Connecting QA with HEP simulations through LTD

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Junta de Castilla y León

VNiVERSiDAD DSALAMANCA

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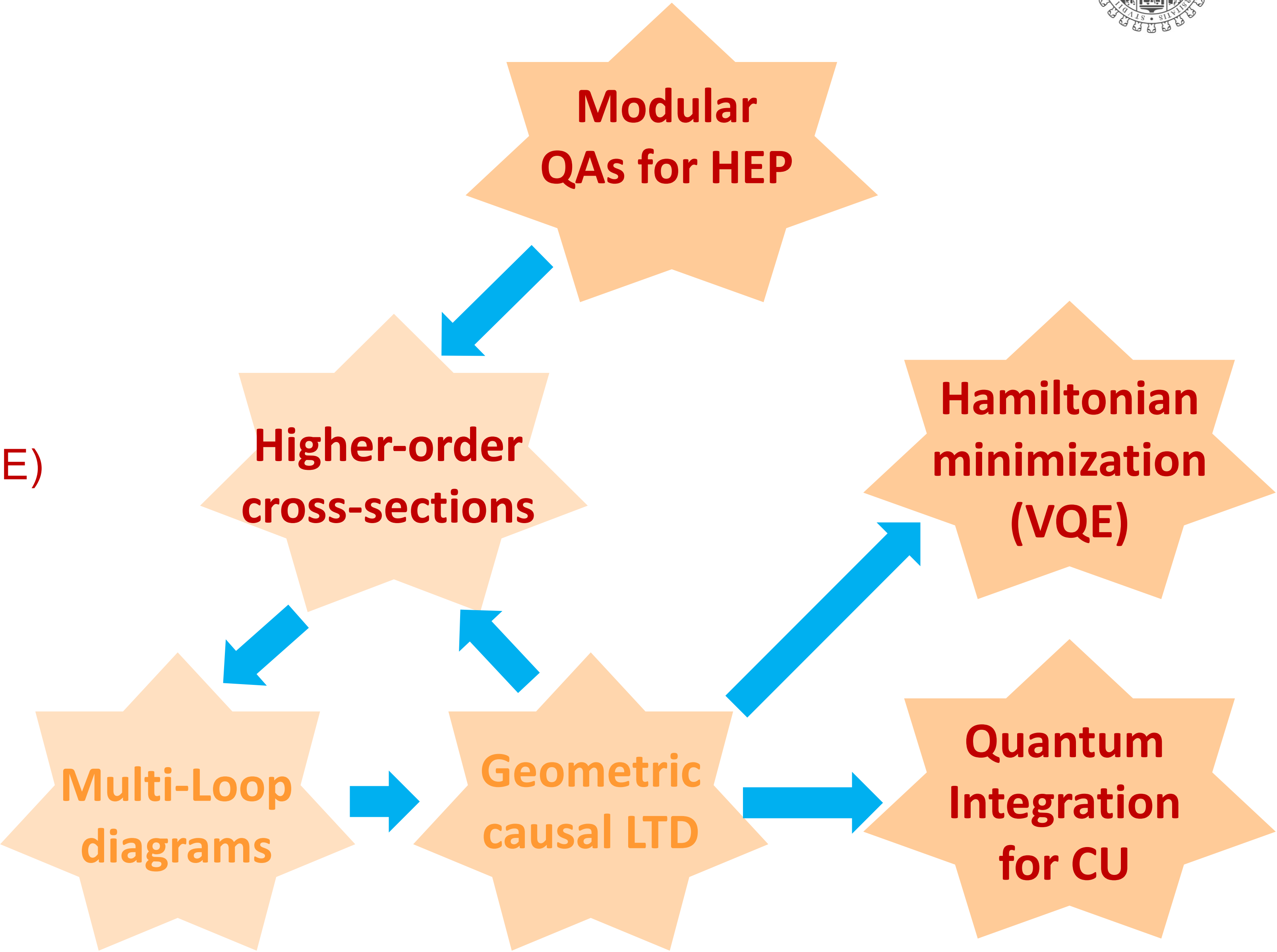
Index

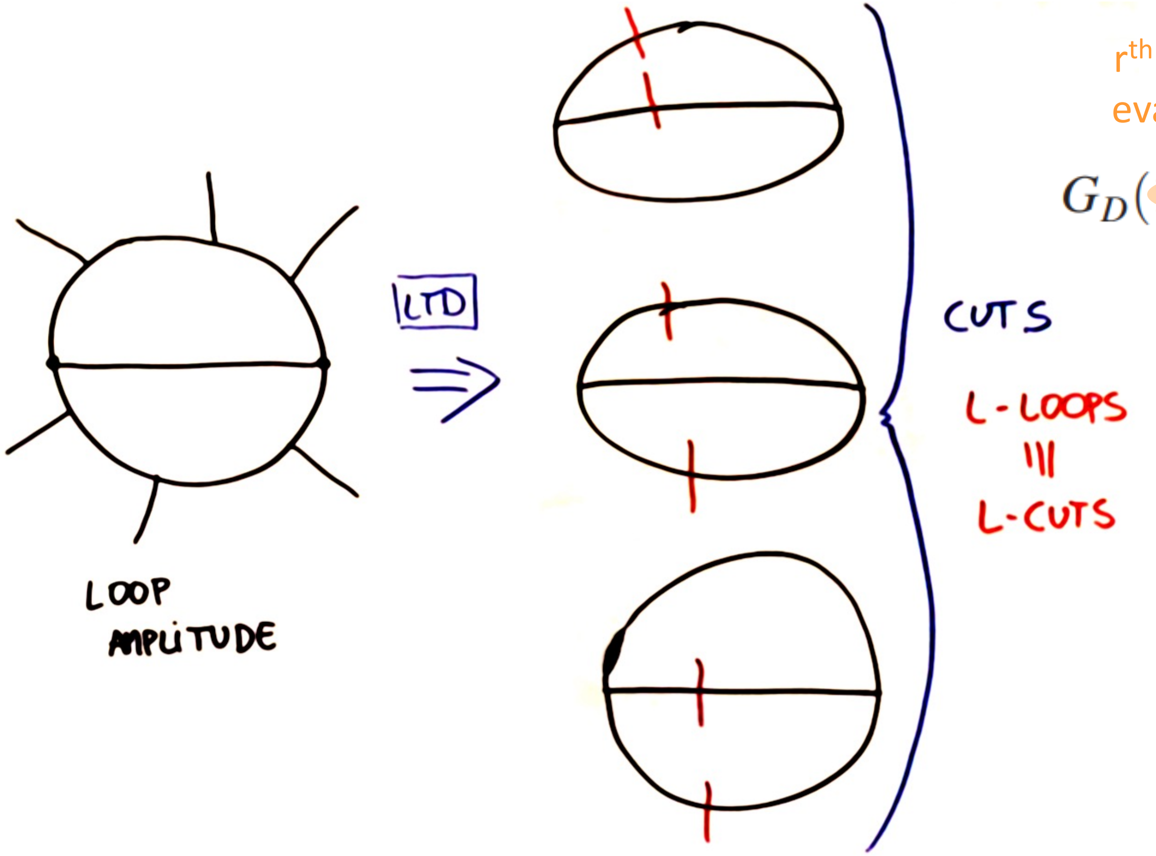
1. Motivation: Loop-Tree Duality
    A. Graphical Causality
    B. Causal Unitarity
    C. Finite Integrals
2. Exploiting LTD to develop QA
    A. Hamiltonian minimization (VQE)
    B. QA for Causal Unitarity
    C. Modular QAs for HEP
3. Conclusions



Introducing the (modern) Loop-Tree Duality

- **Main strategy**: Iterate Cauchy's theorem to *open loops into trees*
- Energy component is removed by using **Cauchy's residue theorem**
- **Multiloop require to iterate ("nest") the procedure (remove all the energy components)**

Rodrigo & collaborators, PRL 124 (2020) 21, 211602; JHEP 12 (2019) 163; JHEP 01 (2021) 069; JHEP 02 (2021) 112; JHEP 04 (2021) 129



r^{th} residue evaluation

Remaining sets (no residue evaluation)

$$G_D(1,\ldots,r;n) = -2\pi\imath \sum_{i_r\in r} \mathrm{Res}(G_D(1,\ldots,r-1;r,n), \mathrm{Im}(\eta\cdot q_{i_r})<0)$$

Sum over all the elements of the r^{th} set

$(r-1)^{th}$ dual function

Depends on integration variables (q_i)

FEYNMAN REP.

$$\xrightarrow[q]{} = \underbrace{\frac{1}{q_0 - q_0^{(+)}} - \frac{1}{q_0 + q_0^{(+)}}}$$

$$q_0^{(+)} = \sqrt{|\vec{q}|^2 + m^2 - i0}$$

ON-SHELL ENERGY

Master formula $$\mathcal{A}_N^{(L)}(1,\ldots,L+k) = \sum_{\sigma\in\Sigma} \int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{\mathcal{N}_\sigma(\{q_{r,0}^{(+)}\},\{p_{j,0}\})}{x_{L+k}} \times \prod_{i=1}^{k} \frac{1}{-\lambda_{\sigma(i)}} + (\sigma \leftrightarrow \bar{\sigma})$$

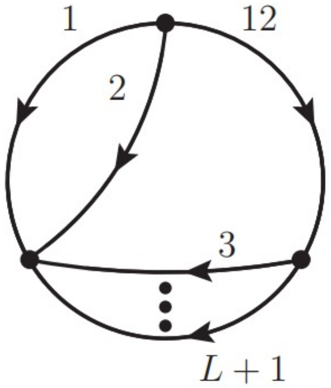
Set of entangled thresholds

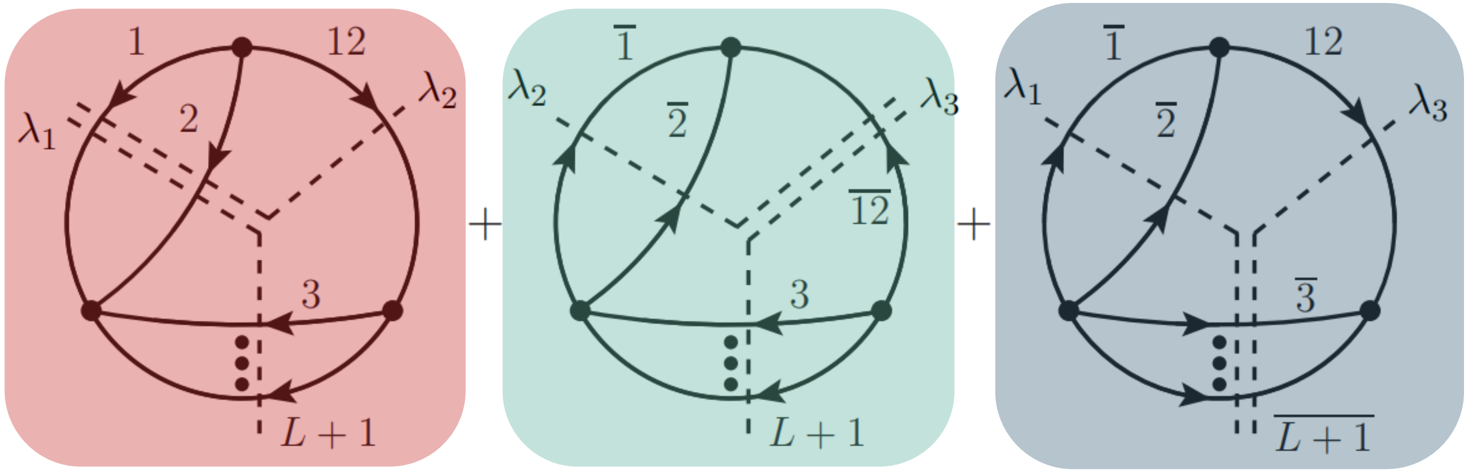
Products of *k* causal propagators

- **Graphical interpretation in terms of entangled thresholds**
  1. **Each causal** propagator represents a **threshold** of the diagram
  2. **Each diagram** contains **several thresholds**
  3. **The causal representation involves products of (*compatible*) thresholds

More details in: JHEP 01 (2021) 069; JHEP 04 (2021) 129; JHEP 04 (2021) 183; Eur.Phys.J.C 81 (2021) 6, 514

Causal denominators (λ) are associated to *cut lines* in the diagrams: momenta flow must be adjusted to be compatible

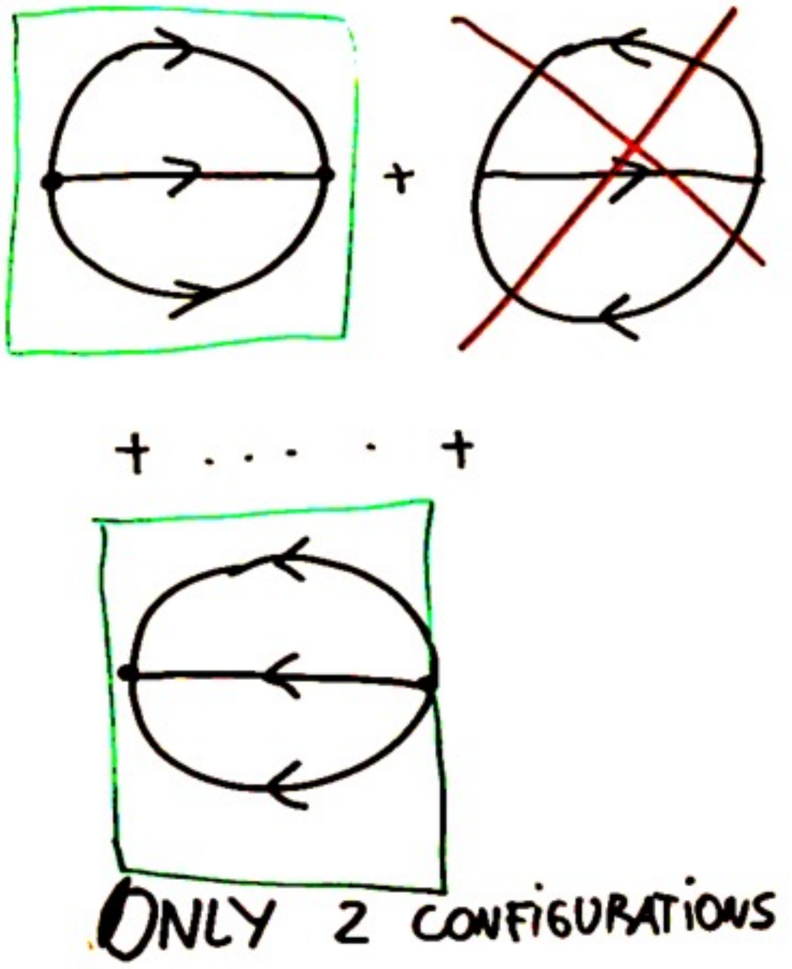




$$\mathcal{A}_{\rm NMLT}^{(L)}(1,2,\ldots,L+2) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

$$\boxed{\lambda_{i_1\cdots i_n \bar{a}} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} - p_{a,0}^{(+)}}$$

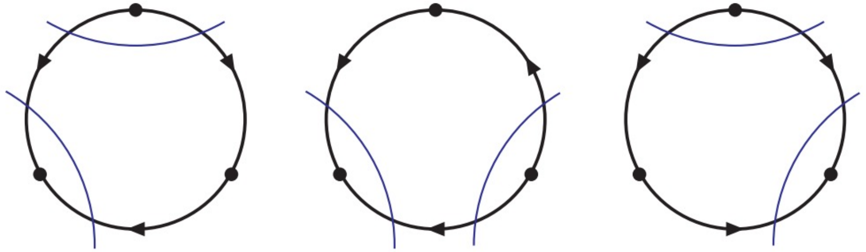
Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129



- ***Geometrical rules look into several properties of the graph***

  "If the graph can be cut in tree-level pieces (thresholds) with consistent momentum flow (entangled thresholds), then we have a causal representation"

- **IMPORTANT:** **Causal configurations MUST FULLFIL the causal-compatible flux condition → Bootstrapping!!**

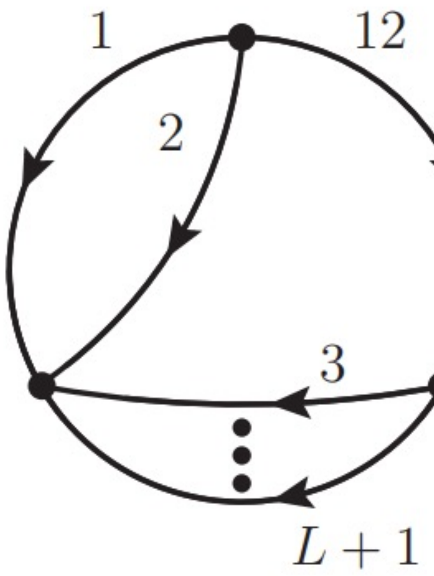


- **From graph theory:** *"Directed oriented graphs that can be split into binary partitions fulfilling (*) have no closed directed cycles"*

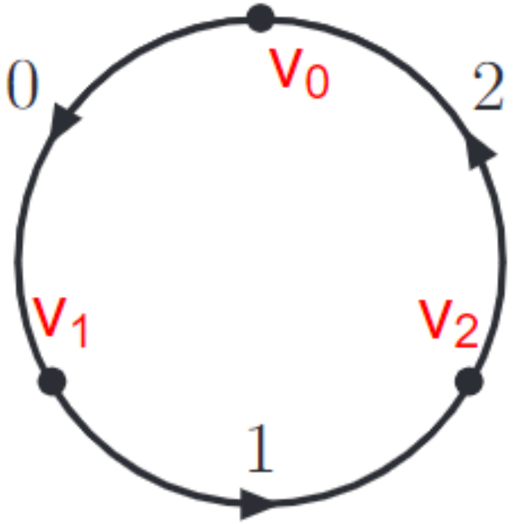
Non-cyclical configurations = Causal flux

More detailed explanation arXiv:2102.05062 [hep-ph] & arXiv:2105.08703 [hep-ph]

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129



Topologically equivalent diagrams



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- *Geometrical rules look into several properties of the graph*

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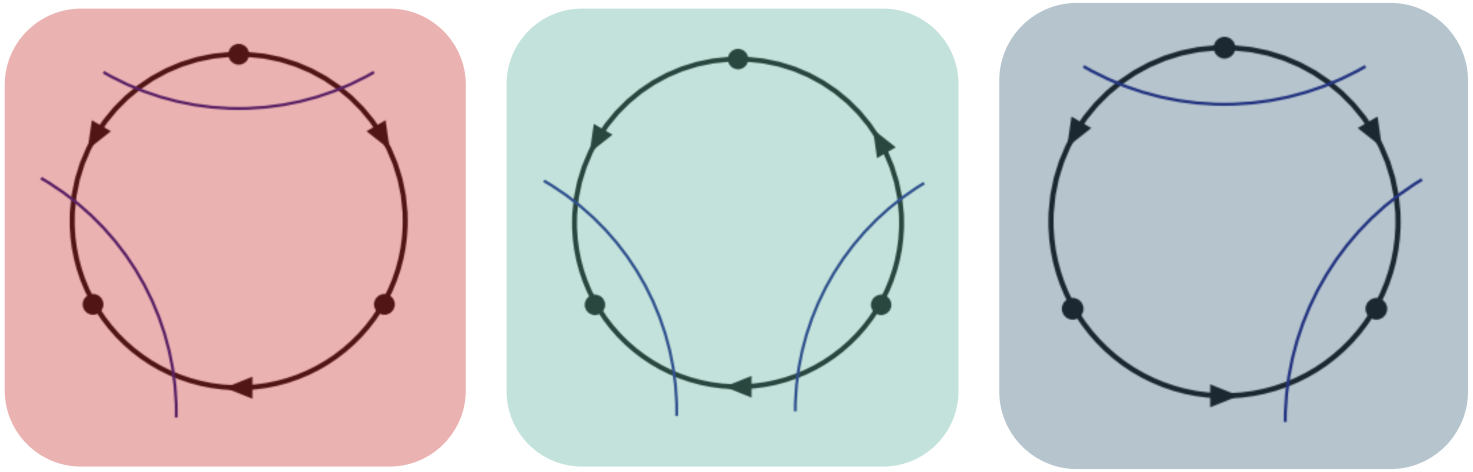


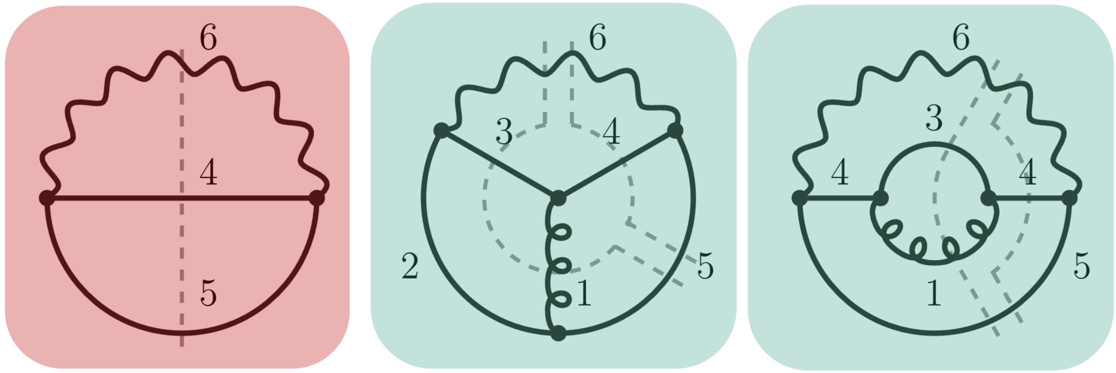
$$= \int_{\vec{\ell}_1} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1 \lambda_2} + \frac{1}{\lambda_2 \lambda_3} + \frac{1}{\lambda_3 \lambda_1} \right)$$

Aguilera-Verdugo et al, JHEP 01 (2021) 069
Ramírez Uribe et al, JHEP 04 (2021) 129

Causal Unitarity

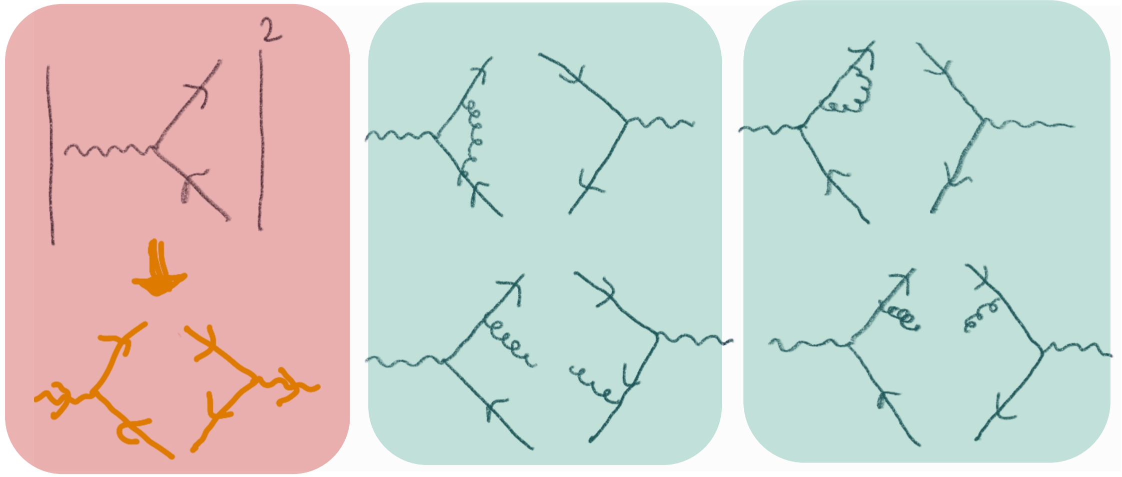
- Graphical causality motivates a connection between cross-sections and vacuum diagrams
- **Novel technology to compute cross-sections: <u>Causal Unitarity (CU)</u>**

$$\gamma^* \to q\bar{q}(g)$$



Tree-level (eq. 2-loop)

Next-to-leading order (eq. 3-loop)



- Within CU, real, virtual and IR counter-terms are simultaneously generated from multiloop vacuum diagram
- **Fully local cancellation of physical singularities!**

Developed by LTD Collaboration: Phys.Rev.Lett. 133 (2024) 21, 211901; JHEP 01 (2025) 103

Rodrigo et al, arXiv:2404.05491 & 2404.05492 [hep-ph]

- **Residues of vacuum diagrams on selected causal thresholds ➡ Generate h.o. terms**
- BONUS: Phase-space measure is automatically generated (including momentum conservation)
- **Results are numerically stable thanks to local cancellation of singularities**

Loop integration variables

$$d\Lambda = \prod_{j=1}^{\Lambda-1} d\Phi_{\ell_j} = \prod_{j=1}^{\Lambda-1} \mu^{4-d} \frac{d^{d-1}\ell_j}{(2\pi)^{d-1}}$$

Phase-space measure

$$\tilde{\Delta}_{i_1 \cdots i_n \bar{a}} = 2\pi\, \delta(\lambda_{i_1 \cdots i_n \bar{a}})$$

$$\lambda_{i_1 \cdots i_n \bar{a}} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} - p_{a,0}^{(+)}$$

Master Formula (decay rates)

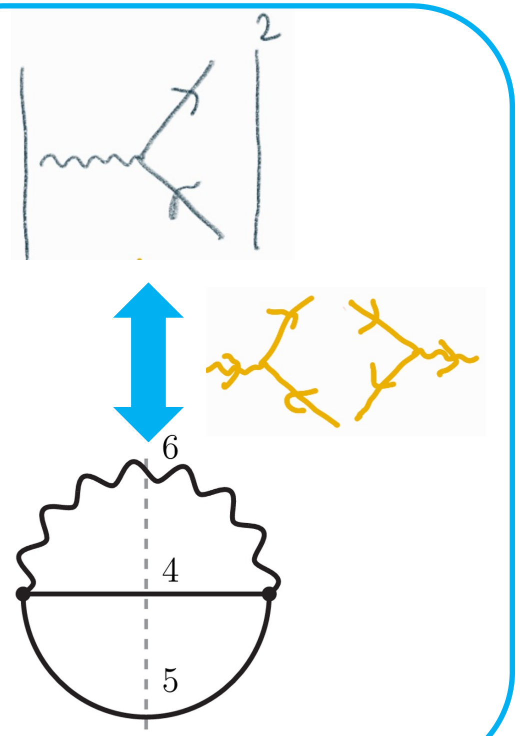
$$\boxed{d\Gamma_a^{(k)} = \frac{d\Lambda}{2m_a} \sum_{(i_1 \cdots i_n a) \in \Sigma} \mathcal{A}_{\mathrm{D}}^{(\Lambda,\mathrm{R})}(i_1 \cdots i_n a)\, \mathcal{O}_{i_1 \cdots i_n}\, \tilde{\Delta}_{i_1 \cdots i_n \bar{a}}}$$

$$\mathcal{A}_{\mathrm{D}}^{(\Lambda,\mathrm{R})}(i_1 \cdots i_n a) = \mathcal{A}_{\mathrm{D}}^{(\Lambda)}(i_1 \cdots i_n a) - \mathcal{A}_{\mathrm{UV}}^{(\Lambda)}(i_1 \cdots i_n a)$$

Observable definition
(e.g. kinematics cuts)

$$\mathcal{A}_{\mathrm{D}}^{(\Lambda)}(i_1 \cdots i_n a) = \mathrm{Res}\left(\frac{x_a}{2} \mathcal{A}_{\mathrm{D}}^{(\Lambda)}, \lambda_{i_1 \cdots i_n a}\right)$$

Residue of the causal representation
of unrenormalized vacuum diagram



Rodrigo et al, arXiv:2404.05491 & 2404.05492 [hep-ph]

Causal Unitarity

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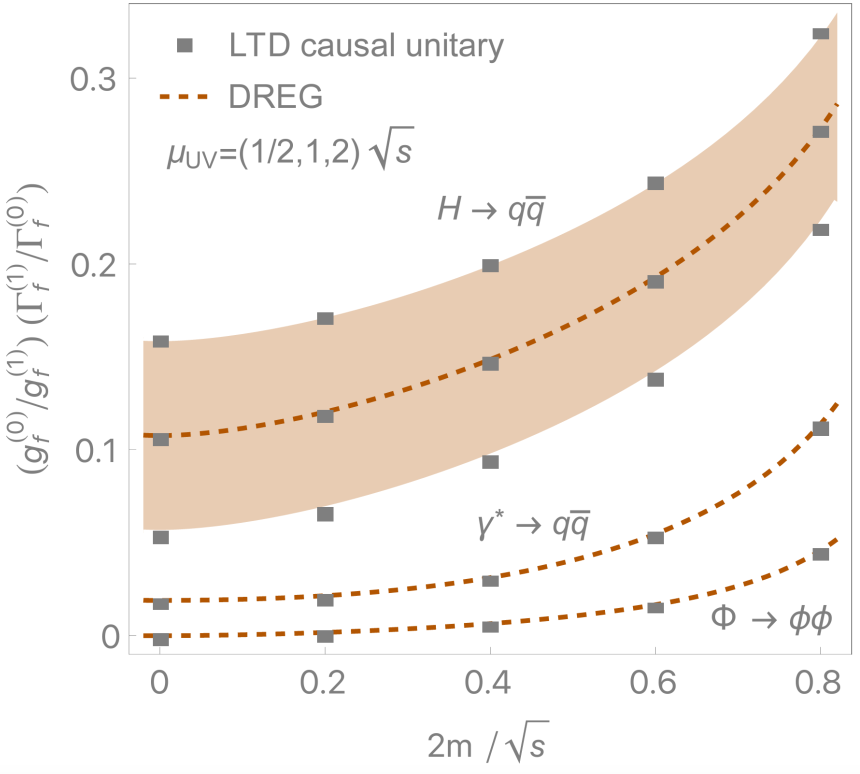
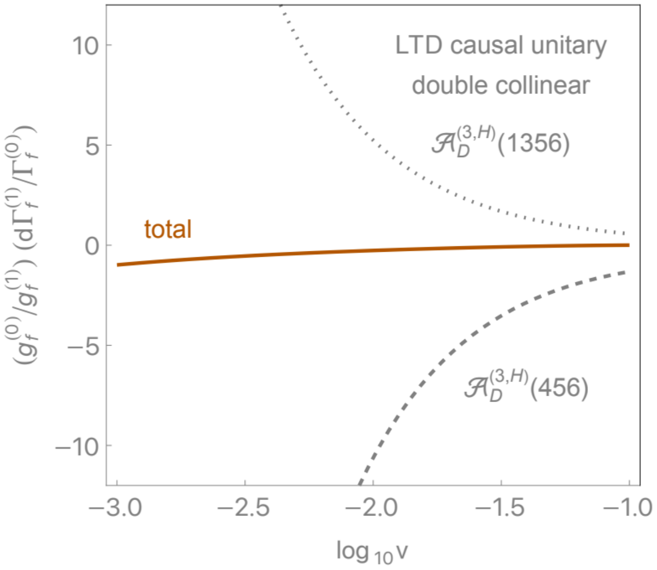
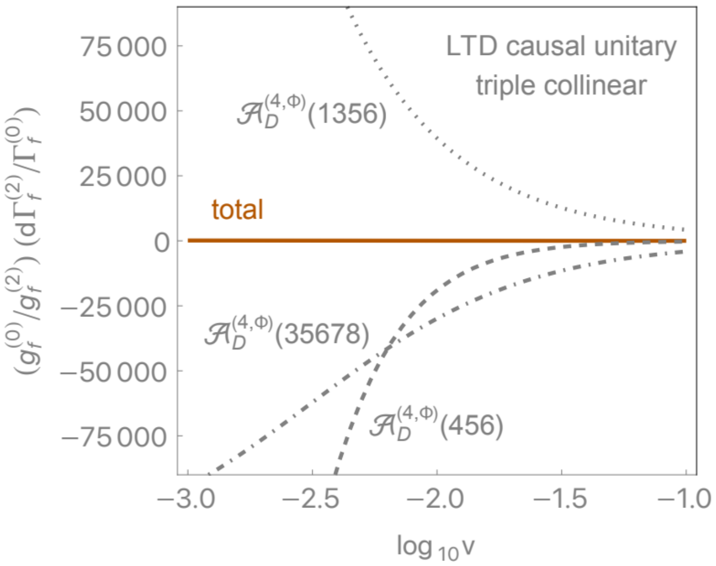






NLO (3 loops)

NNLO (4 loops)

Local cancellation of collinear singularities between phase-space residues with different numbers of final-state particles

Numerical implementation of 1->2 decay processes at NLO as a function of the mass

Perfect agreement with fully local LTD-based FDU results: JHEP 10 (2016) 162; JHEP 08 (2016) 160; JHEP 02 (2016) 044

Rodrigo et al, arXiv:2404.05491 & 2404.05492 [hep-ph]

Finite integrals: exploiting LTD and causality

- **Why finite Feynman integrals?** Because they are less challenging for numerical calculations and could be used as a basis to express other integrals.
- **Why LTD could offer an advantage?** Because LTD causal representation eases the identification of singularities, allowing to neutralize them.
- **Based on the LTD causal representation, we explored three strategies.**

1. Numerator Ansatz Strategy

Step 1: Construct the Ansatz

Define target integrand numerator with arbitrary coefficients

$$\mathcal{N} = \underbrace{\alpha_0}_{\text{arbitrary coefficients}} + \alpha_1 \underbrace{\ell_1 \cdot p_1}_{\text{loop and external momenta}} + \ldots$$

Step 2: Map to LTD

Transform target integrand into its LTD representation

$$\underbrace{f(q_{i_s,0}^{(+)})}_{\text{This is a function of on-shell energies}}$$

Step 3: Nullify Residues

Impose constraints to force collinear residues to vanish

$$\mathrm{Res}\left(\underbrace{\mathcal{A}_{\mathrm{D}}^{(\Lambda)}}_{\text{LTD integrand}}, \underbrace{\lambda_{i_r \cdots i_2;\bar{i}_1}}_{\text{collinear causal denominator}}\right) = 0$$

Step 4: Solve System

Generate a system of linear equations for the coefficients of the monomials in the on-shell energies given by the residues

$$\alpha_i = \cdots \leftarrow \text{determine arbitrary coefficients}$$

Rodrigo et al, arXiv:2603.18691 [hep-ph]
G. Rodrigo's talk at Loops and Legs 2026

Extracted from G. Rodrigo's talk, LL'26

Finite integrals: exploiting LTD and causality

2. LTD-intrinsic Numerator Ansatz Strategy

Step 1:
Construct the Ansatz

Define target integrand numerator with arbitrary coefficients

$$\mathcal{N} = f(\alpha_i, q^{(+)}_{i_s,0})$$

arbitrary coefficients

on-shell energies

Step 2:
Map to LTD

Transform target integrand into its LTD representation

$$f(q^{(+)}_{i_s,0})$$

This is a function of on-shell energies

Step 3:
Nullify Residues

Impose constraints to force collinear residues to vanish

$$\mathrm{Res}\left(\mathcal{A}^{(\Lambda)}_{\mathrm{D}}, \lambda_{i_r\cdots i_2;\bar{i}_1}\right) = 0$$

LTD integrand

collinear causal denominator

Step 4:
Solve System

Generate a system of linear equations for the coefficients of the monomials in the on-shell energies given by the residues

$$\alpha_i = \cdots \quad \leftarrow$$

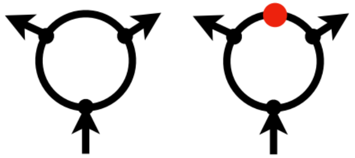
determine arbitrary coefficients

The numerator ansatz is defined in term of on-shell energies

The coefficients are such that the numerators can be expressed in terms of causal thresholds

$$\mathcal{N}^{(1,\mathrm{UV})|d}_{(1,1,1)} = \mathcal{N}^{(1,\mathrm{UV})|d+2}_{(2,1,1)} = \alpha_0 \lambda_{31;\bar{1}}\lambda_{12;\bar{2}}$$

$$\mathcal{N}^{(1,\mathrm{UV})|d}_{(2,1,1)} = \alpha_0 \lambda^2_{31;\bar{1}}\lambda^2_{12;\bar{2}}$$



LIMITATION: Adding a numerator could worsen UV behaviour. Also, there are some singularities that do not occur simultaneously, thus the numerator proposed artificially enhance the UV scaling.

Rodrigo et al, arXiv:2603.18691 [hep-ph]
G. Rodrigo's talk at Loops and Legs 2026

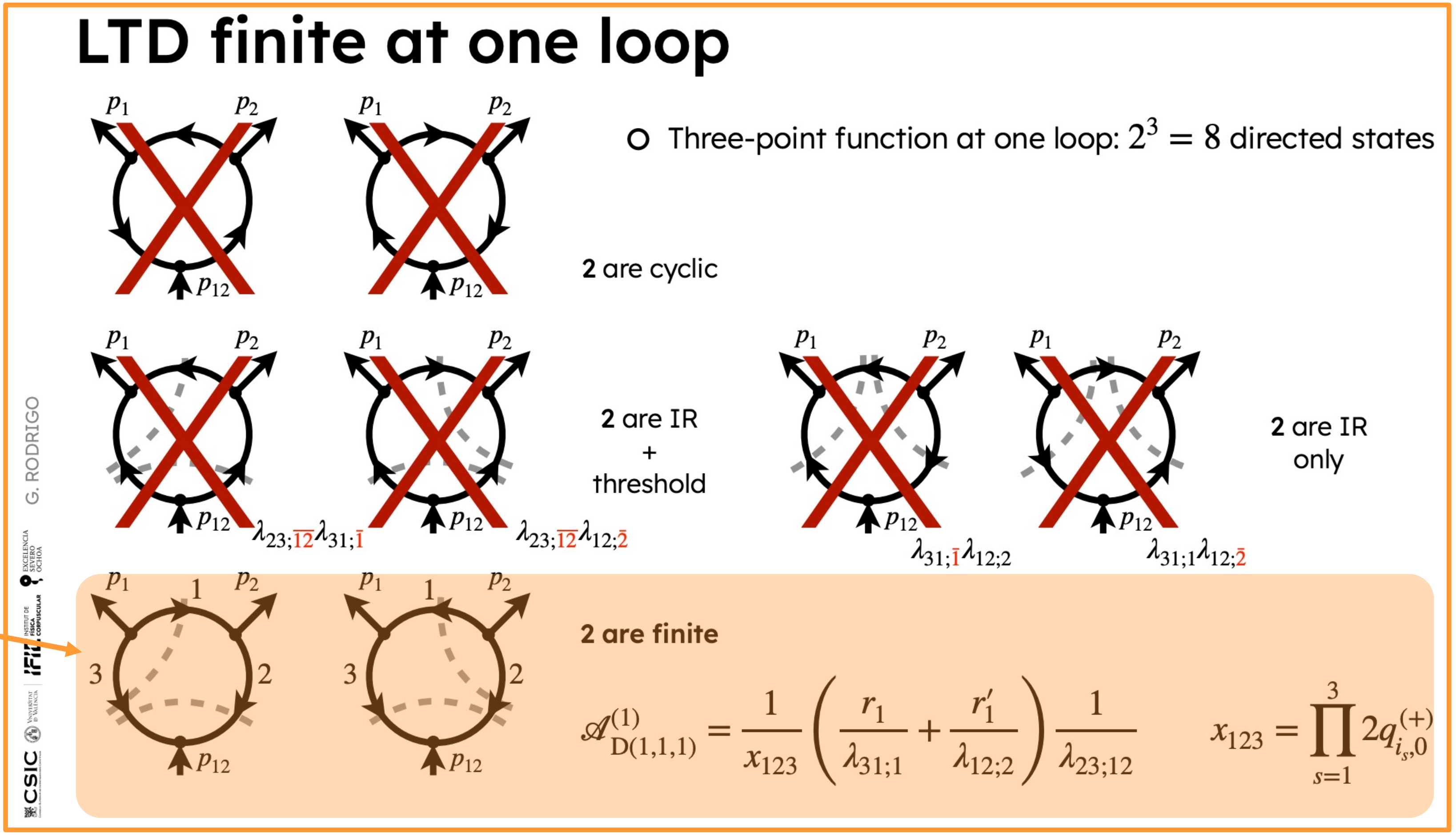
Extracted from G. Rodrigo's talk, LL'26

3. LTD Causal Decomposition Strategy

ALTERNATIVE: Exploit causal decomposition to remove IR and threshold singular contributions.

$$\boxed{\lambda_{i_1 \cdots i_n \bar{a}} = \sum_{s=1}^{n} q_{i_s,0}^{(+)} - p_{a,0}^{(+)}}$$

If external particles have a fixed momentum flow, we can isolate non-singular causal propagators.



Rodrigo et al, arXiv:2603.18691 [hep-ph]

MULTILOOP CASE: Partial fractioning could be required to isolate non-singular terms.

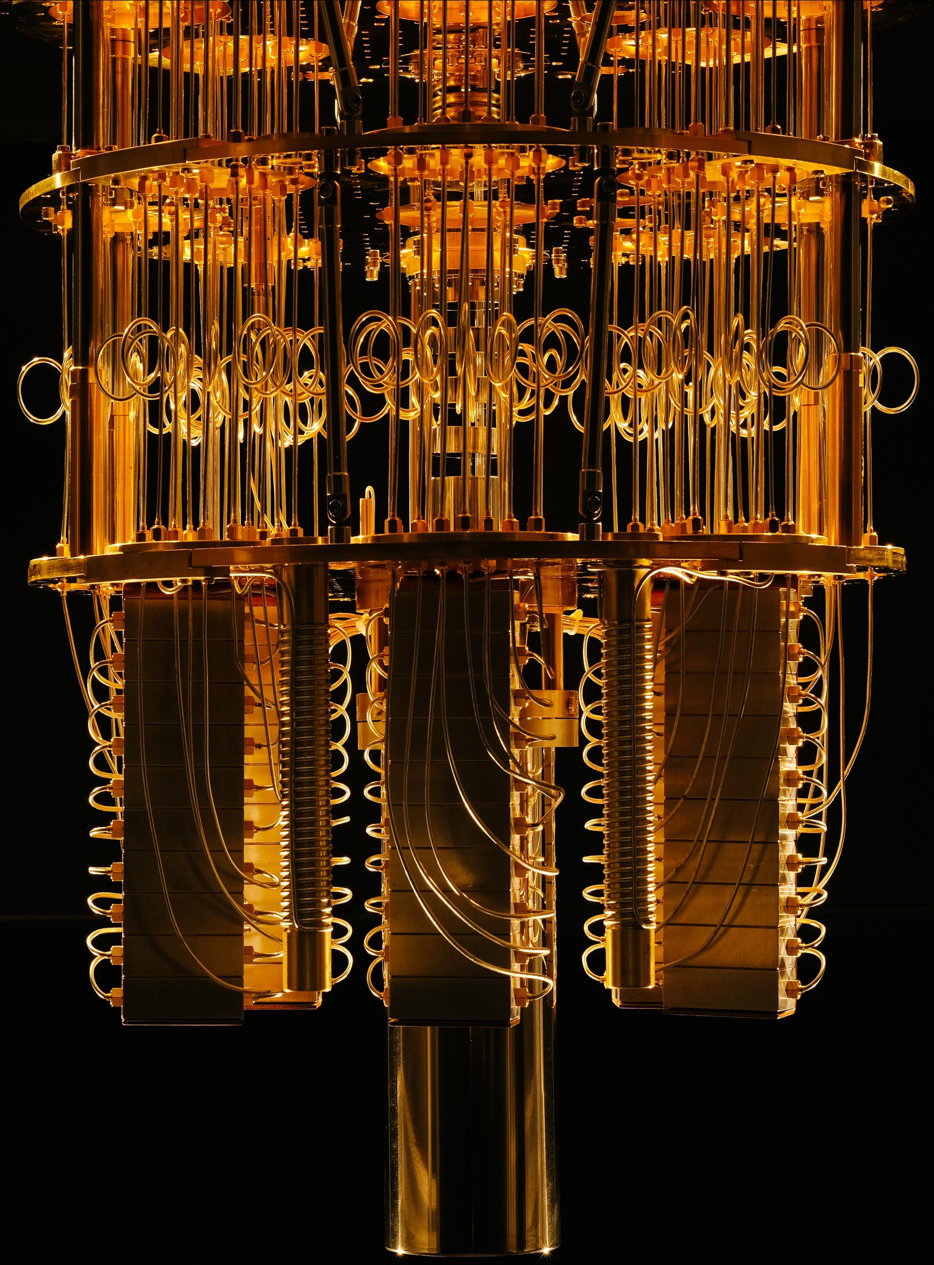
Extracted from G. Rodrigo's talk, LL'26

- In order to apply quantum computing in HEP, we MUST CODIFY OUR CALCULATIONS in a language that quantum computer naturally understand.
- **LTD and causal LTD could offer this classical-quantum bridge.**
- **We explored three "quantum approaches" for HEP:**

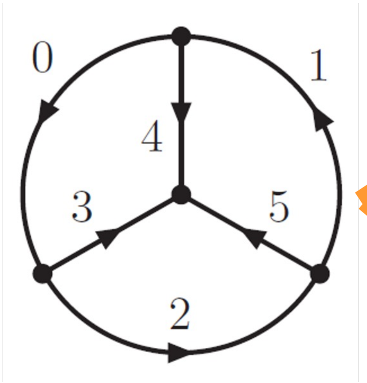
Quantum minimization (causal rep. bootstrap)

Quantum integration (cross-sections with CU)

Quantum encoding (modular HEP simulations)



- Geometrical information is codified in the **adjacency matrix**

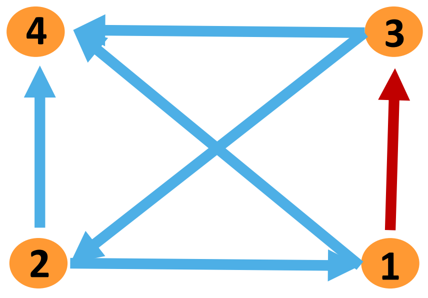


N2MLT topology:
Mercedes-Benz diagram

```
(*Configuracion momentos*)
NumeroVertices = 4; Orden = NumeroVertices - 1;
Eq[1] = {q[6] + q[4] - q[1] + p[1]};
Eq[2] = {q[1] + q[5] - q[2] + p[2]};
Eq[3]  = {q[3] - q[6] + q[2] + p[3]};
Eq[4] = {-q[3] - q[4] - q[5] - (p[1] + p[2] + p[3])};
```

$$\left(\begin{array}{ccccc|cccc} -1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & -1 & -1 & -1 \end{array}\right)$$

Vertex matrix
(from the conservation equations)



$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & \cdots \\ \neg a_{12} & 0 & a_{23} & \cdots \\ \neg a_{13} & \neg a_{23} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency matrix
(how are vertices connected)

- **Useful properties:**

$$f_3(A) \equiv \Sigma_{i_1,i_2,i_3}^{i_1\neq i_2\neq i_3} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_1} \neq 0$$ ⟺ There are triangles

$$f_4(A) \equiv \Sigma_{i_1,i_2,i_3,i_4}^{i_1\neq i_2\neq i_3\neq i_4} a_{i_1 i_2} \times a_{i_2 i_3} \times a_{i_3 i_4} \times a_{i_4 i_1} \neq 0$$ ⟺ There are boxes

In general, there are N-cycles **iff** $f_N(A)$ is non zero

A graph with associated adjacency matrix A is acyclic if:

$$\sum_{n=1}^{V} \mathrm{tr}(A^n) = 0$$

Clemente et al, PRD 108 (2023) 9, 096035
Sborlini, PRD 104 (2021) 3, 036014

Quantum minimization: causal rep. bootstrap

- Exploit the **adjacency matrix** to build a **Hamiltonian** → **Ground state = Acyclic graph**
- **1st approach:** Penalize oriented cycles using projectors to build the **loop Hamiltonian**

$$H_G = \sum_{\gamma \in \Gamma_{G_0}} \prod_{e \in \gamma} \pi_e^{s(e;G_0)}$$

Orientation (0 or 1)

Cycles (within graph)

Edges (within cycle)

Projector

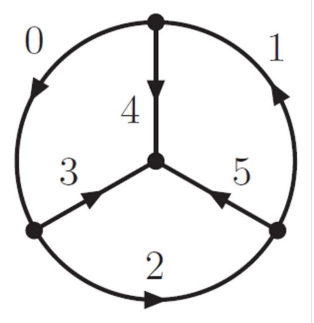
with

$$\pi_e^0 \equiv |0\rangle\langle 0|_e = \frac{1}{2}(I+Z)_e$$

$$\pi_e^1 \equiv |1\rangle\langle 1|_e = \frac{1}{2}(I-Z)_e$$

0 original direction

1 reversed direction



Need to fix a initial orientation, but results are independent of this choice!!

- **2nd approach (BETTER):** Promote adjacency matrix to operator, and *trace over all possible cycles*

$$A \equiv \sum_{e \equiv (v_0, v_1) \in \bar{E}} \left[\sigma_{v_0}^{-} \pi_e^0 \sigma_{v_1}^{+} + \sigma_{v_1}^{-} \pi_e^1 \sigma_{v_0}^{+} \right]$$

Edge = (origin, end)

Pauli +/- operators

$$\boxed{H_G = \sum_{n=1}^{M_G} \mathrm{Tr}_V[A^n]}$$

Trace over vertex space

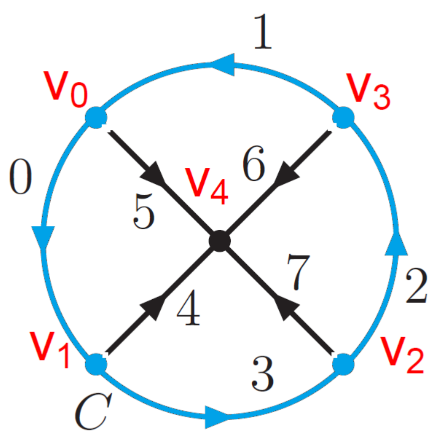
n = cycle length

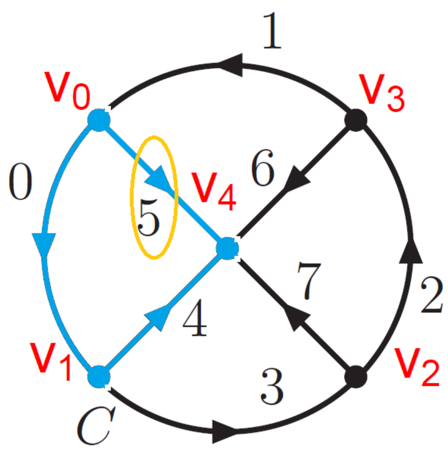
More details arXiv:2210.13240 [hep-ph]

Minimizing H, we find the acyclic graphs (0 energy)

Clemente et al, PRD 108 (2023) 9, 096035

- Exploit the **adjacency matrix** to build a **Hamiltonian** → **Ground state = Acyclic graph**





A few comments:

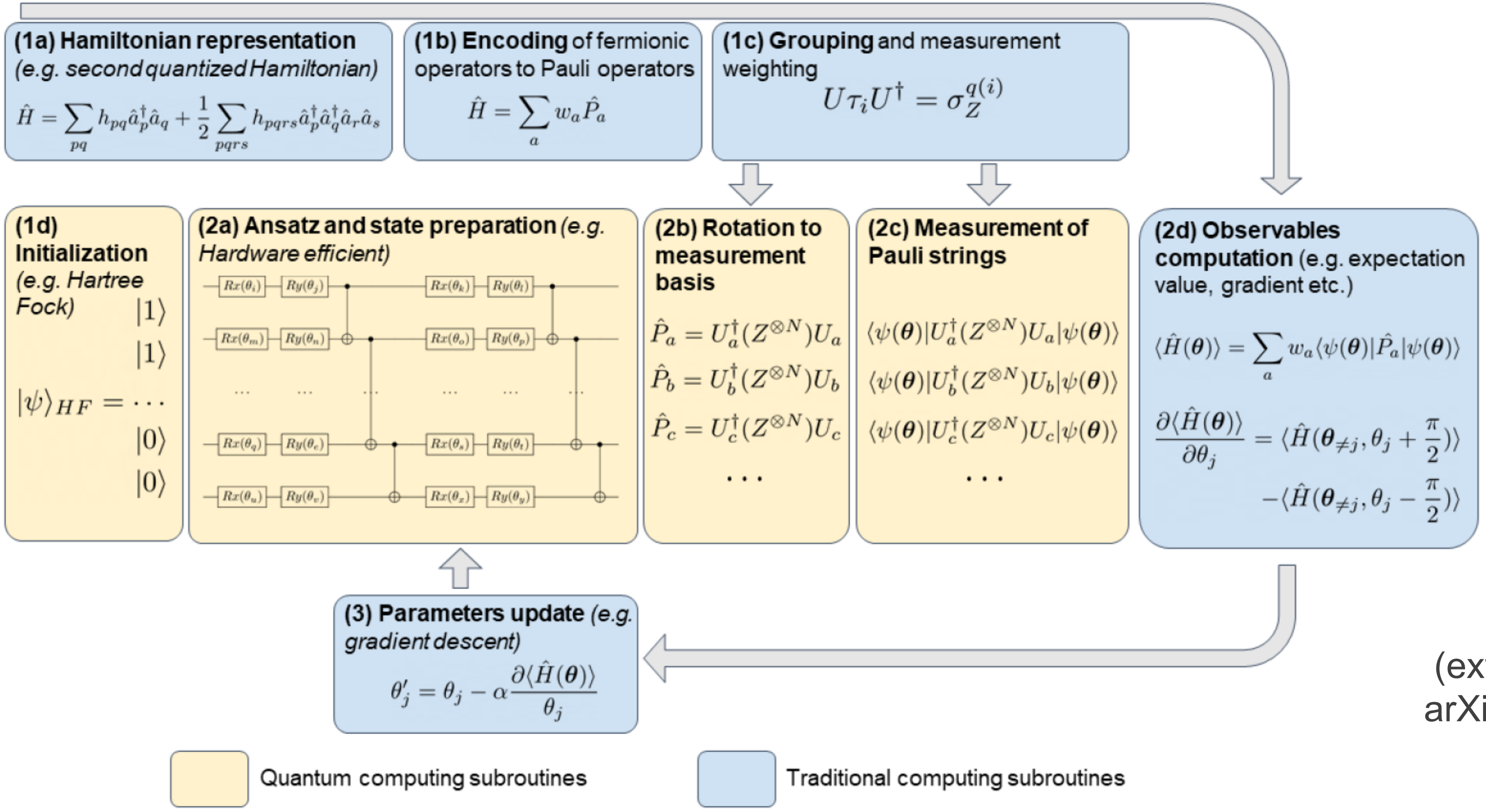
- **For each cycle**, we have **1 penalization term**
- **0 (1) corresponds to the original (reversed)** direction w.r.t. the starting oriented graph
- Scaling depends on the connectivity of the graph: **lower bound (1) independent on the number of vertices V**

Clemente et al, PRD 108 (2023) 9, 096035

$$
\begin{aligned}
H_{\text{Top.C}} = {} & \boxed{4\pi_0^0\,\pi_1^0\,\pi_2^0\,\pi_3^0} + 4\pi_0^1\,\pi_1^1\,\pi_2^1\,\pi_3^1 + \boxed{3\pi_0^1\,\pi_4^1\,\pi_5^0} + 5\pi_1^0\,\pi_2^0\,\pi_3^0\,\pi_4^1\,\pi_5^0 + 3\pi_0^0\,\pi_4^0\,\pi_5^1 \\
& + 5\pi_1^1\,\pi_2^1\,\pi_3^1\,\pi_4^0\,\pi_5^1 + 4\pi_0^1\,\pi_1^1\,\pi_4^1\,\pi_6^0 + 4\pi_2^0\,\pi_3^0\,\pi_4^1\,\pi_6^0 + 3\pi_1^1\,\pi_5^1\,\pi_6^0 + 5\pi_0^0\,\pi_2^0\,\pi_3^0\,\pi_5^1\,\pi_6^0 \\
& + 4\pi_0^0\,\pi_1^0\,\pi_4^0\,\pi_6^1 + 4\pi_2^1\,\pi_3^1\,\pi_4^0\,\pi_6^1 + 3\pi_1^0\,\pi_5^0\,\pi_6^1 + 5\pi_0^1\,\pi_2^1\,\pi_3^1\,\pi_5^0\,\pi_6^1 + 5\pi_0^1\,\pi_1^1\,\pi_2^1\,\pi_4^1\,\pi_7^0 \\
& + 3\pi_3^0\,\pi_4^1\,\pi_7^0 + 4\pi_1^1\,\pi_2^1\,\pi_5^1\,\pi_7^0 + 4\pi_0^0\,\pi_3^0\,\pi_5^1\,\pi_7^0 + 3\pi_2^1\,\pi_6^1\,\pi_7^0 + 5\pi_0^0\,\pi_1^0\,\pi_3^0\,\pi_6^1\,\pi_7^0 \\
& + 5\pi_0^0\,\pi_1^0\,\pi_2^0\,\pi_4^0\,\pi_7^1 + 3\pi_3^1\,\pi_4^0\,\pi_7^1 + 4\pi_1^0\,\pi_2^0\,\pi_5^0\,\pi_7^1 + 4\pi_0^1\,\pi_3^1\,\pi_5^0\,\pi_7^1 + 3\pi_2^0\,\pi_6^0\,\pi_7^1 \\
& + 5\pi_0^1\,\pi_1^1\,\pi_3^1\,\pi_6^0\,\pi_7^1
\end{aligned}
$$

More details arXiv:2210.13240 [hep-ph]

- *Once we built the Hamiltonian, we use VQE to extract causal-compatible terms*
- **QUANTUM PART**: **Evaluation of the Hamiltonian applied to an ansatz (parametrized quantum circuit)**
- **CLASSICAL PART**: **Modification of the parameters, through minimization algorithms**

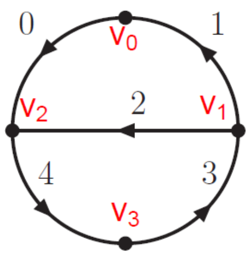


VQE pipeline

(extracted from J. Tilly et al, arXiv:2111.05176 [quant-ph])

J. Tilly et al (2022), Phys. Rep. 986, 1-128

1. Our implementation with Qiskit: **Real Amplitudes** (ansatz) **+ COBYLA** (optimizer)
2. Improved results with **multi-run VQE (MrVQE)**: set a selection **threshold**, collect **solutions** and modify the Hamiltonian with **penalization terms**



Two eloop
5-point

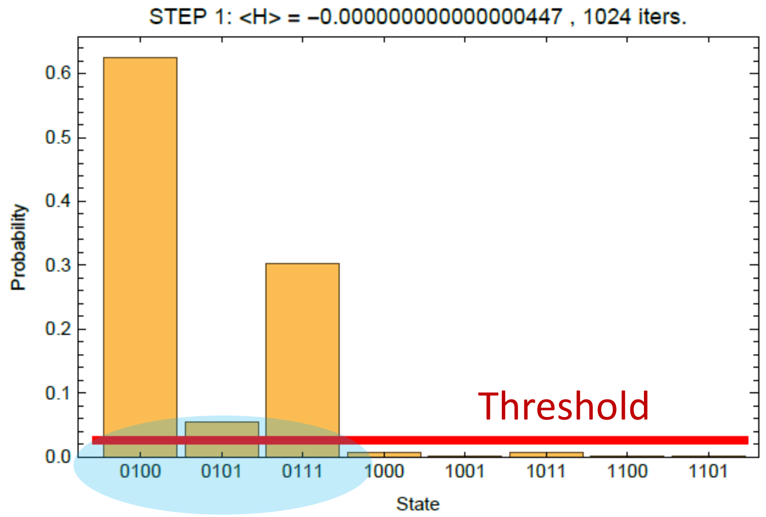
$$\begin{aligned} H_{E/\{e_0\}} &= 4\, I_4 \otimes I_3 \otimes I_2 \otimes I_1 + 2\, I_4 \otimes I_3 \otimes I_2 \otimes Z_1 - I_4 \otimes I_3 \otimes Z_2 \otimes I_1 \\ &- I_4 \otimes I_3 \otimes Z_2 \otimes Z_1 + I_4 \otimes Z_3 \otimes I_2 \otimes I_1 + I_4 \otimes Z_3 \otimes I_2 \otimes Z_1 \\ &+ 2\, I_4 \otimes Z_3 \otimes Z_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes I_1 + Z_4 \otimes I_3 \otimes I_2 \otimes Z_1 \\ &+ 2\, Z_4 \otimes I_3 \otimes Z_2 \otimes I_1 + 3\, Z_4 \otimes Z_3 \otimes I_2 \otimes I_1 + Z_4 \otimes Z_3 \otimes I_2 \otimes Z_1 \end{aligned}$$

Reduced Hamiltonian

Classical methods

Complete set of solutions

$$\{|0011\rangle, |0100\rangle, |0101\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1011\rangle, |1100\rangle, |1101\rangle\}$$



$$\mathcal{S}_1 = \{|0100\rangle, |0101\rangle, |0111\rangle\}$$

Selected solutions (1st run)

$$|\psi^{(1)}\rangle = \sum_j c_j^{(1)} |\phi_j\rangle$$

Approximated ground state found by VQE

$$\mathcal{S}_1 = \{|\phi_j\rangle \,|\, |c_j^{(1)}|^2 > \lambda\}$$

and subset of terms above the selection threshold

$$\Pi^{(1)} = \sum_{|\phi_l\rangle \in \mathcal{S}_1} b_l^{(1)} |\phi_l\rangle\langle\phi_l|$$

Penalization term ...

$$H^{(1)} = H^{(0)} + \Pi^{(1)}$$

... to be added to the Hamiltonian for next run!!

Clemente et al, PRD 108 (2023) 9, 096035

Quantum minimization: causal rep. bootstrap

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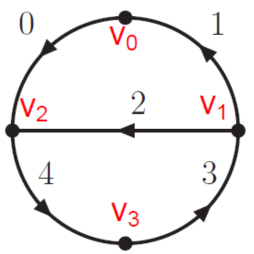

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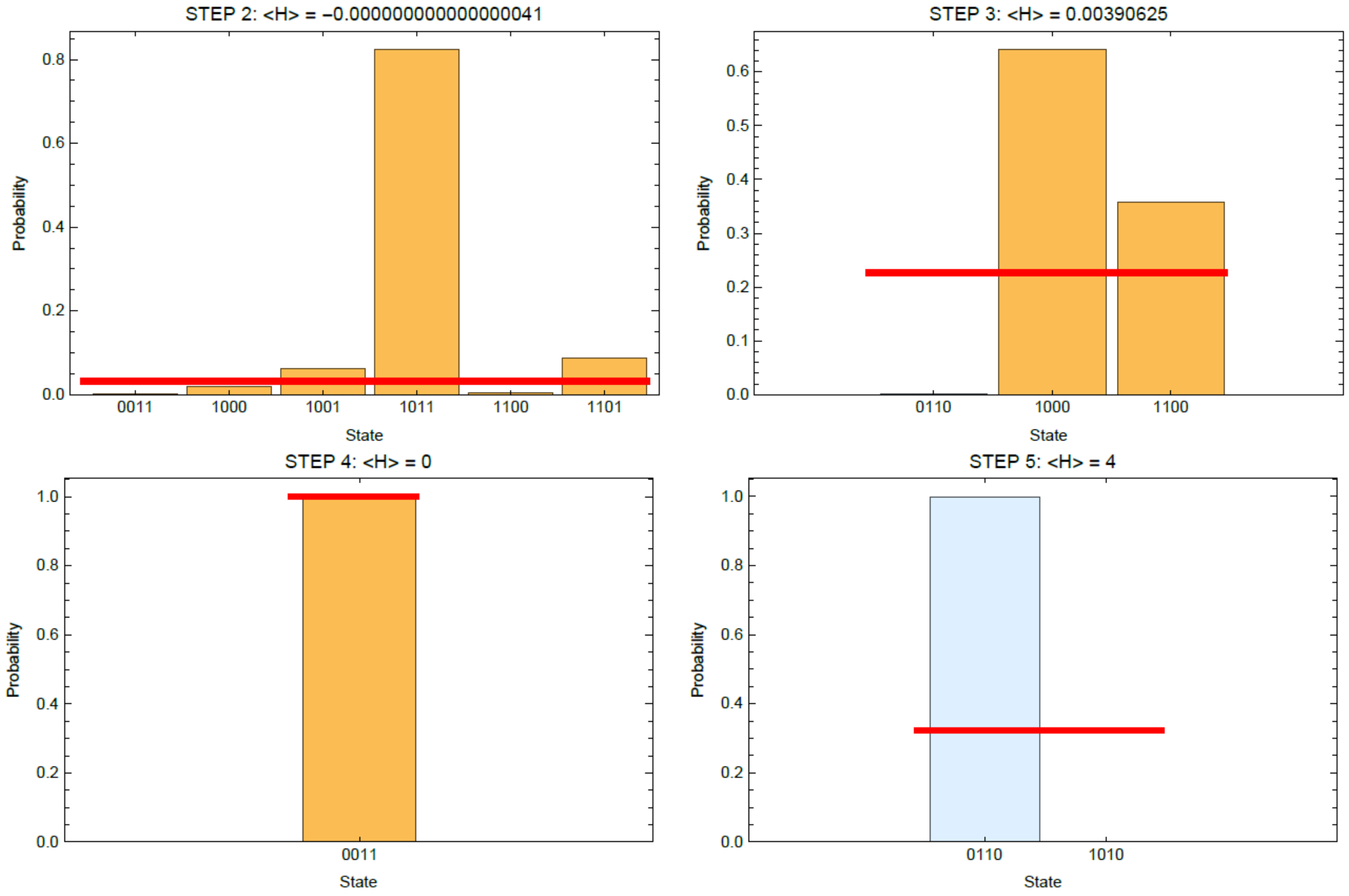
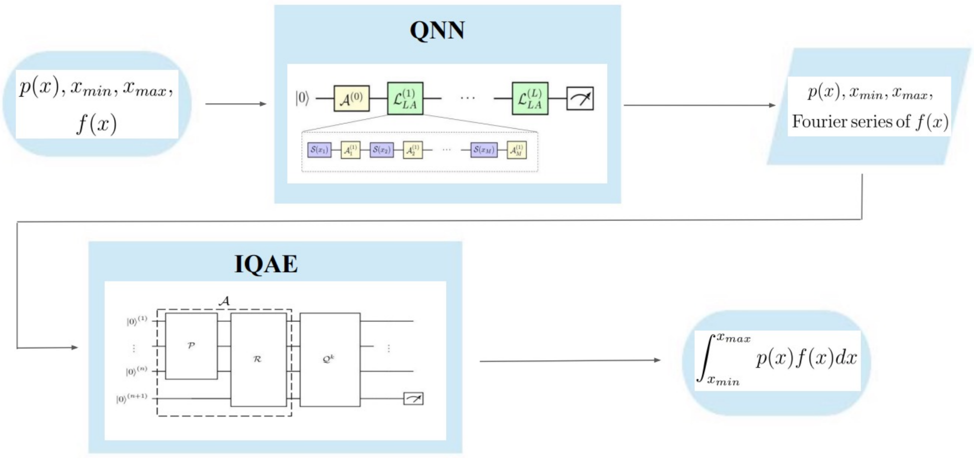


- We collect solutions step by step, till the algorithm converges (if <H> >1)
- Recently improved by the development of a novel warm-started QAOA method (EP25383186 patent filed on 31/10/2025)

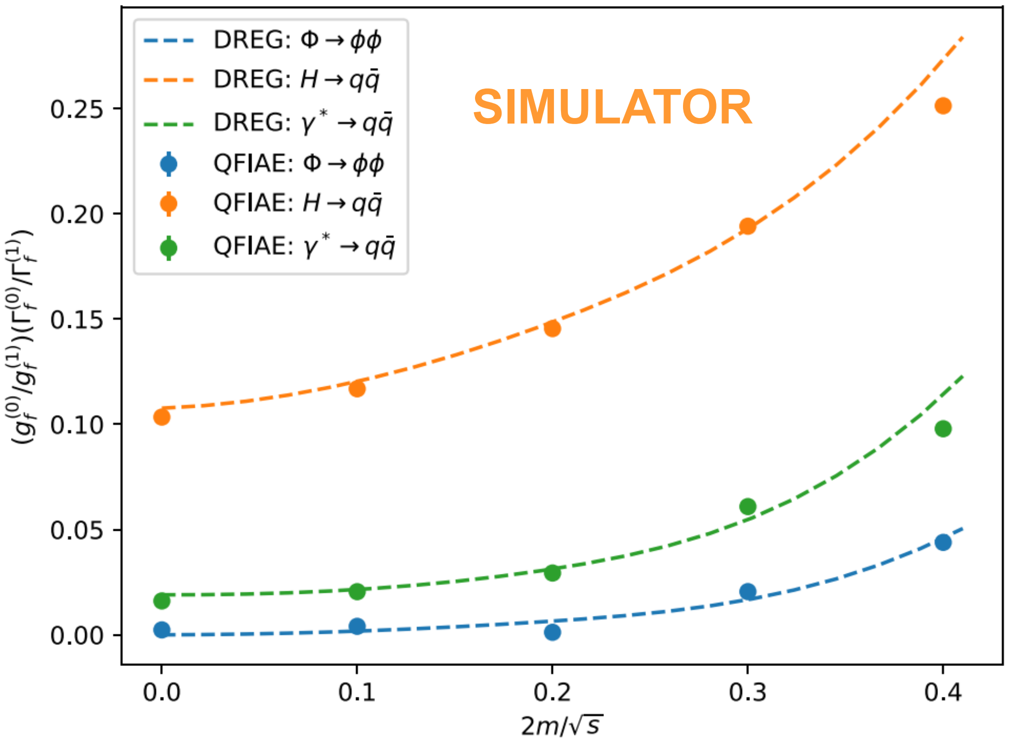
Clemente et al, PRD 108 (2023) 9, 096035

Quantum Integration: cross-sections from CU

- Quantum Fourier iterative amplitude estimation (QFIAE) applied to LTD CU.
- QFIAE is aimed to integrate multidimensional functions through Fourier series decomposition, codified into a Quantum Neural Network (QNN) → **Simplify the target function operator in Iterative Quantum Amplitude Estimation (IQAE)**



QFIAE pipeline (from arXiv:2305.01686 [quant-ph])



NLO 1->2 total decay rate as a function of mass

J. J. Martinez de Lejarza et al, Quantum Sci.Technol. 10 (2025) 2, 025026

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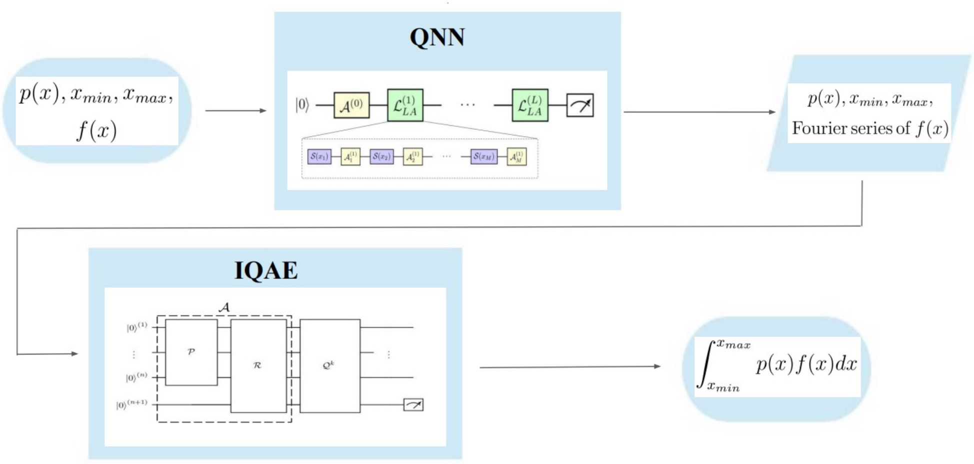

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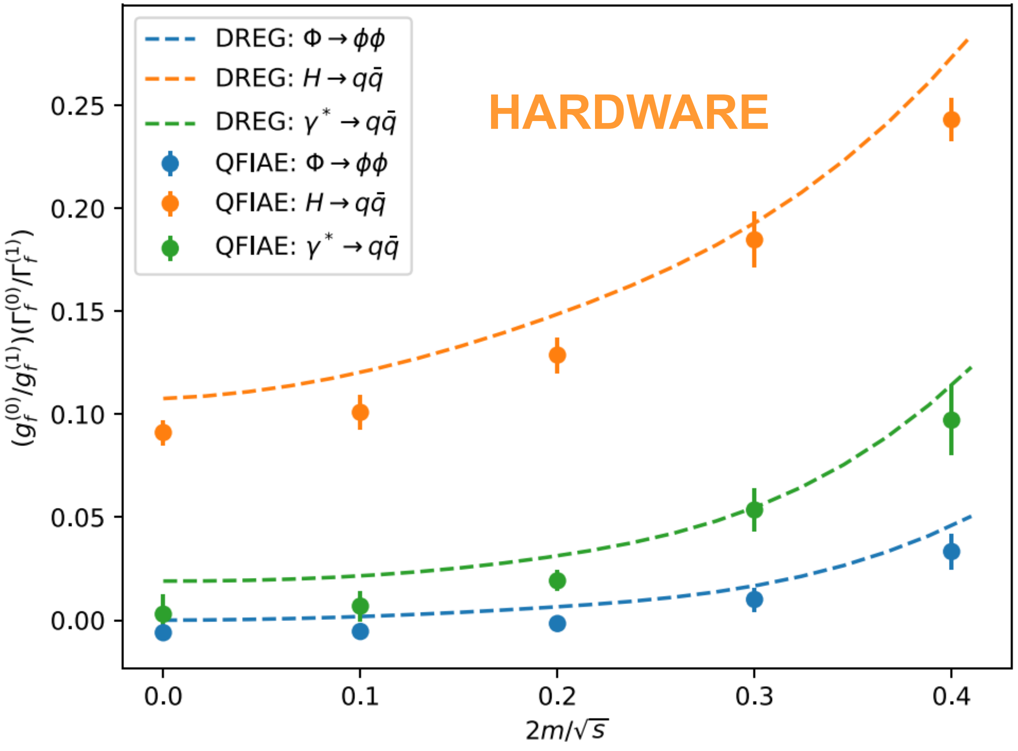
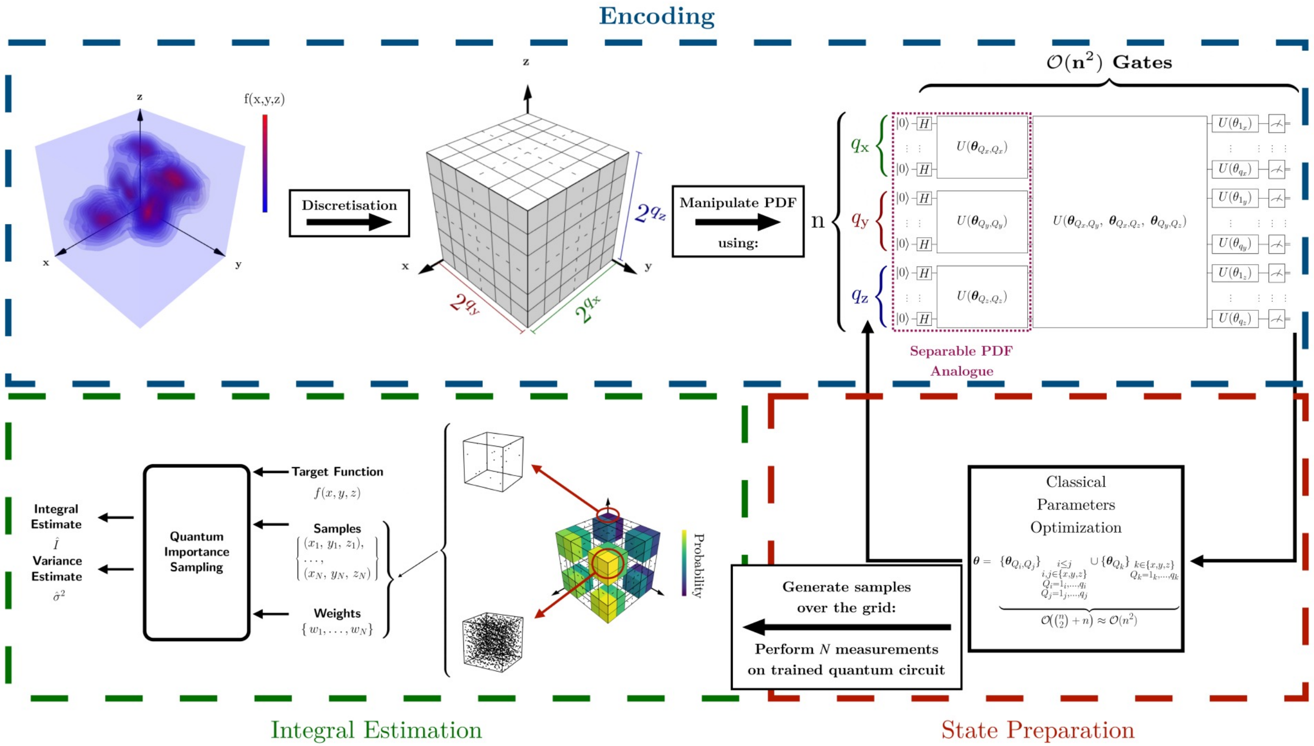
Hardware: IBMQ Mumbai (27-qubit superconducting device), executed with Qiskit



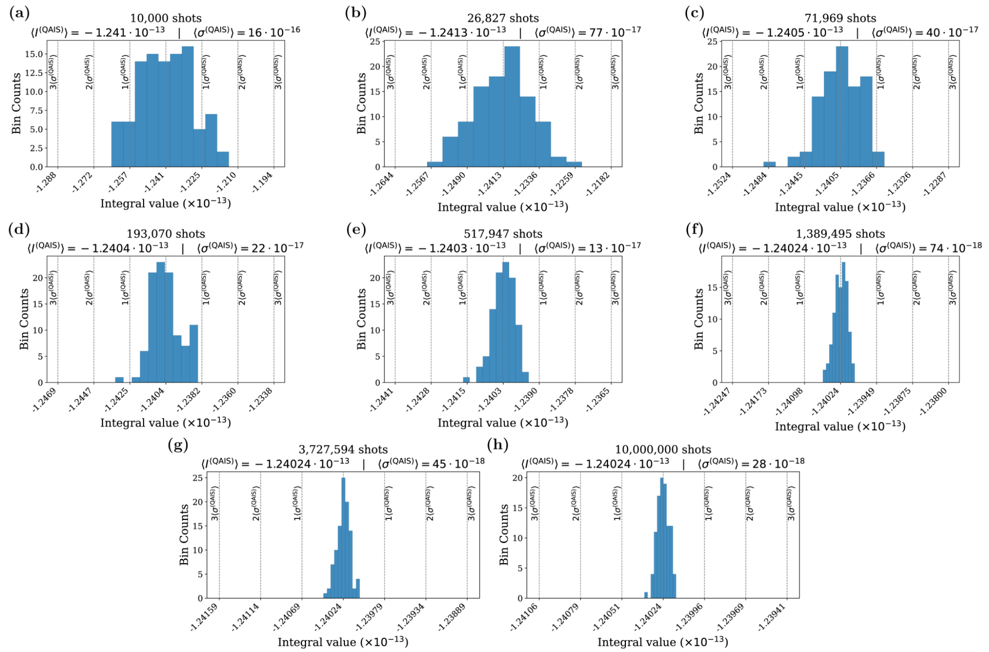
NLO 1->2 total decay rate as a function of mass

J. J. Martinez de Lejarza et al, Quantum Sci.Technol. 10 (2025) 2, 025026

- Another method developed by our group is the Quantum Adaptive Importance Sampling (QAIS)
- Inspired by VEGAS, QAIS aims to learn a given distribution with a parametrized circuit (PQC). Quantum entanglement is useful for reproducing correlations among variables.
- Measuring the PQC allows to determine where to evaluate the target function.



QAIS pipeline (from arXiv:2506.19965 [quant-ph])



Estimation of the value of a one-loop five-point function, for different number of shoots

K. Pyretzidis et al, arXiv:2506.19965 [quant-ph]

Quantum Encoding: modular circuits for HEP

- **IDEA:** Cross-section calculations involve several ingredients (PDFs/FFs, scattering amplitudes, phase-space) ➡ **Each of them can be mapped into a quantum circuit!**

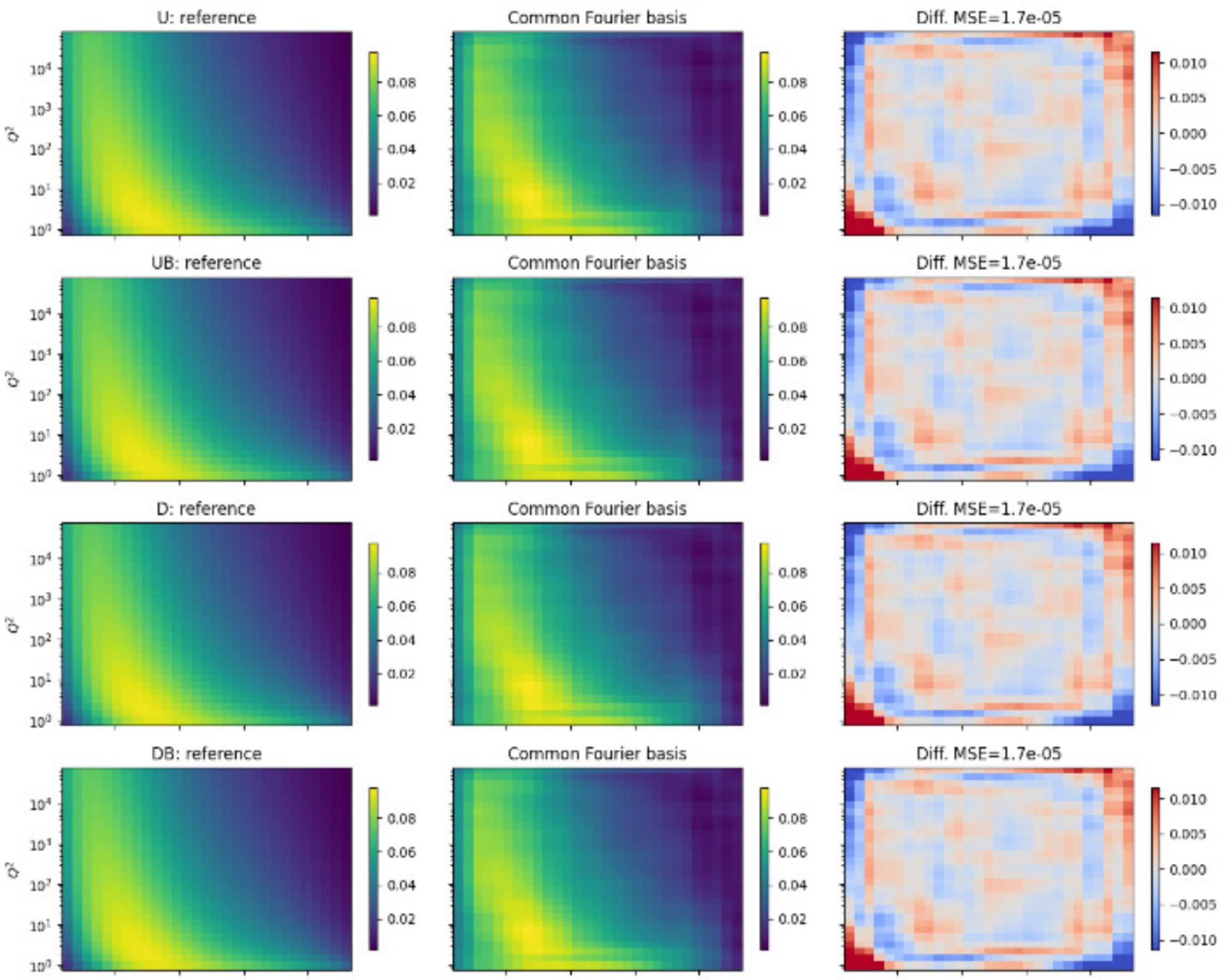
- **We consider FF encoding into variational quantum circuits (VQC),** inspired by previous works (Carrazza et al, Phys. Rev. D 103, 034027 (2021) and Martínez de Lejarza et al, Commun.Phys. 8 (2025) 1, 448)

- We put emphasis in three aspects:
  1. **Exploiting entanglement to incorporate flavour correlations**
  2. **Generate a single object (VQC) containing the full PDF/FF set (i.e. for all flavours)**
  3. *Preparing a framework that can be directly used by FF/PDF fitting communities (preliminar)*

Rentería-Estrada et al, *work in progress*

PRELIMINARY RESULTS

- **Features of our approach:**
  - 1-qubit per flavour
  - We used a PQC for each flavour
  - We added an entanglement block to account for flavour correlation.
  - Then, we optimize the circuit parameters (both in the individual flavour PQC as in the entanglement layer)
  - **Results are fully consistent with z-Q^2 profile of DSS14 FFs.**



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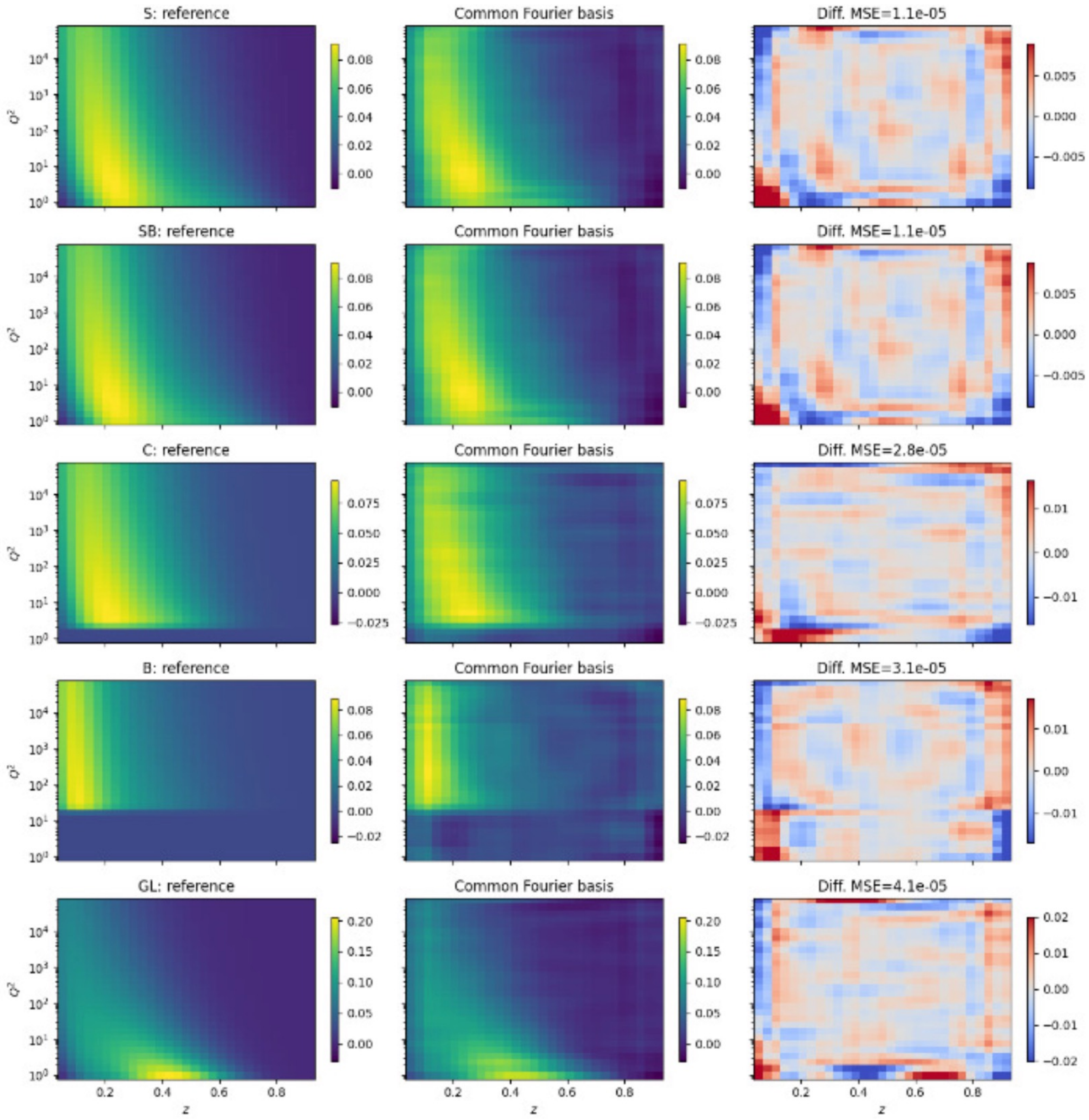

Rentería-Estrada et al, *work in progress*

Conclusions

- **Modern Loop-Tree Duality (LTD) formulation naturally incorporates causality**
- **LTD + Graph theory ➡ New strategy to compute cross-sections from multi-loop vacuum diagrams (Causal Unitarity)**
- **LTD + Causality ➡ New ways to generate finite integrals**

- **Quantum algorithms** can be adapted to solve HEP problems, but they must be translated into a quantum language.
  1. **Quantum minimization:** VQE/warm-started QAOA applied to bootstrap causal representation
  2. **Quantum integration:** QFIAE and QAIS used to estimate loop integrals
  3. **Quantum encoding:** PDFs/FFs codified into QC, similar for scattering amplitudes (w.i.p.)


THANKS!