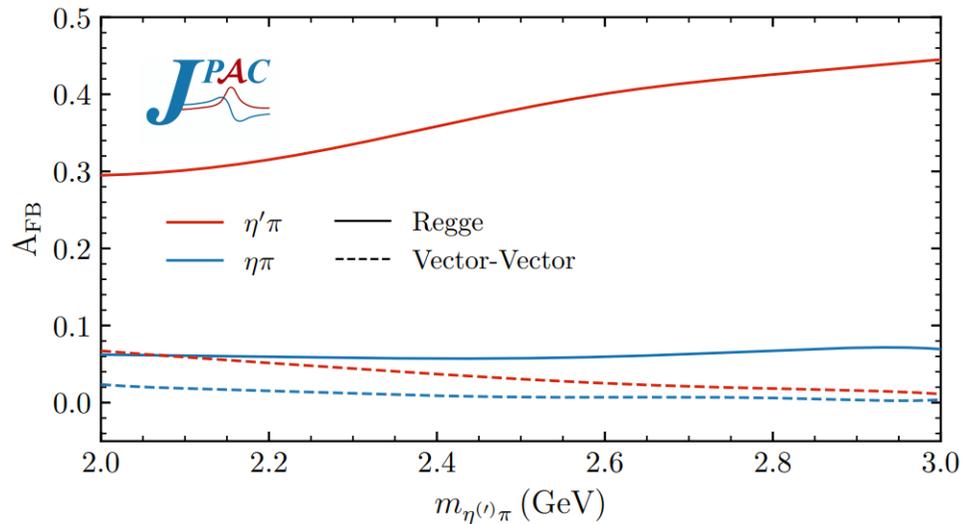
* Predictions for GlueX energy $E_\gamma = 8.5$ GeV



→ Clear forward and backward peaks

* Forward-backward asymmetry

$$A_{\text{FB}}(m_{\eta\pi}^2) = \frac{F(m_{\eta\pi}^2) - B(m_{\eta\pi}^2)}{F(m_{\eta\pi}^2) + B(m_{\eta\pi}^2)}$$



→ Larger for $\eta'\pi$ (but no Pomeron here)

→ Stronger exotic partial waves in $\eta'\pi$

00 Motivation

01 Remarks on Scattering and Regge Theory

02 Pion exchange in pion photoproduction

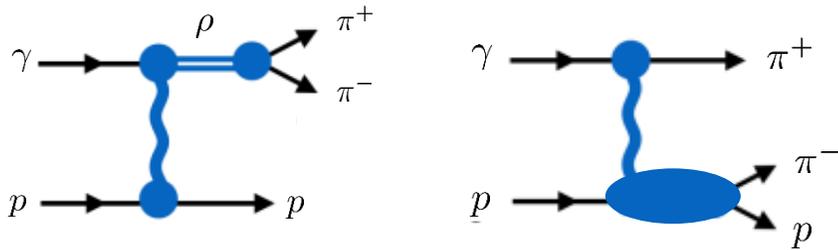
03 Photoproduction of $\eta^{(\prime)}\pi$ in the double-Regge region

04 other studies

05 Conclusions

Two pion photoproduction in the ρ region

Regge-based model fitted to angular moments from CLAS

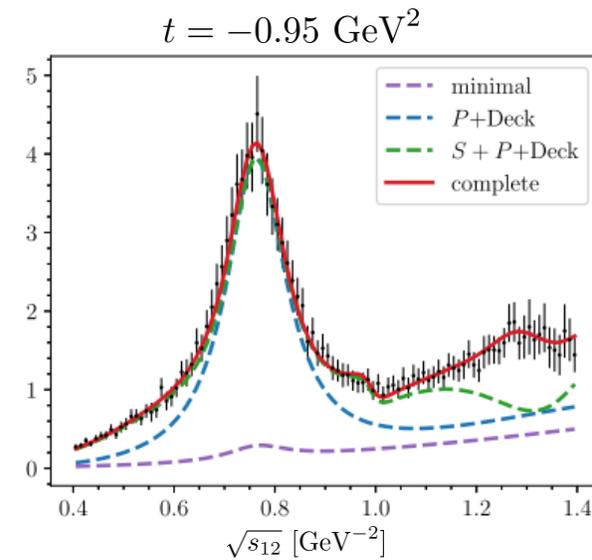
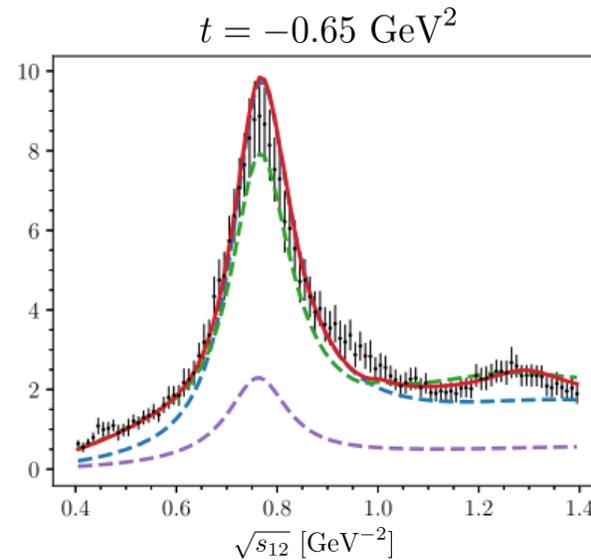
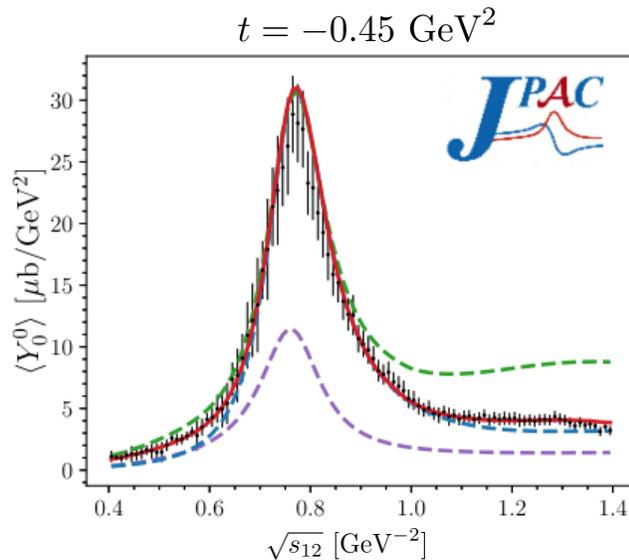


* Resonant component: $f_0(500)$, $\rho(770)$, $f_0(980)$, $f_2(1270)$, $f_0(1370)$ produced through Pomeron and natural parity Regge exchanges

* Non-resonant component (Deck mechanism)

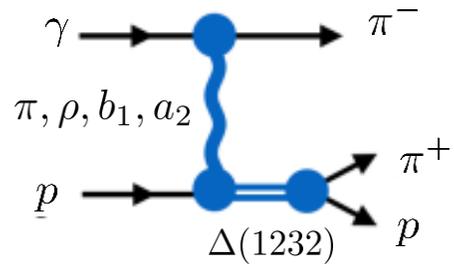
$$\langle Y_M^L \rangle = \sqrt{4\pi} \int d\Omega^H \frac{d\sigma}{dt d\sqrt{s_1} 2d\Omega^H} \text{Re}\{Y_M^L(\Omega^H)\}$$

→ Much more physics than just Pomeron exchange



Data: Phys.Rev.D 80, 072005 (2009)

Regge-based model fitted to spin-density matrix elements (SDMEs) from GlueX



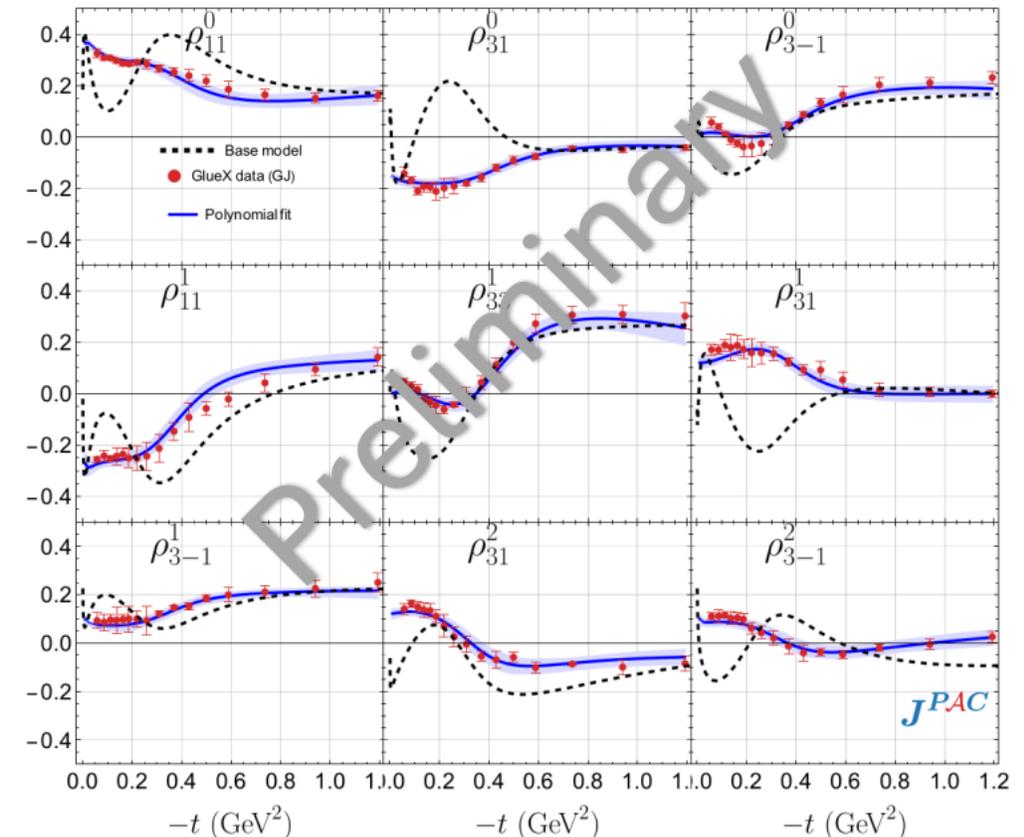
- * 2018 JPAC model fitted to cross section and beam asymmetry data from SLAC (relative phases unconstrained) [Phys.Lett.B 779, 77\(2018\)](#)
- * SDMEs allow for a revisited model (more flexible)

$$W(\Omega_{\pi^+}, \Phi) = 2N \left\{ \rho_{33}^0 \sin^2 \theta + \rho_{11}^0 \left(\frac{1}{3} + \cos^2 \theta \right) + \dots \right\}$$

$$\rho_{\lambda_\Delta \lambda'_\Delta}^0 = \frac{1}{2N} \sum_{\lambda_\gamma \lambda_N} T_{\lambda_\gamma \lambda_N \lambda_\Delta}(s, t) T_{\lambda_\gamma \lambda_N \lambda'_\Delta}^*(s, t)$$

⋮

- Better understanding of the production amplitudes
- Extraction of (universal) Regge couplings



GLUEX Data: [Phys.Lett.B 863, 139639 \(2025\)](#)

Dashed lines: old JPAC model

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- * **Many more ongoing studies:**
 - More complicated final states: $\pi^+\pi^-$, K^+K^- , $\pi\Delta$, b_1p , $b_1\Delta$, $\rho\Delta$, ...
 - In parallel with GlueX/CLAS12 experimental analyses

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