Laplace space for cosmological correlators

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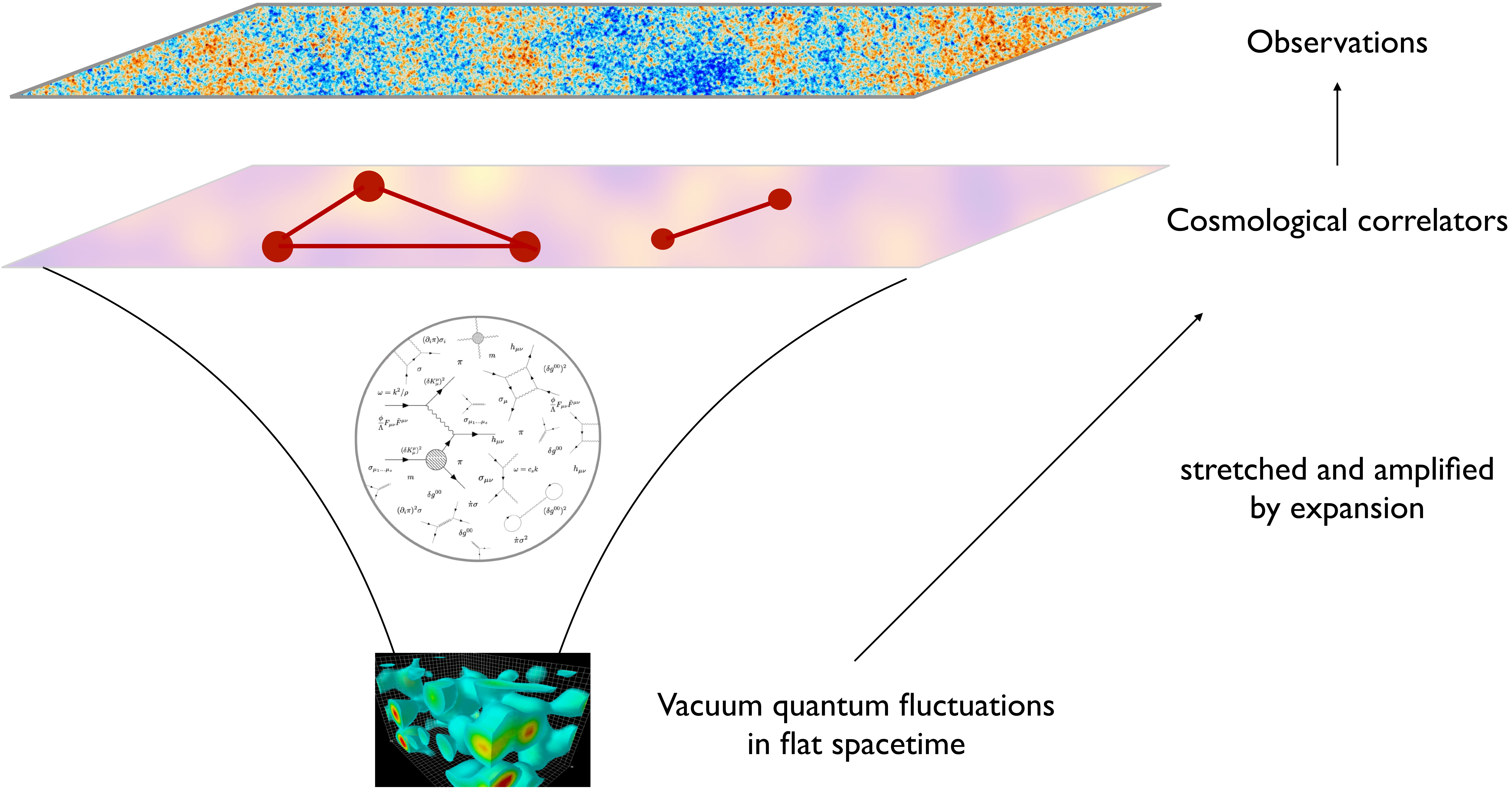
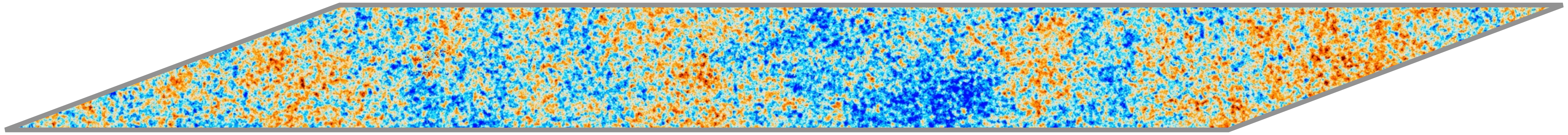
Belrhali, Poisson, RP 2606.27309, 2606.27311, 2608.23243

Inflation

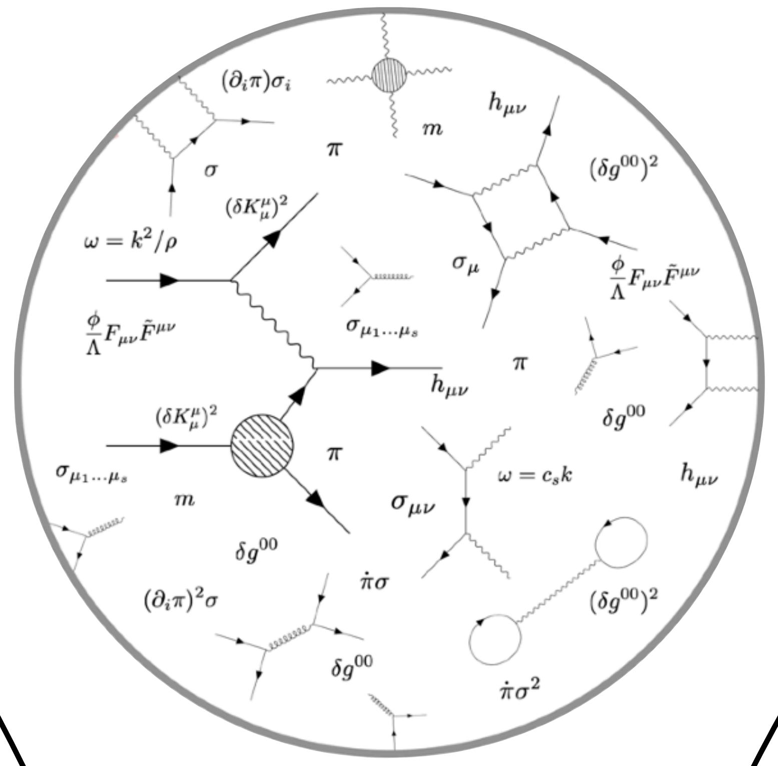

Observations
Cosmological correlators
stretched and amplified
by expansion
Vacuum quantum fluctuations
in flat spacetime

Inflation



Observations

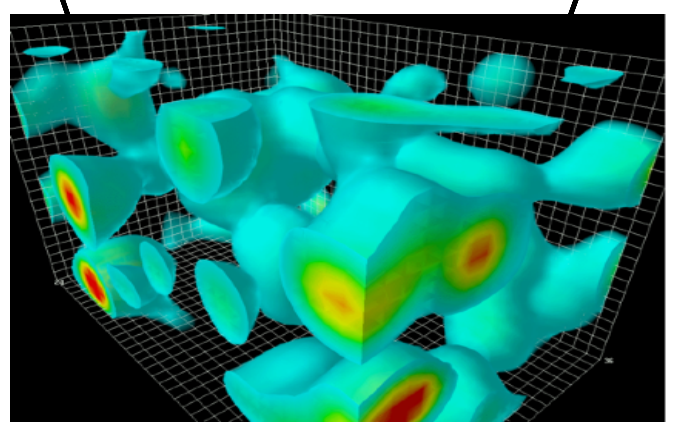
Cosmological correlators



Breaking of time-translation invariance

core mechanism: particle production

core difficulty: no plane wave like amplitudes



Vacuum quantum fluctuations
in flat spacetime

Fourier-Laplace

Space:

Fourier

$$f(\vec{x}) \sim \int \mathrm{d}\vec{k}\, e^{i\vec{k}\cdot\vec{x}} f(\vec{k})$$

Translation invariance makes it optimal
but not required

Time:

Laplace!

$$g(z = k\tau) \sim \int_1^\infty \mathrm{d}\lambda\, e^{-i\lambda z} \tilde{g}(\lambda)$$

'Fourier, but for conformal-time half-line'

Fourier-Laplace

$$g(z = k\tau) \sim \int_1^\infty \mathrm{d}\lambda\, e^{-i\lambda z} \tilde{g}(\lambda)$$

plane wave superposition, with Laplace kernel encoding:

spacetime geometry, field content, dynamics

What do we gain?

1. Universal structure, any model
2. Conceptual insight: cosmology from flat space
3. Computational power

Outline

I. Context and motivation

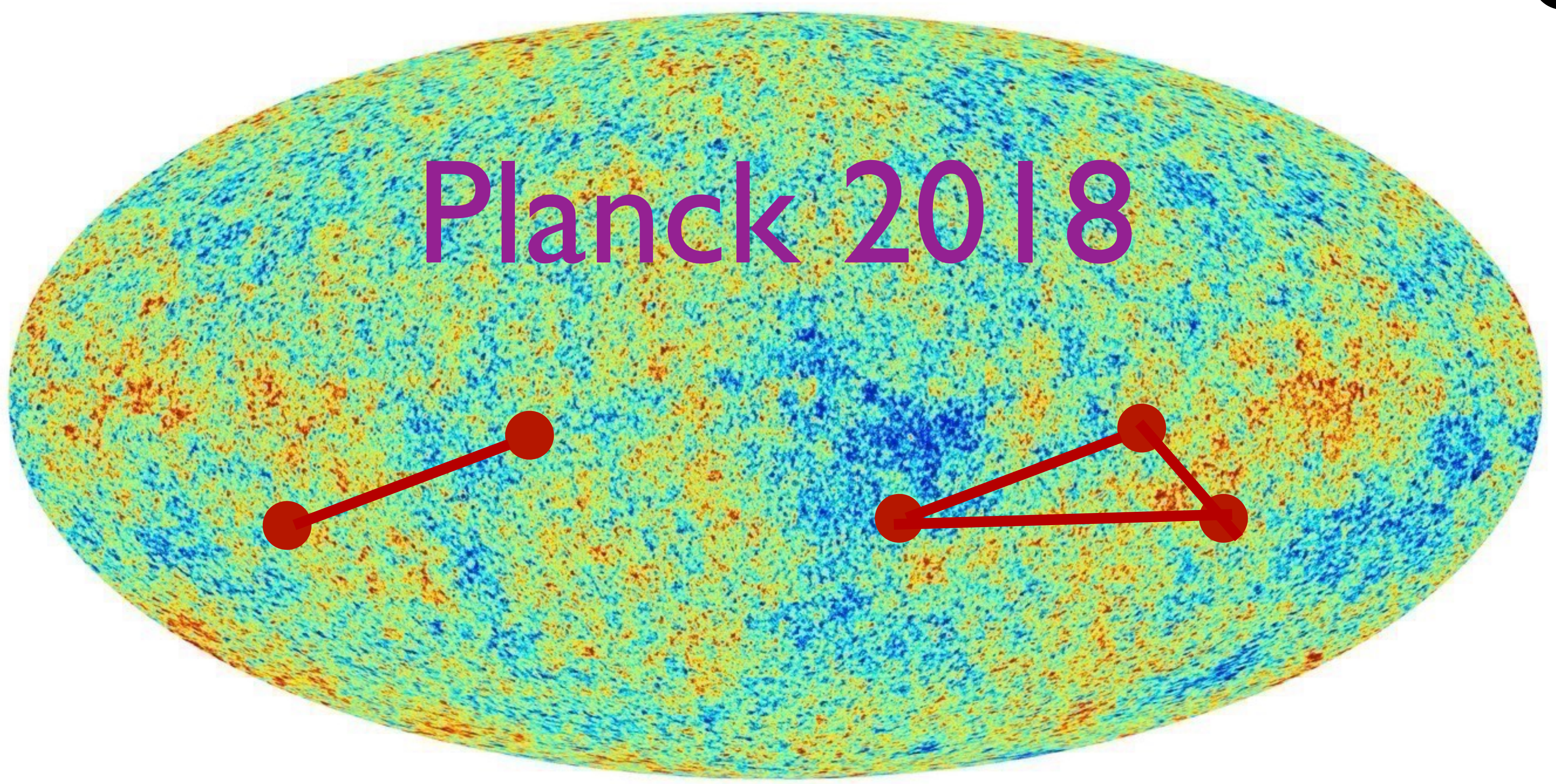
II. Core mechanism

III. Weak mixing

IV. Strong mixing

I. Context and motivation

Context



Most economical explanation: single-field slow-roll

HEP perspective: at best emergent approximate description

New physics? non-Gaussianities / cosmological correlators

Cosmology

Particle physics

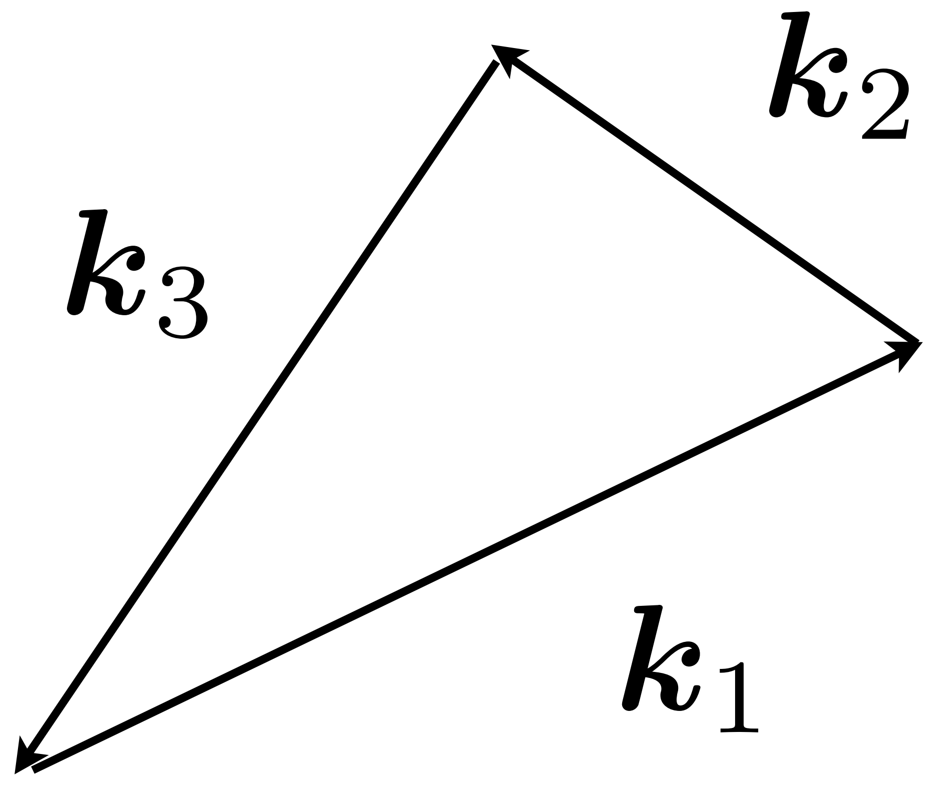
Goal:

Identify degrees of freedom, mass, dispersion relation, spin, interactions

Establish a standard model of inflation

3-point function

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle \sim \delta^{(3)}\left(\sum \boldsymbol{k}_i\right) \mathcal{P}_\zeta^2 \, \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2}$$



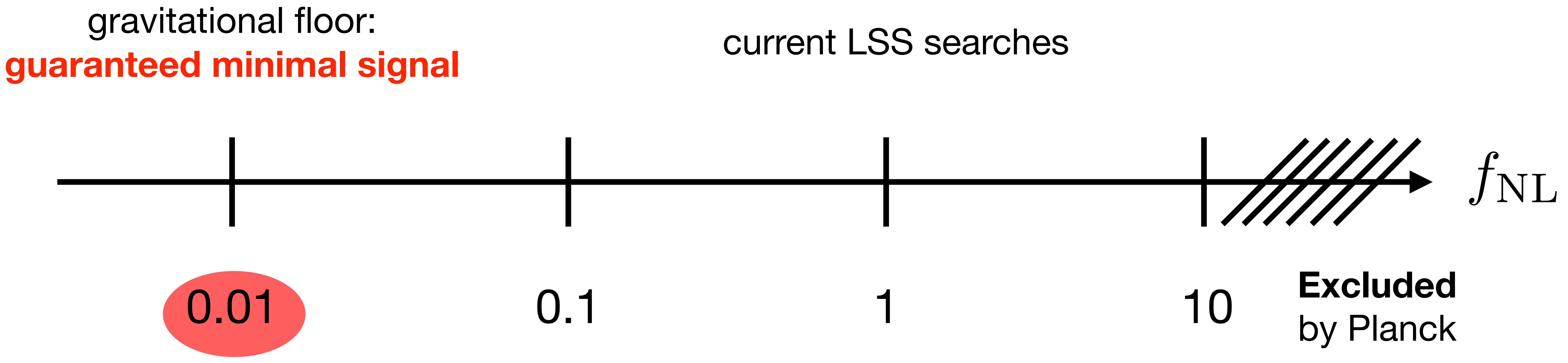
ζ primordial density fluctuations

Amplitude $S \sim f_{\rm NL}$

Scale-dependence (overall size)

Shape dependence (configuration of triangles)

Observational perspective

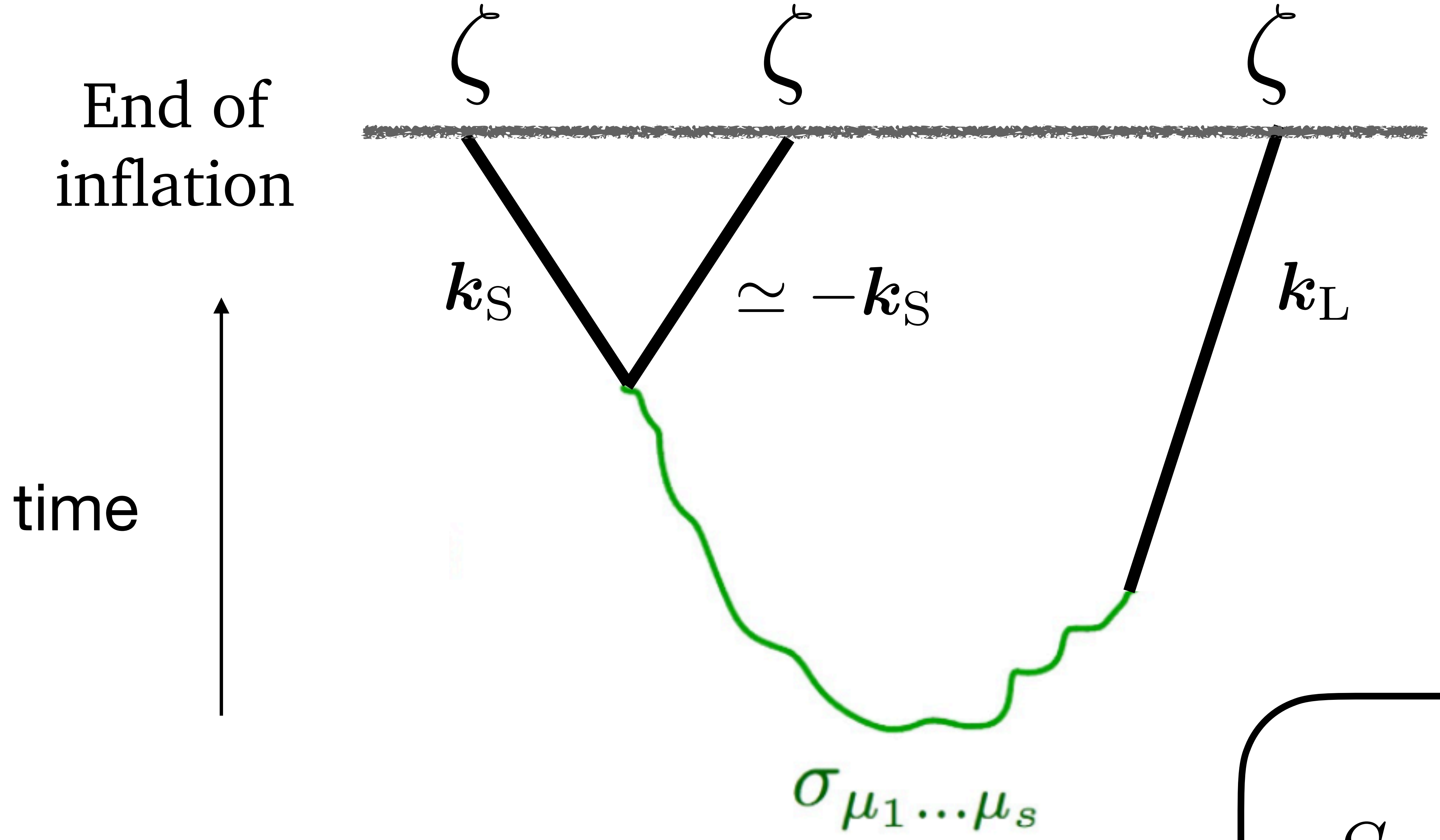


Clear science case

non-minimal level of NG	$\longrightarrow$	new physics
minimal level of NG	$\longrightarrow$	milestone discovery
no detection	$\longrightarrow$	inflation is ruled out

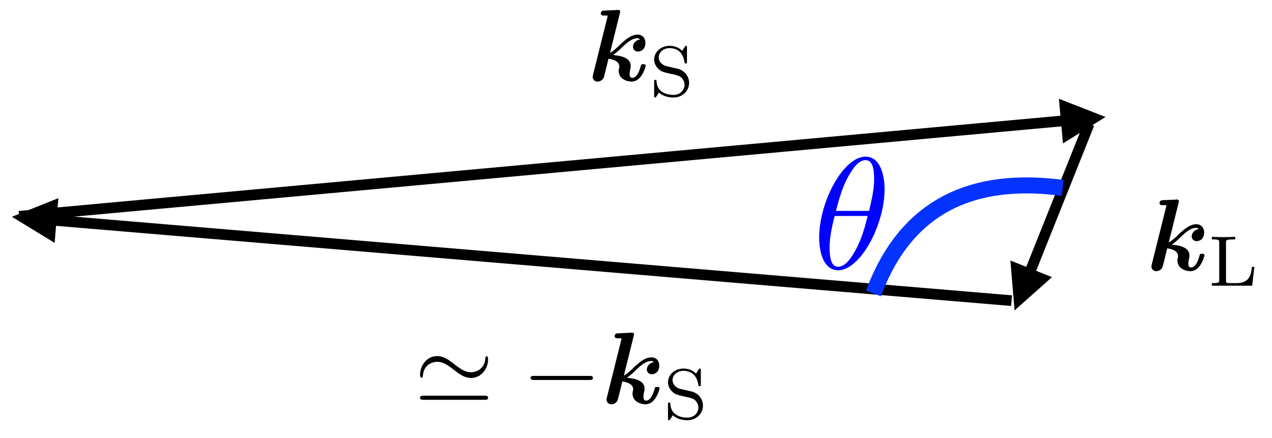
Cosmological collider

see talks from
David Stefanyszyn & Xi Tong



All particles are spontaneously produced by expansion of universe

and leave imprints in cosmological correlators



$$\kappa = k_{\mathrm{L}}/k_{\mathrm{S}} \ll 1$$

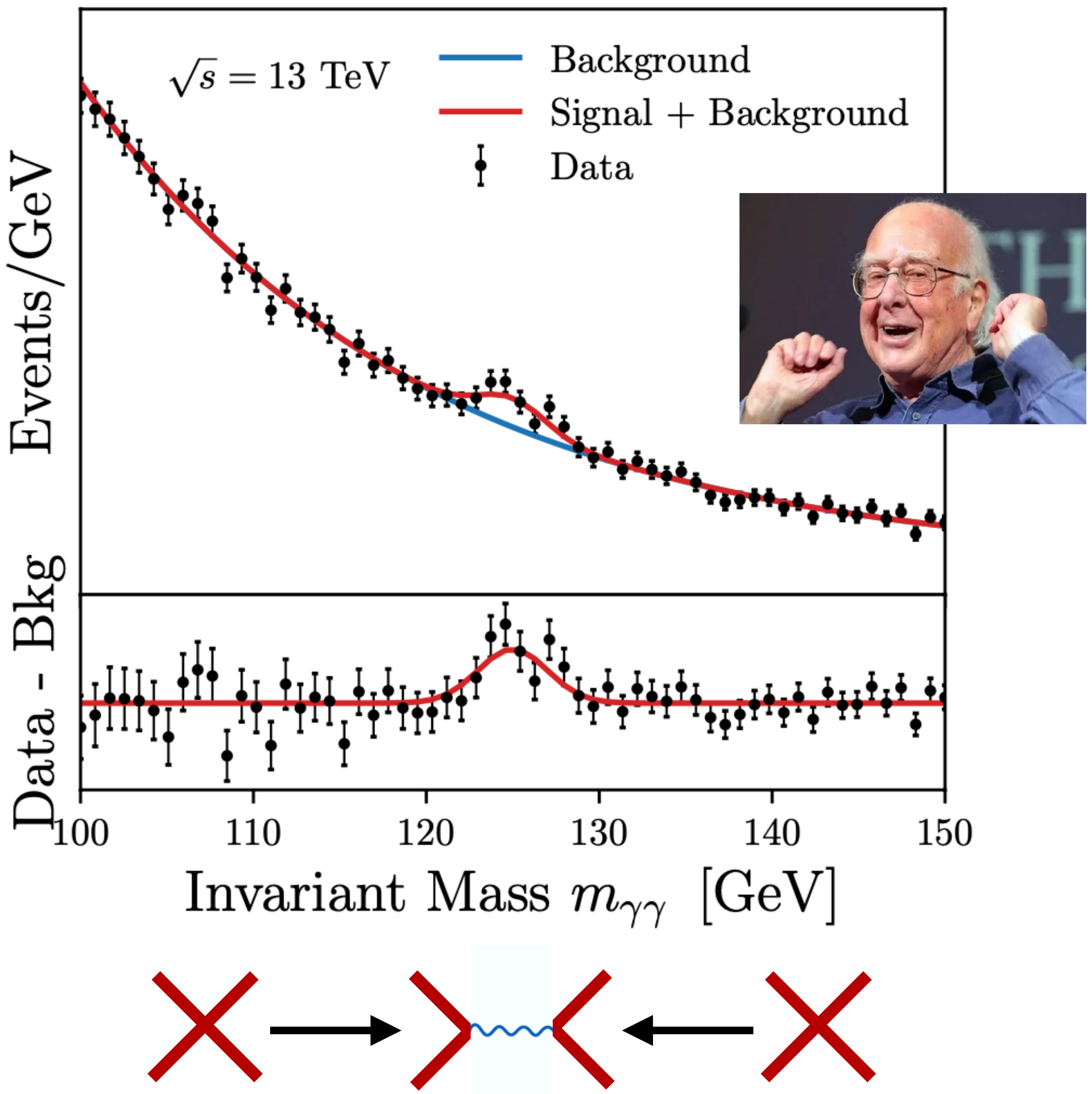
$$S \propto \kappa^{1/2} e^{-\pi m/H} \cos\left(\frac{m}{H}\log(\kappa) + \varphi\right) P_S(\cos\theta)$$

Mass & Spin

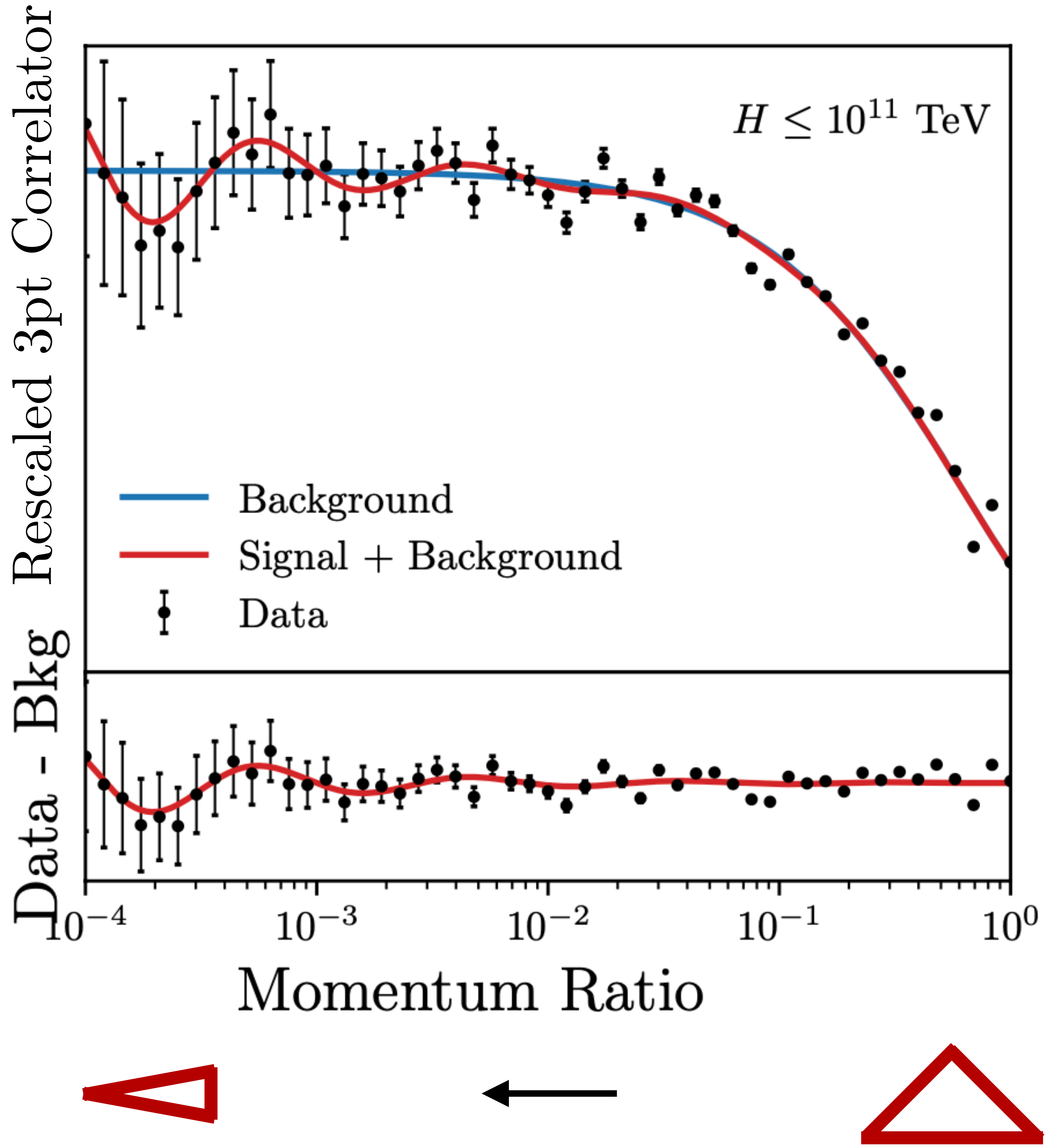
of heavy exchanged particle

Cosmological collider

Particle physics



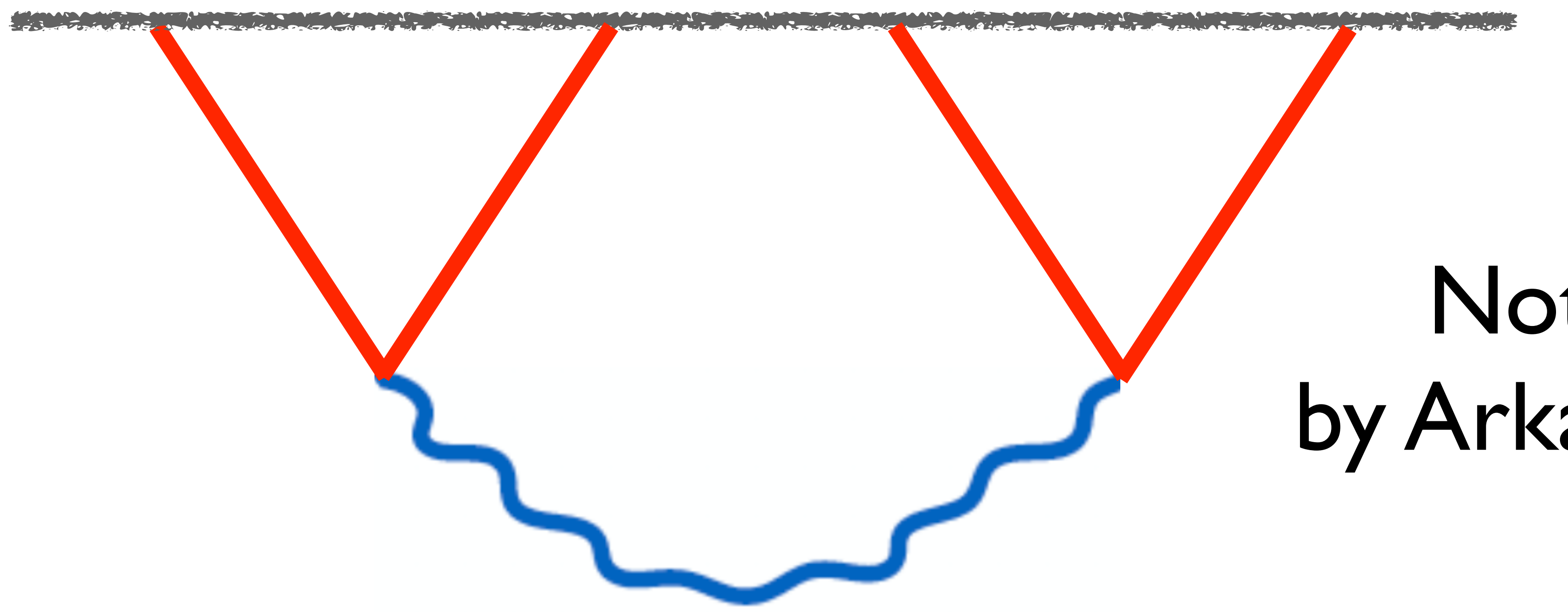
Cosmology



Credit: Denis Werth

Cosmological collider

Possibility to detect particles billion times more massive than with terrestrial colliders



Not until **2018**
by Arkani-Hamed et al

Cosmological correlators much harder than flat space amplitudes

~~Lorentz invariance~~

Intense activity

Theory

Bootstrap (dS and boostless)

Mellin-Barnes

Kinematic flow

Kontorovich-Lebedev-Fourier

Cosmological flow

…

Pheno

Dispersion relation

Multi-field collider

Spinning

4-pt

Loops

…

Data

CMB-BEST

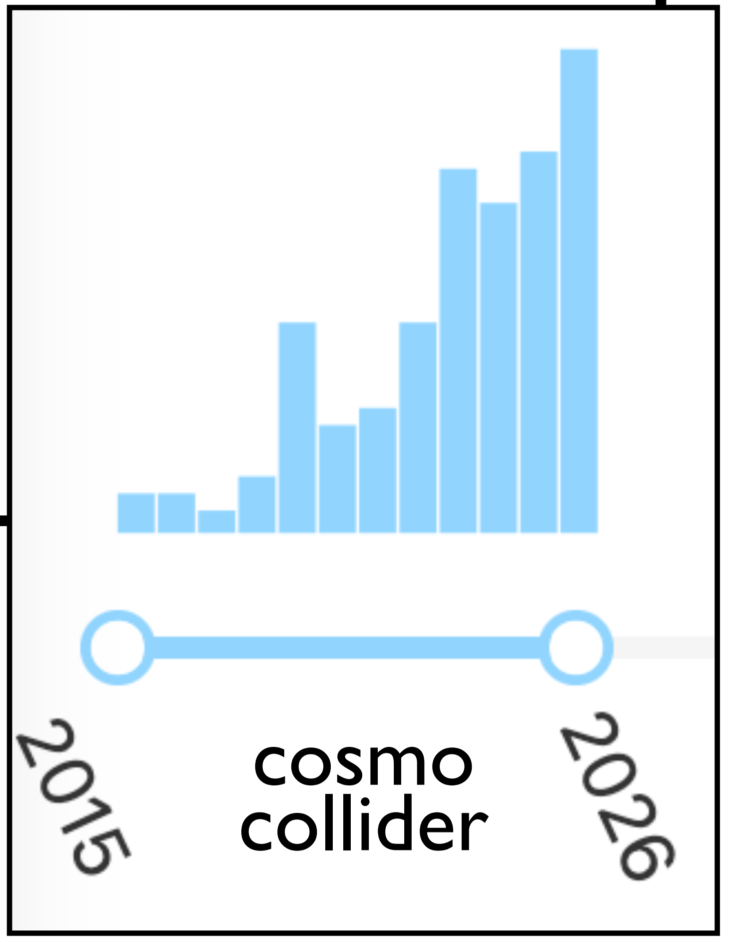
PolySpec

Planck, BOSS

Bias

Lensing

…



Lots of **progress**, but still many **limitations**:

mostly tree-level, some loops, mostly scale-invariant and weak mixing

II. Core mechanism

Laplace transform: recap

time domain $\longrightarrow$ complex frequency domain

$$f(t) \longrightarrow \mathcal{L}[f](s) = \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$$

inversion

$$f(t) = \frac{1}{2i\pi} \int_{c-i\infty}^{c+i\infty} e^{st} \mathcal{L}[f](s)\,\mathrm{d}s$$

Daily use in engineering: linear differential equations, keeping boundary conditions

Surprisingly **underused in cosmology**

Laplace approach for inflation

$$\hat{\mathcal{F}}(\lambda) = \int_{-\infty(1+i\epsilon)}^{0} \mathrm{d}z\, e^{-i\lambda z}\mathcal{F}(z) \longrightarrow \hat{\mathcal{F}}(\lambda) = i\int_0^\infty \mathrm{d}z\, e^{-\lambda z}\mathcal{F}(-iz)$$

mode function

Wick rotation

Laplace transform

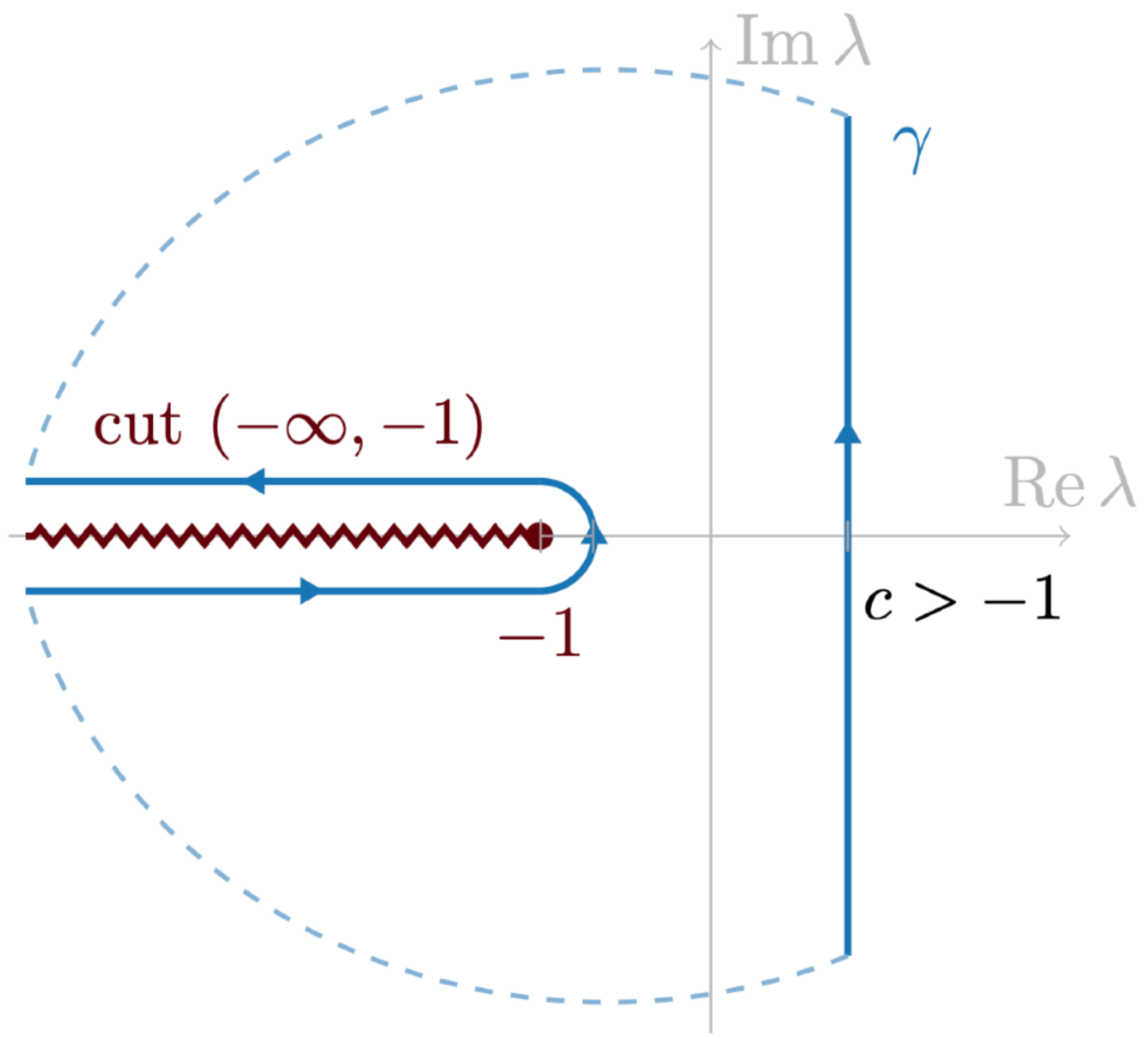
inversion

$$i\mathcal{F}(-iz) = \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}\lambda}{2i\pi} e^{\lambda z}\hat{\mathcal{F}}(\lambda)$$

Analytic structure inherited from Bunch-Davies:

$$\mathcal{F}(z) \underset{z\to-\infty}{\sim} e^{-iz} \longrightarrow$$ branch point at $\lambda = -1$

Laplace approach for inflation



$$\mathcal{F}(z) = \frac{1}{2\pi}\int_1^\infty \mathrm{d}\lambda\, e^{-i\lambda z}\left[\mathrm{Disc}_{\lambda'}\hat{\mathcal{F}}(\lambda')\right]_{\lambda'=-\lambda}$$

Dual Bunch-Davies point plane wave Laplace-space kernel

Easy mapping between bulk dynamics and Laplace-space dynamics

$$\mathcal{L}[z\,\psi](\lambda) = -\partial_\lambda \mathcal{L}[\psi](\lambda) \qquad \mathcal{L}[\psi'](\lambda) = \lambda\,\mathcal{L}[\psi](\lambda) - \psi(0)$$

Laplace approach for inflation

$$\mathcal{F}(z) = \frac{1}{2\pi}\int_{1}^{\infty} \mathrm{d}\lambda\, e^{-i\lambda z} \left[\mathrm{Disc}_{\lambda'}\hat{\mathcal{F}}(\lambda')\right]_{\lambda'=-\lambda}$$

Dual Bunch-Davies point   plane wave   Laplace-space kernel

Flat-space $e^{i(\vec{k}\cdot\vec{x}-\lambda k\tau)}$

$\longrightarrow$

$$m_\lambda^2 = \omega^2 - k^2 = k^2(\lambda^2 - 1)$$

Cosmological mode: superposition of flat-space plane waves, with masses m_λ

- Support $\lambda \geq 1$: <u>spectral condition</u>
- Massless <u>threshold</u> $\lambda = 1$ controls <u>early times and singularities</u>
- <u>Tail</u> $\lambda \to \infty$ controls <u>late times and particle production</u>

III. Weak mixing

Massive scalar field in de Sitter

	time domain	Laplace frequency domain
Dynamics	$z^2\,\mathcal{F}'' + 2z\,\mathcal{F}' + \left(z^2 + \mu^2 + \frac{1}{4}\right)\mathcal{F} = 0$	$(\lambda^2 - 1)\,\hat{\mathcal{F}}''(\lambda) + 2\lambda\,\hat{\mathcal{F}}'(\lambda) + \left(\mu^2 + \frac{1}{4}\right)\hat{\mathcal{F}}(\lambda) = 0$
Boundary conditions	$\alpha e^{-iz} + \cancel{\beta e^{iz}}$ commutation relation	regularity at $\lambda = 1$ behaviour near singular point $\lambda = -1$
Solution	$\mathcal{F}(z) = \dfrac{H^{(1)}_{i\mu}(-z)}{\sqrt{-z}}$	$\hat{\mathcal{F}}(\lambda) = \sqrt{2\pi}\, e^{\frac{\pi\mu}{2} + \frac{i\pi}{4}} \dfrac{P_{i\mu-1/2}(\lambda)}{i\cosh(\pi\mu)}$

$\mu = \sqrt{m^2/H^2 - 9/4}$

$$\boxed{\frac{H^{(1)}_{i\mu}(-z)}{\sqrt{-z}} = -\sqrt{\frac{2}{\pi}}\, e^{\frac{\pi\mu}{2} + \frac{i\pi}{4}} \int_1^\infty \mathrm{d}\lambda\, e^{-i\lambda z}\, P_{i\mu-1/2}(\lambda)}$$

‘Hankel as plane waves’

Cosmological correlators

time domain

$$\langle \zeta^n \rangle \sim \iint [\text{Hankel}] \times [\text{plane waves}]$$

nested
time integrals

in-in diagrammatic rules

Laplace frequency domain

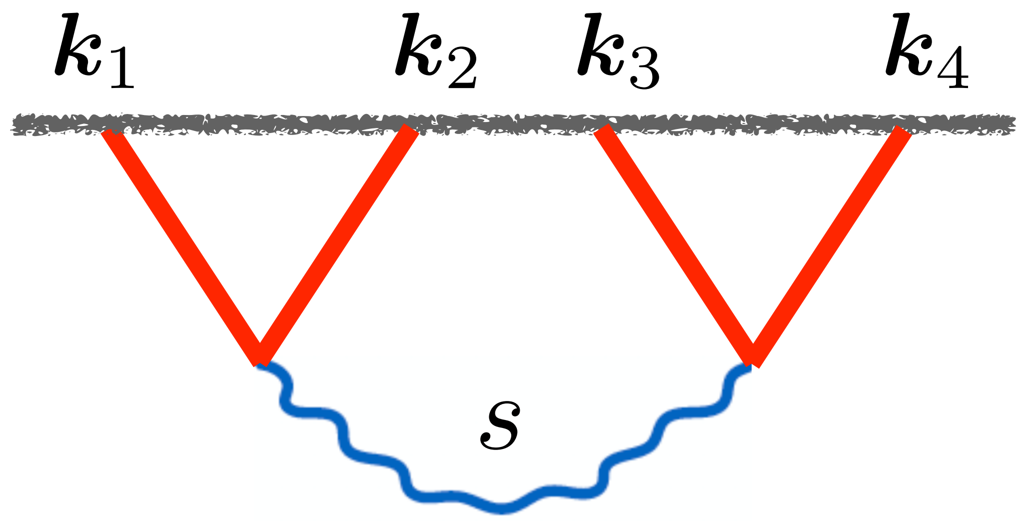
$$\langle \zeta^n \rangle \sim \iint [\text{Legendre}] \times [\text{flat-space correlators}]$$

frequency
integrals

Laplace diagrammatic rules

cosmological correlators from flat space

Massive single exchange. Conceptual



$$\sim \int_1^\infty \int_1^\infty \mathrm{d}\lambda\, \mathrm{d}\tilde{\lambda}\; P_{i\mu-1/2}(\lambda)\, P_{i\mu-1/2}(\tilde{\lambda}) \left[\frac{1}{\lambda_T + \lambda - \tilde{\lambda} - i\epsilon}\right] \times \left[\frac{1}{\lambda_u + \lambda} + \frac{1}{\lambda_v + \lambda}\right]$$

$$\lambda_u = \frac{k_1 + k_2}{s}\,,\ \lambda_v = \frac{k_3 + k_4}{s}\,,\ \lambda_T = \lambda_u + \lambda_v$$

total energy denominator

partial energy denominators

$\lambda_T \to 0$ total-energy singularity from $\lambda \to 1, \tilde{\lambda} \to 1$

$\lambda_{u,v} \to -1$ partial-energy singularities from $\lambda \to 1$

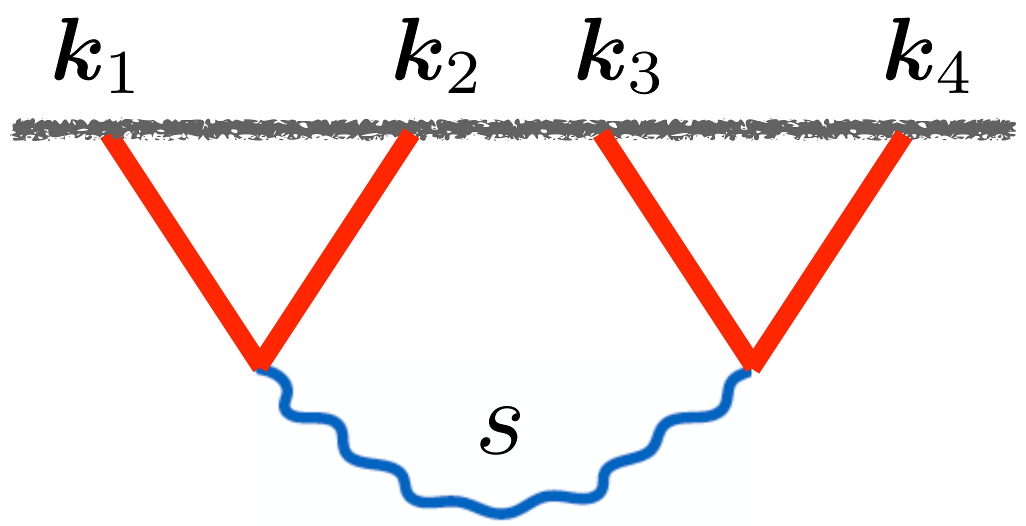
Singularity structure of correlators $\longleftrightarrow$ Flat-space poles reaching Bunch-Davies point $\lambda = 1$

Universal mechanism

Massive single exchange. Computational

Computation with standard complex analysis

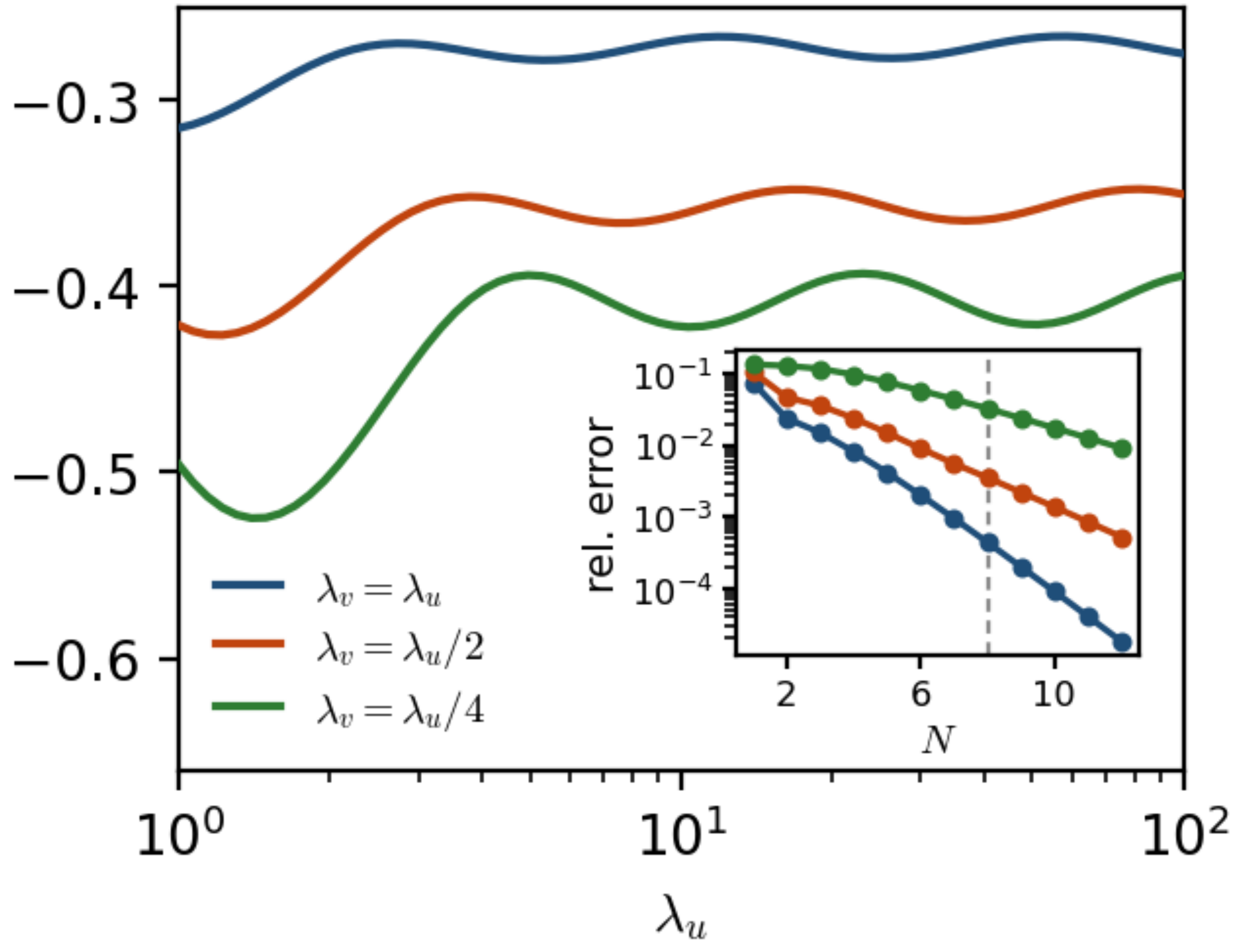
dispersive integral & expansion of kernel at infinity



$$\sim \sum_{n,m=0}^{\infty} a_n(\lambda_u^m + \lambda_v^m)\, {}_2F_1(\{n,m\},\{\lambda_u+\lambda_v\})$$

Single rapidly convergent series throughout kinematic domain

Cosmological collider signal + EFT background at once



Broad range of applicability

- wavefunction coefficients ✓
- derivative interactions ✓
- reduced sound speeds $\lambda_i \to c_{s_i}\lambda_i$ ✓
- spin-1 field with chemical potential ✓
- time-dependent couplings ✓

…

twisted Whittaker as plane waves

$$(iz)^{\alpha} W_{i\tilde{\kappa},i\mu}(2iz) = \int_1^{\infty} \mathrm{d}\lambda\, e^{-i\lambda z} \left(\frac{2}{\lambda^2-1}\right)^{\alpha+1} \times \left(\frac{\lambda+1}{\lambda-1}\right)^{i\tilde{\kappa}} {}_2\tilde{F}_1\left(\begin{matrix} -\frac{1}{2}-\alpha+i\mu,\ -\frac{1}{2}-\alpha-i\mu \\ -\alpha-i\tilde{\kappa} \end{matrix}; \frac{1-\lambda}{2}\right)$$

IV. Strong mixing

Motivation

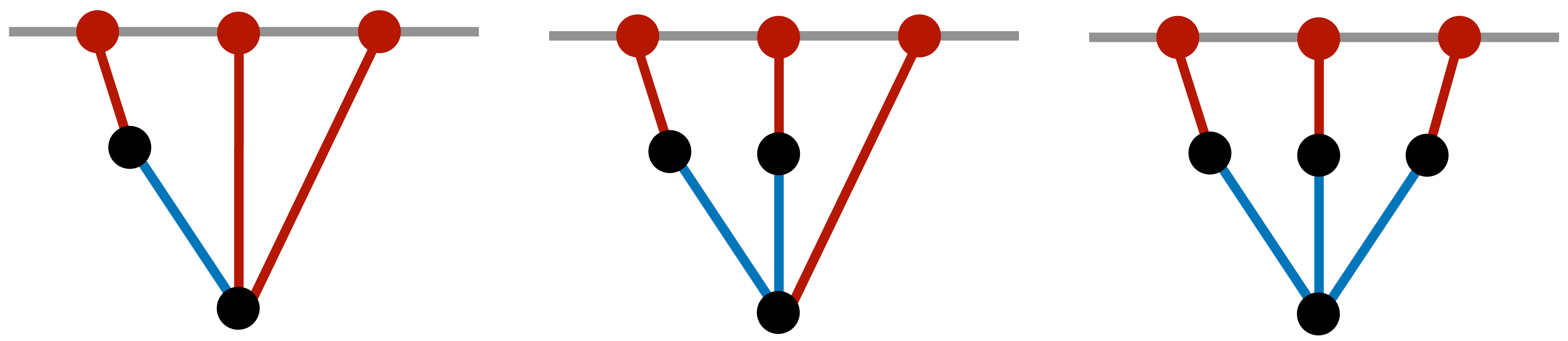
$$\mathcal{L}/a^3 = \frac{1}{2}\dot{\pi}_c^2 - \frac{(\partial_i \pi_c)^2}{2a^2} + \frac{1}{2}\dot{\sigma}^2 - \frac{(\partial_i \sigma)^2}{2a^2} - \frac{1}{2}m^2\sigma^2 + \rho\,\dot{\pi}_c\sigma$$

mixing / turn in field space

Usual weak mixing treatments

$$\mathcal{L}^{(2)} = \mathcal{L}^{(2)}_{\text{diag}} + \mathcal{L}^{(2)}_{\text{mix}}$$

Known mode functions



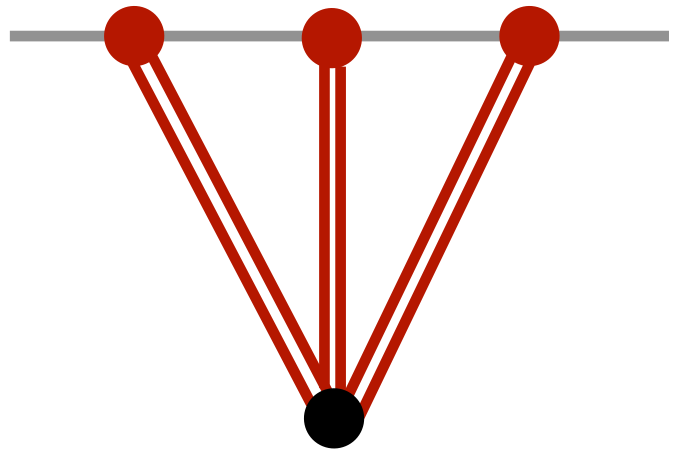
diagrams of increasing complexity

weak signal

Strong mixing

Unknown matrix of mode functions

Numerical solution, e.g. CosmoFlow



effective mass $m^2_{\text{eff}} = m^2 + \rho^2$

strong signal

limited understanding

Pinol, RP, Werth
2312.06559

Linear theory from Laplace space

Quantisation

$$X_{\boldsymbol{k}} = \sum_{a=\pm 1} \Big[X_a(z)\, \hat{b}_{a,\boldsymbol{k}} + X_a^*(z)\, \hat{b}^\dagger_{a,-\boldsymbol{k}} \Big] \qquad X = \pi_c, \sigma$$

Laplace representation

$$\sigma_a(z) \sim z^2 \int_1^\infty \mathrm{d}\lambda \; B_a(\lambda)\, e^{-i\lambda z}$$

Discontinuity of Laplace transform obeys simple Legendre equation

$$\boxed{B_a(\lambda) \sim P^{-a\, i\, \frac{\rho}{H}}_{i\mu_{\mathrm{eff}} - \frac{1}{2}}(\lambda)}$$

minor modifications wrt uncoupled field!

Other field follows

$$\pi_{c,a}(z) \sim \int_1^\infty \mathrm{d}\lambda \; B_a(\lambda)\, e^{-i\lambda z} \times \mathrm{Polynomial}(\lambda, z)$$

Linear theory from Laplace space

$$\sigma_a(z) \sim z^2 \int_1^\infty \mathrm{d}\lambda\, B_a(\lambda)\, e^{-i\lambda z} \qquad \pi_{c,a}(z) \sim \int_1^\infty \mathrm{d}\lambda\, B_a(\lambda)\, e^{-i\lambda z} \times \mathrm{Polynomial}(\lambda, z)$$

Related recent works Huenupi et al 2606.18248, 2607.14529 ; Pinol 2607.15251 ; Wang & Wang 2607.14891

Power spectrum in simple closed form for arbitrary mass and mixing

Recovers all known limits

Bispectrum from Laplace space

$$\underset{\text{from } \sigma^3}{\langle \pi_c^3 \rangle} \sim \iiint_1^\infty \prod_{i=1}^{3} \mathrm{d}\lambda_i \, B_{a_i}(\lambda_i) \, \frac{1}{K^3} \,, \quad K = \sum_i k_i \lambda_i$$

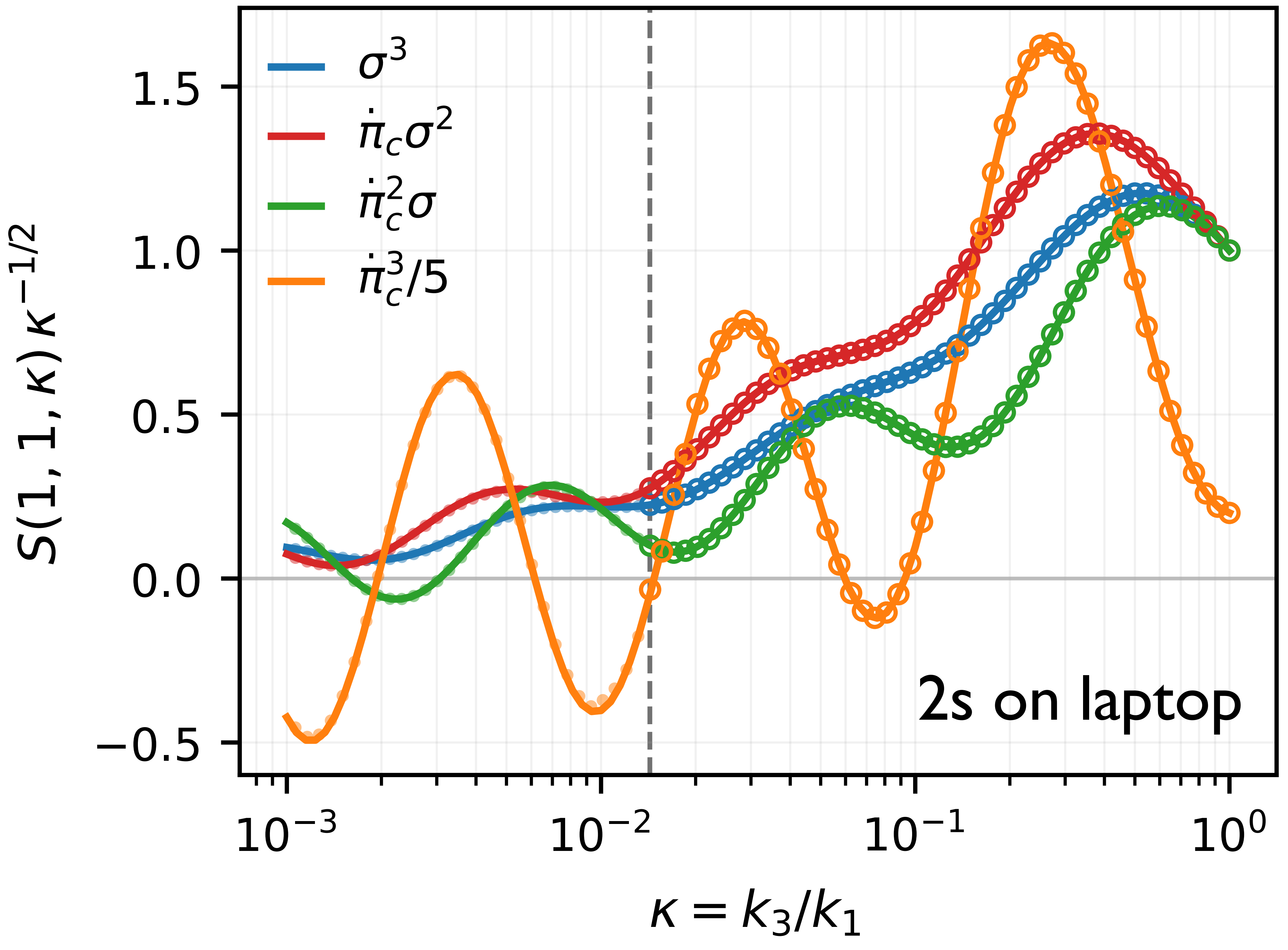
deformed total energy

1. Frequency integral of softest leg

2. Map resulting 2d integral to unit square: Euler integrals giving Beta functions

$\longrightarrow$

Rapidly convergent series



∘ CosmoFlow results

Philcox 2607.18369

Cosmological collider

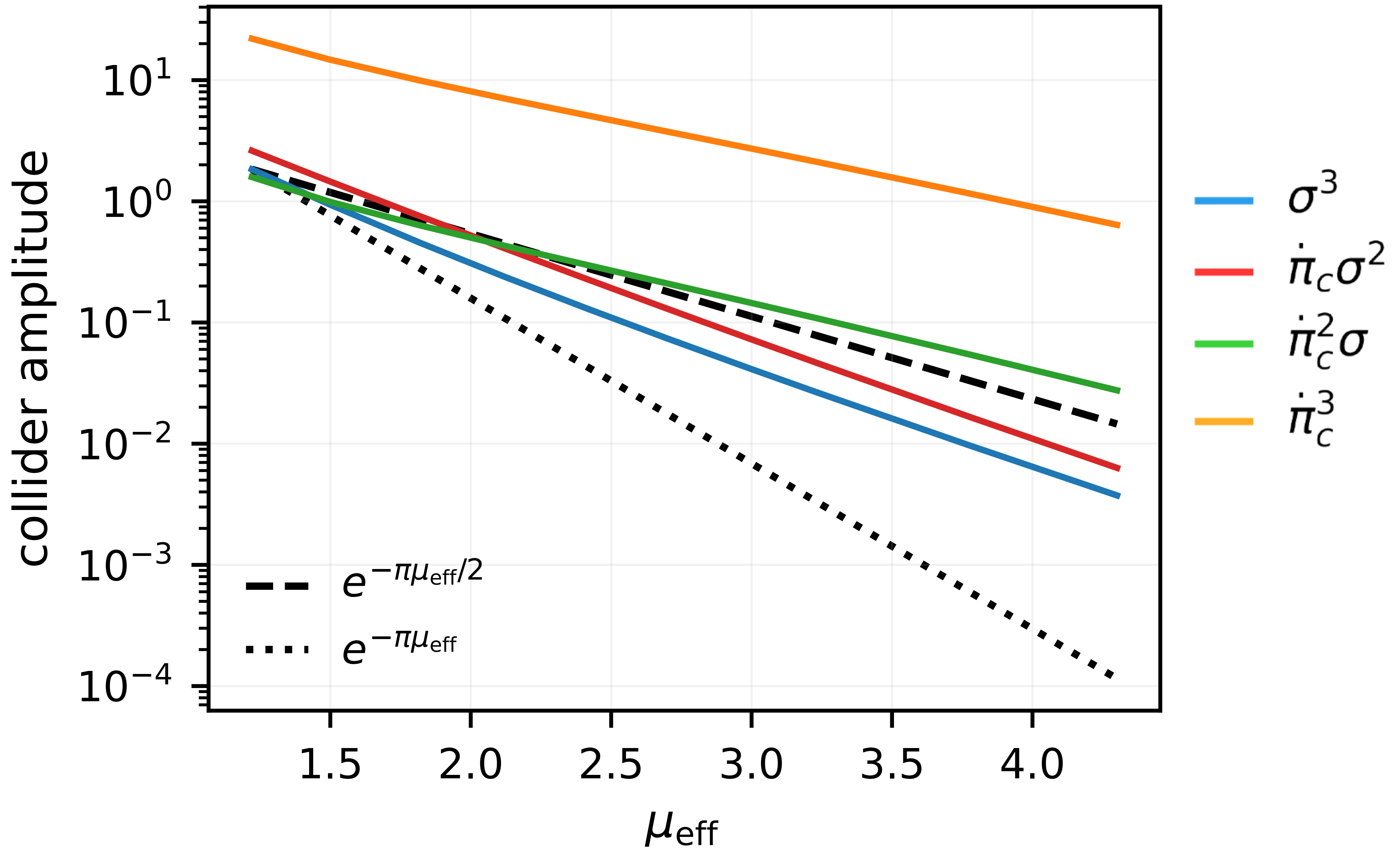
$$S_{\rm col} \simeq \mathcal{A}\,\kappa^{1/2}\cos\left(\mu_{\rm eff}\ln\kappa + \varphi\right)$$

$$\mathcal{A} \;\propto\; \mu_{\rm eff}^{n}\, e^{-\pi\mu_{\rm eff}/2}$$

- half Boltzmann suppression

- grows with number of $\dot{\pi}_c$

  (energy injection at pair creation)



Observational target:

$\dot{\pi}_c^3$ at strong mixing

Large overall non-Gaussianity

Cosmological collider dominant over EFT background

Conclusion

Laplace space: new language for QFT in cosmological background

- Universal: same structure for all fields & interactions, including strong mixing
- Conceptual: cosmology from flat space, rooted in equivalence principle
- Computational: single-exchange in all kinematics at once, analytical bispectrum at strong mixing

Perspectives:

- Other theories, 4-pt, loops
- Analytic structure of correlators from flat space counterparts?
- Other background, black holes?