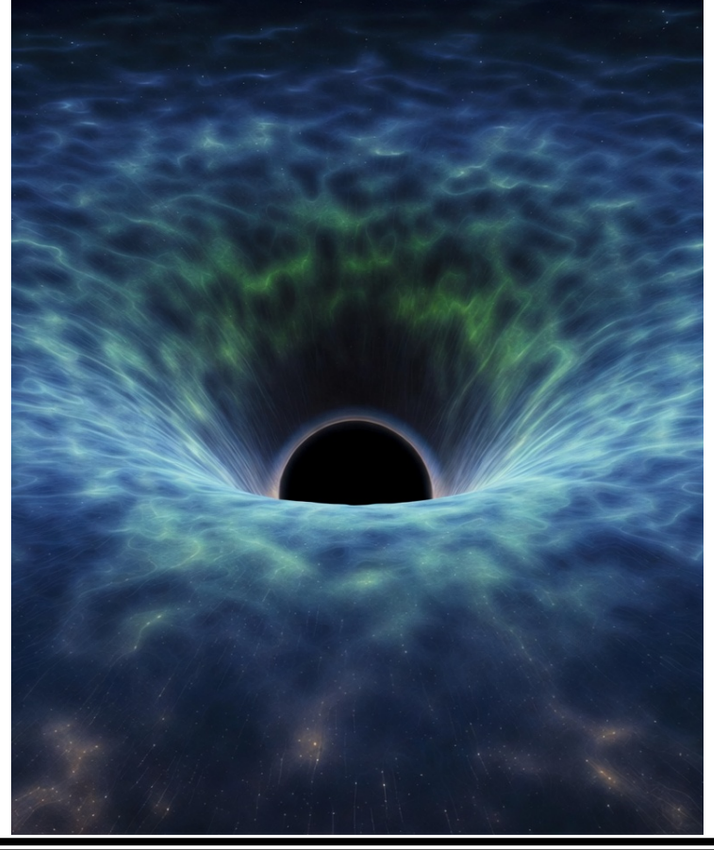


Paris-Saclay
Astroparticle Symposium
2026

26 October 2026 to
13 November 2026

Institut Pascal



On the Bondi Accretion of a Self-Interacting Complex Scalar Field

Alexander Vikman

28.10.2026

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Ministry of Education, Youth and Sports of the Czech Republic

FZU Institute of Physics of the Czech Academy of Sciences

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$\dot{\varphi} \simeq m$

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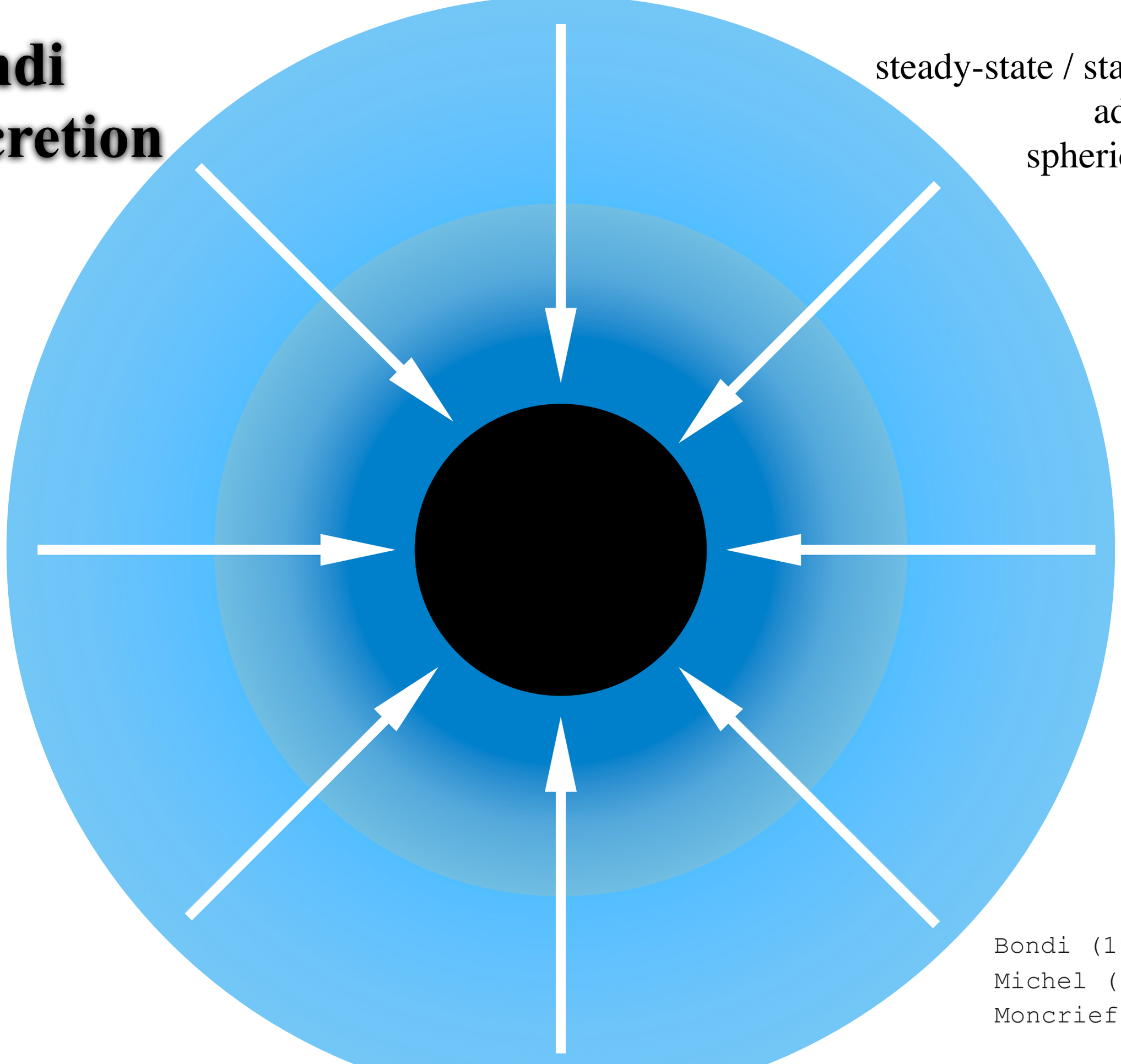
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Bondi Accretion

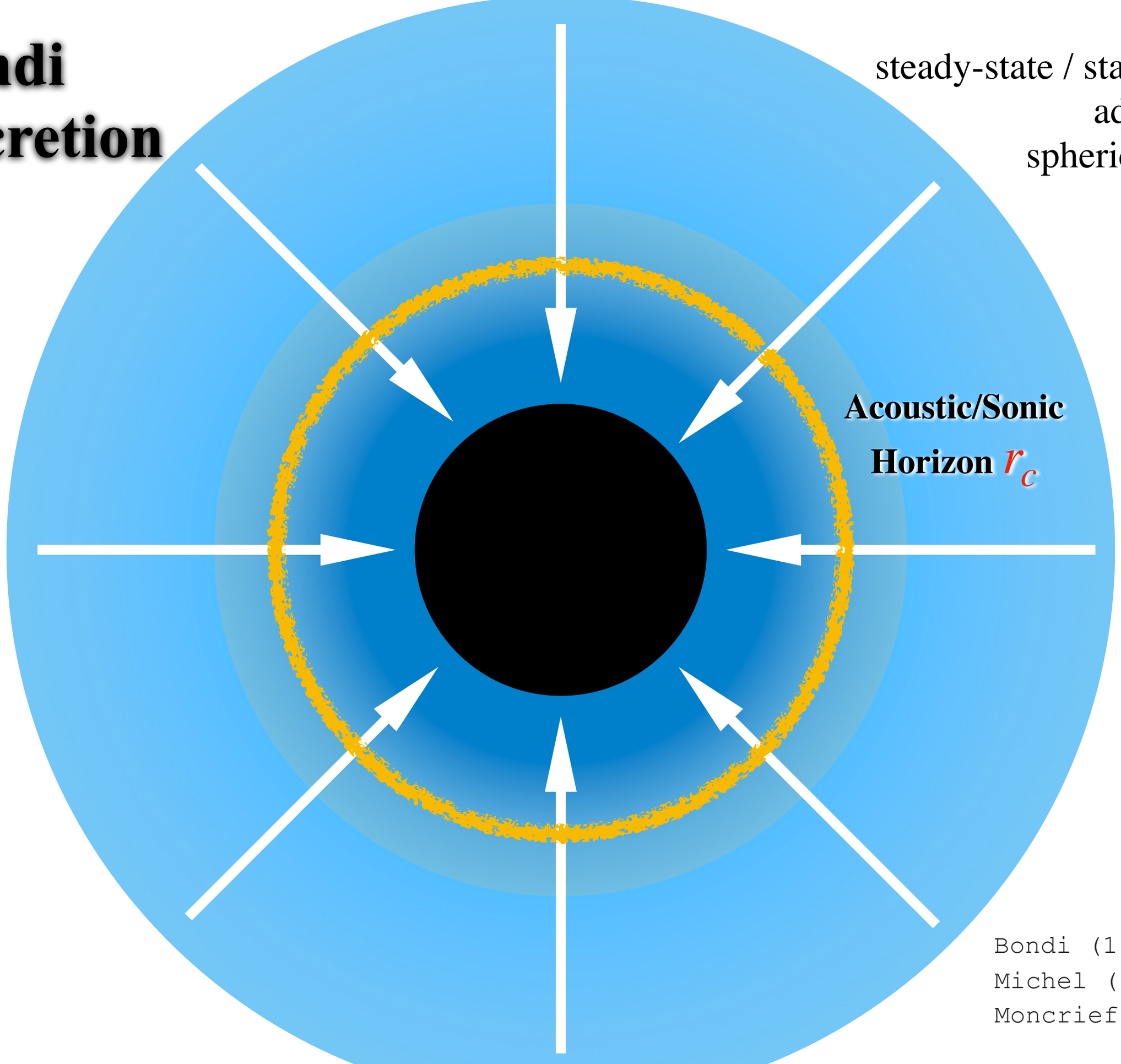
steady-state / stationary,
adiabatic,
spherical flow



Bondi (1952)
Michel (1971)
Moncrief (1980)

Bondi Accretion

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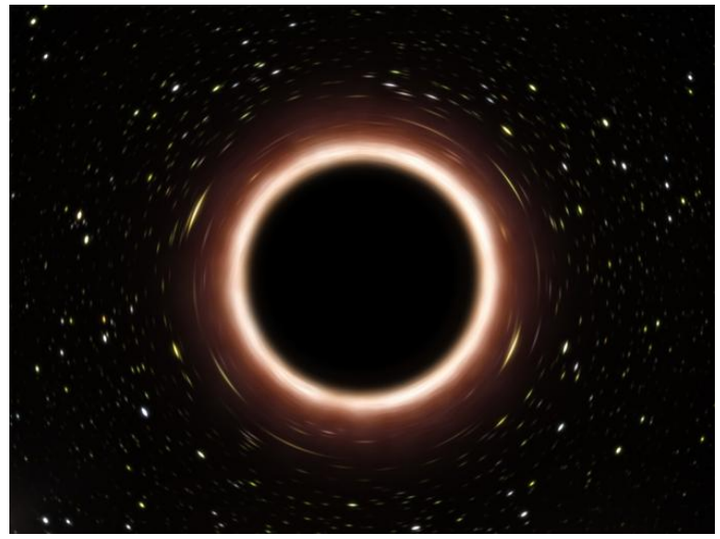


Acoustic/Sonic Horizon r_c

Bondi (1952)
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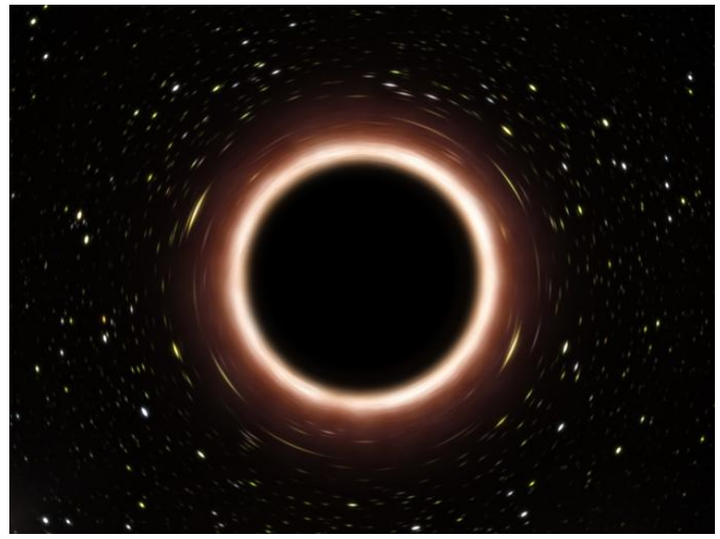
Accretion of Cosmological Condensates onto Black Holes

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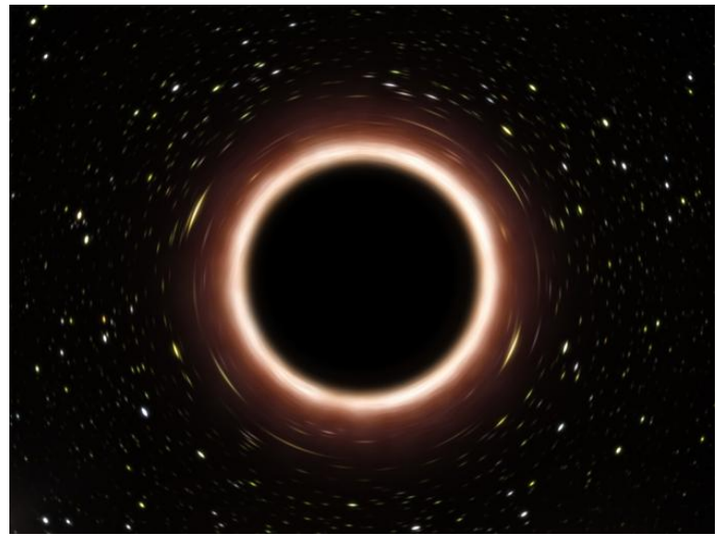
Accretion of Cosmological Condensates onto Black Holes

- Bose-Einstein condensate/superfluid of light DM



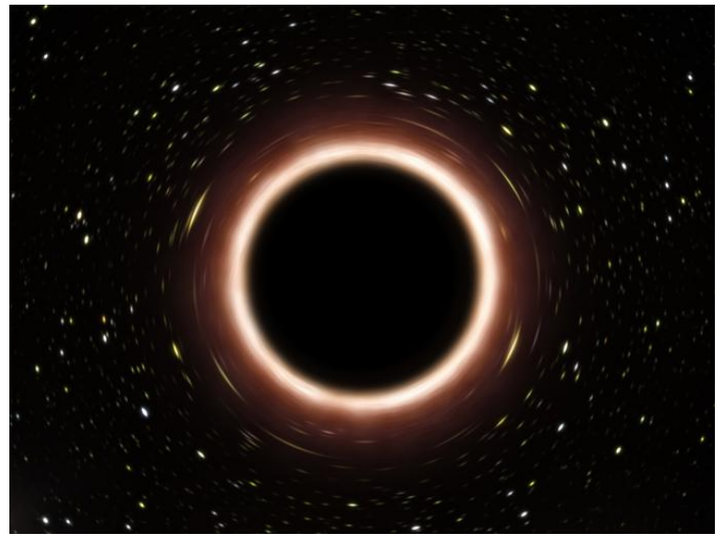
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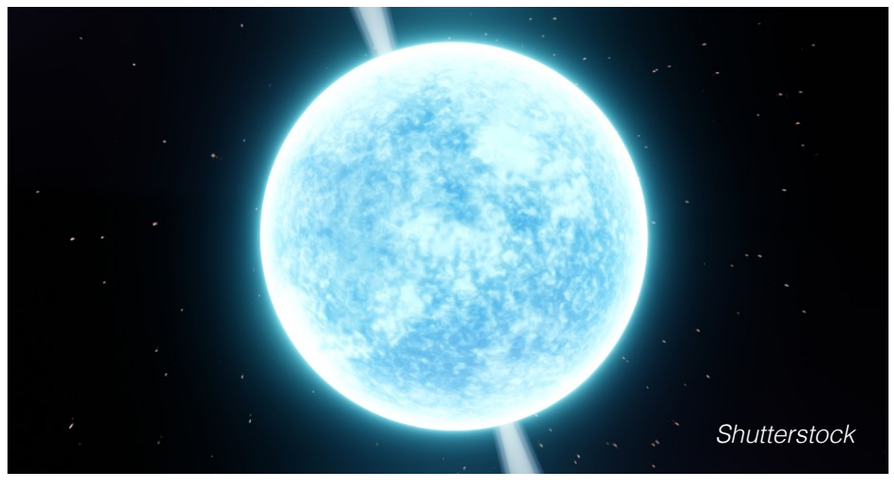
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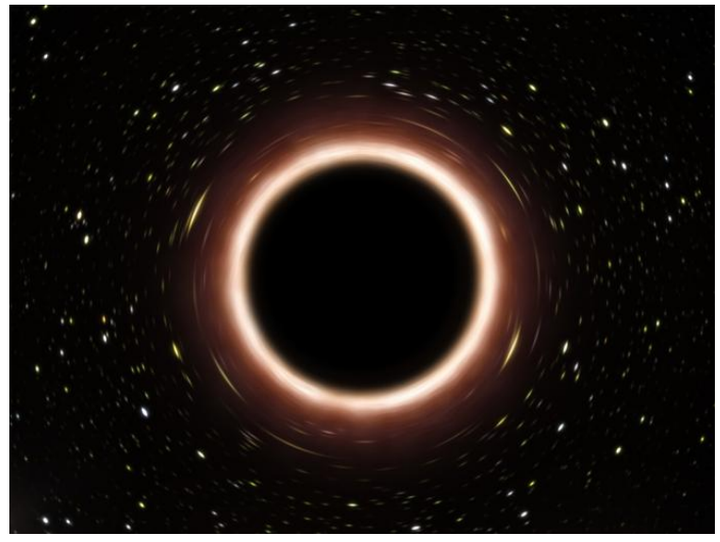
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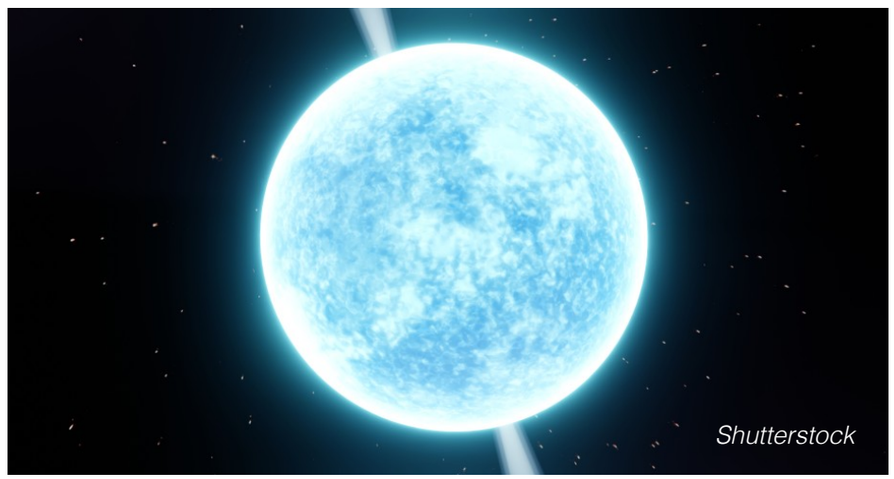


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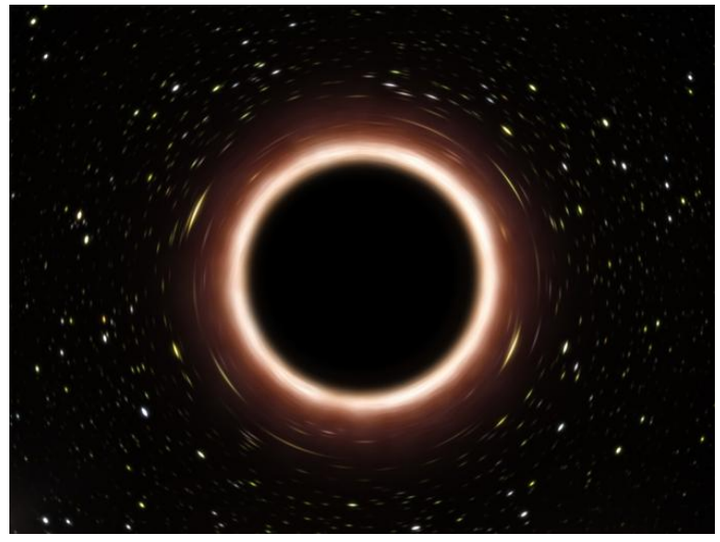


Accretion of Compact Source Material onto Primordial Black Holes at the Centre

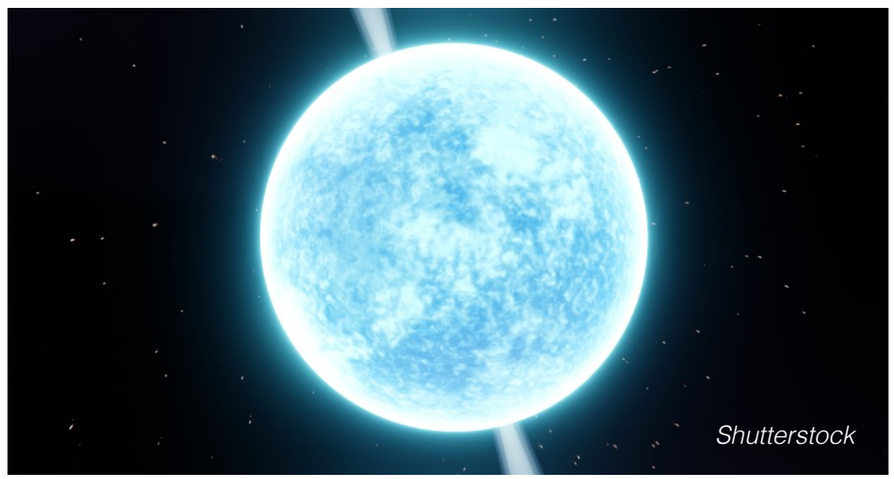


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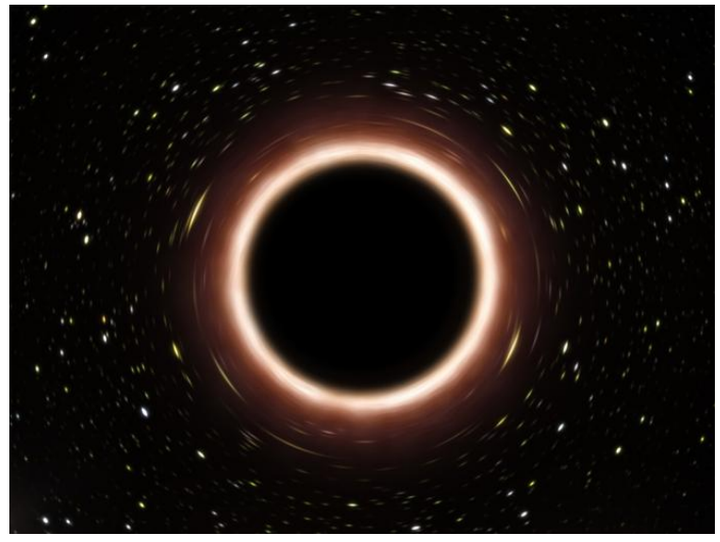


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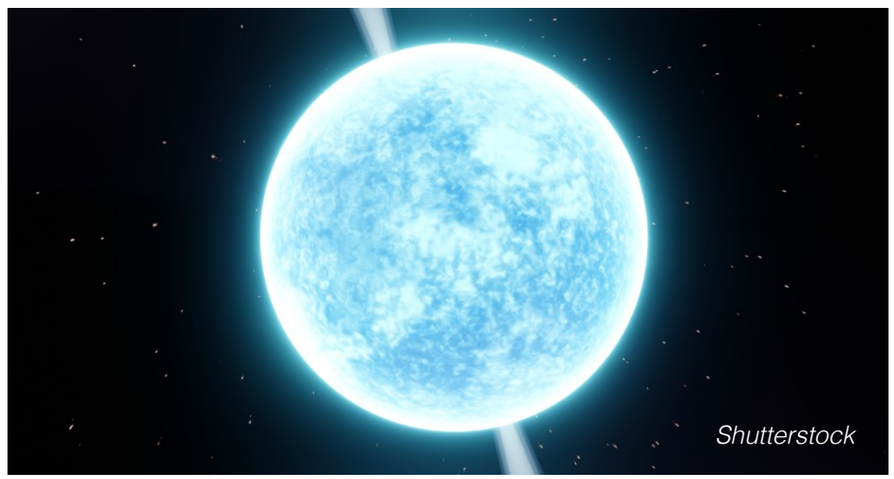
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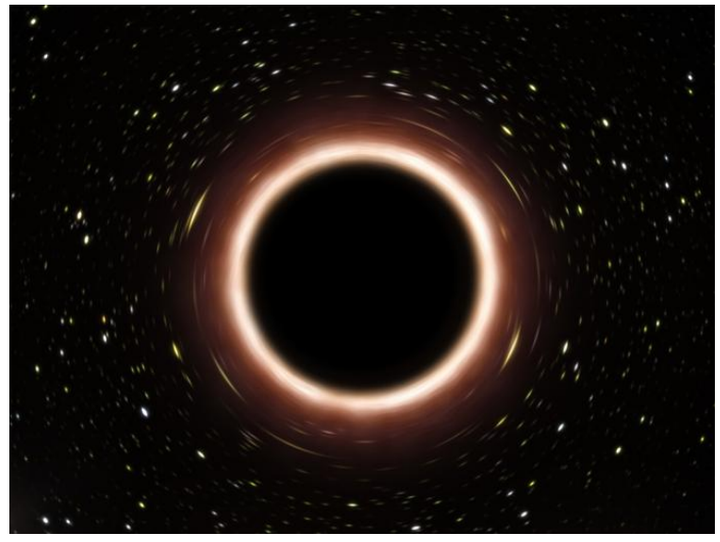
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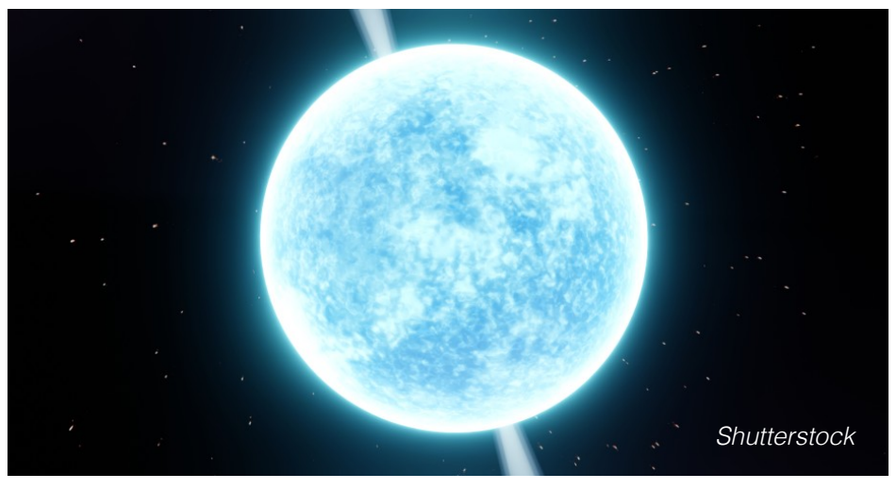
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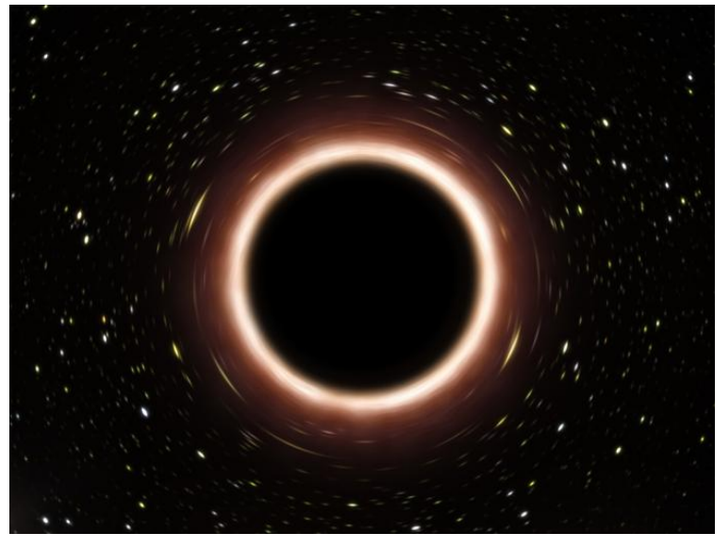
Accretion of Compact Source Material onto Primordial Black Holes at the Centre

- Condensates with stiff equations of state inside a neutron star
- Boson Star (or Q Ball) material



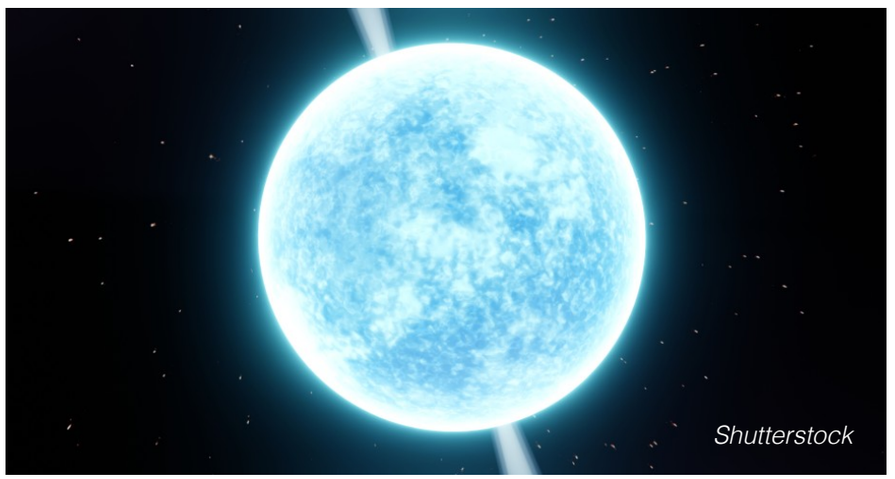
Accretion of Cosmological Condensates onto Black Holes

- Bose-Einstein condensate/superfluid of light DM



Accretion of Compact Source Material onto Primordial Black Holes at the Centre

- Condensates with stiff equations of state inside a neutron star
- Boson Star (or Q Ball) material
- Condensates in Dwarf Stars



Canonical, Complex U(1)-Globally-Symmetric Scalar Field

$$S_{\mathrm{cs}}[\Psi, \Psi^*, g] = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \, \partial_\nu \Psi^* - V(|\Psi|) \right]$$

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$$V\left(\rho\right)=\frac{1}{2}m^2\rho^2+\frac{1}{4}\lambda\rho^4\quad\textbf{with }\lambda>0\textbf{, but }\;m^2\textbf{ can be positive or }\textit{\textbf{negative}}$$

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$$V_0=\theta(-m^2)\frac{m^4}{4\lambda}\equiv\theta(-m^2)\varrho_m$$

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Eddington-Finkelstein and Field Ansatz

Schwarzschild Black Hole $\quad ds^2 = f(r)\,dt^2 - f^{-1}(r)\,dr^2 - r^2 d\Omega^2$

$$f = 1 - \frac{r_S}{r} \qquad r_S = 2G_N M$$

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Ansatz for Spherically Symmetric Stationary Configuration

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leaves the EMT

$$T_{\mu\nu} = \partial_\mu \rho \, \partial_\nu \rho + \rho^2 \partial_\mu \varphi \, \partial_\nu \varphi - g_{\mu\nu} \left[\frac{1}{2} \left(\partial \rho \right)^2 + \frac{1}{2} \rho^2 \left(\partial \varphi \right)^2 - V \left(\rho \right) \right]$$

to be dependent only on radius due to shift-symmetry of the phase field φ

Current Conservation

$$\partial_v \left(\sqrt{-g} J^v \right) + \partial_r \left(\sqrt{-g} J^r \right) = 0$$

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$$J^r=-\dot{\varphi}_0\rho^2\left(1+fW\right) \quad \Rightarrow \quad W\left(r,\rho\right)$$

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$$W\left(r,\rho\right)$$

$$X=\frac{\dot{\varphi}_0^2}{f}\left[1-\frac{(1-f)^4\mathscr{F}^2}{r_S^4\rho^4}\right]$$



$$\rho_S=\frac{\sqrt{\mathscr{F}}}{r_S}$$

Dimensionless quantities

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$$\sigma = \frac{\sqrt{\lambda}\,\rho}{\dot{\varphi}_0}$$

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Energy Density Scale

$$\dot{\varphi}_0^4$$

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$$\lambda$$

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nuclear condensate onto PBH

$$\xi \sim \frac{10^{20} g}{M_{Pl}} \frac{100\,\textbf{MeV}}{M_{Pl}} \sim 10^5$$

Dimensionless quantities

Energy Density Scale

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fluid-like DM onto solar mass BH

$$\xi \sim \frac{10^{33} g}{M_{Pl}} \frac{10^{-20}\,\textbf{eV}}{M_{Pl}} \sim 10^{-10}$$

$$\xi = r_S\,\dot{\varphi}_0$$

$$\beta^2 = \frac{\lambda \mathcal{F}}{\xi^2} \quad \Rightarrow \quad \sigma_S = \beta$$

Accretion Rate

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Applicability of test Field

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Applicability of test Field

1. $\dot{M}\dfrac{1}{H} \ll M$ **where** $H^2 \simeq G\dfrac{\dot{\varphi}_0^4}{\lambda}$

Accretion Rate

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Applicability of test Field

1. $\dot{M}\dfrac{1}{H} \ll M$ **where** $H^2 \simeq G\dfrac{\dot{\varphi}_0^4}{\lambda}$  $\lambda \gg G^3 M^2 \dot{\varphi}_0^4 \simeq \xi^2 G \dot{\varphi}_0^2$

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3. $\frac{M}{r^3} \geq \frac{\dot{\varphi}_0^4}{\lambda}$ **where this broken** $r_K \simeq \left(\frac{\lambda M}{\dot{\varphi}_0^4}\right)^{1/3} \gg r_s$ $\qquad \lambda \gg G^2 \dot{\varphi}_0^4$

Applicability of test Field II

$$\lambda \gg \xi^2 \frac{\dot{\varphi}_0^2}{M_{Pl}^2} = \left(\frac{M}{M_{Pl}}\right)^2 \left(\frac{\dot{\varphi}_0}{M_{Pl}}\right)^4$$

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$$\lambda \gg \left(\frac{10^{20}\,\mathbf{g}}{10^{-5}\,\mathbf{g}}\right)^2 \left(\frac{100\,\mathbf{MeV}}{10^{22}\,\mathbf{MeV}}\right)^4 = 10^{-30}$$

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heavy fluid-like DM onto $10^9 M_\odot$ BH

$$\lambda \gg \left(\frac{10^{9} \times 10^{33}\,\mathbf{g}}{10^{-5}\,\mathbf{g}}\right)^2 \left(\frac{1\,\mathbf{eV}}{10^{28}\,\mathbf{eV}}\right)^4 = 10^{-18}$$

Master Equation, Boundary Problem

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Equation of state neglecting ∂X, "Perfect Fluid"

$$p(\epsilon)_{[+]} = \frac{4}{9}\varrho_m \left(\sqrt{1 + \frac{3}{4}\frac{\epsilon}{\varrho_m}} - 1 \right)^2 \qquad \text{for} \quad m^2 > 0$$

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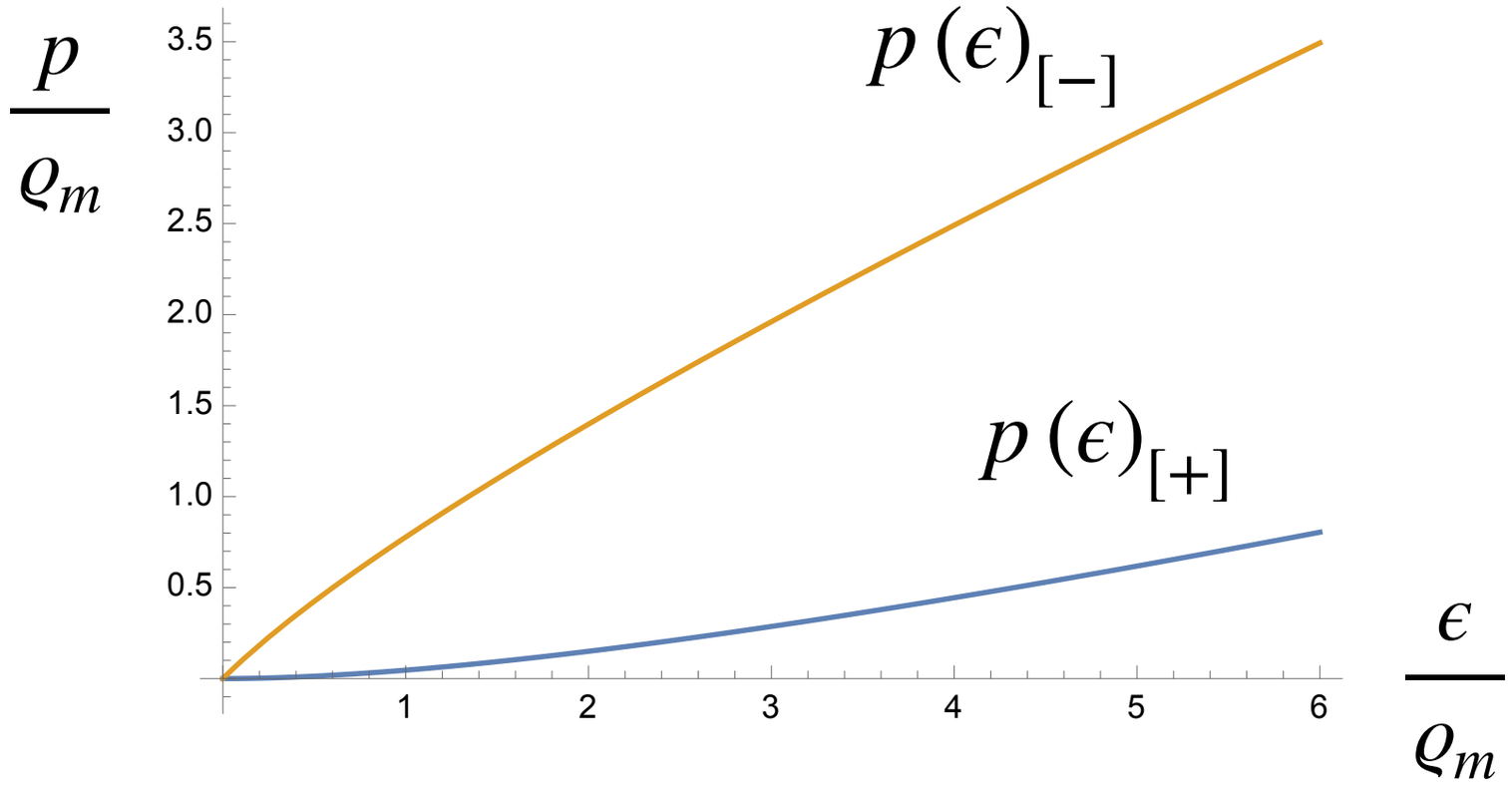
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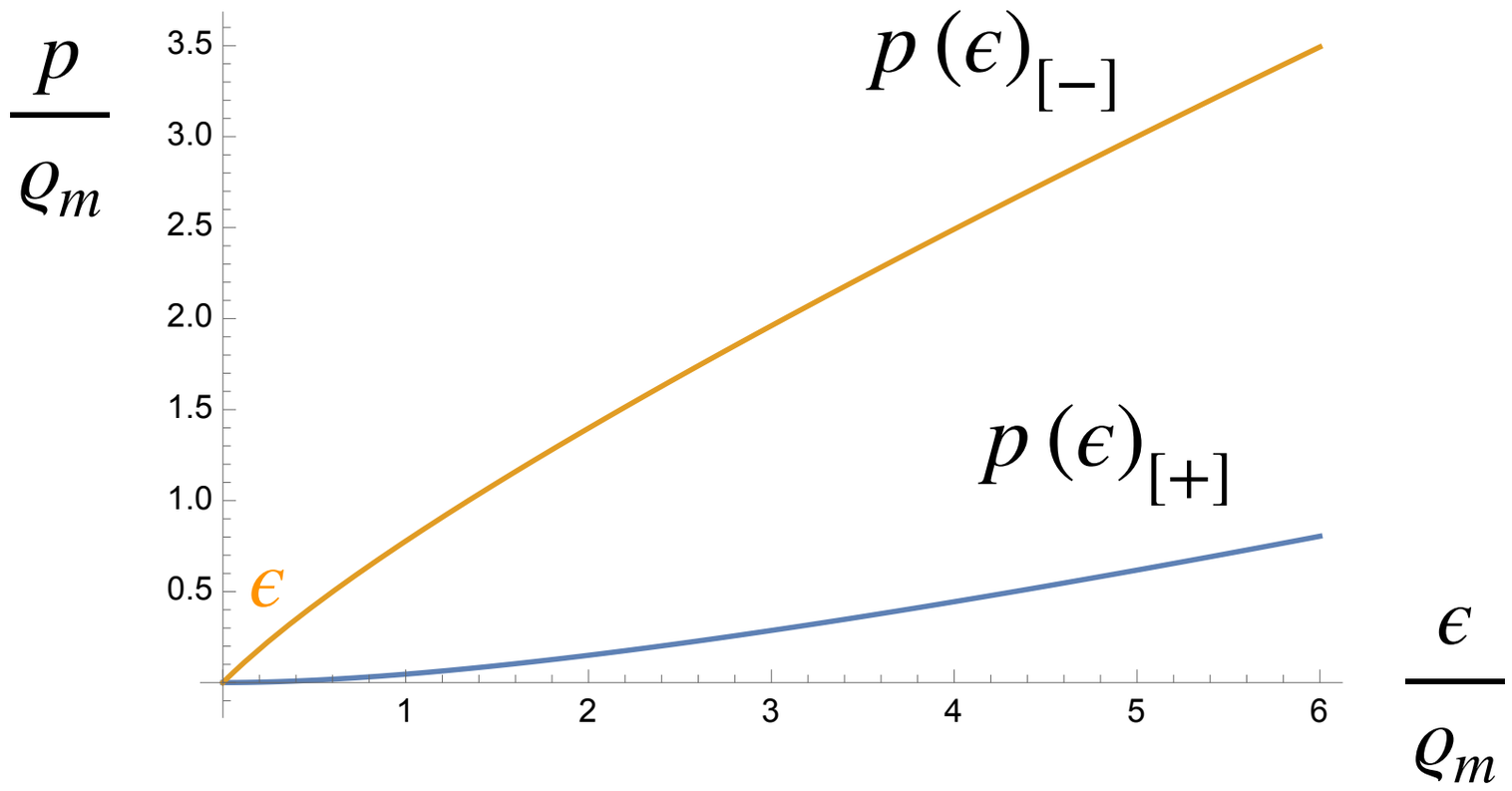
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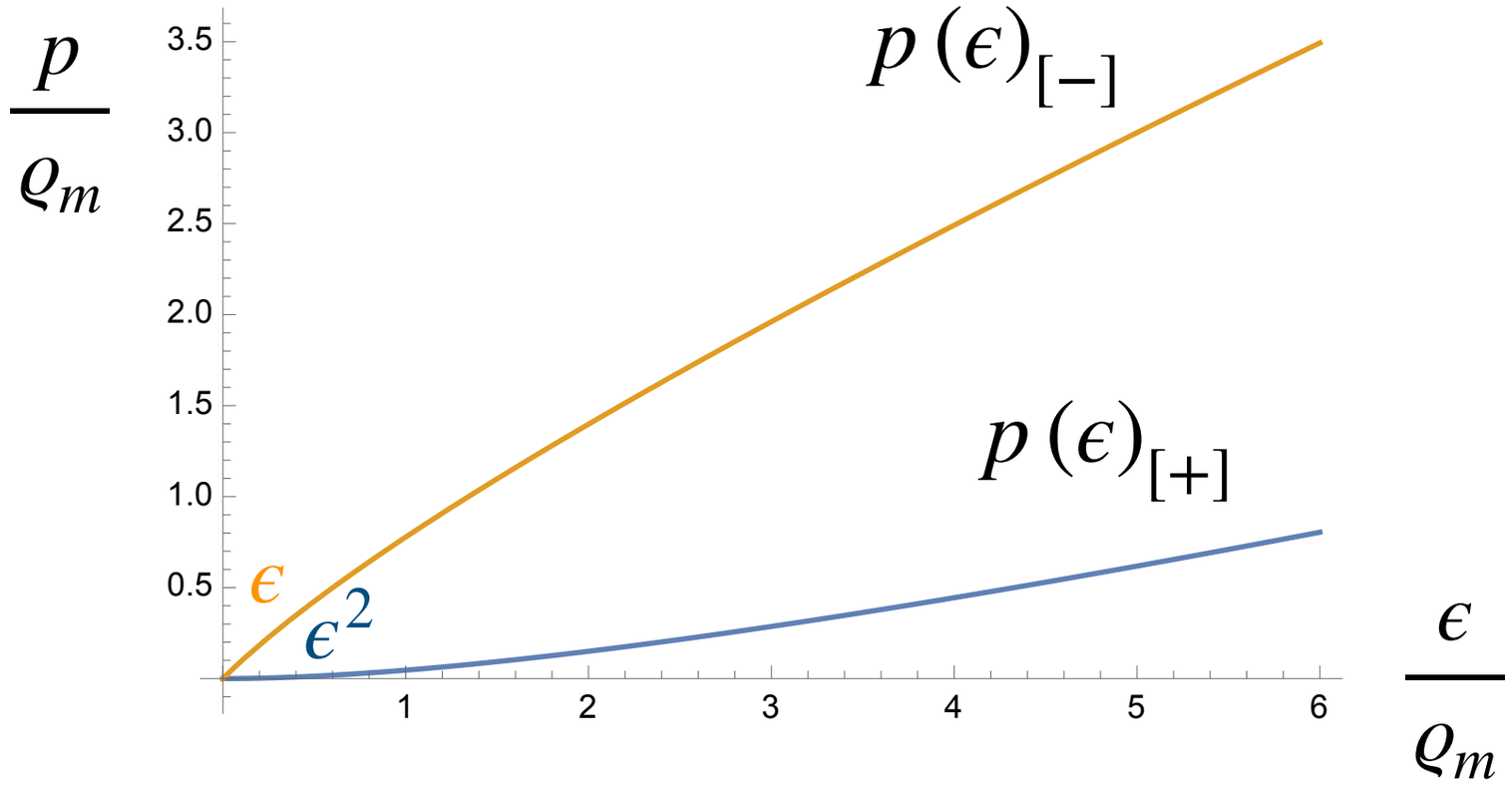
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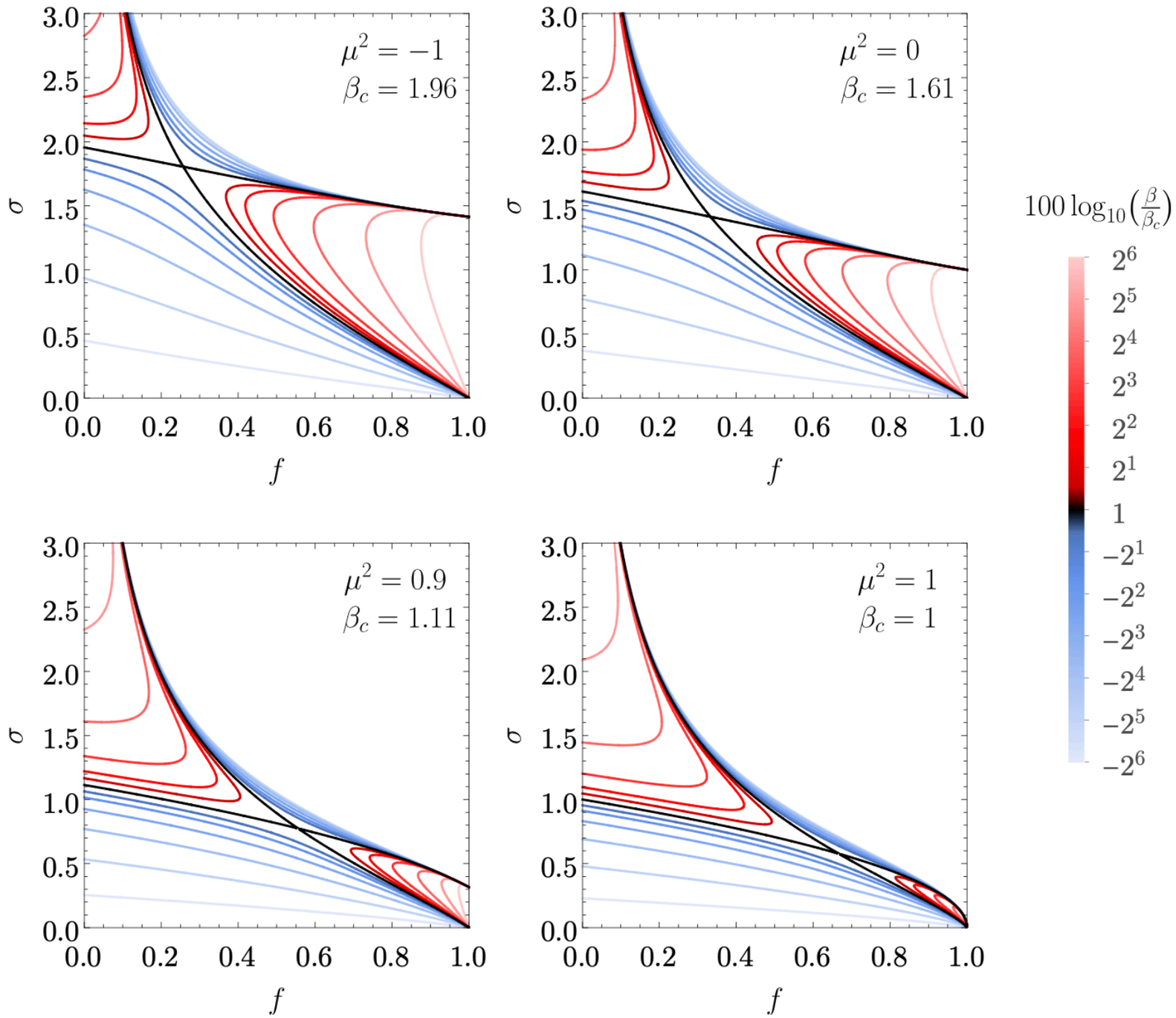
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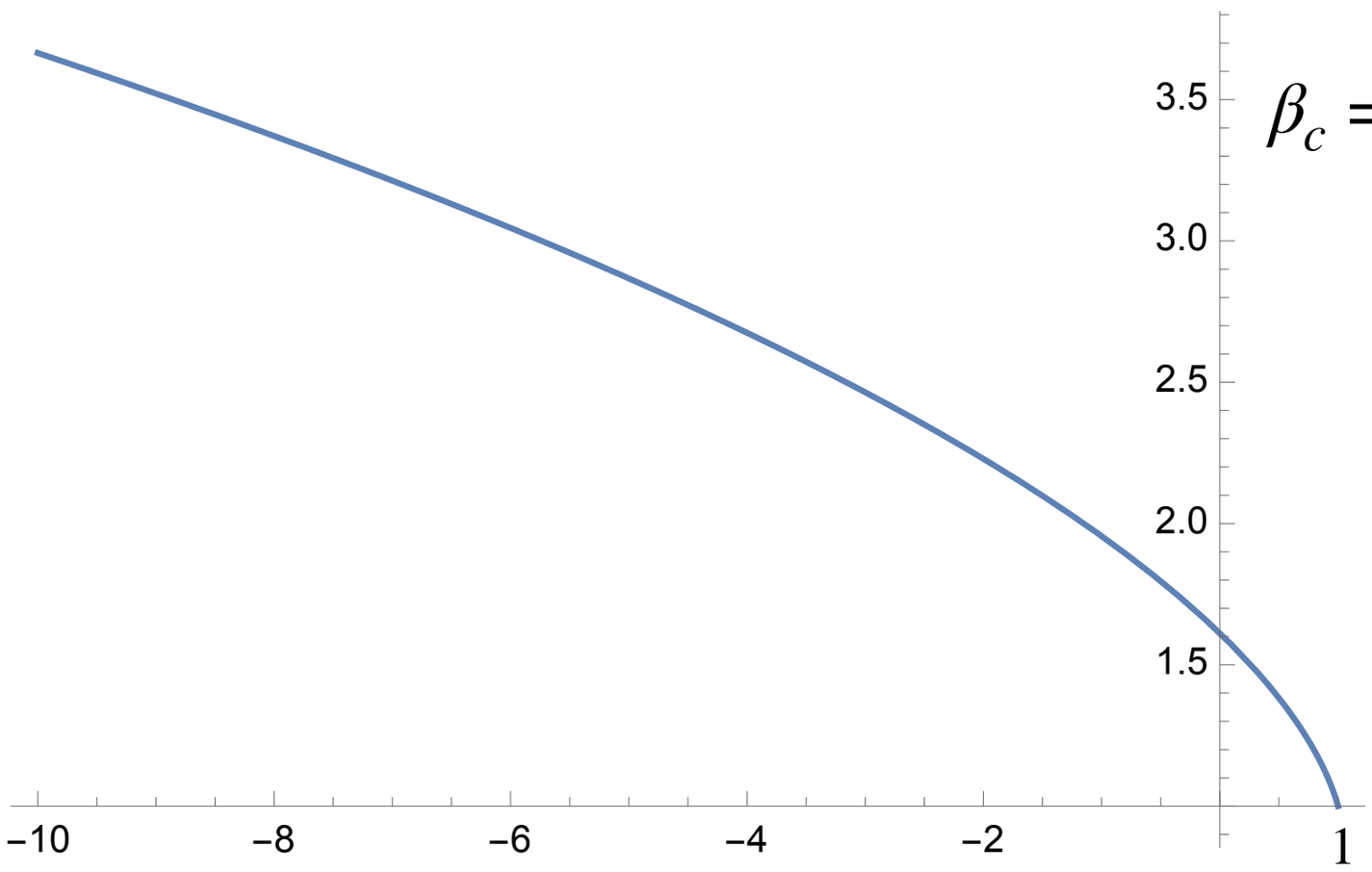
Solutions, Critical Point, for $\xi \to \infty$

$$\mathcal{U}'(\mu^2, \beta^2; f, \sigma) = \mu^2\sigma + \sigma^3 - \frac{\sigma}{f}\left[1 - \frac{(1-f)^4\beta^4}{\sigma^4}\right] = 0$$



modulos value ρ_S on horizon

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$$\beta_c = \frac{\sqrt{\lambda}\,\rho_S}{\dot{\varphi}_0}$$

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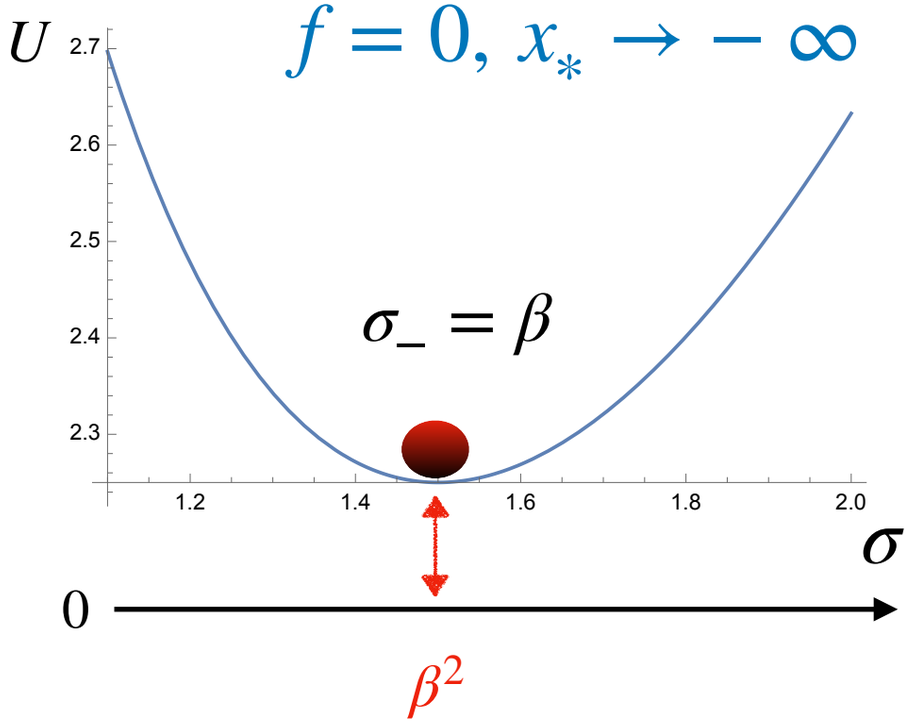
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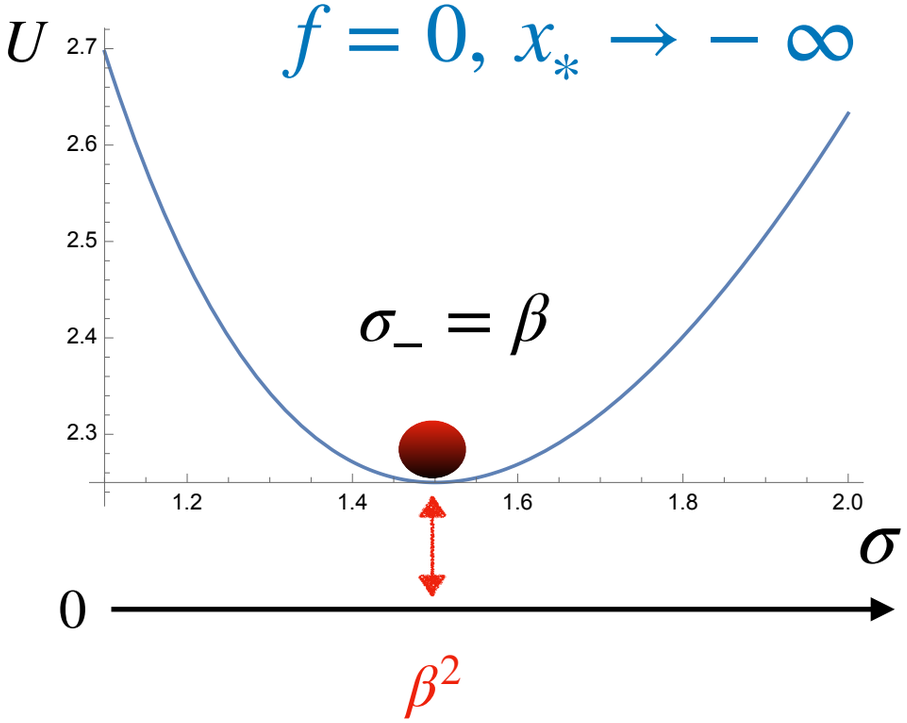
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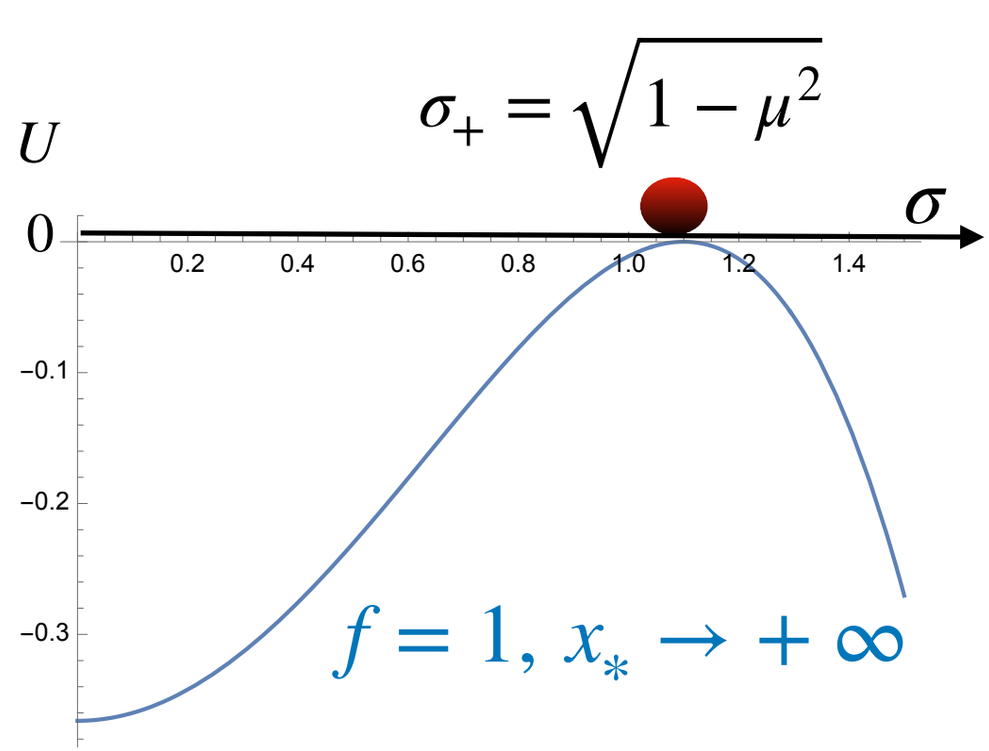
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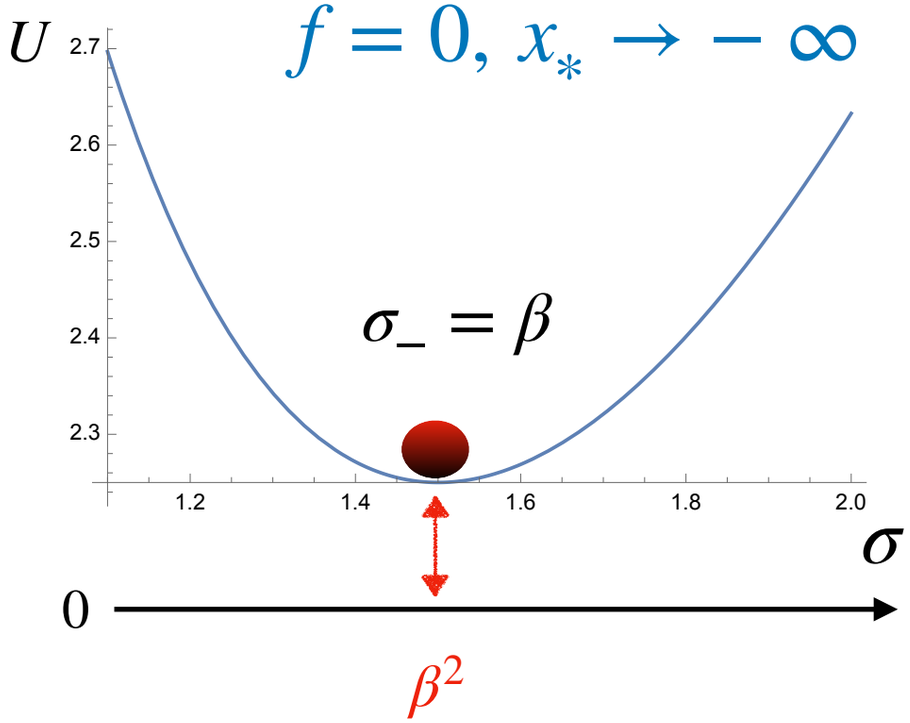


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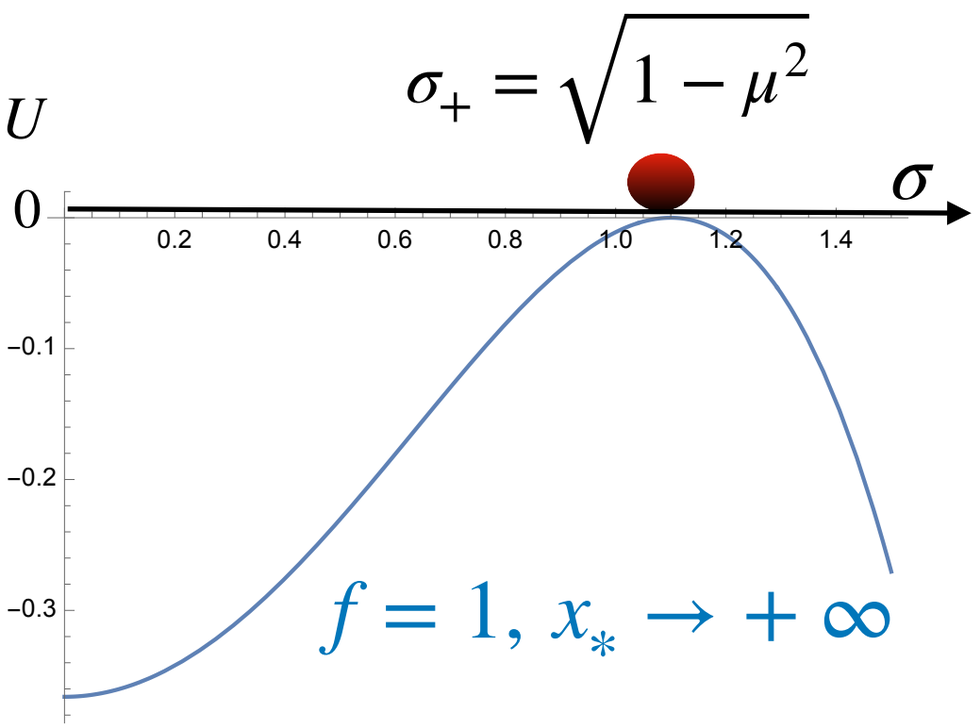
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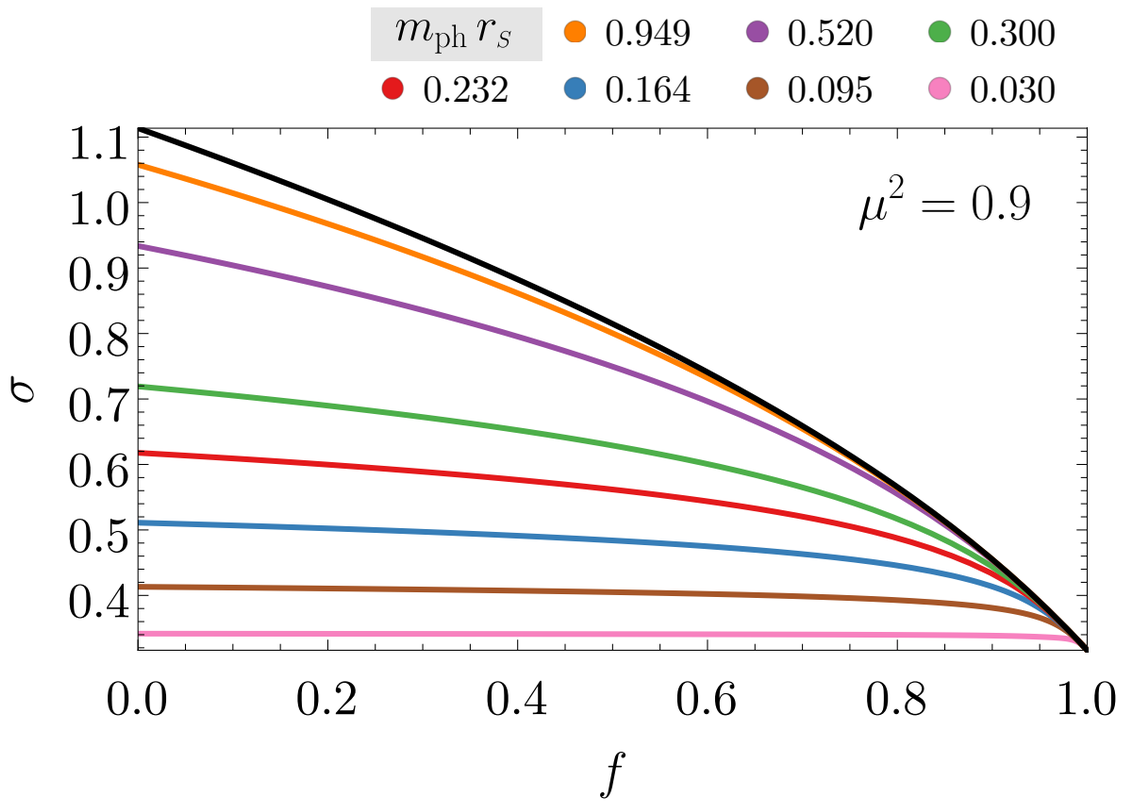
the lighter is the particle, the further it will fly

accretion is maximal for k-essence/P(X)



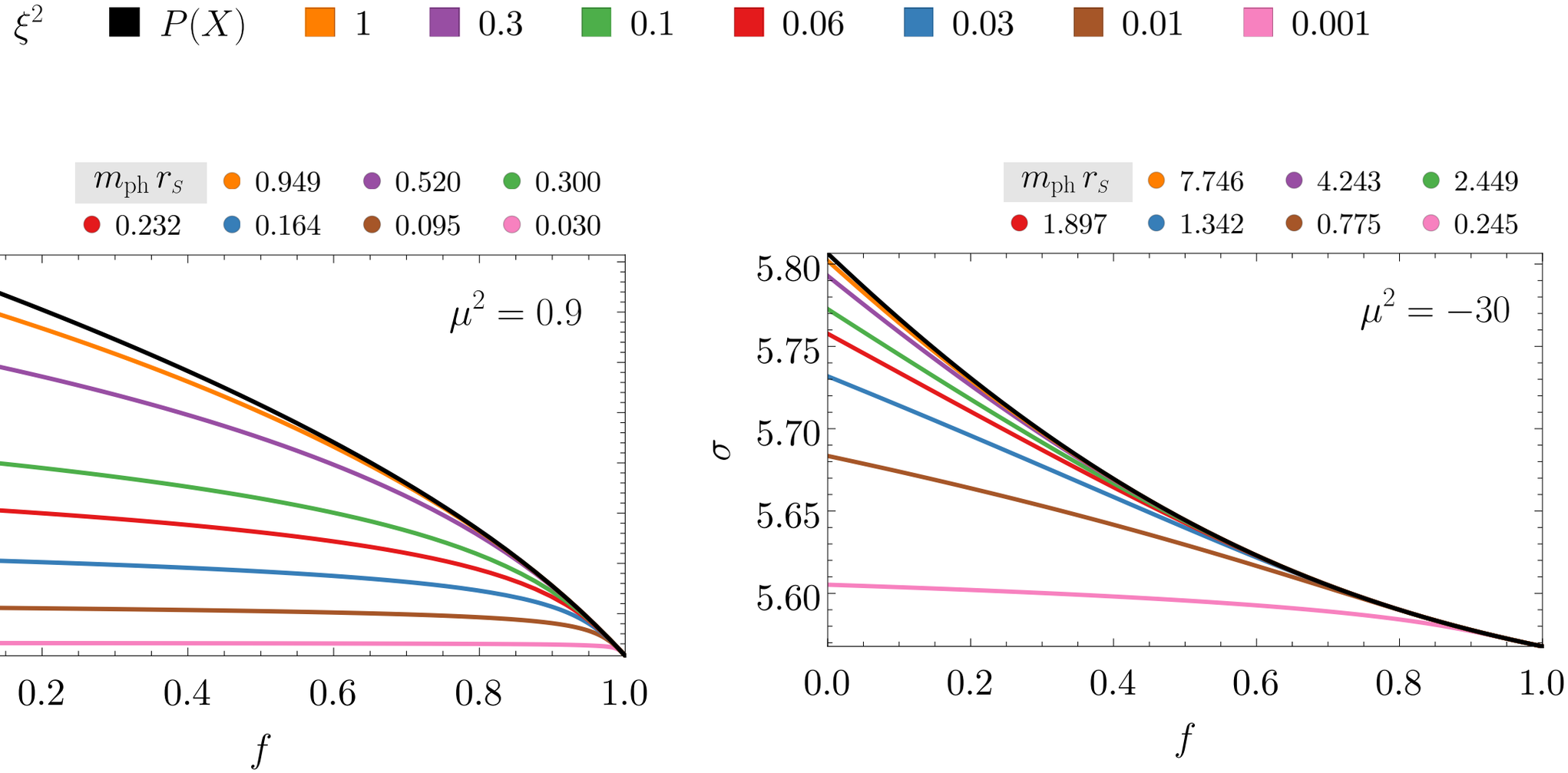
Field Profile $\sigma(f)$

ξ^2 $P(X)$ 1 0.3 0.1 0.06 0.03 0.01 0.001



$$\sigma = \frac{\sqrt{\lambda}}{\dot{\varphi}_0}\,\rho \qquad f = 1 - \frac{r_S}{r} \qquad \mu^2 = \frac{m^2}{\dot{\varphi}_0^2} \qquad \xi = r_S\,\dot{\varphi}_0$$

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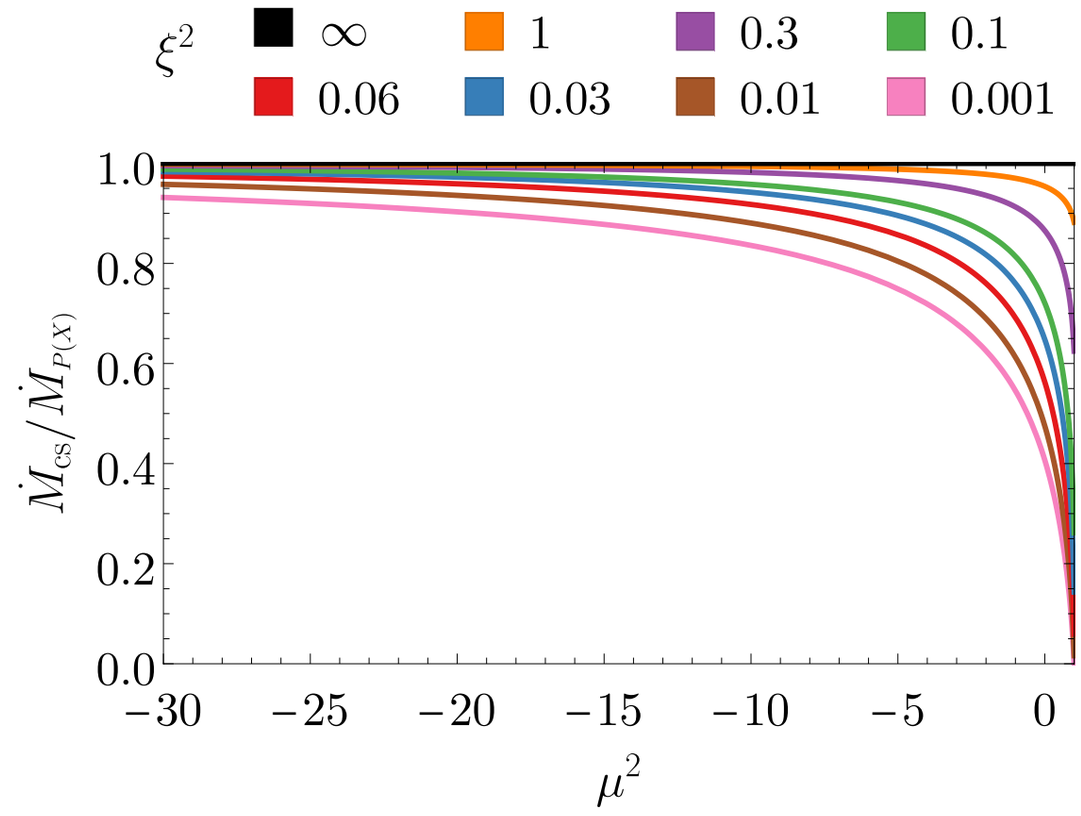
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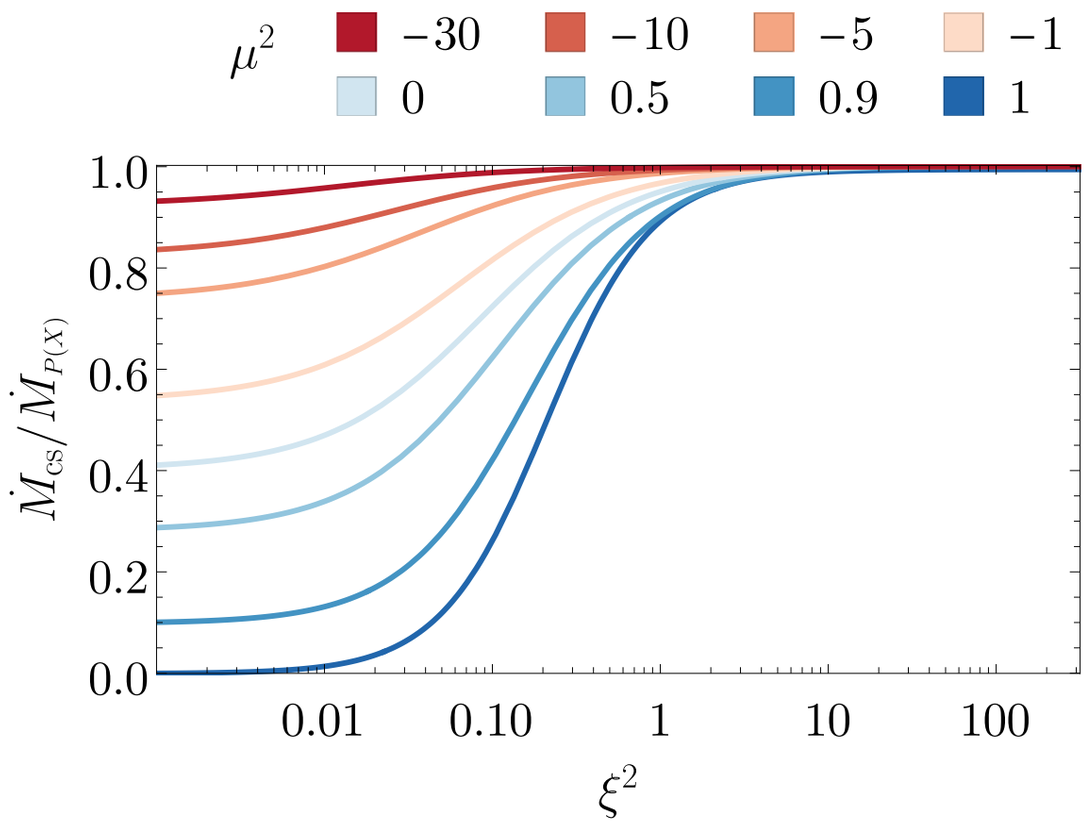
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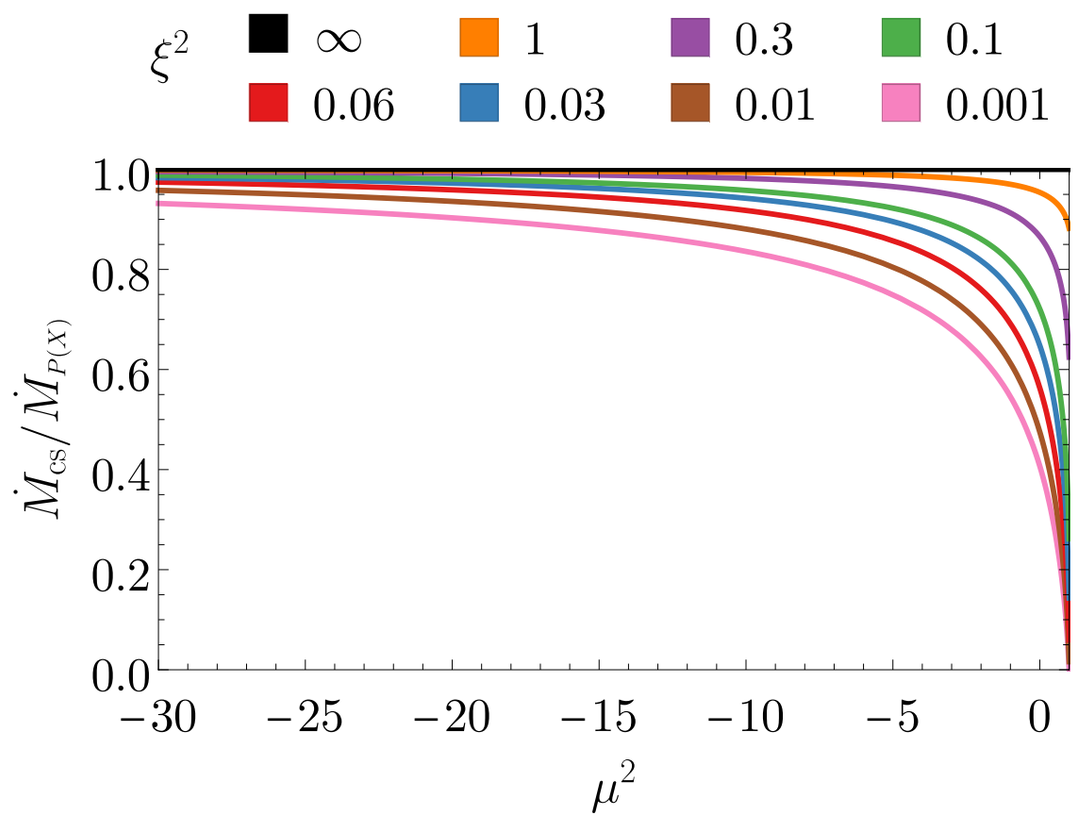


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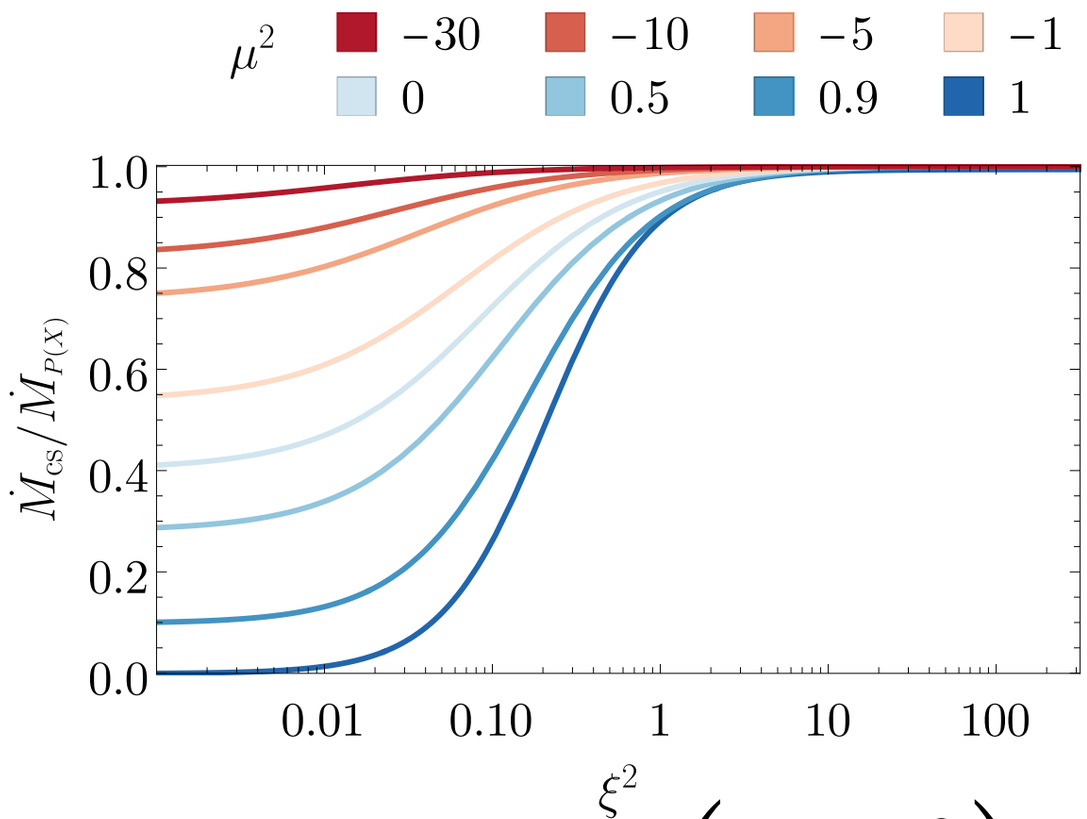


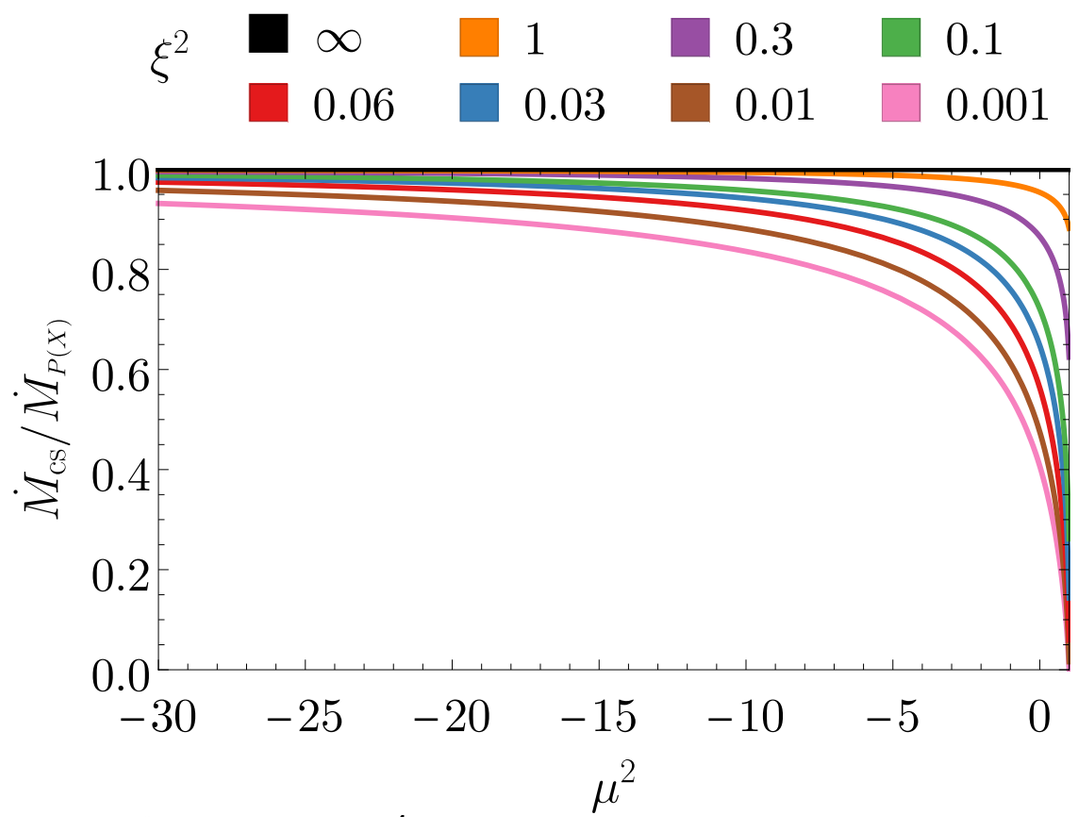


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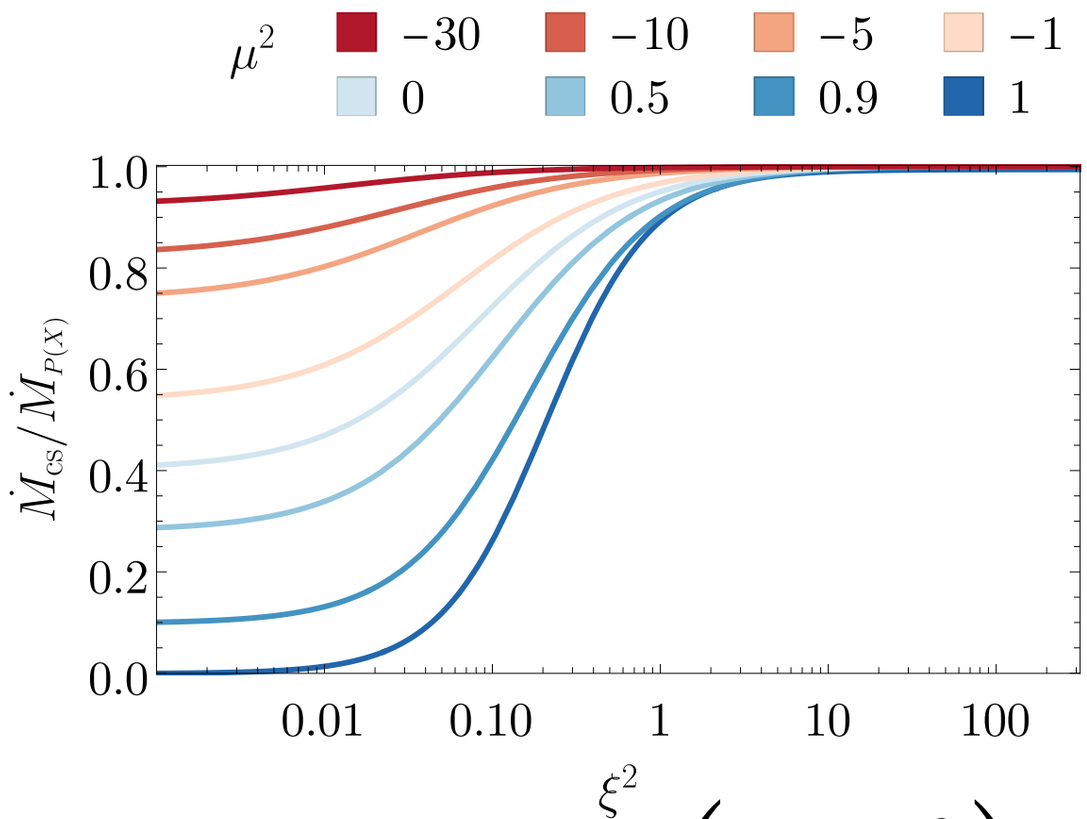


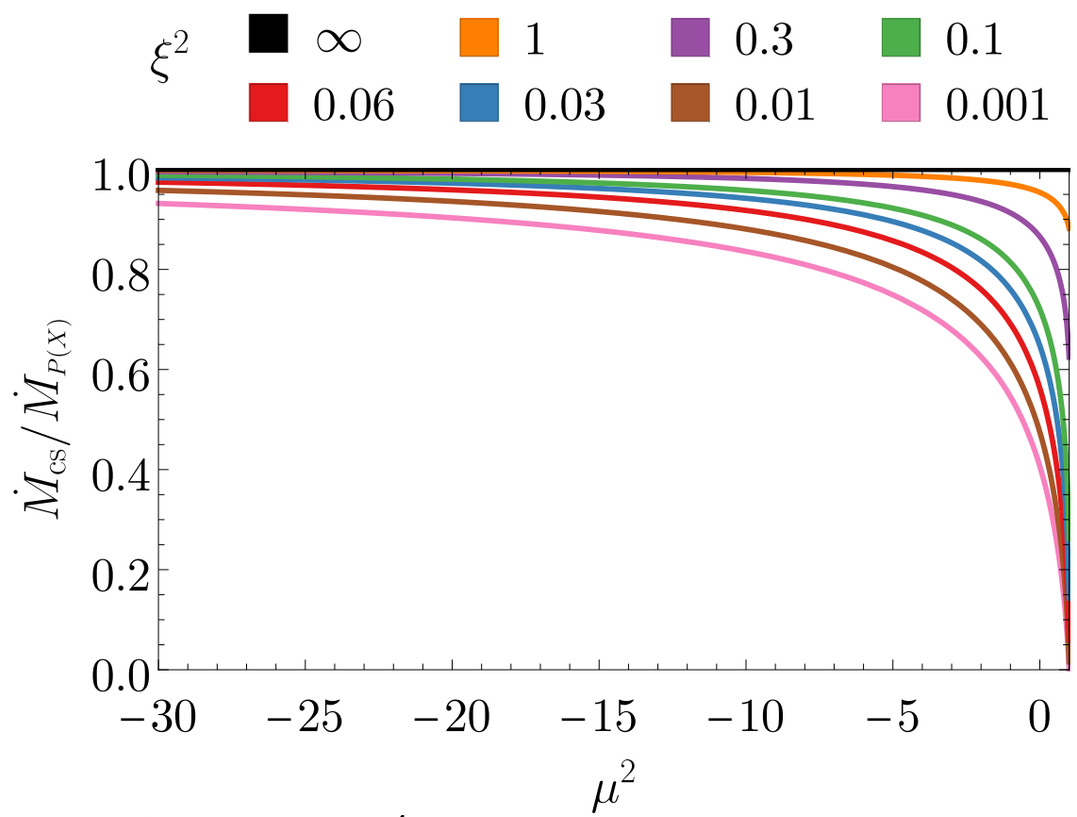
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Thanks a lot for attention!