Particle Physics in the Early Universe

Paris, September 17$^{\text{th}}$ 2026

Lucas Pinol, CNRS researcher, *LPENS, Paris*

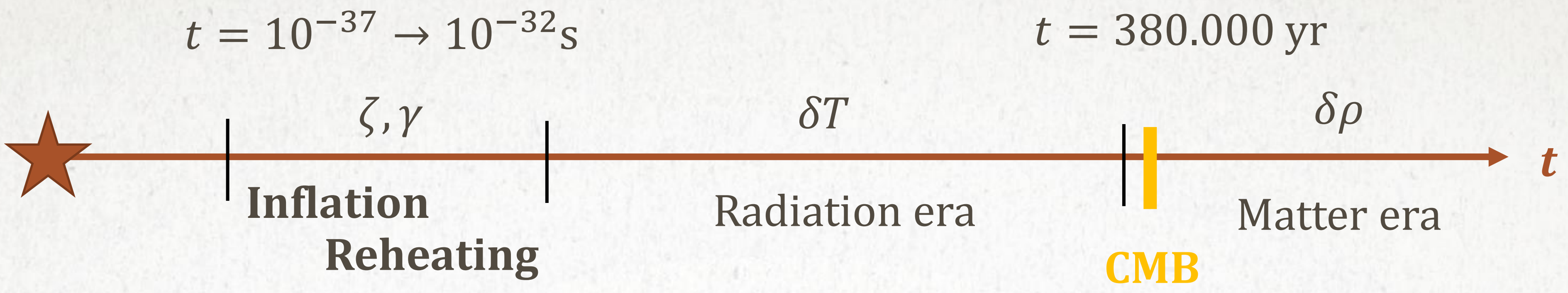
the primordial curvature perturbation

ONE-LOOP FREEZING OF ζ

AND RENORMALIZATION OF THE EFT OF INFLATIONARY FLUCTUATIONS

[M. Braglia, LP 2504.07926]
[M. Braglia, LP 2504.13136]
[M. Braglia, S. Cespedes, LP 2603.12216]
...

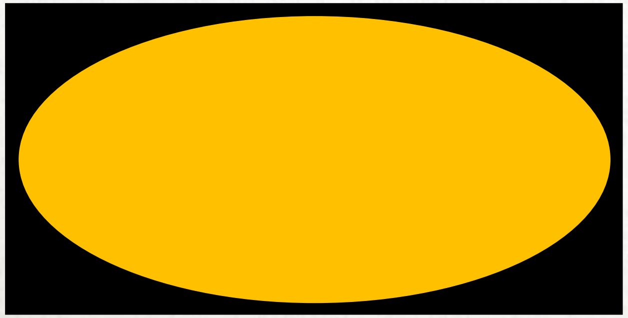
Early universe: the cosmological context



Scalar fluctuations ~ density fluctuations → $\zeta(t, \vec{x})$

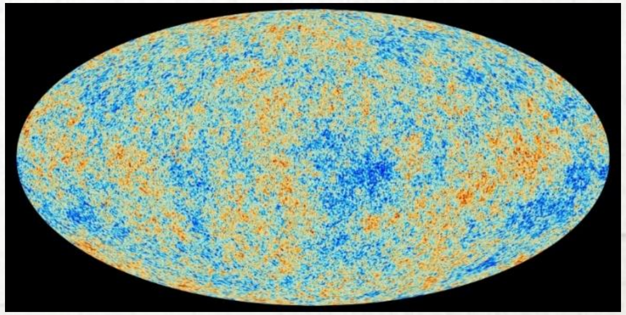
Tensor fluctuations ~ gravitational waves → $\gamma(t, \vec{x})$

Cosmic Microwave Background (CMB)



Penzias-Wilson (1964)

motivates → **Inflation** → predicts



Planck (2013)

Early universe: the high-energy viewpoint

Unique framework: general relativity + quantum field theory + precision data

CMB $\rightarrow \sqrt{\langle \zeta^2 \rangle} = (4.57 \pm \mathbf{0.02}) \times 10^{-5}$

$\log(E/\mathrm{GeV})$

Natural units: $\hbar = c = 1$ and the only dimension is energy

19 — M_{Pl} Planck mass

M_{s} String scale

16 — M_{GUT} Grand Unified Theories

14 —

??? — **H** **Inflation**

Scale of inflation is unknown
Particle content is unknown

3 — Standard Model, LHC

Accessible experiments on Earth

Inflation sensitive to high energies

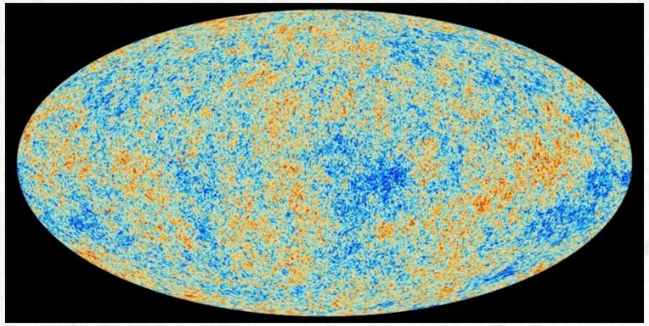
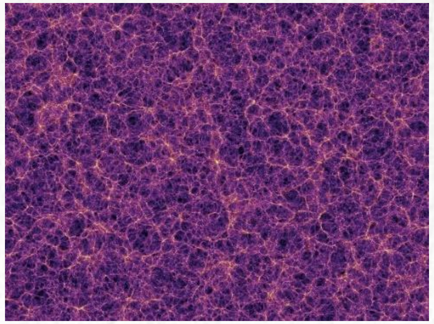
+

Precision data (current and future)

=

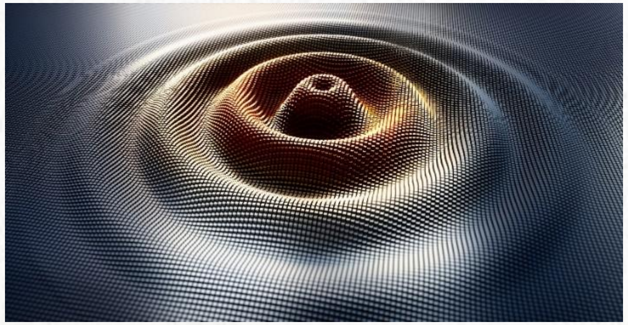
Formidable opportunity to test high-energy physics beyond the reach of terrestrial experiments

Early universe: the observational probes

Current $=$  $+$  (scalars)

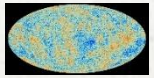
Cosmic Microwave Background

Large-Scale Structures

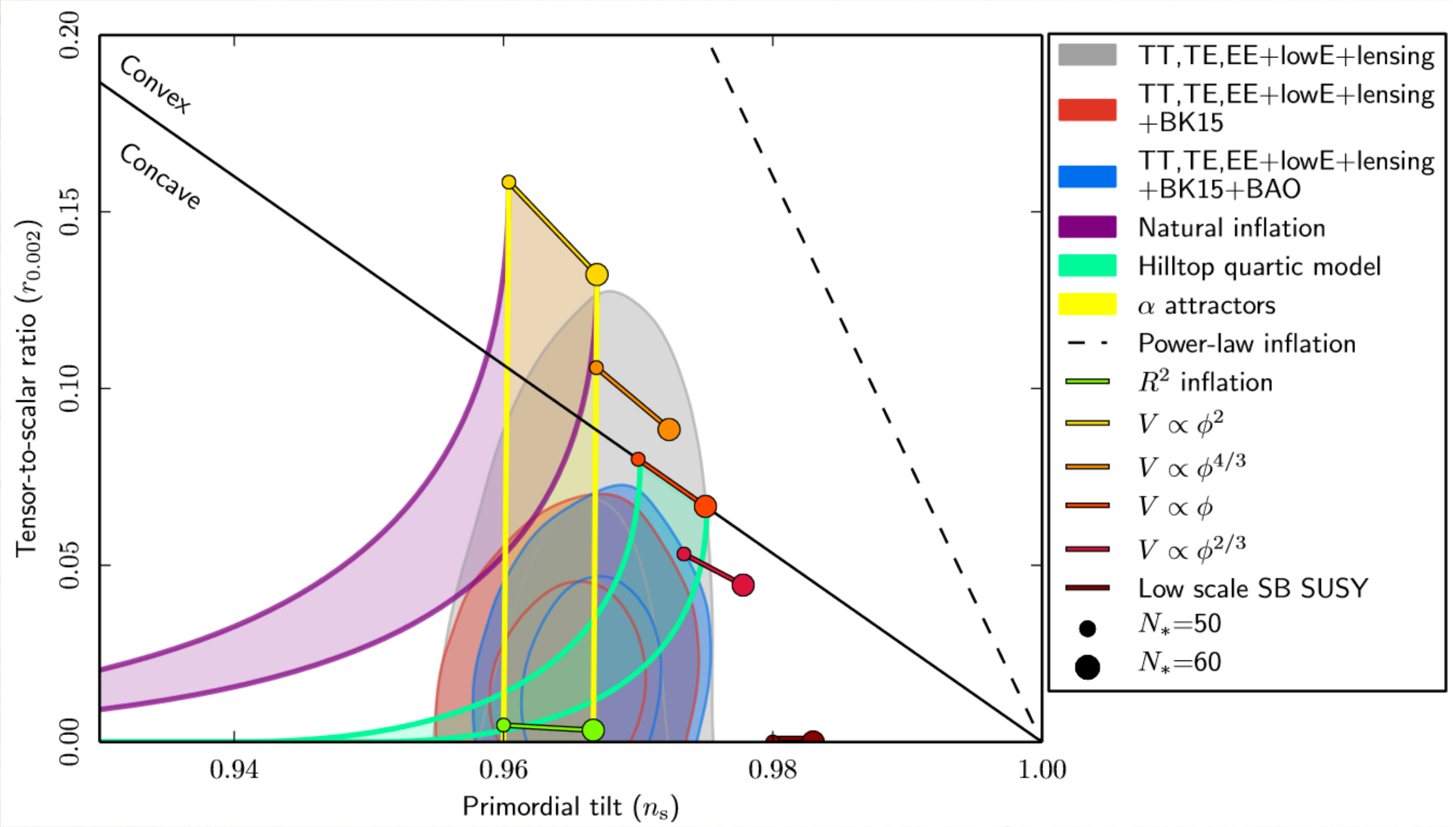
Future $=$  (tensors) ?

Primordial Gravitational Waves

Objects of study $=$ **primordial correlations functions**

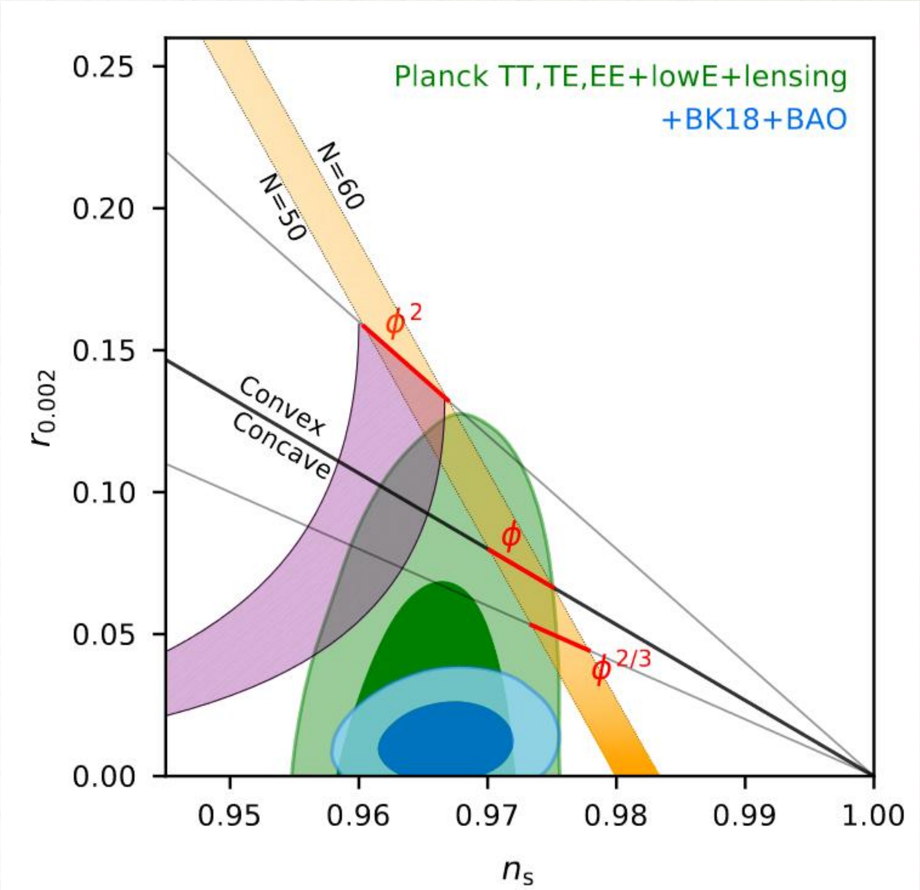
e.g. $\langle \zeta^2 \rangle \rightarrow \Big\langle$  $^2 \Big\rangle$; $\langle \gamma^2 \rangle \rightarrow \Big\langle$  $^2 \Big\rangle$; $\langle \zeta^3 \rangle \rightarrow \Big\langle$  $^3 \Big\rangle$; etc.

Early universe: current CMB constraints



[Planck 2018]

$n_s = 0.9649 \pm 0.0042$



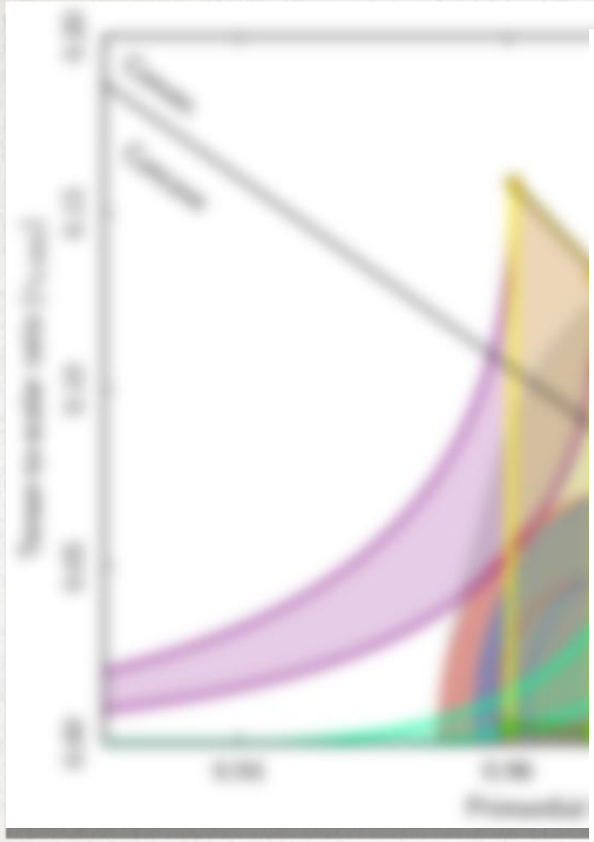
[BICEP-Keck 2022]

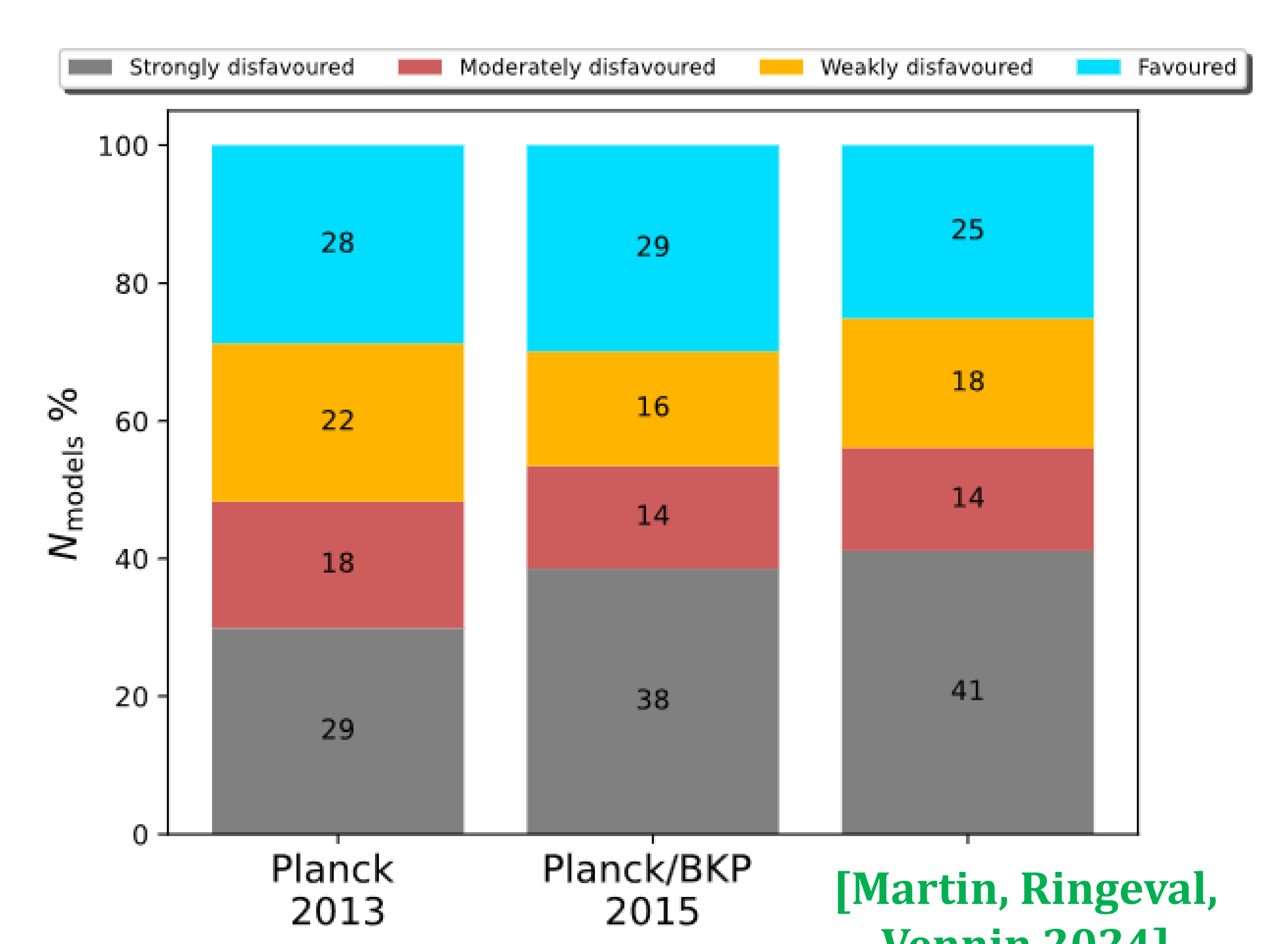
$r < 0.032$

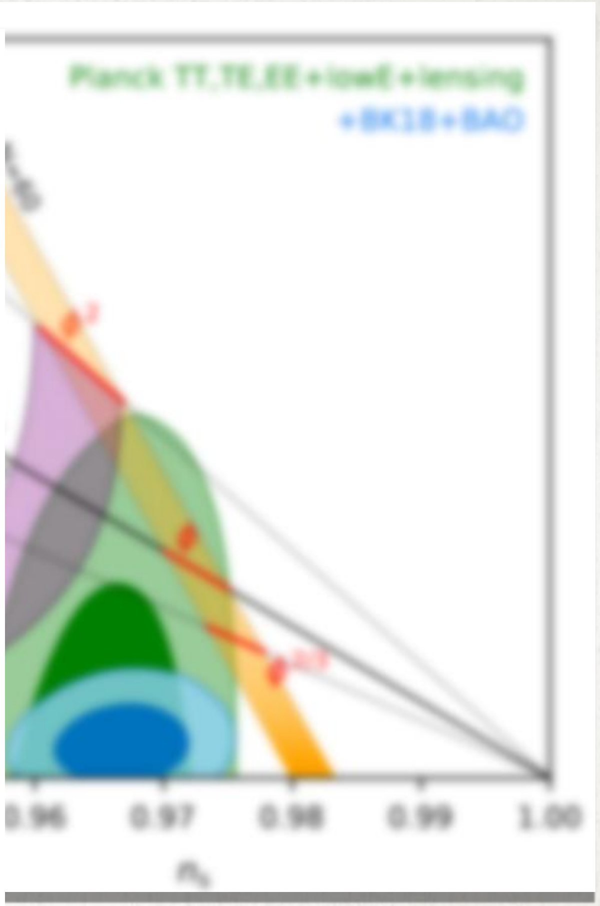
Many minimal single-field slow-roll models are now excluded

Great improvement!
« Planck-BICEP-Keck constraints »

Early universe: current CMB constraints







[Martin, Ringeval, Vennin 2024]

And indeed observations are winning over models

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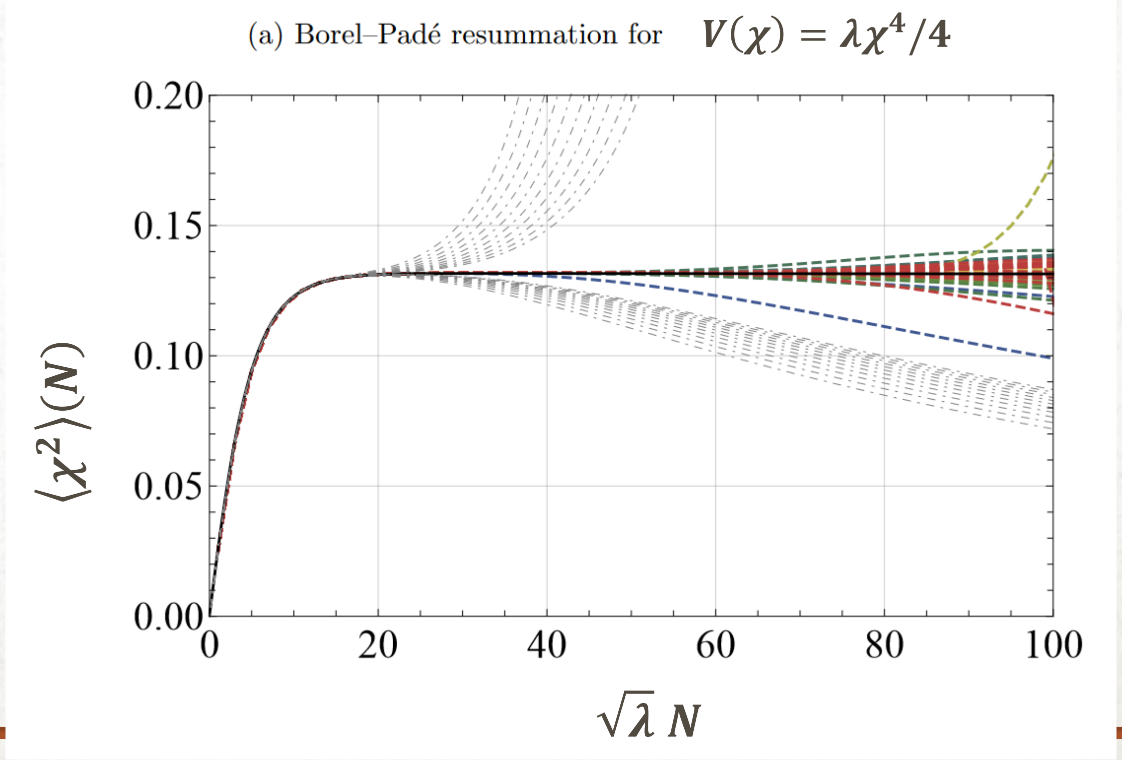
I. FROM TEST FIELDS TO ζ

Secular divergences
Stochastic inflation
Robustness of inflation



YET ANOTHER WEINBERG THEOREM

[Weinberg 2005]

- In-in formalism at any order in the interactions:

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t)\right] \cdots\right]\right]\right\rangle , \qquad (2)$$

[Weinberg 2006]

- Given the mode functions of the linear perturbation theory, and their commutators, he proves:

Weinberg theorem

The late-time, $t \to +\infty$, behavior of any correlation function, at any order in the interaction Hamiltonian, is proportional to the scale factor $a(t)$ at a power of <u>at most 0</u>.

SECULAR DIVERGENCES

What is left? $\rightarrow$ Logarithmic time divergences $\lim_{t\rightarrow+\infty}\langle Q(t)\rangle \ni \log a(t)\,, \log a(t)^2\,, \ldots$

Are secular divergences common?

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Are secular divergences common?

- Massive mode functions decay strictly faster than $a(t)^0$ $\rightarrow$ **no secular divergence**
- Derivative interactions introduce additional negative factors of $a(t)$ $\rightarrow$ **no secular divergence**

$\rightarrow$ **Mostly relevant to massless fields with non-derivative (e.g. test field) or mixed interactions (e.g. gravity)**

Are they physical?

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Are they physical?

It will be interesting to see if the power series in $\log a(t)$ encountered in calculating cosmological correlation functions at time t [...], can be summed [...]. [Weinberg 2006]

STOCHASTIC INFLATION FOR TEST FIELDS

An example where the secular growth is physical and indeed can be summed

- Consider a test massless interacting scalar field minimally coupled to gravity: $\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}\chi^4$
- Suppose background spacetime is de Sitter: $\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2Ht}\mathrm{d}\vec{x}^2$

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- Two-point function at one loop reads: $\langle \chi^2 \rangle(t) = \left(\frac{H}{2\pi}\right)^2 \log a(t) \left[1 - \frac{\lambda}{6\pi^2} \log a(t)^2 + \cdots \right]$

[Tsamis, Woodard 2005]

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$$\langle\chi^2\rangle(t) = \left(\frac{H}{2\pi}\right)^2 \log a(t) \left[1 - \frac{\lambda}{6\pi^2}\log a(t)^2 + \frac{\lambda^2}{20\pi^4}\log a(t)^4 + \cdots\right] \underset{t\to\infty}{\longrightarrow} \sqrt{\frac{3}{2\pi^2}}\frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{H^2}{\sqrt{\lambda}}$$

[Starobinsky, Yokohama 1994]

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[Starobinsky, Yokohama 1994]

You can get any order, first closed formula in [Honda, Jinno, LP, Tokeshi 2023]

But... The series is divergent! $a_k \sim k!$

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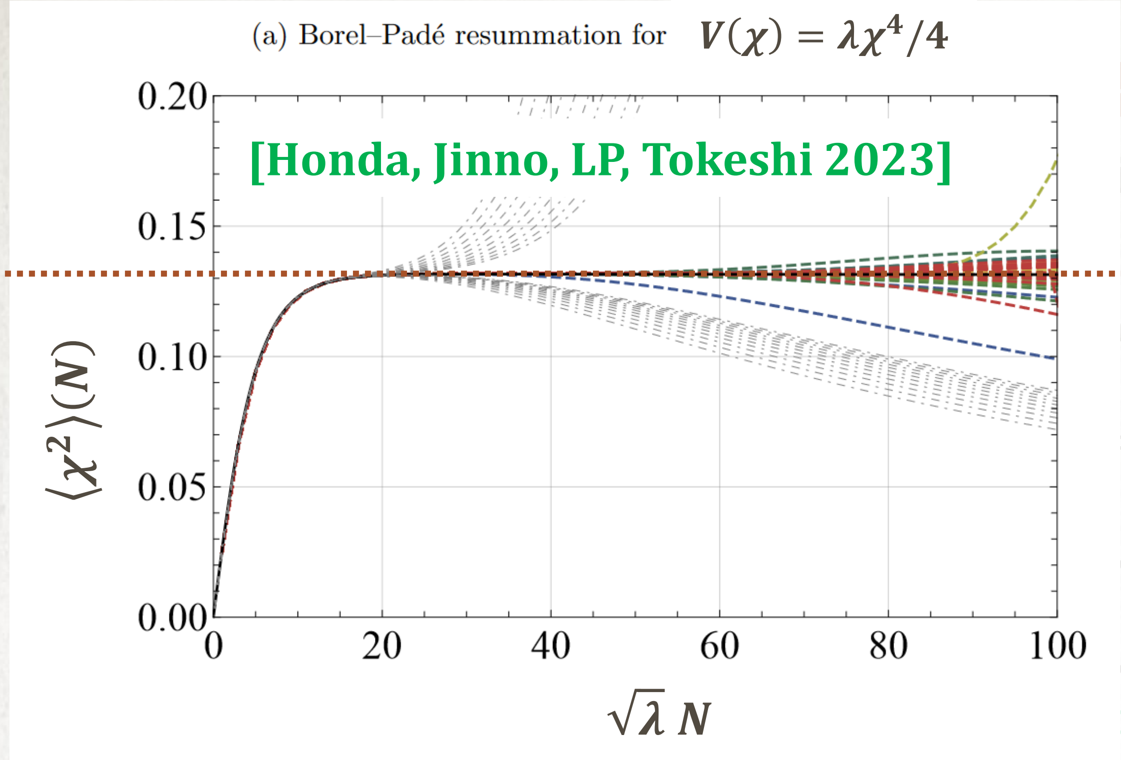
The divergent series can be summed via - the equilibrium PDF [Starobinsky, Yokohama 1994]

[Honda, Jinno, LP, Tokeshi 2023]



TEST FIELDS

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imally coupled to gravity: $\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}\chi^4$

$= -dt^2 + e^{2Ht}d\vec{x}^2$

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The divergent series can be summed via - the equilibrium PDF **[Starobinsky, Yokohama 1994]**

- Borel summation **[Honda, Jinno, LP, Tokeshi 2023]**

transition regime included!

ROBUSTNESS OF INFLATION

Does the curvature perturbation develop secular divergences?

- Inflationary spacetime is classically and linearly stable
- First non-linear classical correction → primordial non-Gaussianities

[Maldacena 2003]

$\boldsymbol{B_\zeta^0(k_1, k_2, k_3)}$ *No secular divergences*

$\boldsymbol{t \to \infty}$

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle (t) = (2\pi)^3 \delta^{(3)}\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right) B_\zeta(k_1, k_2, k_3; t)$$

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- First non-linear quantum correction: $\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\right\rangle(t) = (2\pi)^3\delta^{(3)}\left(\vec{k}_1+\vec{k}_2\right)\left[P_\zeta^{\mathrm{tree}}(k_1;t) + P_\zeta^{1-\mathrm{loop}}(k_1;t)+\cdots\right]$

$\boldsymbol{t\to\infty}$ $\boldsymbol{P_\zeta^0(k_1)}$ $\boldsymbol{t\to\infty}$??? $\boldsymbol{t\to\infty}$

No secular divergences *Are there secular divergences?*

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- Imagine $P_\zeta\ (k_1;t) \ni \log a(t)^N$ →
  - ❖ Inflationary spacetime secularly unstable
    could actually provide a natural end
  - ❖ Breakdown of perturbation theory?
    the framework to make calculations becomes wrong
  - ❖ Loss of predictivity through reheating and Big Bang cosmology
    absence of adiabatic limit implies model dependence

II. THE EFT OF INFLATIONARY FLUCTUATIONS

Model-(in)dependence

Free theory

Leading cubic and quartic gravitational interactions

Matter content + symmetries $\longrightarrow$ **Interactions (some fixed by non-linearly realized symmetries)**

EFT FOR THE ADIABATIC FLUCTUATION

The unitary gauge

- Inflation ↔ time-translation symmetry breaking: $H_0 \to H(t)$

We can expect a (pseudo) Nambu-Goldstone boson

extrinsic spatial curvature

We build an EFT from Lorentz-violating blocks: $g^{\mu\nu}, g^{0\mu}, g^{00}, \boldsymbol{K_{ij}}, \boldsymbol{R_{ij}^{(3)}}, \nabla_\mu, f(t)$

intrinsic spatial curvature

This is called the **unitary gauge**: the NG boson is hidden in the metric

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This is an EFT for the fluctuations only

$\neq$

Covariant EFT "a la Weinberg" $R + R^2 + \cdots$

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- Most general unitary gauge Lagrangian compatible with an FLRW background:
  [Creminelli et al. 2006] [Cheung et al. 2008]

$$S = \int \mathrm{d}^4x \sqrt{-g}\,\mathcal{L}\,, \quad \text{with}$$

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\left(\dot{H} + 3H^2\right)\right] + M_{\rm Pl}^2 \dot{H} g^{00} + F^{(2)}\left(\delta g^{00}, \delta K_{ij}, \delta^{(3)}R_{ij}; \nabla_\mu; t\right)$$

EFT FOR THE ADIABATIC FLUCTUATION

The derivative expansion

- Higher number of derivatives brings higher powers of H/Λ in the observables

- Given a sought precision, one can truncate the EFT to some derivative order

- Non-derivative order: $F^{(2)} \supset \sum_{n=2}^{\infty} M_n^4 \frac{(\delta g^{00})^n}{n!}$

- Higher orders: $F^{(2)} \supset -\bar{M}_1^3 \delta g^{00} \delta K - \bar{M}_2^2 \delta K_{ij} \delta K^{ij} - \bar{M}_3^2 \delta K^2 + \bar{m}_1^2 (\nabla^0 \delta g^{00})^2 + \bar{m}_1^2 \delta g^{00} \delta R^{(3)} + \cdots$

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Example: Suppose there is a unique scale Λ at which new physics enter (background excitation, new particle, etc.)

$$\delta K \sim H \times \delta g, \delta R^{(3)} \sim H^2 \times \delta g \quad \rightarrow \quad M_2^4 (\delta g^{00})^2 \sim \Lambda^4 \times \delta g^2$$

$$\bar{M}_1^3 \delta g^{00}\, \delta K \sim \Lambda^3 H \times \delta g^2$$

$$\bar{M}_3^2 \delta K^2 \sim \Lambda^2 H^2 \times \delta g^2$$

smaller

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[Braglia, LP 2504.07926]
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In these works, (for now) we
- **truncate** (theory assumption) at leading non-derivative order
- **fine-tune** (model dependence) $M_3 = 0 = M_4$

EFT FOR THE ADIABATIC FLUCTUATION

The Stückelberg trick

$$M_2(t)^4 \frac{(\delta g^{00})^2}{2}$$

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\left(\dot{H} + 3H^2\right)\right] + M_{\rm Pl}^2 \dot{H} g^{00} + \overbrace{F^{(2)}\left(\delta g^{00}, \delta K_{ij}, \delta^{(3)}R_{ij}; \nabla_\mu; t\right)}$$

Stückelberg trick = introduce explicitly the Goldstone boson π to restore diffeomorphism invariance

- Define $\pi(t, \vec{x})$ such that $t + \pi(t, \vec{x})$ is invariant under full diffeomorphisms

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- Still a gauge freedom → flat gauge → $\zeta = -H\pi + (\dot{H}\pi^2)/2 - (\ddot{H}\pi^3)/6 + \cdots$
- $t \to t + \pi(t,\vec{x}) \;\; \Rightarrow \; \delta g^{00} \to \delta g^{00}_{\rm flat} + 2\delta g^{0i}_{\rm flat}\partial_i\pi \underbrace{- 2\dot{\pi} - \dot{\pi}^2 + (\partial\pi)^2/a^2}$

non-linearly realized symmetry (consequence of GR diffeomorphism invariance)

EFT FOR THE ADIABATIC FLUCTUATION

The Stückelberg trick

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\left(\dot{H} + 3H^2\right)\right] + M_{\rm Pl}^2 \dot{H} g^{00} + \overbrace{F^{(2)}\left(\delta g^{00}, \delta K_{ij}, \delta^{(3)} R_{ij}; \nabla_\mu; t\right)}^{\boxed{M_2(t)^4 \frac{\left(\delta g^{00}\right)^2}{2}}}$$

Stückelberg trick = introduce explicitly the Goldstone boson π to restore diffeomorphism invariance

- Define $\pi(t,\vec{x})$ such that $t + \pi(t,\vec{x})$ is invariant under full diffeomorphisms
- Still a gauge freedom → flat gauge → $\zeta = -H\pi + (\dot{H}\pi^2)/2 - (\ddot{H}\pi^3)/6 + \cdots$
- $t \to t + \pi(t,\vec{x}) \;\Rightarrow\; \delta g^{00} \to \underbrace{\boldsymbol{\delta g^{00}_{\rm flat}}}_{\textit{lapse } \delta N} + 2\underbrace{\boldsymbol{\delta g^{0i}_{flat}}}_{\textit{shift } \delta N^i}\partial_i \pi - 2\dot{\pi} - \dot{\pi}^2 + (\partial\pi)^2/a^2$
- Decoupling limit: **lapse** and **shift** after Stückelberg give contributions $\propto \epsilon \ll 1$ → negligible in the EFT

$$\epsilon = -\dot{H}/H^2$$
$$\eta = \dot{\epsilon}/(\epsilon H)$$
$$\eta_2 = \dot{\eta}/(\eta H)$$

EFT FOR THE ADIABATIC FLUCTUATION

Summary

- Starting point EFT: m... ...metries and no matter fields
- Assum...
- Fine-tuning...
- Decoupling limit (technical assumption, not required): $\epsilon \ll 1$

Primordial non-Gaussianities:

$$f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth} \propto \left(\frac{1}{c_s^2} - 1\right) \; ; \quad f_{\rm NL}^{\rm loc} \propto (-\eta - s)$$

Non-observations: $c_s^2 > 0.02$

[Braglia, LP 2504.07926]

$$\mathcal{L}^{\pi,(3)}_{\rm decoup.} = -\frac{\epsilon H^3 M_{\rm Pl}^2}{c_s^2}\left[\frac{f_0 c_s^2}{H}\dot{\pi}\frac{(\partial_i \pi)^2}{a^2} + \frac{f_1}{H}\dot{\pi}^3 + f_2 \dot{\pi}^2 \pi + f_3 c_s^2 \pi \frac{(\partial_i\pi)^2}{a^2}\right],$$

$$\text{with} \quad f_0 = \frac{1}{c_s^2} - 1\,, \quad f_1 = c_s^2 - 1\,, \quad f_2 = -\eta - s(1 - c_s^2)\,, \quad \text{and} \quad f_3 = \eta$$

Matches:

- $P(X,\phi)$ Lagrangians

[Burrage, Ribeiro, Seery, 2011]

- Multi-field Lagrangians after integrating out isocurvature fluctuations

[Garcia-Saenz, LP, Renaux-Petel, 2019]

[LP 2020]

EFT FOR THE ADIABATIC FLUCTUATION

Summary

$$\epsilon = -\dot{H}/H^2$$
$$\eta = \dot{\epsilon}/(\epsilon H)$$
$$\eta_2 = \dot{\eta}/(\eta H)$$

- Starting point EFT: most general Lagrangian compatible with symmetries and no matter fields
- Assumption about the UV physics: a single high-energy scale Λ for new physics
- Fine-tuning (technical assumption, not required): $M_3 = 0 = M_4$

$$\mathcal{L}^{\pi,(4)}_{\text{decoup.}} = \frac{\epsilon H^4 M_{\rm Pl}^2}{c_s^2}\left\{\frac{g_0 c_s^2}{H^2}\left[\frac{(\partial_i\pi)^2}{a^2}\right]^2 + \frac{g_1 c_s^2}{H^2}\dot{\pi}^2\frac{(\partial_i\pi)^2}{a^2} + \frac{g_2}{H^2}\dot{\pi}^4 + \frac{g_3}{H}\pi\dot{\pi}^3 + \frac{g_4 c_s^2}{H}\pi\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} + g_5\pi^2\dot{\pi}^2 + g_6 c_s^2\pi^2\frac{(\partial_i\pi)^2}{a^2}\right\},$$

[Braglia, LP 2504.07926]

with
$$g_0 = \frac{1}{4}\left(\frac{1}{c_s^2}-1\right), \quad g_1 = -\frac{1}{2}\left(\frac{1}{c_s^2}-1\right), \quad g_2 = \frac{1-c_s^2}{4}, \quad g_3 = (1-c_s^2)(\eta+s),$$
$$g_4 = -\left(\frac{1}{c_s^2}-1\right)(\eta+s), \quad g_5 = \frac{\eta(\eta+\eta_2)}{2} + (1-c_s^2)s\,(\eta+s+s_2), \quad g_6 = -\frac{\eta(\eta+\eta_2)}{2}$$

$$\epsilon = -\dot{H}/H^2$$
$$\eta = \dot{\epsilon}/(\epsilon H)$$
$$\eta_2 = \dot{\eta}/(\eta H)$$

EFT FOR THE ADIABATIC FLUCTUATION

Free theory and interactions → "Feynman-like" rules

$$\mathcal{L}^{\pi,(2)}_{\text{decoup.}} = \frac{\epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right] \text{ with } \frac{1}{c_s^2} - 1 = \frac{2M_2^4}{\epsilon H^2 M_{\text{Pl}}^2}$$

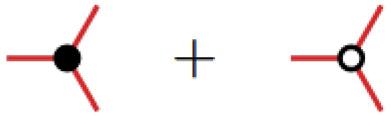
Speed of sound $0 < c_s^2 \leq 1$

$$\pi_k^I(\tau) = \frac{1}{\sqrt{4\epsilon c_s k^3} M_{\text{Pl}}}(1 + ic_s k\tau)\, e^{-ic_s k\tau}$$

Mode functions / propagators

$$a\mathcal{H}^{(3)}_{\text{int}} = -a^2\epsilon\eta H^3 M_{\text{Pl}}^2\left[\frac{1}{c_s^2}\pi\pi'^2 - \pi(\partial_i\pi)^2\right]$$

$$= -\mathcal{L}^{(3)}$$



Leading cubic gravitational interactions

those surviving in the limit $c_s^2 \to 1$, $\epsilon \ll \eta$

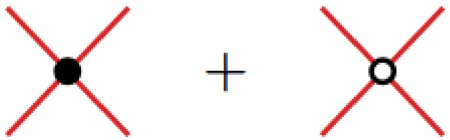
$$a\mathcal{H}^{(4)}_{\text{int}} = \frac{a^2}{2}\epsilon\eta H^4 M_{\text{Pl}}^2\left[\frac{\eta - \eta_2}{c_s^2}\pi^2\pi'^2 + (\eta + \eta_2)\pi^2(\partial_i\pi)^2\right]$$

Leading quartic gravitational interactions

those surviving in the limit $c_s^2 \to 1$, $\epsilon \ll \eta, \eta_2$

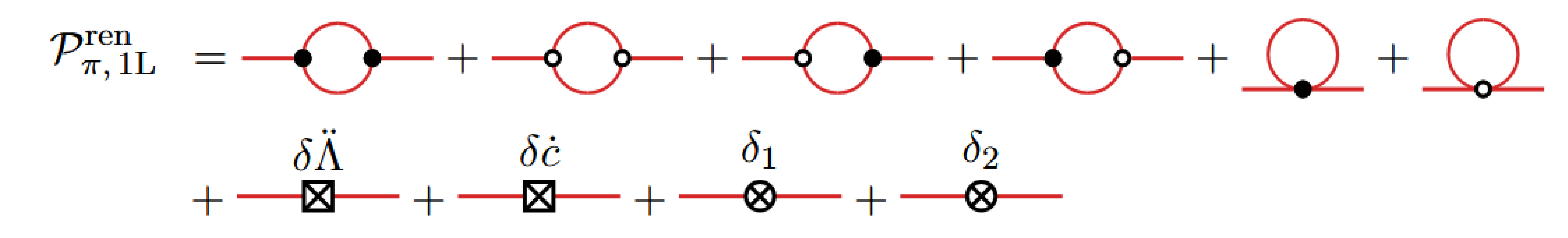
⚠ $\neq -\mathcal{L}^{(4)}$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]



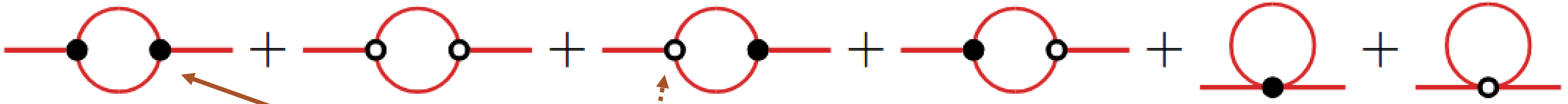
III. LOOP CALCULATION

Bare contributions
Renormalization: tadpoles and UV divergences
Discussion



BARE CONTRIBUTIONS

UV divergences and dimensional regularization



$$a\mathcal{H}_{\rm int}^{(3)} = -a^2 \epsilon\eta H^3 M_{\rm Pl}^2 \left[\frac{1}{c_s^2}\pi\pi'^2 - \pi(\partial_i\pi)^2\right]$$

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All these diagrams are IR-divergent + UV-divergent + secular-divergent

To manipulate finite quantities, we **regulate** them →

- Comoving IR cutoff in Fourier space $k > \Lambda_{\rm IR}$
- Dimensional regularization $d = 3 + \delta$ spatial dimensions
- Finite external super-horizon time $x = -p\,\tau$

BARE CONTRIBUTIONS

UV divergences and dimensional regularization

Modifies both the mode functions and the Fourier integrals

$$\pi_k(\tau) = \frac{\sqrt{\pi} e^{i\pi\delta/4} c_s^{-(1+\delta)/2}}{2\sqrt{2\epsilon}} \frac{1}{M_{\rm Pl}} \left(\frac{H}{\mu}\right)^{\delta/2} \frac{(-c_s k \tau)^{(3+\delta)/2}}{k^{(3+\delta)/2}} H^{(1)}_{(3+\delta)/2}(-c_s k \tau)$$

$$\int \frac{\mathrm{d}^3 \vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \int \frac{\mathrm{d}^d k}{(2\pi)^d} f(\vec{k}) = \Omega_{d-3} \int_0^\infty \mathrm{d}k\, k^{d-1} \int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta \int_0^\pi \mathrm{d}\varphi \sin^{d-3}\varphi\, f(k, \theta, \varphi)$$

All these diagrams are IR-divergent + UV-divergent + secular-divergent

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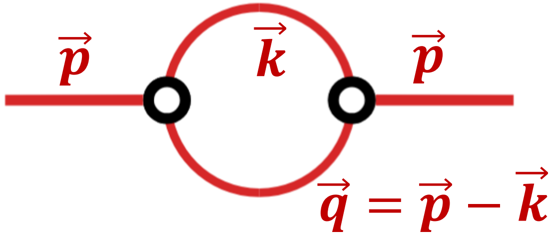
BARE CONTRIBUTIONS

Strategy and results



- Use dimensionless variables, expand the mode function to linear order in δ, perform time and angular integrals, **[Senatore, Zaldarriaga 2009]**

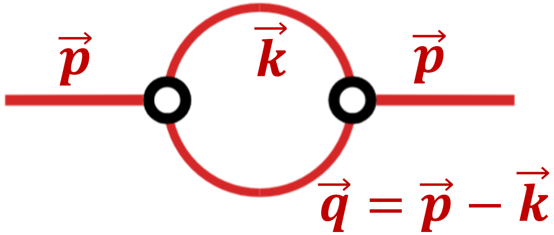
  introduce a fake UV cutoff to expand the integrand at infinity and extract the pole, introduce a comoving IR cutoff **[Ballesteros, Gambín Egea, Riccardi 2024]**

Also presented by Riccardi at "Looping in the primordial universe" workshop I co-organized at CERN in Nov. 2024

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

BARE CONTRIBUTIONS

Strategy and results



$$x \equiv -p\tau$$

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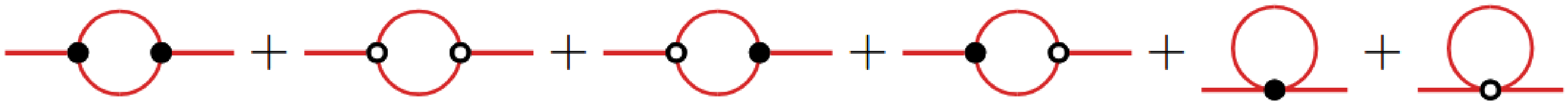
- $\mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{bare}}(x) =$

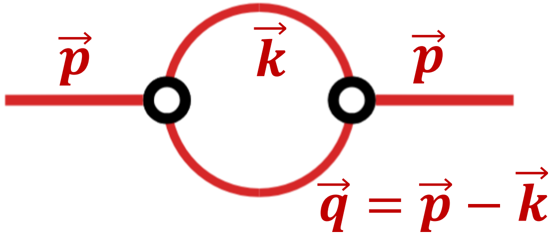


$$= \mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{b,UV}}(x) + \mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{b,fin}}(x) + \mathrm{IR\ divergences}$$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

BARE CONTRIBUTIONS

Strategy and results



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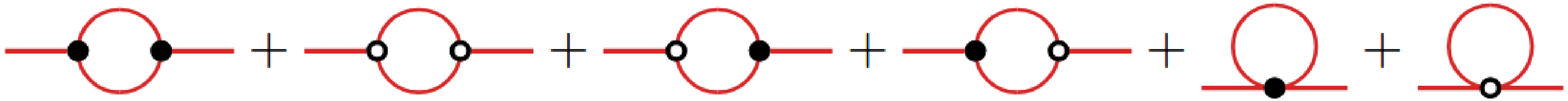
- $\mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{bare}}(x) =$ 

$$= \boldsymbol{\mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{b,UV}}(x)} + \mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{b,fin}}(x) + \mathrm{IR\ divergences}$$

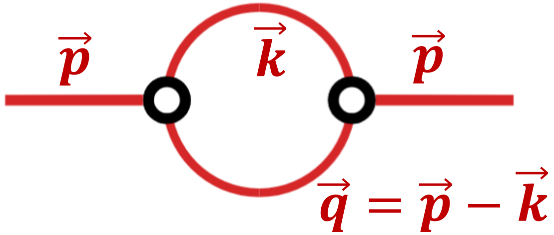
$$\boldsymbol{\mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{b,UV}}(x)} = -\frac{1}{\delta}(1 + c_s^2 x^2)\mathcal{P}_{\pi,0}^{\mathrm{tree}} H^2 \,\frac{\eta(\eta - \eta_2)}{4}$$

→ **The UV divergence is time-dependent!**

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

BARE CONTRIBUTIONS

Strategy and results



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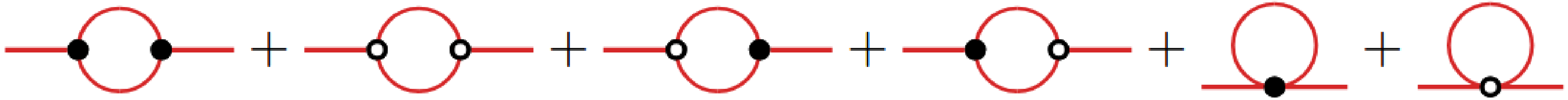
- $\mathcal{P}^{\text{bare}}_{\pi,1\text{L}}(x) =$ 

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$$\lim_{x\to 0} \boldsymbol{\mathcal{P}^{\text{b,fin}}_{\pi,1\text{L}}(x)} \sim -\mathcal{P}^{\text{tree}}_{\pi,0} H^2 \,\frac{\eta(2\eta - \eta_2)}{2} \log(x)$$

Sum of individual secular divergences not vanishing!

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Cancelling UV divergences

- We need quadratic counterterms to absorb the **time-dependent** UV divergences

At lowest order in derivatives we can renormalize the speed of sound $\left(\frac{1}{c_s^2}-1\right)\dot{\pi}^2=\left(\frac{1}{c_{s,\mathrm{ren}}^2}-1\right)\dot{\pi}^2+\delta_{c_s^2}\dot{\pi}^2$

The higher-order derivatives are degenerate, a minimal set is **[Bordin, Cabass, Creminelli, Vernizzi 2017]**

$$\mathcal{L}\supset-\frac{\bar{M}_3^2}{2}\delta K^2+\frac{m_3^2}{2}h^{ij}\left(\nabla_i\delta g^{00}\right)\left(\nabla_j\delta g^{00}\right)\to-\delta_1\frac{(\partial^2\pi)^2}{a^4}-\delta_2\frac{(\partial_i\dot{\pi})^2}{a^2}+\dots$$

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- Counterterms contribute as: $\mathcal{P}_{\pi,\,1\mathrm{L}}^{\mathrm{c.t.},(\delta_i)}(x) = \sum_{i=c_s^2,1,2}$ δ_i  $= \sum_i \delta_i\, f_i(x)$

We can tune the δ_i so as to cancel UV divergences **at all times**: $\delta_1 = c_s^2\delta_2, \qquad \delta_2 = -\frac{1}{\delta}\frac{H^2}{M_{\mathrm{Pl}}^2}\frac{\eta(\eta-2\eta_2)}{8\pi^2 c_s}$

$\delta_{c_s^2} = 0$

[Braglia, LP 2504.07926]
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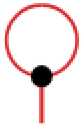
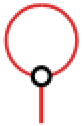
- **No additional secular divergence…** $\delta_{c_s^2}=0$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Tadpole cancellation must be enforced

- We forgot a set of diagrams! **[Pimentel, Senatore, Zaldarriaga 2012]**

$\langle \pi_{\vec{p}}(\tau) \rangle'_{\text{bare}} =$  $+$  $\neq 0$

[Braglia, LP 2504.07926]
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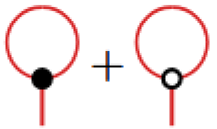
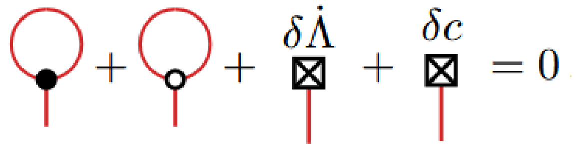


$$\neq 0$$

- We introduce one-point counterterms to cancel them, in the unitary gauge so they respect the EFT symmetries

$$a\mathcal{H}^{(1)}_{\rm c.t.} = a^4 M_{\rm Pl}^2\, \delta\dot{\Lambda}\, \pi - 2a^4\, \delta c\, \dot{\pi}$$



$$\mathcal{L} \supset -g^{00}\delta c(t) - M_{\rm Pl}^2 \delta\Lambda(t)$$

[Braglia, LP 2504.07926]
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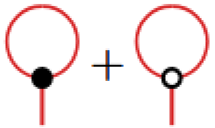

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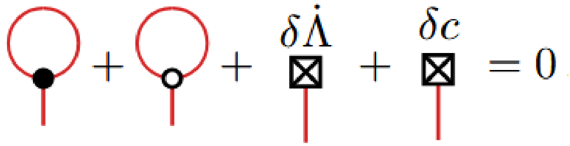

$$a\mathcal{H}^{(2)}_{\mathrm{c.t.}} = \frac{a^4 M_{\mathrm{Pl}}^2}{2}\, \delta\ddot{\Lambda}\, \pi^2 - 2a^4\, (\delta\dot{c} - \eta H\, \delta c)\, \pi\dot{\pi}$$

Non-linearly realized symmetries imply quadratic counterterms are necessarily induced !

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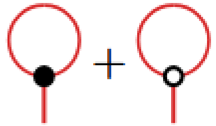
$\langle \pi_{\vec{p}}(\tau) \rangle'_{\text{bare}} =$  $\neq 0$

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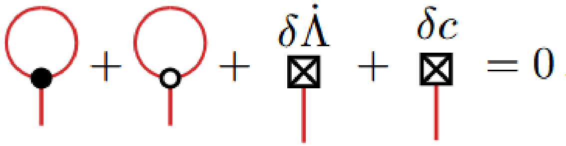

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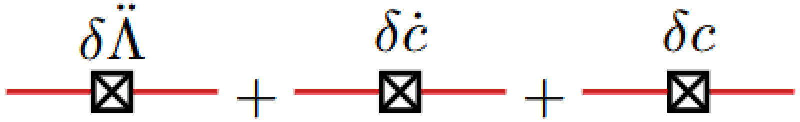
Non-linearly realized symmetries imply quadratic counterterms are necessarily induced !

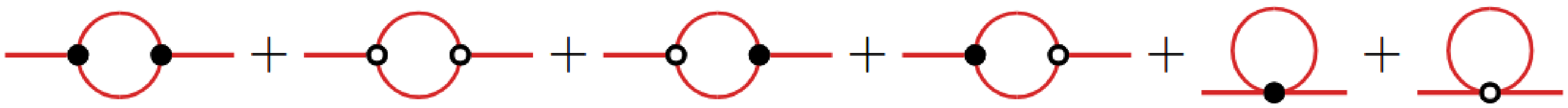
- New contributions to the power spectrum $\mathcal{P}^{\text{c.t.},(\Lambda,c)}_{\pi,\,1\text{L}}(x) =$ 

$$\lim_{x\to 0} \mathcal{P}^{\text{c.t.},(\Lambda,c)}_{\pi,\,1\text{L}}(x) \sim \mathcal{P}^{\text{tree}^2}_{\pi,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2) \log x.$$

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

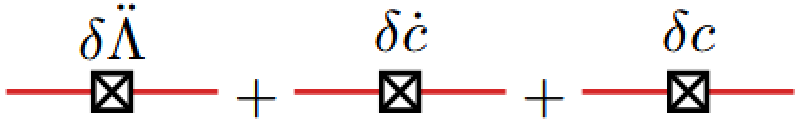
REMEMBER

$$\lim_{x\to 0}\mathcal{P}^{\mathrm{bare}}_{\pi,\,1\mathrm{L}}(x)\sim-\mathcal{P}^{\mathrm{tree}^2}_{\pi,\,0}H^2\frac{1}{2}\eta(2\eta-\eta_2)\log x\,.$$



NOW

$$\lim_{x\to 0}\mathcal{P}^{\mathrm{c.t.},(\Lambda,\,c)}_{\pi,\,1\mathrm{L}}(x)\sim\mathcal{P}^{\mathrm{tree}^2}_{\pi,\,0}H^2\frac{1}{2}\eta(2\eta-\eta_2)\log x\,.$$



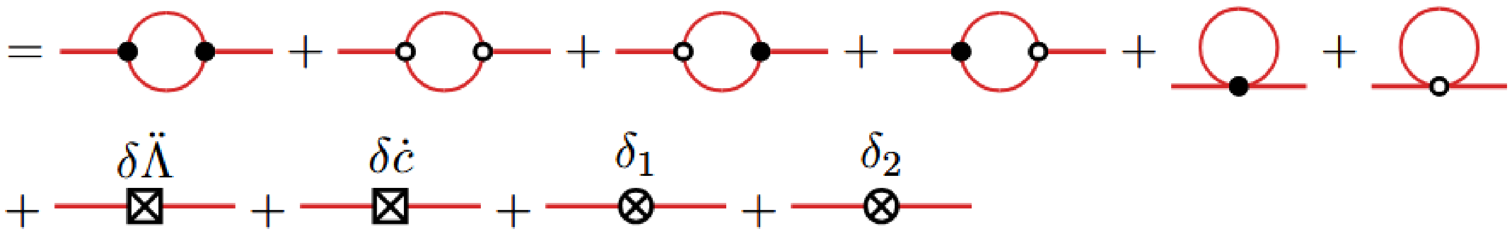
THEIR SUM IS LATE-TIME CONVERGENT!

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

RENORMALIZATION

Total result

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]



The result on super-horizon scales is:

$$\frac{\mathcal{P}^{\mathrm{ren}}_{\pi,1\mathrm{L},0}}{\mathcal{P}^{\mathrm{tree}}_{\pi,0}} = \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_\eta}\right)^2\left[\frac{269}{160} - \frac{29}{16}\log(2) - \frac{1}{4}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}\right)\right]$$
$$+ \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_{\eta_2}}\right)^2\left[\frac{11}{24} + \frac{1}{2}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}\right)\right] + \text{IR divergences}$$

with the strong coupling scales $\Lambda_\eta^2 = M_{\mathrm{Pl}}^2 \frac{\epsilon c_s}{\eta^2}$ and $\Lambda_{\eta_2}^2 = M_{\mathrm{Pl}}^2 \frac{\epsilon c_s}{\eta\eta_2}$

PS: We can safely match $\pi(t,\vec{x})$ to $\zeta(t,\vec{x})$ on super-horizon scales

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Total result

[Braglia, LP 2504.07926]
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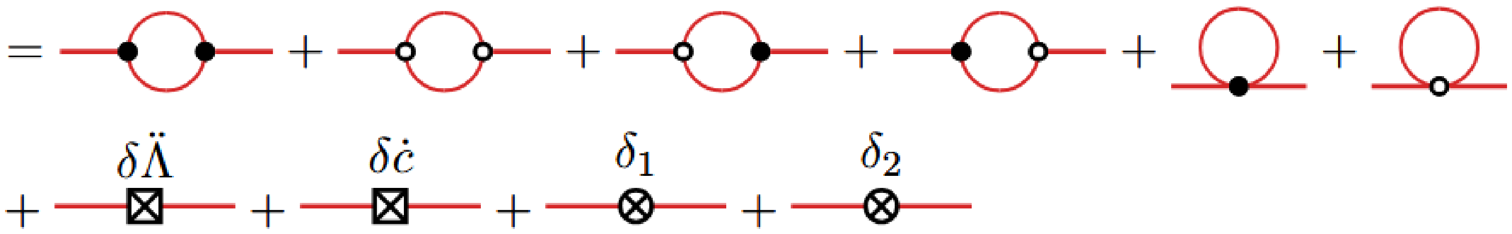

Successful perturbative renormalization of the EFT of inflation at order H^2/Λ^2

The result on super-horizon scales is:

$$\ldots - \frac{1}{4}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}\right)\Bigg]$$

$$+ \frac{\ldots}{8\pi^2}\left(\ldots\Lambda_{\eta_2}\right)\left[\ldots\frac{1}{2}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}\right)\right] + \text{IR divergences}$$

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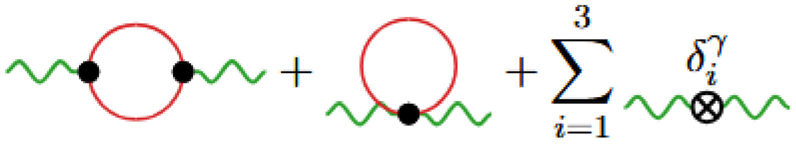
PS: We can safely match $\pi(t,\vec{x})$ to $\zeta(t,\vec{x})$ on super-horizon scales

DISCUSSION

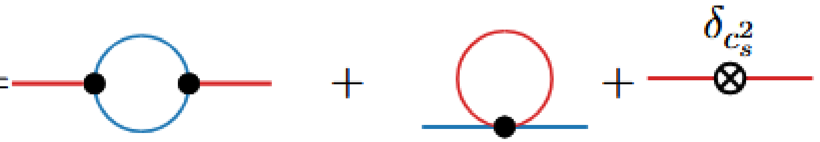
What's done, what's not done

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

Consistent with [Ballesteros, Gambín Egea, Riccardi 2024] in the limit $c_s \to 1$
No tadpole, no apparent secular divergence

- Scalar-induced tensor two-point function: $\mathcal{P}^{\mathrm{ren}}_{\gamma,\,1\mathrm{L}}(x) =$ 

- Massive (conformal) scalar field correction to the one-loop curvature perturbation:

$\mathcal{P}^{\mathrm{ren}}_{\pi,\,1\mathrm{L\ from}\ s}(x) =$ 

- All cases are scale-invariant scenarios: running $\log(H/\mu)$ is not observable

→ Strongly scale-dependent cases with striking small-scale phenomenology remain to be thoroughly studied

- Relax technical assumption of the decoupling limit → anti-derivative interactions $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$

- Explore different scenarios: heavy fields with any mass, different dispersion relations, excited initial states, etc.

DISCUSSION

What's done, what's not done

[Braglia, LP 2504.07926]
[Braglia, LP 2504.13136]

- Scalar-induced tensor two-point function:
- Massive (conformal) scalar field correction to the one-loop curvature perturbation:
- **All cases are scale-invariant scenarios: running $\log(H/\mu)$ is not observable**

→ **Strongly scale-dependent cases with striking small-scale phenomenology remain to be thoroughly studied**
[Braglia, Cespedes, LP 2603.12216]

- Relax technical assumption of the decoupling limit → anti-derivative interactions $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$

Perturbatively scale-dependent scenarios

- Explore different scenarios: heavy fields with any mass, different dispersion relations, excited initial states, etc.

BONUS

First scale-dependent renormalization of inflation

$$x \equiv -p\tau$$

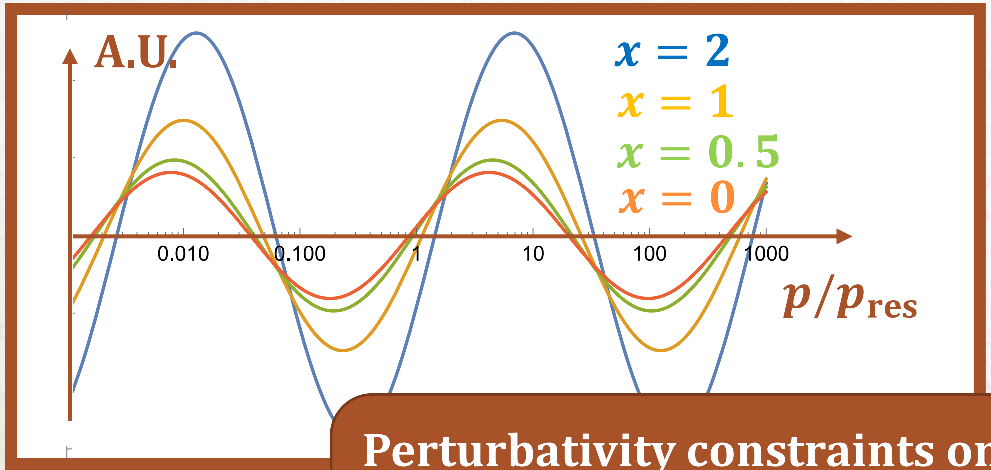
[Braglia, Cespedes, LP 2603.12216]

Repeat the analysis with interactions that violate slow-roll $f(\tau) \rightarrow$ ***resonant features, sharp features, etc.***

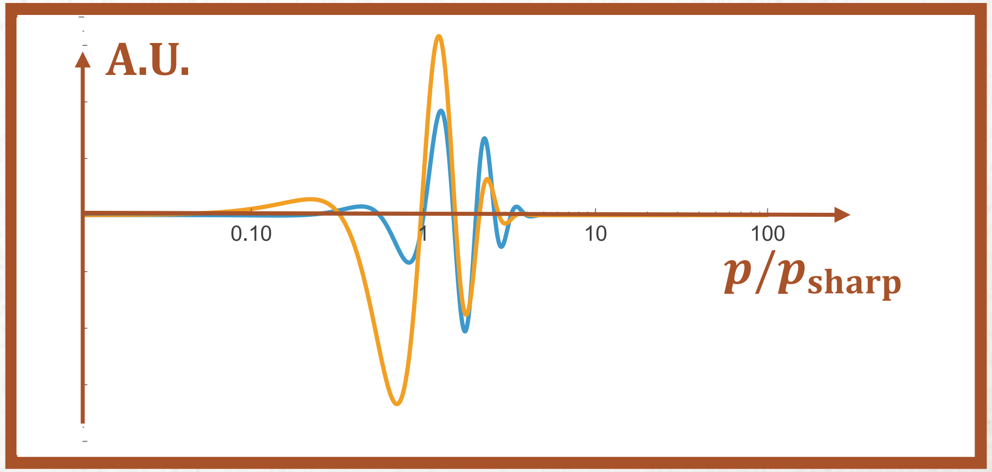
$$\mathcal{P}_{\pi,1\mathrm{L}}^{\mathrm{bare,UV}}(p,x) = f(p,x)\left(\frac{1}{\delta} + \log\left(\frac{H}{\mu}\right)\right)$$

The UV divergences are scale-dependent (p) and time-dependent (x) but **they can still be absorbed** by local higher-derivative counterterms

Resonant feature 1-loop power spectrum (time and scale dependence)



Sharp feature **1-loop** vs. **tree** level power spectrum (scale-dependence at late times)



Perturbativity constraints on:

- **Frequency of the resonant feature**
- **Duration of the sharp feature**

BACKUP SLIDES

A BRIGHT FUTURE

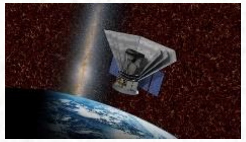
Approximate budgets
Total $\simeq$ 4 billions \$

Towards a standard model of inflation

400M\$
DESI

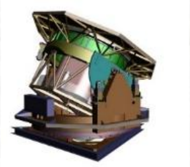
600M\$
Euclid

400M\$
SPHEREx

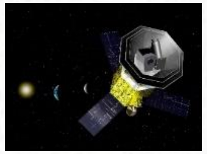


200M\$
Simons Observatory

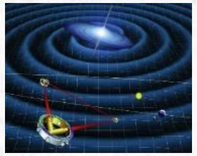
250M\$
CMB-S4

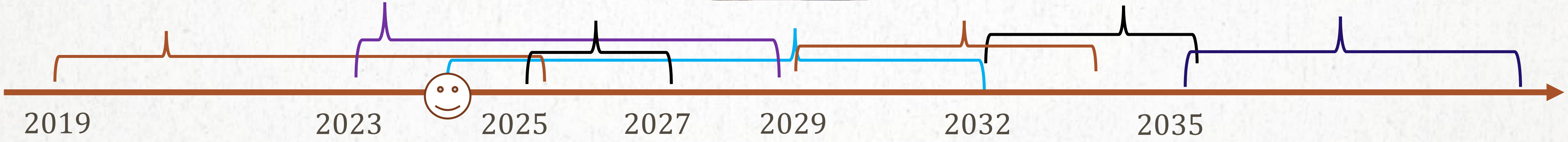


500M\$
LiteBIRD



1700M\$
LISA





<u>Exciting era for primordial cosmology</u>

But discoveries = data + interpretation

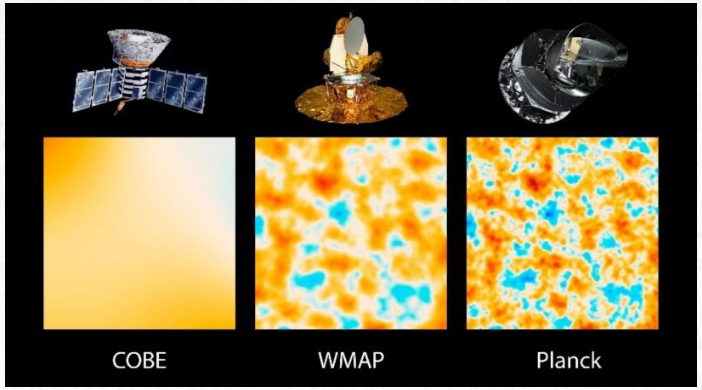
Cosmic Microwave Background

CMB

1964

Penzias-Wilson

1992 2006 2018

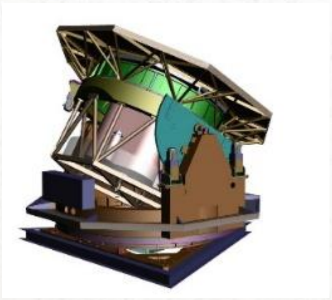


[credits: NASA]

Simons Observatory (2024-?)

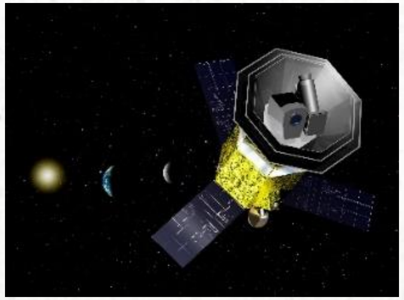
The coming one (Chile)

CMB-S4 (2029-?)

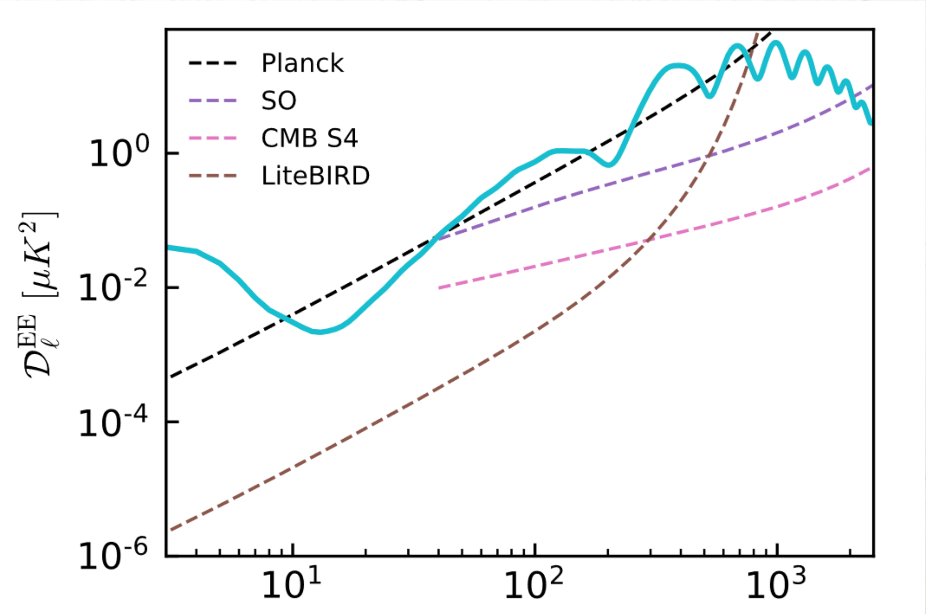


Multiple telescopes
Better foreground removal

LiteBIRD (2032-?)

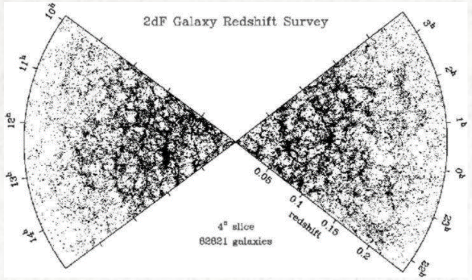


Satellite
Much better for low-ℓ

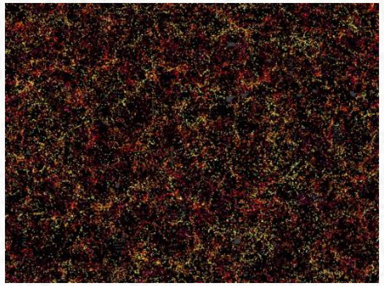


Large-Scale Structures

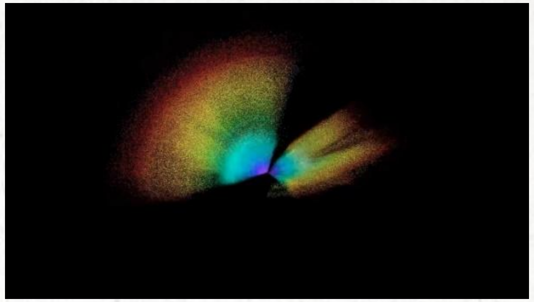
2-dFGRS (~2000)



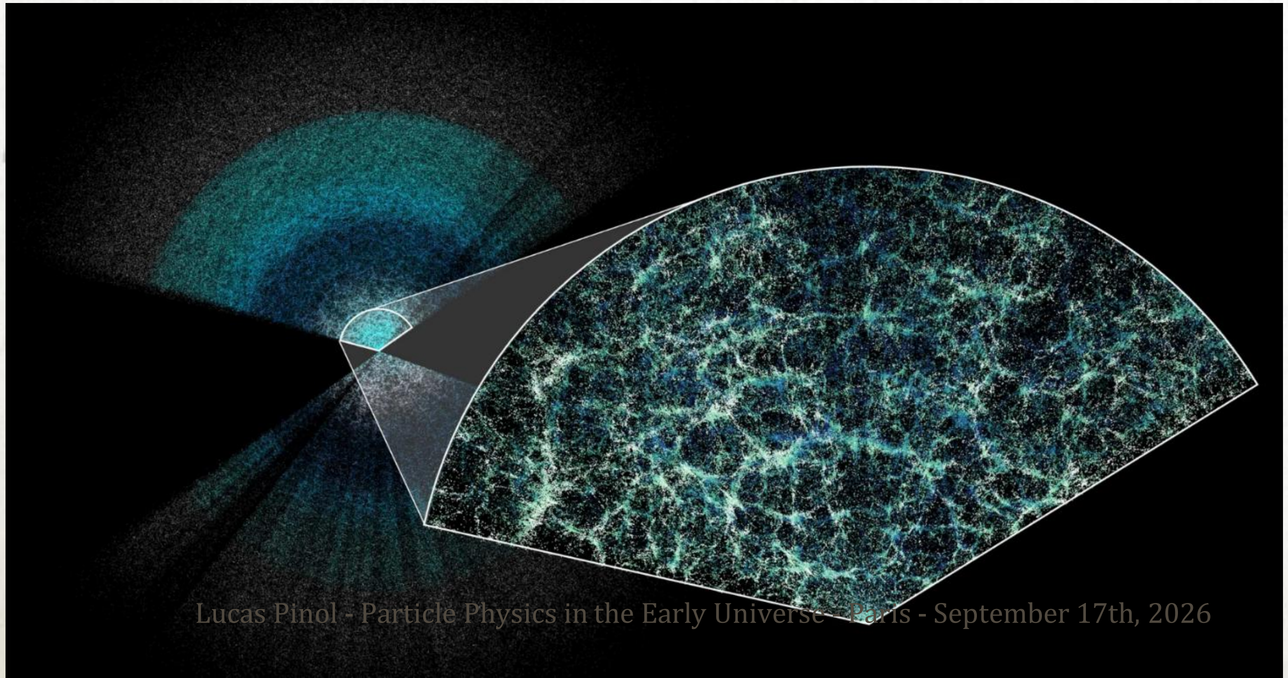
BOSS (2009-2014)



e-BOSS (2014-2020)



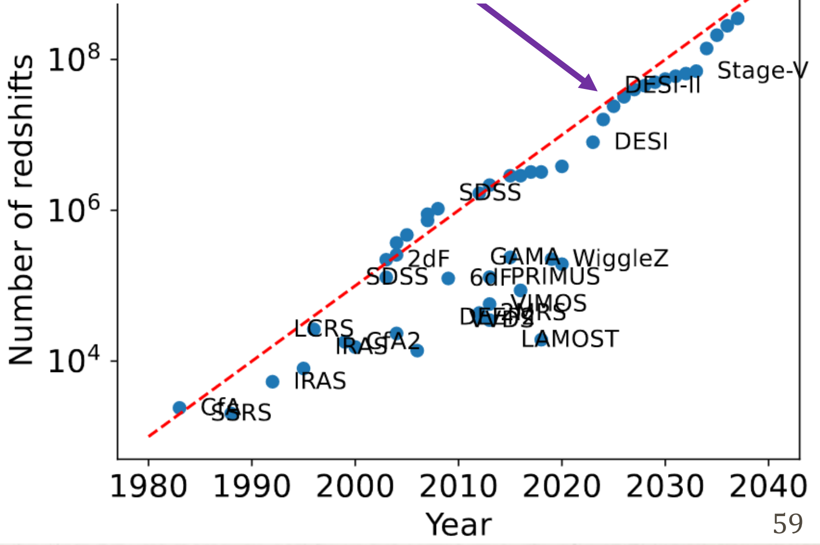
DESI (2020-?)



Euclid (2023-?)

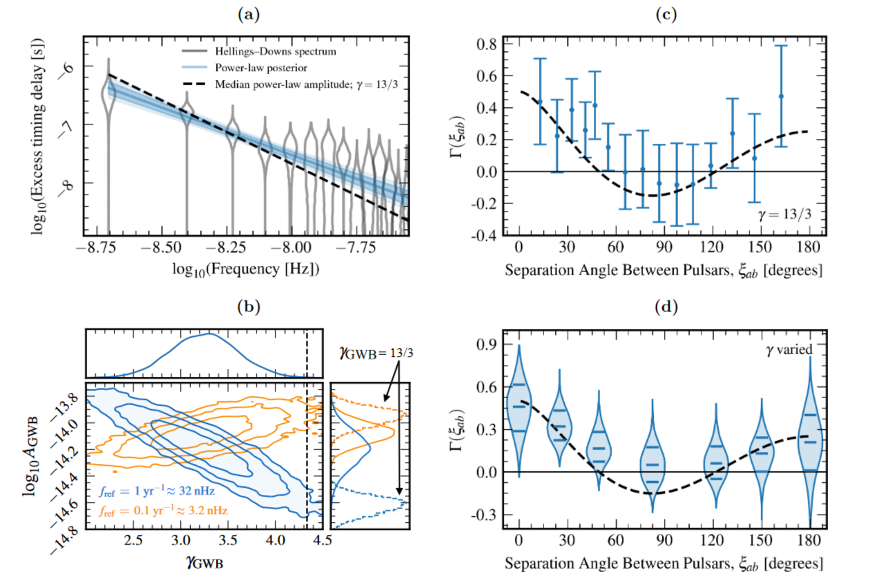
PUMA, MegaMapper, …

Schlegel, Ferraro et al. 2022

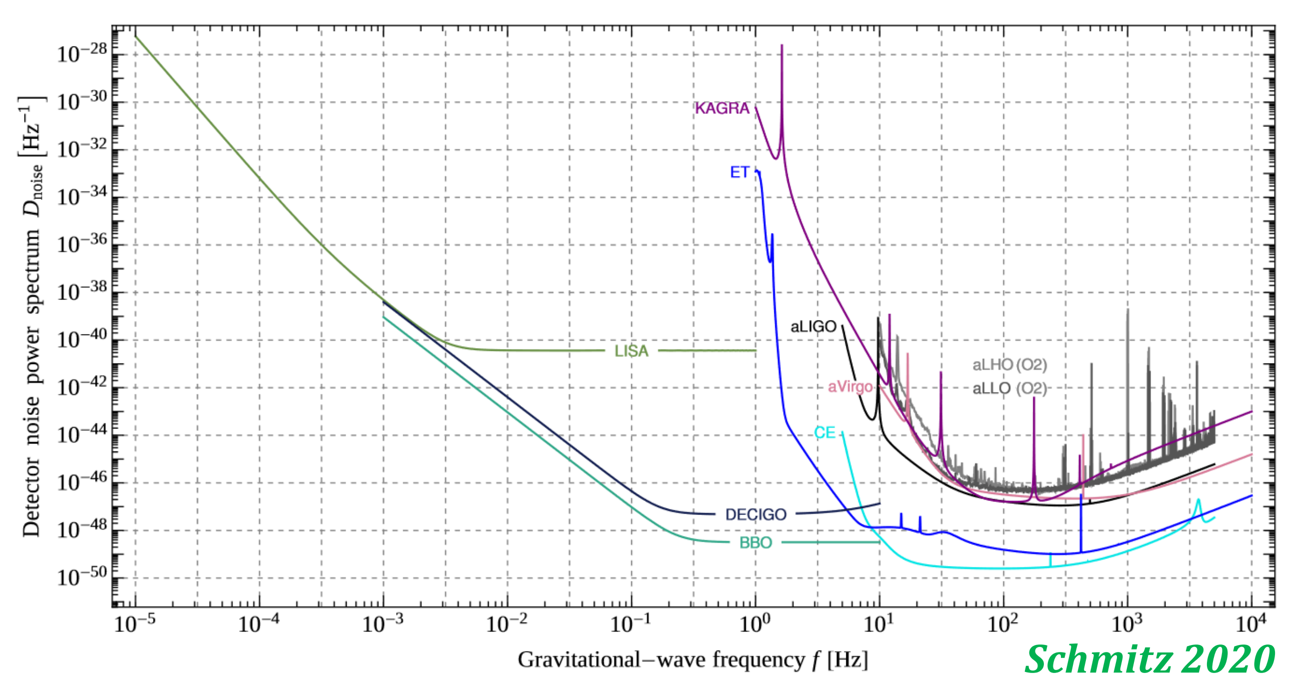


Gravitational-Wave Backgrounds

IPTA (2012-?)



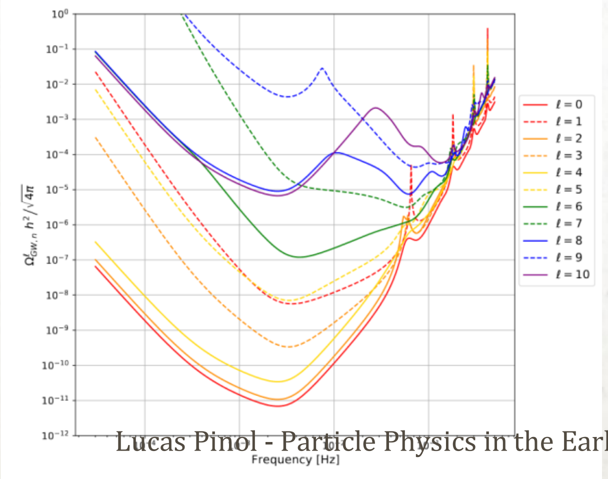
"No GW background" hypothesis is excluded at 3σ



Schmitz 2020

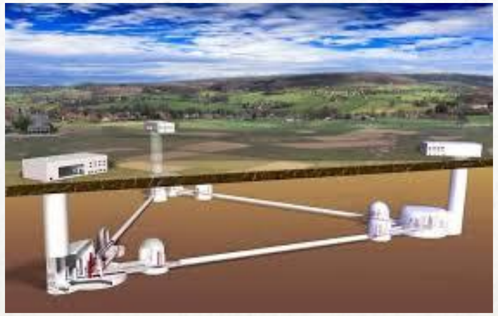
Europe

Anisotropies in **LISA (2034-?)**



Europe

Einstein Telescope

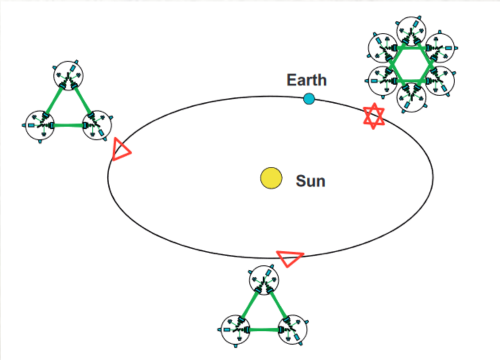


USA

Cosmic Explorer

Japan *USA+Europe*

DECIGO, BBO



THE CURVATURE PERTURBATION AT ONE LOOP

Conceptual and technical difficulties

- Gravity is (perturbatively) dynamical →
  - ❖ Gauge ambiguity
    gauge-invariant predictions must be obtained
  - ❖ GR constraints must be solved
    metric components are non-dynamical but non-zero
  - ❖ Gravitational interactions are mixed
    (anti-)derivative / non-derivative, e.g. $\zeta\dot{\zeta}^2$, $\zeta\partial_i\zeta\partial_i\partial^{-2}\zeta$, etc.

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- Backreaction on the background must be (perturbatively) accounted for

- Three kinds of divergences:
  - ❖ Secular divergences: $\log a(t)$
  - ❖ UV divergences: $\Lambda_{\mathrm{UV}}^2, \Lambda_{\mathrm{UV}}, \log \Lambda_{\mathrm{UV}}$
  - ❖ IR divergences: $\log \Lambda_{\mathrm{IR}}$

$$\int \mathrm{d}t_1 \int \mathrm{d}t_2 \; [H(t_2), [H(t_1), Q(t)]]$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \boldsymbol{f(k)} \xrightarrow{\boldsymbol{k\to\infty}} \frac{\mathbf{1}}{\boldsymbol{k}} + \frac{\mathbf{1}}{\boldsymbol{k^2}} + \frac{\mathbf{1}}{\boldsymbol{k^3}} + \cdots$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \, \boldsymbol{f(k)} \xrightarrow{\boldsymbol{k\to 0}} \frac{\mathbf{1}}{\boldsymbol{k^3}} + \frac{\mathbf{1}}{\boldsymbol{k^2}} + \cdots$$

THE CURVATURE PERTURBATION AT ONE LOOP

Conceptual and technical difficulties

- Gravity is (perturbatively) dynamical → ❖ Gauge ambiguity
  gauge-invariant predictions must be obtained

- Backre

- Three l]]

How to address all that at once?

<u>The effective field theory of inflationary fluctuations is an exceptional framework</u>

$\frac{1}{k^3} + \cdots$

❖ IR divergences: $\log \Lambda_{\mathrm{IR}}$ $\int \frac{}{(2\pi)^3} f(k)$

$k \to 0$ $\frac{1}{k^3} + \frac{1}{k^2} + \cdots$

EFT FOR THE ADIABATIC FLUCTUATION

The decoupling limit

- Lapse and shift are non-dynamical (no $\delta \dot{g}^{00}_{\text{flat}}$, no $\delta \dot{g}^{0i}_{\text{flat}}$) but non-zero

- In principle, constraints must be solved → $\delta g^{00}_{\text{flat}} = 2\epsilon H \pi$

- I want to neglect those, e.g. in $(\delta g^{00}|_{\text{linear}})^2 \rightarrow \left(-2\dot{\pi} + \delta g^{00}_{\text{flat}}\right)^2$

the decoupling limit

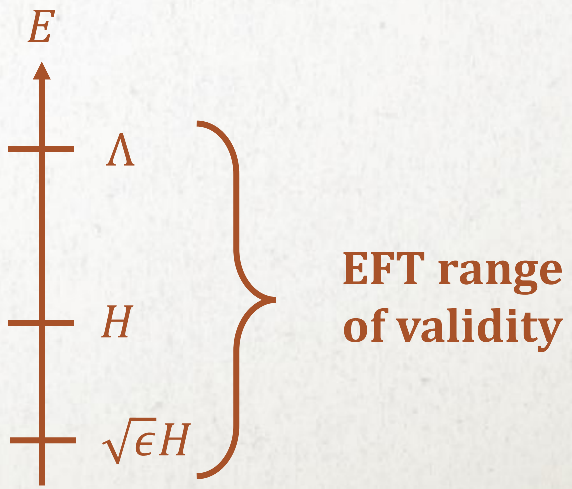
- $\dot{\pi} \sim \omega \times \pi \quad \rightarrow \quad \boldsymbol{\omega^2 \gg \epsilon H^2}$

  At high enough energies, the NG boson alone describes the whole dynamics

- <u>Remarks</u>: using the decoupling limit around horizon crossing $\omega \sim H \quad \rightarrow \quad \epsilon \ll 1$
  no constraints on η and its time derivatives

- On super-horizon scales $\omega \ll H$ and the decoupling ceases to be valid

  Not a problem if an adiabatic limit is reached and ζ has become constant



EFT FOR THE ADIABATIC FLUCTUATION

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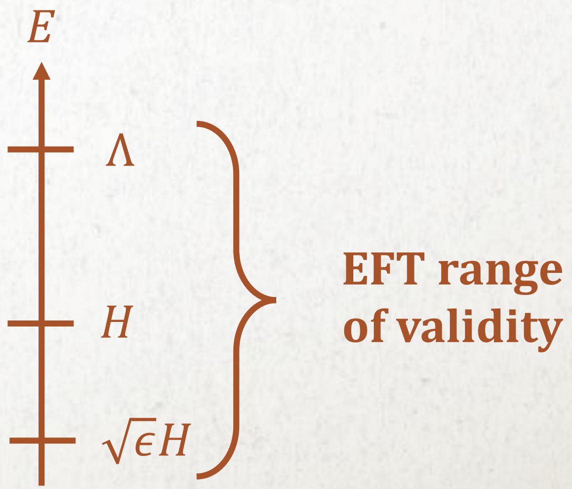
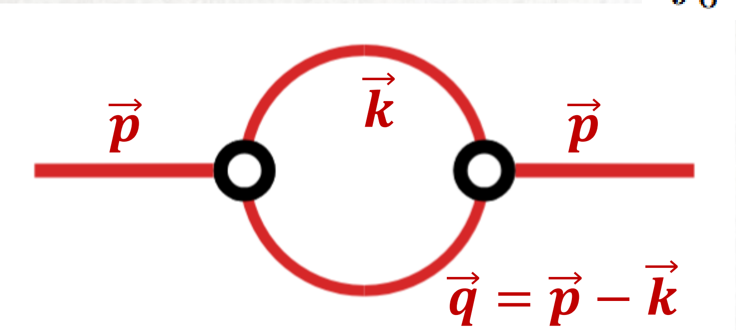


BARE CONTRIBUTIONS

Strategy

- use dimensionless variables $$\int_0^\infty \mathrm{d}t\, t^\delta \int_{-1}^{1} \mathrm{d}s \int \mathrm{d}\tau_1 \int \mathrm{d}\tau_2\, f(\delta,\, t,\, s,\, \tau_1,\, \tau_2)$$



$$v = \frac{k}{p}, \qquad u = \frac{q}{p}$$

$$v = \frac{t - s + 1}{2}, \qquad u = \frac{t + s + 1}{2}$$

- worst divergence is $1/\delta$ so we expand the mode function to linear order in δ **[Senatore, Zaldarriaga 2012]**
- perform time (τ_1, τ_2) and angular (s) integrals $$\int_0^\infty \mathrm{d}t\, t^\delta\, \left[F_0(t) + \delta \cdot F_1(t)\right]$$
- introduce a fake cutoff $t_{\mathrm{UV}} \to \infty$ to expand $F_1(t)$ in $[t_{\mathrm{UV}}, \infty[$ and extract the pole **[Ballesteros, Gambín Egea, Riccardi 2024]**
- introduce a comoving IR cutoff t_{IR} $$\left(\int_0^{t_{\mathrm{IR}}} + \int_{t_{\mathrm{IR}}}^{t_{\mathrm{UV}}} + \int_{t_{\mathrm{UV}}}^{\infty}\right) \mathrm{d}t\, t^\delta\, \left[F_0(t) + \delta \cdot F_1(t)\right]$$

II. THE EFT OF INFLATIONARY FLUCTUATIONS

3. Multiple scalars

EFT FOR MUTIFIELD FLUCTUATIONS

[Senatore, Zaldarriaga 2010] [Baumann, Green 2011] [Noumi, Yamaguchi, Yokoyama 2013]

shift-symmetric (massless)

one additional field, little pheno

one additional field, discovery of the "cosmological collider" signal for massive fields

[LP 2024]

any number of fields, no shift symmetry, insist on nonlinearly realized symmetries and decoupling limit

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \ldots$

Write all possible combinations

EFT FOR MUTIFIELD FLUCTUATIONS

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Write all possible combinations (**no tapdoles**):

1. At **first** order in $\mathcal{S}^\alpha$

$$\cancel{b_\alpha^{(0)} \mathcal{S}^\alpha} + b_\alpha^{(1)} \delta g^{00} \mathcal{S}^\alpha + \cdots + b_\alpha^{(n)} (\delta g^{00})^n \mathcal{S}^\alpha + \cdots$$
$$\cancel{c_\alpha^{(0)} g^{0\mu} \partial_\mu \mathcal{S}^\alpha} + c_\alpha^{(1)} (\delta g^{00}) g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \cdots + c_\alpha^{(n)} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \cdots$$

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

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$$\cancel{c_\alpha^{(0)}g^{0\mu}\partial_\mu\mathcal{S}^\alpha} + c_\alpha^{(1)}(\delta g^{00})g^{0\mu}\partial_\mu\mathcal{S}^\alpha + \cdots + c_\alpha^{(n)}(\delta g^{00})^n g^{0\mu}\partial_\mu\mathcal{S}^\alpha + \cdots$$

Some adiabatic-entropic cubic interactions **fixed** by nonlinearly realized symmetries, others **not fixed**:

$$\begin{cases} b_\alpha^{(1)}\dot{\pi}\mathcal{S}^\alpha + c_\alpha^{(1)}\dot{\pi}\dot{\mathcal{S}}^\alpha \\ -b_\alpha^{(1)}\dfrac{(\partial\pi)^2}{a^2}\mathcal{S}^\alpha - c_\alpha^{(1)}\left[\dfrac{(\partial\pi)^2}{a^2}\dot{\mathcal{S}}^\alpha + \dot{\pi}\dfrac{(\partial\pi\cdot\partial\mathcal{S}^\alpha)}{a^2}\right] + \left(4b_\alpha^{(2)} - b_\alpha^{(1)}\right)\dot{\pi}^2\mathcal{S}^\alpha + \cdots \end{cases}$$

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

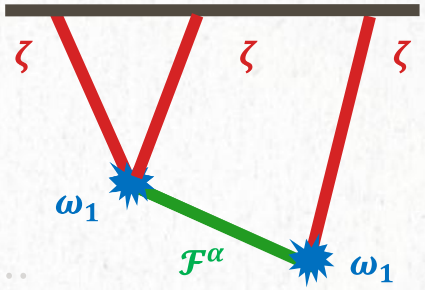
Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \dots$

Write all possible combinations (**no tapdoles**):

1. At first order in $\mathcal{S}^\alpha$

$$\cancel{b_\alpha^{(0)} \mathcal{S}^\alpha} + b_\alpha^{(1)} \delta g^{00} \mathcal{S}^\alpha + \dots + b_\alpha^{(n)} (\delta g^{00})^n \mathcal{S}^\alpha + \dots$$
$$\cancel{c_\alpha^{(0)} g^{0\mu} \partial_\mu \mathcal{S}^\alpha} + c_\alpha^{(1)} (\delta g^{00}) g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \dots + c_\alpha^{(n)} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \dots$$



Brute-force in-in calculation

[LP, Aoki, Renaux-Petel, Yamaguchi 2021]

« Inflationary flavor oscillations » with mixing angles from the mass matrix

"bending" coupling in NLSM $b_\alpha^{(1)} \dot{\pi} \mathcal{S}^\alpha \to \omega_1 \dot{\zeta} \mathcal{F}^1$

Some adiabatic-entropic cubic interactions fixed by **quadratic** interactions by symmetries:

$$\begin{cases} b_\alpha^{(1)} \dot{\pi} \mathcal{S}^\alpha + c_\alpha^{(1)} \dot{\pi} \dot{\mathcal{S}}^\alpha \\ -b_\alpha^{(1)} \dfrac{(\partial \pi)^2}{a^2} \mathcal{S}^\alpha - c_\alpha^{(1)} \left[\dfrac{(\partial \pi)^2}{a^2} \dot{\mathcal{S}}^\alpha + \dot{\pi} \dfrac{(\partial \pi \cdot \partial \mathcal{S}^\alpha)}{a^2} \right] + \left(4 b_\alpha^{(2)} - b_\alpha^{(1)} \right) \dot{\pi}^2 \mathcal{S}^\alpha + \cdots \end{cases}$$

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \ldots$

New single-exchange diagrams

Write all possible combinations (**no tapdoles**):

1. At first order in $\mathcal{S}^\alpha$

$$\cancel{b_\alpha^{(0)} \mathcal{S}^\alpha} + b_\alpha^{(1)} \delta g^{00} \mathcal{S}^\alpha + \cdots + b_\alpha^{(n)} (\delta g^{00})^n \mathcal{S}^\alpha + \cdots$$

$$\cancel{c_\alpha^{(0)} g^{0\mu} \partial_\mu \mathcal{S}^\alpha} + c_\alpha^{(1)} (\delta g^{00}) g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \cdots + c_\alpha^{(n)} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha + \cdots$$

Absent in NLSM and $P(X^{IJ}, \vec{\phi})$

Some adiabatic-entropic cubic interactions fixed by **quadratic** interactions by symmetries:

$$\begin{cases} \boldsymbol{b_\alpha^{(1)}} \dot{\pi} \mathcal{S}^\alpha + \boldsymbol{c_\alpha^{(1)}} \dot{\pi} \dot{\mathcal{S}}^\alpha \\ -\boldsymbol{b_\alpha^{(1)}} \dfrac{(\partial\pi)^2}{a^2} \mathcal{S}^\alpha - \boldsymbol{c_\alpha^{(1)}} \left[\dfrac{(\partial\pi)^2}{a^2} \dot{\mathcal{S}}^\alpha + \dot{\pi} \dfrac{(\partial\pi \cdot \partial \mathcal{S}^\alpha)}{a^2} \right] \end{cases} + \left(\boldsymbol{4b_\alpha^{(2)}} - \boldsymbol{b_\alpha^{(1)}} \right) \dot{\pi}^2 \mathcal{S}^\alpha + \cdots$$

EFT FOR MUTIFIELD FLUCTUATIONS

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Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \ldots$

Write all possible combinations:

2. At **second** order in $\mathcal{S}^\alpha$

$$d_{\alpha\beta}^{(0)} \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + d_{\alpha\beta}^{(n)} (\delta g^{00})^n \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$e_{\alpha\beta}^{(0)} g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + e_{\alpha\beta}^{(n)} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$f_{\alpha\beta}^{(0)} g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + f_{\alpha\beta}^{(n)} (\delta g^{00})^n g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + \bar{f}_{\alpha\beta}^{(0)} \left(g^{0\mu} \partial_\mu \mathcal{S}^\alpha\right)\left(g^{0\nu} \partial_\nu \mathcal{S}^\beta\right) + \cdots$$

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \ldots$

Write all possible combinations:

2. At second order in $\mathcal{S}^\alpha$

$$d^{(0)}_{\alpha\beta} \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + d^{(n)}_{\alpha\beta} (\delta g^{00})^n \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$e^{(0)}_{\alpha\beta} g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + e^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$f^{(0)}_{\alpha\beta} g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + f^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + \bar{f}^{(0)}_{\alpha\beta} \left(g^{0\mu} \partial_\mu \mathcal{S}^\alpha\right) \left(g^{0\nu} \partial_\nu \mathcal{S}^\beta\right) + \cdots$$

f is a kinetic term, symmetric positive definite real 2-form $\rightarrow -\sum_\alpha \partial_\mu \mathcal{S}^\alpha \partial^\mu \mathcal{S}^\alpha$

$\bar{f}$ breaks boosts, symmetric so can be co-diagonalized $\rightarrow \sum_\alpha \left(\frac{1}{c_\alpha^2} - 1\right) \left(\dot{\mathcal{S}}^\alpha\right)^2$ **Absent in NLSM and $P\left(X^{IJ}, \vec{\phi}\right)$**

[LP, Renaux-Petel, Werth 2023]

→ **symmetry: $\left(1/c_\alpha^2 - 1\right) \dot{\mathcal{S}}^\alpha (\partial\pi \cdot \partial\mathcal{S}^\alpha)/a^2$**

EFT FOR MUTIFIELD FLUCTUATIONS

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Write all possible combinations:

2. At second order in $\mathcal{S}^\alpha$

$$d^{(0)}_{\alpha\beta} \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + d^{(n)}_{\alpha\beta} (\delta g^{00})^n \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$e^{(0)}_{\alpha\beta}\, g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + e^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$f^{(0)}_{\alpha\beta}\, g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + f^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + \bar{f}^{(0)}_{\alpha\beta} \left(g^{0\mu} \partial_\mu \mathcal{S}^\alpha\right)\left(g^{0\nu} \partial_\nu \mathcal{S}^\beta\right) + \cdots$$

d defines a mass matrix and its anti-symmetric component vanishes $\rightarrow m^2_{(\alpha\beta)}\, \mathcal{S}^\alpha \mathcal{S}^\beta$

$\hookrightarrow$ **inflationary flavor oscillations are expected more generically than in NLSM**

EFT FOR MUTIFIELD FLUCTUATIONS

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Write all possible combinations:

2. At second order in $\mathcal{S}^\alpha$

$$d^{(0)}_{\alpha\beta} \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + d^{(n)}_{\alpha\beta} (\delta g^{00})^n \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$e^{(0)}_{\alpha\beta} g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots + e^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta + \cdots$$
$$f^{(0)}_{\alpha\beta} g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + f^{(n)}_{\alpha\beta} (\delta g^{00})^n g^{\mu\nu} \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta + \cdots + \bar{f}^{(0)}_{\alpha\beta} \left(g^{0\mu} \partial_\mu \mathcal{S}^\alpha\right)\left(g^{0\nu} \partial_\nu \mathcal{S}^\beta\right) + \cdots$$

$$e^{(0)}_{\alpha\beta} g^{0\mu} \partial_\mu \mathcal{S}^\alpha \mathcal{S}^\beta = e^{(0)}_{(\alpha\beta)} \partial_t\left(\mathcal{S}^\alpha \mathcal{S}^\beta\right) + e^{(0)}_{[\alpha\beta]} \dot{\mathcal{S}}^\alpha \mathcal{S}^\beta + \cdots \quad \rightarrow \Omega_{[\alpha\beta]} \dot{\mathcal{S}}^\alpha \mathcal{S}^\beta$$

Torsion and higher-order curvatures

Purely many-fields effect not explored so far...

→ **symmetry: $\boldsymbol{\Omega_{\alpha\beta} \mathcal{S}^\beta (\partial\pi \cdot \partial \mathcal{S}^\alpha)/a^2}$**

EFT FOR MUTIFIELD FLUCTUATIONS

[LP 2024]

Define the non-adiabatic sector: $N-1$ scalar fluctuations $\mathcal{S}^\alpha$

We add non-adiabatic blocks to the EFT: $\mathcal{S}^\alpha, \partial_\mu \mathcal{S}^\alpha, \mathcal{S}^\alpha \mathcal{S}^\beta, \partial_\mu \mathcal{S}^\alpha \partial_\nu \mathcal{S}^\beta, \ldots$

Write all possible combinations:

3. At **third** order in $\mathcal{S}^\alpha$

$$i^{(0)}_{\alpha\beta\gamma} \mathcal{S}^\alpha \mathcal{S}^\beta \mathcal{S}^\gamma + \cdots + i^{(n)}_{\alpha\beta\gamma} (\delta g^{00})^n \mathcal{S}^\alpha \mathcal{S}^\beta \mathcal{S}^\gamma + \cdots$$

…

Triple-exchange diagrams

EFT FOR MUTIFIELD FLUCTUATIONS

What have we learned?

- We can describe multifield fluctuations without relying on a model

- Non-linearly realized symmetries fix cubic interactions to quadratic ones, like in the single-field case

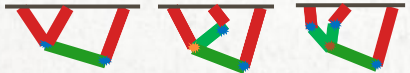
- The resulting interactions are very generic, recovering NLSM and extending beyond

- It can be used to make predictions (power spectrum, non-Gaussianities) straightforward and systematic

  → The "Cosmological Flow" **[Werth, LP, Renaux-Petel 2023]**
  [LP, Renaux-Petel, Werth 2023]
  [LP, Renaux-Petel, Werth 2024]

- The Wilson coefficients are constrained by requirements of perturbativity, weak coupling and naturalness

- Non-Gaussian phenomenology shared by all multifield scenarios independently of the model



$$f_{\rm NL}^{\rm eq} \sim \underbrace{\left(\frac{1}{c_s^2}-1\right)A}_{\textbf{Single-field result}} + \underbrace{\omega[\omega B + \alpha C + \mu D]}_{\to \, \mathbf{0 \text{ for heavy fields: }} m > (\text{few})H}$$

$$f_{\rm NL}^{\rm sq}(\kappa) \sim \underbrace{\kappa f_{\rm NL}^{\rm eq}} + \underbrace{\sqrt{\kappa}\,\omega\left[\omega \sum_i E_i \cos\left(\frac{m_i}{H}\ln\kappa\right) + \alpha \times \cdots + \cdots\right]}_{\textbf{Cosmic spectroscopy}}$$