Simulating perturbative QCD with Quantum Computing

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Progress in algorithms and numerical tools for QCD - Orsay, France

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universität freiburg

Outline:

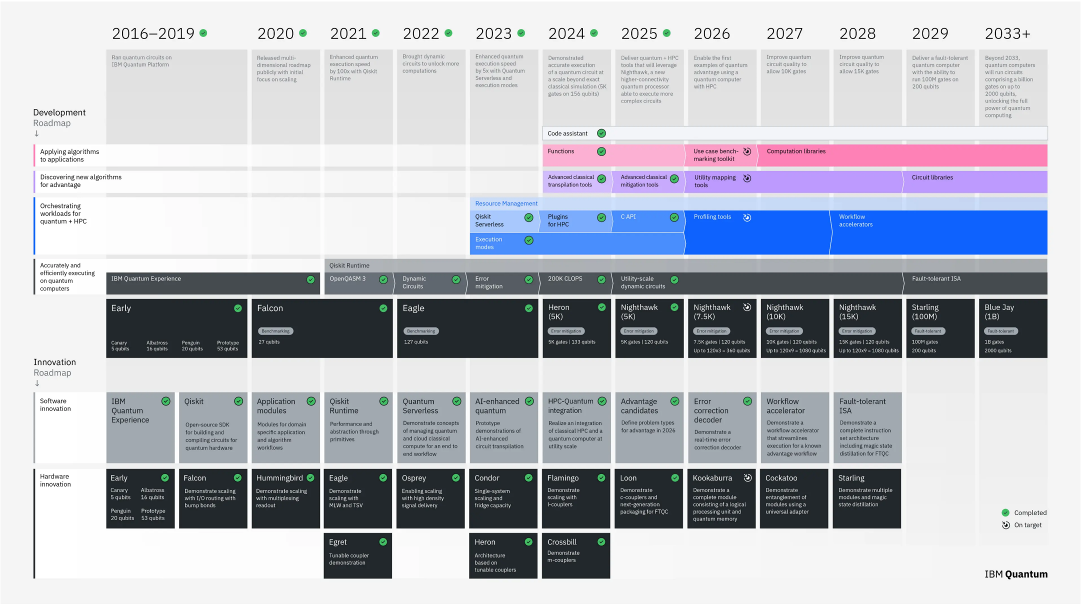
Why quantum computing and high-energy physics?

- Basics of quantum computing
  $\rightarrow$ How to build a quantum circuit

- Context of high-energy physics

How?

- Applications
  $\rightarrow$ Cross section $\rightarrow$ quantum integration
  $\rightarrow$ Amplitudes in QCD $\rightarrow$ dedicated quantum algorithms



2016–2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2033+
Ran quantum circuits on IBM Quantum Platform
Released multi-dimensional roadmap publicly with initial focus on scaling
Enhanced quantum execution speed by 100x with Qiskit Runtime
Brought dynamic circuits to unlock more computations
Enhanced quantum execution speed by 5x with Quantum Serverless and execution modes
Demonstrated accurate execution of a quantum circuit at a scale beyond exact classical simulation (5K gates on 156 qubits)
Deliver quantum + HPC tools that will leverage Nighthawk, a new higher-connectivity quantum processor able to execute more complex circuits
Enable the first examples of quantum advantage using a quantum computer with HPC
Improve quantum circuit quality to allow 10K gates
Improve quantum circuit quality to allow 15K gates
Deliver a fault-tolerant quantum computer with the ability to run 100M gates on 200 qubits
Beyond 2033, quantum computers will run circuits comprising a billion gates on up to 2000 qubits, unlocking the full power of quantum computing
Development Roadmap
Code assistant
Applying algorithms to applications
Functions
Use case benchmarking toolkit
Computation libraries
Discovering new algorithms for advantage
Advanced classical transpilation tools
Advanced classical mitigation tools
Utility mapping tools
Circuit libraries
Orchestrating workloads for quantum + HPC
Resource Management
Qiskit Serverless
Plugins for HPC
C API
Profiling tools
Workflow accelerators
Execution modes
Accurately and efficiently executing on quantum computers
Qiskit Runtime
IBM Quantum Experience
OpenQASM 3
Dynamic Circuits
Error mitigation
200K CLOPS
Utility-scale dynamic circuits
Fault-tolerant ISA
Early
Canary 5 qubits
Albatross 16 qubits
Penguin 20 qubits
Prototype 53 qubits
Falcon
Benchmarking
27 qubits
Eagle
Benchmarking
127 qubits
Heron (5K)
Error mitigation
5K gates | 133 qubits
Nighthawk (5K)
Error mitigation
5K gates | 120 qubits
Nighthawk (7.5K)
Error mitigation
7.5K gates | 120 qubits
Up to 120x3 = 360 qubits
Nighthawk (10K)
Error mitigation
10K gates | 120 qubits
Up to 120x9 = 1080 qubits
Nighthawk (15K)
Error mitigation
15K gates | 120 qubits
Up to 120x9 = 1080 qubits
Starling (100M)
Fault-tolerant
100M gates
200 qubits
Blue Jay (1B)
Fault-tolerant
1B gates
2000 qubits
Innovation Roadmap
Software innovation
IBM Quantum Experience
Qiskit
Open-source SDK for building and compiling circuits for quantum hardware
Application modules
Modules for domain specific application and algorithm workflows
Qiskit Runtime
Performance and abstraction through primitives
Quantum Serverless
Demonstrate concepts of managing quantum and cloud classical compute for an end to end workflow
AI-enhanced quantum
Prototype demonstrations of AI-enhanced circuit transpilation
HPC-Quantum integration
Realize an integration of classical HPC and a quantum computer at utility scale
Advantage candidates
Define problem types for advantage in 2026
Error correction decoder
Demonstrate a real-time error correction decoder
Workflow accelerator
Demonstrate a workflow accelerator that streamlines execution for a known advantage workflow
Fault-tolerant ISA
Demonstrate a complete instruction set architecture including magic state distillation for FTQC
Hardware innovation
Early
Canary 5 qubits
Albatross 16 qubits
Penguin 20 qubits
Prototype 53 qubits
Falcon
Demonstrate scaling with I/O routing with bump bonds
Hummingbird
Demonstrate scaling with multiplexing readout
Eagle
Demonstrate scaling with MLW and TSV
Osprey
Enabling scaling with high density signal delivery
Condor
Single-system scaling and fridge capacity
Flamingo
Demonstrate scaling with l-couplers
Loon
Demonstrate c-couplers and next-generation packaging for FTQC
Kookaburra
Demonstrate a complete module consisting of a logical processing unit and quantum memory
Cockatoo
Demonstrate entanglement of modules using a universal adapter
Starling
Demonstrate multiple modules and magic state distillation
Egret
Tunable coupler demonstration
Heron
Architecture based on tunable couplers
Crossbill
Demonstrate m-couplers
Completed
On target
IBM Quantum

Why quantum computing and high-energy physics?

- Basics of quantum computing
  $\rightarrow$ How to build a quantum circuit

Literature:

- *Quantum Computation and Quantum Information*, Nielsen and Chuang
- *Programming Quantum Computers*, Johnston, Harrigan, and Gimeno-Segovia

Back to quantum mechanics...

$\rightarrow$ Any state can be written as $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$,

$$\text{with } |\alpha|^2 + |\beta|^2 = 1$$

→ In quantum mechanics (and in quantum computing), any operation A is unitary

$$\psi \xrightarrow{A} \psi'$$

$$A\,|\psi\rangle = |\psi'\rangle \;\text{ with } A^{\dagger}A = AA^{\dagger} = \mathrm{Id}$$

$$|\psi\rangle \;—\!\!\!—\boxed{A}—\!\!\!—\; |\psi'\rangle$$

Example of gates (I)

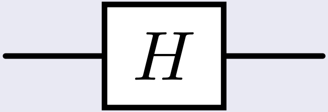
Pauli-X (X)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\psi = \alpha\,|0\rangle + \beta\,{\color{orange}|1\rangle} \rightarrow \ \psi' = \alpha\,{\color{orange}|1\rangle} + \beta\,|0\rangle$$

Hadamard (H)

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

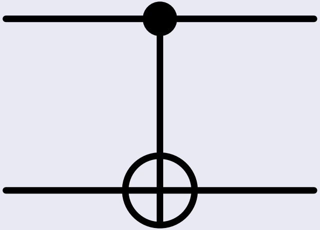


$$\psi = \alpha\,|0\rangle + \beta\,{\color{orange}|1\rangle} \rightarrow \psi' = \frac{\alpha+\beta}{\sqrt{2}}\,|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}\,{\color{orange}|1\rangle}$$

Example of gates (II)

Controlled not (CNOT, CX)

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$|00\rangle \to |00\rangle$; $|01\rangle \to |01\rangle$;
$|10\rangle \to |11\rangle$; $|11\rangle \to |10\rangle$.

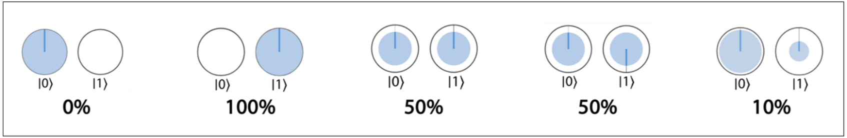
- If 0 $\to$ nothing happens; If 1 $\to$ Pauli-X (X)!
- *Control* qubit (top) and *target* qubit (bottom)

Measurement process

→ State: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

→ Probabilities: $\begin{cases} \text{for} & |0\rangle : \alpha^2 \\ \text{for} & |1\rangle : \beta^2 \end{cases}$ with $|\alpha|^2 + |\beta|^2 = 1$!

→ Measurement = squaring/collapse the amplitude!



NB: Probability obtained after many measurements/shots!

→ α and β encode information that can be measured/computed!

- Live example with QISKIT [open-source software development kit from IBM]

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Quantum-computing applications

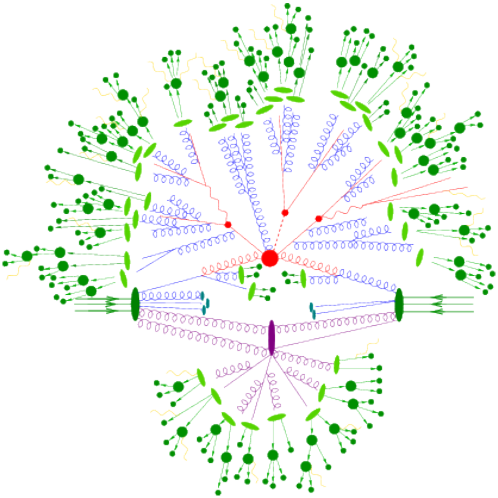
- Key parameters of quantum computers:
  - Number of qubits
    $\rightarrow$ amount of operations in parallel
  - Depth/number of gate operations
    $\rightarrow$ amount of operations in sequence (before decoherence/noise)
  - Error rates
    $\rightarrow$ Fault-tolerant era $\sim$ 2030 (depending on the technology)
- Expressibility is a combination of number of qubits and depth
- Implementation of a problem in quantum computing
  $=$ implementation with low-level language with restrictions (unitary operations)

Why quantum computing and high-energy physics?

- Context of high-energy physics

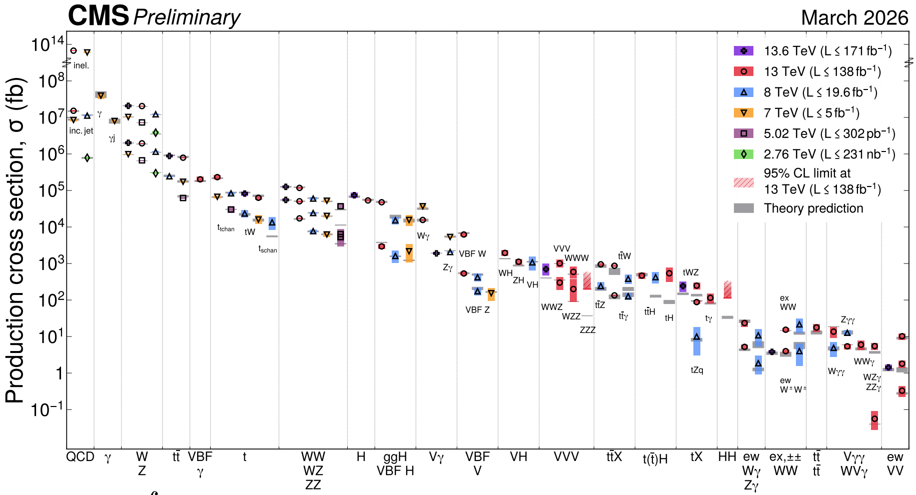
Life at the LHC (on the theory side...)

- Factorisation principle,
  aka "divide and conquer":
  - high-energy (perturbative)
    $\rightarrow$ Hard-scattering, parton-shower
  - low-energy (non-perturbative)
    $\rightarrow$ parton distribution function, hadronisation, underlying events, ...
- **Focus of the presentation: high energy!**



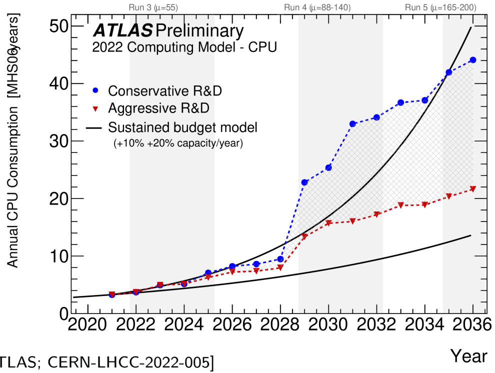
[source: Sherpa]

- Comparison between theory and experiment to learn about particle physics!



$$\rightarrow \quad \sigma \propto \int \mathrm{d}\Phi |\mathcal{M}|^2 \quad \text{with} \quad \mathrm{d}\Phi : \text{ phase space/momentum integration}$$

Computing problem in high-energy physics



[ATLAS; CERN-LHCC-2022-005]

$\rightarrow$ Event generation:

$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1} for ATLAS

[Buckley; 1908.00167], [Valassi et al.; 2004.13687]

- Solutions: GPU & AI

[Borowka et al.; 1811.11720], [Carrazza et al.; 2002.12921, 2009.06635, 2106.10279], [Bothmann et al.; 2106.06507], [Cruz-Martinez, De Laurentis, MP; 2502.07060], [Seymour, Sule; 2403.08692, 2511.19633], [Valassi; 2510.05392], [Bothmann et al.; 2604.03511]

- Can quantum computing be of any use in HEP?
  - $\rightarrow$ to compute things faster/more efficiently?
  - $\rightarrow$ to compute new things?
- Quantum computing provides a natural framework for QFT calculations?

How?

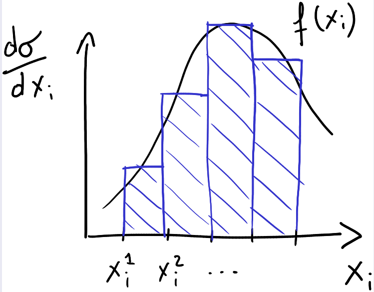
- Applications
  $\rightarrow$ Cross section $\rightarrow$ quantum integration
  $\rightarrow$ Amplitudes in QCD $\rightarrow$ dedicated quantum algorithms

$$\sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n \, f(x_1, \cdots, x_n)\, \Theta(g(x_1, \cdots, x_n))$$

- Cross section:

$$\sigma = \sum_{i,l} c_{il} \frac{\mathrm{d}\sigma}{\mathrm{d}x_l^i}$$

- **Integrating = guessing the values of a function at specific points (Riemann sum)**

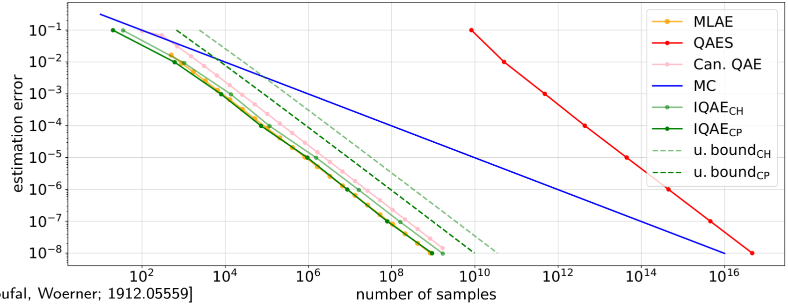


Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$], M: number of applications of $\mathcal{A}$



[Grinko, Gacon, Zoufal, Woerner; 1912.05559]

Resulting estimation error for $a = 1/2$ and 95% confidence level with respect to the required total number of oracle queries.

Idea to apply QAE to high-energy physics [Agliardi, Grossi, MP, Prati; 2201.01547]
$\rightarrow$ Followed by [Martínez de Lejarza et al.; 2204.06496, 2401.03023, 2409.12236,2506.19965], [Williams, MP; 2502.14647]

Two-step procedure

1. Encode the $a_i = f(x_i)$ of the integrand f into the state $\mathcal{A}|0\rangle = \sum_i a_i|\Psi_i\rangle$
2. Guess (=integrate) the a_i using QAE

Encode the a_i

- Analytical expression of the integrand
  $\rightarrow$ $f(x) = (x^2+1)/(x^2-m_{\mathrm{W}}^2)$ with f evaluated for each point
  $\rightarrow$ Quantum arithmetic [Williams, MP; 2502.14647]
- Numerical implementation of the integrand
  $\rightarrow$ algorithm providing a numerical value of f at each point
  $\rightarrow$ next application

How?

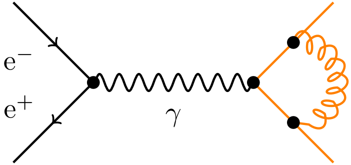
- Applications
  $\rightarrow$ Cross section $\rightarrow$ quantum integration
  $\rightarrow$ Amplitudes in QCD $\rightarrow$ dedicated quantum algorithms

Quantum implementation of QCD amplitude

→ Numerical implementation of integrand

$$\mathcal{M} \sim \sum \mathrm{Tr}\left(T^{a_1} \ldots T^{a_n}\right) \mathcal{K}(1, \ldots, n)$$

- Kinematic part: made of spinors and tensors (and kinematic invariants)
- Colour part: made of SU(3) generators of QCD



Colour algebra in QCD - crash course

- $[T^a, T^b] = \mathrm{i} f_{abc} T^c$.
- $T^a, T^c, \ldots$: SU(3) generators
- Gell-Mann matrices

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^7 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

⚠ T^a are not unitary!

$\rightarrow$ In our example, colour factor: $T^a_{ij} T^a_{jk} = C_F \delta_{ik}$

Quantum implementation of colour

$\rightarrow$ Implementation of the (discrete) colour into qubits:

- Gluon: 8 colours $\rightarrow$ 3 qubits ($2^3 = 8$)
- Quark: 3 colours $\rightarrow$ 2 qubits ($2^2 = 4$)

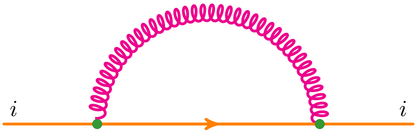
Make non-unitary matrices unitary!

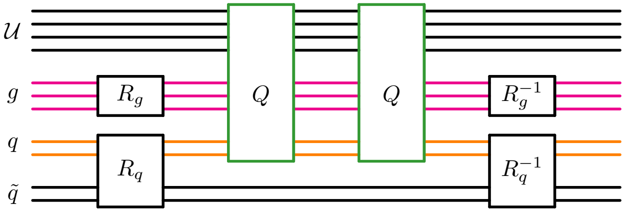
$\rightarrow$ Extend dimension and modify them

$$\overline{T^1} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^2} = \frac{1}{2}\begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^3} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^4} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

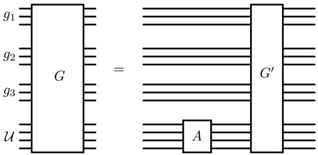
$$\overline{T^5} = \frac{1}{2}\begin{pmatrix} 0 & 0 & -\mathrm{i} & 0 \\ 0 & 1 & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^6} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^7} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^8} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

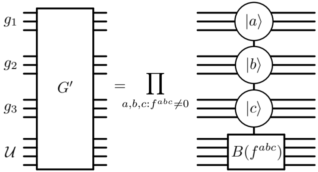
Example



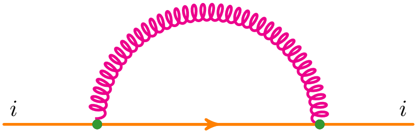


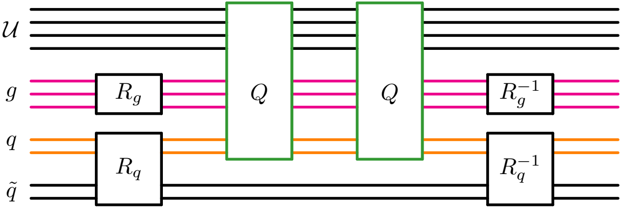
- One-to-one correspondence between Feynman diagram and circuit
- Gates for *qqg* (Q) and ggg (G) vertices to simulate QCD (colour) interaction



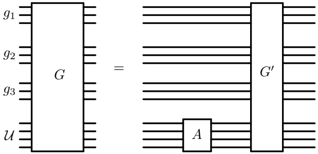


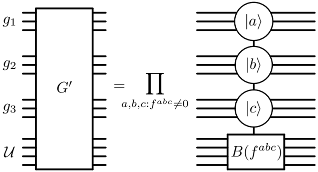
Example

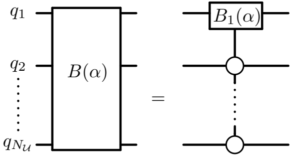




$$\langle\Omega|_{\mathcal{U}} \prod_{i=1}^{N_{ops}} \{B(\alpha_i)A\} |\Omega\rangle_{\mathcal{U}} = \prod_{i=1}^{N_{ops}} \alpha_i$$

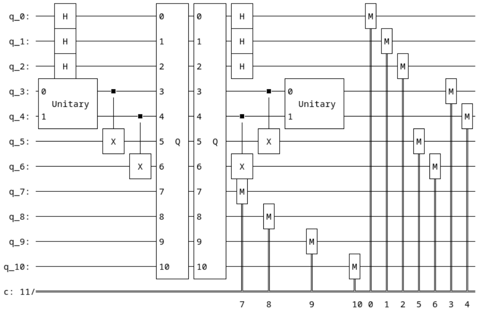






$\rightarrow$ Trace defined in $|00000000000\rangle = |0_{11}\rangle$ state: $|\psi\rangle = \frac{\mathcal{C}}{\mathcal{N}}|0_{11}\rangle + \ldots$

$$\Rightarrow \frac{27415}{N_{\text{shots}}{=}1000000} \sim \left(\frac{\mathcal{C}}{\mathcal{N}} = \frac{(N_c{=}3)C_F}{N_c^{n_q=1}(N_c^2-1)^{n_g=1}} \right)^2$$

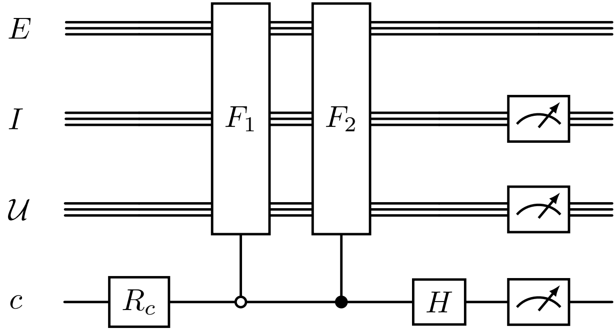


Total counts are: {'00000010101': 226, '00010010011': 342, '00000010110': 225, '00000001101': 696, '00010010010': 362, '00000010001': 872, '00010001101': 2006, '00000001100': 643, '00010001100': 2006, '00010000110': 1051, '00000001111': 638, '00000010010': 904, '00010000010': 1057, '00100010101': 52353, '00010010100': 3342, '00100000101': 6942, '00010000111': 1046, '00000010111': 210, '00100001101': 36223, '00100010111': 51877, '00000001110': 643, '00010010000': 4838, '00000010100': 5421, '00010000100': 1035, '00100000000': 107280, '00010000011': 1075, '00100000111': 6795, '00100001011': 145043, '00100010010': 10275, '00010000000': 65548, '00000000000': 27415, '00010001111': 2031, '00100000110': 7004, '00000001011': 2551, '00100001111': 36471, '00010010111': 8866, '00010000101': 1077, '00100010110': 52220, '00010010110': 9080, '00100010100': 185856, '00100001100': 36173, '00010010101': 8860, '00100010000': 14129, '00100001110': 36925, '00100000100': 6858, '00100010011': 10182, '00100000001': 6950, '00010001110': 2018, '00100000011': 6983, '00000010011': 815, '00010001011': 7957, '00010010001': 340, '00100010001': 10163, '00010000001': 1092, '00100000010': 7010}

$\rightarrow$ Colour factors encoded in one single state $|0_{11}\rangle$ (as needed for QAE)
$\rightarrow$ Any colour factor can be computed

[Chawdhry, MP; 2303.04818]

→ Interference between two amplitudes (F_1 and F_2)

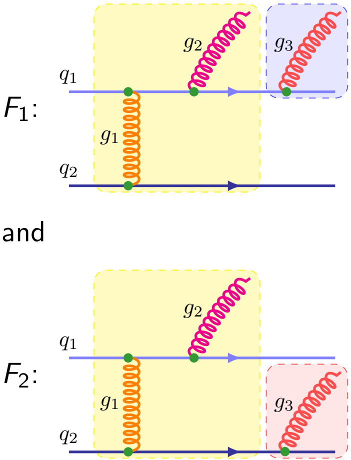


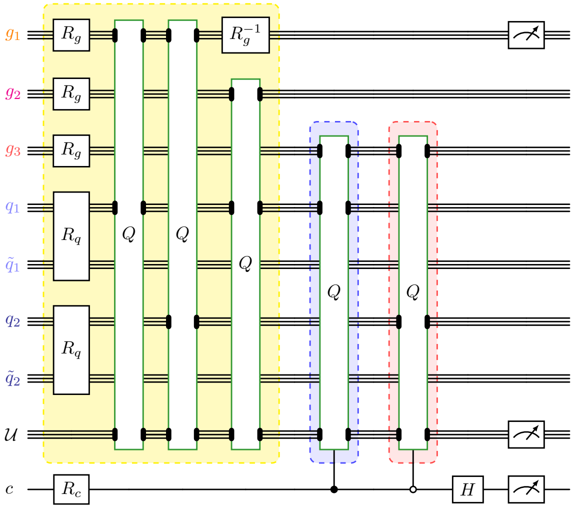
→ c is an equal superposition of F_1 and F_2

→ Making the measurement ($\sim$ square/collapse the amplitude):

$$|\mathcal{M}|^2 = |F_1 + F_2|^2 = |F_1|^2 + |F_2|^2 + 2\,\mathrm{Re}\left\{F_1.F_2^*\right\}$$

→ Common building block factored out

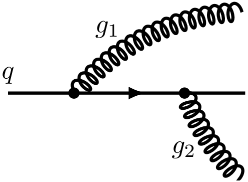




[Chawdhry, MP, Williams; 2507.07194]

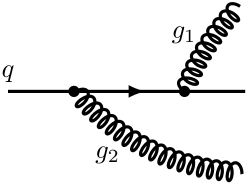
Common building block factored out + simple operation to obtained second amplitude
→ Hint for quadratic improvement with respect to classical approach

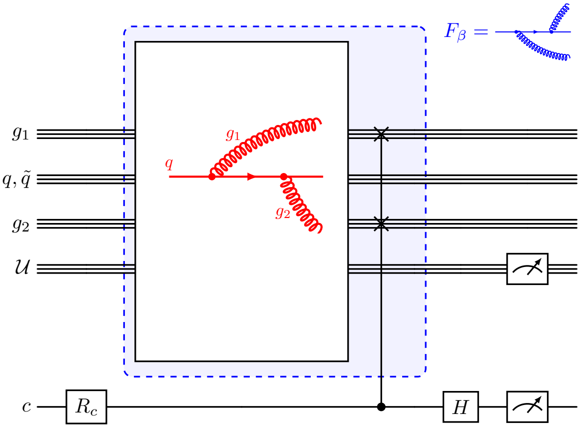
F_1:



and

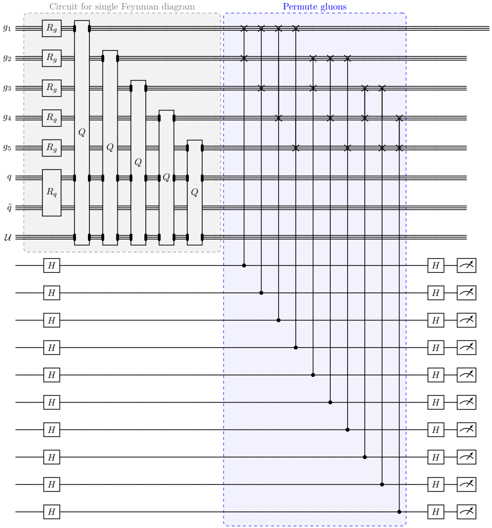
F_2:

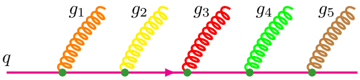




[Chawdhry, MP, Williams; 2507.07194]

→ Application to IBM 120 qubits superconducting with squared lattice (Nightawk generation) [Chawdhry, Pascuzzi, MP; in preparation]





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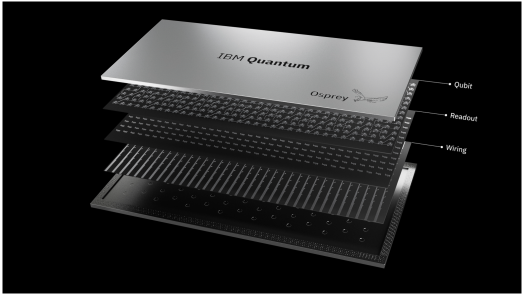

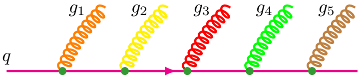

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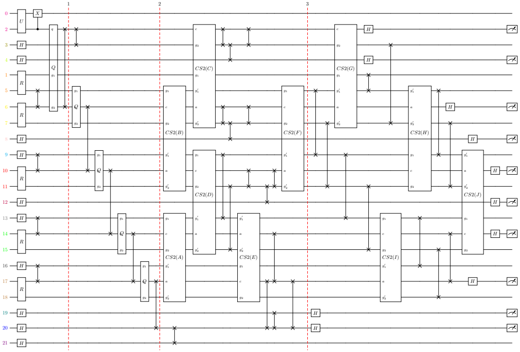

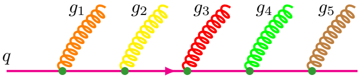

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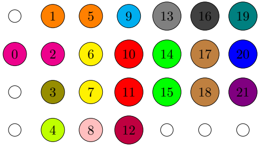

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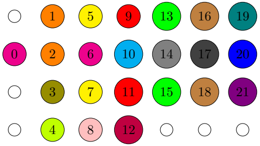

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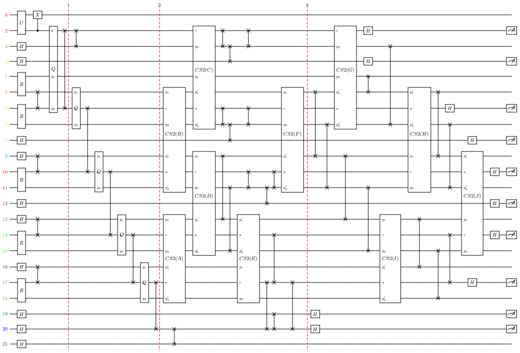

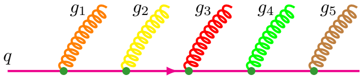

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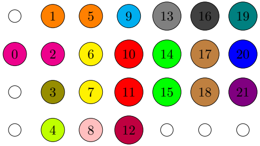

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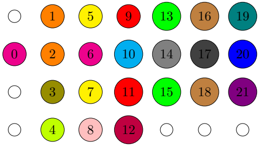


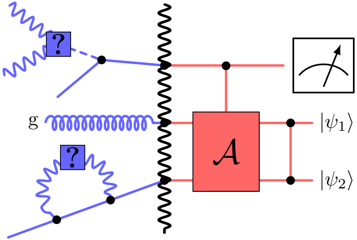
Conclusion

Quantum simulation of High-Energy physics

→ High-energy physics: computationally-intensive field

- **GPU & AI**: Technical, general-purpose approaches
- **Quantum computing**: Tailored to quantum systems (at very high energies)

- Several interesting applications already
- Powerful quantum computers becoming available
  → entering era with non-trivial applications
- Exciting emerging field with many new ideas!



BACK-UP

Reviews

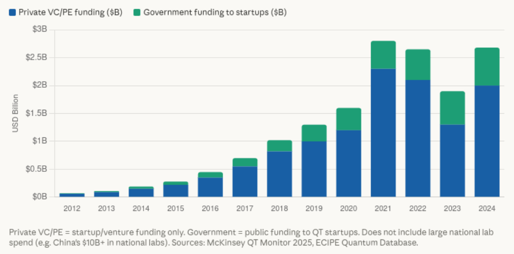
- [Gray, Terashi; Gray:2022fou] (selected topics), [Delgado et al.; 2203.08805] (Snowmass), [Klco et al.; 2107.04769] (lattice), [Di Meglio et al.; 2307.03236] (CERN Quantum Initiative)

Selected references

- Amplitude/loop integrals: [Ramirez-Uribe et al.; 2105.08703], [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Chawdhry, MP; 2303.04818], [Chawdhry, MP, Williams; 2507.07194], [Bashore, Moretti, Vitos; 2507.14252], [Ochoa-Oregon et al.; 2508.04019]
- Monte Carlo (integration): [Bravo-Prieto et al; 2110.06933], [Agliardi, Grossi, MP, Prati; 2201.01547], [Martínez de Lejarza et al.; 2401.03023], [Martínez de Lejarza, Cieri, Rodrigo; 2204.06496], [Cruz-Martinez, Robbiati, Carrazza; 2308.05657], [Williams, MP; 2502.14647], [Martínez de Lejarza et al.; 2409.12236], [Pyretzidis, Martínez de Lejarza, Rodrigo; 2506.19965]
- Parton shower: [Bauer, de Jong, Nachman, Provasoli; 1904.03196, 1901.08148], [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Williams, Malik, Spannowsky, Bepari; 2109.13975], [Deliyannis et al.; 2203.10018], [Chigusa, Yamazaki; 2204.12500], [Gustafson, Prestel, Spannowsky, Williams; 2207.10694], [Bauer, Chigusa, Yamazaki; 2310.19881]
- Others: [Perez-Salinas, Cruz-Martinez, Alhajri, Carrazza: 2011.13934], [Bauer, Freytsis, Nachman; 2102.05044], [Martínez de Lejarza et al.; 2503.16073]

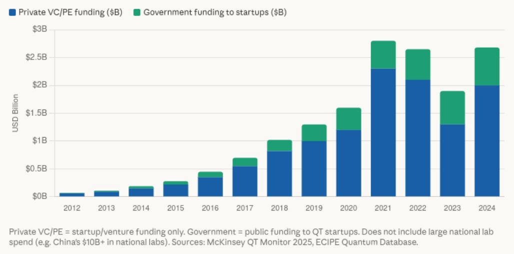
Context in quantum computing

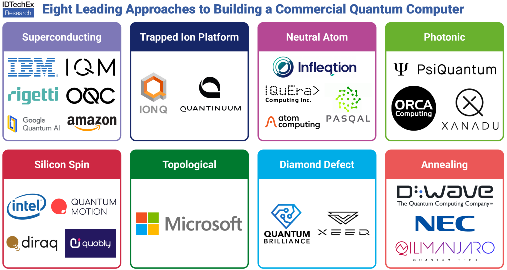
- Great commercial interest



Context in quantum computing

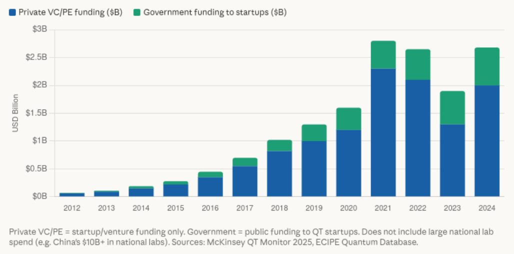
- Great commercial interest
- Various physics approaches (from both big players and start-ups)

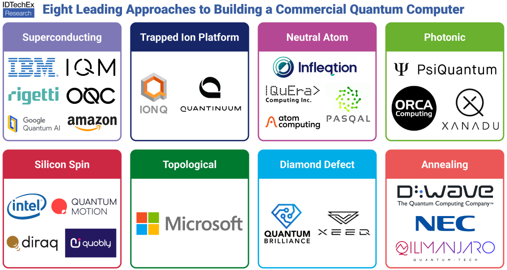


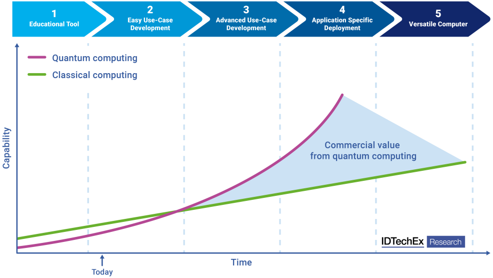


Context in quantum computing

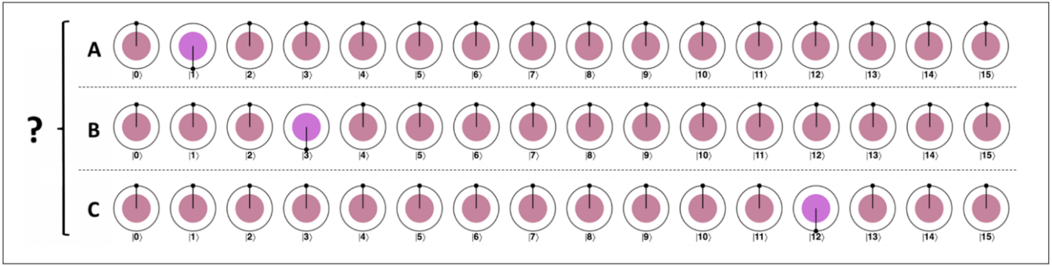
- Great commercial interest
- Various physics approaches (from both big players and start-ups)
- Critical moment





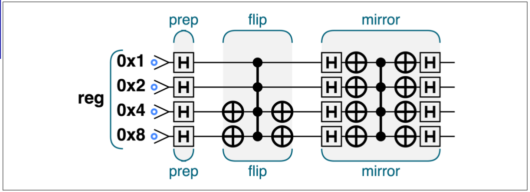


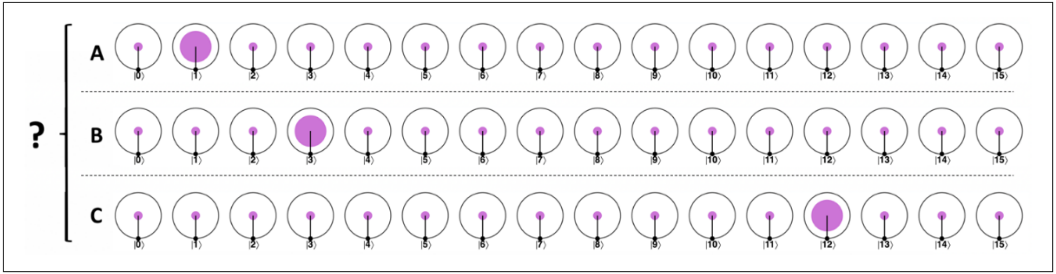
- Example (from [Johnston, Harrigan, Gimeno-Segovia; Programming Quantum Computers])



→ What solution is contained in our quantum register?

→ Applying a Grover iteration





Grover algorithm/iteration

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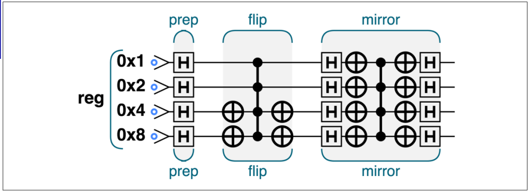

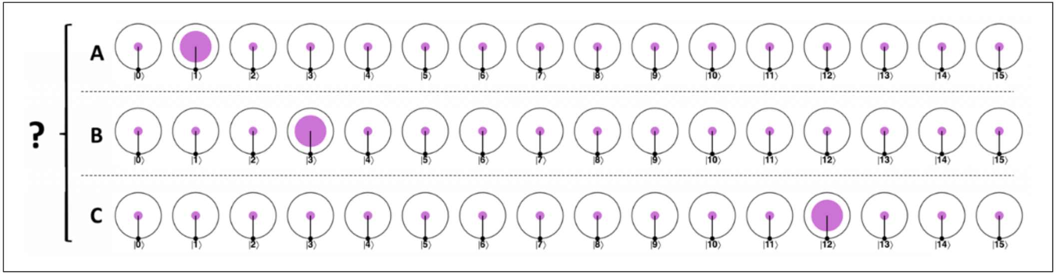

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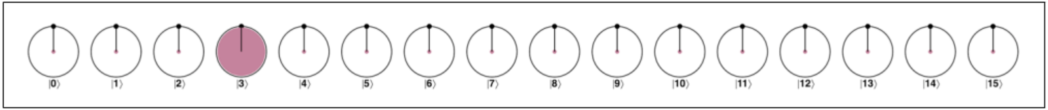

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Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator [Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

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Quantum arithmetics + use of QUANTINUUM Quantum Monte Carlo Integrator

[Akhalwaya et al.; 2308.06081], based on [Herbert; 2105.09100]

- 2D integral

$$\int \prod_{i=1}^{N_I} \mathrm{d}x_i \frac{\sum_{S_k \in I} \alpha_k \prod_{j \in S_k} x_j^{n_j}}{\prod_{p=1}^{N_P} (x_p - M_{op}^2)^2 + M_{op}^2 \Gamma_{op}^2} \quad \rightarrow \quad \int_0^s \int_0^{s-s_2} \mathrm{d}s_1 \mathrm{d}s_2 \frac{s_1^2 s - s_1 M_T^2 s + s_1 M_T^2 s_2}{(s_2 - M_\mathrm{W}^2)^2 + (M_\mathrm{W} \Gamma_\mathrm{W})^2}$$

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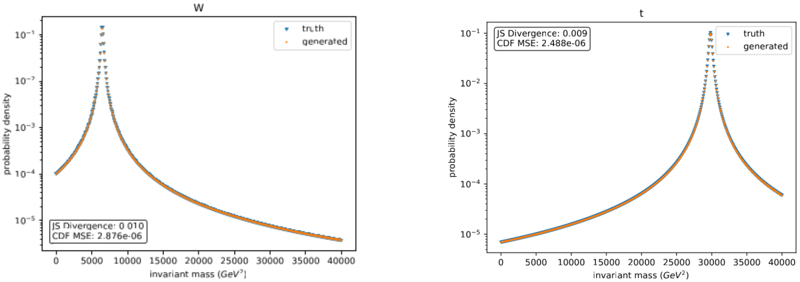
- Building blocks → Propagator (using Fourier expansion) [Rosenkranz et al.; 2405.21058]



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- Resource estimate → Fault-tolerant era

Compilation	Resource	Metric	Precision 10%	1%	0.1%
NISQ	Number of qubits	Largest across circuits	28	28	28
	CX gates	Total number across circuits	7.39×10^7	6.15×10^8	5.09×10^9
		Total depth across circuits	4.34×10^7	3.61×10^8	2.99×10^9
		Number in largest circuit	3.39×10^7	2.71×10^8	1.20×10^9
		Depth of largest circuit	1.99×10^7	1.59×10^8	7.03×10^8
	All gates	Total number across circuits	1.50×10^8	1.24×10^9	1.03×10^{10}
		Total depth across circuits	8.02×10^7	6.67×10^8	5.52×10^9
		Number in largest circuit	6.86×10^7	5.49×10^8	2.42×10^9
		Depth of largest circuit	3.68×10^7	2.94×10^8	1.30×10^9
Fault tolerant	Number of qubits	Largest across circuits	41	41	41
	T gates	Total number across circuits	3.39×10^9	4.08×10^{10}	2.72×10^{11}
		Total depth across circuits	3.29×10^9	3.95×10^{10}	2.63×10^{11}
		Number in largest circuit	1.65×10^9	2.14×10^{10}	6.97×10^{10}
		Depth of largest circuit	1.60×10^9	2.07×10^{10}	6.75×10^{10}

Quantum integration

Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

$\rightarrow$ Definition of a piece-wise function with $f(x_i) = a_i$.

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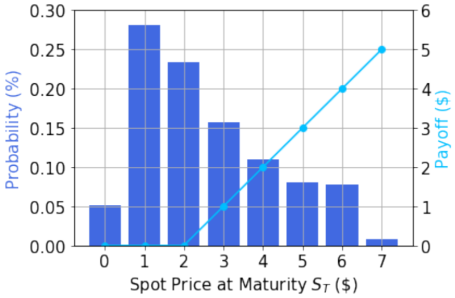
Initially used in finance for simple functions in 1D [Zoufal, Lucchi, Woerner; 1904.00043]
[Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]



$$I = \int \mathrm{d}x\, f(x) g(x)$$

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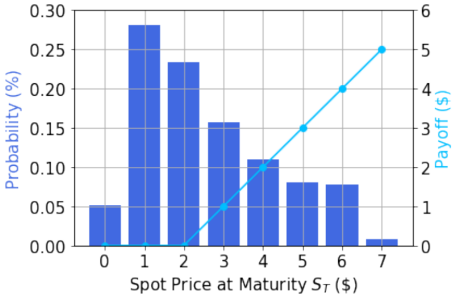
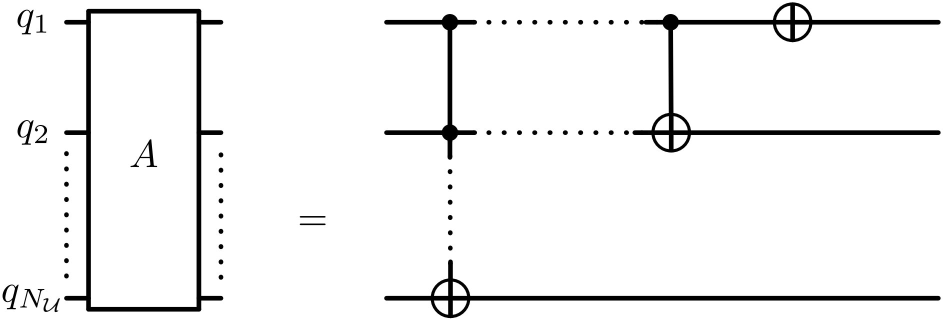


[Zoufal, Lucchi, Woerner; 1904.00043]

$$I = \int \mathrm{d}x\, f(x) g(x)$$

- In finance:
  - f: probability
  - g: payoff
- In high-energy physics:
  - f: $|\mathcal{M}|^2$
  - g: $\Theta(\Phi - \Phi_c)$


q_1
q_2
$q_{N_\mathcal{U}}$
A
=

$$B_1(\alpha) = \begin{pmatrix} \sqrt{1-|\alpha|^2} & \alpha \\ -\alpha & \sqrt{1-|\alpha|^2} \end{pmatrix} \tag{1}$$

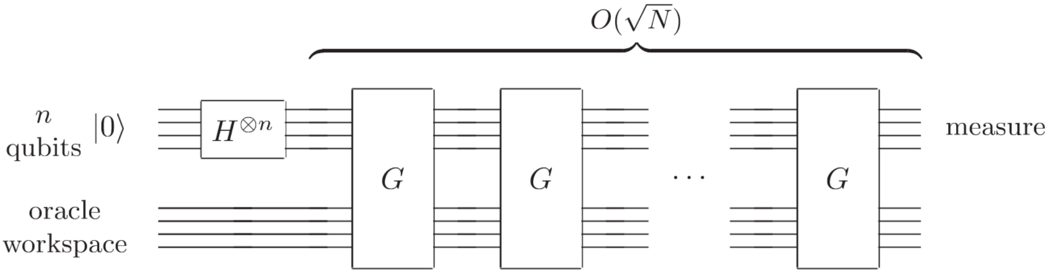
$$B(\alpha)A\,|k\rangle = \begin{cases} \alpha\,|0\rangle + \sqrt{1-|\alpha|^2}\,|1\rangle & \text{if } k=0 \\ |k+1\rangle & \text{if } 0<k<2^{N_{\mathcal{U}}}-1 \\ \sqrt{1-|\alpha|^2}\,|0\rangle - \alpha\,|1\rangle & \text{if } k=2^{N_{\mathcal{U}}}-1 \end{cases} \tag{2}$$

$$\langle\Omega|_{\mathcal{U}} B(\alpha)A|\Omega\rangle_{\mathcal{U}} = \alpha \tag{3}$$

$$\langle\Omega|_{\mathcal{U}} \prod_{i=1}^{N_{ops}} \{B(\alpha_i)A\}\,|\Omega\rangle_{\mathcal{U}} = \prod_{i=1}^{N_{ops}} \alpha_i \tag{4}$$

Grover algorithm/iteration

- Very general quantum algorithm
- Quadratic speed up
  $\rightarrow \mathcal{O}(\sqrt{N})$ operations instead of $\mathcal{O}(N)$
- Most famous example: unstructured database search



Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$]

M: number of applications of $\mathcal{A}$

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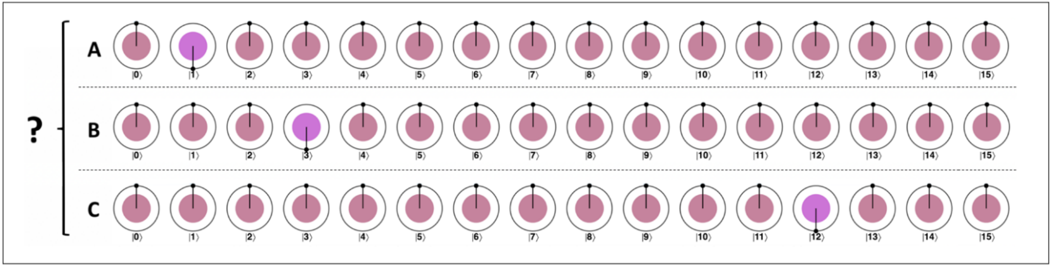
M: number of applications of $\mathcal{A}$

→ What the (orignal) algorithm provides:

- An estimate: $\tilde{a} = \sin^2(\tilde{\theta}_a)$
  with $\tilde{\theta}_a = y\pi/M$, $y \in \{0, ..., M-1\}$, and $M = 2^n$
- A success probability (that can be increased by repeating the algorithm)
- A bound: $|a - \tilde{a}| \leq \mathcal{O}\left(1/M\right)$

Grover algorithm/iteration

- Example (from [Johnston, Harrigan, Gimeno-Segovia; Programming Quantum Computers])



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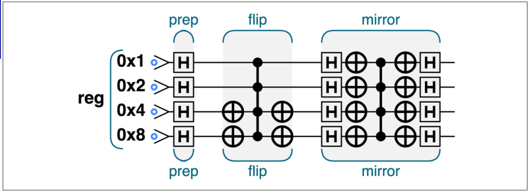

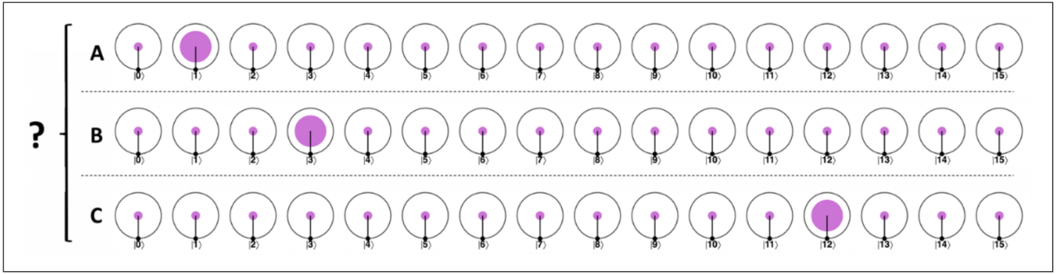

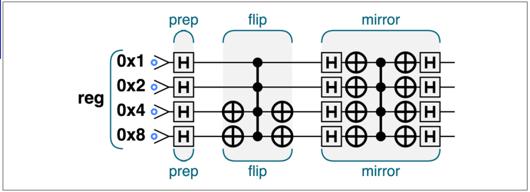

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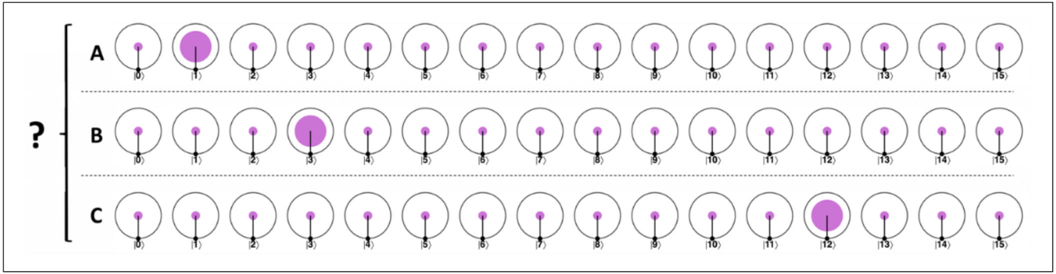

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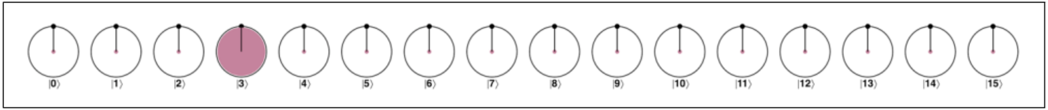

Applications

- **$e^+e^- \to q\bar{q}$** (in QED) $\to$ 1D problem

$$\sigma \sim \int_{-1}^{1}\int_{0}^{2\pi} \mathrm{d}\cos\theta \mathrm{d}\phi \left(1+\cos^2\theta\right)$$

- **$e^+e^- \to q\bar{q}'W$** $\to$ 2D problem

$$\sigma \sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}}\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{2\pi} \mathrm{d}\Phi_3 \left|\mathcal{M}_{\mathrm{e}^+\mathrm{e}^-\to q\bar{q}'\mathrm{W}}\right|^2$$
$$\sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}} \mathrm{d}\tilde{\Phi}_3 \left|\mathcal{M}'\right|^2$$

with $\mathcal{M}' = \mathcal{M}_{\mathrm{e}^+\mathrm{e}^-\to q\bar{q}'\mathrm{W}}\left(\cos\theta_1 = 0,\ \phi_1 = \pi/2,\ \phi_2 = \pi/2\right)$.

Loading of distribution / encoding into qubits

Encoding the distribution to be integrated into qubits (coefficients of states)

- Exact loading [Shende, Bullock, Markov, quant-ph/0406176] (resource intensive)
- Using quantum machine learning (qGAN) [Zoufal, Lucchi, Woerner; 1904.00043] (not exact)

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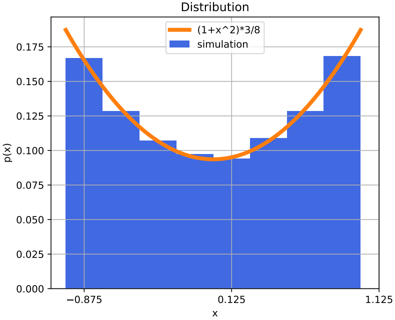
- Example: Exact loading - $1 + x^2$



$\rightarrow$ 3 qubits: $2^3 = 8$ bins

Integration - $1 + x^2$

- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75; 0]$	0.345	-3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5; 0]$	0.215	-5.86	0.201	1.15	0.203	0.0986	0.203	-0.0161
$[-0.25; 0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

Integration - $1 + x^2$

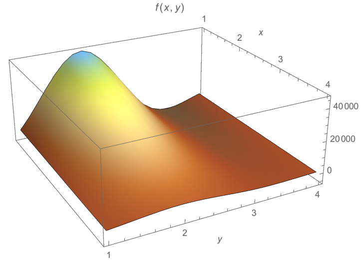
- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

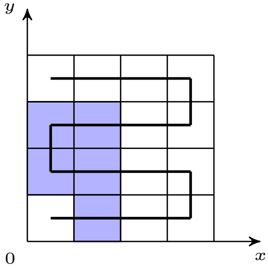
Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75;0]$	0.345	-3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5;0]$	0.215	-5.86	0.201	1.15	0.203	0.0986	0.203	-0.0161
$[-0.25;0]$	0.112	-17.1	0.0939	1.87	0.0960	-0.284	0.0957	-0.0389

- Non-matching boundary of integration

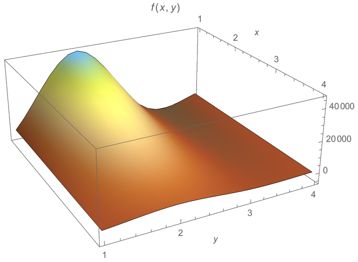
Qubits number	$[-0.7;0.6]$				$[-0.625;0.375]$			
	high stat.		exact		high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
3	0.402	-28.0	0.406	-27.1	0.296	-28.1	0.299	-27.5
4	0.463	-17.0	0.468	-16.0	0.408	-1.07	0.412	5.96×10^{-3}
5	0.527	-5.46	0.532	-4.62	0.408	-1.07	0.412	5.96×10^{-3}
6	0.542	-2.76	0.547	-1.81	0.408	-1.07	0.412	5.96×10^{-3}

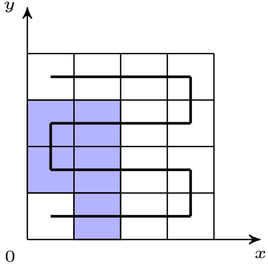
Integration - 2D [$e^+e^- \to \mathbf{q\bar{q}}'W$ with angles fixed]





Integration - 2D [$e^+e^- \to \mathbf{q\bar{q}}'W$ with angles fixed]





Qubits number	Grid dim.	$\mathcal{S}_1$ σ	$\mathcal{S}_1$ $\delta[\%]$	$\mathcal{S}_2$ σ	$\mathcal{S}_2$ $\delta[\%]$
4	4×4	0.55	0	0.70	-4.1
5	5×5	0.52	-4.92	0.53	-26.6
6	6×6	0.47	-14.1	0.79	9
6	7×7	0.62	-14.4	0.70	-3.0
6	8×8	0.55	0	0.78	7.6

$\mathcal{S}_1$: matching boundary of integration

$\mathcal{S}_2$: no matching boundary of integration

[Agliardi, Grossi, MP, Prati; 2201.01547]

Working but control of uncertainty crucial!

Remarks

- Use Qiskit (IBM python software) subroutines and noiseless quantum simulation (perfect quantum computer)
- For present application, too many qubits for test on real hardware
  $\rightarrow$ 4 qubits for representation $\rightarrow$ 9 total qubits
  $\rightarrow$ 6 qubits for representation $\rightarrow$ 13 total qubits
- Largest quantum computer on IBM quantum experience:
  7 qubits previously (127 qubits now)
  $\rightarrow$ Simulators can go up to 5000 qubits

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Summary

- First application of quantum integration in HEP
- Theoretical quadratic speed-up
- Main challenge: error estimate

[Williams, MP; 2502.14647]

Development roadmap

QUANTINUUM

	2020	2023	2025	2027	2029
SYSTEMS:	SYSTEM MODEL H1	SYSTEM MODEL H2	QUANTINUUM HELIOS	QUANTINUUM SOL	QUANTINUUM APOLLO
PHYSICAL QUBITS:	20	56	96	192	1000's
PHYSICAL 2-QUBIT GATE ERROR:	1×10^{-3}	1×10^{-3}	$< 5 \times 10^{-4}$	$< 2 \times 10^{-4}$	1×10^{-4}
LOGICAL QUBITS:		> 12	~ 50	~ 100	100's
LOGICAL ERROR RATES:		1×10^{-3}	$< 10^{-4}$	$\sim 10^{-5}$	1×10^{-5} to 1×10^{-10}*

*analysis based on recent literature in new, novel error correcting codes predict that error could be as low as 1E-10 in Apollo (ref: arXiv:2403.16054, arXiv:2308.07915)