Testing α-attractor models of inflation by CMB and reheating temperature

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A. Ghoshal, P. Kozów, MO, S. Pokorski, JCAP 09 (2026) 78
M. Marciniak, MO, S. Pokorski, Acta Phys. Polon. B57 (2026) 6 + work in progress

Outline

- Motivation for α-attractor models of inflation
- Classes of α-attractor models
- Reheating temperature and consistency condition
- Non-perturbative effects
- Predictions of α-attractor models vs results of CMB experiments
- Summary

Motivation for α-attractor models of inflation

Kallosh-Linde-Roest 2013; Ferrara-Kallosh-Linde-Porrati 2013; Ferrara-Kallosh 2016; Kallosh-Linde-Wrase-Yamada 2017; Kallosh-Linde 2022, 2025, 2026; ...

- **Seven-disk manifolds described by seven complex fields appear in M-theory, string theories and maximal supergravity**
  When some conditions are fulfilled one scalar may be identified with the inflaton and has kinetic term related to hyperbolic geometry with constant curvature

- **Kähler potential has the form (in disk or half-plane variable):**

$$K(Z,\overline{Z}) = -3\alpha \log(1 - Z\overline{Z}) \qquad \text{with} \qquad Z = \tanh\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_P}\right) e^{i\theta}$$

$$K(T,\overline{T}) = -3\alpha \log(T + \overline{T}) \qquad \text{with} \qquad T = \exp\left(-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_P}\right) + i\theta$$

- **Kähler curvature equals $R_K = -2/(3\alpha)$ with $3\alpha \in \{1,2,3,4,5,6,7\}$**

- **Simple potentials for Z or T field lead to several classes of α-attractor models**

Potentials regular at the border of the moduli space

- **T-models:** $(Z\overline{Z})^n$ $\Rightarrow$ $V_{\mathrm{T}}(\phi) = \Lambda_{\mathrm{inf}}^4 \left(\tanh\left[\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\mathrm{P}}}\right]\right)^{2n}$
- **E-models:** $\left(1 - \frac{T+\overline{T}}{2}\right)^{2n}$ $\Rightarrow$ $V_{\mathrm{E}}(\phi) = \Lambda_{\mathrm{inf}}^4 \left(1 - \exp\left[-\sqrt{\frac{2}{3\alpha}}\frac{\phi}{M_{\mathrm{P}}}\right]\right)^{2n}$

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Potentials with divergent derivative at the border of the moduli space

- **P-models:** $\frac{1}{1 + \left(\ln \frac{T+\overline{T}}{2}\right)^{-2n}}$ $\Rightarrow$ $$V_{\mathrm{P}}(\phi) = \Lambda_{\mathrm{inf}}^4 \frac{\phi^{2n}}{\phi^{2n} + \left(\sqrt{\frac{3\alpha}{2}} M_{\mathrm{P}}\right)^{2n}}$$

Classes of α-attractor models

Potentials regular at the border of the moduli space

- **T-models:** $(Z\overline{Z})^n$ $\quad\Rightarrow\quad$ $V_{\rm T}(\phi) = \Lambda^4_{\rm inf}\left(\tanh\left[\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right]\right)^{2n}$
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Potentials singular at the border of the moduli space (examples)

$$V_{\rm S(ln)T}(\phi) = \Lambda^4_{\rm inf}\tanh^{2n}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\left[1+\delta\ln^m\left(\cosh^2\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\right)\right]$$

$$V_{\rm S(p)T}(\phi) = \Lambda^4_{\rm inf}\tanh^{2n}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\left[1+\delta\cosh^{2m}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\right]$$

Parameters of inflation models as functions of CMB observables

Usually one calculates predictions for values of CMB observables for a given model of inflation and given values of its parameters

$$n_s(k) = 1 - M_{\mathrm{P}}^2 \left(3 \left(\frac{\partial_\phi V(\phi)}{V(\phi)} \right)^2 - 2 \frac{\partial_\phi^{(2)} V(\phi)}{V(\phi)} \right) \Bigg|_{\phi=\phi_k}$$

$$r(k) = 8 M_{\mathrm{P}}^2 \left(\frac{\partial_\phi V(\phi)}{V(\phi)} \right)^2 \Bigg|_{\phi=\phi_k} \qquad A_s(k) = \frac{V(\phi_k)}{12\pi^2 M_{\mathrm{P}}^6 \left(\frac{\partial_\phi V(\phi)}{V(\phi)} \right)^2 \Big|_{\phi=\phi_k}}$$

In some models it is possible and convenient to reverse these relationships and calculate values of the model parameters necessary to obtain given values of CMB observables.
For example in T-models

$$\alpha = \frac{64 n^2}{3r} \frac{1}{\xi^2 - 1} \qquad \Lambda_{\mathrm{inf}}^4 = \frac{3\pi^2}{2} M_{\mathrm{P}}^4 \, r A_s \left(\frac{\xi + \sqrt{\xi^2 - 1} + 1}{\xi + \sqrt{\xi^2 - 1} - 1} \right)^{2n}$$

$$\phi_k = M_{\mathrm{P}} \sqrt{\frac{32}{r\left(\xi^2 - 1\right)}} \, n \ln\left(\xi + \sqrt{\xi^2 - 1} \right)$$

where $\xi = n \left(8 \, \frac{1 - n_s}{r} - 1 \right)$

- Results of CMB experiments are used to determine bounds on parameters characterizing the CMB radiation corresponding to some conveniently chosen comoving pivot scale or wavenumber k_*: $A_s(k_*)$, $n_s(k_*)$, $r(k_*)$, $\alpha_s(k_*)$
  - $k_* = 0.05\, a_0\, \mathrm{Mpc}^{-1}$ is the most common choice ($0.002\, a_0\, \mathrm{Mpc}^{-1}$ was also used)
- Perturbations with such wavenumber generated during inflation was inflated and became super-horizon when the following condition was satisfied

$$k_* = a_{k_*} H_{k_*}$$

a_{k_*} and H_{k_*} are values of the cosmic scale factor and the Hubble parameter at that time

Reheating temperature and consistency condition

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- The above condition may be rewritten in the form

$$0 = \ln\left(\frac{k_*}{a_{k_*} H_{k_*}}\right) = \ln\left(\frac{a_{\mathrm{end}}}{a_{k_*}} \frac{a_{\mathrm{RH}}}{a_{\mathrm{end}}} \frac{a_0}{a_{\mathrm{RH}}} \frac{k_*}{a_0 H_{k_*}}\right)$$

  where a_{end}, a_{RH} and a_0 are values of the cosmic scale factor at the end of inflation, at the end of reheating and today, respectively

Reheating temperature and consistency condition

- Number of e-folds of consecutive phases of the evolution of the universe:
  - $N_{k_*} = \ln\left(\frac{a_{\rm end}}{a_{k_*}}\right)$ – inflation after the scale k_* crossed the horizon
  - $N_{\rm RH} = \ln\left(\frac{a_{\rm RH}}{a_{\rm end}}\right)$ – reheating
  - $N_{\rm RD+MD} = \ln\left(\frac{a_0}{a_{\rm RH}}\right)$ – standard evolution after the end of reheating (RD + MD)
- Equation from the previous slide may be rewritten as the following consistency condition relating these numbers of e-folds

$$\boxed{N_{k_*} + N_{\rm RH} + N_{\rm RD+MD} = \ln\left(\frac{a_0 H_{k_*}}{k_*}\right)}$$

Reheating temperature and consistency condition

- N_{k_*} depends on
  - model of inflation (inflaton potential)
  - values of CMB observables
- $N_{\mathrm{RD+MD}}$ depends on
  - reheating temperature T_{RH}
- N_{RH} depends on
  - model of inflation and values of CMB observables (energy density at the end of inflation)
  - reheating temperature T_{RH}
  - mechanism of reheating

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  - The same inflaton potential describes both inflation and reheating (plateau of the potential important for inflation, part of the potential around its minimum important for reheating)
  - Transfer of inflaton energy to energy of SM plasma described by a decay constant Γ which for a given model is in one-to-one correspondence with T_{RH}

Reheating temperature and consistency condition

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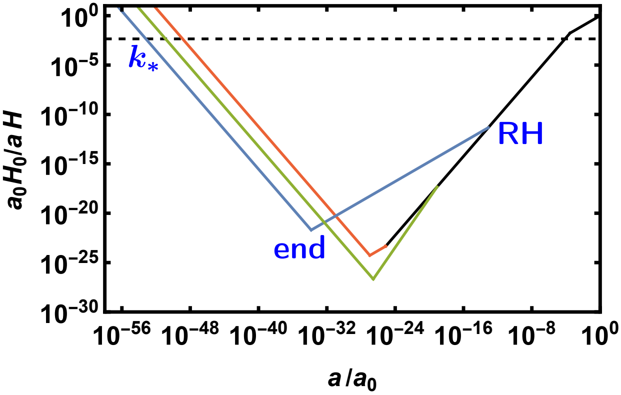
  - Transfer of inflaton energy to energy of SM plasma described by a decay constant Γ which for a given model is in one-to-one correspondence with $T_{\rm RH}$
- All important numbers of e-folds for a given model of inflation depend only on
  - values of CMB observables
  - reheating temperature $T_{\rm RH}$

Reheating temperature and consistency condition

$$N_{k_*} + N_{\rm RH} + N_{\rm RD+MD} = \ln\left(a_0 H_{k_*}/k_*\right)$$



For a given model of inflation and given values of CMB observables (model + values of its parameters)

- value of $T_{\rm RH}$ is fixed by the consistency condition
- shorter reheating period
  $\updownarrow$
  higher $T_{\rm RH}$ (bigger Γ)
- Maximal possible $T_{\rm RH}$ corresponds to instantaneous reheating

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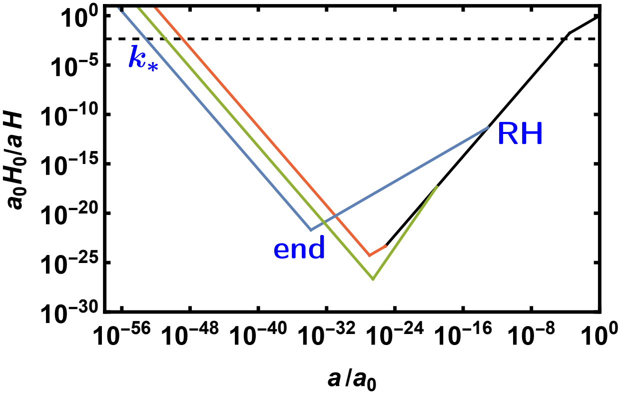

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The reheating temperature is bounded

- upper bound from consistency (value depends mainly on r, $\mathcal{O}(10^{15})$ GeV for big r)
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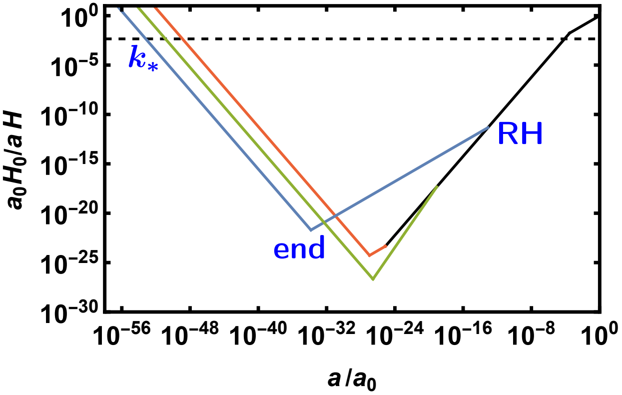

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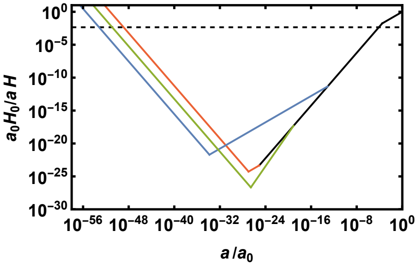
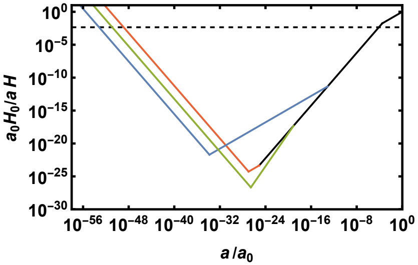
Bound on $T_{\rm RH}$ translate into bounds on N_{k_*}
(some common choices of N_{k_*} are (partially) inconsistent)



This schematic drawing corresponds to (over)simplification in which each period of the evolution (straight segment) is described by a constant equation of state parameter w:

$w_{\rm inf} = -1$; $w_{\rm RD} = \frac{1}{3}$; $w_{\rm MD} = 0$;
$w_{\rm RH} = \frac{n-1}{n+1}$ **for inflaton potential dominated (during reheating) by the term ϕ^{2n}**



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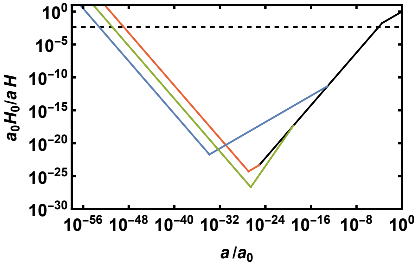
- Number of e-folds during inflation (after scale k_* crossed the horizon) N_{k_*} is obtained from

$$N_{k_*} = -\frac{1}{M_{\rm P}^2}\int_{\phi_{k_*}}^{\phi_{\rm end}} \frac{V(\phi)}{\partial_\phi V(\phi)}\,{\rm d}\phi$$

e.g. for T-models: $N_{k_*} = \frac{8n}{r(\xi^2-1)}\left(2\xi - \frac{16+r(\xi^2-1)+\sqrt{16+r(\xi^2-1)}\sqrt{r(\xi^2-1)}}{2\left(\sqrt{16+r(\xi^2-1)}+\sqrt{r(\xi^2-1)}\right)}\right)$



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- Number of e-folds during reheating $N_{\rm RH}$ is obtained by solving appropriate set of Boltzmann equations

Approximating inflaton potential during reheating by a monomial $V \propto \phi^{2n}$ one gets the following set of Boltzmann equations

$$\boxed{\dot{\rho_\phi} = -3(1+w)H\rho_\phi - \Gamma\rho_\phi \qquad\qquad \dot{\rho_R} = -4H\rho_R + \Gamma\rho_\phi}$$

with $w = \frac{n-1}{n+1}$ and $H^2 = \frac{\rho_\phi + \rho_R}{3M_{\rm P}^2}$

Using (standard) approximation $H^2 = \frac{\rho_\phi}{3M_{\rm P}^2}$ and changing the independent variable from the cosmic time to the number of e-folds (after the end of inflation), $N = a(t)/a_{\rm end}$, we obtain the set of equations

$$\boxed{\frac{\rm d}{{\rm d}N}\rho_\phi = -3(1+w)\rho_\phi - \sqrt{3}M_{\rm P}\Gamma\sqrt{\rho_\phi} \qquad\qquad \frac{\rm d}{{\rm d}N}\rho_R = -4\rho_R + \sqrt{3}M_{\rm P}\Gamma\sqrt{\rho_\phi}}$$

which may be solved (to simplify notation we introduce $\gamma = \frac{\Gamma}{3(1+w)H_{\rm end}}$)

$$\begin{aligned}
\rho_\phi(N) &= \rho_{\rm end}\left[(1+\gamma)e^{-\frac{3}{2}(1+w)N} - \gamma\right]^2 \\
\rho_R(N) &= \rho_{\rm end}\,3(1+w)\gamma\left[\frac{2(1+\gamma)}{5-3w}\left(e^{-\frac{3}{2}(1+w)N} - e^{-4N}\right) - \frac{\gamma}{4}\left(1 - e^{-4N}\right)\right]
\end{aligned}$$

N_{RH}

- Transition between reheating and RD is smooth
- The most natural way to define the end of reheating is the condition $\rho_\phi = \rho_R$
  - In full analogy to the usual definition of the RD/MD border: $\rho_R = \rho_M$
  - With such definition the reheating temperature, $T_{\mathrm{RH}} = T_\times$, is the temperature of the SM plasma at the very beginning of its domination (over inflaton energy)
  - Easily related to less natural (but often used) definitions e.g. $T_{H=\Gamma} \approx \frac{\sqrt{3}(5-3w)^{1/4}}{(2+\sqrt{9-3w})^{1/2}} T_\times$

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- Solutions of the Boltzmann equations give

$$T_{\mathrm{RH}} = T_\times \approx \sqrt{\Gamma M_{\mathrm{P}}} \left(\frac{45}{2g_*}\right)^{1/4} \left(\frac{2+\sqrt{9-3w}}{(5-3w)\pi}\right)^{1/2}$$

$$N_{\mathrm{RH}} = N_\times \approx \frac{2}{3(1+w)} \ln\left(\frac{2(1+\gamma)(5-3w)}{\gamma\left(16+3(1+w)\sqrt{9-3w}\right)}\right)$$

  - Results obtained in the approximation $e^{-4N} \ll e^{-\frac{3}{2}(1+w)N}$ so not accurate for very rapid reheating
  - Very rapid (almost instantaneous) reheating is quite simple and no detailed analysis of such process is needed

Non-perturbative effects

- Boltzmann equations describe smooth and continuous flow of energy from the oscillating inflaton field
  $\Rightarrow$ Equation of state parameter during reheating w_{RH} changes smoothly from the initial value of $\frac{n-1}{n+1}$ (value corresponding to oscillating inflaton field) towards $\frac{1}{3}$ (plasma of relativistic SM particles)

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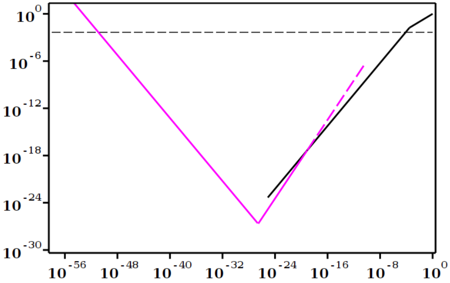
- Lattice simulations necessary to analyse quantitatively the impact of fragmentation
  - Fragmentation leads to qualitatively different modification for $n > 2$ and $n < 2$
  - and has practically no effect on models with:
    $n = 1$ (no fragmentation in quadratic potential)
    $n = 2$ (for quartic potential $w_{\rm inflaton} = \frac{1}{3}$ even before fragmentation)

Results of fragmentation quite simple for models with $n > 2$ so $\frac{n-1}{n+1} > \frac{1}{3}$



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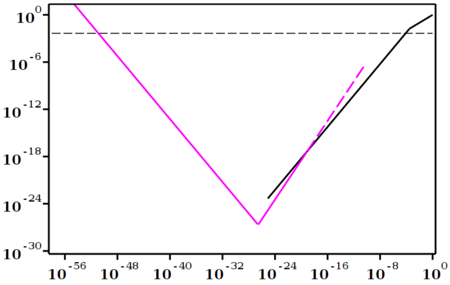

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Fragmentation in models with $n > 2$

Reheating temperatures smaller than some critical value

lead to the same predictions

for values of CMB observables as this critical (model dependent) T

Garcia-Gross-Mambrini-Olive-Pierre-Yoon 2023; Ellis-Garcia-Olive-Verner 2025

Non-perturbative effects

- Results of fragmentation for models with $n < 2$ is different
  - $w_{\text{inflaton}} = \frac{n-1}{n+1} < \frac{1}{3}$
    $\Rightarrow$ energy density of inflaton particles produced during fragmentation red-shifts faster than the energy density of the remaining inflaton condensate
  - $\Rightarrow$ inflaton equation of state parameter w_{inflaton} during fragmentation increases from initial $\frac{n-1}{n+1}$ towards relativistic $\frac{1}{3}$ but after fragmentation it decreases back towards $\frac{n-1}{n+1}$

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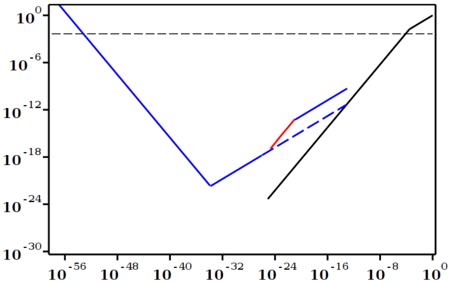

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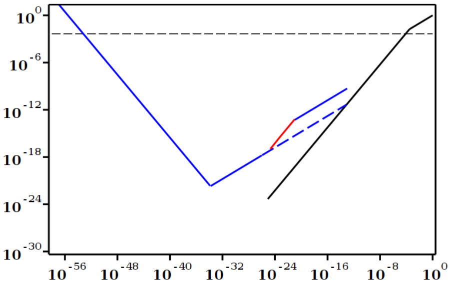

- Case by case lattice simulations are necessary
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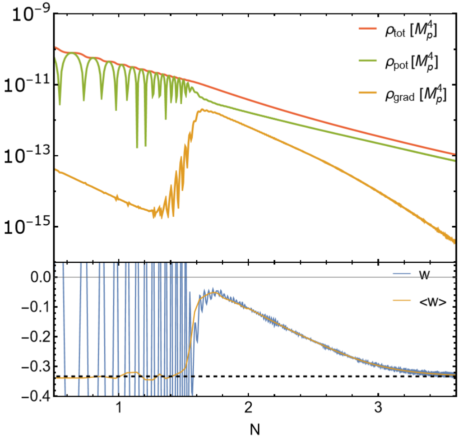
- Example of fragmentation for $n = \frac{1}{2}$ $\longrightarrow$



Reheating temperature in the range: $\mathbf{10\,MeV} \leq T_\times \leq \mathbf{1.6 \cdot 10^{15}\,GeV}$

1σ	2σ	Planck+BICEP/Keck+DESI (P-BK-D)
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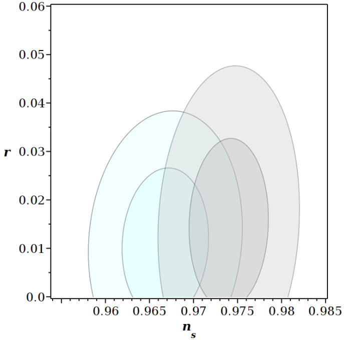
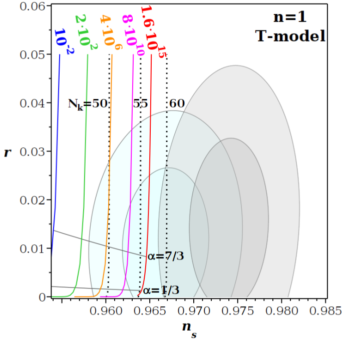


Reheating temperature in the range: $\color{blue}{10\,\mathrm{MeV}} \lesssim T_\times \lesssim \color{red}{1.6\cdot 10^{15}\,\mathrm{GeV}}$

1σ 2σ **Planck+BICEP/Keck+DESI (P-BK-D)**

1σ 2σ **Planck+BICEP/Keck+DESI+ACT (P-BK-D-ACT)**



Reheating temperature in the range: $10\,\mathrm{MeV} \le T_\times \le 1.6 \cdot 10^{15}\,\mathrm{GeV}$

1σ 2σ Planck+BICEP/Keck+DESI (P-BK-D)

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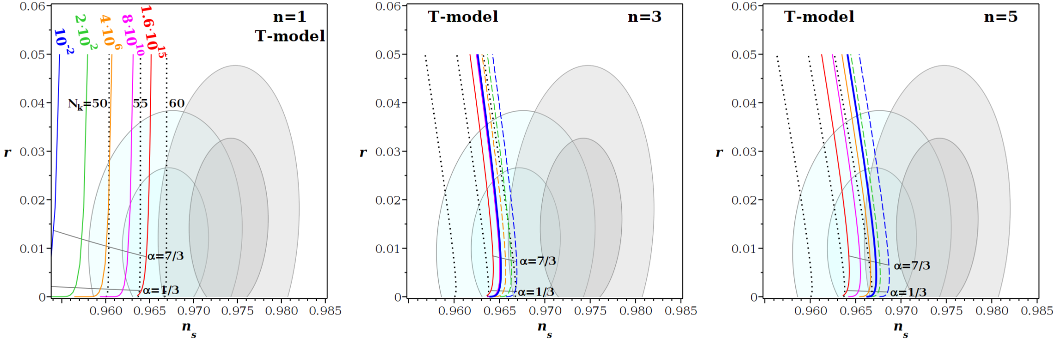
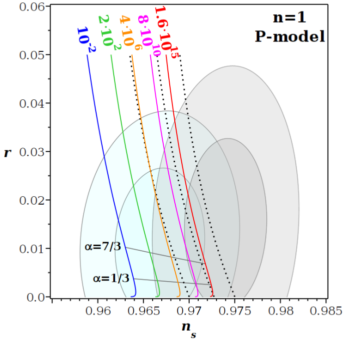


- Typically predictions of T-models are outside 2σ region of P-BK-D-ACT bounds
- Very low $T_\times$ excluded in T-model with $n = 1$
- r in the range $10^{-3} \div 10^{-2}$ for supergravity motivated vales of α

Results for E-models are quite similar

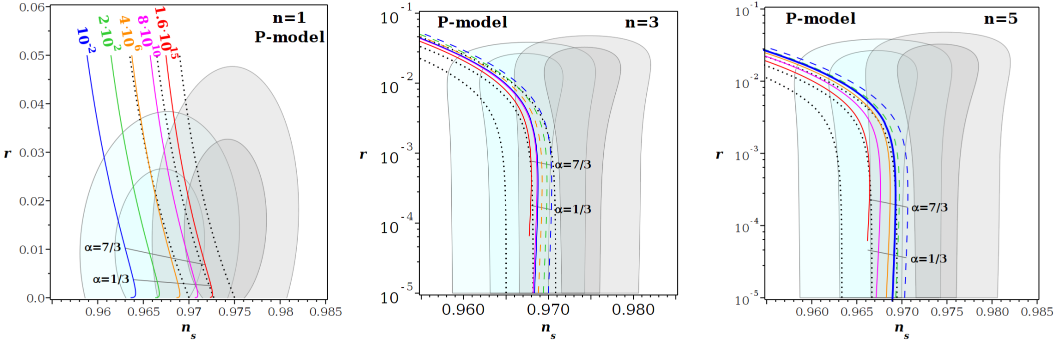
Ellis-Garcia-Olive-Verner 2025

P-models



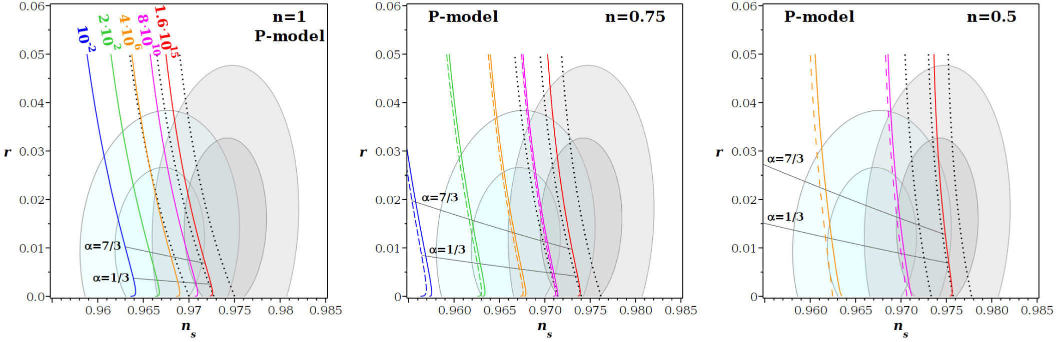
n=1
P-model
10^{-2}
$2\cdot10^{2}$
$4\cdot10^{6}$
$8\cdot10^{10}$
$1.6\cdot10^{15}$
α=7/3
α=1/3
r
n_s
0.06
0.05
0.04
0.03
0.02
0.01
0.0
0.96
0.965
0.97
0.975
0.98
0.985

P-models



- **P-model with $n = 1$:**
  - consistent at 1σ level with P-BK-D data for full range of reheating temperature $10\,\mathrm{MeV} \lesssim T_\times \lesssim 10^{15}\,\mathrm{GeV}$
  - consistent at 1σ level with both sets of experimental data for reheating temperature in the range $10^{10}\,\mathrm{GeV} \lesssim T_\times \lesssim 10^{15}\,\mathrm{GeV}$
- **P-model with $n > 1$:**
  - consistent with P-BK-D-ACT data only at 2σ level
  - values of r for supergravity motivated α are below $\mathcal{O}(10^{-3})$ and decrease with n

P-models with $n < 1$



For P-models with $n < 1$ decreasing n results in:

- slightly bigger values of n_s for very high reheating temperature;
- smaller (much smaller) values of n_s for medium (low) reheating temperature;
- stronger lower bounds on $T_\times$ resulting from CMB experimental data;
- (non-perturbative fragmentation weakens the last two properties but only slightly)
- bigger r for supergravity motivated models.

Singular T-models

Two types of singular α-attractor T-models:

- Logarithmic singularity at the border of supergravity moduli space:

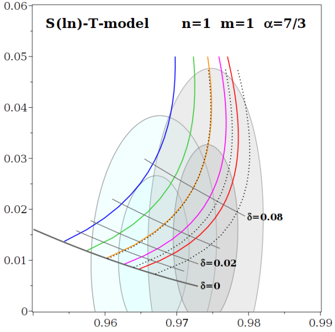
$$V(\phi) = V_0 \tanh^{2n}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\left[1+\delta \ln^{m}\left(\cosh^2\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\right)\right]$$

- Power-low singularity at the border of supergravity moduli space:

$$V(\phi) = V_0 \tanh^{2n}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\left[1+\delta \cosh^{2m}\left(\sqrt{\frac{1}{6\alpha}}\frac{\phi}{M_{\rm P}}\right)\right]$$

One additional exponent, m or $2m$, in the definition of a model
One additional parameter δ, $\delta > 0$ in singular models

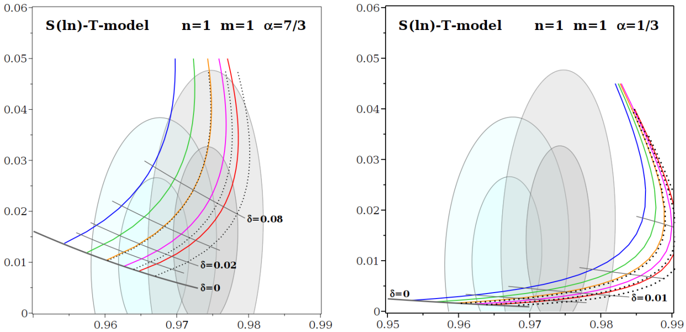
Linde et al 2017; Kallosh-Linde 2025



The simplest ($n = 1$, $m = 1$) logarithmically singular T-model

- **for $\alpha = \frac{7}{3}$ and $0.01 \lesssim \delta \lesssim 0.2$ gives predictions for n_s in nice agreement with the P-BK-D-ACT data and relatively big values of r (up to present exp. bounds)**

Singular T-models



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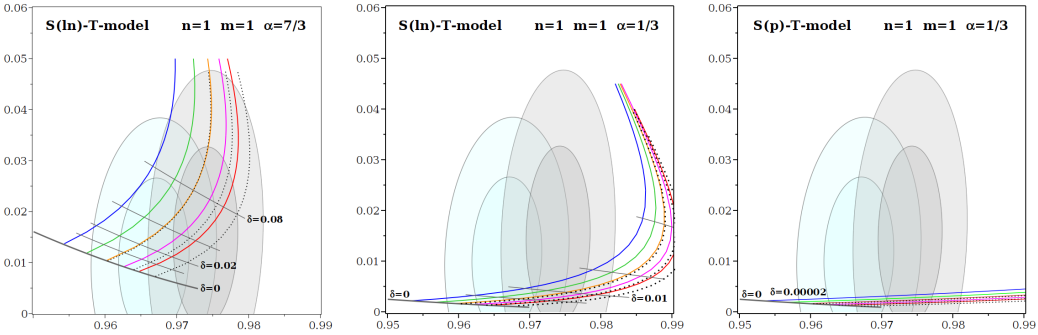
- predicted values of n_s increase with decreasing α
- for $\alpha = \frac{1}{3}$ experimental bounds may be fulfilled only for small values of r and δ

Singular T-models



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- predicted values of n_s increase with decreasing α
- for $\alpha = \frac{1}{3}$ experimental bounds may be fulfilled only for small values of r and δ

In many versions of singular models predictions for n_s are too high (often $n_s > 1$) even for very small values of δ

Summary

- Predictions for CMB observables compared with experimental bounds for several classes of supergravity motivated α-attractor models of inflation:

  T-models, E-models, P-models and singular T-models investigated
- Consistency condition used to determine reheating temperature $T_\times$ for each model
- Lattice simulations performed in order to take into account non-perturbative fragmentation of oscillating inflaton field

Summary

- Predictions for CMB observables compared with experimental bounds for several classes of supergravity motivated α-attractor models of inflation:

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- Consistency condition used to determine reheating temperature $T_\times$ for each model
- Lattice simulations performed in order to take into account non-perturbative fragmentation of oscillating inflaton field

- Model-independent limits on $T_\times$ (lower from BBN, upper from inflation energy scale) in many cases lead to quite strong bounds on the spectral index n_s
  - The resulting allowed range for the number of e-folds during inflation, N_{k_*}, is substantially different from the often assumed one $50 \leq N_{k_*} \leq 60$
- For some models experimental bounds on CMB observables result in limits on $T_\times$ stronger than the model-independent ones (e.g. models with $n < 1$)
- P-models, especially for $n \leq 1$, may accommodate slightly bigger maximal values and much smaller minimal values of n_s as compared to T- and E-models
- Singular T-models may accommodate much bigger values of n_s
  - in many cases n_s becomes too big
  - one of the simplest models nicely agrees with P-BK-D-ACT data