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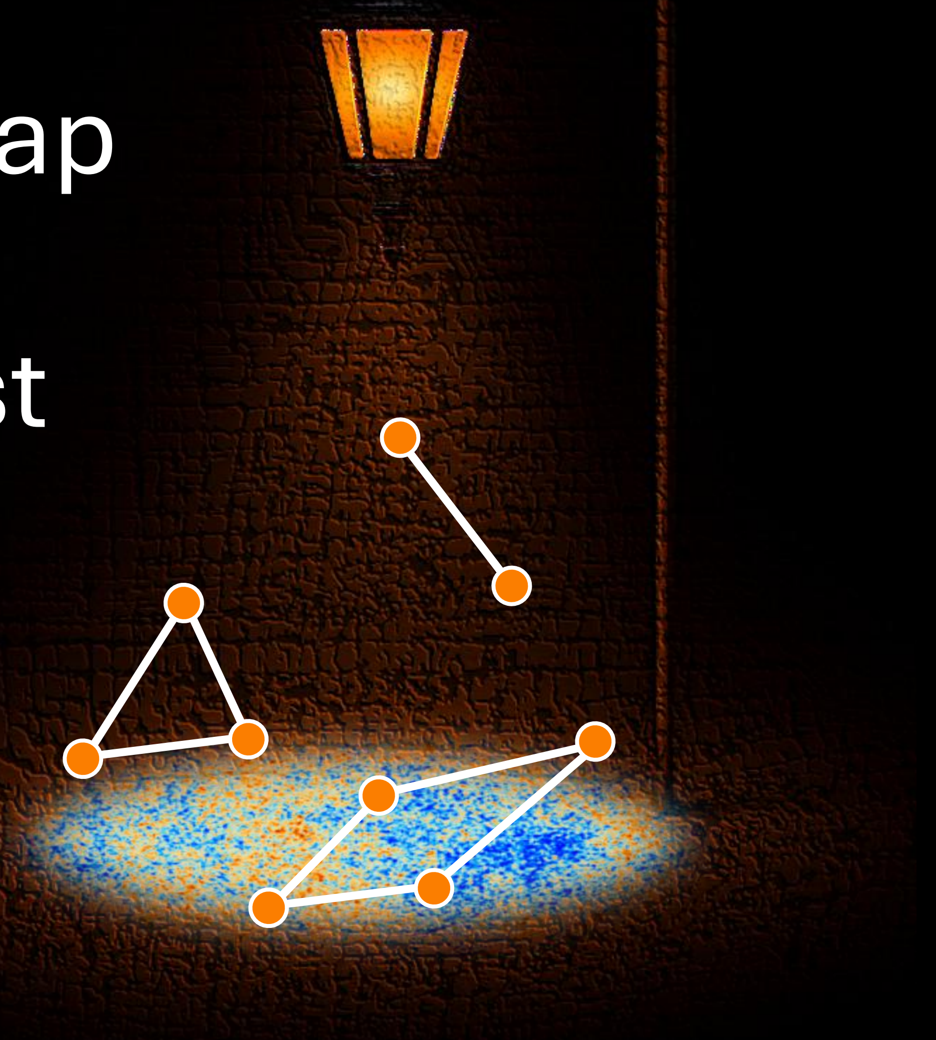
Cosmological Bootstrap beyond the Symmetric Lamppost



Xi Tong(童曦)

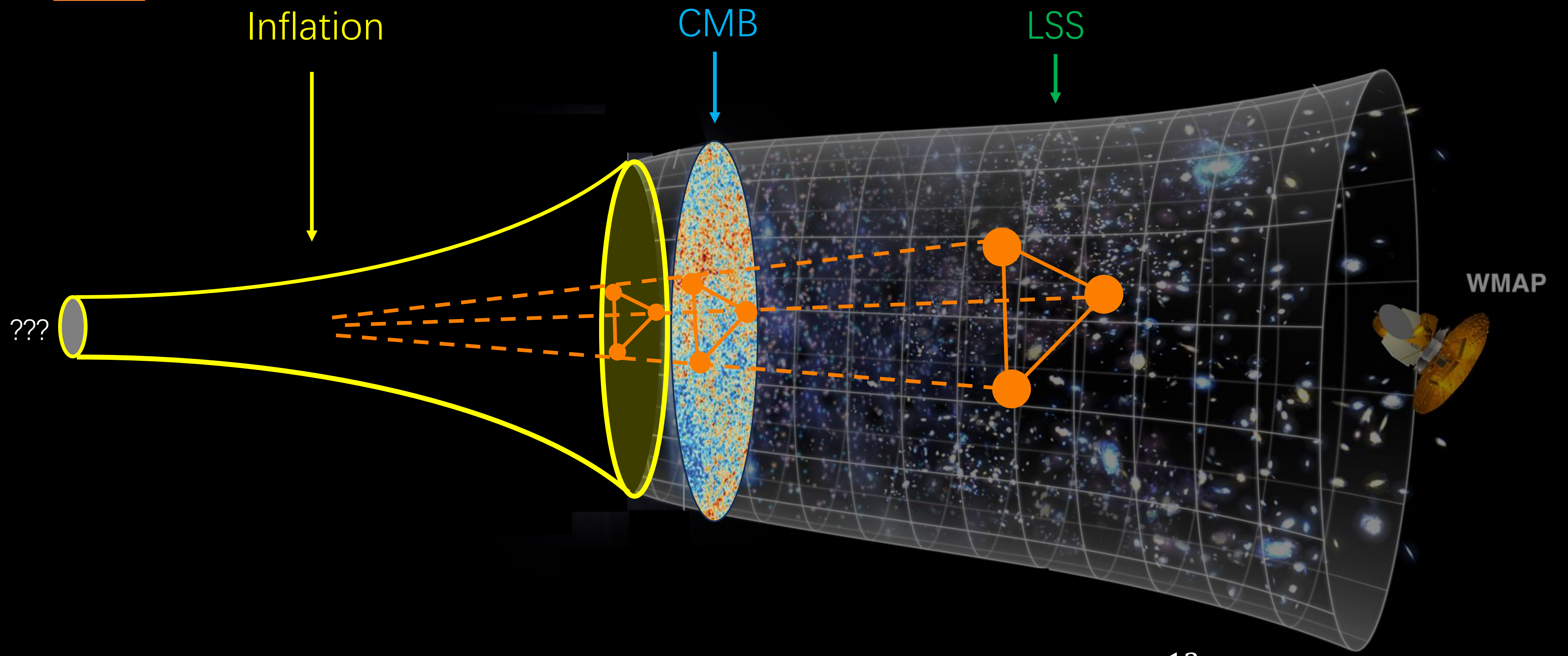
DAMTP → UvA

- Based on
  2511.00152 (JHEP)
  2506.01555 (JCAP)

- With Sadra Jazayeri, Yuhang Zhu, Zhehan Qin, Sébastien Renaux-Petel, and Denis Werth

Outline

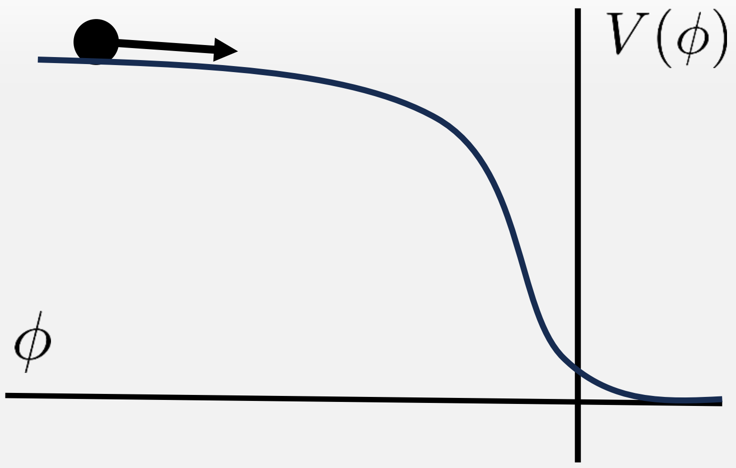
- Introduction – Inflationary correlators
- Symmetries and beyond
- A **boundary bootstrap** of symmetry breaking
    - ~~Dilation~~
- Summary

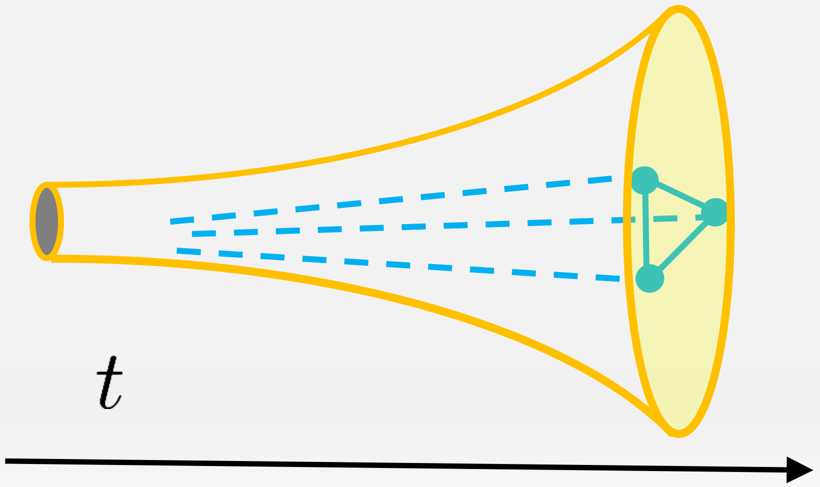


❖ **Cosmological correlators** probe physics of inflation @ $H \leq 10^{13}$ GeV
(particle spectrum, interactions, symmetries, etc.)

NASA/WMAP Science Team

What are the inflationary correlators?





Inflationary action $S[\phi, g_{\mu\nu}, \cdots]$

⇩

Background $\phi_0(t)\ ,\ a(t)$

⇩

Perturbations $(\delta\phi\ ,\ \delta g_{\mu\nu}) \leftrightarrow (\zeta\ ,\ \gamma_{ij})$

⇩

Late-time correlators

$$B_n \equiv \lim_{t\to\infty} \langle \varphi(t, \mathbf{k}_1) \cdots \varphi(t, \mathbf{k}_n) \rangle_{\text{inf}}$$

$$\varphi = \zeta, \gamma_{ij}$$

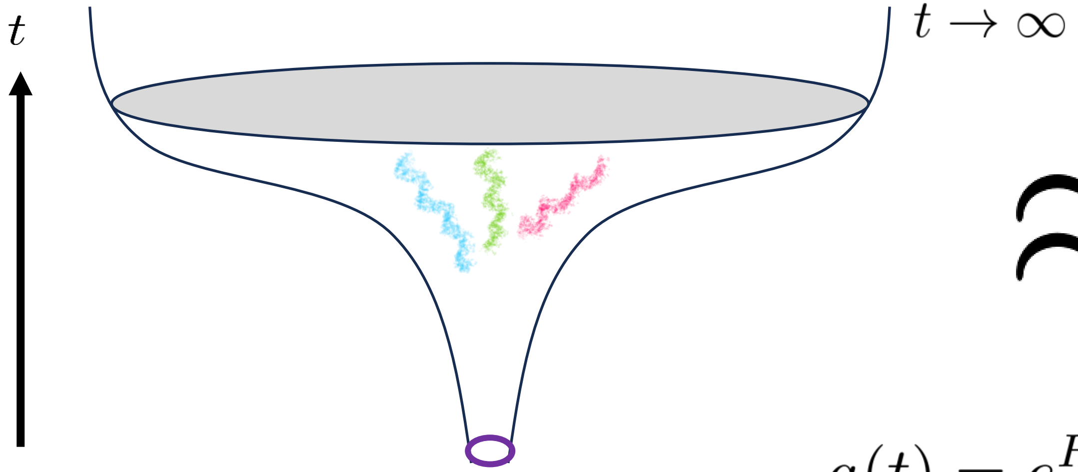
Inflation ≈ de Sitter

Inflationary late-time correlators

$$\lim_{t\to\infty}\langle\varphi(t,\mathbf{k}_1)\cdots\varphi(t,\mathbf{k}_n)\rangle_{\text{inf}}$$

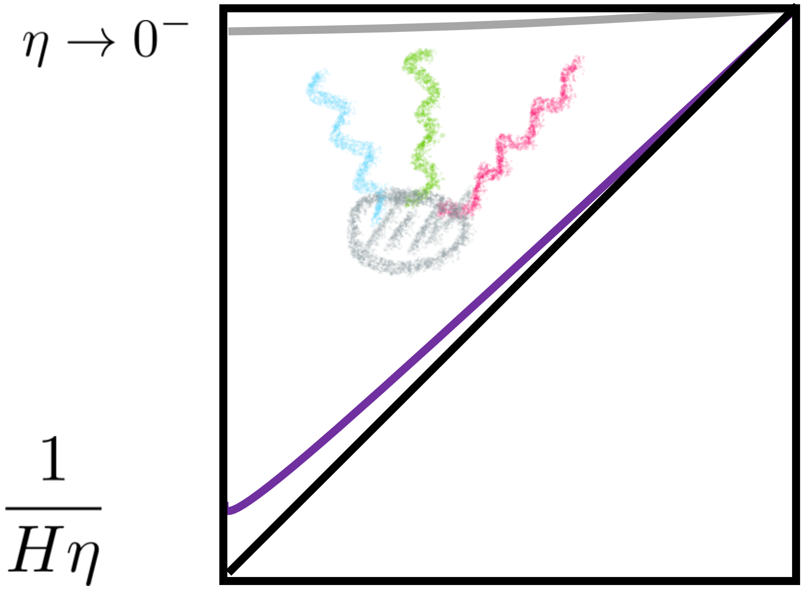
dS boundary correlators

$$\lim_{\eta\to 0^-}\langle\varphi(\eta,\mathbf{k}_1)\cdots\varphi(\eta,\mathbf{k}_n)\rangle_{\text{dS}}$$



$$\approx$$

$$a(t) = e^{Ht} = -\frac{1}{H\eta}$$



dS correlators are highly constrained by symm.

❖ dS isometries

$$SO(4,1) = \begin{cases} \text{3 trans.} & \mathbf{x} \to \mathbf{x} + \mathbf{c} \\ \text{3 rots.} & \mathbf{x} \to R\mathbf{x} \\ \text{1 dilation} & (\eta, \mathbf{x}) \to (\lambda\eta, \lambda\mathbf{x}) \\ \text{3 boosts} & (\eta, \mathbf{x}) \to \cdots \end{cases}$$

❖ Ward identities

$$B_n \propto \delta^3(\sum \mathbf{k})$$

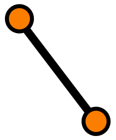
$$B_n = B_n(\mathbf{k}_\mathrm{a} \cdot \mathbf{k}_\mathrm{b})$$

$$B_n \sim k^{-3(n-1)}$$

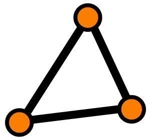
$$\mathcal{D}_i B_n = 0 \ , \ \ i = 1, 2, 3$$

❖ Ward ids **completely fix $n = 2, 3$**

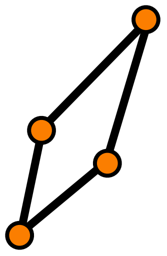
and **highly constrain $n = 4$**



$$B_2' = A_s k^{-3}$$



$$B_3' = \sum \text{Appell-}F_4 \left(\cdots \middle| \frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$



$$B_{4,s}'^{\text{tree}} = \sum c_{mn} u^m v^n + {}_2F_1(\cdot|u) \times {}_2F_1(\cdot|v)$$

$$u \equiv \frac{s}{k_1 + k_2} \ , \ \ v \equiv \frac{s}{k_3 + k_4}$$

$$s \equiv |\mathbf{k}_1 + \mathbf{k}_2|$$

[Arkani-Hamed, Baumann, Lee & Pimentel, 2018]

Inflation $\neq$ de Sitter

- However, inflation is **NOT** just dS – the uniform bg. selects a **rest frame** & a **preferred clock**

$$\langle\phi(t,\mathbf{x})\rangle = \boxed{\phi_0(t)} \xrightarrow{\text{in general}} \text{3 } \cancel{\text{boosts}}\text{, 1 } \cancel{\text{dilation}}$$

$$\approx \boxed{\dot{\phi}_0 t} \xrightarrow{\text{slow-roll}} \text{3 } \cancel{\text{boosts}}\text{, 1 dilation}$$

- Fewer symmetries implies **less control** over the correlators

$$\text{\# of symmetries} = \begin{cases} 10-3-1=6 \\ 10-3=7 \end{cases}$$

	B_2	B_3	B_4
# of DoFs	6	9	12
Fixed?	✗ ✓	✗ ✗	✗ ✗

Less control = more to wish for ☺

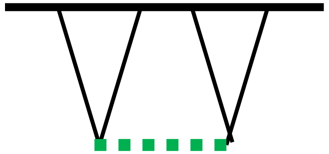
❖ Symm.-breaking correlators are also **more phenomenologically interesting**

✓ More particle production

$$n_\sigma = e^{-2\pi M/H} \xrightarrow[\kappa = \frac{\dot{\phi}_0}{\Lambda_c}]{\text{boost-breaking chemical potential}} e^{-2\pi(M-\kappa)/H}$$

[Sou, **Tong** & Wang, 2021]
[See Marco & Ruth's talks]

✓ Larger cosmological collider signals

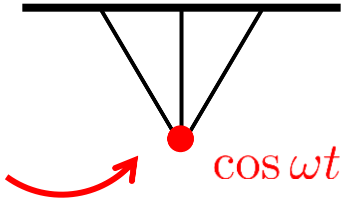


$$\Rightarrow B_4 \sim e^{-\pi(M-\kappa)/H} \times \left(\frac{k_L}{k_S}\right)^{3/2 \pm iM/H}$$

[Wang & Xianyu, 2019]
[**Tong** & Xianyu, 2022]
[See Sebastien's talk]

✓ New shapes e.g. resonant NG

Dilation-breaking from e.g. axion-monodromy

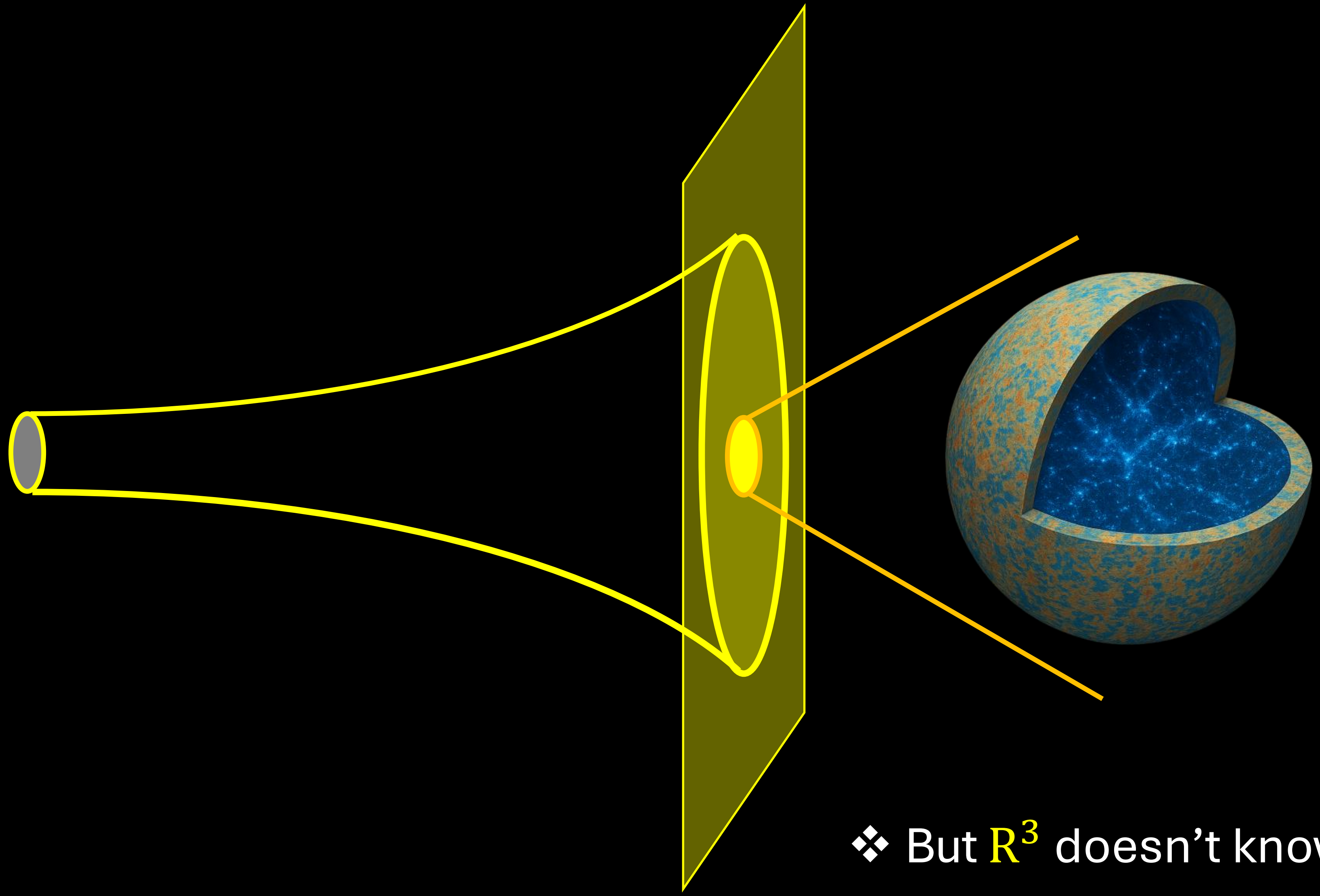


$$\Rightarrow B_3 \sim \omega^n k_T^{-6 \pm i\omega}$$

[Flauger & Pajer, 2010]
[Duaso Pueyo & Pajer, 2023]

One more motivation *à la* “bootstrap”...

Motivation from a “boundary bootstrap” ?



- In cosmology,

  time → spatial kinematics

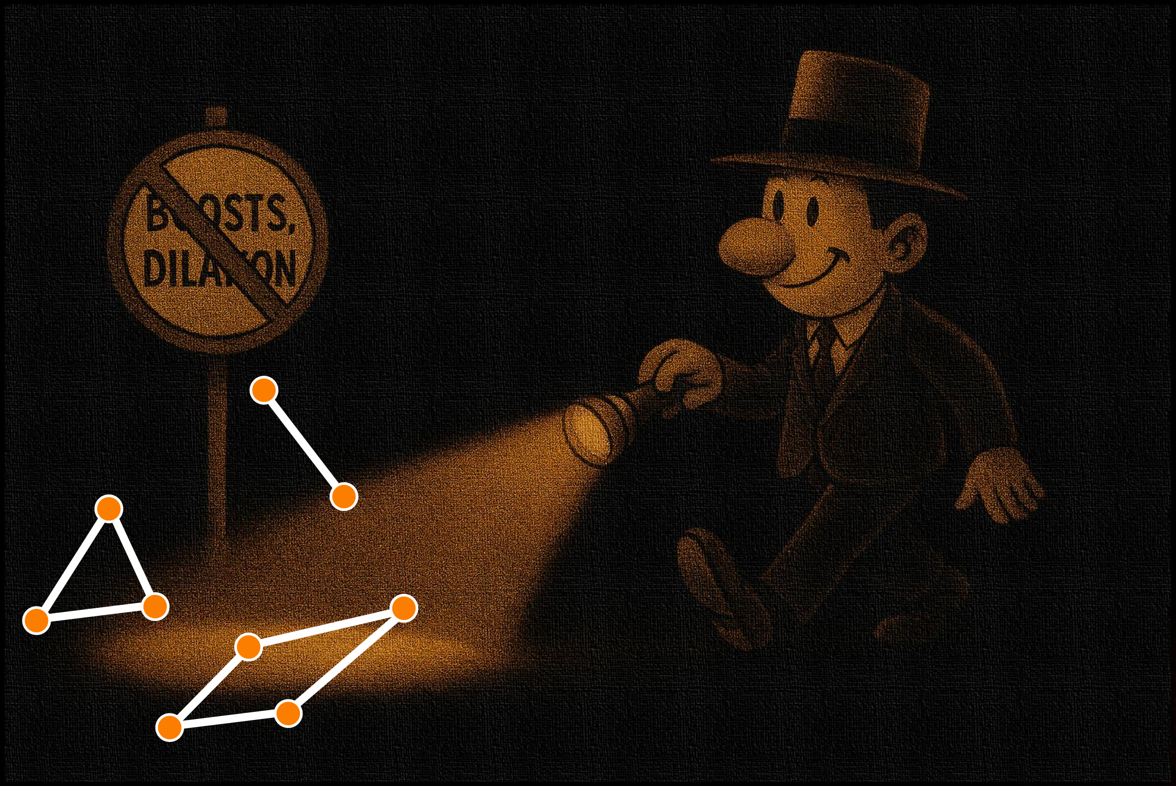
- CMB/LSS trace back to the 3d “boundary”

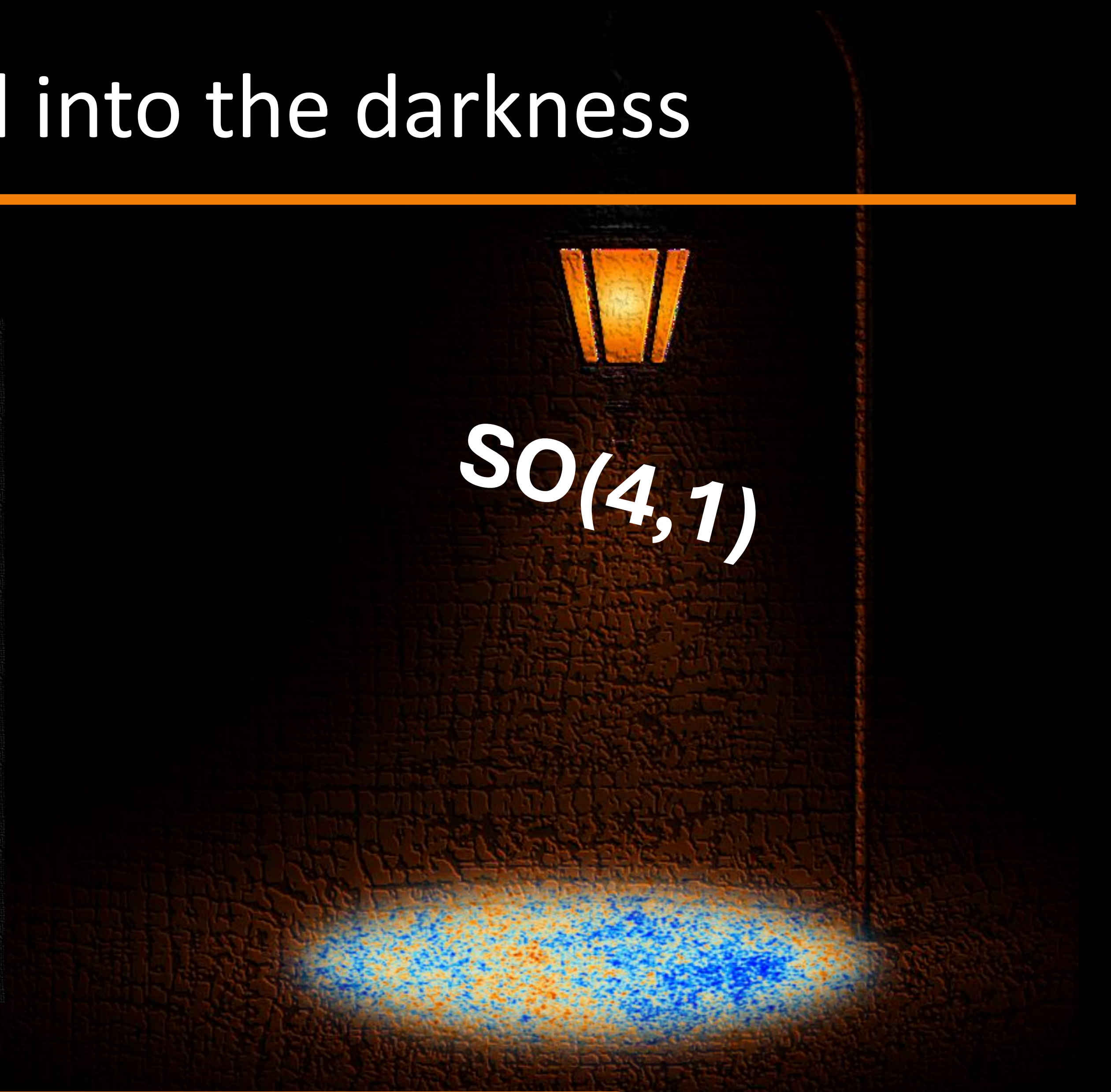
- Bootstrap eq. expressed in $\mathbf{k} \in \mathrm{R}^3$

$$\widehat{\mathcal{E}}[B_n](\mathbf{k}_1, \cdots, \mathbf{k}_n) = 0$$

- But R^3 doesn't know about boosts & dilation

Out of the symmetries and into the darkness

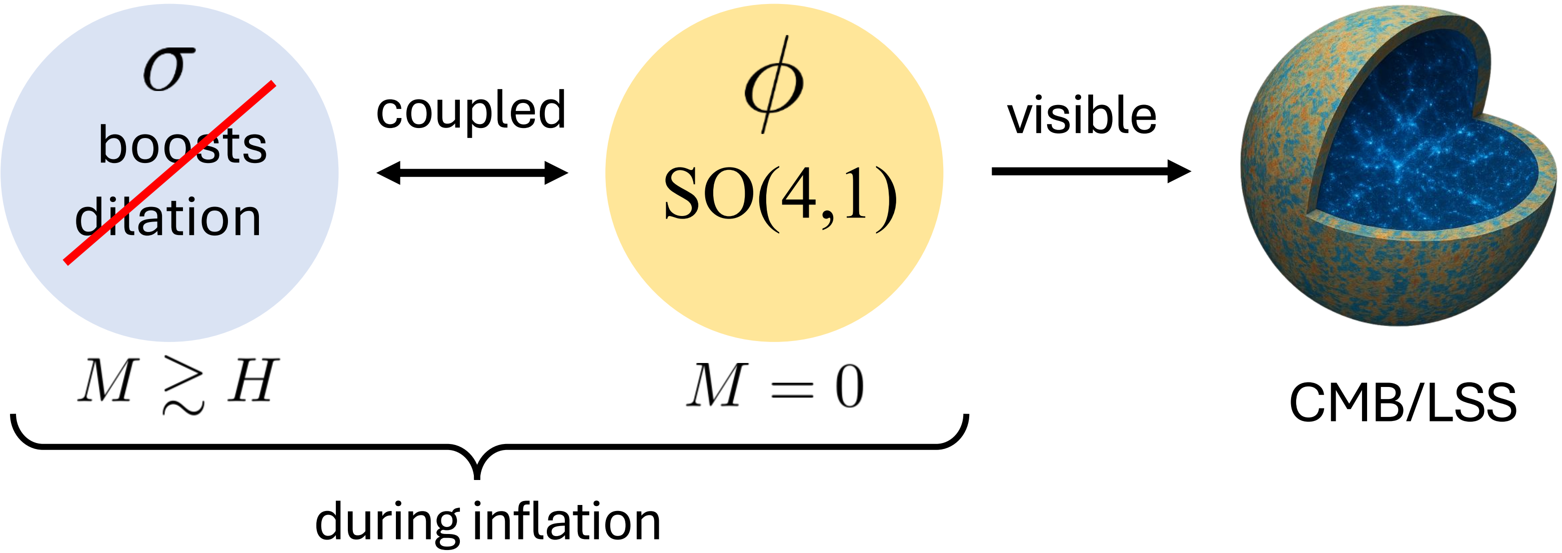




- A **systematic** and **non-perturbative** boundary bootstrap?

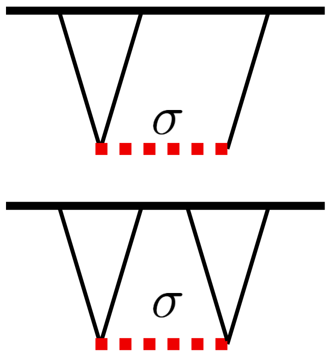
Out of the symmetries and into the darkness

❖ Consider a **restricted** symmetry-breaking set up



❖ Goal: find & solve boundary bootstrap eqs.

$$\widehat{\mathcal{E}}[B_n^{\phi}](\mathbf{k}_1, \cdots, \mathbf{k}_n) = 0\,? \quad , \quad n = 3, 4$$



Symmetry-breaking dynamics in the σ-sector

[Jazajeri, **Tong** & Zhu, 2025]

- Boost-breaking dispersion: $\omega_\sigma^2 = \alpha_0 + \alpha_1 k + \alpha_2 k^2$

Rest mass M^2

local iff spinning

$\kappa\, \mathbf{k} \cdot \mathbf{S}$ (chem ptl. e.g. $\frac{\phi_0(t)}{\Lambda_c}\sigma_{\mu\nu}\widetilde{\sigma}^{\mu\nu}$)

Sound speed $c_s^2 \neq 1$

- Most general **unitary**, **boost-** & **dilation-breaking** EoM up to 2 derivatives

$$\left[\eta^2\partial_\eta^2 - 2\eta\partial_\eta + \left(M^2(\eta) + \kappa(\eta)k\eta + c_s^2(\eta)k^2\eta^2\right)\right]\sigma_{\mathbf{k}}(\eta) = 0$$

Dilation breaking *à la* primordial features [See Matteo's talk]

[Jazajeri, **Tong** & Zhu, 2025]

❖ First study an **oscillating mass** alone

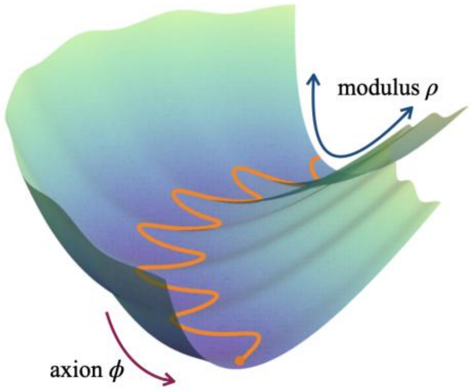
$$\left[\eta^2\partial_\eta^2 - 2\eta\partial_\eta + \left(M^2(\eta) + k^2\eta^2\right)\right]\sigma_{\mathbf{k}}(\eta) = 0$$

$$\downarrow$$

$$M_0^2(1 + g^2\cos\omega t)\ , \quad g^2 < 1$$

- Motivated from axion-monodromy inflation

$$V(\phi,\sigma) \supset \frac{1}{2}g^2 M_0^2 \sigma^2 \cos\frac{\phi}{f}$$



[Image credit: Pajer, Wang & Zhang, 2024]

- General time dep. can always be Fourier expanded

$$M^2(t) = M_0^2\left(1 + g^2\int \mathrm{d}\omega\, \rho_\omega \cos\omega t\right)$$

Traditional case (dilation-invariant): PDEs

- Bootstrap strategy: Integrate the EoM

KG dynamics in the bulk

Kinematic constraint on the bdy.

$$0 = \int \mathrm{d}\eta\, e^{ik\eta}[\Box - M^2]\sigma \quad = \quad [\Delta_k + M^2] \int \mathrm{d}\eta\, e^{ik\eta}\sigma = 0$$

$$\eta^n \leftrightarrow (-i\partial_k)^n$$

$$\partial_\eta^n \leftrightarrow (ik)^n$$

- A **partial differential equation (PDE)**

$$(\Delta_{12} + M_0^2)\, F_4(k_{12}, k_{34}, s) = \frac{1}{k_{12} + k_{34}}$$

with $\quad \Delta_{12} \equiv (k_{12}^2 - s^2)\partial_{k_{12}}^2 + 2k_{12}\partial_{k_{12}} - 2$

[See Sebastien’s talk]

Our case (dilation-breaking): IDEs

- Oscillating mass translates to a **fractional derivative**

$$0 = \int \mathrm{d}\eta\, e^{ik\eta}[\Box - M^2(\eta)]\sigma \quad = \quad [\Delta_k + ???] \int \mathrm{d}\eta\, e^{ik\eta}\sigma = 0$$

$$M_0^2\left(1 + \frac{g^2}{2}(-\eta)^{i\omega} + \text{c.c.}\right)$$

- An **integro-differential equation (IDE)**

$$(\Delta_{12} + M_0^2)\, F_4(k_{12}, k_{34}, s) = \frac{1}{k_{12} + k_{34}} + \int_0^\infty \mathrm{d}q\, K(q)\, F_4(k_{12} + q, k_{34}, s)$$

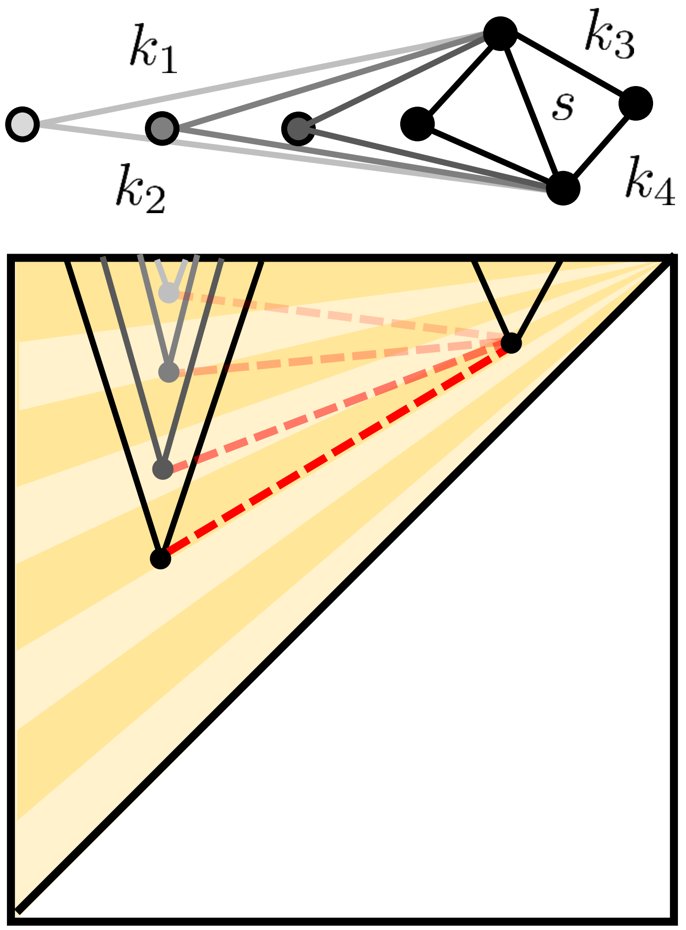
Memory kernel: $K(q) = -\frac{1}{2} g^2 M_0^2 \frac{e^{-\pi\omega/2}}{\Gamma(i\omega)} \frac{1}{q} (-q\eta_0)^{+i\omega} + (\omega \leftrightarrow -\omega)$

- General time dependences + $\kappa(\eta)\ ,\ c_s^2(\eta)$ $\Rightarrow$ same IDE, different memory kernel

Kinematic non-locality from special moments in time

[Jazajeri, **Tong** & Zhu, 2025]

$$(\Delta_{12} + M_0^2)\, F_4(k_{12}, k_{34}, s) = \frac{1}{k_{12} + k_{34}} + \int_0^{\infty} \mathrm{d}q\, K(q)\, F_4(k_{12} + q, k_{34}, s)$$



❖ Why the kinematic non-locality?

✓ Dialing kinematics on the bdy. = dialing time in the bulk

✓ Broken dilation = extra **ref time** (mass phase)

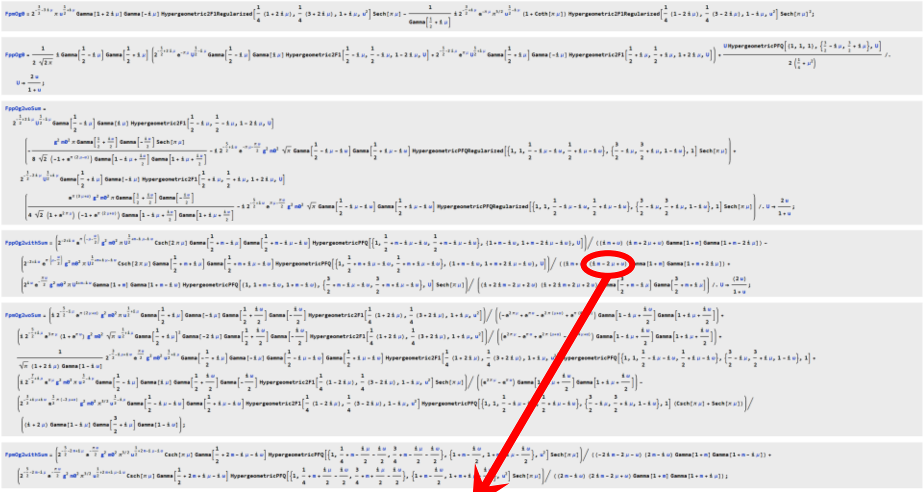
✓ Non-locality in kinematics = **memory** in the bulk

Analytical bootstrap: results

$$(\Delta_{12} + M_0^2)\, F_4(k_{12}, k_{34}, s) = \frac{1}{k_{12}+k_{34}} + \int_0^\infty \mathrm{d}q\, K(q)\, F_4(k_{12}+q, k_{34}, s)$$

[Jazajeri, **Tong** & Zhu, 2025]

✓ Perturbative sol. up to $\mathcal{O}(g^2)$



$$\boxed{/\,(\mathrm{i}\ m-2\ \mu+\omega)} \propto \frac{g^2}{\omega - 2\mu}$$

breakdown of PT ?!

✓ Non-pert. scaling exponent near the **narrow resonances**

$$F_4 \sim \left(\frac{k_{12}}{s}\right)^{-1/2+\lambda_n \pm i\mu}$$

Floquet analysis of the boundary IDE shows

$$\lambda_1 = \frac{g^2}{4\mu}\left(\mu^2 + \frac{9}{4}\right), \quad \omega = 2\mu$$

$$\lambda_2 = \frac{g^4\mu}{8}\left(1 + \frac{9}{4\mu^2}\right)^2, \quad \omega = \mu$$

Numerical bootstrap: FDM

[Jazajeri, **Tong** & Zhu, 2025]

✓ **Finite difference** approach

- Change of var. $\left[\left(1-e^{-2r}\right)\partial_r^2+\left(1+e^{-2r}\right)\partial_r+\left(\mu^2+\frac{1}{4}\right)\right]F(r;x_0)=\frac{1}{1+e^r}+\int_r^\infty \mathrm{d}r'\,\mathcal{K}(r,r';x_0)\,F(r';x_0)$

$$k_{12}/s\equiv e^r\ ,\quad r\in[0,\,\infty)$$

- Discretisation $r\in[0,L]\to r_i=\frac{i}{N}L\ ,\ i=0,1,\cdots,N$

$$\partial_r F(r)\to\frac{F(r_{i+1})-F(r_{i-1})}{2L/N}\ ,$$

$$\partial_r F(0)\to\frac{F(r_1)-F(r_0)}{L/N}\ ,\quad \partial_r F(L)\to\frac{F(r_N)-F(r_{N-1})}{L/N}\ ,$$

$$\partial_r^2 F(r)\to\frac{F(r_{i+1})-2F(r_i)+F(r_{i-1})}{L^2/N^2}\ ,$$

$$\partial_r^2 F(0)\to\frac{F(r_2)-2F(r_1)+F(r_0)}{2L^2/N^2}\ ,\quad \partial_r^2 F(L)\to\frac{F(r_N)-2F(r_{N-1})+F(r_{N-2})}{2L^2/N^2}\ .$$

$$\int_r^L \mathrm{d}r'\mathcal{K}(r,r')F(r')\to\frac{L}{2N}\mathcal{K}(r_i,r_i)F(r_i)+\sum_{j=i+1}^{N-1}\frac{L}{N}\mathcal{K}(r_i,r_j)F(r_j)+\frac{L}{2N}\mathcal{K}(r_i,r_N)F(r_N)$$

- Boundary condition: matching with PT

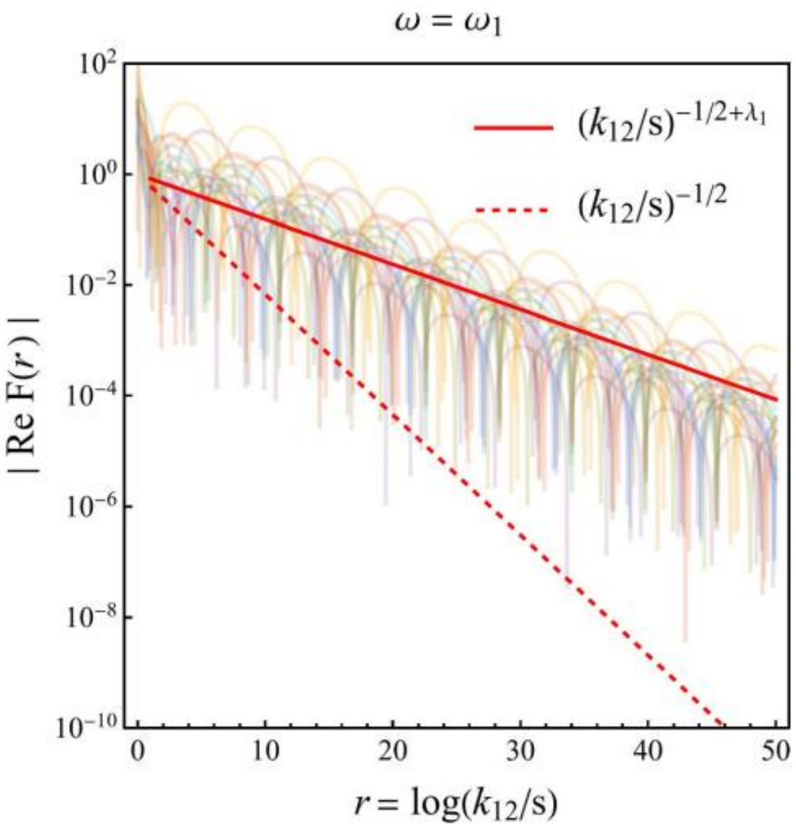
$$F(r_{\mathrm{I}})=F^{(g^2)}(r_{\mathrm{I}})\ ,$$
$$F(r_{\mathrm{II}})=F^{(g^2)}(r_{\mathrm{II}})$$

(accuracy bottleneck)

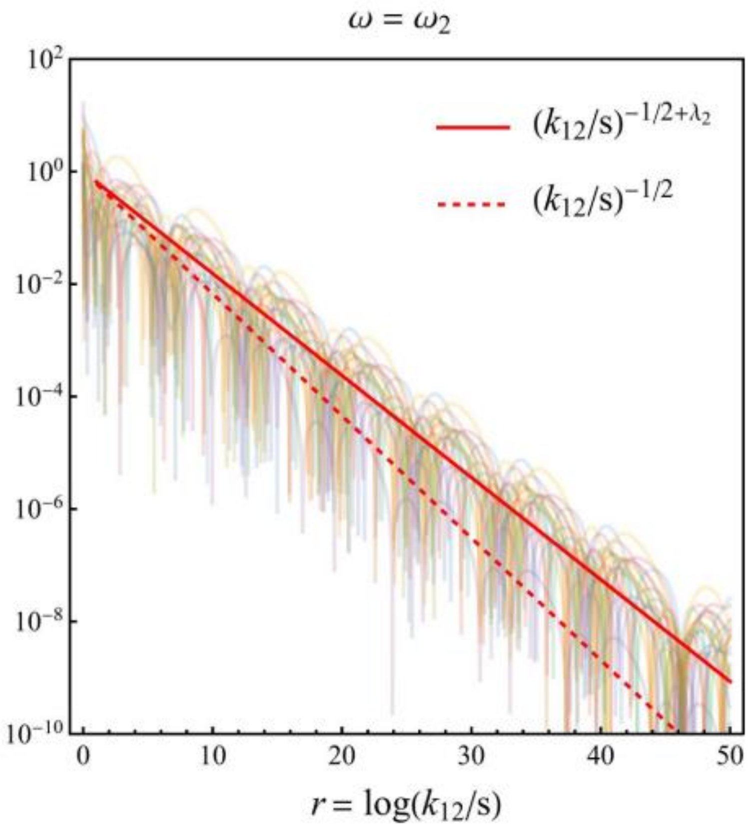
Numerical bootstrap: results

[Jazajeri, **Tong** & Zhu, 2025]

- The primary resonance



- The secondary resonance



- 20 random boundary conditions
- Perfect match with the boundary analysis

Summary

- ❖ Inflationary correlators are useful probes of new physics

- ❖ dS symmetries are powerful

- ❖ Inflation breaks dS symmetries

- ❖ A boundary bootstrap beyond the symmetric lamppost

  - Memory effect ⇒ an IDE is needed in general

$$\mathcal{D}F = \int KF$$

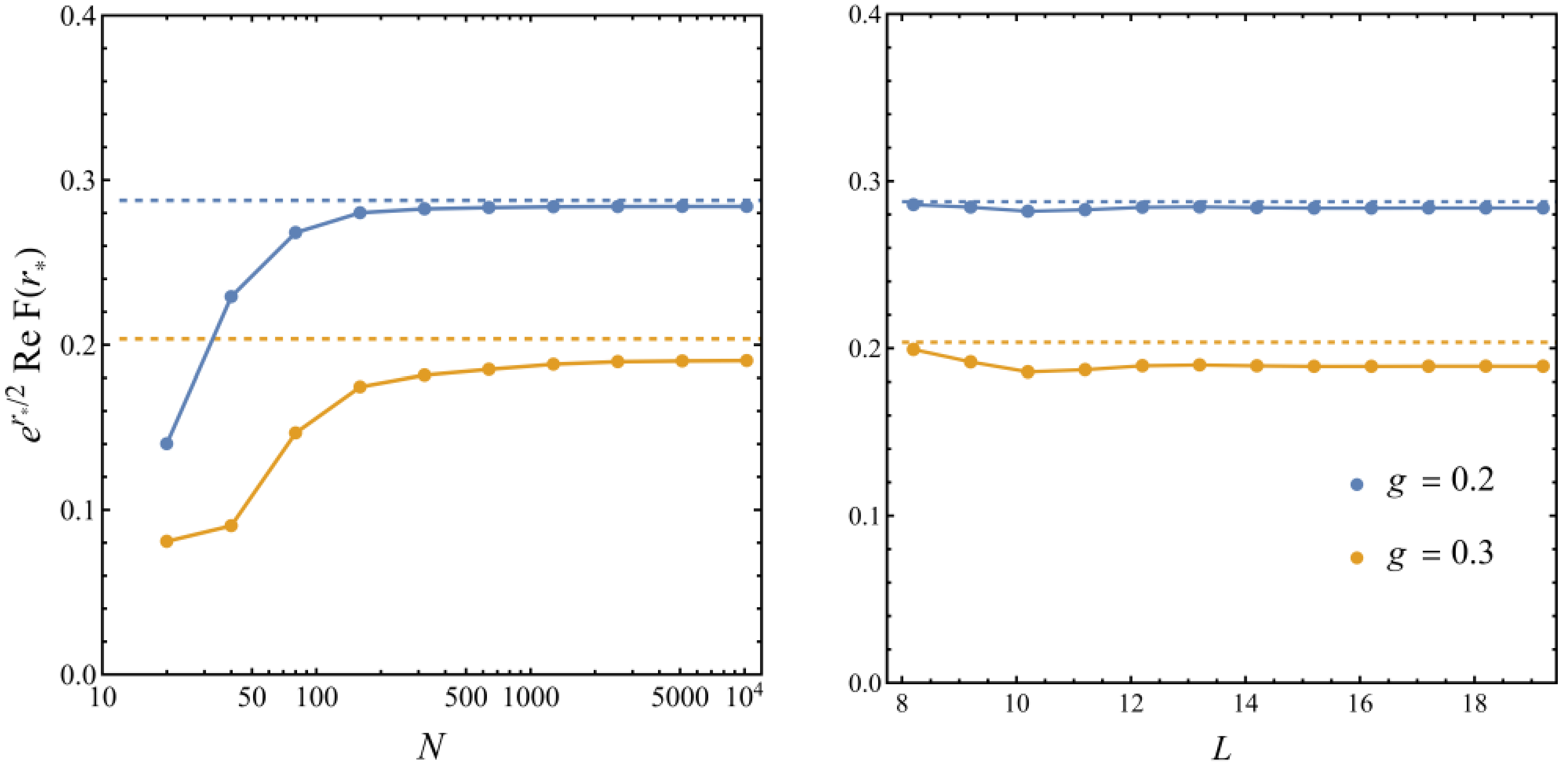
  - Non-perturbative resonances resummed


Thank you for listening!

Back ups

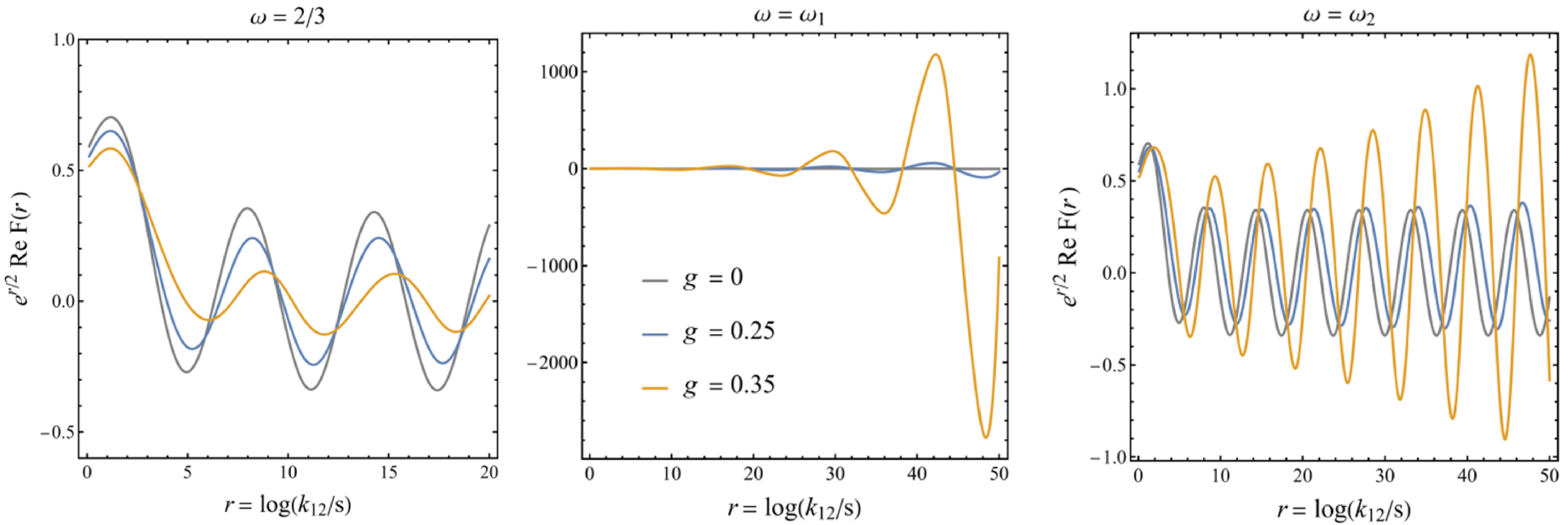
Case II: Numerical cosmological bootstrap

✓ Numerical convergence



Case II: Numerical cosmological bootstrap

- Off-resonance
- The primary resonance
- The secondary resonance



Case I: boost-breaking only

[Qin, Renaux-Petel, **Tong**, Werth & Zhu, 2025]

❖ Boost-breaking dispersion: $\omega_\sigma^2 = \alpha_0 + \alpha_1 k + \alpha_2 k^2$

Rest mass M^2

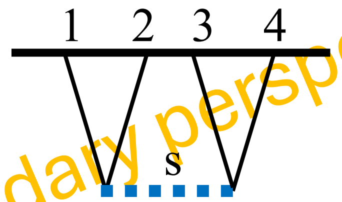
Sound speed $c_s^2 \neq 1$

local iff spinning

$\kappa\, \mathbf{k} \cdot \mathbf{S}$ (chem ptl. e.g. $\frac{\phi_0(t)}{\Lambda_c} \sigma_{\mu\nu} \tilde{\sigma}^{\mu\nu}$)

❖ EoM: $$\left[\eta^2 \partial_\eta^2 - 2\eta\partial_\eta + \left(M^2 \pm 2\kappa k\eta + c_s^2 k^2 \eta^2\right)\right] \sigma_{\mathbf{k}}^{\pm}(\eta) = 0$$

❖ 4pt diagram



$$= \sum_{\pm,\, \text{SK ids.}} \int_{-\infty}^{0} \mathrm{d}\eta_1 \mathrm{d}\eta_2 \hat{V}_1(K_1 K_2) G_{\sigma,\mathbf{s}}^{\pm}(\eta_1, \eta_2) \hat{V}_2(K_3 K_4)$$

A boundary perspective?

Case I: lifting to the boundary

[Qin, Renaux-Petel, **Tong**, Werth & Zhu, 2025]

- Essentially we are dealing with: $f_3^{\pm}(k_{12}, s) \sim \int_{-\infty}^{0} \mathrm{d}\eta\, e^{ik_{12}\eta} \sigma_{\mathbf{s}}^{\pm}(\eta)$
- Consider integrating the EoM

$$\int_{-\infty}^{0} \mathrm{d}\eta\, e^{ik_{12}\eta} \left[\eta^2 \partial_\eta^2 - 2\eta\partial_\eta + \left(M^2 \pm 2\kappa k\eta + c_s^2 k^2 \eta^2\right)\right] \sigma_{\mathbf{k}}^{\pm}(\eta) = 0$$

- ***<u>Key trick: IBP and trade time with energy</u>***

$$\int \mathrm{d}\eta\, \eta^n e^{ik_{12}\eta} = (-i\partial_{k_{12}})^n \int \mathrm{d}\eta\, e^{ik_{12}\eta}$$

- An ***<u>ordinary differential eq</u>***. for boundary kinematics:

$$\underbrace{\left[(k_{12}^2 - c_s^2 s^2)\partial_{k_{12}}^2 + (2 \pm i\kappa) k_{12}\partial_{k_{12}} + (M^2 - 2)\right]}_{\Delta_{12}^{\pm}} f_3(k_{12}, s) = 0$$

Case I: Solving the boundary ODE

[Qin, Renaux-Petel, **Tong**, Werth & Zhu, 2025]

- Similarly for 4pts – two **ODEs**

$$\Delta^{\pm}_{12} F^{\pm}_{4}(k_{12}, k_{34}, s) = \Delta^{\pm}_{34} F^{\pm}_{4}(k_{12}, k_{34}, s) = \frac{1}{k_{12} + k_{34}}$$

- Boundary conditions come from **BD vac.** & **causality**

$$\lim_{k_{12}, k_{34} = c_s s} F_4 = \text{fin.} \qquad \text{(no folded poles)}$$

$$\lim_{s \to 0} F_4 = f_3 \times f_3 \qquad \text{(soft non-analyticities factorise)}$$

- **Exact analytical sol.** ($c_s \neq 1, \kappa \neq 0$) as hypergeometric series

$$F_4 = \sum^{\infty} {}_3\mathrm{F}_2 + {}_2\mathrm{F}_1 \times {}_2\mathrm{F}_1$$

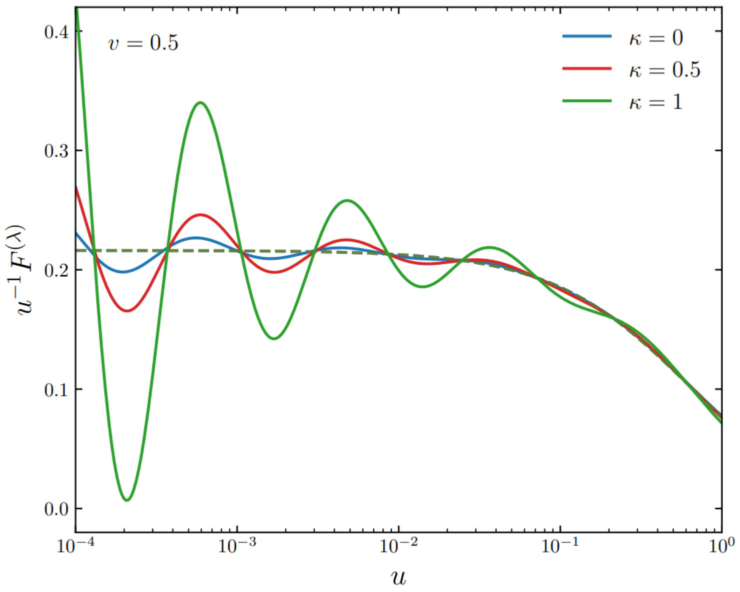
- ✓ Change of var.
- ✓ Series resummation
- ✓ Cross check with spectral decomp.

Case I: Phenomenology

[Qin, Renaux-Petel, **Tong**, Werth & Zhu, 2025]

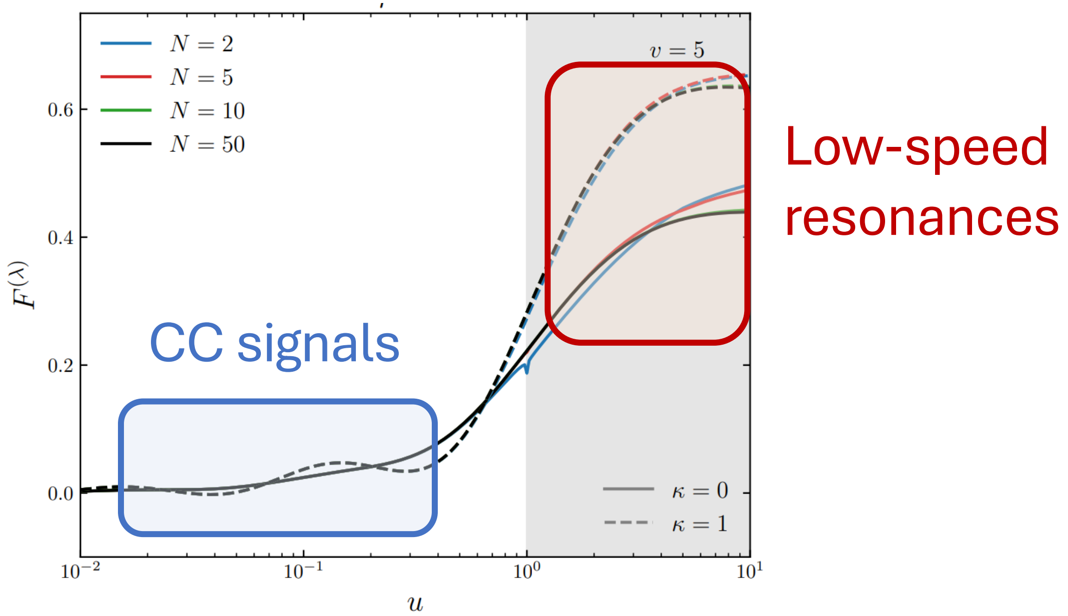
- ❖ Enhanced cosmological collider signals

$c_s = 1, \kappa \neq 0$



- ❖ New waveforms

$c_s > 1, \kappa \neq 0$



- ❖ Level splitting

$$(f_{\rm NL}^{\rm CC})_{\rm dS} \sim e^{-\pi\mu}$$

$$(f_{\rm NL}^{\rm CC})_{\cancel{\rm boost}} \sim \begin{cases} e^{-\pi(\mu\pm\kappa)} \\ e^{-\pi(\mu\pm\kappa)/2} \\ e^{-\pi\mu} \end{cases}$$