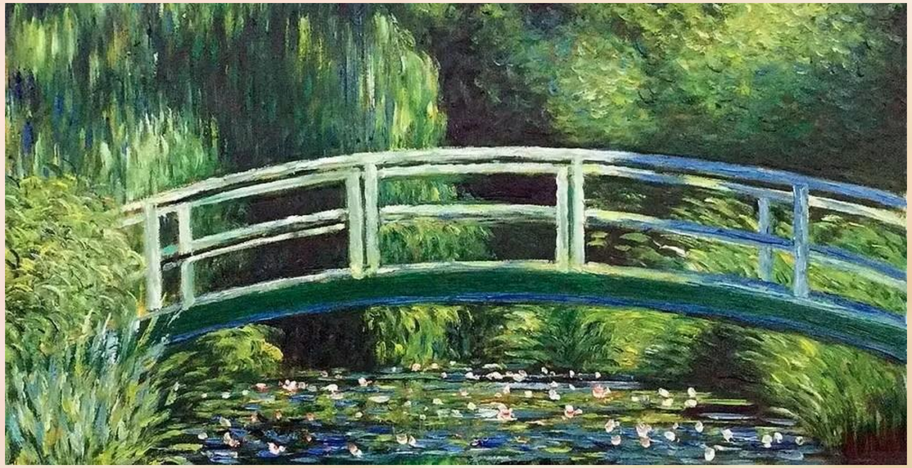
Gauge anomalies on shell and collinear factorization

Edoardo Alviani, IJCLab – Université Paris-Saclay

alviani@ijclab.in2p3.fr

Based on work 2509.03368 in collaboration with Adam Falkowski

A river can be crossed with a bridge or a boat.

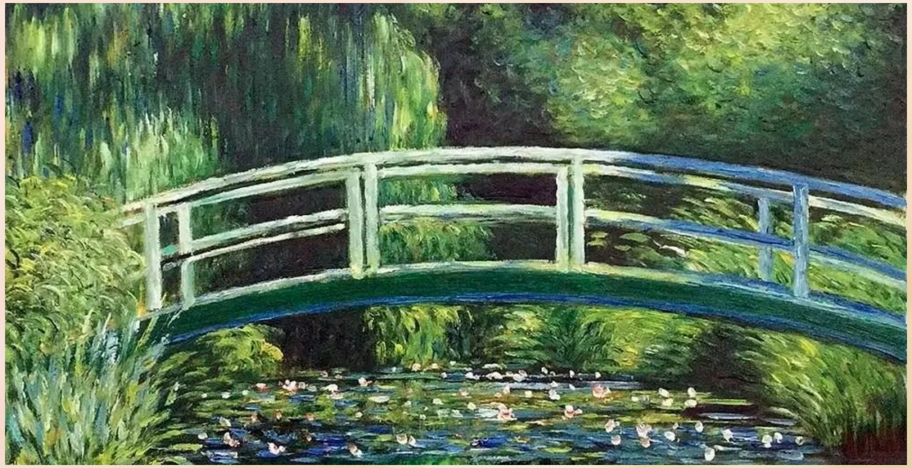


A storm calls for

Resistance to wind fluctuations

Resistance to water waves fluctuations

A river can be crossed with a bridge or a boat.



A storm calls for

Resistance to wind fluctuations

Resistance to water waves fluctuations

In the same way any theory has to be **resistant to quantum fluctuations** regardless of the formalism we use to navigate it.

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

Why???

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

[*Fermion masses and mixing angles from gauge symmetries,* Ibanez, Ross]

[*Yukawa matrices from a spontaneously broken abelian symmetry,* Dudas, Pokorski, Savoy]

Why???

- Gauge anomalies must cancel when gaugeing flavour symmetries
- On-shell understanding of the Green-Schwarz mechanism in anomalous-$U(1)$ flavour models

1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

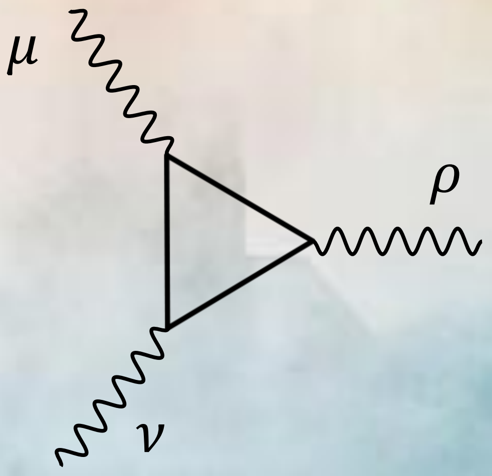
Necessary: prevents the existence of negative norm states

1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states



Anomaly: breaking of **a** symmetry upon computing loop correlators.

Not problematic in principle.

1. Introducing the question: quantum fields

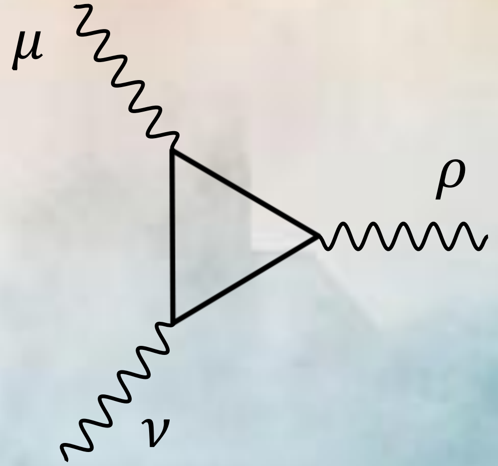
Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states

At one-loop, gauge symmetry (as Ward identities) is lost

∂_μ



$\neq 0$

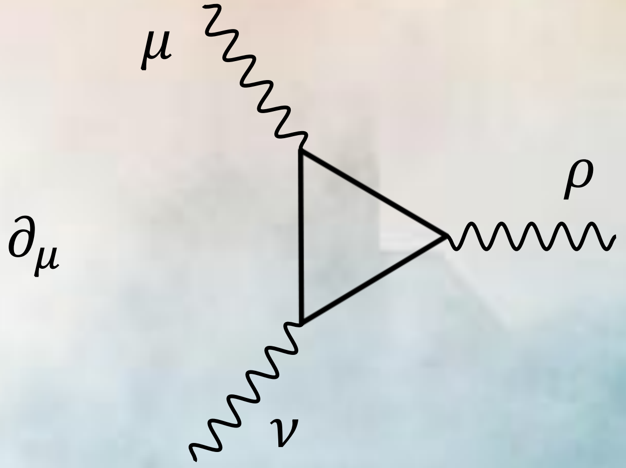
1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states

At one-loop, gauge symmetry (as Ward identities) is lost unless **gauge anomaly cancellation conditions** are satisfied

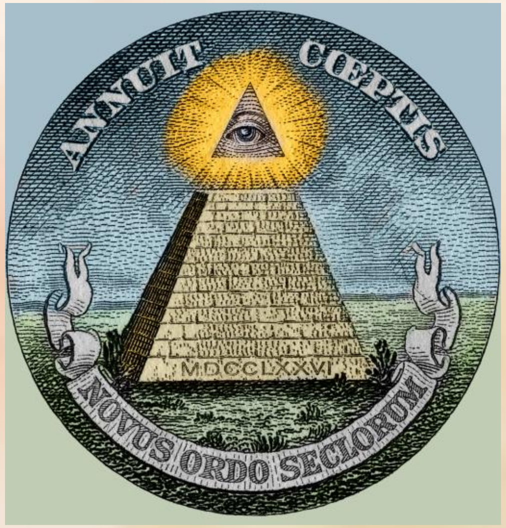


$\neq 0$

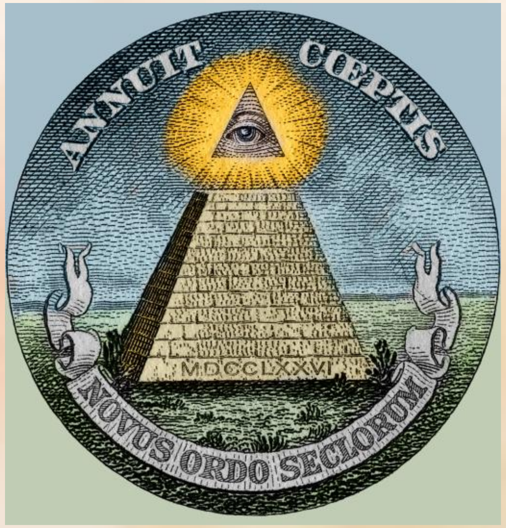
$$\sum_n Q_n^3 = 0$$

Physical constraints on the spectrum of the theory

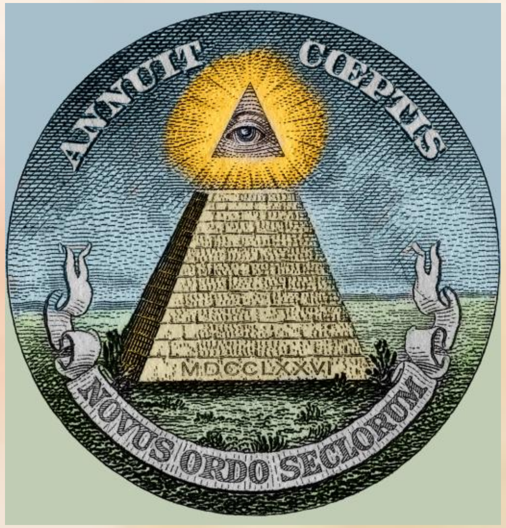
1. Excursus: Amplitudes



1. Excursus: Amplitudes



1. Excursus: Amplitudes



JOIN THE AMPLITUDE CULT!

1. Excursus: S-matrix

JOIN THE AMPLITUDE CULT

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i\,H_0 t_f}\, e^{-i\,H(t_f - t_i)} e^{-i\,H_0 t_i} \quad = 1 + i\,T$$

1. Excursus: S-matrix

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i H_0 t_f} e^{-i H(t_f - t_i)} e^{-i H_0 t_i} \quad = 1 + i\,T$$

Amplitudes are its matrix elements: $< f|T\,|i> = (2\pi)^4 \delta^{(4)}\left(\sum p_f - \sum p_i\right) M(i \to f)$

1. Excursus: S-matrix

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i H_0 t_f} e^{-i H (t_f - t_i)} e^{-i H_0 t_i} \quad = 1 + i\,T$$

Amplitudes are its matrix elements: $< f|T\,|i> = (2\pi)^4 \delta^{(4)}\left(\sum p_f - \sum p_i\right) M(i \to f)$

Poincaré invariance: M depends only on s_{ij} (and eventual polarizations)

1. Excursus: S-matrix

JOIN THE AMPLITUDE CULT

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i H_0 t_f} \, e^{-i H (t_f - t_i)} e^{-i H_0 t_i} \quad = 1 + i\, T$$

Amplitudes are its matrix elements: $< f|T\, |i> = (2\pi)^4 \delta^{(4)}\left(\sum p_f - \sum p_i\right) M(i \to f)$

Poincaré invariance: M depends only on s_{ij} (and eventual polarizations)

Locality: $M(s_{ij})$ is analitytic in s_{ij} except from branch cuts and simple poles

1. Excursus: S-matrix

JOIN THE AMPLITUDE CULT

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i H_0 t_f} e^{-i H(t_f - t_i)} e^{-i H_0 t_i} \quad = 1 + i\,T$$

Amplitudes are its matrix elements: $< f|T\,|i> = (2\pi)^4 \delta^{(4)}\left(\sum p_f - \sum p_i\right) M(i \to f)$

Poincaré invariance: M depends only on s_{ij} (and eventual polarizations)

Locality: $M(s_{ij})$ is analitytic in s_{ij} except from branch cuts and simple poles

[*Mizera*]

Crossing symmetry: relates

1. Excursus: S-matrix

JOIN THE AMPLITUDE CULT

S-matrix: unitary operator relating states at past and future infinity:

$$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} e^{i H_0 t_f} e^{-i H(t_f - t_i)} e^{-i H_0 t_i} \quad = 1 + i\, T$$

Amplitudes are its matrix elements: $< f|T\,|i> = (2\pi)^4 \delta^{(4)}\left(\sum p_f - \sum p_i\right) M(i \to f)$

Poincaré invariance: M depends only on s_{ij} (and eventual polarizations)

Locality: $M(s_{ij})$ is analitytic in s_{ij} except from branch cuts and simple poles

Crossing symmetry: relates

[*Mizera*]

Little group symmetry: can we exploit it manifestly?

1. Excursus: On shell methods

$$p_\mu \sigma^\mu = \begin{pmatrix} p_0 - p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 + p_3 \end{pmatrix}$$

$$\det(p_\mu \sigma^\mu) = p^2 = 0$$

$$p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = |p>_\alpha [p|_{\dot{\alpha}}$$

with $[p|_{\dot{\alpha}} = (|p>_\alpha)^*$
, $\alpha = 1{,}2$

JOIN THE AMPLITUDE CULT!

1. Excursus: On shell methods

$$p_\mu \sigma^\mu = \begin{pmatrix} p_0 - p_3 & p_1 - i p_2 \\ p_1 + i p_2 & p_0 + p_3 \end{pmatrix}$$

$$\det(p_\mu \sigma^\mu) = p^2 = 0$$

Make little group covariance manifest, aka packages spin well

$$p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = |p>_\alpha [p|_{\dot{\alpha}}$$

with $[p|_{\dot{\alpha}} = (|p>_\alpha)^*$,
$\alpha = 1,2$

JOIN THE AMPLITUDE CULT!

$$|p> \to z|p>$$
$$[p| \to z^{-1}[p|$$

1. Excursus: On shell methods

$$p_\mu \sigma^\mu = \begin{pmatrix} p_0 - p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 + p_3 \end{pmatrix}$$

$$\det(p_\mu \sigma^\mu) = p^2 = 0$$

Make little group covariance manifest, aka packages spin well

$$p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = |p>_\alpha [p|_{\dot{\alpha}}$$

with $[p|_{\dot{\alpha}} = (|p>_\alpha)^*$,
$\alpha = 1,2$

$|p> \to z|p>$
$[p| \to z^{-1}[p|$

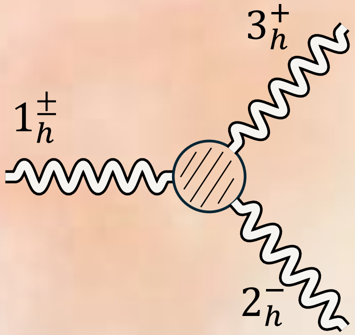
JOIN THE AMPLITUDE CULT!

Scattering amplitudes are **naturally expressed** in terms of these variables.

$$M(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} g_h < 12 >^{h_3-h_1-h_2} < 23 >^{h_1-h_2-h_3} < 13 >^{h_2-h_1-h_3}, & h_1 + h_2 + h_3 < 0 \\ g_a [12]^{h_1+h_2-h_3} [23]^{h_2+h_3-h_1} [13]^{h_1+h_3-h_2}, & h_1 + h_2 + h_3 \geq 0 \end{cases}$$

1. Excursus: On shell methods

$$M(1_h^- 2_h^- 3_h^+) = \frac{1}{M_{Pl}} \frac{< 12 >^6}{< 13 >^2 < 23 >^2}$$

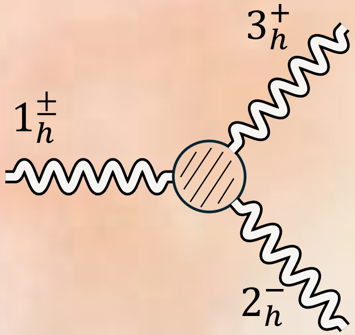


$$G_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2,p_3)} = \left(\frac{i}{8}\right) Sym_{\mu_1\leftrightarrow\nu_1} Sym_{\mu_2\leftrightarrow\nu_2} Sym_{\mu_3\leftrightarrow\nu_3} \left[g_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2)} + 5\ perms\ of\ (1,2,3) \right]$$

$$g_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2)} = \left(\frac{\kappa}{4}\right)\left[\left(\frac{1}{2}\right) p_1^{\mu_3} p_2^{\nu_3} \eta^{\mu_1\mu_2}\eta^{\nu_1\nu_2} - p_1^{\mu_3} p_2^{\mu_1} \eta^{\nu_1\mu_2}\eta^{\nu_2\nu_3} + (p_1\cdot p_2)\left(-\left(\frac{1}{2}\right)\eta^{\mu_1\nu_1}\eta^{\mu_2\mu_3}\eta^{\nu_2\nu_3} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1} - \left(\frac{1}{4}\right)\eta^{\mu_1\mu_2}\eta^{\nu_1\nu_2}\eta^{\mu_3\nu_3} + \left(\frac{1}{8}\right)\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}\right)\right]$$

1. Excursus: On shell methods

$$M(1_h^- 2_h^- 3_h^+) = \frac{1}{M_{Pl}} \frac{< 12 >^6}{< 13 >^2 < 23 >^2}$$



$$G_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2,p_3)} = \left(\frac{i}{8}\right) Sym_{\mu_1\leftrightarrow\nu_1} Sym_{\mu_2\leftrightarrow\nu_2} Sym_{\mu_3\leftrightarrow\nu_3} \left[g_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2)} + 5\, perms\ of\ (1,2,3)\right]$$

$$g_3^{(\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3)(p_1,p_2)} = \left(\frac{\kappa}{4}\right)\left[\left(\frac{1}{2}\right) p_1^{\mu_3} p_2^{\nu_3} \eta^{\mu_1\mu_2}\eta^{\nu_1\nu_2} - p_1^{\mu_3} p_2^{\mu_1} \eta^{\nu_1\mu_2}\eta^{\nu_2\nu_3} + (p_1\cdot p_2)\left(-\left(\frac{1}{2}\right)\eta^{\mu_1\nu_1}\eta^{\mu_2\mu_3}\eta^{\nu_2\nu_3} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1} - \left(\frac{1}{4}\right)\eta^{\mu_1\mu_2}\eta^{\nu_1\nu_2}\eta^{\mu_3\nu_3} + \left(\frac{1}{8}\right)\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}\right)\right]$$

Great for:
- finding bases of higher dimensional operators
- deriving higher point tree amplitudes from factorization and recursion
- simplifying loop matching and operator mixing via unitarity
- making chirality and helicity selection rules manifest

1. Excursus: On shell methods

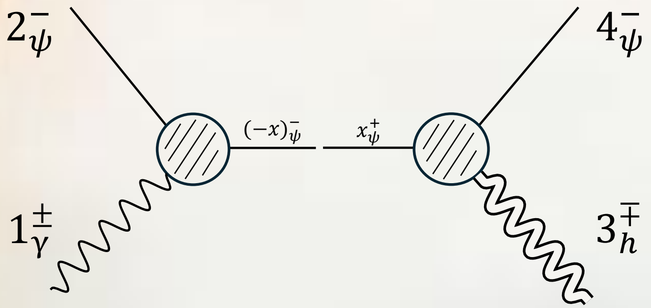
$$M_4 = \frac{R_s}{s} + \frac{R_t}{t} + \frac{R_u}{u} + \frac{R_{st}}{st} + \frac{R_{su}}{su} + \frac{R_{tu}}{tu} + \frac{R_{stu}}{stu} + C$$

JOIN THE AMPLITUDE CULT!

1. Excursus: On shell methods

$$M_4 = \frac{R_s}{s} + \frac{R_t}{t} + \frac{R_u}{u} + \frac{R_{st}}{st} + \frac{R_{su}}{su} + \frac{R_{tu}}{tu} + \frac{R_{stu}}{stu} + C$$

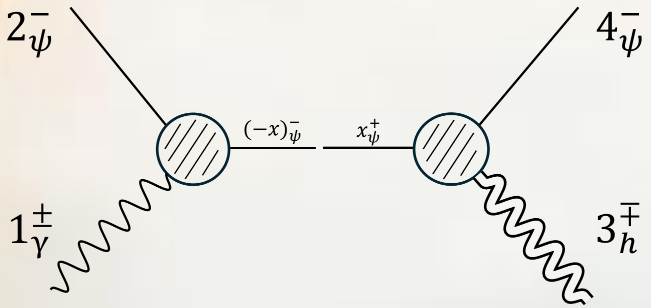
$$Res[M_4, s=0] = \lim_{s\to 0} M_4\, s = R_s + \frac{R_{st}}{t} + \frac{R_{su}}{u} + \frac{R_{stu}}{tu} =$$



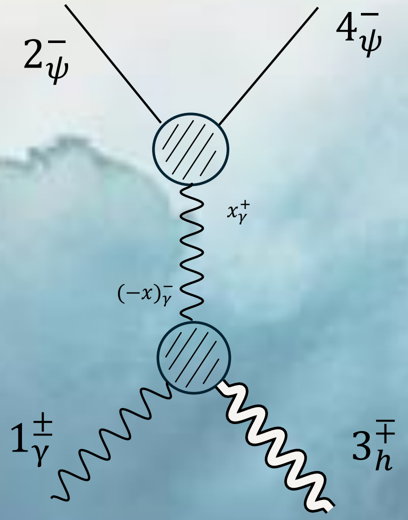
1. Excursus: On shell methods

$$M_4 = \frac{R_s}{s} + \frac{R_t}{t} + \frac{R_u}{u} + \frac{R_{st}}{st} + \frac{R_{su}}{su} + \frac{R_{tu}}{tu} + \frac{R_{stu}}{stu} + C$$

$$Res[M_4, s=0] = \lim_{s\to 0} M_4\, s = R_s + \frac{R_{st}}{t} + \frac{R_{su}}{u} + \frac{R_{stu}}{tu} =$$



$$Res[M_4, t=0] = \lim_{t\to 0} M_4\, t = R_t + \frac{R_{st}}{s} + \frac{R_{tu}}{u} + \frac{R_{stu}}{su} =$$

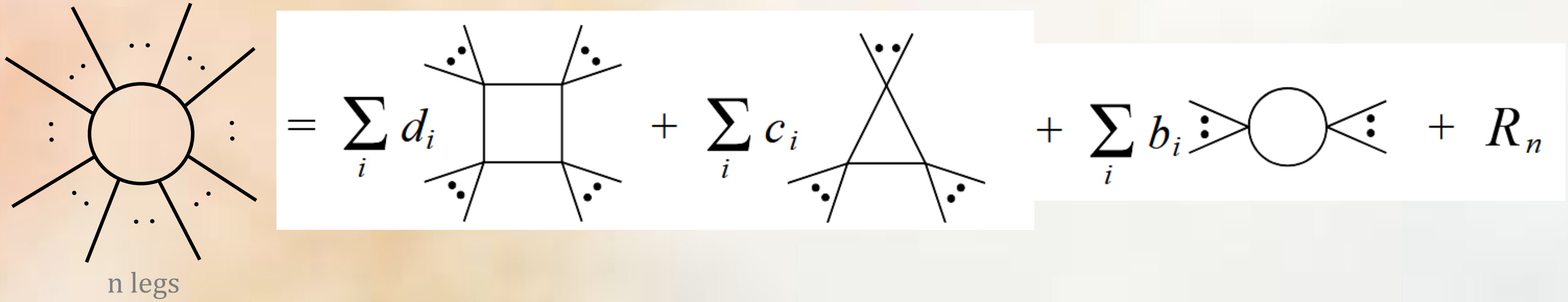


1. Excursus: Unitarity cuts

JOIN THE AMPLITUDE CULT

We can reconstruct loop amplitudes just by recycling tree-level informations

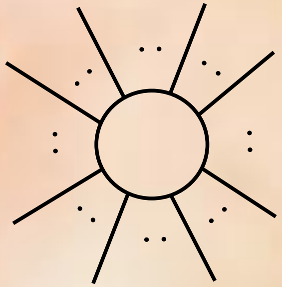
$A_n^{1-loop} =$

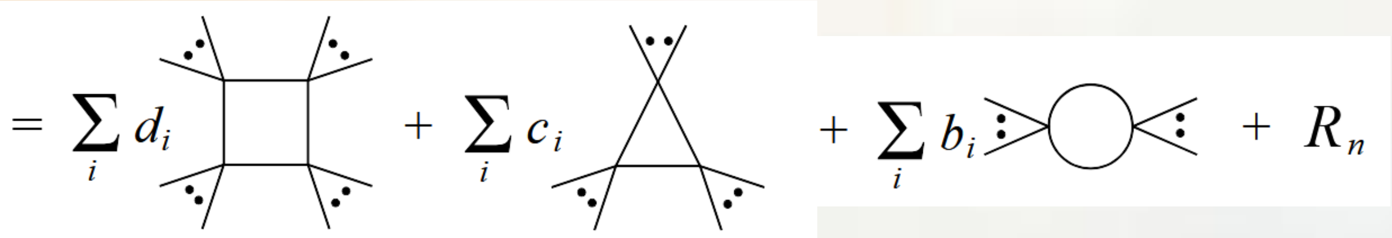


1. Excursus: Unitarity cuts

We can reconstruct loop amplitudes just by recycling tree-level informations

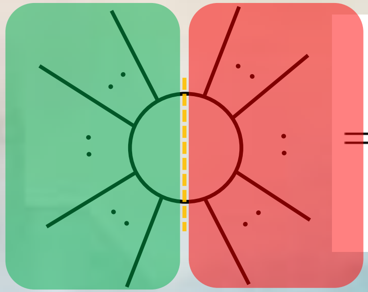
$A_n^{1-loop} =$

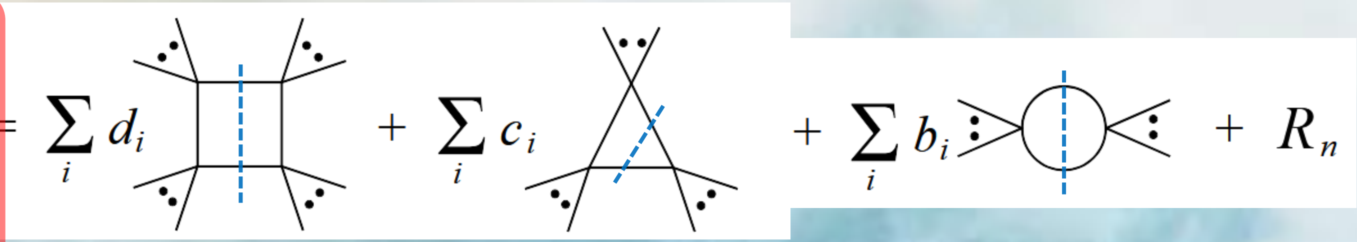




Unitarity cut: replace two propagators with delta functions putting them on-shell

$Disc_{s_{ij}}[A_n^{1-loop}] =$



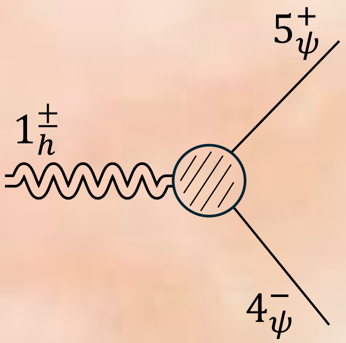


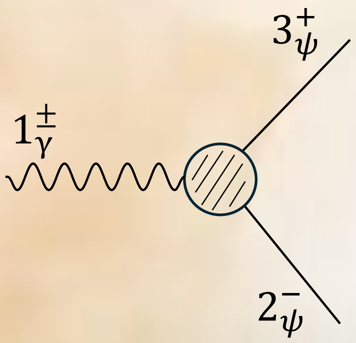
This is computed in terms of the tree-level amplitudes on the left and the right of the cut

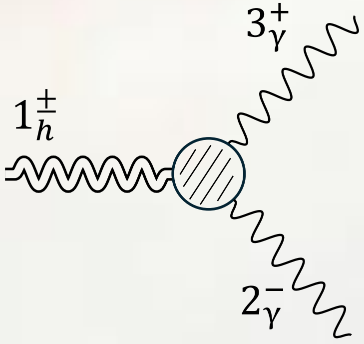
Each cut gives you some coefficients

1. Introducing the question: on-shell methods

We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.

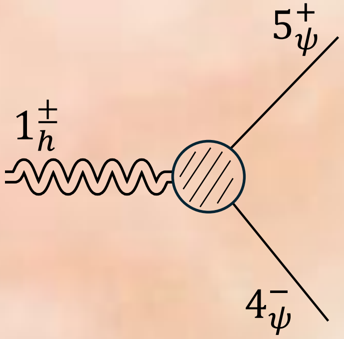


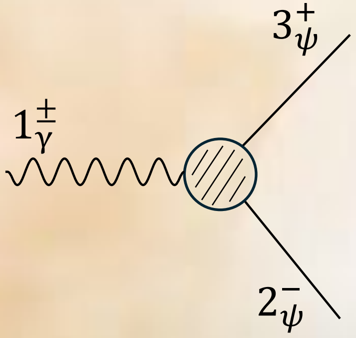


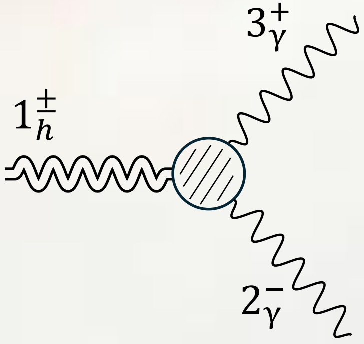


1. Introducing the question: on-shell methods

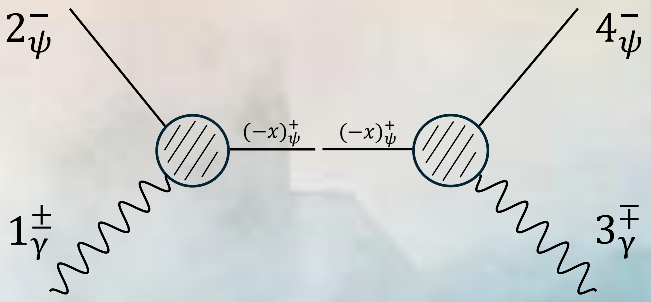
We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.

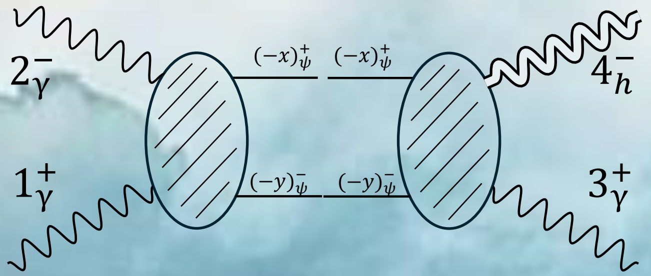






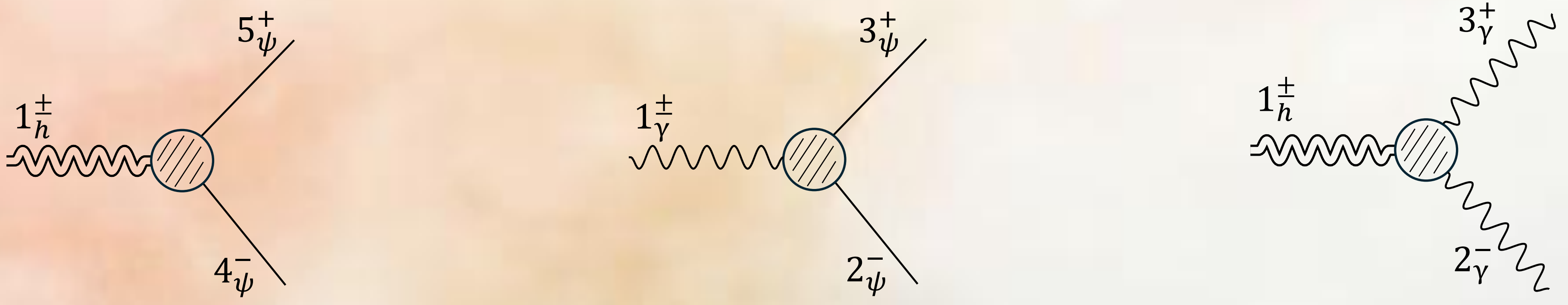
Higher-point and loop-amplitudes can be constructed from a set of three-point ones by unitarity





1. Introducing the question: on-shell methods

We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.



Higher-point and loop-amplitudes can be constructed from a set of three-point ones by unitarity

Gauge invariance is bypassed from the get-go

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum.

On-shell formulation of QFT bypasses gauge invariance.

Where are such constraints from in the on-shell formulation?

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum.

On-shell formulation of QFT bypasses gauge invariance.

Where are such constraints from in the on-shell formulation?

Earlier work [Y. Huang, D. McGady] suggests that they are hidden in a tension between unitarity and locality

The question was also discussed from other groups

[Q. Bonnefoy, S. De Angelis, E. Gendy, C. Grojean, J. Roosmale Nepveu]

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum

On-shell formulation of QFT bypasses gauge invariance

Where are such constraints from in the on-shell formulation?

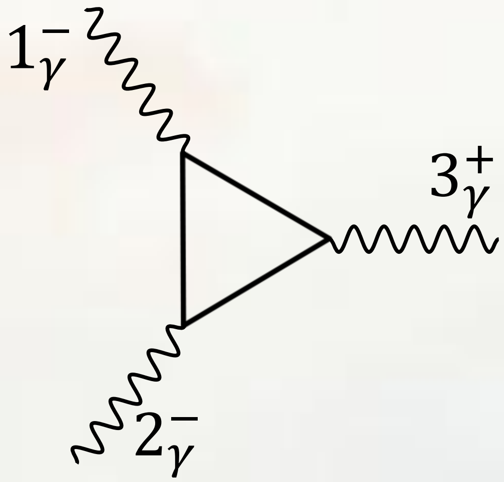
Earlier work [Y. Huang, D. McGady] suggests that they are hidden in a tension between unitarity and locality

The question was also discussed from other groups [Q. Bonnefoy, S. De Angelis, E. Gendy, C. Grojean, J. Roosmale Nepveu]

Answer: **anomalies lead to a breakdown of collinear factorization**

2. Gravity as a probe for anomalies

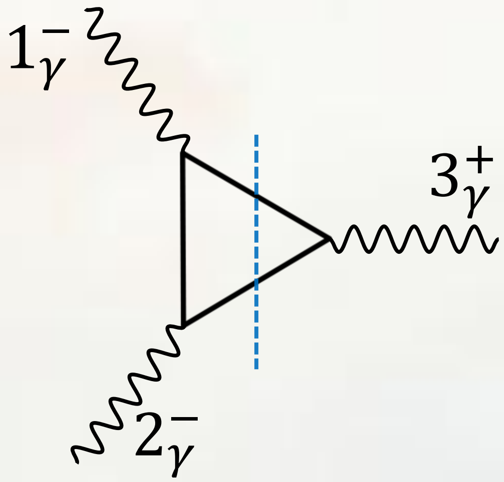
Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.



$\sim \sum_n Q_n^3$

2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.

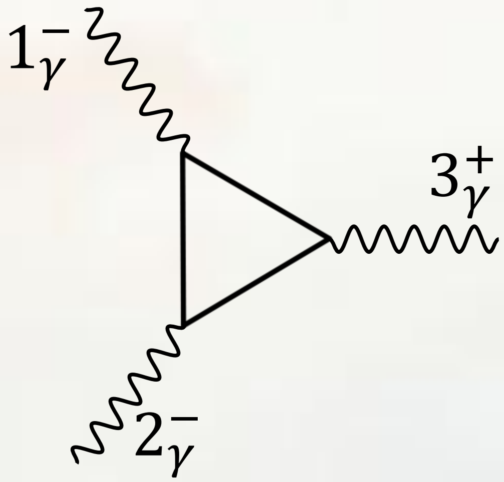


$$\sim \sum_n Q_n^3$$

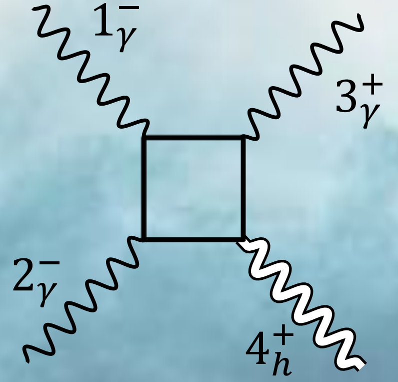
2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.

Certain four-point amplitudes probe the anomaly, thanks to gravity. These are computable on-shell, up to rational terms.



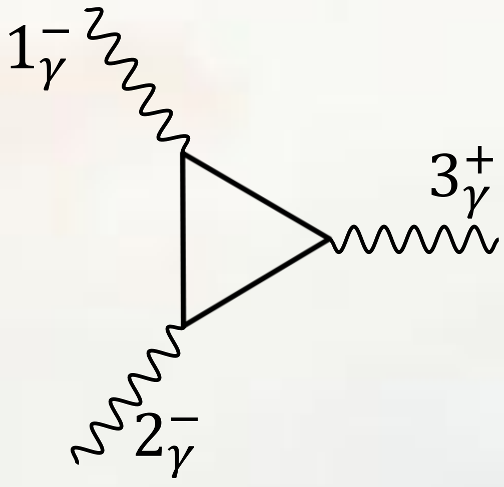
$$\sim \sum_n Q_n^3$$



$$\sim \sum_n Q_n^3$$

2. Gravity as a probe for anomalies

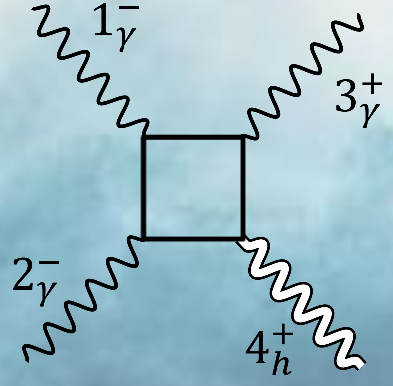
Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.



$$\sim \sum_n Q_n^3$$

Certain four-point amplitudes probe the anomaly, thanks to gravity. These are computable on-shell, up to rational terms.

$$\begin{aligned}
\mathcal{M}^{(1)}[1_\gamma^- 2_\gamma^- 3_\gamma^+ 4_h^+] &= M_{3\gamma h} + R_{3\gamma h}, \\
M_{3\gamma h} &\equiv -\left[\sum_{n=1}^{N} s_n Q_n^3\right] \frac{\sqrt{2} e^3}{16\pi^2 M_{\text{Pl}}} \langle 12\rangle^2 [34]^2 [4|p_1 p_2|4] F_{3\gamma h}(s,t), \\
F_{3\gamma h}(s,t) &\equiv \frac{1}{s^3}\left\{4\log\left(\frac{t}{u}\right) + \frac{t-u}{s}\left[\log\left(\frac{t}{u}\right)^2 + \pi^2\right]\right\}\Big|_{u=-s-t}.
\end{aligned}$$

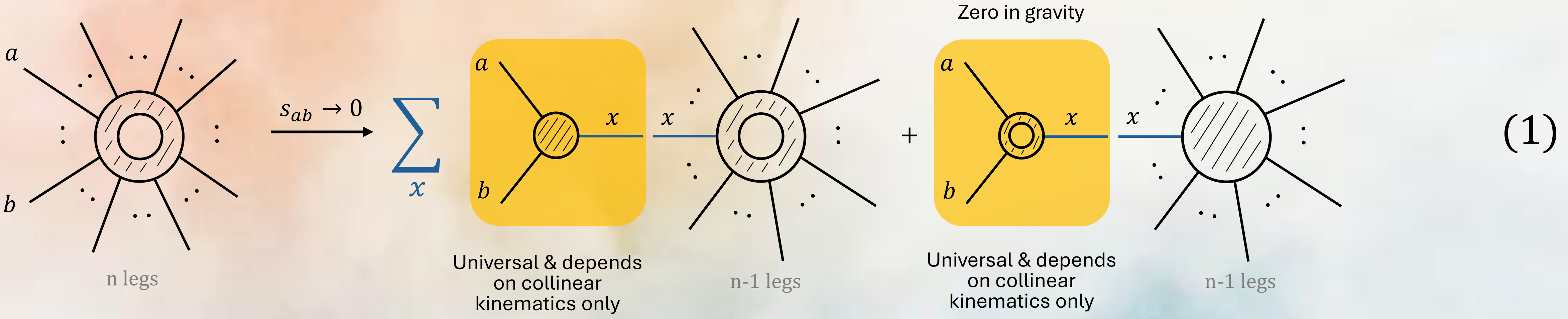


$$\sim \sum_n Q_n^3$$

3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

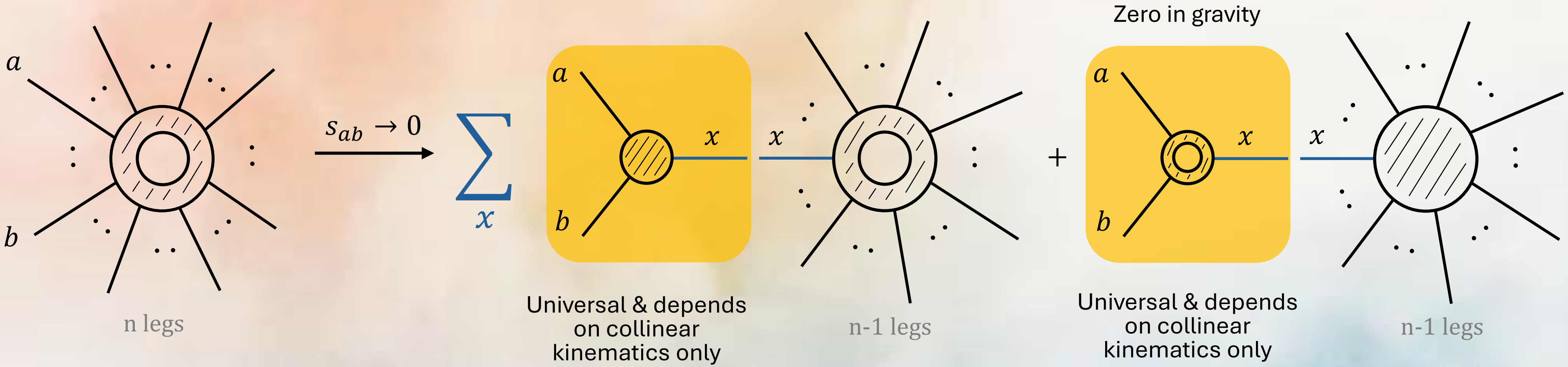
[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]



3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]



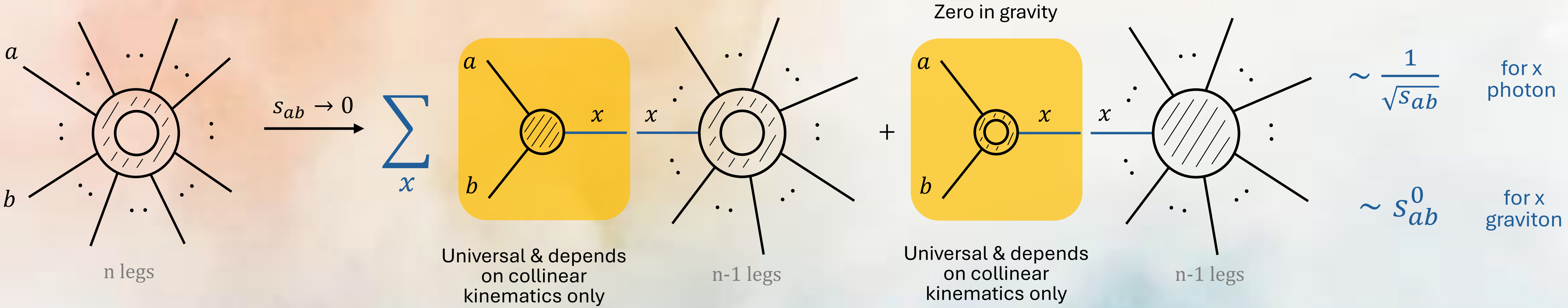
(1)

Crucially, this factorization holds for $n \geq 5$

3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]

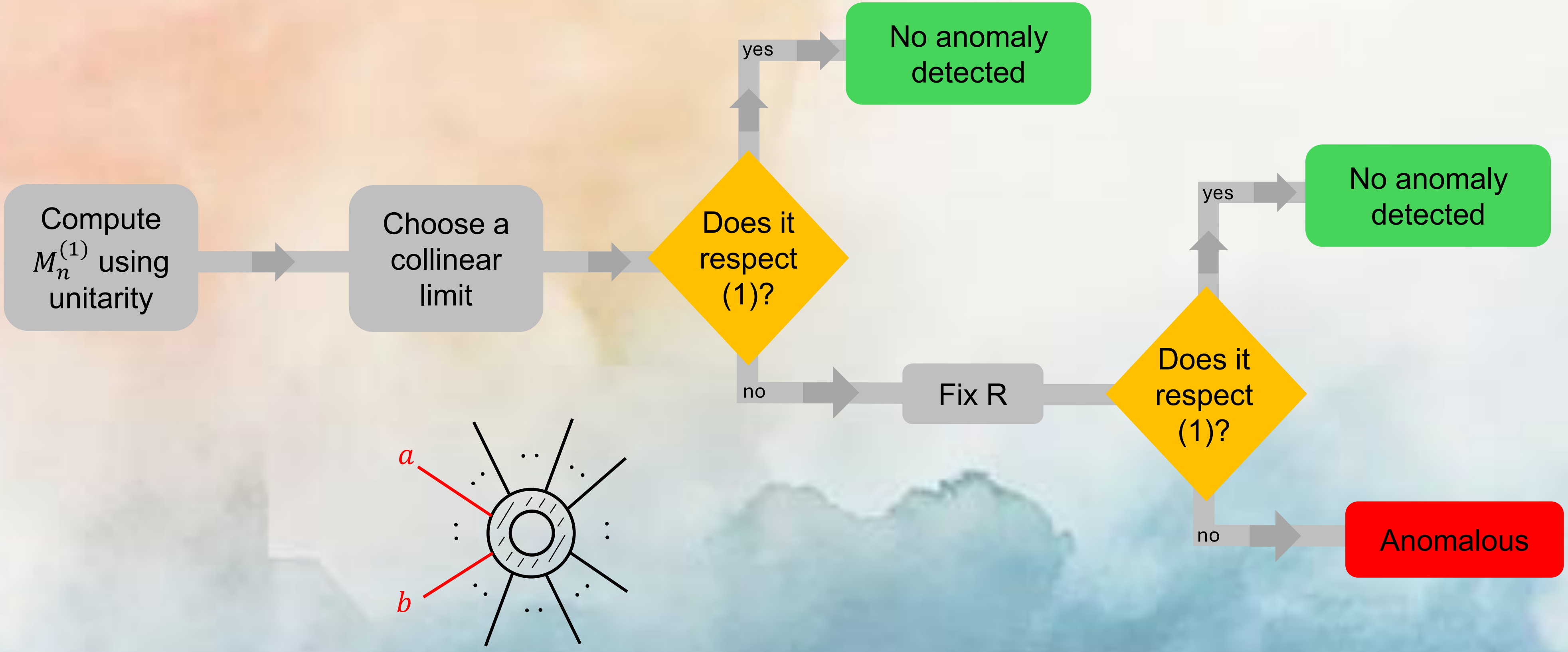


Crucially, this factorization holds for $n \geq 5$

Any deviation from this pattern leads to an inconsistency.

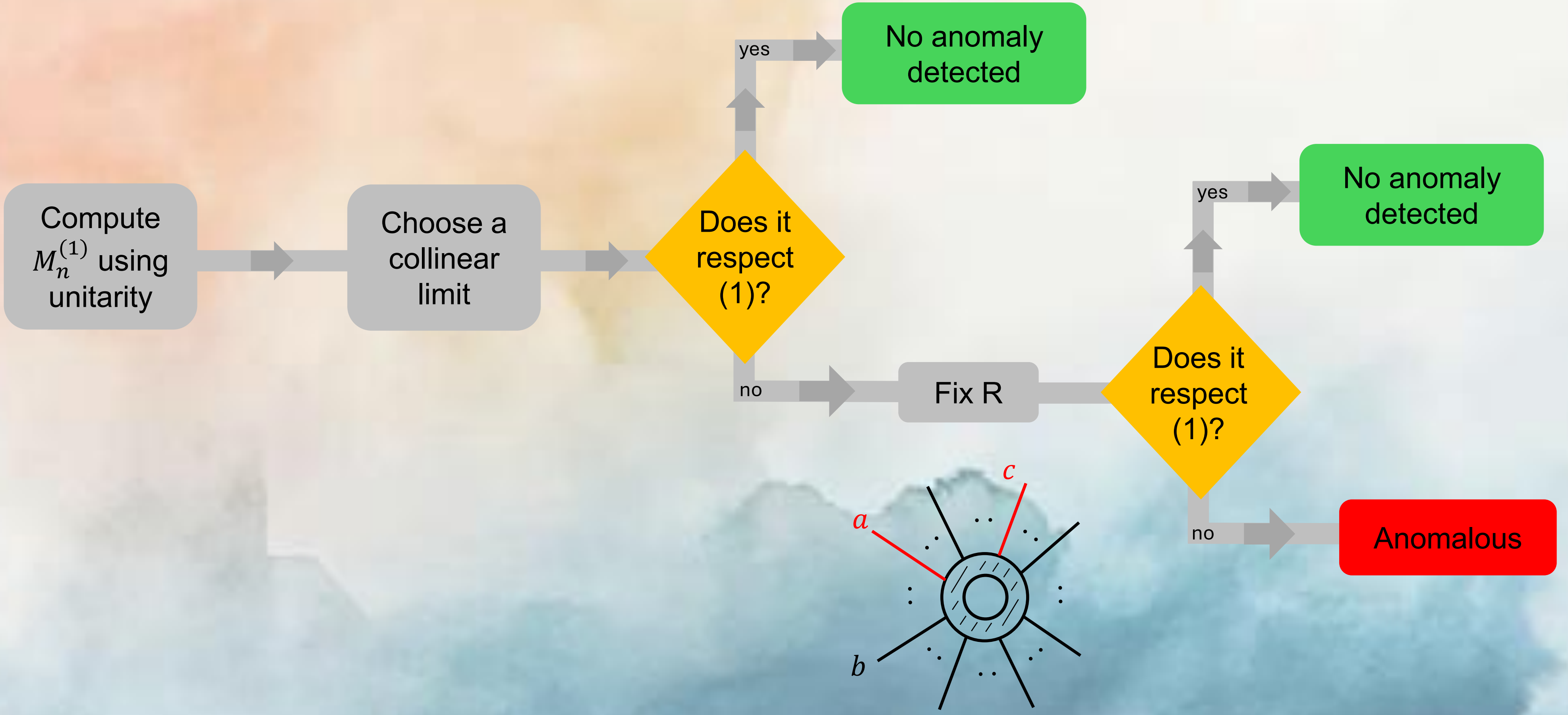
3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell



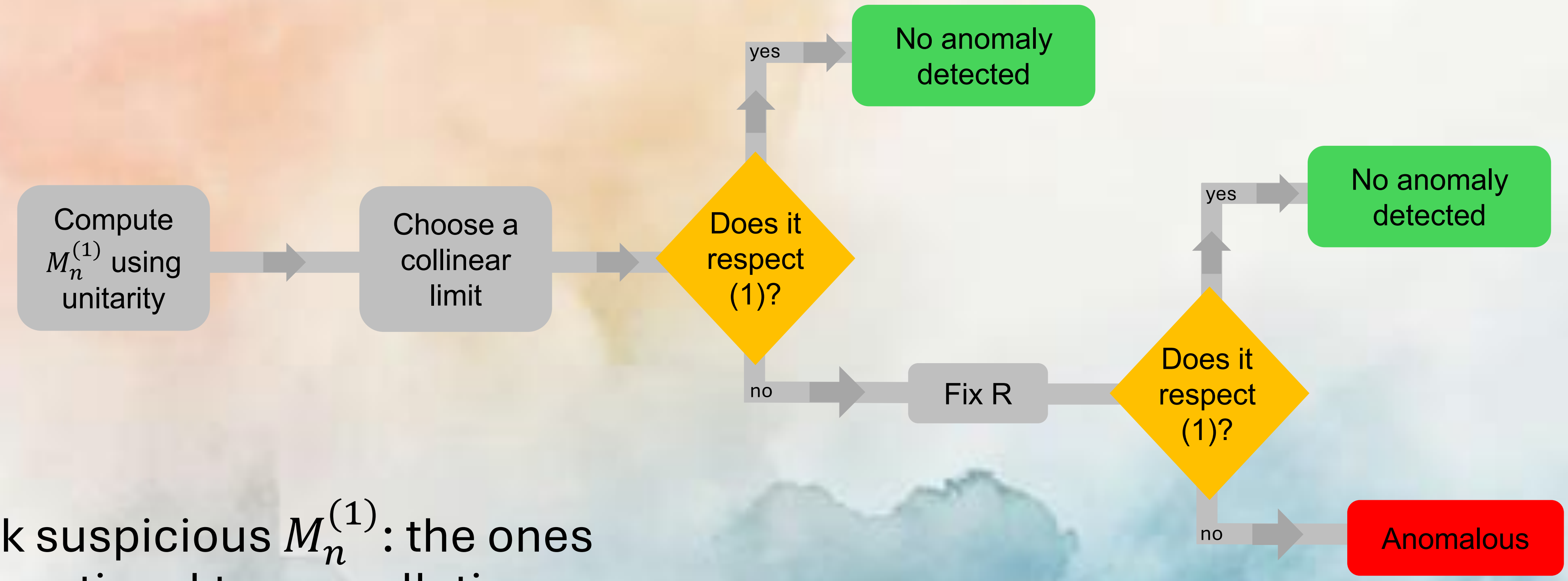
3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell



3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell

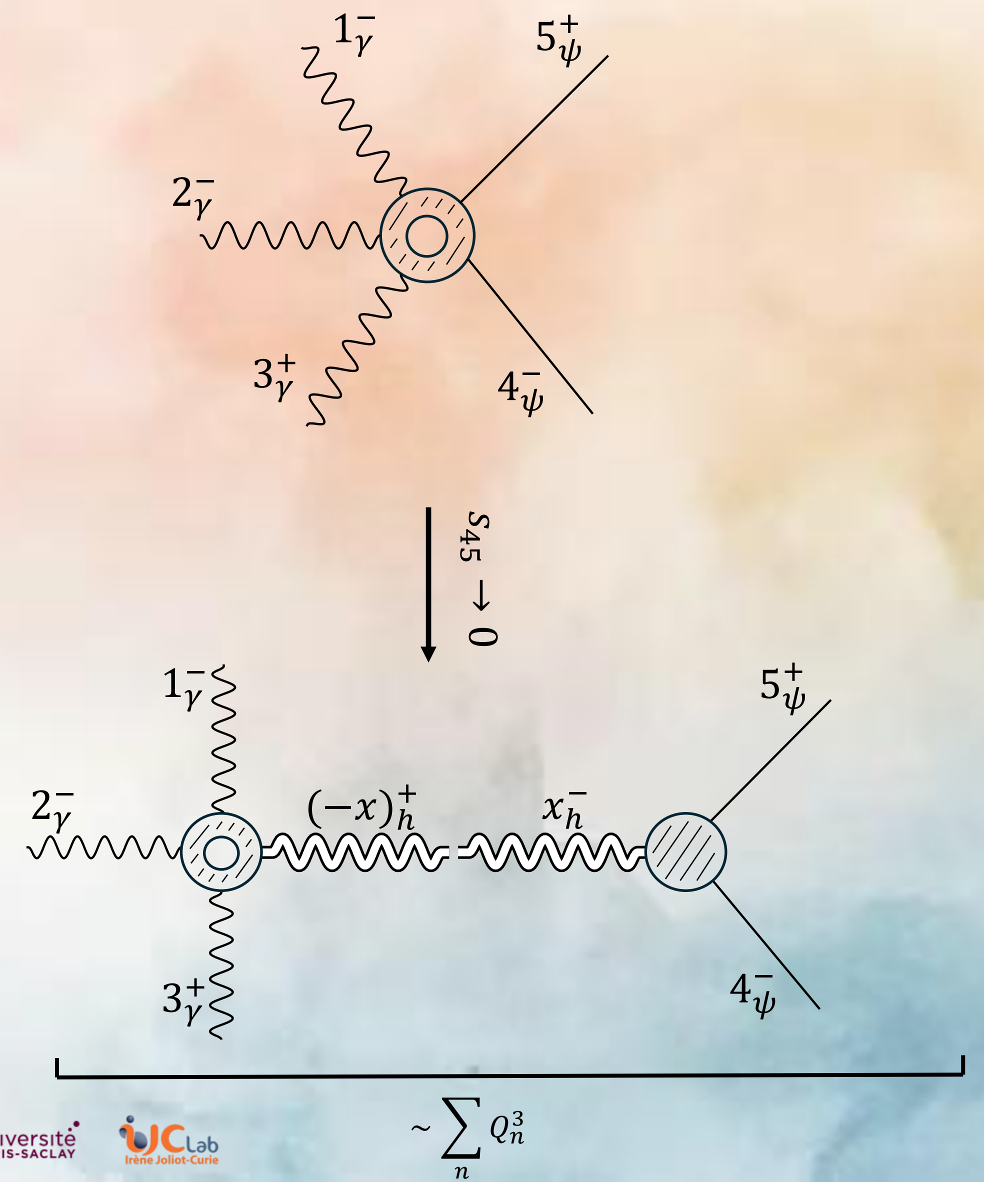


Pick suspicious $M_n^{(1)}$: the ones proportional to cancellation conditions (e.g. $\sum_n Q_n^3$)

3. Attention span saving break

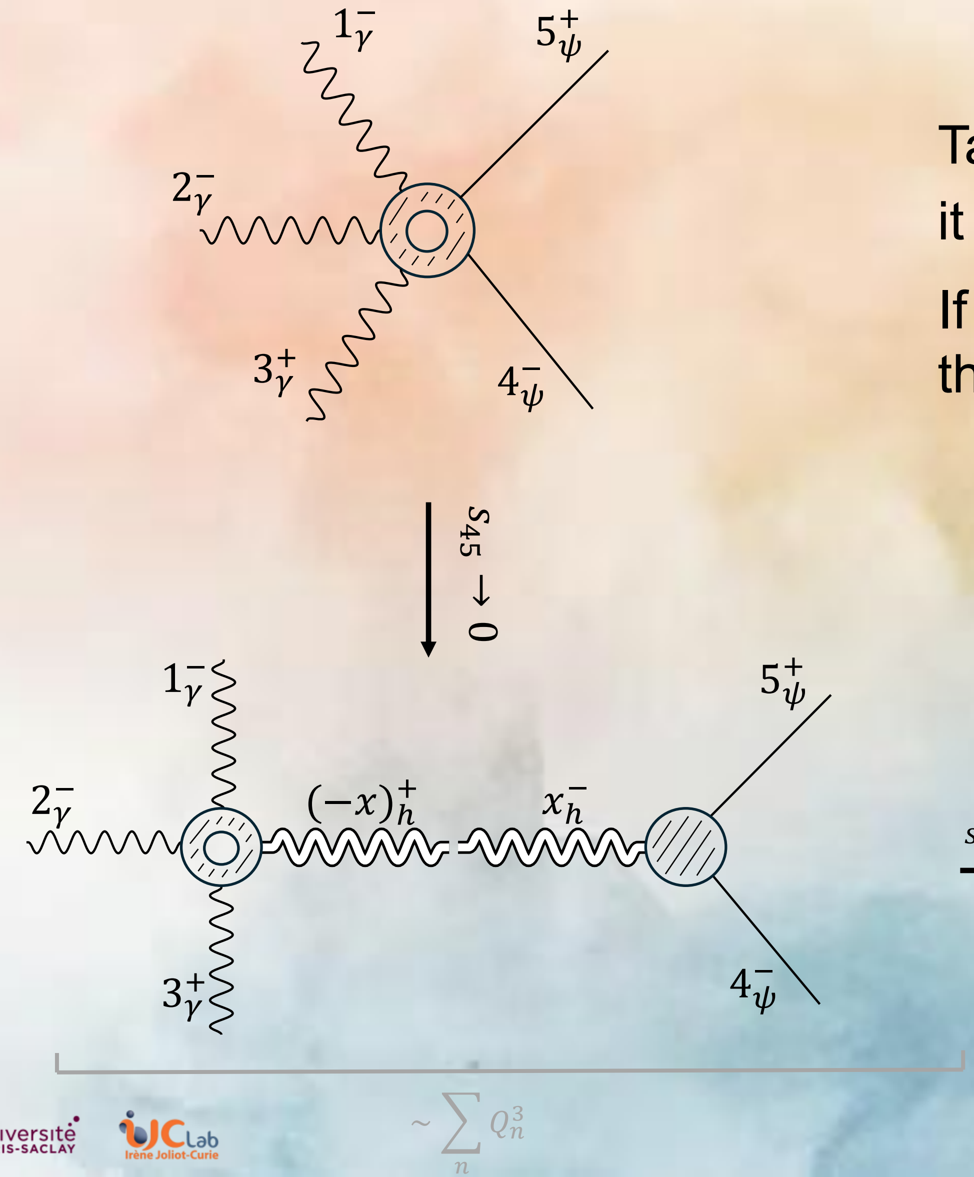
- On shell methods → gauge symmetry is dropped
- Gauge anomaly cancellation conditions → hidden somewhere
- Collinear factorization → broken in some channels
- Restore it with a rational term → breaks it in other channels

4. Gauge anomalies on-shell: $U(1)^3$



Start from a five point amplitude and take a collinear limit to expose the structure of interest.

4. Gauge anomalies on-shell: U(1)^3



Take a second collinear limit to test the pole structure. Does it scale as $\frac{1}{\sqrt{s_{12}}}$ or s_{12}^0?

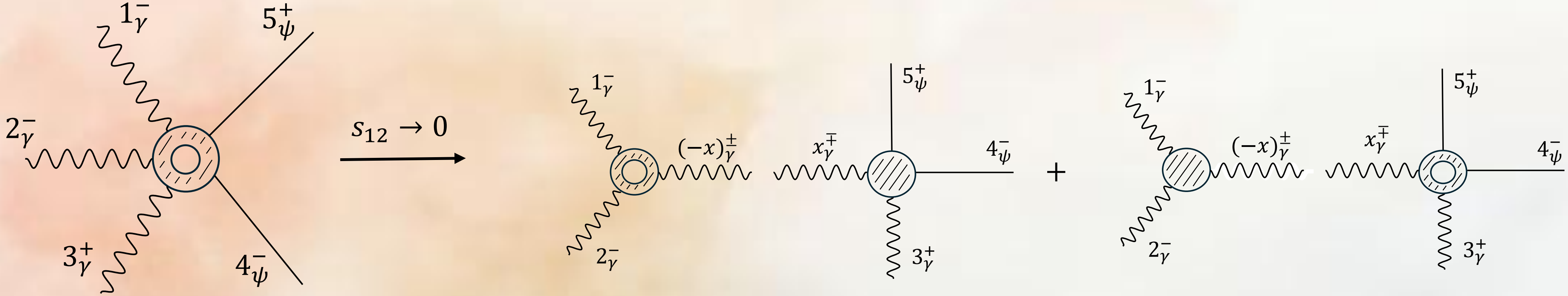
If yes, can we **interpret this as a collinear factorization** of the 5-point amplitude?

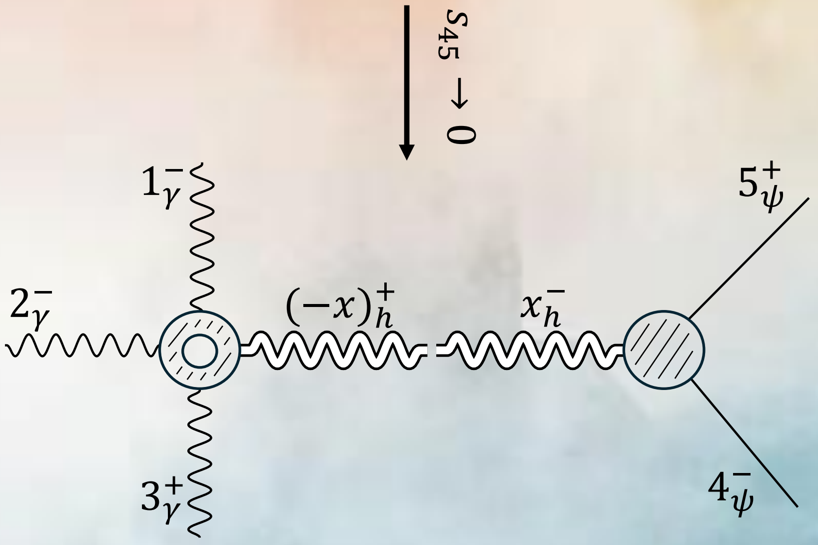
$$\xrightarrow{s_{12} \to 0} \underbrace{A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} \right.}_{\sim \frac{1}{\sqrt{s_{12}}}} \left. + O({s_{12}}^0, {s_{45}}^1) \right) + R$$

$$S_{QED} = \frac{<12>^2 [35]^3 < 3|p_2 \sigma|4] < 45>^2}{s_{12} s_{45}} \sim \sqrt{s_{12}} + O(s_{12})$$

$$A = \frac{-\sqrt{2} e^3}{16 \pi^2 M_{Pl}^2}$$

4. Gauge anomalies on-shell: $U(1)^3$





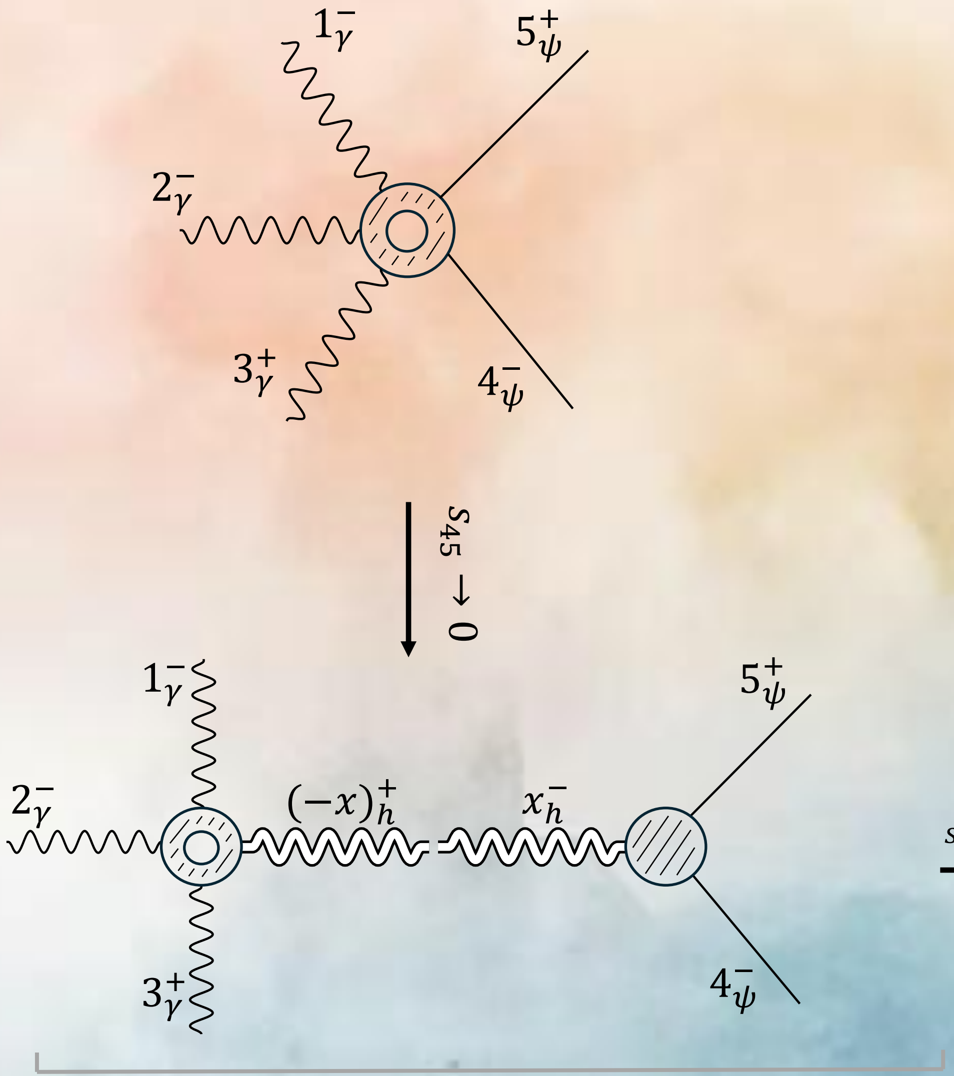
$\sim \frac{1}{\sqrt{s_{12}}}$

$$\xrightarrow{s_{12} \to 0} A\, S_{QED}\left(\frac{\sum_n Q_n^3}{s_{12}s_{13}} + O({s_{12}}^0, {s_{45}}^1)\right) + R$$

$\sim \sum_n Q_n^3$

The **collinear interpretation fails**: it leads to non universal splitting functions.
We can still save the amplitude by **adjusting** R.

4. Gauge anomalies on-shell: $U(1)^3$



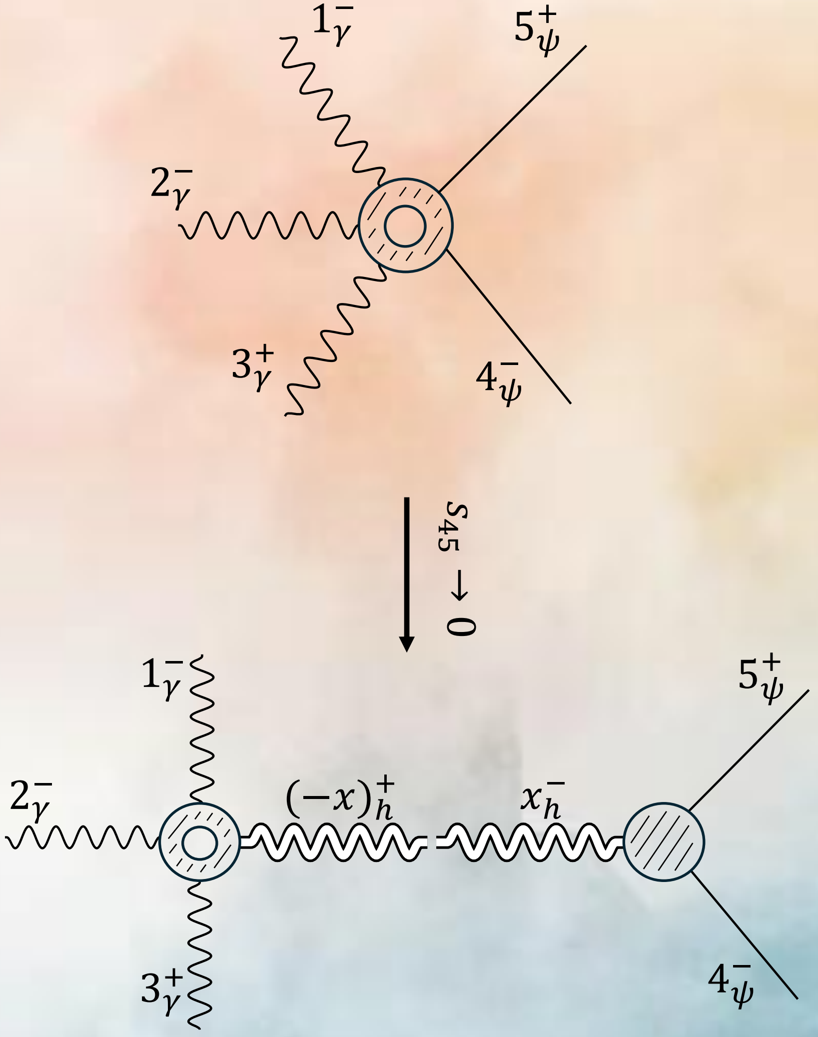
$\sim \sum_n Q_n^3$

R **cancels** the s_{12} singularity

$$R = A \sum_n Q_n^3 \; S_{QED} \frac{1}{s_{12} s_{13}}$$

$$\xrightarrow{s_{12} \to 0} \quad A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \sim O(\sqrt{s_{12}})$$

4. Gauge anomalies on-shell: $U(1)^3$



$\sim \sum_n Q_n^3$

R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

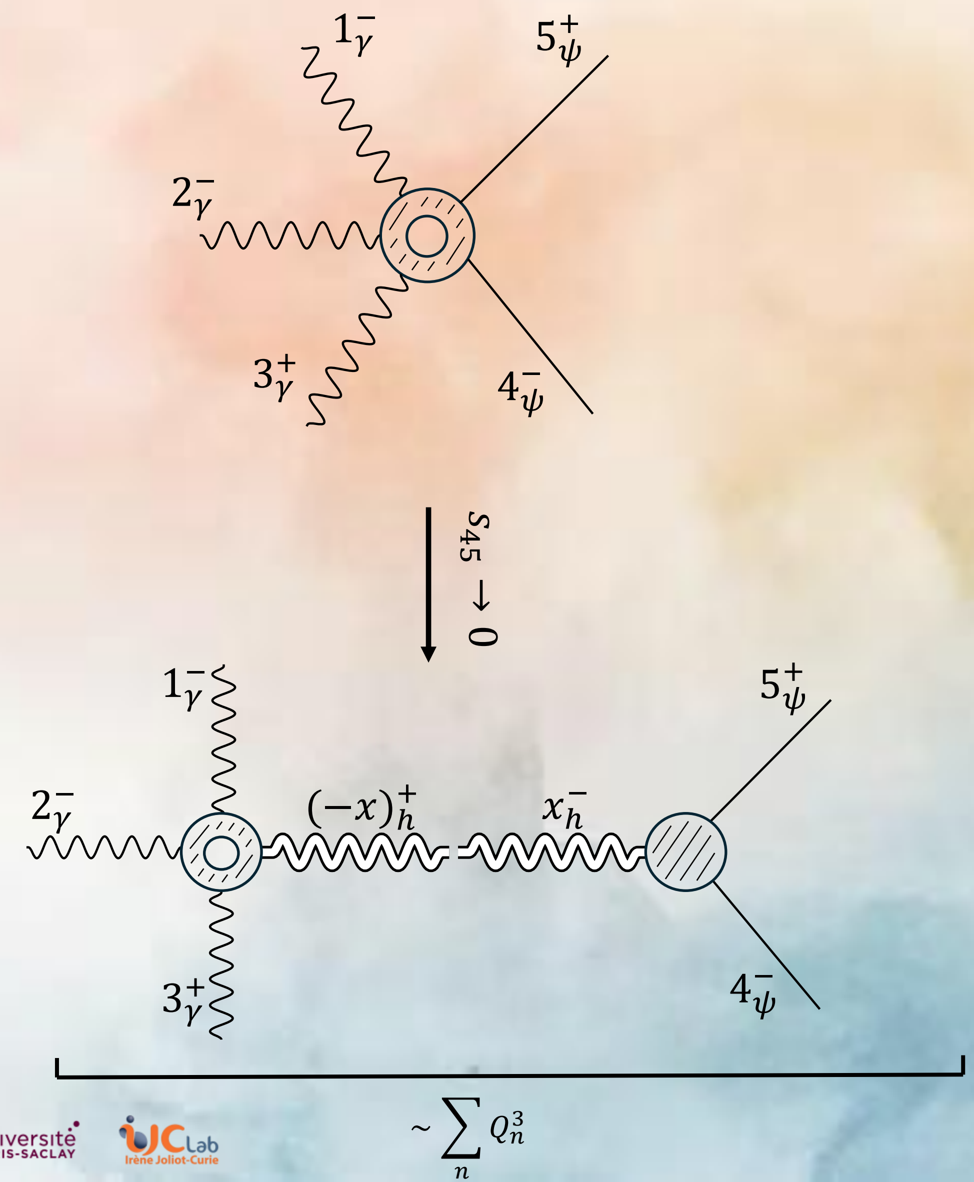
$s_{13} \to 0$: $R = A \sum_n Q_n^3 \, S_{QED} \frac{1}{s_{12} s_{13}} \sim \frac{1}{s_{13}}$

$s_{12} \to 0$: $A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \sim O(\sqrt{s_{12}})$

$$S_{QED} = \frac{<12>^2 [35]^3 <3|p_2\sigma|4] <45>^2}{s_{12} s_{45}} \sim \sqrt{s_{12}} + O(s_{12})$$

$$A = \frac{-\sqrt{2} e^3}{16\pi^2 M_{Pl}^2}$$

4. Gauge anomalies on-shell: $U(1)^3$



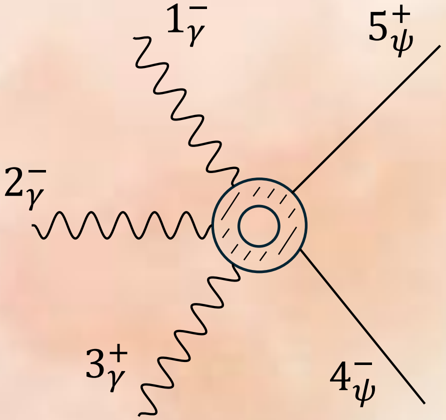
R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

The amplitude is inconsistent, unless this piece is identically zero. This means imposing

$$\sum_n Q_n^3 = 0$$

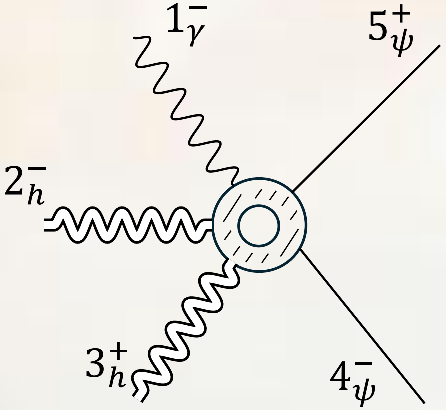
Anomaly cancellation conditions follow from restoring collinear factorization

4. Gauge anomalies on-shell: general



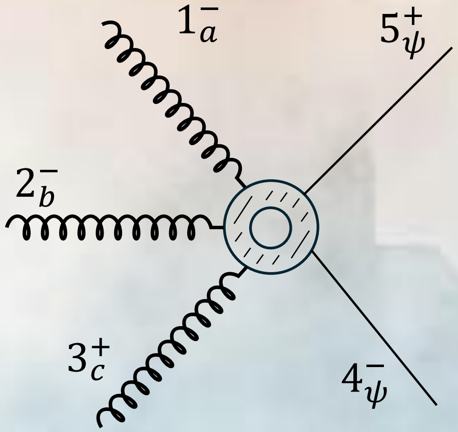
U(1)^3

$$\sum_n Q_n^3 = 0$$



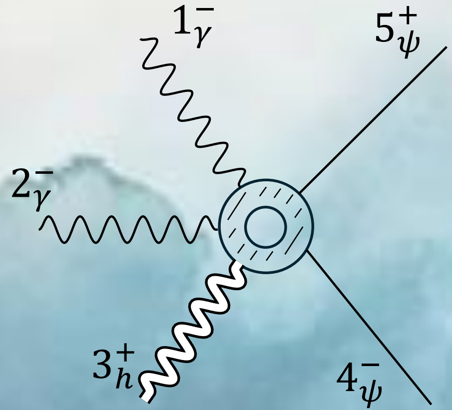
U(1)-gravitational

$$\sum_n Q_n = 0$$



SU(3)^3

$$d^{abc} = 0$$



$$\sum_n Q_n^2 \neq 0$$

5. Outlook

In this work we showed how:

- Gauge anomalies appear on shell as breakdown of collinear factorization
- Restoring the factorization yields gauge anomaly cancellation conditions

For the future:

- Include Green-Schwarz formalism: how new DOF restore factorization
- Study global anomalies via a gravity probe
- Possible on-shell proof of the Adler-Bardeen theorem

5. Outlook

In this work we showed how:

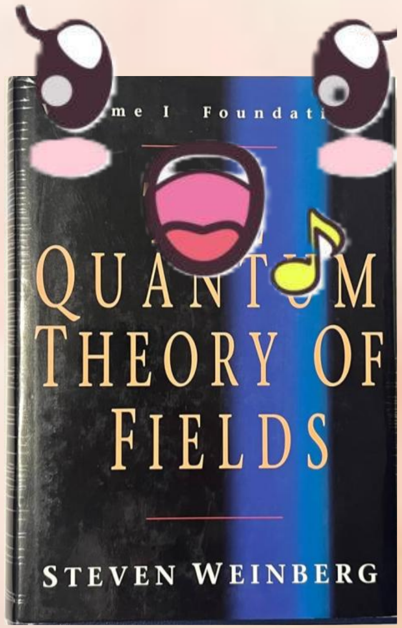
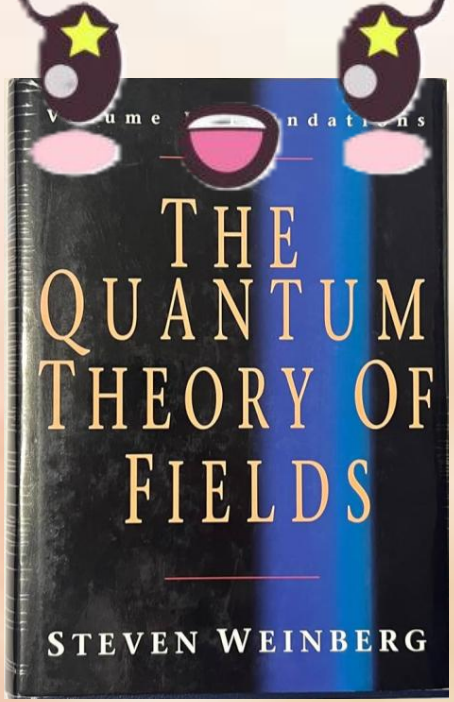
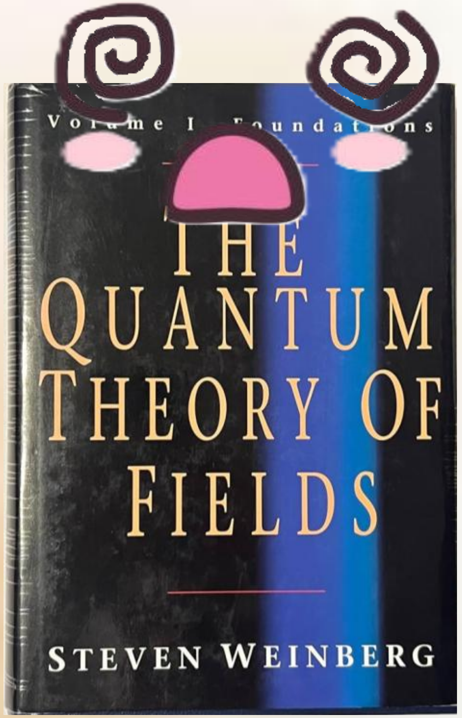
- Gauge anomalies appear on shell as breakdown of collinear factorization
- Restoring the factorization yields gauge anomaly cancellation conditions

For the future:

- Include Green-Schwarz formalism: how new DOF restore factorization
- Study global anomalies via a gravity probe
- Possible on-shell proof of the Adler-Bardeen theorem

JOIN THE AMPLITUDE CULT!

And I thank you!


QUANTUM
THEORY OF
FIELDS
STEVEN WEINBERG

THE
QUANTUM
THEORY OF
FIELDS
STEVEN WEINBERG

Volume I Foundations
THE
QUANTUM
THEORY OF
FIELDS
STEVEN WEINBERG


Volume I Foundations
THE QUANTUM THEORY OF FIELDS
STEVEN WEINBERG