Developments in gravitational particle production

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– cosmological gravitational particle production:

- always there / dark matter production

- nuisance for model building

– complicated in realistic settings

- time dependent masses

- initial conditions

- Bogolyubov vs Starobinsky

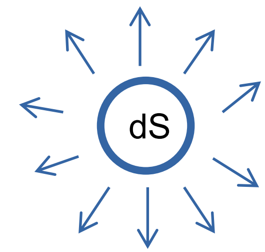
- fermion mixing

- ...

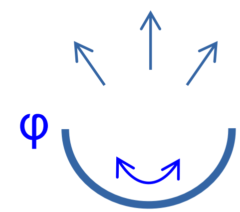
OL 2210.02293

Particle production in the Early Universe:

- during inflation

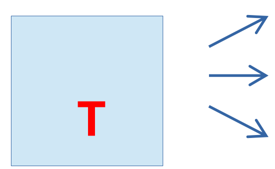


- via inflaton oscillations



- inflaton decay

- thermal emission (freeze-in)



Stable (very) <u>weakly</u> coupled particles accummulate!

Earlier work:

Parker ’69

Grib, Mamaev ’69

Zeldovich, Starobinsky ’71

Ford, …
…
…

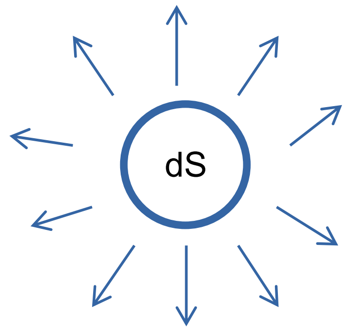
Chung, Kolb, Riotto (superheavy DM, isocurvature)

Ema, Jinno, Mukaida, Nakayama (metric oscillations)

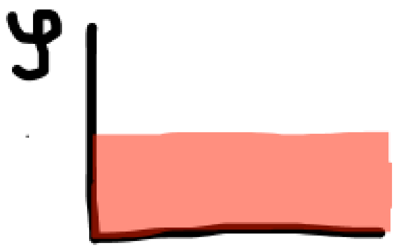
Mambrini, Olive (graviton exchange approximation)

Bogolyubov vs Starobinsky

| **in** > , | **out** > states



EOM: $$\chi_k'' + \omega_k^2\, \chi_k = 0$$



$$\beta_k = i(\chi_k^{\mathrm{out}\,\prime}\chi_k^{\mathrm{in}} - \chi_k^{\mathrm{out}}\chi_k^{\mathrm{in}\,\prime})$$

$$\langle 0^{\mathrm{in}}|N^{\mathrm{out}}|0^{\mathrm{in}}\rangle = V\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,|\beta_k|^2$$

???

evolve the condensate
after inflation

Particle production via classical gravity

Parker, Grib, Zeldovich, Starobinsky,... 60-70's

Expanding Friedmann Universe :

$$g_{\mu\nu} = a^2(\eta)\,\mathrm{diag}(1,-1,-1,-1)$$

(*conformal time* η : $dt = a\,d\eta$)

Free scalar field :

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - V(\Phi)$$

$$V(\Phi) = \frac{1}{2}m^2\Phi^2$$

Rescaled scalar :

$$\phi = a(\eta)\,\Phi$$

Mode expansion and EOM :

$$\phi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[a_{\mathbf{k}}\,\chi_k(\eta)\,e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\,\chi^*_k(\eta)\,e^{-i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$\chi''_k + \omega_k^2\,\chi_k = 0 \quad , \quad \omega_k^2(\eta) = k^2 + a^2(\eta)\,m^2 + \left(\frac{1}{6}-\xi\right) a^2(\eta)\,R(\eta)$$

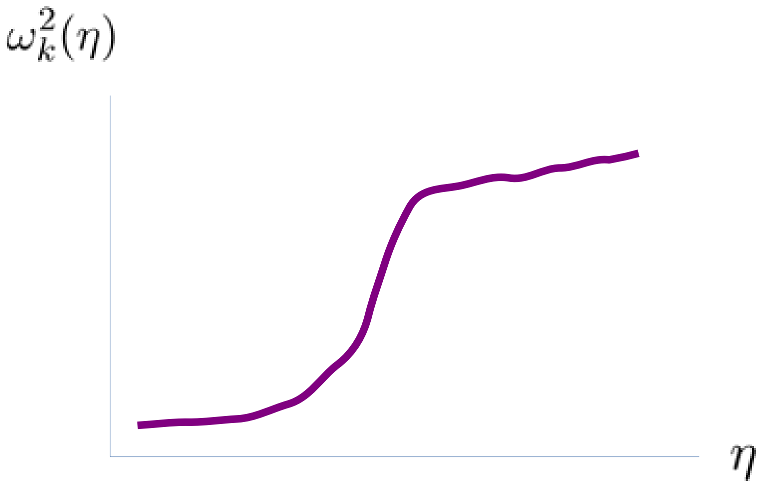
particle production!

To define a particle, need adiabatically flat space-time and Fock space vacuum

$$\chi_k'' + \omega_k^2\,\chi_k = 0$$

$$\eta \to -\infty \quad : \qquad \omega_k^2(\eta) \;\to\; k^2$$

$$\eta \to +\infty \quad : \qquad \omega_k^2(\eta) \;\to\; a^2(\eta)\, m^2$$



Asymptotic solutions :
("Minkowski-like")

$$\chi_k^{\rm in}(\eta \to -\infty) \to \frac{e^{-i\int^{\eta} d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\,,$$

$$\chi_k^{\rm out}(\eta \to +\infty) \to \frac{e^{-i\int^{\eta} d\eta'\omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}}\,.$$

Bunch-Davies vacuum :
(no particles initially)

$$a_{\mathbf{k}}^{\rm in}\,|0^{\rm in}\rangle = 0$$

Keeping the field intact, can choose different expansion modes,

$$\phi(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[a_{\mathbf{k}}\, \chi_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}\, \chi^*_k(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

Bogolyubov transform :

$$\begin{cases} \chi^{\rm in}_k(\eta) = \alpha_k\, \chi^{\rm out}_k(\eta) + \beta_k\, \chi^{{\rm out}\,*}_k(\eta) \\ a^{\rm in}_{\mathbf{k}} = \alpha^*_k\, a^{\rm out}_{\mathbf{k}} - \beta^*_k\, a^{{\rm out}\,\dagger}_{-\mathbf{k}} \end{cases} \Rightarrow \quad a^{\rm out}_{\mathbf{k}}\, |0^{\rm in}\rangle \neq 0$$

Particle number at late times:

$$\langle 0^{\rm in} | N^{\rm out} | 0^{\rm in} \rangle = V \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\beta_k|^2$$

$$\beta_k = i(\chi^{\rm out}_k{}'\chi^{\rm in}_k - \chi^{\rm out}_k \chi^{\rm in}_k{}')$$

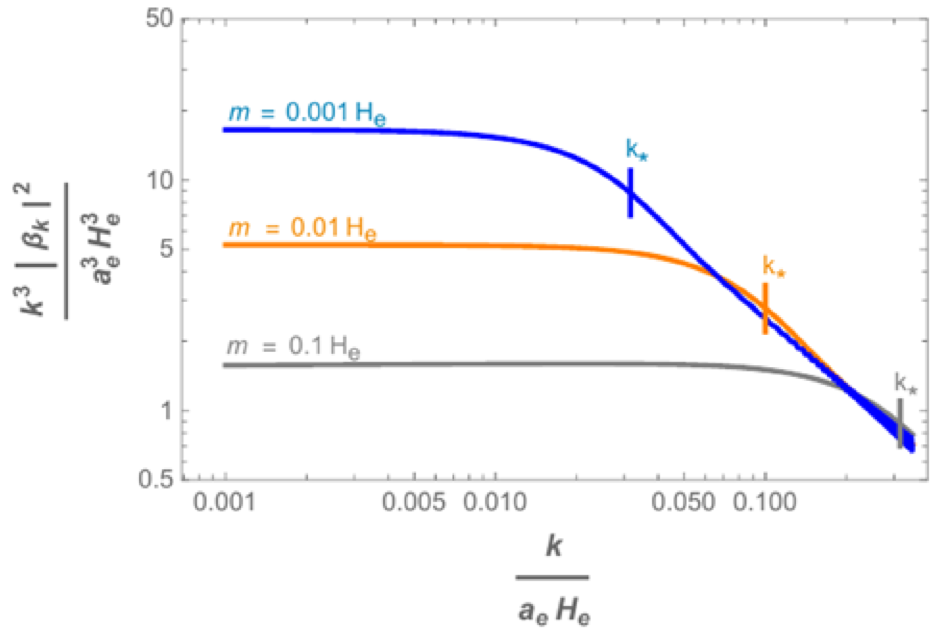
Light scalar :

$$n \sim H^3 \;\; / \text{ scale factor}^3 \qquad\qquad (?)$$

10^{30}

Light scalar :

$$n \sim H^3 \;/\; \text{scale factor}^3 \quad \text{x} \quad \mathbf{(\,H\,/\,m\,)^{5/2}}$$



Feiteira, OL' 25

WHY ?

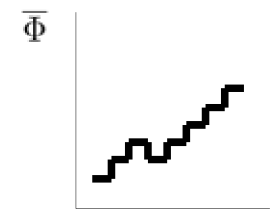
Starobinsky

$$\Phi = \overline{\Phi}(t,\mathbf{r}) + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\theta(k-\epsilon a(t)H)\,\left(a_{\mathbf{k}}\chi_k(t)\,e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}}\chi^*_k(t)\,e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

long wavelength *short wavelength*

Random walk:

$$\frac{d}{dt}\langle\overline{\Phi}^2\rangle = -\frac{2}{3}\,\frac{m^2}{H}\,\langle\overline{\Phi}^2\rangle + \frac{H^3}{4\pi^2}$$



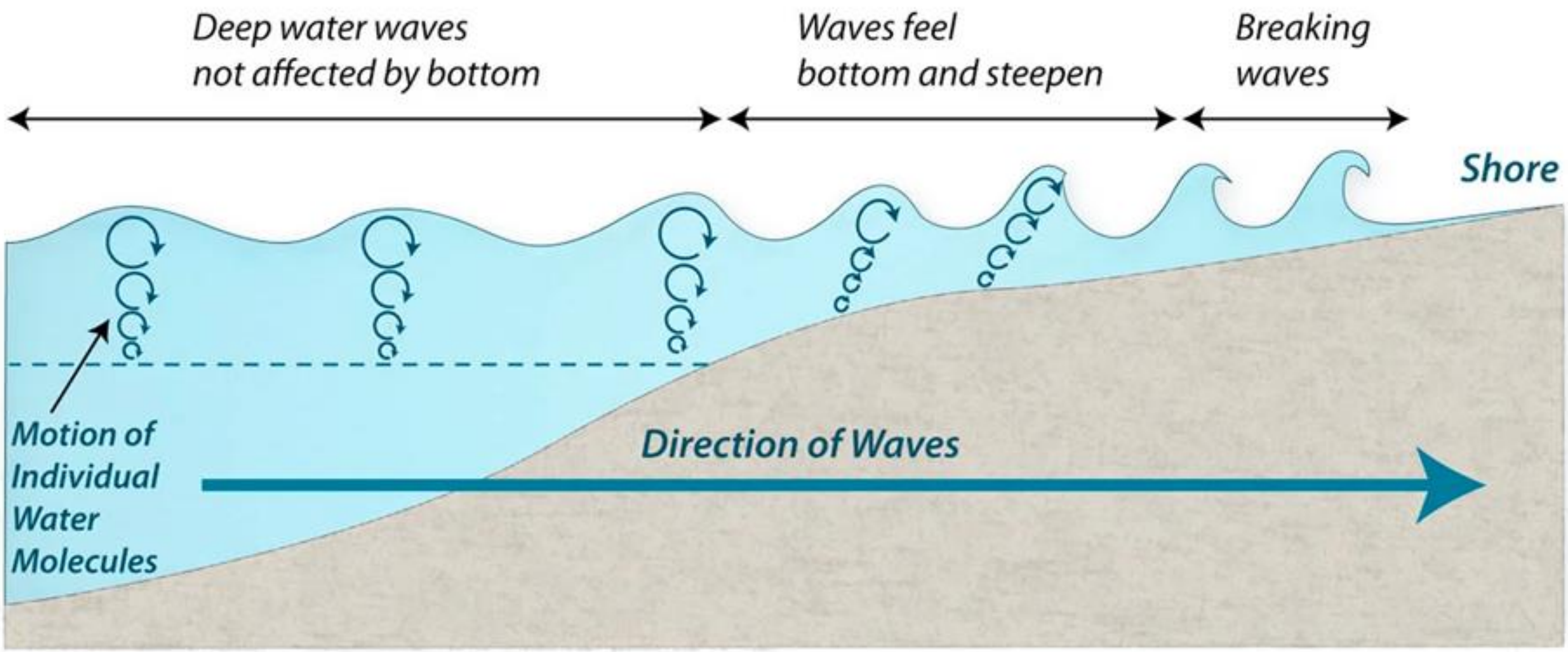
$$\boxed{\langle\overline{\Phi}^2\rangle = \frac{3H^4}{8\pi^2 m^2} + \left(\langle\overline{\Phi}^2\rangle_0 - \frac{3H^4}{8\pi^2 m^2}\right)\,e^{-\frac{2m^2}{3H}\,(t-t_0)} \to \frac{3}{8\pi^2}\,\frac{H^4}{m^2}}$$

anomalously large average field at the end of inflation !

$$\langle\Phi^2\rangle_{\rm IR} = \frac{1}{a^2(\eta)}\int^{\epsilon a H}\frac{d^3\mathbf{k}}{(2\pi)^3}\,\left|\chi^{\rm in}_k(\eta)\right|^2 = \frac{3}{8\pi^2}\,\frac{H^4}{m^2} \quad\Longrightarrow\quad a^3 n = \frac{3\kappa^2\,a_e^3}{4\pi^2}\,\frac{H_e^{11/2}}{m^{5/2}}$$

Inflationary particle production:

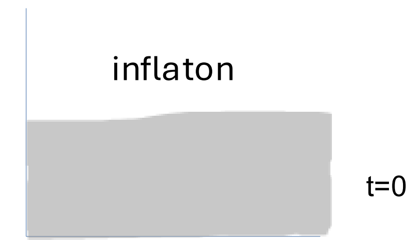
fluctuations $\longrightarrow$ *particles*



On initial conditions

“Standard” Bogolyubov : $a_{\mathbf{k}}^{\text{in}} \, |0^{\text{in}}\rangle = 0$

Starobinsky : $\langle \overline{\Phi}^2 \rangle_0$



non-Bunch-Davies!

$$\langle \overline{\Phi}^2 \rangle = \frac{3H^4}{8\pi^2 m^2} + \left(\langle \overline{\Phi}^2 \rangle_0 - \frac{3H^4}{8\pi^2 m^2} \right) e^{-\frac{2m^2}{3H}\,(t-t_0)} \to \frac{3}{8\pi^2}\,\frac{H^4}{m^2}$$

Starobinsky → Bogolyubov for $t >> H/m^2$

- Equilibration is extremely slow for low *m*
- **Inflation is finite, initial conditions matter !**
- correct variable = condensate at the end of inflation $\langle \overline{\Phi}^2 \rangle$

Particle abundance:

$\overline{\Phi} \longrightarrow$ particles when **H ~ m**

$$\boxed{Y \simeq 0.07 \times \frac{\overline{\Phi}^2}{m^{1/2} M_{\rm Pl}^{3/2}}}$$

(radiation domination)

$$H_e \lesssim \overline{\Phi} \lesssim M_{\rm Pl}$$

Example:

H ~ 10^{13} GeV , m > 1 GeV $\Rightarrow$ **Y > 10^7 x Y_{obs} !**

(minimal assumptions, standard cosmology, \`free' scalar)

More sensible solution:

low reheating temperature T_R

$$Y \simeq 0.07 \times \frac{1}{\Delta} \frac{H_e^{1/2}\,\overline{\Phi}^2}{m M_{\rm Pl}^{3/2}}$$

(matter domination)

$$\Delta \equiv \sqrt{\frac{H_e}{H_R}} \simeq \frac{T_{\rm inst}}{T_R} \gg 1$$

(e.g. T_R < GeV)

T_R ~ inflaton-SM coupling

General scaling result :

$$\boxed{Y \propto \frac{\overline{\Phi}^2}{m_{\text{eff}}^{1/2}} \times \left(\frac{H_R}{m_{\text{eff}}}\right)^{\gamma}}$$

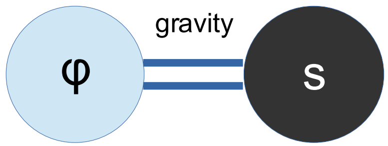
γ = 0 (radiation dom.) or ½ (matter dom.)

Includes

- weak self-coupling λ
- small non-minimal coupling to gravity ξ
- general initial conditions

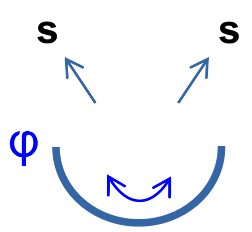
Losing control: quantum gravity effects

Quantum gravity induces all gauge invariant operators
(*with unknown coefficients)*



Effective field theory regime ***after inflation*** $\varphi \lesssim M_{\rm Pl}$:

$$\boxed{\frac{\phi^4 s^2}{M_{\rm Pl}^2}\ ,\quad \frac{\phi^6 s^2}{M_{\rm Pl}^4}\ ,\quad \frac{\phi^8 s^2}{M_{\rm Pl}^6}\ ,\ \ldots}$$



Example:

H ~ 10^{13} GeV , m > 1 GeV , Wilson coef. ~ 1 $\Longrightarrow$ **Y > 10^{16} x Y_{obs} !**

$\Downarrow$

the coefficients must be tiny (10^{-8}) or T_R low

String theory landscape

stable / long-lived
weakly interacting
scalar
(m >> eV)

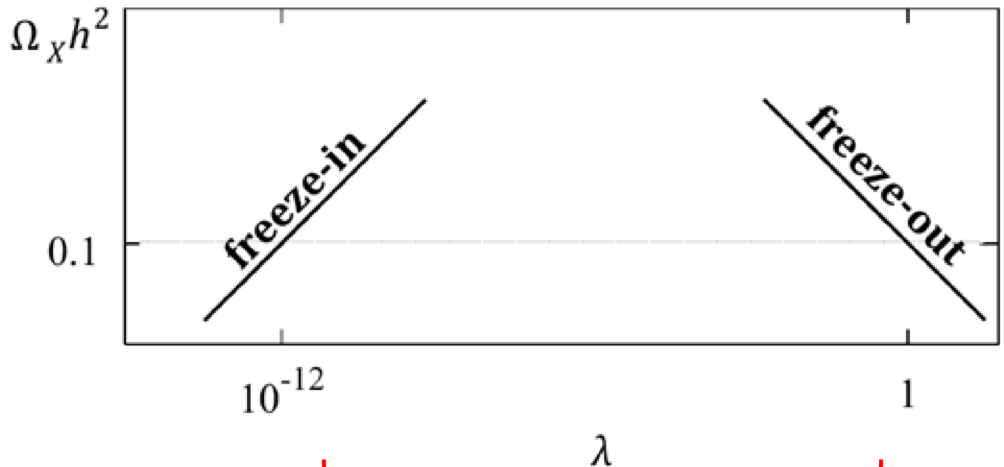
↓

totally dark Universe

Low T_R benefits:

Cosme,Costa,OL '23

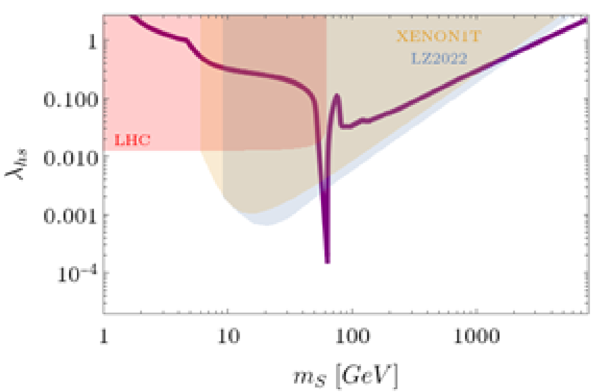
FIMP → WIMP



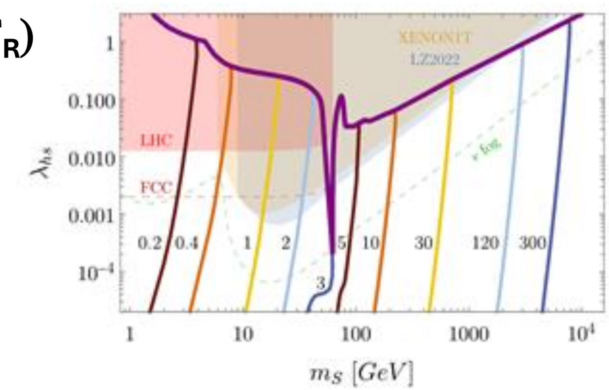
freeze-in at low T_R (stronger coupling)

Example:

Arcadi,Costa,Goudelis,OL '24



(m, λ) → (m, λ, T_R)



Your favorite DM model is not ruled out, just cool it…

Developments in fermion production

Parker ' 71
Chung et al.'11

EOM:

$$(i\gamma^\mu \partial_\mu - a(\eta)M)\,\Psi = 0 \quad \Rightarrow \quad Y \propto \left(\frac{M}{M_{\rm Pl}}\right)^{3/2}$$

However, in reality (SM) all masses are environment-dependent, $M_f = \frac{1}{\sqrt{2}} Y_f \langle h \rangle$:

$$\boxed{M = M(\eta)}$$

Feiteira, Koutroulis, OL, Pokorski '25

$$\Downarrow$$

$Y \sim Y_{naive} \times 10^{20}$

More: Fotis Koutroulis' talk

Time-dependent fermion mixing

Neutrino-like fermions:

$$\mathcal{M} = \begin{pmatrix} 0 & m(\eta) \\ m(\eta) & M \end{pmatrix} \quad , \qquad m(\eta) = Y_\nu \, v(\eta)/\sqrt{2}$$

EOM:

$$\boxed{\left[\mathbb{1} \times i \, \not{\partial} - a(\eta) \, \mathcal{M}(\eta) \right] \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0}$$

For a single fermion:

$$\Psi(x) = \sum_i \left(a_i U_i + b_i^\dagger V_i \right)$$

mode function

ladder operator

$$U_{\mathbf{k},s}(\eta, \mathbf{x}) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{(2\pi)^{3/2}} \underbrace{\begin{pmatrix} u_{A,k}(\eta) \\ s \, u_{B,k}(\eta) \end{pmatrix}}_{\text{time-dep.}} \otimes \underbrace{h_s(\hat{\mathbf{k}})}_{\text{spin}}$$

Define $$\Phi \equiv \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\Phi(x) = \sum_s \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{x}} \left(a^1_{\mathbf{k}s}\, \mathbf{u}^1_{\mathbf{k}s} + a^2_{\mathbf{k}s}\, \mathbf{u}^2_{\mathbf{k}s}\right) \otimes h_s(\hat{\mathbf{k}}) + e^{-i\mathbf{k}\cdot\mathbf{x}} \left(b^{1\,\dagger}_{\mathbf{k}s}\, \mathbf{v}^1_{\mathbf{k}s} + b^{2\,\dagger}_{\mathbf{k}s}\, \mathbf{v}^2_{\mathbf{k}s}\right) \otimes h_s(-\hat{\mathbf{k}}) \right]$$

$\mathbf{u}^1_{\mathbf{k}s}, \mathbf{u}^2_{\mathbf{k}s}, \mathbf{v}^1_{\mathbf{k}s}, \mathbf{v}^2_{\mathbf{k}s}$ are 4-vectors solving the EOM :

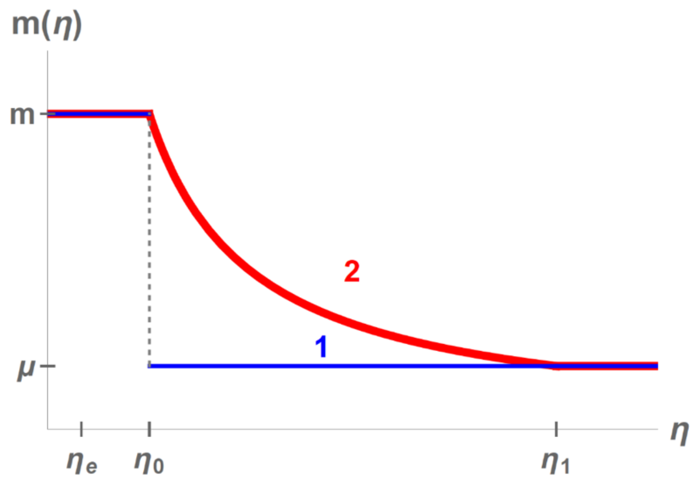
$$i\partial_\eta\, \mathbf{f} = \begin{pmatrix} 0 & ks & am & 0 \\ ks & 0 & 0 & -am \\ am & 0 & aM & ks \\ 0 & -am & ks & -aM \end{pmatrix} \mathbf{f} = H\, \mathbf{f}$$

Flavorful Bogolyubov transform:

$$\mathbf{a} \to \tilde{\mathbf{a}} = \mathcal{O}^* \mathbf{a} \qquad \text{with} \qquad \mathbf{a} = \begin{pmatrix} a^1 \\ b^{1\dagger} \\ a^2 \\ b^{2\dagger} \end{pmatrix}_\alpha$$

$$\tilde{\mathcal{N}}_{\text{tot}} = \sum_\alpha \left(\tilde{a}^{1\dagger}\tilde{a}^1 + \tilde{a}^{2\dagger}\tilde{a}^2 + \tilde{b}^{1\dagger}\tilde{b}^1 + \tilde{b}^{2\dagger}\tilde{b}^2 \right)_\alpha$$

Mass time-dependence:



$$\text{production stage}: \quad m(0)$$

$$\text{present stage}: \quad \frac{m(\infty)^2}{M} \lll m(0)$$

Features:

- Lorentz-flavor conversion
- (Currently) **massless states efficiently produced (!)**

Upper bound on abundance:

$$Y \lesssim 10^{-3} \times \left(\frac{m}{M_{\rm Pl}}\right)^{3/2} \lesssim 10^{-11}$$

- **The gravity of gravity is underappreciated**

- ***Weighs in*** **on non-thermal DM models (often unfavorably)**

- **Progress on gravity effects in realistic settings**

Back-up

Fermion production via QG ops:

$$\frac{\mathcal{C}}{M_{\rm Pl}}\,\phi^2\,\bar{\Psi}\Psi$$

Abundance constraint:

$$\mathcal{C} \lesssim 10^{-4}\,\Delta_{\rm NR}^{1/2}\,\frac{M_{\rm Pl}^{3/4}H_e^{1/4}}{\phi_0}\,\sqrt{\frac{\rm GeV}{M}}$$

Example:

H ~ 10^{13} GeV , m > 1 GeV $\Longrightarrow$ $\mathcal{C} \leq 10^{-5}$