Constraining the gluon Sivers function with unpolarized protons

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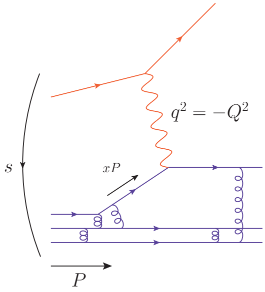
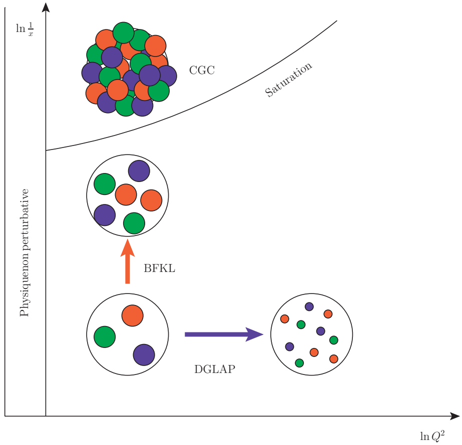
CPHT

Heavy Ion & Hadronic Physics Meeting, Orsay

Accessing the partonic content of hadrons with an electromagnetic probe

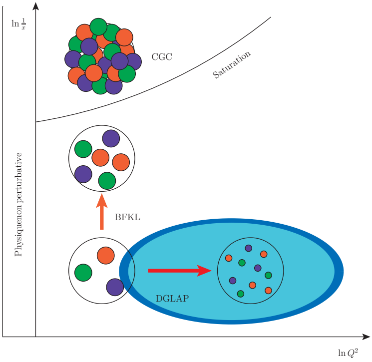


Electron-proton collision (parton model)

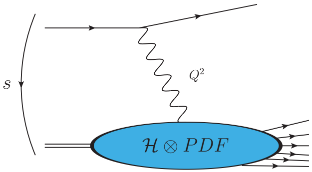


QCD at moderate x

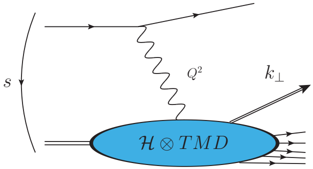
$$Q^2 \sim s$$



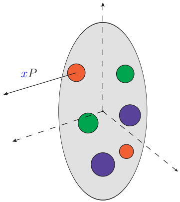
Parton Distributions

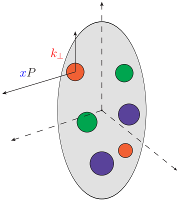


Parton Distribution Fonction (PDF)

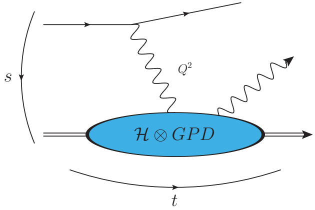


Transverse Momentum Dependent distributions (TMD)

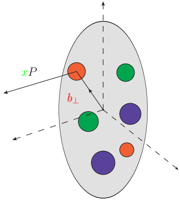


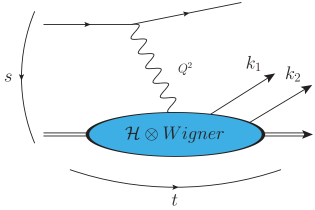


Parton Distributions

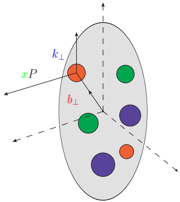


Generalized Parton Distribution (GPD)





Generalized Transverse Momentum Dependent distributions (GTMD)



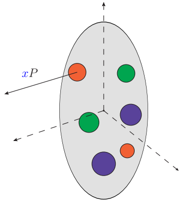
Operator definition for parton distributions

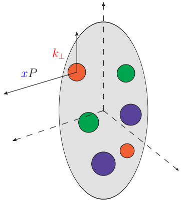
Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \left| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \right| P \right\rangle$$

Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4 z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$





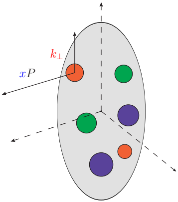
Operator definition for parton distributions

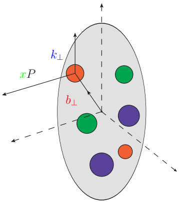
TMD distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4z\delta(z^-)e^{ixP^-z^+ + i(k_\perp \cdot z_\perp)}\left\langle P \left| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \right| P \right\rangle$$

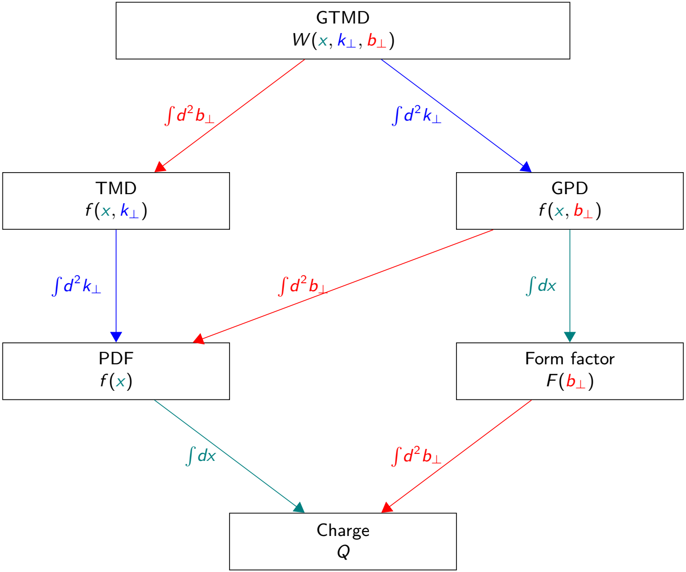
Generalized TMD distribution

$$\mathcal{F}(x, k_\perp, \Delta) \propto \int d^4z\delta(z^-)e^{ixP^-z^+ + i(k_\perp \cdot z_\perp)}\left\langle P+\Delta \left| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \right| P \right\rangle$$





The family tree of parton distributions



Leading twist gluon TMD distributions

Hadron pol. \ Parton	Unpolarized	Circular	Linear
Unpolarized	f_1^g	$\emptyset$	$h_1^{\perp g}$
Longitudinal	$\emptyset$	g_{1L}^g	$h_{1L}^{\perp g}$
Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

PDF-spanning

Unpolarized f_1^g

Helicity g_{1L}^g

Naive T-even pure TMDs

Worm-gear $h_{1L}^{\perp g}, g_{1T}^g$

Pretzelosity $h_{1T}^{\perp g}$

Transversity h_1^g

Naive T-odd pure TMDs

Boer-Mulders $h_1^{\perp g}$

Sivers $f_{1T}^{\perp g}$

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Naive T-odd pure TMDs

Boer-Mulders $h_1^{\perp g}$

Sivers $f_{1T}^{\perp g}$

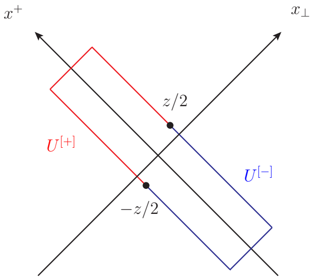
(So-called) non-universality of TMD distributions:

The importance of gauge links

TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



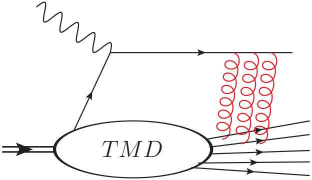
$$q^{[+]}(x, k_\perp) \propto \left\langle P, S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2}, -\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P, S \right\rangle$$

$$q^{[-]}(x, k_\perp) \propto \left\langle P, S \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2}, -\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P, S \right\rangle$$

For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers effect

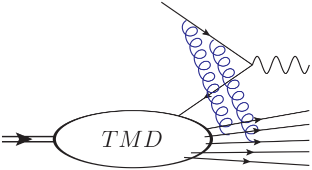
The Sivers effect

SIDIS



Final state interactions: $q^{[+]}$

Drell-Yan

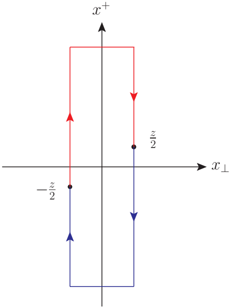


Initial state interactions: $q^{[-]}$

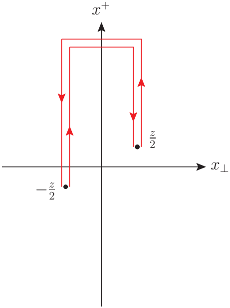
The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions



$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

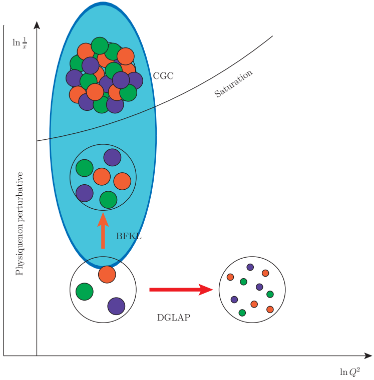


$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

Even more possibilities for gluon TMD distributions!

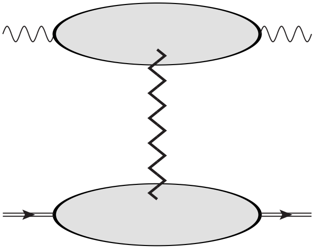
QCD at small x

$$Q^2 \ll s$$



The Pomeron

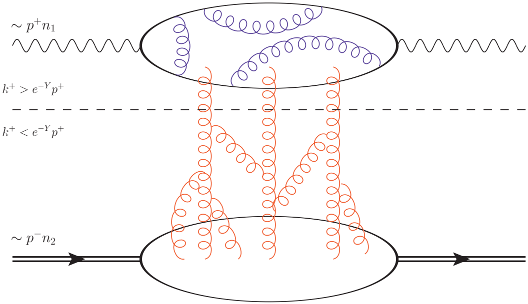
Regge theory: for asymptotic values of s, an effective particle with the quantum numbers of the vacuum is exchanged



Positive C-parity: Pomeron exchange, negative C-parity: Odderon exchange

- How can we understand the Pomeron and the Odderon in perturbative QCD?
- How does it couple to hadrons?

Rapidity separation

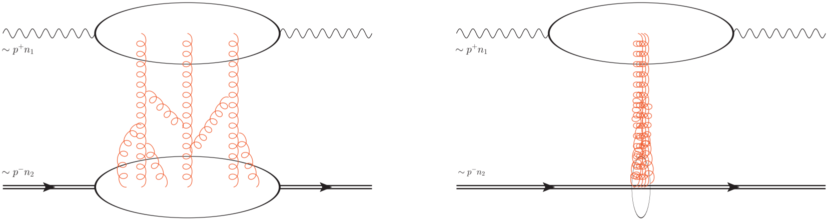


Let us split the gluonic field between "fast" and "slow" gluons

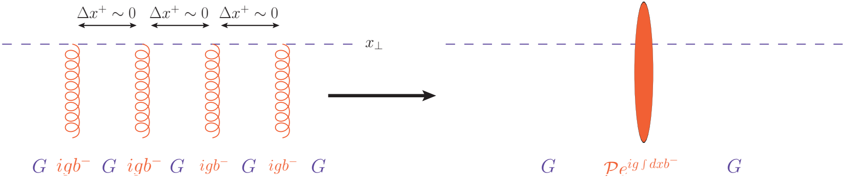
$$\mathcal{A}^{\mu a}(k^+, k^-, \mathbf{k}) = A^{\mu a}_{Y_c}(|k^+| > e^{-Y_c} p^+, k^-, \mathbf{k}) + b^{\mu a}_{Y_c}(|k^+| < e^{-Y_c} p^+, k^-, \mathbf{k})$$

$e^{-Y_c} \ll 1$

Large longitudinal boost to the projectile frame

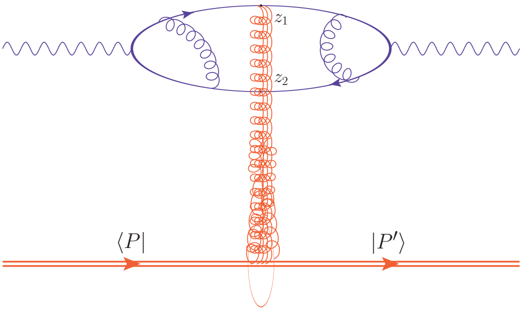


Shockwave approximation



Interactions exponentiate into Wilson lines

Factorized picture



Factorized amplitude

$$\mathcal{A}^{Y_c} = \int d^d\mathbf{x}_1 d^d\mathbf{x}_2\, \Phi^{Y_c}(\mathbf{x}_1,\mathbf{x}_2)\, \langle P'|1 - \frac{1}{N_c}\mathrm{tr}(V_{\mathbf{x}_1}^{Y_c} V_{\mathbf{x}_2}^{Y_c\dagger})|P\rangle$$

Y_c independence: B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

The seemingly incompatible nature of the distributions

Two different kinds of gluon distributions

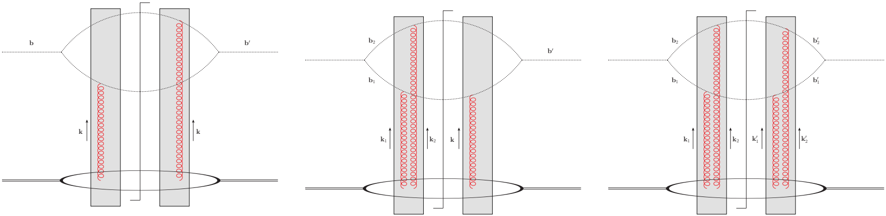
Moderate x distributions	Low x distributions
GTMD, GPD, TMD, PDF...	Dipole scattering amplitude
$\langle P^{(\prime)} \| F^{-i} W F^{-j} W \| P \rangle$	$\langle P^{(\prime)} \| \mathrm{tr}(V_1 V_2^{\dagger}) \| P \rangle$

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB], [RB, Mehtar-Tani]

Generalizes [Dominguez, Marquet, Xiao, Yuan] to all powers

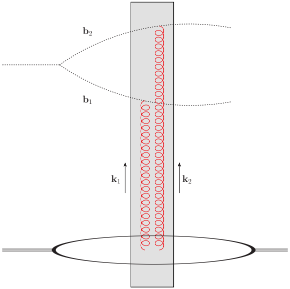


$$
\begin{aligned}
\sigma = &\ \mathcal{H}_2^{ij}(\boldsymbol{k}) \otimes f_2^{ij}(x=0, \boldsymbol{k}) \\
&+ \mathcal{H}_3^{ijk}(\boldsymbol{k}, \boldsymbol{k}_1) \otimes f_3^{ijk}(x=0, x_1=0, \boldsymbol{k}, \boldsymbol{k}_1) \\
&+ \mathcal{H}_4^{ijkl}(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1') \otimes f_4^{ijkl}(x=0, x_1=0, x_1'=0, \boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1')
\end{aligned}
$$

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

[Altinoluk, RB], [RB, Mehtar-Tani]



$$\mathcal{H}^{ij}\left(\boldsymbol{k}_1, \boldsymbol{k}_2\right) \otimes f^{ij}(x=0, \xi=0; \boldsymbol{k}, \boldsymbol{\Delta})$$

Every exclusive low x process probes a GTMD!

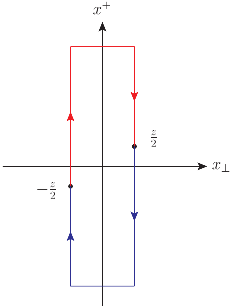
The dipole-type gluon GTMD at small x

Most basic exclusive low x object: the dipole

$$\int \mathrm{d}^2\boldsymbol{v} e^{i(\boldsymbol{k}\cdot\boldsymbol{v})} \left\langle P' \left| \mathrm{tr} V_{\frac{\boldsymbol{v}}{2}} V^\dagger_{-\frac{\boldsymbol{v}}{2}} - N_c \right| P \right\rangle$$

$$\propto \frac{\left(\boldsymbol{k}^i + \frac{\boldsymbol{\Delta}^i}{2}\right)\left(\boldsymbol{k}^j - \frac{\boldsymbol{\Delta}^j}{2}\right)}{\left(\boldsymbol{k} + \frac{\boldsymbol{\Delta}}{2}\right)^2 \left(\boldsymbol{k} - \frac{\boldsymbol{\Delta}}{2}\right)^2} \bar{u}_{P'} \left[\delta^{ij} F^g + i\epsilon^{ij} G^g + \tau^{ijkl} H_T^{gkl}\right]_{x=0,\xi\simeq 0} u_P$$

- F^g: unpolarized GTMD correlator
  $F^g_{1,1}, F^g_{1,2}, F^g_{1,3}, F^g_{1,4}$
- G^g: polarized GTMD correlator
  $G^g_{1,1}, G^g_{1,2}, G^g_{1,3}, G^g_{1,4}$
- H^g: transversity GTMD correlator
  $H^g_{1,1}, H^g_{1,2}, H^g_{1,3}, H^g_{1,4},$
  $H^g_{1,5}, H^g_{1,6}, H^g_{1,7}, H^g_{1,8}$



The dipole-type gluon GTMD at small x

Most basic exclusive low x object: the dipole

$$\int \mathrm{d}^2\boldsymbol{v}\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{v})} \left\langle P' \left| \mathrm{tr} V_{\frac{\boldsymbol{v}}{2}} V^{\dagger}_{-\frac{\boldsymbol{v}}{2}} - N_c \right| P \right\rangle$$

$$\propto \frac{\left(\boldsymbol{k}^i + \frac{\boldsymbol{\Delta}^i}{2}\right)\left(\boldsymbol{k}^j - \frac{\boldsymbol{\Delta}^j}{2}\right)}{\left(\boldsymbol{k}+\frac{\boldsymbol{\Delta}}{2}\right)^2\left(\boldsymbol{k}-\frac{\boldsymbol{\Delta}}{2}\right)^2} \bar{u}_{P'} \left[\delta^{ij} F^g + i\epsilon^{ij} G^g + \tau^{ijkl} H_T^{gkl}\right]_{x=0,\xi\simeq 0} u_P$$

Remarkable properties of dipole GTMDs at small x

- $F^g \Leftrightarrow H^g$: saturated hadrons contain as many gluon pairs with opposite helicity as ones with the same helicity [RB, Altinoluk]
- $F^g \Leftrightarrow G^g$: maximal entanglement of helicity and Orbital Angular Momentum in saturated hadrons [Bhattacharya, RB, Hatta]
- Observables can be written in terms of unpolarized (F) distributions only

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$$\propto \frac{1}{\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}} \bar{u}_{P'} \left[F^g_{1,1} + i\frac{\sigma^{i-}}{P^-}\left(\boldsymbol{k}^i F^g_{1,2} + \boldsymbol{\Delta}^i F^g_{1,3}\right) + i\frac{\sigma^{ij}\boldsymbol{k}^i\boldsymbol{\Delta}^j}{M^2} F^g_{1,4}\right] (0,0,\boldsymbol{k},\boldsymbol{\Delta})\, u_P$$

At small x, $F^g_{1,4} = 0$. At small ξ,

$$F^g_{1,1} = \mathrm{Re}\left(F^g_{1,1}\right) + i\frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2}\mathrm{Im}\left(F^g_{1,1}\right)$$

$$F^g_{1,2} = \frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2}\mathrm{Re}\left(F^g_{1,2}\right) + i\mathrm{Im}\left(F^g_{1,2}\right)$$

$$F^g_{1,3} = \mathrm{Re}\left(F^g_{1,3}\right) + i\frac{\boldsymbol{k}\cdot\boldsymbol{\Delta}}{M^2}\mathrm{Im}\left(F^g_{1,3}\right)$$

so at small $x, \xi, \boldsymbol{\Delta}$,

$$\int \mathrm{d}^2 \boldsymbol{v} \mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{v})} \left\langle P' \simeq P \left| \mathrm{tr} V_{\frac{\boldsymbol{v}}{2}} V^\dagger_{-\frac{\boldsymbol{v}}{2}} - N_c \right| P \right\rangle$$

$$\propto \frac{1}{\boldsymbol{k}^2} \bar{u}_{P'} \left[\mathrm{Re}\left(F^g_{1,1}\right) - \frac{\sigma^{i-}}{P^-}\boldsymbol{k}^i \mathrm{Im}\left(F^g_{1,2}\right)\right] (0, \xi \simeq 0, \boldsymbol{k}, \boldsymbol{\Delta} \simeq \boldsymbol{0})\, u_P$$

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$$\propto \frac{1}{\boldsymbol{k}^2} \bar{u}_{P'} \left[\mathrm{Re}\left(F^g_{1,1}\right) - \frac{\sigma^{i-}}{P^-} \boldsymbol{k}^i \mathrm{Im}\left(F^g_{1,2}\right) \right] (0, \xi \simeq 0, \boldsymbol{k}, \boldsymbol{\Delta} \simeq \boldsymbol{0})\, u_P$$

Known forward limits:

$$\mathrm{Re}\left(F^g_{1,1}\right)(x, 0, \boldsymbol{k}, \boldsymbol{0}) = xf(x, \boldsymbol{k}^2)$$

$$\mathrm{Im}\left(F^g_{1,2}\right)(x, 0, \boldsymbol{k}, \boldsymbol{0}) = -\frac{1}{2} x f_{1T}^{\perp g}(x, \boldsymbol{k}^2)$$

so finally

$$\int \mathrm{d}^2\boldsymbol{v} e^{i(\boldsymbol{k}\cdot\boldsymbol{v})} \left\langle P' \simeq P \left| \mathrm{tr} V_{\frac{\boldsymbol{v}}{2}} V^{\dagger}_{-\frac{\boldsymbol{v}}{2}} - N_c \right| P \right\rangle$$

$$\propto \frac{1}{\boldsymbol{k}^2} \bar{u}_{P'} \left[xf(x, \boldsymbol{k}^2) + \frac{1}{2} \frac{\sigma^{i-}}{P^-} \boldsymbol{k}^i x f_{1T}^{\perp g}(x, \boldsymbol{k}^2) \right]_{x=0} u_P$$

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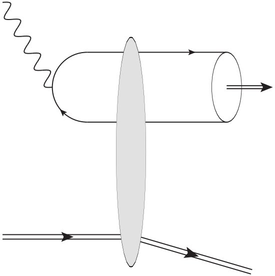
Spin structure:

$$\boldsymbol{k}^i \bar{u}_{P,h'} \sigma^{i-} u_{P,h} \propto (\boldsymbol{k} \times \boldsymbol{h})_z \delta_{h,-h'}$$

Access to transverse spin with an unpolarized proton beam via the Sivers GTMD, forbidden in inclusive processes involving a C-odd t-channel exchange (Odderon)

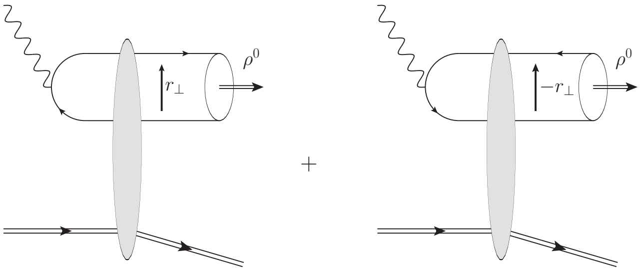
Deeply Virtual Meson Production



DVMP, the Pomeron and the Odderon

DVMP and the Pomeron(s)

Pomeron exchange: C odd meson production

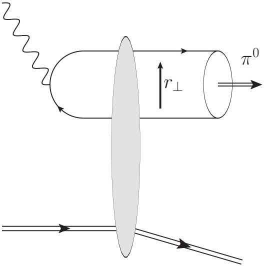


$$\frac{1}{2}\left[\mathrm{tr}\left(V_{b+\frac{r}{2}}V^\dagger_{b-\frac{r}{2}}\right) + \mathrm{tr}\left(V_{b-\frac{r}{2}}V^\dagger_{b+\frac{r}{2}}\right)\right] - N_c$$

$$\propto \frac{1}{\boldsymbol{k}^2}\bar{u}_{P'}\left[xf(x,\boldsymbol{k}^2)\right]_{x=0}u_P$$

DVMP and the Odderon(s)

Odderon exchange: C even meson production



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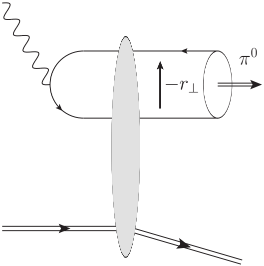

$$\frac{1}{2}\left[\mathrm{tr}\left(V_{b+\frac{r}{2}}U^\dagger_{b-\frac{r}{2}}\right) - \mathrm{tr}\left(V_{b-\frac{r}{2}}V^\dagger_{b+\frac{r}{2}}\right)\right]$$

$$\propto \frac{1}{\boldsymbol{k}^2}\bar{u}_{P'}\left[\frac{\sigma^{i-}}{2P^-}\boldsymbol{k}^i x f_{1T}^{\perp g}(x,\boldsymbol{k}^2)\right]_{x=0} u_P$$

Probing the Sivers function

[RB, Hatta, Szymanowski, Wallon]

Thanks to the Odderon/GTMD equivalence, the cross section for exclusive π^0 electroproduction at small x and small t with unpolarized lepton and proton beams is a direct probe for the gluon Sivers function

$$\frac{d\sigma}{d\xi dQ^2 d\,|t|} \simeq (2\pi)^3 \frac{\alpha_{\rm em}^2 \alpha_s^2 f_\pi^2}{8\xi N_c M^2 Q^2}(1 - y + \frac{y^2}{2}) \times \left[\int_0^1 dz \frac{\phi_\pi(z)}{z\bar{z}Q^2} \int d\boldsymbol{k}^2 \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + z\bar{z}Q^2} x f_{1T}^{\perp}(x,\boldsymbol{k}^2)\right]^2 .$$

We can thus understand the gluonic content of the transversely polarized protons without polarizing the proton beam.

Conclusions

- GTMD distributions are what allows to match standard parton distributions and semi-classical descriptions of small x physics
- Exclusive physics at small x is particularly interesting to understand spin physics without going beyond the eikonal approximation
- Processes with Odderon exchange constitute a direct probe for the gluonic content of transversely polarized protons, with unpolarized proton beams

A very important question

BUT HOW DOES IT COMPARE

WITH THE

SPECTACULAR RESULTS

OF

CHIRAL PERTURBATION THEORY ???