Cosmological gravitational particle production

University of Helsinki



Duarte Feiteira

Supervisor: Prof. Oleg Lebedev

Duarte Feiteira, Oleg Lebedev; Cosmological gravitational particle production: Starobinsky vs Bogolyubov, uncertainties, and issues; ***JCAP* 07 (2025) 003**

Duarte Feiteira, Fotis Koutroulis, Oleg Lebedev, Stefan Pokorski; Quark, lepton and right-handed neutrino production via inflation; ***JCAP 03 (2026) 030***

Duarte Feiteira, Oleg Lebedev, Vinícius Oliveira; Warm dark matter from freeze-in at stronger coupling; ***Phys. Lett. B 877 (2026) 140486***

Duarte Feiteira, Vinícius Oliveira; Warm fermionic dark matter from freeze-in at stronger coupling; ***arXiv 2606.20815***

Outline

1. **Motivation**
2. **Scalar production via inflation**
3. **Fermion production via inflation**
4. **Conclusions**



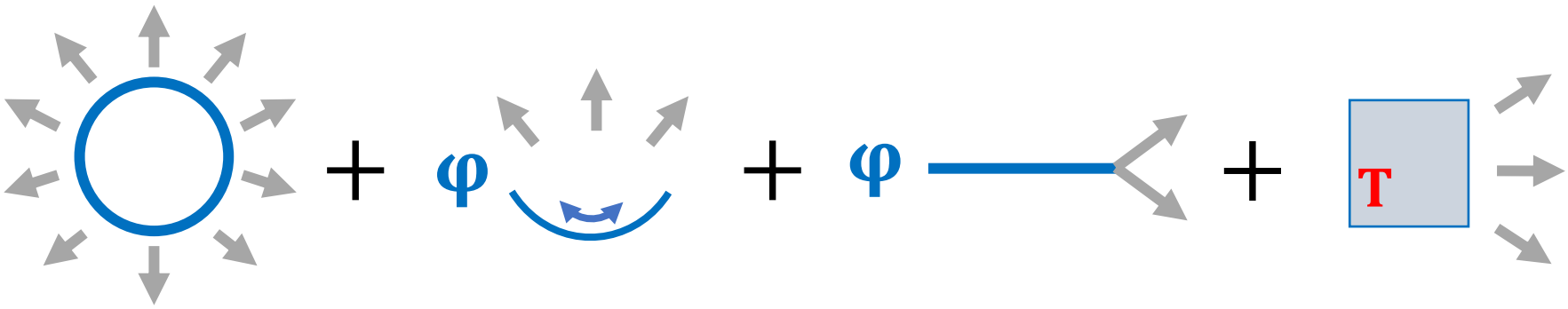
Oulanka National Park, Finland

Motivation

Lebedev 2210.02293

- Gravitational particle production is crucial for **non-thermal dark matter** studies.

- **Add up** all production mechanisms:



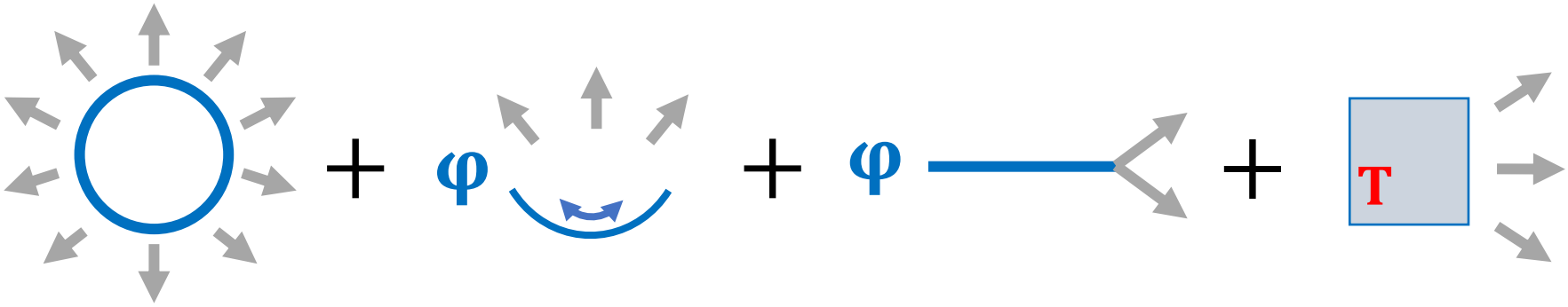
During inflation + **Via inflaton oscillations** + **Inflaton decay** + **Thermal emission (freeze-in)**

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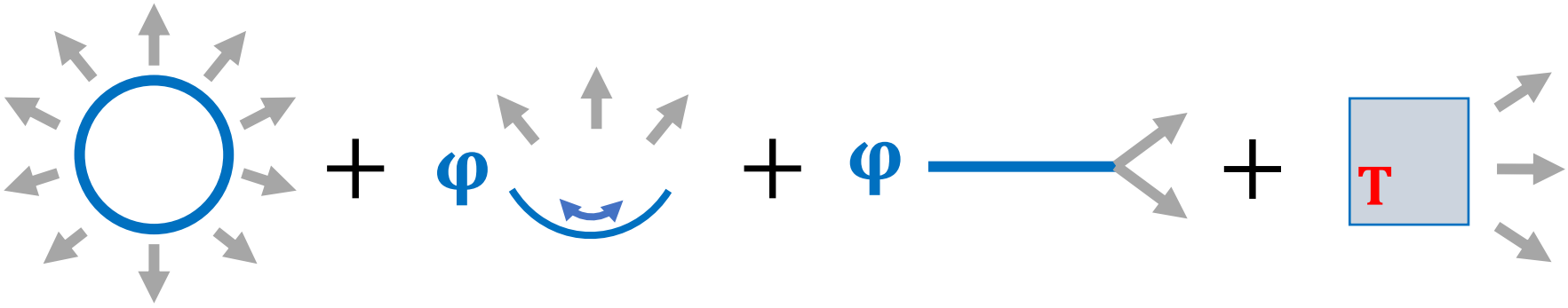
Irreducible background for other production mechanisms.

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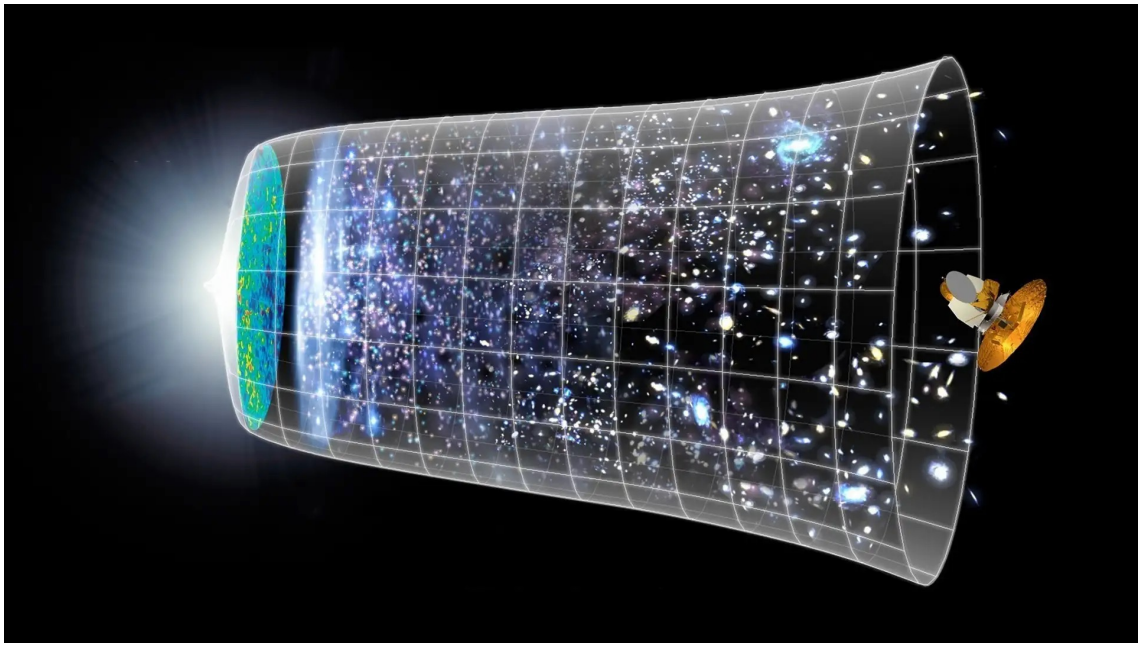
Inflaton decay

Thermal emission (freeze-in)

Irreducible background for other production mechanisms.

- **Dark relics:** if particle number is conserved (free or very weakly interacting particles), dark relics produced in inflation survive up to the present day.

Scalar production via inflation

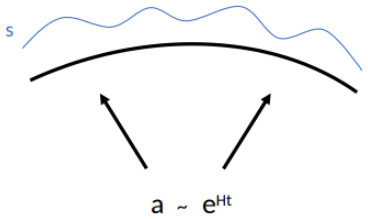


Timeline of the universe based on the Big Bang theory and inflation models. NASA/WMAP.

Bogolyubov coefficient approach

- Condensate of a **scalar spectator field** with fluctuations.

$$\phi(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{\dagger}\,\chi_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$



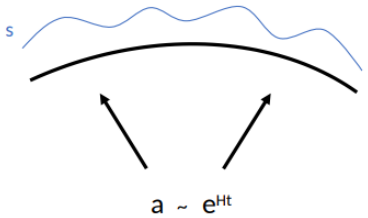
Parker, Grib, Zeldovich, Starobinsky. 60-70's

$$\chi_k''(\eta) + \omega_k^2(\eta)\,\chi_k(\eta) = 0, \qquad \omega_k^2(\eta) = k^2 + a^2(\eta)\,m^2 + \left(\frac{1}{6} - \xi\right)a^2(\eta)R(\eta)$$

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- Vacua:

$$\eta \to -\infty \qquad\qquad \eta \to +\infty$$

$$a^{\text{in}}_{\mathbf{k}}|0^{\text{in}}\rangle = 0 \qquad\qquad a^{\text{out}}_{\mathbf{k}}|0^{\text{out}}\rangle = 0$$

$$\downarrow \qquad\qquad \downarrow$$

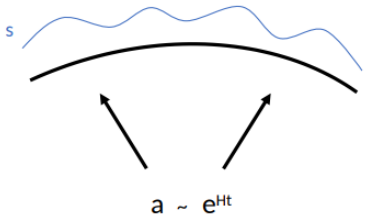
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Bunch-Davies (no particles) — Radiation/matter domination

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Bunch-Davies (no particles) — Radiation/matter domination

- Bogolyubov coefficient and particle number density:

$$\beta_k = i\left(\chi_k^{\text{out}\,\prime}\,\chi_k^{\text{in}} - \chi_k^{\text{out}}\,\chi_k^{\text{in}\,\prime}\right), \qquad a^3 n_k \equiv \frac{k^3}{2\pi^2}\left|\beta_k\right|^2$$

Bogolyubov coefficient approach

- **Analytical results:**

Inflation followed by **radiation dominated** epoch:

$$a^3 n = \frac{3\kappa^2 a_e^3}{4\pi^2} \boxed{\frac{H_e^{11/2}}{m^{5/2}}}$$

Very large number

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Field size at the end of inflation: $\langle \Phi^2 \rangle = \frac{3}{8\pi^2} \frac{H^4}{m^2}$

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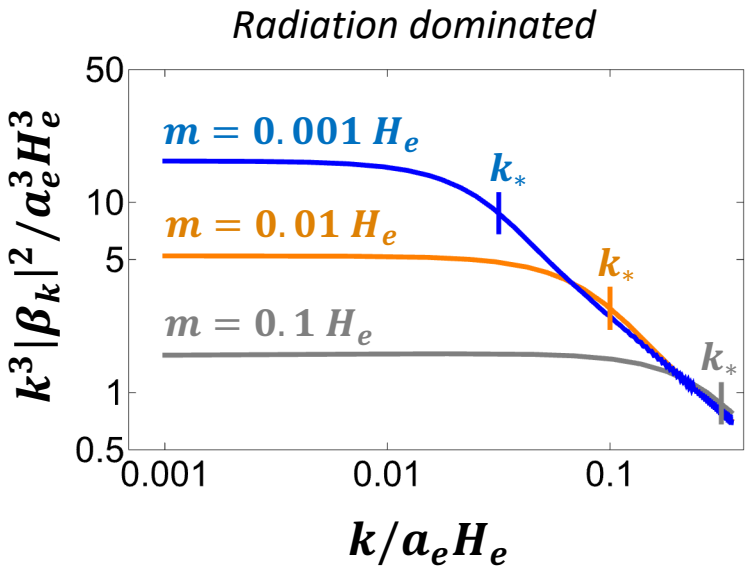
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- **Numerical results:**



Starobinsky approach

Starobinsky ‘86

$$\Phi(t,\mathbf{x}) = \bar{\Phi}(t,\mathbf{x}) + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\theta(k-\epsilon a(t)H)\left[a_{\mathbf{k}}\,\chi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{r}} + a_{\mathbf{k}}^{\dagger}\,\chi_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{r}}\right]$$

Long wavelength

Short wavelength

Starobinsky approach

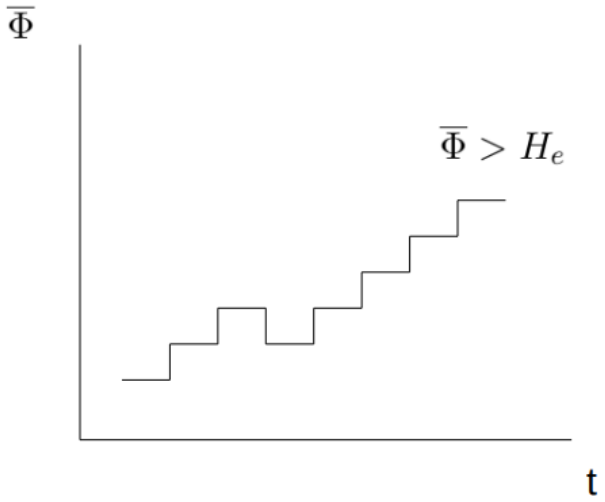
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- Random walk:



$$\frac{d}{dt}\langle\bar{\Phi}^2\rangle = -\frac{2}{3}\frac{m^2}{H}\langle\bar{\Phi}^2\rangle + \frac{H^3}{4\pi^2}$$

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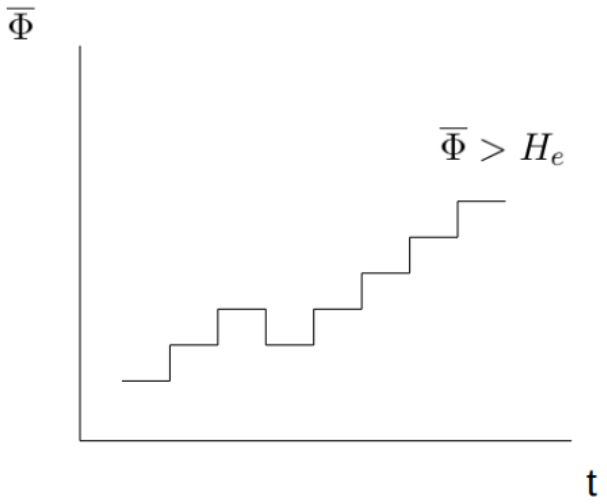
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$$\langle\bar{\Phi}^2\rangle = \frac{3}{8\pi^2}\frac{H^4}{m^2} + \left(\langle\bar{\Phi}^2\rangle_0 - \frac{3}{8\pi^2}\frac{H^4}{m^2}\right)e^{-\frac{2m^2}{3H}(t-t_0)}$$

Pre-inflationary initial conditions

Duration of inflation

- **Infinitely long inflation:** agrees with Bogolyubov approach.
- **Finite inflation:** strong dependence on the initial conditions.

Starobinsky approach

- When $H \sim m$: condensate $\bar{\Phi}$ is **converted into particles**: $\boldsymbol{a^3 n = a_e^3 \frac{H_e^{3/2}}{2m^{1/2}} \bar{\Phi}^2}$

- For **exponentially long inflation**: $\bar{\Phi}^2 \longrightarrow \frac{3}{8\pi^2}\frac{H^4}{m^2}$

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- Abundance of stable particles produced by inflation cannot exceed that of dark matter.

$$Y \leq 4.4\times10^{-10}\frac{\mathrm{GeV}}{m}, \qquad Y = \frac{n}{s_{SM}}, \qquad s_{SM} = \frac{2\pi^2 g_*}{45}T^3$$

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- **Matter domination:** $\boldsymbol{Y \simeq 0.07\times \frac{1}{\Delta}\frac{H_e^{1/2}\bar{\Phi}^2}{m\, M_{PL}^{3/2}} \Rightarrow \bar{\Phi} < \Delta^{1/2}\frac{5\times10^9}{(H_e/\mathrm{GeV})^{1/4}}\,\mathrm{GeV}}$ $\Rightarrow$ $\boxed{\boldsymbol{T_R \lesssim \mathrm{GeV}}}$

$$\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{inst}}{T_R} \gg 1$$

Overproduction of particles

- Huge production of dark relics during and after inflation:

Dilution of the relics + Non thermal production

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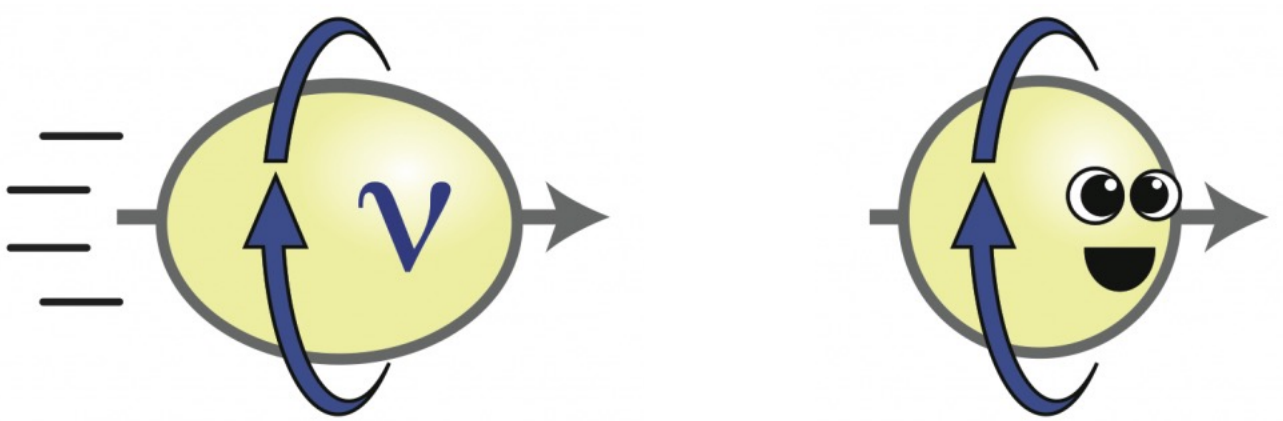


Production of the right amount of DM
through **freeze-in at stronger coupling**

- Dilution factor: $\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{inst}}{T_R}$ ➡ **Low T_R**

- All we know: $\boldsymbol{T_R > 4\ \mathrm{MeV}}$ (Hannestad 2004)

Fermion production via inflation



Fermion production

- **Dirac equation in curved space**:

$$(i\,\gamma^{\alpha}\nabla_{\alpha} - M)\,\Psi = 0$$

$$(i\,\gamma^{\alpha}\partial_{\alpha} - a(\eta)M)\,\Psi = 0$$

$$i\partial_{\eta}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} aM & k \\ k & -aM \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

Bunch-Davies boundary conditions at $\eta \to -\infty$.

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- **Two decoupled second order ODE:**

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- **Bogolyubov coefficient**:

$$\beta_k = \text{phase} \times \left(u_{A,k}^{\text{out}}u_{B,k}^{\text{in}} - u_{B,k}^{\text{out}}u_{A,k}^{\text{in}}\right)$$

Time dependent mass

- **Time-dependent mass**:

  Fermion masses are not constant in the early Universe – depend on the Higgs dynamics.

Higgs field h:

Inflation

Φ_0 ⇒ H_e

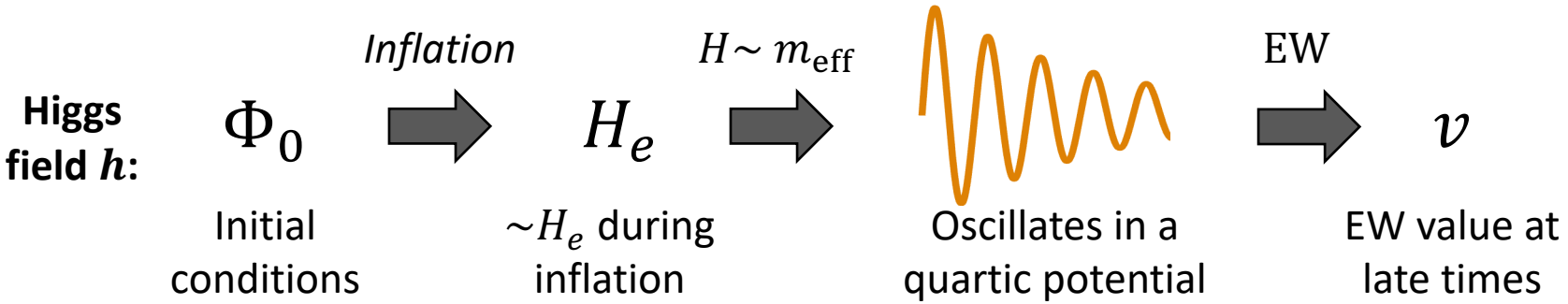
Initial conditions

$\sim H_e$ during inflation

Time dependent mass

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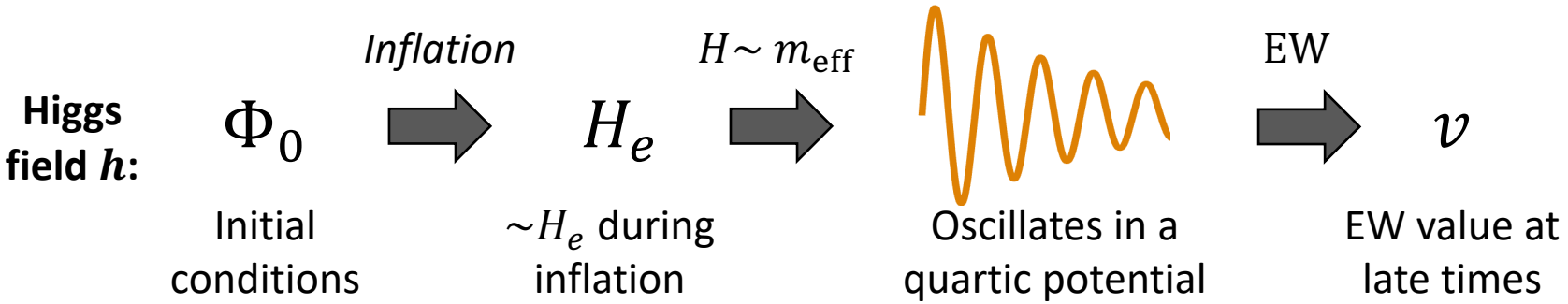
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- **Fermion masses in the SM**:

$$M_f = \frac{1}{\sqrt{2}} Y_f \langle h \rangle$$

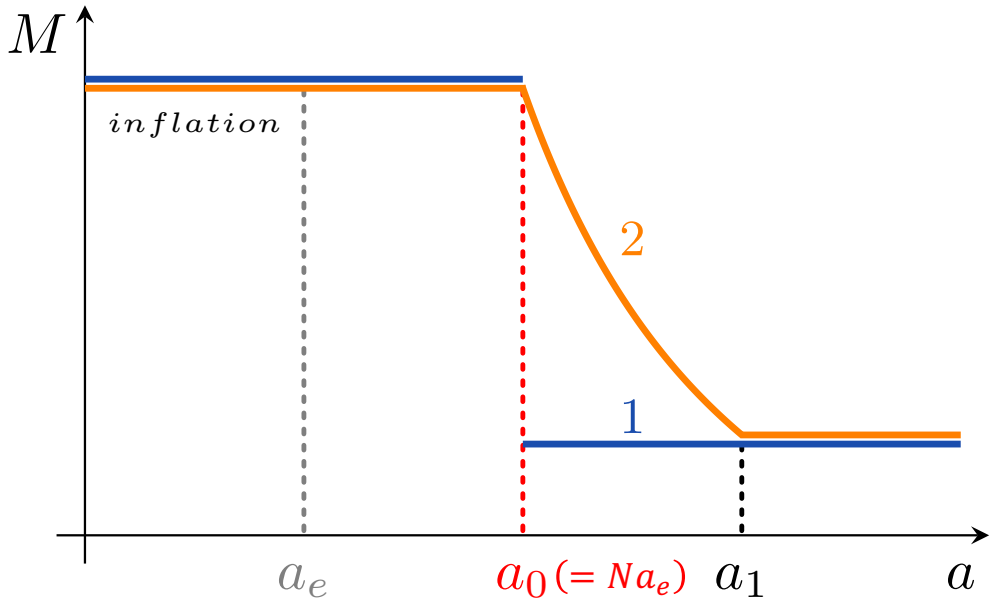
- **Higgs field takes on large values in the Early Universe: SM fermions very heavy.**

$$\langle h^2 \rangle \to 0.1 \frac{H^2}{\sqrt{\lambda_h}} \Rightarrow \langle h \rangle \approx H_e$$

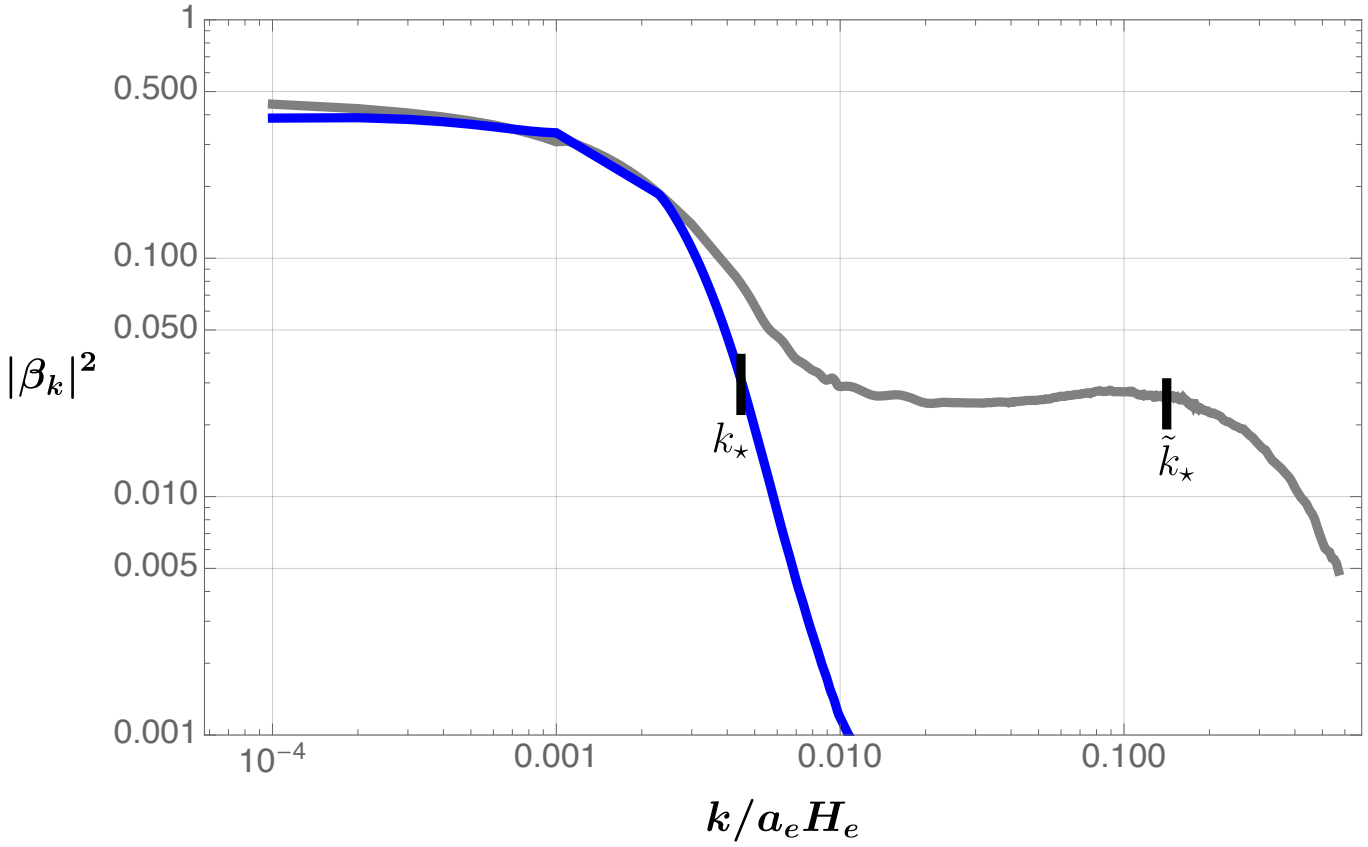
Mass variation

1. **Step function mass variation**:
   Large mass M during inflation and after, until $H \sim m_{\text{eff}}$;
   Small mass m afterwards.

2. **Thermal mass variation:**
   Large mass M during inflation and after, until $H \sim m_{\text{eff}}$;
   Thermal mass $M \propto T \propto 1/a$ due to interaction with the thermal bath ;
   Small mass m at late times.



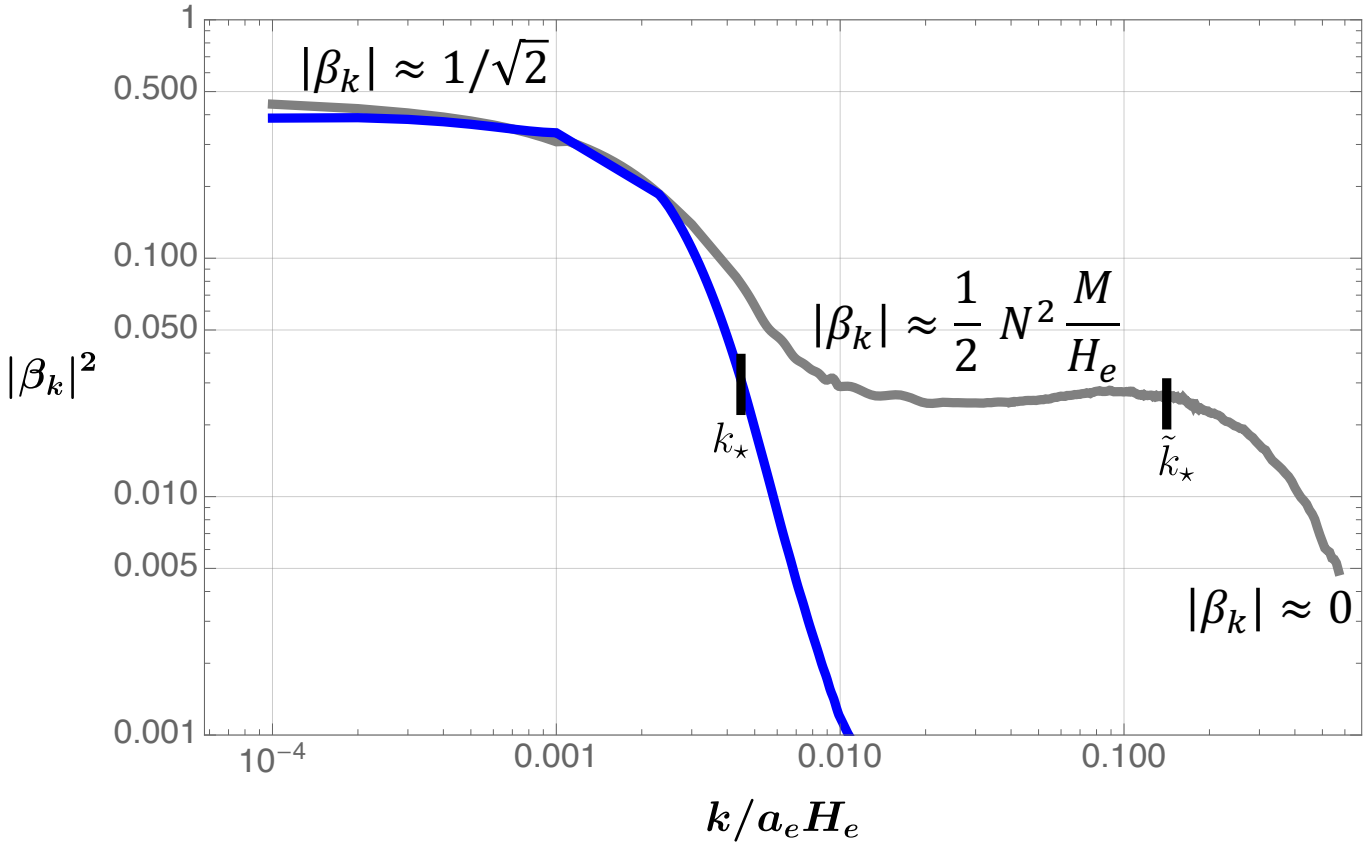
Step function mass variation



Blue: constant fermion mass;

Grey: Step function fermion mass.

Step function mass variation



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$$k_* = \sqrt{2ma_e^2 H_e}\,, \qquad \tilde{k}_* = \sqrt{2Ma_e^2 H_e}$$

➡ **Particle abundance up to $10^{19}\ Y_f^2$ times larger than for constant fermion mass.**

Right-handed neutrino production

- **New setting:**

  ν_R couples to a singlet scalar s, which, unlike the Higgs field, can have a very small self coupling.

- **Consequences:**

  ν_R production from condensate decay is suppressed;
  Thermalization is precluded;
  Gravitational production can be the leading source of ν_R.

Right-handed neutrino production

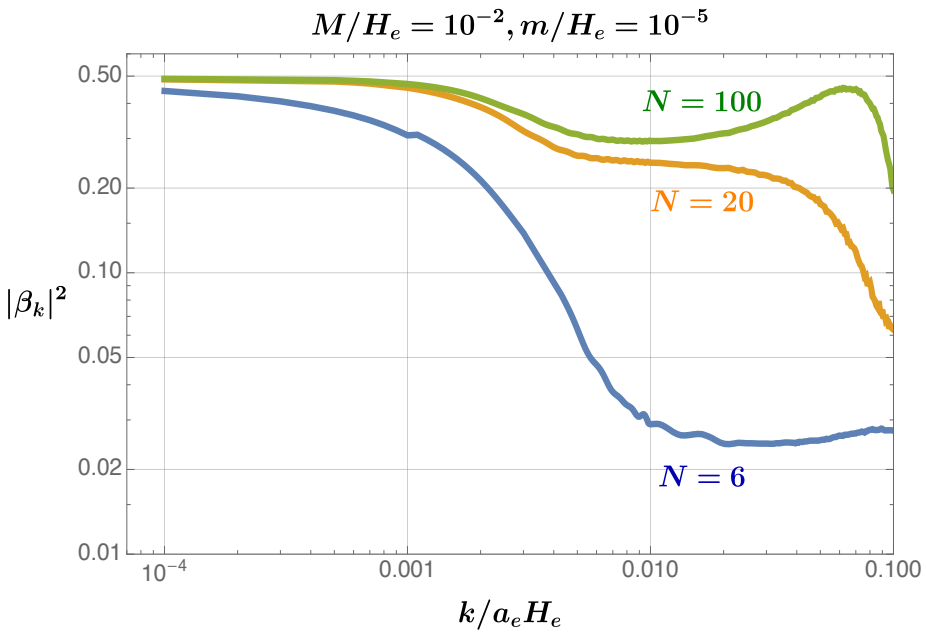
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  Gravitational production can be the leading source of ν_R.

- The scalar condensate can be long-lived: $N\sqrt{M/H_e} \gtrsim 1$.



Conclusions

- **Huge production of stable scalars** via classical gravity.
- Unknown $\langle \overline{\Phi}^2 \rangle_0$: **non-removable uncertainty** in the relic abundance.
- **Infinitely long inflation**: Bogolyubov and Starobinsky give the **same particle number**.

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- **Infinitely long inflation**: Bogolyubov and Starobinsky give the **same particle number**.
- **Variable fermion mass:** fermion production **orders of magnitude larger**.
- Not relevant for reheating, relevant for **right-handed neutrino production.**
- Stable dark relic with $m \ll H$: $\boldsymbol{T_R \lesssim \mathrm{GeV}} \Rightarrow$ **Freeze-in at stronger coupling.**

Thank you!



Helsinki, Finland

Backup slides



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Bogolyubov coefficient approach

- **Analytical results:**

Inflation followed by **matter dominated** epoch:

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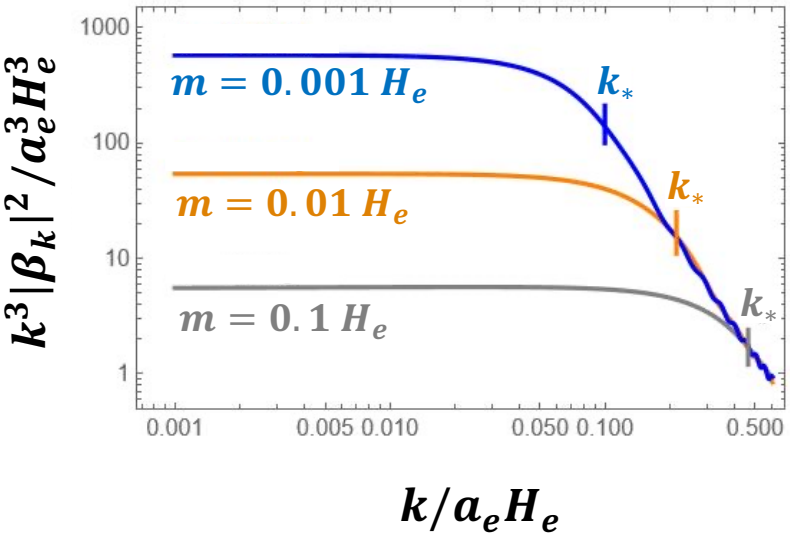
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Very large number

- **Numerical results:**

Matter dominated



Constraints on dark relics: general result

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- General result for inflationary scalar production:

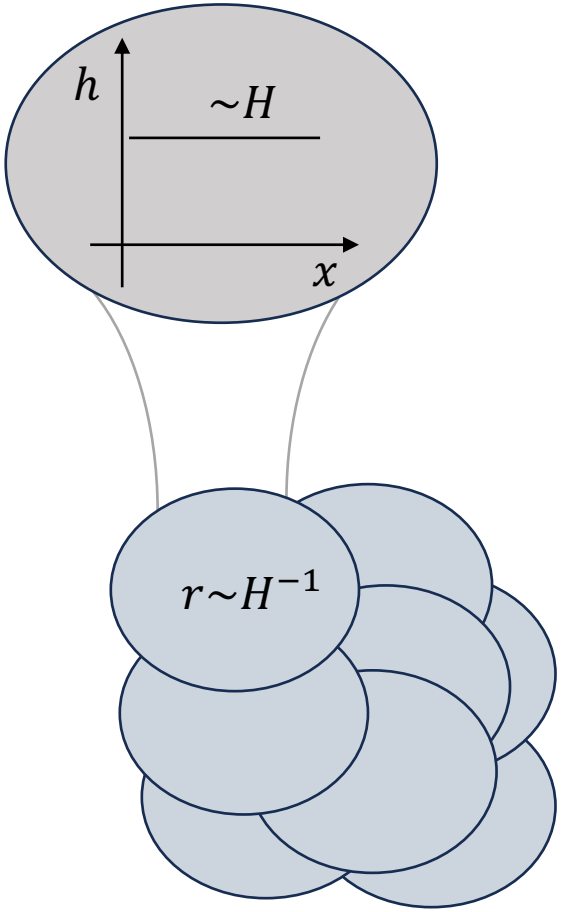
$$Y \propto \frac{\bar{\Phi}^2}{m_{\mathrm{eff}}^{1/2}} \left(\frac{H_R}{m_{\mathrm{eff}}} \right)^{\gamma}$$

- Radiation domination: $\gamma = 0$;
- Matter domination: $\gamma = 1/2$.
- This general expression includes:

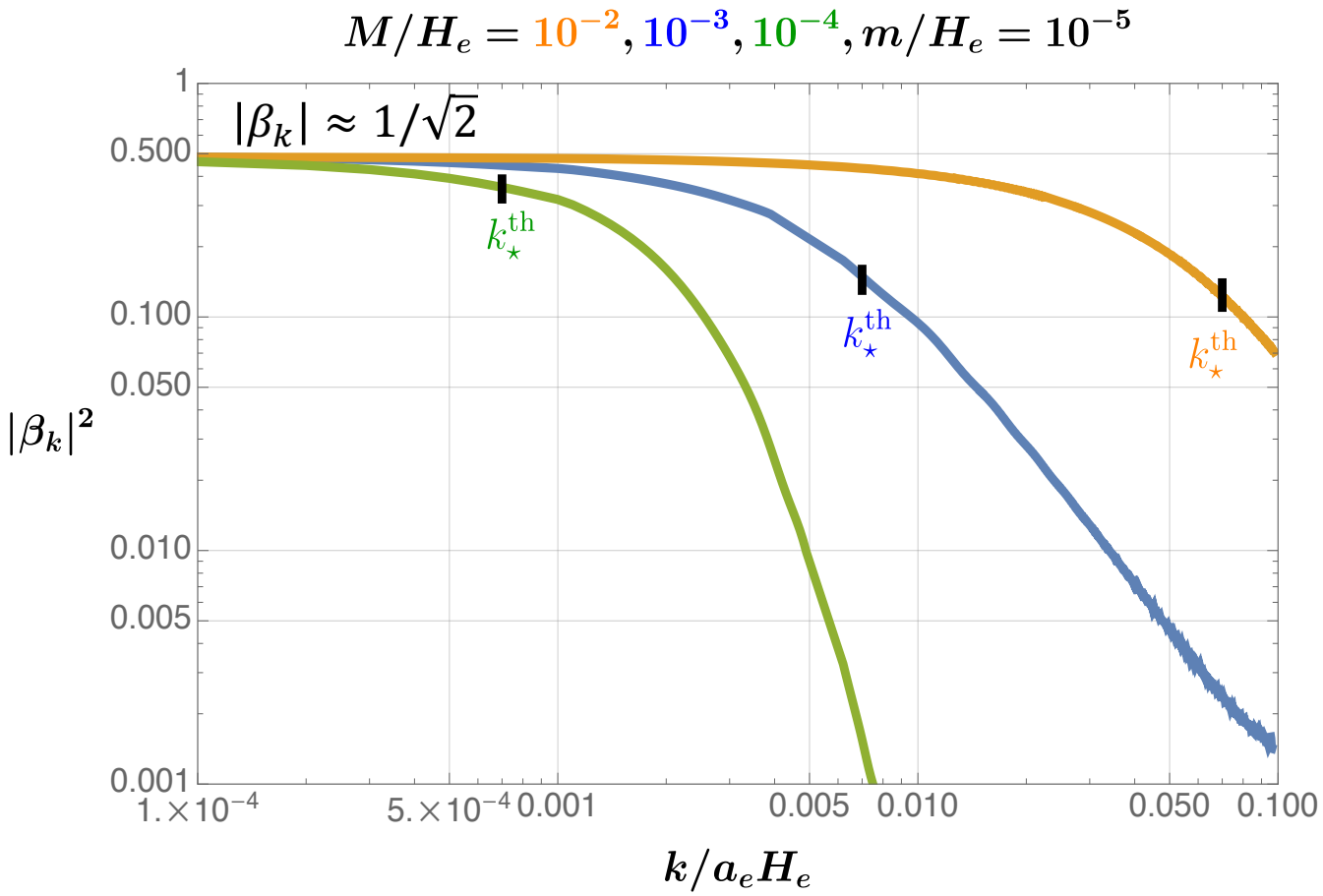
  Weak self-coupling λ;
  Small non-minimal coupling to gravity ξ;
  General initial conditions (included in $\bar{\Phi}$).

Time dependent mass

- Fermion masses are determined by the long wavelength component of the Higgs field, which is approximately constant at each Hubble patch.

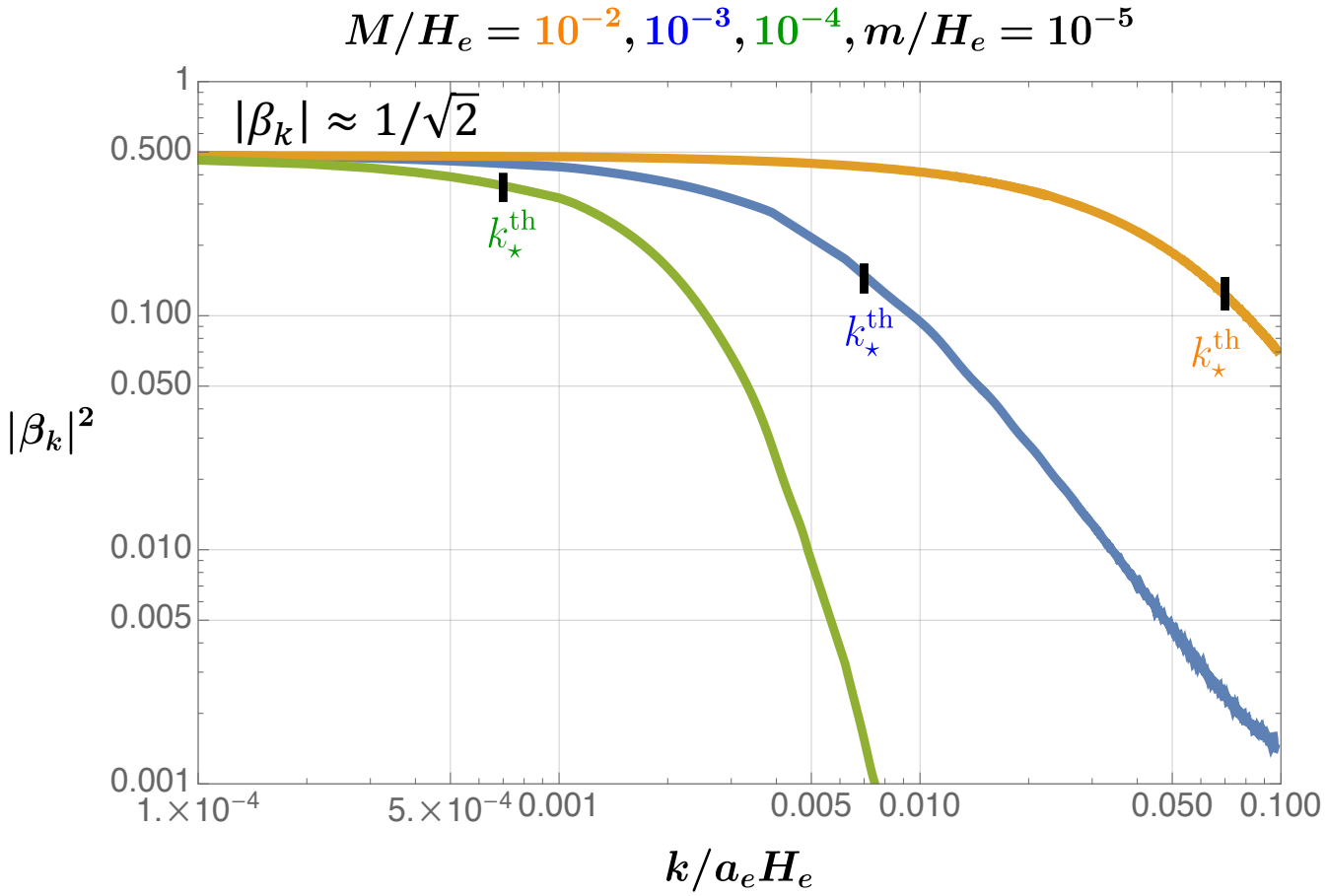


Thermal mass variation



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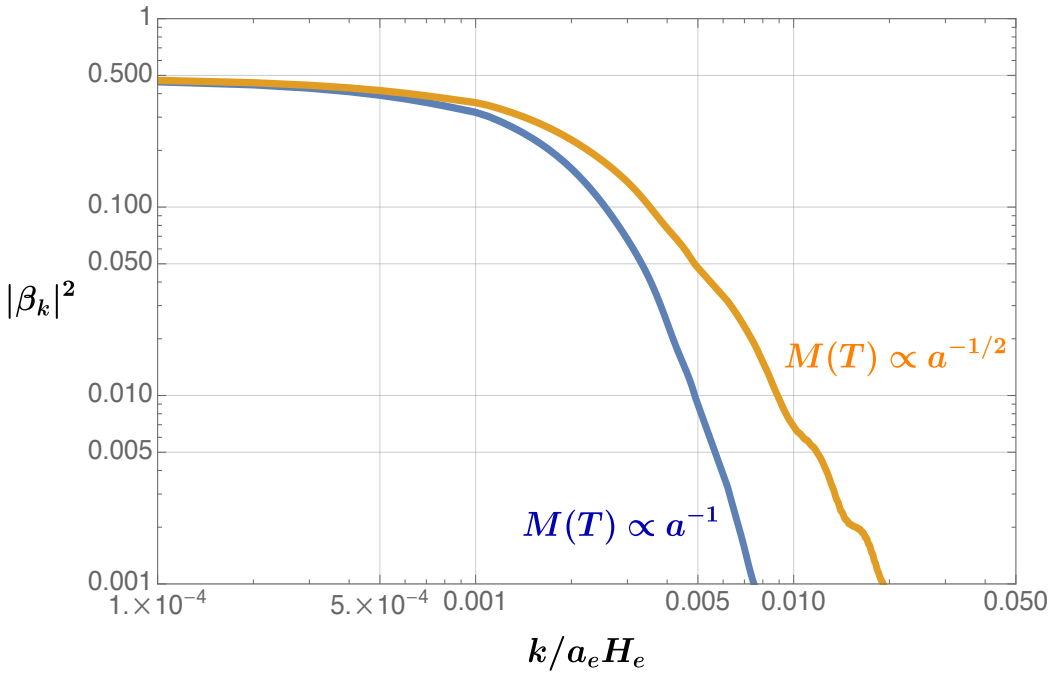
Particle abundance: $Y \sim 10^{-3}\, N^3 \dfrac{M^3}{(M_{\rm Pl}\, H_e)^{3/2}}$ ➡ $\left(N\sqrt{\dfrac{M}{H_e}}\right)^{-1}$ times larger than for step function mass.

Variations of the assumptions

- **Different mass scaling**:

  If the SM bath is subdominant in the energy balance, its energy density scales as $1/a^2$, $M(a) \propto a^{-1/2}$.

  Effect on the Bogolyuobov coefficient is insignificant.



Variations of the assumptions

- **Sharp features in $M(\eta)$:**

  $M(\eta)$ can change abruptly at early times due to fast non-perturbative effects.

  Thermal mass could grow larger than that induced by the Higgs condensate.

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- **Sharp features in $M(\eta)$:**

$M(\eta)$ can change abruptly at early times due to fast non-perturbative effects.

Thermal mass could grow larger than that induced by the Higgs condensate.

$$M(\eta) = M\,\theta(\eta_0 - \eta) + \mathbf{M}\,\theta(\eta - \eta_0)$$

Small correction as long as $N\sqrt{\frac{\mathbf{M}}{H_e}} \ll 1$.

β_k is primarily determined by the thermal mass, which decreases slowly over a long period, rather than abrupt mass variations at $\eta \lesssim \eta_0$.

Quark and lepton production

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- **Sharp effective mass decrease:**

$$n_f^{\text{fast}} \sim O(10)\, N_c Y_f^{7/2}\, H_e^3 \frac{a_e^3}{a^3}$$

Quark and lepton production

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Energy density of the produced fermions is tiny compared to the inflaton. This contribution is unimportant for reheating.

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- **Slow effective mass decrease:**

$$n_f^{\text{slow}} \sim \alpha\, g^3 N_c T_{\text{max}}^3 \frac{a_e^3}{a^3}$$

where $M_f(T) = \alpha\, gT/\sqrt{6}$.

No dependence on $\boldsymbol{Y_f}$. Fermion production is almost democratic and determined by the gauge coupling.

Top-quark production

- $M_t \sim H_e$: $M_f \ll H_e$ approximation breaks down.

- Significant **cancellation in the Bogolyubov** coefficient even at small momenta.

- Effect **exponentially sensitive** to the fermion mass.

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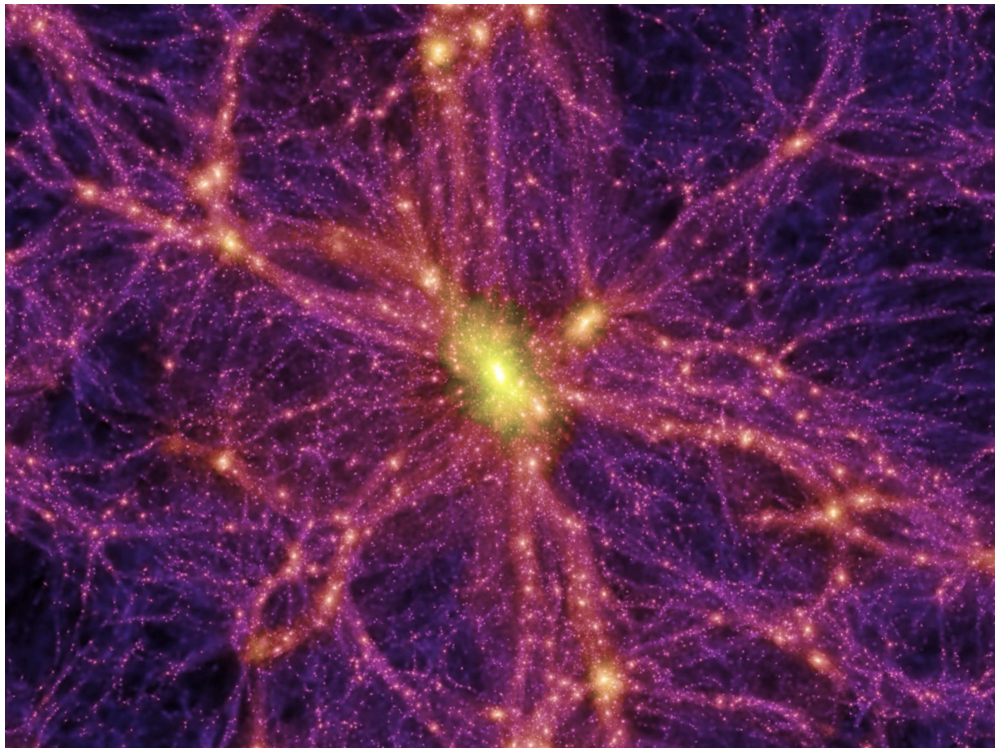
$$n_f^{\text{fast}} \sim 10^{-11}\, N_c\, H_e^3\, \frac{a_e^3}{a^3}$$

Production suppressed compared to $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{\tau}$; but more efficient than $\boldsymbol{u}$, $\boldsymbol{d}$, $\boldsymbol{e}$.

$$n_f^{\text{slow}} \sim 10^{-4}\, N_c\, H_e^3\, \frac{a_e^3}{a^3}$$

For large $T_{\max}$, **abundance is flavor universal**: all SM fermion thermal masses are of the same order of magnitude.

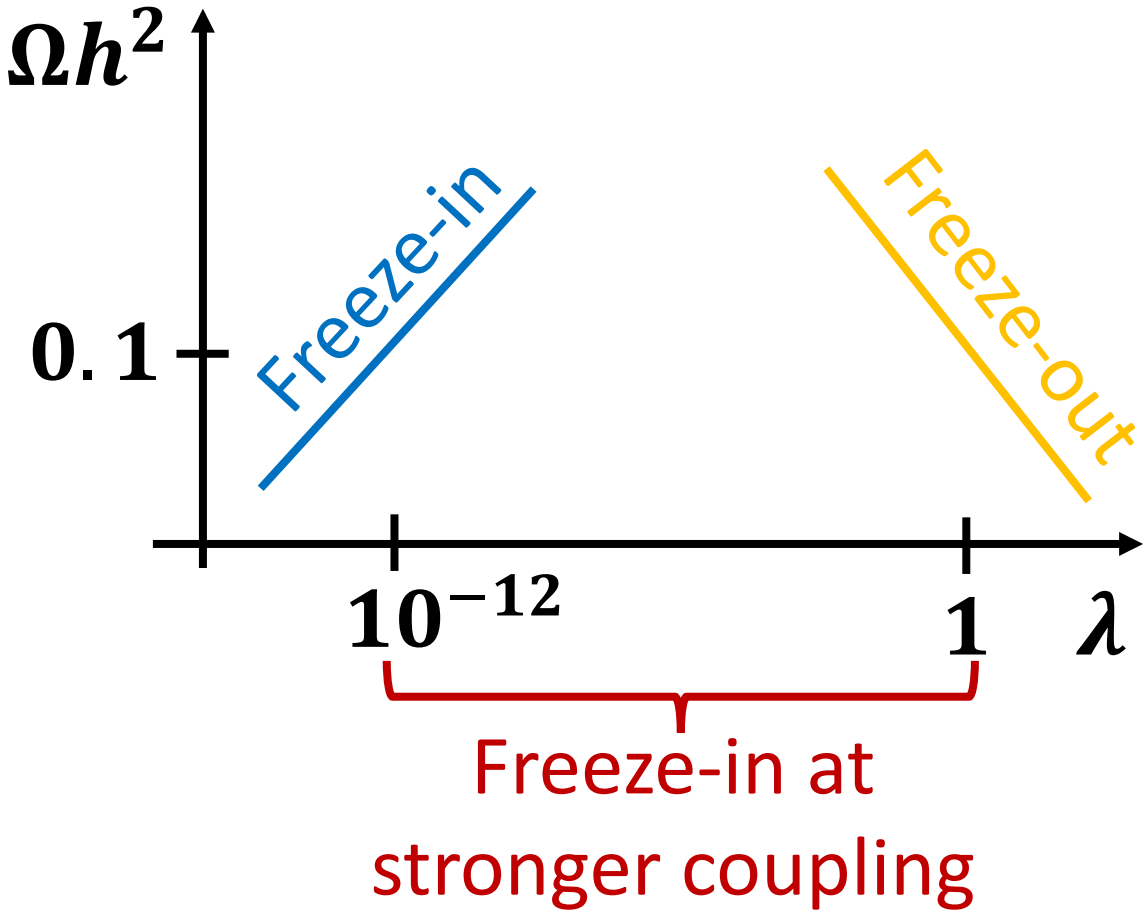
Low temperature freeze-in



Simulation of the DM distribution in the Universe. Volker Springel, Max Planck Institute for Astrophysics, et al.

Low temperature freeze-in

- **High temperature freeze-in – very small couplings.**
- **Low temperature freeze-in – large couplings provided that $T_R \ll m$.**
- Freeze-in at stronger coupling fills the **gap between high temperature freeze-in and freeze-out**:



Higgs portal to DM

- Simplest model: scalar DM with small coupling to Higgs

$$V(s) = \frac{1}{2}\lambda_{hs}\, s^2 H^\dagger H + \frac{1}{2} m^2 s^2$$

- DM production reaction:

$$h + h \rightarrow s + s$$

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- Boltzmann equation:

$$\dot{n} + 3Hn = \Gamma(h_i h_i \rightarrow ss) - \Gamma(ss \rightarrow h_i h_i)$$

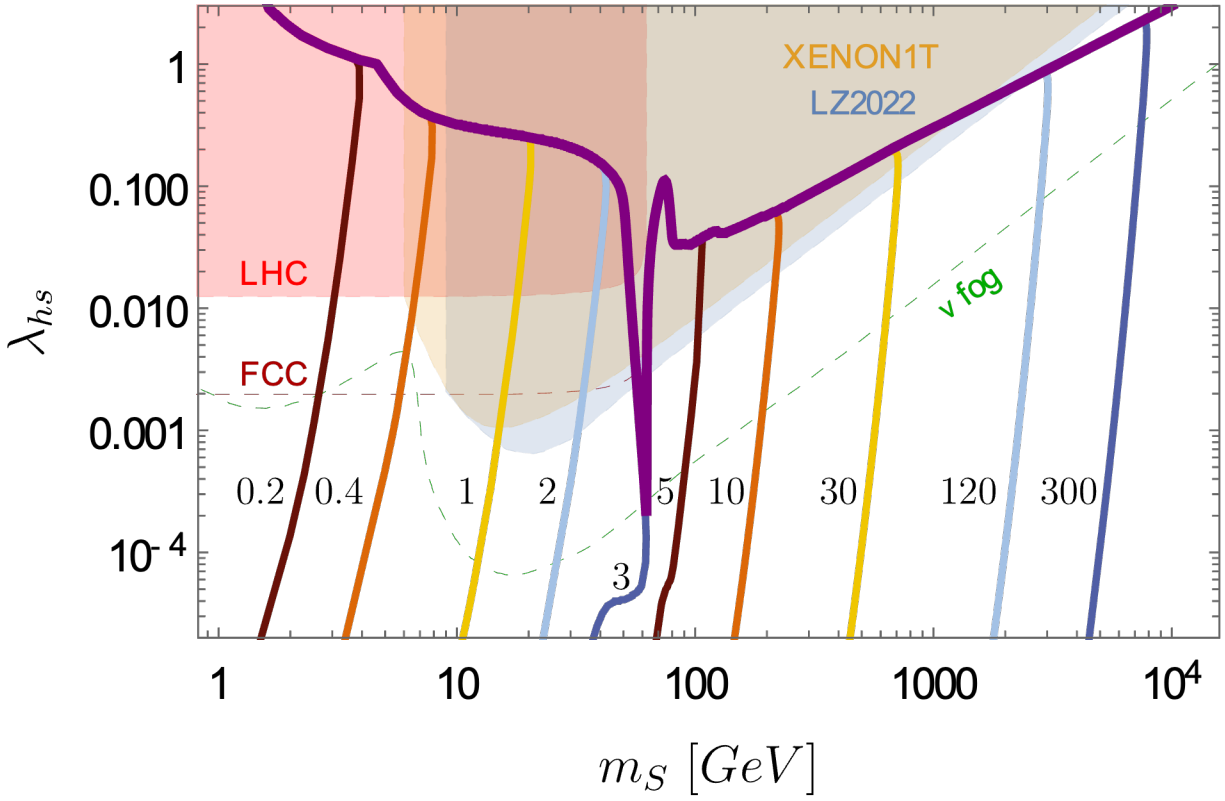
$$\lambda_{hs} \simeq 3\times10^{-11}\, e^{m/T_R} \sqrt{\frac{T_R}{m}}$$

$\Rightarrow$ **Large coupling for $T_R \ll m$**

Constraints

Arcadi, Costa, Goudelis, Lebedev
2405.03760

- Solving Boltzmann equation numerically, we can plot the curves reproducing the correct relic abundance in parameter space:



- New input: treat unknown T_R as a free parameter.

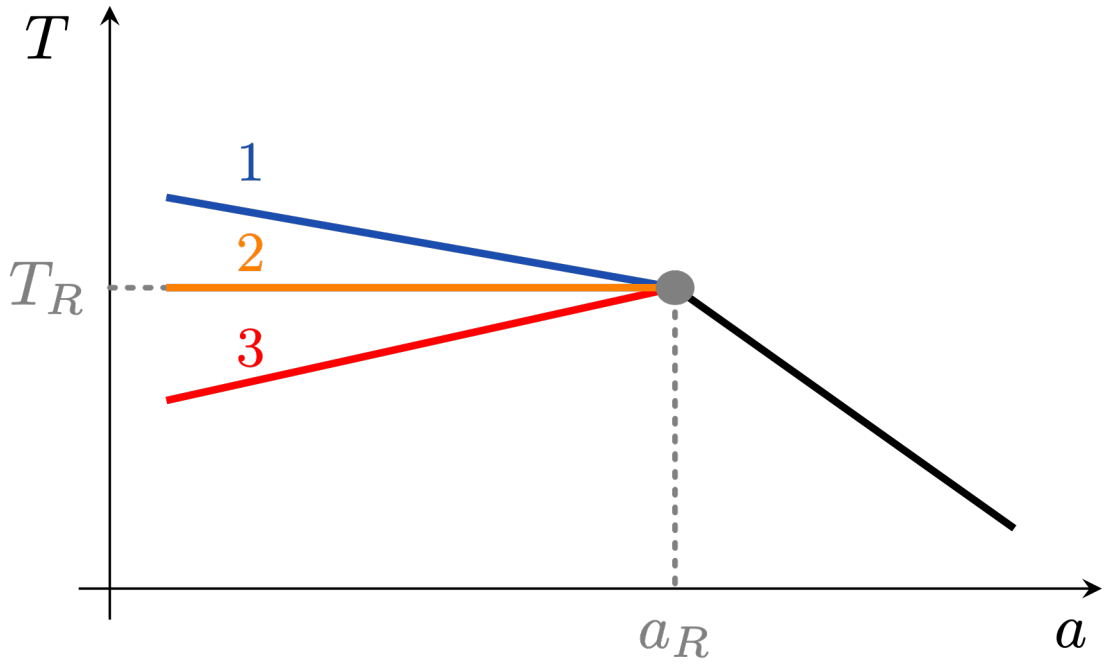
- Extension to lower DM masses – **warm dark matter**.

Temperature in the early Universe

- General case of SM sector production from the decay of a field χ:

$$\dot{\rho} + 4H\rho = \Gamma_\chi \rho_\chi \,,$$
$$H = H_0/a^m,$$
$$\rho_\chi = \rho_\chi^0/a^n.$$

$$a \gg 1 \Rightarrow \rho(a) \to \frac{\Gamma_\chi \rho_\chi^0}{(4-n+m)H_0} \frac{1}{a^{n-m}} \Rightarrow T \propto a^{-\frac{n-m}{4}}$$



Warm dark matter through freeze-in

- Warm DM is very light – Boltzmann suppression due to the **SM bath temperature being lower than the mass of SM particles** responsible for DM production.

- Higgs portal models: $\boldsymbol{\pi^a}$ **and** $\boldsymbol{\mu^{\pm}}$ in most of the parameter space of interest.



- For **very low temperatures**, $T \sim 10$ MeV, π^a and $\mu^{\pm}$ contributions become irrelevant due to strong Boltzmann suppression: production **dominated by** $\boldsymbol{e^{\pm}}$ channel.

DM distribution function

- Key quantity for our analysis: DM momentum distribution function $f(|\mathbf{p}|, t)$.

$$\frac{\partial f}{\partial t} - H|\mathbf{p}|\frac{\partial f}{\partial |\mathbf{p}|} = \frac{1}{p_0}\mathcal{C}(p,t)$$

$$\Rightarrow f(|\mathbf{p}|, t) = \int dt' \frac{\mathcal{C}(p', t')}{{p_0}'}, \qquad \mathbf{p}' = \mathbf{p}(t)\frac{a(t)}{a(t')}$$

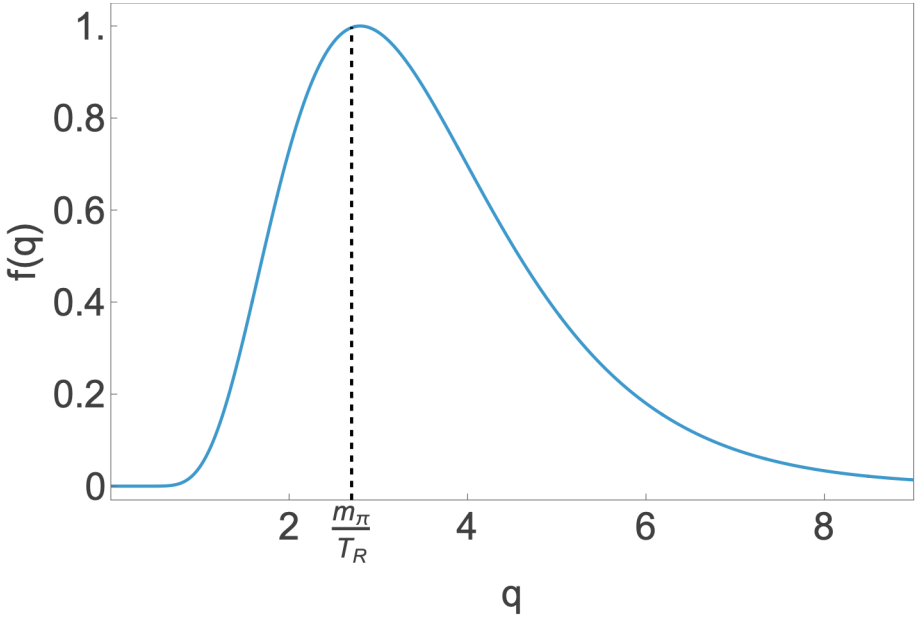
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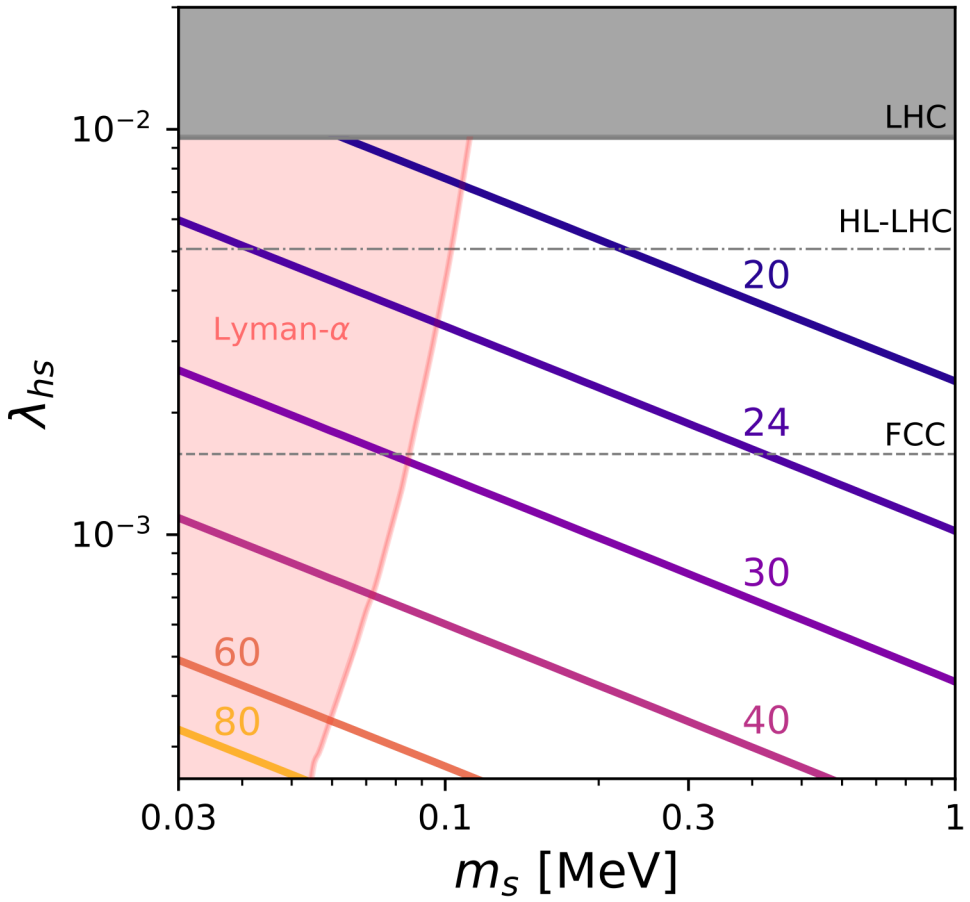
- Non-relativistic pions – most DM is produced at T_R with energy m_π.



Parameter space constraints

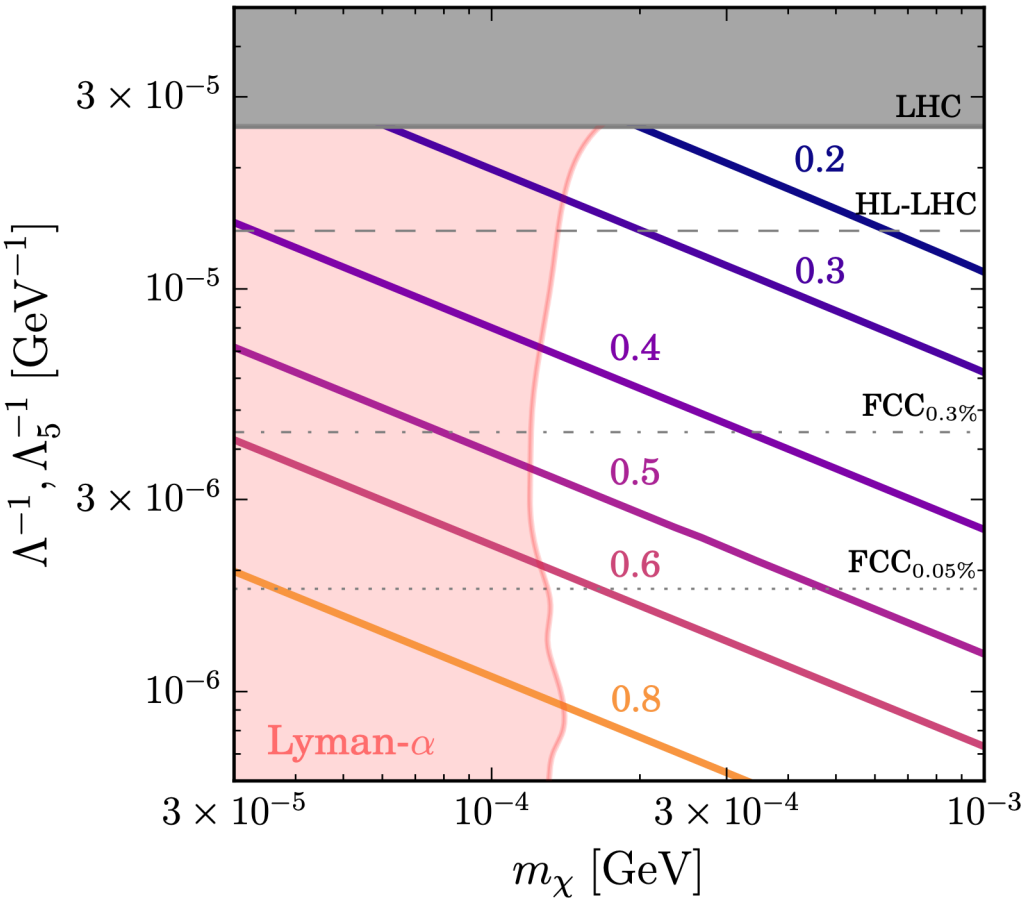
- Constraints **stronger for small T_R** – less time for "red-shifting" the momentum to small values at the structure formation epoch

- $m_s < 70 - 100\ \mathrm{keV}$ excluded – **stronger than in usual high-T models** (larger DM comoving momentum and IR suppression of $f(q)$).



Parameter space: Fermionic WDM

$$\Delta\mathcal{L} \sim \frac{1}{\Lambda} H^\dagger H \, \bar{\psi}\psi$$

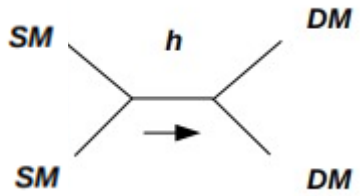


- Higgs decay allows to probe $\boldsymbol{\Lambda \lesssim 2.5 \times 10^5\ \mathrm{GeV}}$.
- Lyman-α imposes $\boldsymbol{m_\chi \gtrsim 100\ \mathrm{keV}}$.

Probes of freeze-in

Lebedev, Morais, Oliveira, Pasechnik
2410.21874

- **Invisible Higgs decay:**

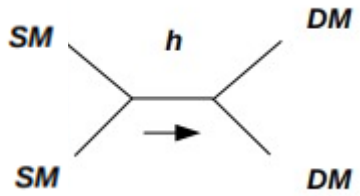


- Large coupling – can be observable.
- LHC already rules out a significant region of the parameter space, requiring the **branching ratio to be lower than 10 %**.
- HL-LHC aims to probe down to 3%. FCC aims to 0.03%.

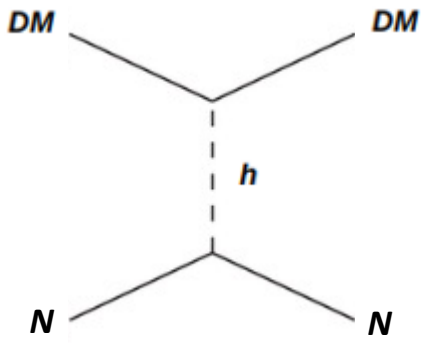
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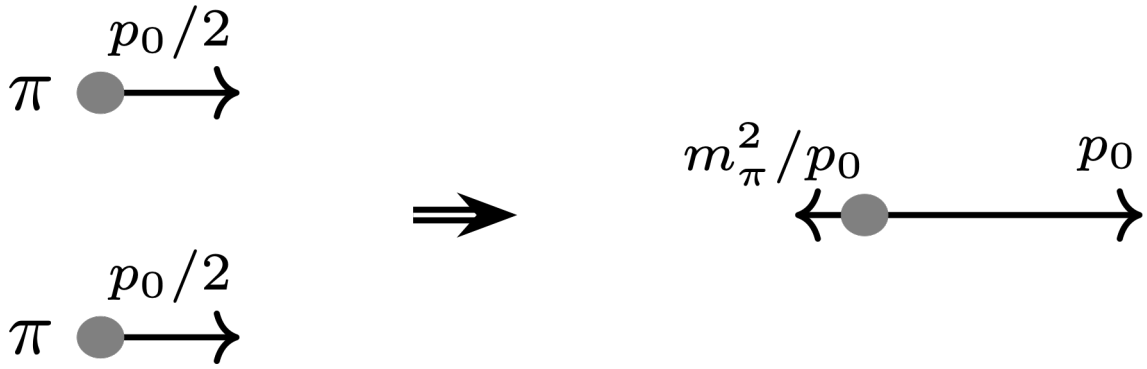
- Large coupling – can be observable.
- LHC already rules out a significant region of the parameter space, requiring the **branching ratio to be lower than 10 %**.
- HL-LHC aims to probe down to 3%. FCC aims to 0.03%.
- **Direct detection:**



- XENON and Darwin can probe freeze-in at stronger coupling down to ν-fog.

Production kinematics

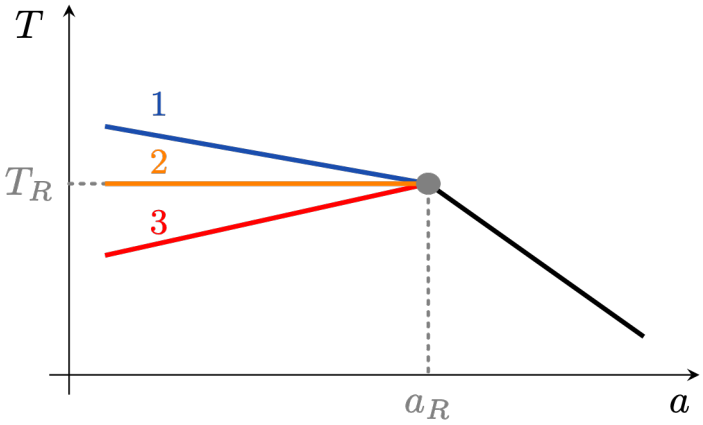
- Consider the regime $m_s \ll T \ll m_\pi$. π^a are mostly non-relativistic – DM energy close to m_π.
- Reaction rate contains Boltzmann factors for the initial state $e^{-E_1/T} e^{-E_2/T}$
- Reaction is the most efficient when $E_1 + E_2 \to \min$, with $E_1 + E_2 \geq p_0$.



$$\Rightarrow \boldsymbol{e^{-E_1/T}\, e^{-E_2/T} = \exp\left(-\frac{p_0}{T} - \frac{m_\pi^2}{p_0 T}\right)}$$

Temperature in the early Universe

1. $\phi \to \mathrm{SM}$
   χ is the inflaton and dominates energy density: $m = n/2$.
   Maximal temperature far above T_R.

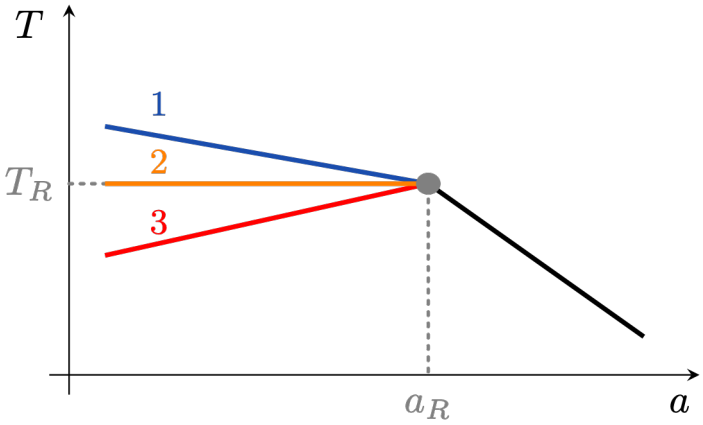


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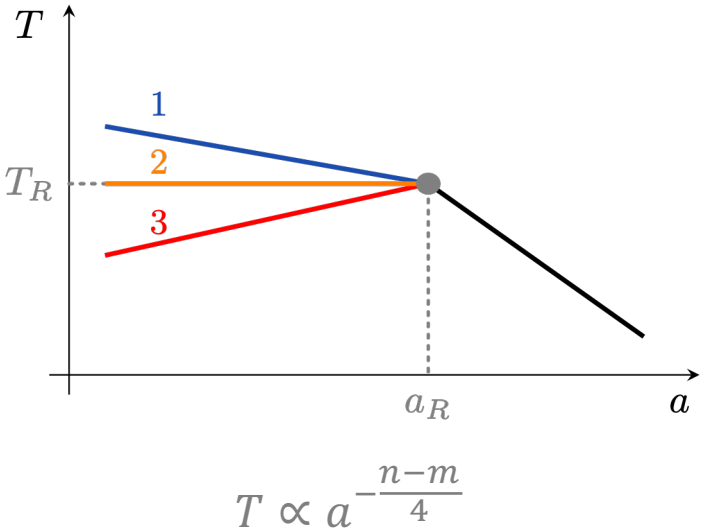


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3. $\phi \to \chi_1\chi_1 \to \chi_2\chi_2\chi_2\chi_2 \to \cdots \to \mathrm{SM}$
   m is constant as long as the energy density is dominated by ϕ: $n - m < 0$.
   Maximal temperature close to T_R.

In cases 2. and 3. the temperature of the SM bath may have never exceeded the MeV scale. This low temperature **does not require non-standard cosmology**.

Temperature in the early Universe

- Consider the case $\phi \rightarrow \chi\chi \rightarrow \mathrm{SM}$

$\dot{\rho} + 4H\rho = \Gamma_\chi \, \rho_\chi \,,$
$H = H_0/a^m,$
$\rho_\chi = \rho_\chi^0/a^n.$

$\dot{\rho}_\chi + 4H\rho_\chi = \Gamma_\phi \, \rho_\phi \,,$
$H = H_0/a^m,$
$\rho_\phi = \rho_\phi^0/a^{n'}.$

$\Rightarrow \rho_\chi \, (a) \propto \frac{1}{a^{n'-m}}$, $n' = 2m$ since ϕ dominates the energy density

$$\Rightarrow \rho_\chi \, (a) \propto \frac{1}{a^m}$$

- Compare now with the first system:

$$\Rightarrow n = m$$

Starobinsky: Details

- Equations of motion for $\bar{\Phi}$ in the slow roll regime, $\ddot{\Phi} \ll H\dot{\Phi}$, to leading order in ε:

$$\dot{\bar{\Phi}} = -\frac{1}{3H}V'(\bar{\Phi}) + f(t, \mathbf{r})$$

Classical force Noise term (stochastic)

$$\langle f(t, \mathbf{r}) f(t', \mathbf{r}) \rangle = \frac{H^3}{4\pi^2}\delta(t - t')$$

- $\bar{\Phi}$ evolves according to the Langevin equation containing classical and stochastic forces.

- Probability of finding a specific value $\bar{\Phi}$ is described by the distribution function $\rho(\bar{\Phi}, t)$, which is subject to the Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \bar{\Phi}^2} + \frac{1}{3H}\frac{\partial}{\partial \bar{\Phi}}\left(V'(\bar{\Phi})\,\rho\right)$$

$$\Rightarrow \frac{d}{dt}\langle \bar{\Phi}^2 \rangle = -\frac{2}{3}\frac{m^2}{H}\langle \bar{\Phi}^2 \rangle + \frac{H^3}{4\pi^2}$$