Quadratic Couplings and Dark Matter

Clare Burrage

University of Nottingham

Outline:

Scalar dark matter with quadratic couplings

Dark matter production and ‘kicks’

Local tests and environmental screening



University of Nottingham

UK | CHINA | MALAYSIA

What is the scale of new physics?

Current Hubble Scale ~ 10^{-42} GeV

1 / Galactic size ~ 10^{-36} GeV

Cosmological Constant scale ~ 10^{-12} GeV

Mass of dark matter particle:

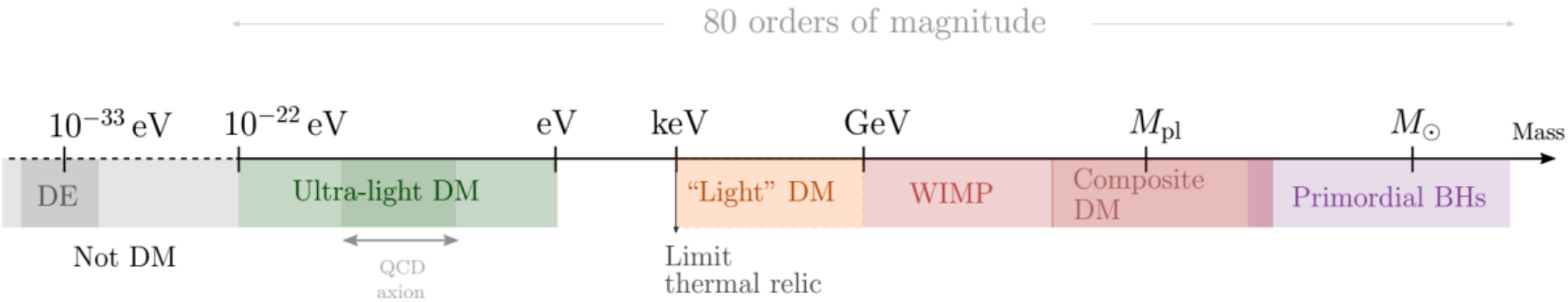


Image Credit: Ferreira. Astron. Astrophys. Rev. 29 (2021) 1, 7

Ultra-Light Scalar Dark Matter

Light and feebly interacting bosons can be dark matter

A scalar field oscillating in a quadratic potential

$$\phi_{\mathrm{homog}}(t) = \phi_\infty \cos(mt + \delta)$$

Gives zero pressure and dark matter energy density

$$\rho_{\mathrm{DM}} = \frac{1}{2} m^2 \phi_\infty^2$$

Preskill, Wise, Wilczek. Phys. Lett.B 120 (1983) 127–132. Abbott, Sikivie. Phys. Lett.B 120 (1983) 133–136. Dine, Fischler, Phys. Lett. B 120 (1983)137–141.
For a review see e.g. Ferreira. Astron. Astrophys. Rev. 29 (2021) 1, 7

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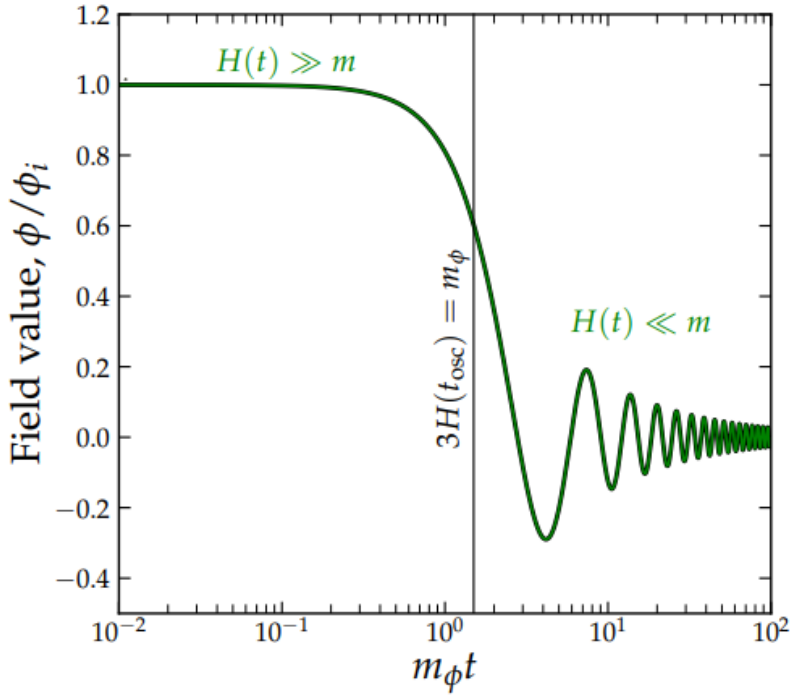
Coherence length today $\sim \frac{1}{m v_0}$

Where v_0 is the virial velocity ~ 200 km/s

Preskill, Wise, Wilczek. Phys. Lett.B 120 (1983) 127–132. Abbott, Sikivie. Phys. Lett.B 120 (1983) 133–136. Dine, Fischler, Phys. Lett. B 120 (1983)137–141.
For a review see e.g. Ferreira. Astron. Astrophys. Rev. 29 (2021) 1, 7

Ultra-Light Scalar Dark Matter

Light and feebly interacting bosons can be dark matter



High occupation numbers mean they can be described as classical fields exhibiting a wave-like behaviour

Figure from lecture notes by O'Hare. Arxiv:2403.17697

Linear Couplings

$$\mathcal{L}_{\rm int} = \frac{\phi}{M_{\rm pl}}\left[\frac{d_e^{(1)}}{4}F_{\mu\nu}F^{\mu\nu} - \frac{d_g^{(1)}\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d,e}\left(d_{m_i}^{(1)} + \gamma_{m_i}d_g^{(1)}\right)m_i\bar{\psi}_i\psi_i\right]$$

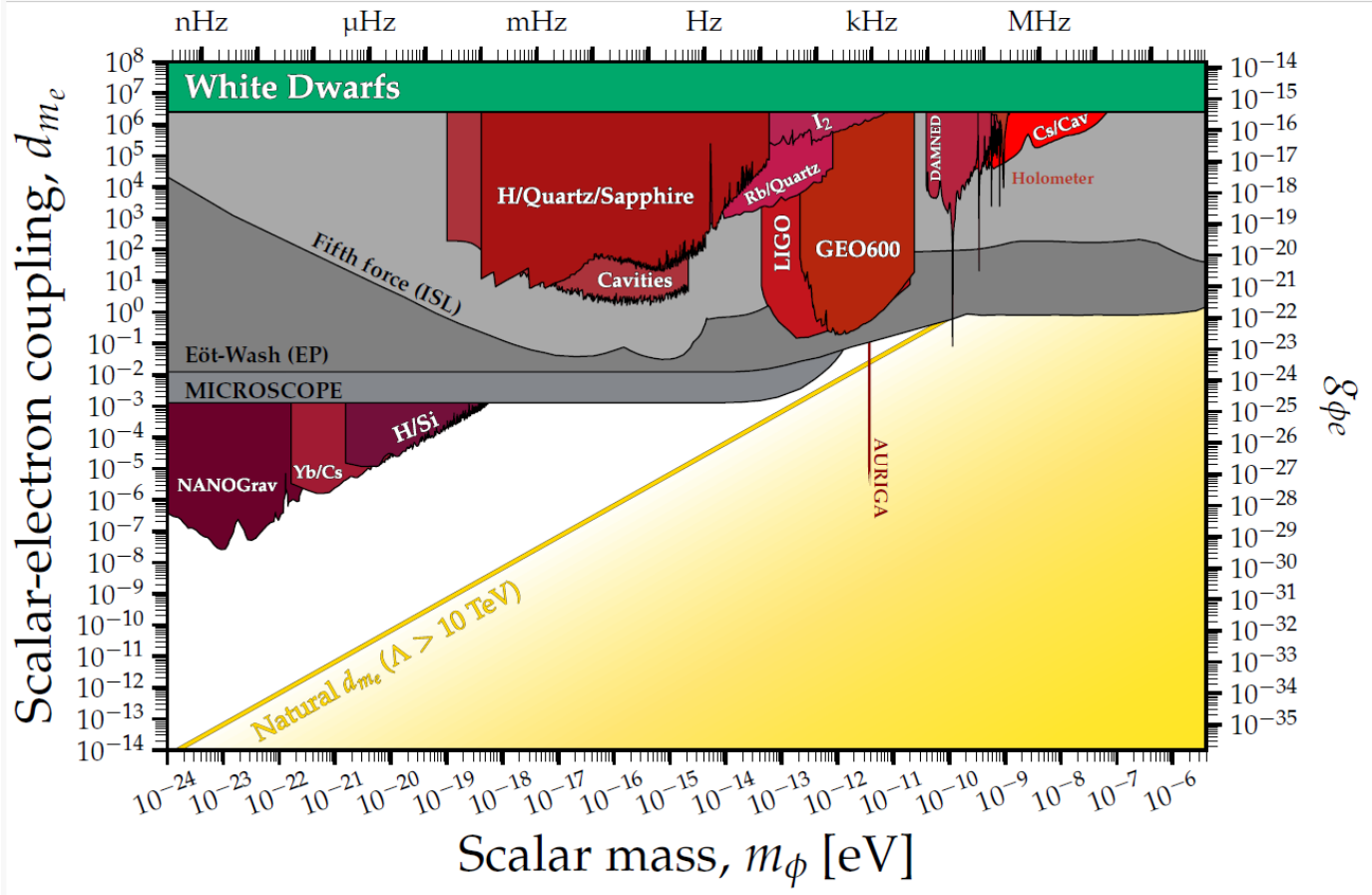


Figure from Axion Limits, C. O'Hare. https://cajohare.github.io/AxionLimits/

Quadratic Couplings

Quadratic interactions with the Standard Model

$$\mathcal{L}_{\rm int} = \frac{\phi^2}{2M_{\rm pl}^2}\left[\frac{d_e}{4}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d,e}\left(d_{m_i}+\gamma_{m_i}d_g\right)m_i\bar{\psi}_i\psi_i\right]$$

Quadratic Couplings

From e.g. symmetric Higgs-portals

$$\mathcal{L} \supset \frac{\phi^2}{2M^2} H^\dagger H$$

Quadratic interactions with the Standard Model

$$\mathcal{L}_{\text{int}} = \frac{\phi^2}{2M_{\text{pl}}^2}\left[\frac{d_e}{4} F_{\mu\nu}F^{\mu\nu} - \frac{d_g \beta_3}{2g_3} F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d,e} \left(d_{m_i} + \gamma_{m_i} d_g\right) m_i \bar{\psi}_i \psi_i\right]$$

Burgess, Pospelov, ter Veldhuis. Nucl. Phys. B 619:709 (2001). O'Connell, Ramsey-Musolf, Wise. Phys.Rev.D 75 037701 (2007). Patt, Wilczek. arXiv: hep-ph/0605188. Arcadi, Djouadi, Raidal. Phys. Rept. 842:1 (2020)

Quadratic Couplings

From (pseudo-scalar) axion couplings
- with small field approximation

Quadratic interactions with the Standard Model

$$\mathcal{L}_{\rm int} = \frac{\phi^2}{2M_{\rm pl}^2}\left[\frac{d_e}{4}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d,e}\left(d_{m_i}+\gamma_{m_i}d_g\right)m_i\bar{\psi}_i\psi_i\right]$$

Hook, Huang. JHEP 06 (2018) 036. Garani, Redi, Tesi JHEP 12 (2021) 139.
Tsai, McGehee, Murayama, Phys. Rev. Lett. 128 (2022) 17 172001.
Bauer, Rostagni. Phys.Rev.Lett. 132 (2024) 10, 101802

Quadratic Couplings

Quadratic interactions with the Standard Model

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Leading to: - Varying fundamental ‘constants’

- Macroscopic couplings to matter density and pressure

$$S = \int d^4x\left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\alpha(\rho - 3p)\phi^2\right)$$

QUADRATIC COUPLINGS IN COSMOLOGY

Quadratic Couplings in Cosmology

Homogeneous cosmological equation of motion for the scalar

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi - \alpha\phi\rho_R\Sigma_i$$

CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Quadratic Couplings in Cosmology

Homogeneous cosmological equation of motion for the scalar

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi - \alpha\phi\rho_R\Sigma_i$$

where

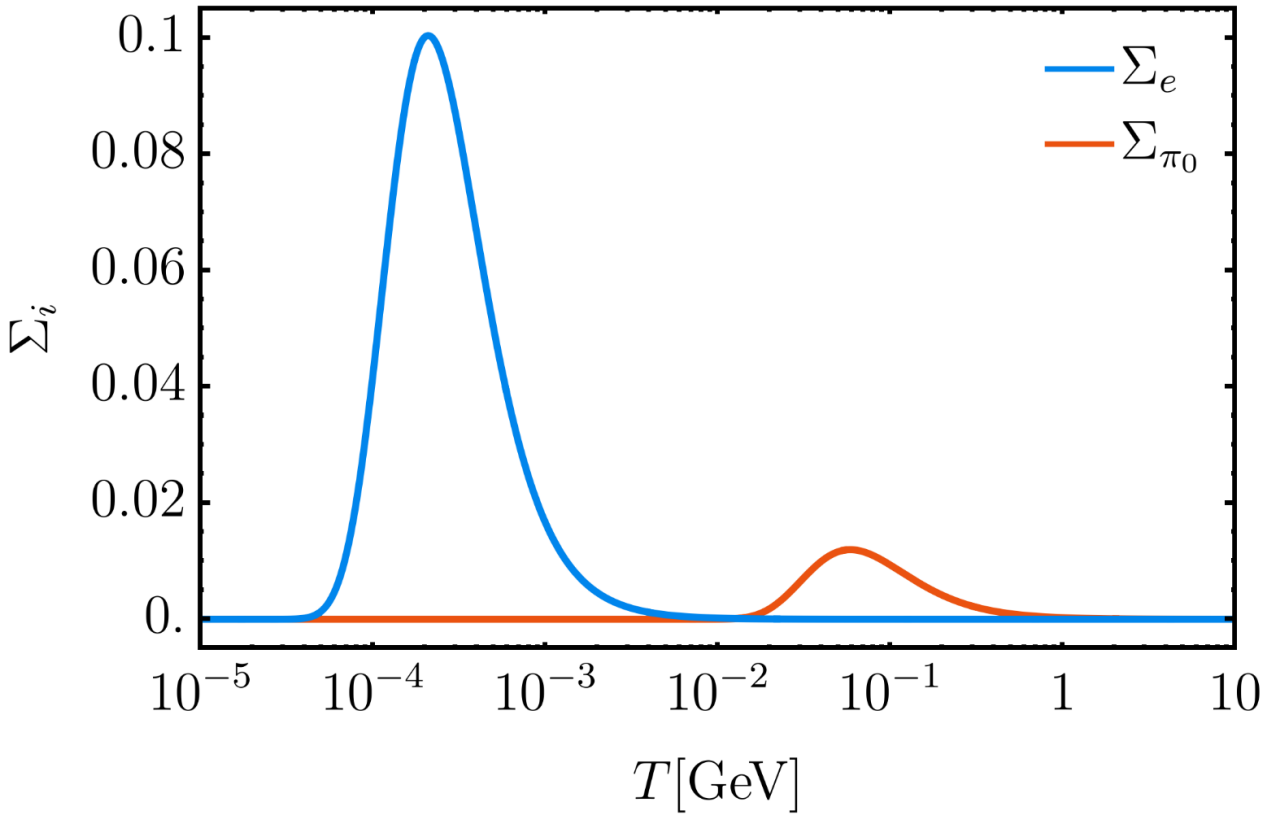
$$\Sigma_i = \frac{\rho_i - 3P_i}{\rho_R}$$

$$\Sigma_i(T) = \frac{15}{\pi^4}\frac{g_i}{g_*(T)}\frac{m_i^2}{T^2}\int_{m_i/T}^{\infty}\frac{\sqrt{u^2 - (m_i/T)^2}}{e^u \pm 1}\,du$$

CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

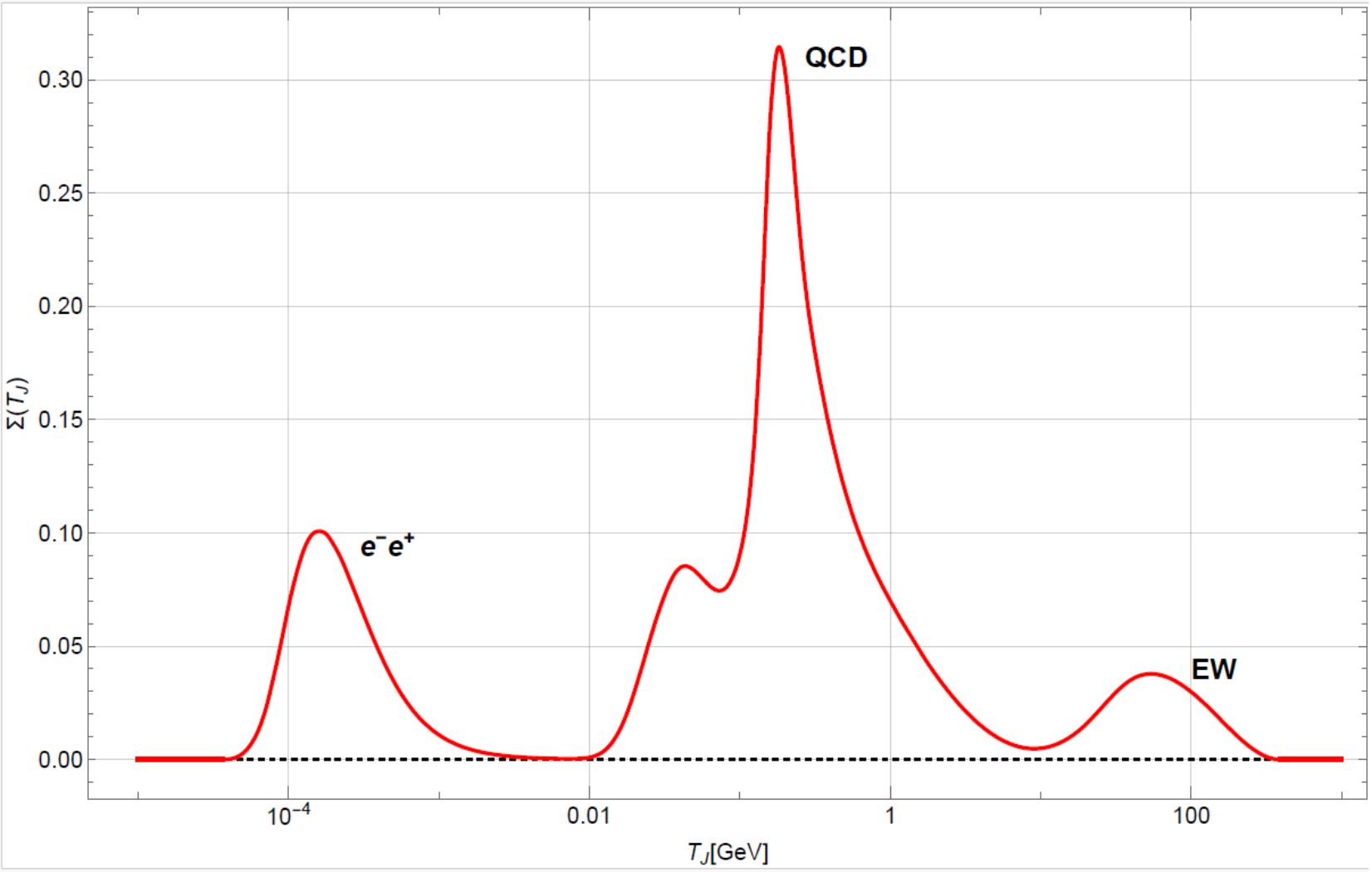
Kicking Function

$$\Sigma_i(T) = \frac{15}{\pi^4}\frac{g_i}{g_*(T)}\frac{m_i^2}{T^2}\int_{m_i/T}^{\infty}\frac{\sqrt{u^2-(m_i/T)^2}}{e^u \pm 1}du$$



CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Kicking Function



QCD kick from Borsanyi et al. arXiv:1606.07494.
Figure produced by Xavier Pritchard.

Effects of the Kick

If the kick occurs while the field is frozen

- Field may start to oscillate

- For negative coupling, field could be destabilised

CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Effects of the Kick

If the kick occurs while the field is frozen

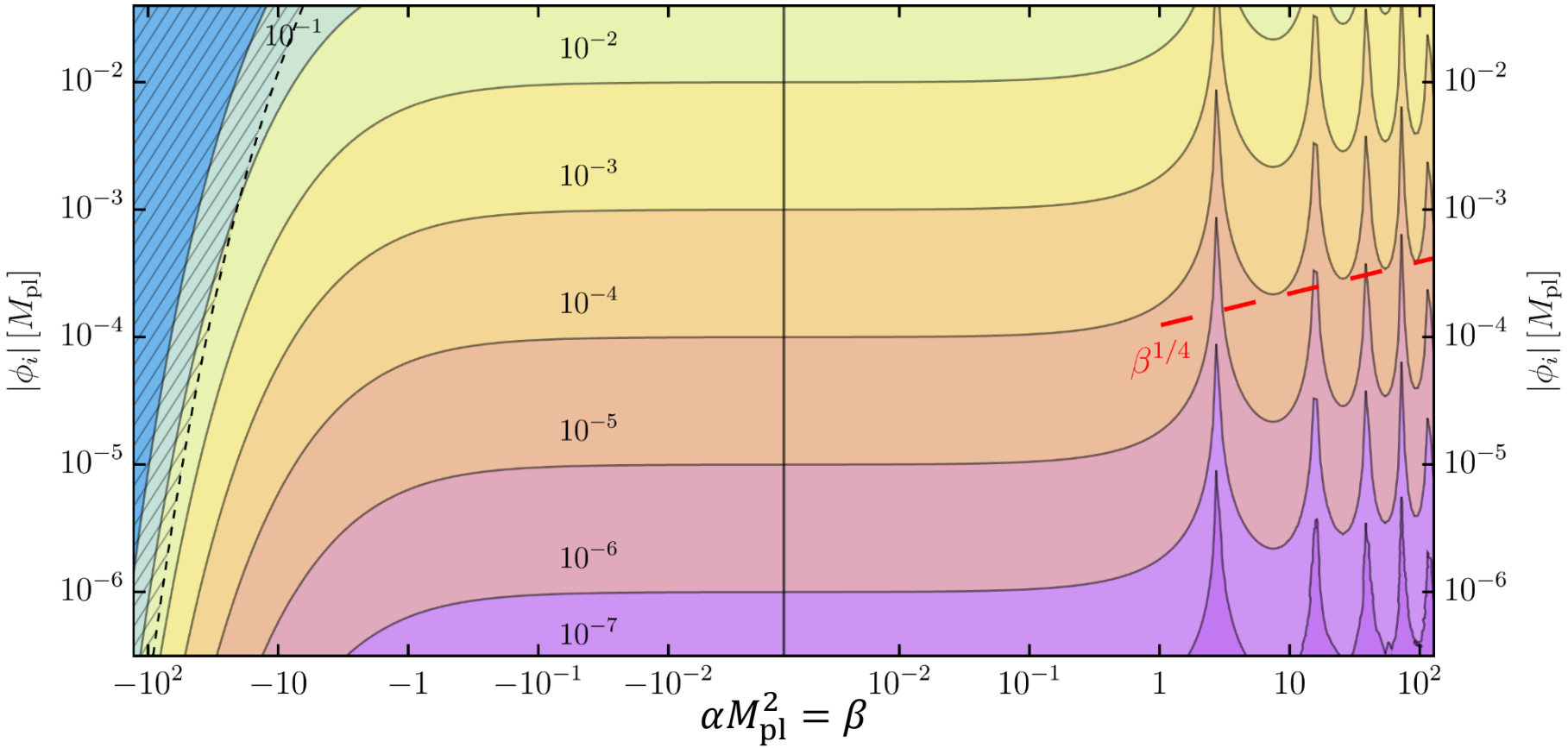
- Field may start to oscillate
- For negative coupling, field could be destabilised

Effect on final dark matter density

$$\langle \rho_f \rangle \propto \begin{cases} \left(\alpha M_{\rm pl}^2\right)^{-1/2} , & \alpha M_{\rm pl}^2 \gg 1 \\ \exp\left(-\alpha M_{\rm pl}^2 \bar{\Sigma}\right) , & \left|\alpha M_{\rm pl}^2\right| < 1 \\ \exp\left(\sqrt{-\alpha M_{\rm pl}^2 \bar{\Sigma}}\right) , & \alpha M_{\rm pl}^2 \ll -1 \end{cases}$$

CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Effects of the Kick



CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Dark QCD-Like Axion Model

Bare axion potential

$$V(a) = -\Lambda^2 \sqrt{1 - \xi \sin^2\left(\frac{a}{2f}\right)}.$$

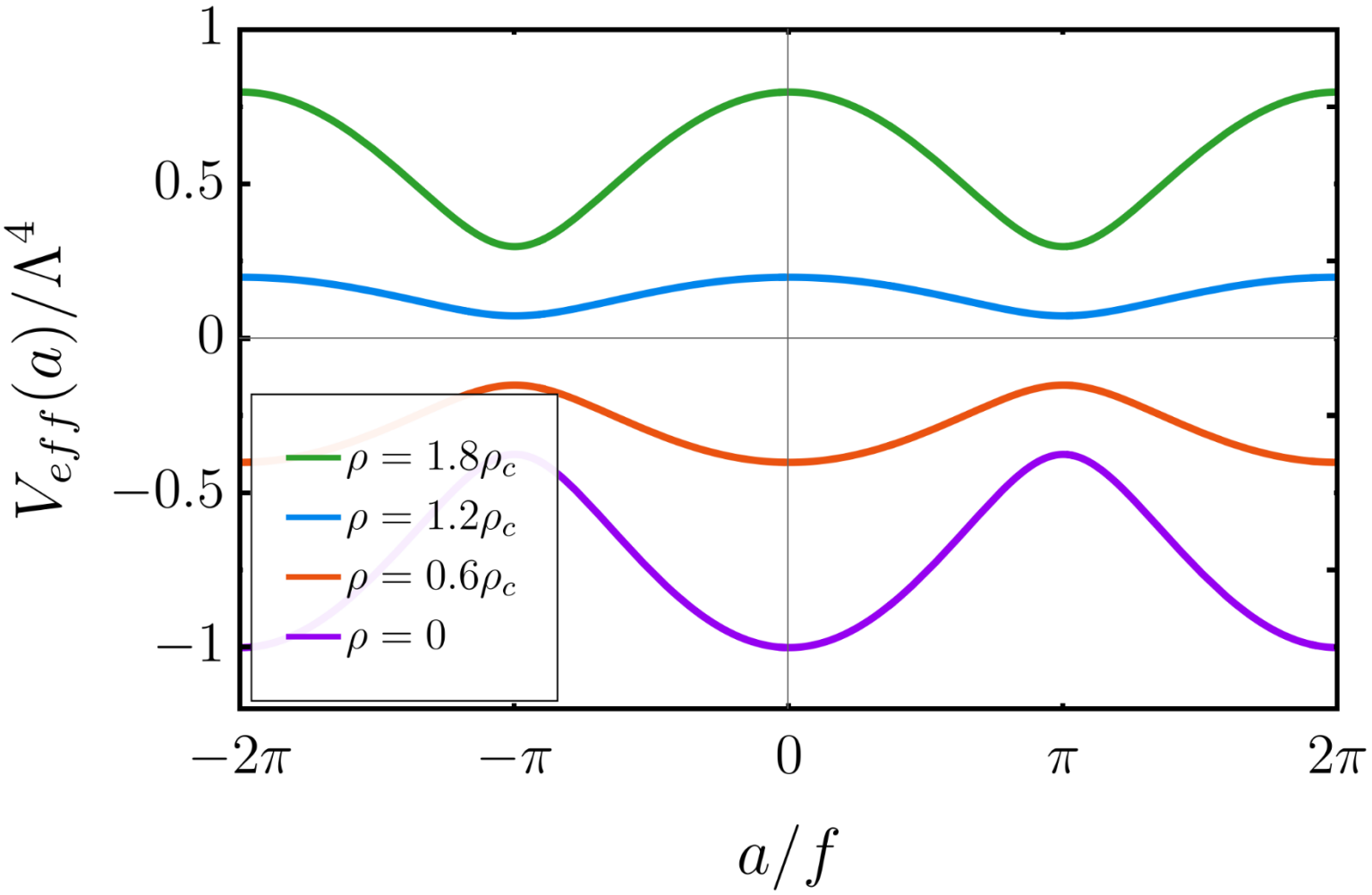
Coupling to density of dark baryons

$$V_b(a) = +2\tilde{\alpha} \sqrt{1 - \xi \sin^2\left(\frac{a}{2f}\right)} T_\mu^{\mu(b)}$$

Gives rise to negative coupling strengths

Grilli di Cortona, Hardy, Pardo Vega, Villadoro. JHEP 01 (2016) 034.
Hook, Huang. JHEP 06 (2018) 036. Garani, Redi, Tesi JHEP 12 (2021) 139.
Tsai, McGehee, Murayama, Phys. Rev. Lett. 128 (2022) 17 172001.
Khoury, Lin, Trodden. Phys. Rev. Lett. 135 (2025) 18 181001.

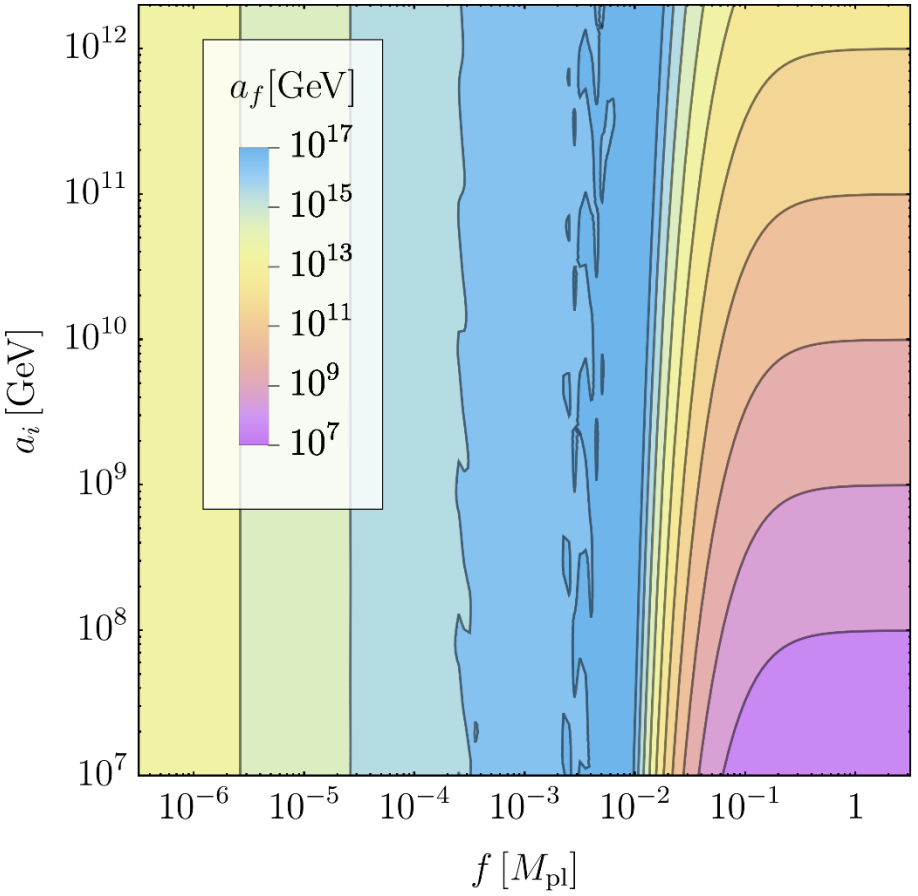
Dark QCD Axion



CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Effect of a dark pion kick

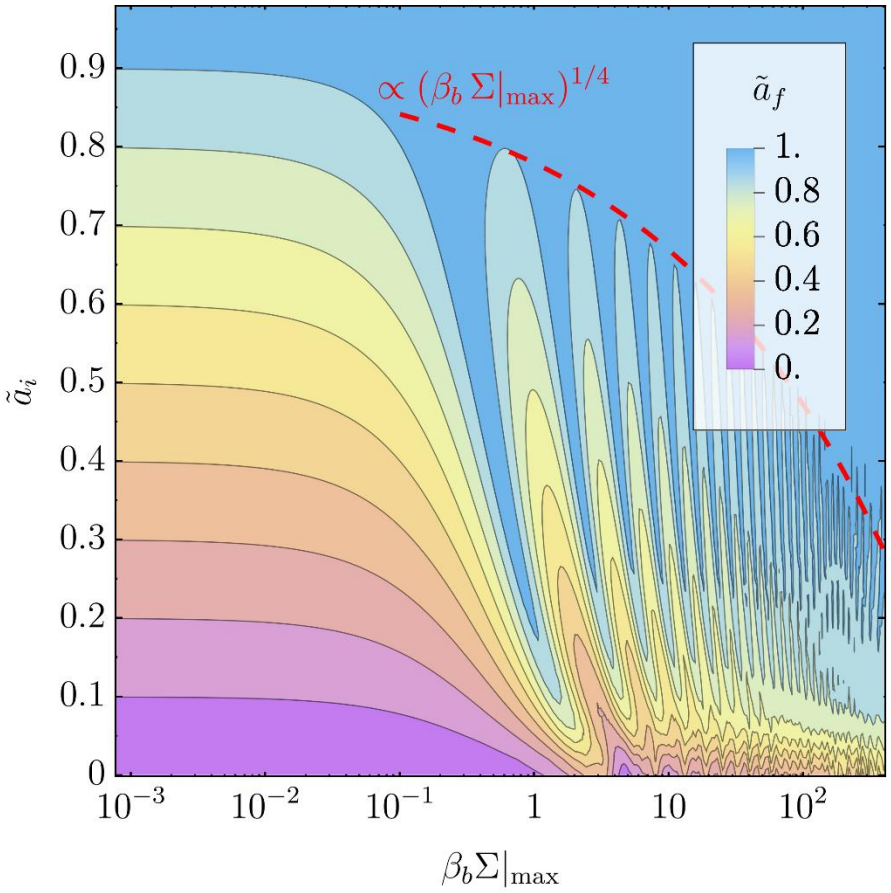
Fixing $\tilde{\alpha} = 1/2$



CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

Effect of a dark pion kick

Rescaling axion, $a = \tilde{\alpha}\pi f$, and allowing coupling, $\beta_b = 3M_{\rm pl}^2\tilde{\alpha}\xi/\pi f^2$, to vary

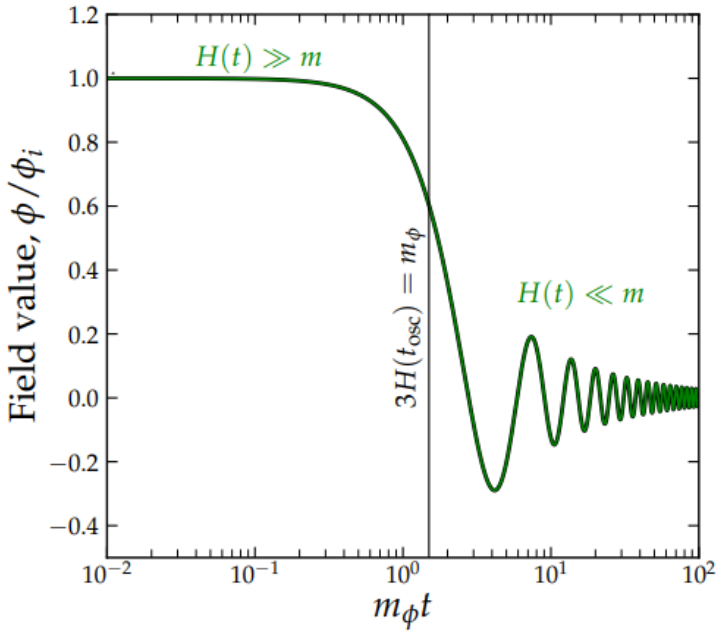


CB, Sevillano Muñoz. Phys.Rev.D 114 (2026) 4, 043541

SEARCHING FOR QUADRATICALLY COUPLED ULDM

Searching for ULDM

Homogeneous background evolution



Can we sense the time evolution of the background field?

Can we see scalar mediated forces?

Figure from lecture notes by O'Hare. Arxiv:2403.17697

Quadratic Couplings

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Leading to:

- Varying fundamental ‘constants’
- Macroscopic couplings to matter density and pressure

$$S = \int d^4x\left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\alpha(\rho - 3p)\phi^2\right)$$

Time Varying Constants

Interactions with matter

$$\mathcal{L}_{\rm int} = \frac{\phi^2}{2M_{\rm pl}^2}\left[\frac{d_e}{4}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d,e}\left(d_{m_i} + \gamma_{m_i}d_g\right)m_i\bar{\psi}_i\psi_i\right]$$

lead to variations in fundamental constants

$$\frac{\delta\alpha}{\alpha} = \frac{d_\gamma\phi^2}{2M_{\rm pl}^2}\,,\ \frac{\delta m_f}{m_f} = \frac{d_{m_f}\phi^2}{2M_{\rm pl}^2}\,,\ \frac{\delta\Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} = \frac{d_g\phi^2}{2M_{\rm pl}^2}$$

Constraints from comparison of atomic clocks

e.g. ^{87}Sr, ^{171}YB$^+$, ^{133}Cs

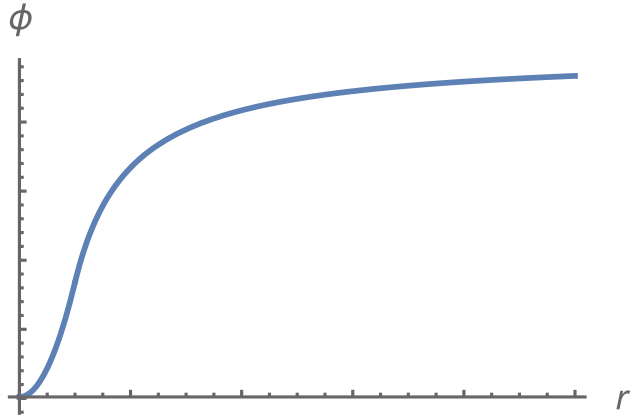
See e.g. Sherrill et al, New J. Phys. 25 093012 (2023)

Scalar Mediated Forces

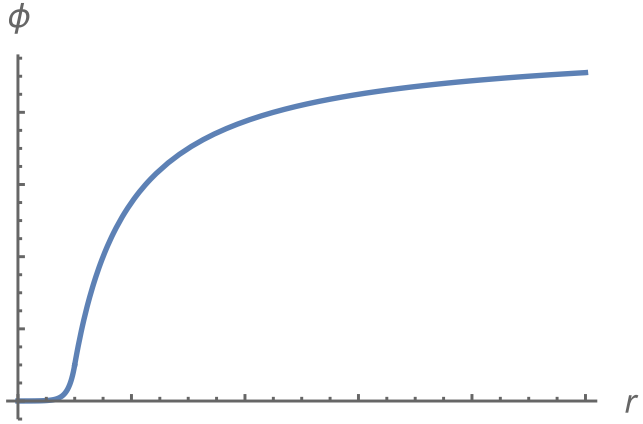
In the presence of a static, spherical dense object

$$\phi_{\rm st}(r,t) = \phi_\infty \cos(mt+\delta) \begin{cases} \dfrac{\cosh(kR)\sinh(kr)}{kr\sinh(kR)} & r \le R \\ 1 - \dfrac{kR - \tanh(kR)}{kr} & r > R \end{cases}$$

where $k = \sqrt{|\alpha|\rho}$



Small kR



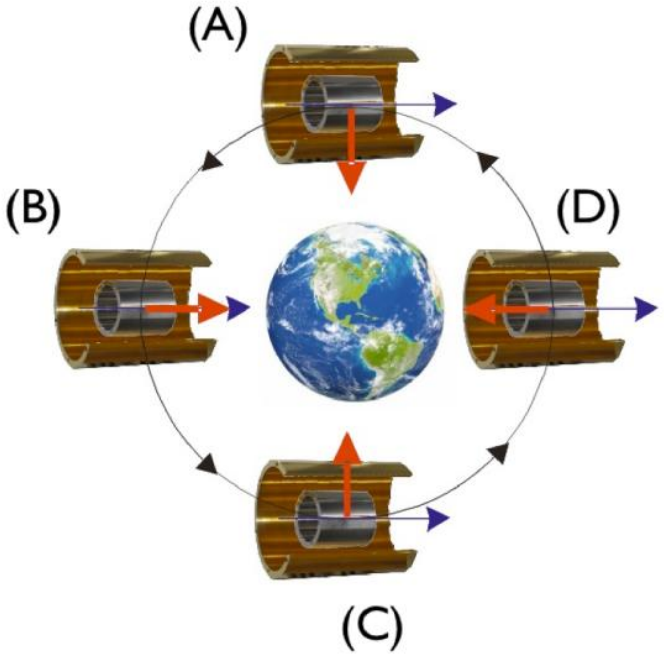
Large kR

Hees, Minazzoli, Savalle, Stadnik, Wolf. Phys. Rev. D 98 no. 6, (2018) 064051
Bauer, Rostagni. Phys. Rev. Lett. 132 (2024) 10 101802. Bauer, Chakraborti. Phys. Rev. D 112 (2025) 10, 103019. CB, Elder, Garcia del Castillo, Jaeckel. Phys.Rev.D 111 (2025) 10, 103526

MICROSCOPE

Eötvös parameter

$$\eta_{A,B} = 2\frac{a_A - a_B}{a_A + a_B}$$



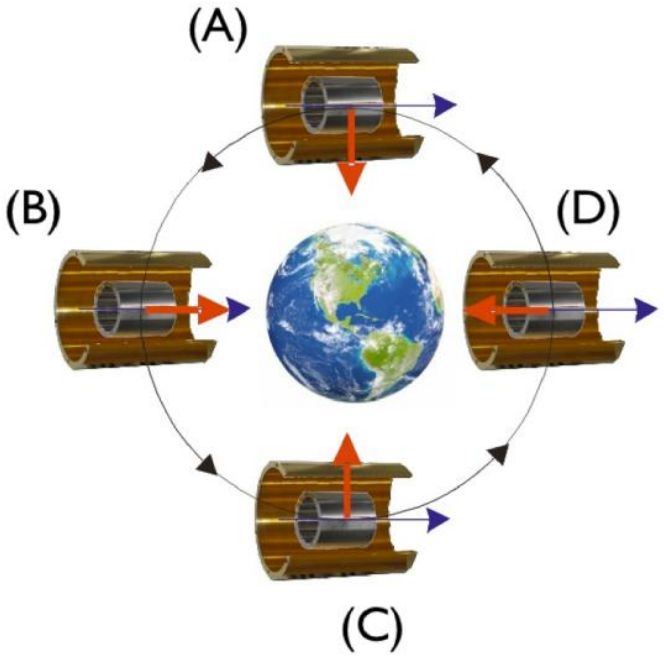
$$\eta(\mathrm{Ti};\mathrm{Pt}) = [-1.5 \pm 2.3 \pm 1.5]\times 10^{-15}$$

Touboul et al. Phys. Rev. Lett. 129, 121102 (2022), Class. Quantum Grav. 39 204009 (2022)

MICROSCOPE

Eötvös parameter

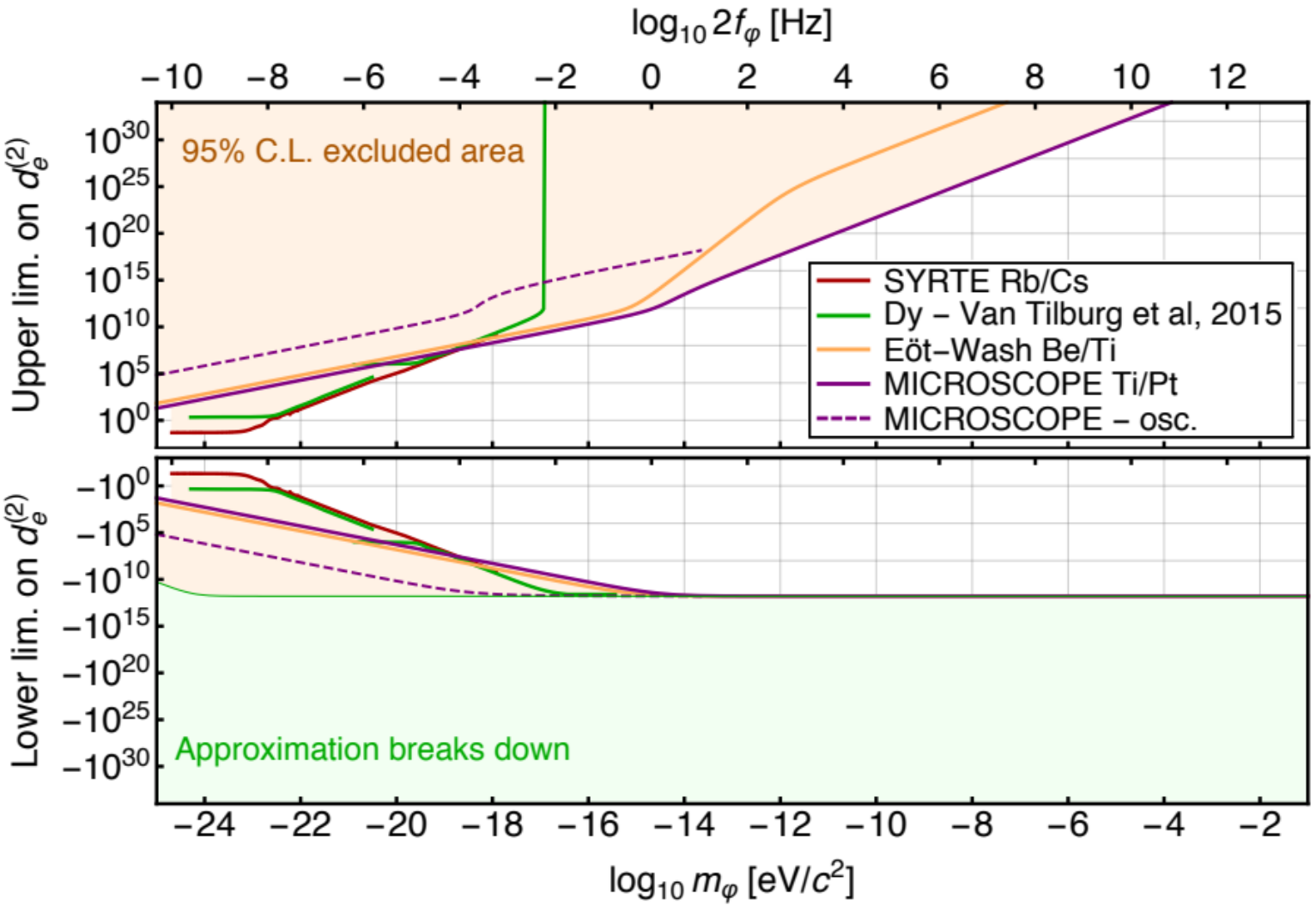
$$\eta_{A,B} = 2\frac{a_A - a_B}{a_A + a_B}$$



Future improvements possible with STEIN

Touboul et al. Phys. Rev. Lett. 129, 121102 (2022), Class. Quantum Grav. 39 204009 (2022)

Current Constraints



Hees, Minazzoli, Savalle, Stadnik, Wolf. Phys. Rev. D 98 no. 6, (2018) 064051

Ultra-Light Scalar Dark Matter

$$S = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\alpha\rho\phi^2 \right)$$

Density dependent effective mass, can lead to short Compton wavelengths

$$\lambda_C = \frac{h}{m_{\mathrm{eff}}c} \approx 10^{-5} \left(\frac{\mathrm{GeV}^{-2}}{\alpha} \right)^{1/2} \left(\frac{\mathrm{g/cm}^3}{\rho} \right)^{1/2} \mathrm{cm}$$

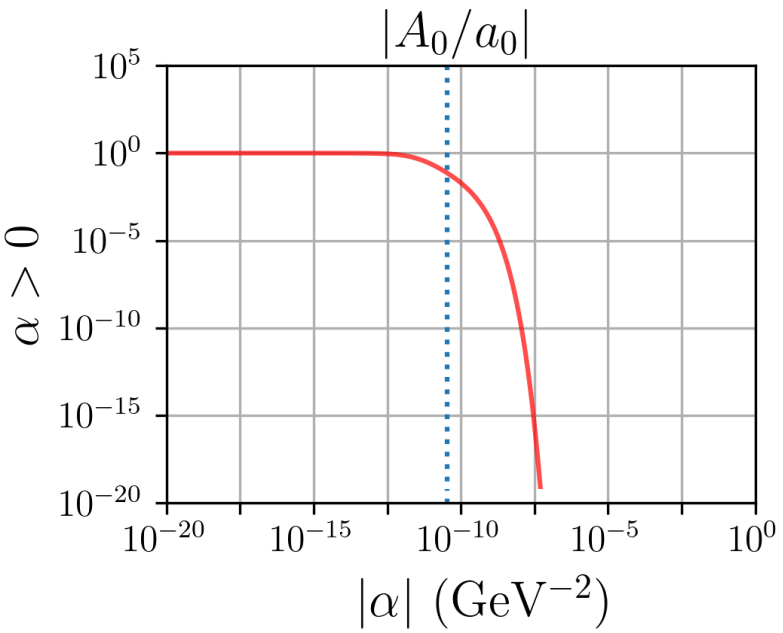
Scalar cavity can suppress the amplitude of dark matter oscillations

CB, MacDonald, Ross, Rybka, Todarello. Phys.Rev.D 113 (2026) 063505

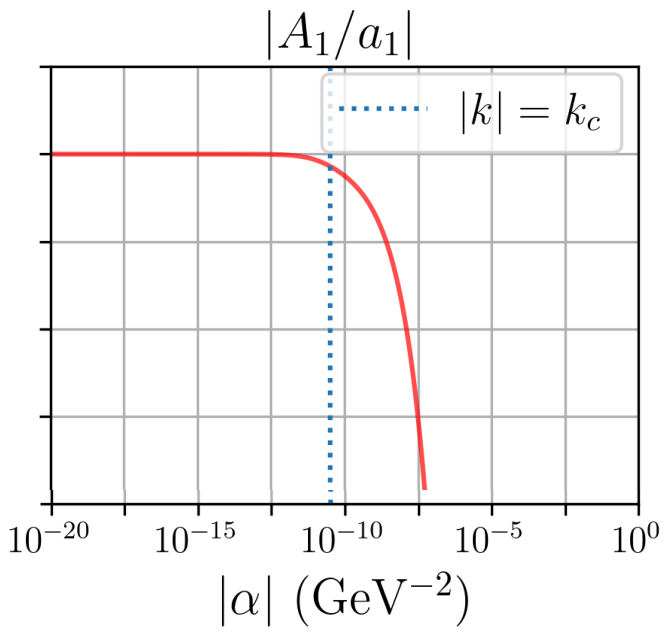
Ultra-Light Scalar Dark Matter

Experimental cavity can suppress the amplitude of dark matter oscillations

Field



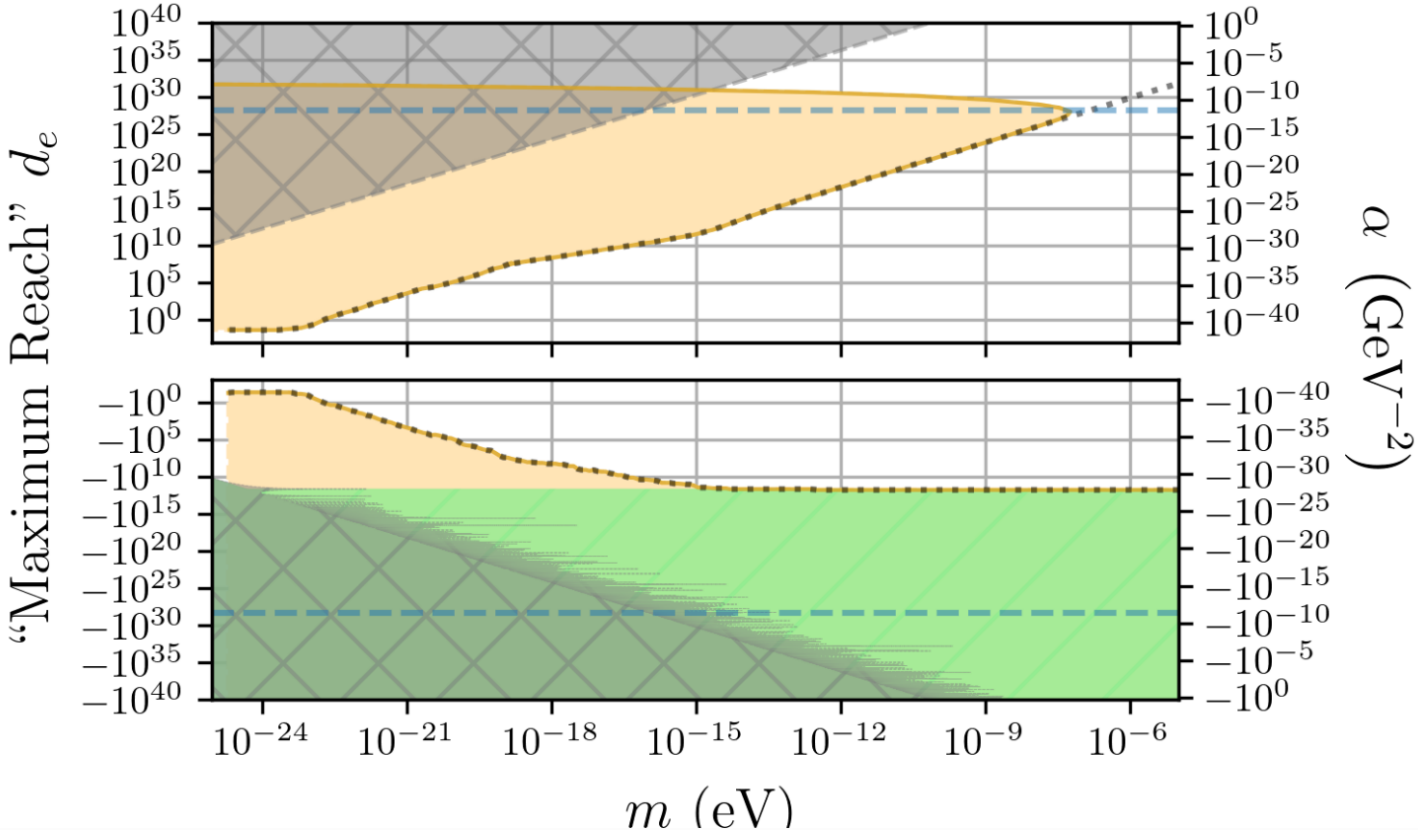
Gradient



CB, MacDonald, Ross, Rybka, Todarello. Phys.Rev.D 113 (2026) 063505

Effects of a Spherical Cavity

Naive rescaling weakens constraints at strong coupling



New opportunities for space-based searches?

CB, MacDonald, Ross, Rybka, Todarello. Phys.Rev.D 113 (2026) 063505

Pericentre precession of LAGEOS II

Laser Geodynamic Satellite II - high-density passive laser reflector in a stable medium Earth orbit



Anomalous pericentre precession constrained to be

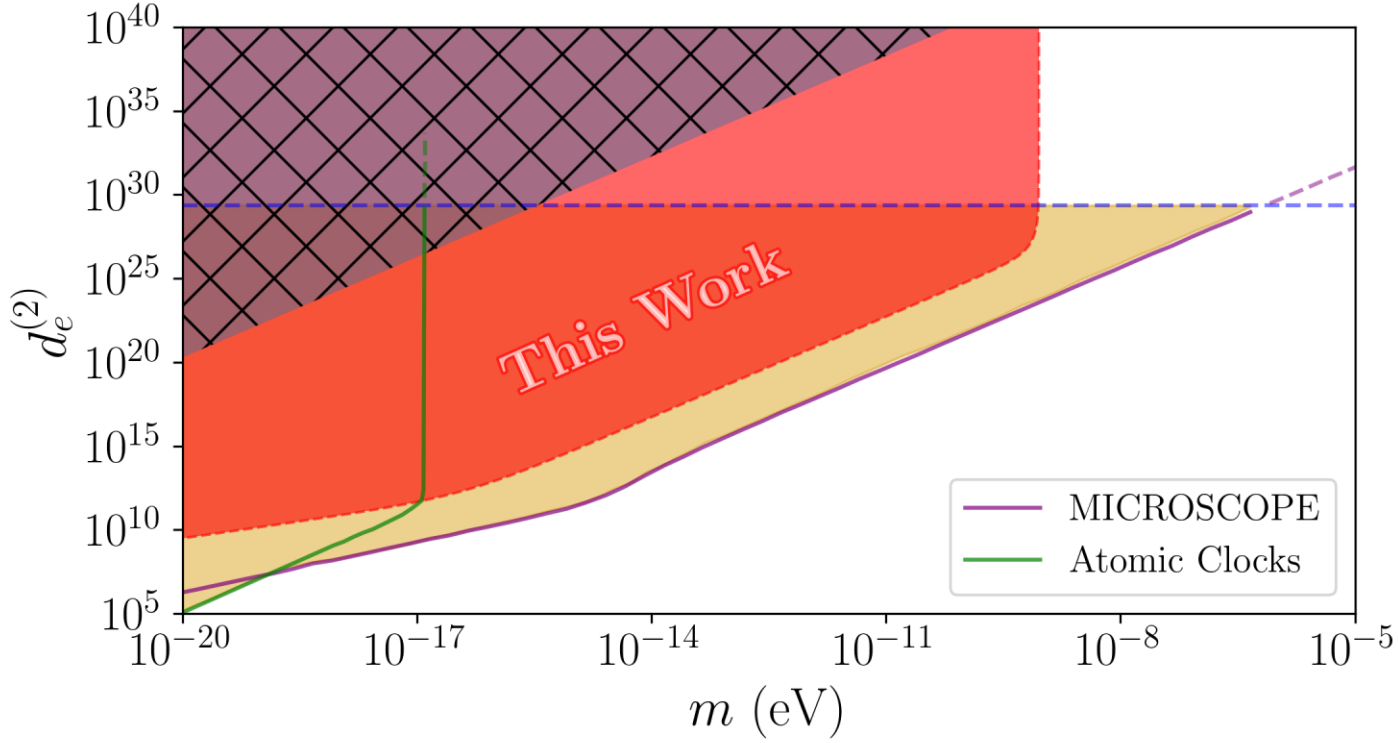
$$\Delta\dot{\omega}_{\phi} \lesssim -181\times10^{-3}\ \mathrm{as}\ /\ \mathrm{yr}$$

Lucchesi, Peron. Phys. Rev. D, 89(8):082002, (2014). Image credit: NASA

Pericentre precession of LAGEOS II

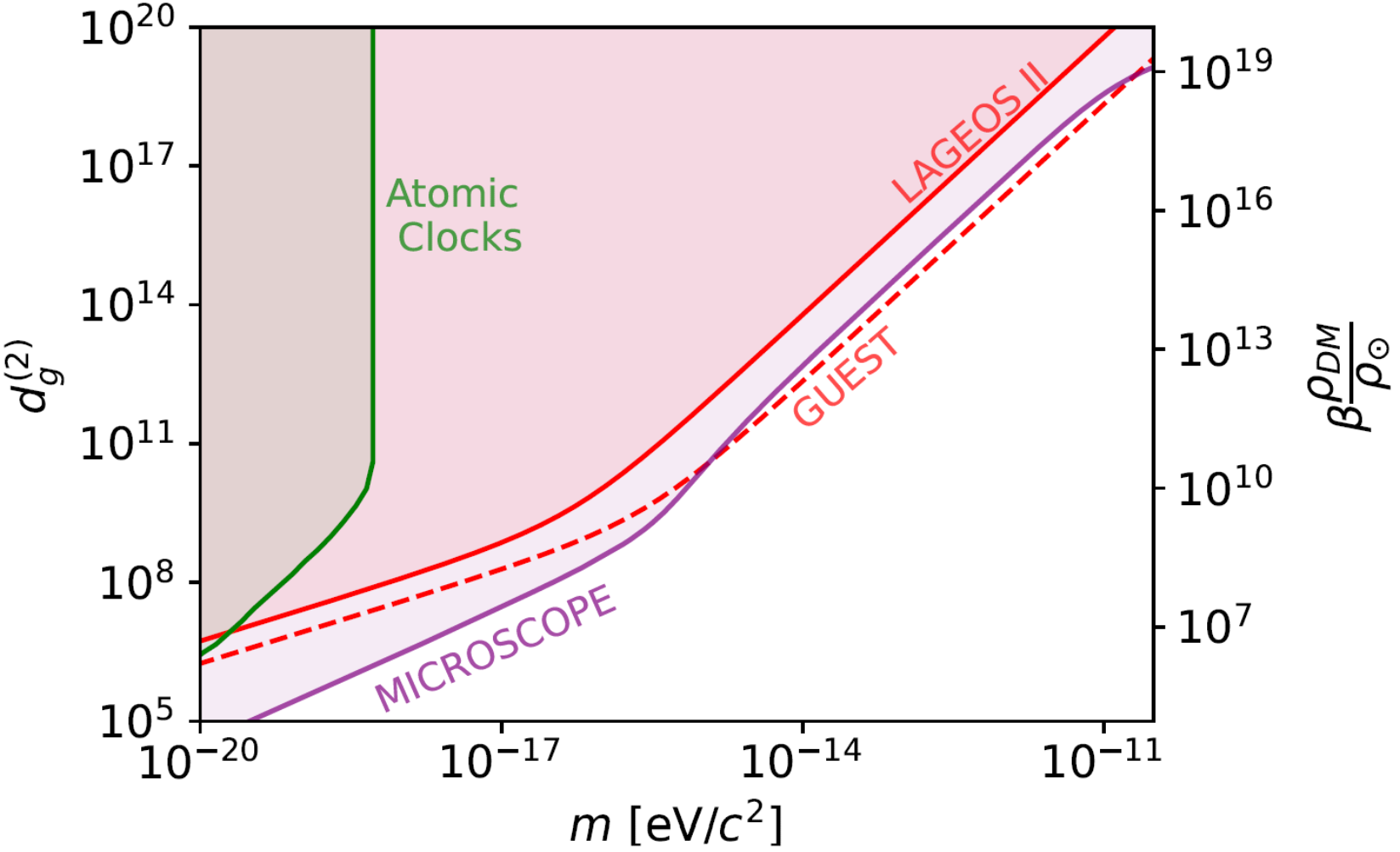
Anomalous pericentre precession constrained to be

$$\Delta\dot{\omega}_\phi \lesssim -181\times10^{-3}\ \text{as}\ /\ \text{yr}$$



CB, Macdonald, Todarello. arXiv:2605.28248

Future Improvements with GUEST



Blas et al. arXiv:2607.18390

Approximations

Neglecting the internal structure of the satellite and the Earth

Treating the Earth as a spherical source

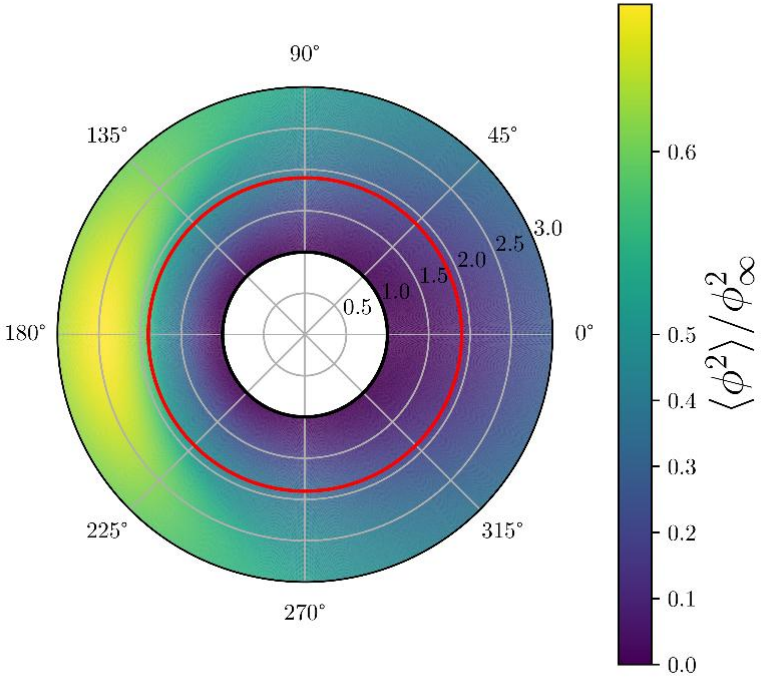
Assuming the scalar profile due to the Earth can be linearised over the size of the satellite

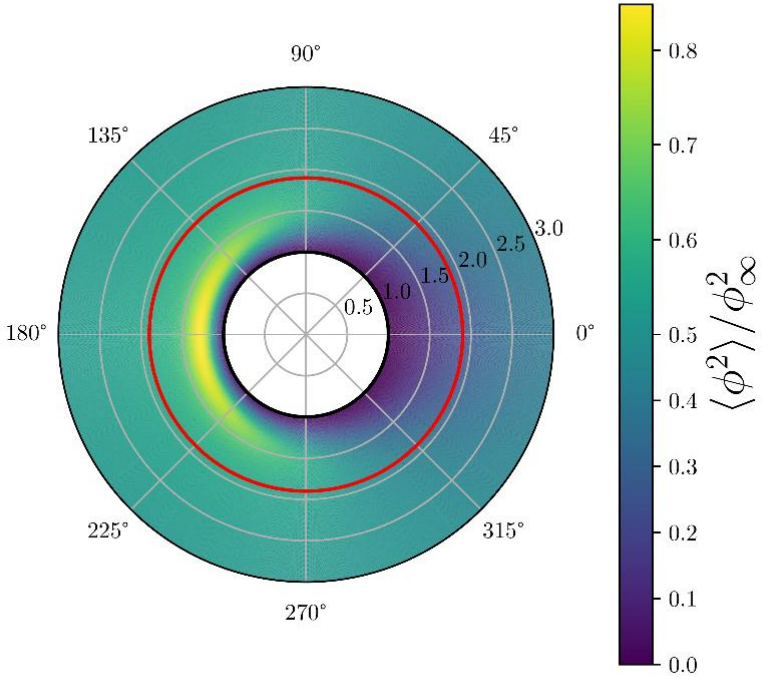
Neglecting the effects of the Sun and the Moon

CB, Macdonald, Todarello. arXiv:2605.28248

Neglecting the dark matter wind?

Modelling the wind as an incoherent ensemble





$m = 2\times10^{-11}$ eV $\qquad$ $m = 10^{-10}$ eV

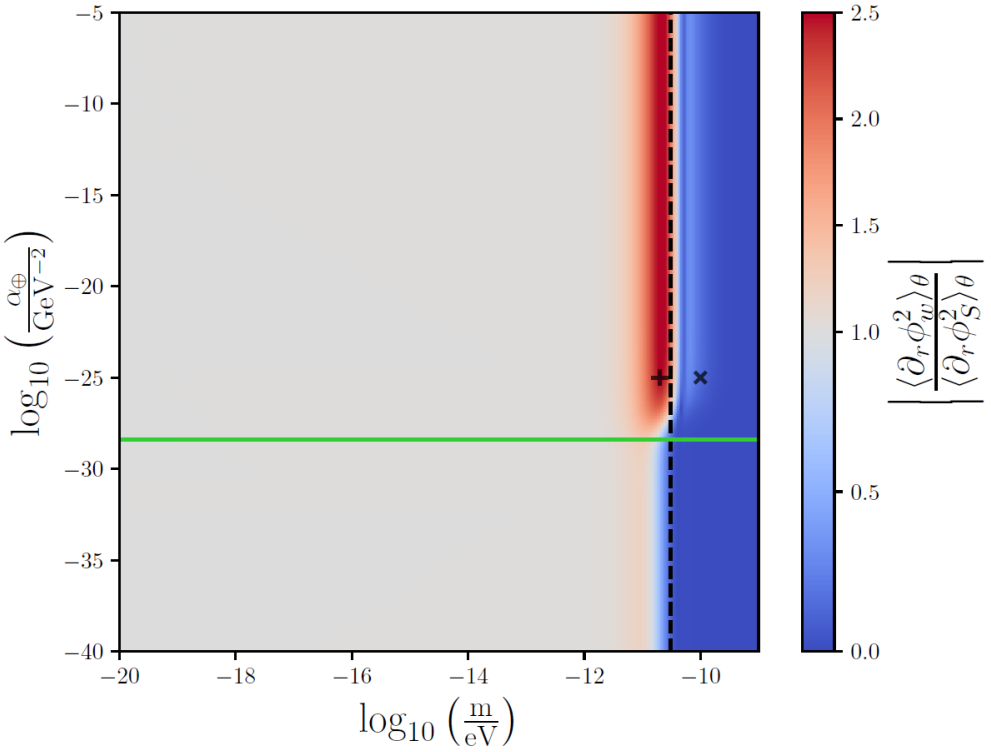
for $\alpha = 10^{-25}\ \mathrm{GeV}^{-2}$

CB, Macdonald, Todarello. arXiv:2605.28248.

See also: Gan, Liu, Liu, Luo, Yu. JHEP 02 (2026) 043

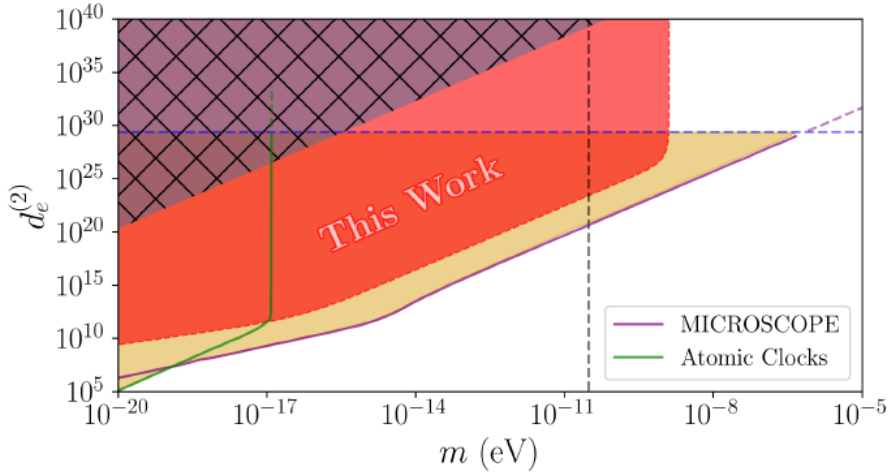
Neglecting the dark matter wind?

Modelling the wind as an incoherent ensemble of field modes

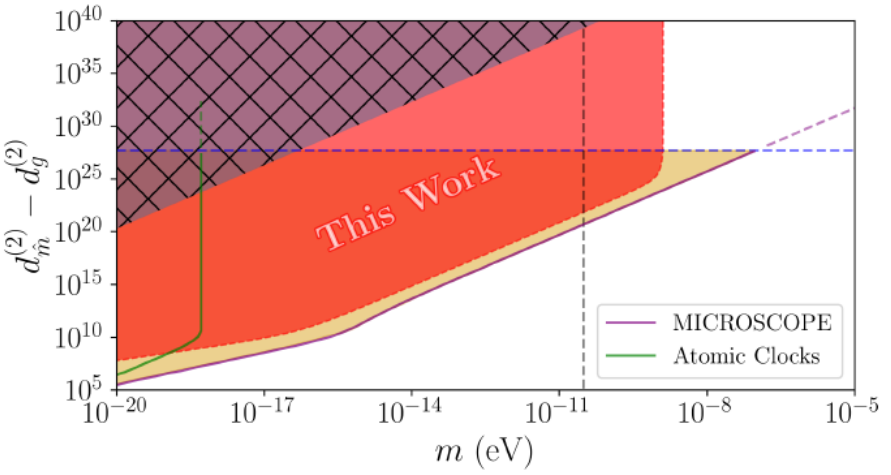


CB, Macdonald, Todarello. arXiv:2605.28248

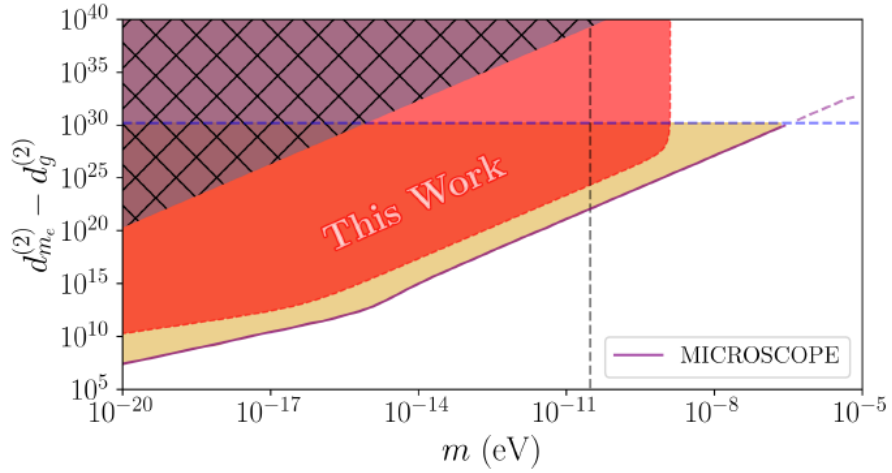
Pericentre precession of LAGEOS II

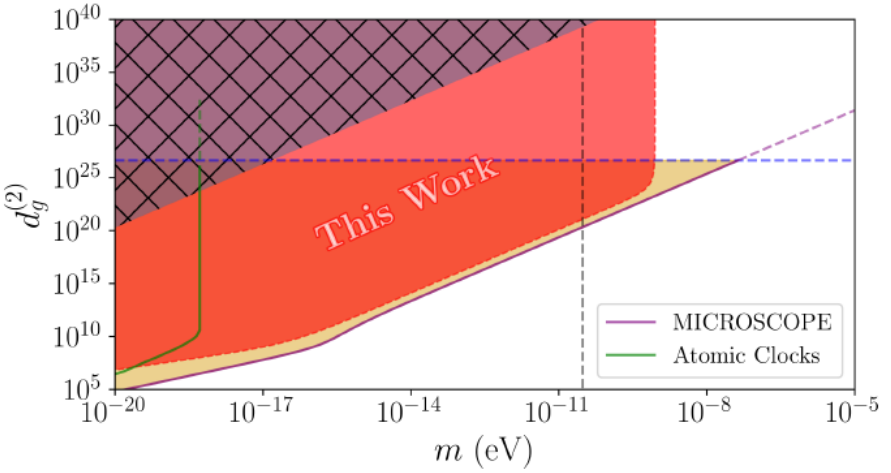


(a)



(b)



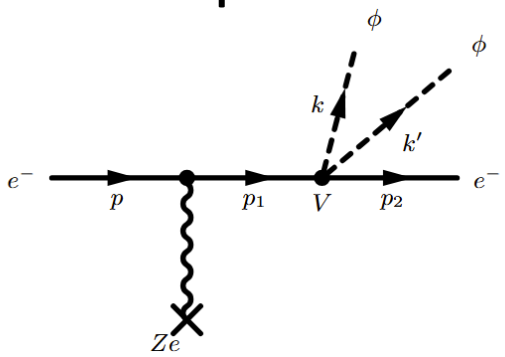


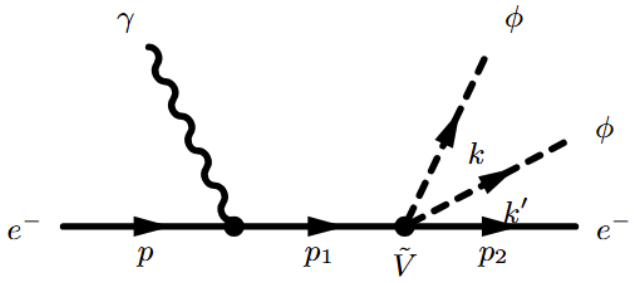
CB, Macdonald, Todarello. arXiv:2605.28248

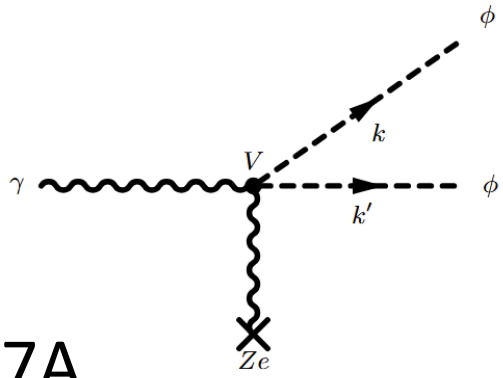
Preliminary: Bounds from Stellar Burning

Production of light scalars in stars provides an additional energy loss channel

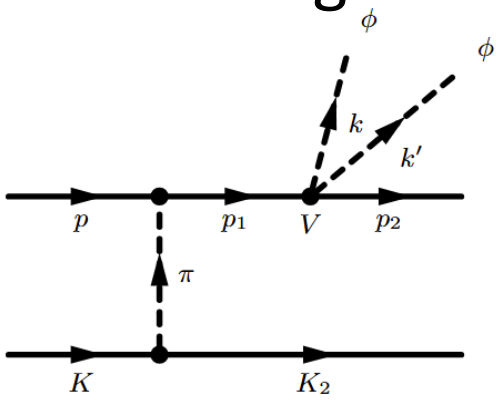
- Bremsstrahlung, Compton scattering and Primakov production in the Sun and Horizontal branch stars





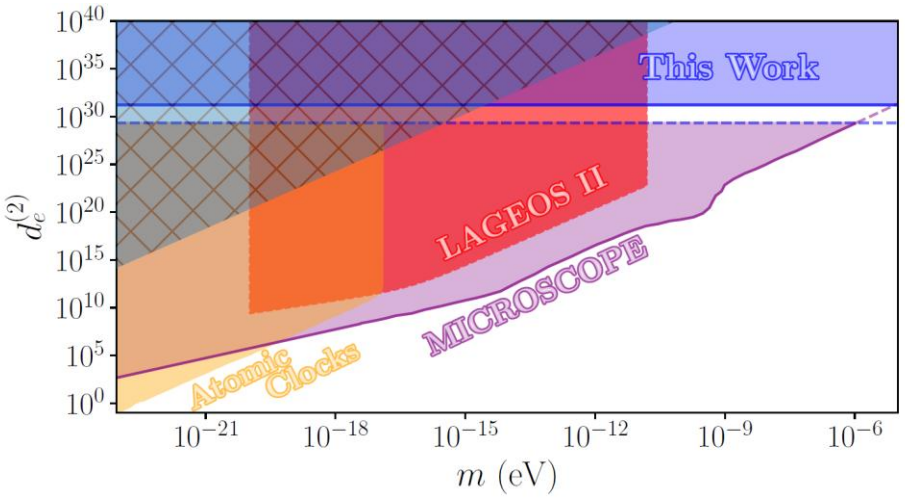


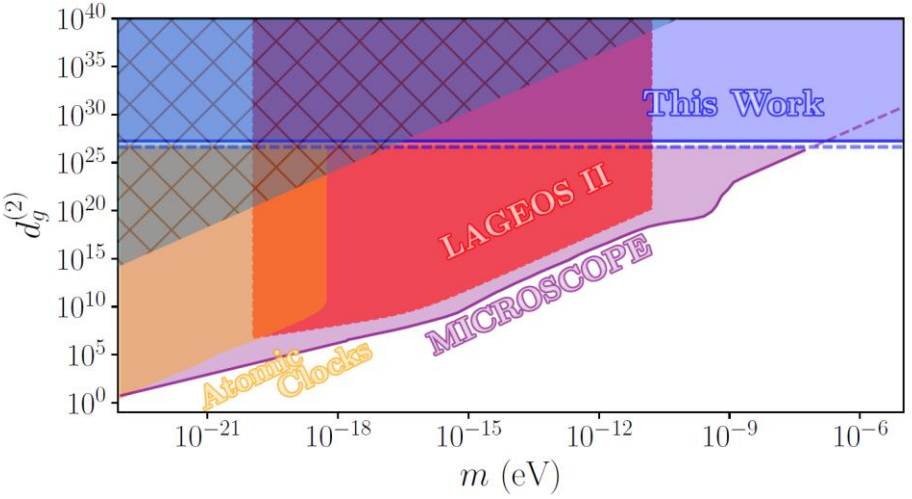
- Via pion exchange in SN1987A

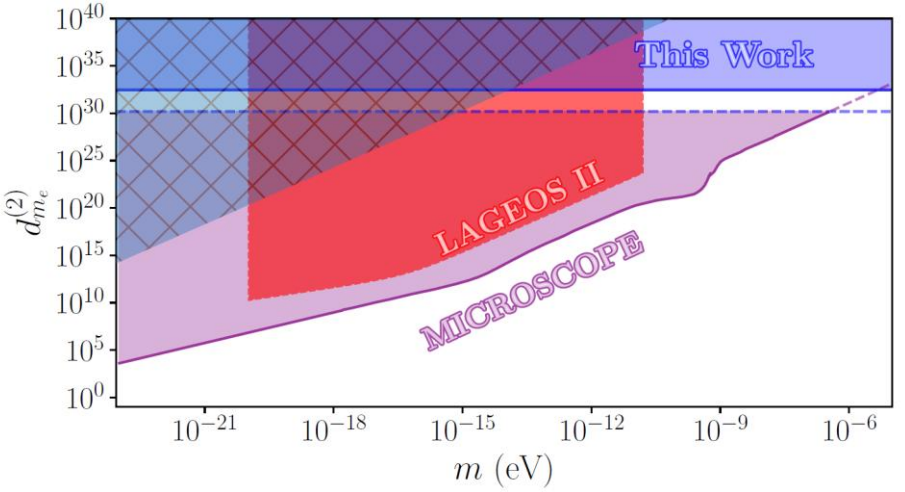


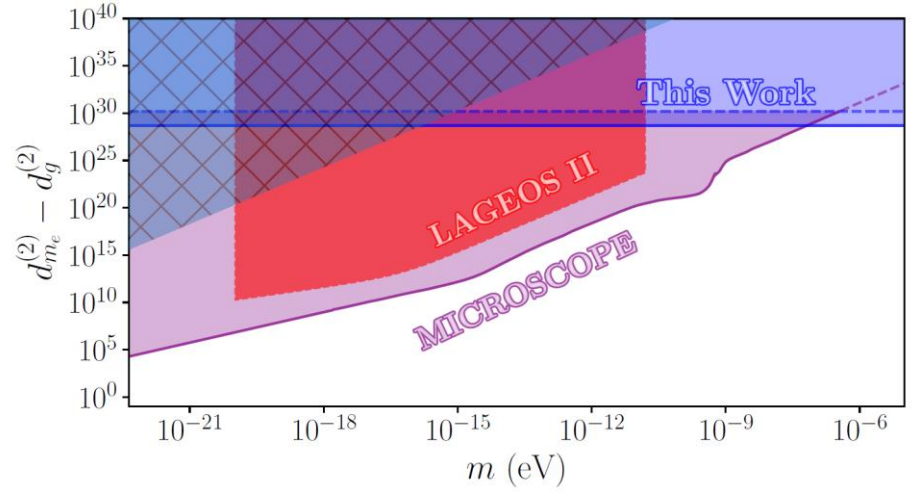
Brax, CB. To appear.

Current Constraints on Quadratic Couplings









Hees, Minazzoli, Savalle, Stadnik, Wolf. Phys. Rev. D 98 no. 6, (2018) 064051

CB, Macdonald, Todarello. arXiv: 2605.28248. Brax, CB. To appear.

Summary

Ultra-light scalar fields are a possible dark matter candidate

When particle species become non-relativistic in the early universe this kicks the dynamics of the scalar field

Changes expected scalar value and dark matter abundance

Environmental dependence can suppress signatures of strongly coupled models in some experiments

But leave opportunities open for others

Compactness

In GR compactness is defined as: $\mathcal{C} = \frac{GM_*}{R} = \frac{\rho R^2}{6M_{\rm Pl}^2}$

Object	Compactness		
Black hole	~1		
Neutron star	0.6		
White dwarf	4×10^{-4}		
Sun	4×10^{-6}		
Milky Way	3×10^{-6}		
Earth	1×10^{-9}		
Asteroid	1×10^{-11}		
ISS	7×10^{-25}		
Apple	4×10^{-27}		
Silica Microsphere	1×10^{-36}		
Atomic Nucleus	1×10^{-40}		

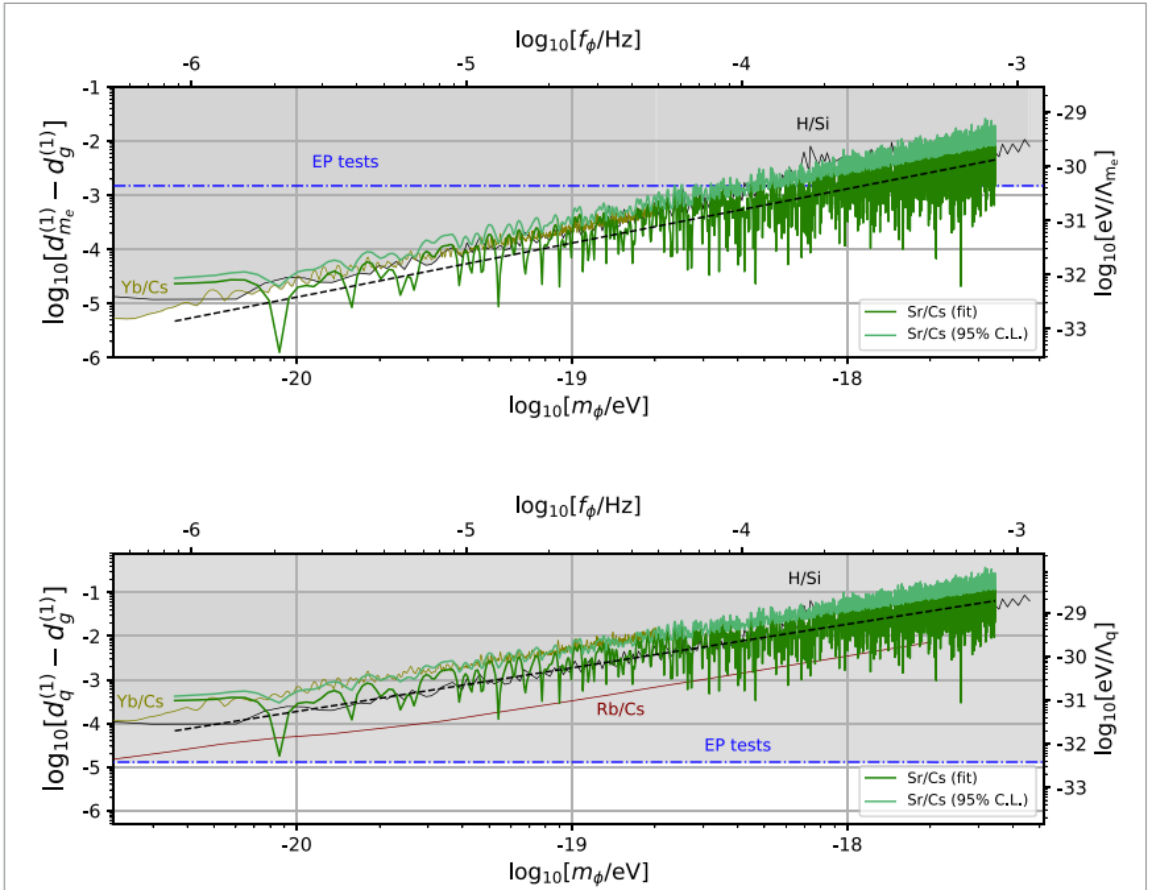
Compactness

Choosing a different normalisation scale: $\tilde{\mathcal{C}} = \frac{\rho R^2}{M^2}$

Object	Compactness	10^{10} GeV	10^{5} GeV
Black hole	~1	3×10^{16}	3×10^{26}
Neutron star	0.6	2×10^{16}	2×10^{26}
White dwarf	4×10^{-4}	1×10^{13}	1×10^{23}
Sun	4×10^{-6}	1×10^{11}	1×10^{21}
Milky Way	3×10^{-6}	9×10^{10}	9×10^{20}
Earth	1×10^{-9}	4×10^{7}	4×10^{17}
Asteroid	1×10^{-11}	4×10^{5}	4×10^{15}
ISS	7×10^{-25}	2×10^{-8}	200
Apple	4×10^{-27}	1×10^{-10}	1
Silica Microsphere	1×10^{-36}	3×10^{-20}	3×10^{-10}
Atomic Nucleus	1×10^{-40}	3×10^{-24}	3×10^{-14}

Atomic Clocks – Constraints on ULDM

$$\phi_{\mathrm{homog}}(t) = \phi_{\infty} \cos(mt + \delta)$$



Sherrill et al, New J. Phys. 25 093012 (2023)

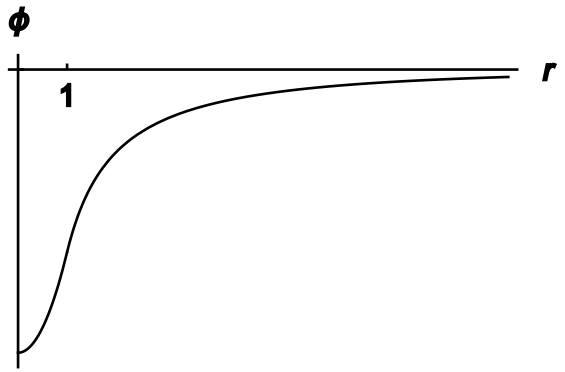
Yukawa Forces

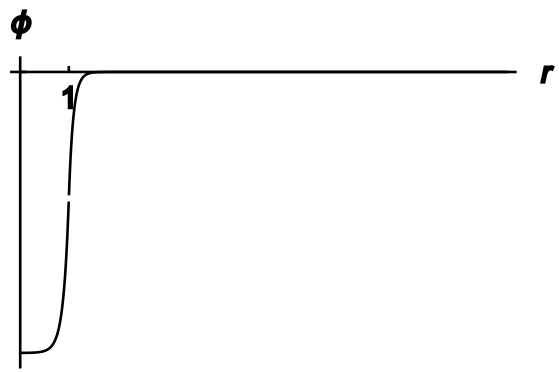
Static Klein-Gordon equation

$$\nabla^2\phi - m^2\phi = \frac{\rho}{M}$$

Spherical object of constant density ρ and radius R

$$\frac{\phi(r)}{M} = -\frac{\rho}{M^2 m^2}\begin{cases} 1 - \dfrac{1+Rm}{\cosh m\,R + \sinh m\,R}\dfrac{\sinh m\,r}{mr}\,, & r \le R \\ \dfrac{e^{m(R-r)}}{mr}\left(\dfrac{Rm\cosh m\,R - \sinh m\,R}{\cosh m\,R + \sinh m\,R}\right)\,, & r > R \end{cases}$$

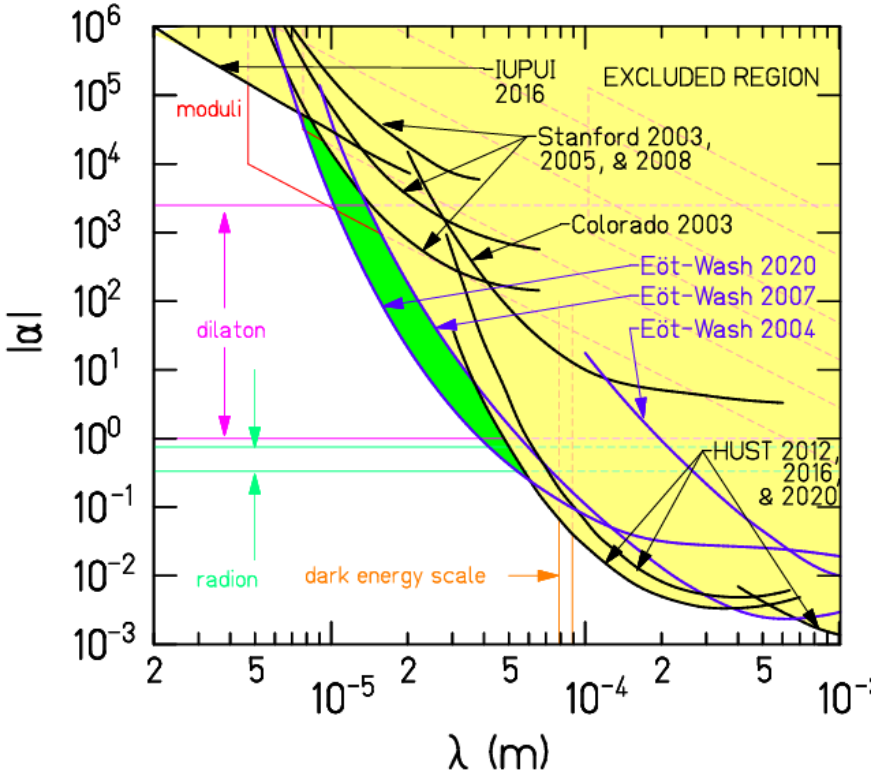


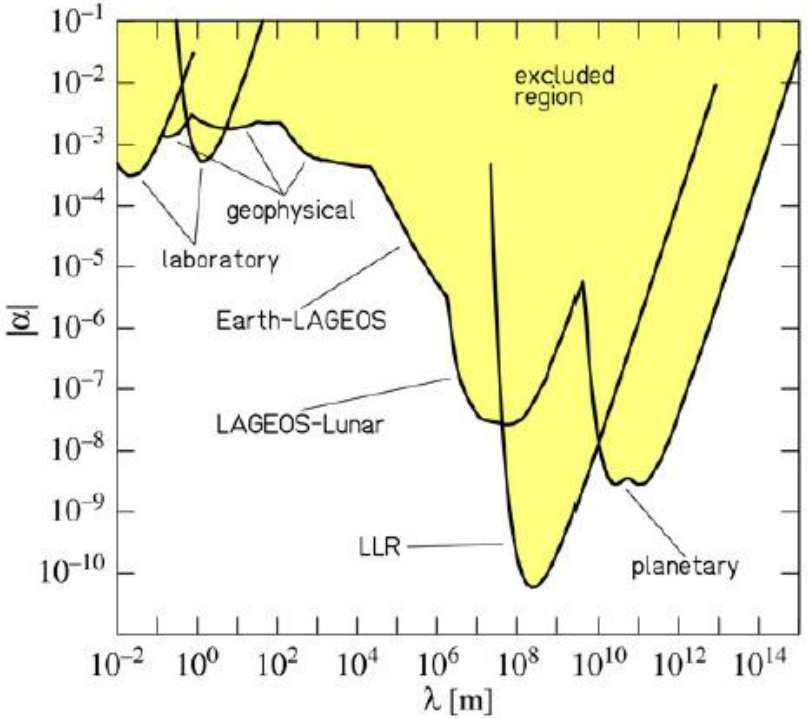


Yukawa Fifth Forces

A long-range Yukawa fifth force is excluded to a high degree of precision in the solar system

$$V(r) = -\frac{G\alpha m_1 m_2}{r} e^{-m_\phi r}$$





Adelberger et al. Prog. Part. Nucl. Phys. 62 (2009) 102. Lee et al. PRL 124 (2020) 10, 101101.