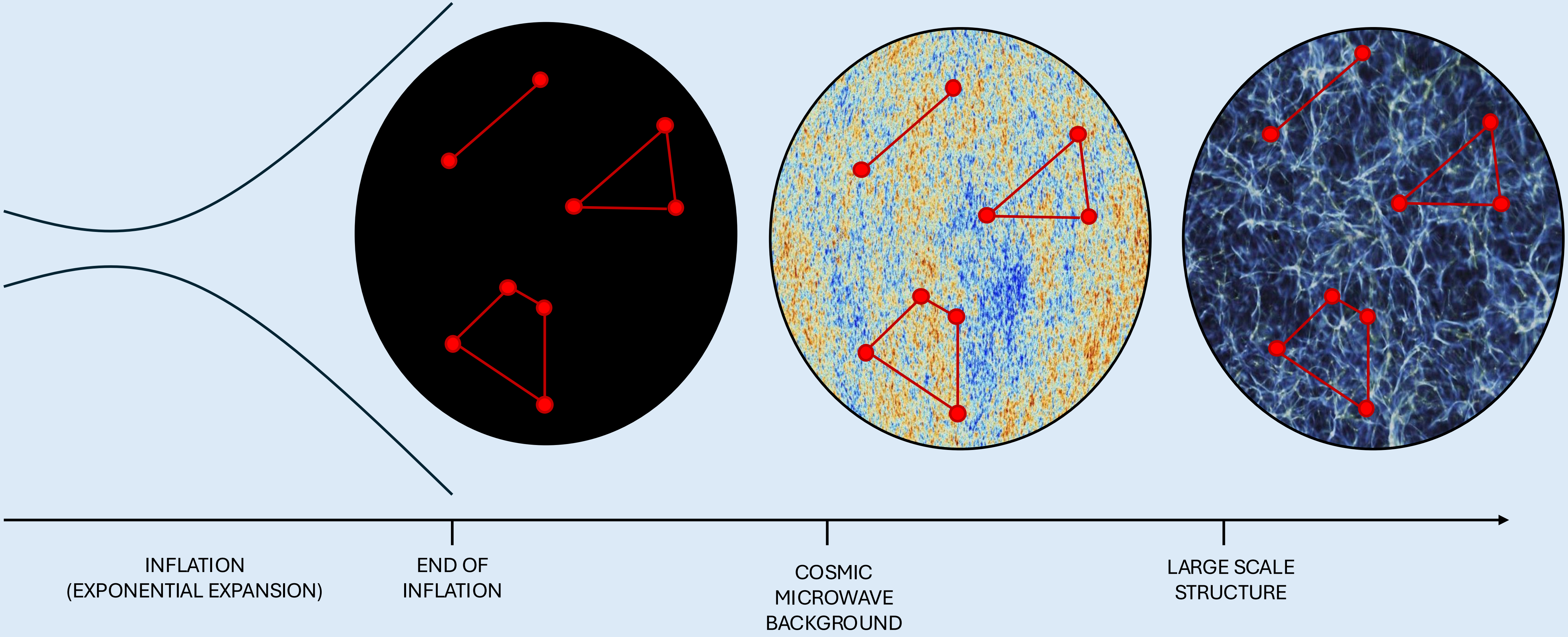
Microcausality and the Cosmological Collider

David Stefanyszyn
University of Nottingham

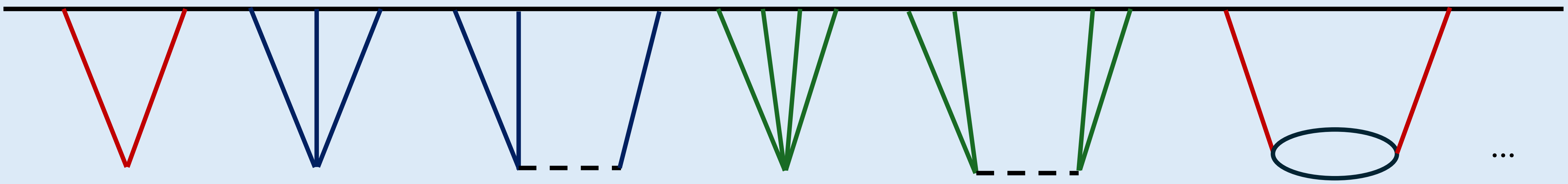
Based on work to appear very soon with:
Trevor Cheung, Carlos Duaso Pueyo and Dong-Gang Wang

In cosmology, we are interested in correlations functions of inflaton and graviton perturbations that are generated during inflation and imprinted on cosmological observables



Cosmological correlators are expectation values of quantum fields (inflaton and graviton perturbations) computed in the Bunch-Davies vacuum and evaluated at the end of inflation

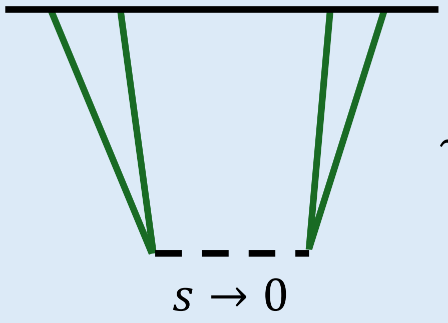
$$\langle \varphi^n \rangle = \langle \Omega | \varphi(\vec{k}_1) \ldots \varphi(\vec{k}_n) | \Omega \rangle$$



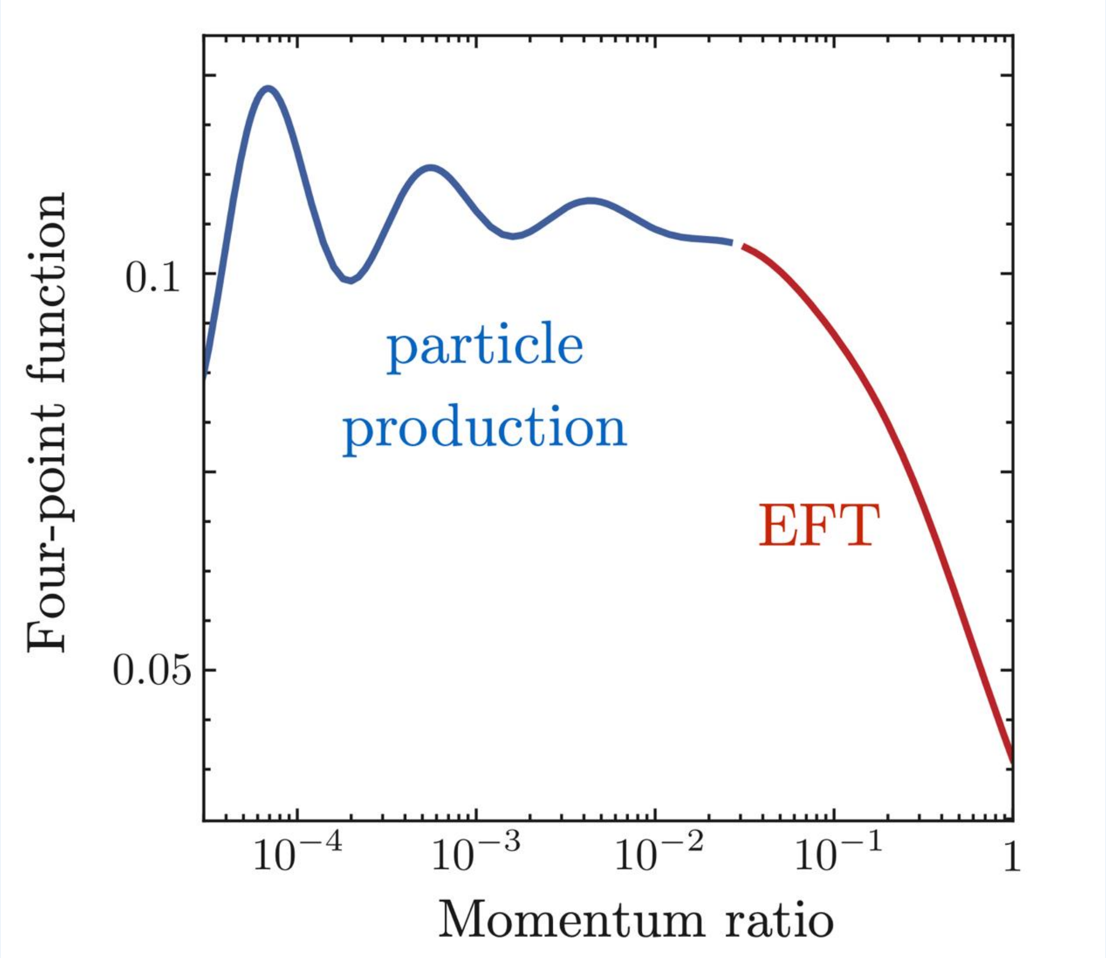
TREES *LOOPS*

In-in formalism, Schwinger-Keldysh path integral, wavefunction of the universe

In general, inflaton/graviton self-interactions give rise to rational functions of the external momenta, whereas exchanges of new heavy fields introduce interesting non-analyticities and oscillations



$$\sim (spin - dependent)\left(\frac{s^2}{k_L k_R}\right)^{i\,\mu}$$



https://github.com/ddbaumann/cosmo-correlators

The signal is a consequence of particle production and so is Boltzman suppressed for m > H

Ways to describe spinning fields in cosmology:

1. Covariant description that on de Sitter space adheres to all maximally symmetric symmetries. For spin-1 this is the massive Proca theory.

2. Describe fluctuations on FRW as irreducible representations of the group of rotations.

3. Use rules of the Effective Field Theory of Inflation and write down the most general quadratic action treating temporal and spatial indices separately.

$$\mathcal{L} = \mathcal{L}(A_\mu)$$

Pimentel and Wang, arXiv:2205.00013

$$\mathcal{L} = \mathcal{L}(A_i)$$

Bordin et al., arXiv:1806.10587

$$\mathcal{L} = \mathcal{L}(A_\mu, A_0)$$

This work

Upon integrating out the temporal mode, non-localities can arise. For example, for massive Proca we have:

Covariant theory: $$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} {F_{\mu\nu}}^2 - \frac{1}{2} m^2 {A_\mu}^2 \right)$$

Constraint equation: $$\nabla^\mu A_\mu = 0 \quad \Longrightarrow \quad A_0 = \frac{k}{k^2 + m^2 a^2(\eta)} \frac{\partial A_L}{\partial \eta} \qquad a(\eta) = -\frac{1}{H\eta}$$

Action for the dynamical modes: $$S \supset \frac{1}{2} \int d\,\eta \int d^3k \left(\frac{m^2 a^2(\eta)}{k^2 + m^2 a^2(\eta)} \left(\frac{\partial A_L}{\partial \eta} \right)^2 - m^2 a^2(\eta) {A_L}^2 \right)$$

The allowed non–localities are those compatible with microcausality

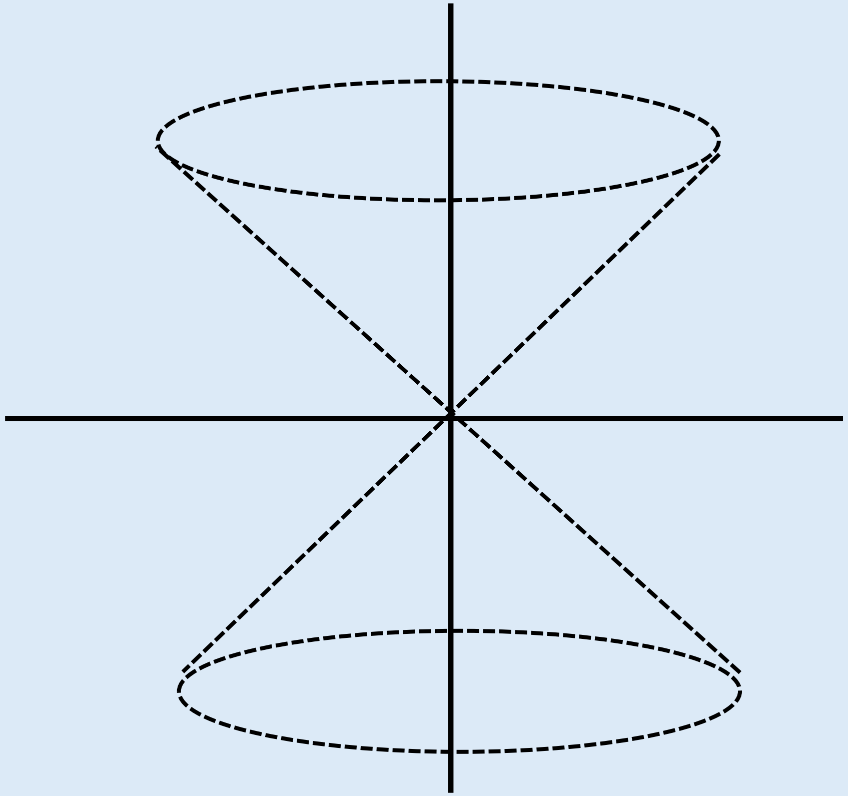
Consider a scalar field coupled to an external source with equation of motion

$$\mathcal{O}(\eta, \boldsymbol{x})\phi(\eta, \boldsymbol{x}) \;=\; J(\eta, \boldsymbol{x}) \qquad \Longrightarrow \quad \phi(\eta, \boldsymbol{x}) = \phi_0(\eta, \boldsymbol{x}) \;+\; \int d\eta' \, d^3\boldsymbol{x}' \, G(\eta, \eta', \boldsymbol{x} - \boldsymbol{x}') J(\eta', \boldsymbol{x}')$$

The relevant Green's function for the response to a source is the retarded Green's function satisfying $G(\eta, \eta', \boldsymbol{x} - \boldsymbol{x}') \;=\; 0, \quad \text{for} \quad \eta < \eta'.$

In a theory obeying relativistic causality, the Green's function also vanishes if the field and the source are spacelike separated

$$G(\eta, \eta', \boldsymbol{x} - \boldsymbol{x}') \;=\; 0, \quad \text{for} \quad |\eta - \eta'| \;<\; |\boldsymbol{x} - \boldsymbol{x}'|.$$



$$\boxed{[\phi(\eta, \boldsymbol{x}) \, , \phi(\eta', \boldsymbol{x}') \,] = \; 0, \quad \text{for} \quad |\eta - \eta'| \;<\; |\boldsymbol{x} - \boldsymbol{x}'|.}$$

The mixed time-momentum representation of the Green's function is defined by

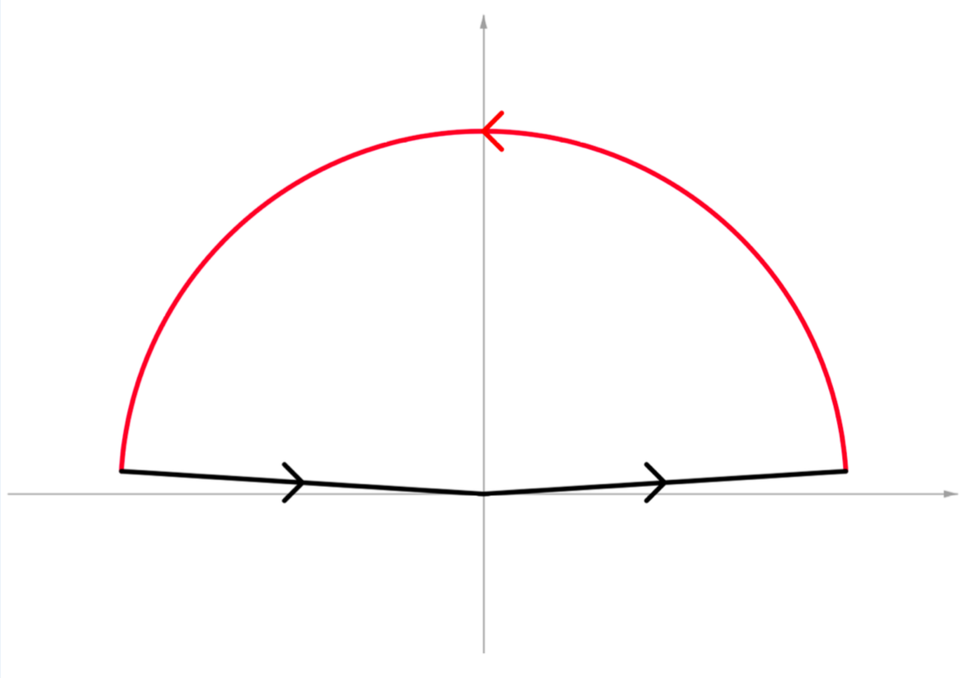
$$G(\eta, \eta', \boldsymbol{x} - \boldsymbol{x}') = \int d^3\boldsymbol{k}\, G(\eta, \eta', \boldsymbol{k})\, e^{i\boldsymbol{k}.(\boldsymbol{x}-\boldsymbol{x}')}$$

The Paley-Weiner theorem then implies that the necessary and sufficient conditions for the Green's function to vanish for spacelike separations are (L. Hui et al., arXiv:2502.04215):

1. The Green's function is an entire function in the three-momentum i.e. it is analytic everywhere in the complex plane of $\boldsymbol{k}$.

2. The Green's function is exponentially bounded by

$$G(\eta, \eta', \boldsymbol{k}) \leq e^{|\mathrm{Im}(\boldsymbol{k})|.|\eta-\eta'|}$$



For a massive spin-1 field propagating during inflation we have

$$G_{ij}(\eta,\eta',\boldsymbol{k}) = G_L(\eta,\eta',k)\frac{k_i k_j}{k^2} + G_T(\eta,\eta',k)\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)$$

Microcausality therefore requires:

1. $G_T(\eta,\eta',k)$ and $G_{\mathrm{L}}(\eta,\eta',k)$ are analytic functions of k^2

2. $G_T(\eta,\eta',k)$ and $G_{\mathrm{L}}(\eta,\eta',k)$ tend to the same limit as $k \to 0$

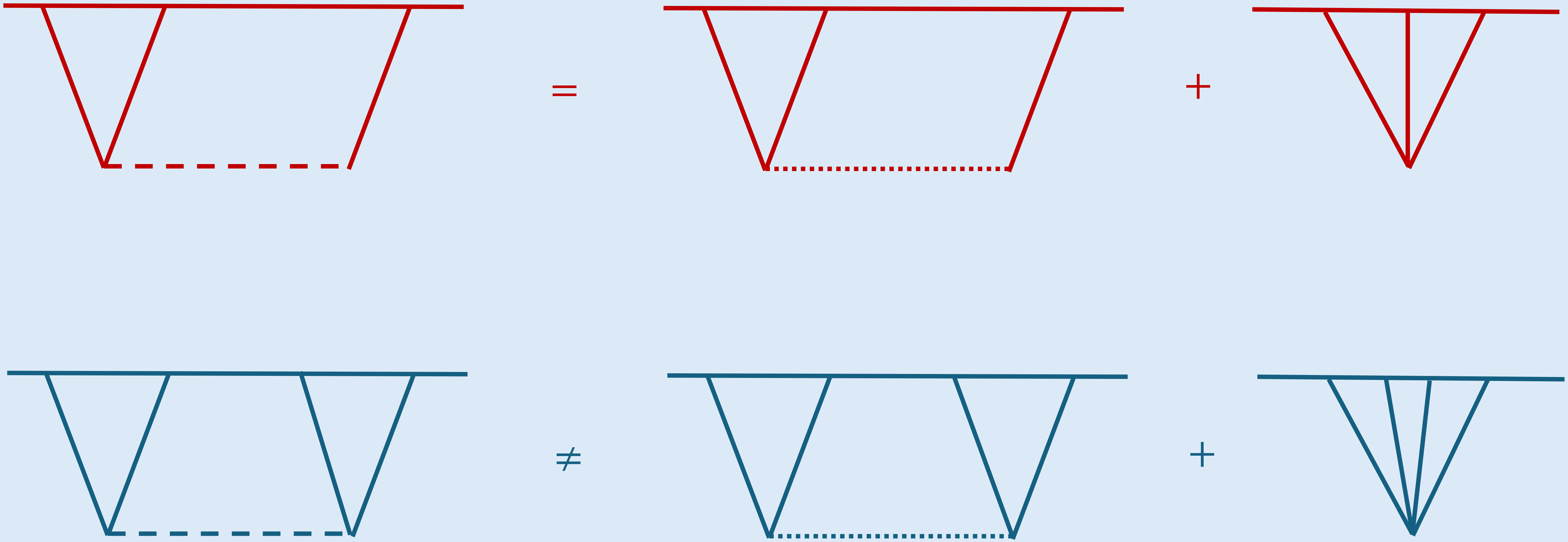
Given a Lagrangian, we can integrate out the non-dynamical mode, compute the Green's functions for the longitudinal and transverse modes, and constrain the theory by demanding conditions 1 and 2 are satisfied.

This is what we found:

Kinetic terms:	${F_{\mu v}}^2$	${F_{0\mu}}^2$	Microcausality forces gauge invariance
Mass terms:	${A_\mu}^2$	${A_0}^2$	Symmetry-breaking mass terms allowed
Mixing term:	$A_0 \nabla^\mu A_\mu$		New inflationary term
Parity-odd term:	$\epsilon^{0\mu v\rho} A_\mu \nabla_v A_\rho$		Unique parity-odd term

The exponentially boundedness then requires all modes to propagate subluminally

New spin-1 vs massive Proca vs FRW spin-1:



Thank you!

For the massive Proca theory in de Sitter space the mode functions are

$$A_L(\eta,k) \;=\; \frac{\sqrt{\pi}H}{4m}\, e^{i\pi(i\mu+1/2)/2}(-\eta)^{\frac{1}{2}}\Big[k\eta\left(H^{(1)}_{i\mu+1}(-k\eta)-H^{(1)}_{i\mu-1}(-k\eta)\right)-H^{(1)}_{i\mu}(-k\eta)\Big]$$

$$A_{\mathrm{T}}(\eta,k) \;=\; \frac{\sqrt{\pi}}{2}\, e^{i\pi(i\mu+1/2)/2}(-\eta)^{\frac{1}{2}}H^{(1)}_{i\mu}(-k\eta) \qquad\qquad \mu \;=\; \sqrt{\frac{m^2}{H^2}-\frac{1}{4}}$$

These mode functions have a branch point at the origin and give rise to the oscillations. These non-analyticities cancel in the Green's function:

$$G_T(\eta,\eta',k) = \frac{i\,\pi\,(\eta\eta')^{1/2}}{2\,sinh(\pi\mu)}\left[\left(\frac{\eta}{\eta'}\right)^{i\mu} C_{i\mu}\left(-\frac{k^2\eta^2}{4}\right)C_{-i\mu}\left(-\frac{k^2\eta^2}{4}\right)\;-(\eta\;\leftrightarrow\;\eta')\right]$$

Where C is the Bessel-Clifford function that is an entire function. G_{L} can be expressed in a similar manner so condition 1 is satisfied.