

Lattice QCD form factors for radiative leptonic decays of heavy mesons

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Progress in Algorithms and Numerical Tools for QCD

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UK Research
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Introduction: Leptonic decays

First, consider D -meson leptonic decay

$$D \rightarrow \mu \nu_\mu$$

The branching fraction is linked to the CKM matrix

$$\mathcal{B} \equiv \frac{\Gamma(D \rightarrow \mu \nu_\mu)}{\Gamma_{\text{total}}} = \tau_D \frac{G_F^2}{8\pi} |V_{cd}|^2 f_D^2 m_\mu^2 M_D \left(1 - \frac{m_\mu^2}{M_D^2}\right)^2$$

Use

$$\mathcal{B} = 3.71(20) \times 10^{-4} \quad \text{BESIII [1]}$$

$$f_D = 212.0(7) \text{ MeV} \quad N_f = 2 + 1 + 1 \text{ [10]}$$

$$\tau_D = 1.033(5) \times 10^{-12} \text{ s} \quad \text{Lifetime [7]}$$

The CKM matrix element is then [7]

$$|V_{cd}| = 0.2181(50)$$

Lattice input at 0.3% precision!

In experiment, one really measures the radiation-inclusive rate

$$\Gamma(\Delta E_\gamma) \equiv \Gamma(D^+ \rightarrow \mu^+ \nu_\mu [\gamma])|_{E_\gamma < \Delta E_\gamma}$$

with ΔE_γ the experimental sensitivity.

$$\Gamma(\Delta E_\gamma) = \Gamma_0 + \Gamma_1(\Delta E_\gamma)$$

We've added two types of QED corrections,

- Loop (virtual) corrections to $\Gamma_0 = \Gamma(D^+ \rightarrow \mu \nu_\mu)$
- Real radiative decay $\Gamma_1(\Delta E_\gamma) = \Gamma(D^+ \rightarrow \mu \nu_\mu \gamma)|_{E_\gamma < \Delta E_\gamma}$

QED corrections are expected to be 1% [1] at vanishing ΔE_γ , and dominant for large E_γ [5]

Introduction: Real radiative decays

For the real radiative decay

$$D^+(p) \rightarrow \mu^+(\ell) \nu_\mu(n) \gamma(k, \lambda)$$

the amplitude $\mathcal{M}_1$ can be split into hadronic and leptonic parts

$$\mathcal{M}_1^{(\lambda)} = \epsilon_\mu^{(\lambda)*}(k) \left(\mathcal{M}_{\text{had}}^\mu + \mathcal{M}_{\text{lep}}^\mu \right)$$

The hadronic part is

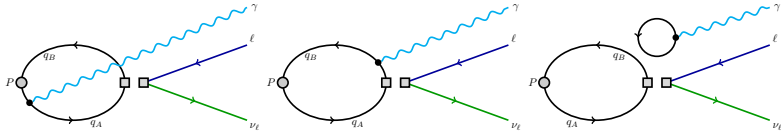
$$\mathcal{M}_{\text{had}}^\mu = \frac{G_F}{\sqrt{2}} V_{cd}^* e H^{\mu\nu}(p, k) \bar{u}(\ell) \gamma_\beta (1 - \gamma_5) v(n)$$

The non-perturbative physics appears in the hadronic tensor

$$H^{\mu\nu}(p, k) = i \int d^4x e^{ik \cdot x} \left\langle 0 \left| T \{ j_{\text{em}}^\mu(x) j_W^\nu(y) \} \right| D^+(p) \right\rangle$$

Introduction: Real radiative decays in Euclidean space

There are three Wick contractions of $H^{\mu\nu}(p, k)$



The hadronic tensor can be computed in Euclidean space [3]

$$H^{\mu\nu}(\mathbf{k}, \mathbf{p}) = \int dt_\gamma d^3x d^3z e^{t_\gamma E_\gamma} e^{i\mathbf{p}\cdot\mathbf{z}} e^{-i\mathbf{k}\cdot\mathbf{x}} \underbrace{\langle 0 | j_{\text{em}}^\mu(x) j_W^\nu(y) \phi^\dagger(z) | 0 \rangle}_{C^{\mu\nu}(x, y, z)}$$

where

$$j_{\text{em}}^\mu(x) = \sum_q e_q \bar{q}(x) \gamma_\mu q(x)$$

$$j_W^\nu(y) = \bar{q}_A(y) \gamma^\nu (1 - \gamma_5) q_B(y)$$

$$\phi = \bar{q}_A(z) \gamma_5 q_B(z)$$

No need for explicit photon field!

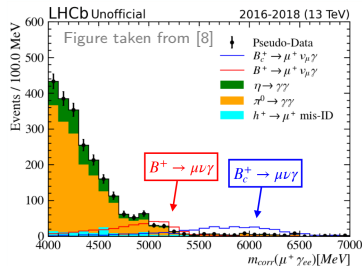
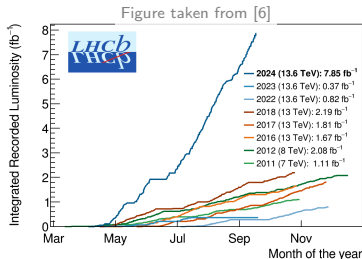
Objective

Study the 1st and 2nd rows of CKM with leptonic decays of

$$D \quad D_s \quad B \quad B_c$$

PDG [7]	Leptonic	Semi-leptonic
$ V_{ub} $	$4.11(39) \times 10^{-3}$	$3.82(20) \times 10^{-3}$
$ V_{cd} $	0.2181(50)	0.2330(136)
$ V_{cs} $	0.984(12)	0.972(7)
$ V_{cb} $	Not seen	$41.1(12) \times 10^{-3}$

Experiments get more precise [9] and look at new channels [8]



Setup: JLQCD DWFs

Action

- All quarks are Möbius $b + c = 2$, $b - c = 1$
- Tree-level improved Symanzik gauge + stout smearing

DWFs preserve chiral symmetry

- Our observable is chiral
- Simpler renormalization

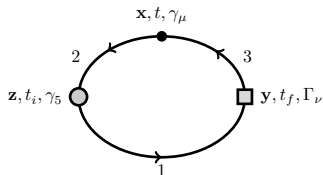
We employ one **JLQCD** $N_f = 2 + 1$ ensemble [4]

Label	β	a [fm]	a^{-1} [GeV]	$L^3 \times T \times L_s$	m_π	m_K	$m_\pi L$
F-ud3-sa	4.47	0.0439(1)	4.494(2)	$64^3 \times 128 \times 8$	284(1)	486(1)	4.0

29 configs, 32 measurements/cnfg. Simulate at six heavy-quarks

am_ℓ	am_s	am_h					
0.003	0.015	0.210476	0.263095	0.328869	0.411086	0.513857	0.642322

Setup: Quark-connected contribution



Assemble Wick contraction

Solve propagators $Q^{(1)}$, $Q^{(2)}$

→ $Z_2 \otimes Z_2$ wall source at $(\mathbf{z}, t_i)$

→ Jacobi smearing

Solve propagator $Q^{(3)}$

→ Point source at $(\mathbf{y} = \mathbf{0}, t_f)$

$$C^{\mu\nu} = \langle \text{tr}[\Gamma^\nu E_\xi \{ Q^{(1)}(\mathbf{y}, t_f; t_i) \gamma_5 Q^{(2)}(t_i; \mathbf{x}, t) \} \gamma^\mu Q^{(3)}(\mathbf{x}, t; \mathbf{y}, t_f)] \rangle_G$$

Kinematics

→ Meson at rest $\mathbf{p} = \mathbf{0}$

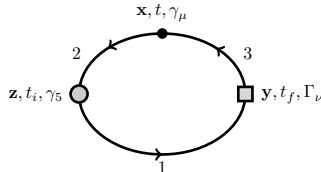
→ Photon $\mathbf{k} = \frac{2\pi}{L} \mathbf{n}$ with Fourier mode $\mathbf{n}$

→ Lepton-neutrino pair $\mathbf{q} = -\mathbf{k}$

Setup: Quark-connected contribution

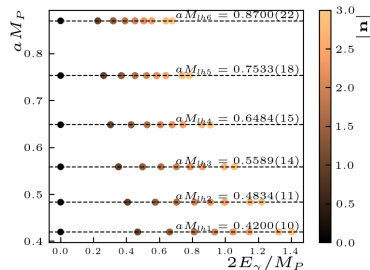
Our setup employs

$$N_q = 8 \quad N_{t_i} = 128 \quad N_{t_f} = 32$$



of propagators N_{prop} grows linearly with # of sources

$$N_{\text{prop}} = N_q \cdot (N_{\text{src}} + N_{\text{snk}}) = 1280$$



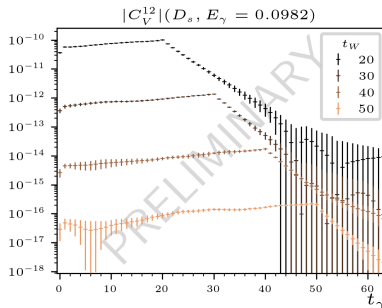
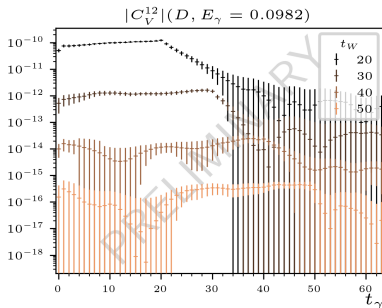
Fourier $|\mathbf{n}| \leq 3$

Three-point functions

Example data for $\mathbf{k} = \frac{2\pi}{L}(0, 0, 1)$, $\mathbf{p} = (0, 0, 0)$

$$C_V^{\mu\nu}(\mathbf{p}, t_W; \mathbf{k}, t_\gamma) = \langle 0 | j_{\text{em}}^\mu(\mathbf{k}, t_\gamma) \bar{q}_A(\mathbf{0}, t_W) \gamma^\nu q_B(\mathbf{0}, t_W) \phi^\dagger(\mathbf{p}, 0) | 0 \rangle$$

where $t_W = t_f - t_i$ and $t_\gamma = t - t_i$

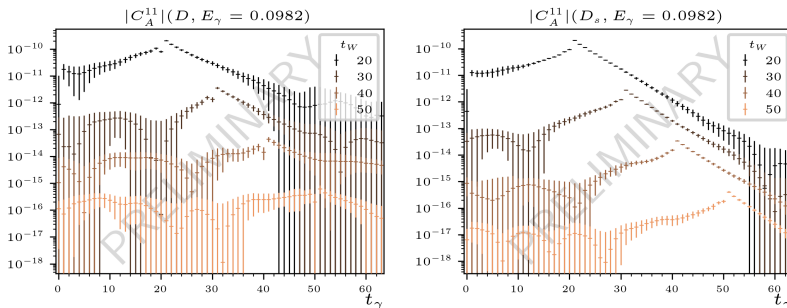


Three-point functions

Example data for $\mathbf{k} = \frac{2\pi}{L}(0, 0, 1)$, $\mathbf{p} = (0, 0, 0)$

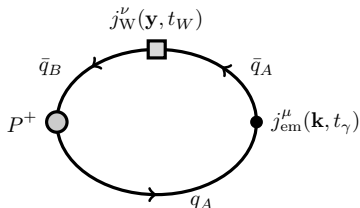
$$C_A^{\mu\nu}(\mathbf{k}, t_\gamma; \mathbf{p}, t_W) = \langle 0 | j_{\text{em}}^\mu(\mathbf{k}, t_\gamma) \bar{q}_A(\mathbf{0}, t_W) \gamma^\nu \gamma_5 q_B(\mathbf{0}, t_W) \phi^\dagger(\mathbf{p}, 0) | 0 \rangle$$

where $t_W = t_f - t_i$ and $t_\gamma = t - t_i$



Second half of the lattice has unphysical time-ordering and vanishes

Exponential fall off



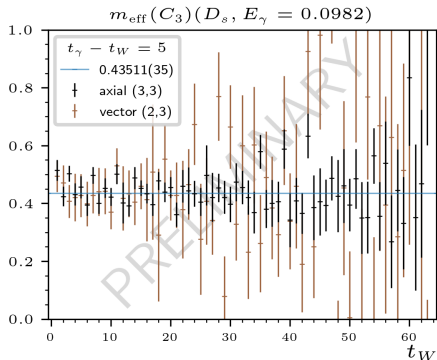
Compare spectrum
obtained from 3pt and 2pt

For $0 < t_W < t_\gamma$:

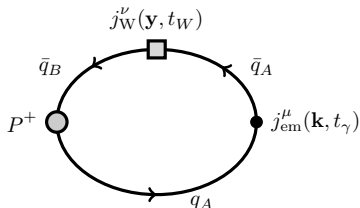
Fix $t_\gamma - t_W > 0$

Vary t_W

Initial meson at rest



Exponential fall off



For $0 < t_W < t_\gamma$:

Fix t_W

Vary t_γ

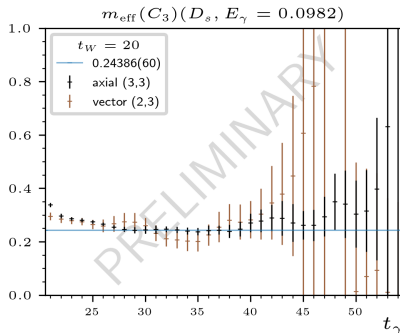
1^{--} meson momentum $\mathbf{k}$

1^{--} state depends on q_A

q	s	Q_0	Q_i
$\rho(770)$	$\phi(1020)$	J/ψ	$\Upsilon(1s)$

Same for $C_V^{\mu\nu}$ and $C_A^{\mu\nu}$

Similar analysis for $0 < t_\gamma < t_W$



D_s vector form factor

$H^{\mu\nu}$ has a form-factor decomposition

$$H^{\mu\nu} = H_V^{\mu\nu} + H_A^{\mu\nu}$$

The vector component depends on F_V

$$H_V^{\mu\nu} = -i\epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma \frac{F_V(x_\gamma)}{M_P} \rightarrow iF_V(x_\gamma) \begin{pmatrix} 0 & k_3 & -k_2 & 0 \\ -k_3 & 0 & k_1 & 0 \\ k_2 & -k_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

F_V depends only on one Lorentz scalar

$$x_\gamma = \frac{2p \cdot k}{M_P^2} \stackrel{\text{CM}}{=} \frac{2E_\gamma}{M_P}$$

Physical decays occur for $0 \leq x_\gamma \leq 1 - (m_\ell/M_P)^2 \sim 1$

Vector form factor

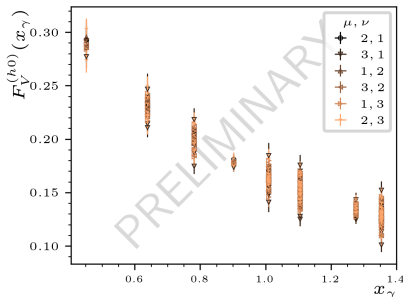
Form the ratio

$$R^{\mu\nu} = e^{(t_\gamma - t_W)E_\gamma} C_V^{\mu\nu}(\mathbf{k}, t_\gamma; \mathbf{p}, t_W) / \sqrt{C_2(t_W)}$$

The vector form factor from any $C^{\mu\nu}$ component is

$$-i\epsilon^{\mu\nu\rho 4} k_\rho f(t_W) F_V(x_\gamma) = \int_0^\infty dt_\gamma R^{\mu\nu}$$

where $f(t_W) \equiv e^{-M_P t_W/2} / \sqrt{2M_P}$



Signal improvement

Use Lorentz covariance

- 1 Select a reference vector k , for example $k = 2\pi(0, 0, 1, 1)/L$
- 2 Find all k' so that

$$k^\mu = \Lambda(R)^\mu_\alpha k'^\alpha \quad \text{where} \quad \Lambda(R) = \begin{pmatrix} R & 0 \\ 0 & 1 \end{pmatrix}$$

R is one of 24 rotations in the cubic group

$$Lk'/2\pi = (\pm 1, 0, 0, 1), (0, \pm 1, 0, 1), (0, 0, \pm 1, 1)$$

- 3 The hadronic tensor $H^{\mu\nu}$ transforms as

$$H^{\mu\nu}(k) = \Lambda(R)^\mu_\alpha \Lambda(R)^\nu_\beta H^{\alpha\beta}(k')$$

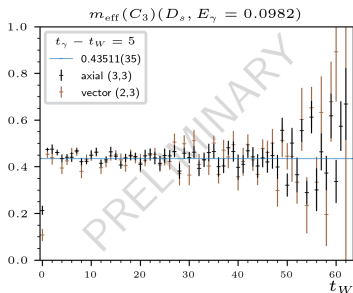
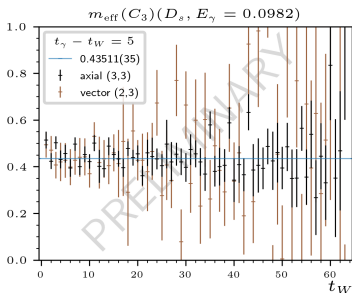
- 4 Average all $H^{\mu\nu}$ related in this fashion

Use T-symmetry

$$C_V^{ij}(\mathbf{k}, t_\gamma; \mathbf{p}, t_W) = -C_V^{ij}(\mathbf{k}, T - t_\gamma; \mathbf{p}, T - t_W)$$

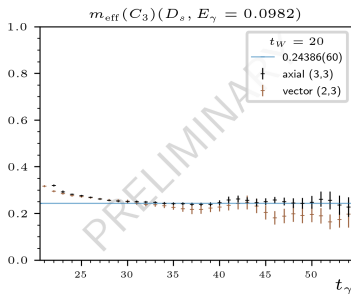
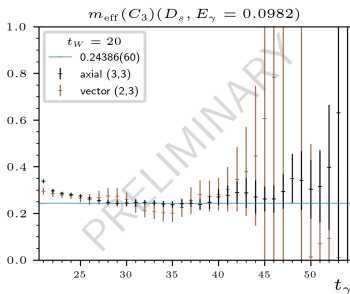
3pt effective mass vs 2pt energy fit

→ Initial meson at rest



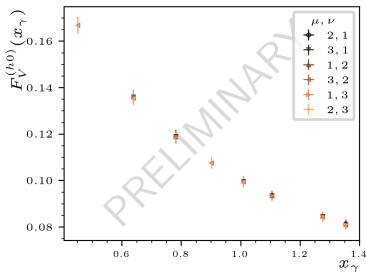
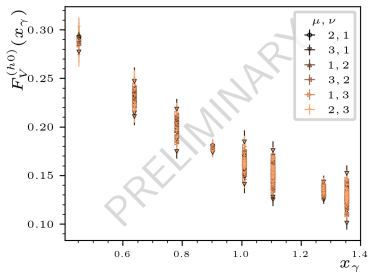
3pt effective mass vs 2pt energy fit

→ 1^{--} meson with momentum $\mathbf{k} = 2\pi(0, 0, 1)/L$



D_s vector form factor

Vector form factor from $C^{\mu\nu}$



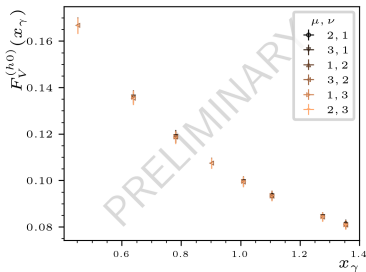
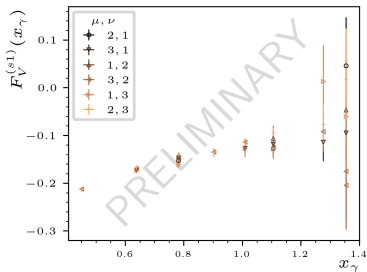
There are two main unexplored systematics:

- Truncation of integral in t_γ
- Excited states/dependence on t_W

Combine fit of 2pt and 3pt functions to extract F_V

D_s vector form factor

Vector form factor from $C^{\mu\nu}$



There are two main unexplored systematics:

- Truncation of integral in t_γ
- Excited states/dependence on t_W

Combine fit of 2pt and 3pt functions to extract F_V

$H^{\mu\nu}$ has a form-factor decomposition

$$H^{\mu\nu} = H_V^{\mu\nu} + H_A^{\mu\nu}$$

The axial component depends on f_P , F_A , H_1 , H_2

$$\begin{aligned} H_A^{\mu\nu} = & f_P \left(\eta^{\mu\nu} + \frac{(2p-k)^\mu (p-k)^\nu}{2p \cdot k - k^2} \right) \\ & + \frac{F_A(x_\gamma)}{M_P} \left[\eta^{\mu\nu} (p \cdot k - k^2) - (p-k)^\mu k^\nu \right] \\ & + \frac{H_1(x_\gamma)}{M_P} \left(\eta^{\mu\nu} k^2 - k^\mu k^\nu \right) \\ & + \frac{H_2(x_\gamma)}{M_P} \frac{k^\mu (p \cdot k - k^2) - k^2 (p-k)^\mu}{(p-k)^2 - M_P^2} (p-k)^\nu \end{aligned}$$

Form the ratio

$$R_A^{\mu\nu}(\mathbf{k}, t_\gamma; t_W) \equiv e^{(t_\gamma - t_W)E_\gamma} C_A^{\mu\nu}(\mathbf{k}, t_\gamma; \mathbf{p} = \mathbf{0}, t_W) / \sqrt{C_2(t_W)}$$

Project with polarization vectors $\epsilon^{(\lambda)}$ to remove H_1 and H_2 terms

$$\epsilon^{(\lambda)}(k) \cdot k = 0 \qquad \epsilon^{(\lambda)} \cdot \epsilon^{(\lambda')} = \delta_{\lambda\lambda'}$$

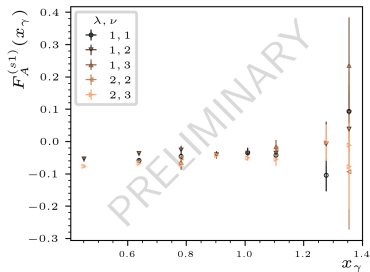
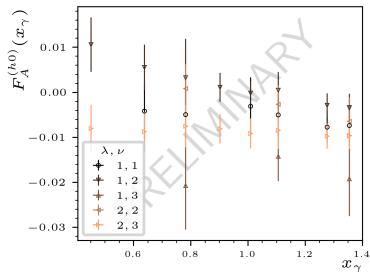
Subtract $\mathbf{k} = \mathbf{0}$ [3] to remove the f_P term

$$\epsilon^{(\lambda)\nu} f'(t_W) F_A(x_\gamma) = \frac{\epsilon_\mu^{(\lambda)}}{x_\gamma} \int_0^\infty dt_\gamma (R_A^{\mu\nu}(\mathbf{k}, t_\gamma; t_W) - R_A^{\mu\nu}(\mathbf{0}, t_\gamma; t_W))$$

where

$$f'(t_W) = \sqrt{M_P/8} e^{-M_P t_W/2}$$

D_s axial form factor



Conclusions and outlook

- QED corrections to leptonic decays are relevant for CKM unitarity tests
- Radiative decays are given by two form factors

$$F_V(2E_\gamma/M_P) \quad F_A(2E_\gamma/M_P)$$

- Thanks to very fine lattice spacings, we can study

$$D \quad D_s \quad B \quad B_c$$

with the same quark action

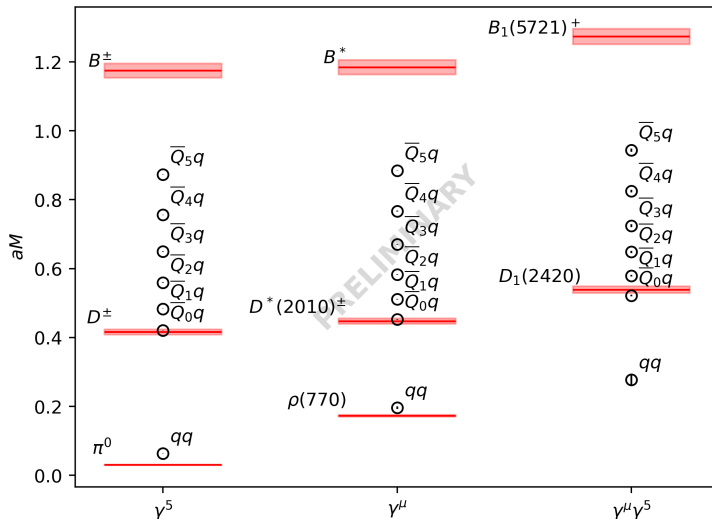
Next steps

- Finalize main analysis
- Study systematics:
 - 1 Excited states t_W
 - 2 Truncation of integral in t_γ
- Reach the physical point with more ensembles

Appendix

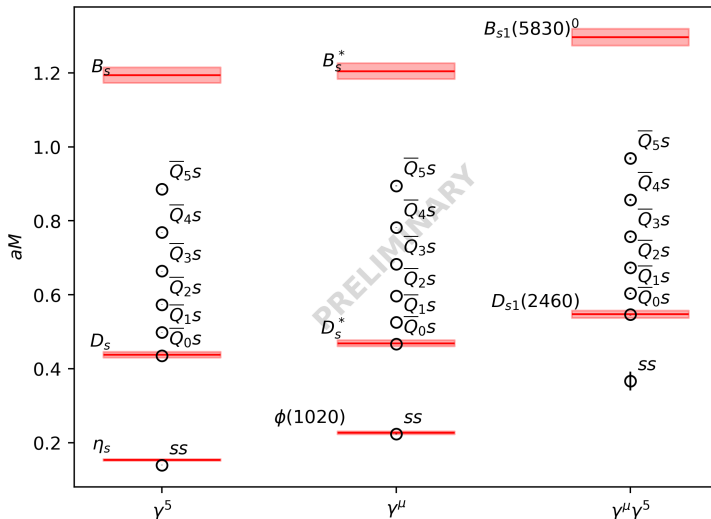
Heavy-light spectrum on F-ud3-sa

Values at finite lattice spacing compared to PDG values [7]



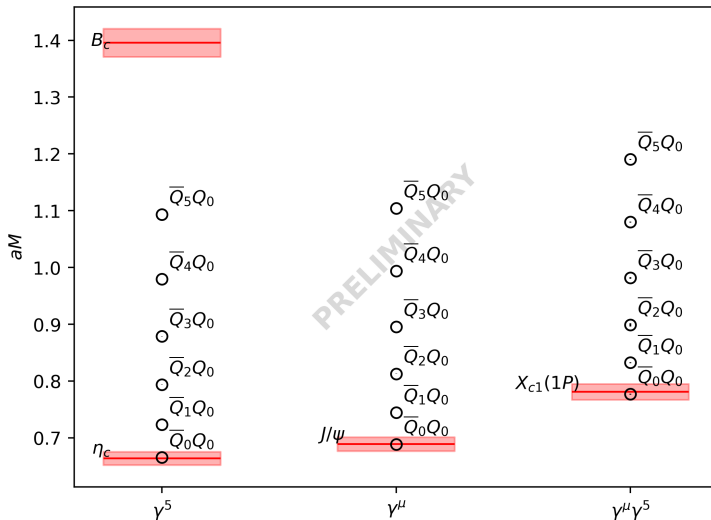
Heavy-strange spectrum on F-ud3-sa

Values at finite lattice spacing compared to PDG values [2, 7]



Heavy-heavy spectrum on F-ud3-sa

Values at finite lattice spacing compared to PDG values [7]



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- [2] Sz. Borsanyi et al. “Leading hadronic contribution to the muon magnetic moment from lattice QCD”. In: *Nature* 593.7857 (2021), pp. 51–55. DOI: 10.1038/s41586-021-03418-1. arXiv: 2002.12347 [hep-lat].

- [3] A. Desiderio et al. “First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons”. In: *Phys. Rev. D* 103.1 (2021), p. 014502. DOI: 10.1103/PhysRevD.103.014502. arXiv: 2006.05358 [hep-lat].
- [4] Brian Colquhoun et al. “Form factors of $B \rightarrow \pi \ell \nu$ and a determination of $|V_{ub}|$ with Möbius domain-wall fermions”. In: *Phys. Rev. D* 106.5 (2022), p. 054502. DOI: 10.1103/PhysRevD.106.054502. arXiv: 2203.04938 [hep-lat].
- [5] R. Frezzotti et al. “Lattice calculation of the D_s meson radiative form factors over the full kinematical range”. In: *Phys. Rev. D* 108.7 (2023), p. 074505. DOI: 10.1103/PhysRevD.108.074505. arXiv: 2306.05904 [hep-lat].

- [6] LHCb collaboration. *Navigating Challenges: LHCb's Milestone Achievements and Intensive Data Collection in 2024*. 2024. URL: <https://ep-news.web.cern.ch/content/navigating-challenges-lhcb-milestone-achievements-and-intensive-data-collection-2024> (visited on 10/14/2025).
- [7] S. Navas et al. "Review of particle physics". In: *Phys. Rev. D* 110.3 (2024), p. 030001. DOI: 10.1103/PhysRevD.110.030001.
- [8] Martino Borsato. *Taming hadronic uncertainties using rare (semi-)leptonic decays*. 2025. URL: https://indico.ijclab.in2p3.fr/event/11668/contributions/39145/attachments/26468/39264/HadronicUnc_MBorsato.pdf (visited on 10/23/2025).

- [9] “Projections for Key Measurements in Heavy Flavour Physics”. In: (Mar. 2025). [arXiv: 2503.24346 \[hep-ex\]](#).
- [10] Y. Aoki et al. “FLAG review 2024”. In: *Phys. Rev. D* 113.1 (2026), p. 014508. DOI: [10.1103/nfzp-p5dn](#). [arXiv: 2411.04268 \[hep-lat\]](#).