Statistical Nuclear Reactions
&
Quantum Pre-Equilibrium Models

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supervisor : Marc DUPUIS *director* : Cyrille DE SAINT JEAN

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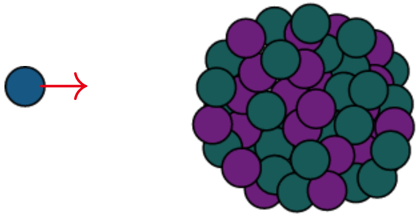
Statistical Nuclear Reactions

Single Resonances



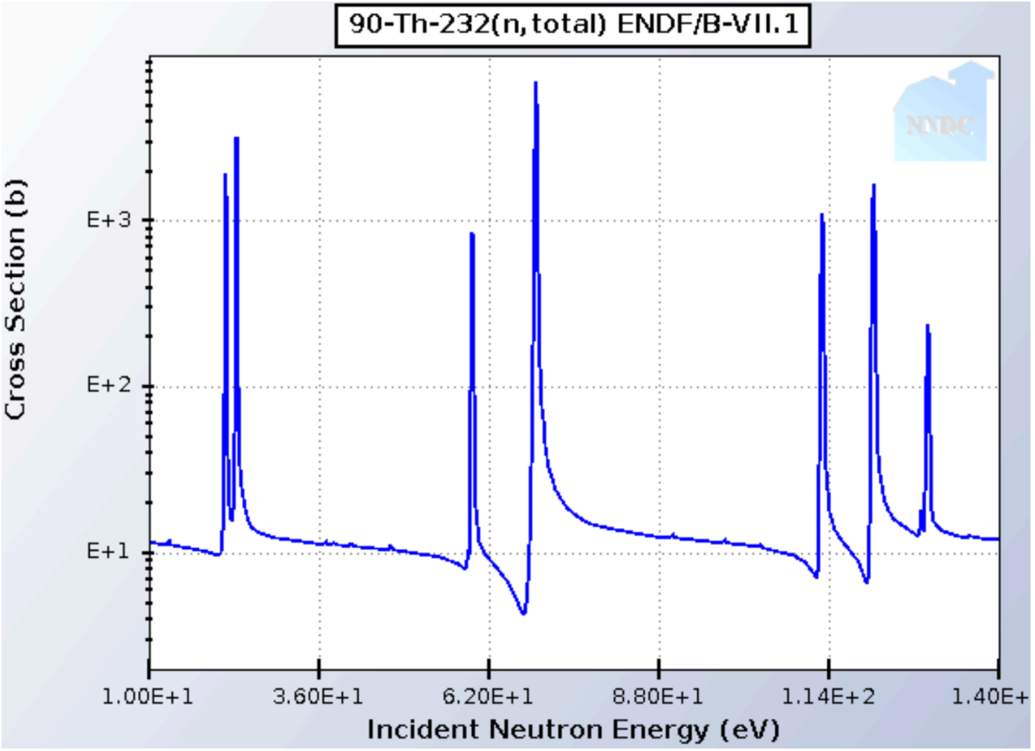
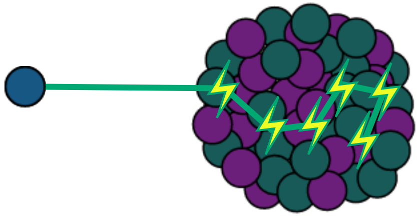
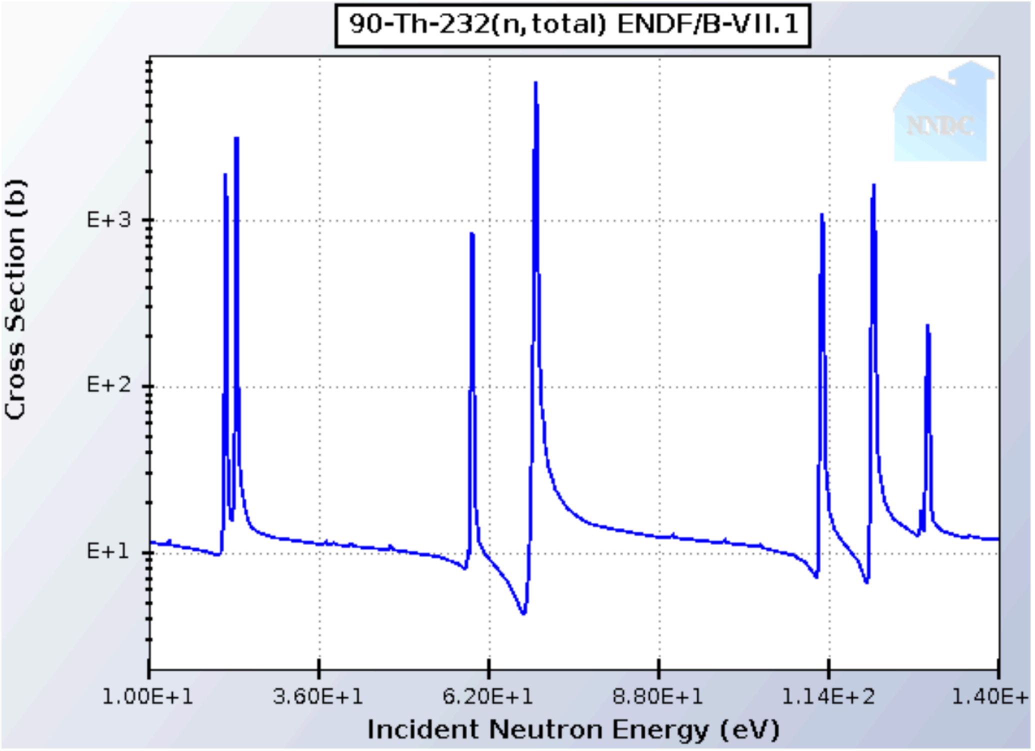


Figure 1: n + ^{232}Th *(from NNDC)*

Single Resonances



Compound Nucleus Formation



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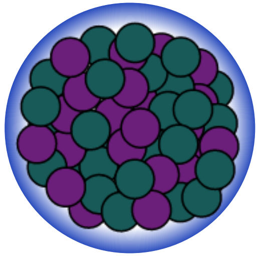

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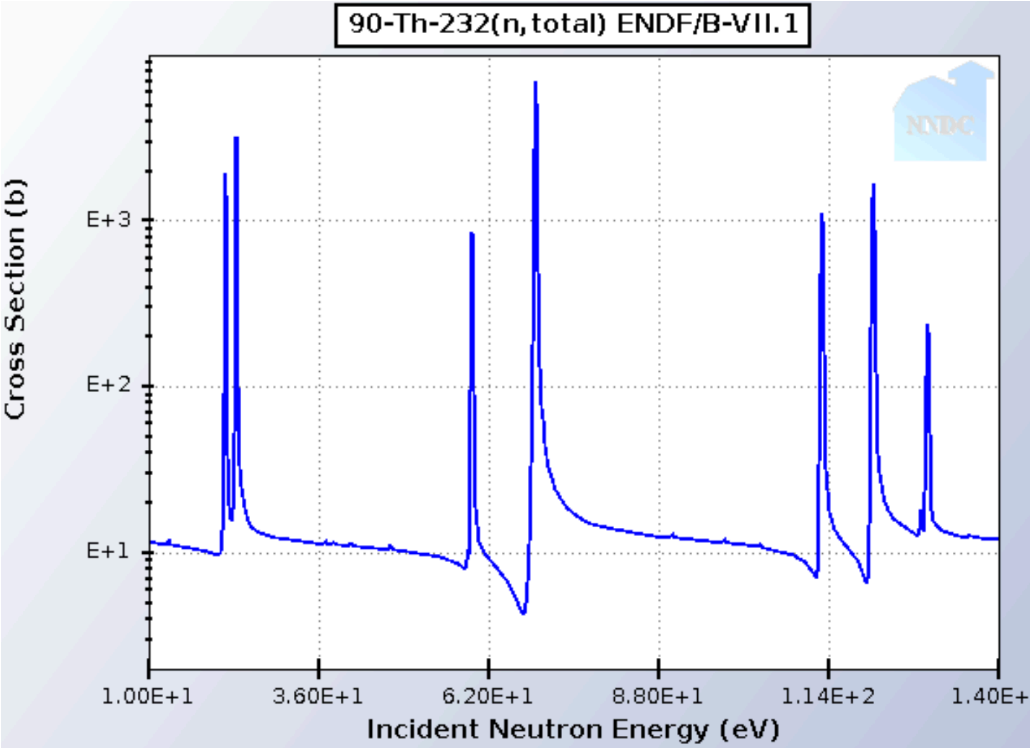
form Quasibound state of lifetime $\sim \frac{1}{\Gamma}$



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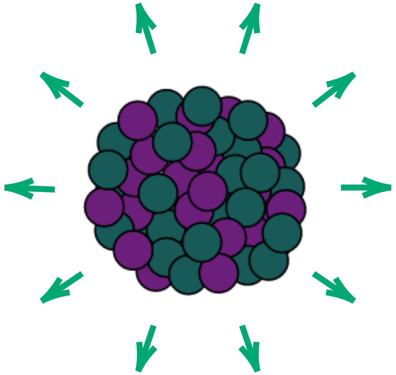

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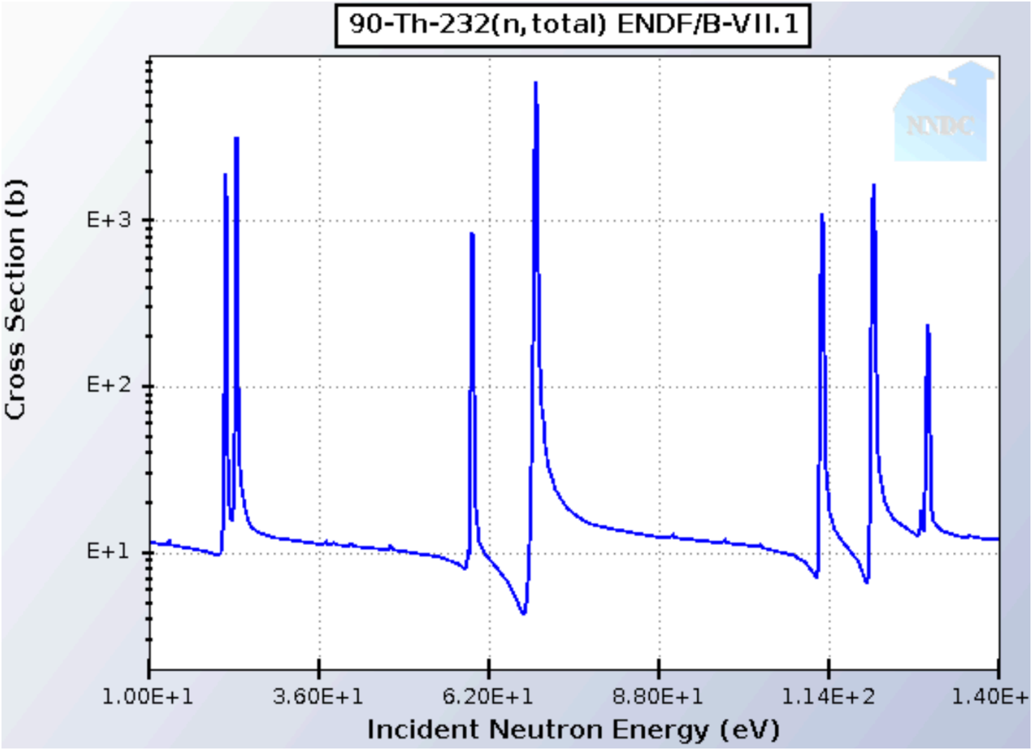
Compound Nucleus Decay



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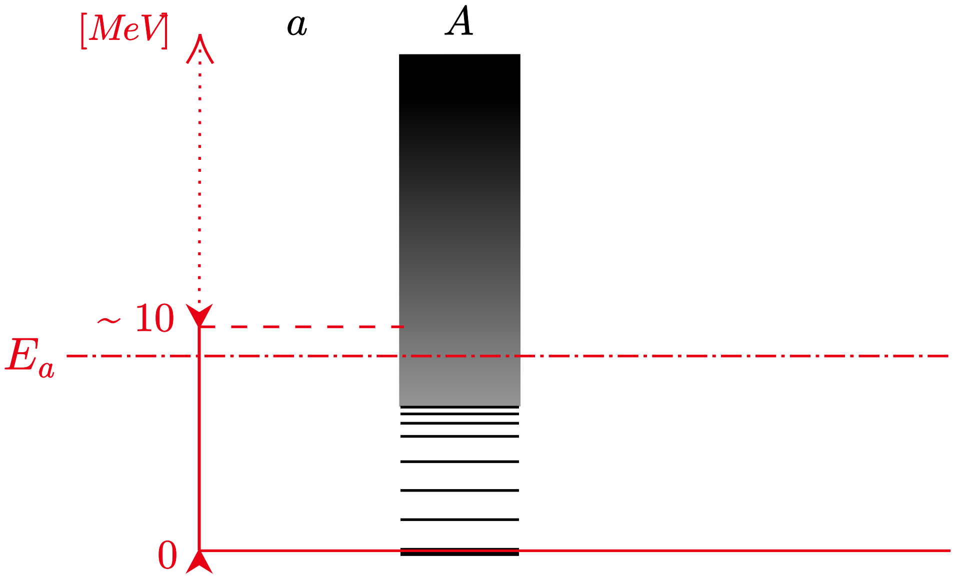
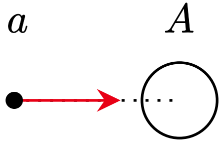
Overlapping Resonances





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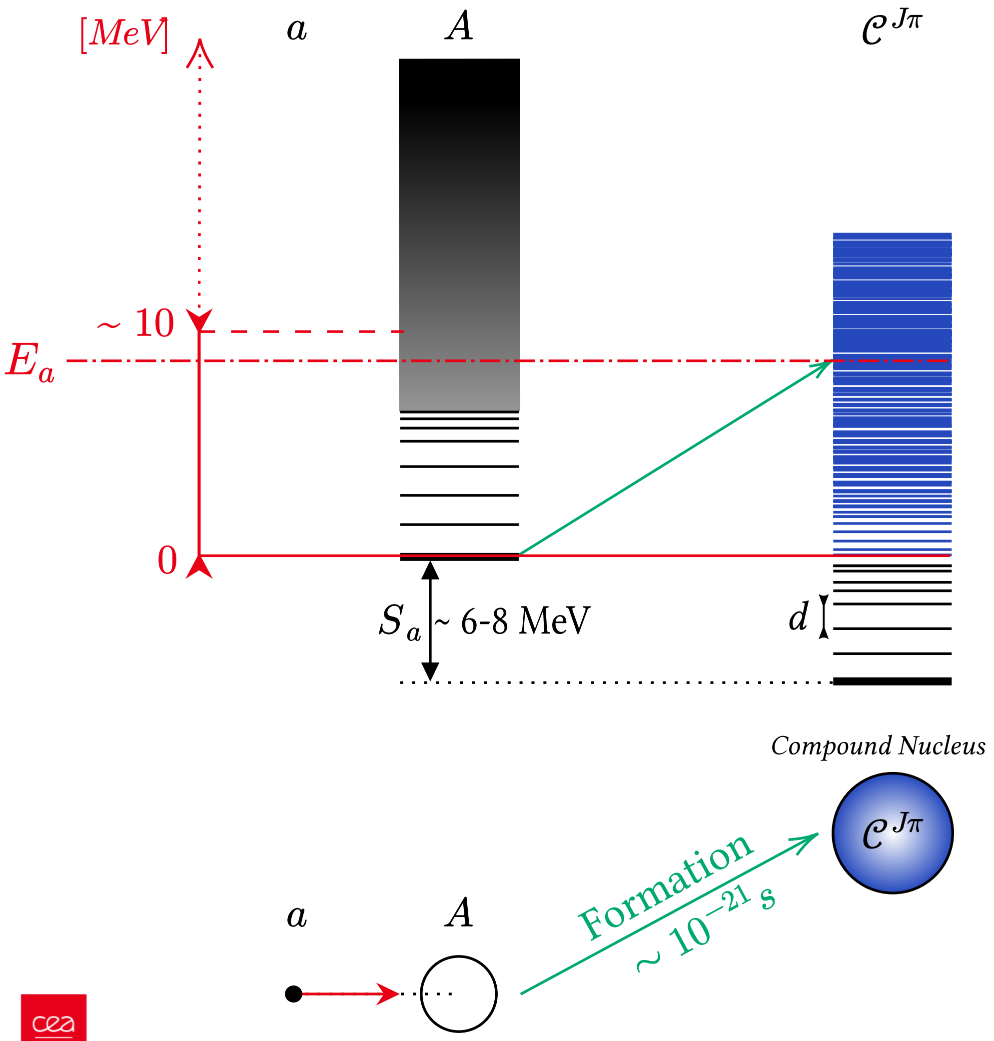

- $\rho \sim e^{\sqrt{cE}}$

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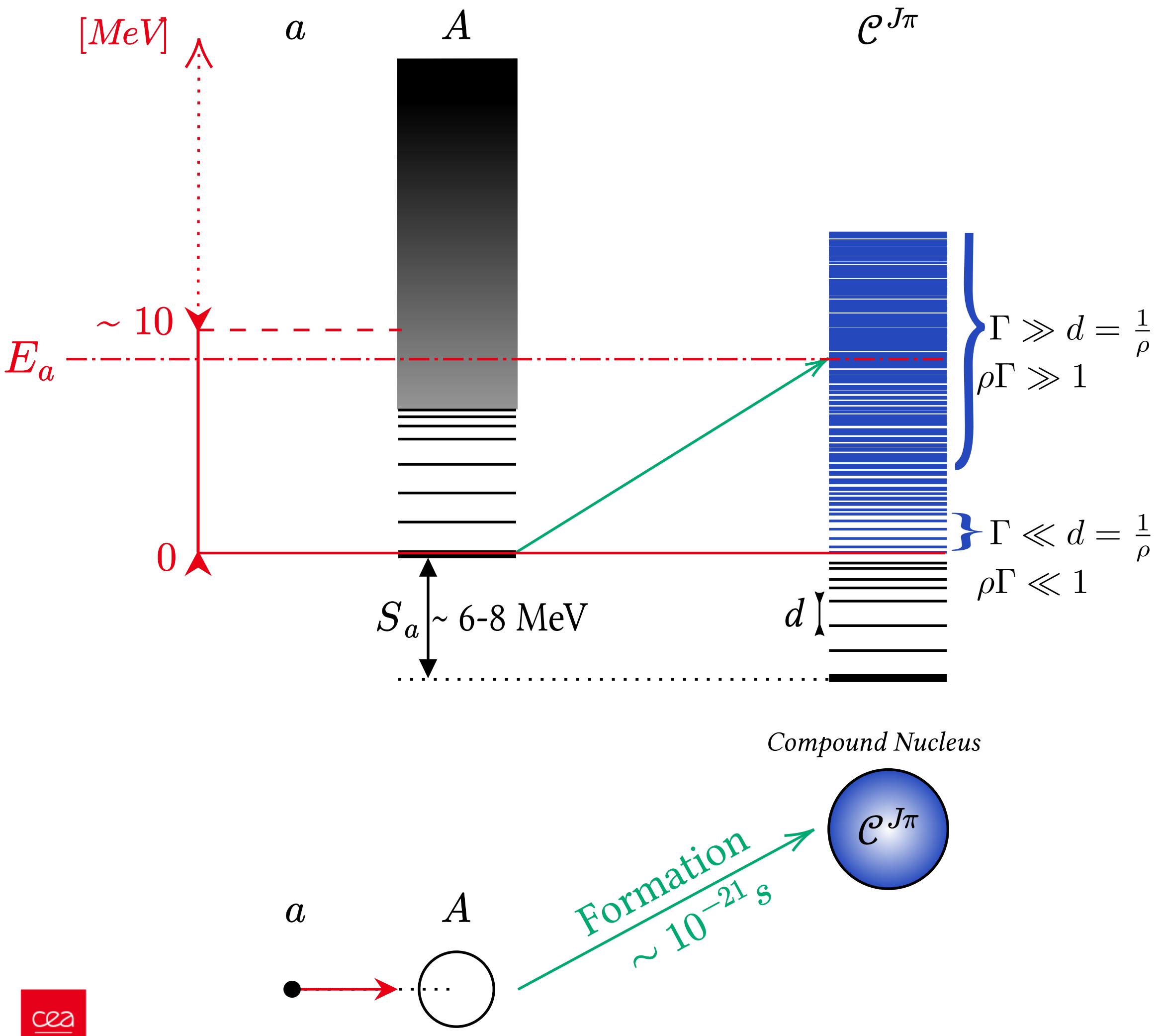


- $\rho \sim e^{\sqrt{cE}}$
  $\hookrightarrow$ Number of open channels Λ grows similarly
- $\rho\Gamma \propto \sum_{\beta=1}^{\Lambda} T_\beta = \Lambda^{\text{eff}}$
  $\hookrightarrow$ semiclassical expansion in $\left(\Lambda^{\text{eff}}\right)^{-1}$
- From single to overlapping resonances

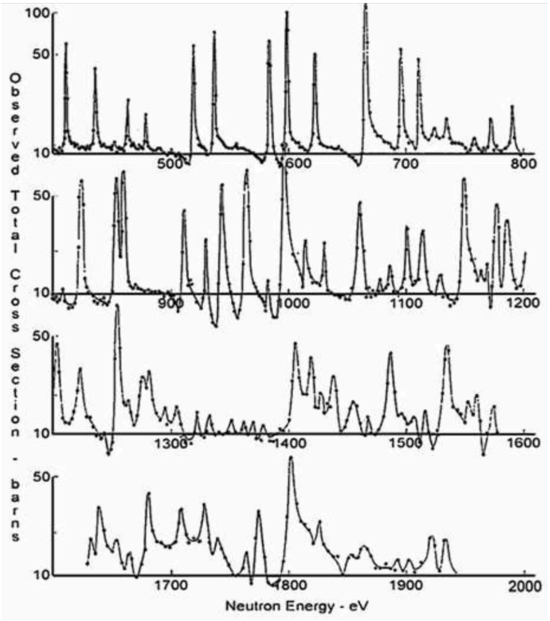
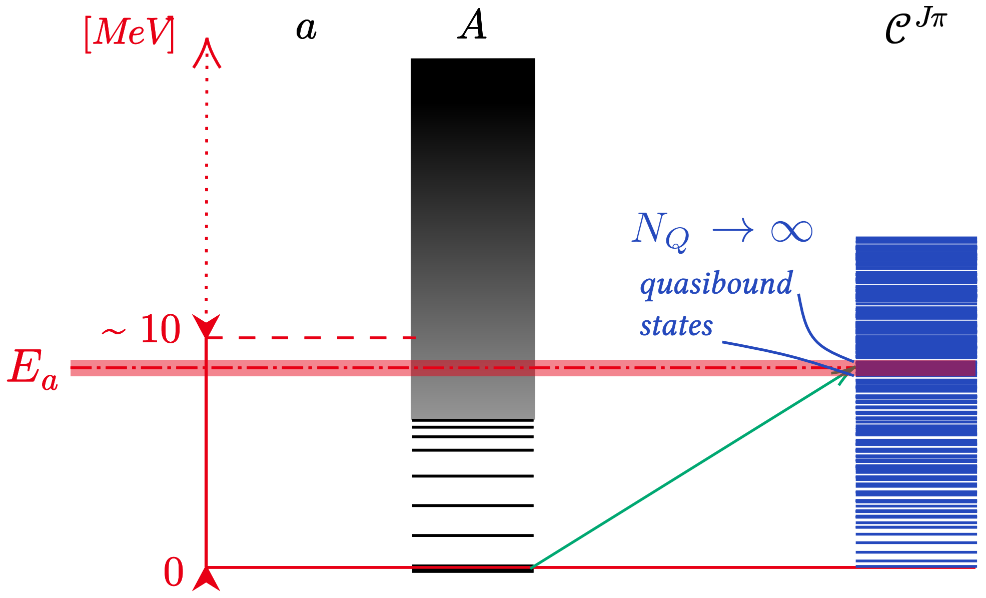


Figure 2: n + ^{238}U *(from [1])*

Average & Fluctuations



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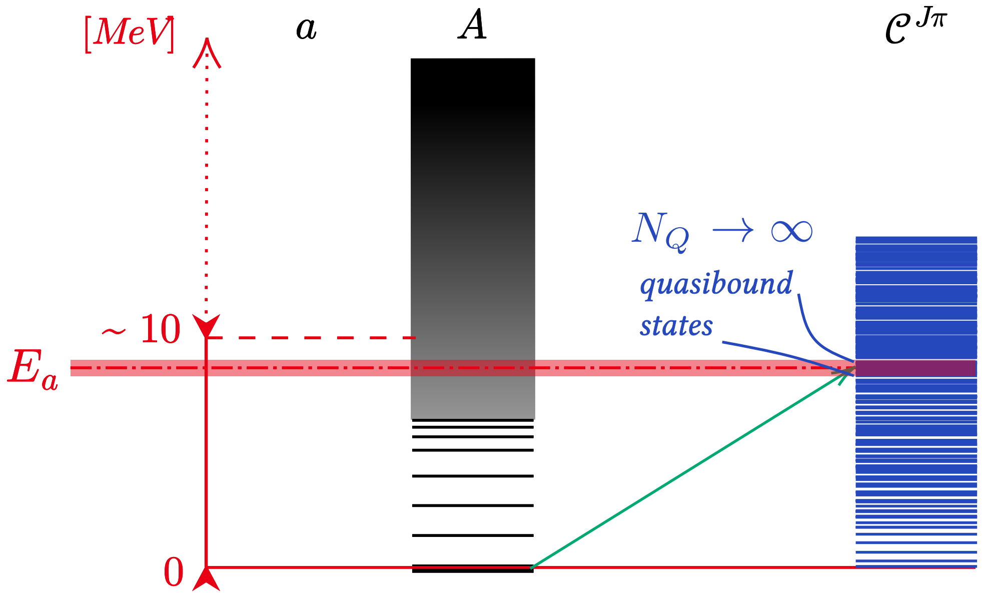

Feshbach projectors

$P(E) = \sum_{\beta=1}^{\Lambda(E)} P_\beta$ onto the $\Lambda(E)$ open channels β

$Q(E) = 1 - P(E)$ onto the N_Q quasibound states & the closed channels

$$|\Psi_\alpha^+\rangle = P|\Psi_\alpha^+\rangle + Q|\Psi_\alpha^+\rangle = |\Psi_\alpha^+\rangle + |\Psi_\alpha^+\rangle$$

$$H^N = H_{PP}^N + H_{QQ}^N + H_{PQ}^N + H_{QP}^N$$

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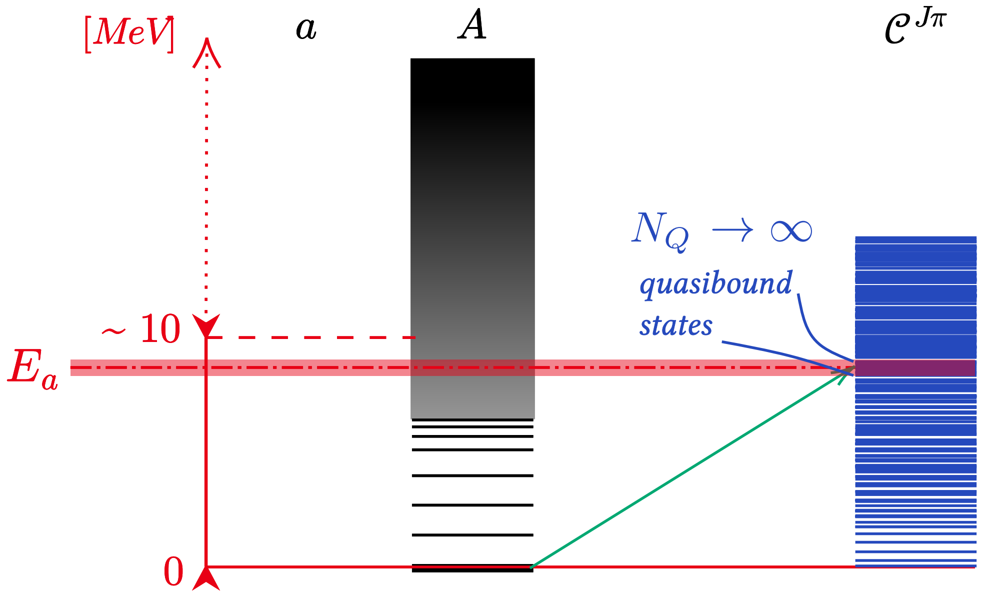

Coupled equations

$$\left[E - H^N_{QQ} - H^N_{QP}\frac{1}{E^+ - H^N_{PP}}H^N_{PQ}\right]|\Psi^+_\alpha\rangle = H^N_{QP}|\psi^+_\alpha\rangle$$

$$\left[E - H^N_{PP} - H^N_{PQ}\frac{1}{E^+ - H^N_{QQ}}H^N_{QP}\right]|\Psi^+_\alpha\rangle = 0$$

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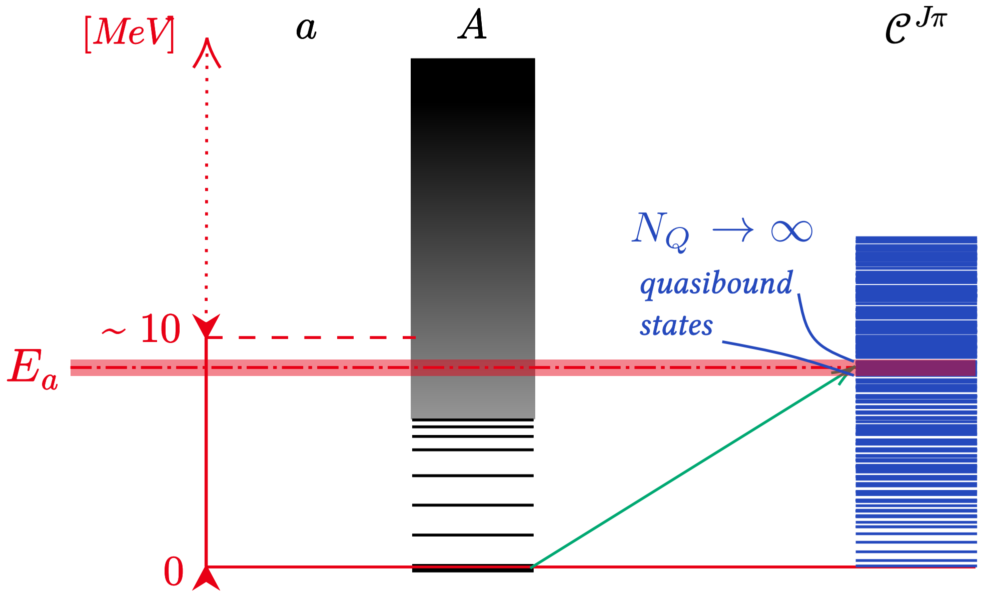

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S-matrix

$$S_{\alpha\beta} = \langle\psi_\beta^-|\psi_\alpha^+\rangle - i2\pi\langle\psi_\beta^-|H_{PQ}^N \frac{1}{\mathcal{D}(E)} H_{QP}^N|\psi_\alpha^+\rangle$$

where $\mathcal{D} = E - H_{QQ}^N - \xi(E) + i\frac{1}{2}\Gamma(E)$

$$= \langle S_{\alpha\beta}\rangle^{\Delta E} + S_{\alpha\beta}^{\text{fl}}$$

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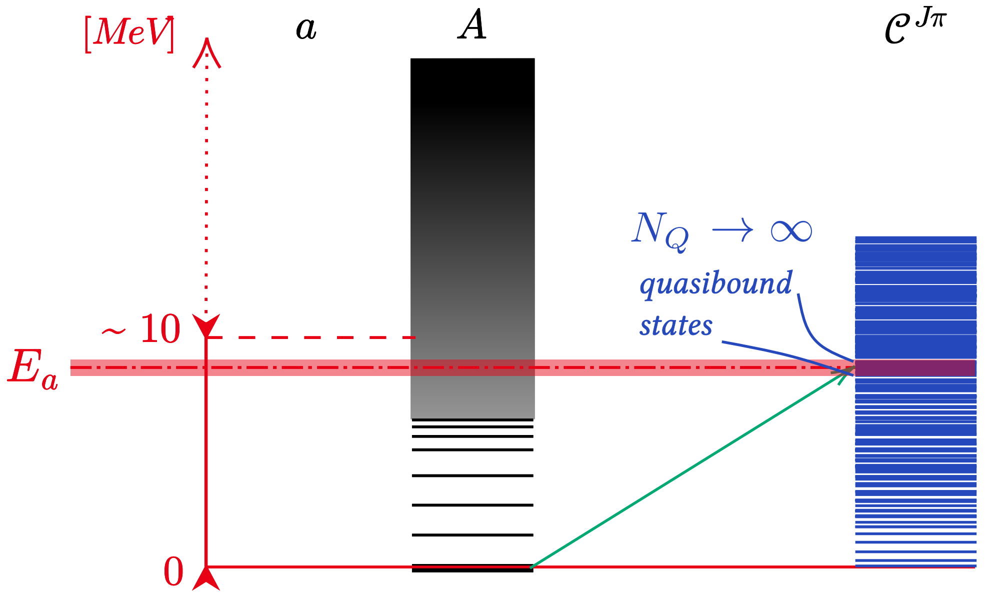

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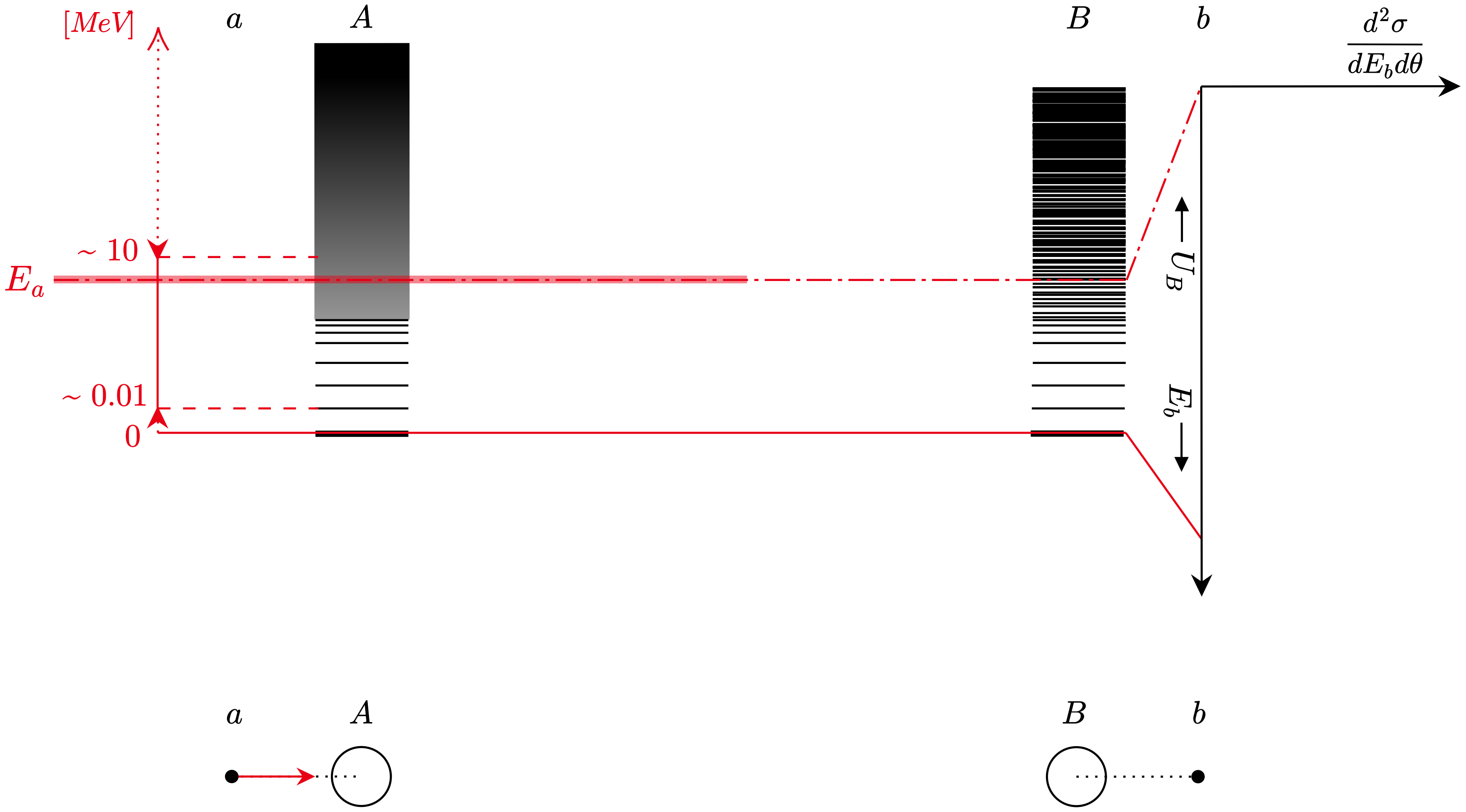
$$= \boxed{\langle S_{\alpha\beta} \rangle^{\Delta E}} + \boxed{S_{\alpha\beta}^{\text{fl}}}$$

Cross section

$$\langle \sigma_{\alpha\beta} \rangle^{\Delta E} \propto \left\langle |S_{\alpha\beta} - \delta_{\alpha\beta}|^2 \right\rangle^{\Delta E}$$

$$\propto \underbrace{\left|\langle S_{\alpha\beta} \rangle^{\Delta E} - \delta_{\alpha\beta}\right|^2}_{\sigma_{\alpha\beta}^{\text{D}}} + \underbrace{\left\langle |S_{\alpha\beta}^{\text{fl}}|^2 \right\rangle^{\Delta E}}_{\sigma_{\alpha\beta}^{\text{CN}}}$$

Nucleon-channels reactions at $E_a \lesssim 10$ MeV



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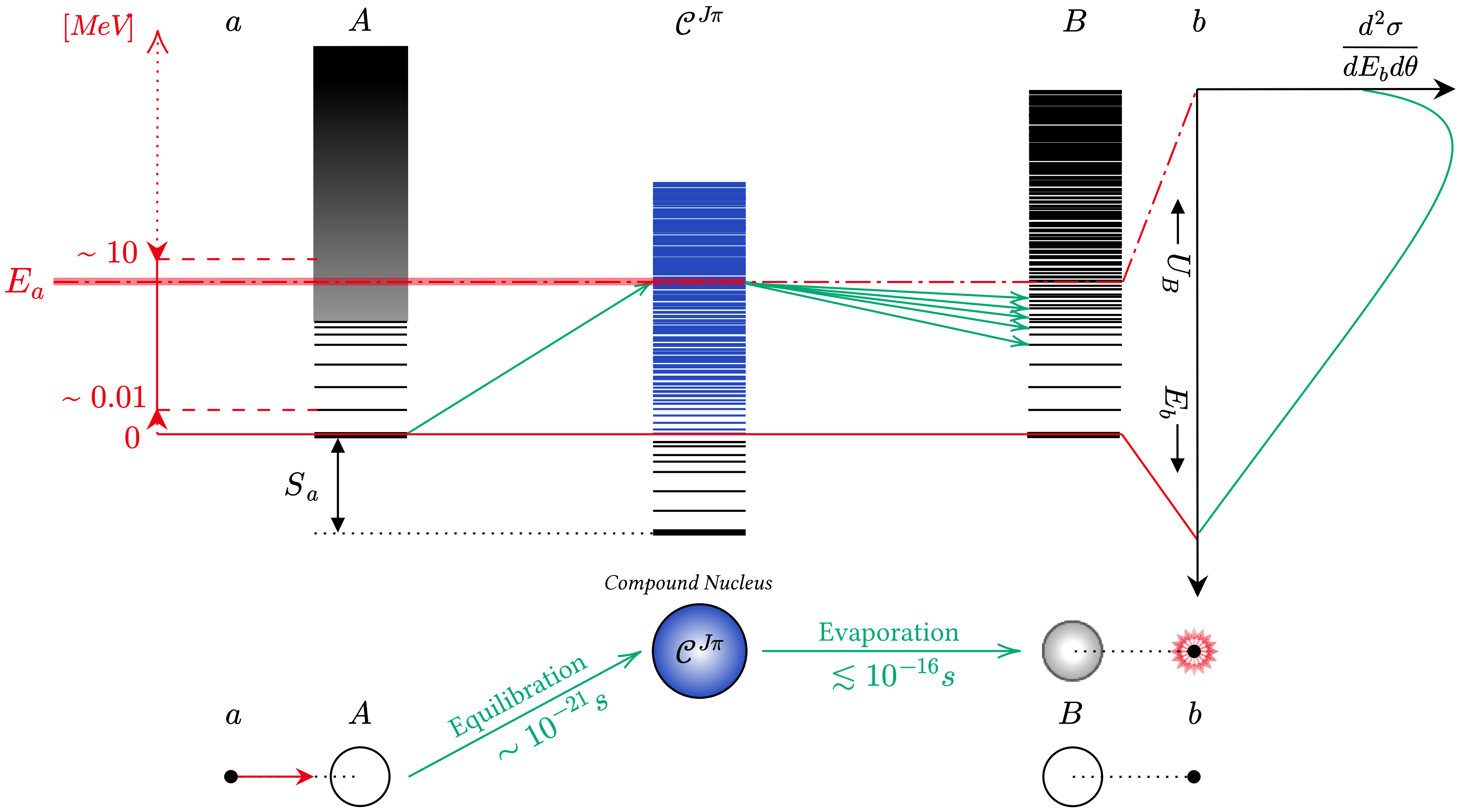

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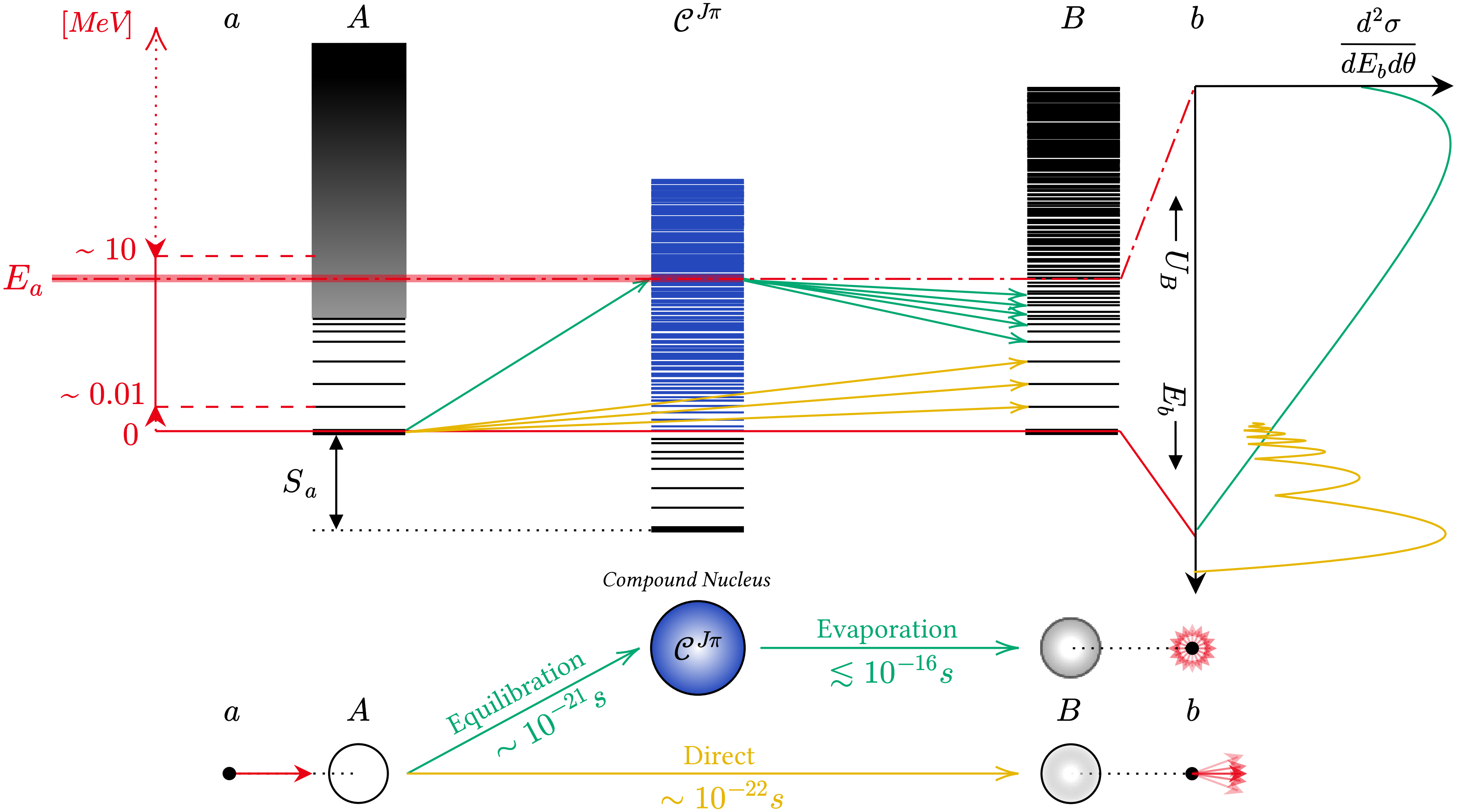
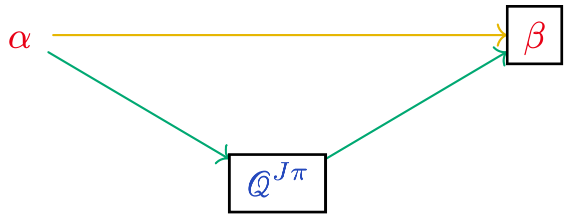


Statistical Reaction Theory



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Why $J\pi$?

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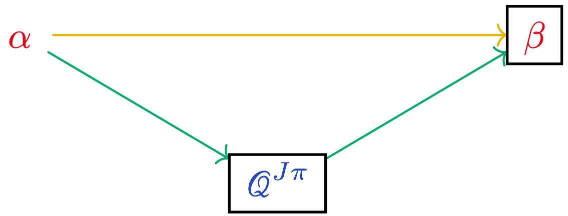

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Chaos in nuclei

$$\#\text{constraints} \ll \#\text{dof} \quad \Rightarrow \begin{cases} \text{regular motion} \\ \underline{\text{chaotic motion}} \end{cases}$$

$$H_{QQ}^N = \begin{pmatrix} H_{QQ}^{NJ_1\pi_1} & & \\ & H_{QQ}^{NJ_2\pi_2} & \\ & & \ddots \end{pmatrix}$$

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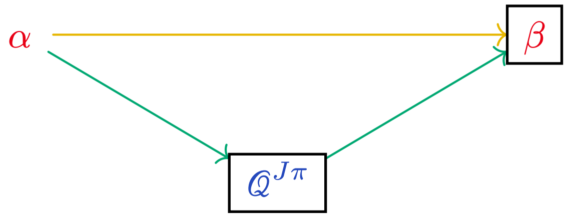

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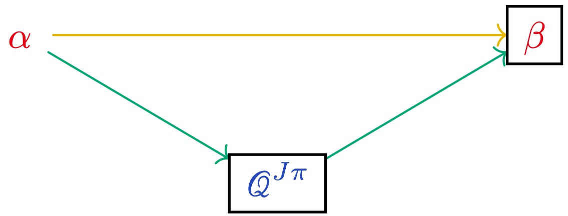

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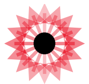
↪ 90° symmetrical emission 

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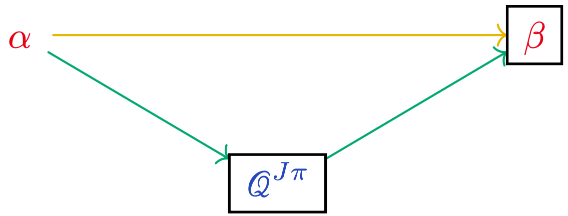

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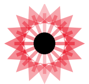
↪ 90° symmetrical emission 

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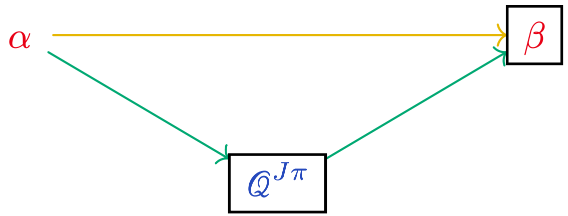
Random Hamiltonians

$$H_{QQ}^{NJ\pi} \to H^{\mathrm{GOE}} \in \left\{ \begin{pmatrix} H_{11} & \cdots & H_{1N_Q} \\ & \ddots & \vdots \\ & & H_{N_Q N_Q} \end{pmatrix} \right\}$$

Statistical Reaction Theory



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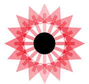
↪ 90° symmetrical emission 

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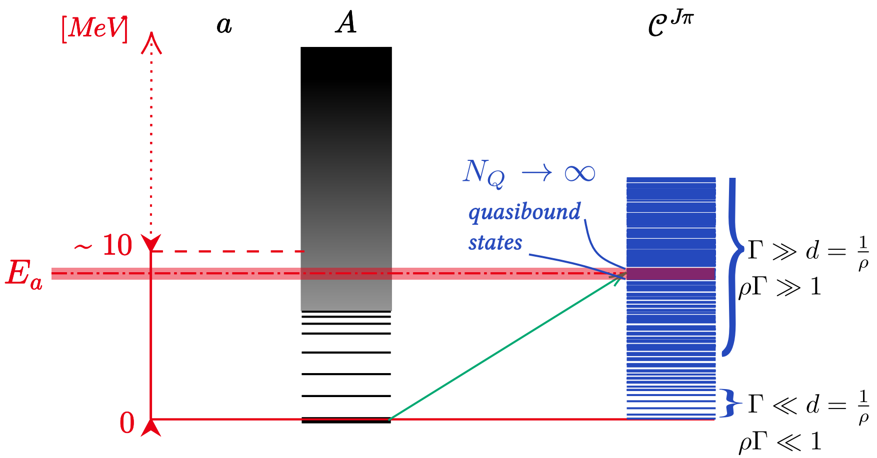
Ergodicity of GOE

$$\langle f(H^{\text{GOE}})\rangle^{\Delta E} \underset{N_Q\to\infty}{\longrightarrow} \overline{f(H^{\text{GOE}})}^N \left(= \mathbb{E}(f(H^{\text{GOE}}))\right)$$

$$\boxed{\left\langle\left|S^{\text{fl}\,J\pi}_{\alpha\beta}\right|^2\right\rangle^{\Delta E} \longrightarrow \overline{\left|S^{\text{fl}\,J\pi}_{\alpha\beta}\right|^2}^N \sim \overline{\frac{1}{\mathcal{D}}\frac{1}{\mathcal{D}}}^N}$$

Statistical Reaction Theory



$$\left\langle \left| S_{\alpha\beta}^{\text{fl}\,J\pi} \right|^2 \right\rangle^{\Delta E} \to \overline{\left| S_{\alpha\beta}^{\text{fl}\,J\pi} \right|^2}^N \sim \overline{\frac{1}{\mathcal{D}}\frac{1}{\mathcal{D}}}^N$$

- Diagrammatic expansion & Partial Resummation
  ✓ leading terms in $\left(\Lambda^{\text{eff}} \propto \rho\Gamma\right)^{-1}$

- Generating functional approach $\overline{\frac{1}{\mathcal{D}}\frac{1}{\mathcal{D}}}^N \propto \frac{\partial^2}{\partial J^2} \ln Z(E,J)|_{J=0}$

  ↪ "replica trick" $\ln(Z) = \lim_{n\to 0} \frac{1}{n}(Z^n - 1)$ ✓

  ↪ ✓ SUSY integration method to normalize $Z(E,0) = 1$
  $\Rightarrow \frac{\partial^2}{\partial J^2} \ln Z(E,J)|_{J=0} = \frac{\partial^2}{\partial J^2} Z(E,J)|_{J=0}$

- SUSY 1985 : ✓ CN (VWZ) [2]

Random Matrix Theory & Nuclear Physics

Symmetries & Chaos

- Which matrix ensemble for which symmetries
  - **3-fold way** by Wigner, Dyson (1950s, 1960s)
  - **10-fold way** (~ Cartan) by Altland, Zirnbauer (1997)
  - **non-Cartan way** for non-Hermitian Hamiltonians (RPA...)

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- **no microscopic information required !!!**

  $\hookrightarrow$ universal ($c \sim d \sim$ GOE)

  $\hookrightarrow$ ~ thermodynamics for spectral statistics

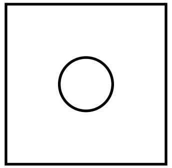


Figure 1: Sinai Billard

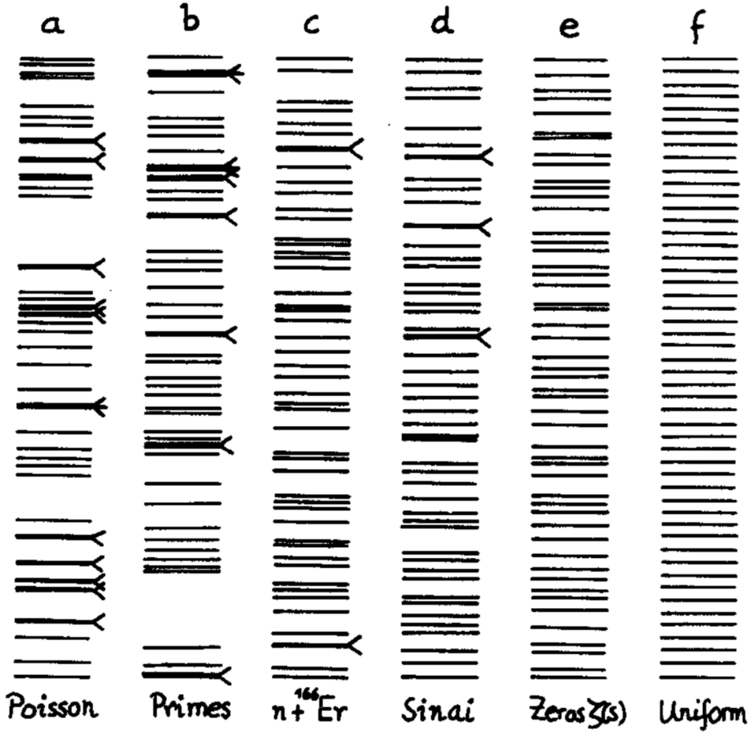


Figure 2: Various unfolded spectra (from [3])

Symmetries & Chaos

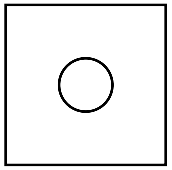
- Which matrix ensemble for which symmetries
  - **3-fold way** by Wigner, Dyson (1950s, 1960s)
  - **10-fold way** (~ Cartan) by Altland, Zirnbauer (1997)
  - **non-Cartan way** for non-Hermitian Hamiltonians (RPA...)

- **no microscopic information required !!!**

  $\hookrightarrow$ universal ($c \sim d \sim$ GOE)

  $\hookrightarrow$ ~ thermodynamics for spectral statistics

- A **quantum** system is **chaotic** if ?
  1. its classical counterpart is chaotic (based on periodic orbits)
  2. it displays random matrix spectral statistics



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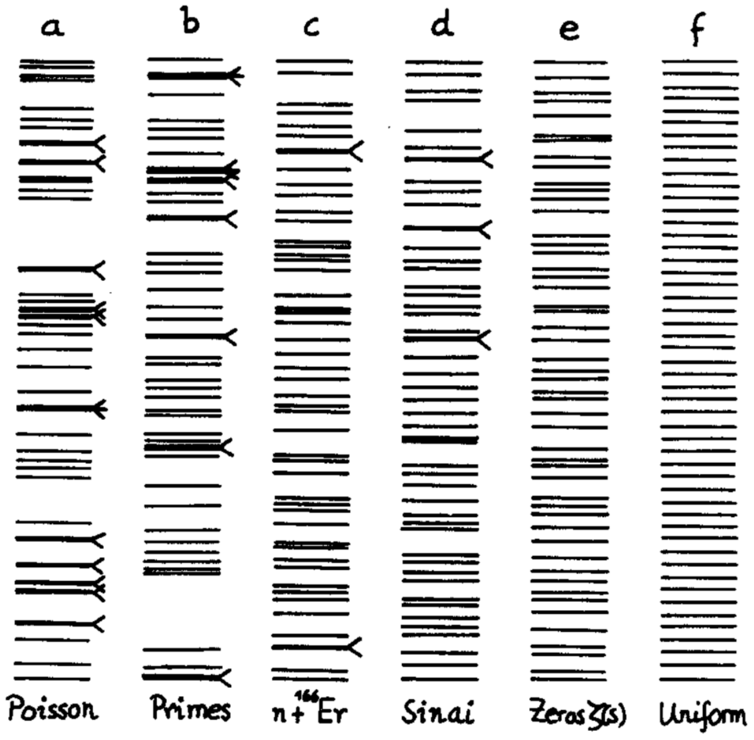


Figure 2: Various unfolded spectra (from [3])

Randomness in Shell-Model

- GOE vs shell-model

$$H_{QQ}^{NJ\pi} \to H^{\mathrm{GOE}} \in \left\{ \begin{pmatrix} H_{11} & \cdots & H_{1N_Q} \\ & \ddots & \vdots \\ & & H_{N_QN_Q} \end{pmatrix} ; \text{iid } H_{\mu\nu} \hookrightarrow \mathcal{N}\left(0, (1+\delta_{\mu\nu})\sigma^2\right) \right\}$$

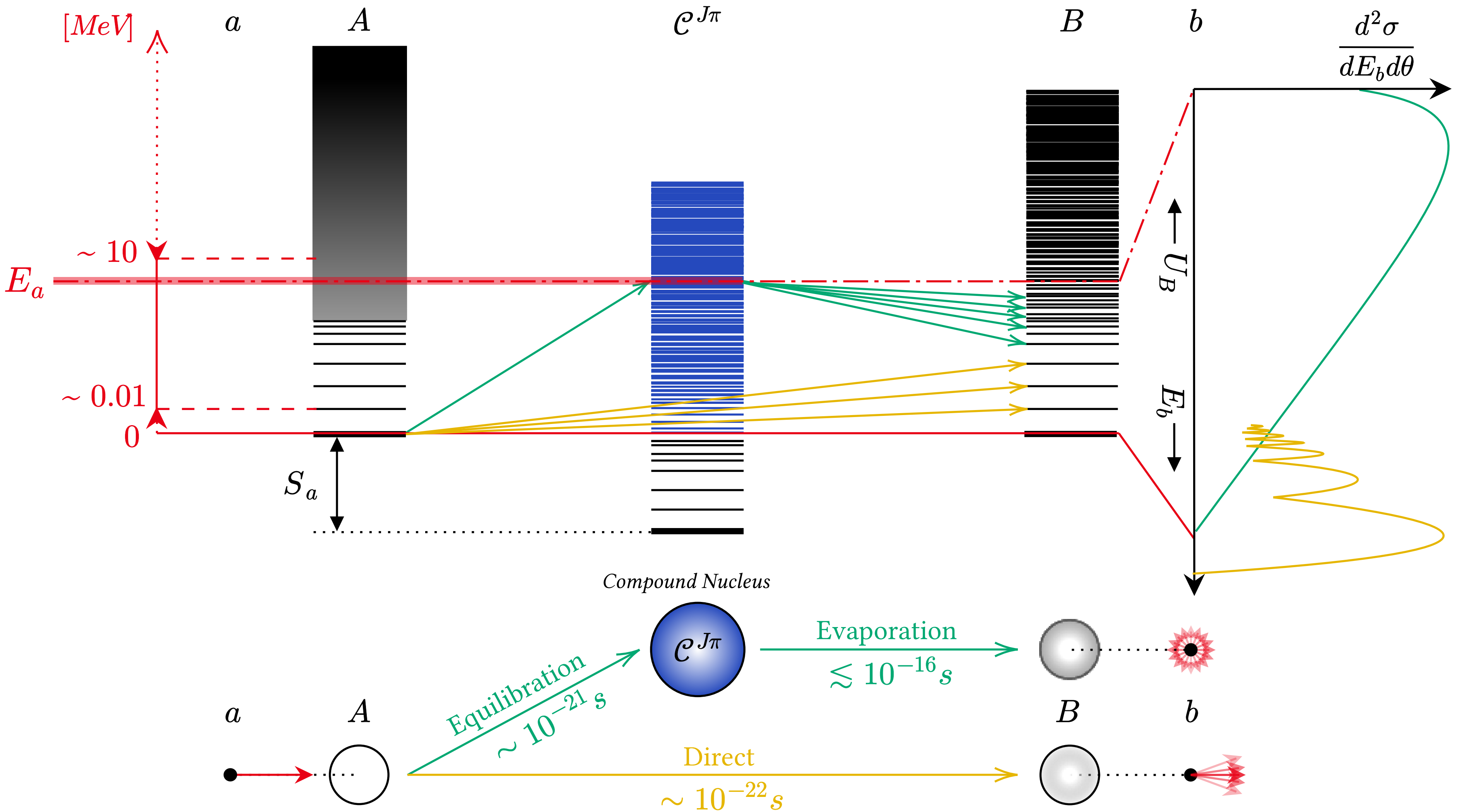
every pair of quasibound states is equally coupled
equivalent '1, ..., N-body' interaction $\longrightarrow$ ✗

- 1970-1971 : TBRE (Two-Body Random Ensemble)
- 1975 : EGOE(k) (Embedded k-body interaction GOE)
- 2015 : SYK model (Dirac $\to$ Majorana fermions)

$$\mathrm{AdS}_2 \longleftrightarrow \mathrm{CFT}_1 = \mathrm{QM}$$
$$\text{black-hole} \longleftrightarrow \text{SYK model} \sim \mathcal{C}^{J\pi}$$

Quantum Pre-Equilibrium Models

Intermediate Structure & Pre-Equilibrium Emission



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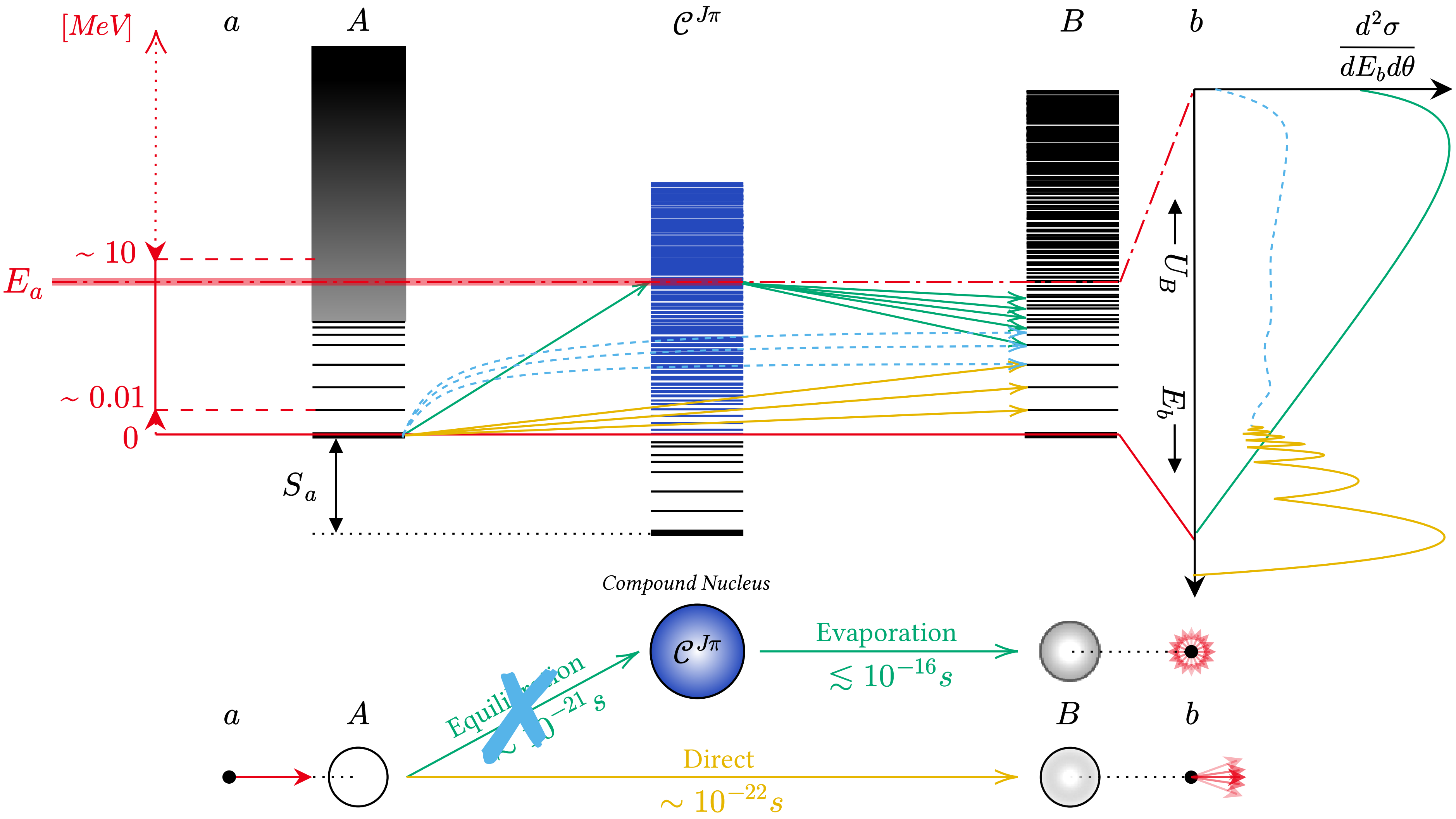
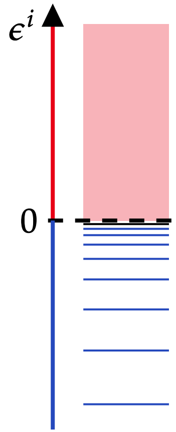


Microscopic Projectors

$$
\begin{aligned}
H^N &= \sum_{i=1}^{N} t(x_i) + V^N \\
&= \underbrace{\sum_{i=1}^{N} \overbrace{[t(x_i) + v_0(x_i)]}^{h_0(x_i)}}_{\text{IPM term}} + \underbrace{V^N - \sum_{i=1}^{N} v_0(x_i)}_{\text{residual interaction}} \\
&= \qquad H^N_{\text{IPM}} \qquad + \qquad V^N_{\text{IPM}}
\end{aligned}
$$

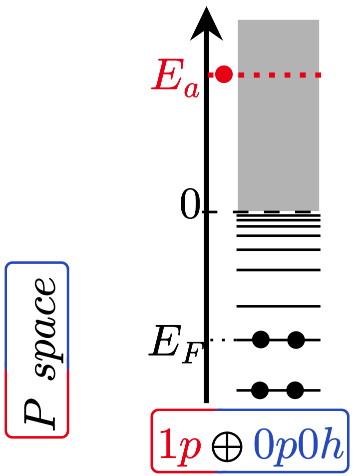
distribute N single-particles in $\mathrm{Sp}(h_0(x_i)) \rightarrow$



$$
\left(\mathscr{P}(E) \xrightarrow[+\text{ closed}]{} \right) \; \mathscr{P} = \left\{\text{scattering states of } H^N_{\text{IPM}}\right\}
$$

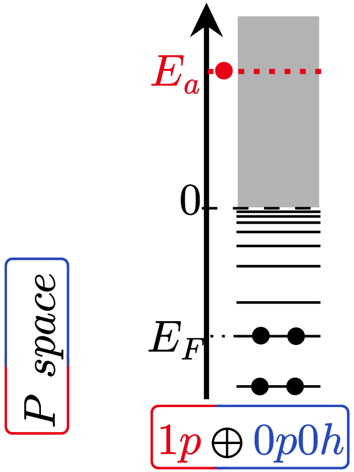
$$
\left(\mathscr{Q}(E) \xrightarrow[-\text{ closed}]{} \right) \; \mathscr{Q} = \left\{\text{bound states of } H^N_{\text{IPM}}\right\} = \left\{\text{quasibound states}\right\}
$$

Pre-equilibrium emission examples



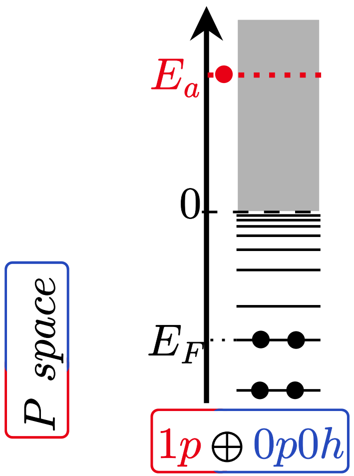
$H^{N-1} + t(N)$

Pre-equilibrium emission examples



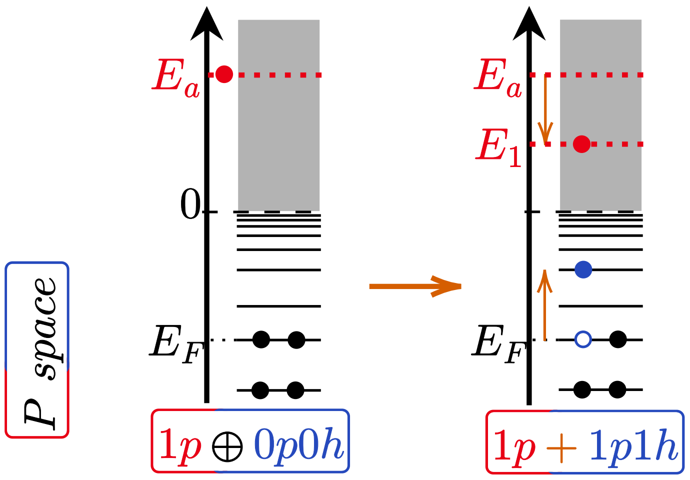
$$H^{N-1} + t(N) \qquad + V(N, \leq (N-1))$$

Pre-equilibrium emission examples



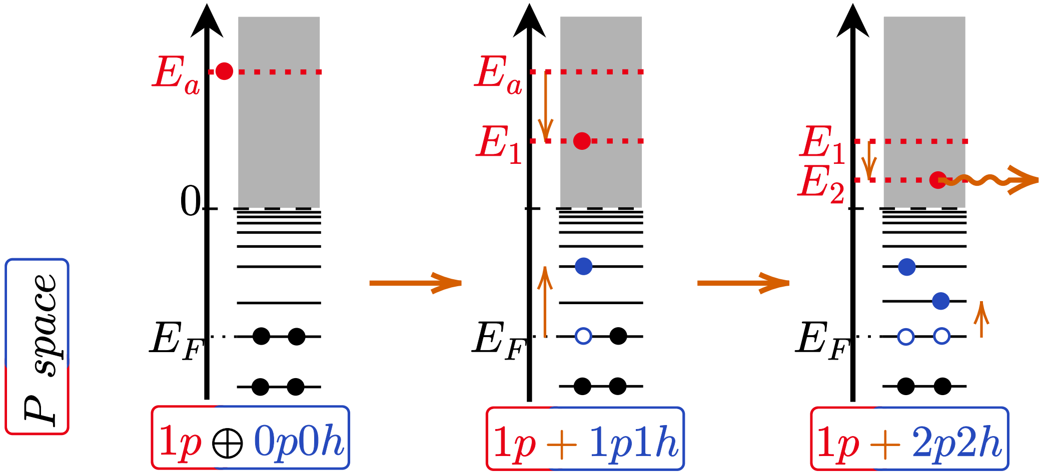
$$H^{N-1} + t(N) \quad + V(N, \leq (N-1)) \\ + u_{\text{opt}}(N) - u_{\text{opt}}(N)$$

Pre-equilibrium emission examples



$H^{N-1} + \mathcal{h}_{\text{opt}}(N) + \mathcal{V}_{\text{opt}}$

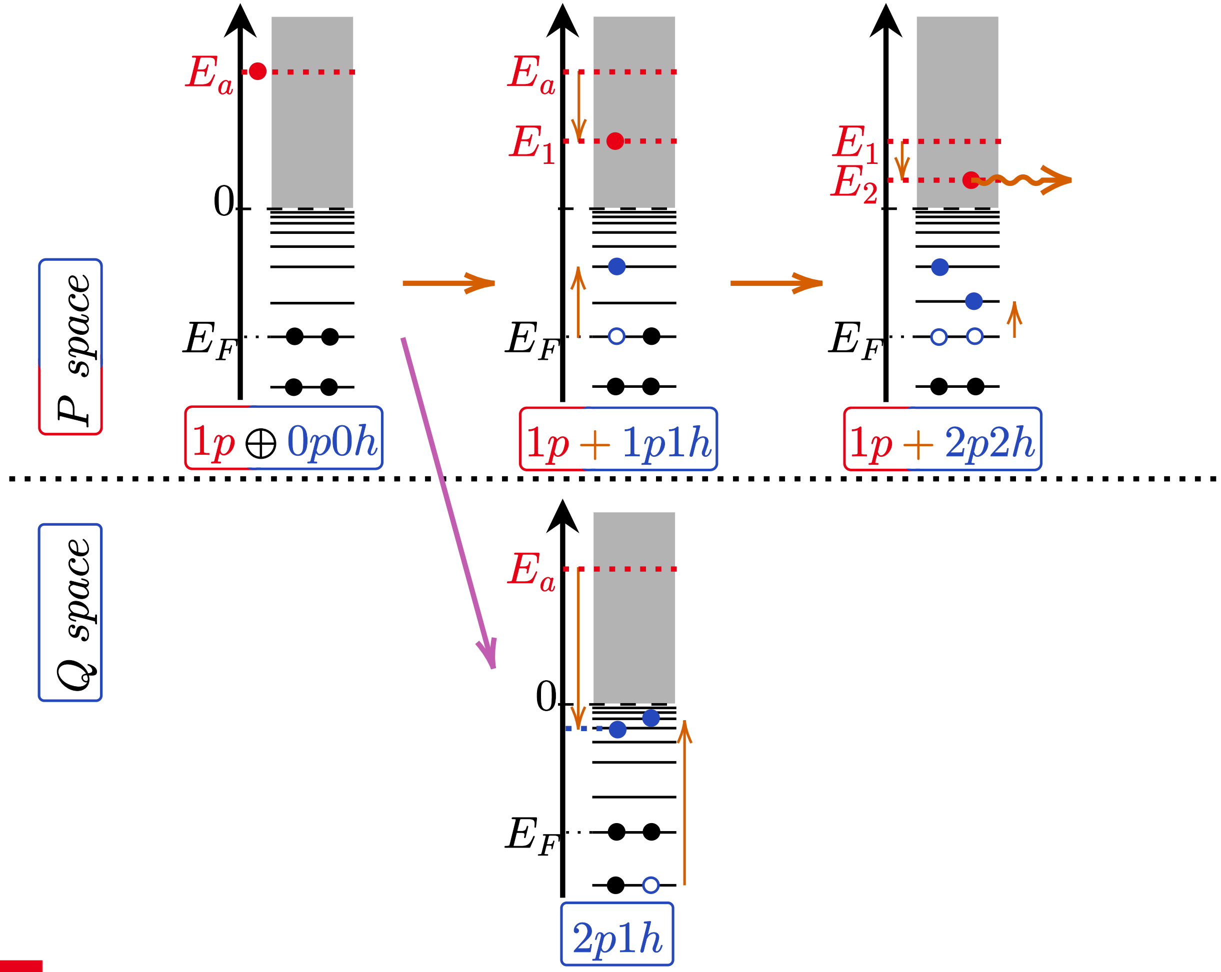
Pre-equilibrium emission examples



$$H^{N-1} + \mathcal{h}_{\text{opt}}(N) + \mathcal{V}_{\text{opt}}$$

- only through P-space emission { $\rightsquigarrow$ } :
  ↪ **Multi-Step Direct** emission
  MSD

Pre-equilibrium emission examples



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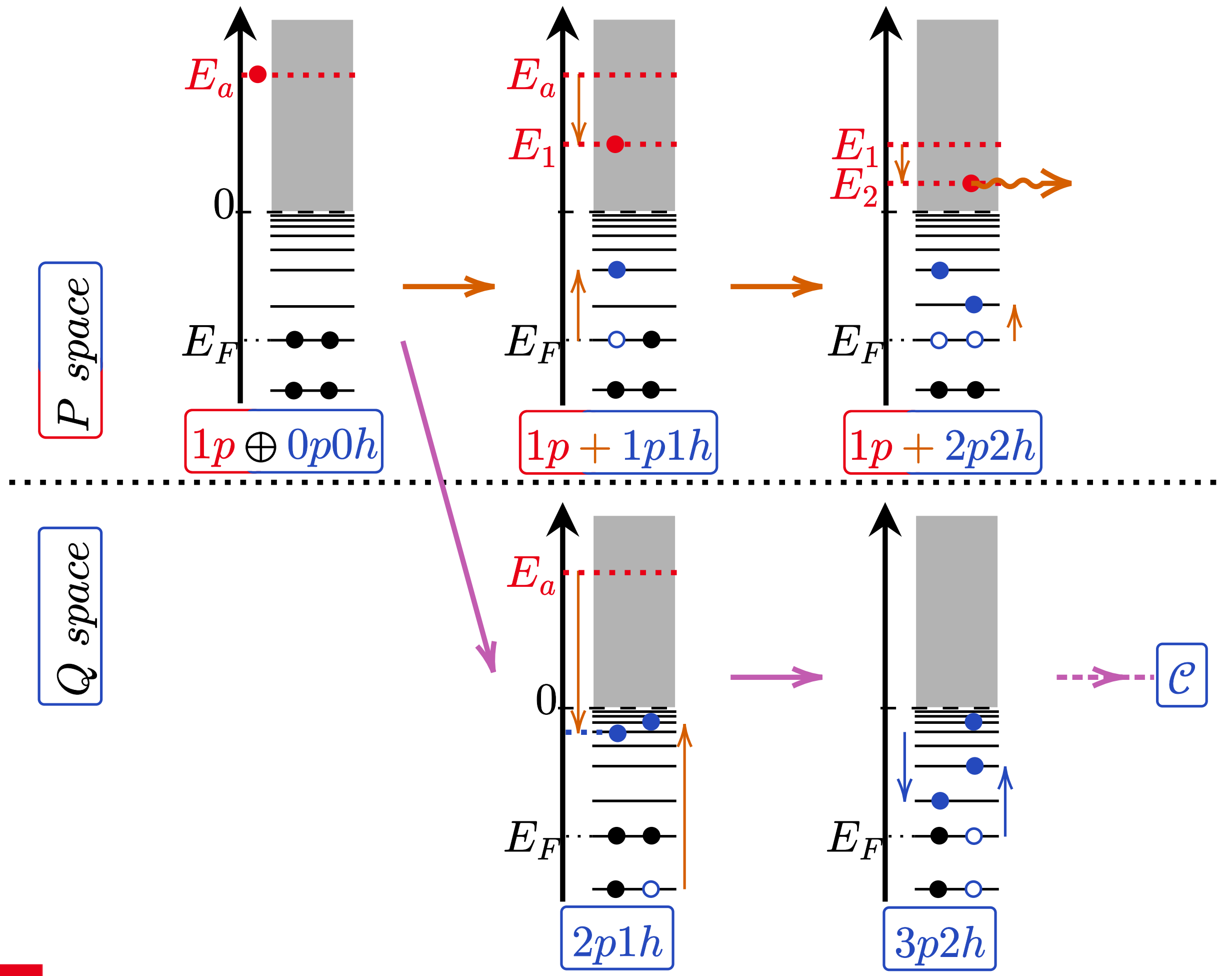

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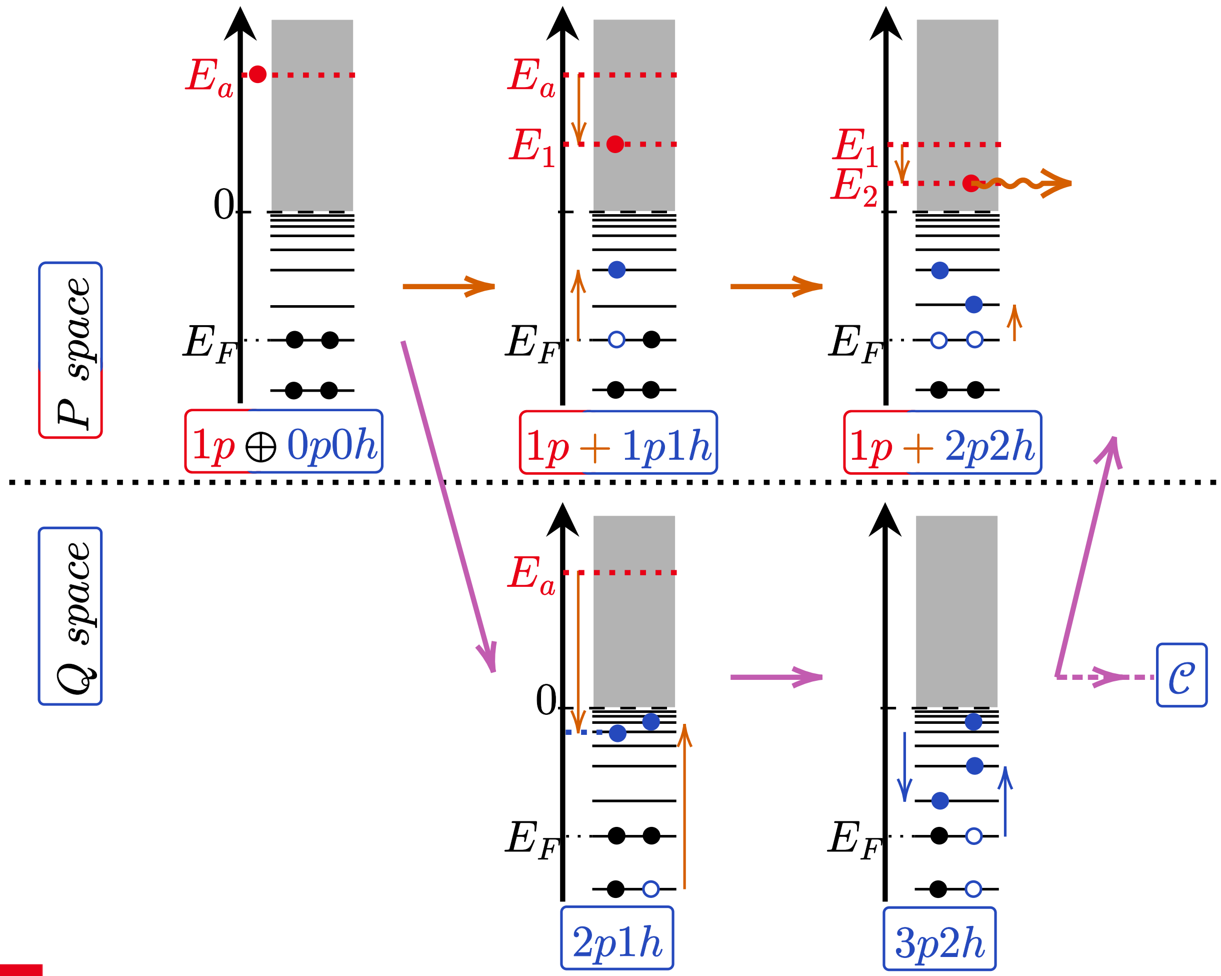

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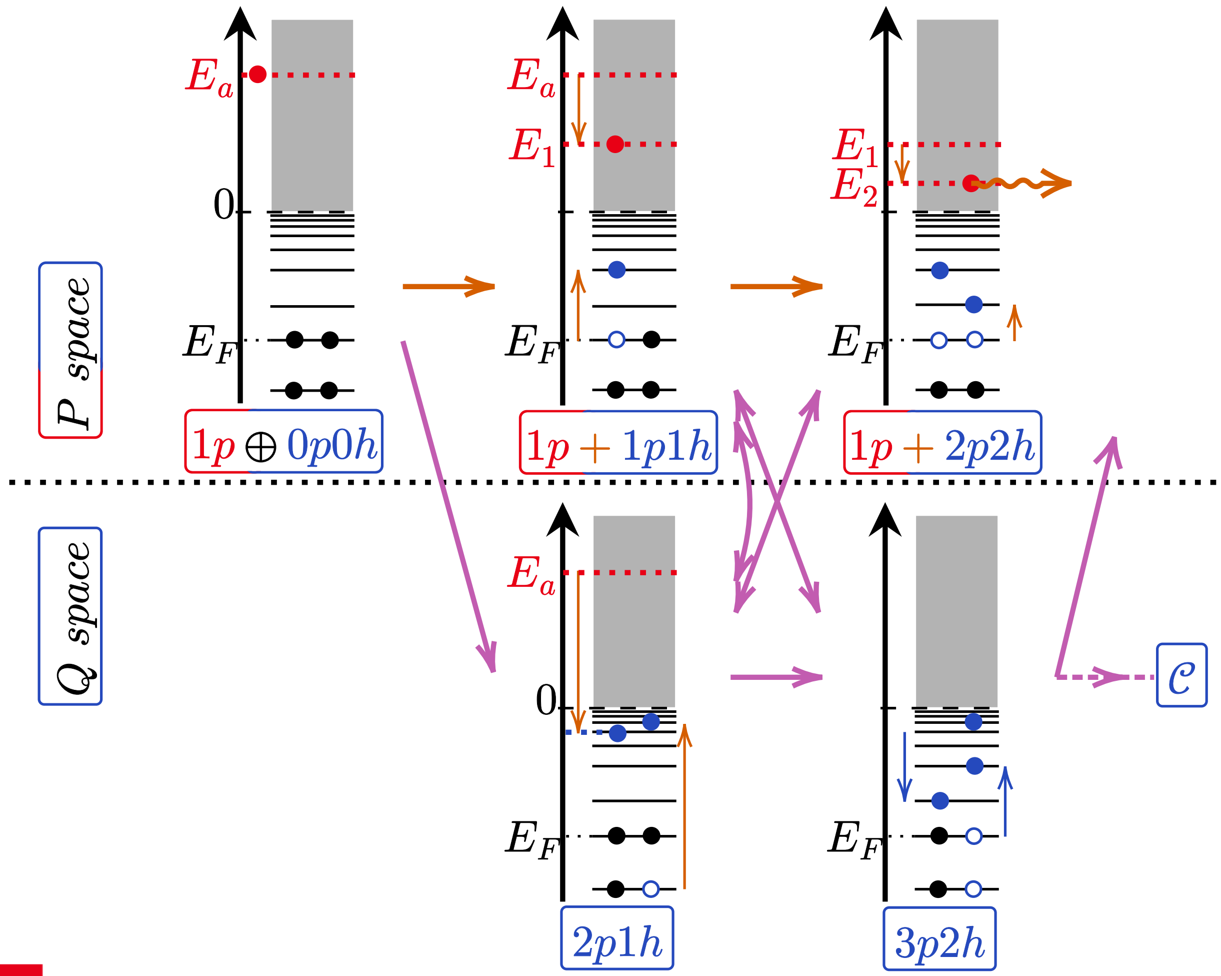
Pre-equilibrium emission examples



$H^{N-1} + \hbar_{\rm opt}(N) + \mathcal{V}_{\rm opt}$

- only through P-space emission $\{\rightsquigarrow\}$:
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- $(P \to Q) = \{\swarrow, \downarrow, \searrow\}$:
  ↪ **gradual absorption**
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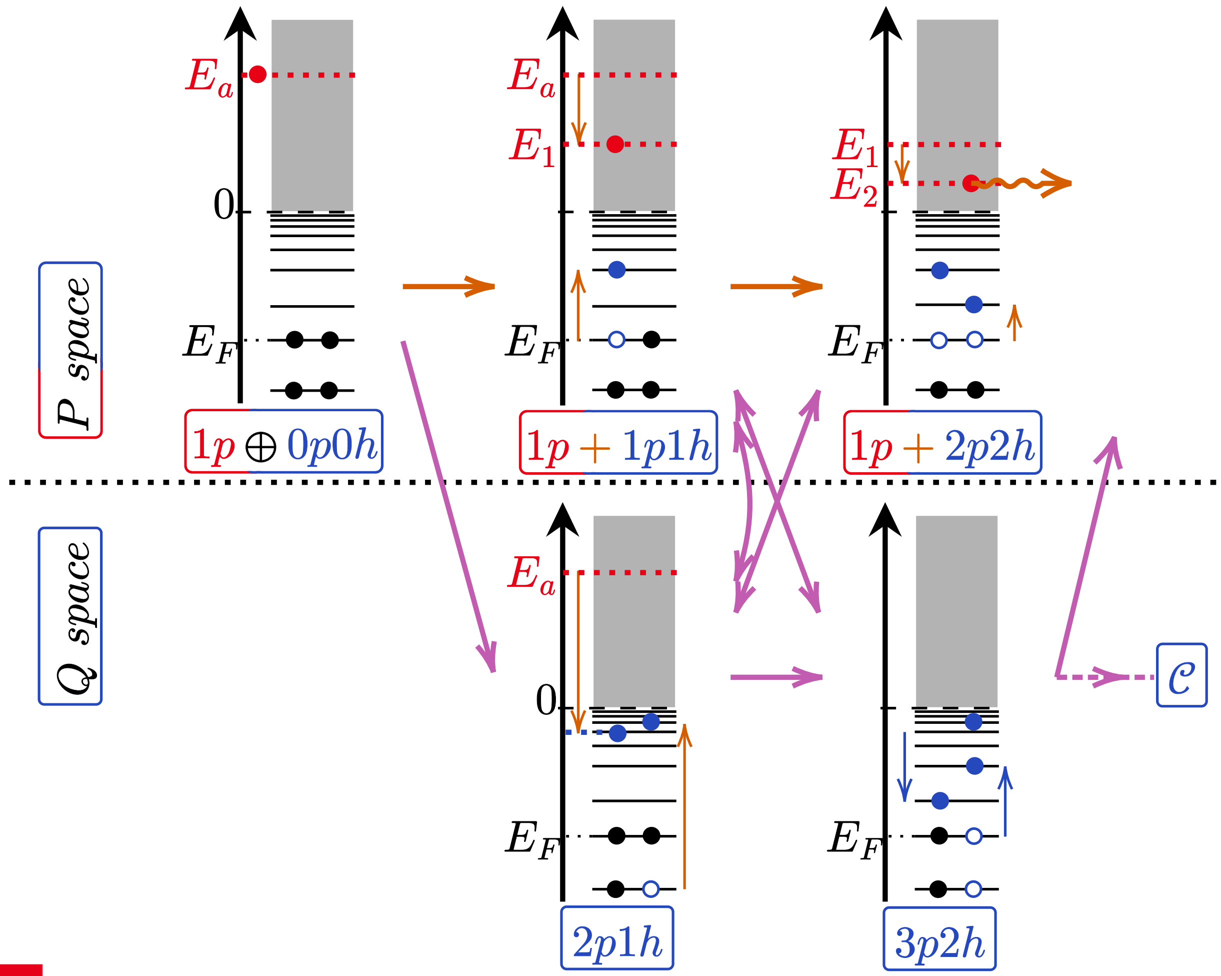

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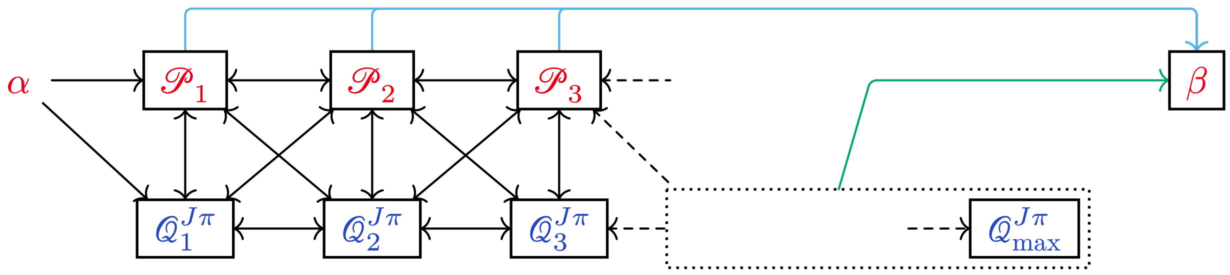


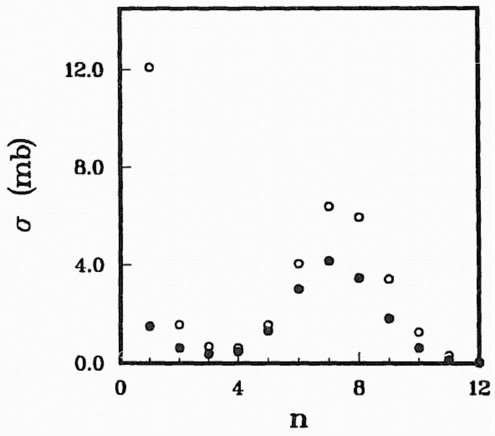
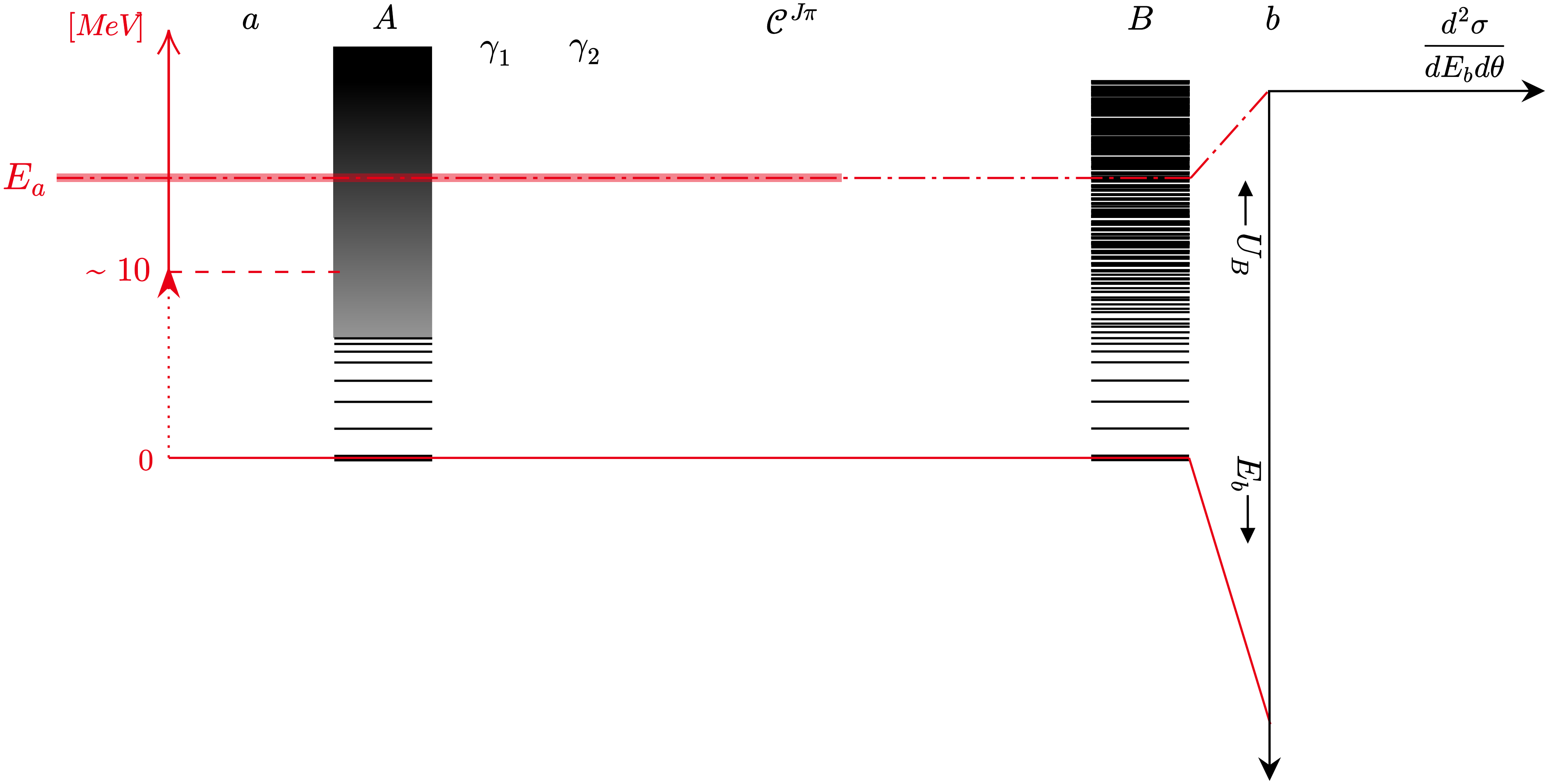


Fig. 4. Total (energy integrated) neutron emission as a function of the class number for two different values of the composite nucleus spin ($J^{\pi} = 2^+$ - solid dots, $J^{\pi} = 12^+$ - open dots).

one GOE $\rightarrow$ many GOEs : how do they couple ?

Nucleon-channels reactions at $10 \lesssim E_a \lesssim 30$ MeV



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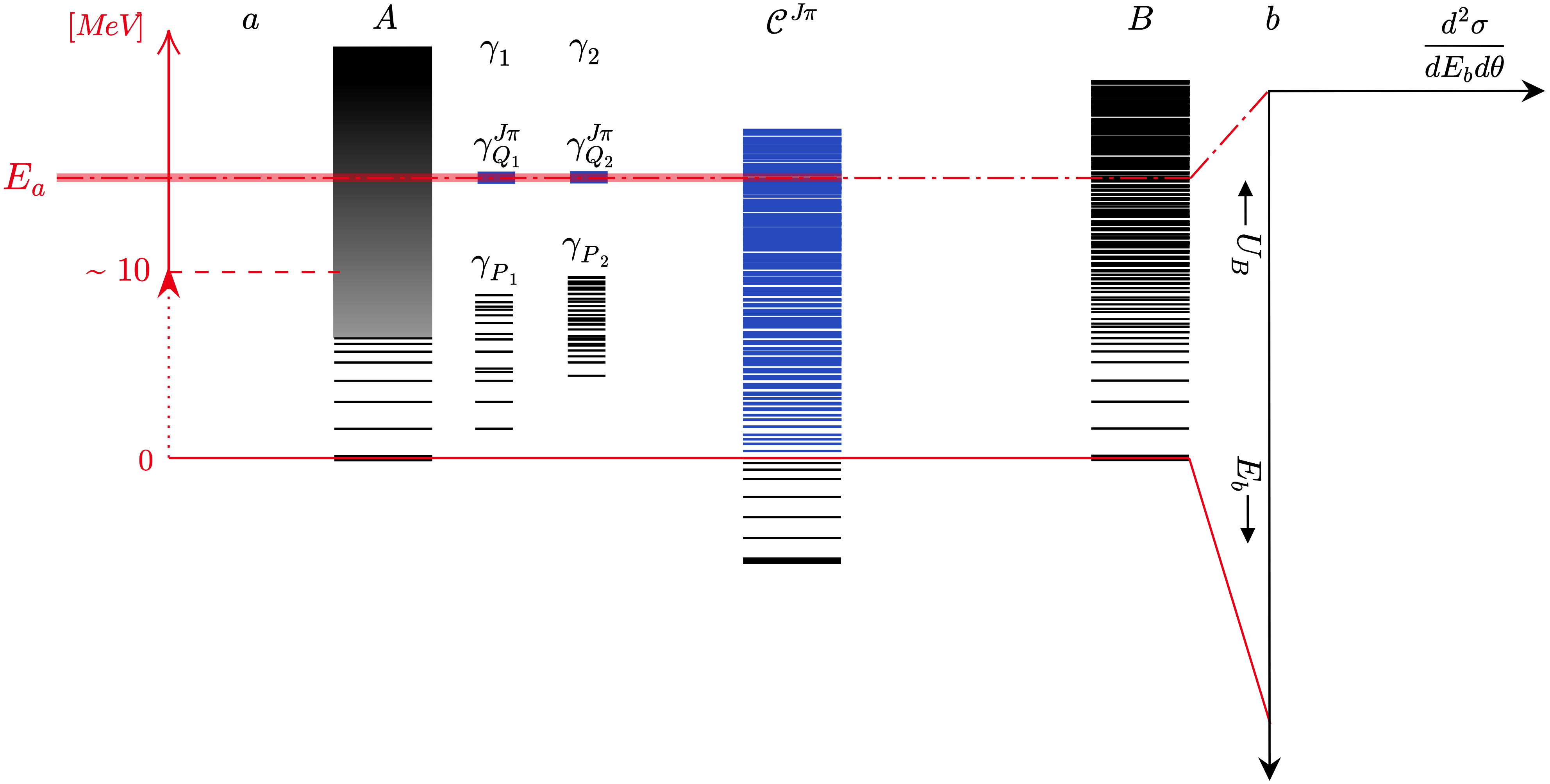

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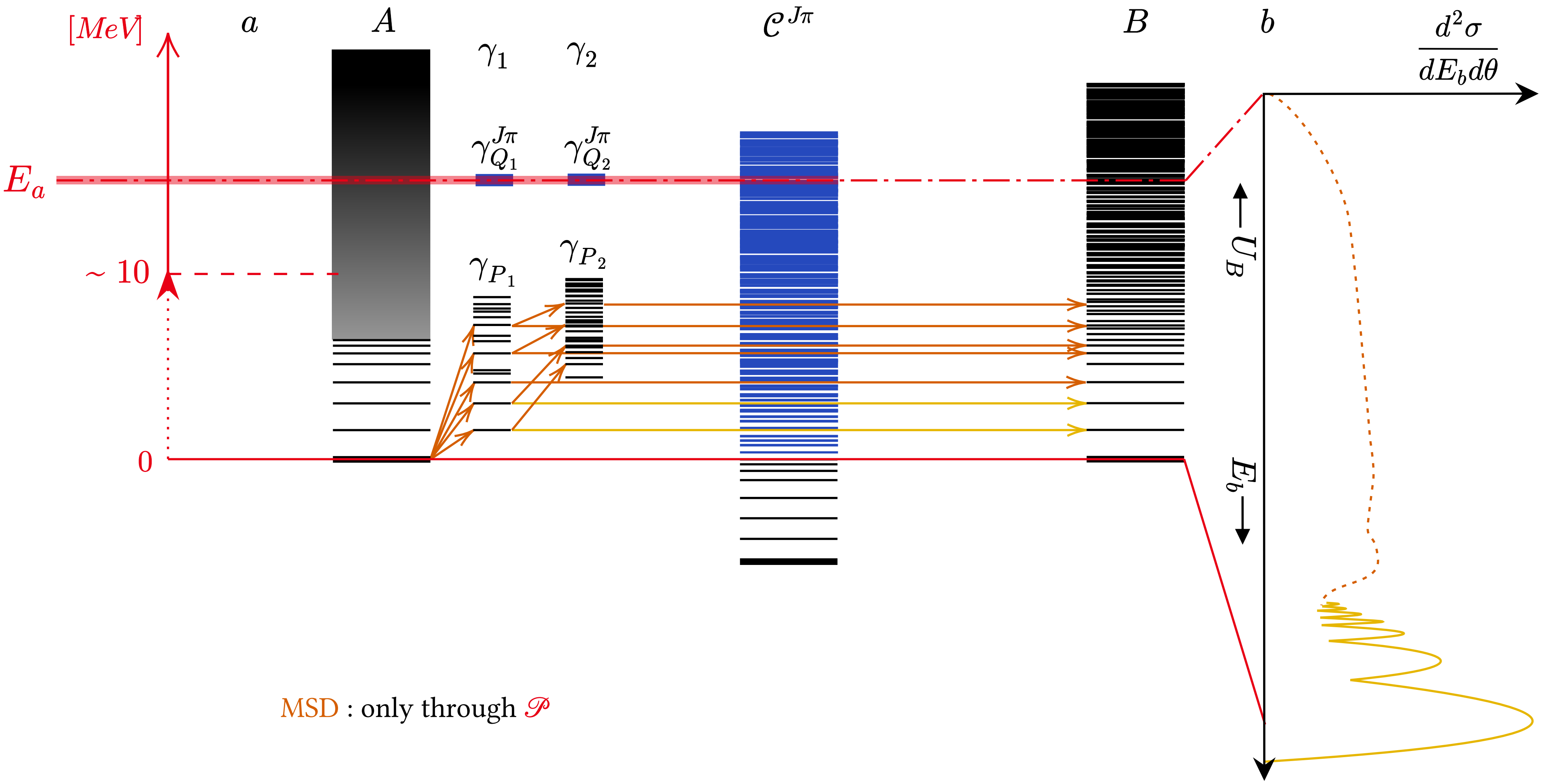

MSD : only through $\mathscr{P}$

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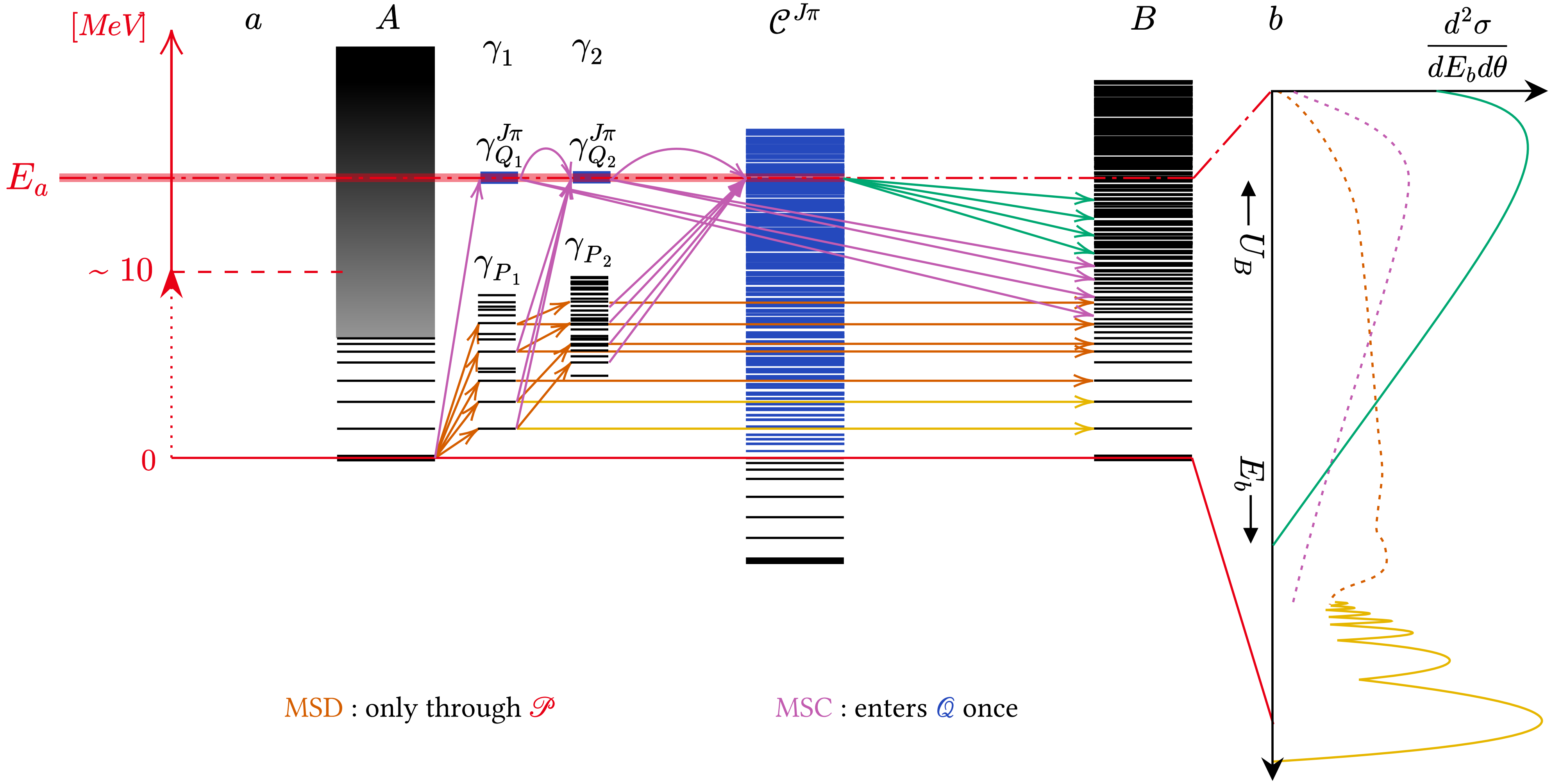

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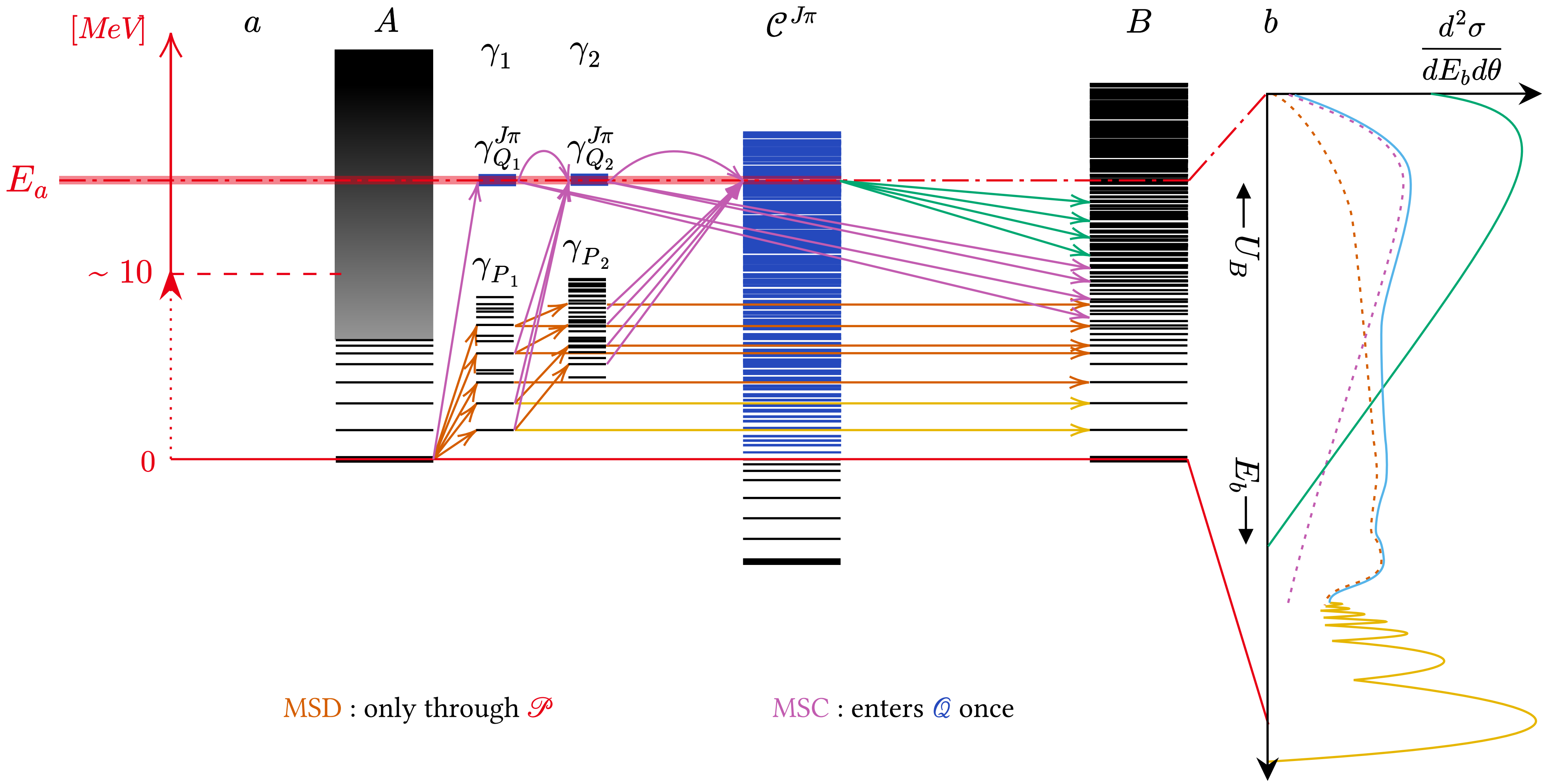

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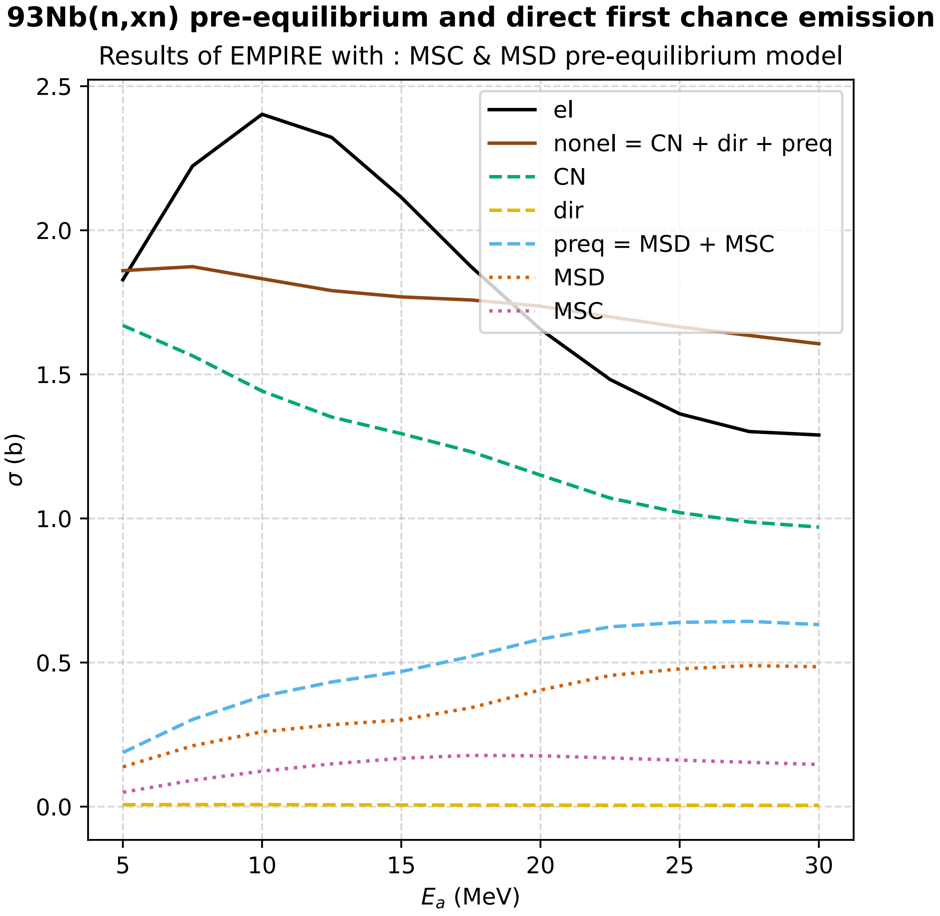
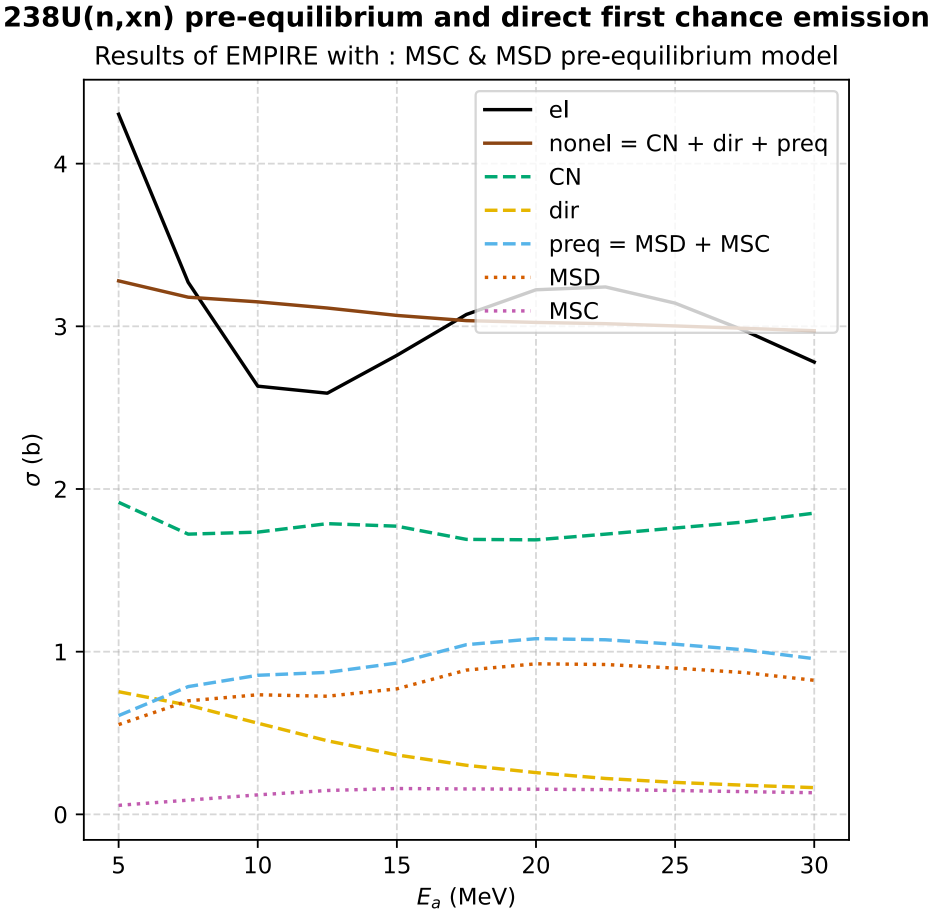
MSD & MSC contribution





"Heidelberg Group" Theory of Pre-Equilibrium

- CN (VWZ) [2]

- MSC (NVWY) [4]
  - weak-coupling limit (equivalent to AWM [5])
    ↪ numerical implementation [6] in the weak-coupling limit
  - and the more general **strong-coupling** limit ⟶ systematically emphasized as more physical
    ↪ **no implementation** (requires enhanced level densities)

- MSD (NWY) [7]
  - assuming $\mathcal{V}_{\mathrm{opt}}$ is 2-body type → **same description of $N-1$ nucleon system as N** → common inputs with NVWY [8]

$$\left\langle \sigma_{\alpha\beta} \right\rangle^{\Delta E} \propto \underbrace{\left| \left\langle S_{\alpha\beta} \right\rangle^{\Delta E} - \delta_{\alpha\beta} \right|^2}_{\sigma_{\alpha\beta}^{\mathrm{MSD}}} + \sum_{J,\pi} \underbrace{\left\langle \left| S_{\alpha\beta}^{J\pi\,\mathrm{fl}} \right|^2 \right\rangle^{\Delta E}}_{\sigma_{\alpha\beta}^{J\pi\ \mathrm{MSC}}}$$

$$\langle X \rangle^{\Delta E} \to \overline{X}^{N} \quad \& \quad \langle X \rangle^{\Delta U_B} \to \overline{X}^{N-1}$$

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  - J, π conservation $\Rightarrow$ correct angular distribution & residual nucleus population
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✗ MSC

  - 1-SD : (Q)RPA for spherical & deformed nuclei
  - 2-SD : - adding multiple (Q)RPA-phonons raises questions
    - "on-shell"-like approximation for intermediate states

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  - $\overbrace{H_0^{N-1} + \boldsymbol{V_0^{N-1}}}^{H^{N-1}} + h_{\text{opt}}(N) + \mathcal{V}_{\text{opt}}$

    $\hookrightarrow V_0^{N-1}$ VS $\mathcal{V}_{\text{opt}}$: MSD equilibration competition
  - extended (Q)RPA models

Any question ?

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