Lattice Simulations of Non-Abelian Gauge Theories: From Algorithms to Composite Higgs Phenomenology

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in collaboration with

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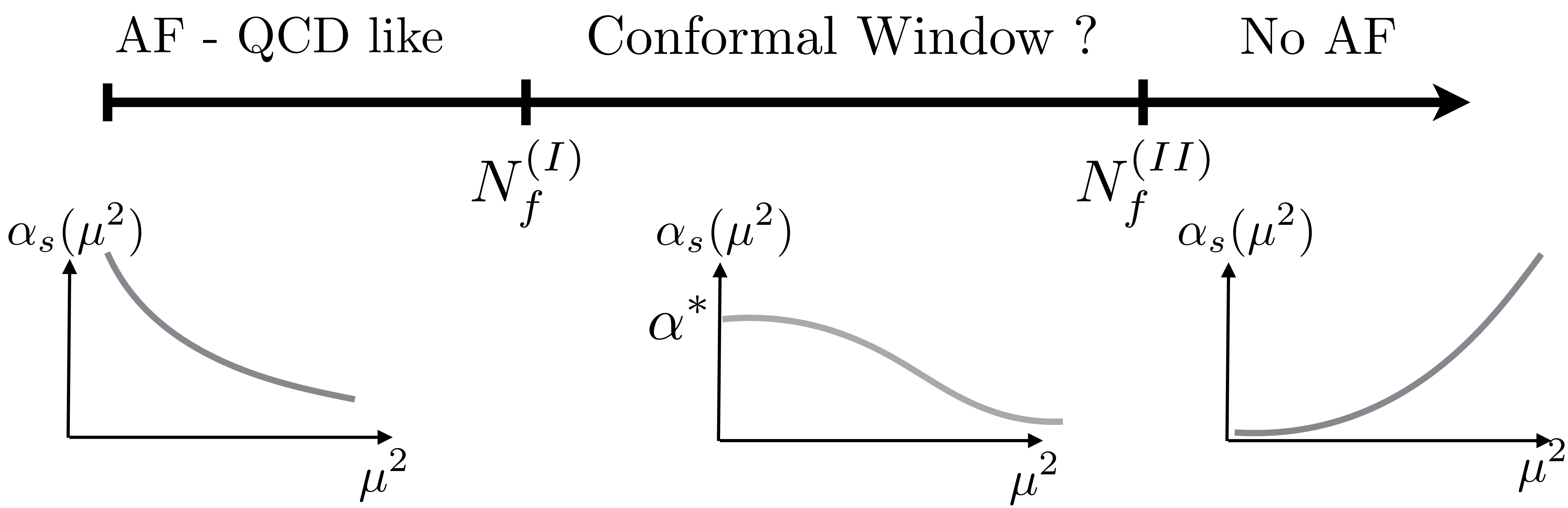
GDR QCD, Orsay - 10 June 2026

UNIVERSITY OF PLYMOUTH

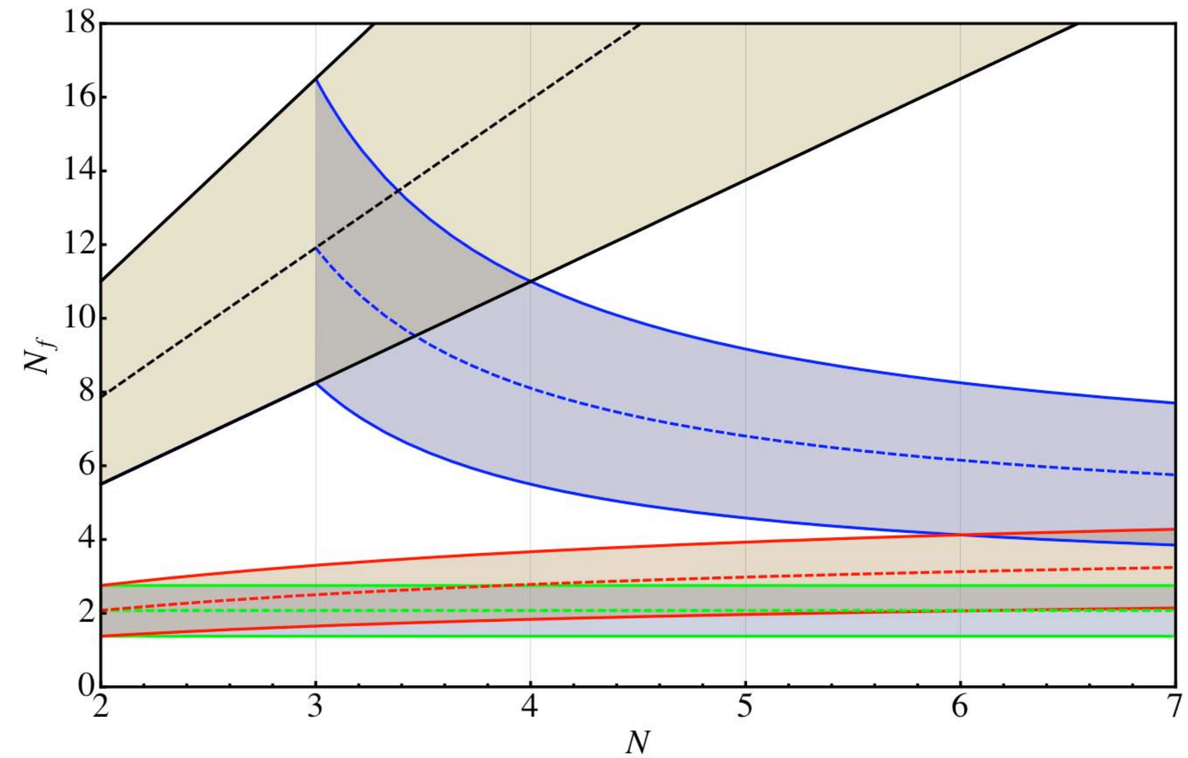
Introduction

The theory landscape



Parameters :

- Gauge group : **SU, SO, Sp**
- Matter representation : **R**
- # of flavour : **N_f**



D. Dietrich, F. Sannino, PRD75 (2007) 085018
Mojaza, Pica, Ryttov, Sannino PRD86 (2012) 076012

« QCD-like » theories

- Confinement + spontaneous chiral symmetry breaking

- Symmetry breaking pattern:

  ★ SU(Nf)xSU(Nf) → SU(Nf) (complex representation)

  ★ SU(2Nf) → SO(2Nf) (real representation)

  ★SU(2Nf) → Sp(2Nf) (pseudo real representation)

- A number of light Goldstone bosons (GBs) expected

- A non trivial spectrum, e.g.:

  ★ 0^{++} & 1^{--}: expected to decay into GBs for sufficiently light GBs

Composite Pseudo-Goldstone Higgs boson

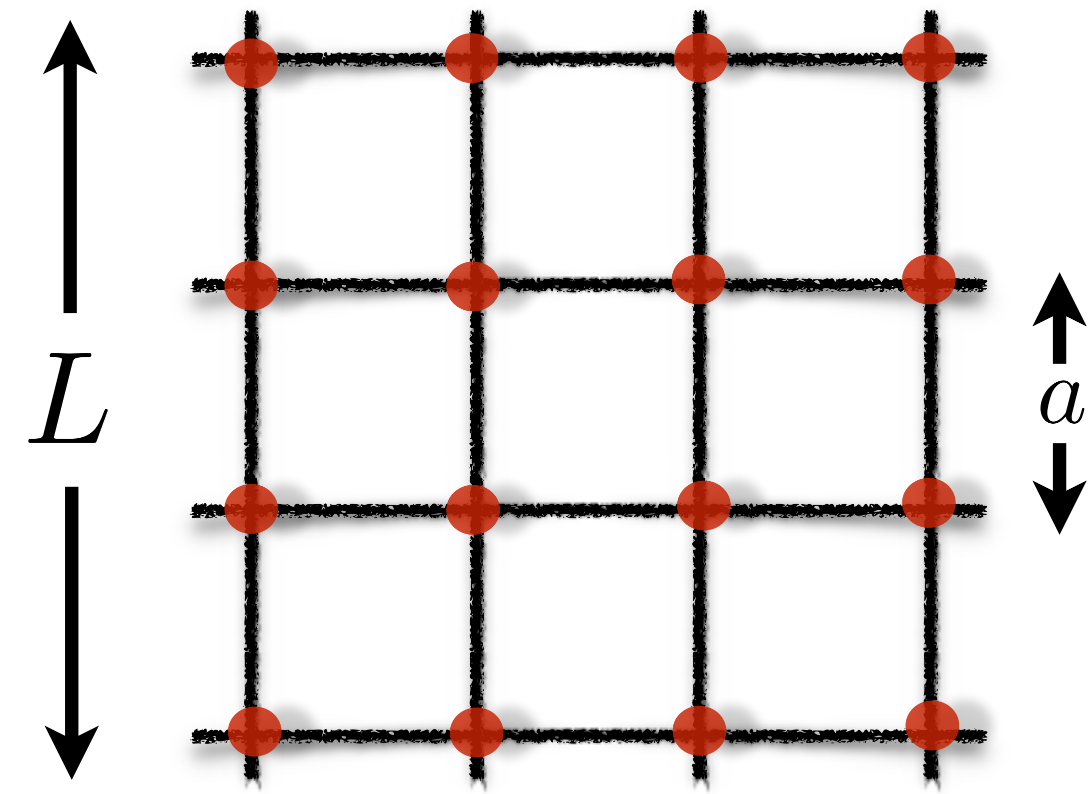
Kaplan & Georgi '84,…

- Ingredients:
  - Replace the Higgs sector of the SM by a new Strongly Interacting theory
  - Consider a new QCD-like sector (i.e with spontaneous chiral symmetry breaking) with at least 4 GBs
  - choose quantum numbers to have a Higgs doublet made of GBs and therefor a light Higgs
  - Add an additional effective coupling to generate/model SM fermion masses.

- Remarks:
  - Loop corrections of EW gauge bosons will generate a potential for the Higgs but will not break EW symmetry.
  - Generically couplings to the EW bosons and SM fermions mimic those of the SM Higgs.

Lattice simulations

Lattice simulations in a nutshell



Roadmap:

- Discretise space-time (euclidean)
- Sample the path integral to calculate the correlation function of the theory
- Predict observable at finite lattice spacing and in a finite volume
- Take the infinite volume and continuum limit and make rigorous predictions

Challenges :

- Sampling of the Botlzmann distribution
- Require HPC ressources

Next generation of lattice simulations: HiRep 2.0

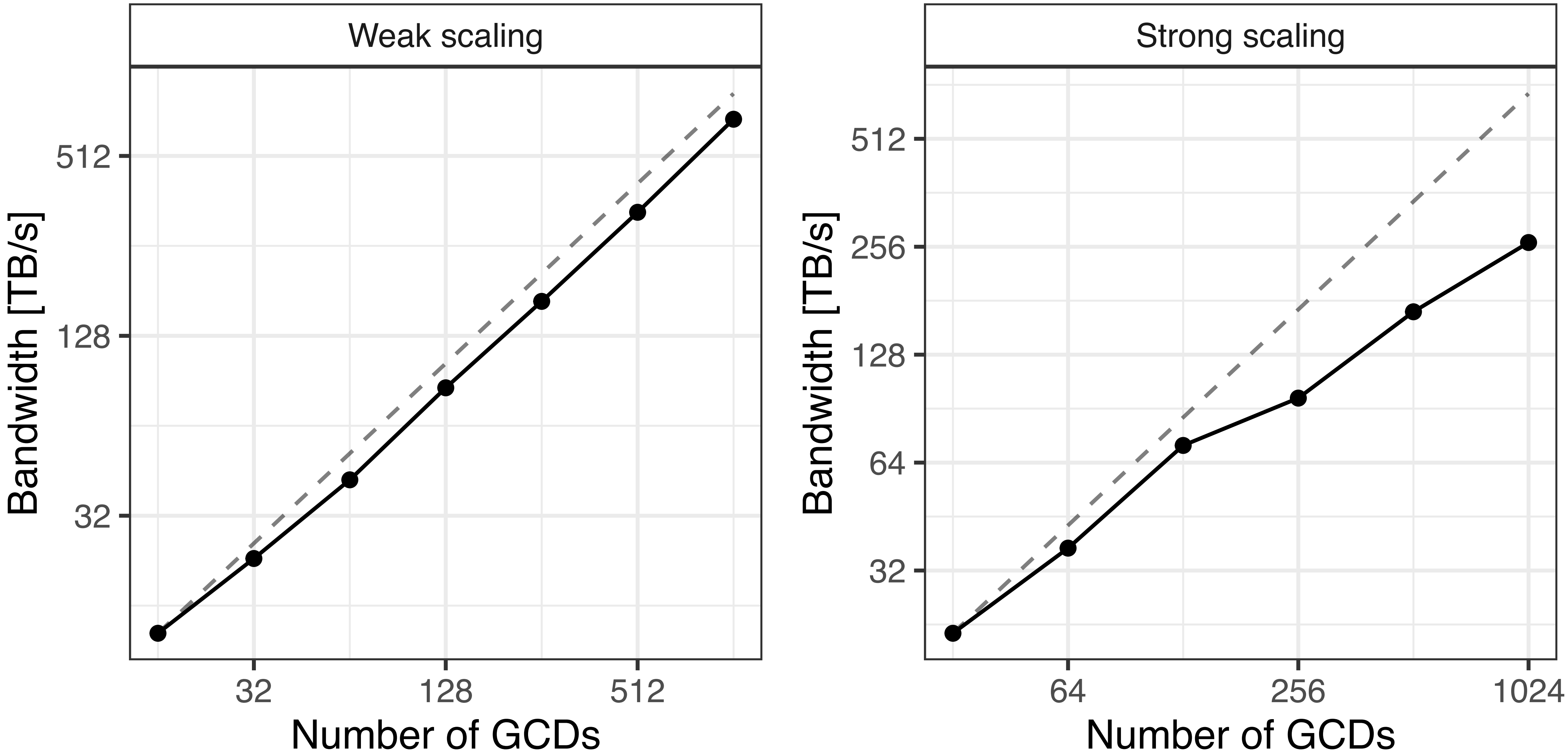
[VD et al, "High-Performance Simulations of Higher Representations of Wilson Fermions". Comput.Phys.Commun. 322 (2026), 110061]

- Target:
  - Generic SU(N), SO(N), (Sp(N))
  - Arbitrary fermion representation
  - Various actions Wilson based actions
  - Contains tools to calculate spectrum, renormalisation constants, scattering amplitudes, matrix elements….
- Implementation:
  - multi-CPUs; multi-GPUs : portability
  - State of art algorithms

HiRep 2.0 : performances

Lattice calculations are memory-bound

GCD :”Graphics Compute Die”



~40% of peak theoretical bandwith

[S. Martins et al., PoS LATTICE2024 (2025) 453 — scaling de HiRep sur LUMI-G]

Applications to Composite Higgs models

The model

SU(2)$_c$ with N$_f$=2 fundamental Dirac flavours

- SU(2) gauge theory with N$_f$ = 2 Dirac fermions in the fundamental representation.

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{U}\gamma^\mu D_\mu U + i\overline{D}\gamma^\mu D_\mu D + \frac{m}{2}Q^T(-i\sigma^2)C\,EQ + \frac{m}{2}\left(Q^T(-i\sigma^2)C\,EQ\right)^\dagger$$

- Pseudo-real irrep of SU(2): **global flavour symmetry is upgraded to SU(4)** :

$$Q \equiv \begin{pmatrix} U_L \\ D_L \\ \widetilde{U}_L \\ \widetilde{D}_L \end{pmatrix} \equiv \begin{pmatrix} U_L \\ D_L \\ -i\sigma_2 C\bar{u}_R^T \\ -i\sigma_2 C\bar{d}_R^T \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

- Infinitesimal SU(4) transformation : $Q \longrightarrow \left(1 + i\sum_{n=1}^{15}\alpha^n T^n\right)$

- Lagrangian invariant if : $ET^n + T^{nT}E = 0$

- Chiral symmetry breaking pattern : ***SU(4) → Sp(4)***

→ **5 Goldtone bosons**

EW embedding

[G. Cacciapaglia & F. Sannino, JHEP 1404,111 (2014)]

- $Q_L = (U_L, D_L)$: $SU(2)_L$ doublet with hypercharge 0
- $\tilde{U}_L, \tilde{D}_L$: $SU(2)_L$ singlet with hypercharge $\pm 1/2$

- Two interesting alignments of the condensate :

$\Sigma_H \equiv E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$: break EW symmetry $\qquad \Sigma_B \equiv \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$: does not break EW

- General case :

$$\Sigma_0 = \cos\theta \; \Sigma_B + \sin\theta \; \Sigma_H$$

- Two limit cases :
  - $\theta = 0$: EW does not break : composite Higgs limit
  - $\theta = \pi/2$: EW breaks + DM candidate : technicolor limit
- at LO : $m_W^2 = 2\, g\, (F_{PS} \, \sin\theta)^2$

The model interpolate between TC and CH

Generic predictions

- 0^{++} and GBs mix lightest state is the Higgs boson
- top quark: shift mass of 0^{++} and GBs

[G. Cacciapaglia & F. Sannino, JHEP 1404,111 (2014)]

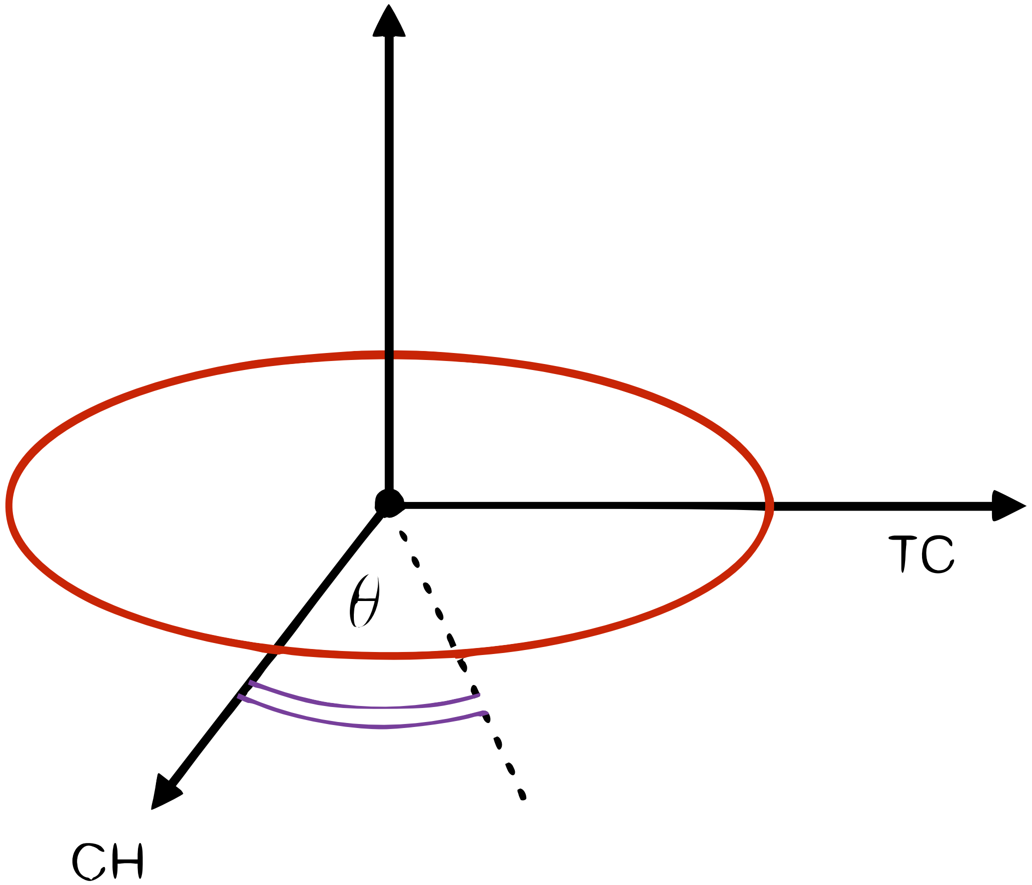
- Model not excluded by LHC data

[Arbey et al - Phys.Rev.D 95 (2017) 1, 015028]

- Higgs couplings $\theta \approx 0$:

$$\frac{g_{WWh_1}}{g^{\rm SM}_{WWh}} = 1 + A\theta + \mathcal{O}(\theta^2)$$

$$\frac{g_{tth_1}}{g^{\rm SM}_{tth}} = 1 + B\theta + \mathcal{O}(\theta^2)$$



A improved approach

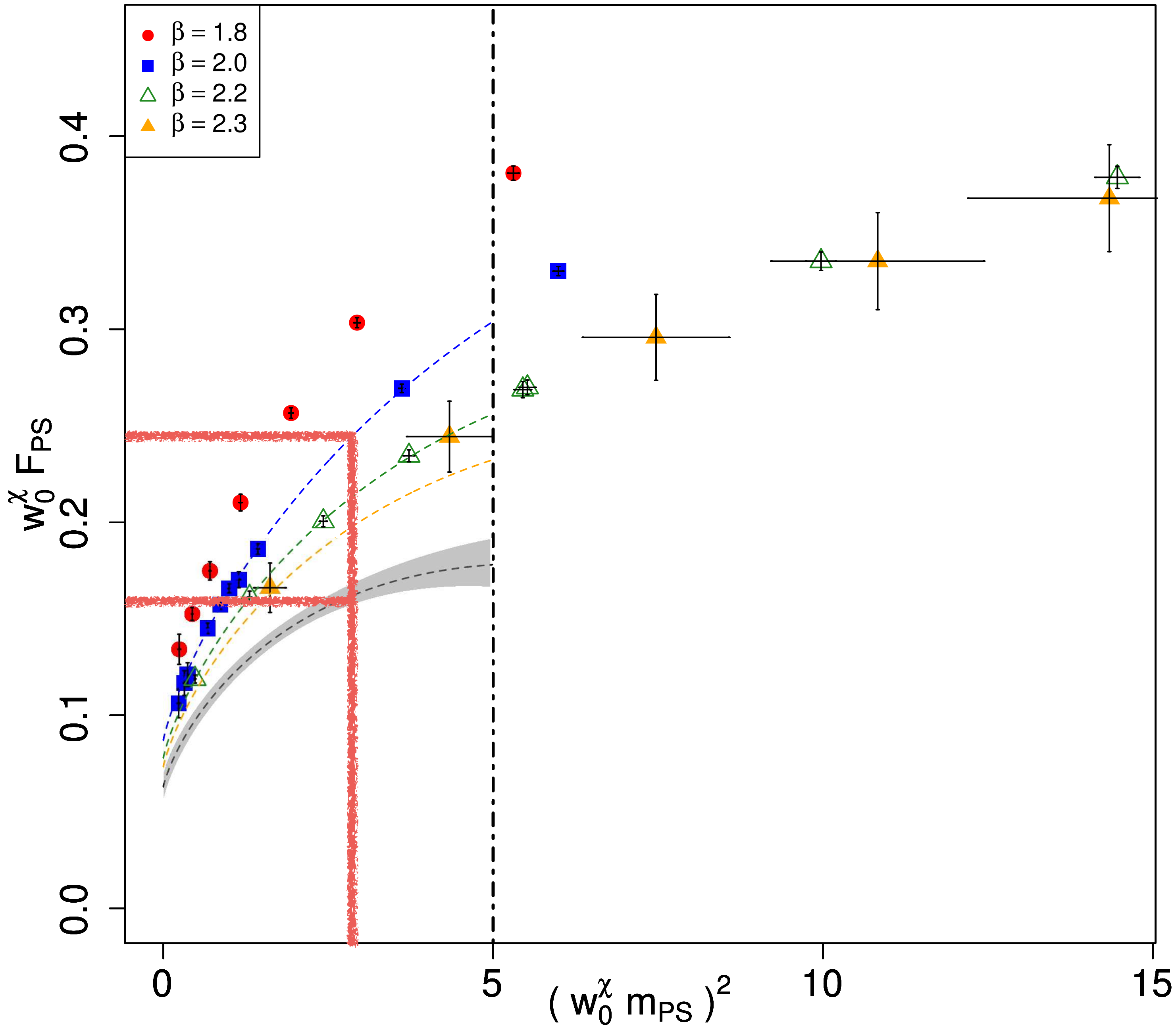
Why an improved action?

R. Arthur, V.D, A. Hietanen, M. Hansen, C. Pica, F. Sannino - Phys.Rev.D 94 (2016) 9, 094507

- Overview of pseudoscalar decay constant f_{PS} for 4 lattice spacing using unimproved Wilson fermions

~0.25

~0.17



~30% discretisation error !

A new Wilson–Dirac operator

[Francis, Fritzsch, Lüscher, Rago - Comput.Phys.Commun. 255 (2020) 107355]

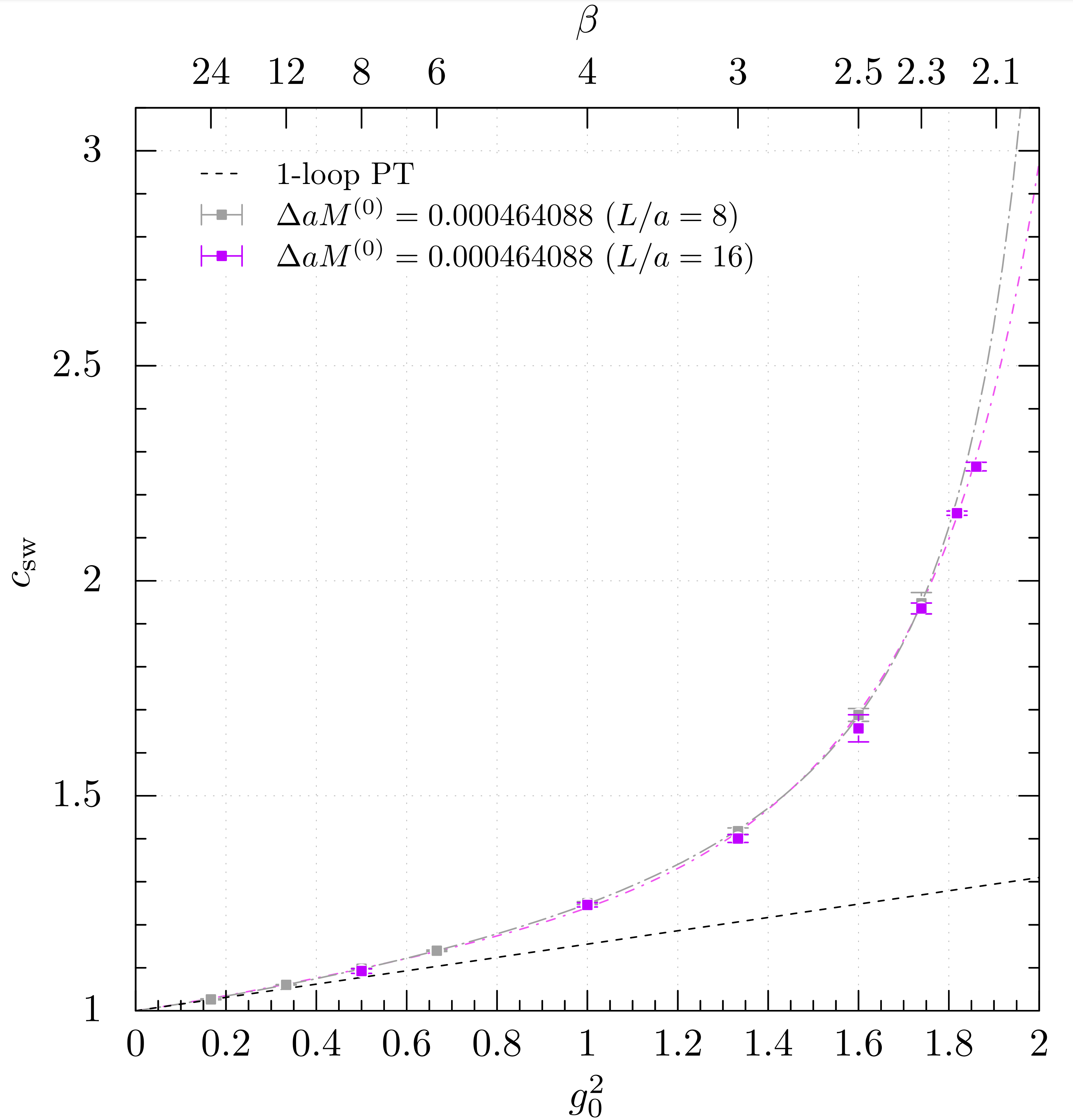
- **Clover improvement** : not protected from arbitrarily small eigenvalues; instabilities in D_{oo}^{-1}

$$D_{\mathrm{ee},m} + D_{\mathrm{oo},m} = (4+m) + c_{\mathrm{sw}} \frac{\mathrm{i}}{4} \sigma_{\mu\nu} F_{\mu\nu}$$

- **Exponential clover** :
  - regulates UV fluctuations;
  - valid Symanzik improvement;
  - guarantees invertibility

$$D_{\mathrm{ee},m} + D_{\mathrm{oo},m} = (4+m_0) \exp\left(\frac{c_{\mathrm{sw}}}{4+m_0} \frac{\mathrm{i}}{4} \sigma_{\mu\nu} F_{\mu\nu} \right)$$

Tuning c_{sw} - final result

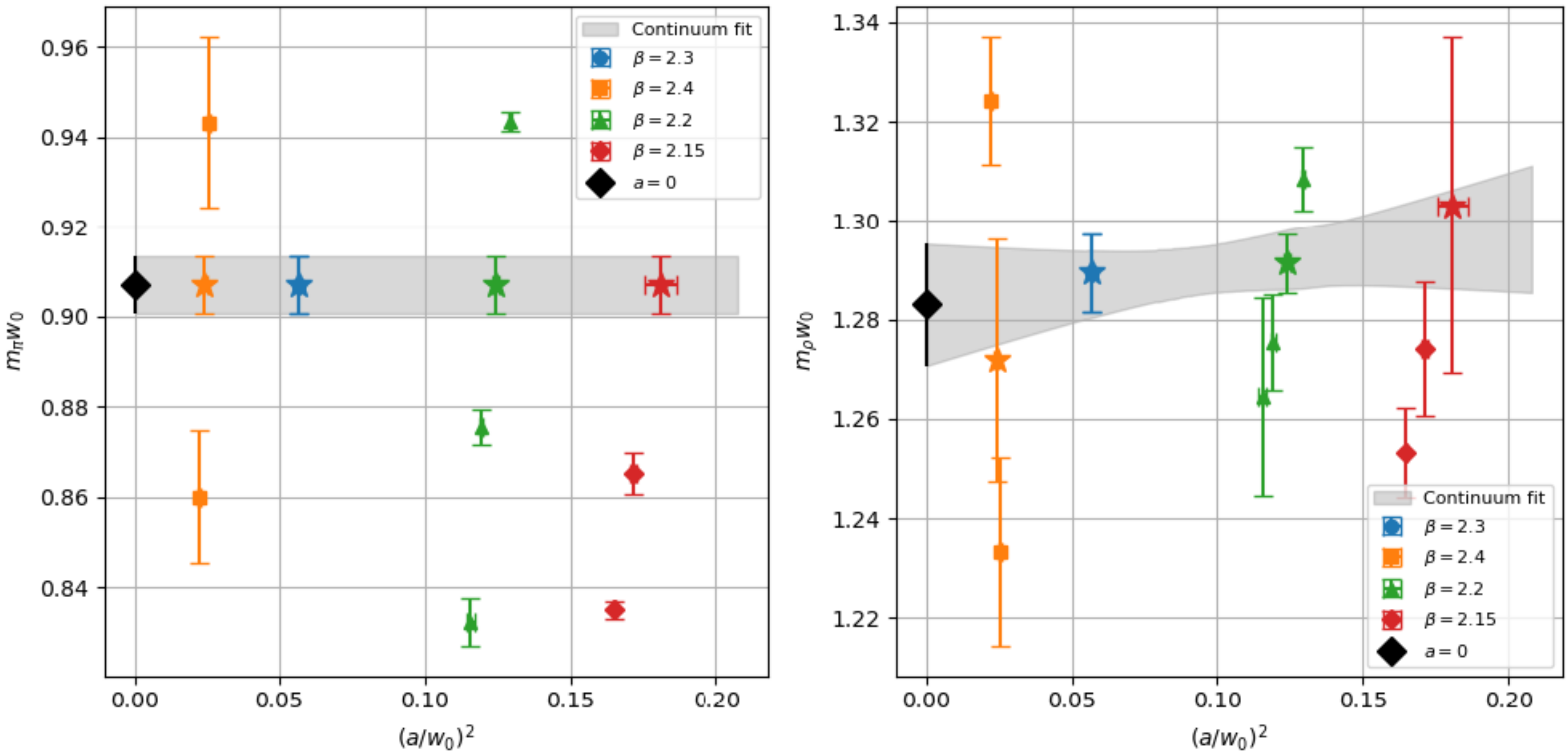


Testing the setup

- **Goal**: control the systematics
- New action: continuum limit at finite mass as a first step
- **Strategy**: continuum limit of f_{PS} and other observables
- Use mixed action setup to avoid non perturbative renormalisation of f_{PS}

→ Twisted mass fermions at maximal twist

Interpolation at fixed mass & continuum limit



- Reference mass (w_0 m_{ps}) = 0.908
- Interpolated values represented by a star

PRELIMINARY

Role of the scalar resonance

The role of the σ

- New sector in isolation features a scalar resonance - the σ
- Electroweak interaction mixes the GBs and the σ
- Expect two scalar resonances at the electroweak scale: the Higgs and a new one
- Resonance is expected to be produced at the LHC similarly to the SM Higgs, i.e. via gluon fusion and vector boson fusion mechanisms

- **Goal**: understand the role of the σ resonance in the phenomenology of of composite Higgs models at scales explored by the LHC

- **Strategy**: determine using lattice simulations the low energy constants of the EFTs that describes the low energy limit of the sector in isolation

The challenge

V. Drach, P. Fritzsch , A Rago. And F. Romero-López, Eur.Phys.J.C 82 (2022) 1, 47

- Calculation of the scattering amplitude in the I=0 channel with tree level clover improved fermions using Lüscher's method
- **Sketch** :
  - Evaluate the 2x2 correlation matrix spanned by the 2 operators

$$\mathcal{O}_{\pi\pi} = \frac{1}{\sqrt{5}}\Big[+\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0 + \Pi_{ud}\Pi_{\bar{u}\bar{d}} + \Pi_{\bar{u}\bar{d}}\Pi_{ud}\Big].$$

$$\mathcal{O}_{\sigma} = \frac{1}{\sqrt{2}}\left[\bar{u}(x)u(x) + \bar{d}(x)d(x)\right]$$

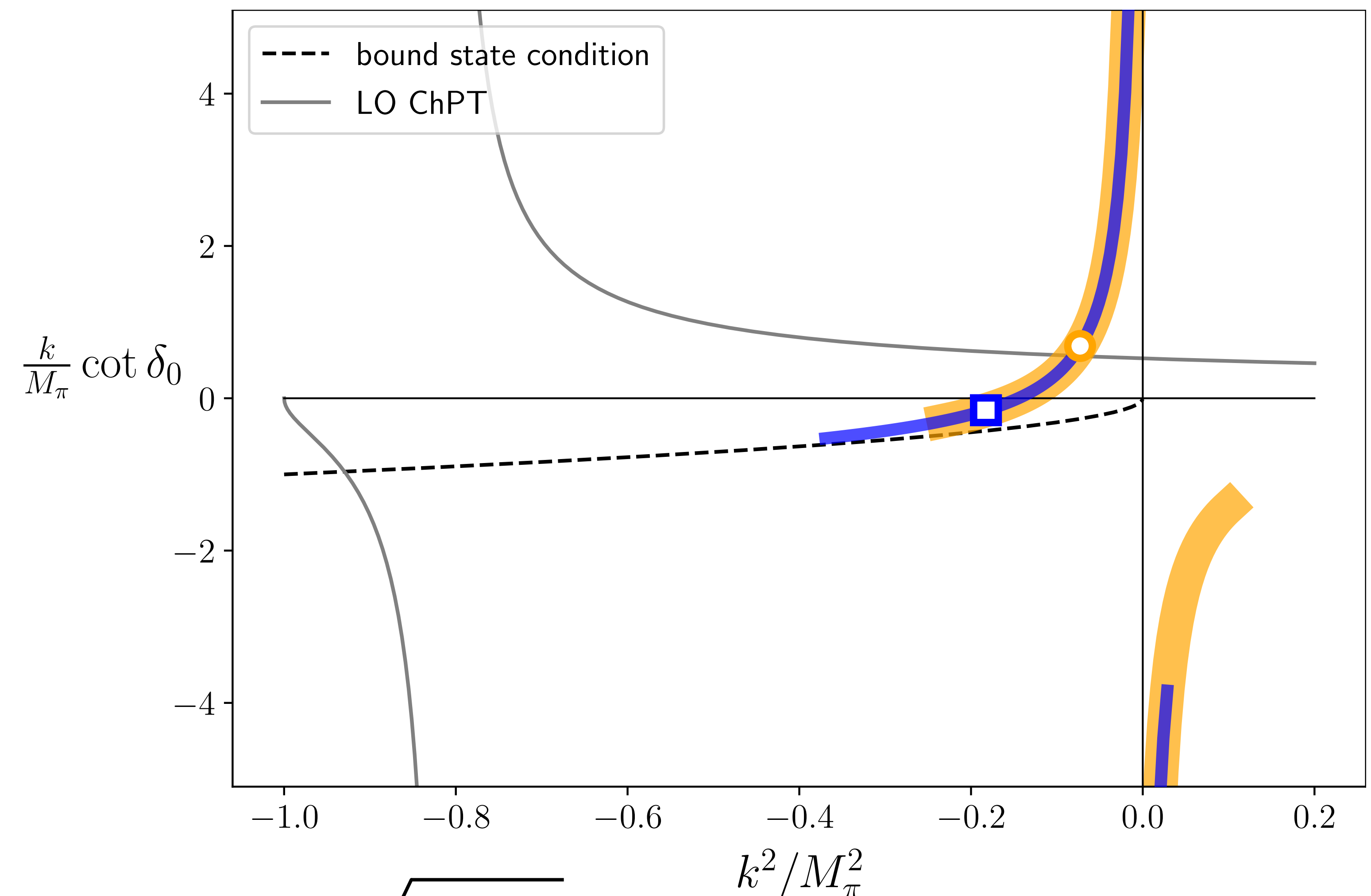
  - Solve the GEVP
  - Eigen-energies are shifted with respect to non-interacting energies -> k is the relative momentum in the center-of-mass (CM) frame
  - finite-volume energy levels are related to the phase shift (quantisation condition)

$$k\cot\delta_0\,(k) = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(\eta^2), \quad \eta = \frac{Lk}{2\pi}$$

The issue

V. Drach, P. Fritzsch , A Rago. And F. Romero-López, Eur.Phys.J.C 82 (2022) 1, 47

- Calculation of the scattering amplitude in the I=0 channel with tree level clover improved fermions using Lüscher's method
- k is the relative momentum in the CMF



- Bound state condition : $k \cot \delta_0 (k) = -\sqrt{-k^2}$

→ suggests σ is stable for $m_v/m_{ps} \sim 2.4$

→ currently running $m_V/m_{ps} \sim 3$ in the new setup.

Conclusion

- Lattice simulation a laboratory to explore non-perturbative dynamics of gauge theories
- HiRep 2.0 : **a polyvalent tool to explore theories beyond QCD**
- $SU(2)_c + N_f{=}2$: UV completion for a CH/TC scenario
- New non-perturbatively improved simulations with exponential clover discretisation - **first continuum limit with improved fermions**
- **First calculation of the scattering amplitude in the singlet channel**
- Challenge: the σ is stable over a large range of mass.
- On-going simulations: targeting $m_V/m_{ps} \sim 3$