Quantum and classical fluctuations in inflation

Guillermo Ballesteros

Universidad Autónoma de Madrid

Instituto de Física Teórica UAM-CSIC

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Outline

1. Three questions
2. Quantum vs *classical* inflationary fluctuations
3. Example 1: Scalar bispectrum
4. Example 2: Scalar-induced tensor modes
5. Excited states and ultra slow-roll
6. Summary (with an answer)

Alejandro Pérez Rodríguez

U. Washington

Jesús Gambín Egea

ICTP, Trieste

arXiv: 2604.18416

Q1: Did inflation happen?

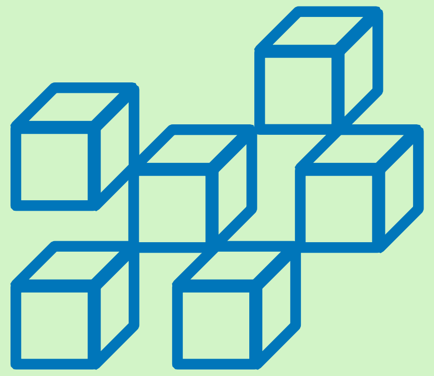
- Primordial tensor modes. CMB B-modes

  $r \sim 10^{-3}$ (LiteBIRD) $\sim 10^{13}\,\mathrm{GeV}$

- At small scales (CMB + Lyman- α $\lesssim 1\ \mathrm{Mpc}^{-1}$)
  tensor and scalar fluctuations could be much larger

Motivation for non-linear perturbation theory and also for non-perturbative analyses (classical lattice computations)

Caravano, Komatsu, Lozanov, Weller 2021
Caravano, Franciolini, Renaux-Petel 2025, 2026
G. Figueroa, Lizarraga, Loayza, Urio, Urrestilla 2026

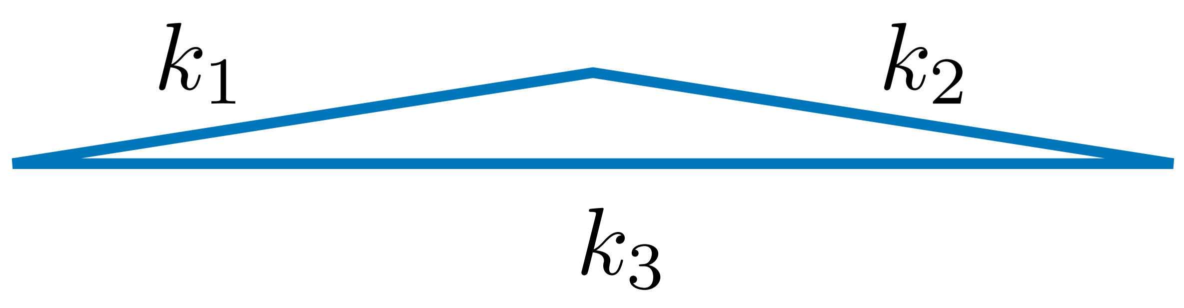


Q2: How do we test the nature (quantum vs classical) of inflation?

- Bell inequalities, Wigner function, ...

  Kiefer, Polarski, Starobinski 1998
  Campo & Parentani 2005
  Maldacena 2016
  Martin & Vennin 2017
  Ireland & Vennin 2026
  Green, Gupta, Zhang 2026

- Pole structure of cosmological correlators



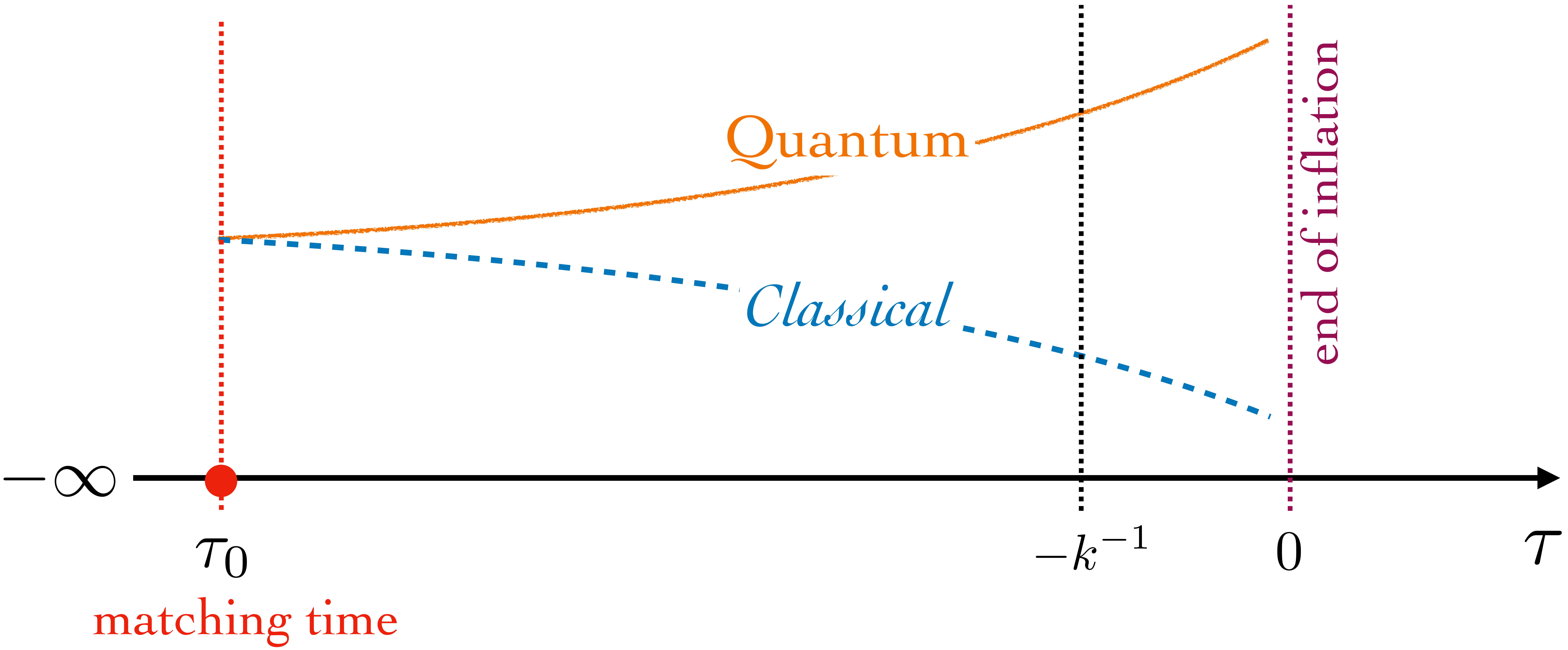
$$k_3 \simeq k_1 + k_2$$

"Poles arise in folded configurations under classical evolution".

(And not under quantum evolution)

Green & Porto 2020

Q3: Can we use classical evolution to describe inflationary fluctuations?



Quantum and classical evolution in the interaction picture

Quantum ^

$$\hat{\mathcal{O}}(t) = \sum_{n=0} \frac{1}{(i\hbar)^n} \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 \cdots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n \left[\left[\left[\hat{\mathcal{O}}_f(t), \hat{H}_I(t_1)\right], \hat{H}_I(t_2)\right], \cdots, \hat{H}_I(t_n)\right]$$

interaction Hamiltonian

free evolution

Classical

$$\mathcal{O}(t) = \sum_{n=0} \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 \cdots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n \{\cdots\{\{\mathcal{O}_f(t), H_I(t_1)\}_0, H_I(t_2)\}_0 \cdots, H_I(t_n)\}_0$$

Quantum and classical evolution in the interaction picture

$$\hat{A}(\tau_1)\hat{B}(\tau_2) = \tfrac{1}{2}\left[\hat{A}(\tau_1), \hat{B}(\tau_2)\right] + \tfrac{1}{2}\left(\hat{A}(\tau_1)\hat{B}(\tau_2) + \hat{B}(\tau_2)\hat{A}(\tau_1)\right)$$

Intrinsically quantum

Insensitive to operator ordering
Classically reproducible

$$\frac{1}{2}\langle[\hat{\Phi}(\tau_1), \hat{\Phi}(\tau_2)]\rangle = \mathrm{Im}[s_k(\tau_1)s_k^*(\tau_2)] \qquad \frac{1}{2}\langle\{\hat{\Phi}(\tau_1), \hat{\Phi}(\tau_2)\}\rangle = \mathrm{Re}[s_k(\tau_1)s_k^*(\tau_2)]$$

For Quantum $\simeq$ *Classical* , necessary condition:

$$\mathrm{Re}[s_k(\tau_1)s_k^*(\tau_2)] \gg \mathrm{Im}[s_k(\tau_1)s_k^*(\tau_2)] \qquad \text{(small commutators)}$$

s_k : free modes

Example 1: Scalar bispectrum

$$S = \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x}\, a^2 M_P^2\, \epsilon \left(\zeta'^2 - \partial\zeta^2 + \frac{\lambda}{3!} \frac{\zeta'^3}{aH} \right)$$

Quantum computation (in-in formalism)

$$(2\pi)^{3/2} \left\langle \textstyle\prod_{i=1}^{3} \hat{\zeta}_{\mathbf{k}_i}(\tau) \right\rangle \equiv \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B_\zeta^Q(\tau; \{k_i\})$$

$$B_\zeta^Q = -\frac{2M_P^2 \epsilon \lambda}{H\hbar} \,\mathrm{Im} \left\{ \int_{-\infty_-}^{\tau} \mathrm{d}\tau'\, a \prod_{i=1}^{3} s_{k_i}(\tau) s_{k_i}'^{*}(\tau') \right\}$$

$$\lim_{\tau \to 0} B_\zeta^Q = \frac{\lambda\, \hbar^2 H^4}{16 \epsilon^2 M_P^4} \frac{1}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3}$$

Example 1: Scalar bispectrum

$$S = \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x}\, a^2 M_P^2\, \epsilon \left(\zeta'^2 - \partial\zeta^2 + \frac{\lambda}{3!} \frac{\zeta'^3}{aH} \right)$$

Classical evolution (Green function)

$$\zeta_{\mathbf{k}_1}(\tau) = \zeta^f_{\mathbf{k}_1}(\tau) - \frac{M_P^2 \epsilon \lambda}{H\hbar} \int_{\tau_0}^{\tau} \mathrm{d}\tau' a(\tau') \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^{3/2}} \mathrm{Im}\{ s_{k_1}(\tau) s'^*_{k_1}(\tau') \}\, \zeta^{f\prime}_{\mathbf{p}} \zeta^{f\prime}_{\mathbf{k}-\mathbf{p}} \Big|_{\tau'}$$

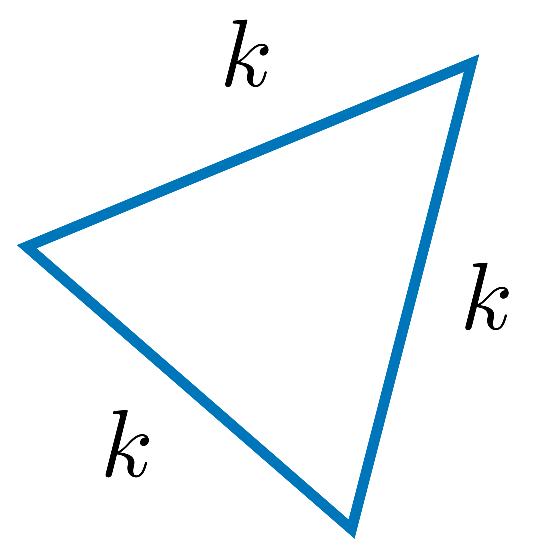
Matching conditions

$$\left\{ \begin{array}{c} \langle \zeta^f_{\mathbf{k}_1} \zeta^f_{\mathbf{k}_2} \zeta^f_{\mathbf{k}_3} \rangle^{\mathrm{Cl}} \neq 0 \\ \langle \alpha_{\mathbf{k}_1} \alpha_{\mathbf{k}_2} \rangle = 0 \quad \text{and} \quad \langle \alpha_{\mathbf{k}_1} \alpha^*_{\mathbf{k}_2} \rangle = \delta(\mathbf{k}_1 - \mathbf{k}_2)/2 \\ \langle \alpha_{\mathbf{k}_1} \alpha_{\mathbf{k}_2} \alpha_{\mathbf{k}_3} \rangle^{\mathrm{Cl}} = \dfrac{i M_P^2 \epsilon \lambda}{H\hbar} \dfrac{\delta\left(\sum_{i=1}^{3} \mathbf{k}_i \right)}{(2\pi)^{3/2}} \displaystyle\int_{-\infty_-}^{\tau_0} \mathrm{d}\tau' a \prod_{i=1}^{3} s'^*_{k_i} \end{array} \right.$$

Example 1: Scalar bispectrum

Difference between Quantum and *Classical* computations

$$\Delta B_\zeta = -\frac{\hbar^2 M_P^2 \epsilon\lambda}{4H}\int_{\tau_0}^{\tau} \mathrm{d}\tau' a \prod_{i=1}^{3} \partial_{\tau'} \underbrace{2\hbar^{-1}\,\mathrm{Im}\{s_k(\tau)s_k^*(\tau')\}}_{\text{Green function}}$$



$$\frac{\Delta B_\zeta}{B_\zeta^Q} = 20 + \frac{1}{8}\mathrm{Re}\left\{81e^{-iz}(z^2-2zi-2)-e^{-3iz}\left(9z^2-6iz-2\right)\right\} \sim e^{2\Delta N}$$

$$z \equiv -k\tau_0 = k/a(\tau_0)H = e^{\Delta N}$$

Exponentially growing difference in the number of e-folds elapsed between the matching time and horizon crossing

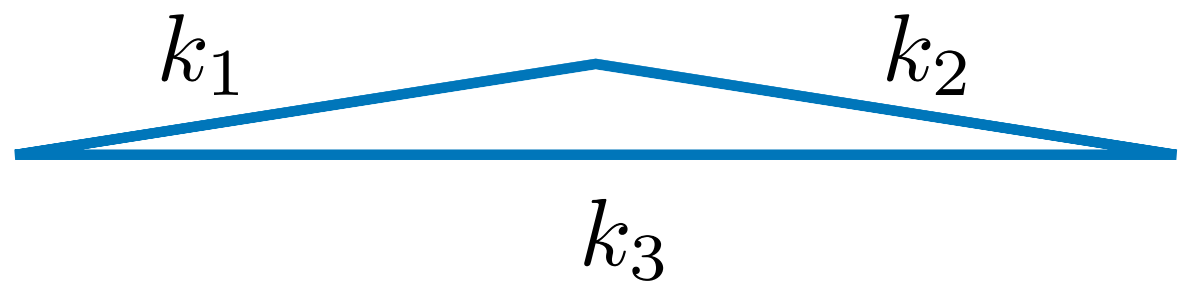
Example 1: Scalar bispectrum

Absence of poles in folded configurations in the *Classical* computation

$$\lim_{\tau\to 0}\Delta B_\zeta = \frac{\lambda H^4\hbar^2}{4\epsilon^2 M_P^4}\,\mathrm{Re}\left\{f_0 - \left[f_3 + \frac{3k_3^4(k_1^2+k_2^2-k_3^2) - 14/3\,k_1^2k_2^2k_3^2}{K_0^3K_1^3K_2^3K_3^3} + \text{perms.}\right]\right\}$$

$$K_0 = \textstyle\sum_i^3 k_i,\; K_j = K_0 - 2k_j,\; j = 1,2,3$$

$$f_i = e^{iK_i\tau_0}(2 - 2iK_i\tau_0 - (K_i\tau_0)^2)/32k_1k_2k_3K_i^3\,,\quad i \;=\; 0,1,2,3$$



$k_3 \simeq k_1 + k_2$

Example 1: Scalar bispectrum

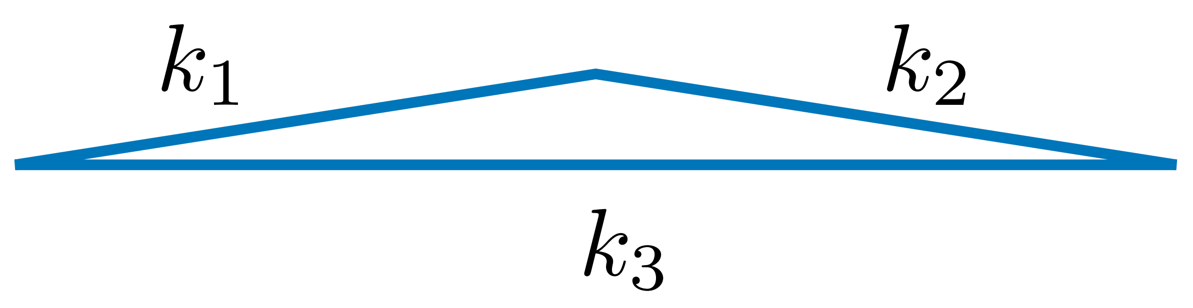
Absence of poles in folded configurations in the *Classical* computation

The kind of integral that appears: $\int_{\tau_0}^{\tau} d\tau' (\tau')^n e^{iK\tau'}$ is regular as $K = k_1 + k_2 - k_3 \to 0$

In Green & Porto 2020 the contributions from τ_0 were assumed to be to zero at $\tau_0 = -\infty$

No clear *classical* prescription to deal with the divergence at $\tau_0 \to -\infty$

In the in-in formalism (Quantum computation) the $i\epsilon$ prescription tames the divergence



$k_3 \simeq k_1 + k_2$

The lack of a " *classical* $i\epsilon$ " prescription impedes reaching a conclusion about the presence/absence of poles in the classical computation from $\tau_0 = -\infty$

Example 2: Scalar-induced tensor modes

Inflaton field minimally coupled to gravity with a standard kinetic term

$$S \supset \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x}\, \frac{a^2(\tau)}{2} \left(h_{ij} - \frac{1}{2} h_{ik} h_{kj} \right) \partial_i \delta\phi\, \partial_j \delta\phi$$

Quantum one-loop computation (in-in formalism)

GB, Gambín Egea, Riccardi 2024

Difference between Quantum and *Classical* computations

$$\Delta \mathcal{P}_h(\tau, k) = \frac{\hbar^2 k^3}{16\pi^2} \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} p^4 \sin^4\theta \int_{\tau_0}^{\tau} \mathrm{d}\tau' a^2 \int_{\tau_0}^{\tau'} \mathrm{d}\tau'' a^2 \underbrace{g_k^h(\tau, \tau') g_k^h(\tau, \tau'') g_p^\phi(\tau', \tau'') g_q^\phi(\tau', \tau'')}_{\text{Product of Green functions}}$$

Again: $\Delta \mathcal{P}_h(\tau, k) \propto (k\tau_0)^2$ for $k\tau_0 \to -\infty$

Exponential difference in the number of e-folds

Excited states

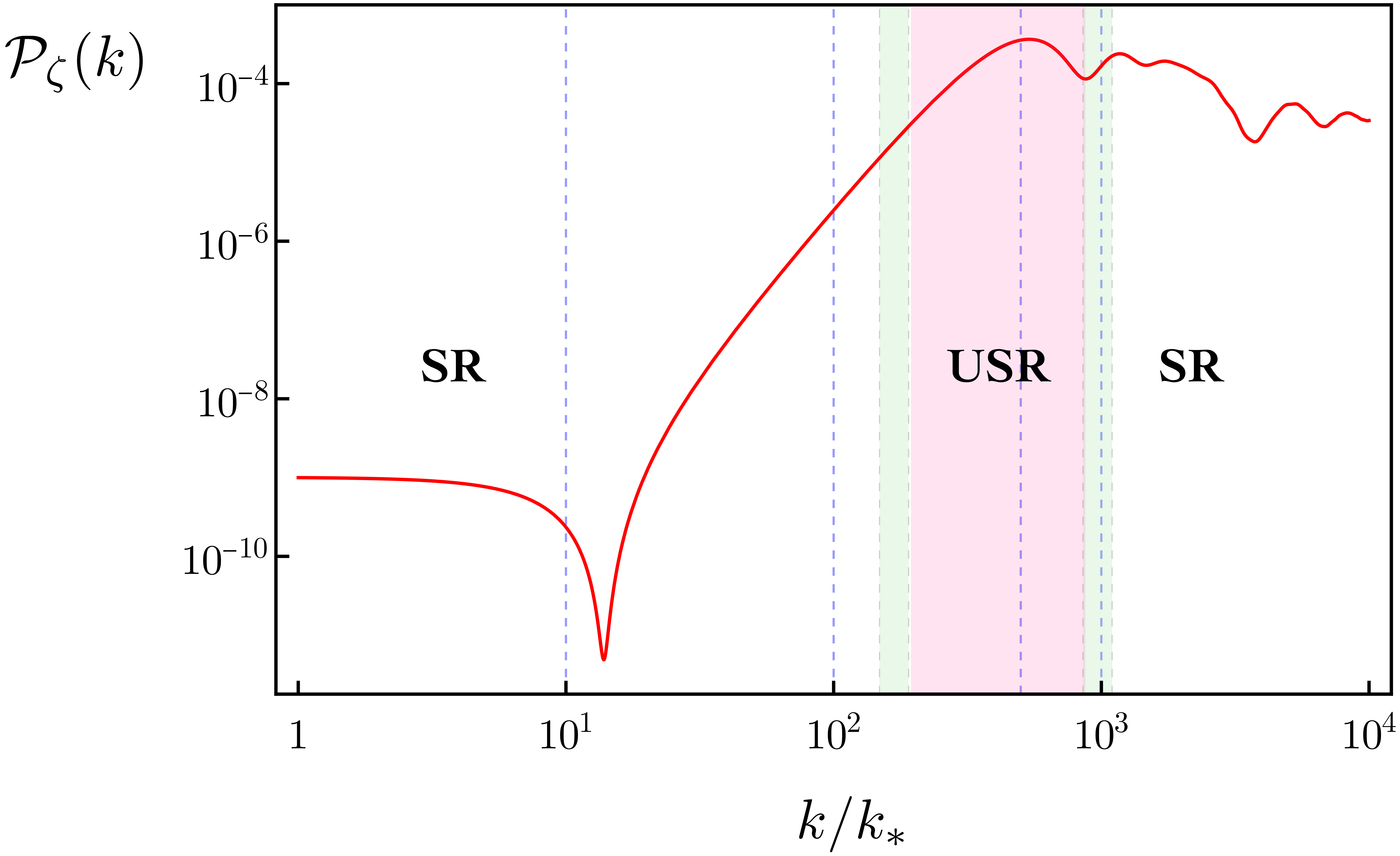
$$s_k^{\mathrm{out}}(\tau) = \cosh r_k \, s_k^{\mathrm{in}}(\tau) + e^{i\theta_k} \sinh r_k \, s_k^{\mathrm{in}*}(\tau)$$ Bogolyubov transformation

$$\frac{\mathrm{Re}[s_k^{\mathrm{out}}(\tau)s_k^{\mathrm{out}*}(\tau')]}{\mathrm{Im}[s_k^{\mathrm{out}}(\tau)s_k^{\mathrm{out}*}(\tau')]} = \frac{\mathrm{Re}[s_k^{\mathrm{out}}(\tau)s_k^{\mathrm{out}*}(\tau')]}{\mathrm{Im}[s_k^{\mathrm{in}}(\tau)s_k^{\mathrm{in}*}(\tau')]} \sim e^{2|r_k|} \gg 1 \qquad (|r_k| \gg 1)$$

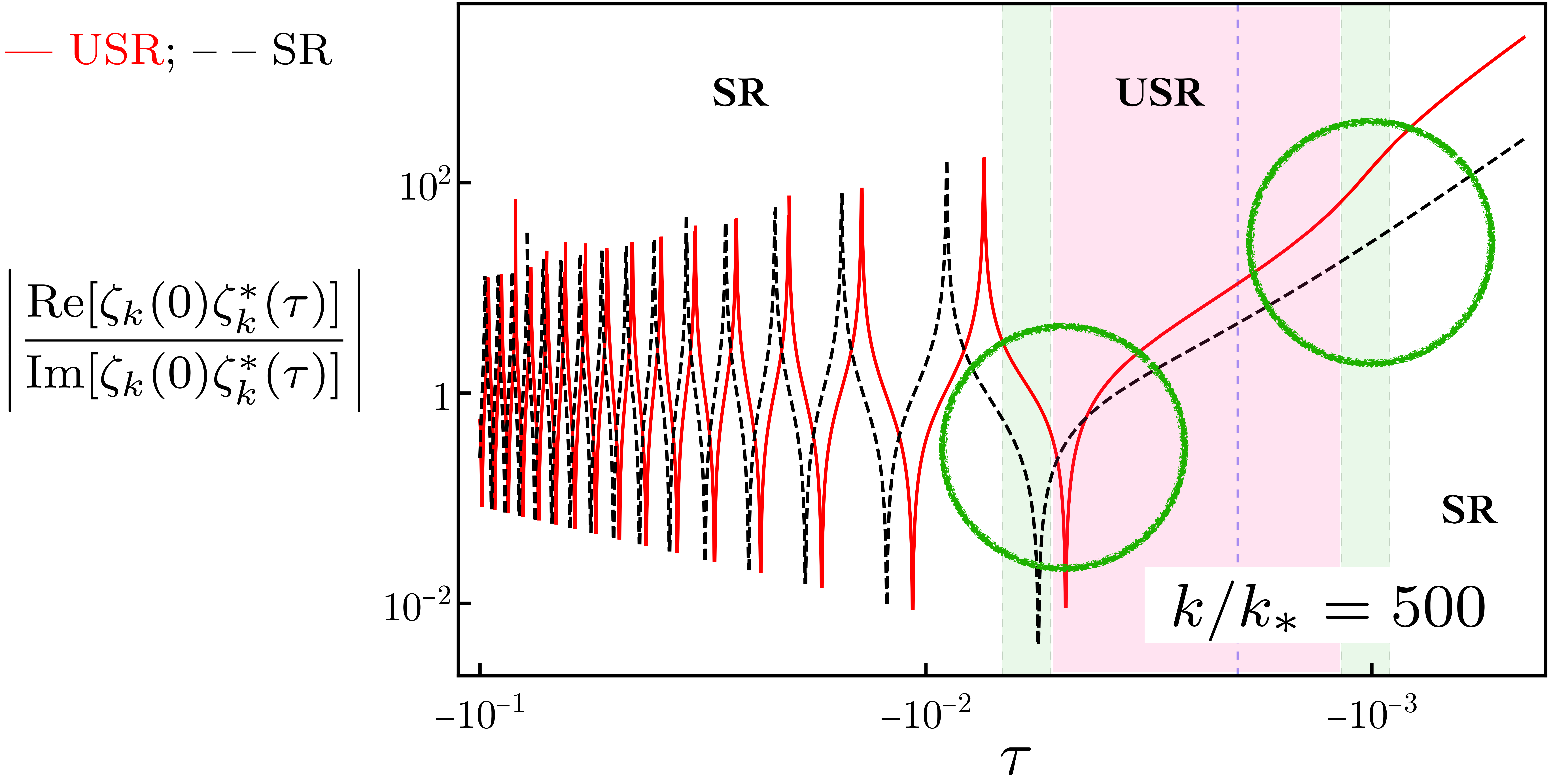
Excited states may, in principle, satisfy the classicality condition

Caveat: it must hold when the observable is generated

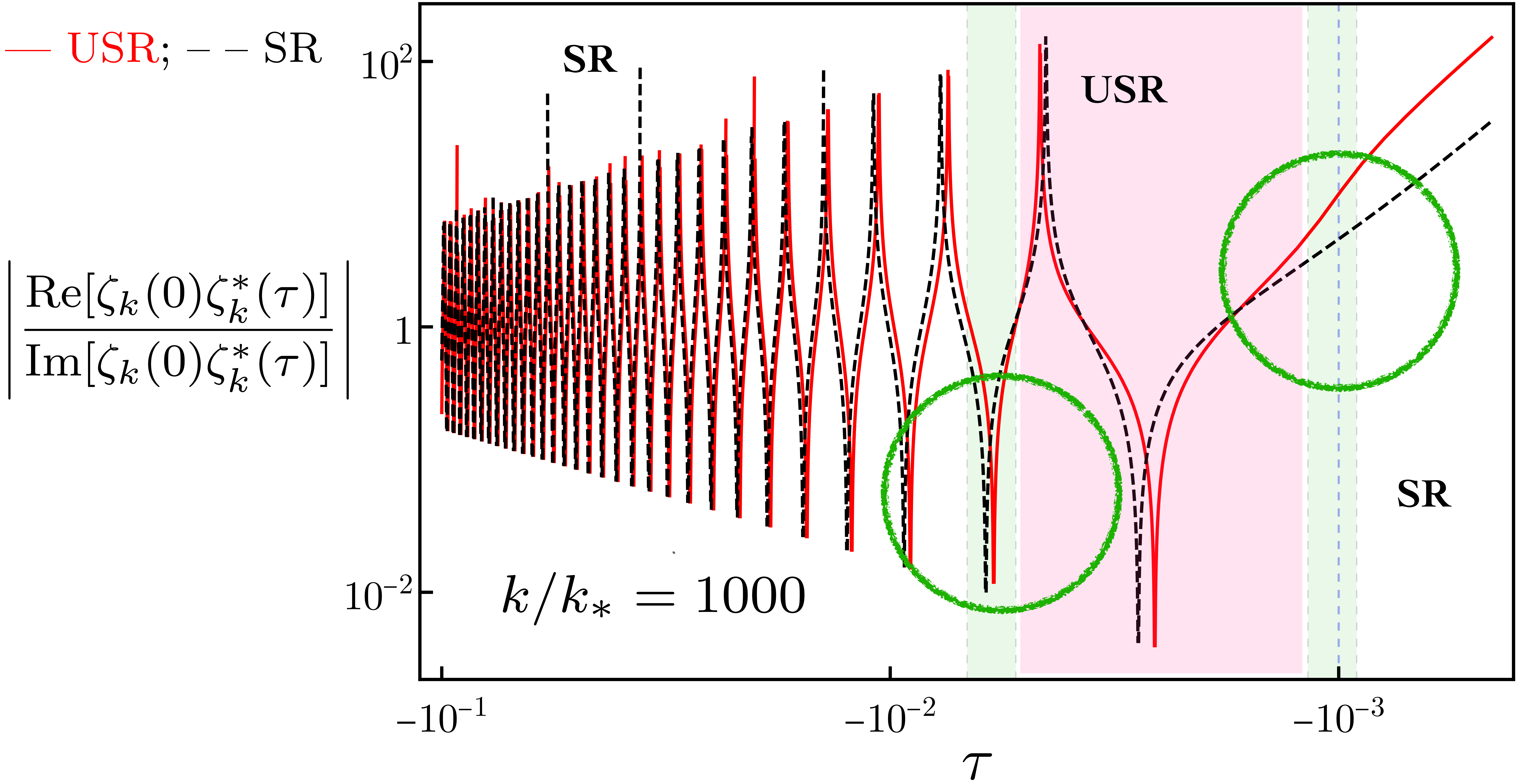
Transient ultra slow-roll



Transient ultra slow-roll



Transient ultra slow-roll



Hiding the dependence on the matching time

Interaction localized in time

$$S = \int \mathrm{d}\tau\, \mathrm{d}^3\mathbf{x}\, a^2 M_P^2\, \epsilon \left(\zeta'^2 - \partial\zeta^2 + \frac{\lambda}{3!}\frac{\zeta'^3}{aH} \right) \quad , \quad \lambda \propto \delta(\tau - \tau_*)$$

$$\frac{\Delta B}{B^{\mathrm{Q}}} = \frac{\sin^2(k\tau_*)}{2\cos(2k\tau_*)+1}$$

There is no dependence on the matching time,
but the Q vs C difference can be arbitrarily large

Summary

In general, classical dynamics does not reproduce (interacting) primordial correlators.

(Even if setting stochastic initial conditions that match quantum statistics at a finite time)

Lattice simulations of inflation need to be interpreted with great care.

In the above setup (matching at finite time), the scalar bispectrum does not develop poles in folded configurations

In-in formalism

$$\langle Q(t)\rangle = \frac{{}_I\langle 0|\, F^{-1}(t,-\infty^+)Q_I(t)F(t,-\infty^-)\,|0\rangle_I}{{}_I\langle 0|\, F^{-1}(t,-\infty^+)F(t,-\infty^-)\,|0\rangle_I} \;=\; {}_I\langle 0|\, F^{-1}(t,-\infty^+)Q_I(t)F(t,-\infty^-)\,|0\rangle_I\,\Big|_{\text{no bubbles}}$$

$$F(t,-\infty^-) = 1 - i\int_{-\infty}^{t}\mathrm{d}t'\,H_I(t'_-) - \int_{-\infty}^{t}\mathrm{d}t'\int_{t'}^{t}\mathrm{d}t''\,H_I(t''_-)H_I(t'_-)$$

$$t_\pm \equiv t(1\pm i\omega)\,,\quad \omega>0$$

$$\begin{aligned}\langle Q(t)\rangle = \langle 0|\,Q_I(t)\,|0\rangle &+ 2\,\mathrm{Im}\Big\{\int_{-\infty}^{t}\mathrm{d}t'\,\langle 0|\,Q_I(t)H_I(t'_-)\,|0\rangle\Big\}\\ &+ 2\,\mathrm{Re}\Big\{\int_{-\infty}^{t}\mathrm{d}t'\int_{t'}^{t}\mathrm{d}t''\,\langle 0|\,\big(H_I(t''_+)Q_I(t) - Q_I(t)H_I(t''_-)\big)\,H_I(t'_-)\,|0\rangle\Big\}\end{aligned}$$

Classical evolution in the interaction picture

$$\mathcal{O}(t) = \sum_{n=0} \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 \cdots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n \, \{\cdots\{\{\mathcal{O}_f(t), H_I(t_1)\}_0, H_I(t_2)\}_0 \cdots, H_I(t_n)\}_0$$

Poisson bracket relative to t_0 :

$$\{A_f(t_1), B_f(t_2)\}_0 \equiv \sum_a \int \mathrm{d}^3\mathbf{k} \left(\frac{\partial A_f(t_1)}{\partial \Phi_{a,\mathbf{k}}(t_0)} \frac{\partial B_f(t_2)}{\partial \Pi_{a,\mathbf{k}}(t_0)} - \frac{\partial A_f(t_1)}{\partial \Pi_{a,\mathbf{k}}(t_0)} \frac{\partial B_f(t_2)}{\partial \Phi_{a,\mathbf{k}}(t_0)} \right)$$

$$\Phi^f_{a,\mathbf{k}}(t) = s_{a,k}(t)\alpha_{a,\mathbf{k}} + s^*_{a,k}(t)\alpha^*_{a,-\mathbf{k}}$$

$$\{\Phi_{a,\mathbf{k}}(t_0), \Pi_{b,\mathbf{p}}(t_0)\}_0 = \delta_{ab}\,\delta(\mathbf{k}-\mathbf{p})$$

$$\alpha_{a,\mathbf{k}} = \mathcal{W}^{-1}_{a,k}\left(\Phi_{a,\mathbf{k}}(t_0)\dot{s}^*_{a,k}(t_0) - \Pi_{a,-\mathbf{k}}(t_0)s^*_{a,k}(t_0)\right)$$

$$\{\alpha_{a,\mathbf{k}}(t_0), \alpha^*_{b,\mathbf{p}}(t_0)\}_0 = \mathcal{W}^{-1}_{a,k}\,\delta_{ab}\,\delta(\mathbf{k}-\mathbf{p})$$

$$\mathcal{W}_{a,k} \equiv s_{a,k}(t_0)\dot{s}^*_{a,k}(t_0) - s^*_{a,k}(t_0)\dot{s}_{a,k}(t_0)$$