

**Particle Physics
in the Early Universe**
14-18 September 2026
Paris & Orsay



Modelling Ultra-Slow-Roll Inflation

PBHs from single field inflation

David Wands

Barnert, Iacconi, Assadullahi, Koyama & Wands, arXiv:2606.28005

Artigas, Frion, Miranda, Vennin & Wands, arXiv:2504.05937

Motivation :

- enhanced primordial power spectra on small scales could leave gravitational relics
 - primordial gravitational waves
 - primordial black holes
- must violate "slow roll" in single-field inflation
 - transient, non-attractor dynamics
 - non-linear perturbations, non-Gaussianity
 - non-Gaussianity & loop corrections
 - quantum diffusion & non-perturbative dynamics
 - lattice simulations & stochastic inflation



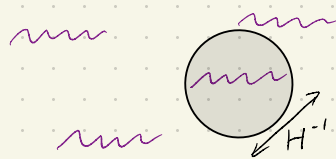
inflation

accelerated expansion ($\ddot{a} > 0$) in v.e.u.

e.g. quasi-de Sitter : $H = \frac{\dot{a}}{a} \approx \text{constant}$

quantum vacuum

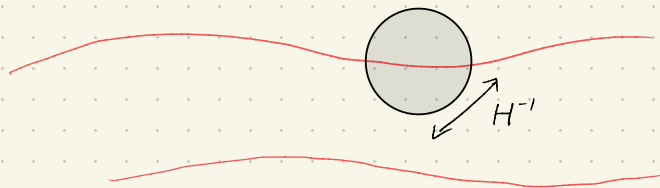
UV ($k \gg aH$)



inflation
→

primordial density fluctuations

IR ($k \ll aH$)



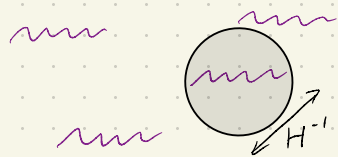
inflation

accelerated expansion ($\ddot{a} > 0$) in v.e.u.

e.g. quasi-de Sitter : $H = \frac{\dot{a}}{a} \approx \text{constant}$

quantum vacuum

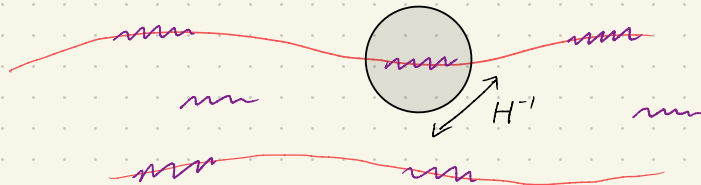
UV ($k \gg aH$)



inflation
 $\longrightarrow$

primordial density fluctuations

IR ($k \ll aH$)



sufficient inflation ($N = \int H dt > 70$)
generates density fluctuations on all scales

classical vs quantum inflation

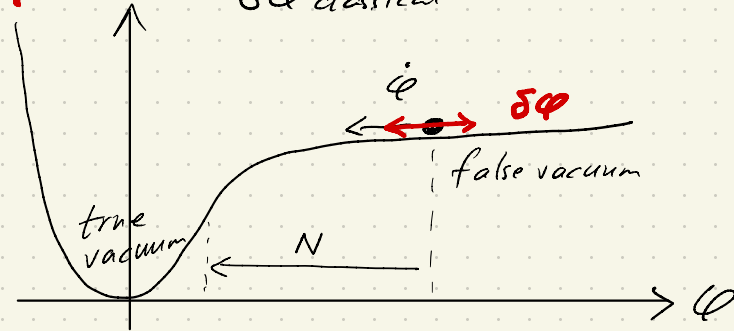
- classical slow roll inflates universe

$$\rightarrow \ln(a) = N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi$$

- quantum field fluctuations perturb local adiabatic expansion

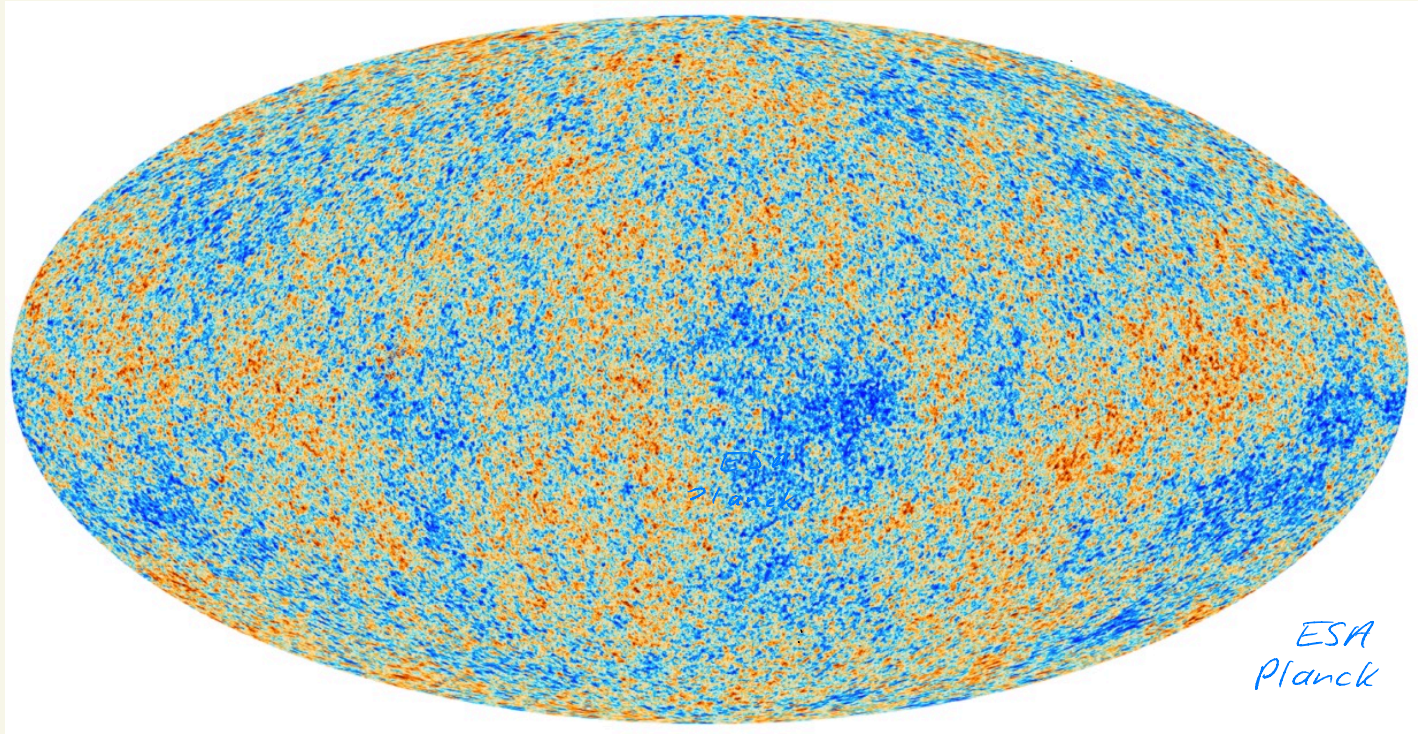
$\rightarrow$ curvature perturbation

$$R = \delta N = H \frac{\delta \phi}{\dot{\phi}} = \frac{\delta \mathcal{L}_{\text{quantum}}}{\delta \mathcal{L}_{\text{classical}}}$$



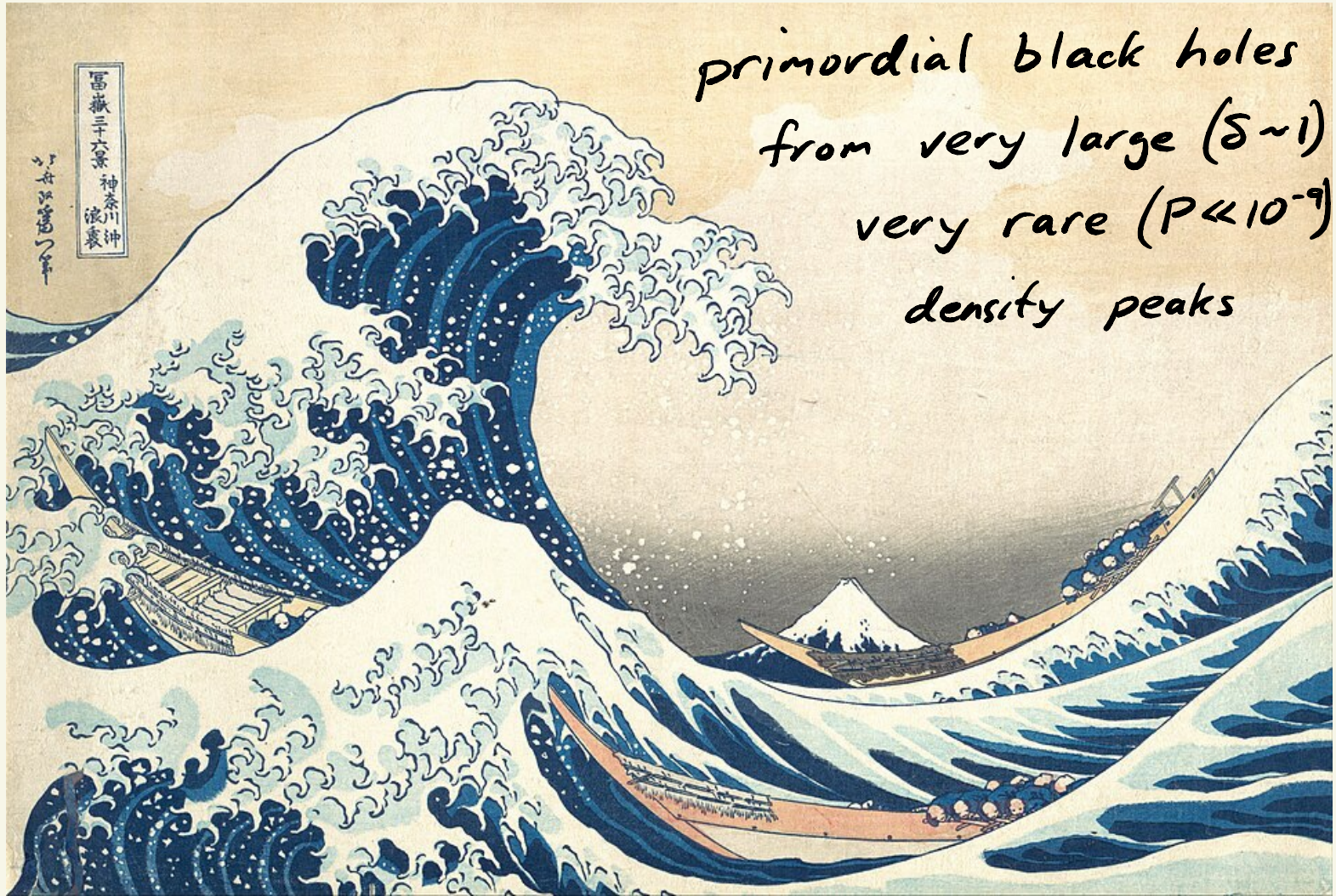
primordial density perturbations from inflation

$$\frac{\Delta T}{T} \approx \frac{2}{5} R \sim 10^{-5} \quad \text{adiabatic, Gaussian, } \approx \text{scale invariant}$$



ESA
Planck

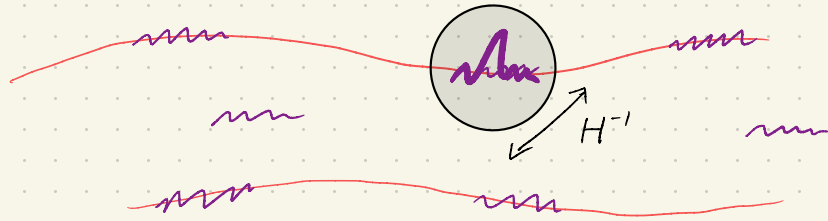
primordial black holes
from very large ($\delta \sim 1$)
very rare ($P \ll 10^{-9}$)
density peaks



Primordial black holes

Zeldovich & Novikov (1967), Hawking (1971)
Hawking & Carr (1974)

- formed by collapse of large density perturbations, $\frac{\delta \rho}{\rho} > \delta_{crit}$
- mass related to horizon mass at formation time in radn. era



$$H^{-1} = 2t$$

$$M_{PBH} \sim \frac{t}{10^{-6} s} M_{\odot}$$

- challenging to model origin during inflation:

non-perturbative, $\delta \sim 1$, and very rare

mass fraction at formation : $\beta(M) \sim 10^{-8} \left(\frac{M_{PBH}}{M_{\odot}} \right)^{1/2} \Omega_{PBH}$

sensitive to primordial non-Gaussianity

ultra-slow-roll inflation

Starobinsky '92; Seto et al. '99

Tsamis & Woodward '03; Kinney '05...

Garcia-Bellido & Ruiz '17...

primordial curvature
power spectrum

$$\langle R^2 \rangle = \left(\frac{H}{\dot{\varphi}} \right)^2 \langle \delta\varphi^2 \rangle$$

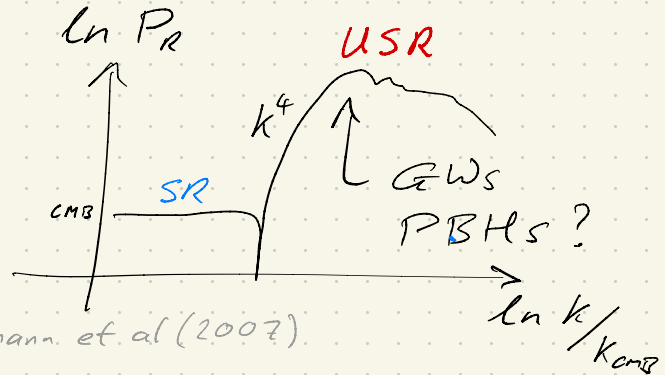
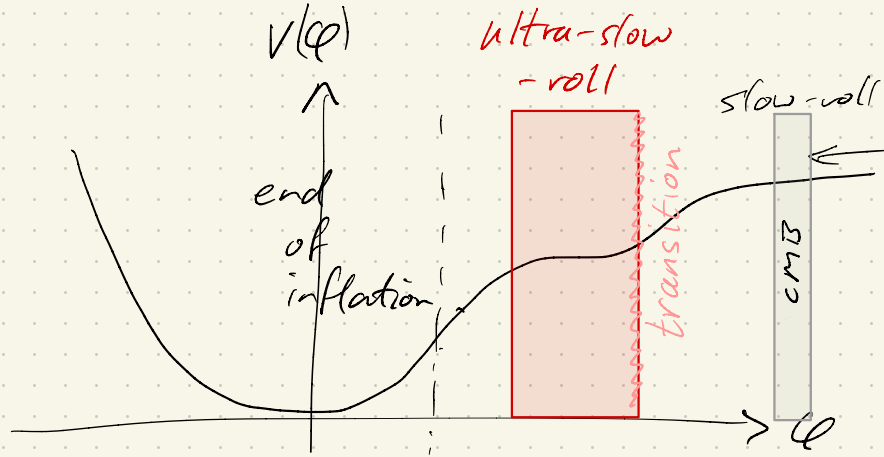
↻ boosted as $\left(\frac{\dot{\varphi}}{H} \right)^2 \rightarrow 0$

* **primordial black holes**

Carr & Hawking '74

* **induced grav. waves**

Matarrese et al., Ananda et al., Baumann et al. (2007)



Scalar field dynamics in FRW cosmology

Klein-Gordon eqn:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

two regimes

(i) **slow-roll** quasi-equilibrium : $|\ddot{\phi}| \ll |3H\dot{\phi}|$

$$\dot{\phi} \simeq -\frac{V'}{3H}, \text{ attractor, } \phi(\phi)$$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

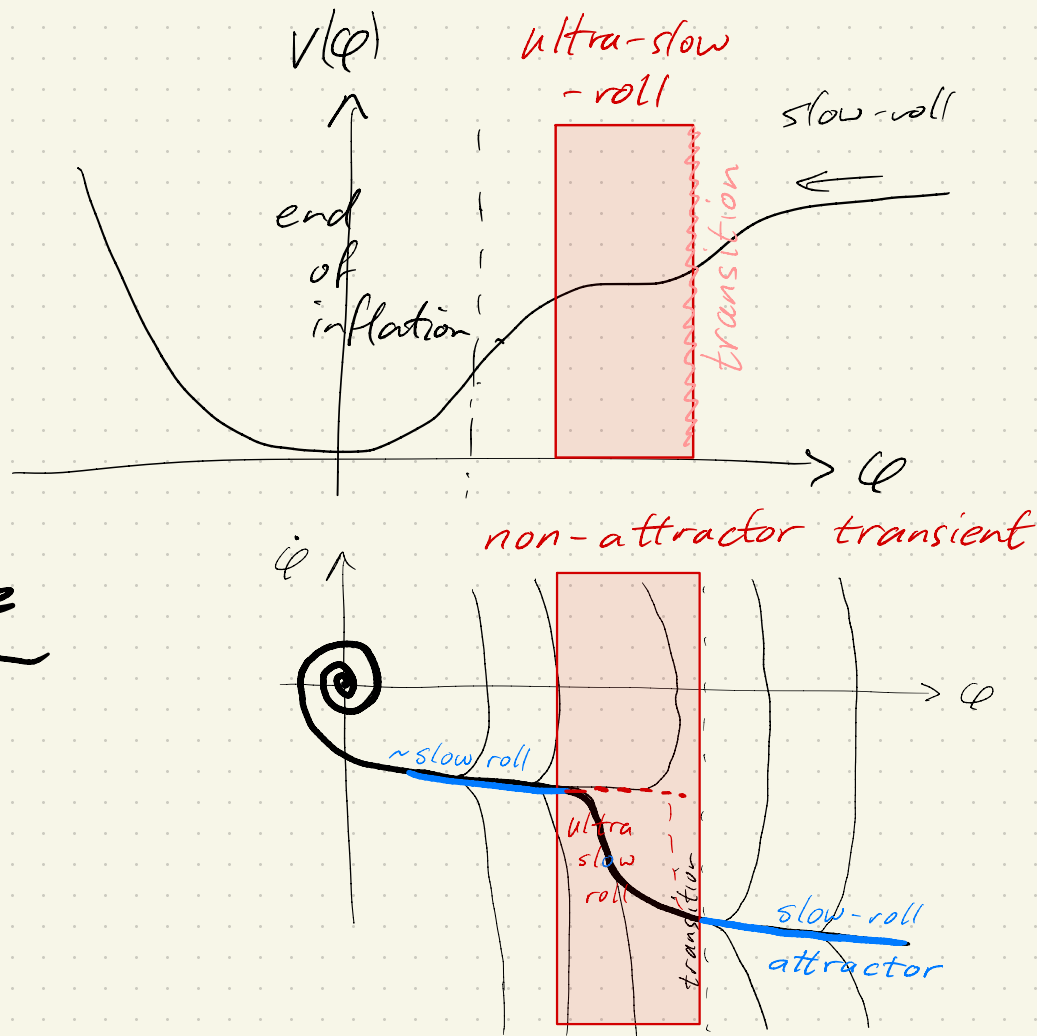
(ii) **ultra-slow-roll** free evolution : $|V'| \ll |3H\dot{\phi}|$

$$\ddot{\phi} \simeq -3H\dot{\phi}, \text{ transient decay in 2D phase space}$$

$$\dot{\phi} \simeq \frac{D}{a^3} \rightarrow 0 \text{ as } a \rightarrow \infty$$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \approx 3$$

USR phase space



non-adiabatic pressure in single-field inflation

FLRW cosmology : $\rho = \frac{1}{2} \dot{\varphi}^2 + V(\varphi)$, $P = \frac{1}{2} \dot{\varphi}^2 - V(\varphi)$

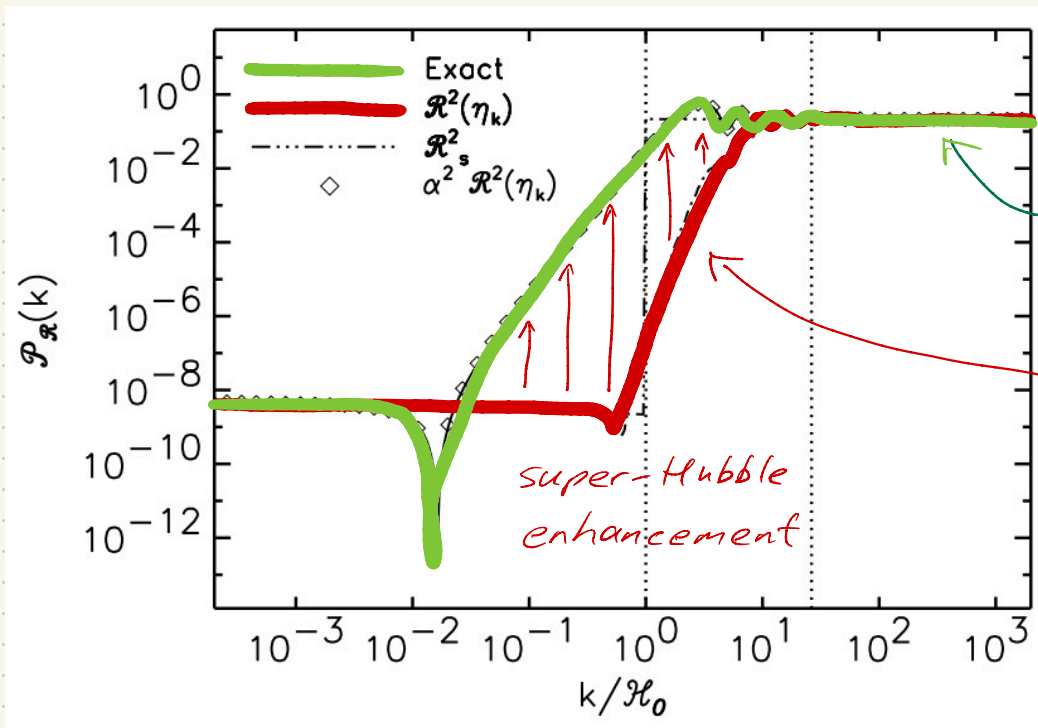
non-adiabatic perturbation : $\delta P_{nad} = \delta P - (\dot{P}/\dot{\rho}) \delta \rho$
 $= 0$ if $\exists$ attractor $\dot{\varphi} = \dot{\varphi}(\varphi)$
on large scales in slow roll

$\neq 0$ on large scales
in ultra slow roll
 $\dot{\varphi} \neq \dot{\varphi}(\varphi)$

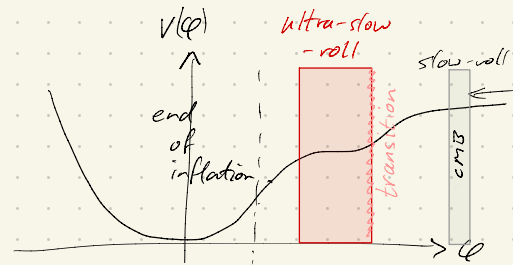
curvature evolution : $\dot{R} = -\frac{3H^2}{2\dot{V}} \delta P_{nad}$

Growth of curvature perturbation on super-Hubble scales

Leach, Sasaki, Wands & Liddle (2001)



USR models: potentials, $V(\phi)$



- inflection point - Higgs inflation, polynomial, exponential...
- localised feature - piecewise linear/quadratic, Gaussian bump, double well...

all require delicate fine-tuning Cole et al (2023)

one part in $10^7 - 10^8$ for sufficient (but not too much!)

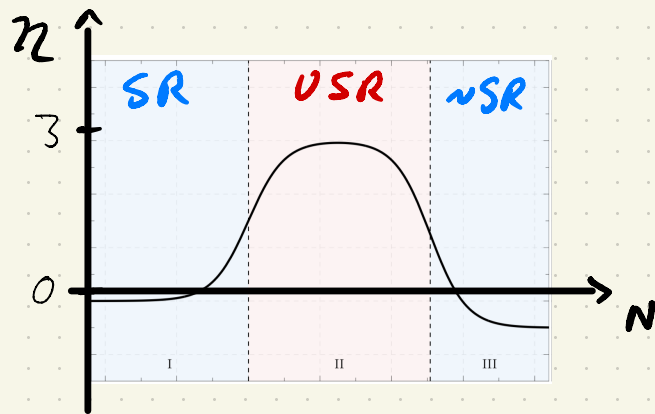
USR duration ($\phi \rightarrow 0$) and hence enhanced power, $P_R \approx 10^{-2}$

can we make generic predictions?

Hubble-flow USR model

Taoso & Urbano (2021)

$$\eta(N) = \frac{\eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{in}}}{\Delta N} \right) \right] + \frac{\eta_{\text{III}} - \eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{out}}}{\Delta N} \right) \right]$$



$$\eta_{\text{I}} = 0, \quad \eta_{\text{II}} = 3, \quad \eta_{\text{III}} = -0.5$$

smooth transition, $\Delta N = 0.5$

where $\eta \equiv -\ddot{\phi} / 3H\dot{\phi}$

$$\eta = -\frac{1}{2} \frac{d \ln \epsilon_1}{dN} + \epsilon_1$$

$$\Rightarrow \epsilon_1(N)$$

$$\epsilon_1 = -\frac{d \ln H}{dN} \Rightarrow H(N)$$

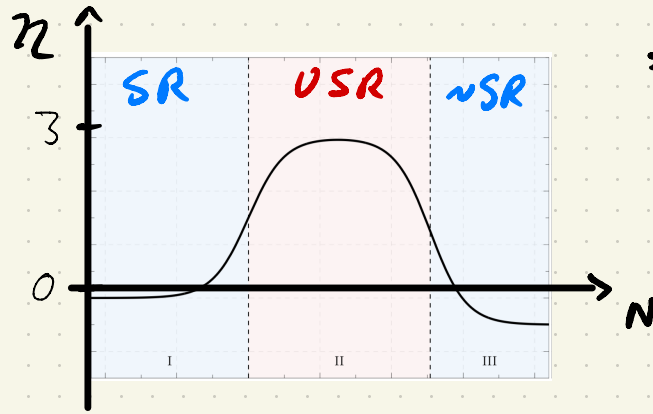
& higher-order SR parameters

$$\epsilon_{i+1} = \frac{d \ln \epsilon_i}{dN}$$

Hubble-flow VSR model

Taoso & Urbano (2021)

$$\eta(N) = \frac{\eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{in}}}{\Delta N} \right) \right] + \frac{\eta_{\text{III}} - \eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{out}}}{\Delta N} \right) \right]$$



$$\eta_{\text{I}} = 0, \quad \eta_{\text{II}} = 3, \quad \eta_{\text{III}} = -0.5$$

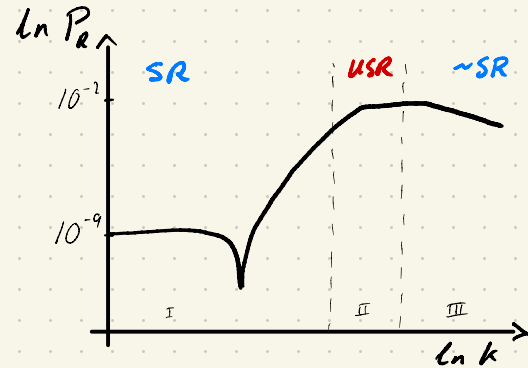
smooth transition, $\Delta N = 0.5$

$\Rightarrow$ linear perturbations, $u = a \delta \varphi$

$$u'' + \left(k^2 - \frac{\mu^2(\epsilon_i)}{a^2 H^2} \right) u = 0$$

where

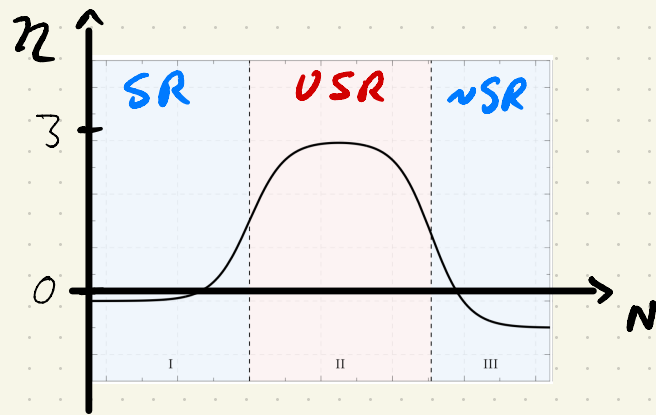
$$\mu^2 = 2 - \epsilon_1 + \frac{3}{2} \epsilon_2 + \left(\frac{1}{4} \epsilon_1 - \frac{1}{2} \epsilon_1 + \frac{1}{2} \epsilon_3 \right) \epsilon_2$$



Hubble-flow USR model

Taoso & Urbano (2021)

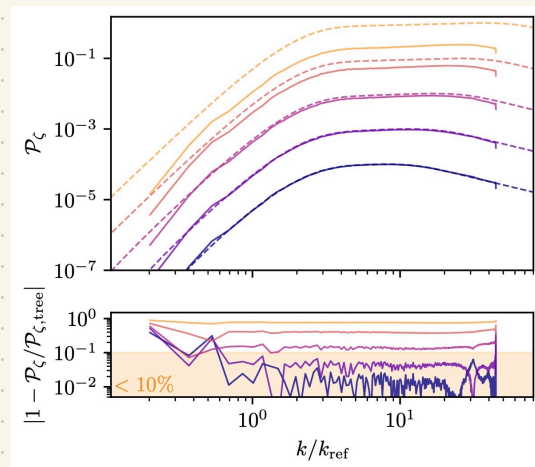
$$\eta(N) = \frac{\eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{in}}}{\Delta N} \right) \right] + \frac{\eta_{\text{III}} - \eta_{\text{II}}}{2} \left[1 + \tanh \left(\frac{N - N_{\text{out}}}{\Delta N} \right) \right]$$



$$\eta_{\text{I}} = 0, \quad \eta_{\text{II}} = 3, \quad \eta_{\text{III}} = -0.5$$

smooth transition, $\Delta N = 0.5$

non-linear power spectrum

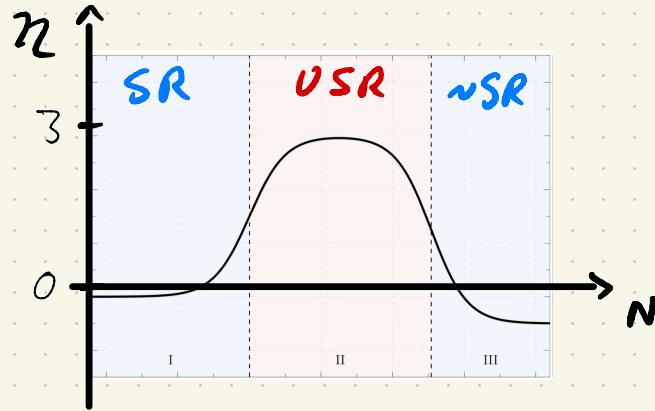


Caravano, Franciolini & Renaux-Petel (2024)

primordial non-Gaussianity, etc

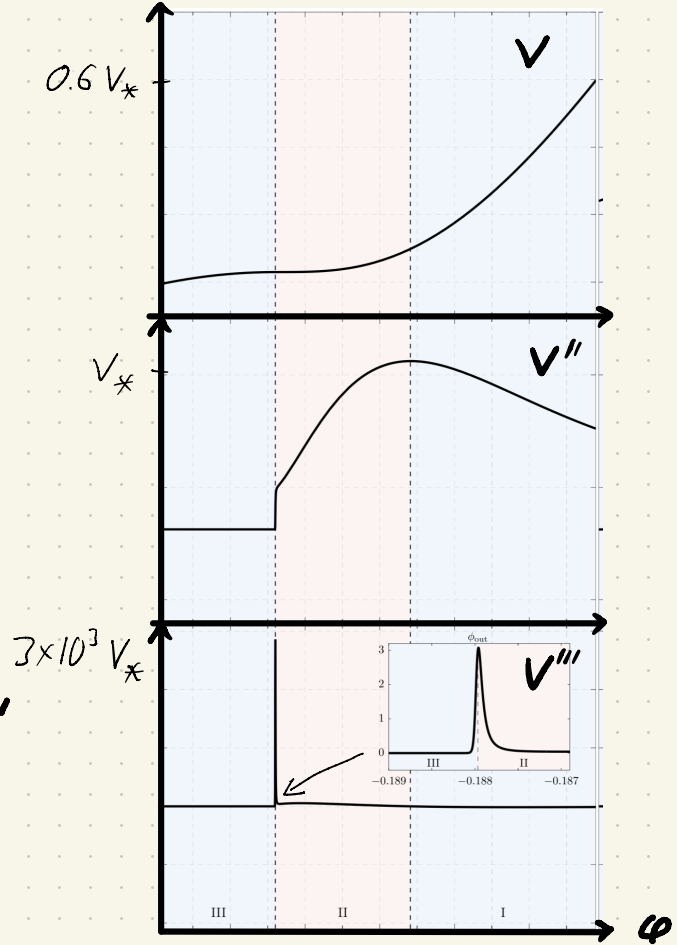
see also Ballesteras et al (2024)

Hubble-flow VSR model



large, sharp
features "artefacts" in V'''
at second transition
 $VSR \rightarrow SR$

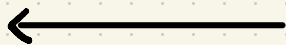
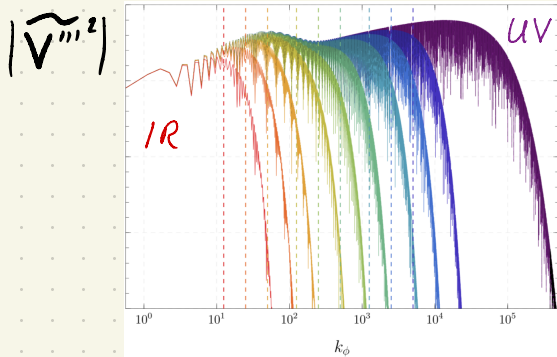
$\Rightarrow$ specific potential



UV-filtered Hubble-flow model

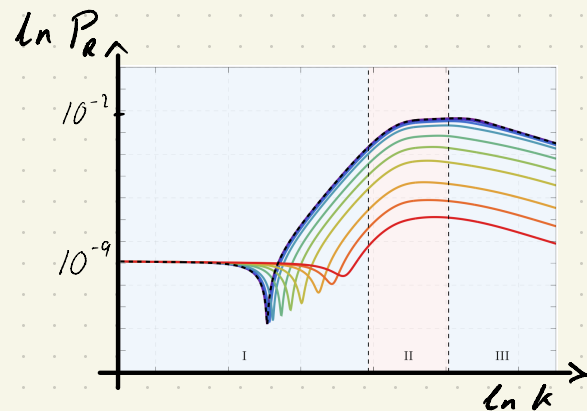
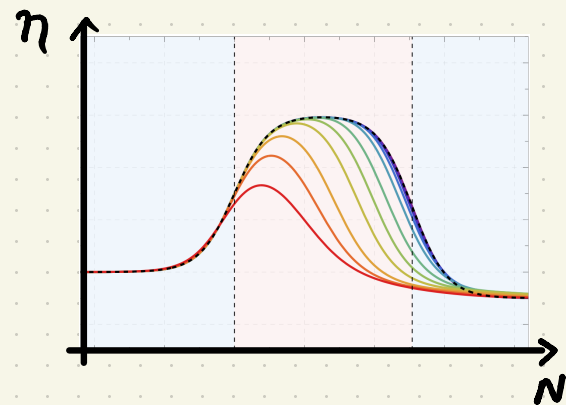
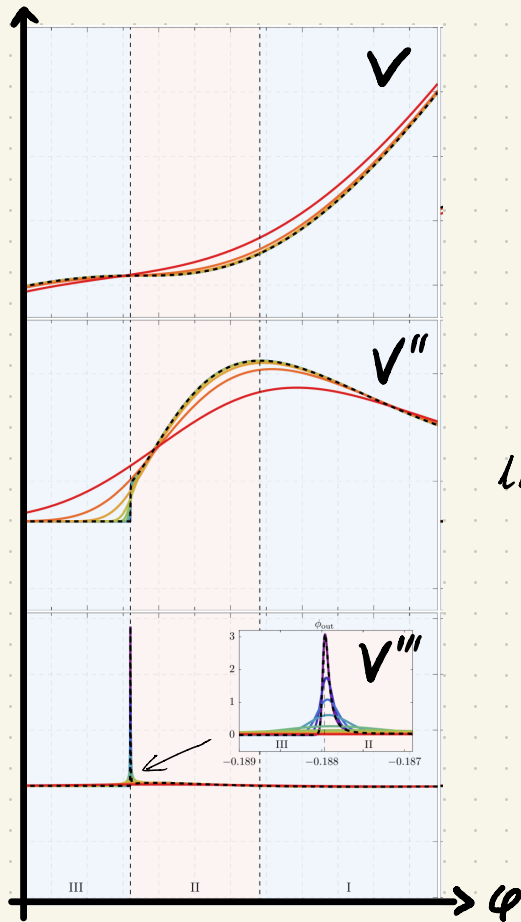
remove sharp features

Fourier transform



$k_{\phi, \max}$

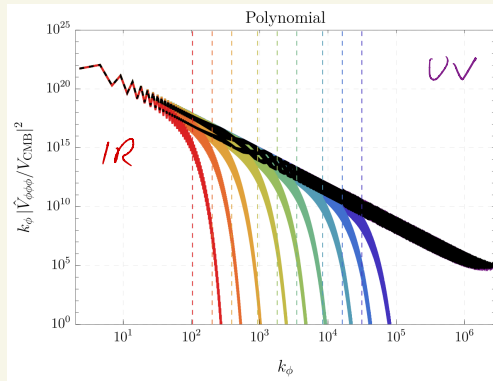
second transition
sensitive to
UV power



UV - filtered Potential model : polynomial $V(\phi)$

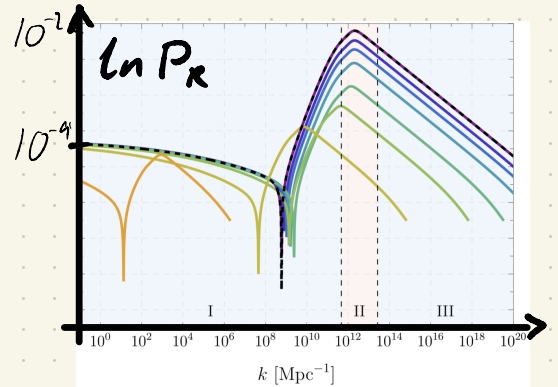
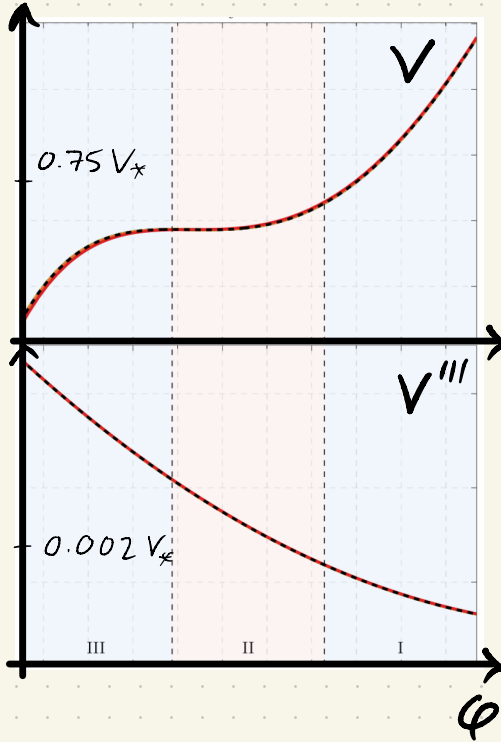
Hertzberg & Yamada (2018)

$|\widetilde{V}'''|^2$



$K_{\phi, \max}$

little UV power



P_R still sensitive to UV filter

what's wrong with $\eta(N)$ models of USR?

(A) smooth $\eta(N) \rightarrow$ smooth $V''(N)$

$$\text{USR: } \eta \approx -\frac{1}{2} \frac{d \ln \epsilon_1}{dN} \approx 3 \rightarrow \epsilon_1 = \frac{1}{2} \left(\frac{d\phi}{dN} \right)^2 \rightarrow 0$$

$\rightarrow$ compressed $V''(\phi)$, large $V'''(\phi)$

(B) forcing USR (transient) $\rightarrow$ SR (attractor) unphysical

attractor + feature $\rightarrow$ transient ✓

transient + feature $\rightarrow$ transient (not attractor!) ^x

USR transient will naturally relax back to SR
in absence of any sharp features

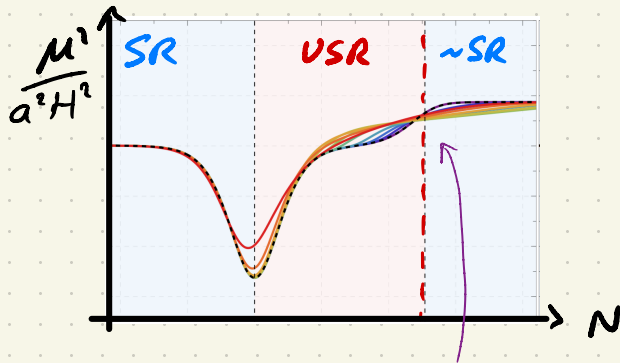
duality invariance of MS mode equation Wands (1999)

$$\mu^2 = \frac{z''}{z} \quad \text{invariant under} \quad z \rightarrow \tilde{z} = z(\eta) \int^\eta \frac{d\eta'}{z(\eta')}$$

e.g., at **USR $\rightarrow$ SR transition** Leach et al (2001)

invariance holds in $V(k)$ USR models (generic feature)

broken in Hubble flow $\eta(N)$ models (not generic)



UV filter restores $\mu^2 \sim \text{constant}$ at transition

loop corrections to power spectrum?

- Kristiano & Yokoyama (2021) claimed enhanced power on small scales, $P_R \sim 10^{-2}$, gave large loop corrections to power spectrum on CMB scales
- many papers followed...
- Kristiano & Yokoyama (2024)
"we find that one loop correction to the power spectrum... vanish if and only if the MS equation for perturbation satisfies Wands duality [invariance]"

Conclusions

ultra-slow-roll dynamics during inflation

→ enhances density perturbations on small scales

→ primordial black holes + grav. waves

- "smooth" Hubble-flow, $n(N)$, models of USR inflation have sharp features in $V'''(\varphi)$ at USR $\rightarrow$ SR
→ large field self-interactions

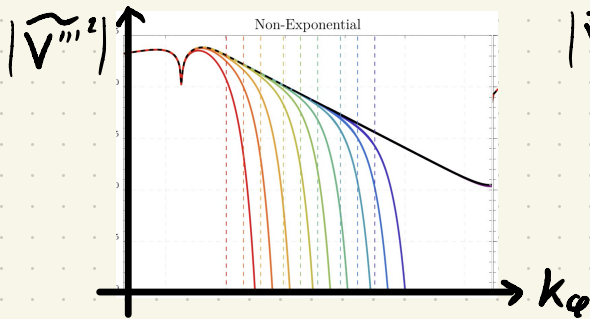
"UV artefacts" Barnert et al, arXiv:2606....

- potential models require finely tuned parameters have smooth $V(\varphi)$ → weak self-interactions
→ invariance of MS mode equation thru USR $\rightarrow$ ~SR
next step, nonlinear perturbations... Work in progress

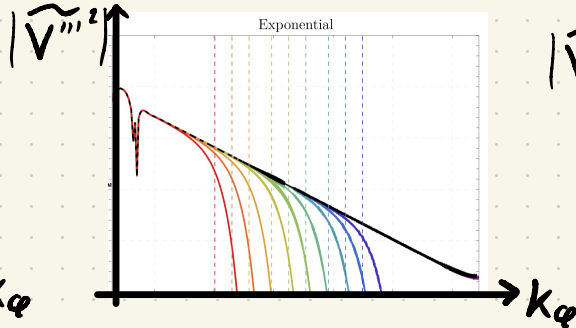
bonus material

more potential models

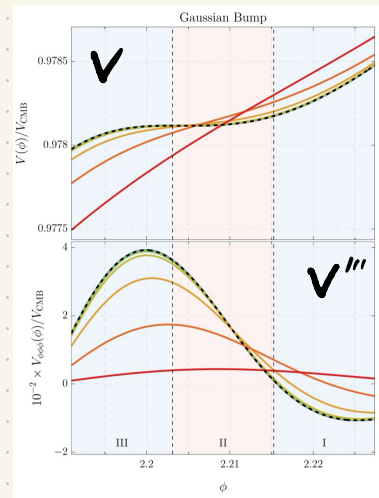
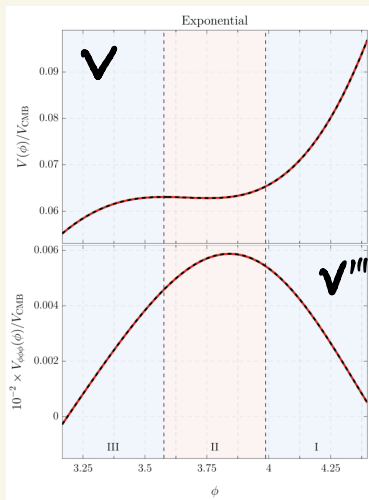
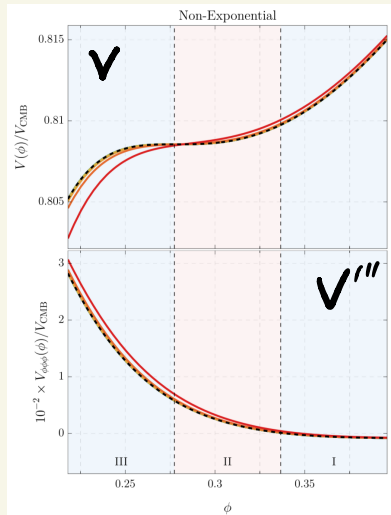
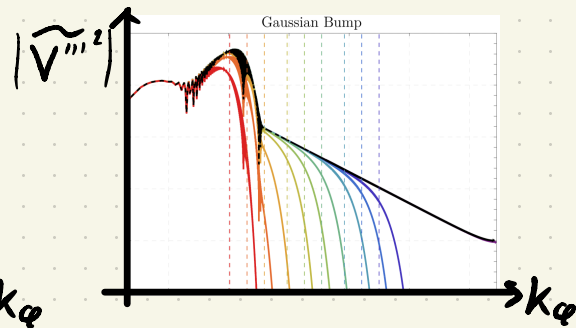
García-Bellido & Morales (2017)



Cicoli, Díaz & Pedro (2018)

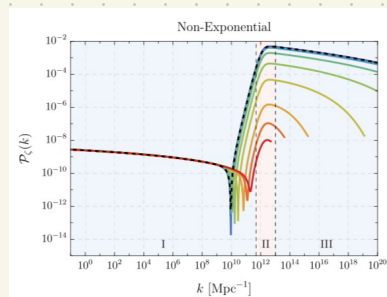
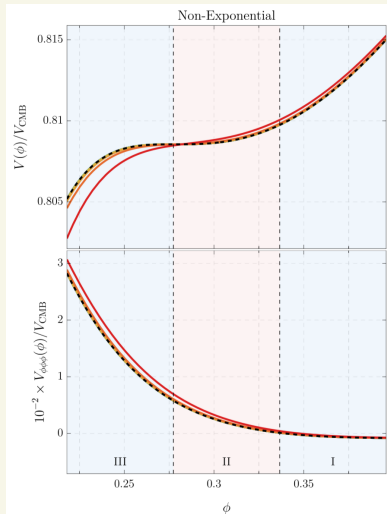


Mishra & Sahni (2020)

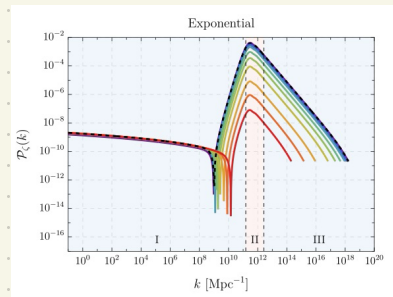
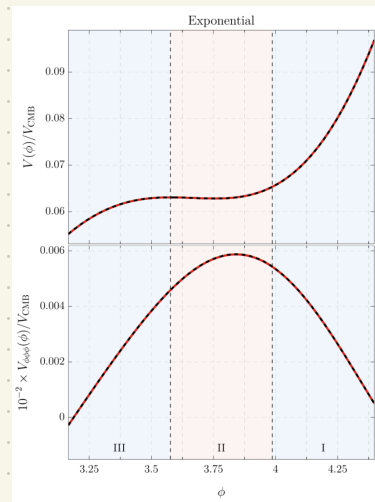


more potential models

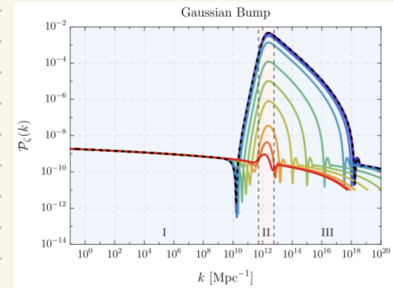
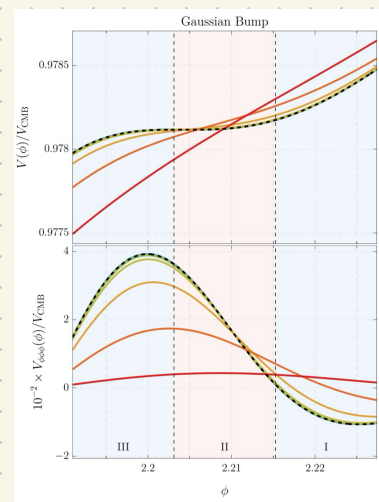
García-Bellido & Morales (2017)



Cicoli, Díaz & Pedro (2018)

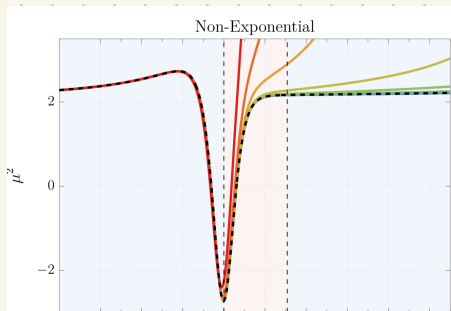


Mishra & Sahni (2020)

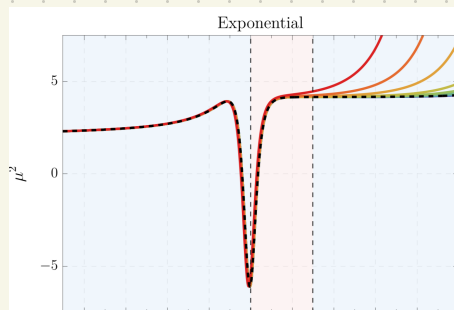


more potential models

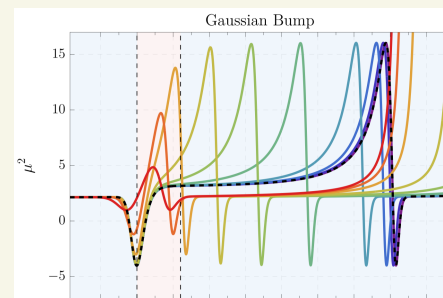
García-Bellido & Morales (2017)



Cicoli, Díaz & Pedro (2018)



Mishra & Sahni (2020)

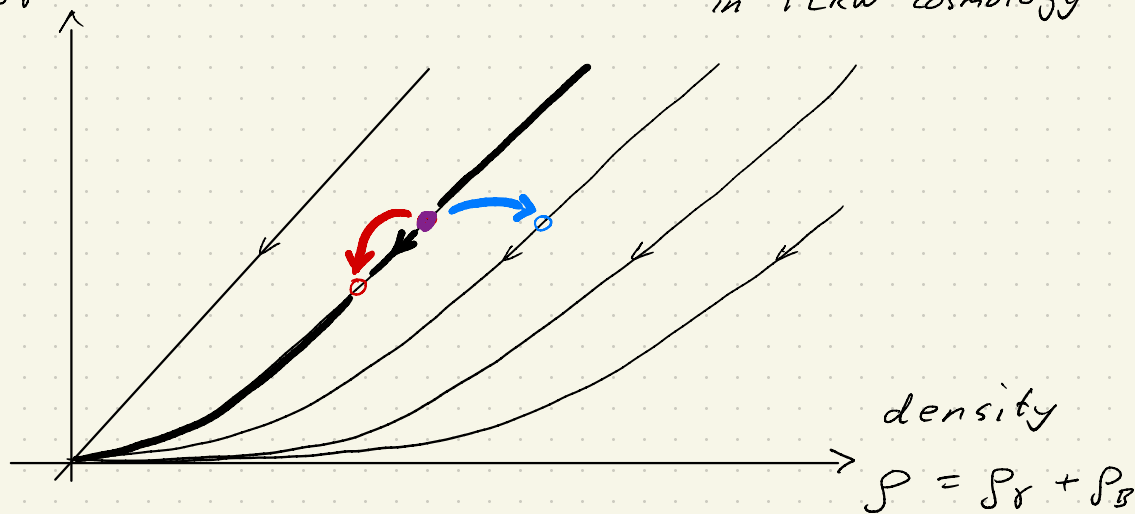


pressure

$$P = \frac{1}{3} P_\gamma$$

photon - baryon phase space

in FLRW cosmology



adiabatic perturbation

$$R = H \frac{\delta P}{\dot{P}} = H \frac{\delta P}{\dot{P}}$$

$$\Rightarrow \delta \left(\frac{n_\gamma}{n_B} \right) = 0$$

non-adiabatic perturbation

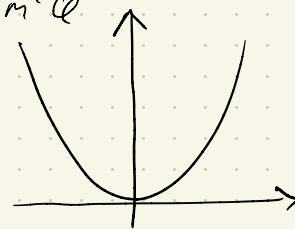
$$S = 3H \left(\frac{\delta P}{\dot{P}} - \frac{\delta P}{\dot{P}} \right)$$

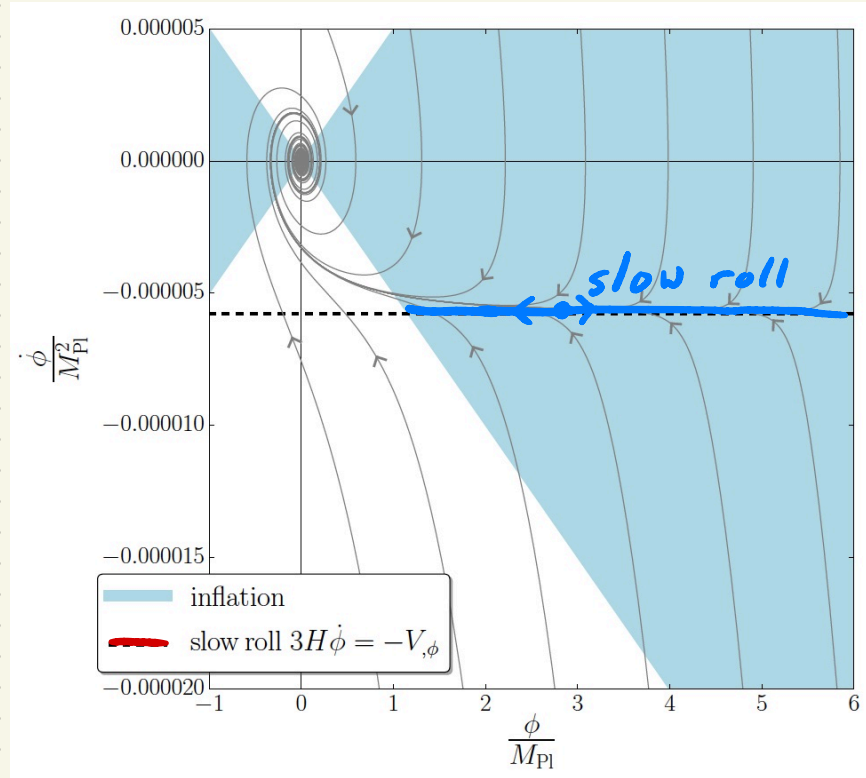
a.k.a. isocurvature,
entropy perturbation.

slow-roll inflation phase space

Grain & Vennin (2017)

e.g. $V = \frac{1}{2} m^2 \phi^2$





adiabatic expansion

$$N = \int H dt$$

adiabatic perturbation

$$R = H \frac{\delta \phi}{\dot{\phi}}$$

$R = \text{constant}$ on super-Hubble scales ($k < aH$)

gauge dependence of large-scale limit

Artigas et al, arXiv:2504.05937

in Newtonian gauge

Bardeen potential $\Phi \rightarrow \bar{\Phi}_0$ as $k \rightarrow 0$

$$\Rightarrow \delta P_{\text{rad}} \sim k^2 \Phi \rightarrow 0 \quad "$$

note: δP_{rad} is gauge invariant

so this is a physical restriction

in spatially flat gauge used in stochastic inflation

allows $\delta P_{\text{rad}} \neq 0$ as $k \rightarrow 0$

necessary for large-scales in ultra-slow roll

Einstein equations in gradient expansion

extrinsic curvature of 3-space : $K^i_j = -\tilde{H} \delta^i_j + A^i_j$

$$G^0_0 \rightarrow 3\tilde{H}^2 = 8\pi G (V(\tilde{\varphi}) + \frac{1}{2}\dot{\tilde{\varphi}}^2) + \mathcal{O}(\nabla^2)$$

$$G^i_i \rightarrow \ddot{\tilde{H}} = -4\pi G \dot{\tilde{\varphi}}^2 + \mathcal{O}(\nabla^2)$$

$$G^i_{j \neq i} \rightarrow \dot{A}^i_j = -3H A^i_j + \mathcal{O}(\nabla^2)$$

$$G^0_i \rightarrow \nabla_i \tilde{H} + 4\pi G \dot{\tilde{\varphi}} \nabla_i \tilde{\varphi} \propto \nabla_j A^j_i + \mathcal{O}(\nabla^3)$$

Salopek & Bond (1990)

gradient expansion on large scales

anisotropic shear evolution: $\dot{A}^i_j = -3H A^i_j$

$$\Rightarrow A^i_j \propto e^{-3 \int H dt} \rightarrow 0$$

momentum constraint neglecting anisotropic shear

$$\delta H + 4\pi G \dot{\varphi} \delta\varphi = \delta H - \frac{\dot{H}}{\dot{\varphi}} \delta\varphi = 0$$

adiabatic perturbations $\Rightarrow \tilde{H} = H(\tilde{\varphi})$

Hamilton-Jacobi solution for perturbations

Artigas et al (2025)

momentum constraint

see also Tanaka & Urakawa (2021)
Tomberg (2023)

$$\delta H + 4\pi G \dot{\varphi} \delta \varphi = \delta H - \frac{\dot{H}}{\dot{\varphi}} \delta \varphi \propto e^{-3 \int H dt}$$

dynamically decays but not gradient suppressed

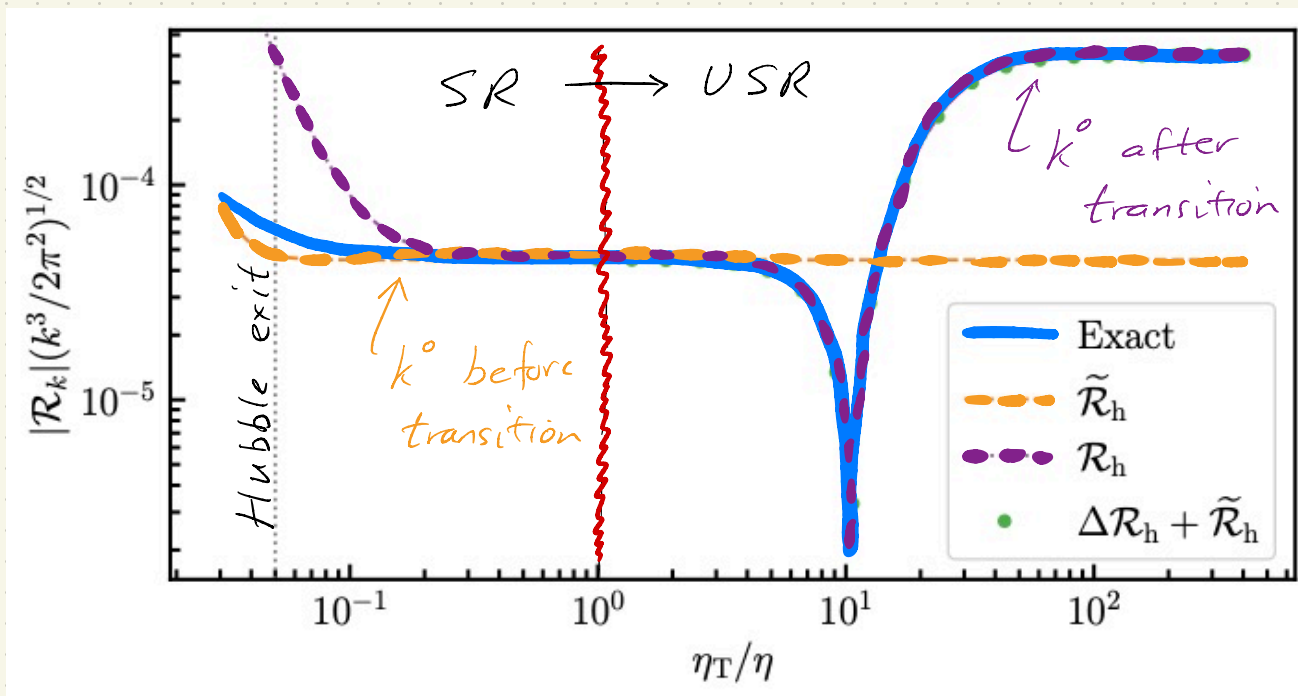
related to non-adiabatic pressure

$$\delta P_{\text{nad}} \propto \frac{v'}{\dot{\varphi}} e^{-3 \int H dt}$$

- e.g.
- slow roll $\dot{\varphi} \approx \text{constant} \Rightarrow \delta P_{\text{nad}} \propto e^{-3 \int H dt} \rightarrow 0$
 $\rightarrow$ Hamilton-Jacobi, $\tilde{H} = H(\tilde{\varphi})$
 - ultra-slow roll $\dot{\varphi} \propto e^{-3 \int H dt} \Rightarrow \delta P_{\text{nad}} \rightarrow \text{const.}$
 $\Rightarrow \tilde{H} = H(\tilde{\varphi}, \dot{\tilde{\varphi}})$ 2D phase space

The separate-universe approach and sudden transitions during inflation

Joseph H. P. Jackson,^a Hooshyar Assadullahi,^{a,b} Andrew D. Gow,^a Kazuya Koyama,^a Vincent Vennin,^{c,a} David Wands^a

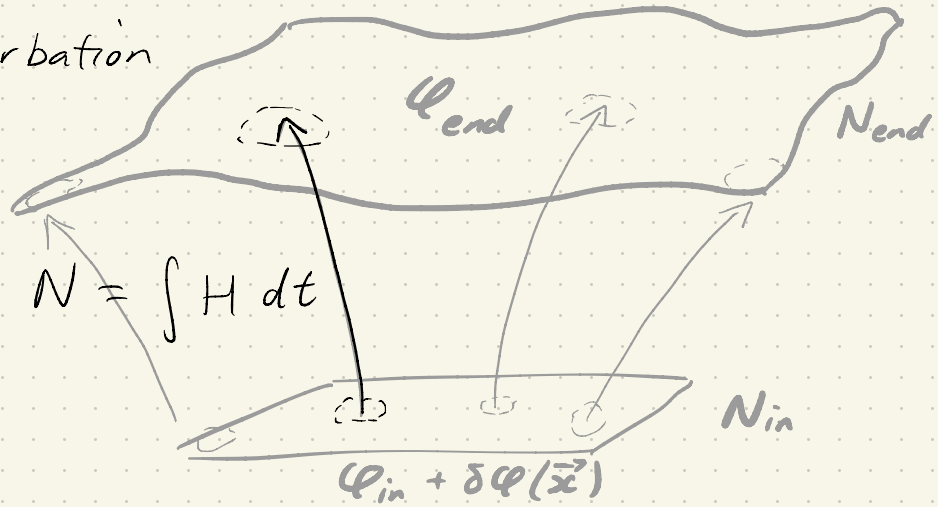


non-perturbative approach: δN formalism

primordial density perturbation

= perturbed expansion

$$\delta N(\vec{x}) = N(\vec{x}) - \bar{N}$$

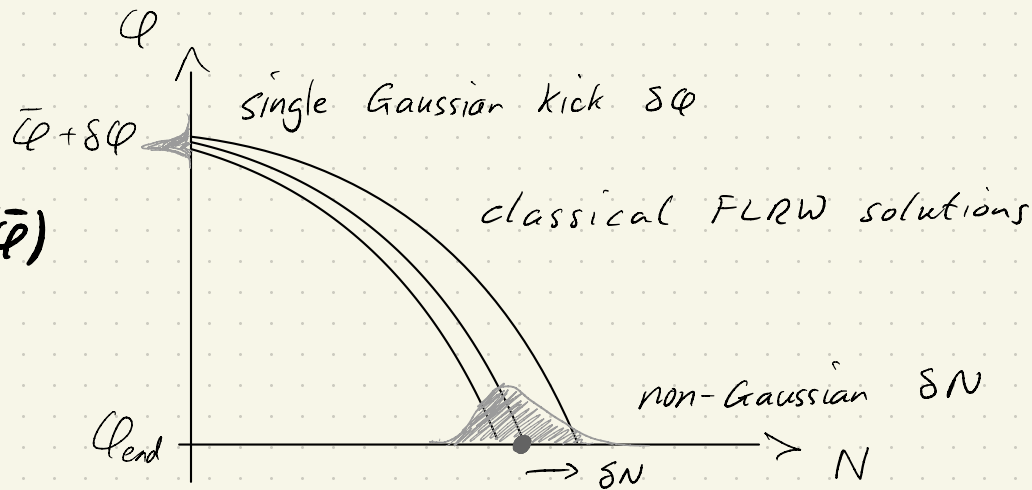


use non-linear FLRW equations on super-Hubble scales
"separate universe approach"

Classical δN

$$\delta N = N(\bar{\varphi} + \delta\varphi) - N(\bar{\varphi})$$

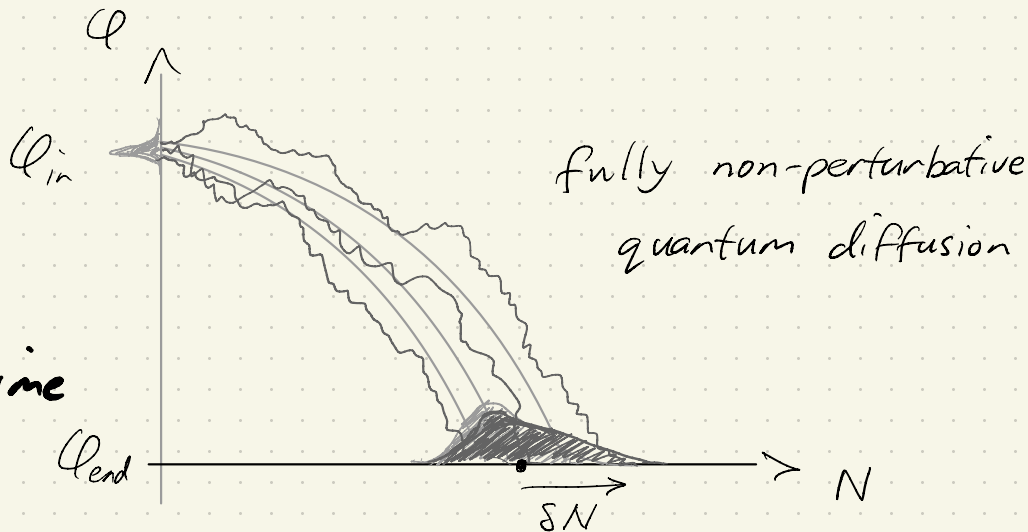
nonlinear classical
solution $N(\varphi)$



Stochastic δN

$$\delta N = N - \langle N \rangle$$

N = stochastic
first passage time
to cross φ_{end}

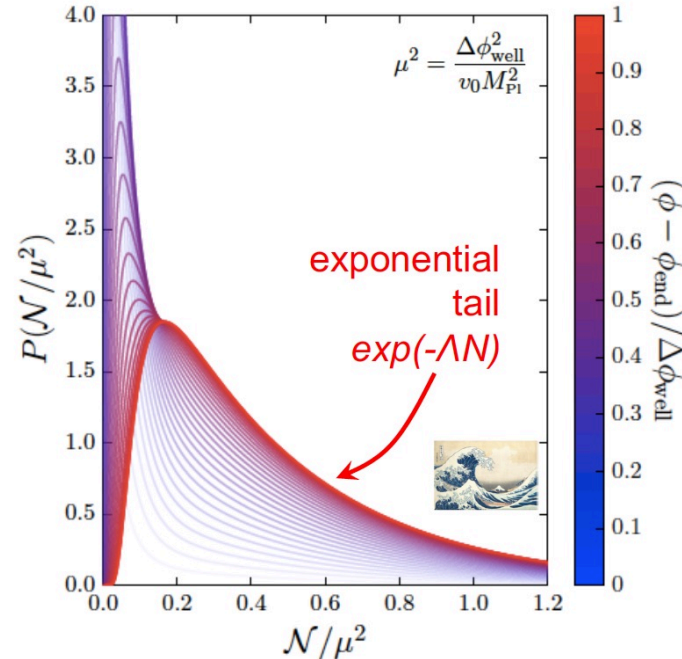
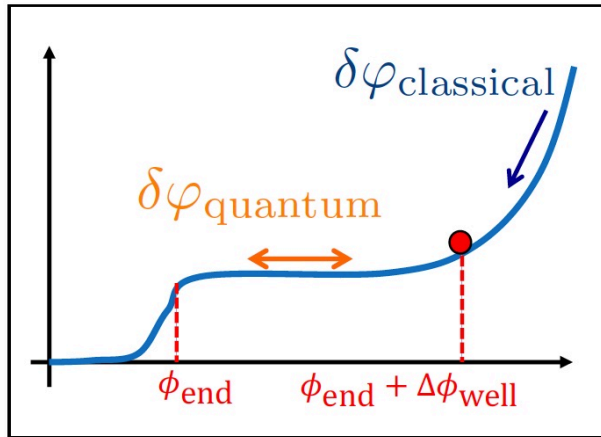


Full probability distribution function

Pattison, Vennin, Assadullahi & Wands (2017)



Stochastic diffusion leads to highly non-Gaussian pdf, $P(N)$:



probability distribution in general slow-roll has an exponential tail

Ezquiaga, Garcia-Bellido & Vennin (2019)

classical USR

take limit $V' \rightarrow 0$

$$\ddot{\varphi} + 3H\dot{\varphi} \approx 0 \Rightarrow \dot{\varphi} \propto e^{-3Ht}$$

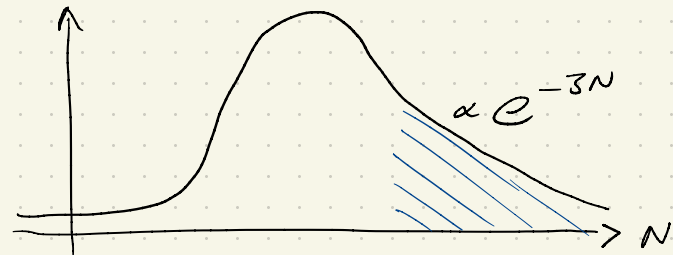
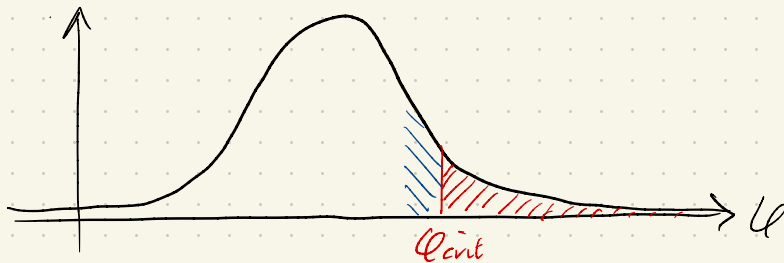
$$N(\varphi, \dot{\varphi}) = -\frac{1}{3} \ln \left(1 + \frac{3H(\varphi - \varphi_{\text{end}})}{\dot{\varphi}} \right)$$

Cai et al 2017

Pi & Sasaki 2022

$$N \rightarrow \infty \quad \text{as} \quad \varphi \rightarrow \varphi_{\text{crit}} = \varphi_{\text{end}} - \frac{\dot{\varphi}}{3H}$$

Gaussian $P(\varphi)$ $\longrightarrow$ exponential tail, $P(N) \propto e^{-3N}$



φ trapped if $\varphi - \varphi_{\text{end}} > -\dot{\varphi}/3H$ (incomplete)

stochastic USR

take limit $V' \rightarrow 0$

$$\ddot{\varphi} + 3H\dot{\varphi} \approx 0 \Rightarrow \dot{\varphi} \propto e^{-3Ht}$$

$$N(\varphi, \dot{\varphi}) = -\frac{1}{3} \ln \left(1 + \frac{3H(\varphi - \varphi_{\text{end}})}{\dot{\varphi}} \right)$$

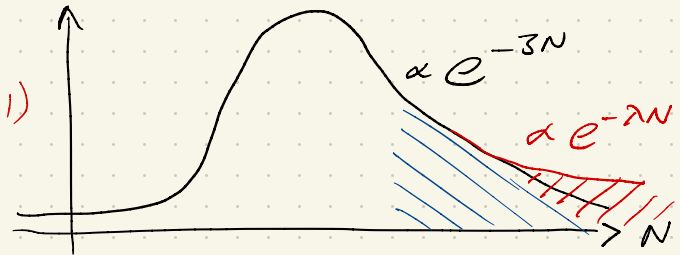
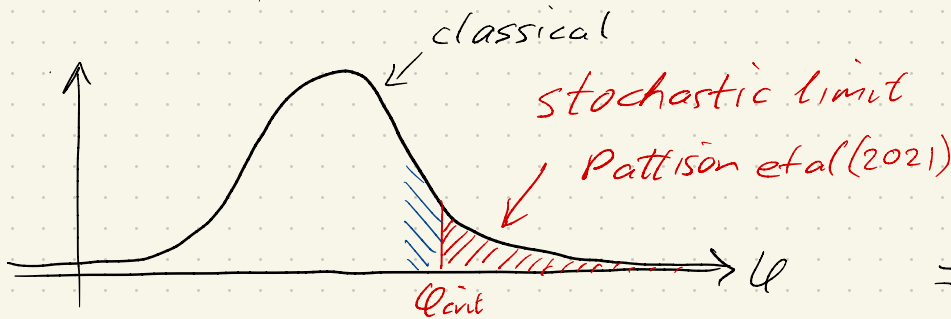
Cai et al 2017

Pi & Sasaki 2022

$$N \rightarrow \infty \quad \text{as} \quad \varphi \rightarrow \varphi_{\text{crit}} = \varphi_{\text{end}} - \frac{\dot{\varphi}}{3H}$$

Gaussian $P(\varphi)$

exponential tails, $P(N) \propto e^{-3N}$



diffusion takes over when $\varphi - \varphi_{\text{end}} > -\dot{\varphi}/3H$

Stochastic Ultra-Slow-Roll Inflation

Fironzjahi et al, Pattison et al (2021)

UV : linear mode functions on sub-Hubble scales

stochastic noise* as modes cross coarse-graining scale

IR : coarse-grained field obeys Langevin equations
for $\mathcal{Q}$ and $\Pi = a^3 \dot{\mathcal{Q}}$

$$\frac{d\mathcal{Q}}{dN} = \frac{\pi}{a^3 H} + \hat{\xi}_{\mathcal{Q}}$$

$$\frac{d\Pi}{dN} = -\frac{a^3 V'}{H} + \hat{\xi}_{\Pi}$$

* squeezed state

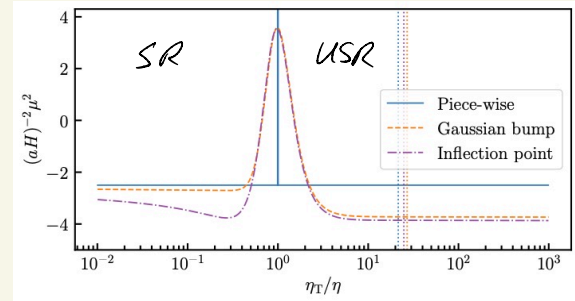
→ correlated noise
 $\hat{\xi}_{\Pi} \propto \hat{\xi}_{\mathcal{Q}}$

but not adiabatic
in USR

non-adiabatic transition SR $\rightarrow$ USR Starobinsky (1992) Jackson et al (2023)

mode equation $u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$

$u_k = z R_k = a \delta\phi_k$



general solution in SR / USR limit

$$\Rightarrow u_k \approx \frac{1}{\sqrt{2k}} \left\{ \alpha_k \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta} + \beta_k \left(1 + \frac{i}{k\eta} \right) e^{ik\eta} \right\}$$

before transition: BD vacuum, $\alpha_k = 1$, $\beta_k = 0$

after transition: particle production on sub-Hubble scales
non-adiabatic perturbations on super-Hubble scales

gradient terms
at sudden transition

$\alpha_k \neq 0$, $\beta_k \propto \left(\frac{k_\tau}{k} \right)^3$ for $k \ll k_\tau$

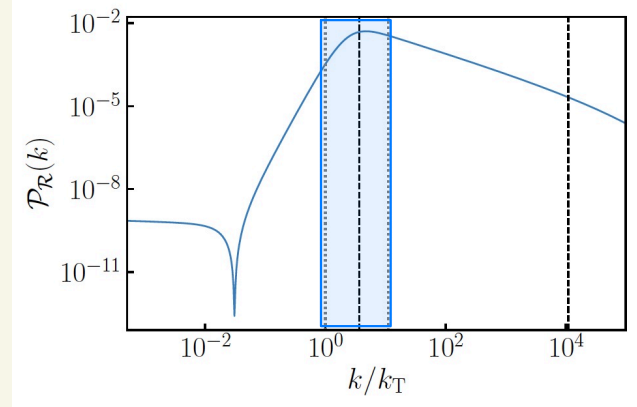
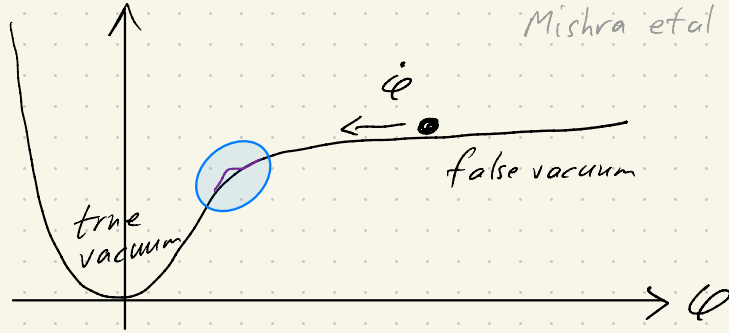
primordial density perturbations in stochastic VSR

single scalar field, $\varphi(t)$

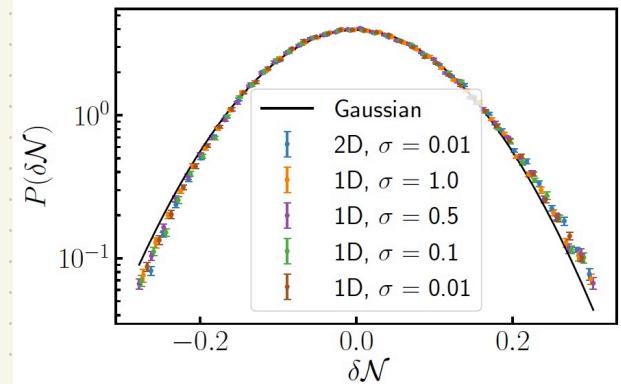
Jackson et al 2410.13683

potential energy, $V(\varphi)$ + tiny bump

Mishra et al (2024)



typical
fluctuations
 $\approx$ Gaussian



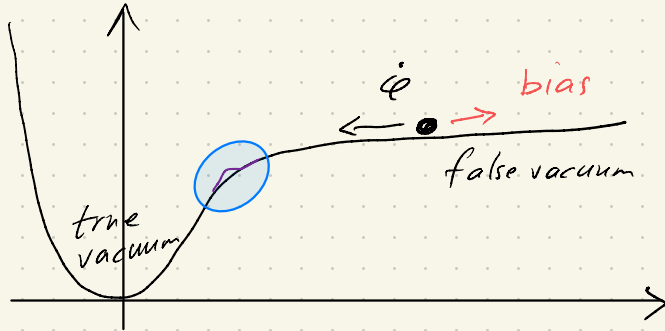
smoothing scale $k = z_{\text{atH}}$

importance sampling in stochastic USR

Jackson et al 2410.13683

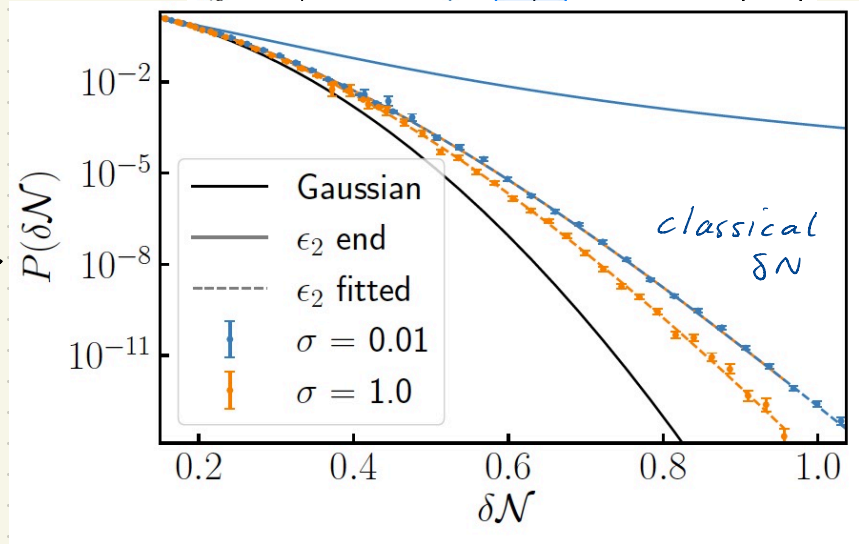
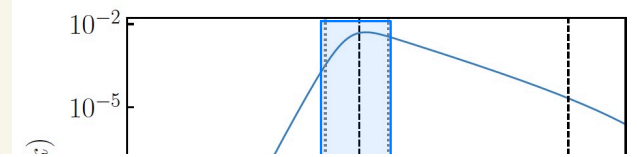
single scalar field, $\varphi(t)$

potential energy, $V(\varphi)$ + tiny bump



bias dynamics

→ rare fluctuations



but

- noise calculated only along classical background
- result depends on coarse-graining scale, $k = 3 \text{ aH}$

