# Nouvelle Cuisine

Sep $16^{th}$

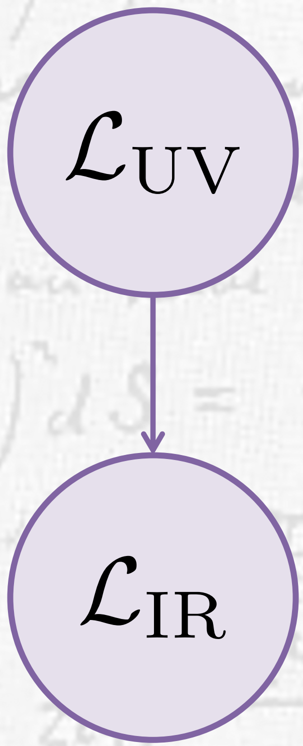
Particle Physics in the Early Universe

Paris and Orsay

Matthew McCullough

# EFT

Whether or not we know a UV theory, we have tools to understand how its parameters may map into the IR EFT.  Dimensional analysis, NDA, **spurion analysis**, symmetries...



# Mapping to IR: Spurion Analysis

Basic idea: In UV or IR theory, treat parameters as if they were fields, or were vanishing. If this were the case, what global symmetries would there be?

$$\mathcal{L}(\text{Fields}, \text{Params})$$

Under these symmetries, the "charges" of the parameters dictate:

- How they enter into physical observables.
- The structure of quantum corrections.
- RG patterns.
- The structure of the IR EFT.

# Spurion Analysis

## Example: The electron mass.

The electron Yukawa is the only parameter in

$$\mathcal{L}_{\mathrm{IR}}$$

which breaks a U(1) electron chiral symmetry. Thus, within the IR it can only be renormalized proportional to itself.

# Spurion Analysis

Example: The electron mass.

Furthermore, if there is only one parameter in

$$\mathcal{L}_{\mathrm{UV}}$$

which breaks the chiral symmetry (and gives rise to the Yukawa), then even in the UV the renormalization of the operator responsible for the IR electron Yukawa is also renormalized proportional to itself.

Can start small, stay small, at all scales.

('t Hooft Natural)

# The Flavour Landscape

If we stick with the classic methods, but avoid old-fashioned traditions, what else is possible for flavour?

# The SU(3)$_{Q,U,D,L,E}$ Landscape


Are we sure we've looked around us?

# Example: Scalar Quartic

Standard lore would have it that in a theory such as

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

the cutoff should be at

$$\Lambda \lesssim \frac{(4\pi)^2}{\lambda}m^2$$

However, consider a U(1) pNGB with explicit breaking by operator of charge "n". EFT is:

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}(\partial\phi)^2 + \epsilon M^2 f^2 \cos\frac{n\phi}{f} \\ \approx& \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\epsilon n^2 M^2\phi^2 + \frac{1}{4!}\epsilon n^4\frac{M^2}{f^2}\phi^4 + \ldots\end{aligned}$$

# Example: Scalar Quartic

EFT is:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \epsilon M^2 f^2 \cos\frac{n\phi}{f}$$

$$\approx \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\epsilon n^2 M^2 \phi^2 + \frac{1}{4!}\epsilon n^4 \frac{M^2}{f^2}\phi^4 + \ldots$$

True cutoff is $\Lambda^2 \lesssim (4\pi)^2 f^2$

Naïve cutoff is $\Lambda^2 \lesssim (4\pi)^2 f^2/n^2$

Naïve estimate underestimates natural scale separation by factor n. Beware minimality!

# What about Flavour?

Fermion Yukawas explicitly break flavour symm

$$\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \to \mathrm{U}(1)^3$$

Yukawa coupling is a spurion in the irrep

$$(\mathbf{3}, \overline{\mathbf{3}})$$

UV scenarios for this include:

- MFV
- Froggatt-Nielsen
- Universal
- Aligned...

All of which correspond to minimal irreps for underlying UV spurion.

# What about Flavour?

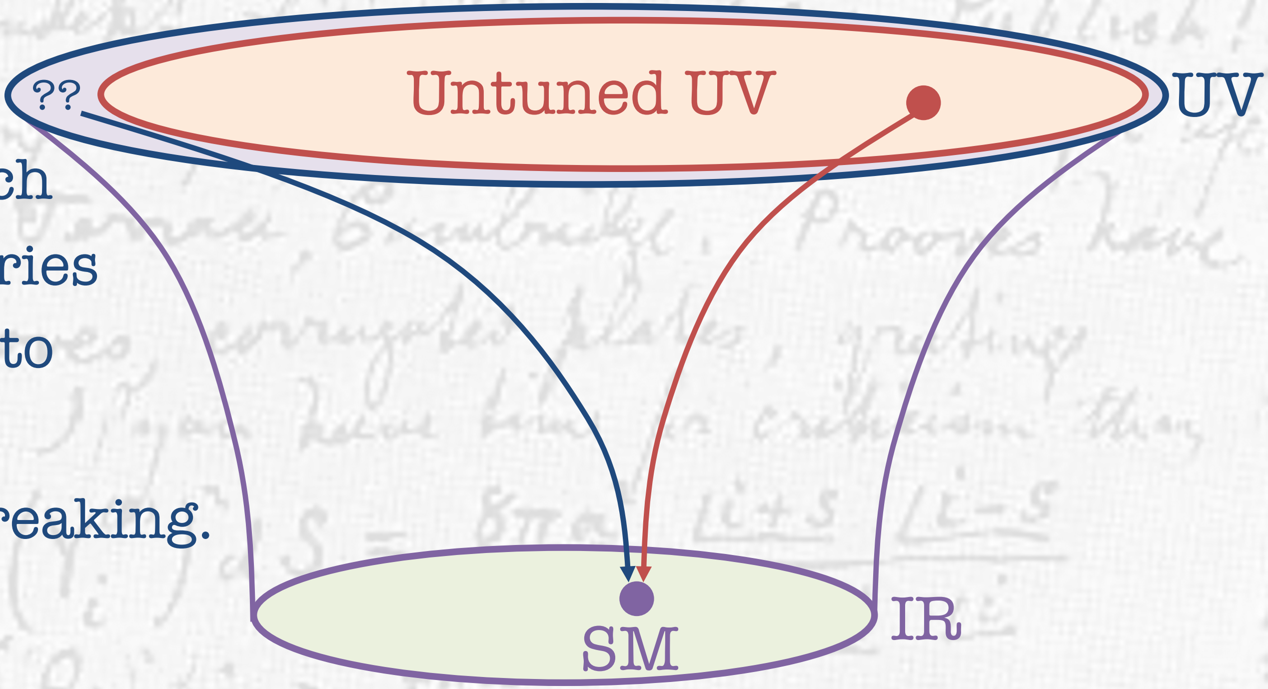
In the UV the SM flavour symmetries could correspond to

a) Spontaneously broken gauge symmetries

b) Accidental global symmetries

c) ...?

The parameters which break those symmetries in the UV will feed into the IR and generate flavour symmetry breaking.




The fact that in the IR the leading violation is by a minimal irrep does not mean that it is broken by a minimal irrep in the UV.

# SM Example: 1

Consider the many faces of the SU(2) gauge symmetry of the SM:

$E \gg M_W$ Gauge Symmetry

$E \sim M_W$ Spontaneously broken gauge symmetry
Dynamical radial mode: Higgs.

$\Lambda_{QCD} \lesssim E \ll M_H$ Global symmetry explicitly broken by spurion in doublet.

$E < \Lambda_{QCD}$ Part of the pion shift symmetry, explicitly broken by spurion in bifundamental.

# BSM Example: 2

Consider the Weinberg operator. Accidentally, lepton number perturbatively conserved in SM. Nothing to conserve it at dim-5:

$$\mathcal{L}_W = \frac{(H \cdot L)(H \cdot L)}{\Lambda}$$

An observation much less considered is that this operator is in a non-minimal irrep of $SU(3)_L$. In fact, it is in the symmetric **6** irrep.

Majorana neutrino masses explicitly break $SU(3)_L$ non-minimally. (Credit: Conversations with Neal Weiner)

# Non-Minimality and Accidents

# Non-minimal Flavour Irreps

Work with Banks, Crawford, Sutherland... Suppose the UV flavour breaking is through a non-minimal spurion? Simple example:

$$\mathcal{Y} \sim \mathbf{6} \times \overline{\mathbf{6}}$$

This irrep transforms as a $\mathcal{Y}^{\alpha\beta}_{ab}$ where upper (lower) indices transform as (anti-) fundamentals.

Whatever happens in the UV, the $(\mathbf{3}, \overline{\mathbf{3}})$ Yukawa can only arise in the IR through at least three insertions:

$$y \sim \mathcal{Y}^2 \overline{\mathcal{Y}} + \ldots \ ,$$

# Non-minimal Flavour Irreps

Consider the entire EFT expansion. In general the full $(\mathbf{3}, \overline{\mathbf{3}})$ Yukawa coupling will be generated by the expansion:

$$y \sim \mathcal{Y}^2\overline{\mathcal{Y}} + \epsilon\left(\mathcal{Y}^4 + \mathcal{Y}\overline{\mathcal{Y}}^3\right) + \epsilon^2\left(\overline{\mathcal{Y}}^5 + \mathcal{Y}^3\overline{\mathcal{Y}}^2\right) + \ldots$$

Consecutive orders can be naturally suppressed as they correspond to higher orders in EFT expansion, captured by $\epsilon$.

Here we are assuming Y is all O(1).

# Non-minimal Flavour Irreps

Consider this example of non-vanishing spurion entries:

| Element | $Q_3$ | $Q_8$ | $q_3$ | $q_8$ |
|---|---|---|---|---|
| $\mathcal{Y}^{11}_{12}$ | 0 | -2 | 2 | 2 |
| $\mathcal{Y}^{23}_{13}$ | -1 | 1 | -1 | -1 |
| $\mathcal{Y}^{13}_{22}$ | 2 | -2 | 1 | -1 |
| $\mathcal{Y}^{23}_{22}$ | 2 | -2 | -1 | -1 |

All considered to be O(1) and charges under the four possible U(1) subgroups are shown.

# Non-minimal Flavour Irreps

For this spurion the EFT expansion:

$$y \sim \mathcal{Y}^2\overline{\mathcal{Y}} + \epsilon\left(\mathcal{Y}^4 + \mathcal{Y}\overline{\mathcal{Y}}^3\right) + \epsilon^2\left(\overline{\mathcal{Y}}^5 + \mathcal{Y}^3\overline{\mathcal{Y}}^2\right) + \ldots$$

Generates a Yukawa of the form:

$$y \approx \begin{pmatrix} \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & 0 \\ 0 & 0 & \mathcal{O}(\epsilon) \\ 0 & \mathcal{O}(\epsilon) & \mathcal{O}(1) \end{pmatrix} + \mathcal{O}(\epsilon^2) \;,$$

With hierarchical eigenvalues:

$$(\mathcal{O}(\epsilon^2), \mathcal{O}(\epsilon), \mathcal{O}(1))$$

Automatically follows from allowed rank and non-Abelian nature.

# Non-minimal Flavour Irreps

Is this just a very complicated FN model? For these non-zero spurion elements:

| Element | $Q_3$ | $Q_8$ | $q_3$ | $q_8$ |
|---|---|---|---|---|
| $\mathcal{Y}^{11}_{12}$ | 0 | -2 | 2 | 2 |
| $\mathcal{Y}^{23}_{13}$ | -1 | 1 | -1 | -1 |
| $\mathcal{Y}^{13}_{22}$ | 2 | -2 | 1 | -1 |
| $\mathcal{Y}^{23}_{22}$ | 2 | -2 | -1 | -1 |

These O(1) entries would have been allowed by Abelian symmetries alone:

| Entry | Combination |
|---|---|
| $y_{21}$ | $\mathcal{Y}^{11}_{12}\mathcal{Y}^{23}_{13}\overline{\mathcal{Y}}^{12}_{11}$, $\mathcal{Y}^{23}_{13}\mathcal{Y}^{23}_{13}\overline{\mathcal{Y}}^{13}_{23}$, $\mathcal{Y}^{23}_{13}\mathcal{Y}^{13}_{22}\overline{\mathcal{Y}}^{22}_{13}$, $\mathcal{Y}^{23}_{13}\mathcal{Y}^{23}_{22}\overline{\mathcal{Y}}^{22}_{23}$ |
| $y_{22}$ | $\mathcal{Y}^{23}_{13}\mathcal{Y}^{13}_{22}\overline{\mathcal{Y}}^{22}_{23}$ |

Would have spoiled hierarchy. True non-Abelian magic. **Not a FN model!**

# Non-minimal Flavour Irreps

There are some outstanding questions.... This spurion

| Element | $Q_3$ | $Q_8$ | $q_3$ | $q_8$ |
|---|---|---|---|---|
| $\mathcal{Y}_{12}^{11}$ | 0 | -2 | 2 | 2 |
| $\mathcal{Y}_{13}^{23}$ | -1 | 1 | -1 | -1 |
| $\mathcal{Y}_{22}^{13}$ | 2 | -2 | 1 | -1 |
| $\mathcal{Y}_{22}^{23}$ | 2 | -2 | -1 | -1 |

Could not arise dynamically as minimum of renormalizable potential. For that either 0, 1, or equal eigenvalues.

Also, top Yukawa enters at $O(Y^3)$. Require UV completion where this is $O(1)$ yet higher powers are suppressed. Possible, but not automatic.

# Phenomenology

Dim-6 SMEFT operators in **8**'s and **27**'s of flavour symmetries, unlike Yukawas, so expect different correlations. Under Abelian subgroups:

$$\left(\bar{Q}\gamma Q\right)^2 : \quad c_{LL} \approx \epsilon^{\frac{|Q_8 - Q_3|}{2}},$$

$$\left(\bar{q}\gamma q\right)^2 : \quad c_{RR} \approx \epsilon^{\frac{|q_8|}{3}},$$

$$\left(\bar{Q}\gamma Q\right)\left(\bar{q}\gamma q\right) : \quad c_{LR} \approx \epsilon^{\frac{1}{6}|3Q_3 - 2q_8 - Q_8| + (Q_3 \bmod 2)}.$$

Whereas for a FN model we have

$$\epsilon^{|aQ_3 + bQ_8 + cq_3 + dq_8|}$$

# Phenomenology

Ultimately this gives rise to a pattern of meson oscillations which is distinct:

| | MFV | AH | FN |
|---|---|---|---|
| | $sd$ | | |
| $c_{LL}$ | $y_c^4 s_{12}^2$ | $\epsilon^2$ | $\dot{\epsilon}^2$ |
| $c_{LR}$ | $y_c^4 s_{12}^2 y_d y_s$ | $\epsilon^1$ | $1$ |
| $c_{RR}$ | $y_c^4 s_{12}^2 y_d^2 y_s^2$ | $1$ | $\dot{\epsilon}^2$ |
| | $bd$ | | |
| $c_{LL}$ | $y_t^4 s_{12}^2 s_{23}^2$ | $\epsilon^2$ | $\dot{\epsilon}^6$ |
| $c_{LR}$ | $y_t^4 s_{12}^2 s_{23}^2 y_d y_b$ | $\epsilon^2$ | $\dot{\epsilon}^2$ |
| $c_{RR}$ | $y_t^4 s_{12}^2 s_{23}^2 y_d^2 y_b^2$ | $\epsilon^2$ | $\dot{\epsilon}^2$ |
| | $bs$ | | |
| $c_{LL}$ | $y_t^4 s_{23}^2$ | $\epsilon^4$ | $\dot{\epsilon}^4$ |
| $c_{LR}$ | $y_t^4 s_{23}^2 y_s y_b$ | $\epsilon^3$ | $\dot{\epsilon}^2$ |
| $c_{RR}$ | $y_t^4 s_{23}^2 y_s^2 y_b^2$ | $\epsilon^2$ | $1$ |

This was for our specific spurion and for a specific FN model.

Unsupressed Kaon mixing in this scenario pushes the flavour scale very high, however alternatives have suppressed mixing.

# Non-Minimality in Power Counting

# EFT and Power Counting

We have good reason to expect lowest orders in EFT momentum expansion to dominate at low energies (Englert, Giudice, Greljo, MM): "Convergence"

$$\mathcal{M}(s) = \int_0^\infty dq^2 \left( \frac{F(q^2)}{s+q^2+i\epsilon} - \frac{F(q^2)}{s-q^2-i\epsilon} \right) + \text{Poly}(s)$$

Consider forward scattering amplitude with Wilson coefficients:

$$b_{n>l/2} = \frac{1}{2n!} \left. \frac{d^{2n}\mathcal{M}(s)}{ds^{2n}} \right|_{s=0} \qquad R_n = M^4 \frac{b_{n+1}}{b_n}$$

From dispersion must satisfy:

$$R_n \leqslant 1$$

# EFT and Power Counting

In the field expansion we have no such constraint. No reason low dimension operators should dominate IR. Thus light fermion masses could be **dominantly** from:

$$\mathcal{O}_{\mathrm{PY}} = c_N \, \frac{|H|^{2N}}{M^{2N}} \, \mathcal{O}_{\mathrm{Yuk}}$$

"Powerful Yukawa": Cohen, MM, Weiner.

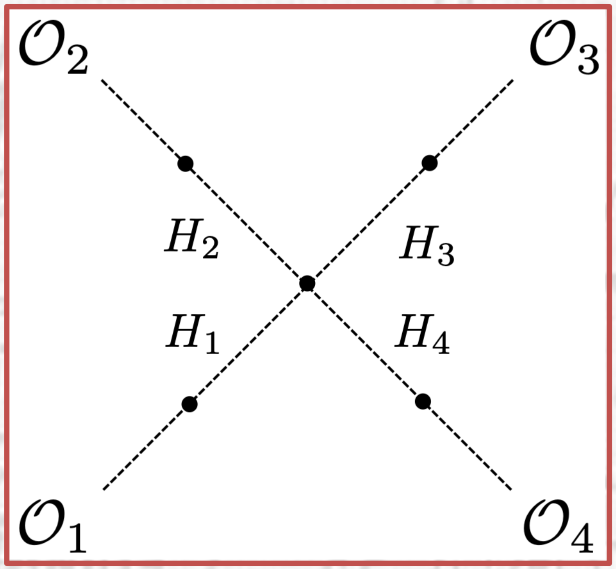
If so, the usual mass-coupling relation would be modified to:

$$y_F^{\mathrm{PY}} = (2N+1) \, \frac{M_F}{v}$$

Consistent with flavour bounds as long as light-fermion philic **or** flavour-blind.

# UV Model

Novel class of "Sprouting Symmetry Breaking" models provide UV-completion:




$$\mathcal{O}_1 = y e^c L \quad ,$$

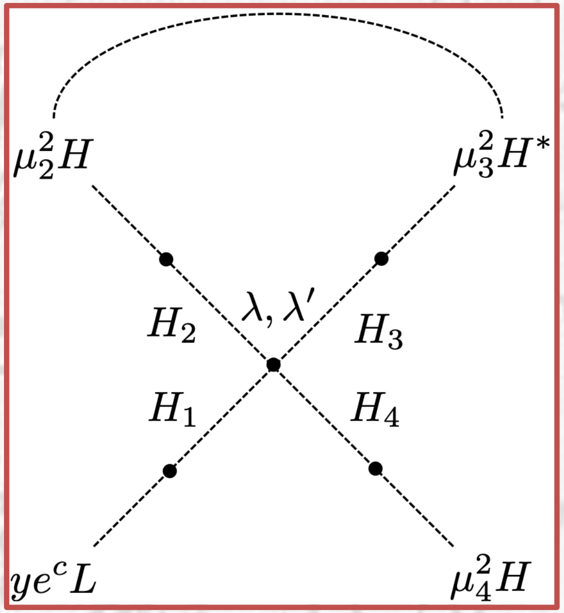
$$\mathcal{O}_{2,4} = \mu^2_{2,4} H \quad ,$$

$$\mathcal{O}_3 = \mu_3^2 H^*$$

Spurion selection rules on the U(1)-symmetry breaking parameters ensure lower-order Yukawa operators not generated at tree-level. Can be iterated to generate higher-dimension operators as leading.

# UV Cutoff

Higher-loop generation of the lower-order operators is calculable (and generic):

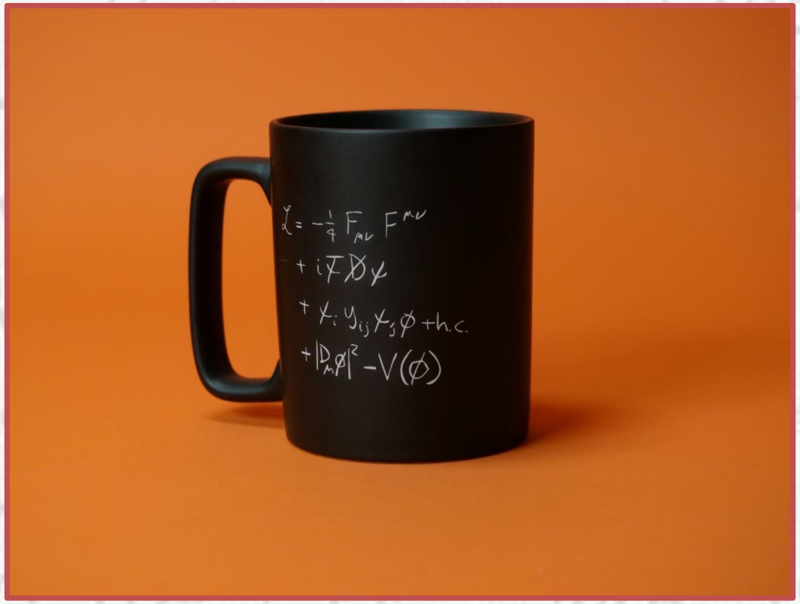



$$\Rightarrow c_N \frac{|H|^{2N-1}}{M^{2N-1}} \mathcal{O}_{\mathrm{Yuk}}$$

For this particular model, N=1, so just generate the usual Yukawa. This will dominate the fermion mass unless the new states are light: M < 2 TeV.

# Powerful Yukawas

Let's not count our chickens.... We cannot yet take this for granted:




It is still possible that e, u, d, s masses do not dominantly originate as in this mug. HL-LHC can test this for strange. In any case would expect new EW states if not.

# Early Universe Cosmology

# Phase Transitions and Non-Minimality

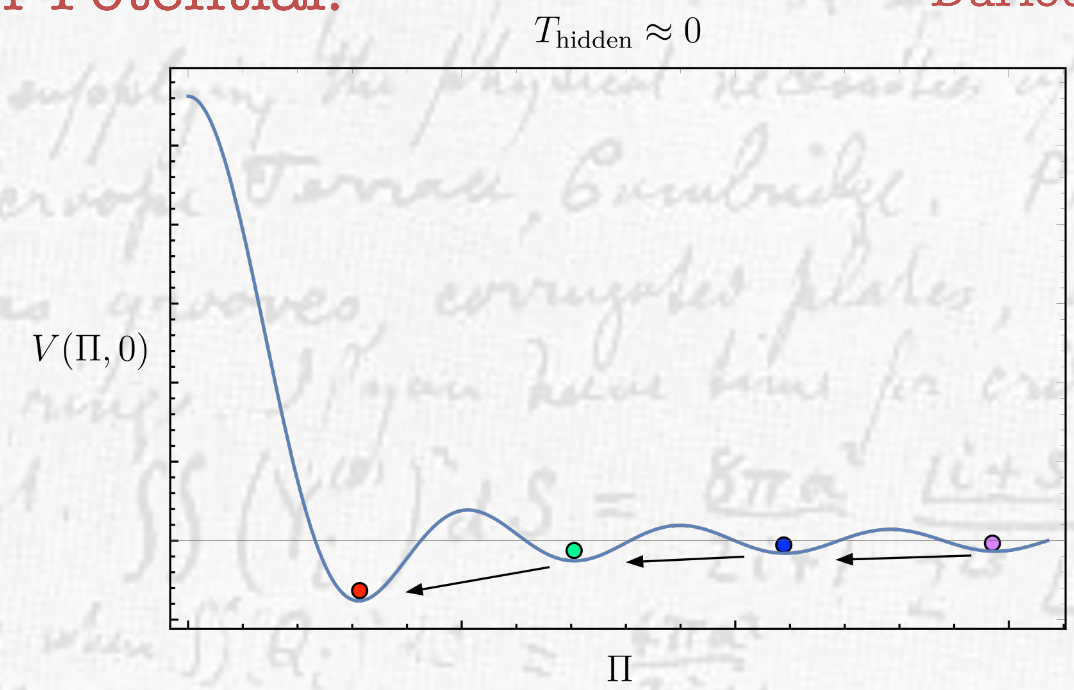
Non-minimality in EFT power counting implies that Higgs potential could have important contributions at higher orders:

$$V = \sum_i c_i \Lambda^4 \left(\frac{|H|}{\Lambda}\right)^{2i}$$

Which generically would impact the nature of the EW phase transition.

# Phase Transitions and Non-Minimality

Pseudo-Nambu Goldstone Bosons arising from spontaneous symmetry breaking with a source of non-minimal irrep explicit symmetry breaking have a Gegenbauer Potential: Durieux, MM, Salvioni.




Which can have highly non-trivial vacuum dynamics and phase transitions. Koutroulis, MM, Merchand, Pokorski, Sakuri.

# Conclusion

Symmetry breaking by minimal irreps may look or sound appealing to us theorists, but are we just drawn to minimality?

Nature likes to break symmetries in non-minimal irreps all the time and isn't that, after all, truly more beautiful?

# Spurion Analysis

## Example 2: The Higgs mass.

The only symmetries that can protect a scalar mass in the absence of additional states are a shift symmetry (non-linearly realized global symm) or conformal symmetry. In

$$\mathcal{L}_{\mathrm{IR}}$$

if the Higgs mass parameter was the only one to break these symmetries, then it could be naturally small, just like the electron Yukawa.

# Spurion Analysis

## Example 2: The Higgs mass.

However, all of its interactions break shift and existence of the UV scale (and RG in IR) break any putative scale symmetry.

Thus there is nothing that can suppress corrections such as

$$\delta m_h^2 \sim \frac{\hbar y_t^2}{(4\pi)^2} M_{UV}^2$$

(Note this cannot be Planck scale, more like string scale...)

# Non-Minimality as a Multiplier

# Non-minimal Flavour Irreps

Banks, Crawford, MM, Sutherland. Suppose the UV flavour breaking is through a non-minimal spurion? Simple example:

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This irrep transforms as a $\mathcal{Y}^{\alpha\beta}_{ab}$ where upper (lower) indices transform as (anti-) fundamentals.

Whatever happens in the UV, the $(\mathbf{3}, \overline{\mathbf{3}})$ Yukawa can only arise in the IR through at least three insertions:

$$y \sim \mathcal{Y}^2 \overline{\mathcal{Y}} + \ldots \ ,$$

# Non-minimal Flavour Irreps

Suppose there is a small hierarchy in some of the elements, such that:

| Element | $\mathcal{Y}^{33}_{33}$ | $\mathcal{Y}^{13}_{13}$ | $\mathcal{Y}^{23}_{22}$ | $\mathcal{Y}^{22}_{23}$ | $\mathcal{Y}^{12}_{12}$ |
|---|---|---|---|---|---|
| Scaling | $\mathcal{O}(\delta)$ | $\mathcal{O}(\delta)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |

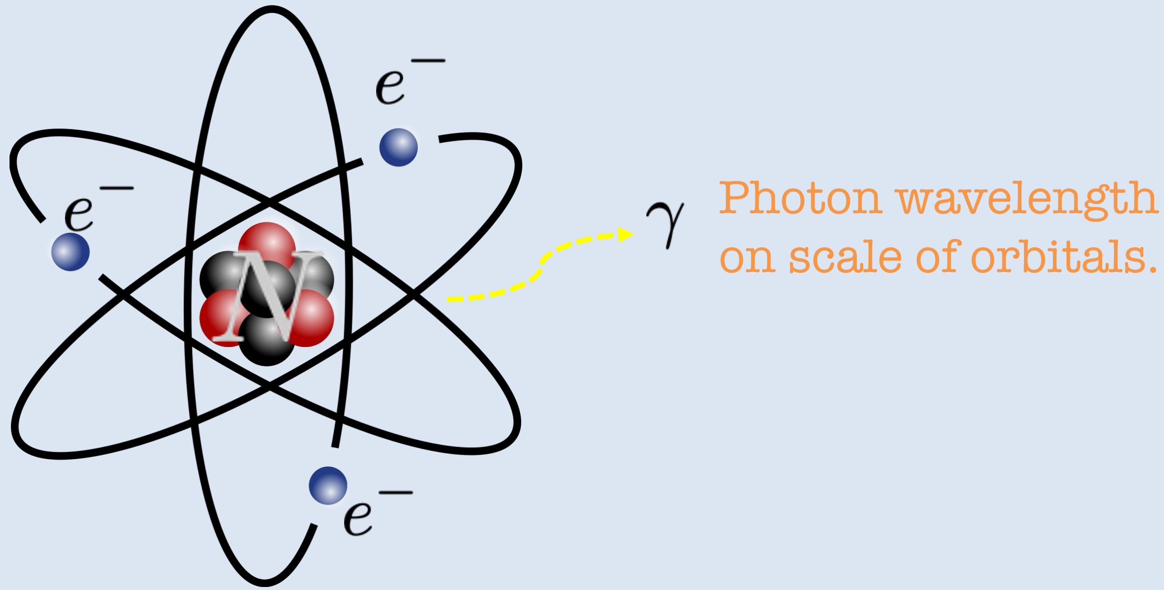
Then the Yukawa coupling is proportional to .

$$y \approx \begin{pmatrix} \mathcal{O}(\delta^2) & 0 & 0 \\ 0 & \mathcal{O}(\delta) & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix}$$

A small hierarchy has been generated as a result of the irrep! Not totally generic though...

# Effective Field Theory Basics

Consider exploring a neutral atom at eV energies:

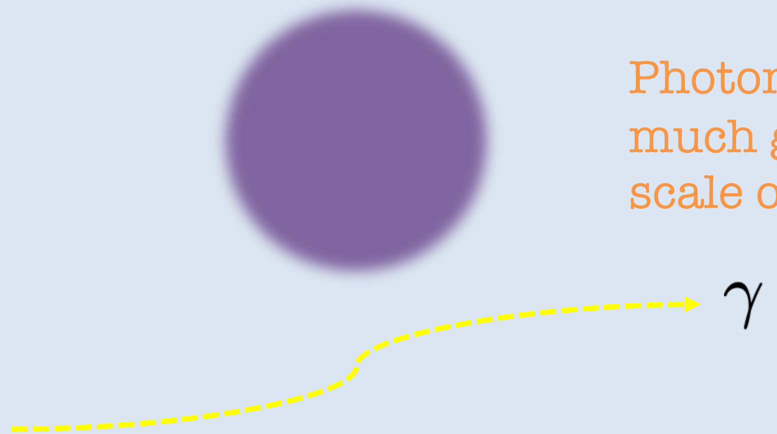



The appropriate theory at this length scale contains the photon, electrons and nucleus:

$$\mathcal{L} = \mathcal{L}(\gamma, e^-, N)$$

# Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:



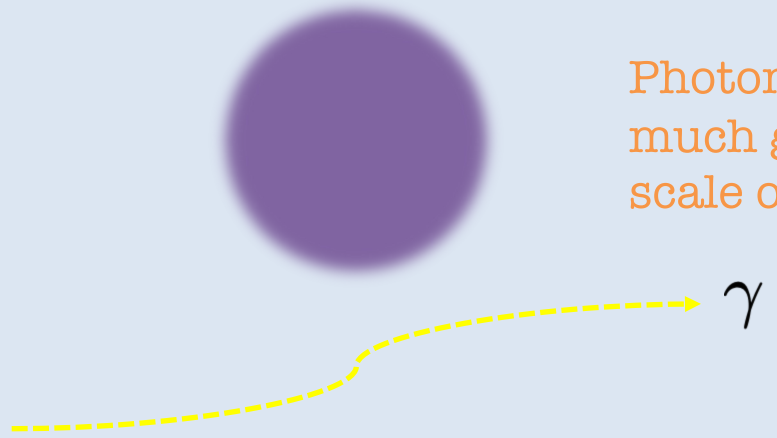

Photon wavelength much greater than scale of orbitals.

The appropriate theory at this length scale contains the photon and neutral atom...

$$\mathcal{L} = \mathcal{L}(\gamma, \chi)$$

# Effective Field Theory Basics

Consider exploring a neutral atom at much lower energies:

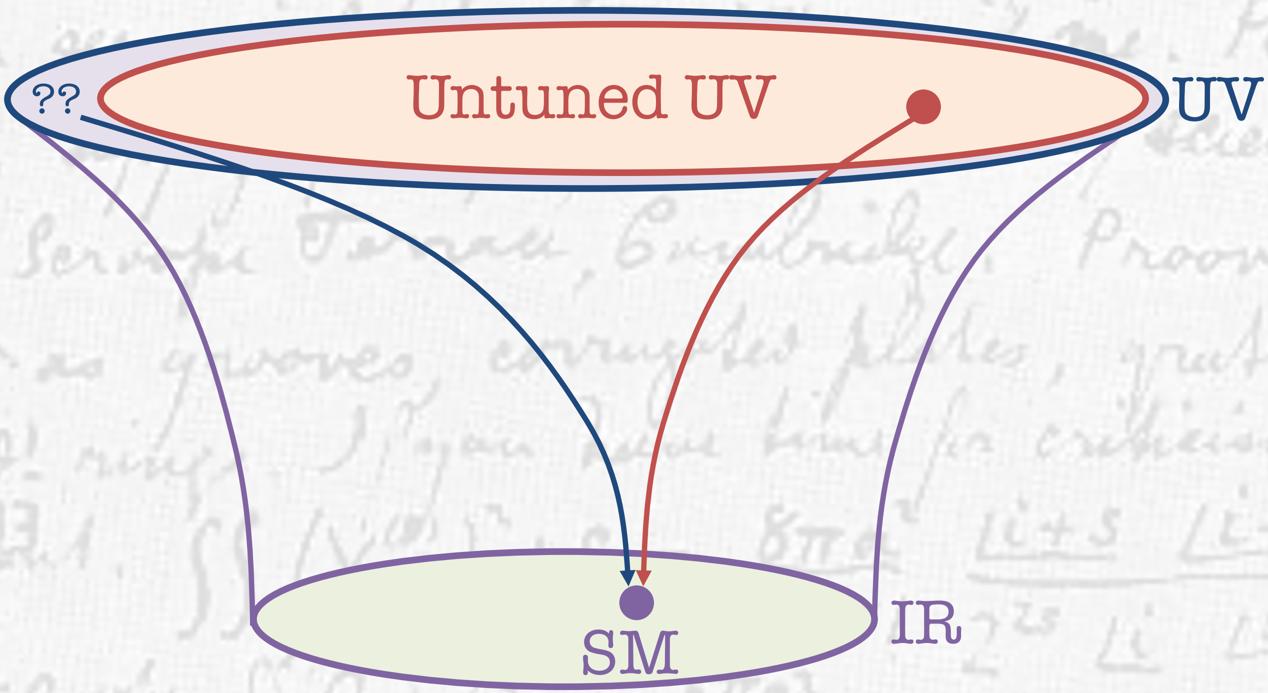



Crucially, the substructure is encoded in "higher dimension operators", like dipoles or Rayleigh…

$$\mathcal{L} = \ldots + \frac{\chi^2}{\Lambda^2} F^{\mu\nu} F_{\mu\nu} + \ldots$$

# And for Particle Physics?

Throughout the development of fundamental science successive layers of EFT have been revealed by more and more powerful microscopes = colliders.




SM cannot describe scattering at ultra-high energy. Must be replaced.

# CKM

CKM can be accommodated as relates to separate

$$\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d$$

rotations. No reason, a priori, that $Y_u$ and $Y_d$ ought to be aligned in some way.

If, on the other hand, we choose different O(1) values, but same texture, for both spurions, get:

$$V_{\mathrm{CKM}} = U_u^L (U_d^L)^\dagger = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon) & \mathcal{O}(1) & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^2) & \mathcal{O}(1) \end{pmatrix},$$

So do find mechanism for hierarchies in CKM.