

Gravitational Waves from Primordial Black Hole Reheating

Yann Gouttenoire

15th September 2026



Particle Physics in the Early Universe

Coorganised by

**Polish Academy of Sciences Scientific Center in Paris,
Institute for Advanced Studies of the University of Warsaw,
IJCLab Orsay**

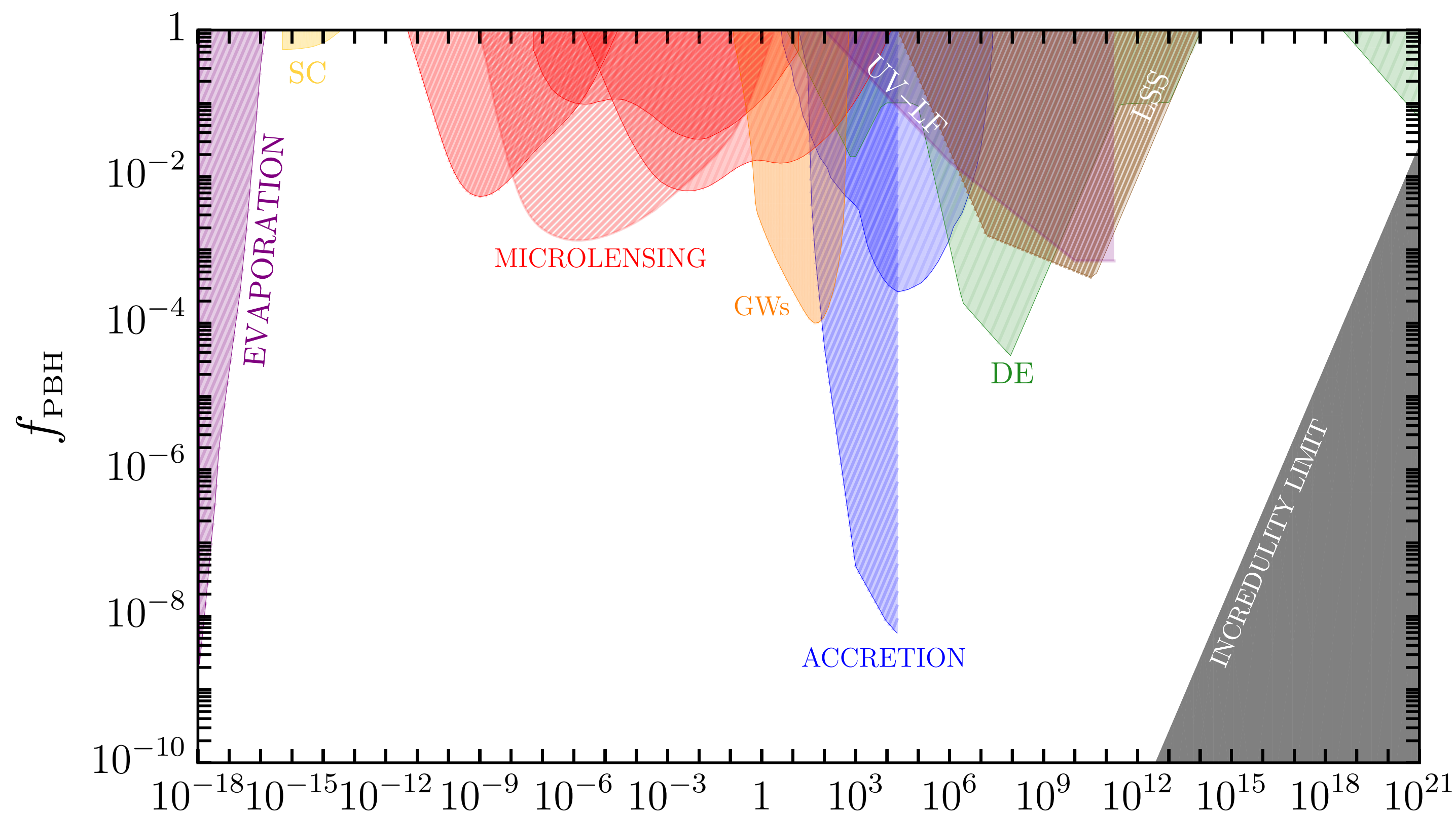


Paris
Centre Scientifique

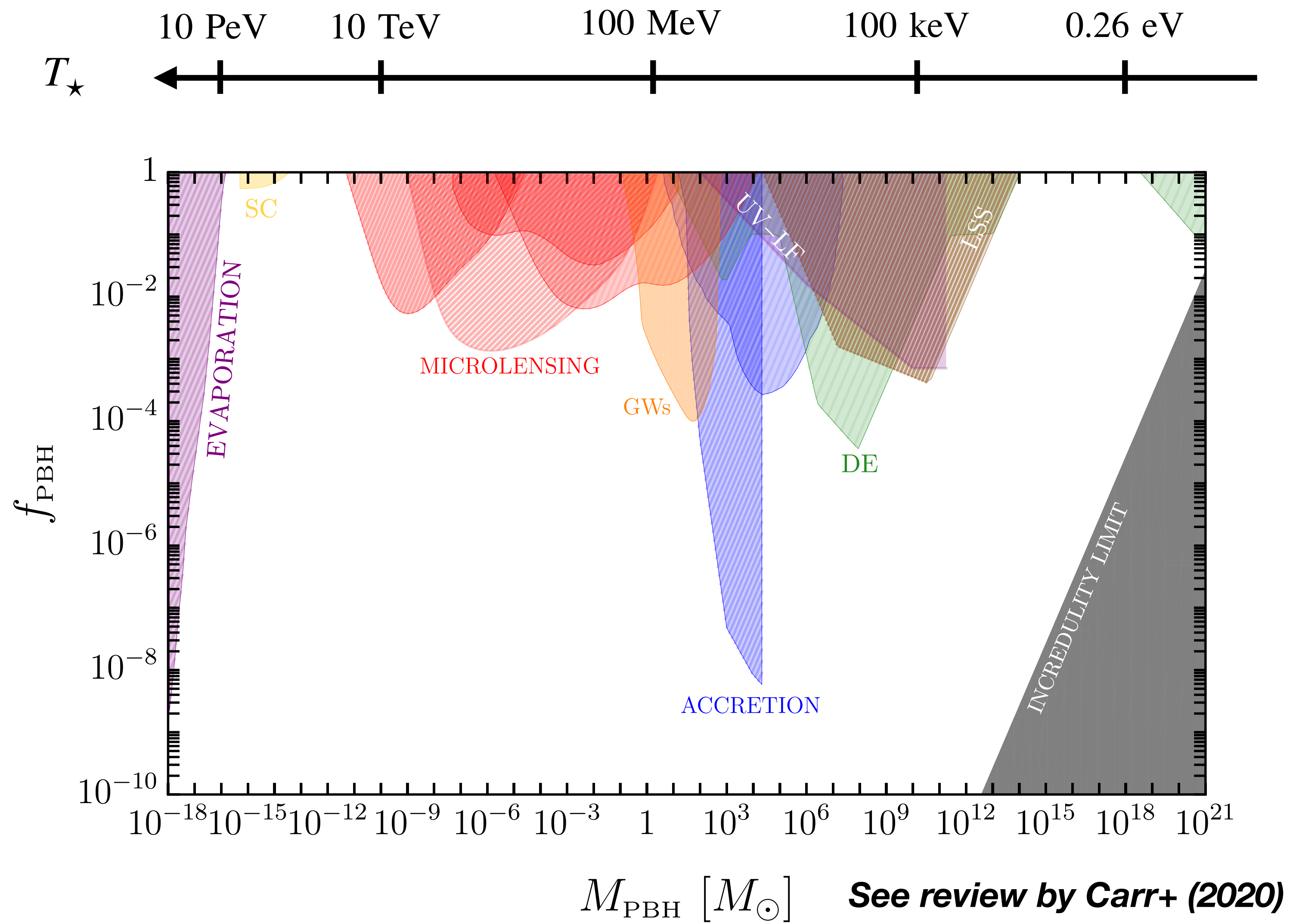


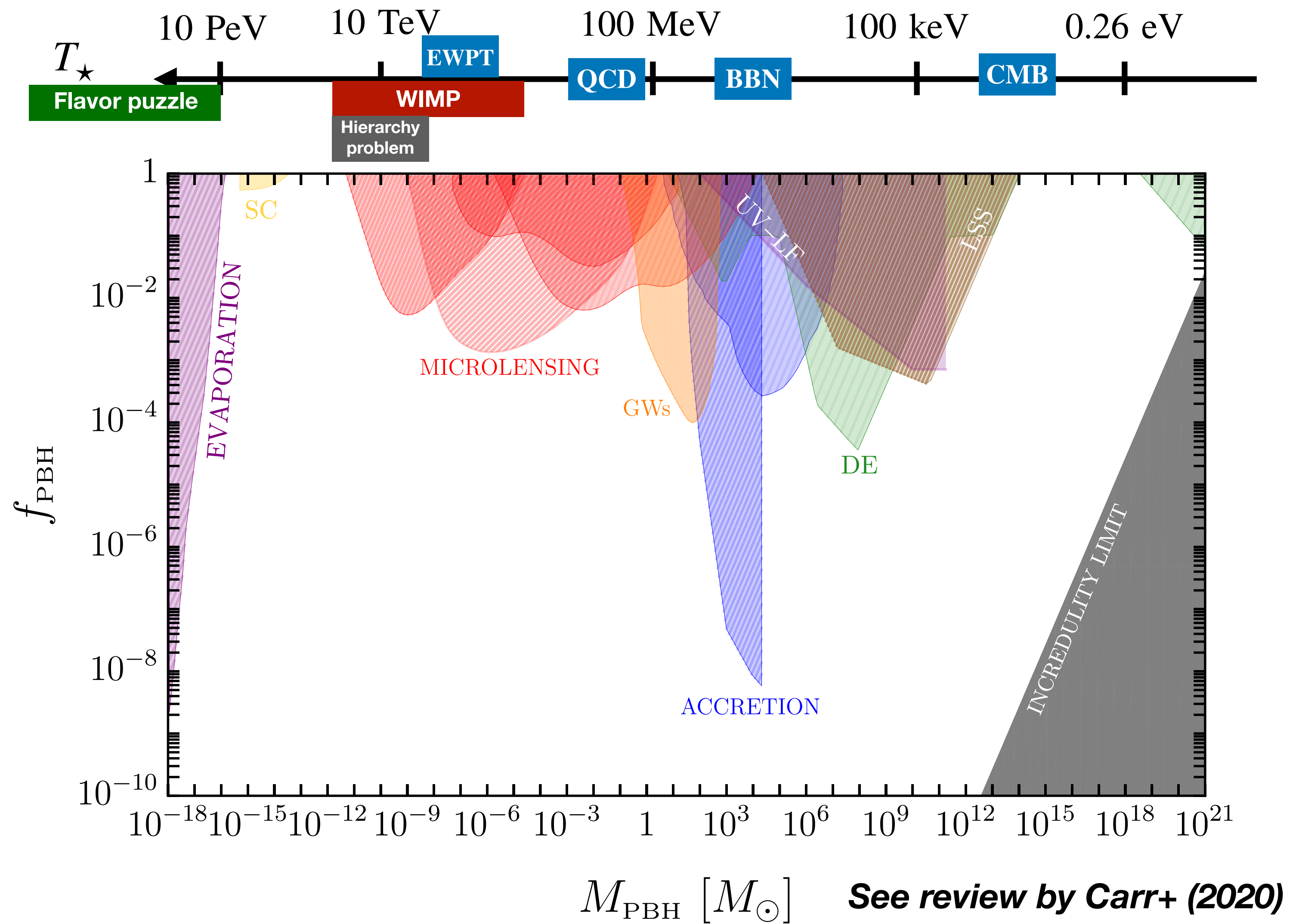
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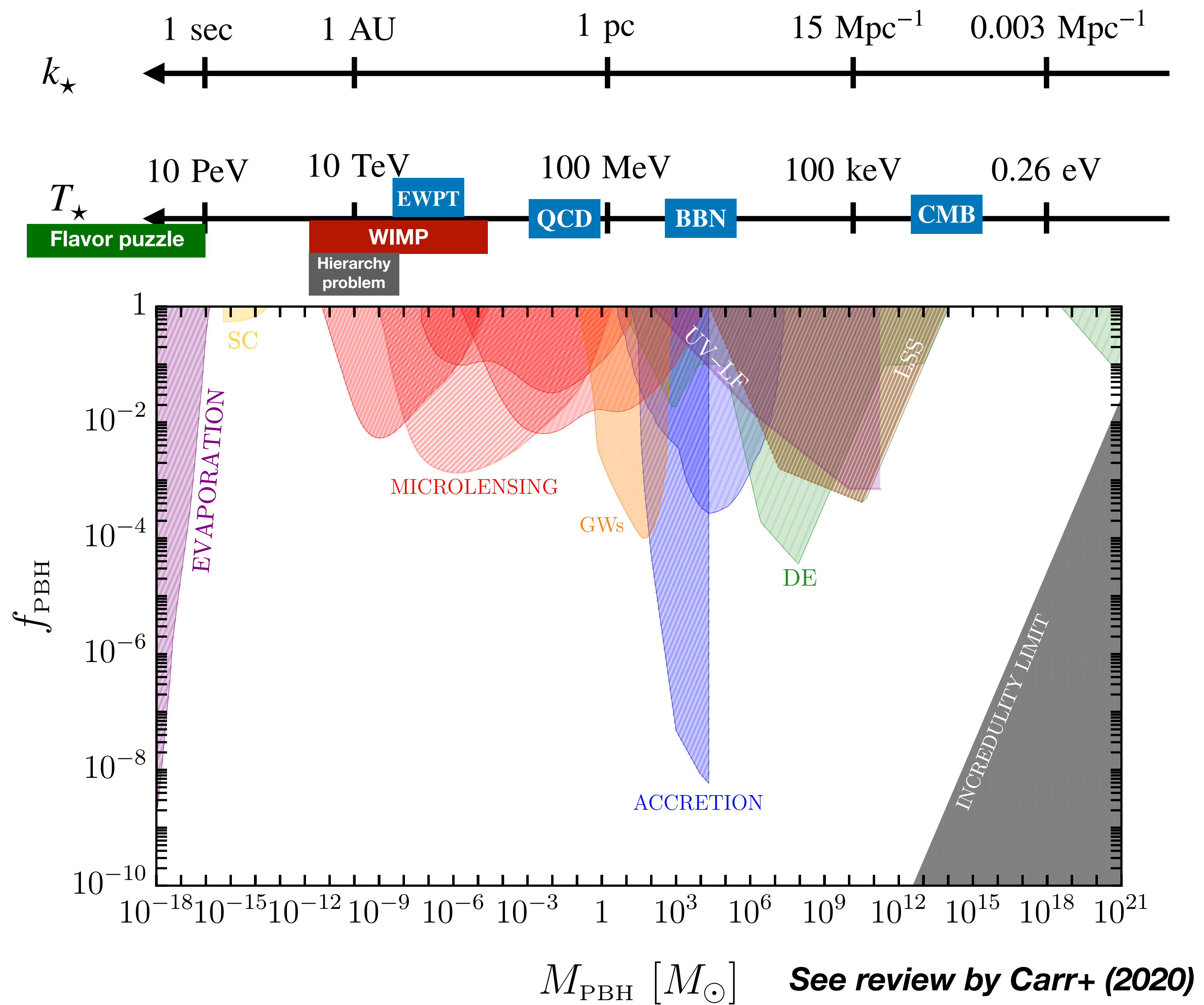




$M_{\text{PBH}} [M_{\odot}]$ **See review by Carr+ (2020)**







Lyman α

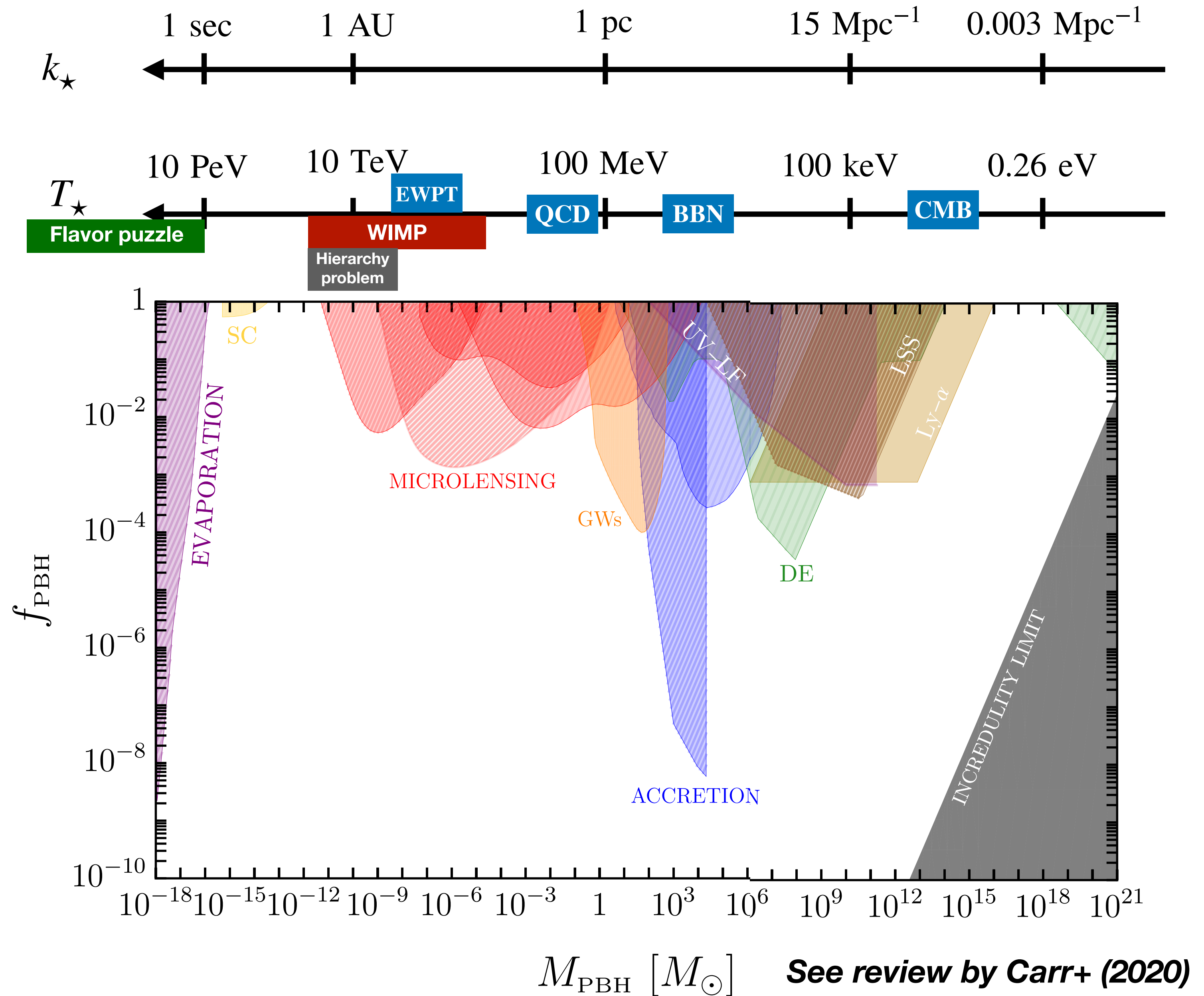
Effective Field Theory Constraints on Primordial Black Holes from the High-Redshift Lyman- α Forest

Mikhail M. Ivanov^{1,*} and Sokratis Trifinopoulos^{1,2,3,†}

¹Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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Lyman – α

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CMB + BAO

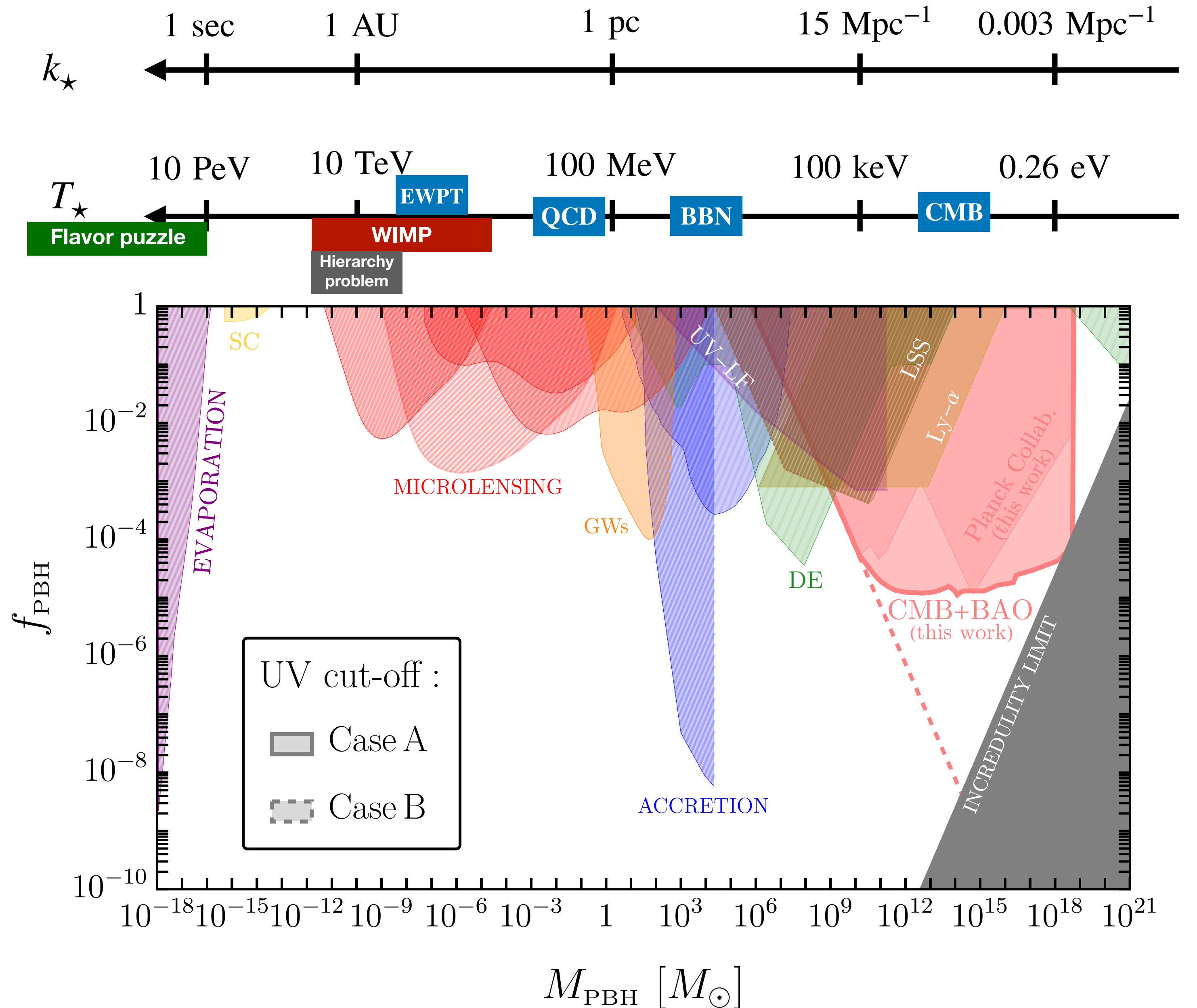
Closing the Mass Window for Stupendously Large Black Holes

Christopher Gerlach^{Ⓜ,1,*} Yann Gouttenoire^{Ⓜ,1,†} Antonio J. Iovino^{Ⓜ,2,3,‡} and Nicholas Leister^{Ⓜ,1,§}

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Lyman – α To appear in PRL

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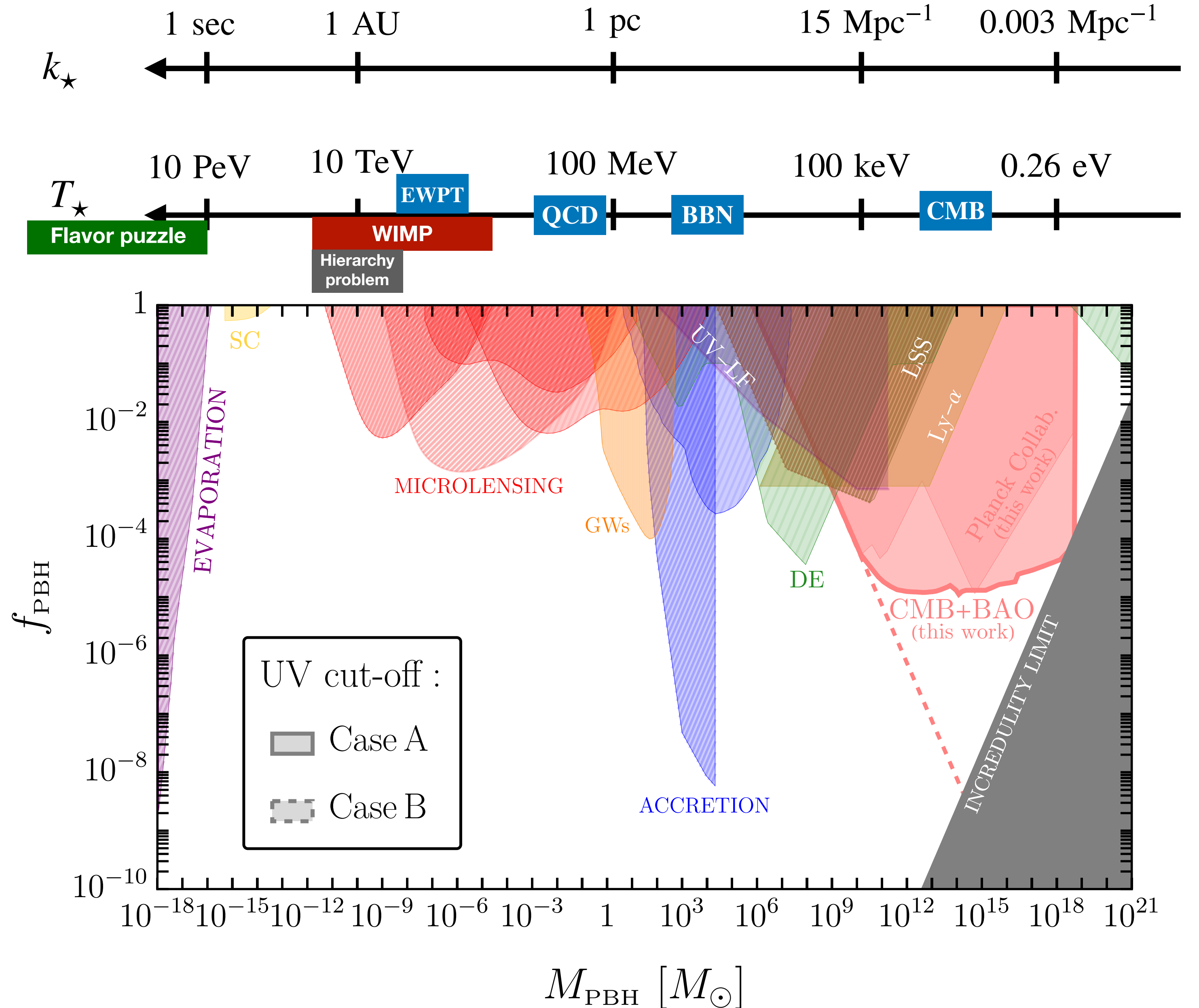
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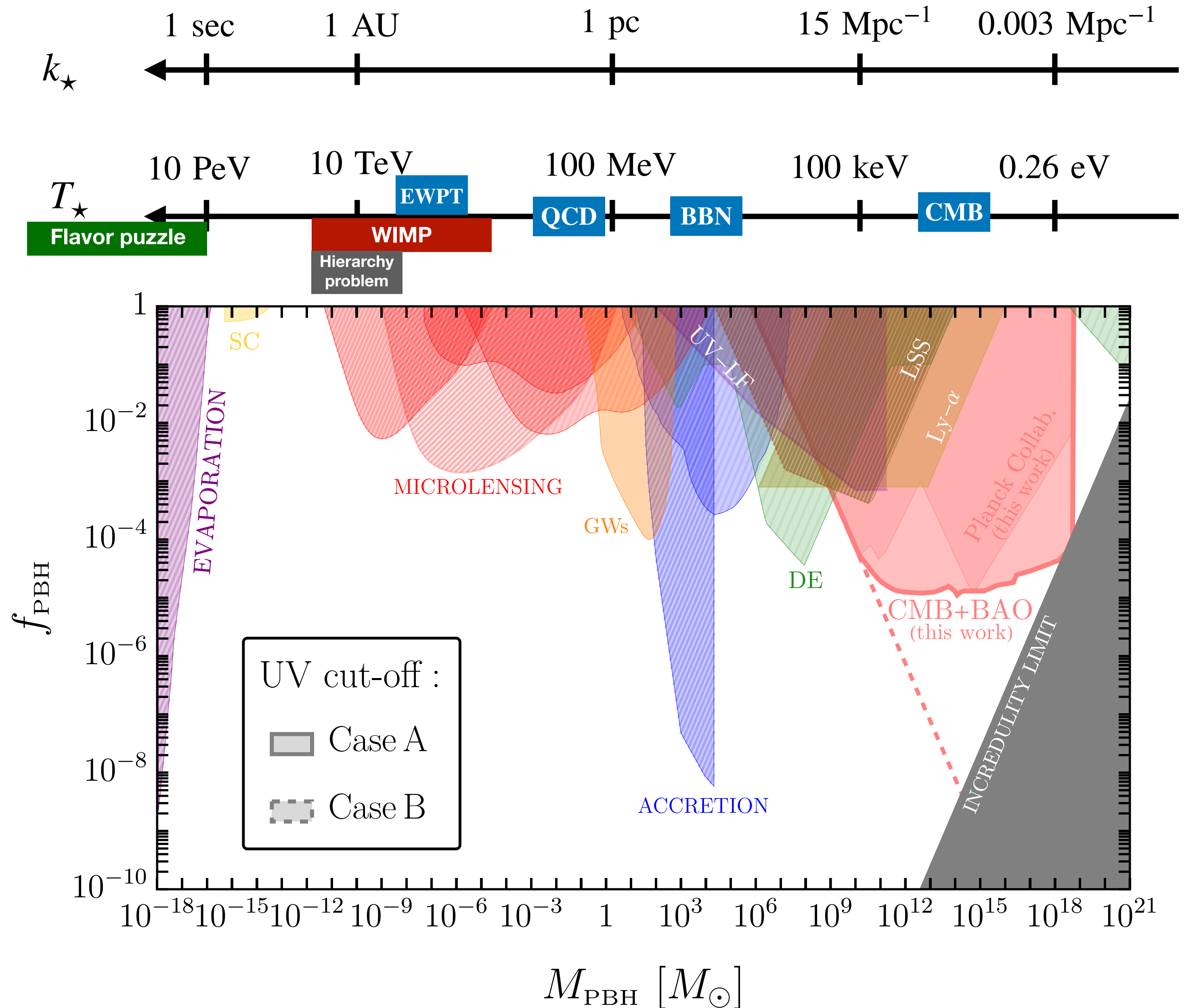
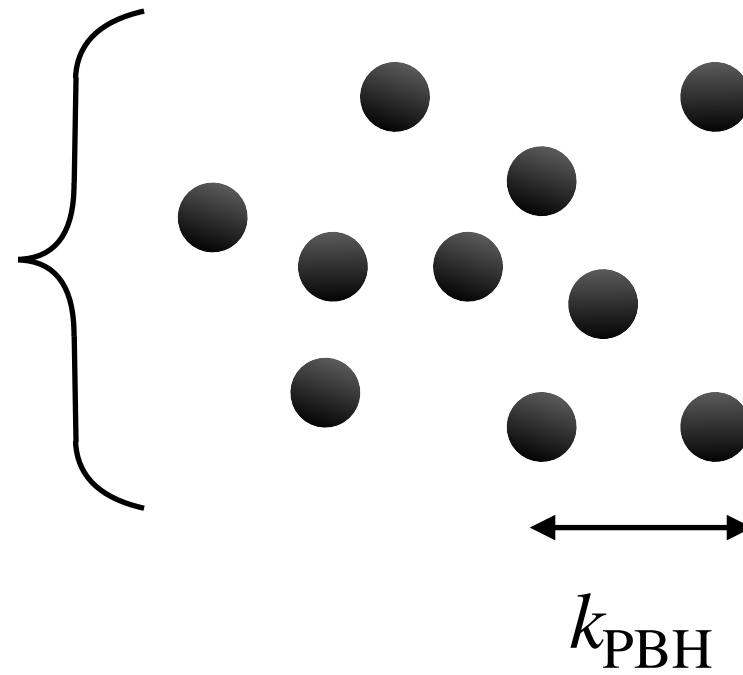
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Gas of PBHs



Lyman – α To appear in PRL

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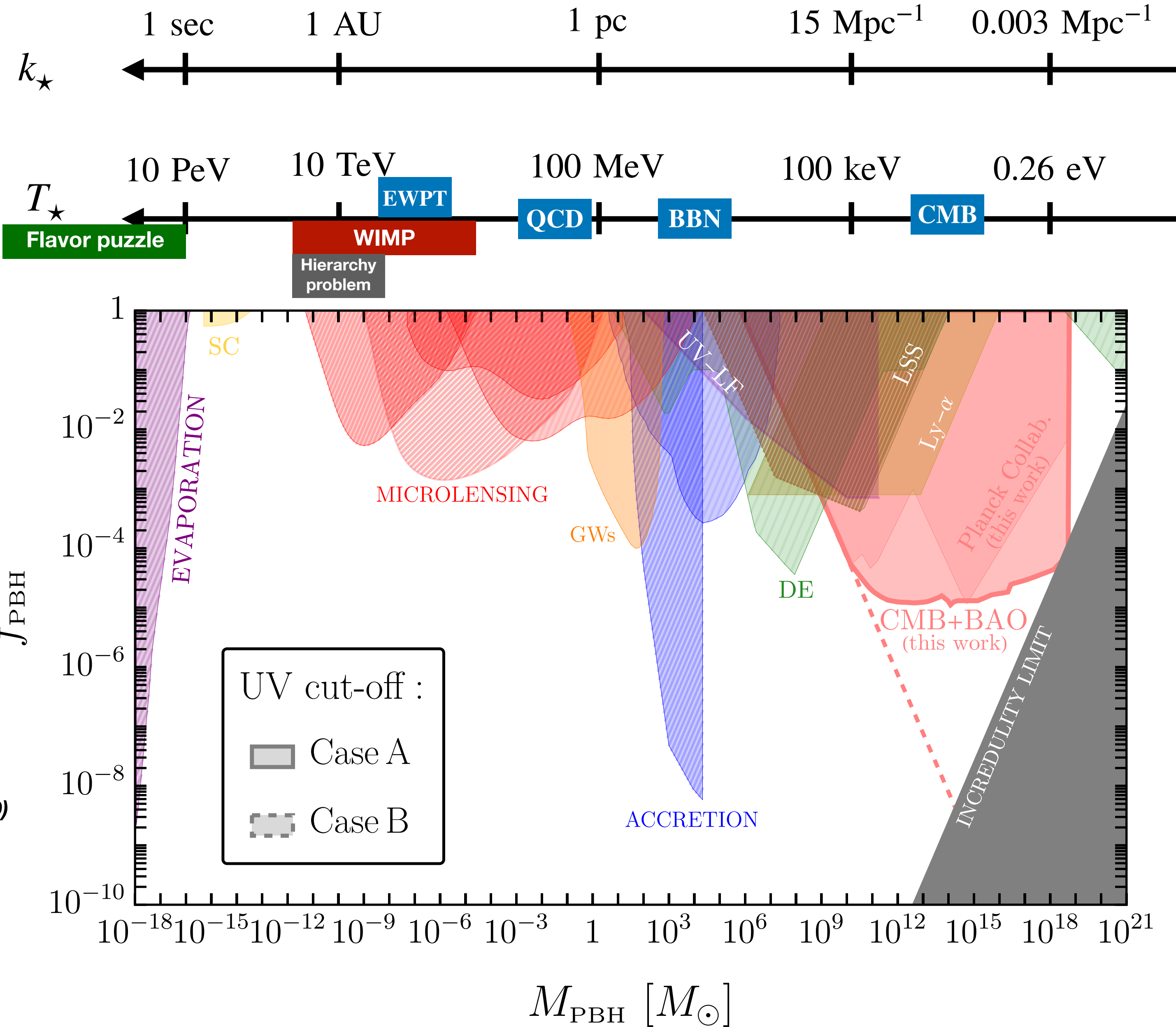
Gas of PBHs

Poisson noise :

$$\mathcal{P}_\delta = f_{\text{PBH}}^2 \left(\frac{k}{k_{\text{PBH}}} \right)^3 \Theta(k_{\text{cut}} - k)$$

Inman, Ali-Haïmoud (1907.08129)

⇒ source isocurvatures



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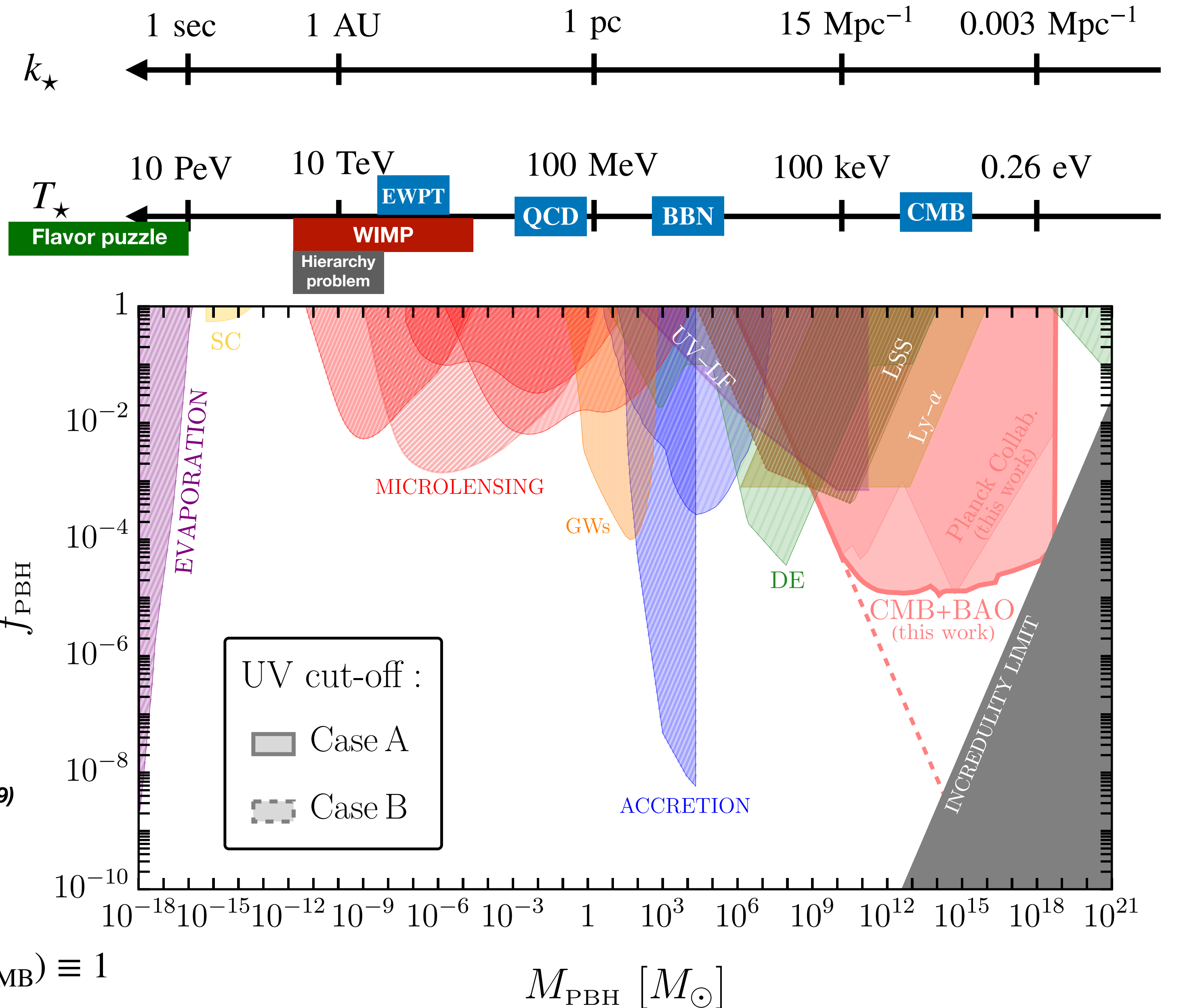
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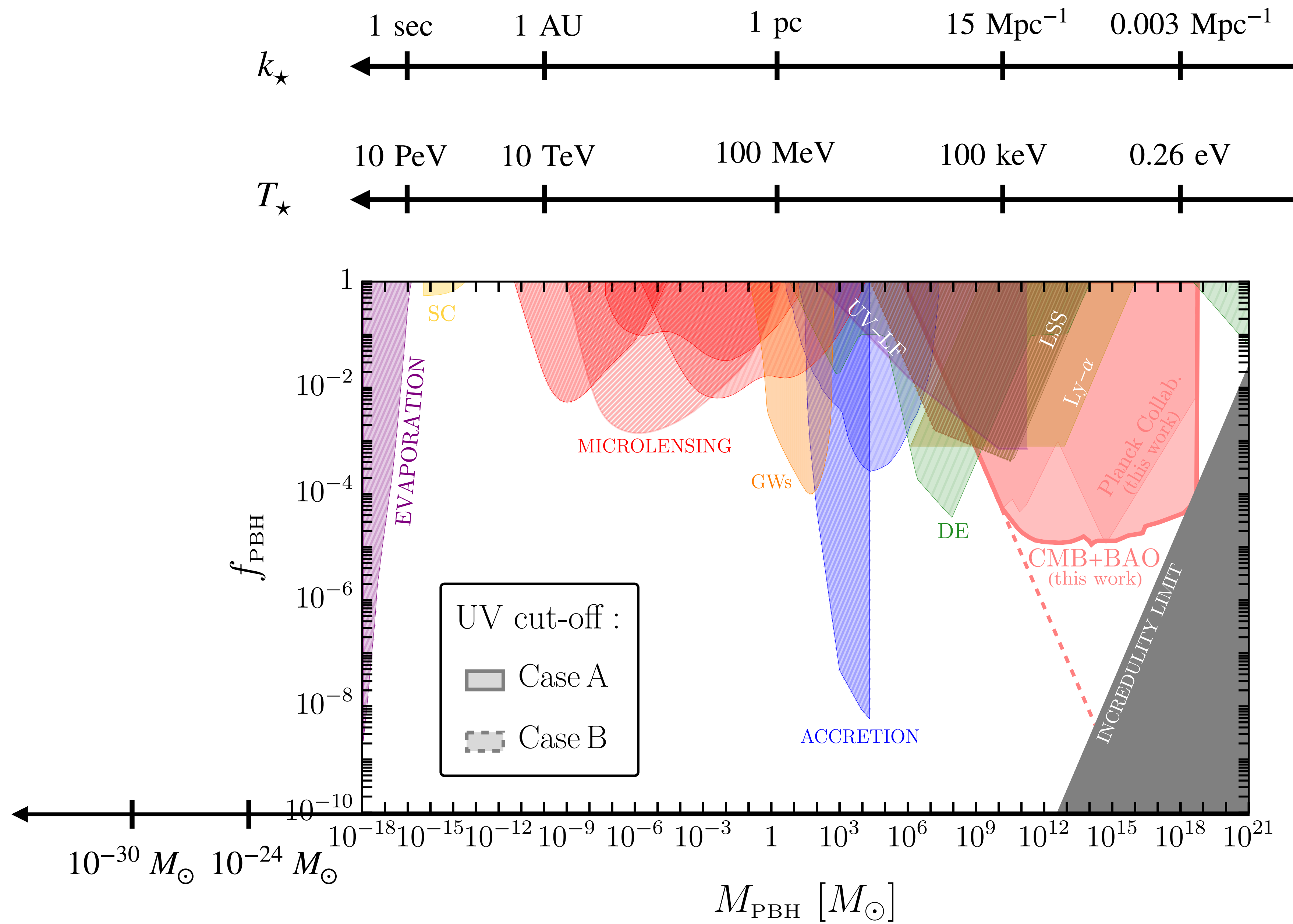
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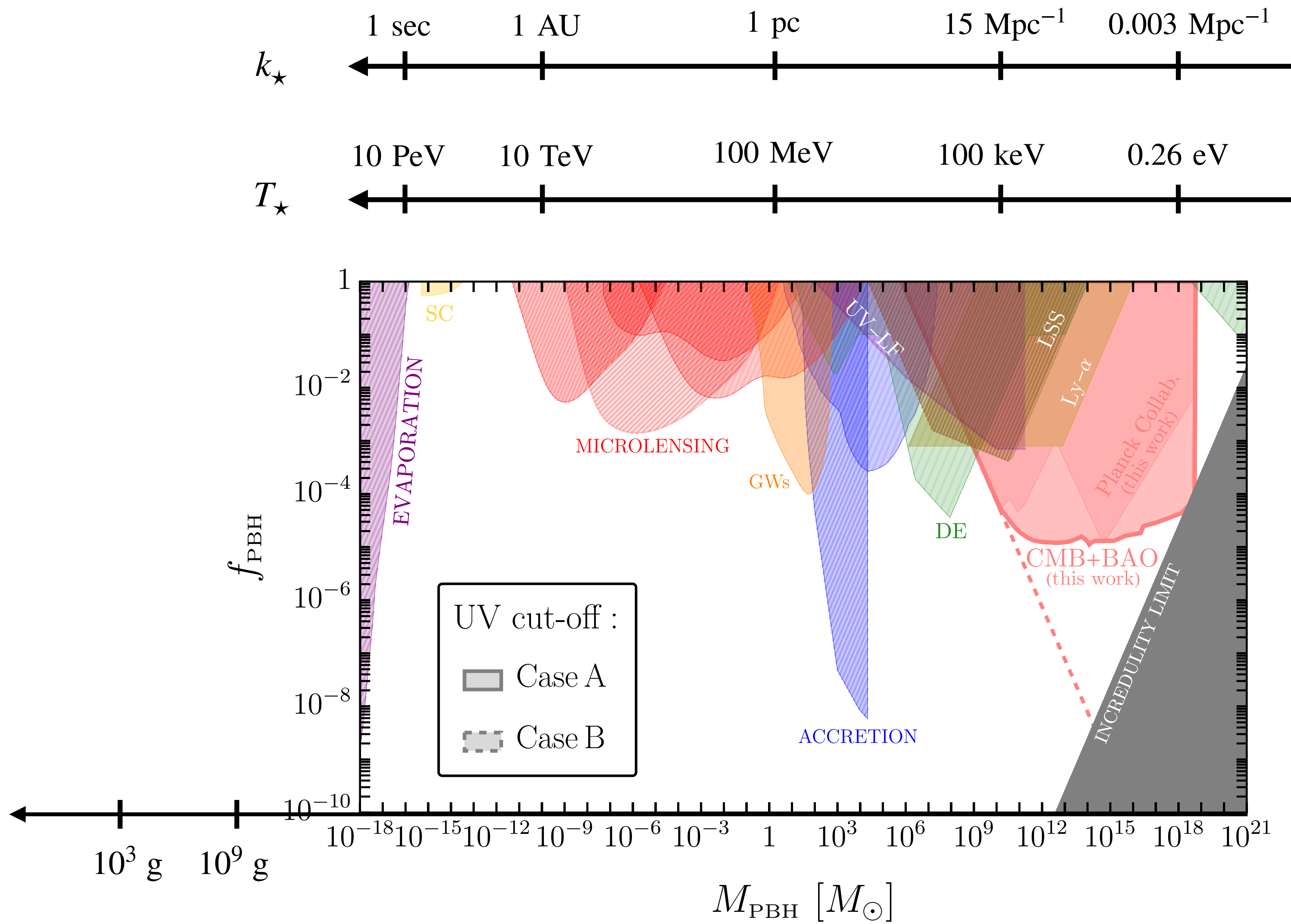
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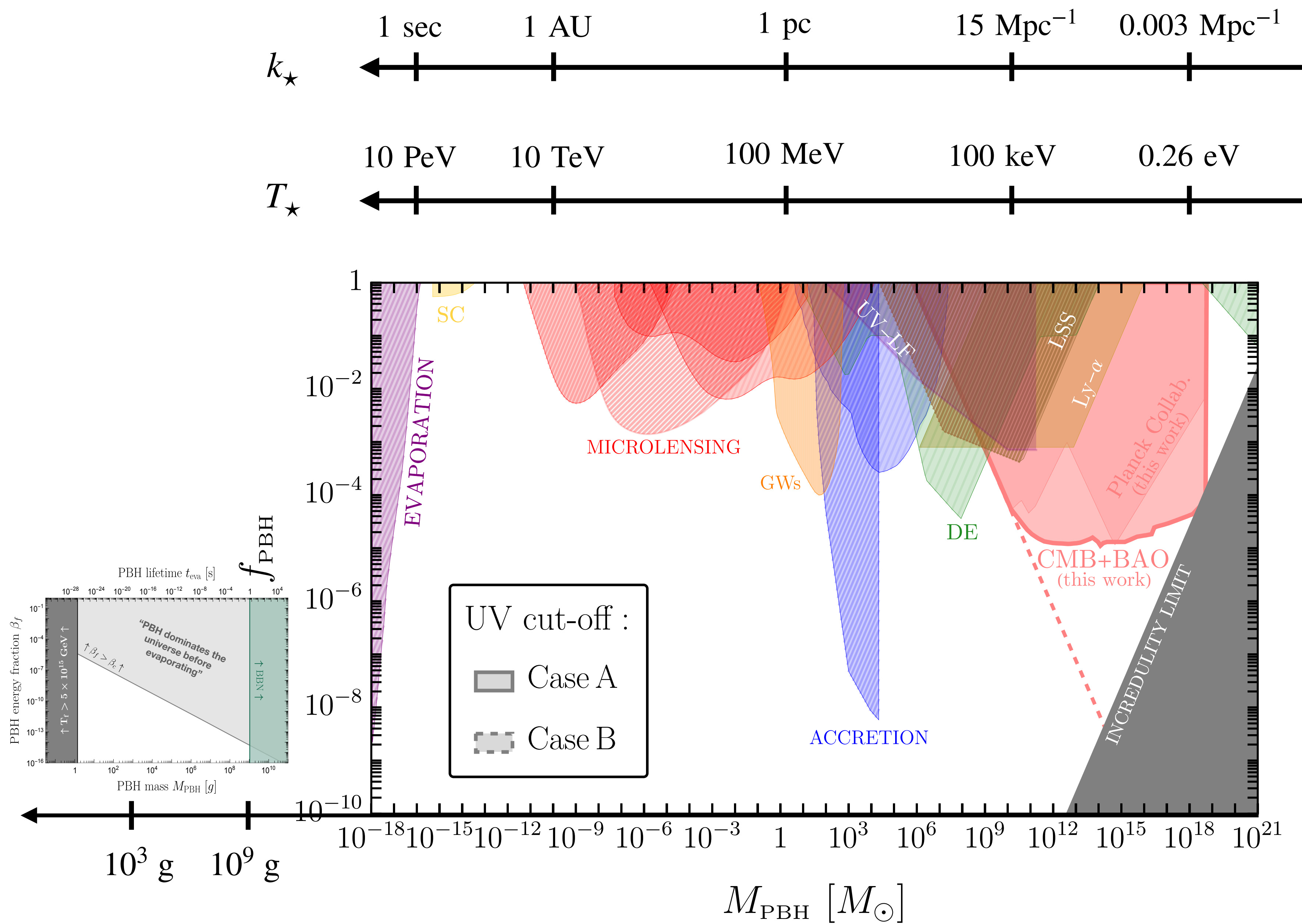
⇒ **source isocurvatures**

$$k_{\text{cut}} = \begin{cases} k_{\text{PBH}}, & \text{(solid line)} \\ k_{\text{NL}}, & \text{(dashed line)} \end{cases} \quad \text{with} \quad \delta_{\text{DM}}(k_{\text{NL}}, t_{\text{CMB}}) \equiv 1$$



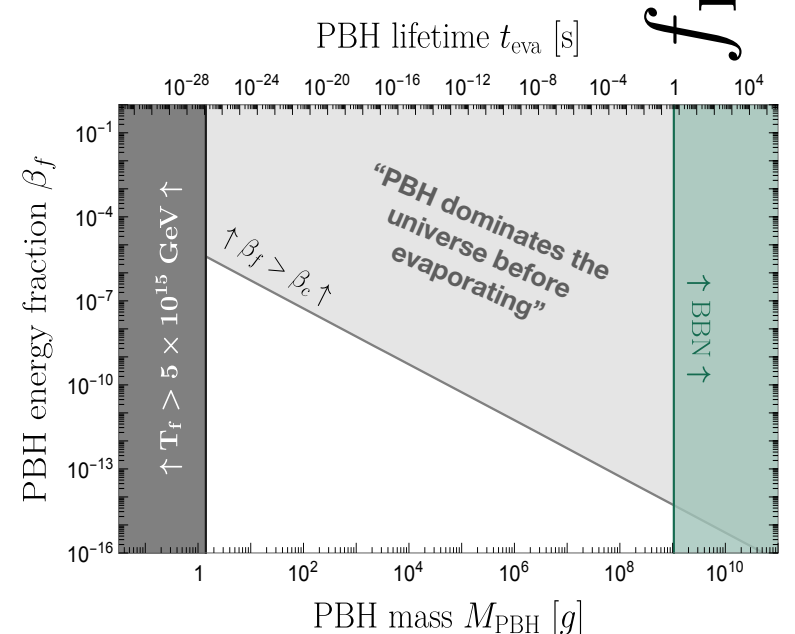
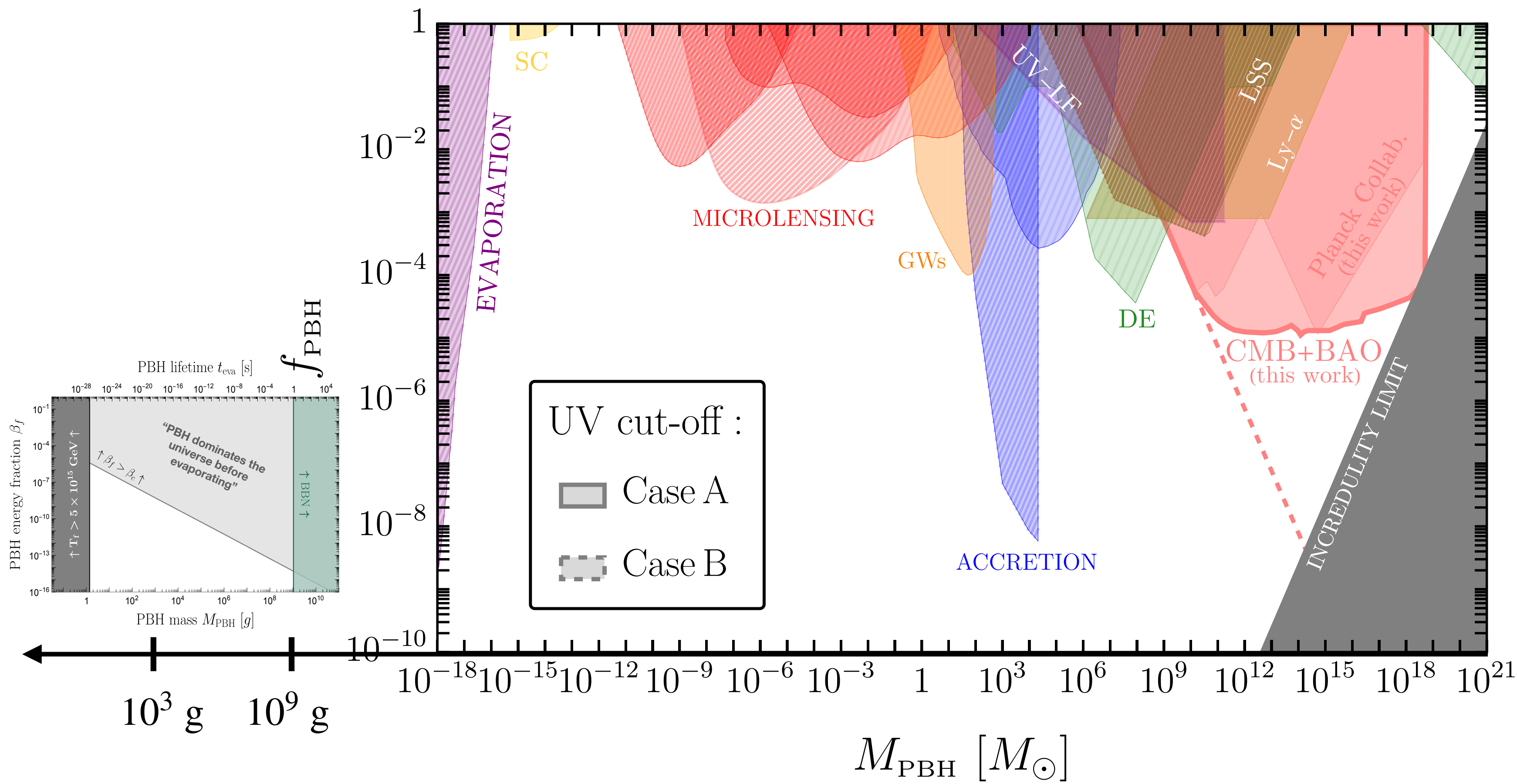
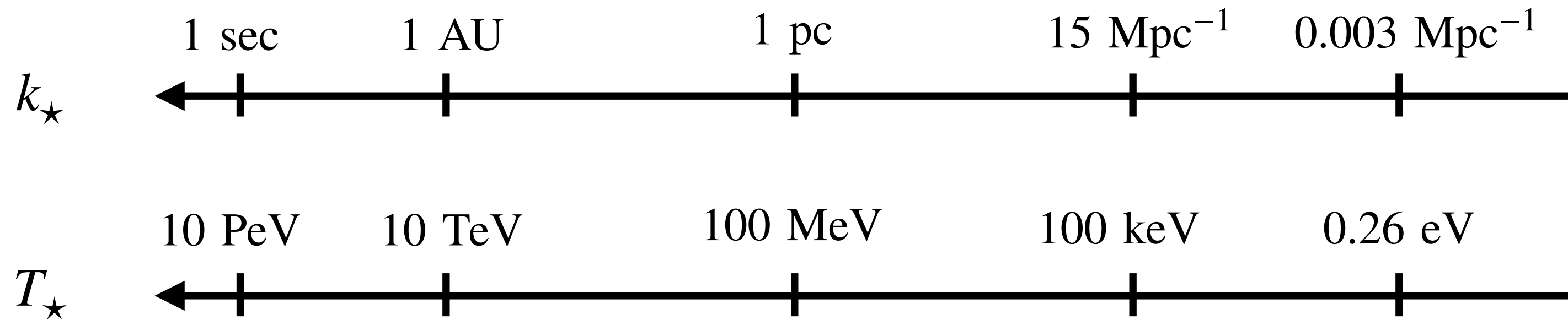
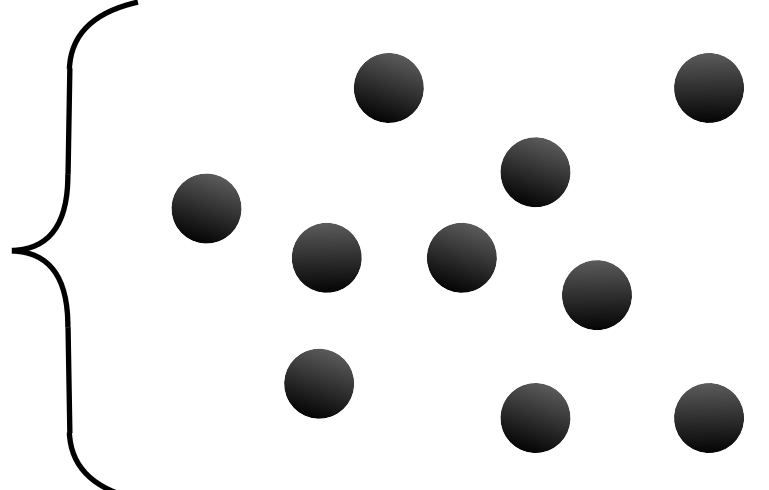






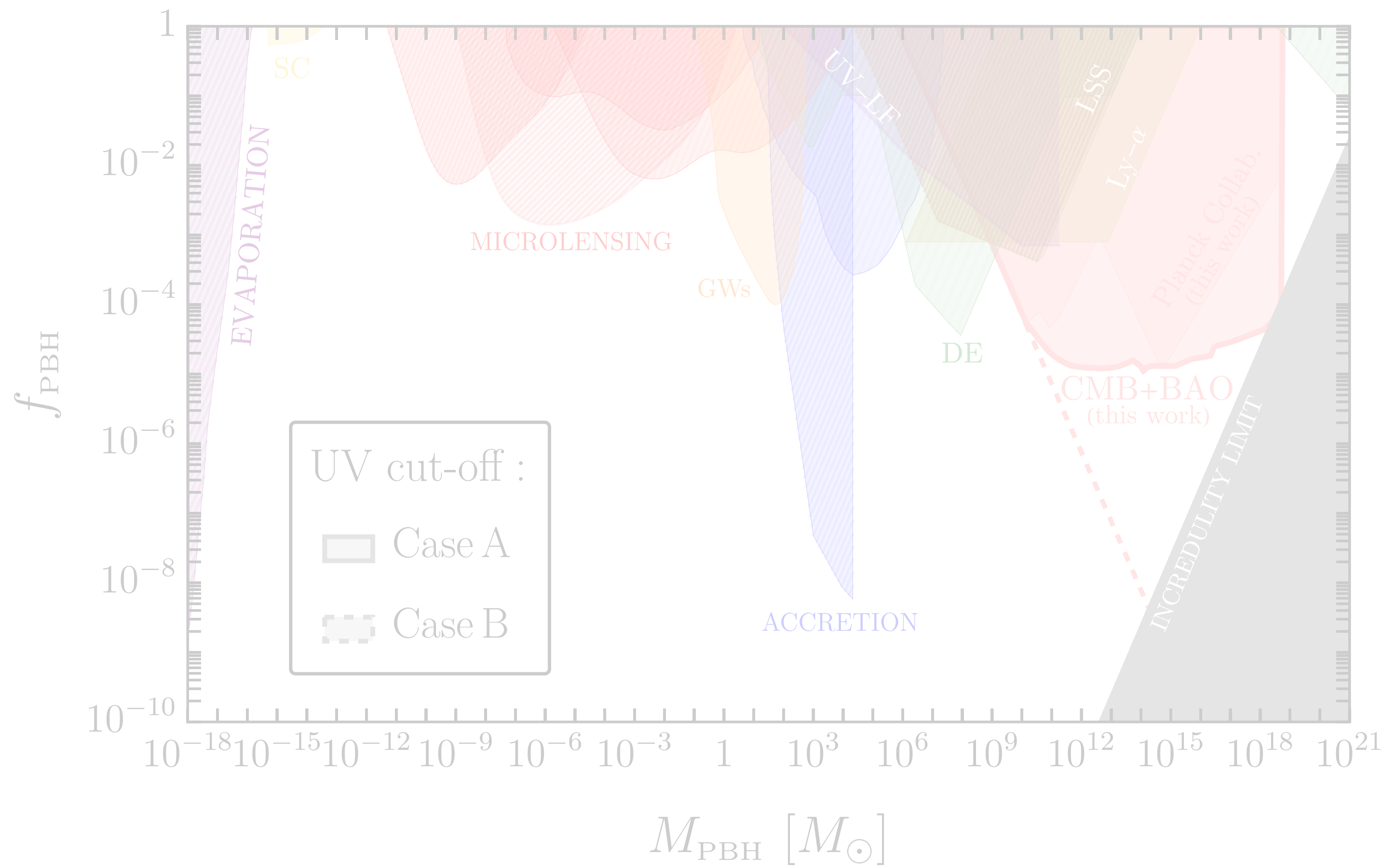
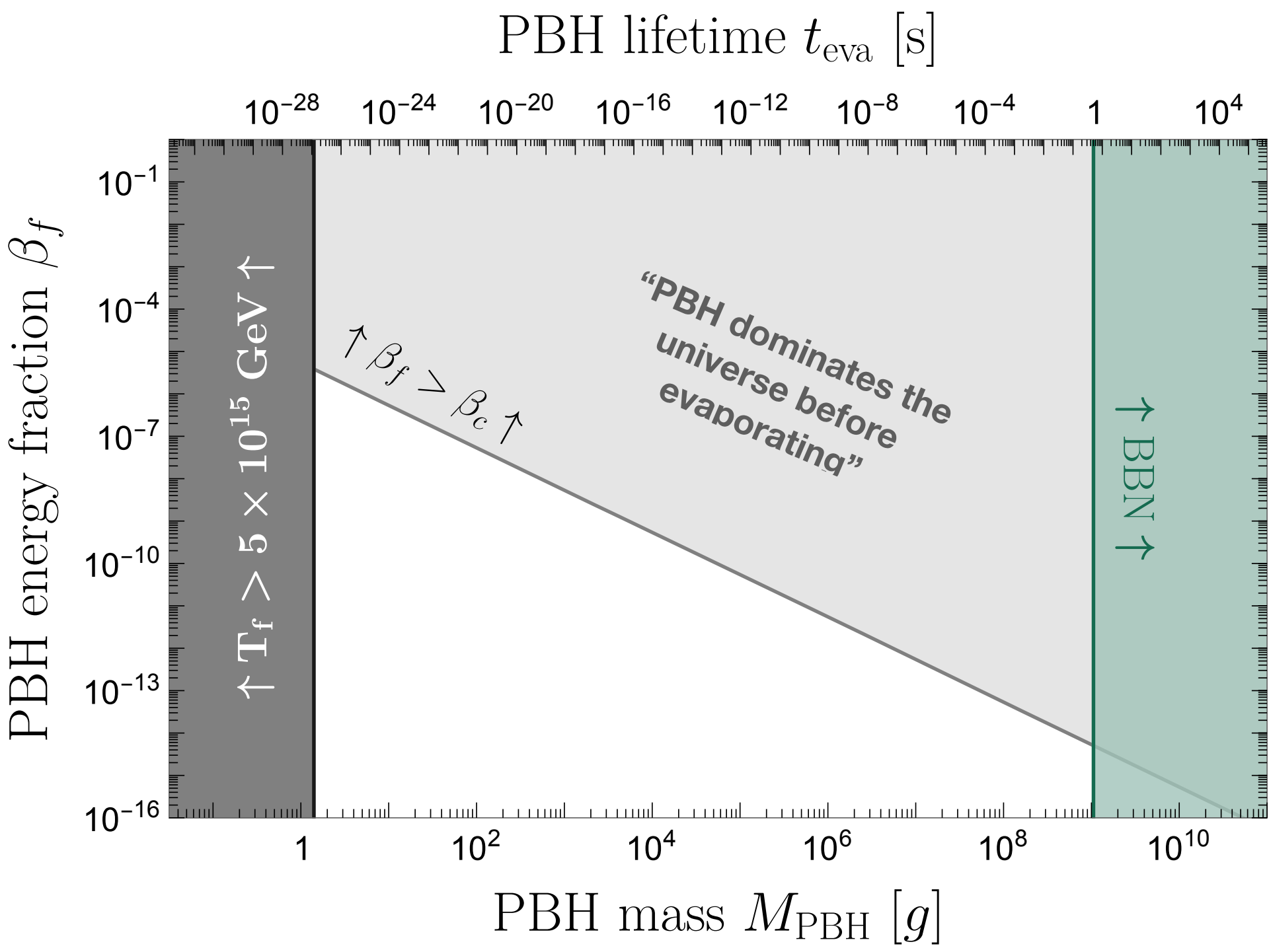
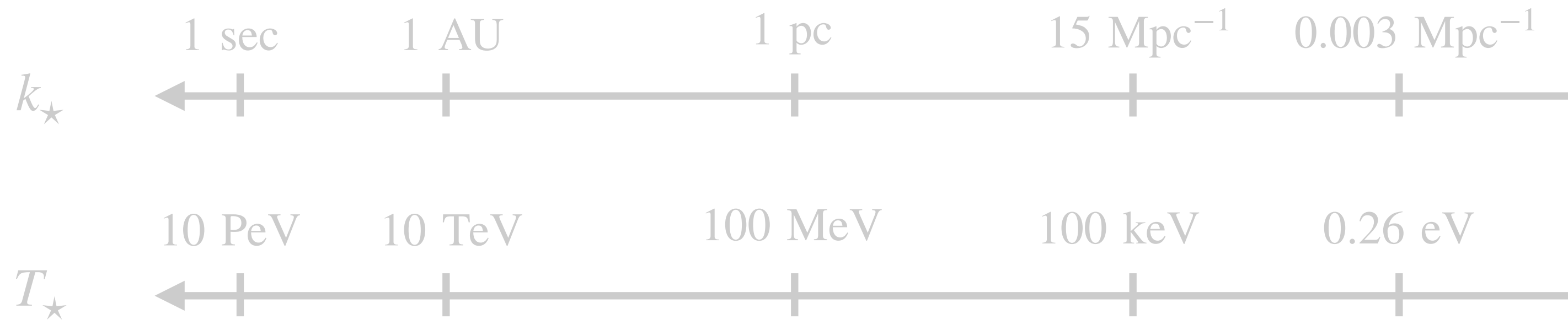
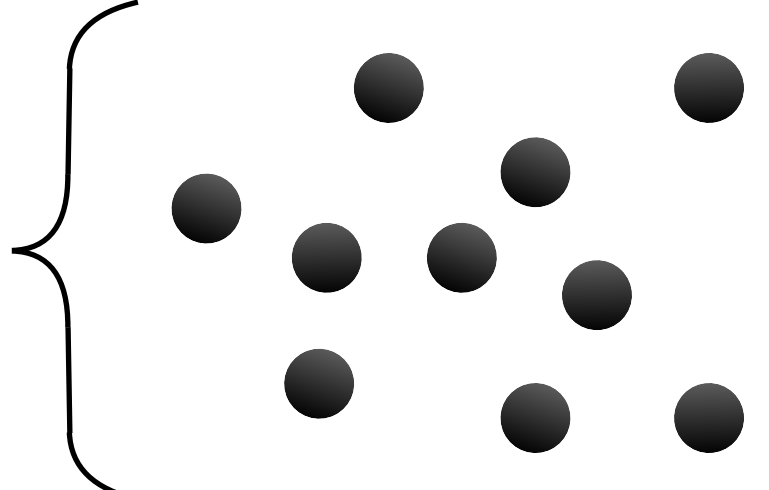
Ultra-light PBHs

Gas of
evaporating PBHs



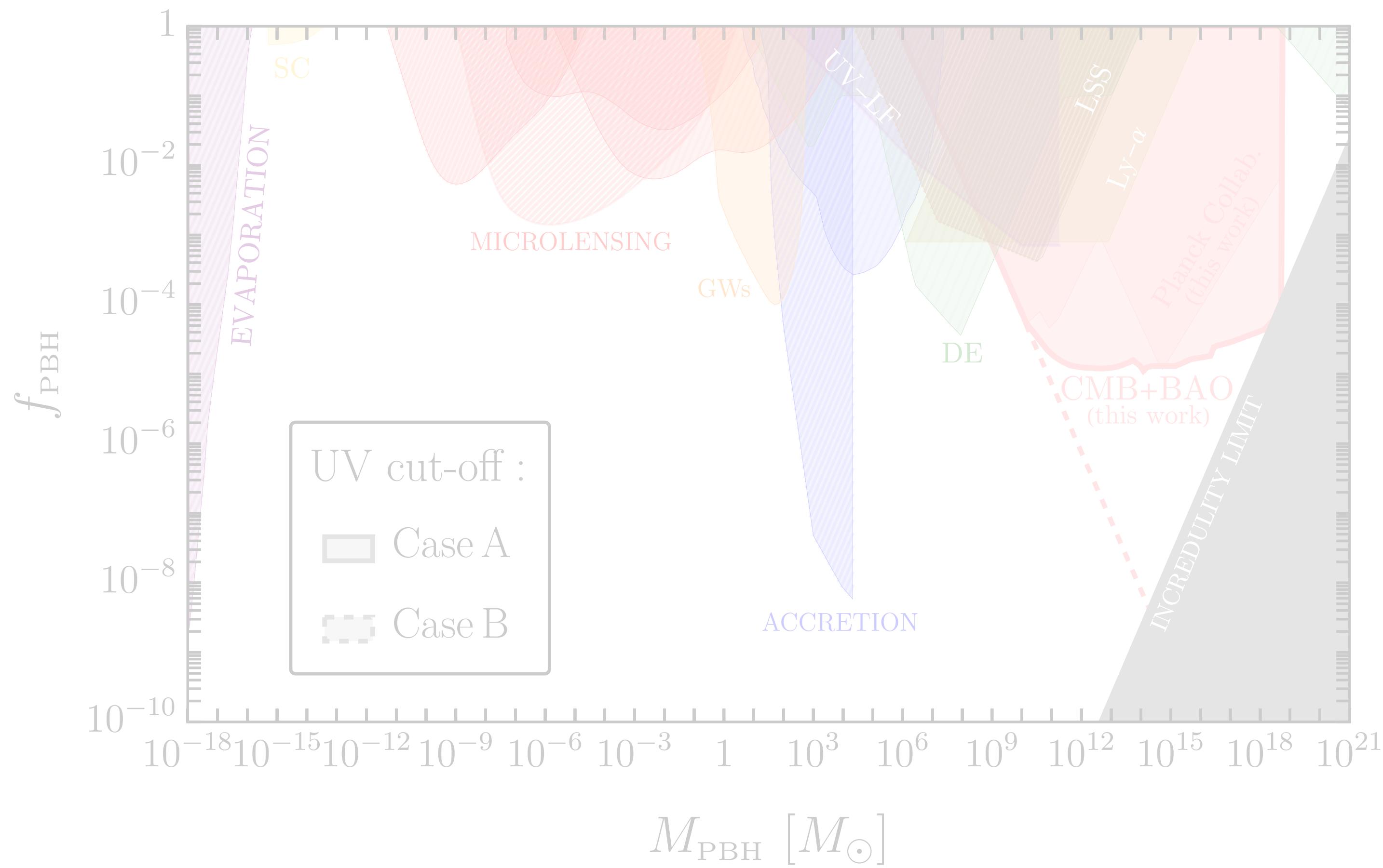
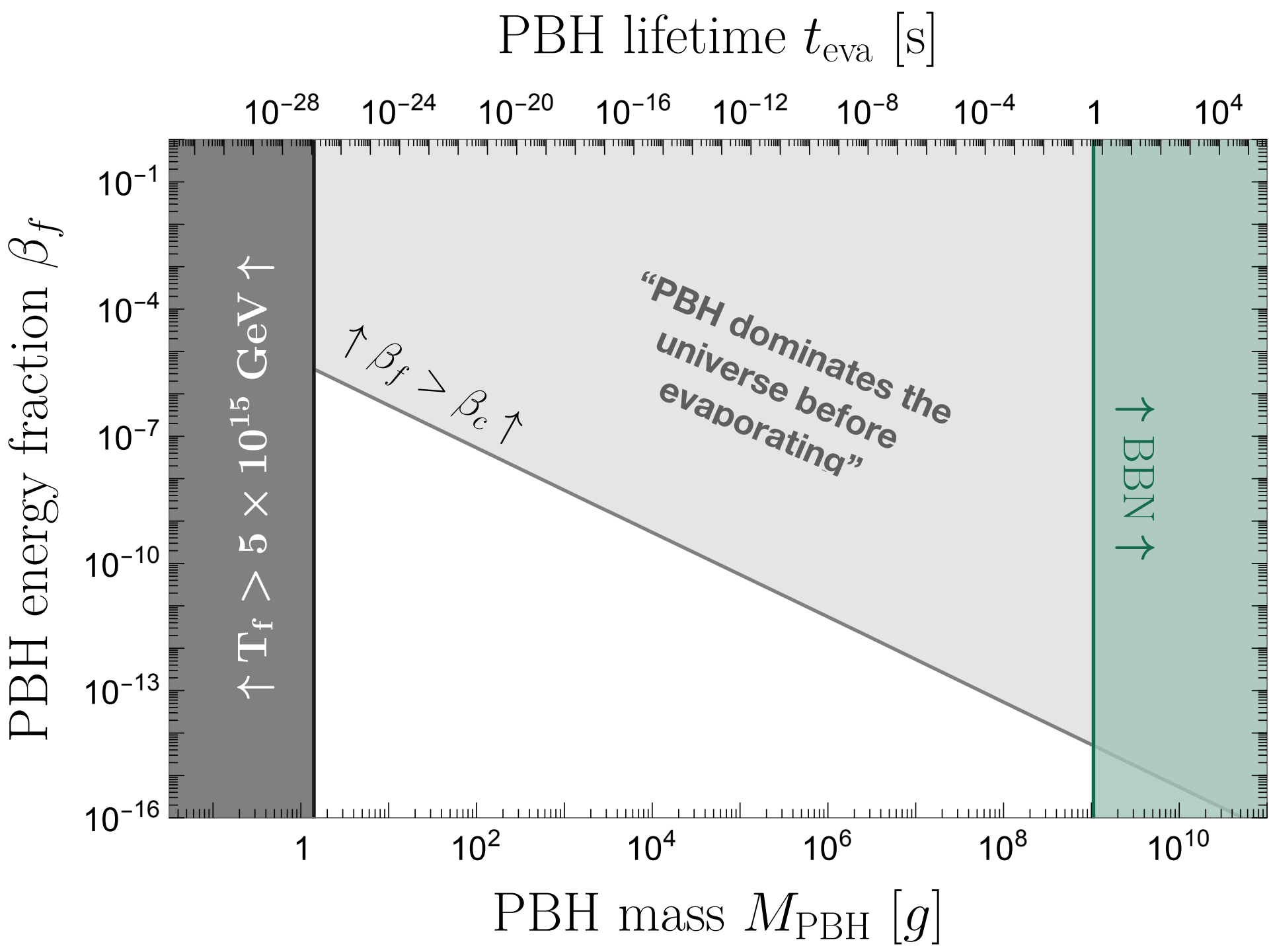
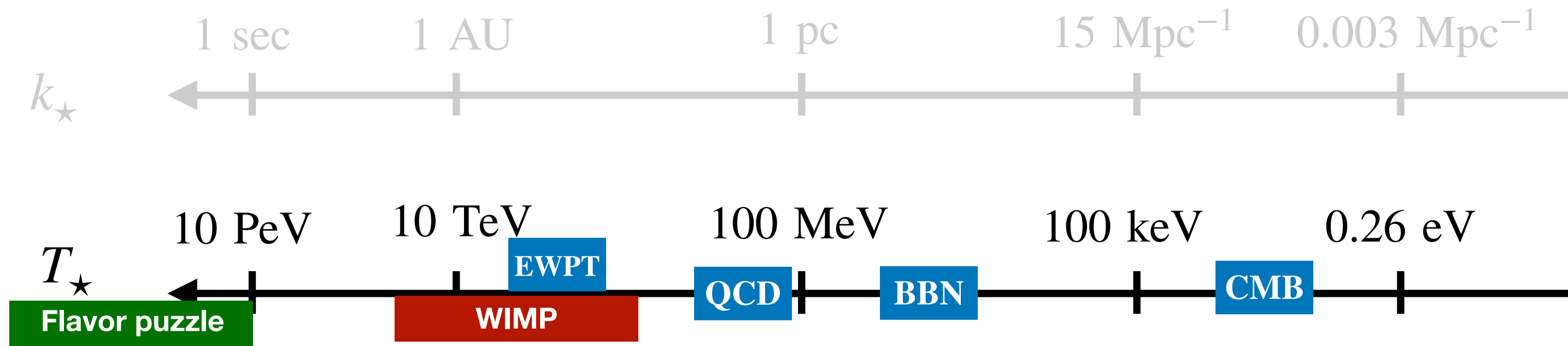
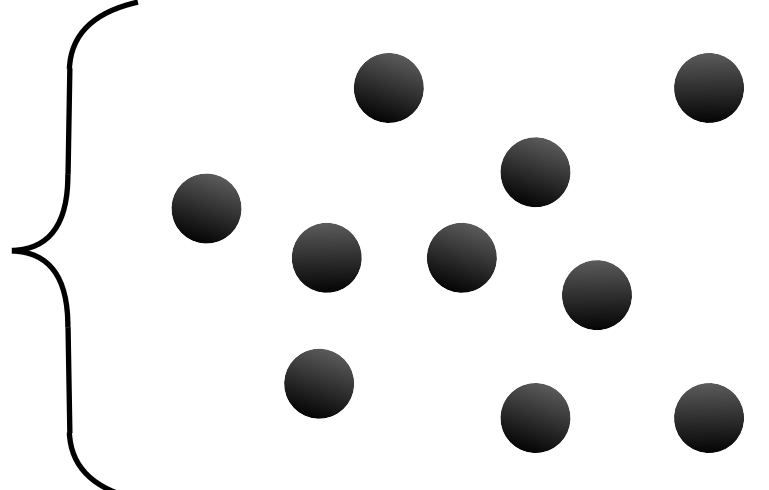
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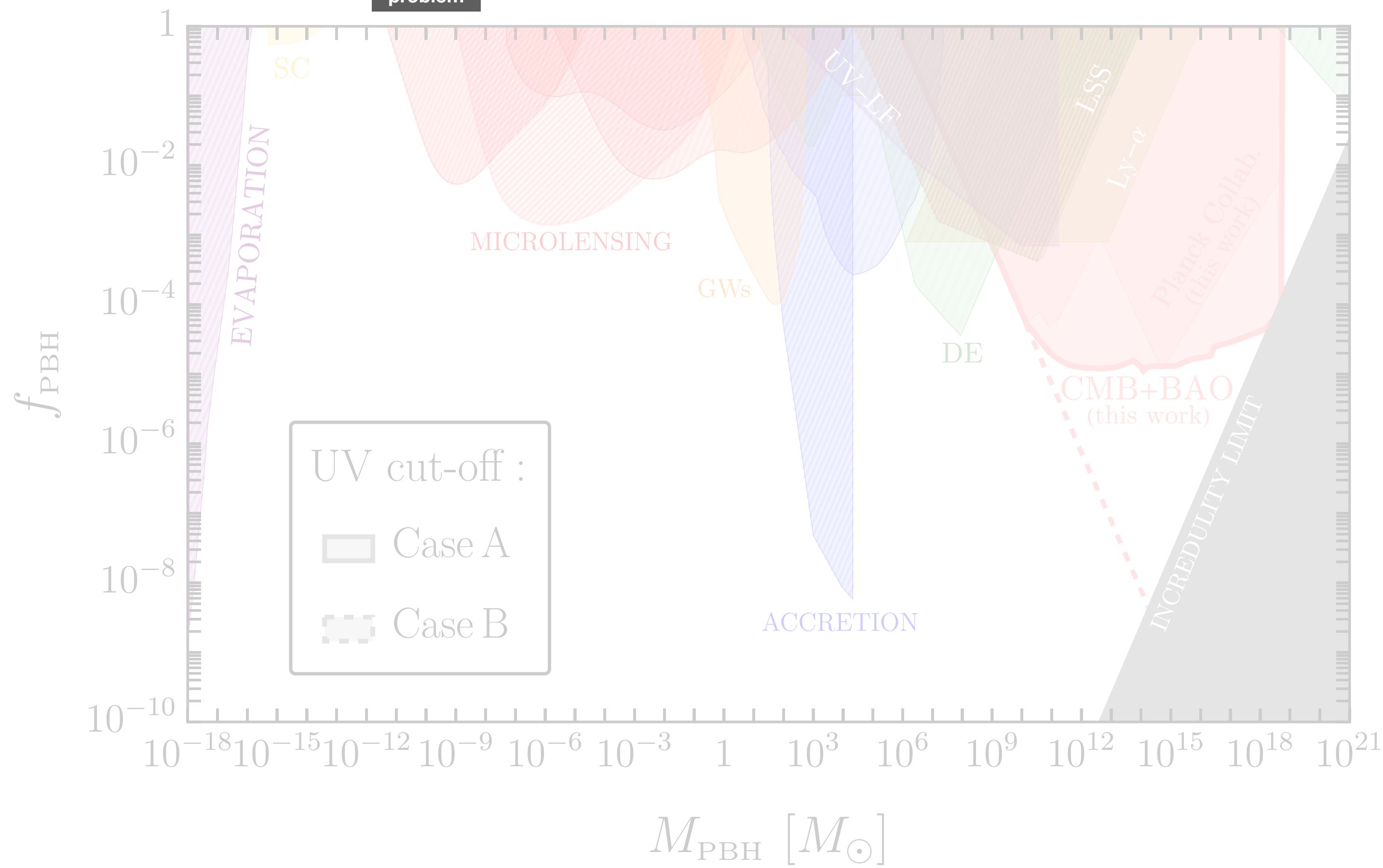
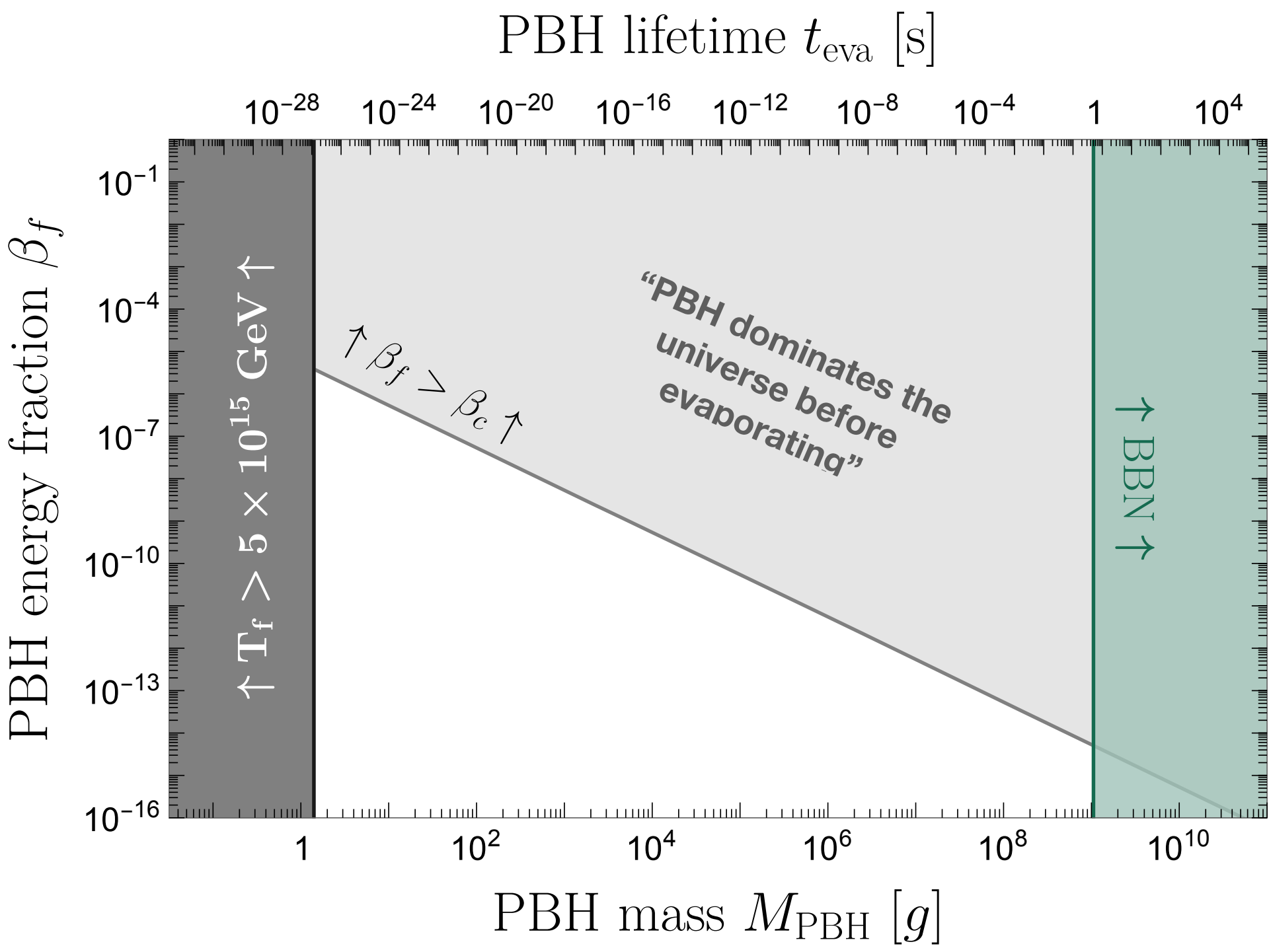
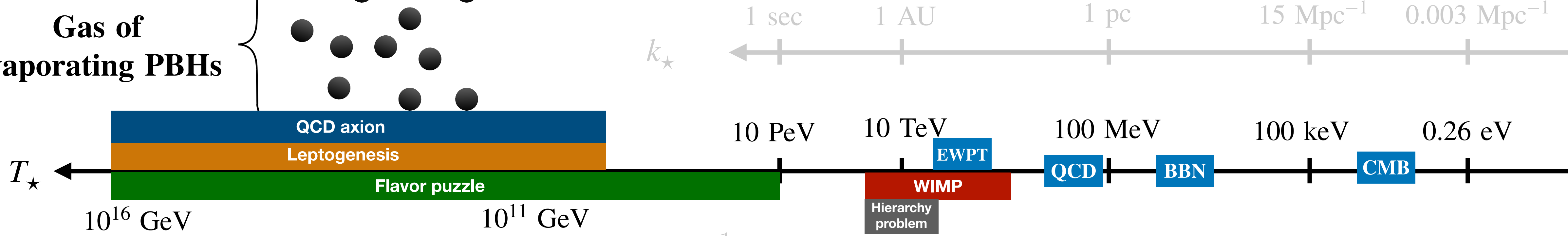
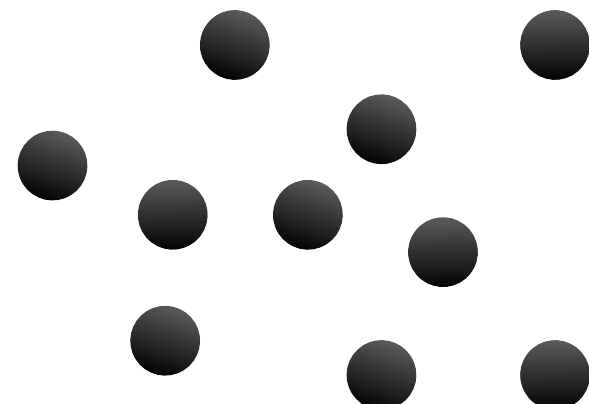
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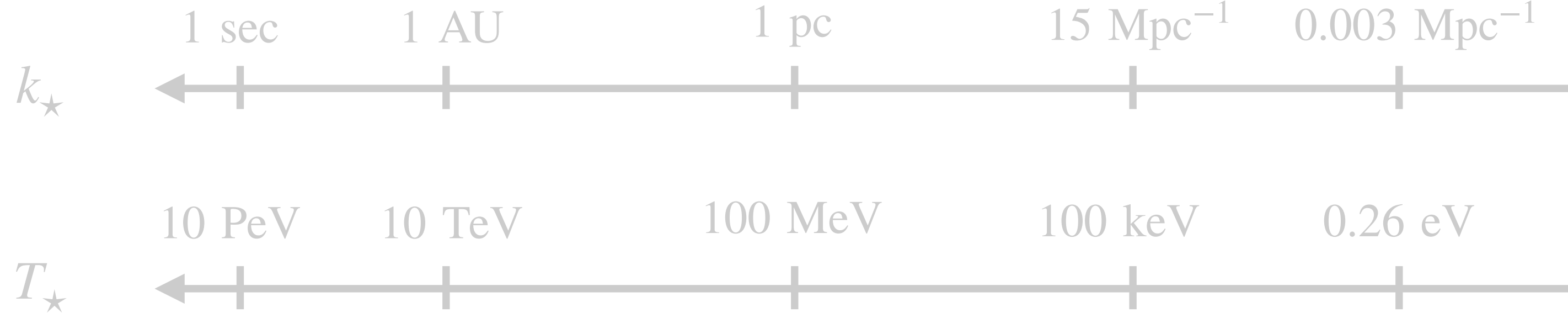
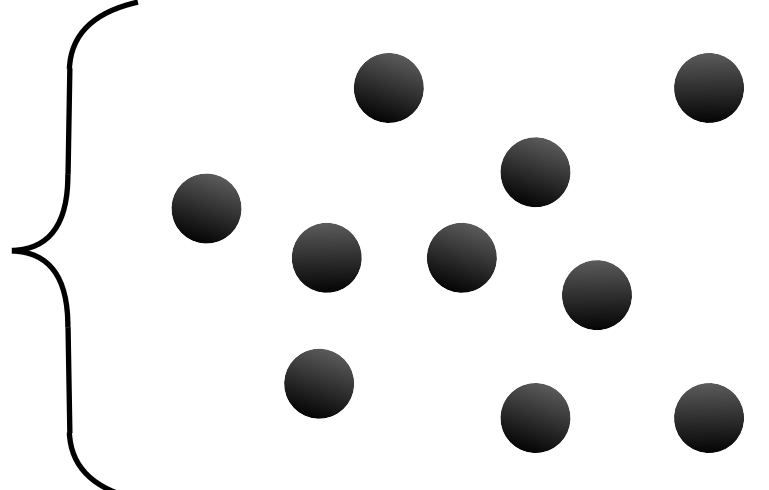
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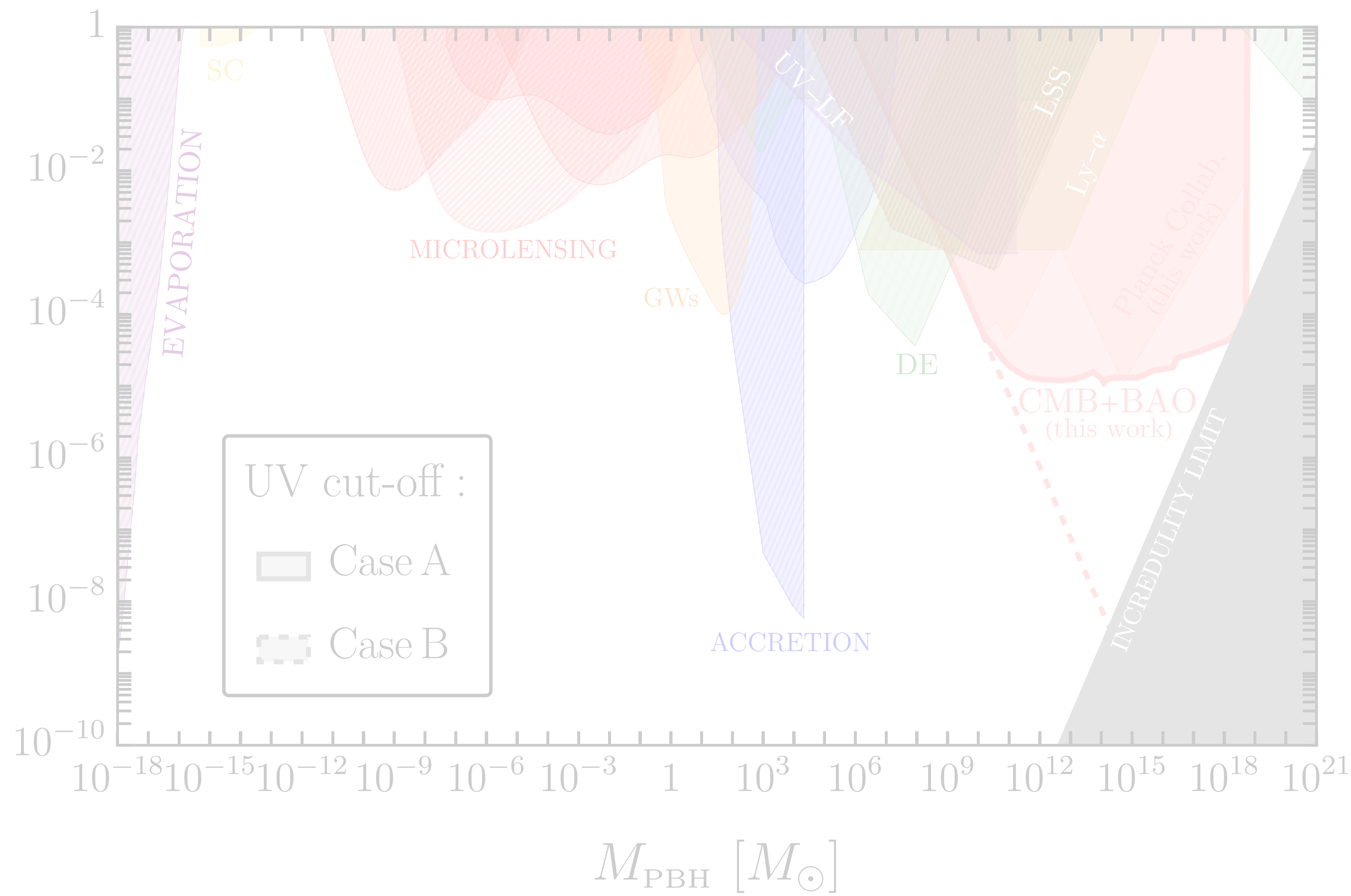
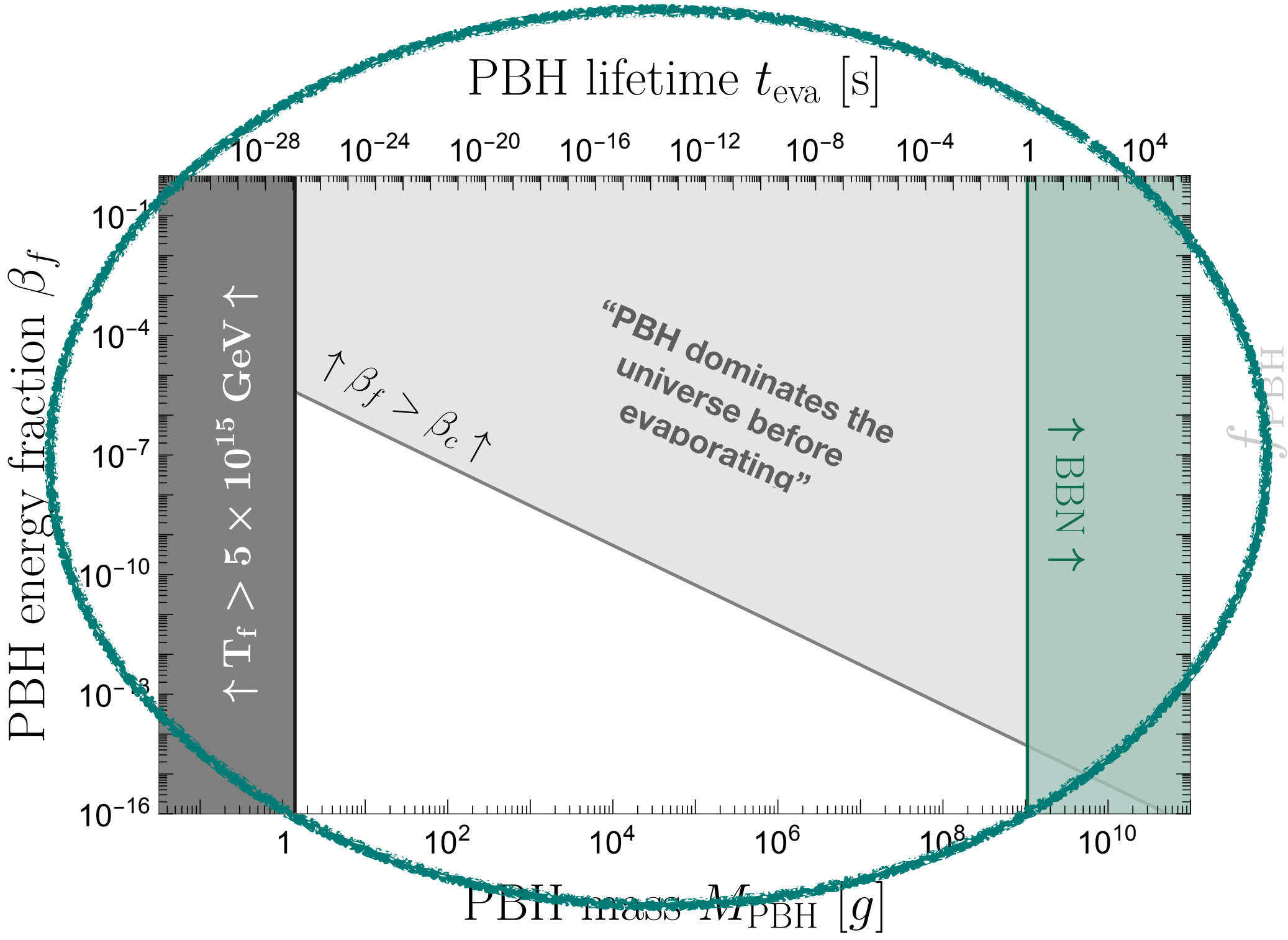


Ultra-light PBHs

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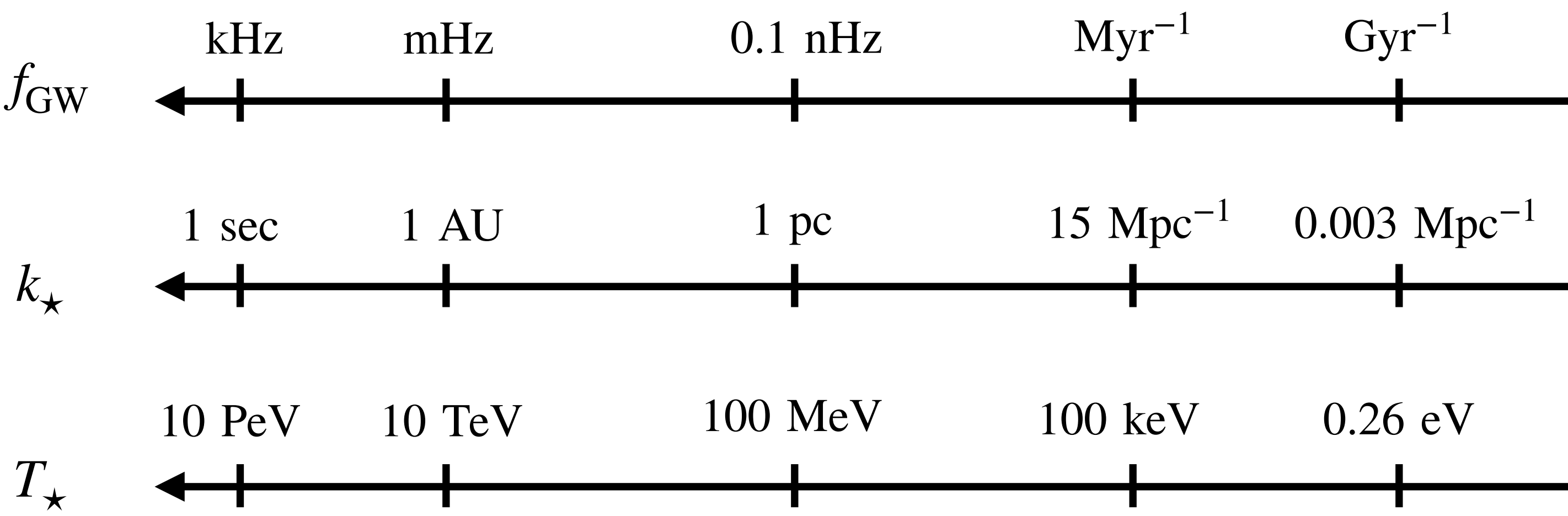
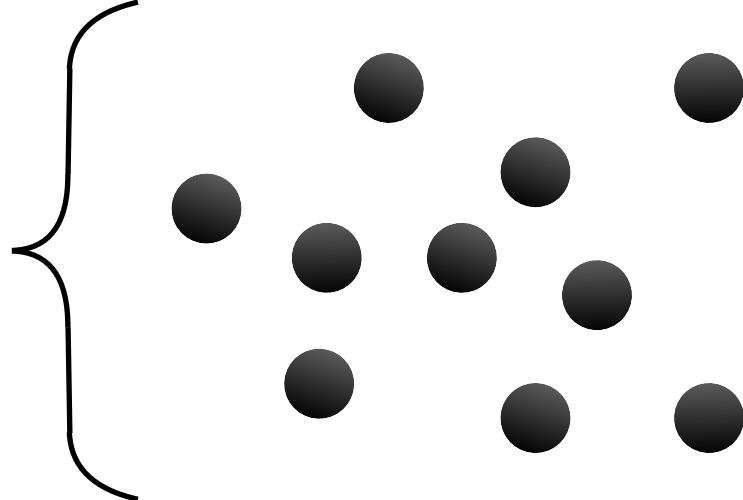


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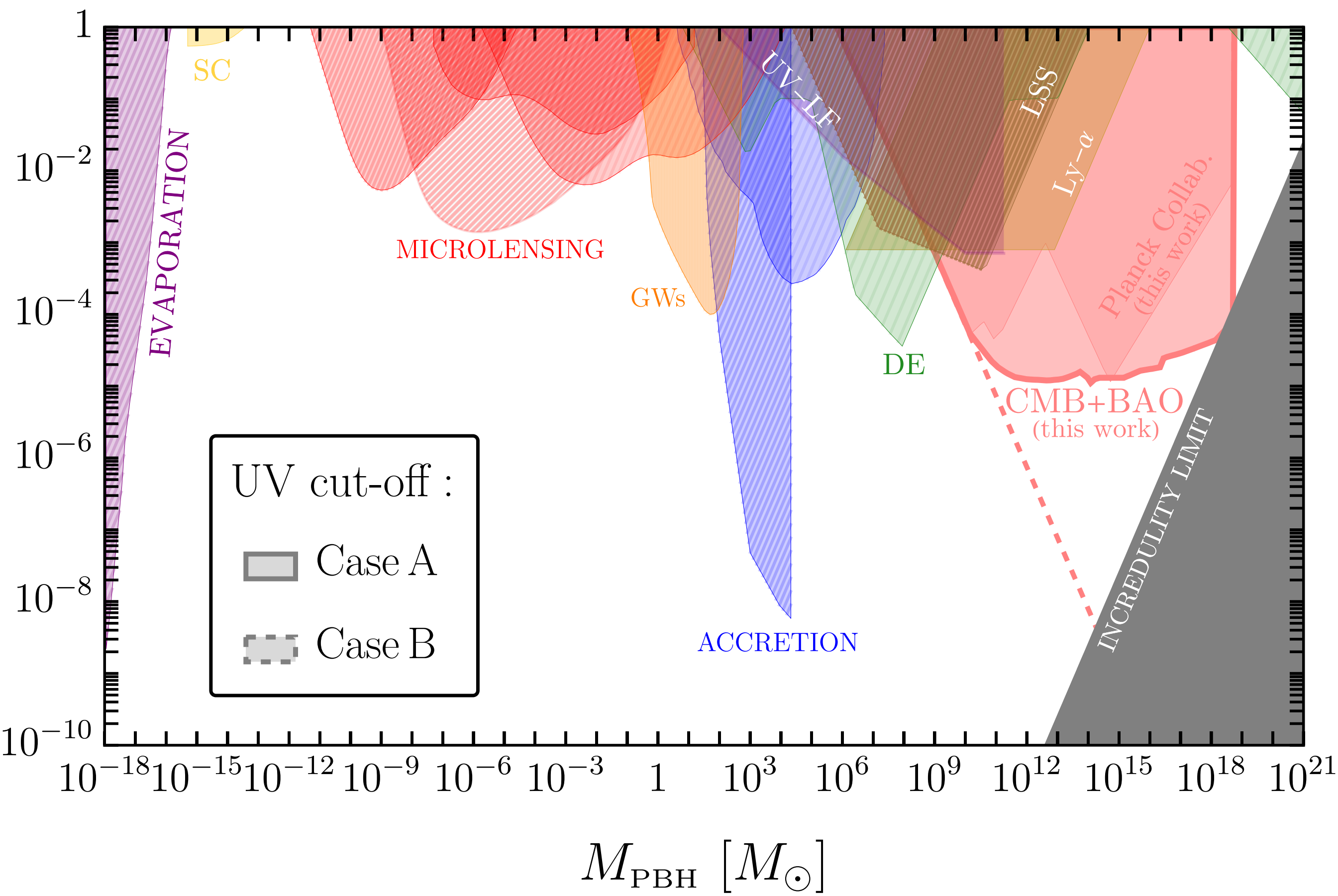
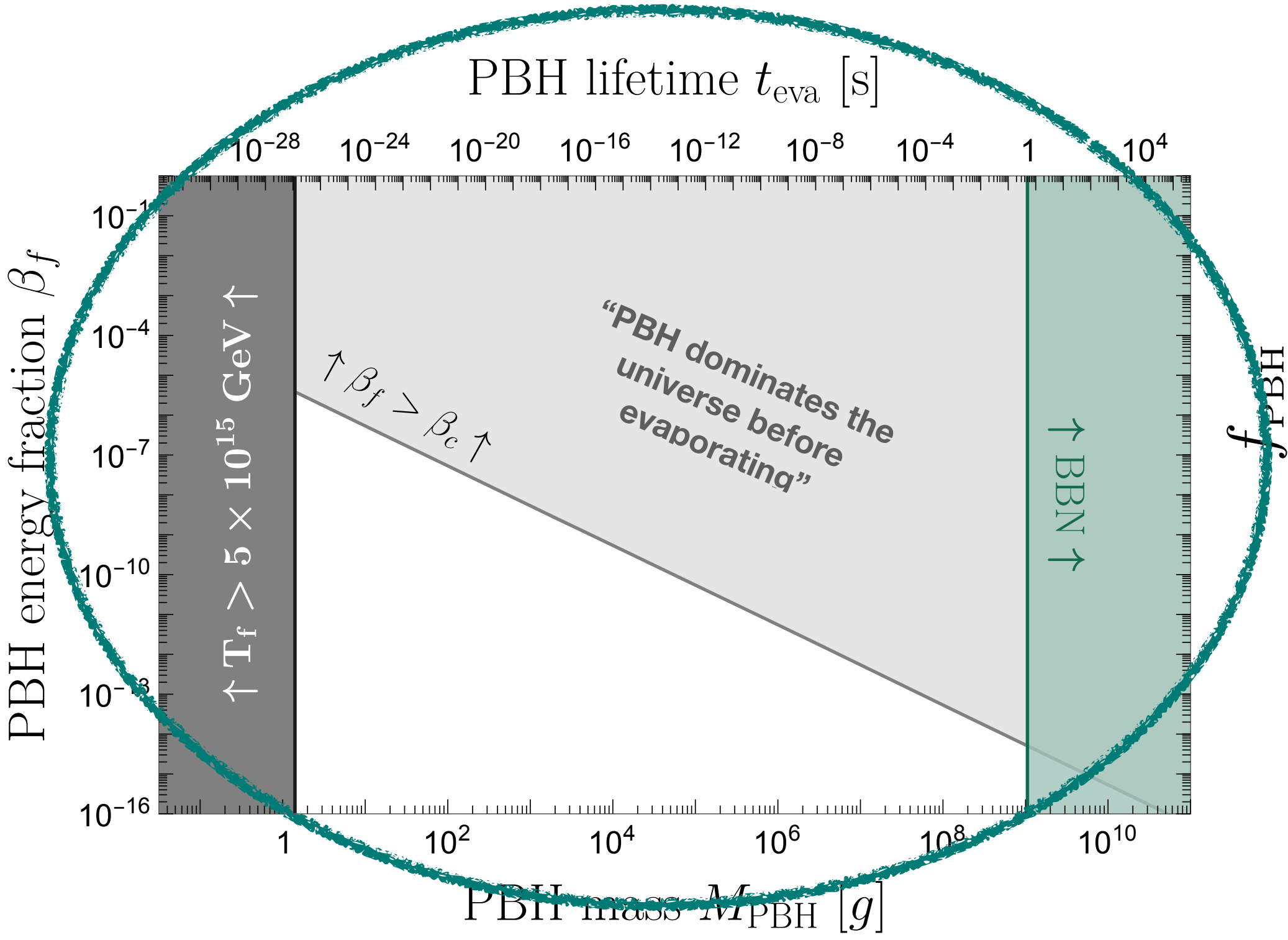


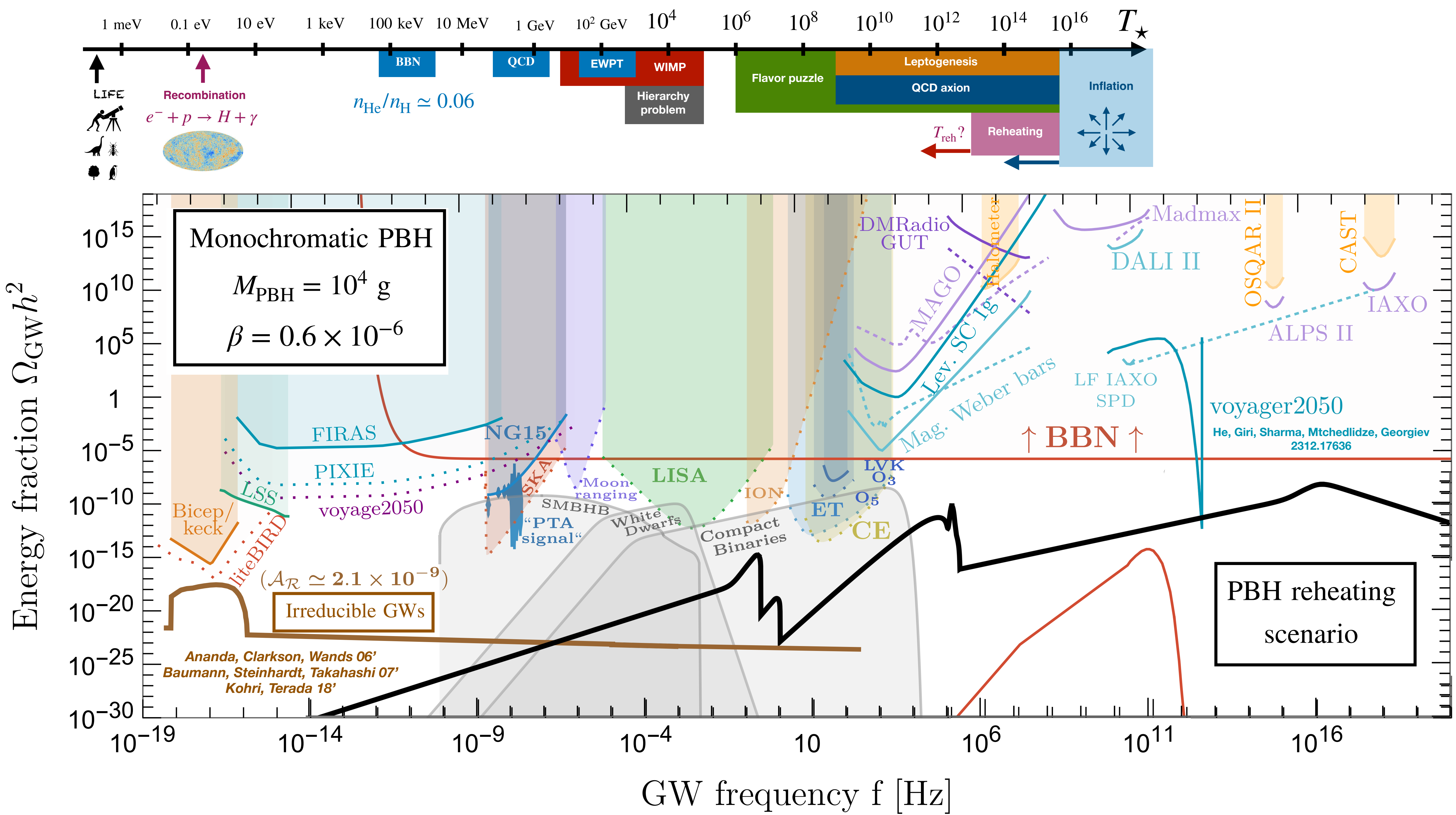
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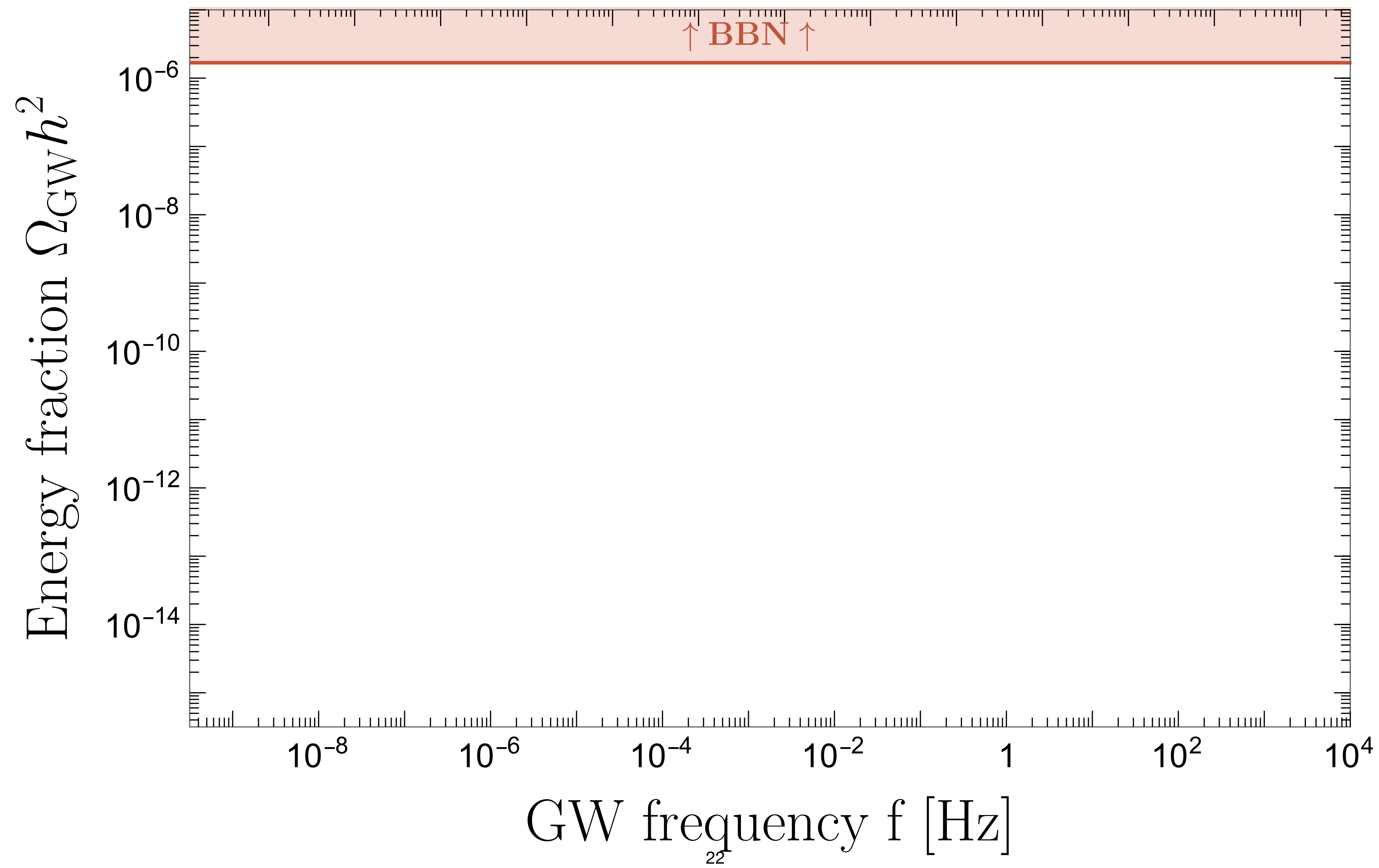
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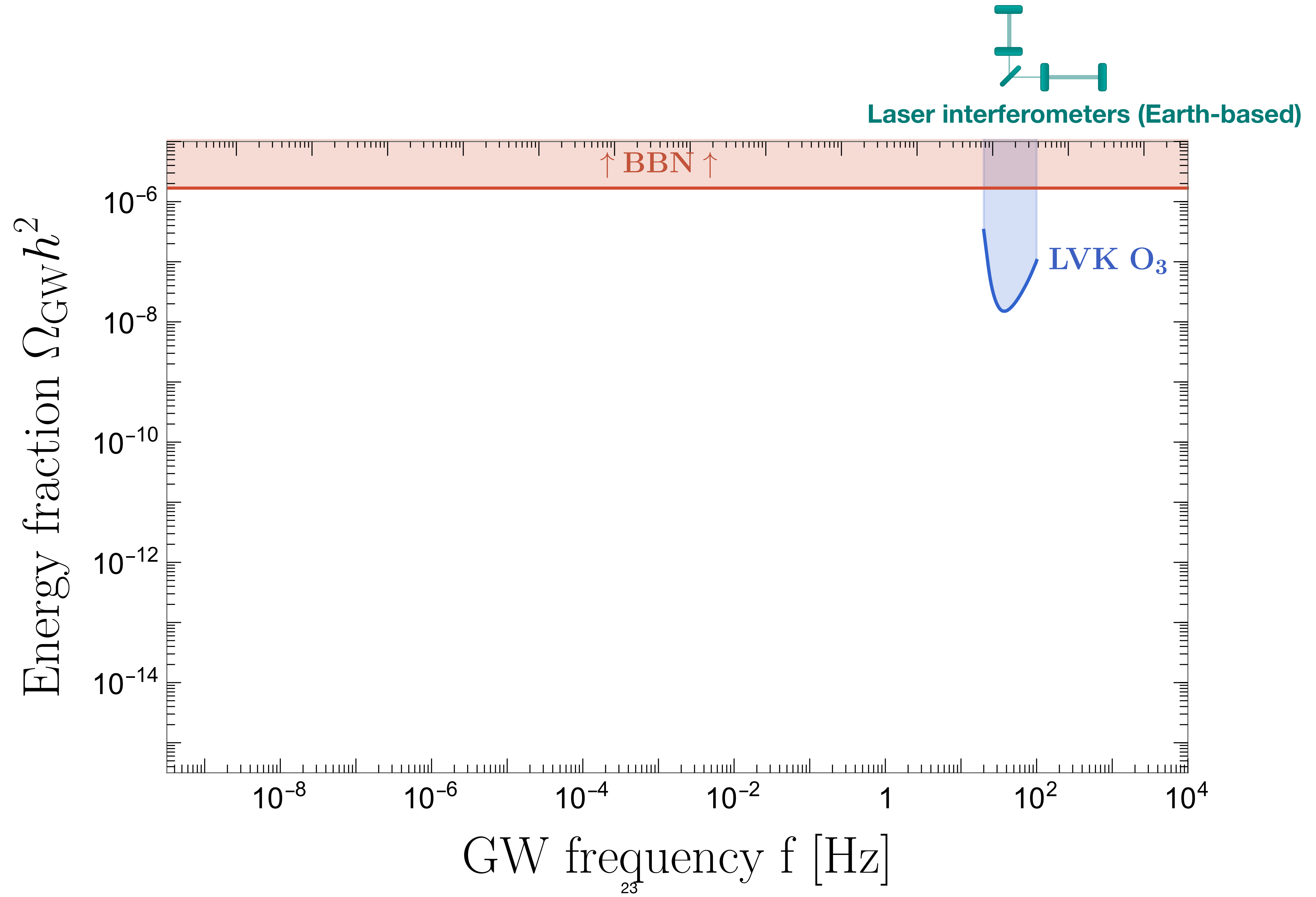


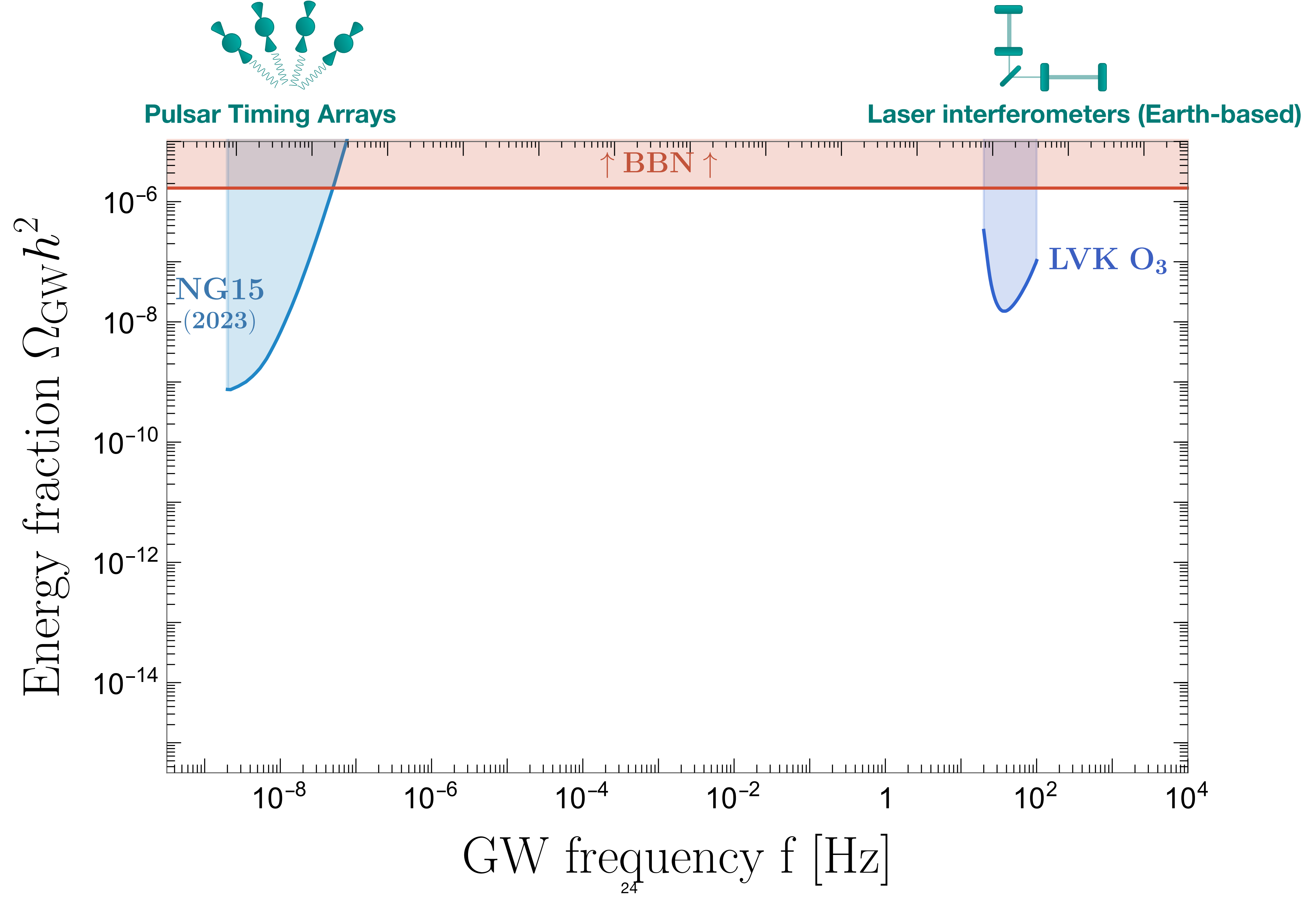
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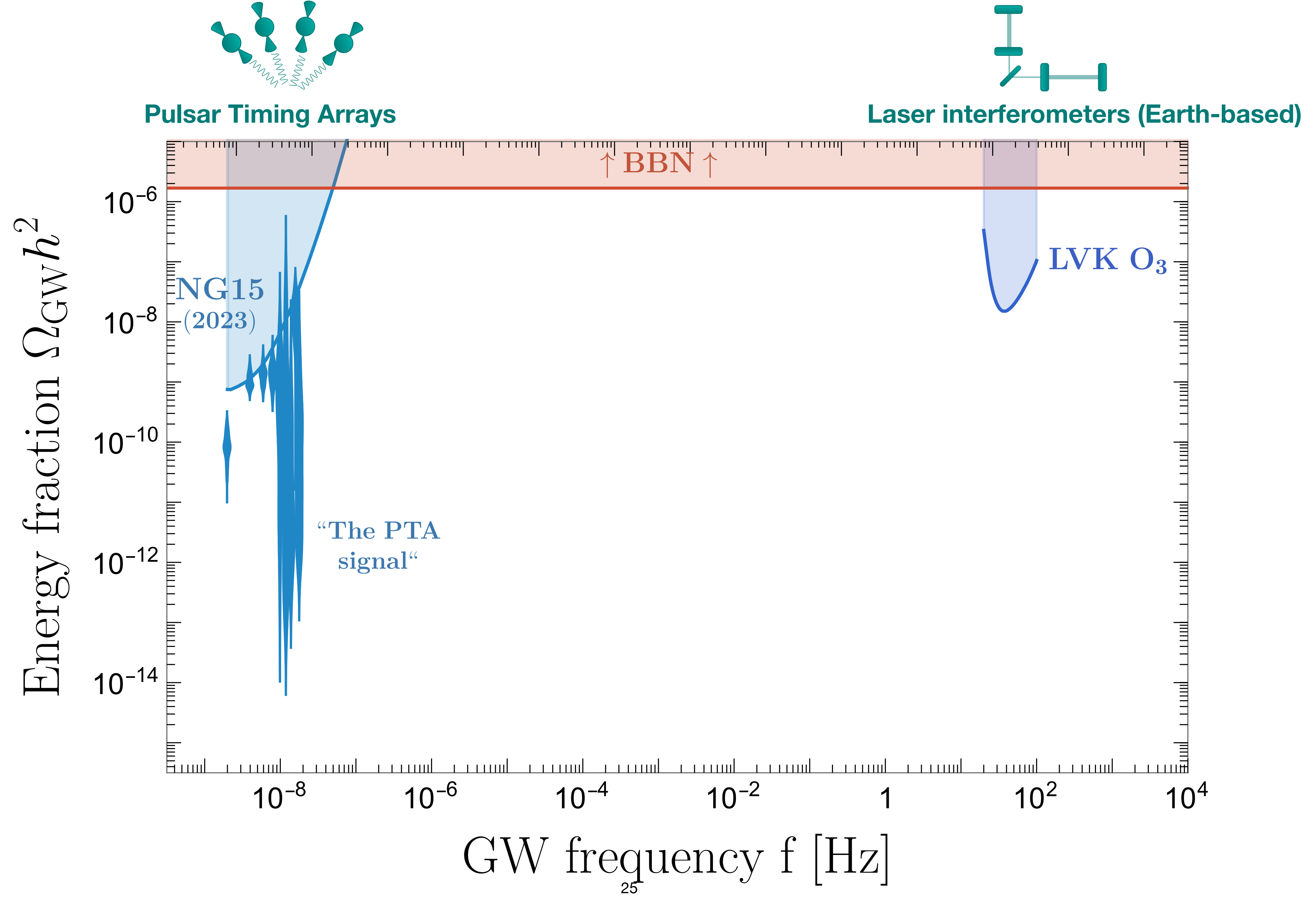


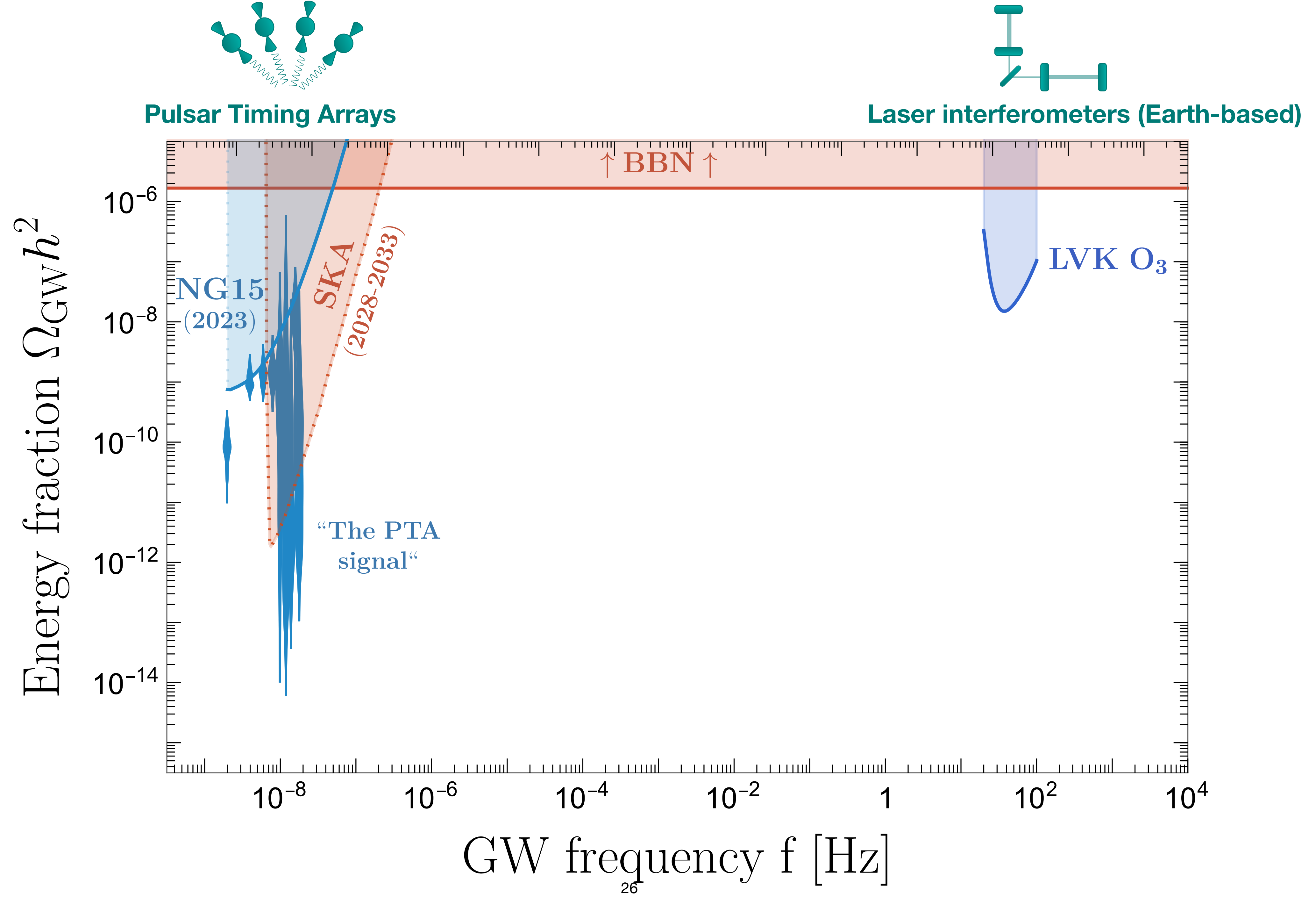


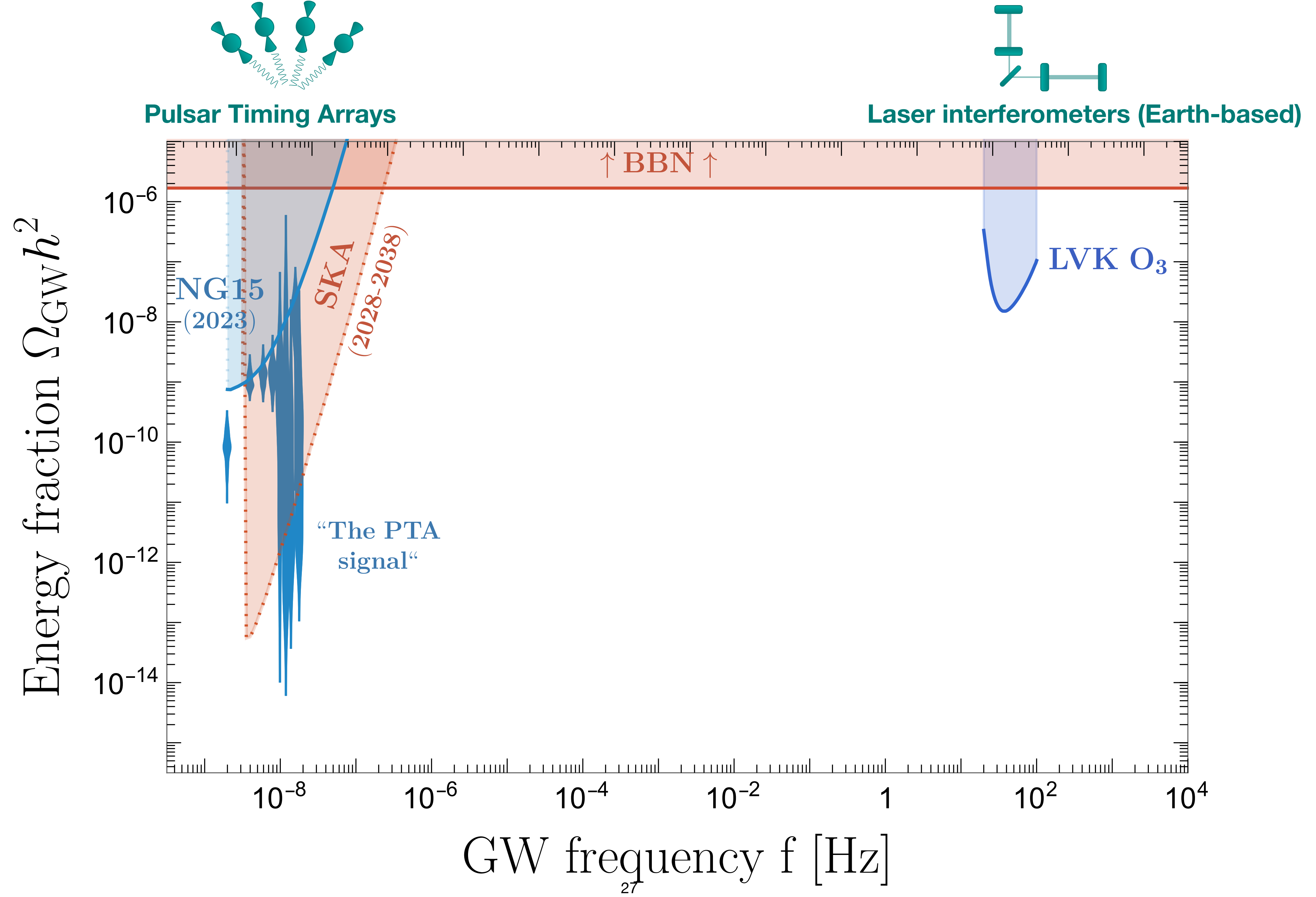


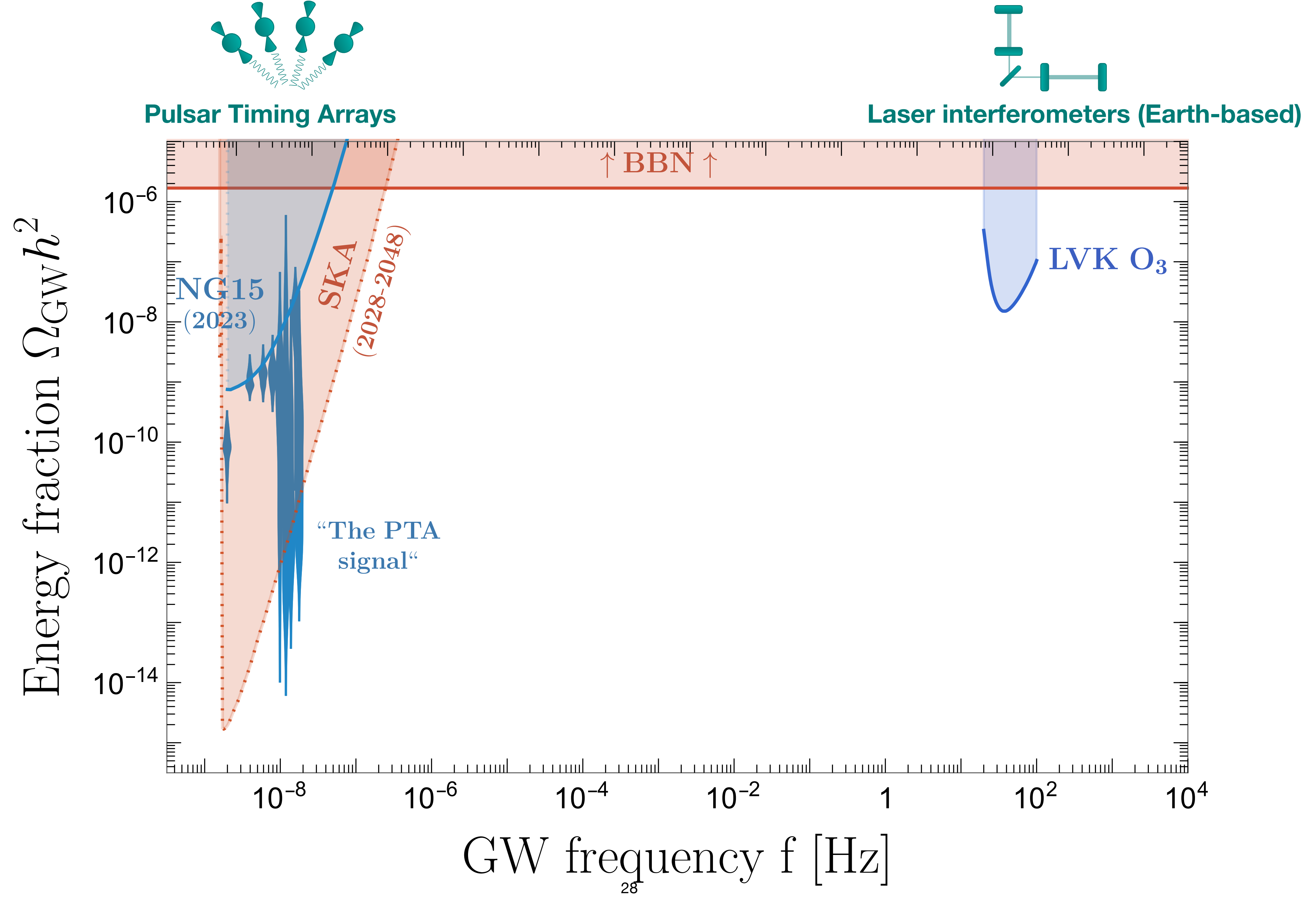


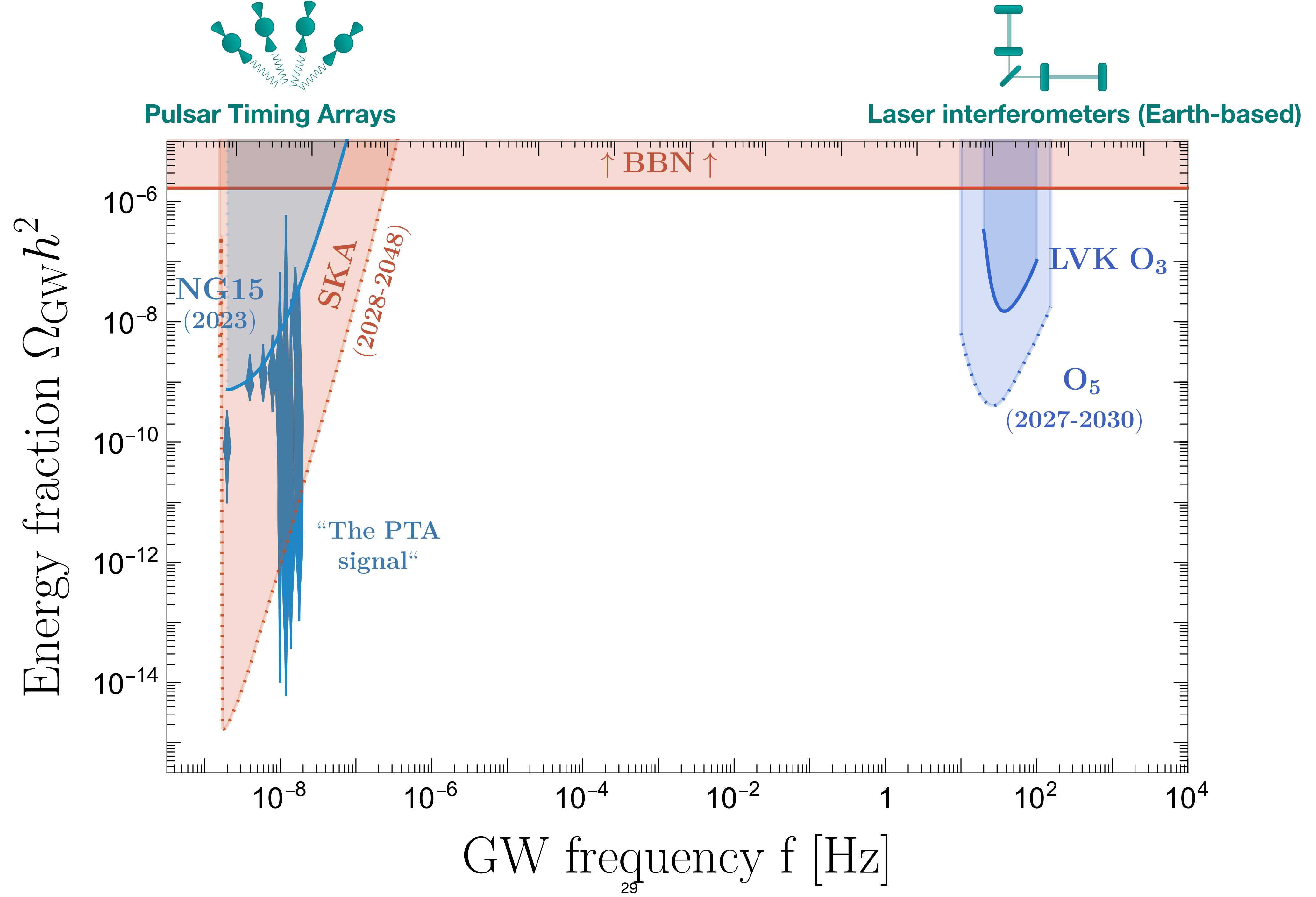


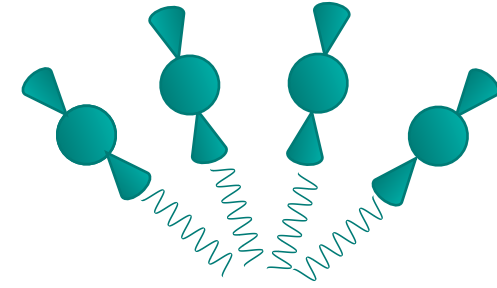




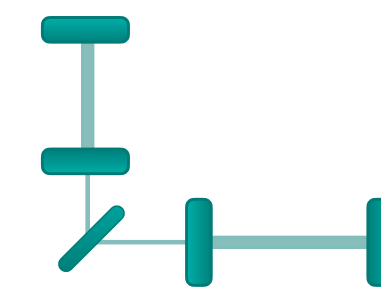




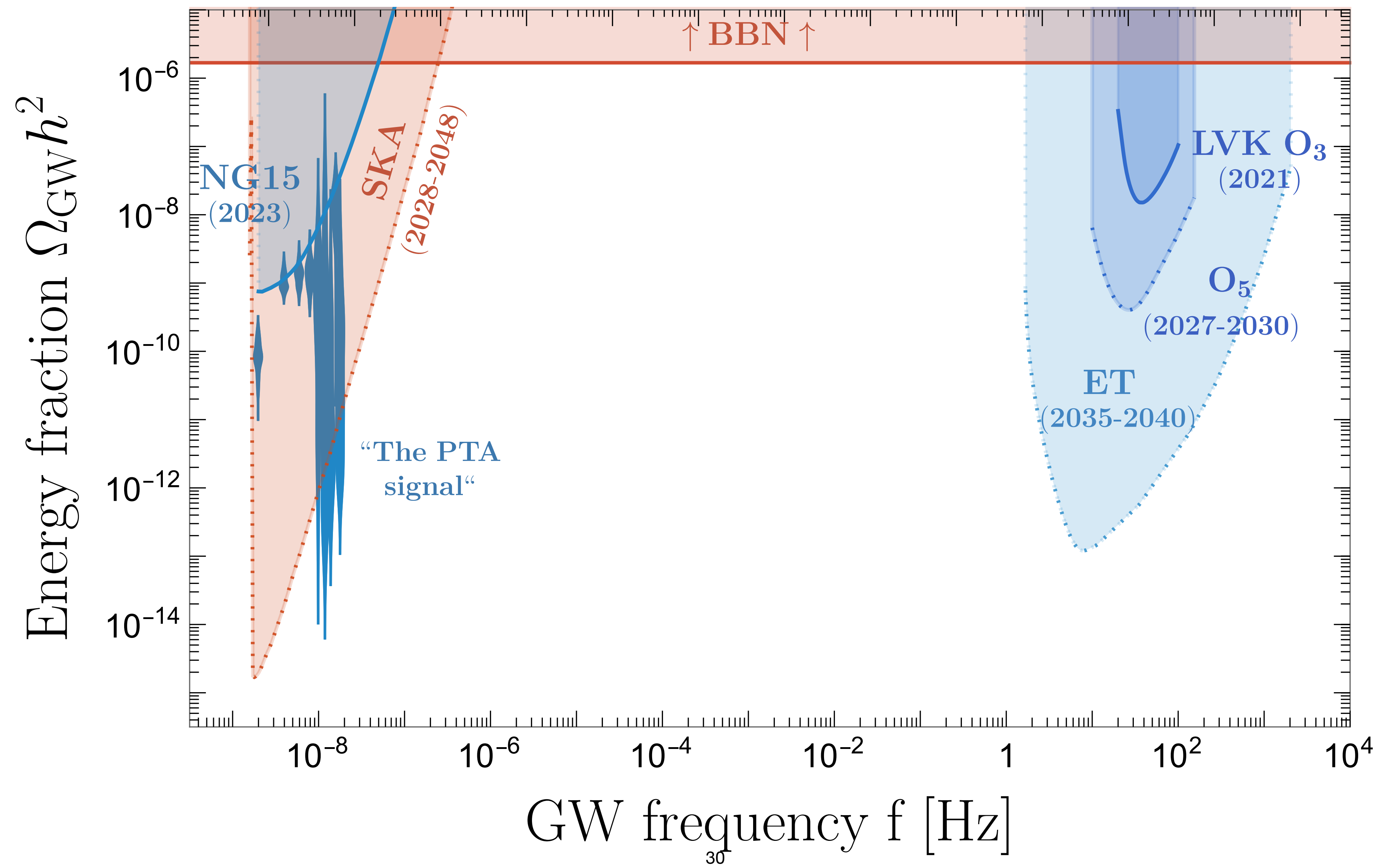


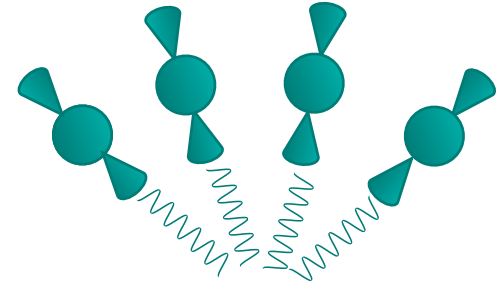


Pulsar Timing Arrays

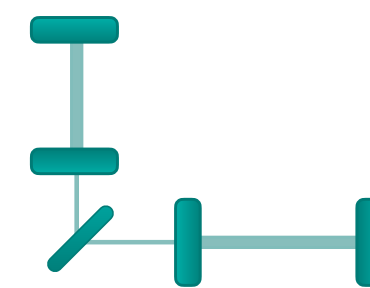


Laser interferometers (Earth-based)

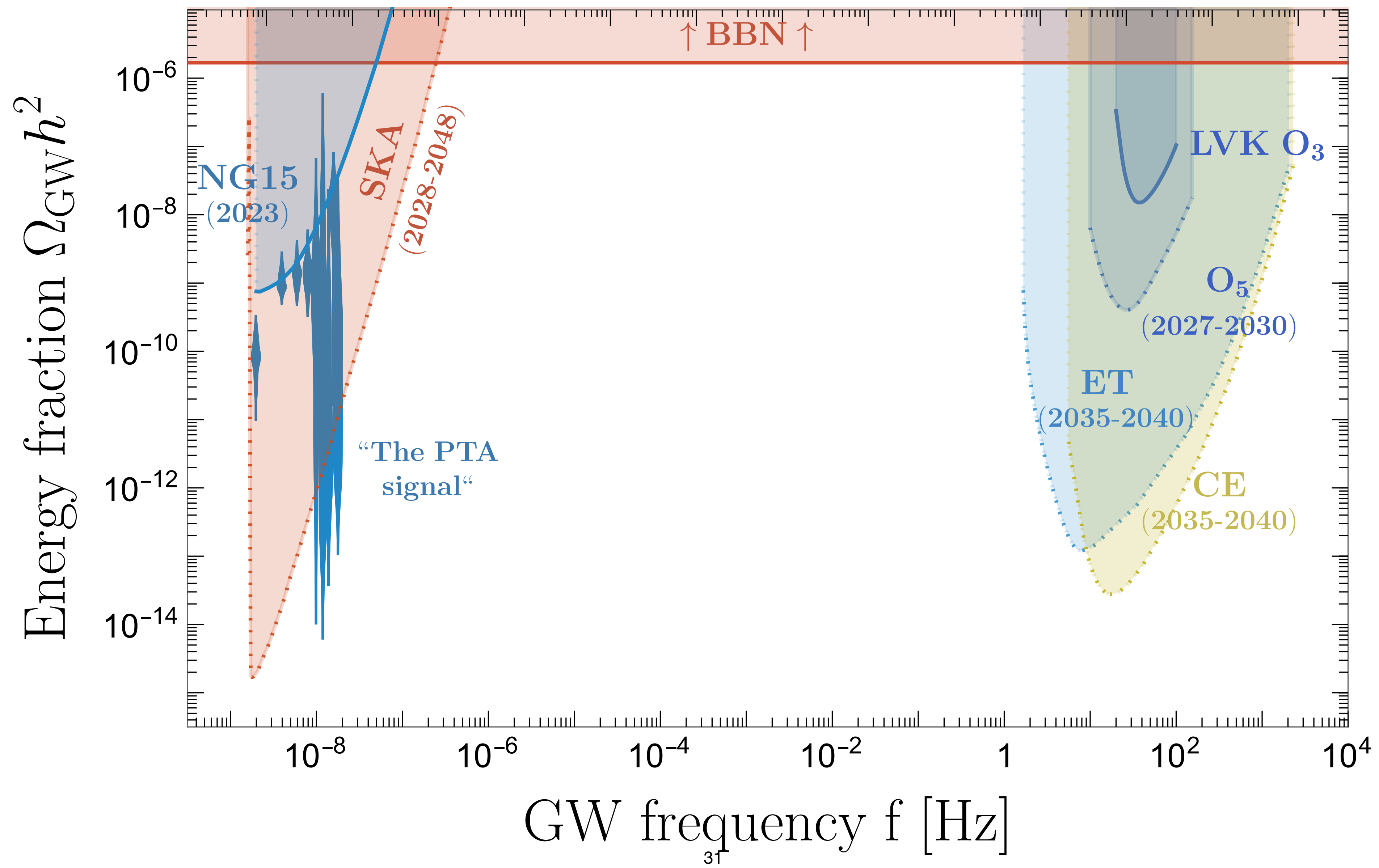


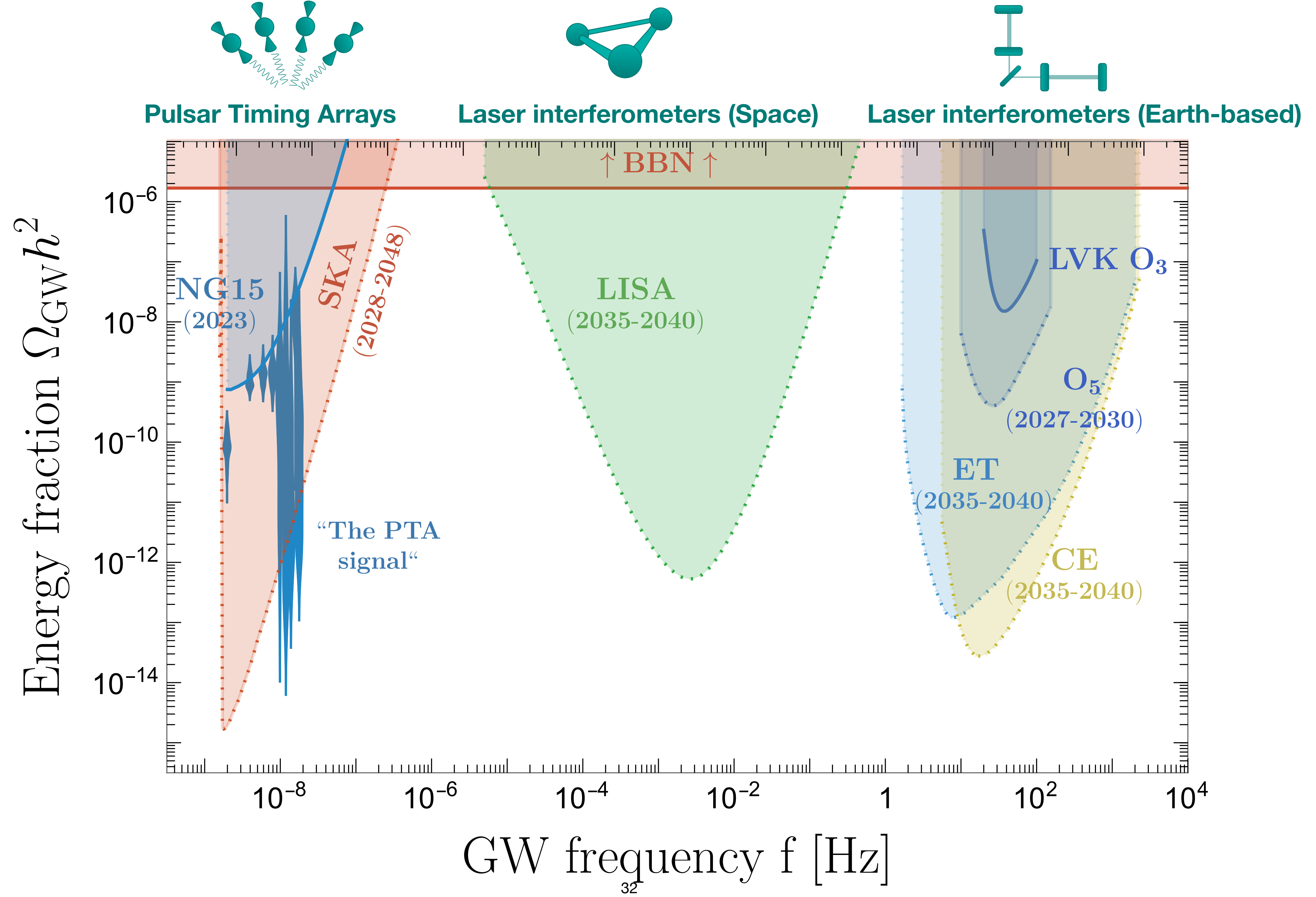


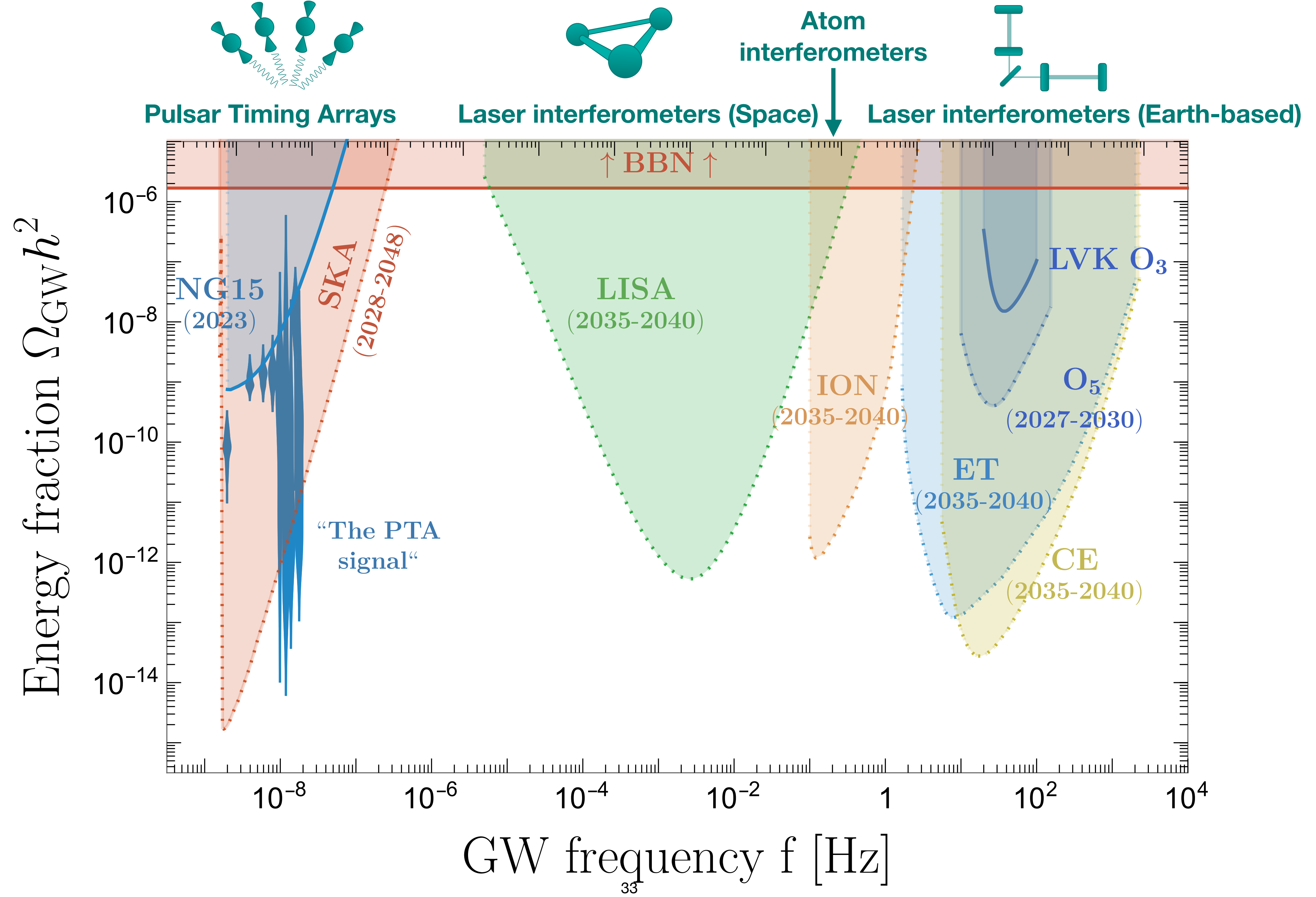
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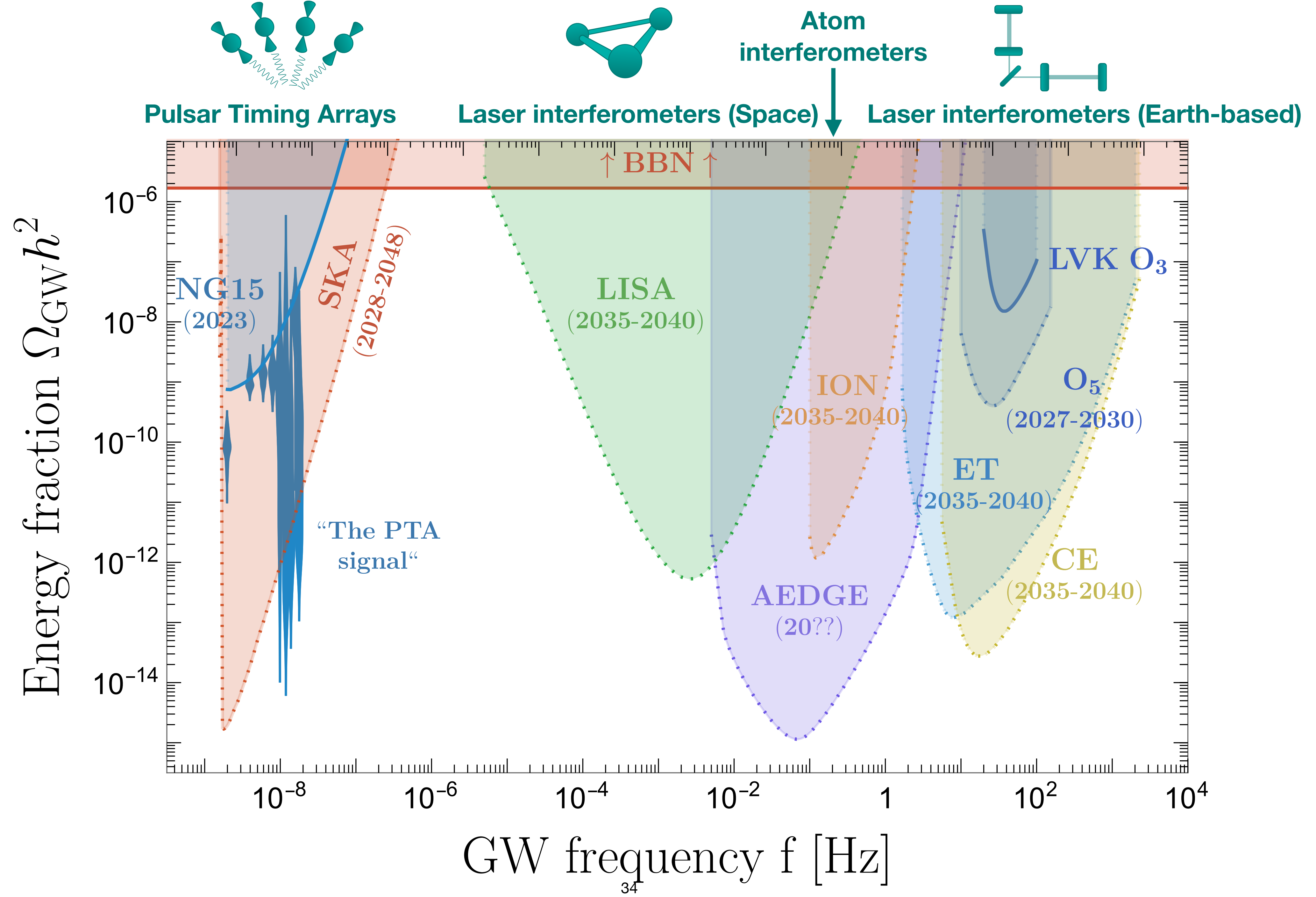


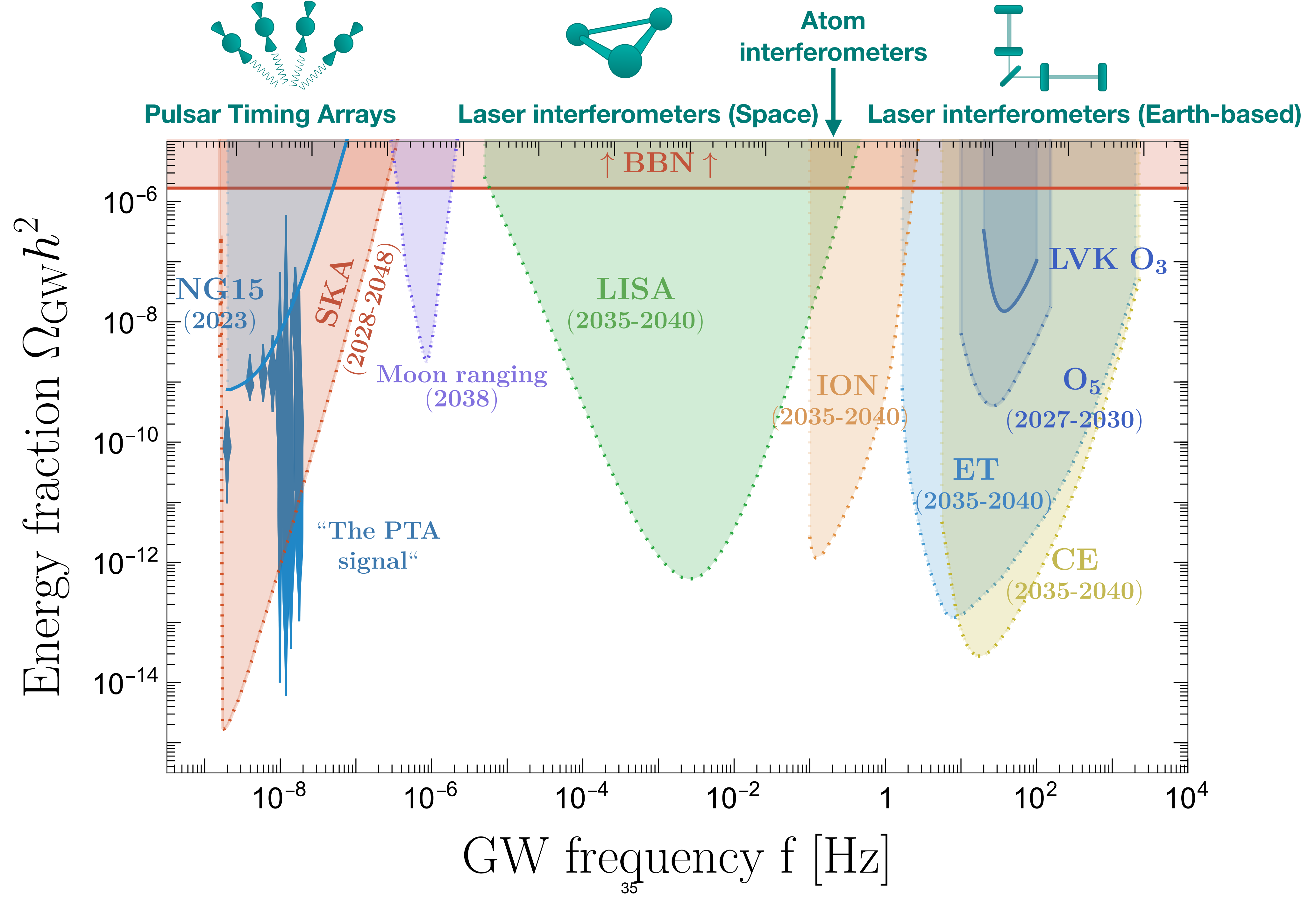
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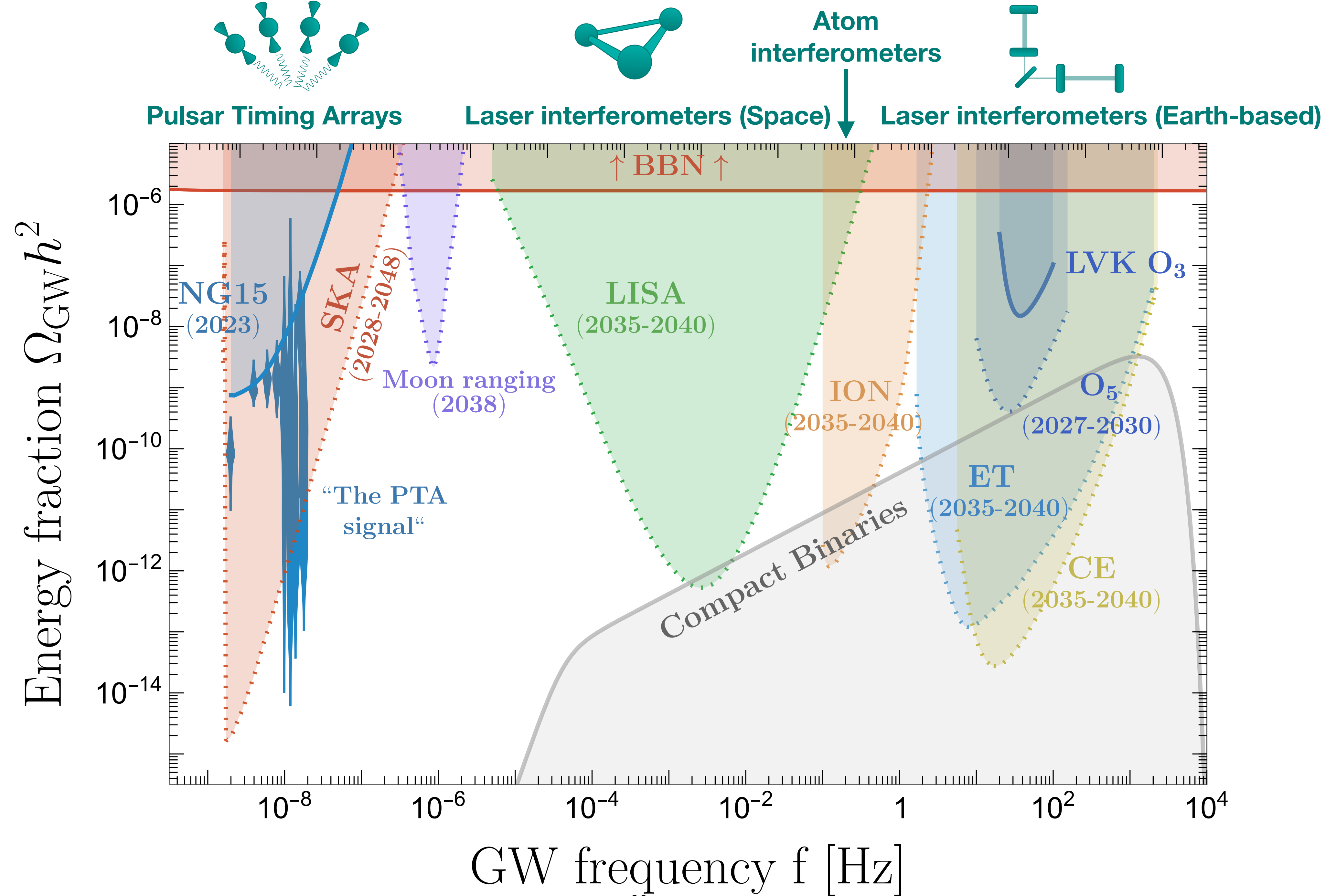


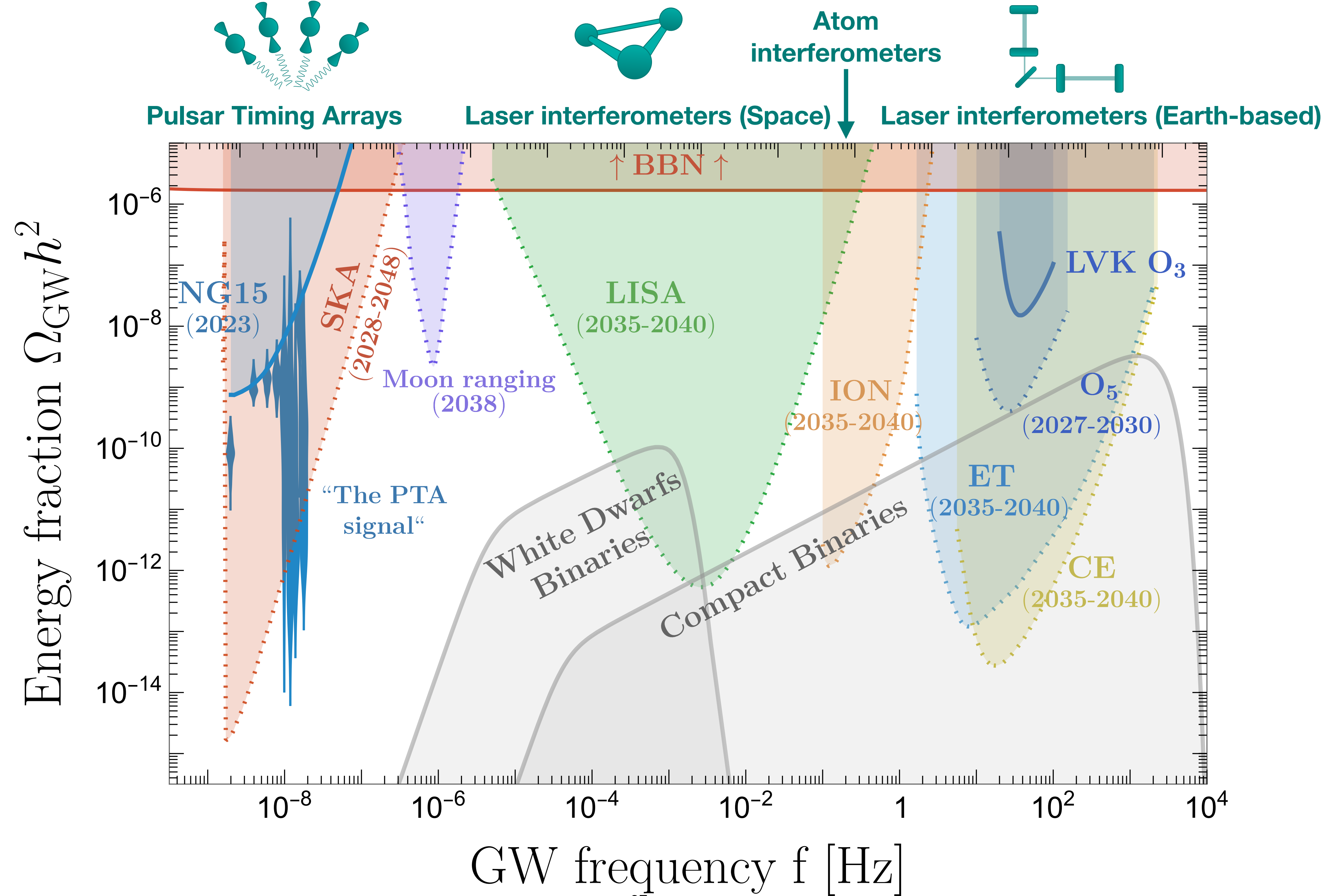


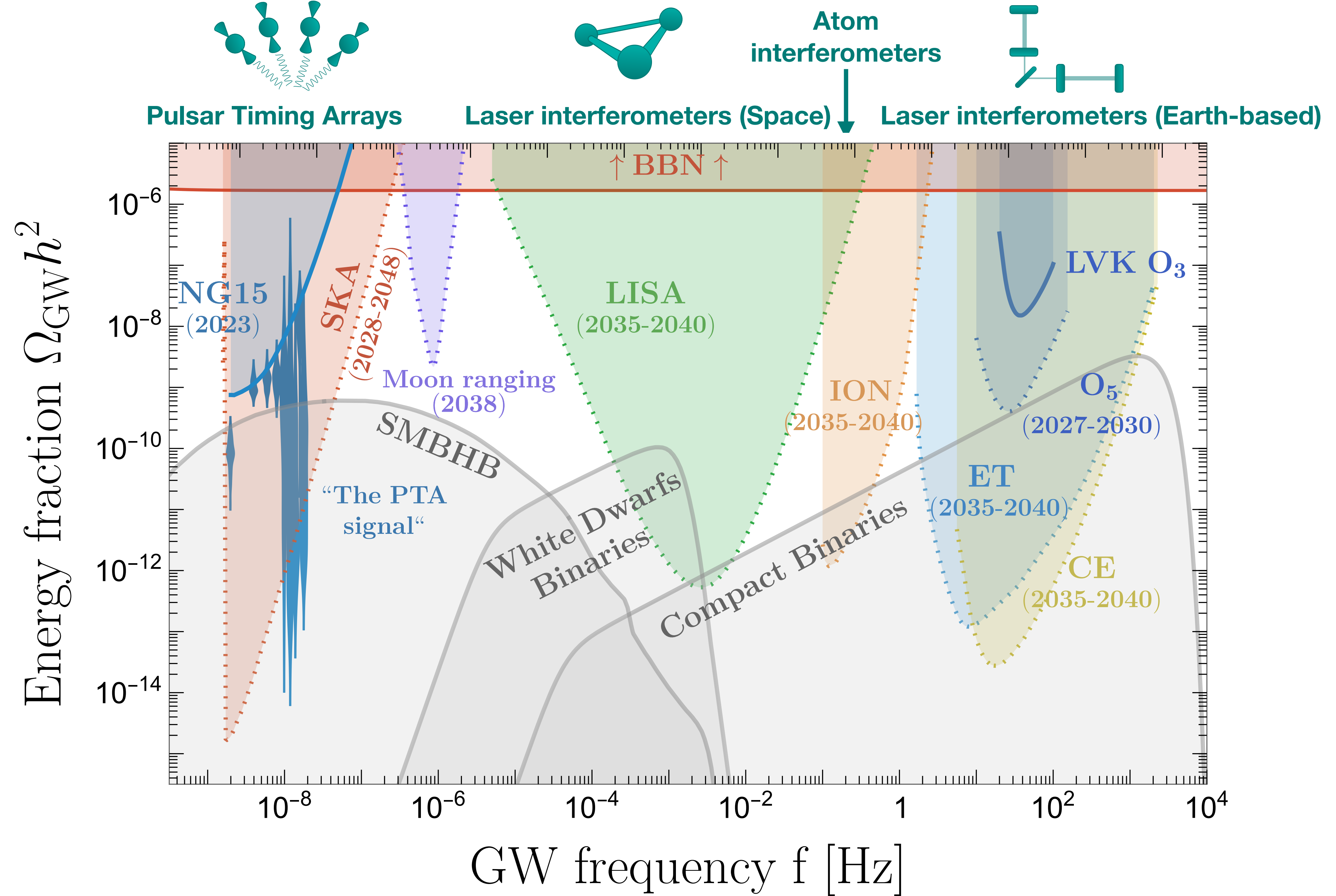


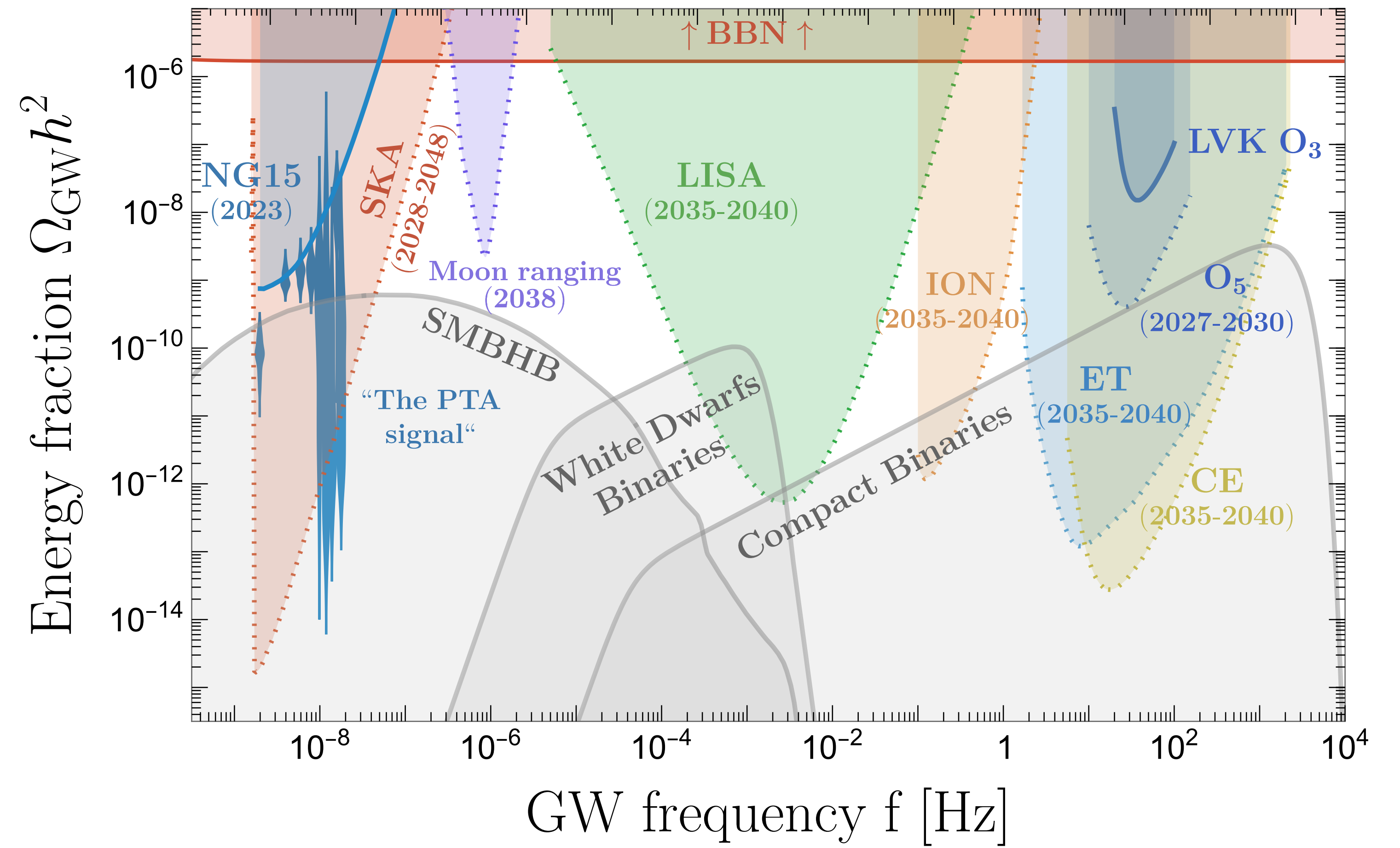
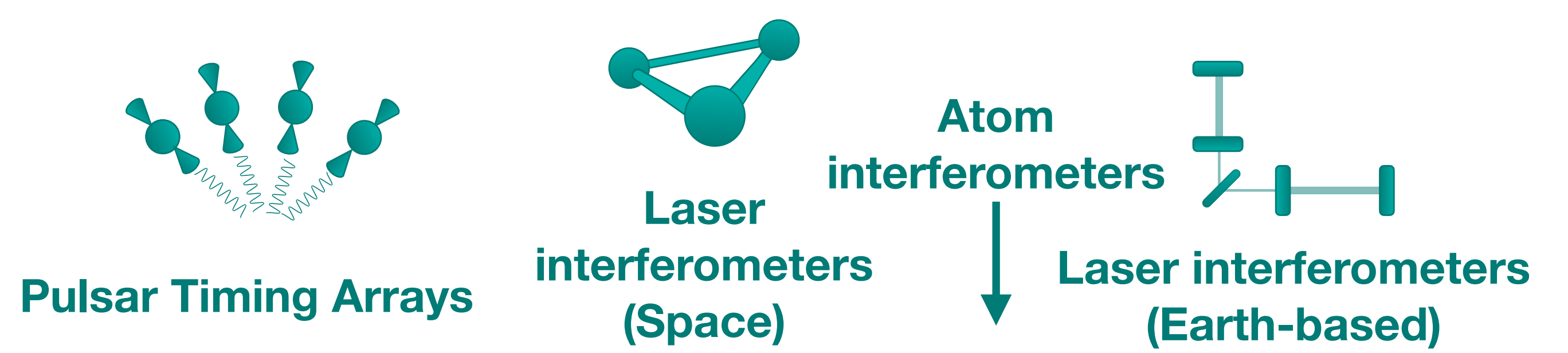


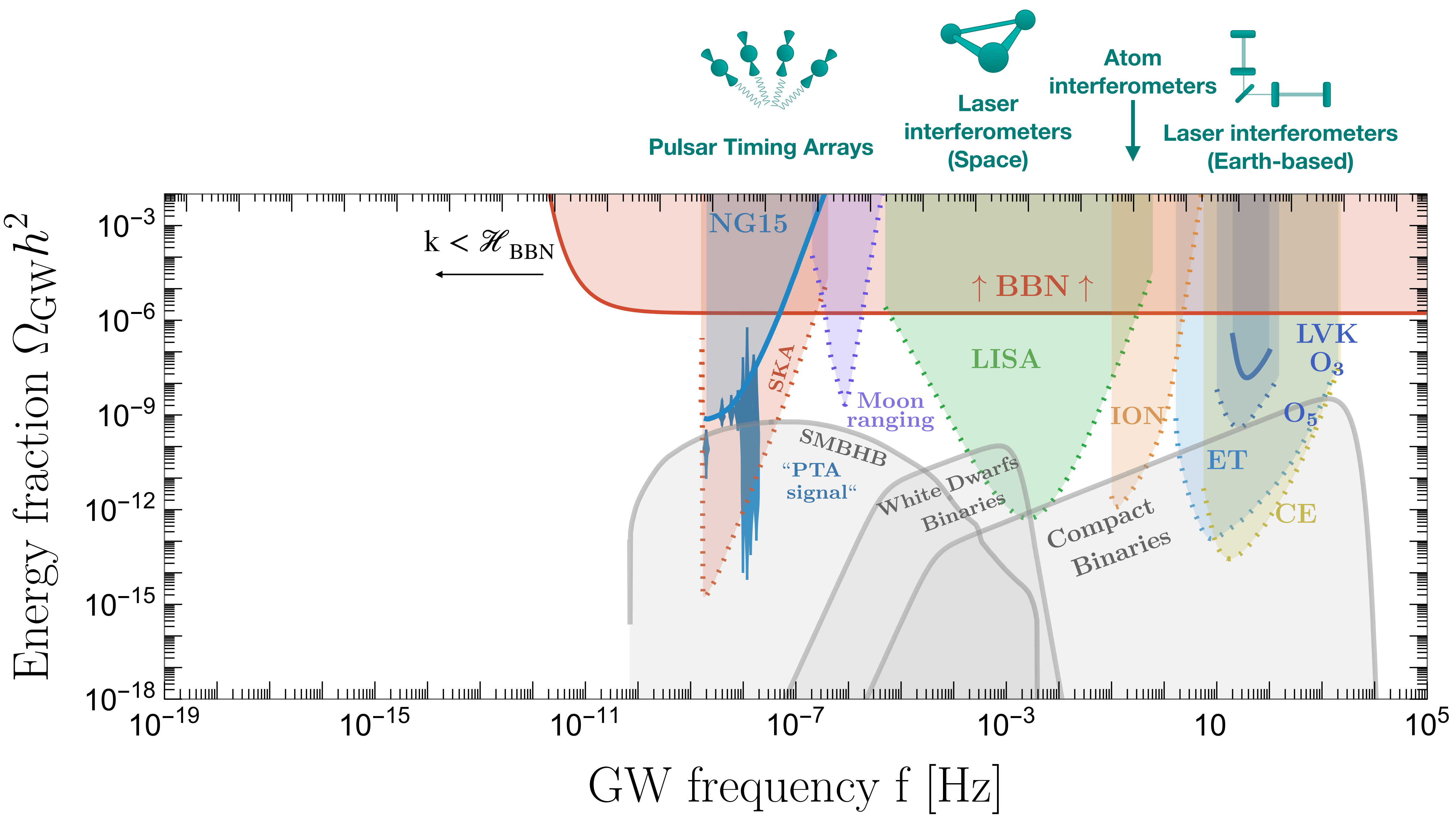


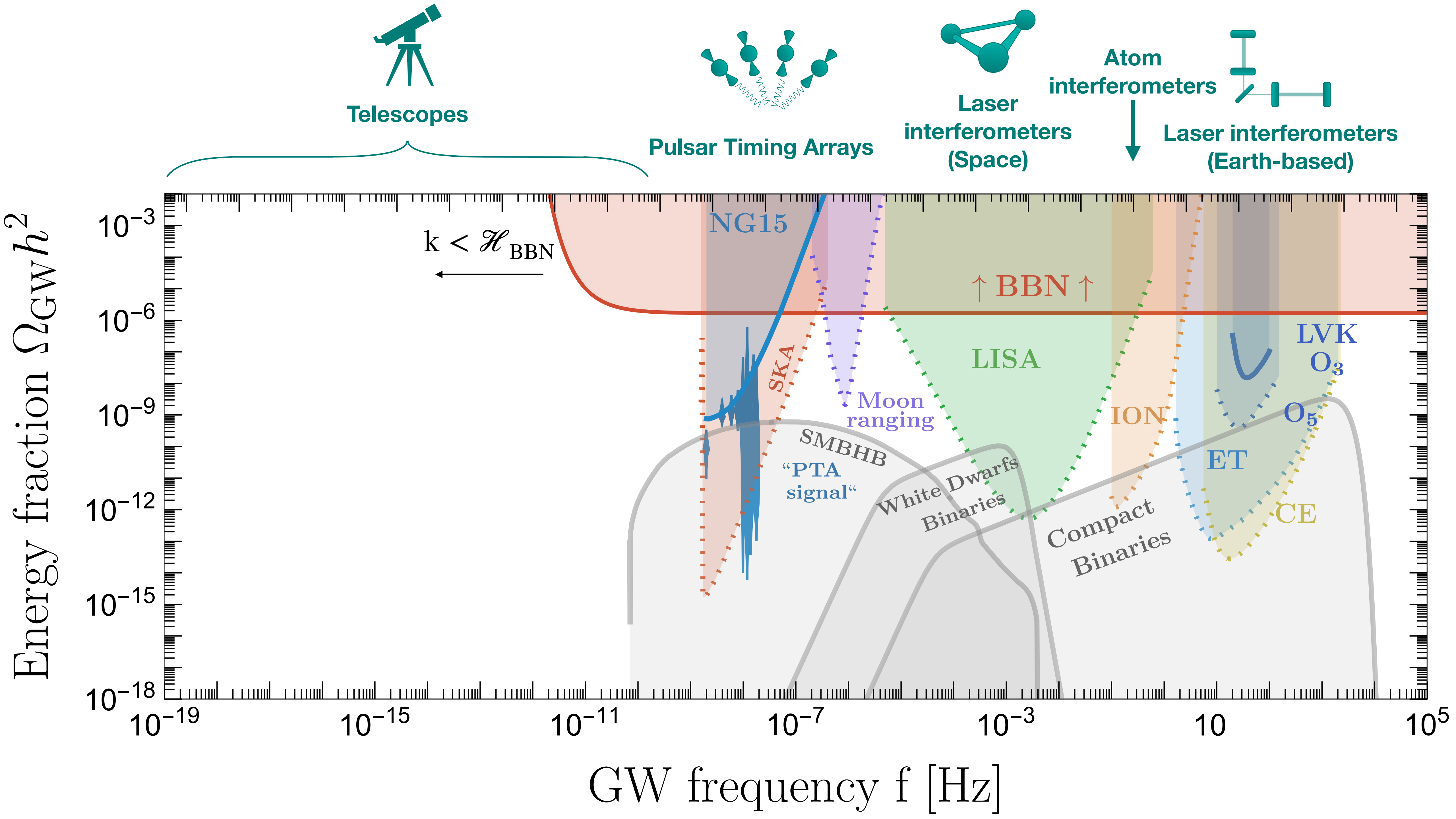


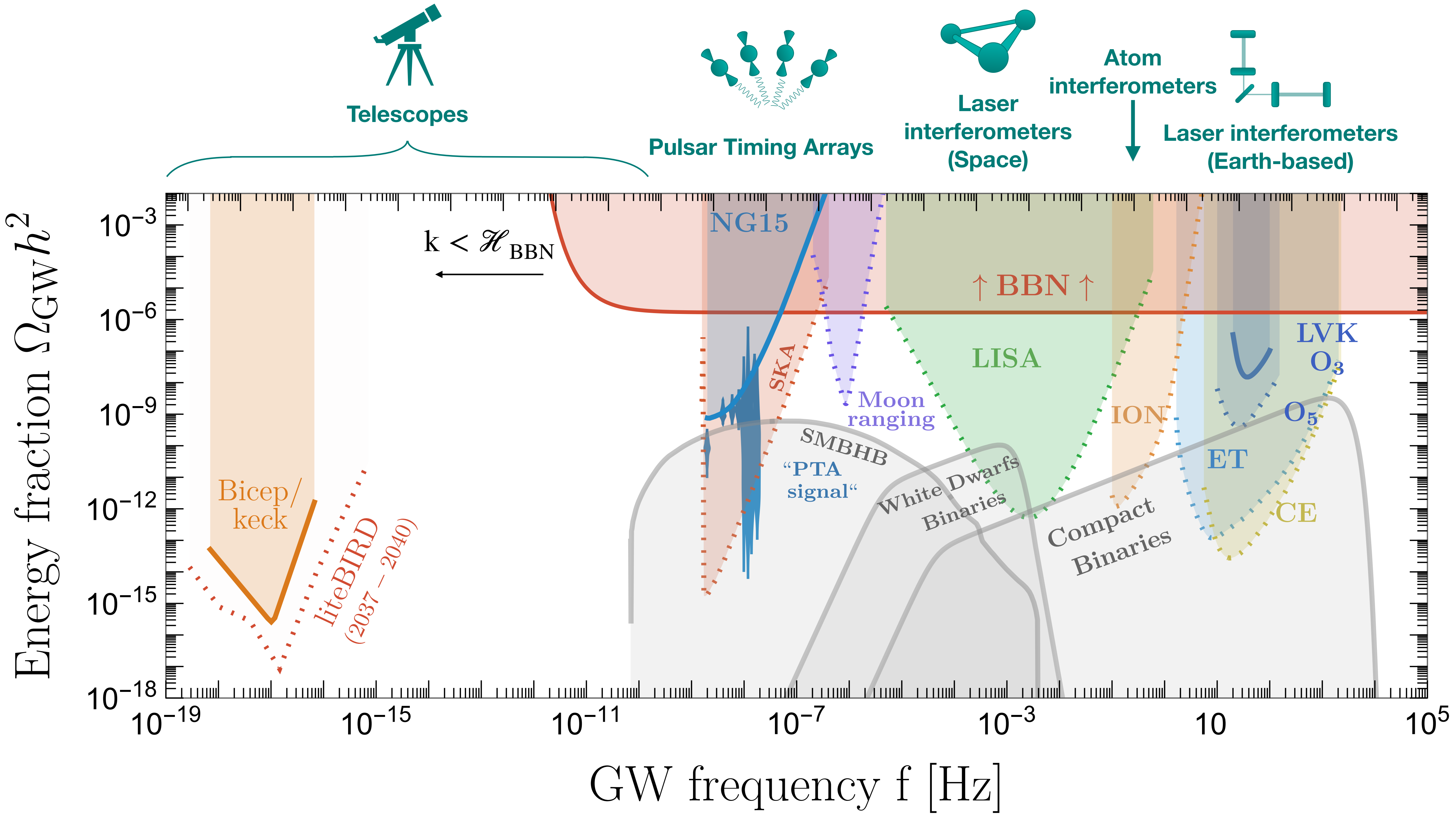


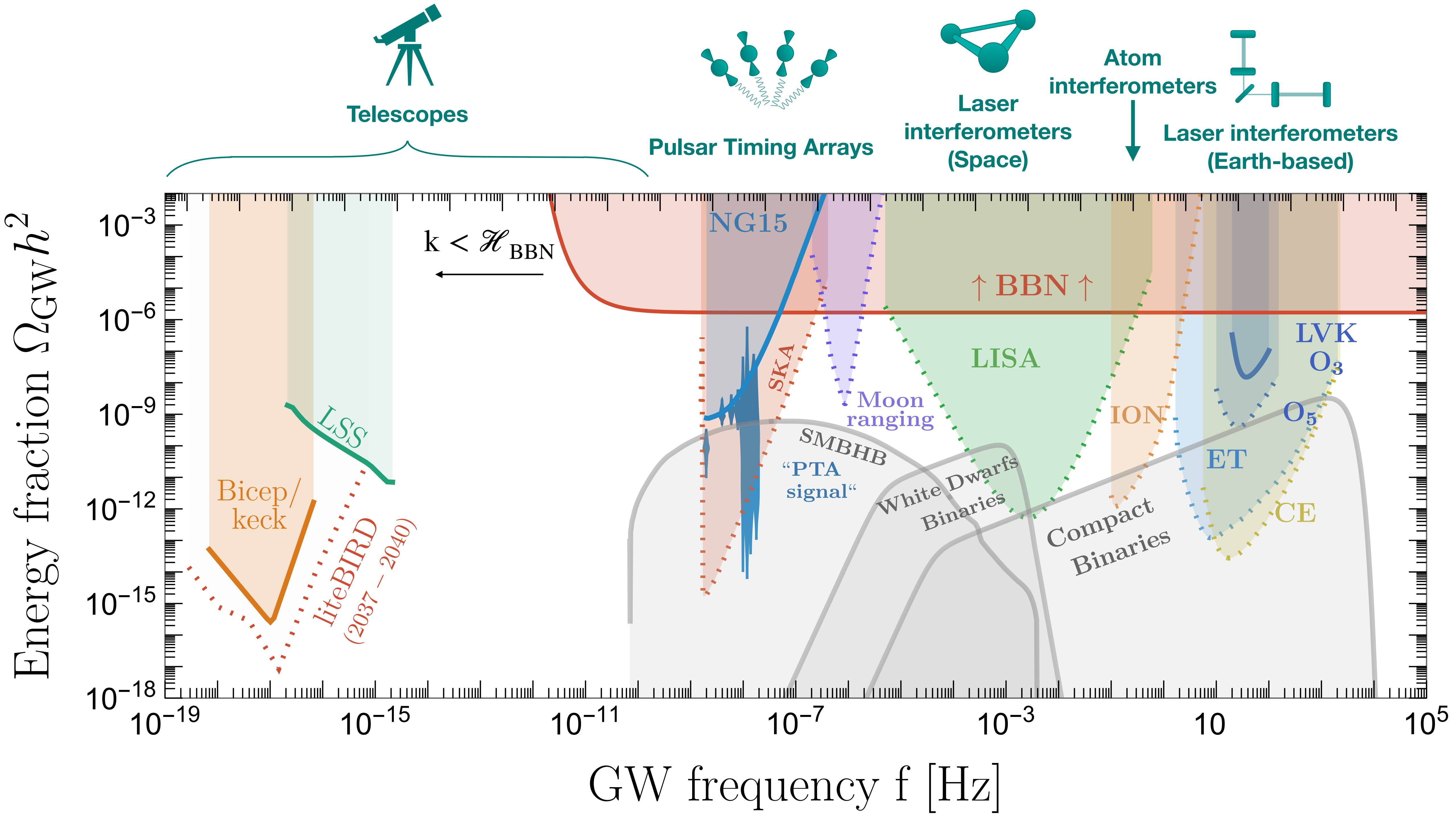


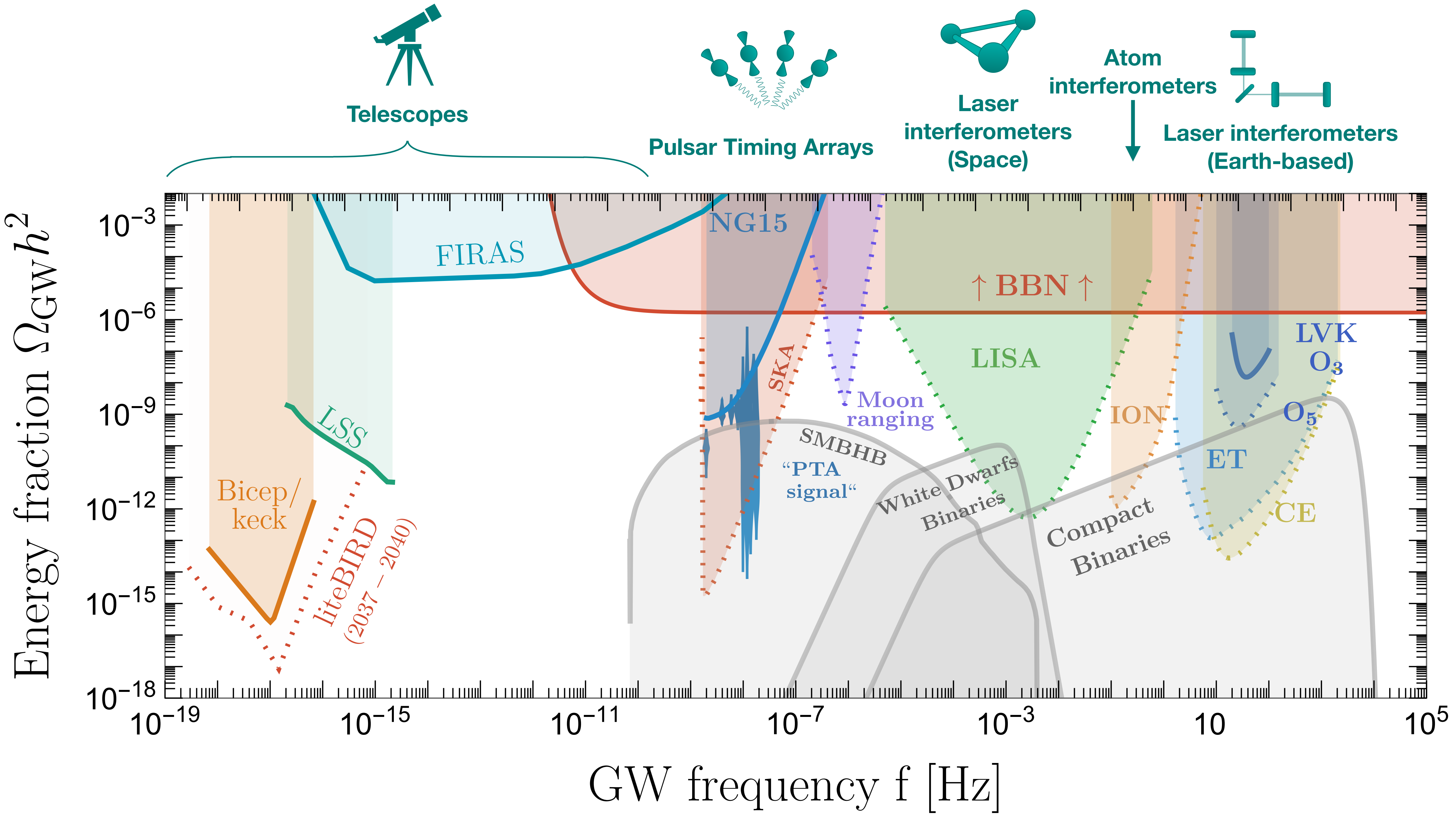


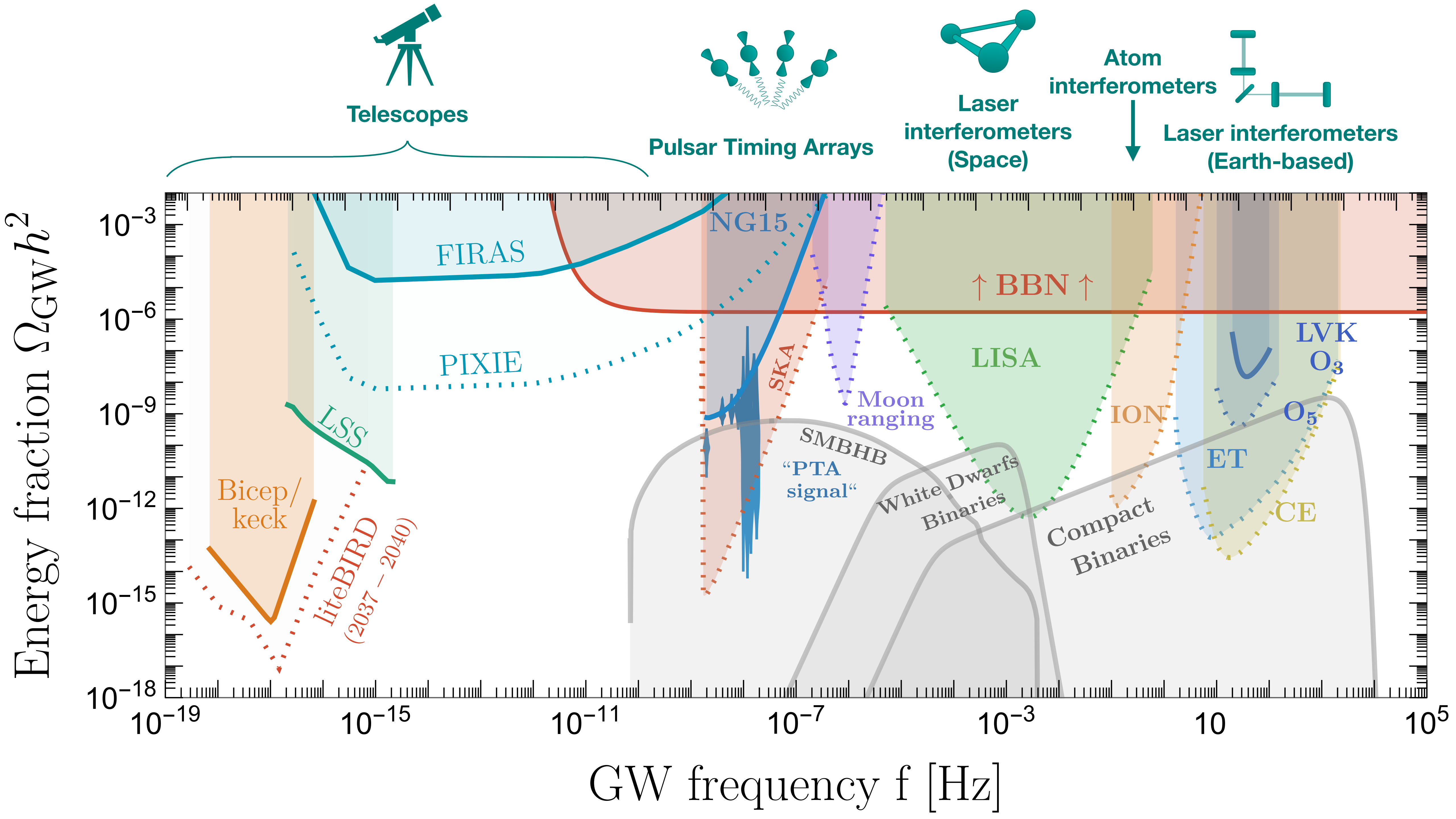


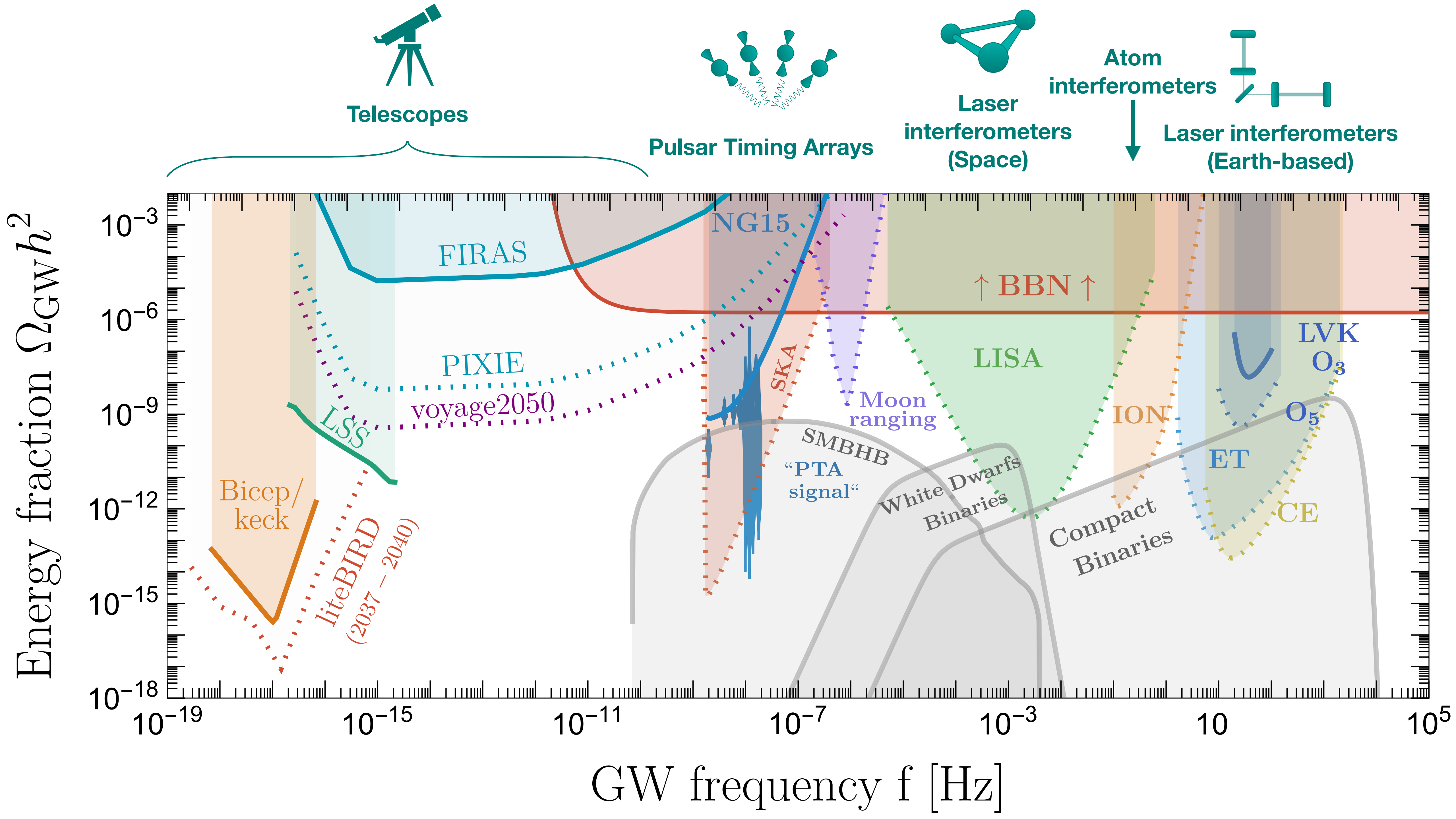


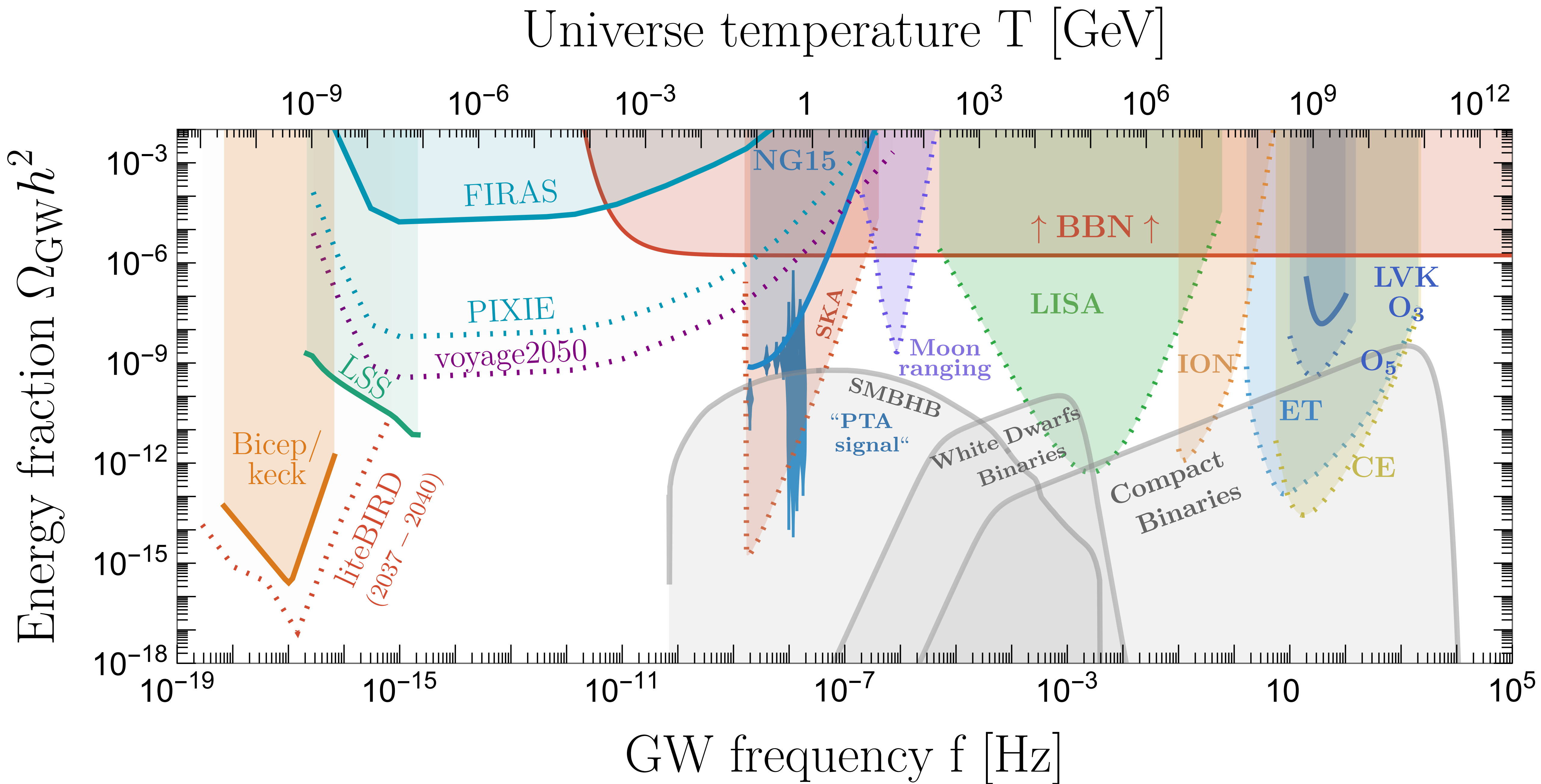




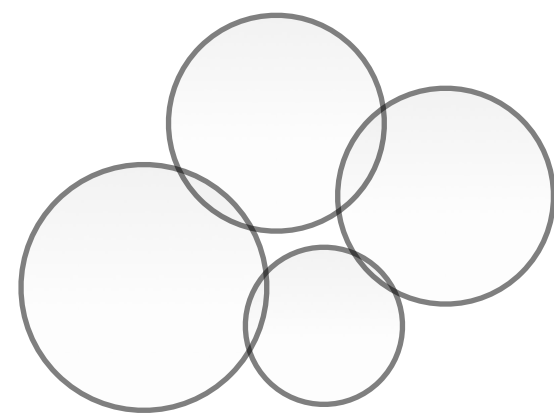






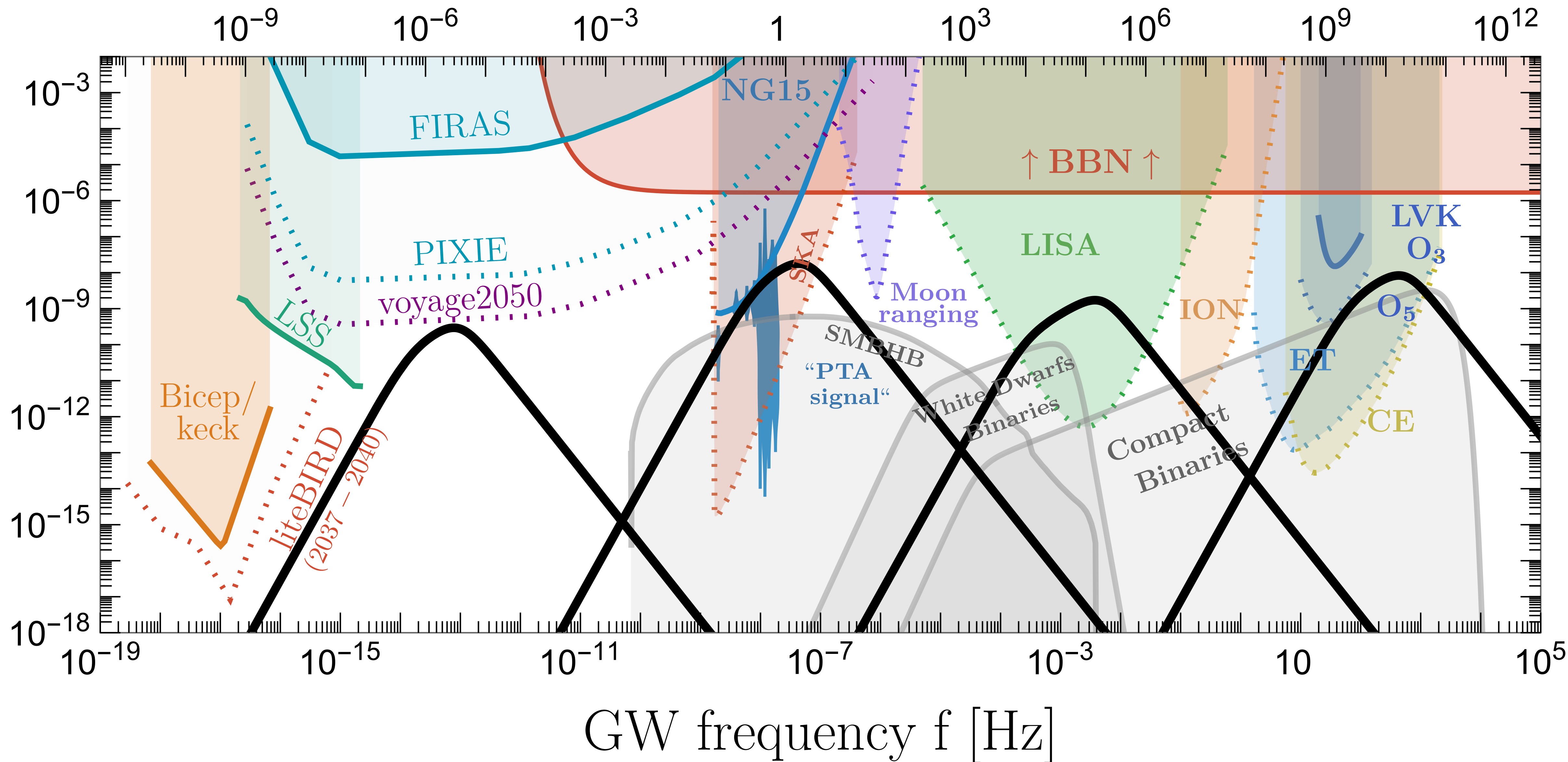


**First-order
phase transition**

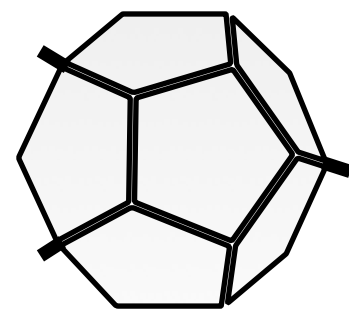


Universe temperature T [GeV]

Energy fraction $\Omega_{\text{GW}} h^2$

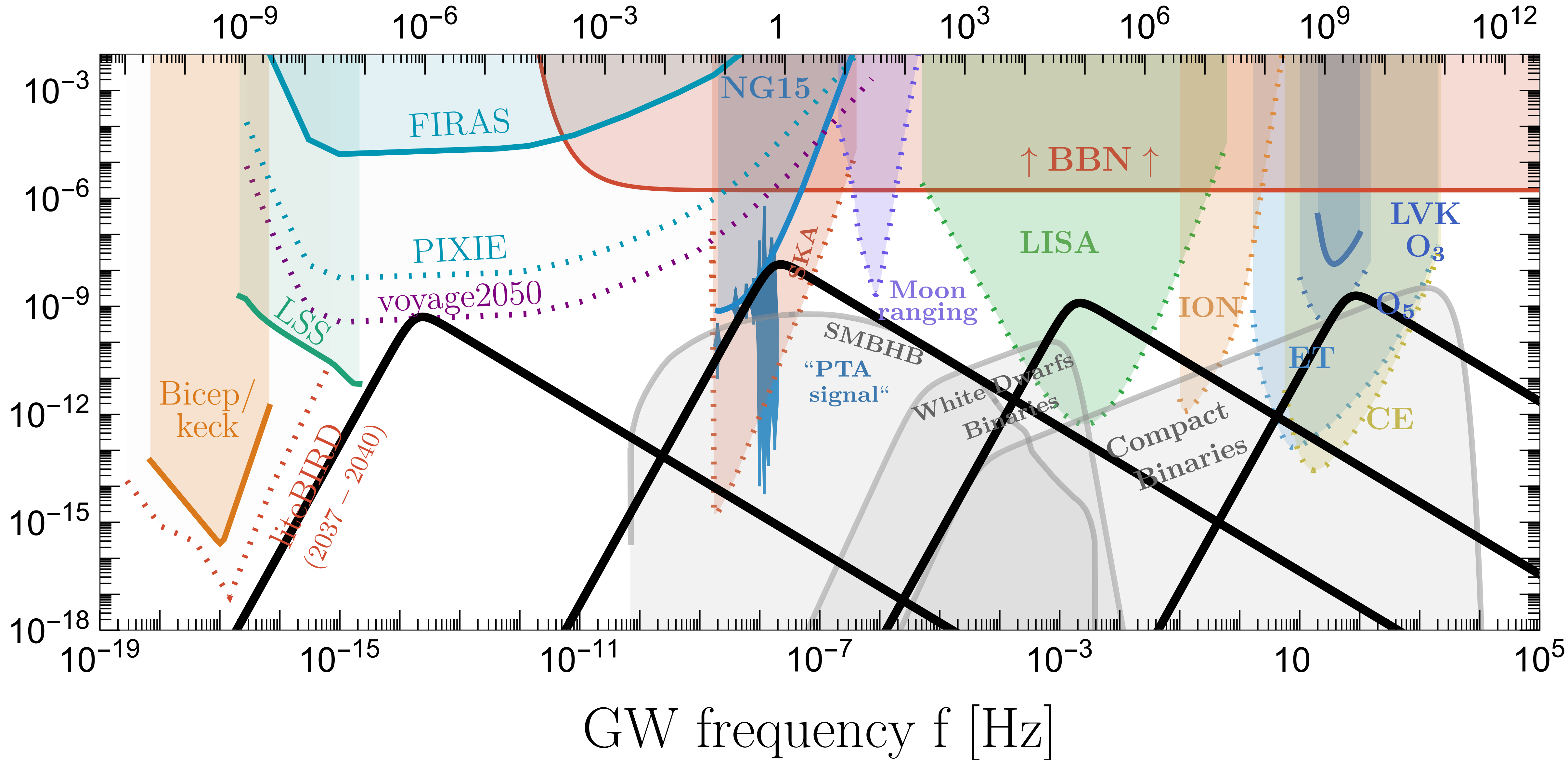


Domain walls

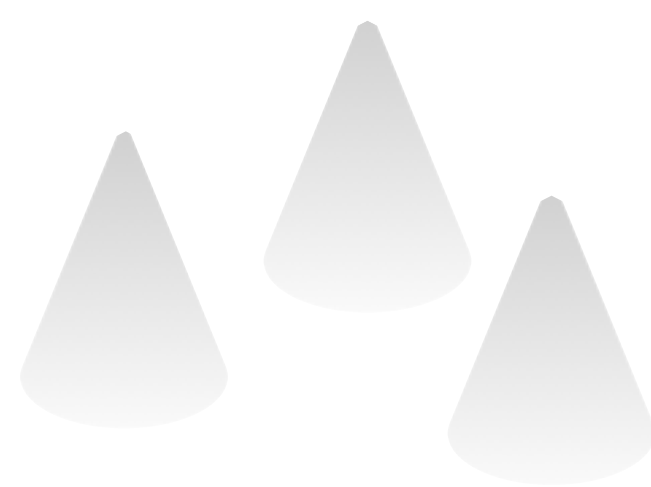


Universe temperature T [GeV]

Energy fraction $\Omega_{\text{GW}} h^2$

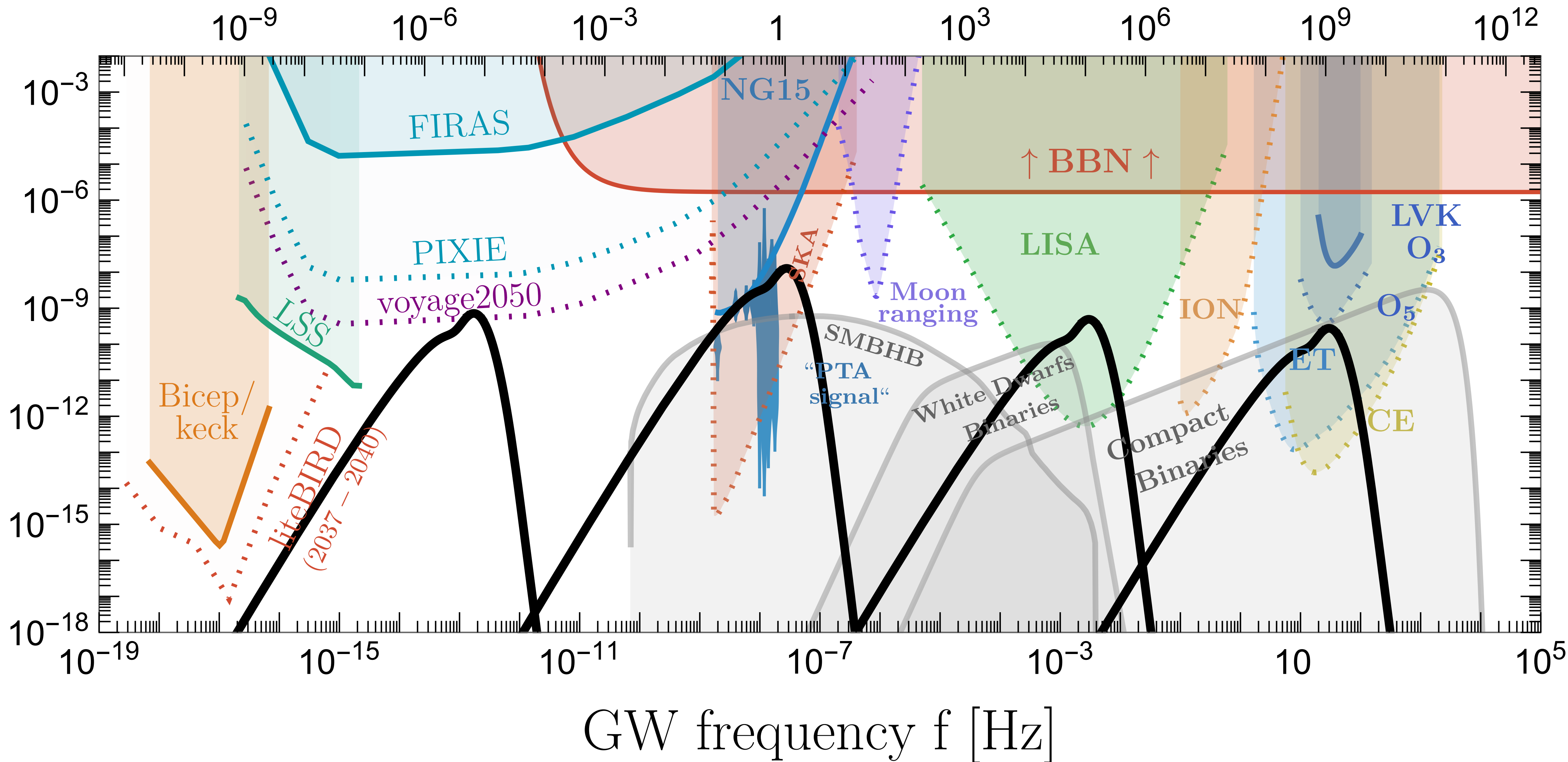


Curvature peaks

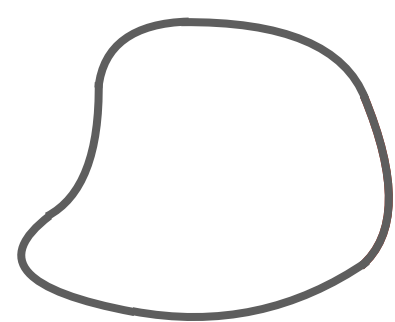


Universe temperature T [GeV]

Energy fraction $\Omega_{\text{GW}} h^2$

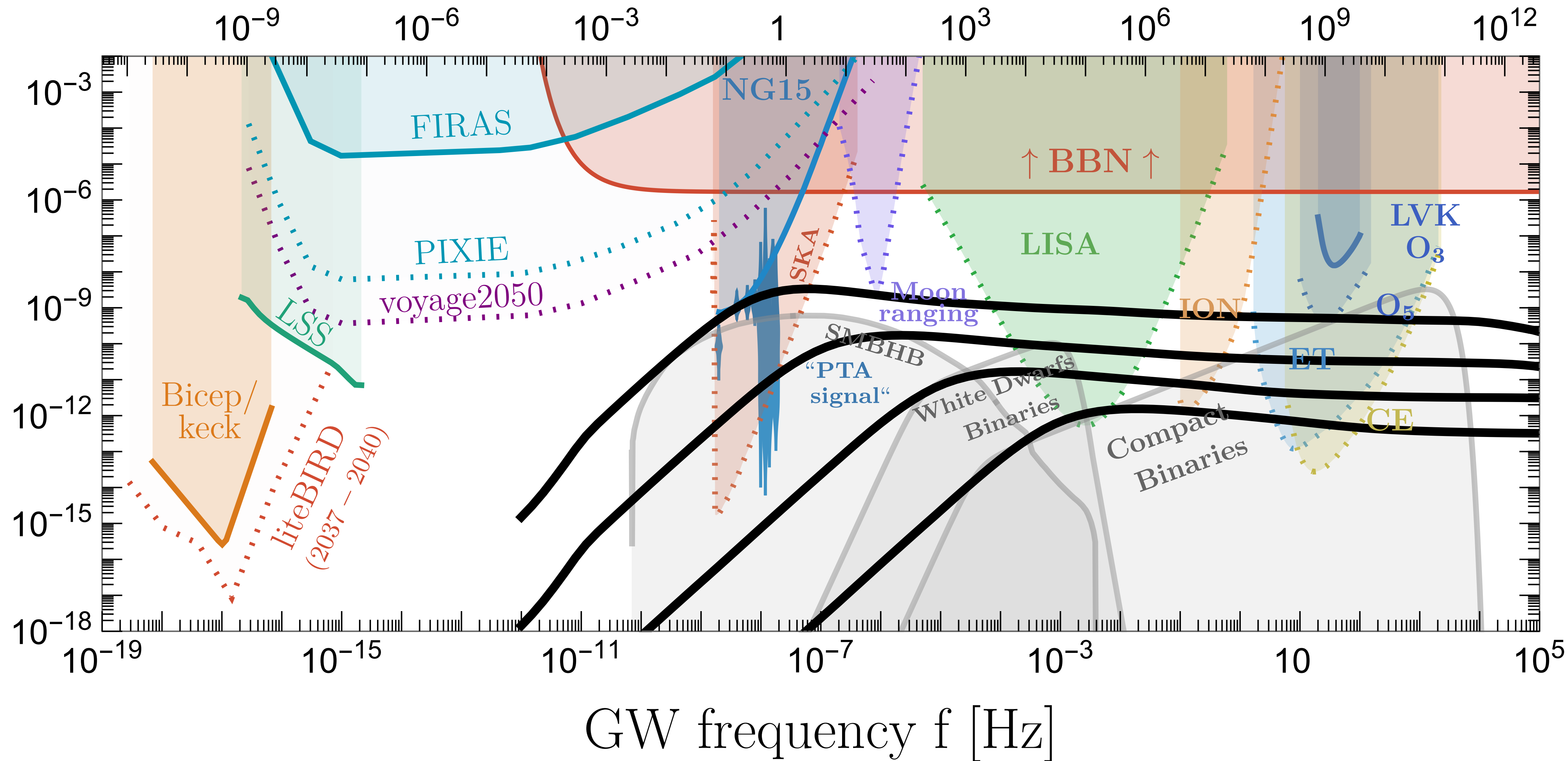


Cosmic strings

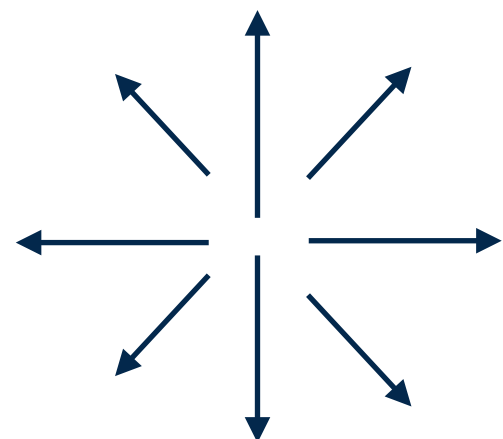


Universe temperature T [GeV]

Energy fraction $\Omega_{\text{GW}} h^2$

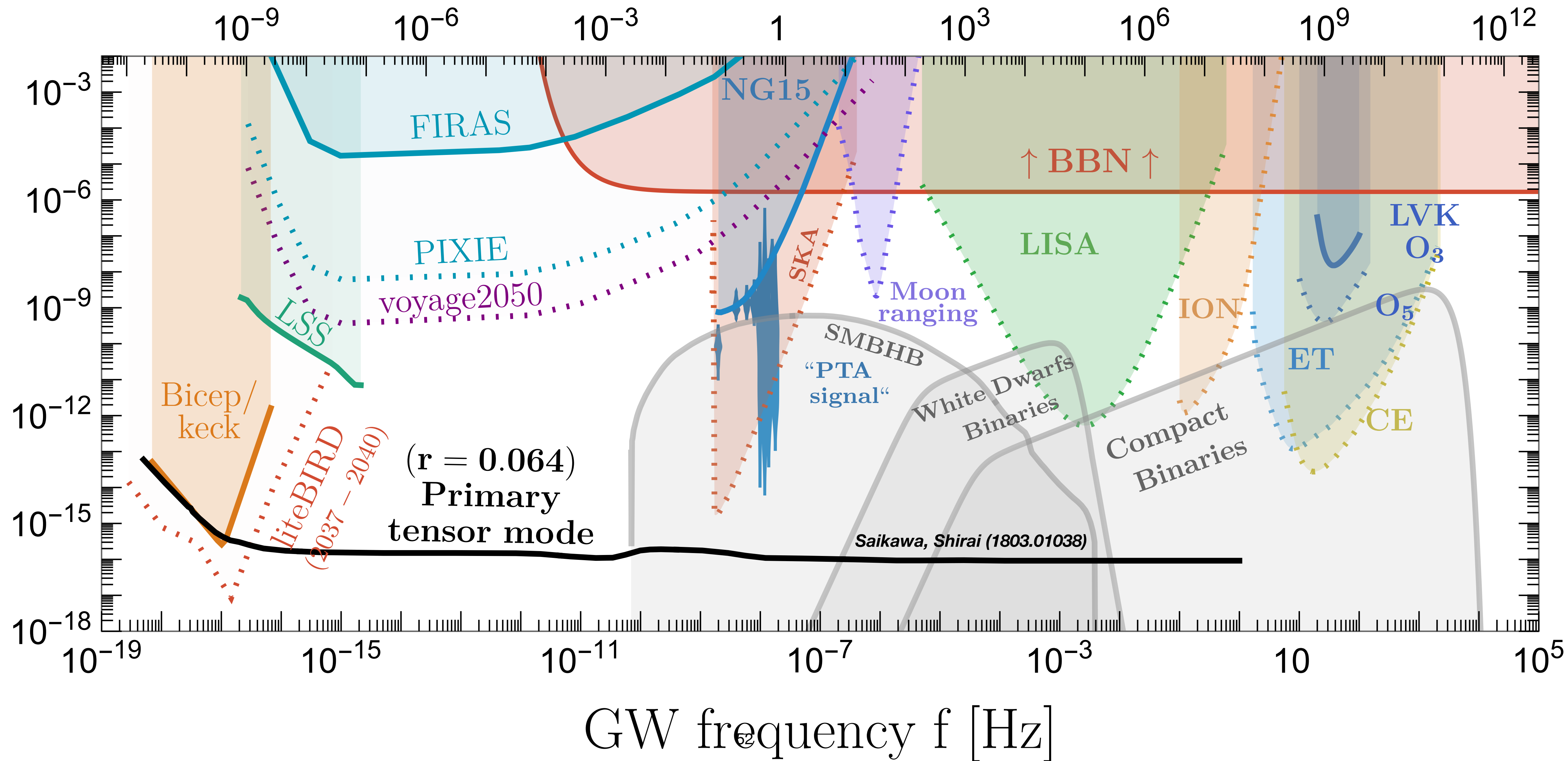


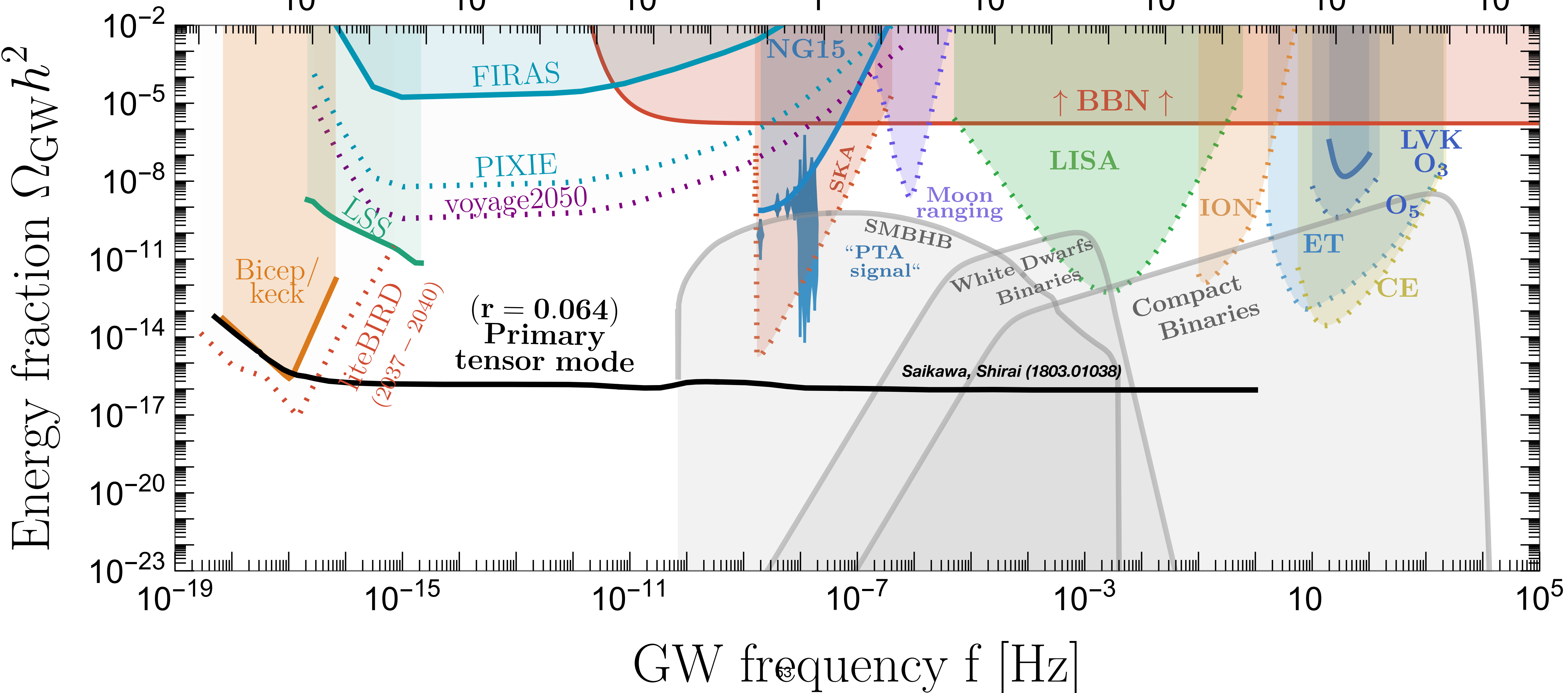
Inflation



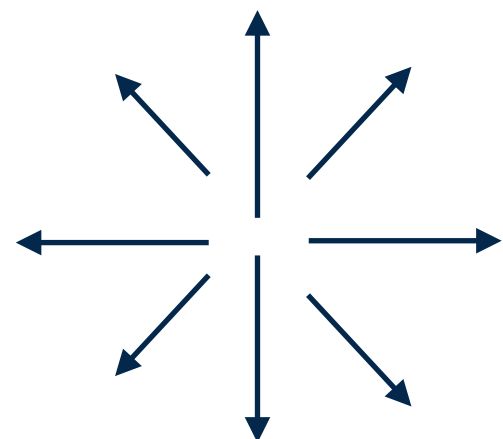
Universe temperature T [GeV]

Energy fraction $\Omega_{\text{GW}} h^2$



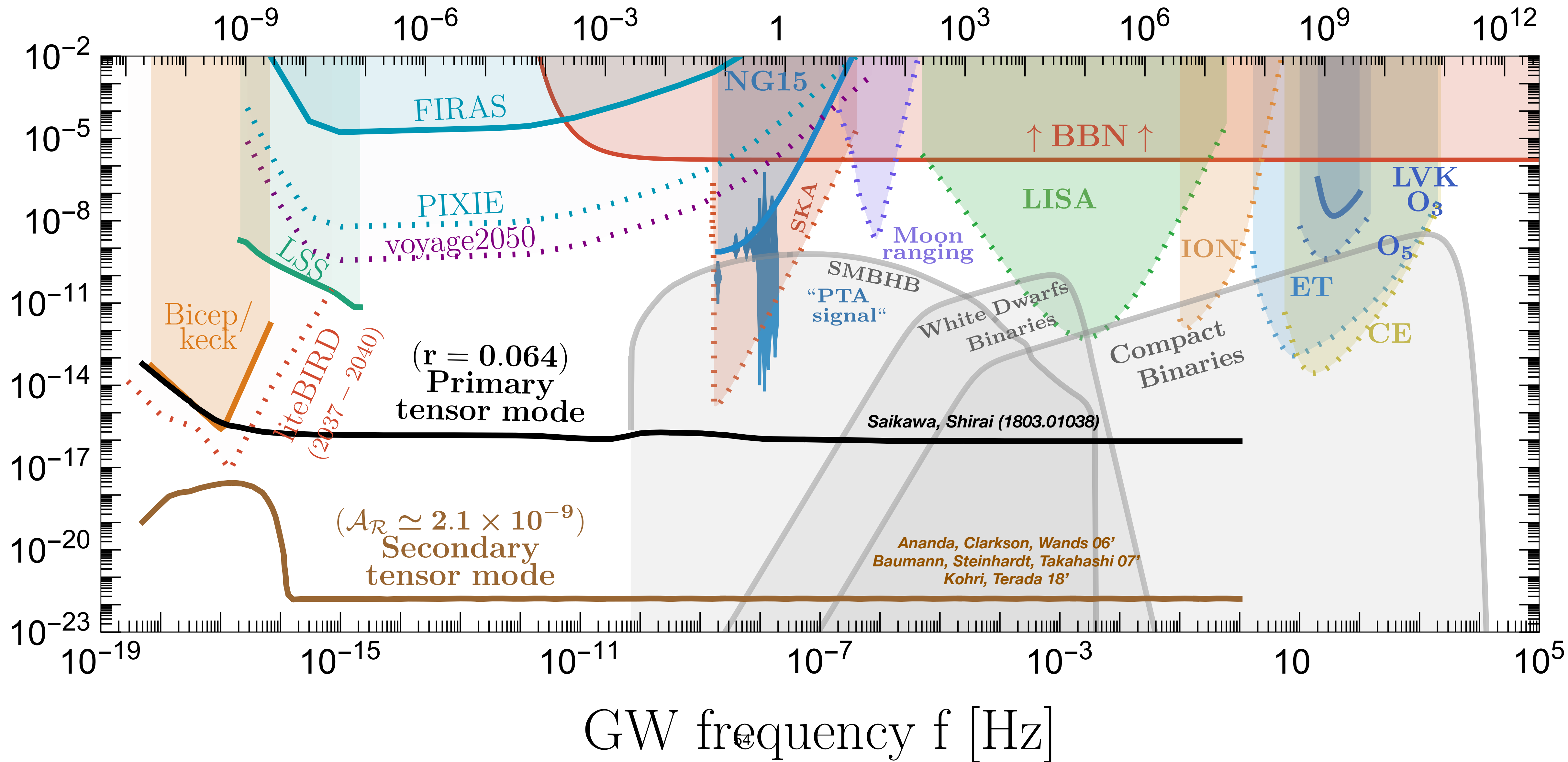
$$10^{-9} \quad 10^{-6} \quad 10^{-3} \quad 1 \quad 10^3 \quad 10^6 \quad 10^9 \quad 10^{12}$$


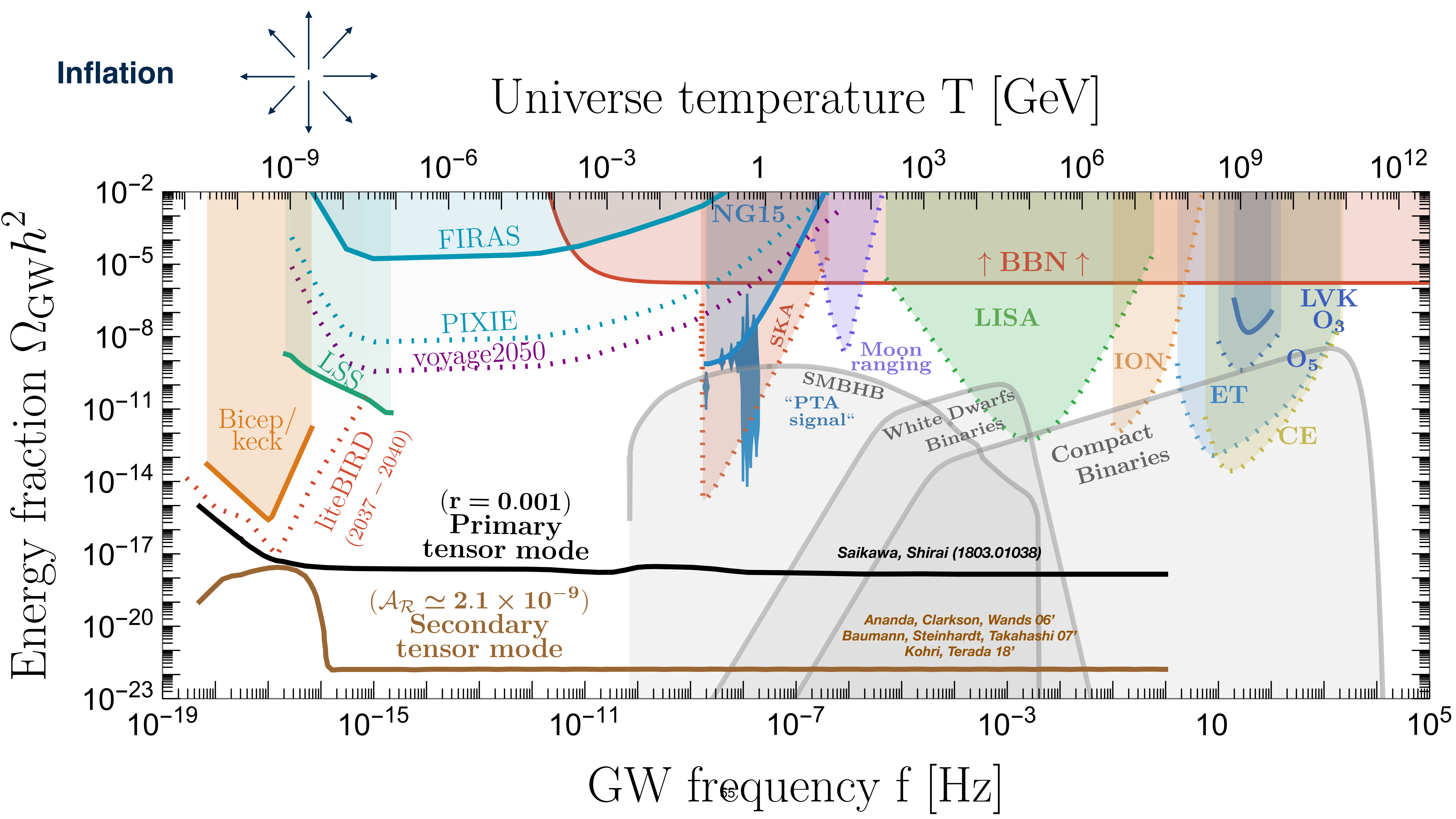
Inflation

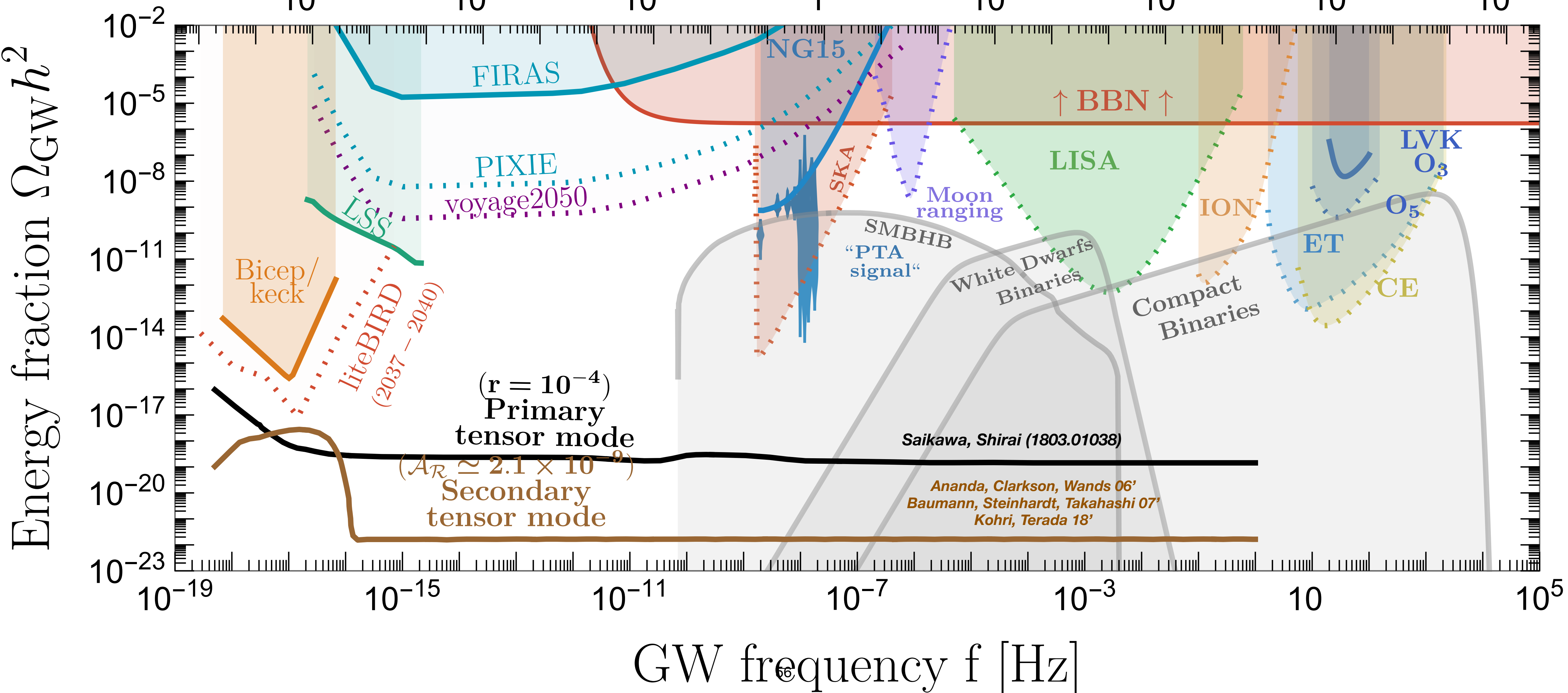


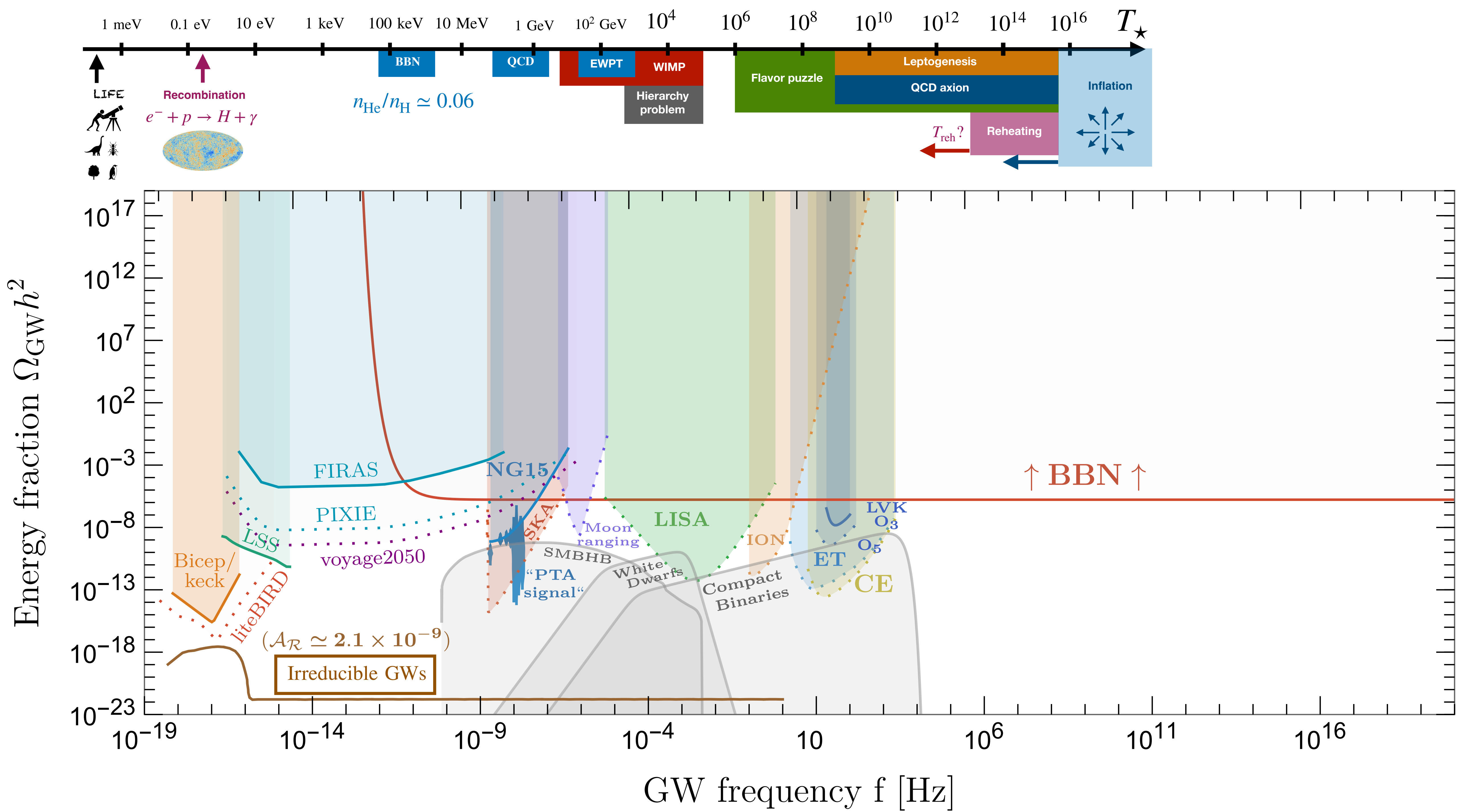
Universe temperature T [GeV]

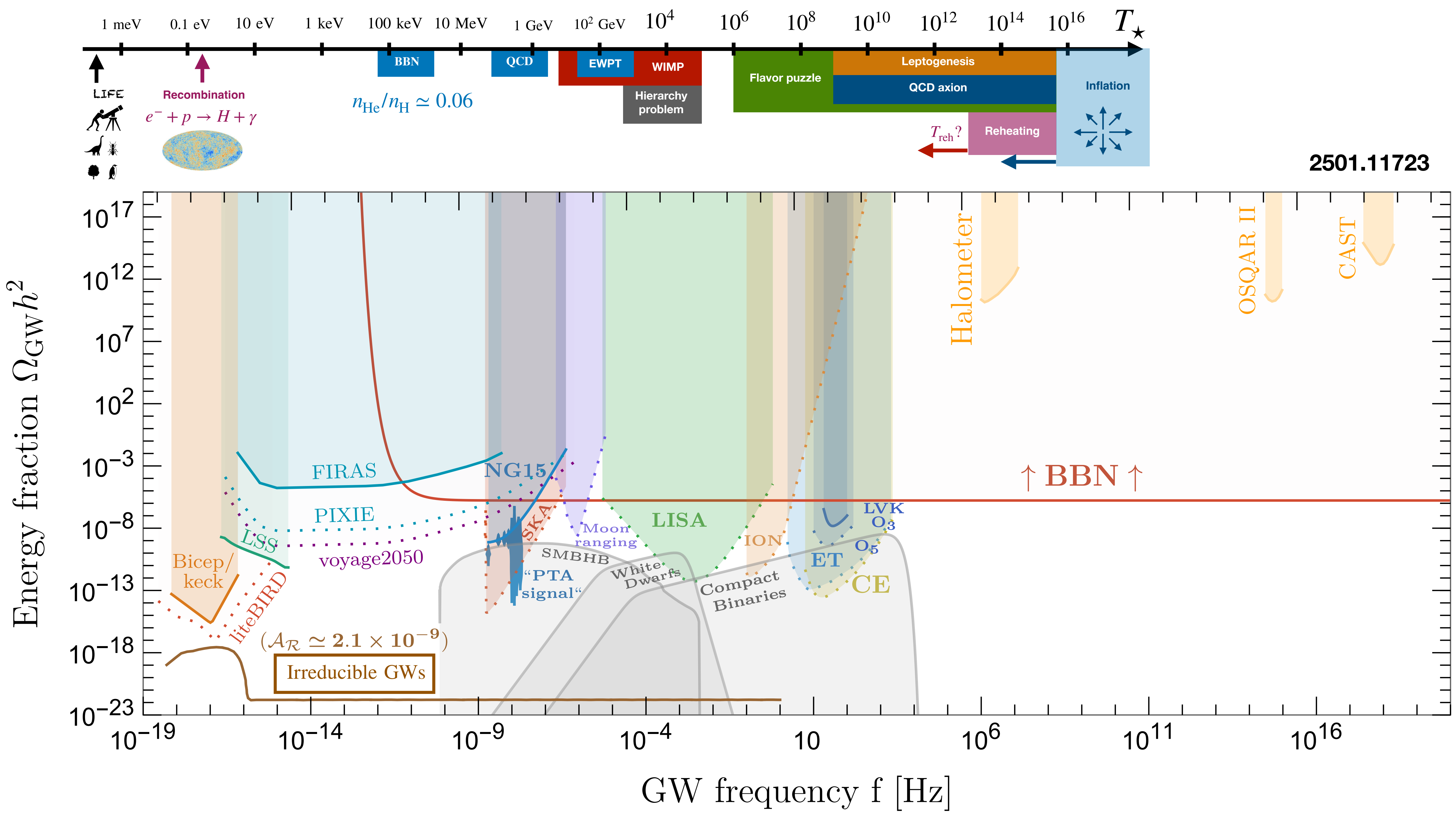
Energy fraction $\Omega_{\text{GW}} h^2$

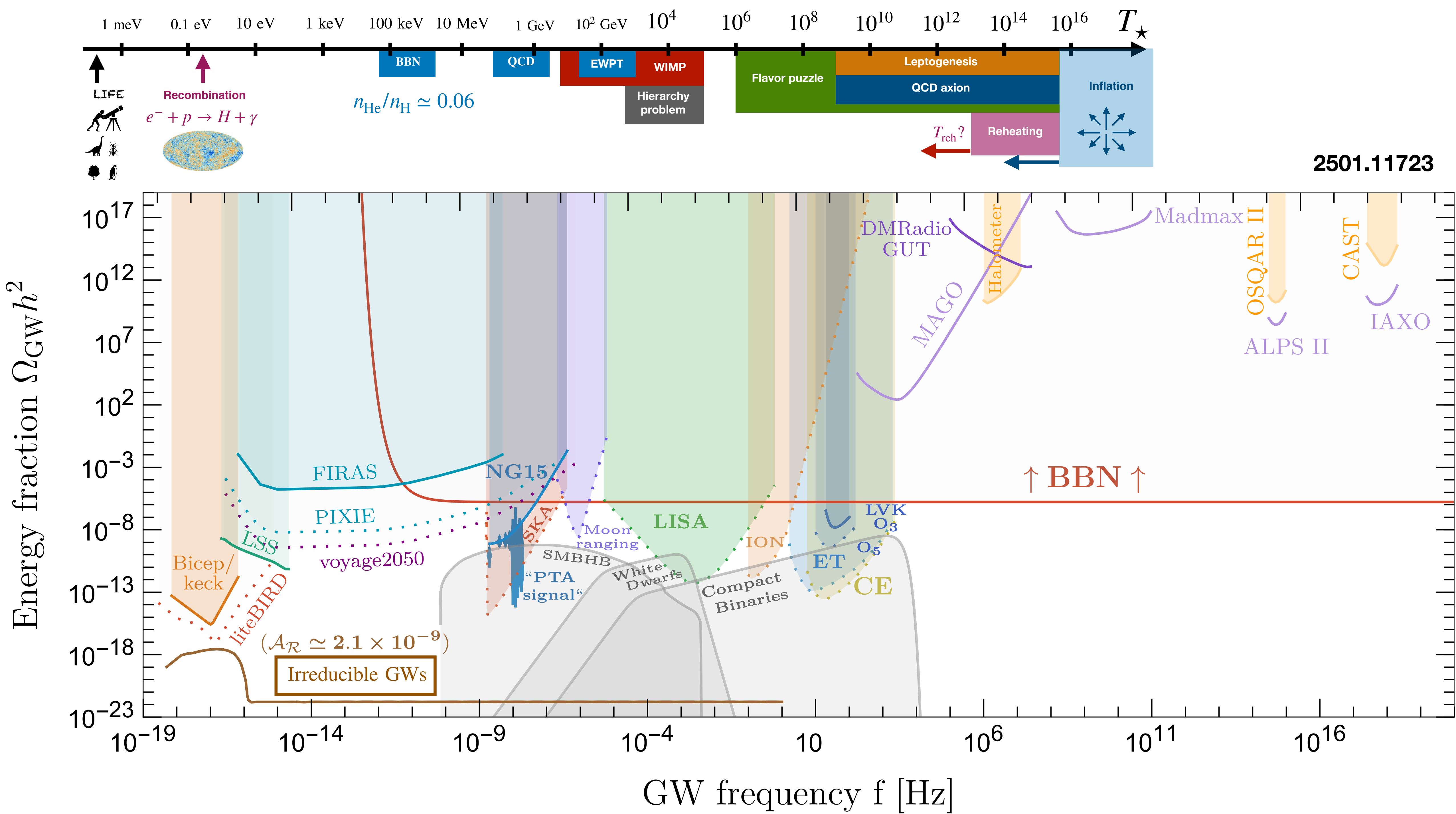


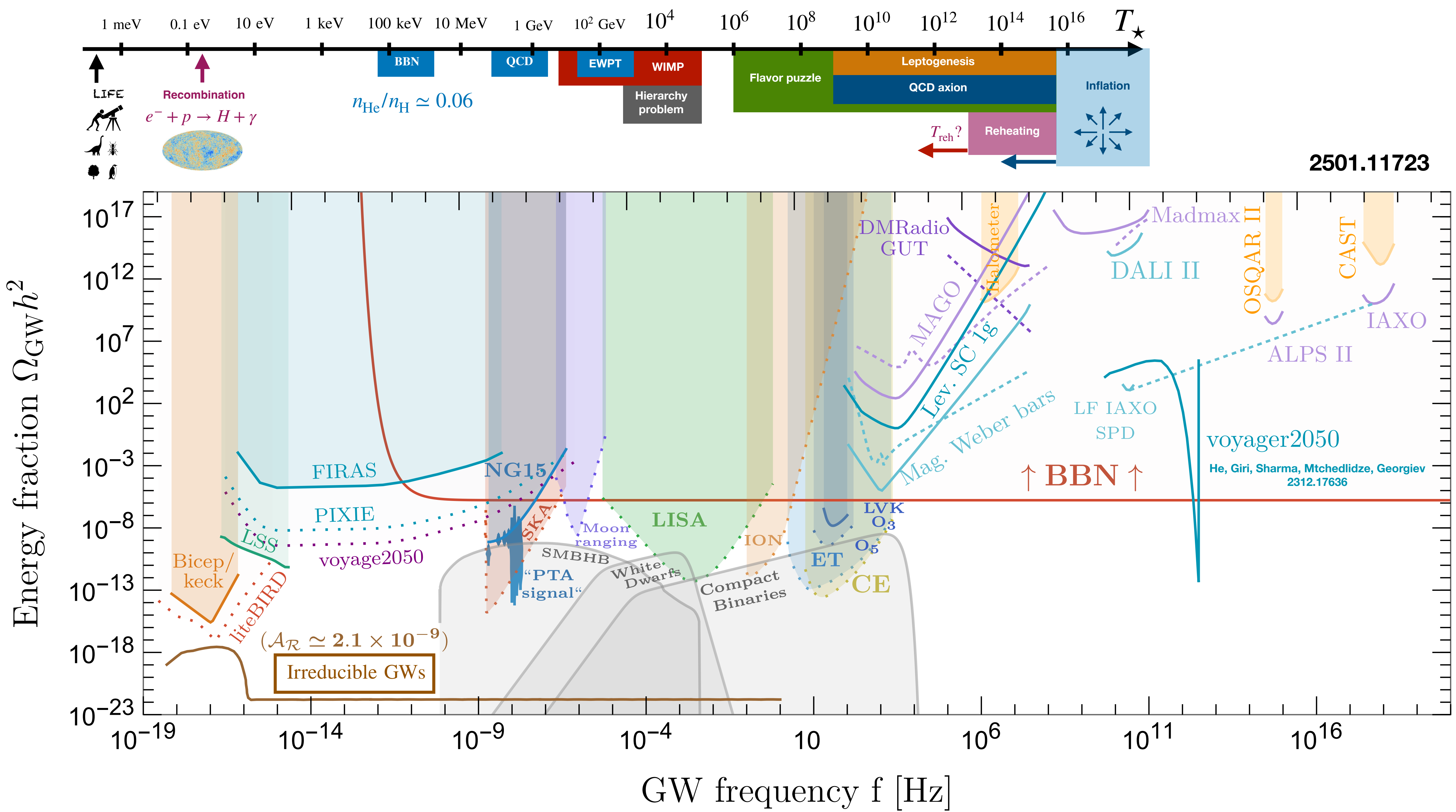


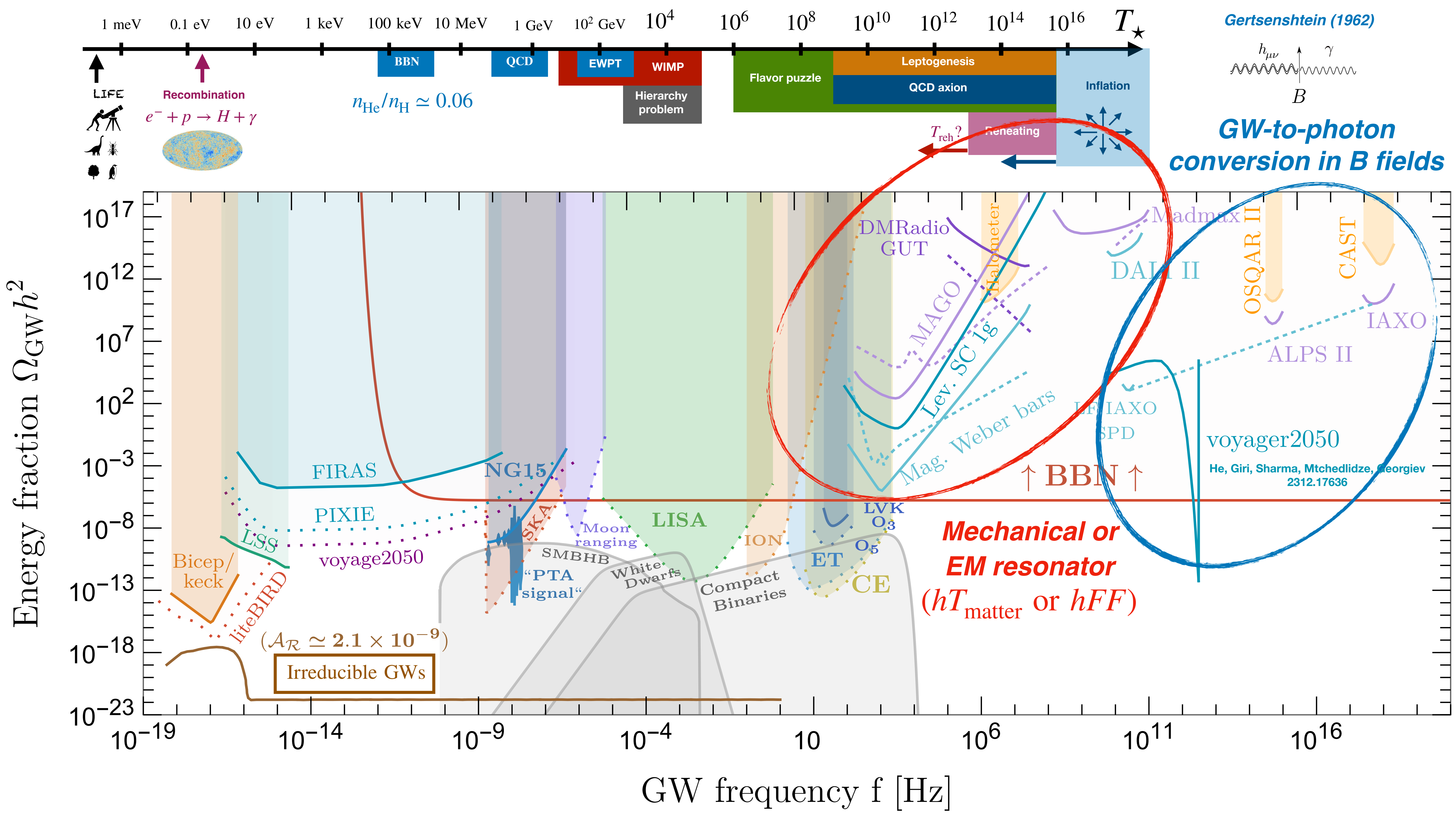
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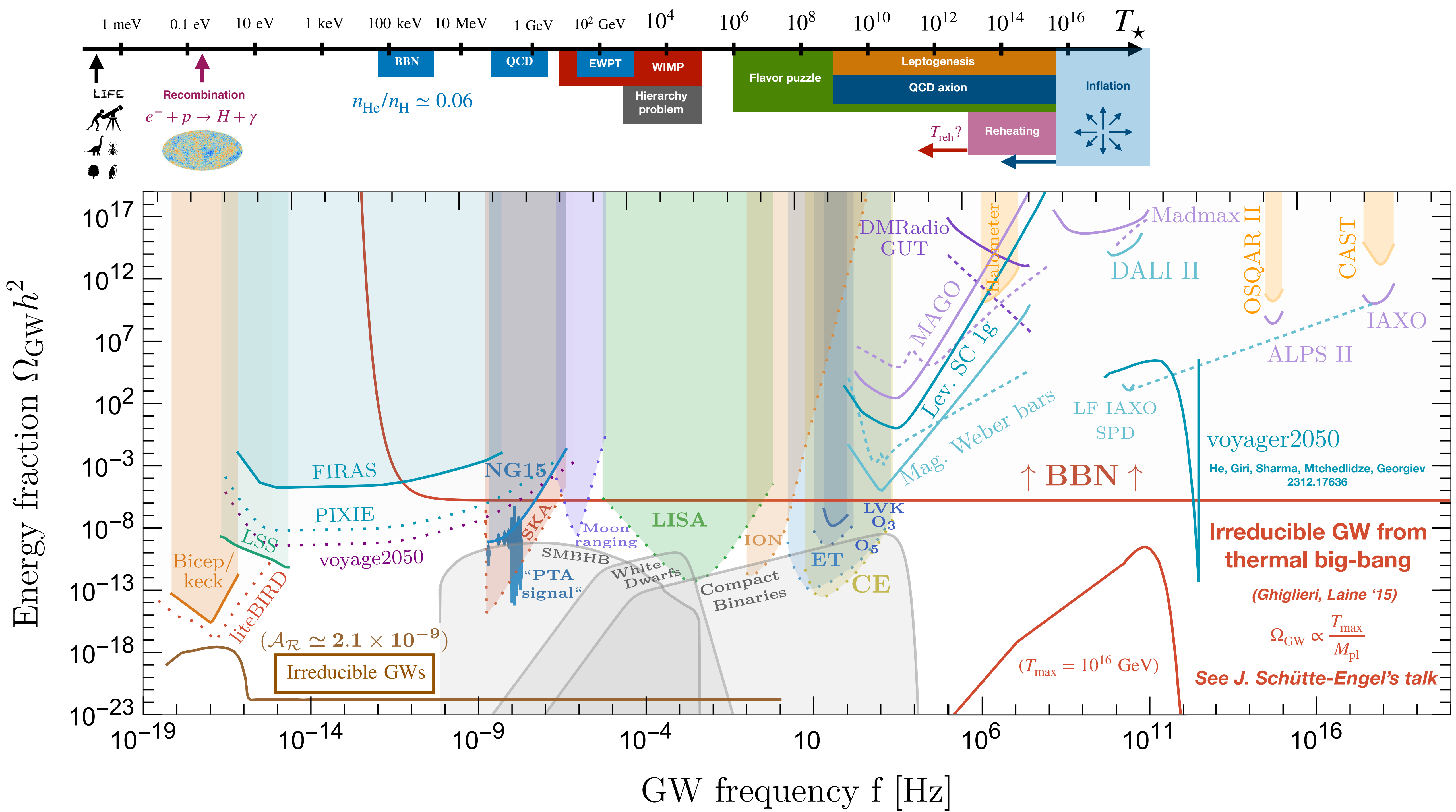


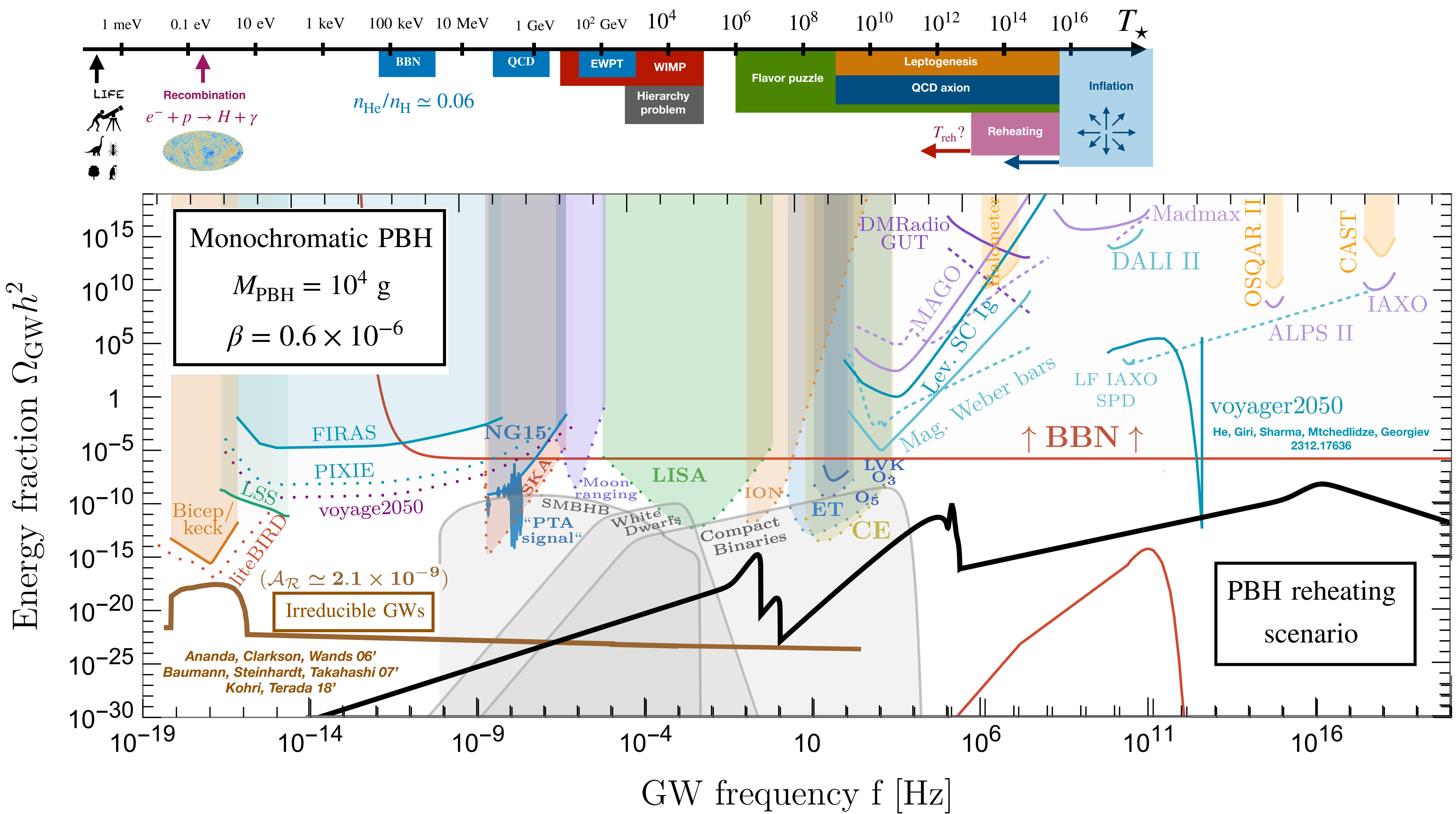


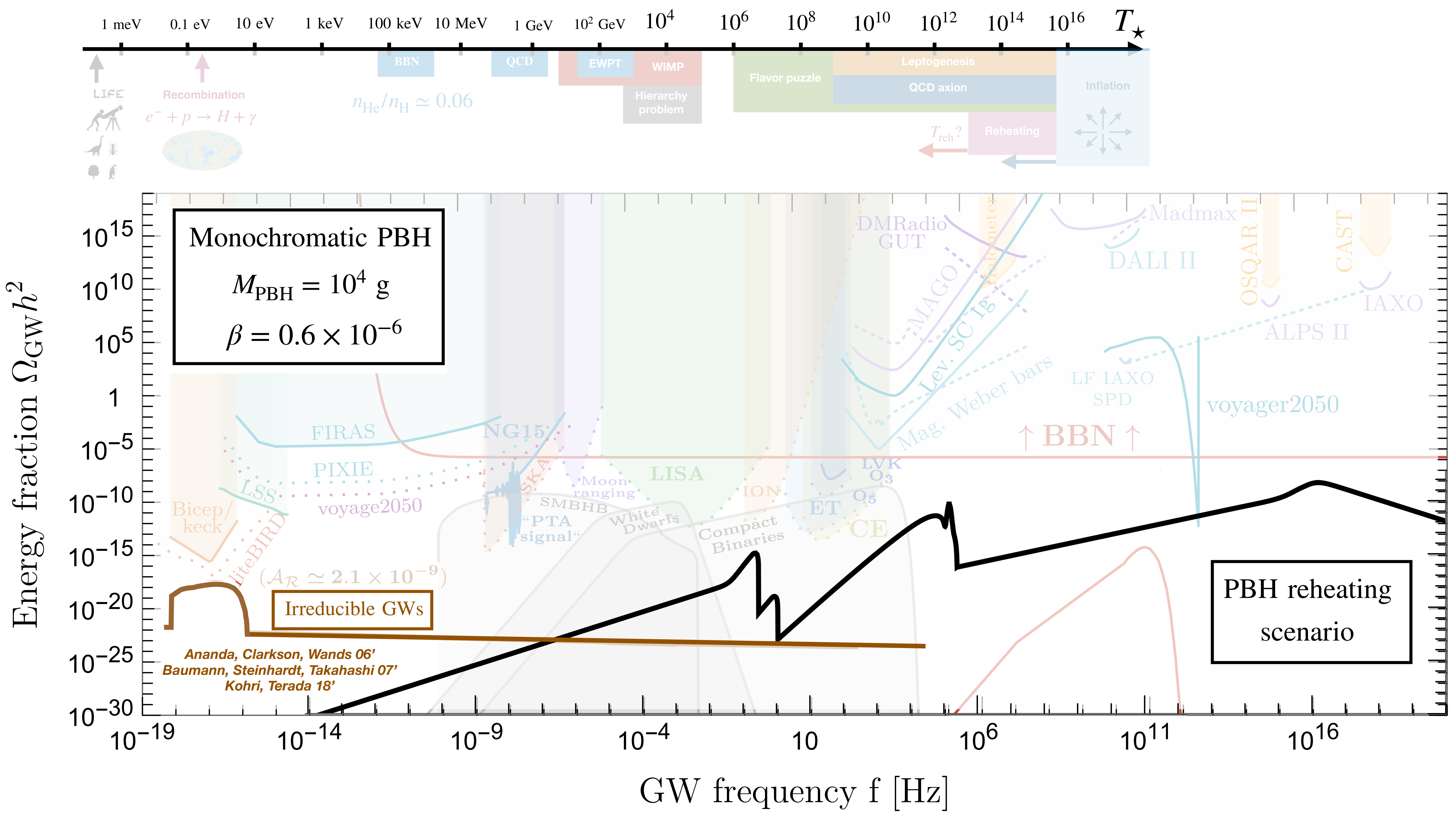


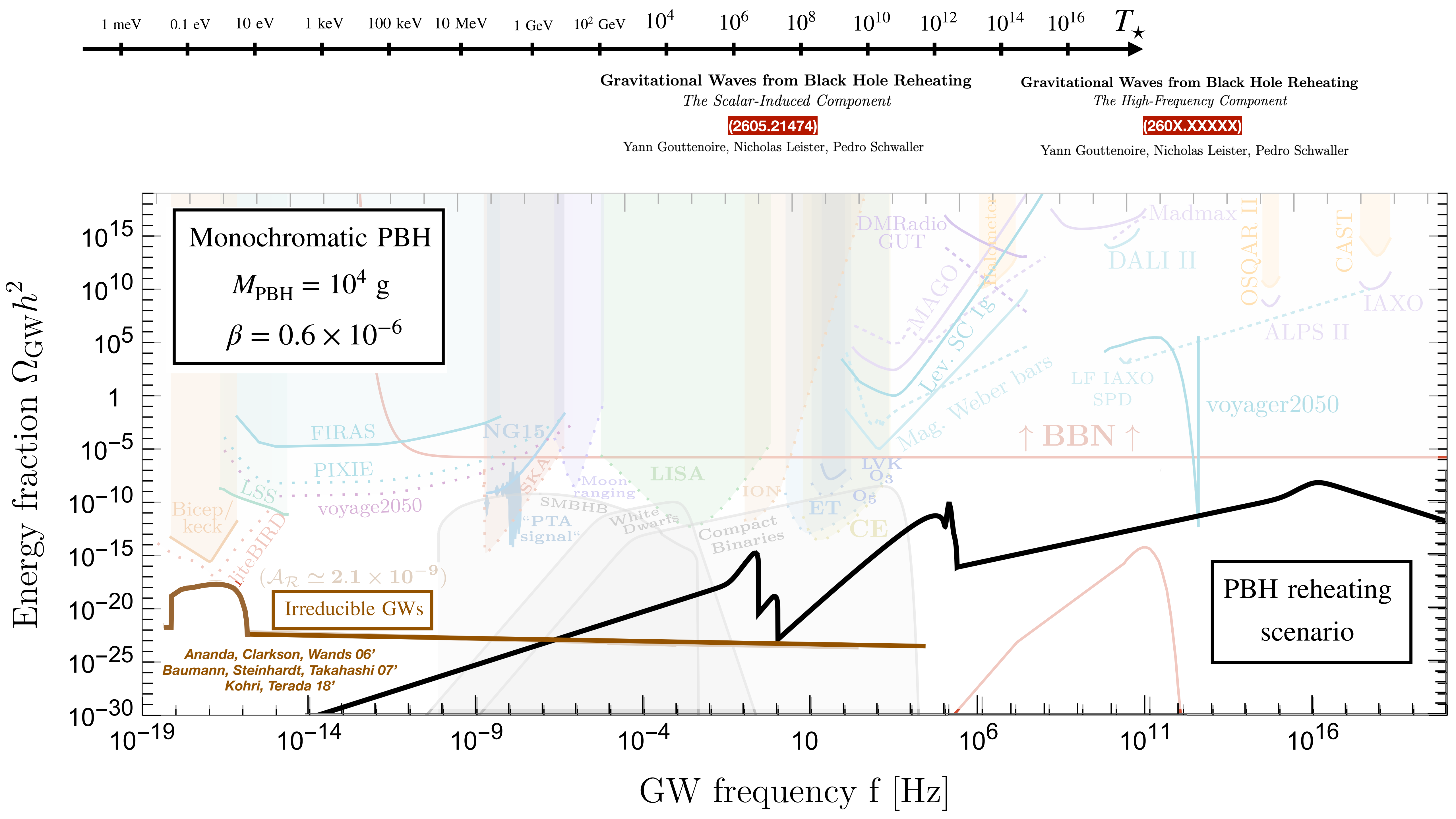


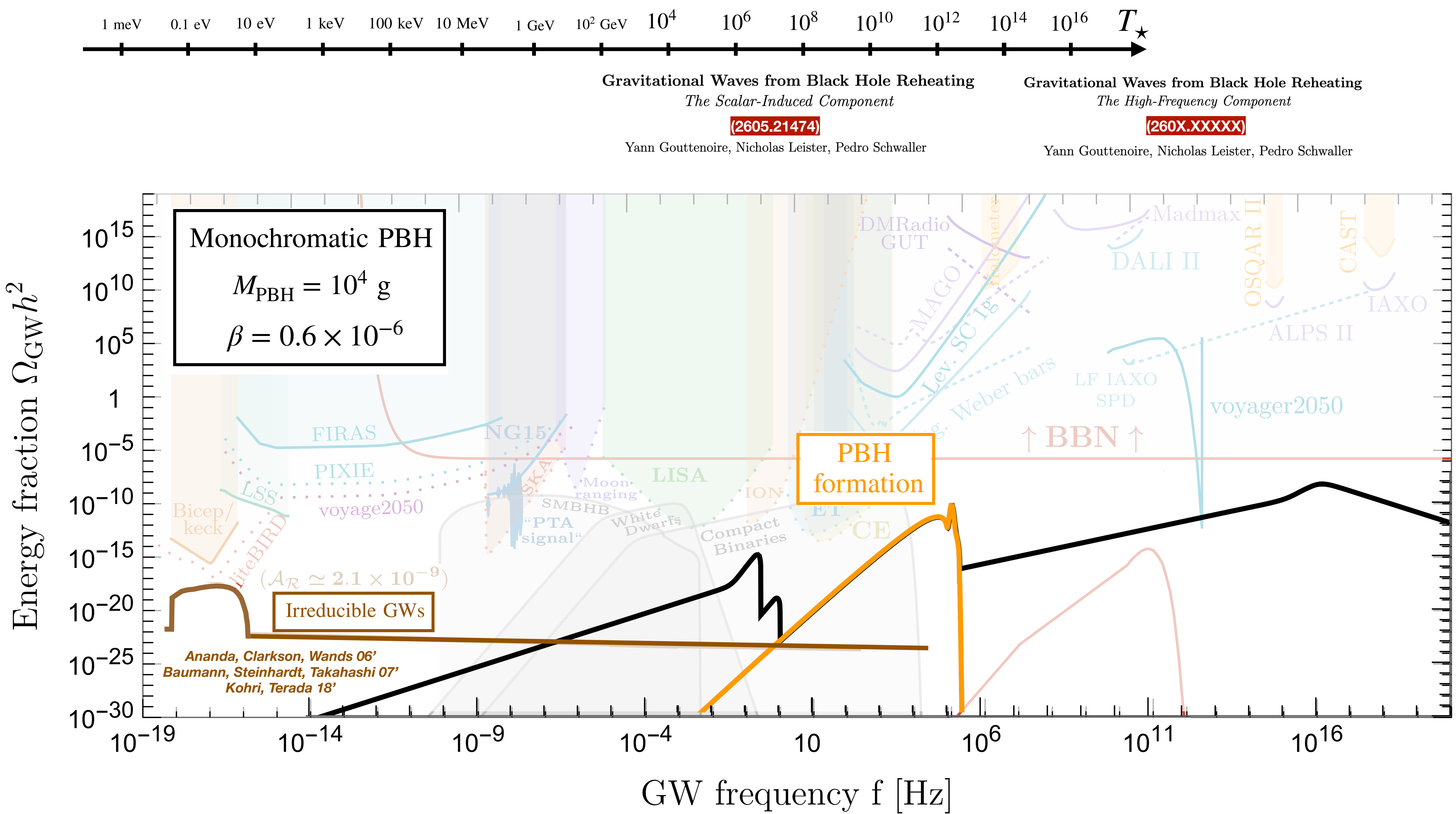


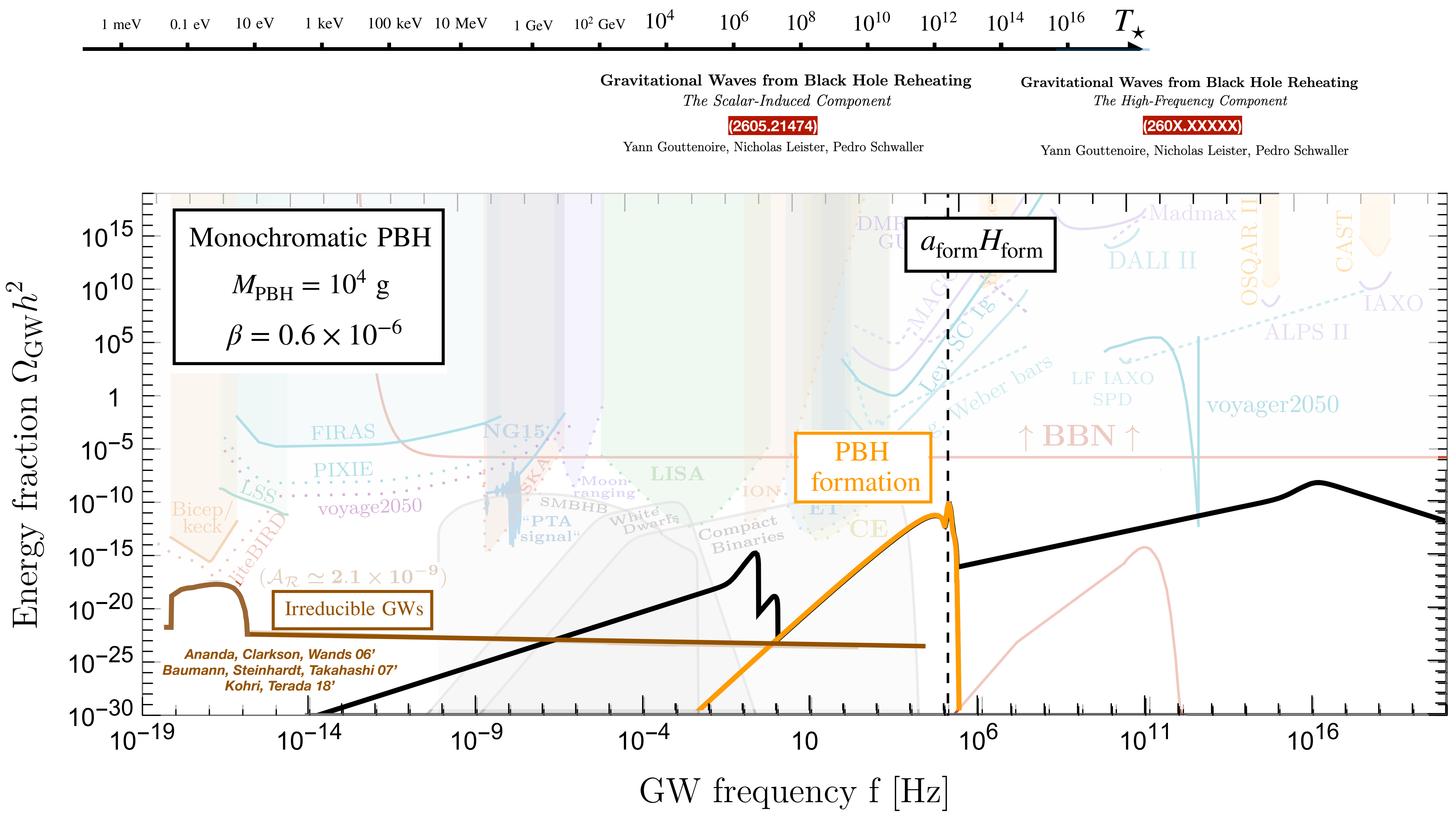


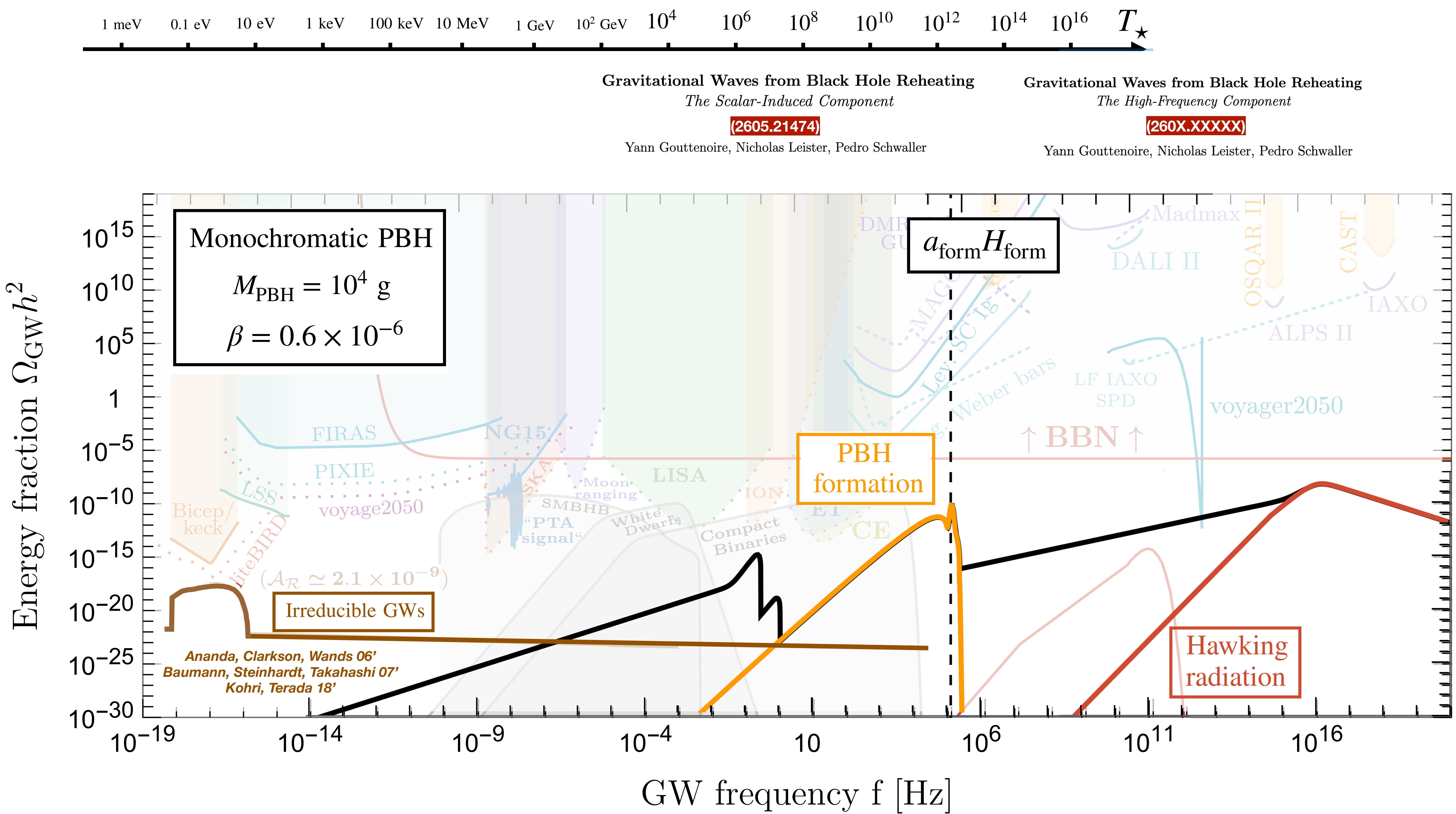


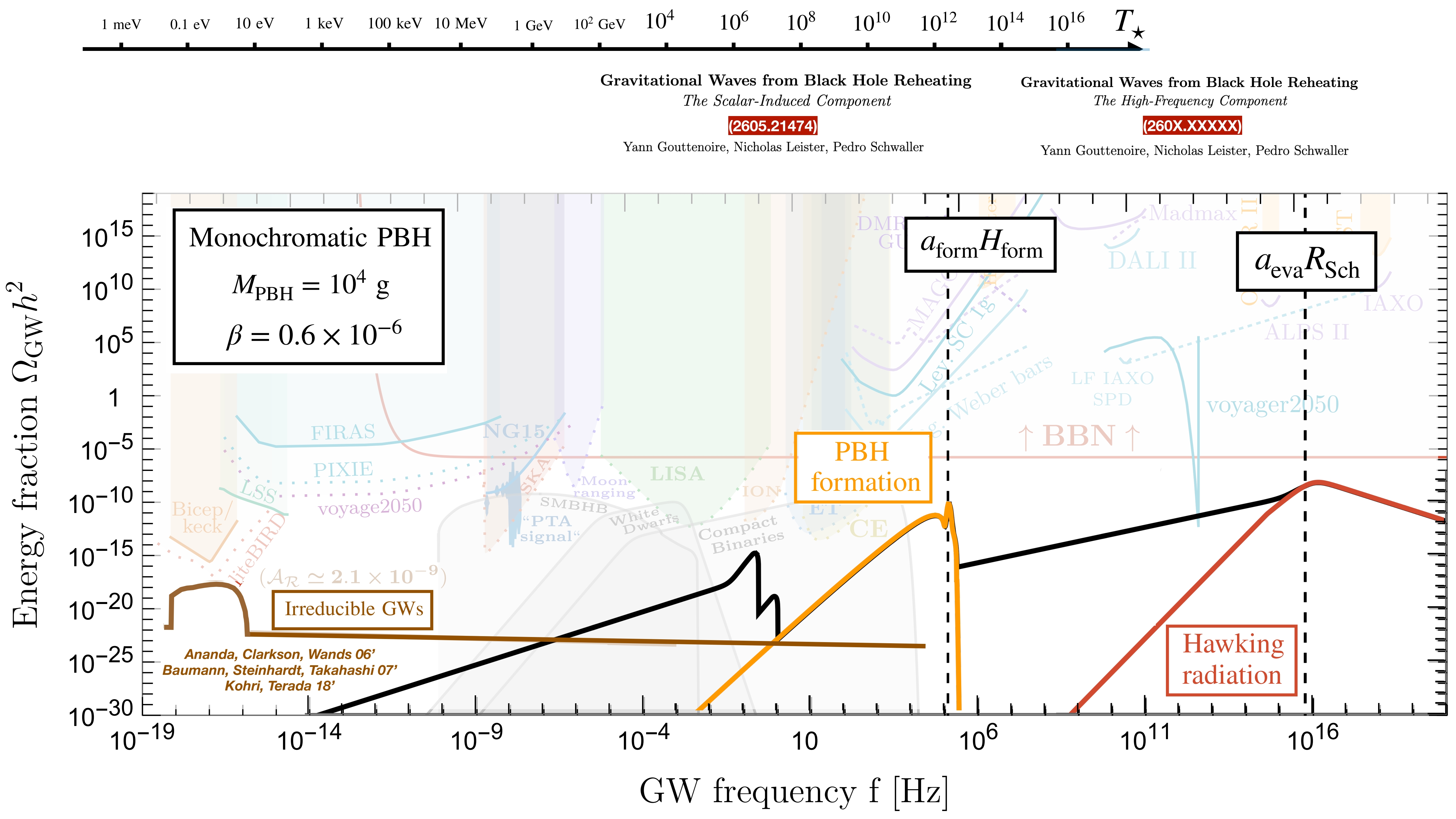


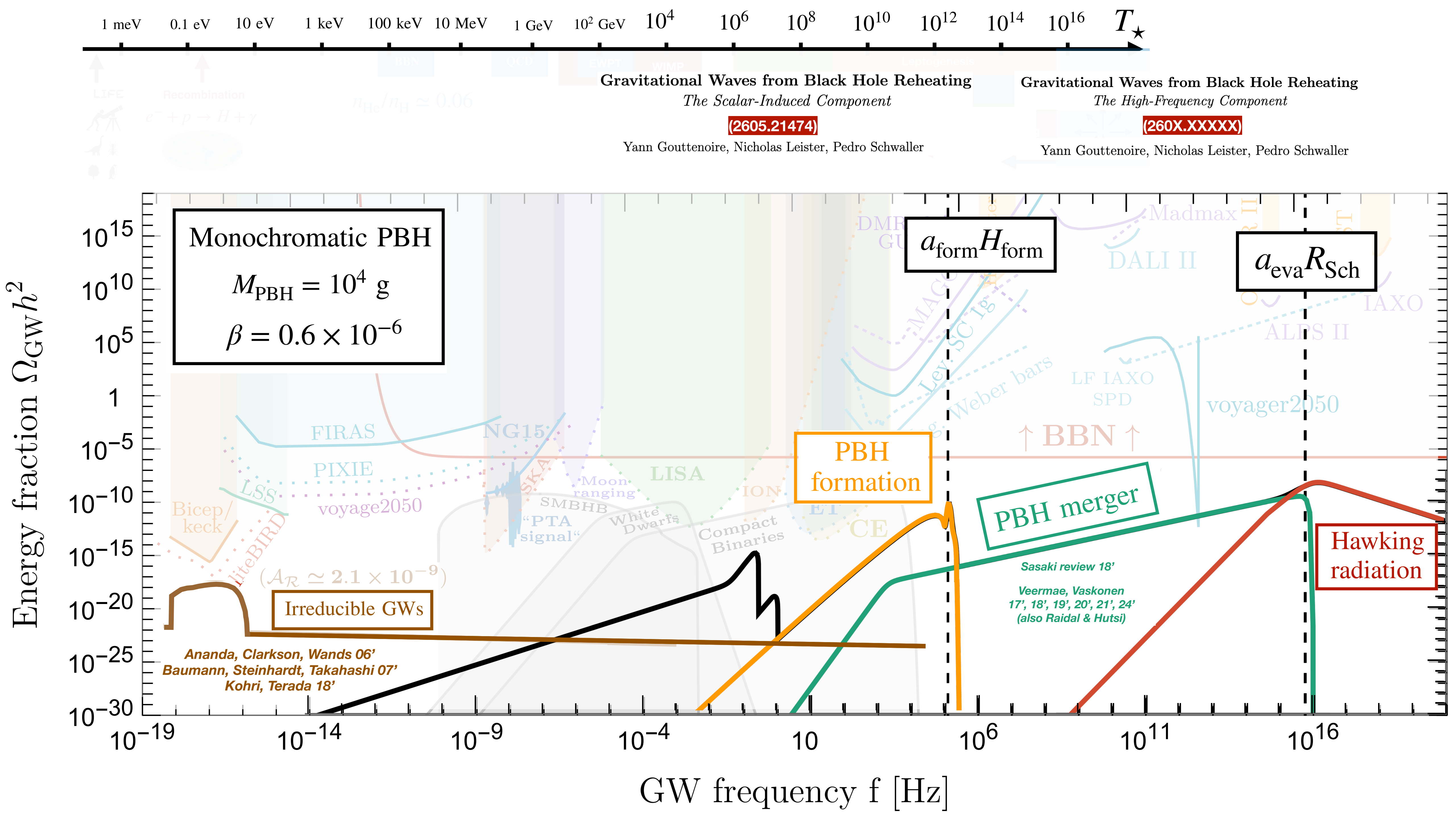


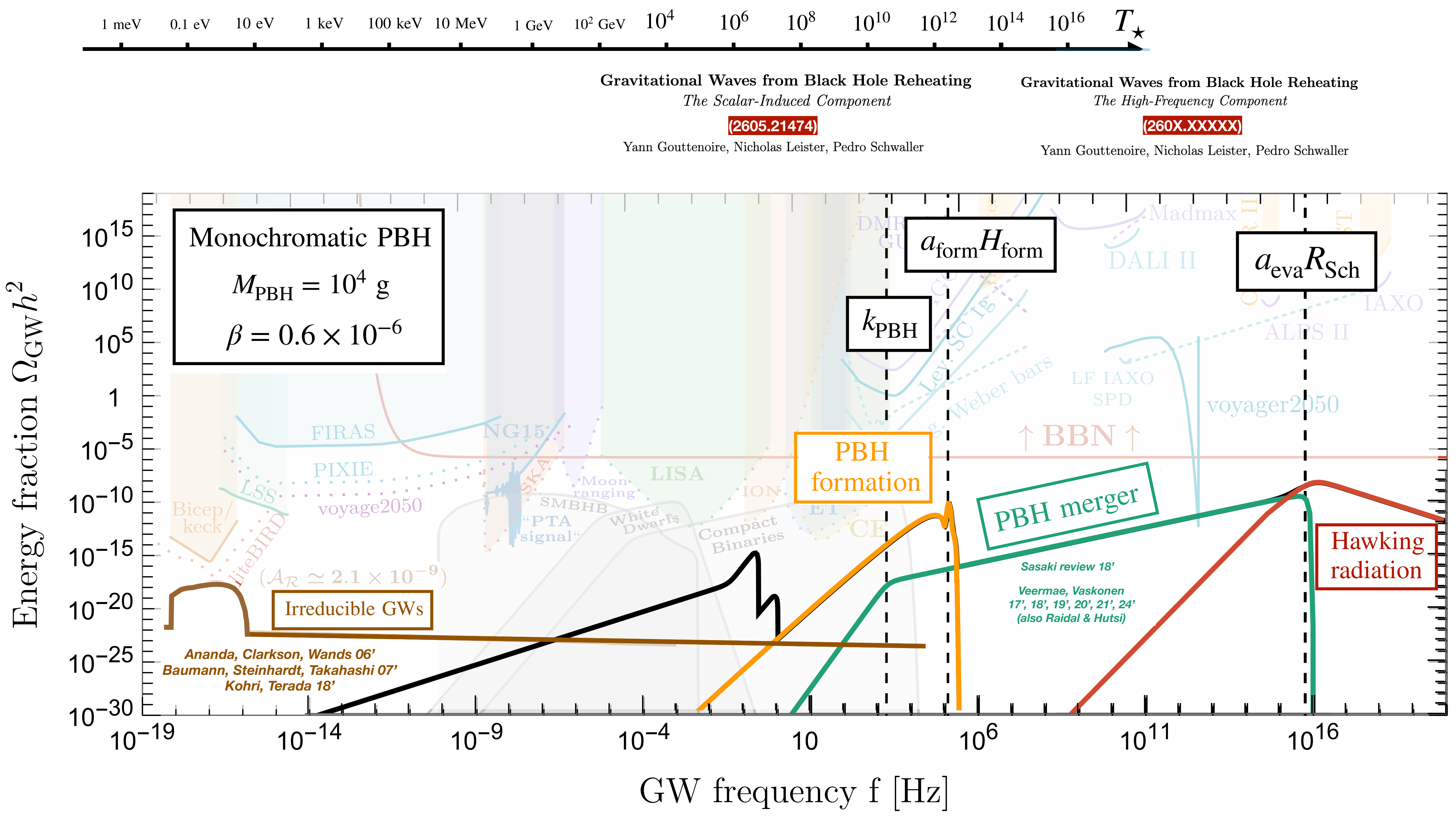


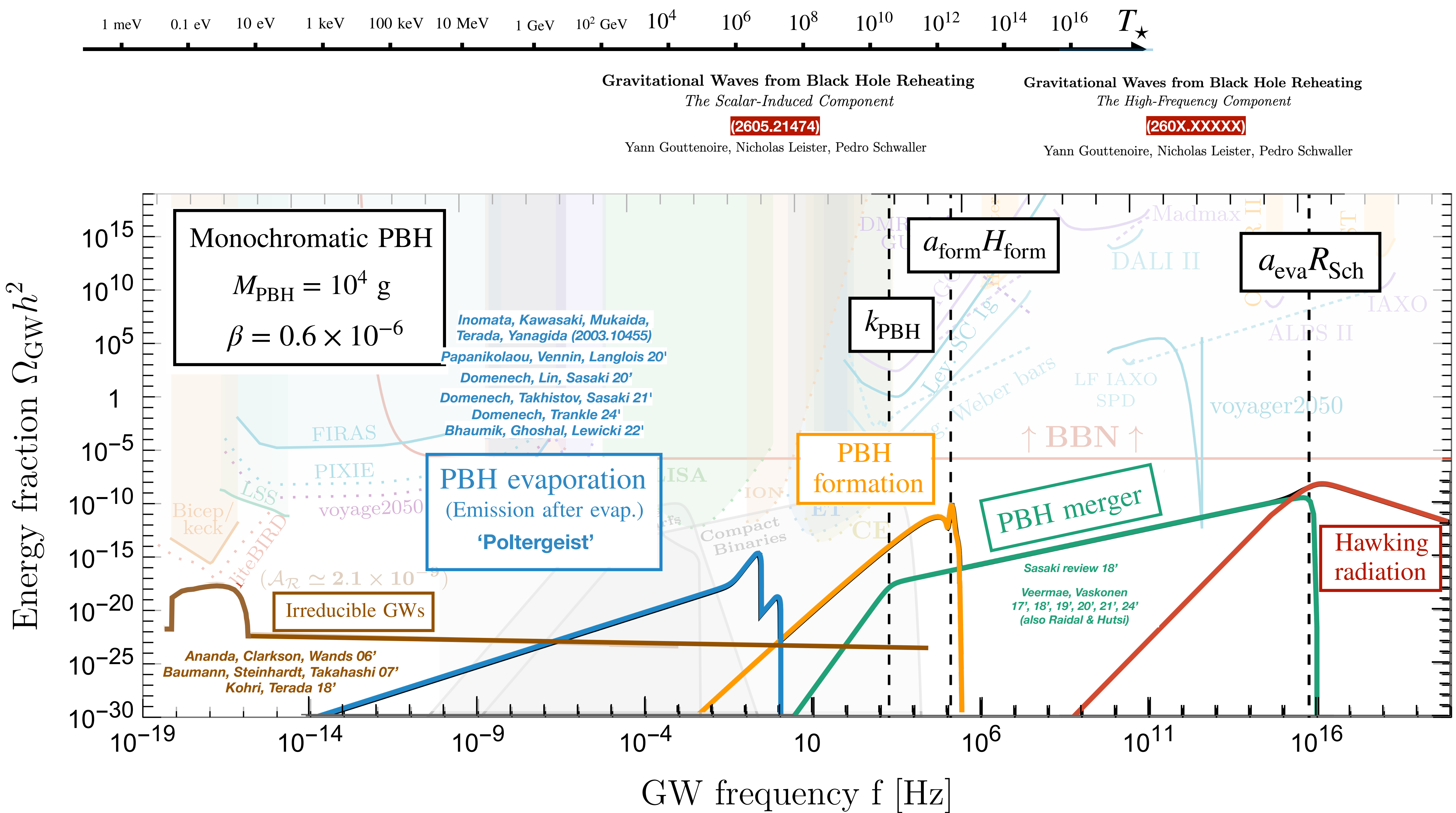


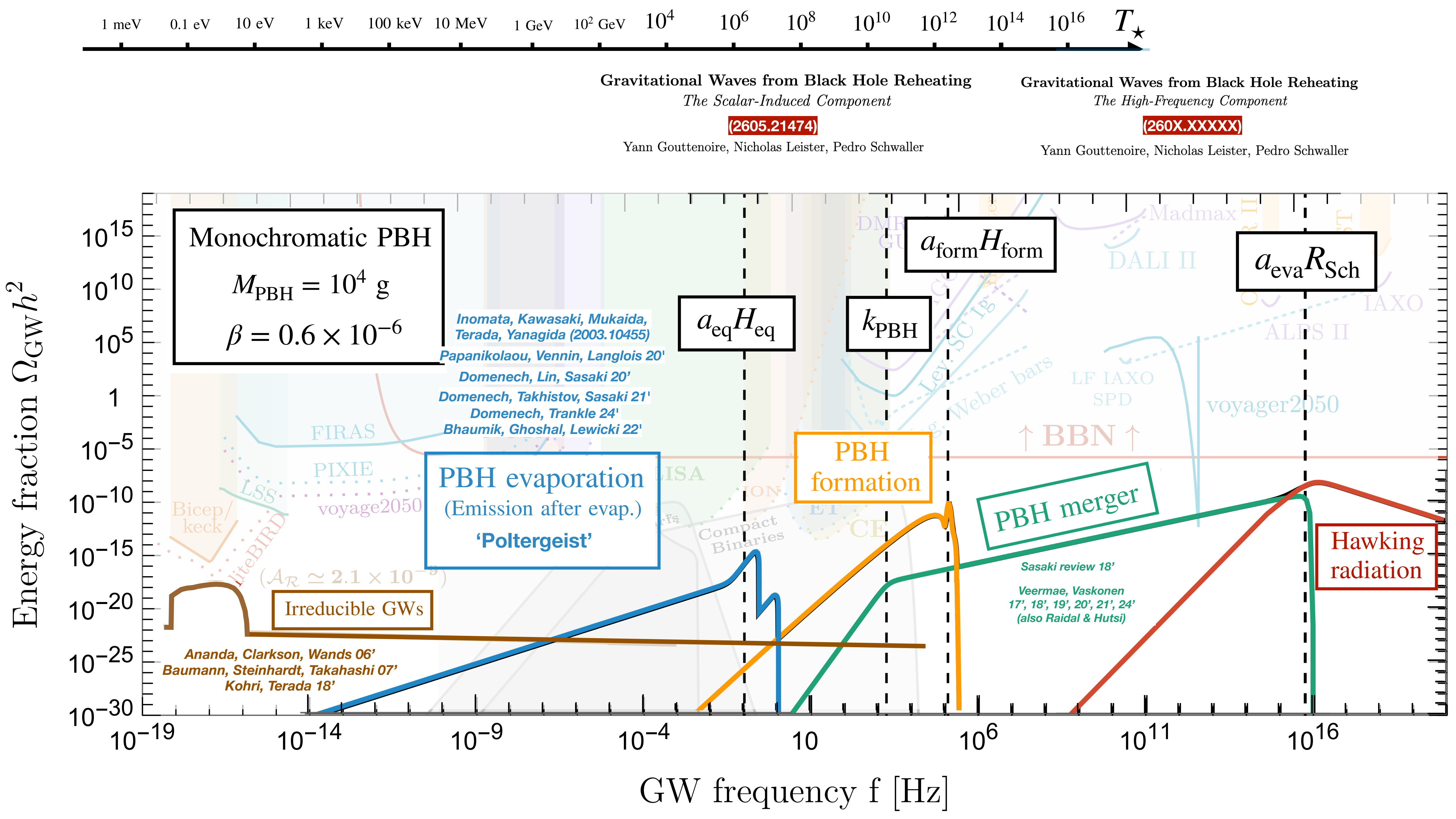


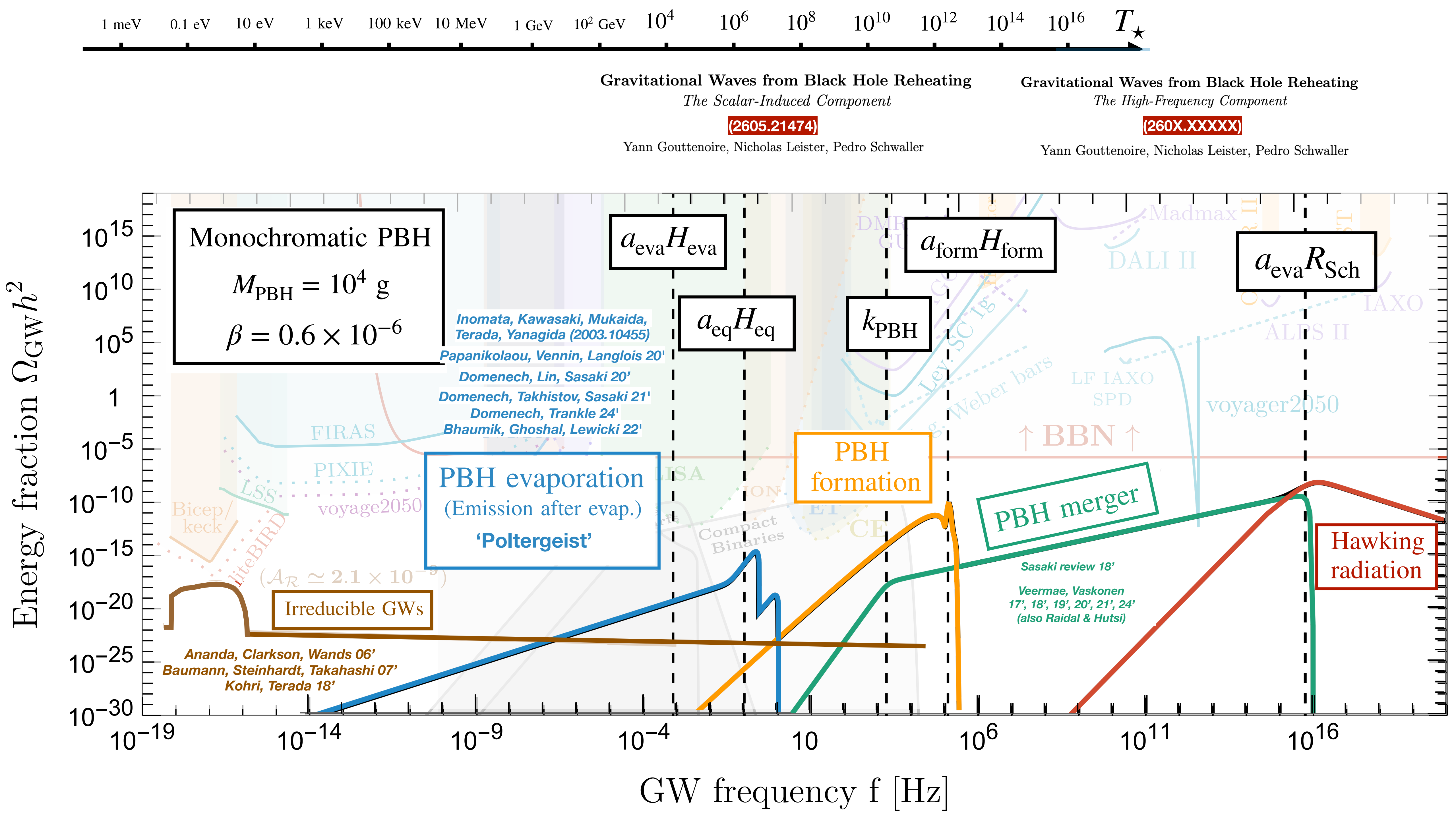


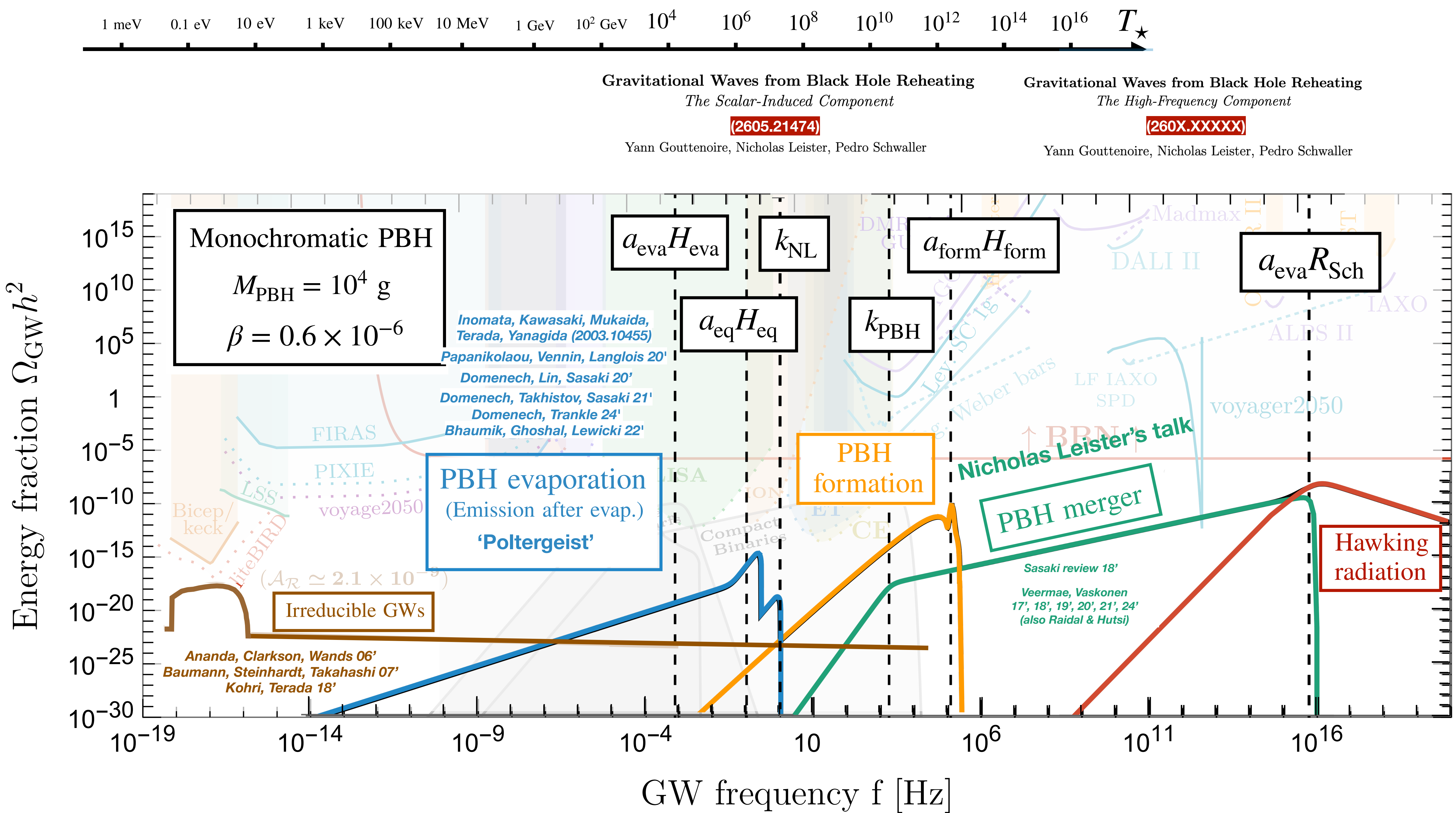


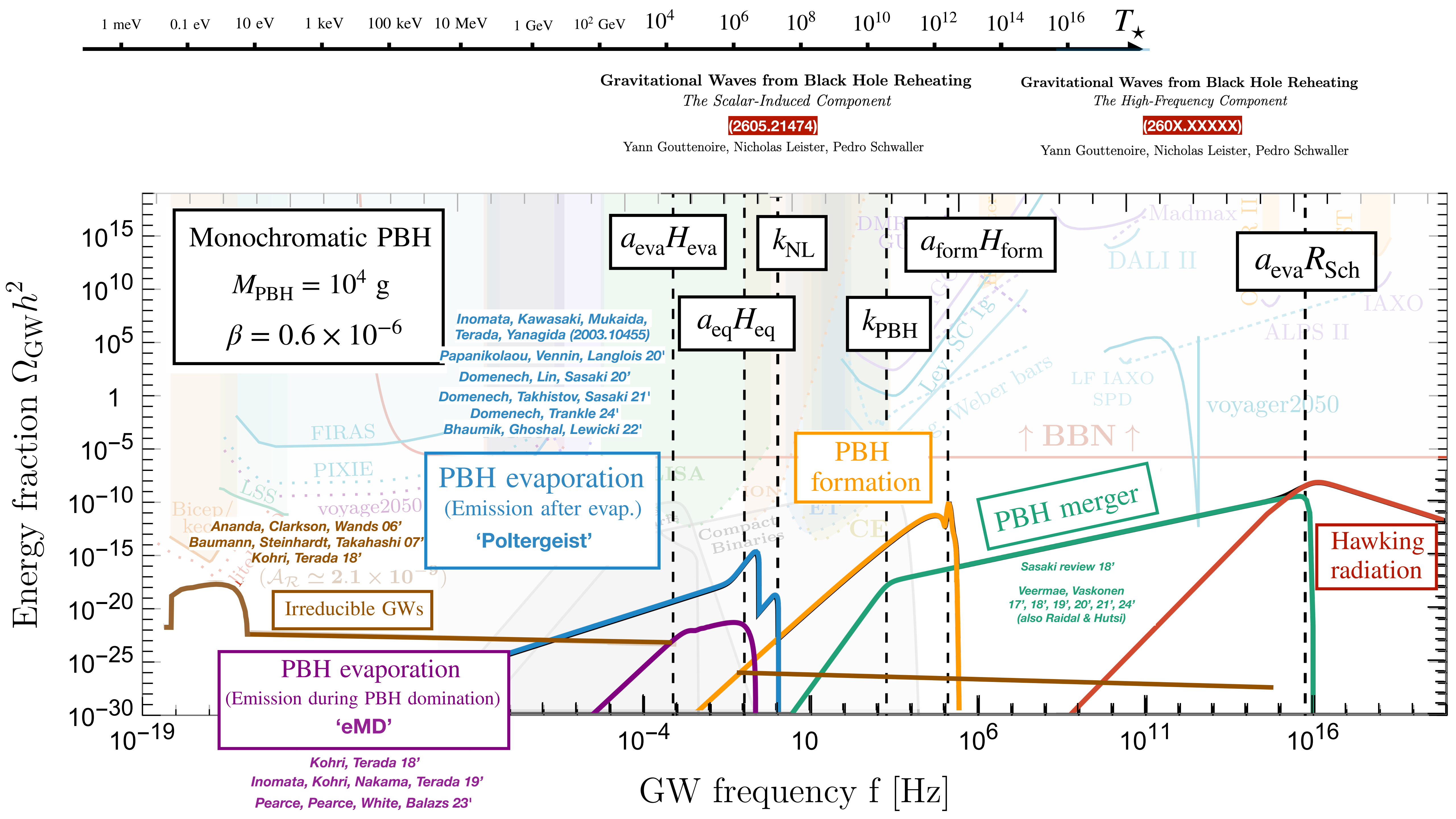


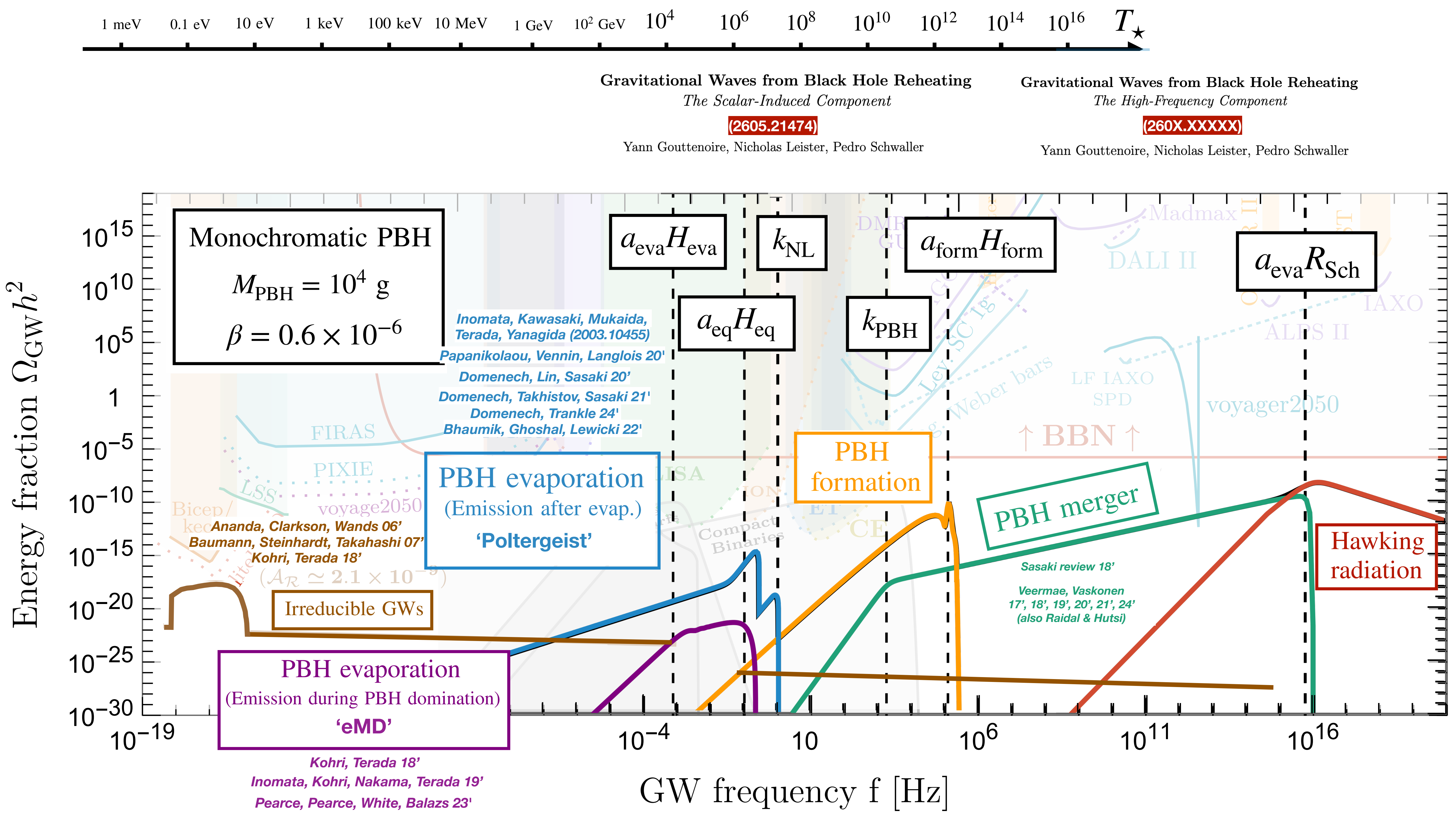










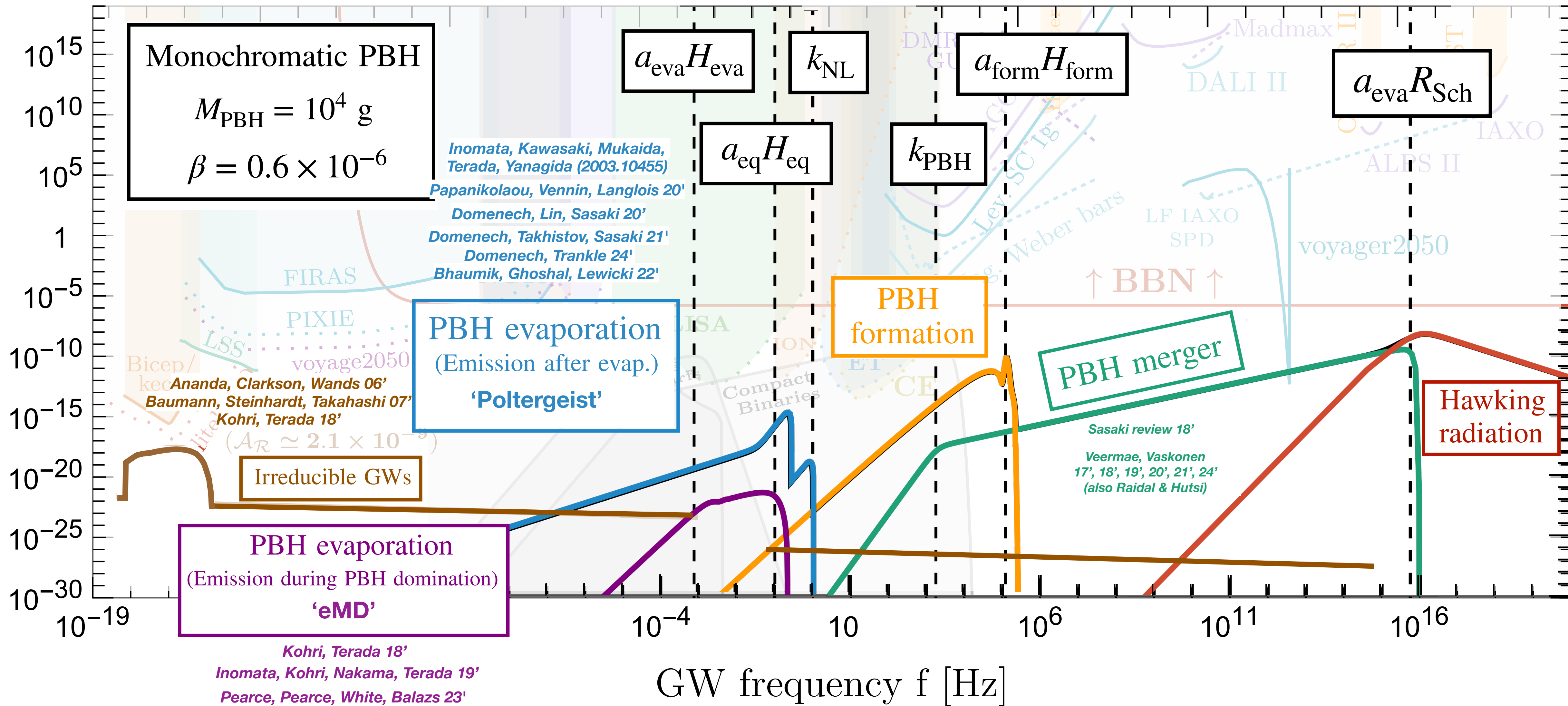


$$\Omega_{\text{GW}}(k) = \frac{k^2}{12 \mathcal{H}^2} 8 \int_0^\infty dv \int_{|1-v|}^{1+v} du \mathcal{K}(u, v) \left[\overline{I}_\Phi^2 \mathcal{P}_\Phi(uk) \mathcal{P}_\Phi(vk) + \overline{I}_S^2 \mathcal{P}_S(uk) \mathcal{P}_S(vk) \right]$$

$$\mathcal{K}(u, v) = \left[\frac{4v^2 - (1+v^2 - u^2)^2}{4uv} \right]^2; \text{ kernels } \overline{I}_X^2 \text{ set by the era; the two uncorrelated seeds add:}$$

— **Adiabatic** — Φ from inflation, CMB-normalised $\mathcal{A}_\mathcal{R} \simeq 2.1 \times 10^{-9}$
 **Isocurvature** — $S \simeq \delta_{\text{PBH}}$, PBH Poisson noise $\mathcal{P}_S = \frac{2}{3\pi} (k/k_{\text{PBH}})^3$

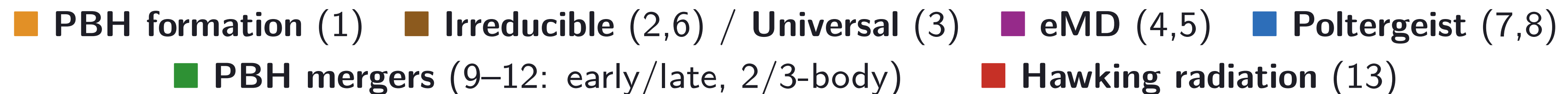
Energy fraction $\Omega_{\text{GW}} h^2$



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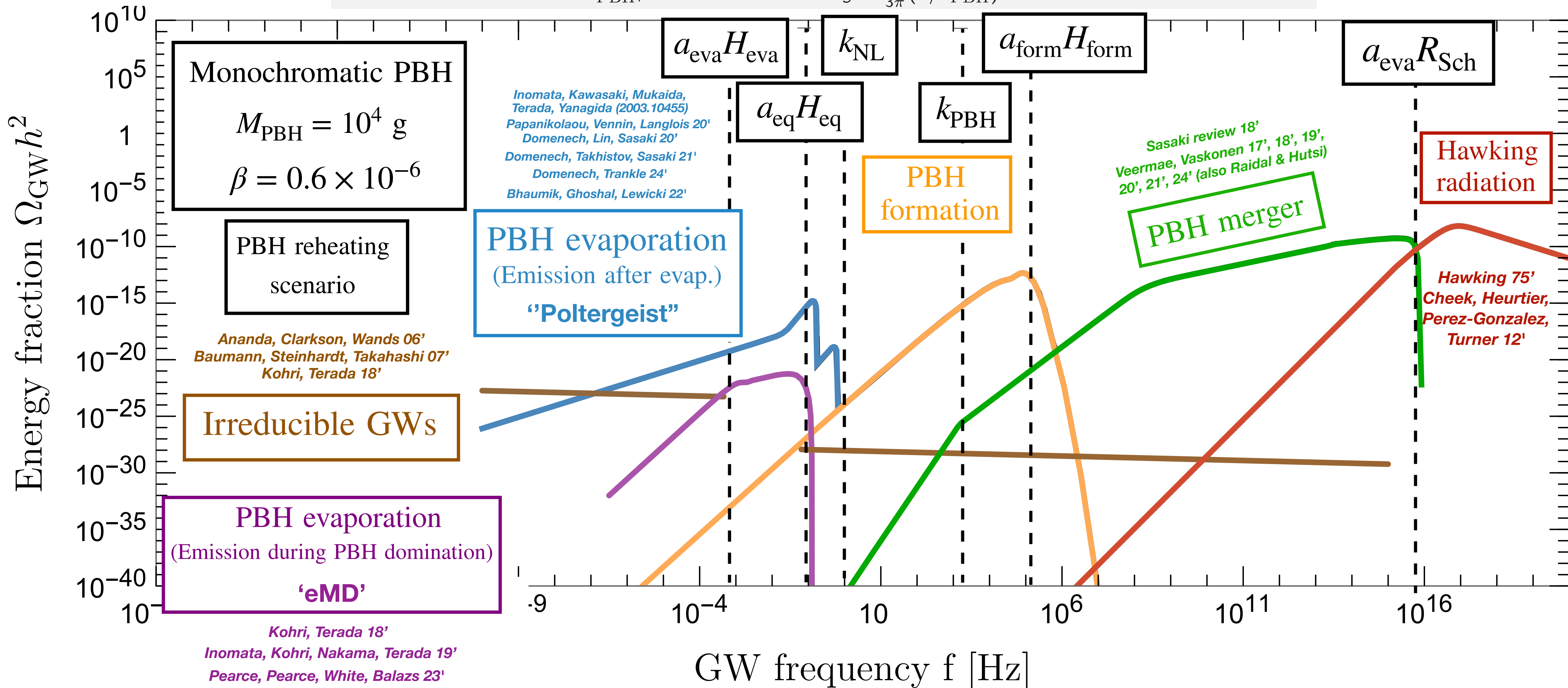


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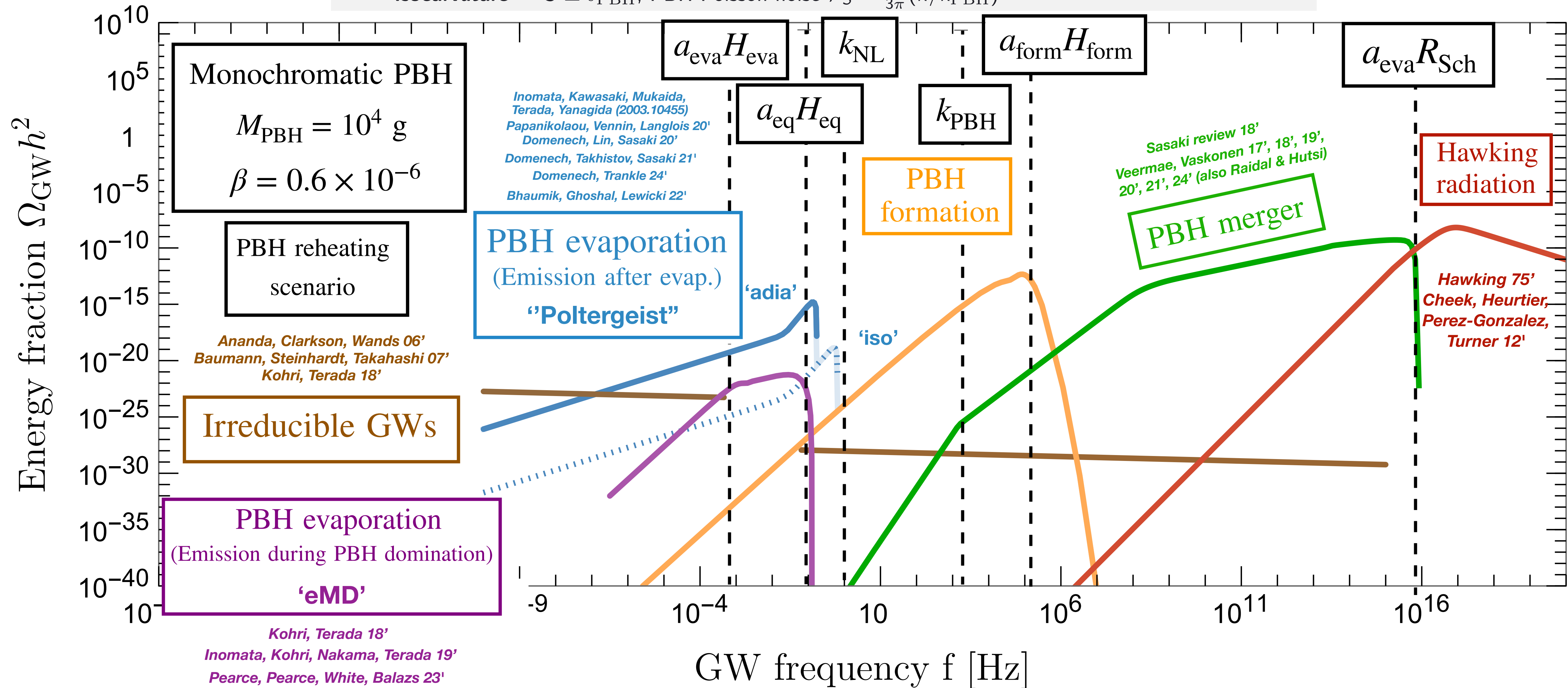


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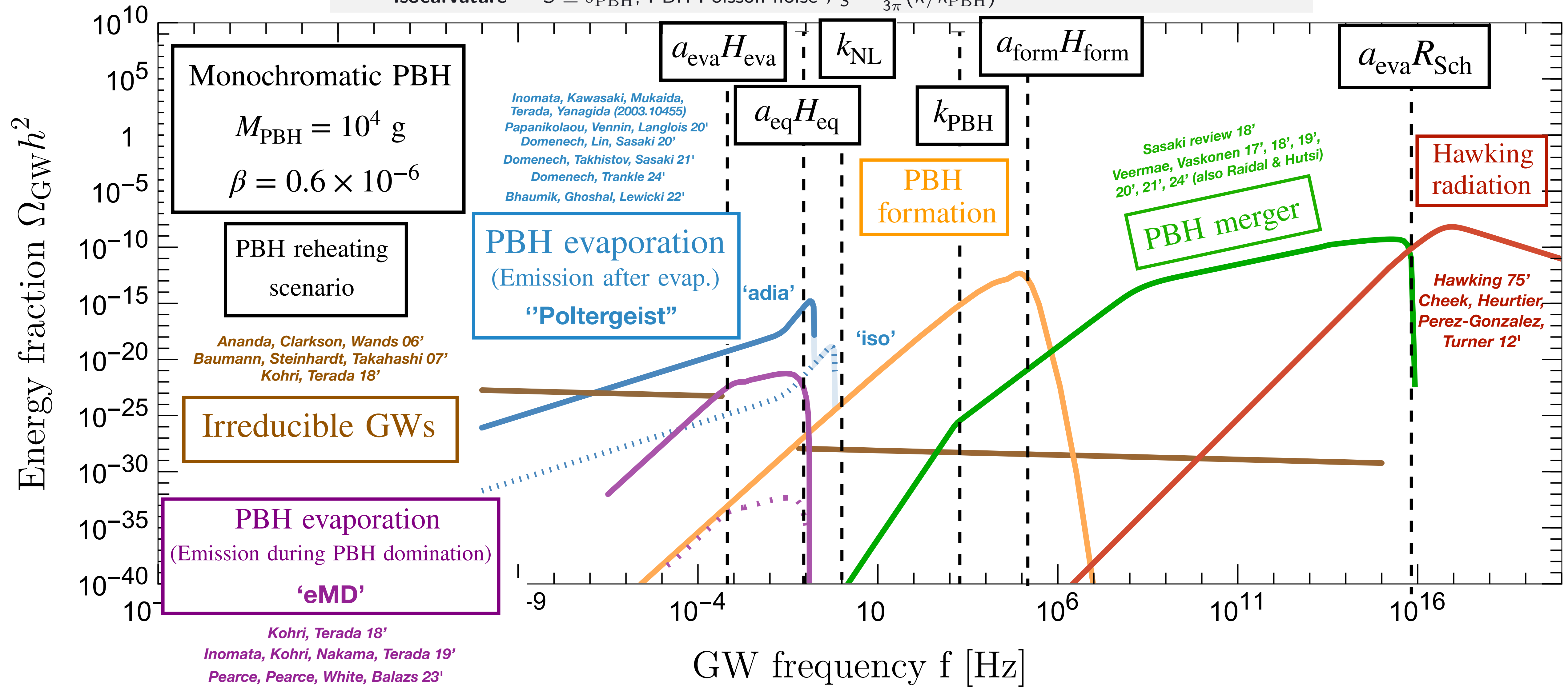
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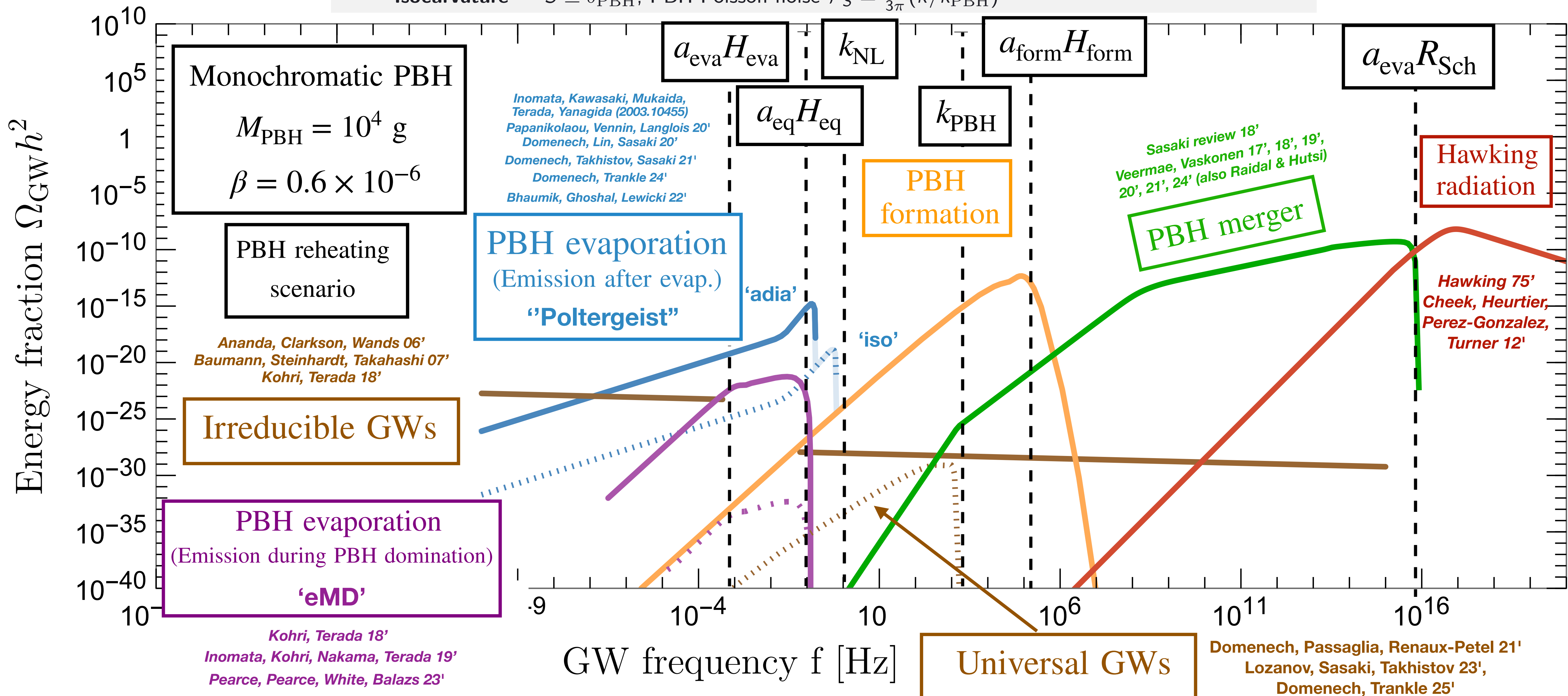
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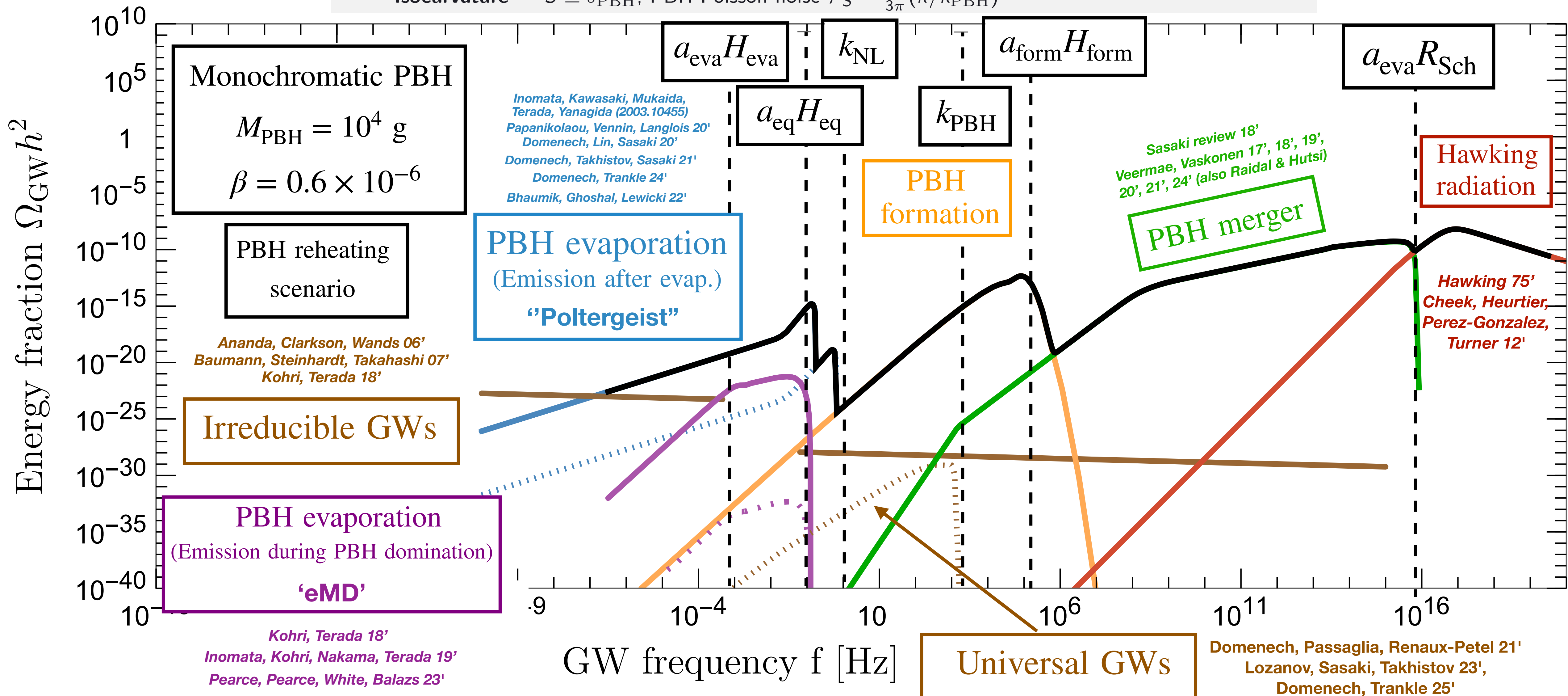
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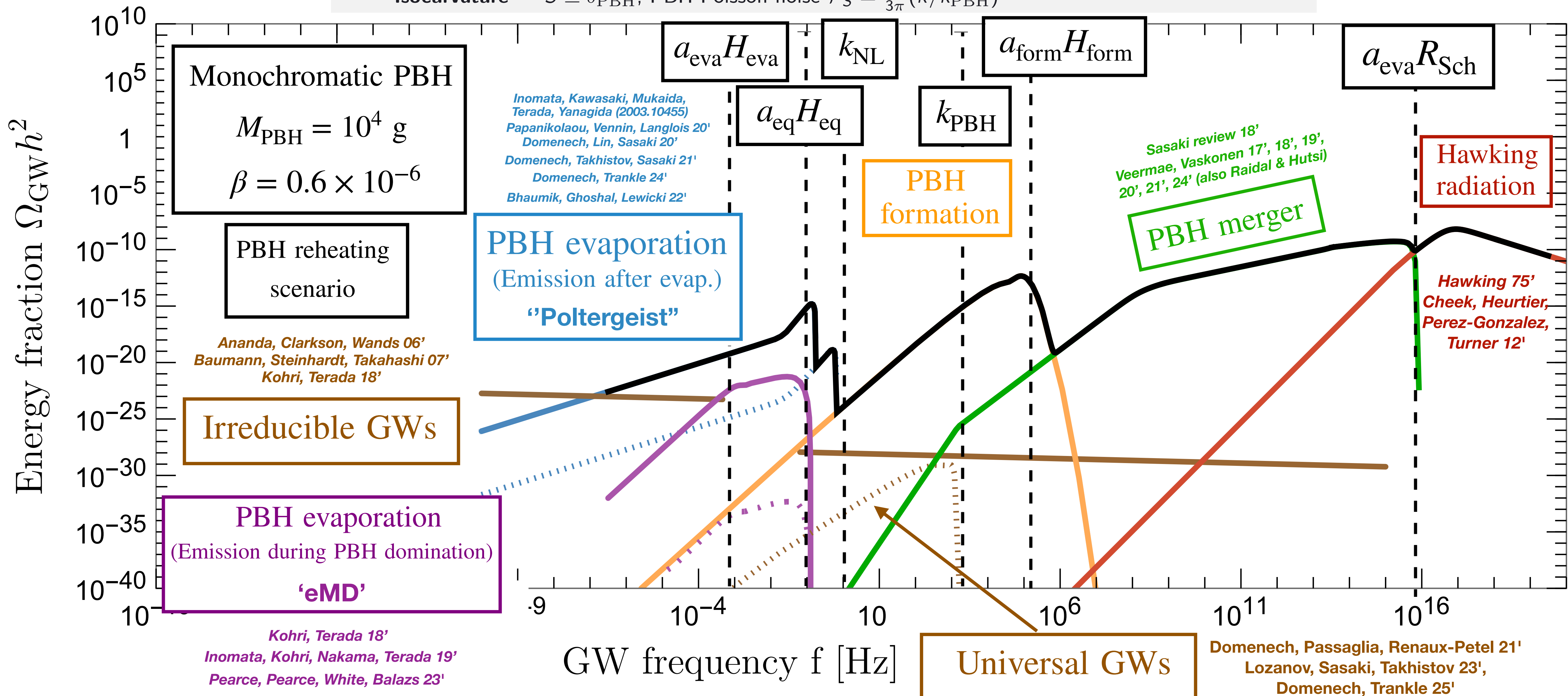
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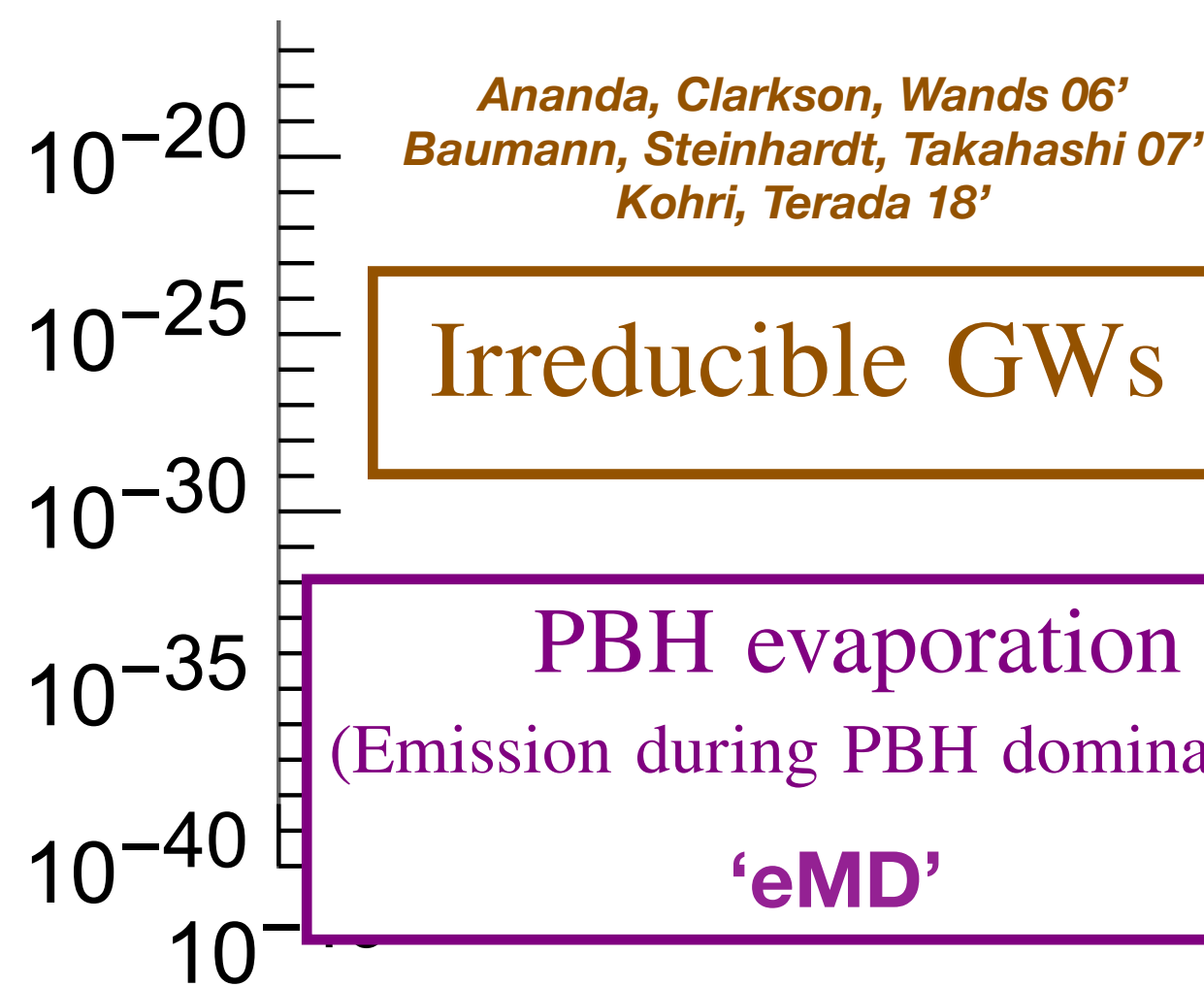
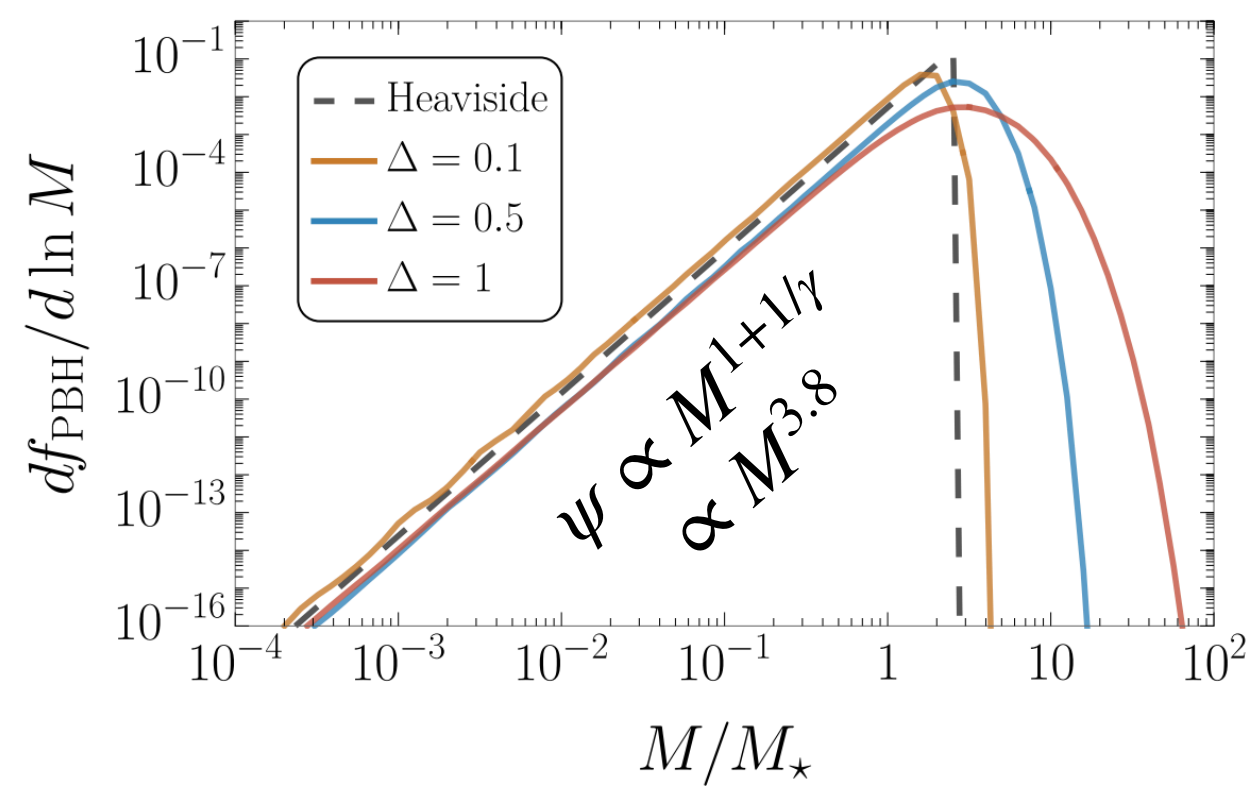
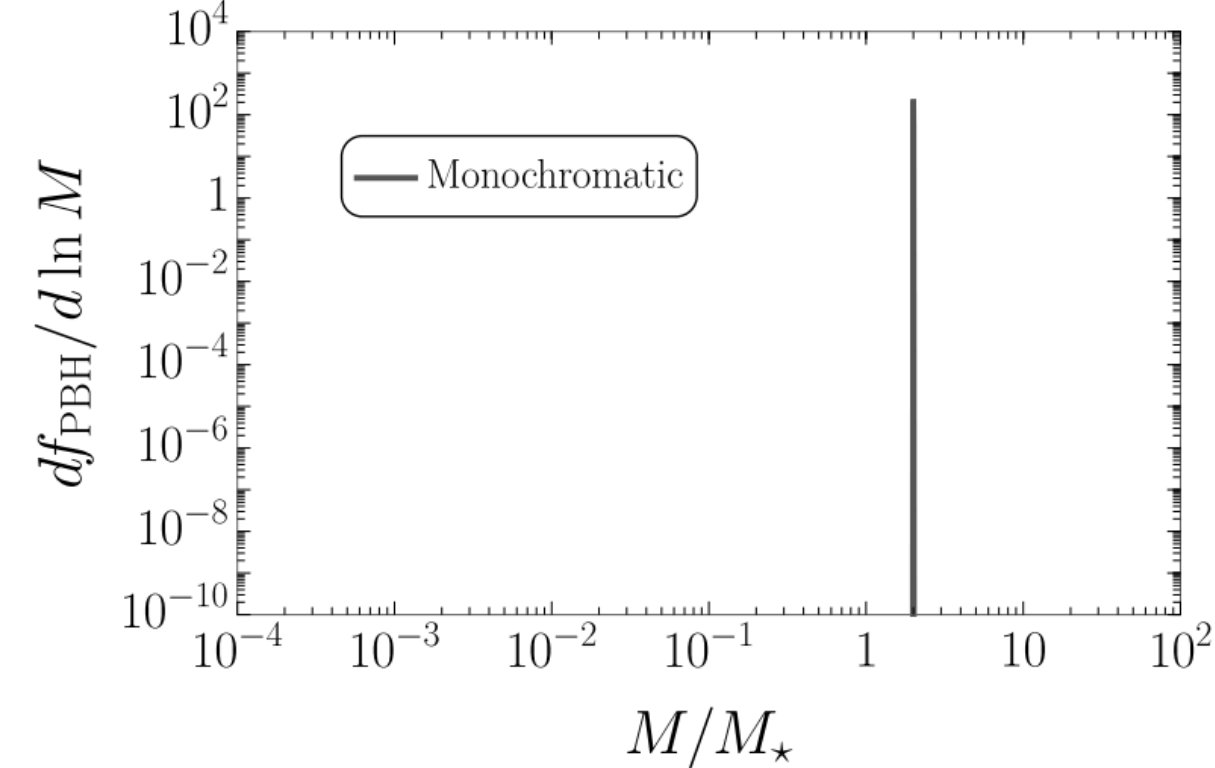
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Energy fraction $\Omega_{\text{GW}} h^2$



Kohri, Terada 18'
Inomata, Kohri, Nakama, Terada 19'
Pearce, Pearce, White, Balazs 23'

$$M_{\text{PBH}} = M_{\text{H}}(\delta - \delta_c)^\gamma$$
$$\gamma \simeq 0.36$$

Choptuik's law
(Choptuik 93')

Gravitational Waves from Black Hole Reheating

The Scalar-Induced Component

(2605.21474)

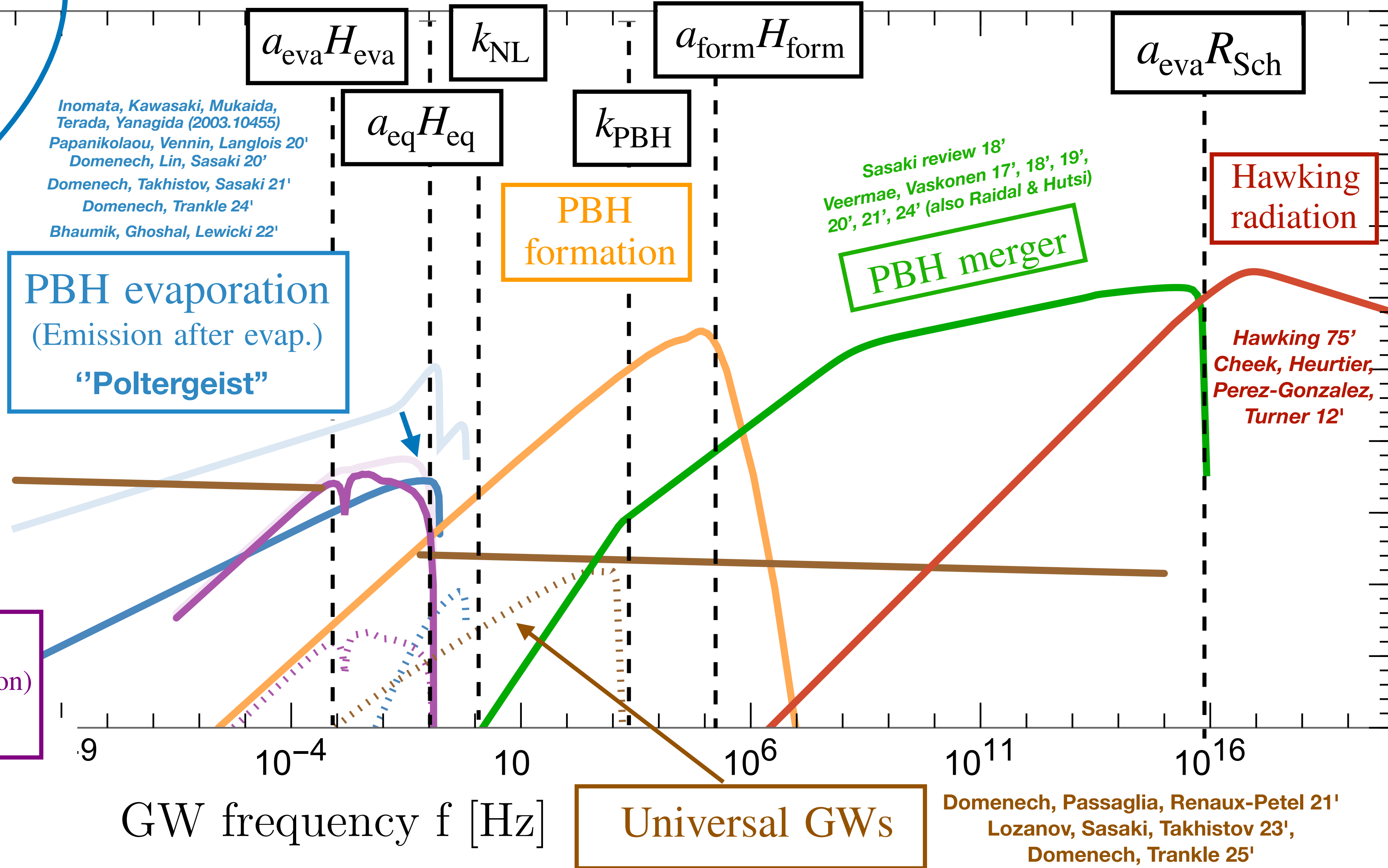
Gravitational Waves from Black Hole Reheating

The High-Frequency Component

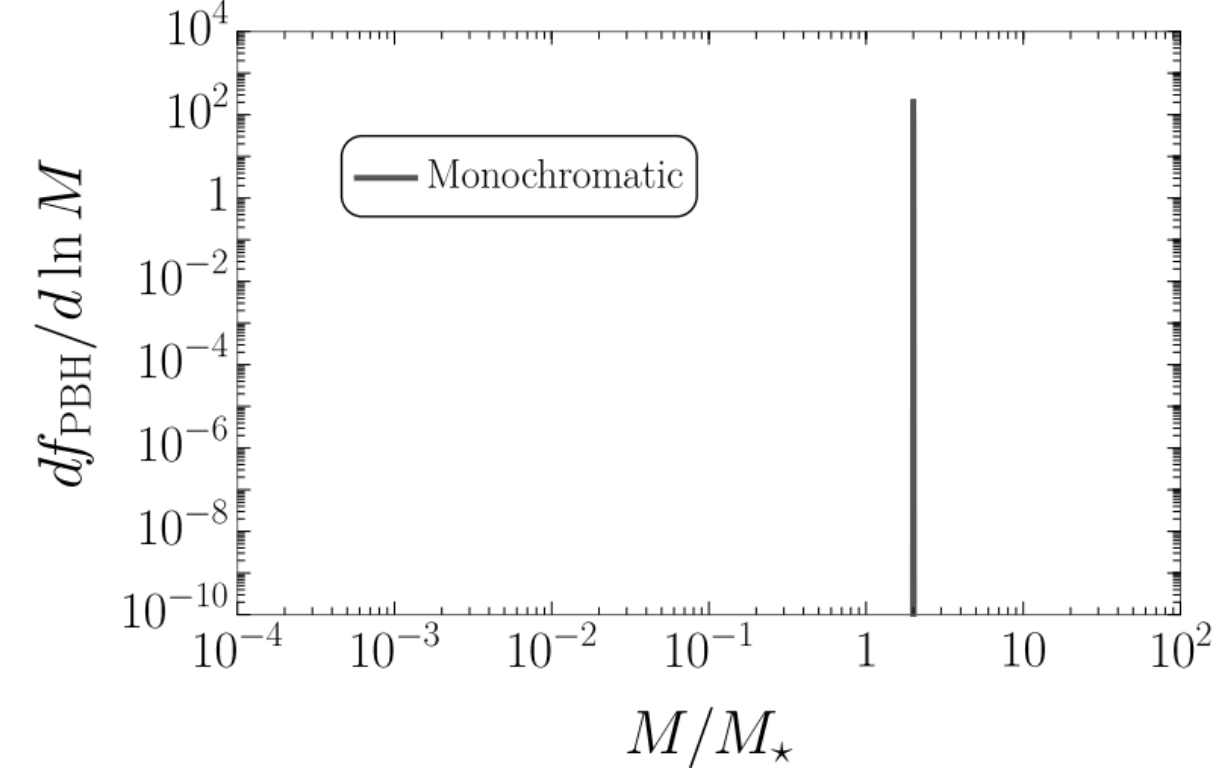
(260X.XXXXX)

Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

$$M_{\text{PBH}} = 10^4 \text{ g}$$
$$\beta = 0.6 \times 10^{-6}$$

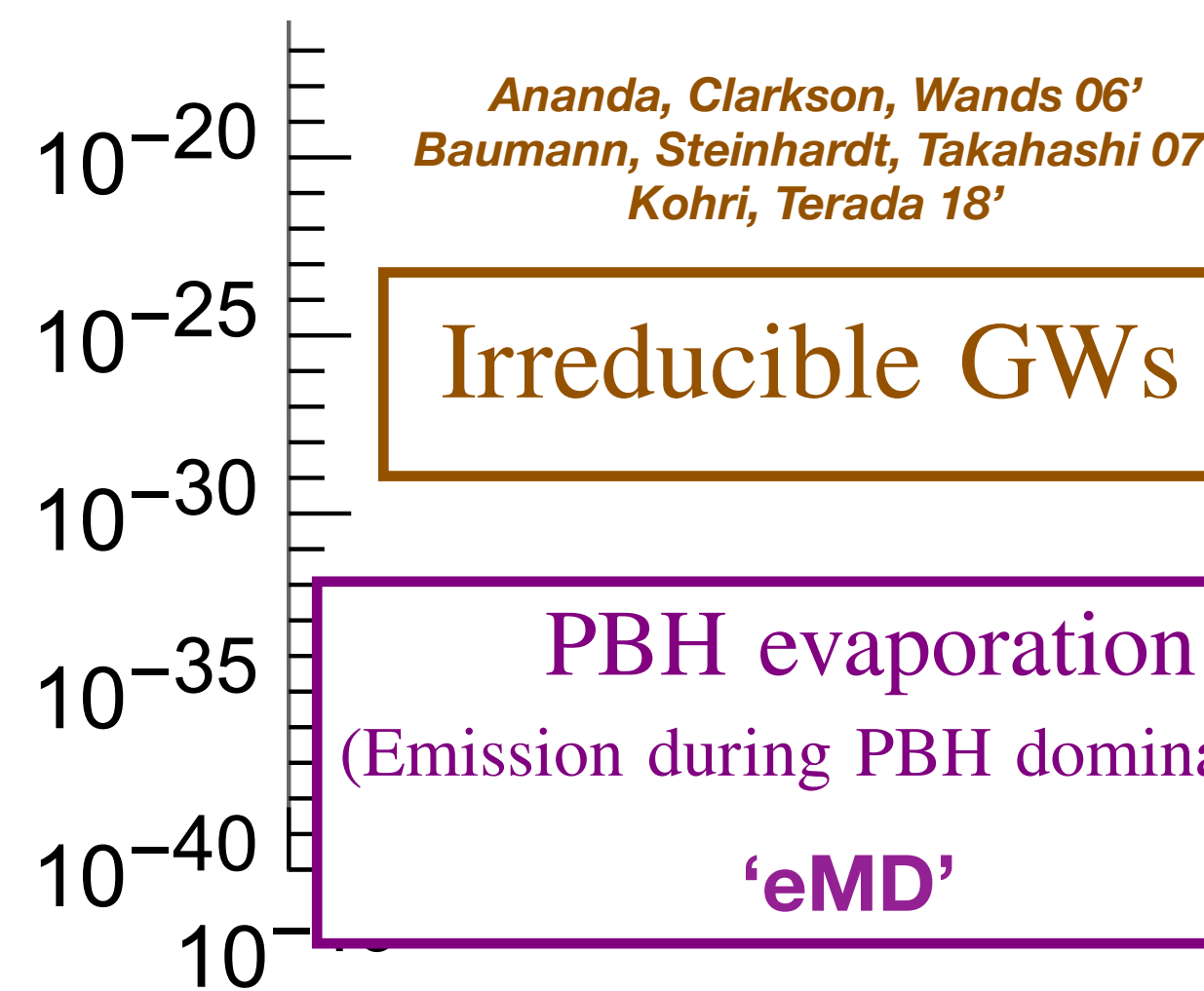
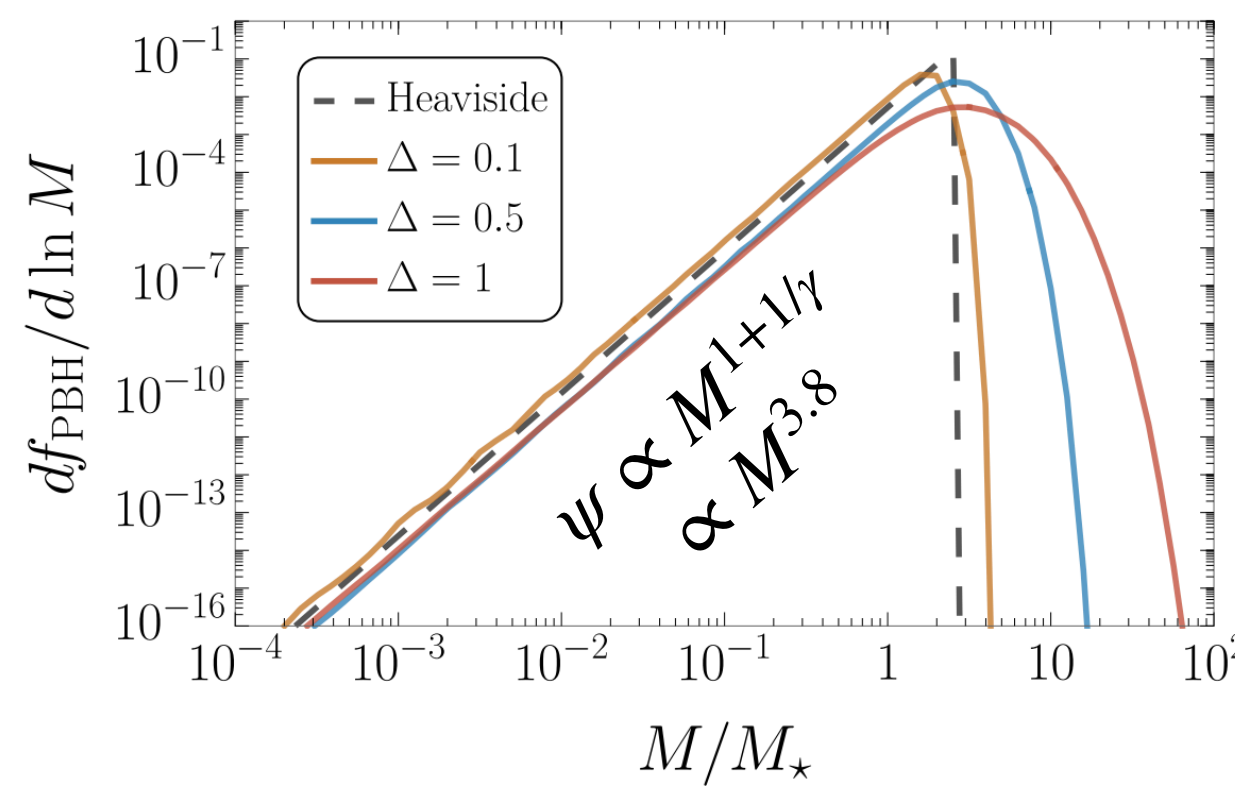


Energy fraction $\Omega_{\text{GW}} h^2$



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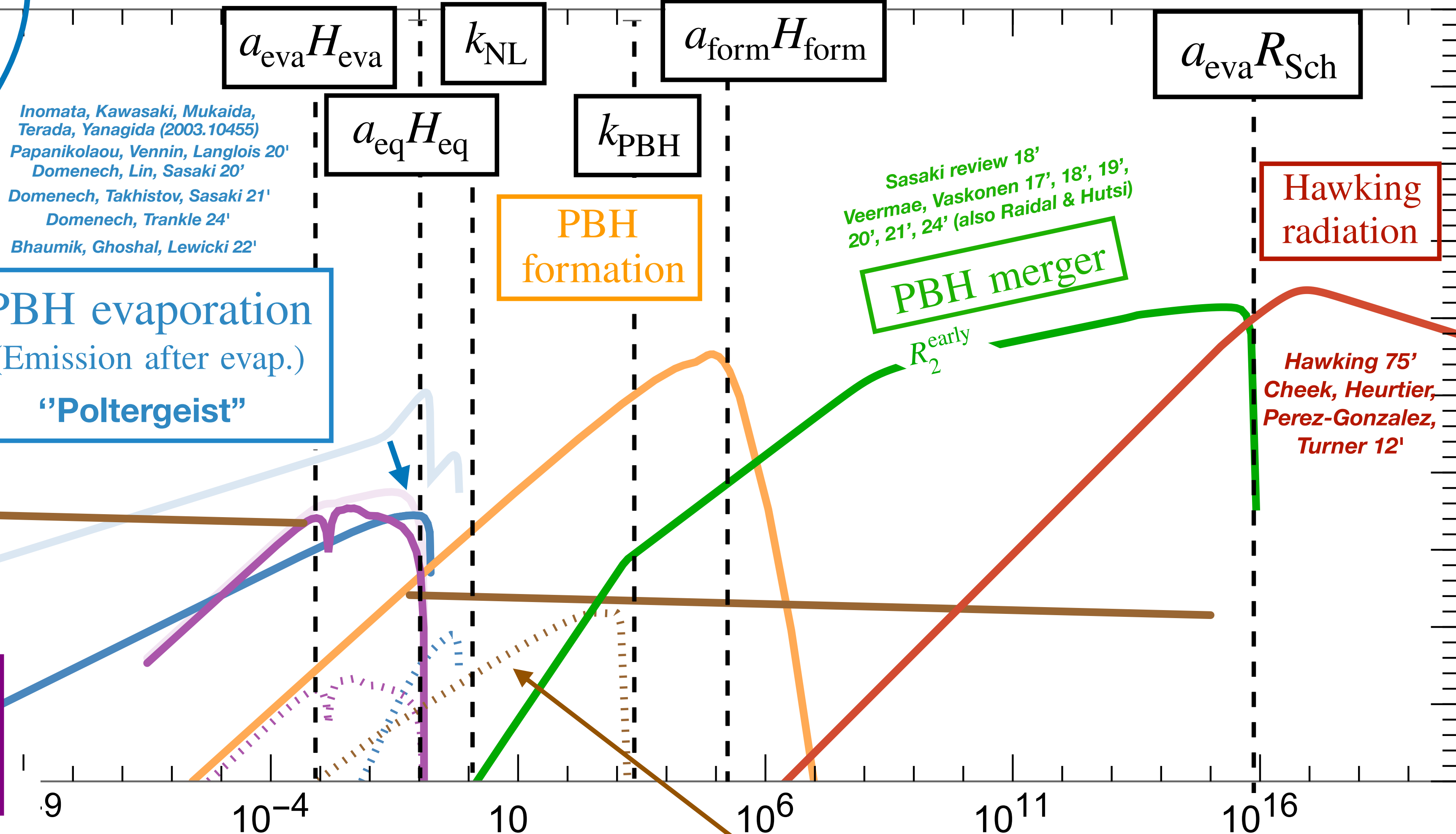


PBH evaporation
(Emission during PBH domination)
'eMD'

Kohri, Terada 18'
Inomata, Kohri, Nakama, Terada 19'
Pearce, Pearce, White, Balazs 23'

PBH evaporation
(Emission after evap.)
"Poltergeist"

Ananda, Clarkson, Wands 06'
Baumann, Steinhardt, Takahashi 07'
Kohri, Terada 18'



Domenech, Passaglia, Renaux-Petel 21'
Lozanov, Sasaki, Takhistov 23',
Domenech, Trankle 25'

Gravitational Waves from Black Hole Reheating

The Scalar-Induced Component

(2605.21474)

Gravitational Waves from Black Hole Reheating

The High-Frequency Component

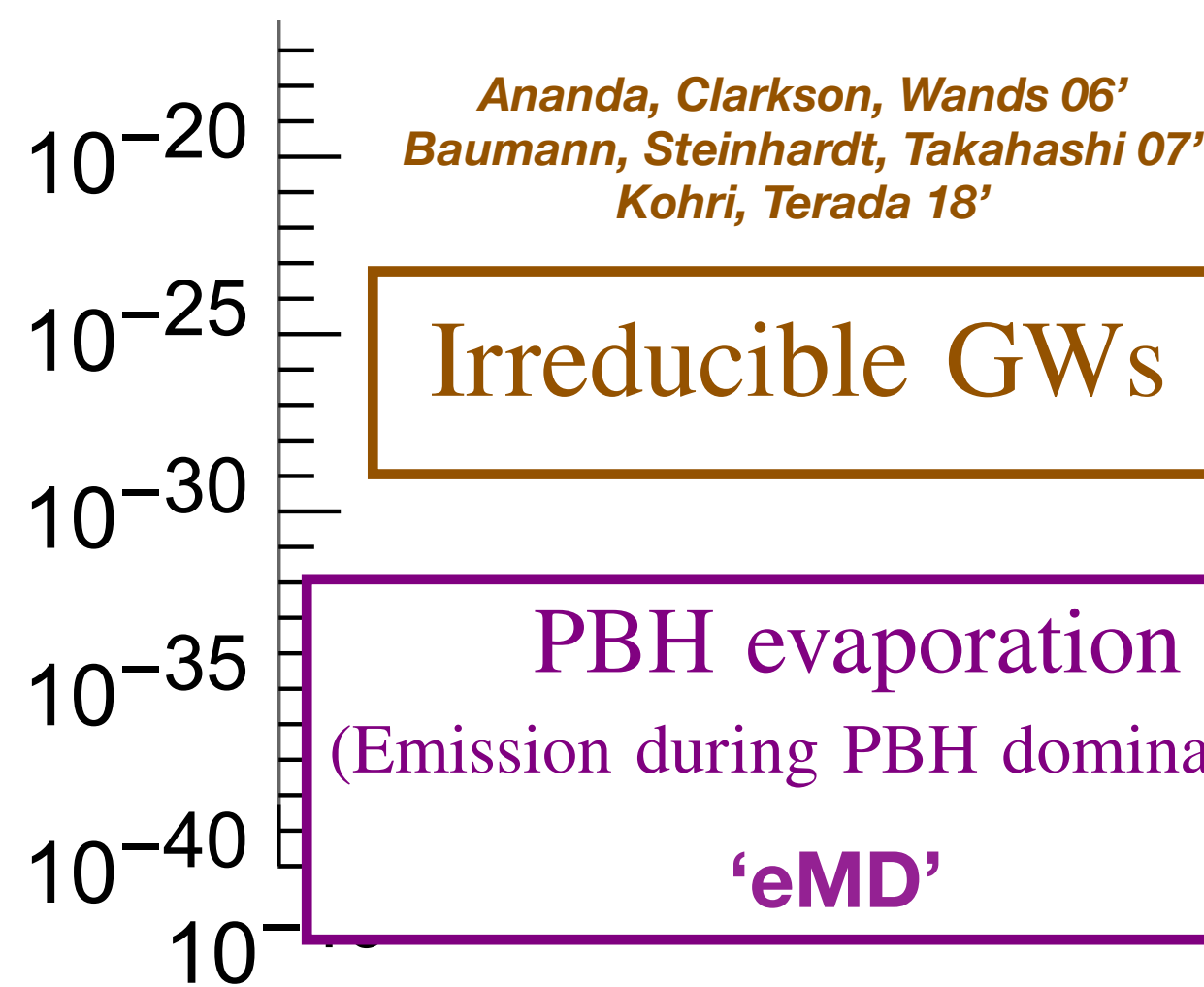
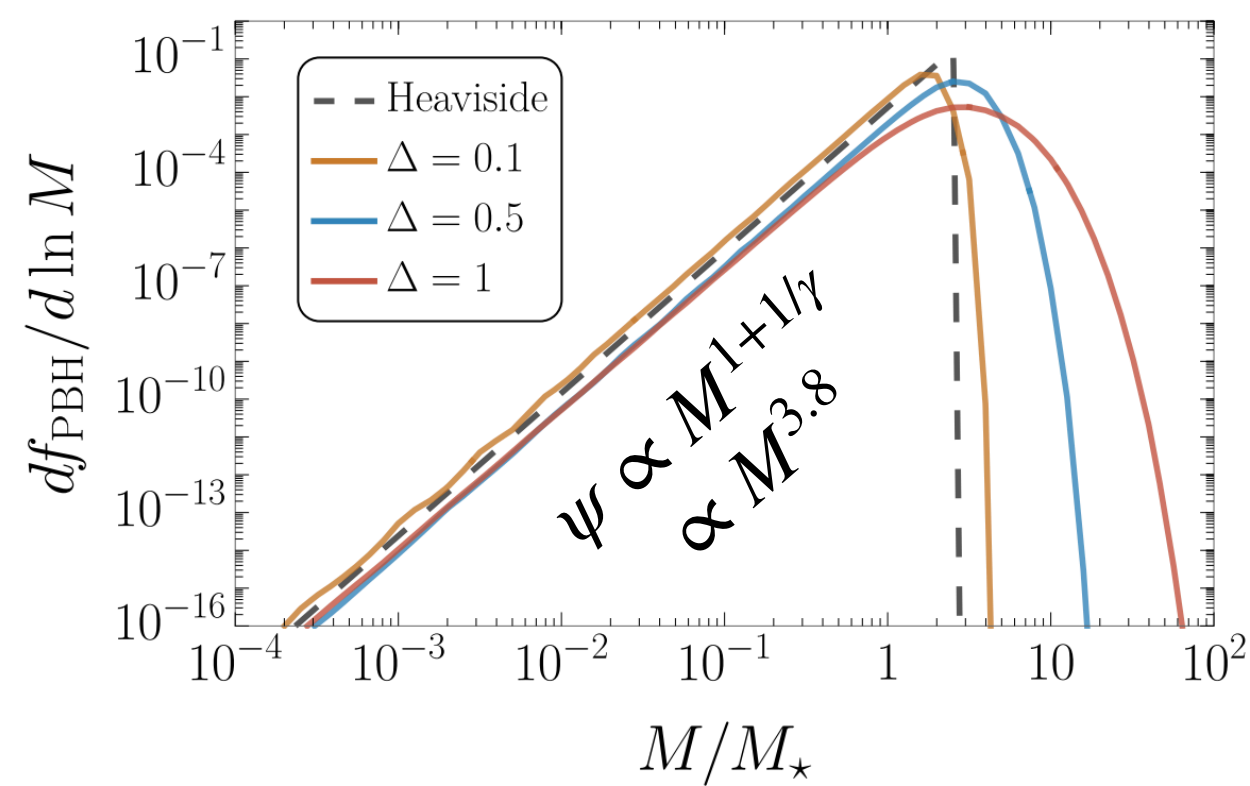
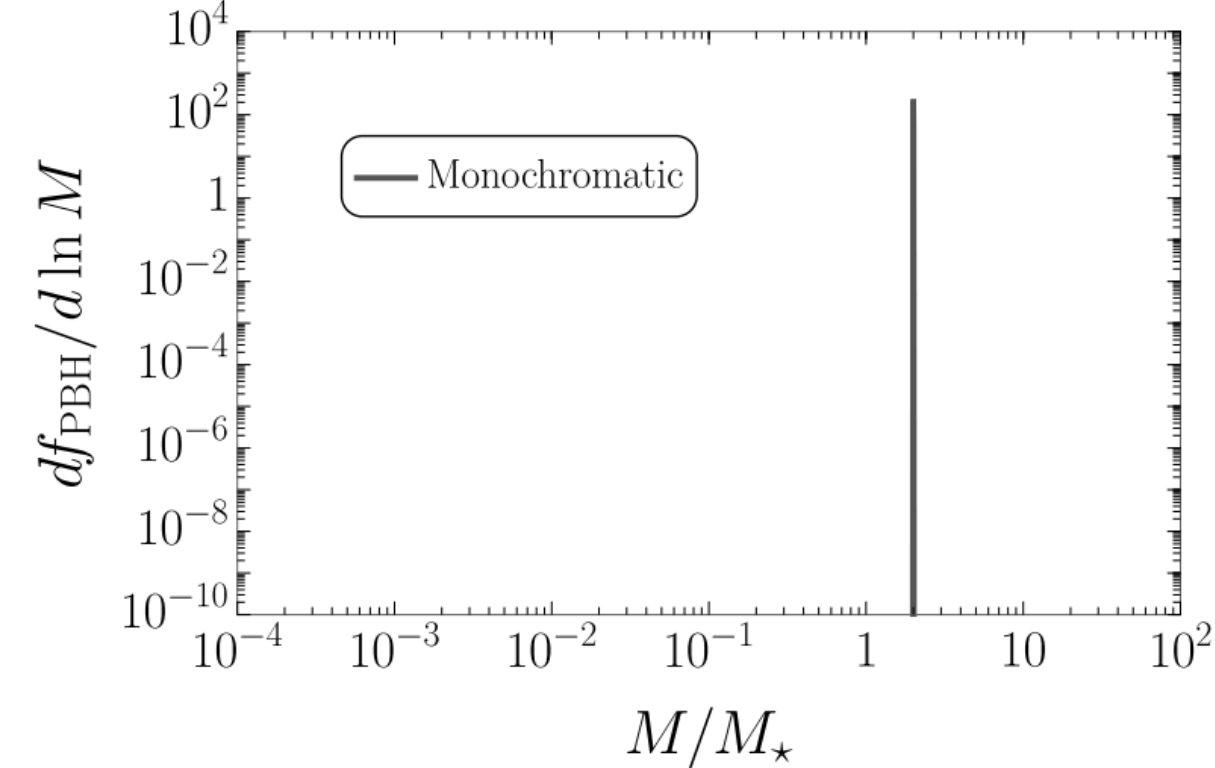
(260X.XXXXX)

Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

$$M_{\text{PBH}} = 10^4 \text{ g}$$
$$\beta = 0.6 \times 10^{-6}$$

Sasaki review 18'
Veermæ, Vaskonen 17', 18', 19',
20', 21', 24' (also Raidal & Hutsi)

Energy fraction $\Omega_{\text{GW}} h^2$



Kohri, Terada 18'
Inomata, Kohri, Nakama, Terada 19'
Pearce, Pearce, White, Balazs 23'

$$M_{\text{PBH}} = M_{\text{H}}(\delta - \delta_c)^\gamma$$
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Choptuik's law
(Choptuik 93')

Gravitational Waves from Black Hole Reheating

The Scalar-Induced Component

(2605.21474)

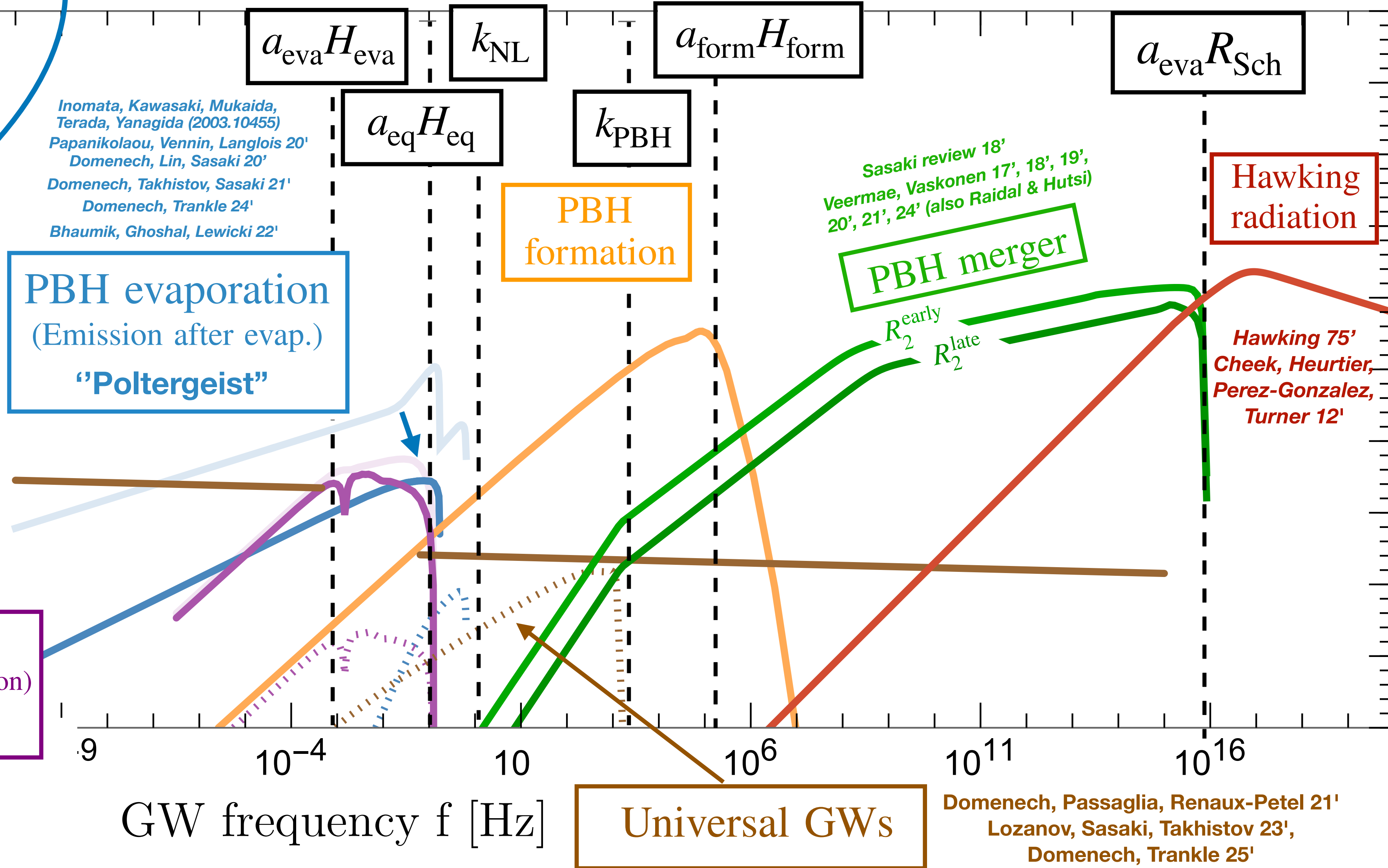
Gravitational Waves from Black Hole Reheating

The High-Frequency Component

(260X.XXXXX)

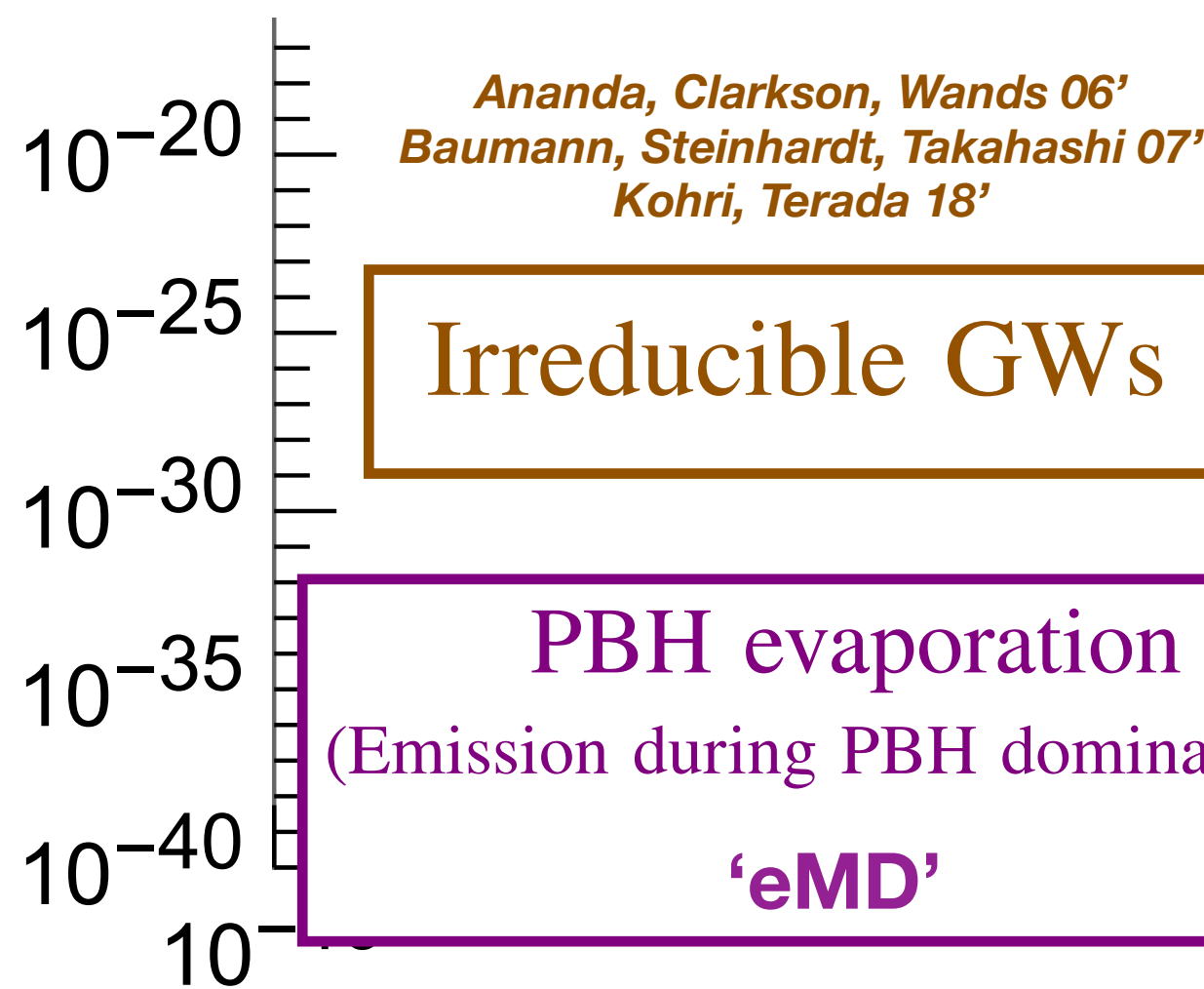
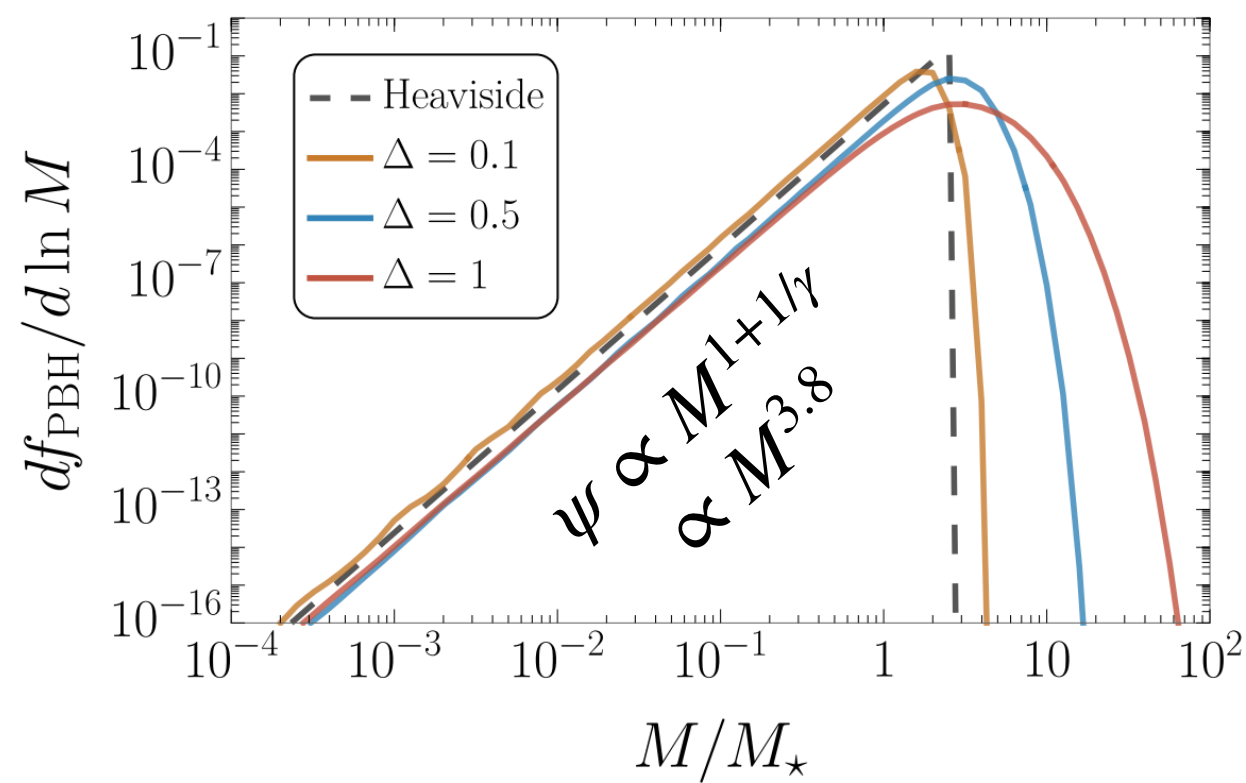
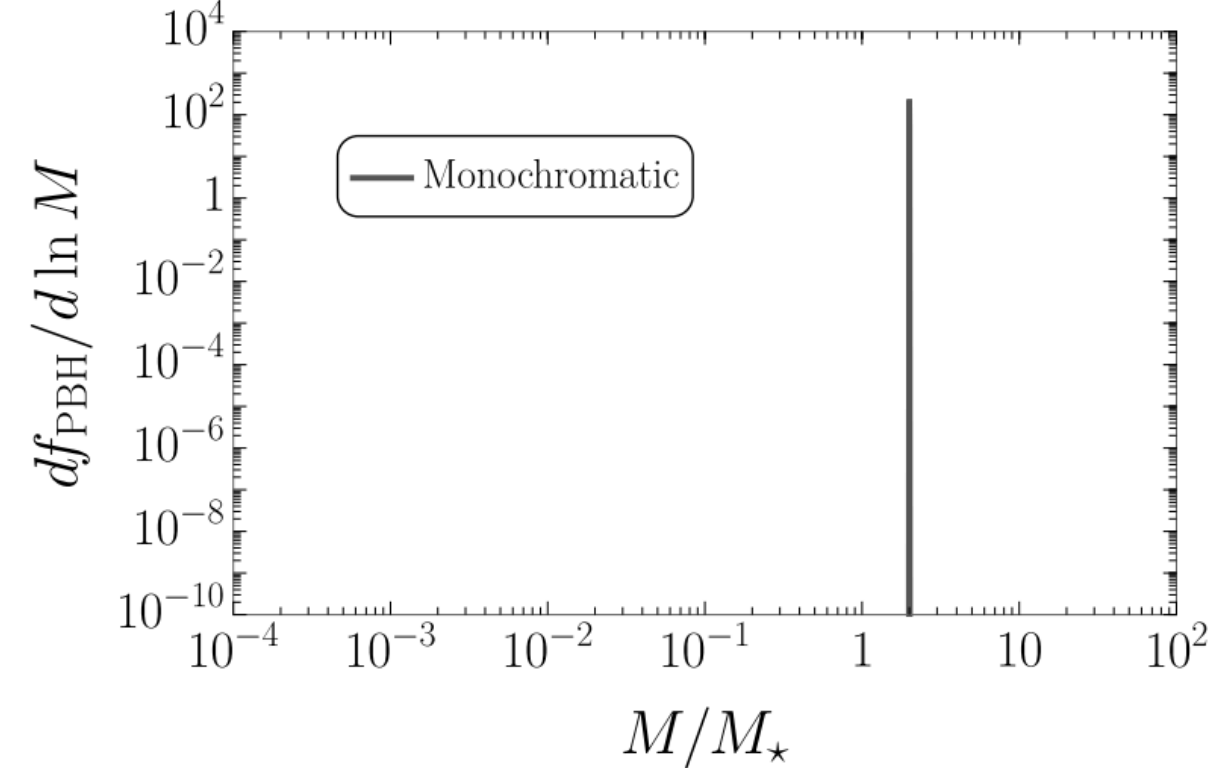
Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

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Domenech, Passaglia, Renaux-Petel 21'
Lozanov, Sasaki, Takhistov 23',
Domenech, Trankle 25'

Energy fraction $\Omega_{\text{GW}} h^2$



Kohri, Terada 18'
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Choptuik's law
(Choptuik 93')

**PBH evaporation
(Emission after evap.)
"Poltergeist"**

*Inomata, Kawasaki, Mukaida,
Terada, Yanagida (2003.10455)*
Papanikolaou, Vennin, Langlois 20'
Domenech, Lin, Sasaki 20'
Domenech, Takhistov, Sasaki 21'
Domenech, Trankle 24'
Bhaumik, Ghoshal, Lewicki 22'

Gravitational Waves from Black Hole Reheating

The Scalar-Induced Component

(2605.21474)

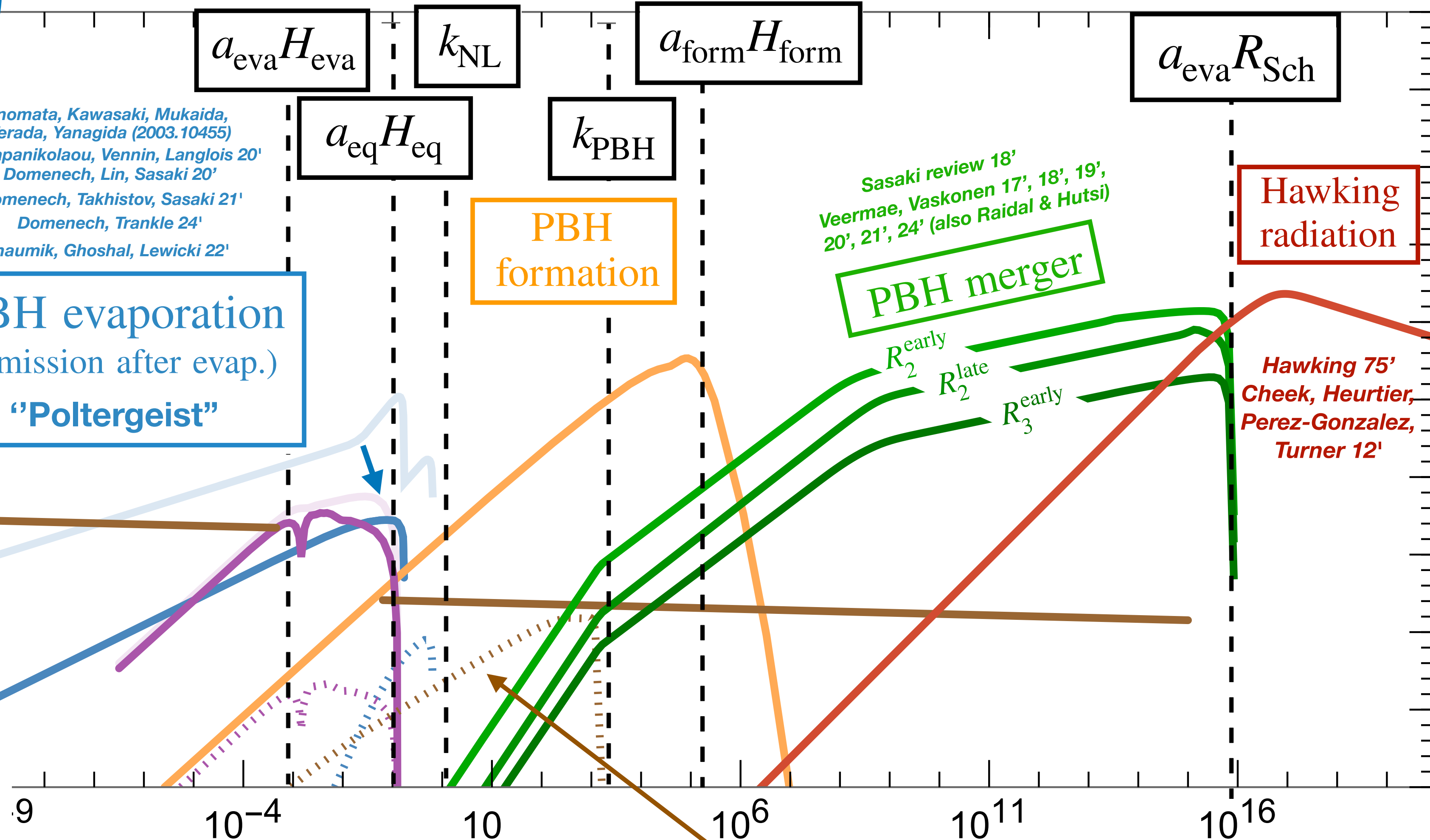
Gravitational Waves from Black Hole Reheating

The High-Frequency Component

(260X.XXXXX)

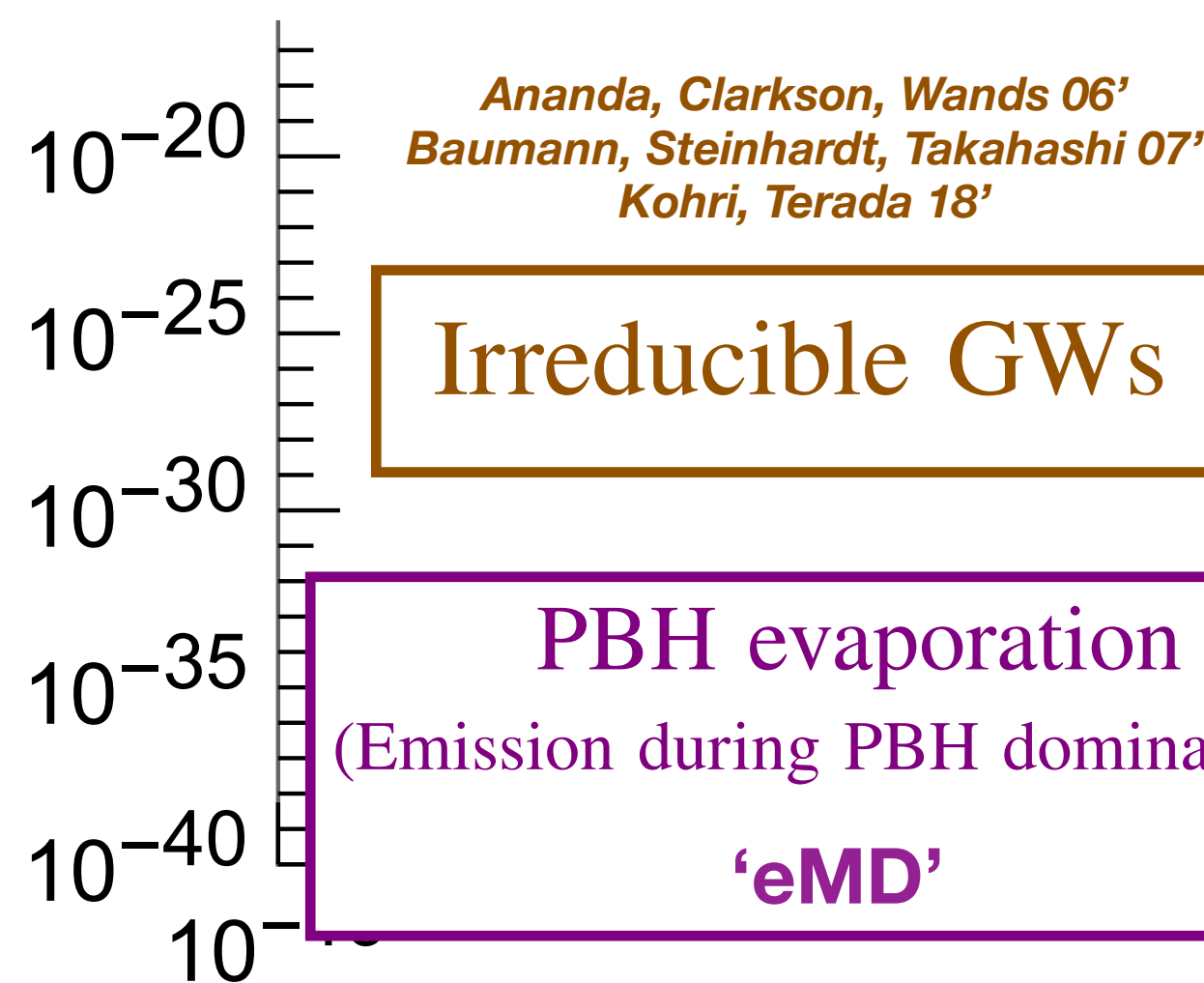
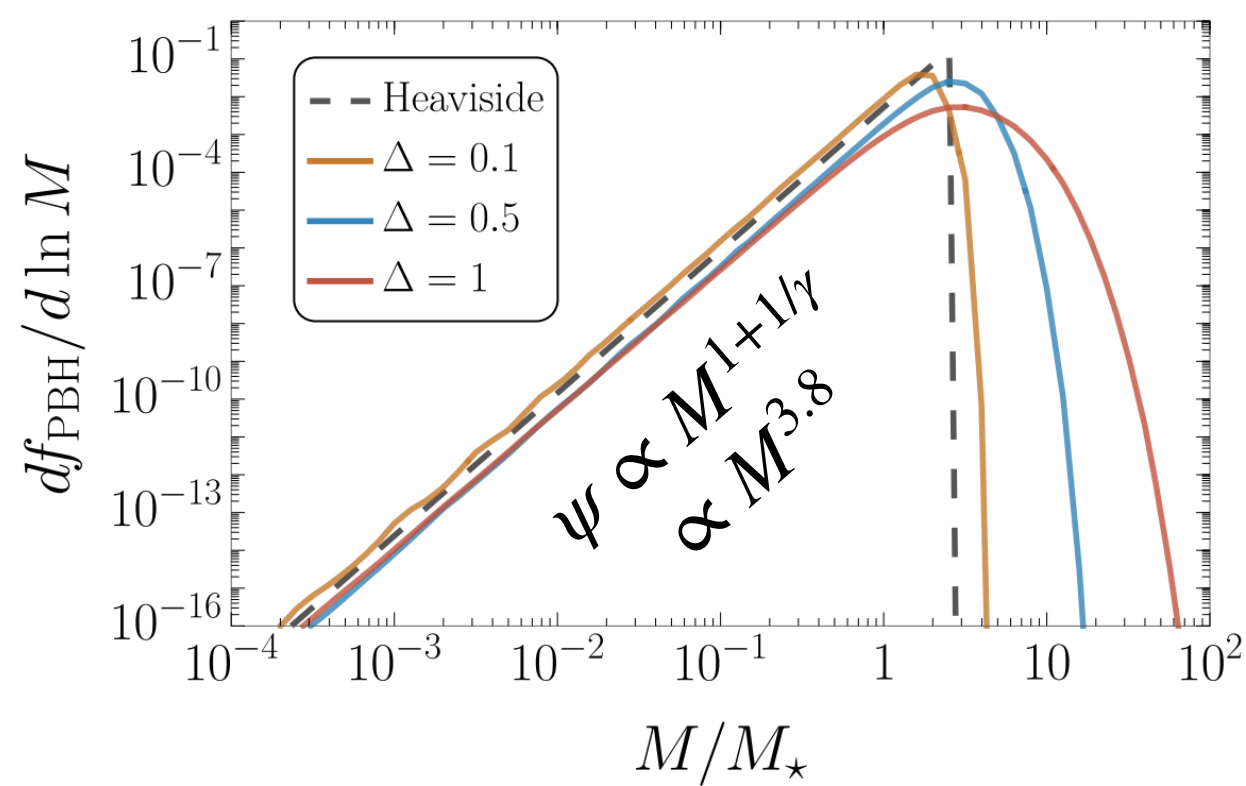
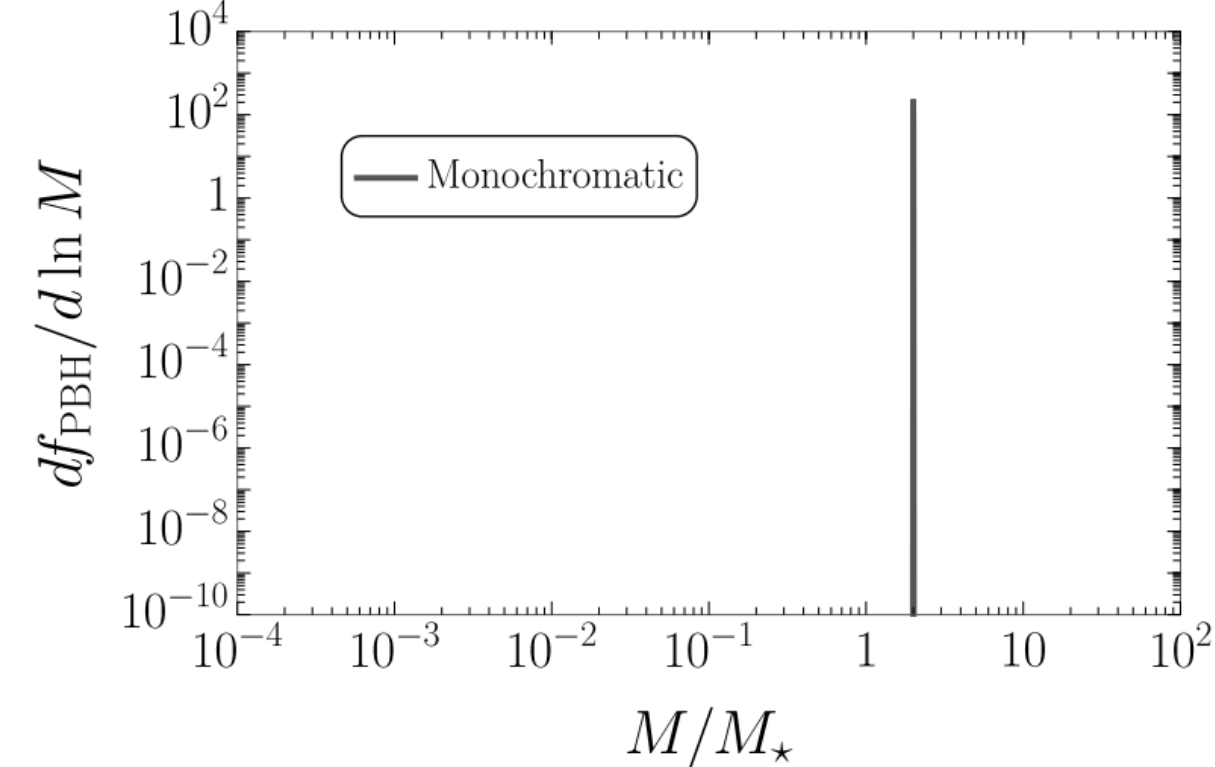
Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

$$M_{\text{PBH}} = 10^4 \text{ g}$$
$$\beta = 0.6 \times 10^{-6}$$



Domenech, Passaglia, Renaux-Petel 21'
Lozanov, Sasaki, Takhistov 23',
Domenech, Trankle 25'

Energy fraction $\Omega_{\text{GW}} h^2$



Kohri, Terada 18'
Inomata, Kohri, Nakama, Terada 19'
Pearce, Pearce, White, Balazs 23'

$$M_{\text{PBH}} = M_{\text{H}}(\delta - \delta_c)^\gamma$$
$$\gamma \simeq 0.36$$

Choptuik's law
(Choptuik 93')

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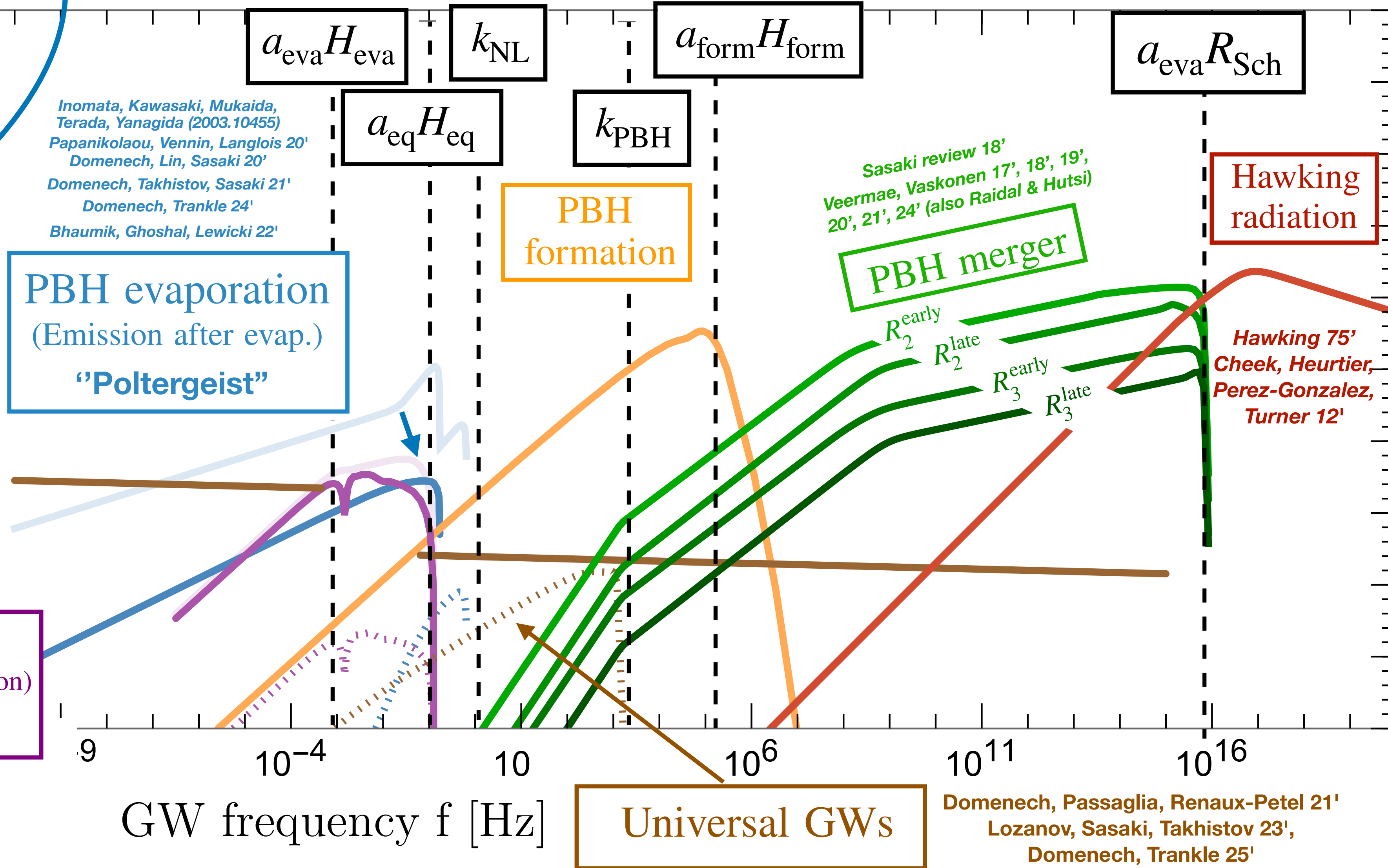
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Gravitational waves from PBH reheating

13 channels

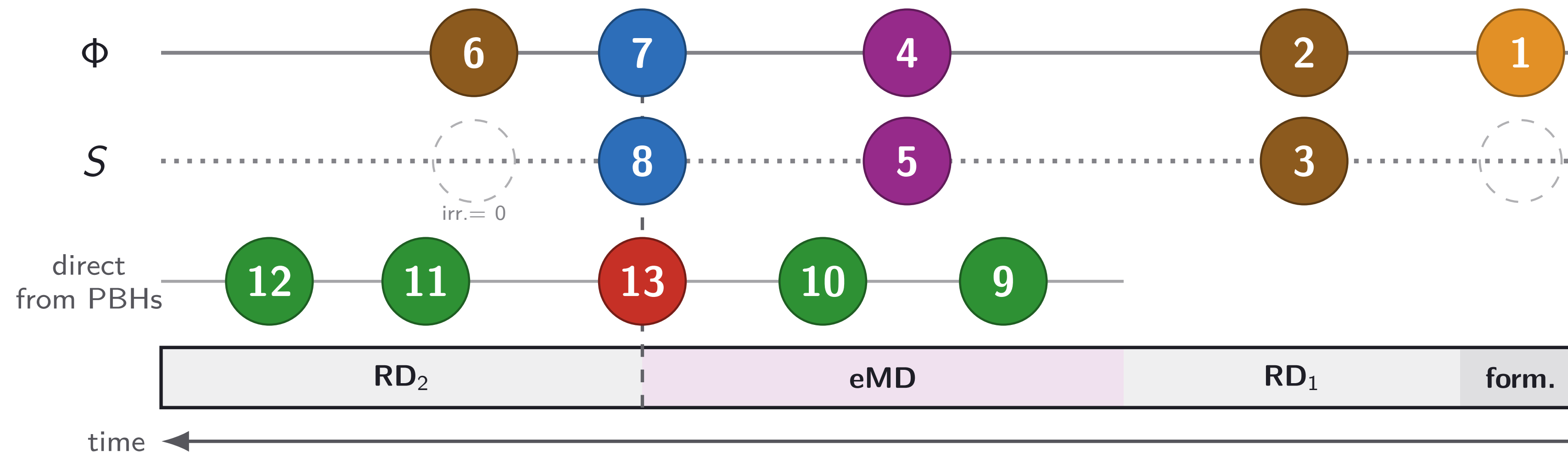
Two routes: **scalar-induced** GWs ($h \propto \Phi^2$, two seeds below) + **direct** emission (PBH binaries, Hawking).

$$\Omega_{\text{GW}}(k) = \frac{k^2}{12 \mathcal{H}^2} 8 \int_0^\infty dv \int_{|1-v|}^{1+v} du \mathcal{K}(u, v) \left[\overline{I}_\Phi^2 \mathcal{P}_\Phi(uk) \mathcal{P}_\Phi(vk) + \overline{I}_S^2 \mathcal{P}_S(uk) \mathcal{P}_S(vk) \right]$$

$$\mathcal{K}(u, v) = \left[\frac{4v^2 - (1+v^2 - u^2)^2}{4uv} \right]^2; \text{ kernels } \overline{I}_X^2 \text{ set by the era; the two uncorrelated seeds add:}$$

— **Adiabatic** — Φ from inflation, CMB-normalised $\mathcal{A}_\mathcal{R} \simeq 2.1 \times 10^{-9}$

..... **Isocurvature** — $S \simeq \delta_{\text{PBH}}$, PBH Poisson noise $\mathcal{P}_S = \frac{2}{3\pi} (k/k_{\text{PBH}})^3$

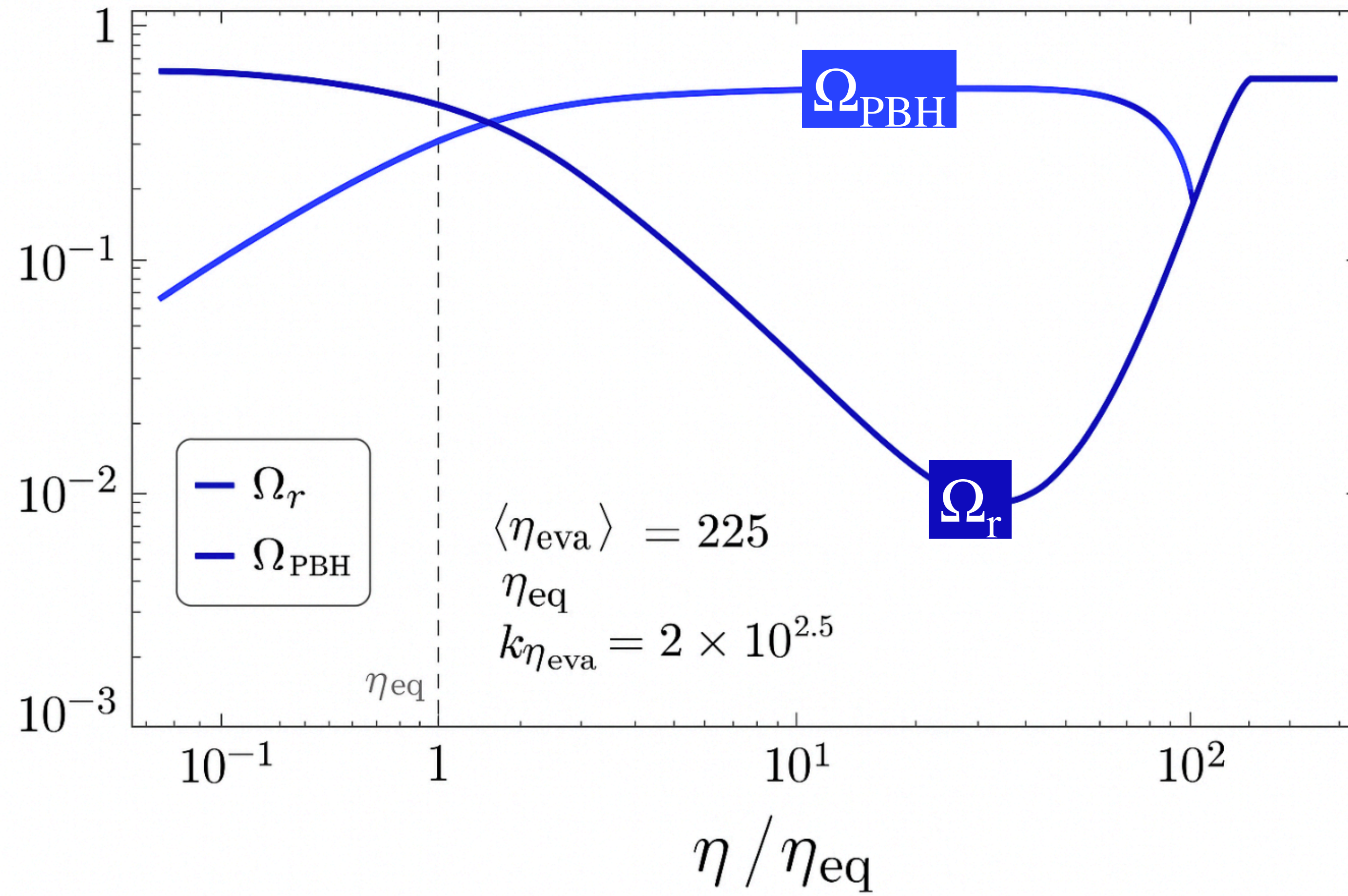


- PBH formation (1)
 ■ Irreducible (2,6) / Universal (3)
 ■ eMD (4,5)
 ■ Poltergeist (7,8)
- PBH mergers (9–12: early/late, 2/3-body)
 ■ Hawking radiation (13)

Background equations

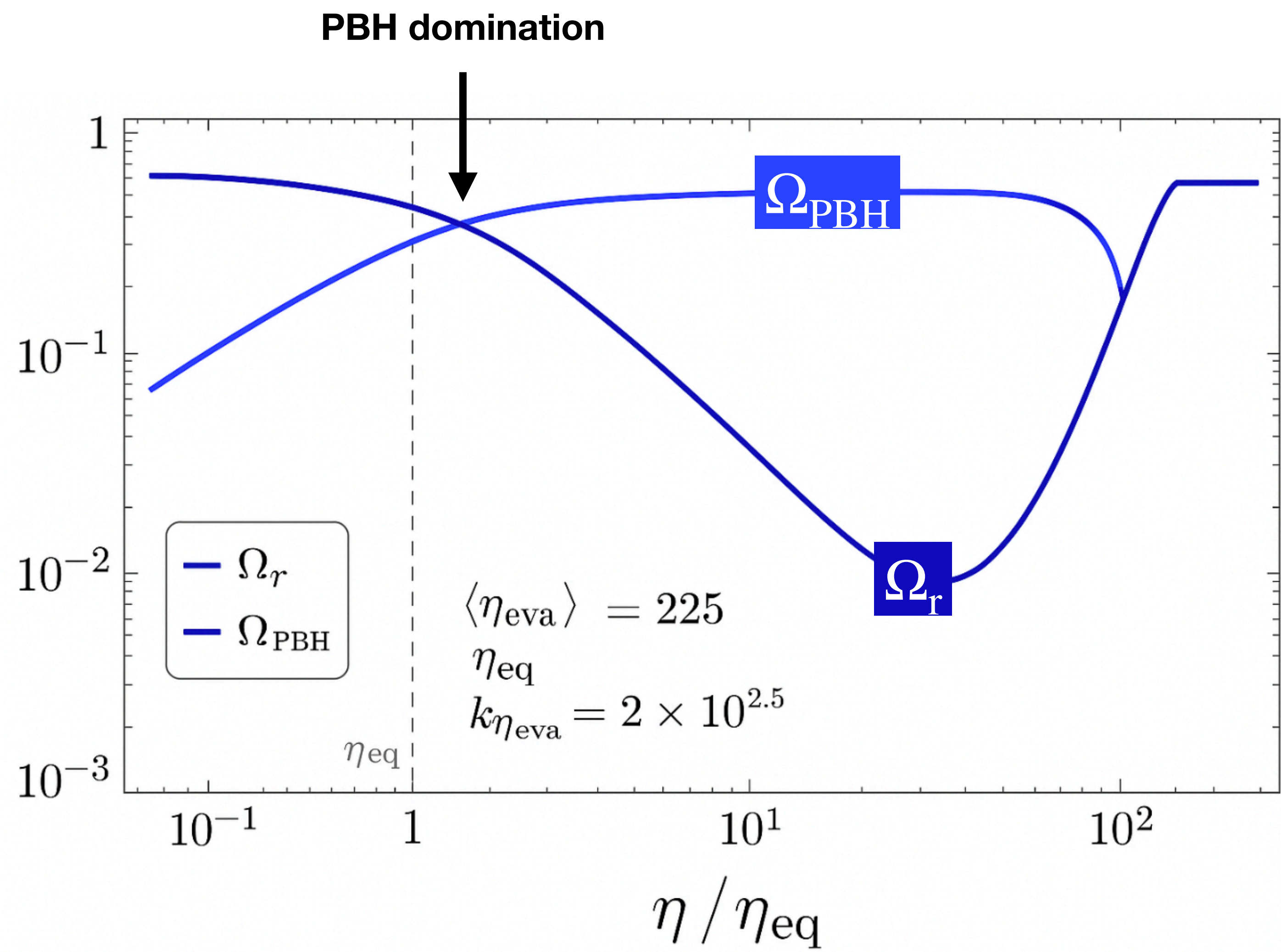
$$\rho'_{\text{PBH}} + 3\mathcal{H}\rho_{\text{PBH}} = -\Gamma\rho_{\text{PBH}}$$

$$\rho'_r + 4\mathcal{H}\rho_r = \Gamma\rho_{\text{PBH}}$$



Background equations

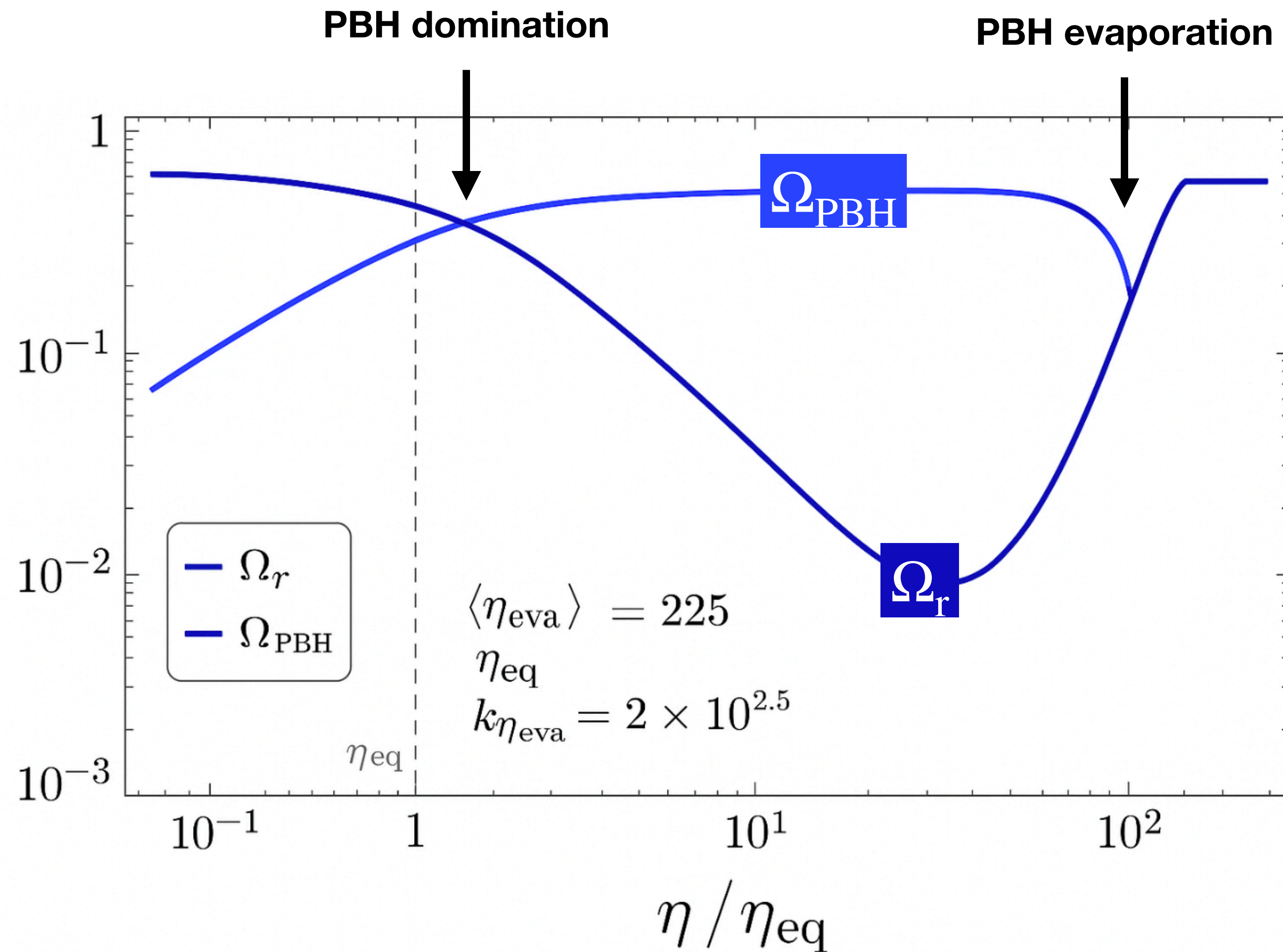
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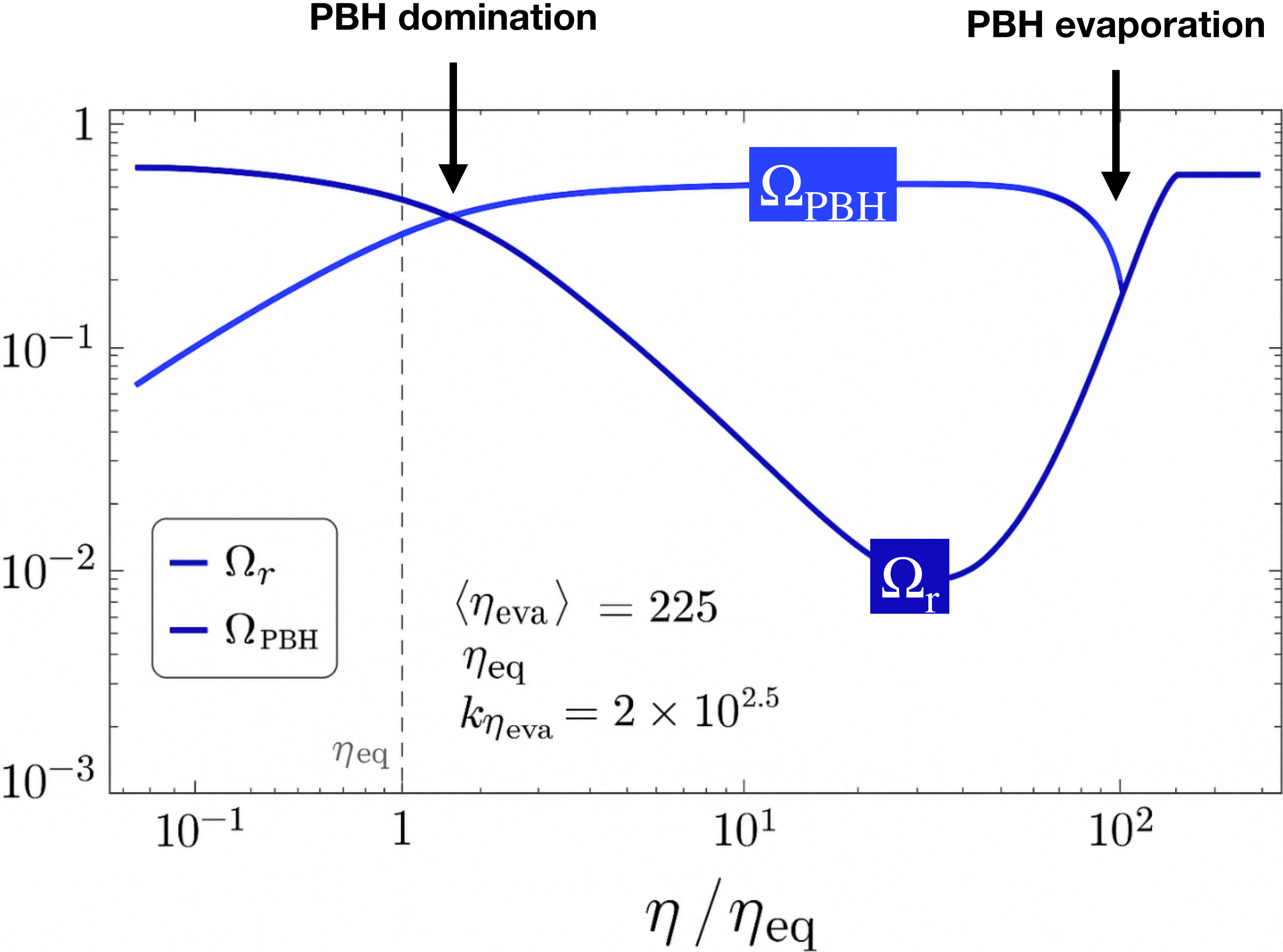
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$$G_{\mu\nu} = \frac{8\pi G}{3} T_{\mu\nu}$$

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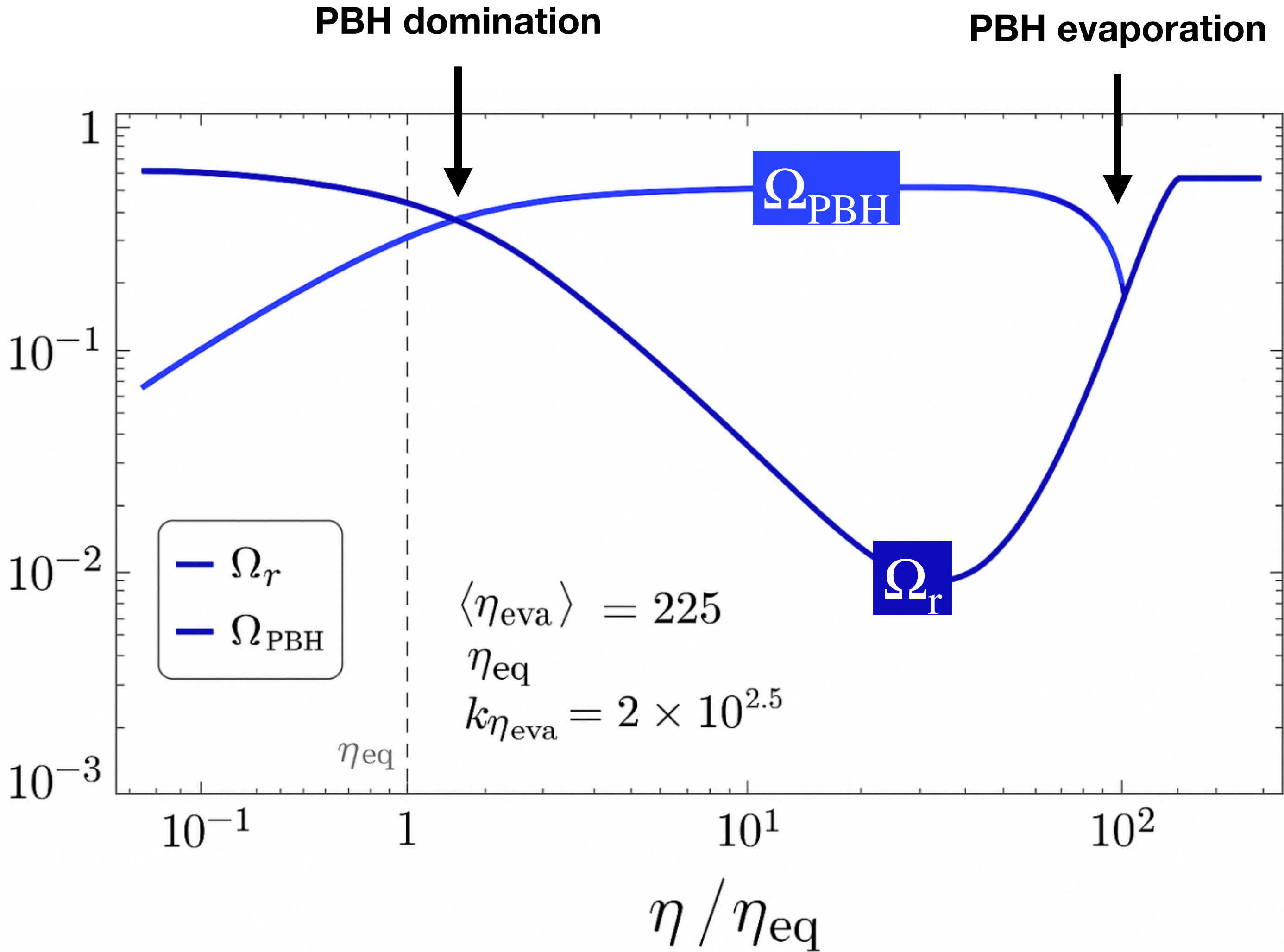


$$ds^2 = a^2(\eta) \left[(1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$

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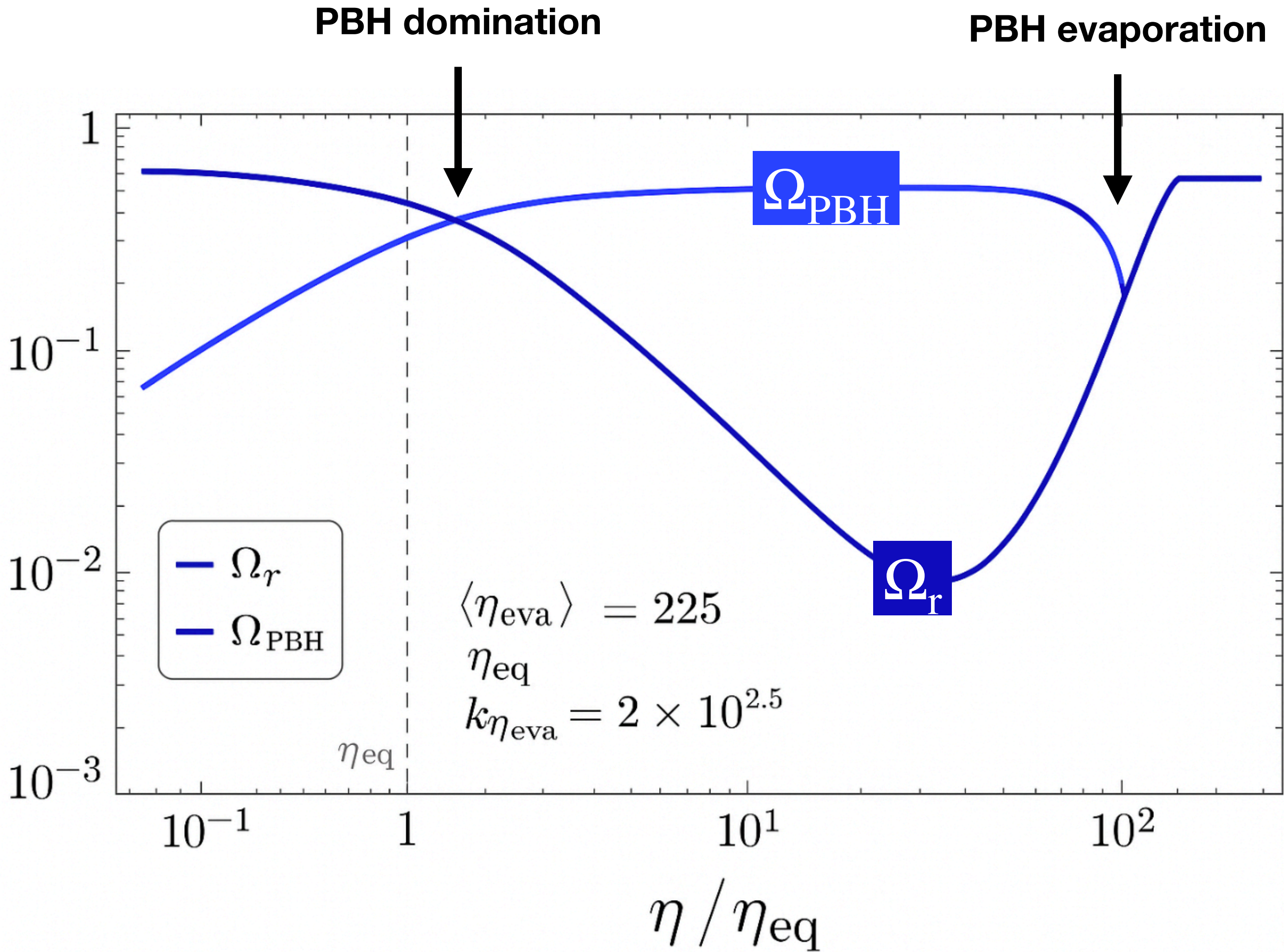
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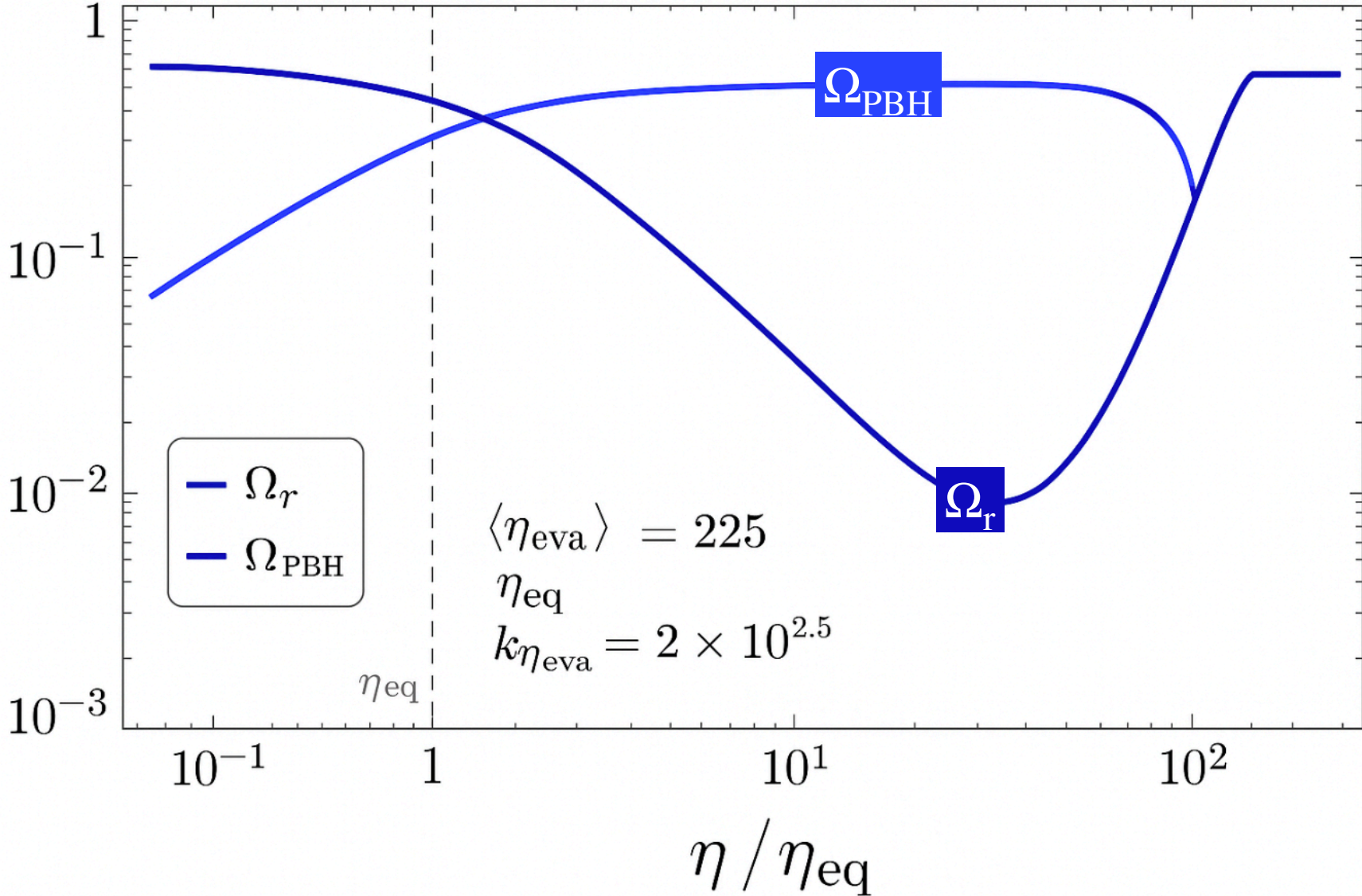
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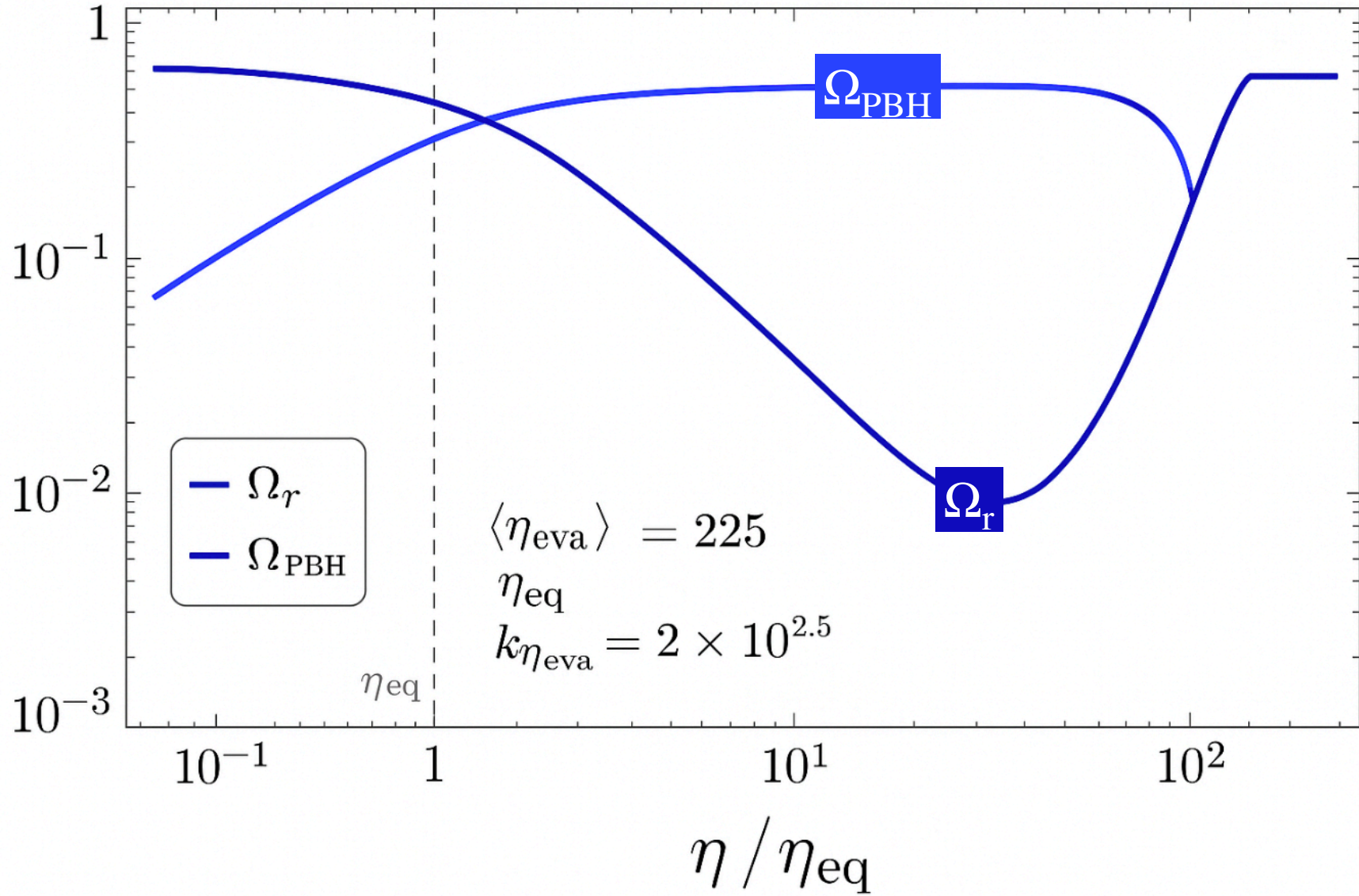


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Scalar perturbation equation

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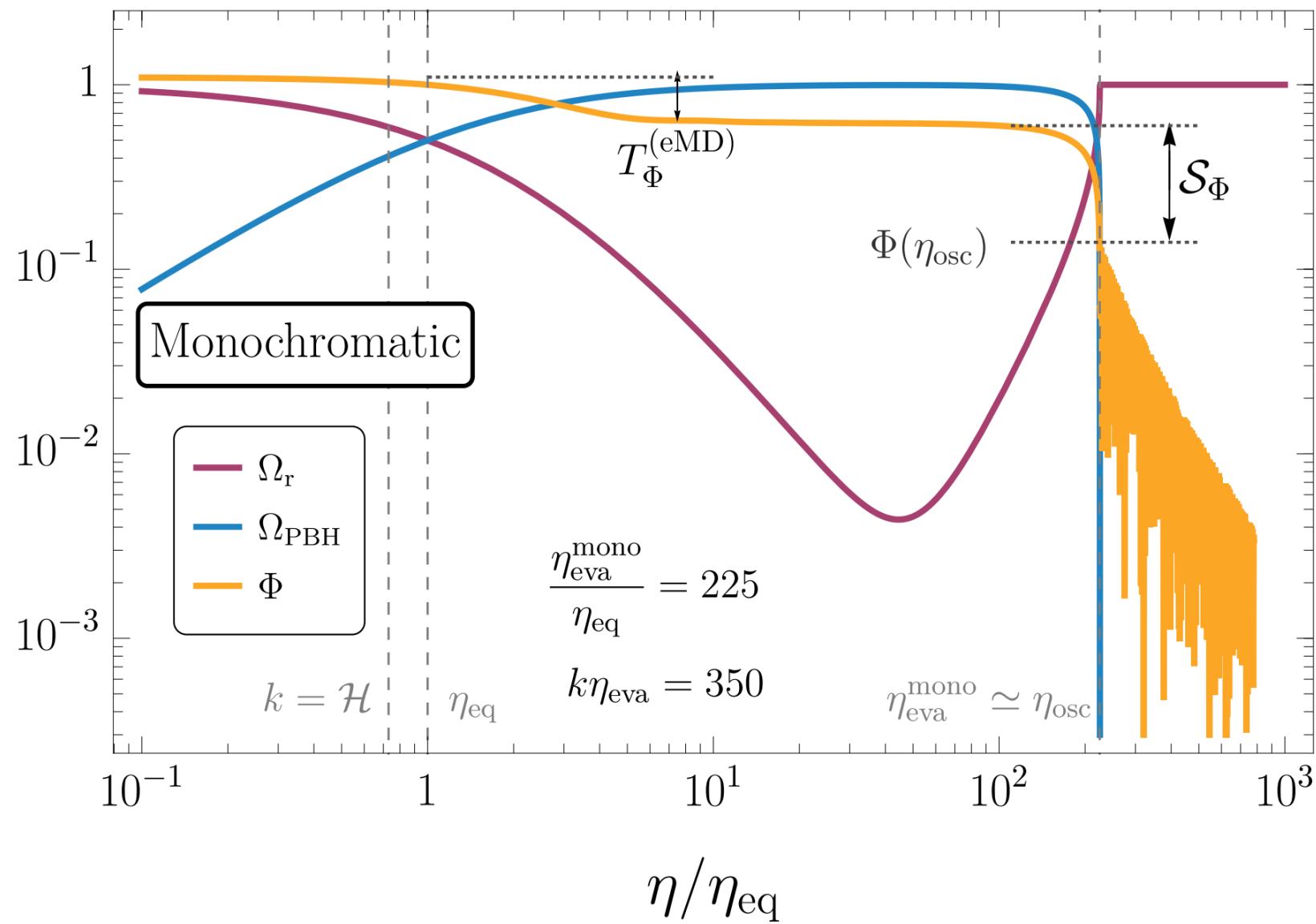
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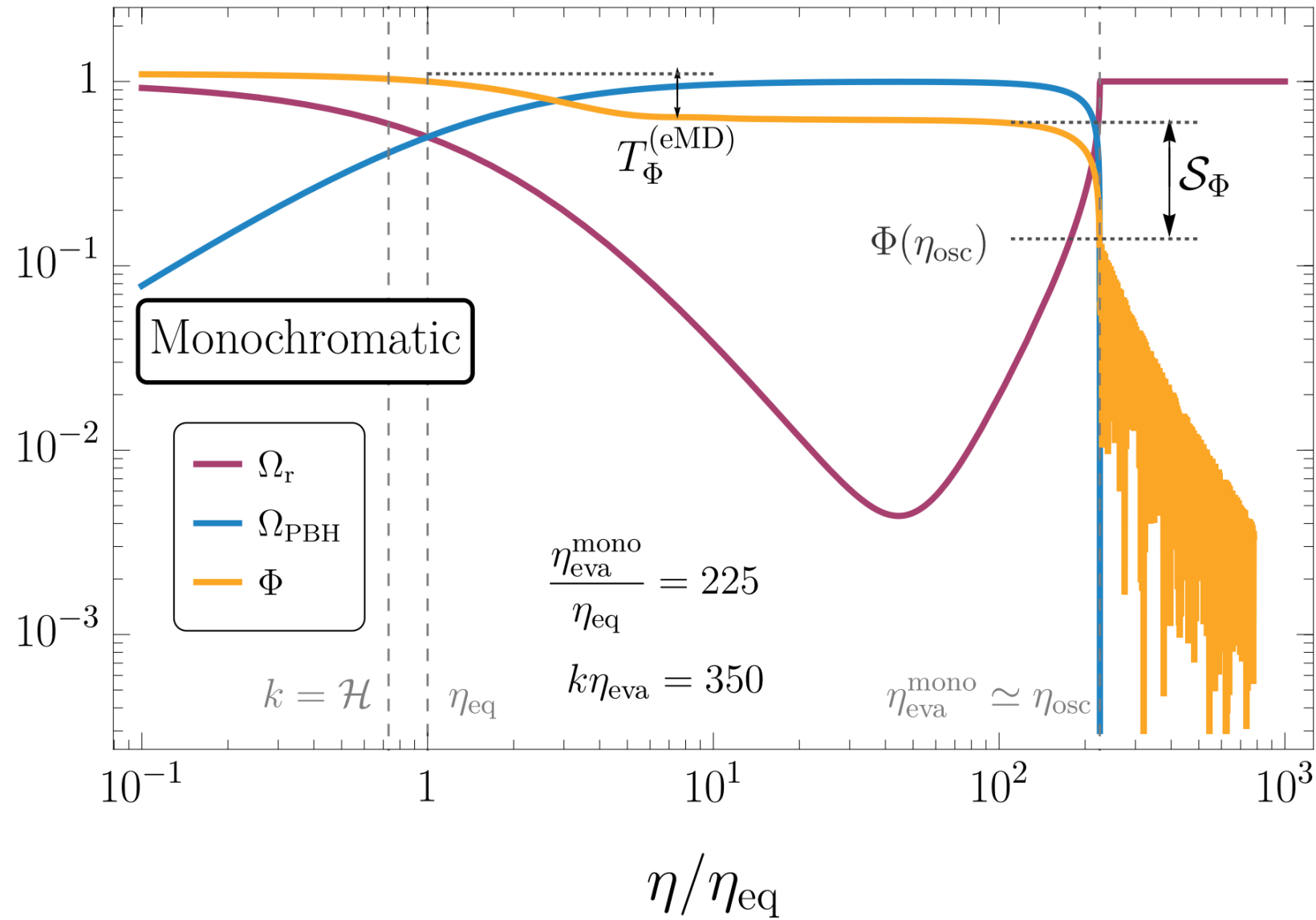
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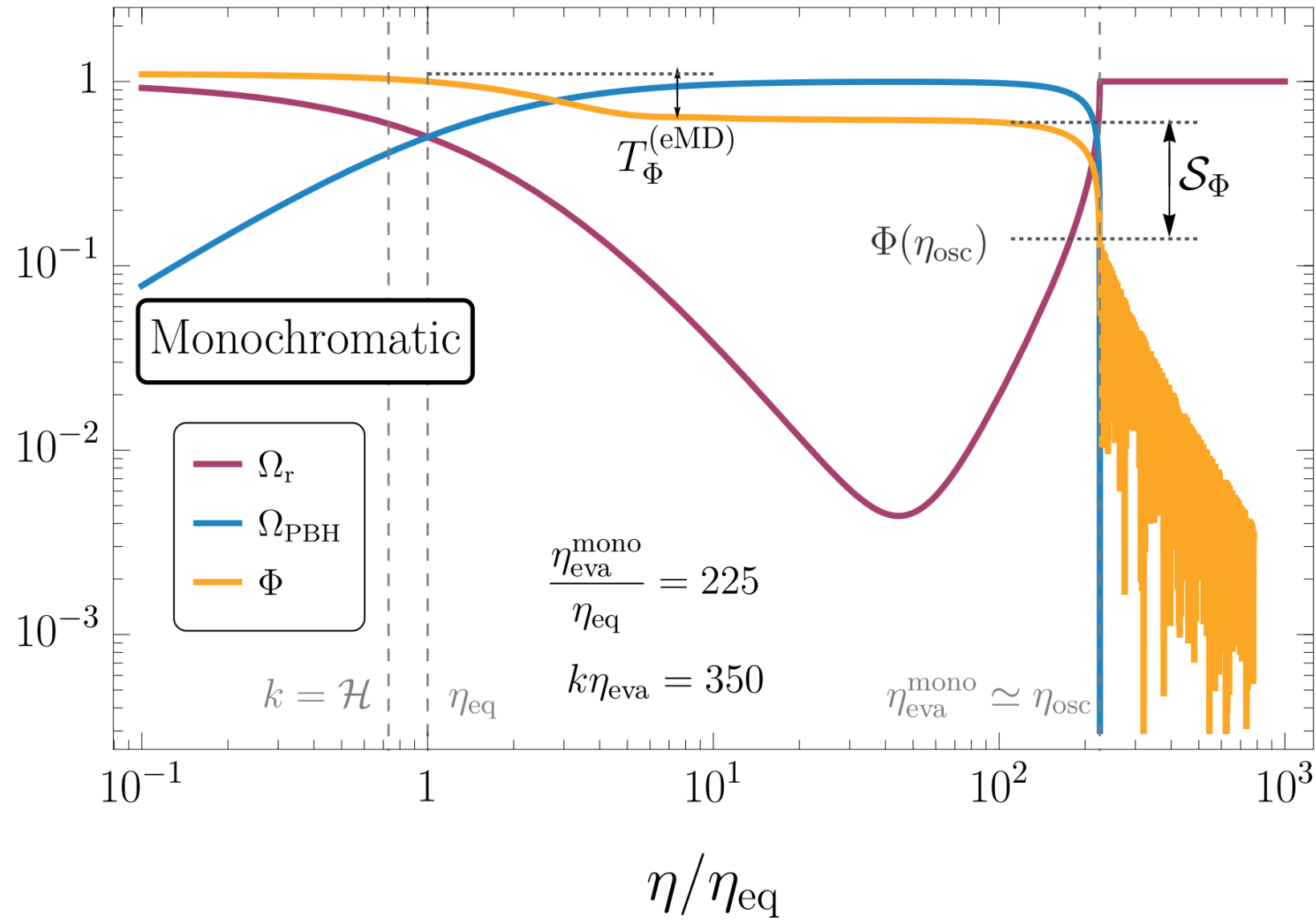
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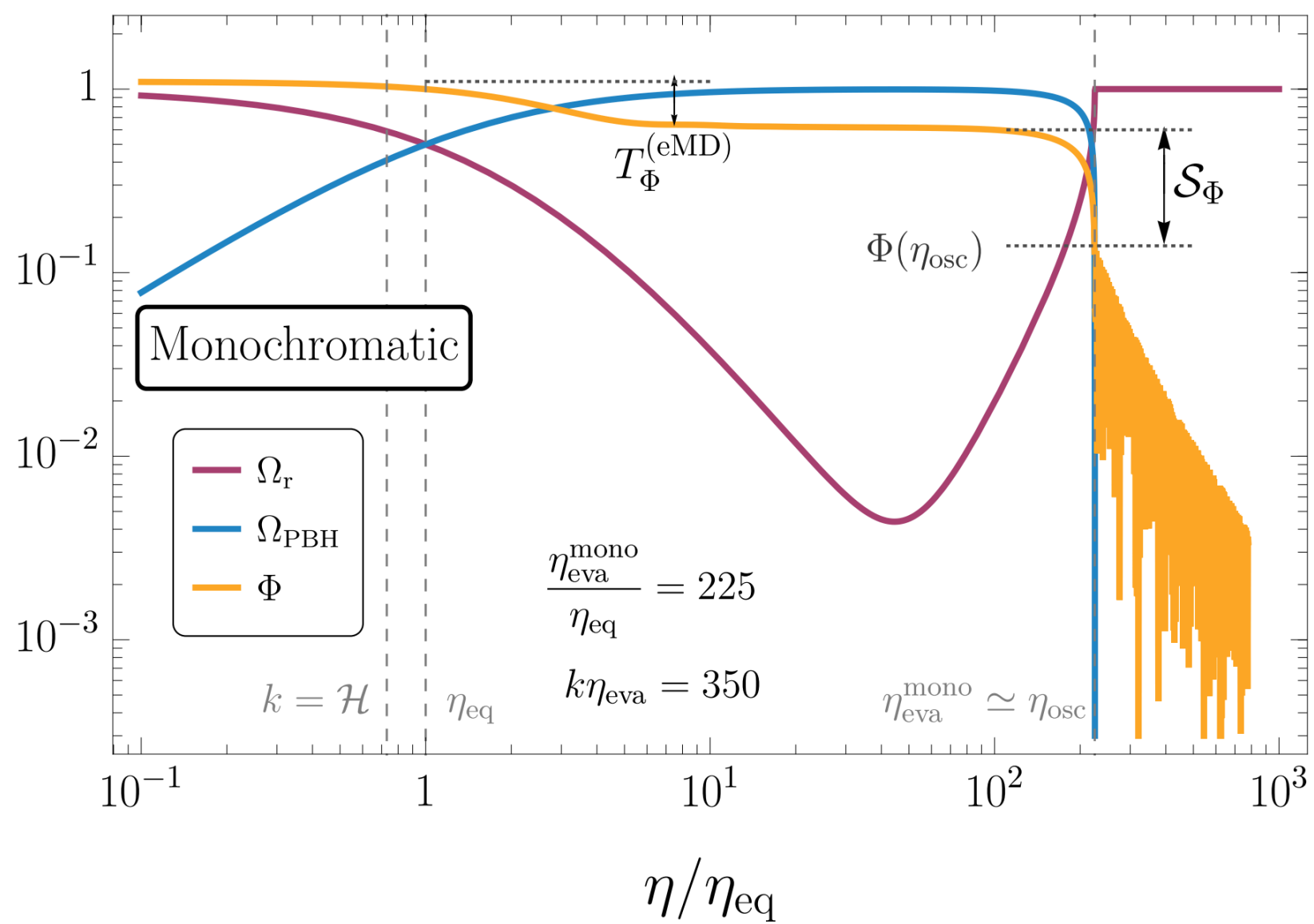
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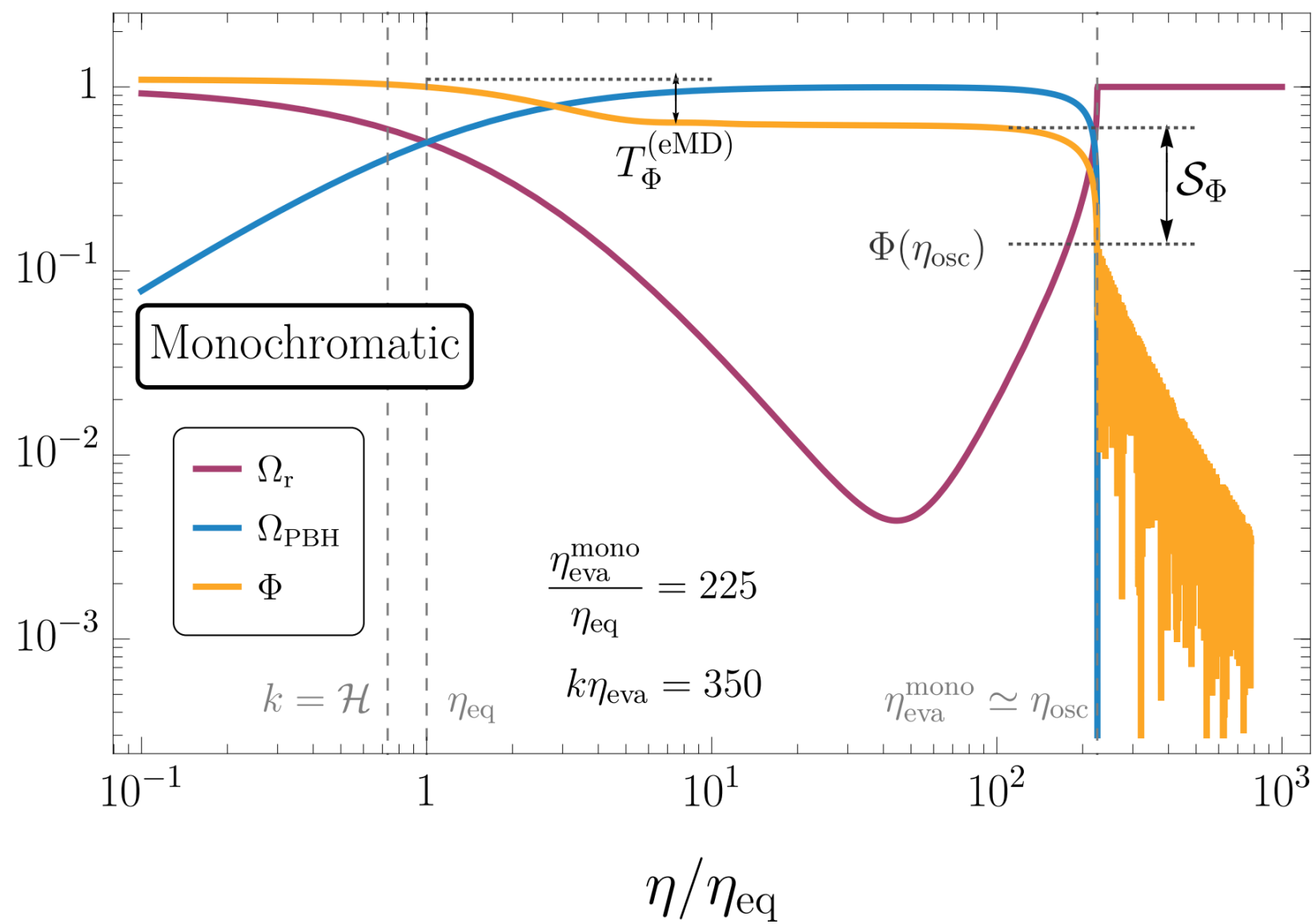
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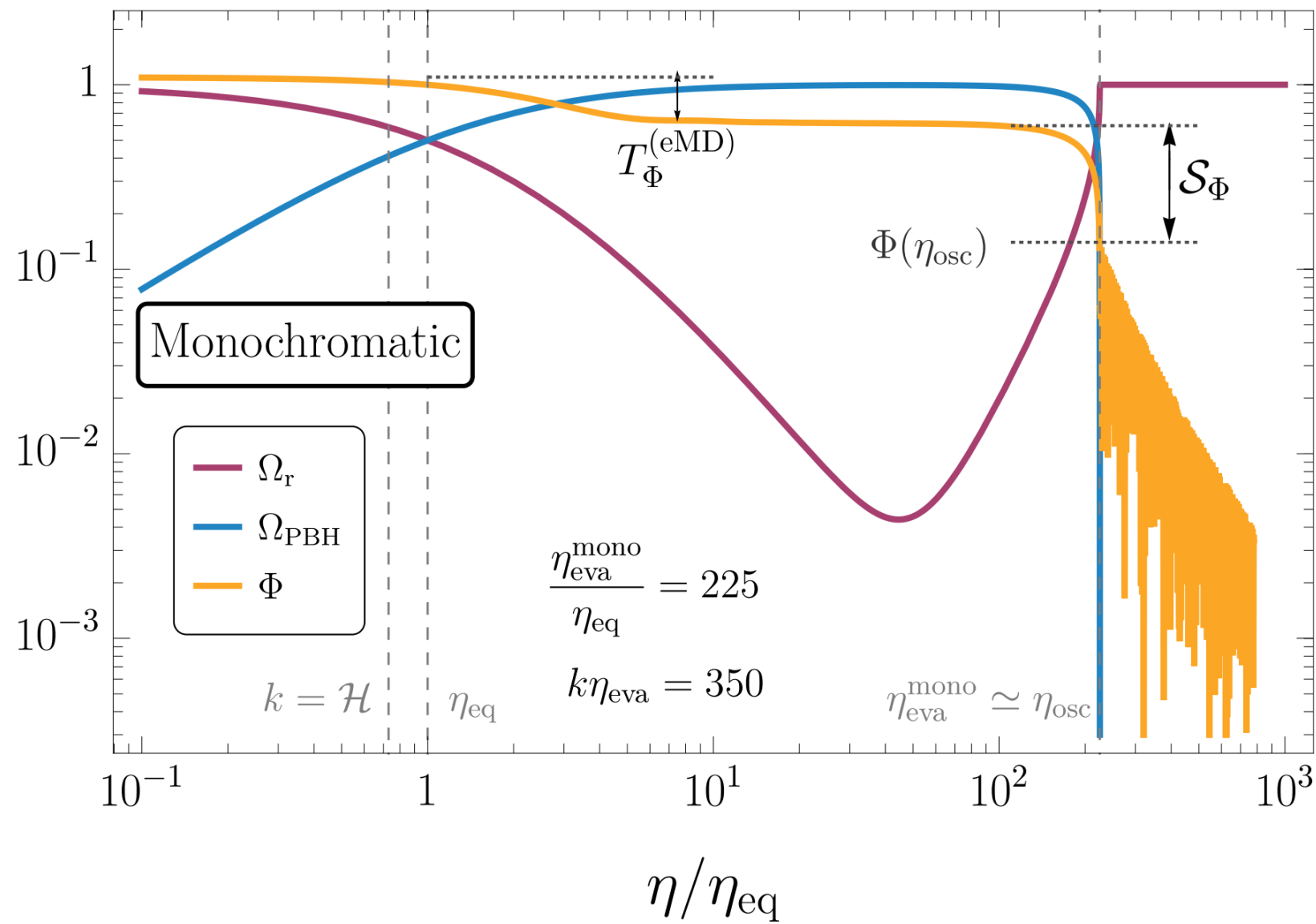
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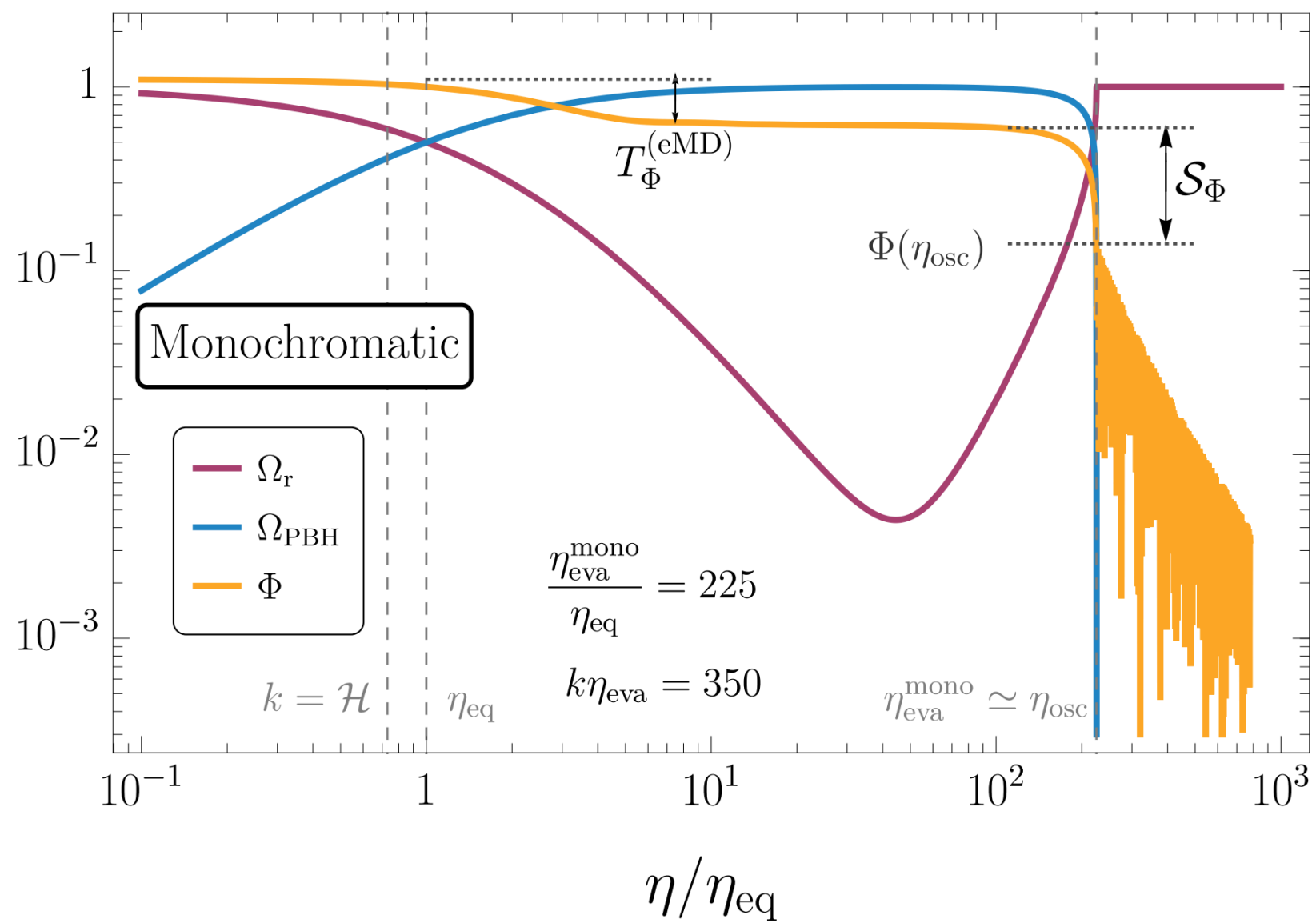
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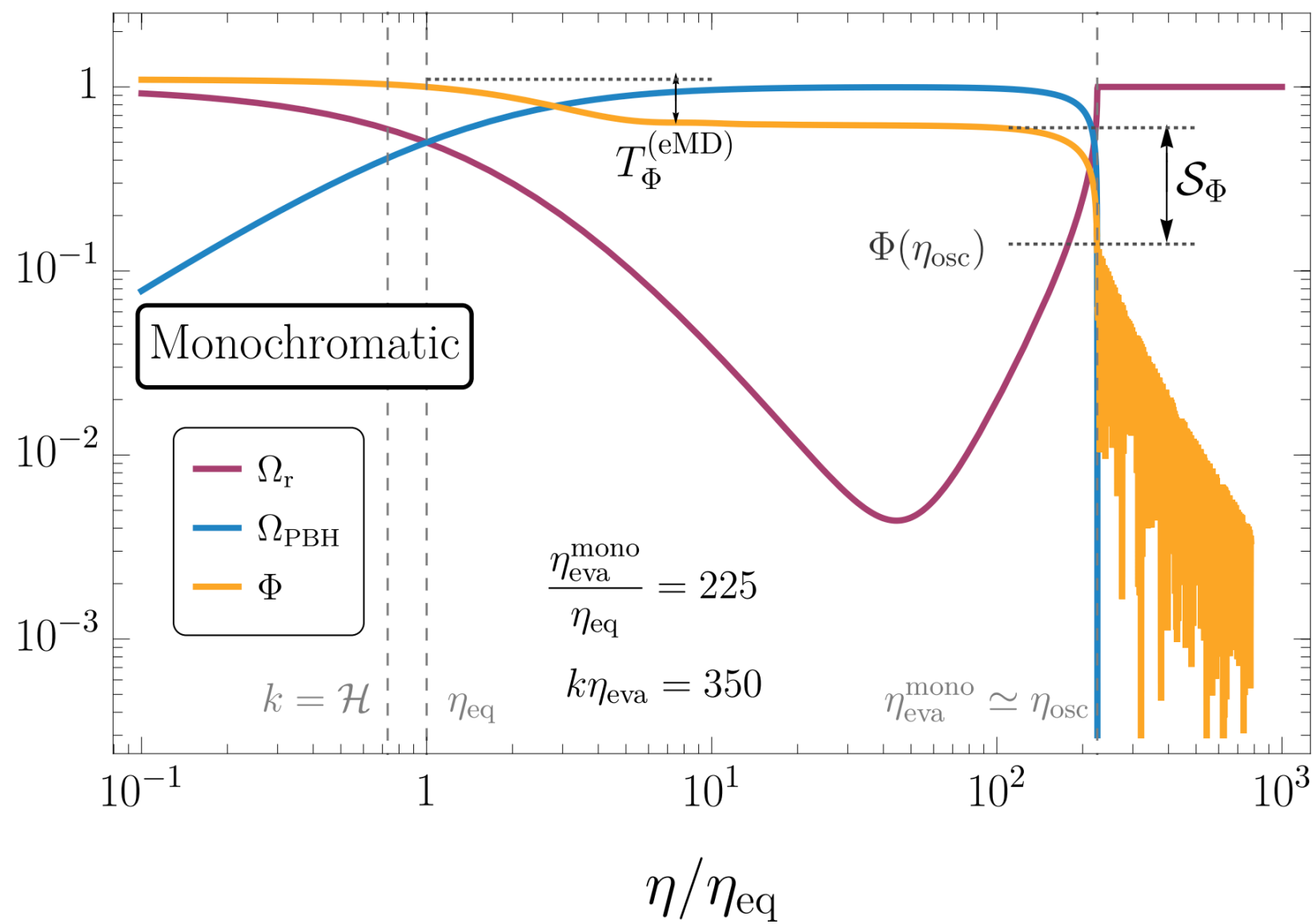
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$$ds^2 = a^2(\eta) \left[(1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$

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$$G_{\mu\nu} = \frac{8\pi G}{3} T_{\mu\nu}$$

Background equations

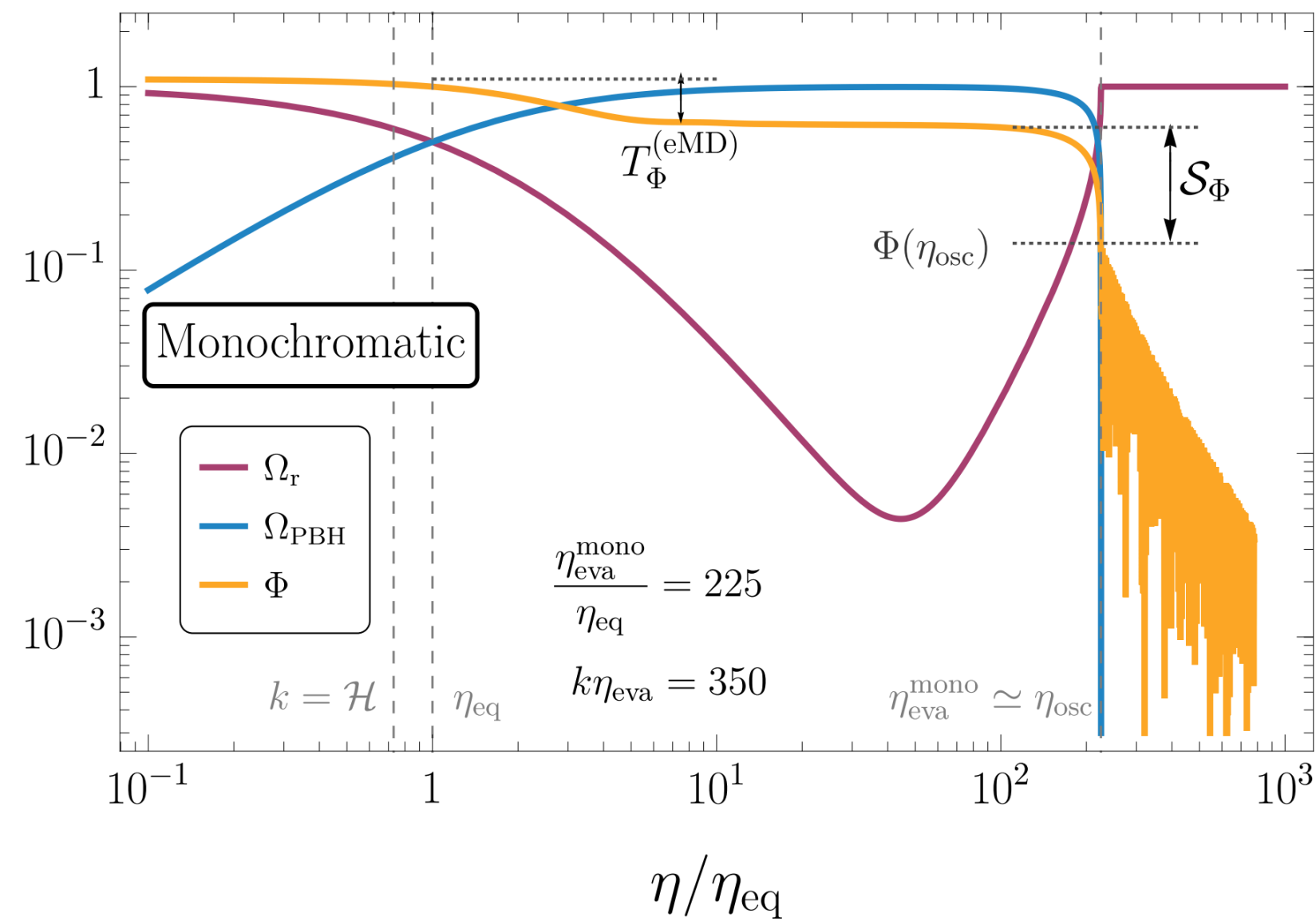
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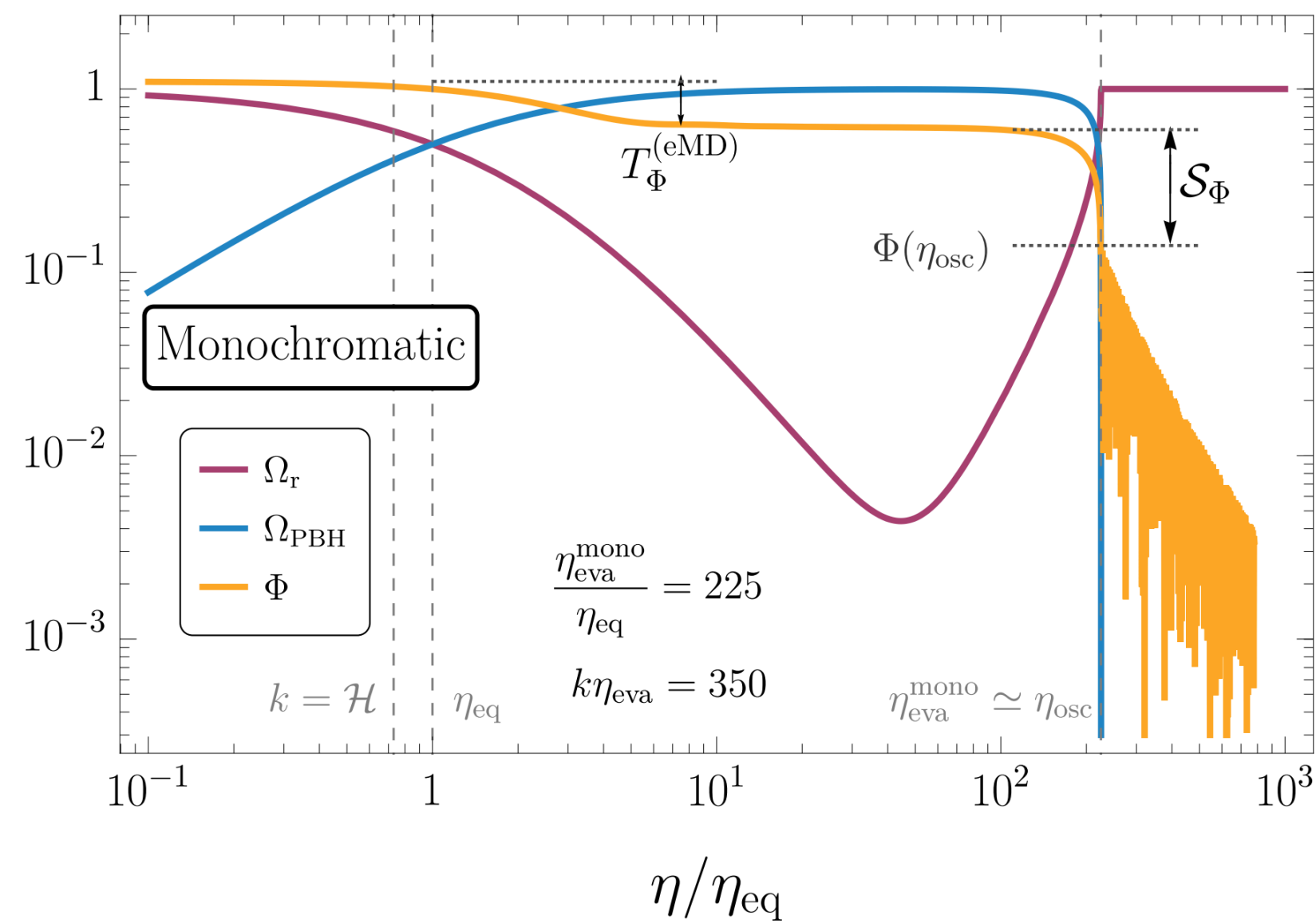
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time



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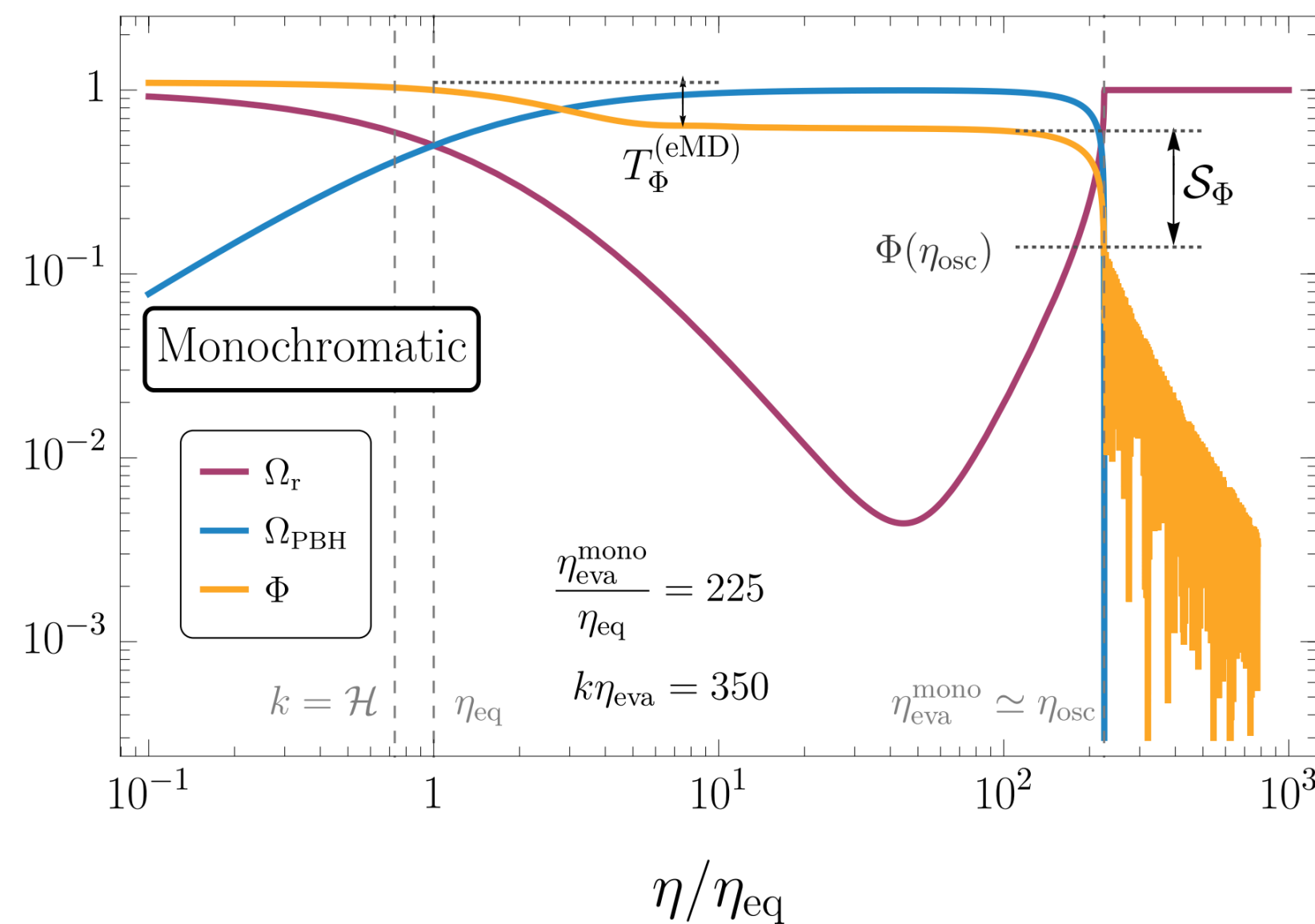
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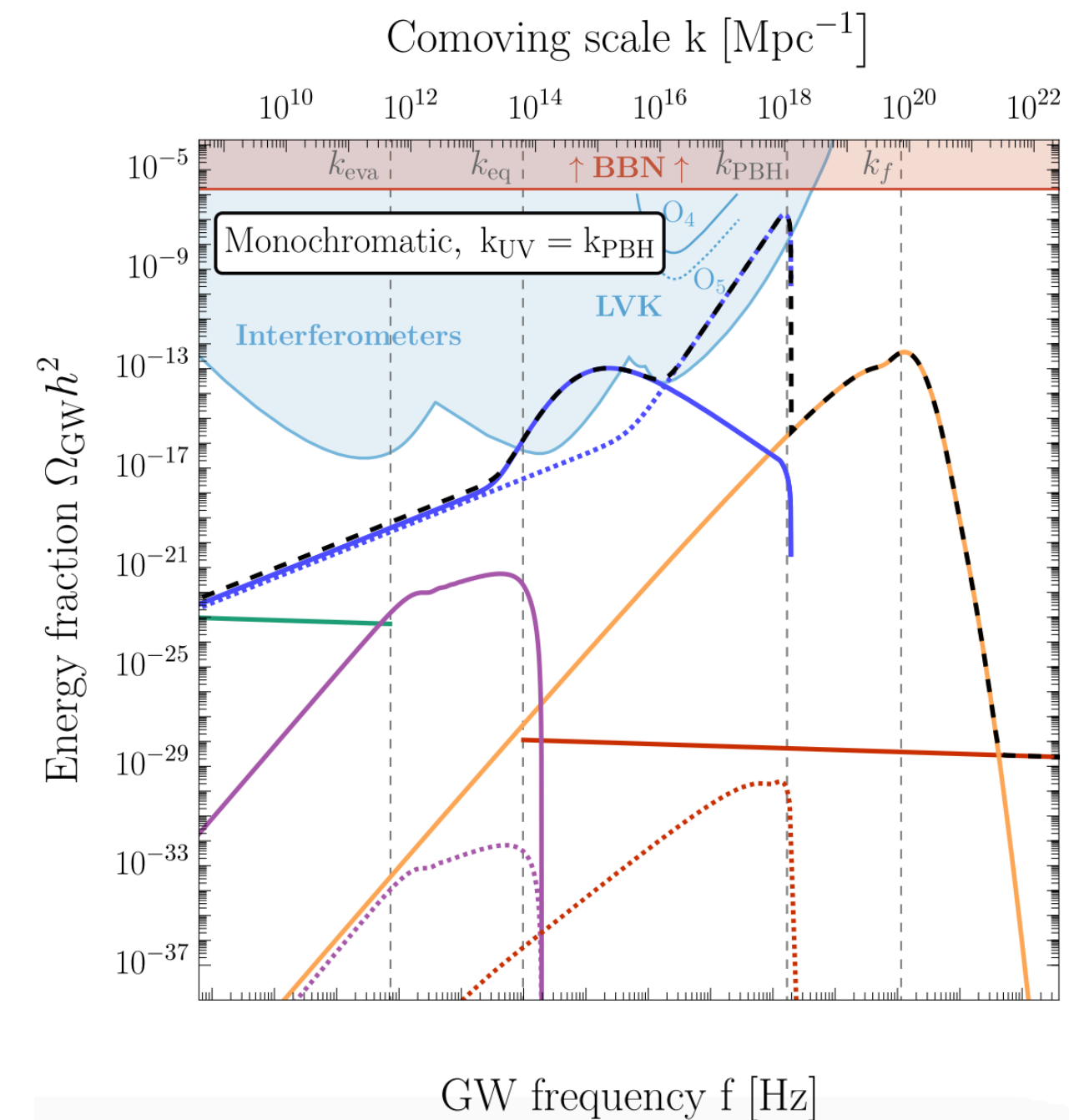
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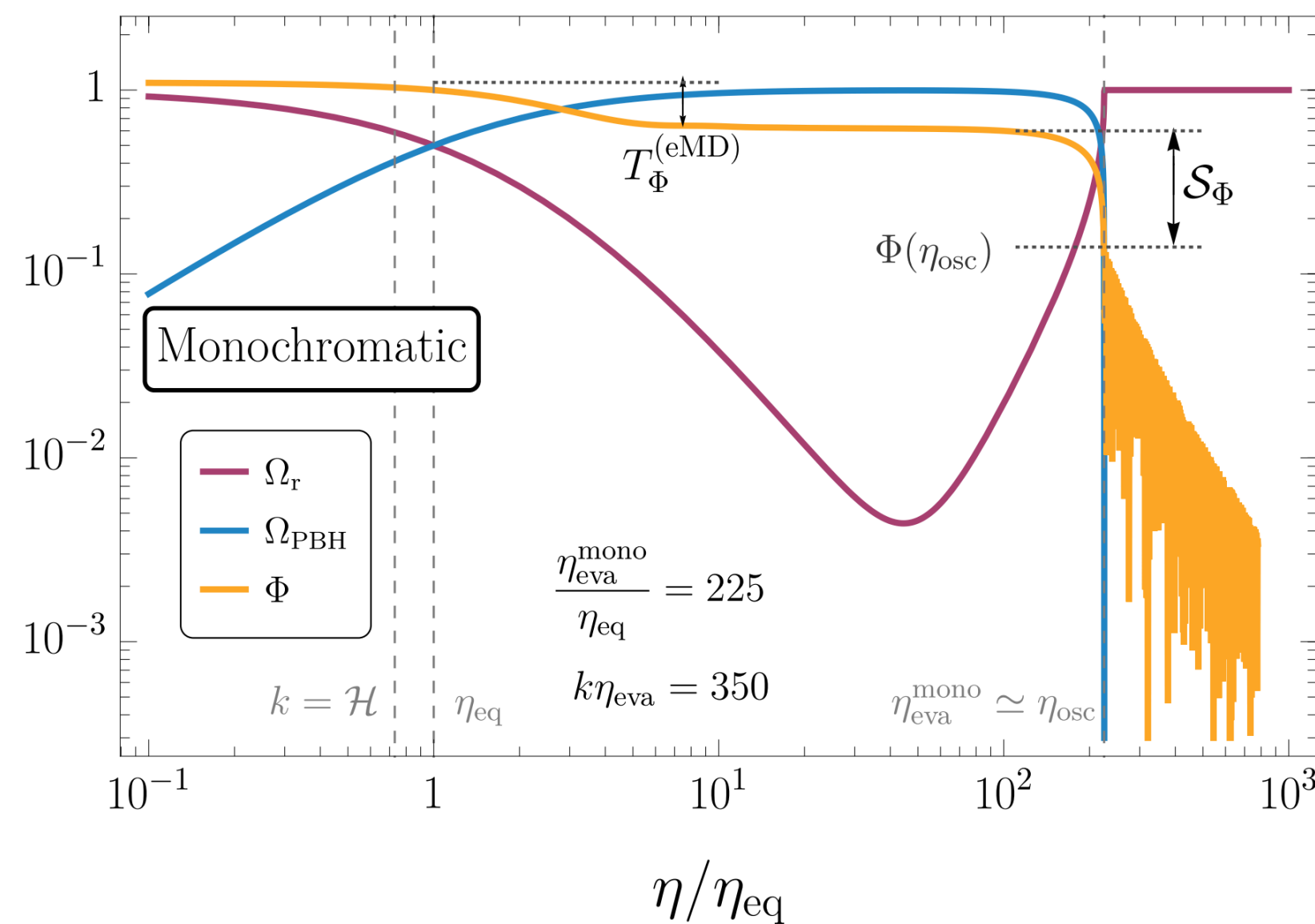
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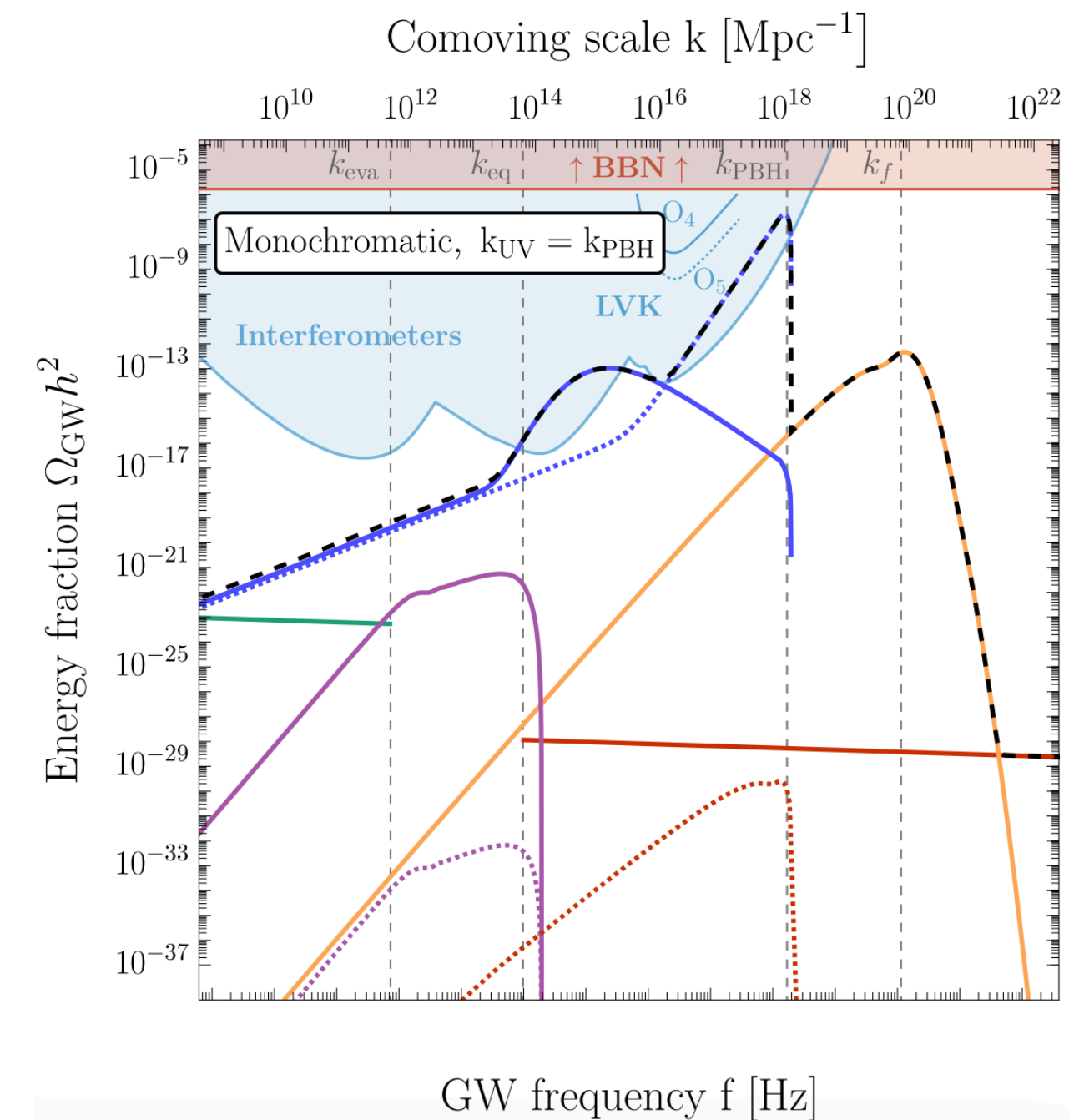
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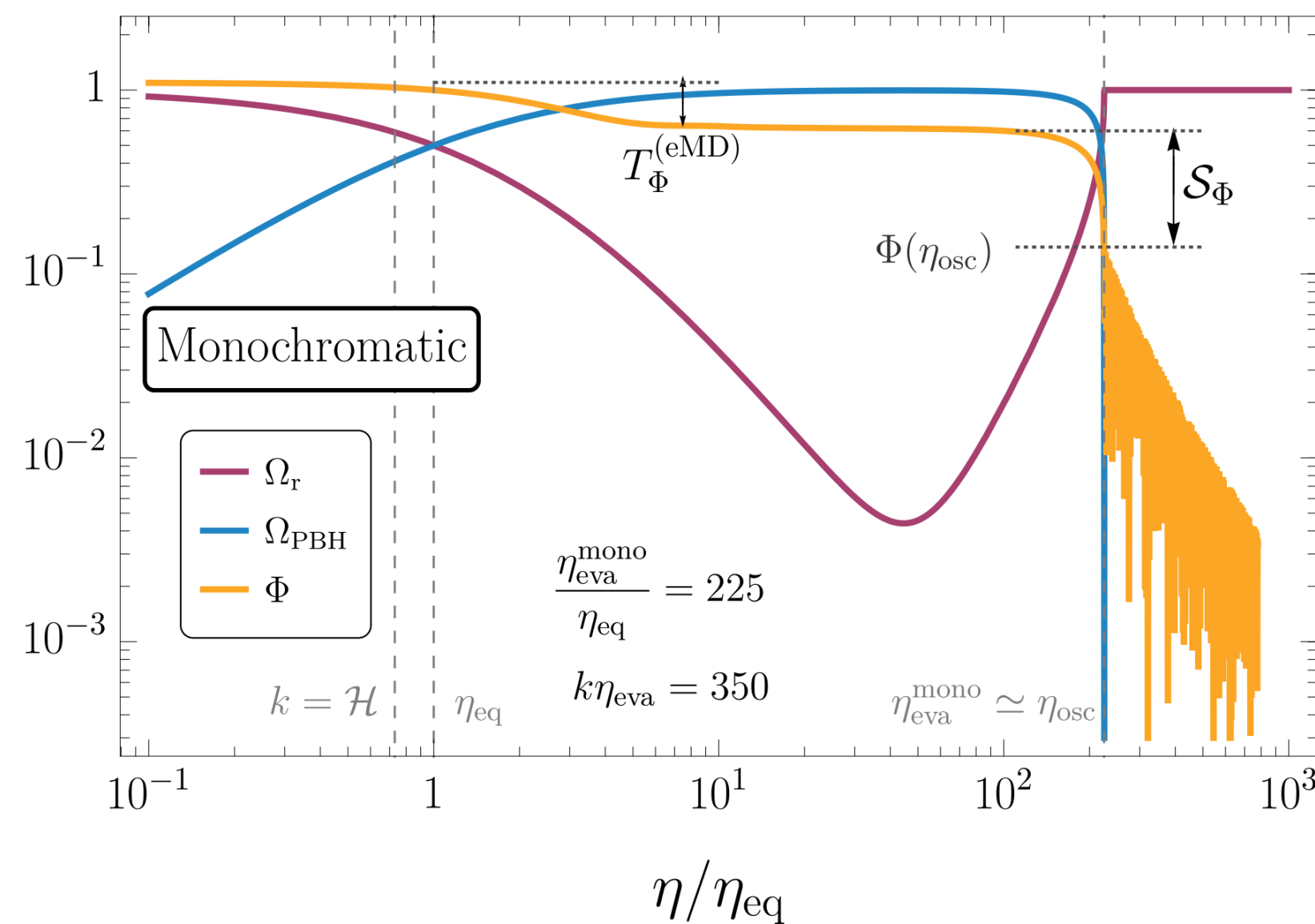
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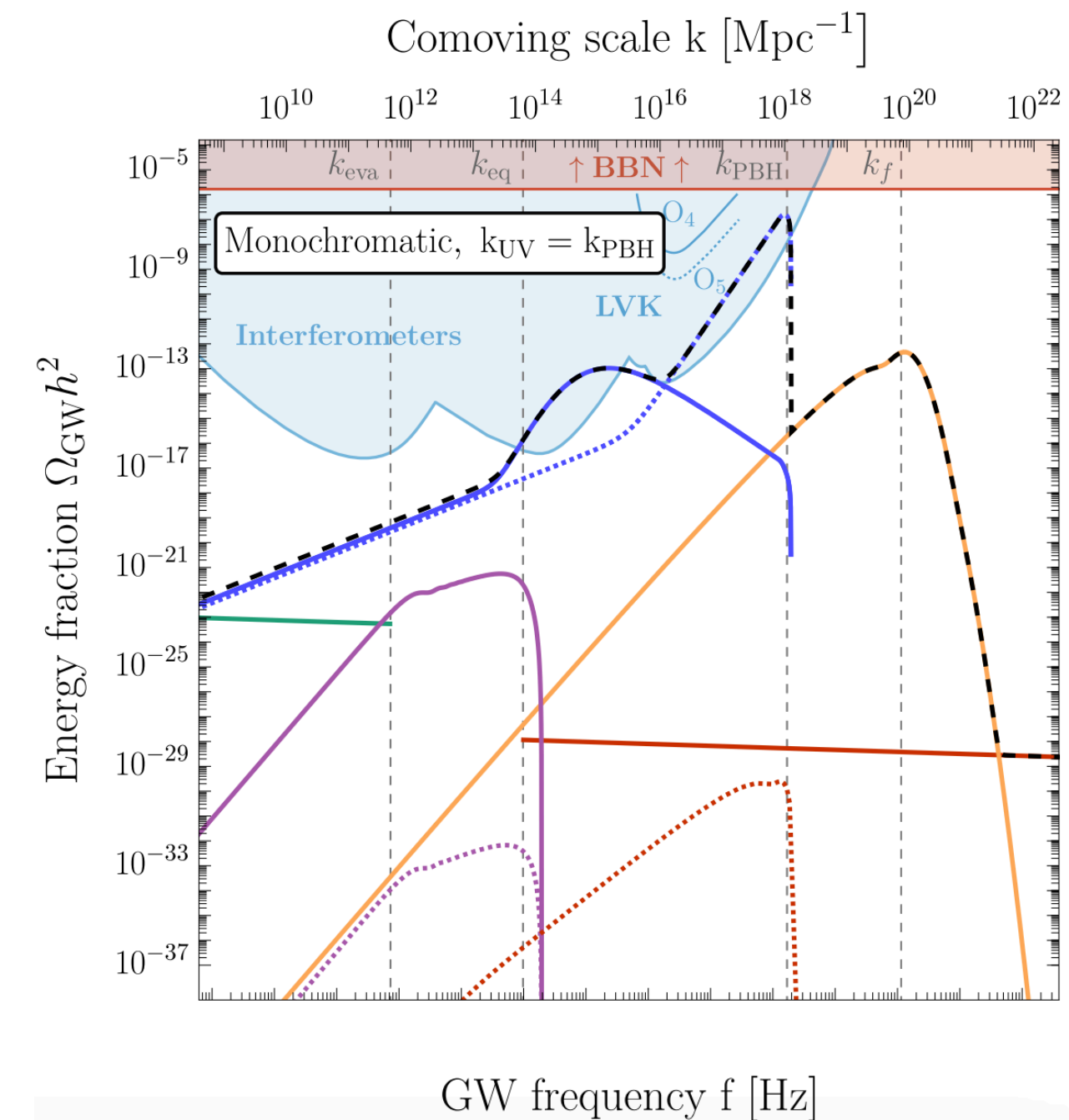
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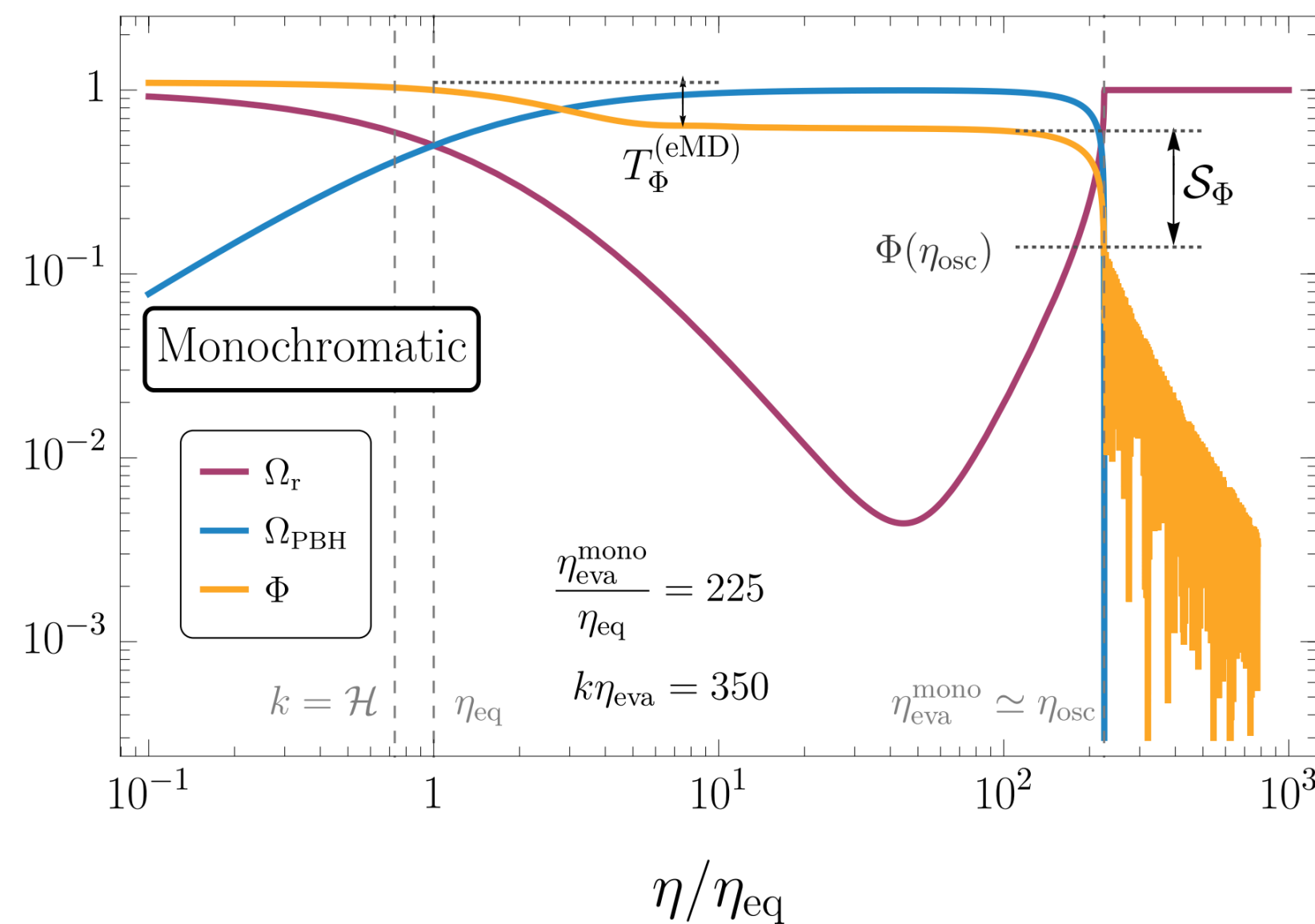
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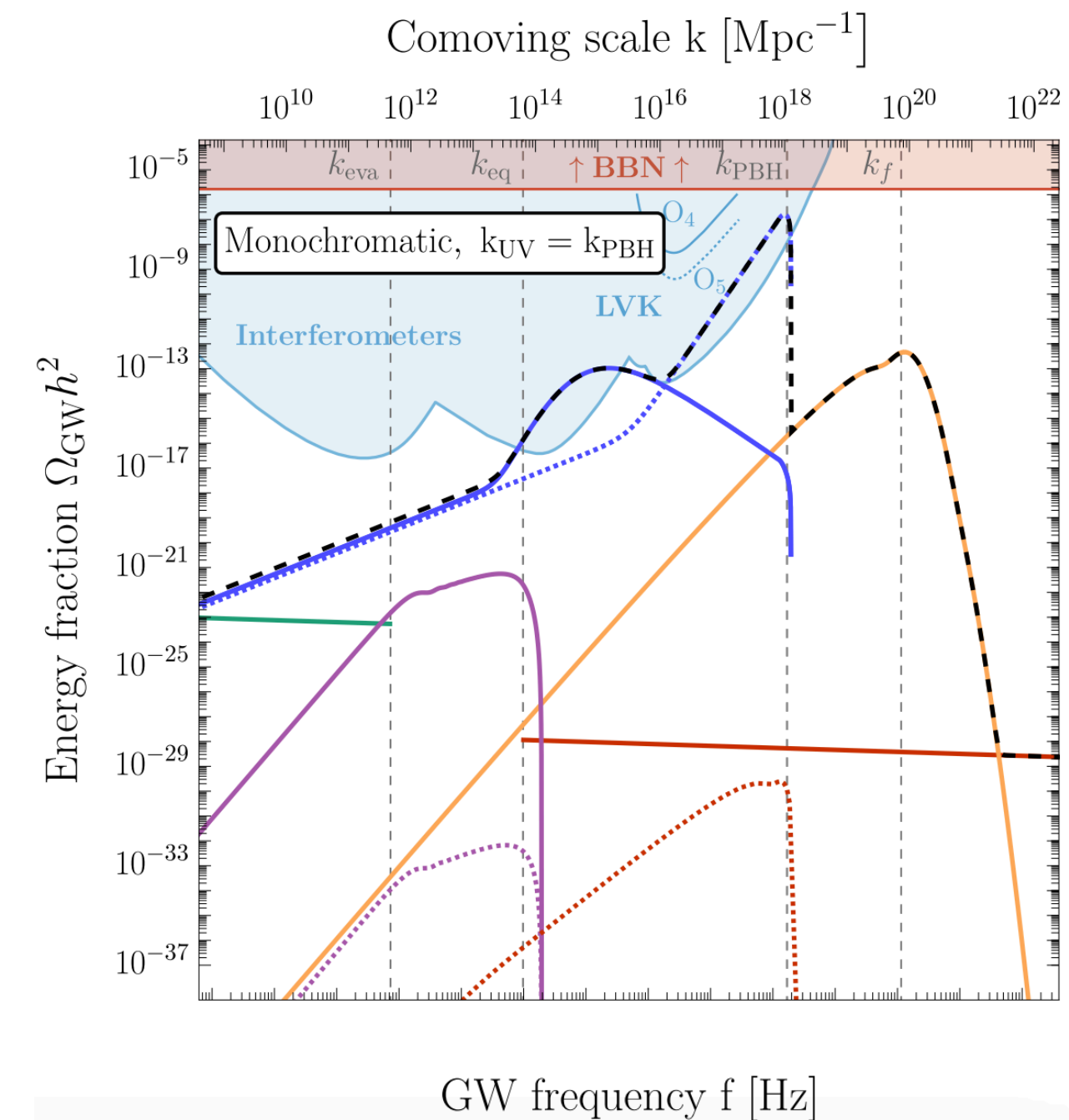
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Tensor perturbation equation

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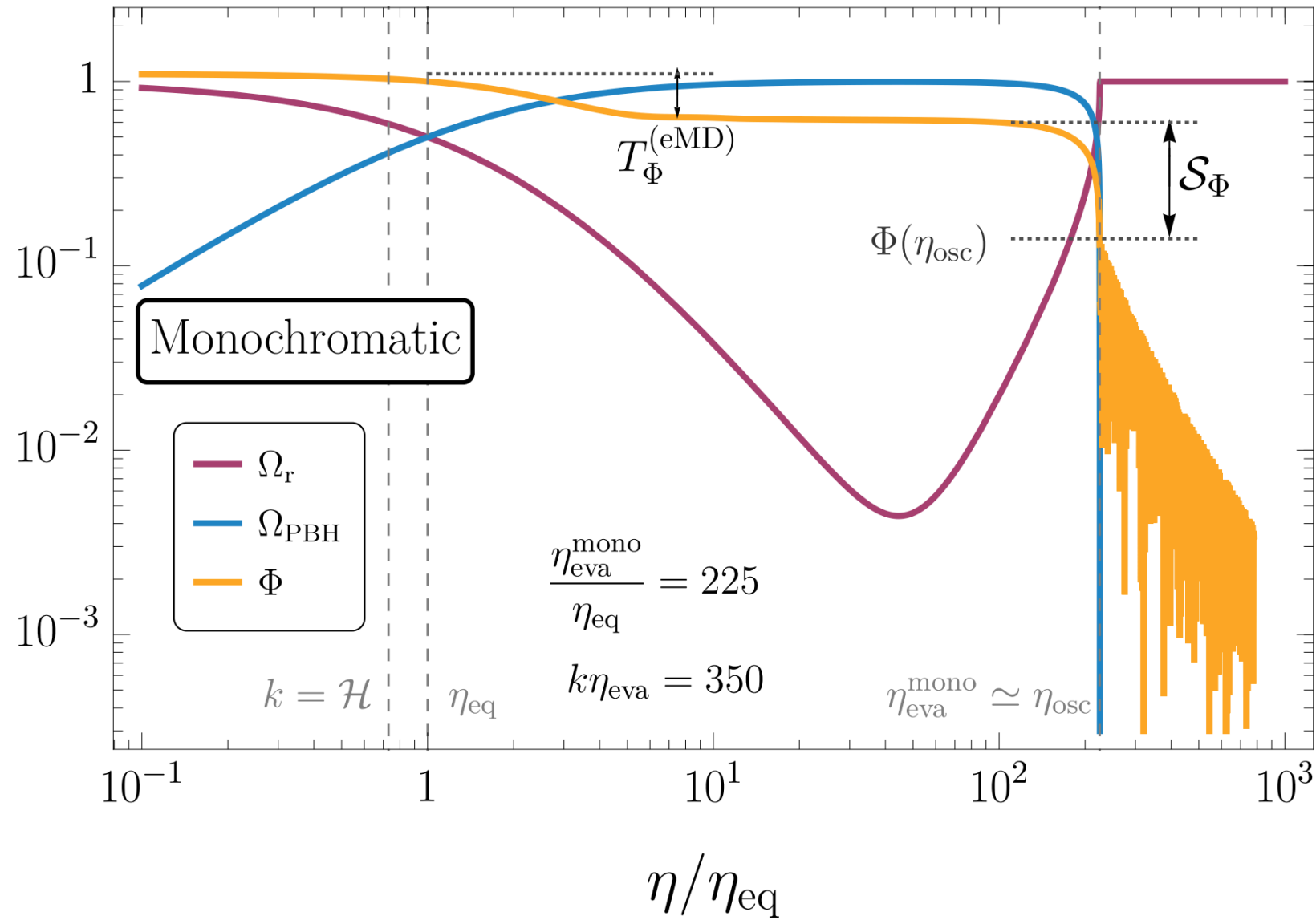
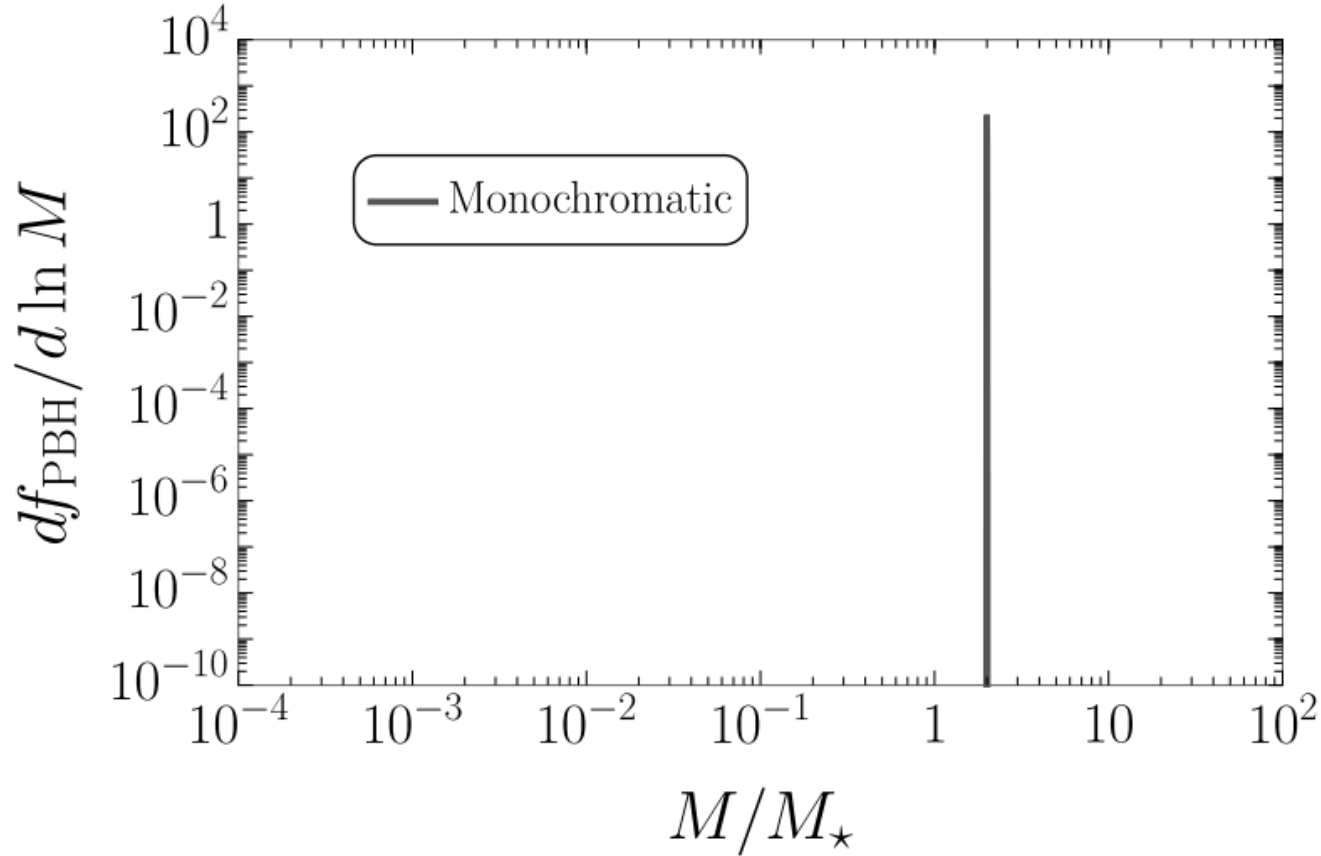
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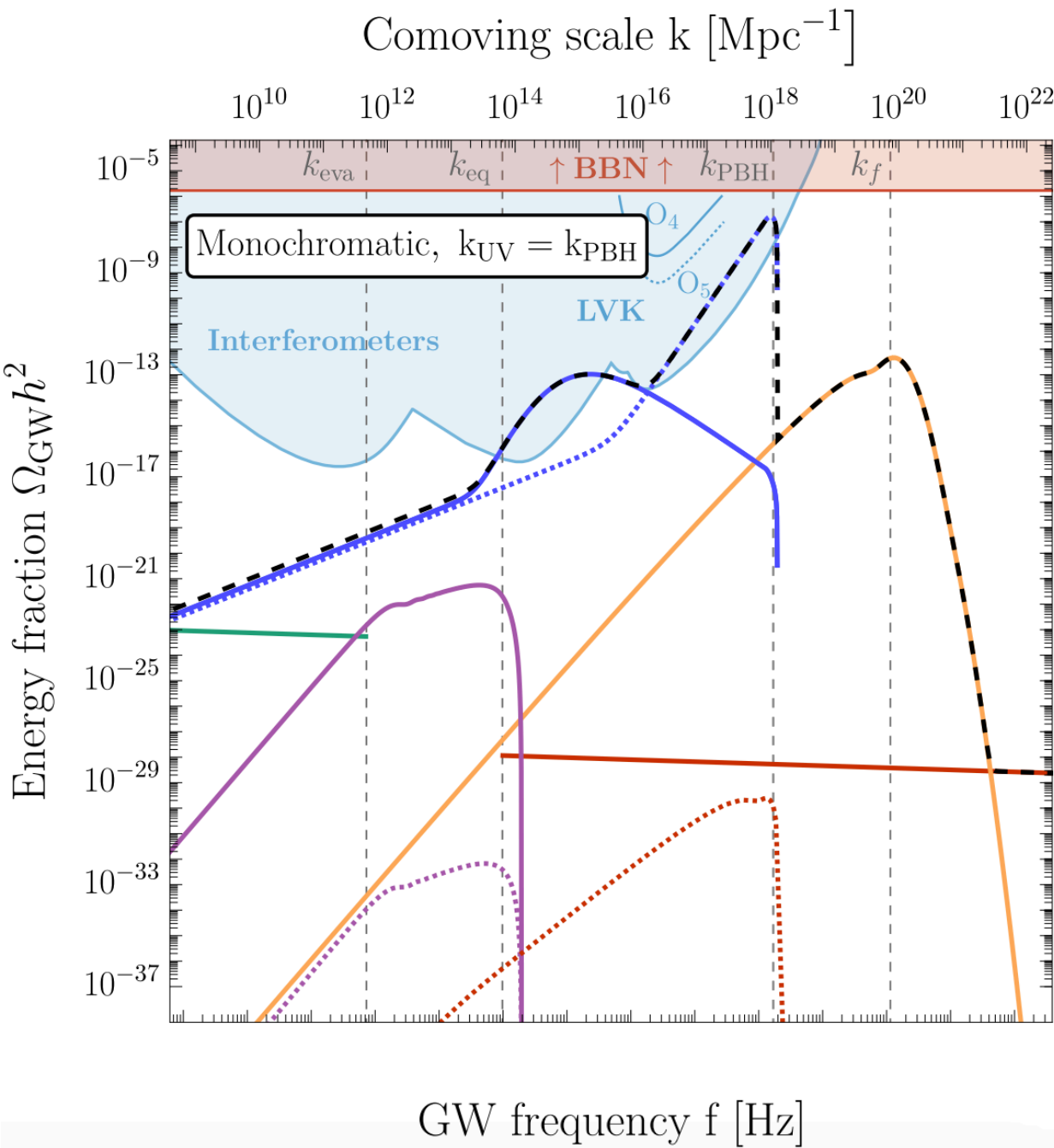
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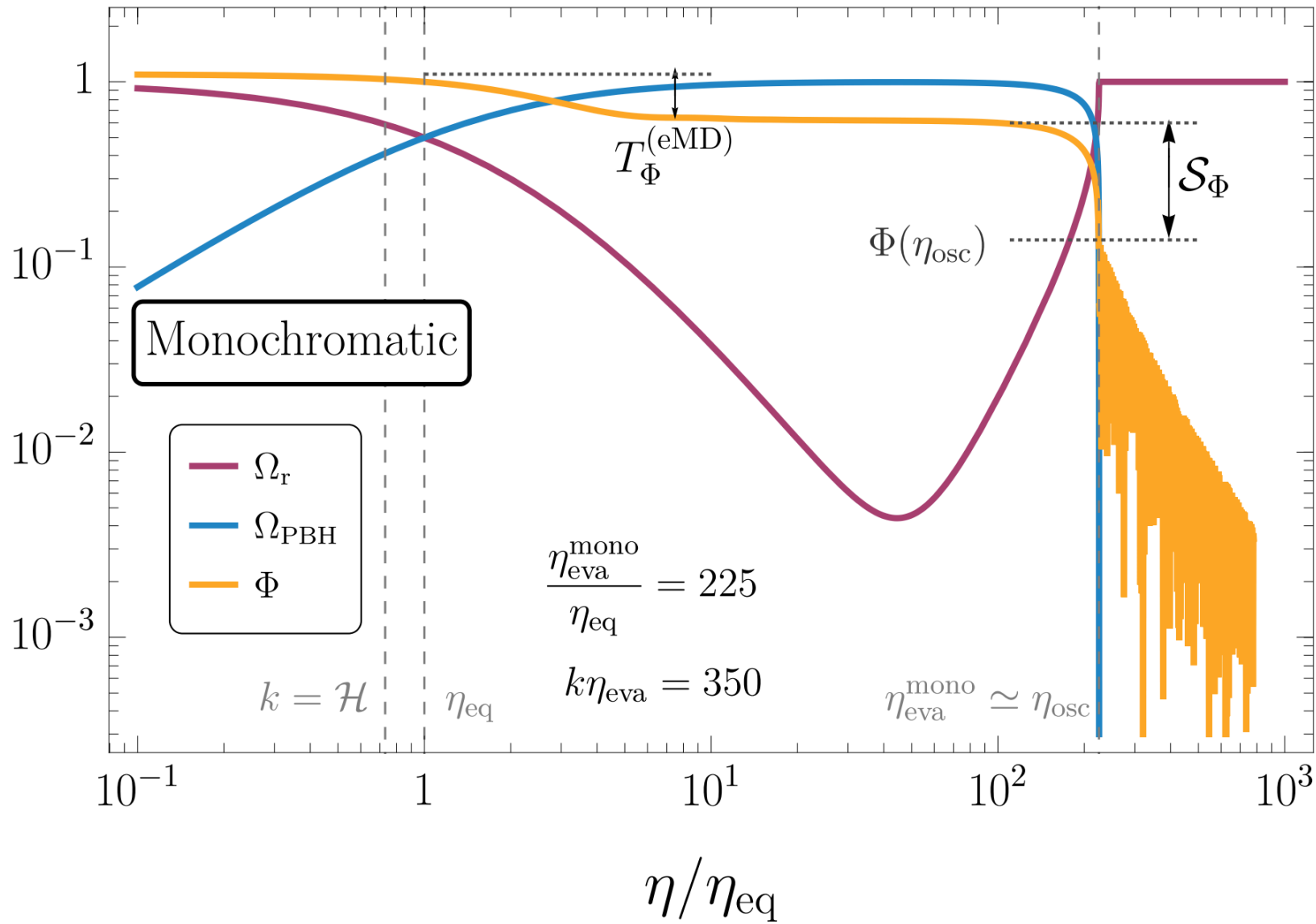
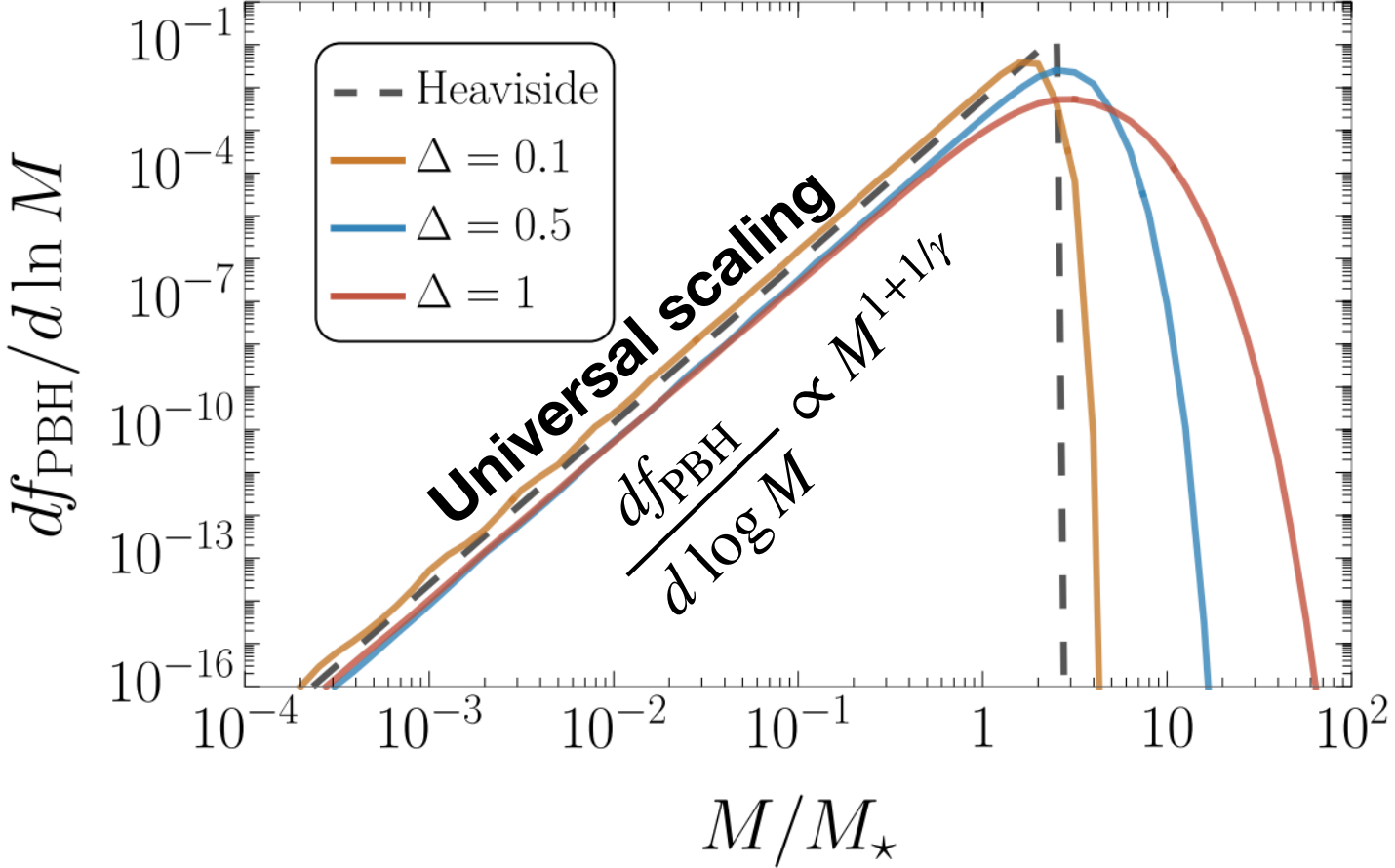
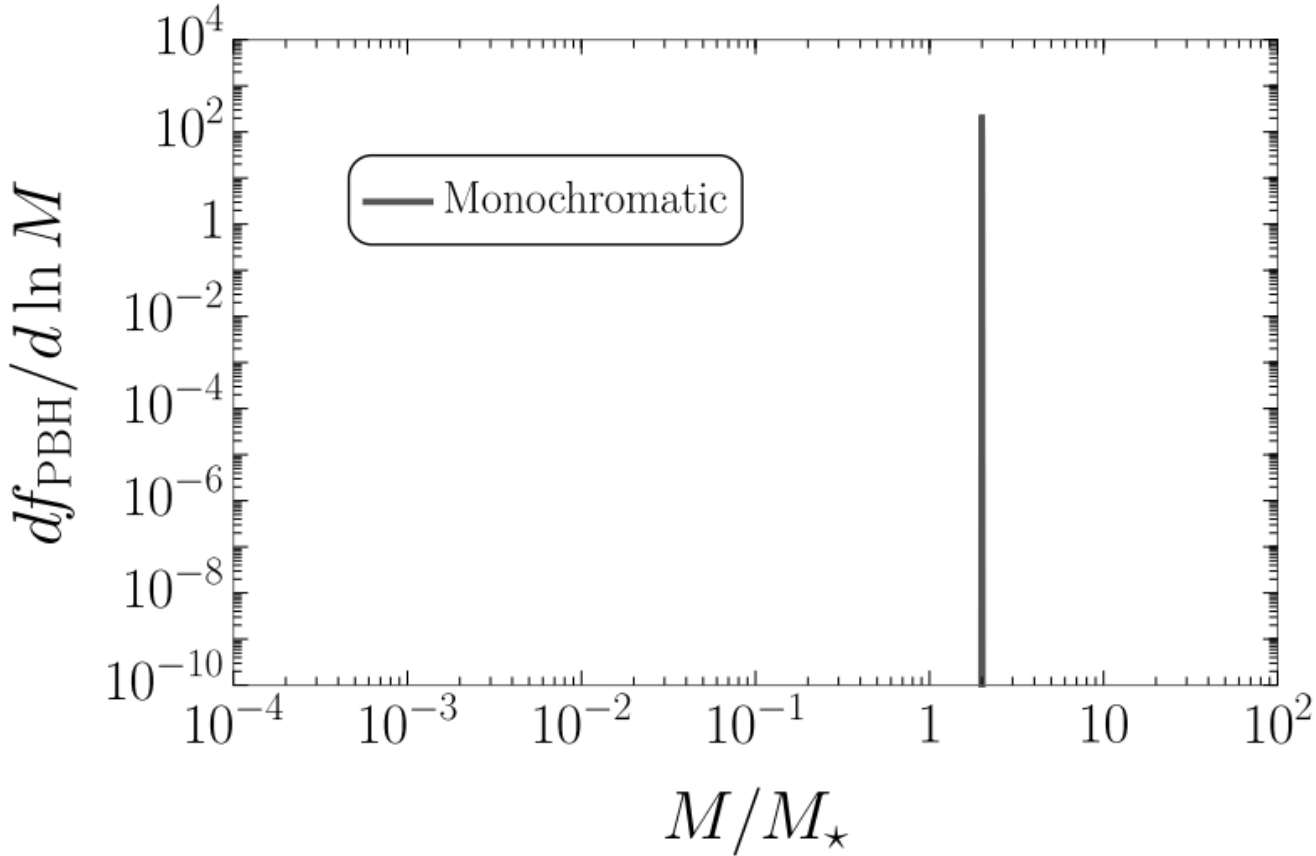
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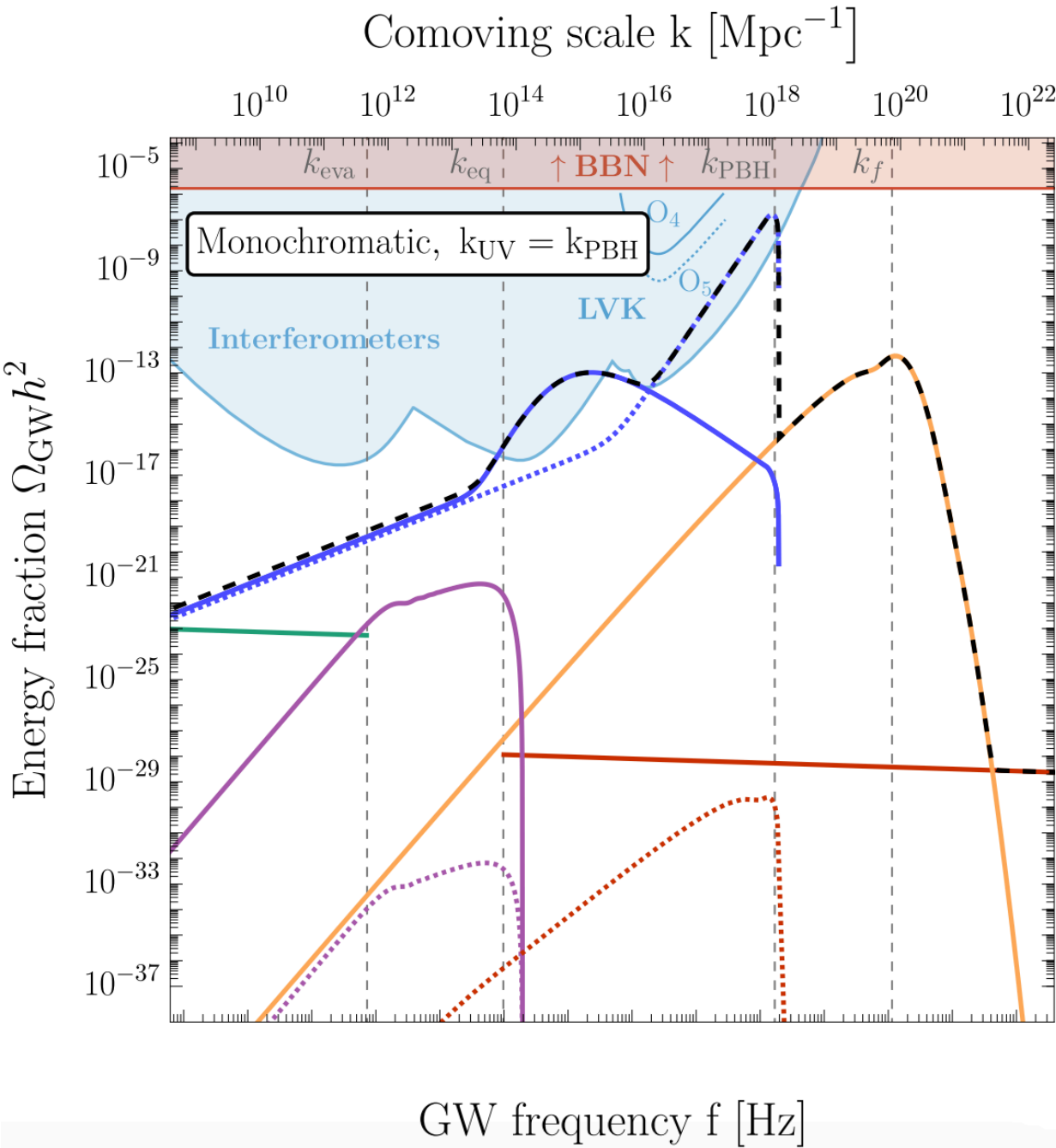
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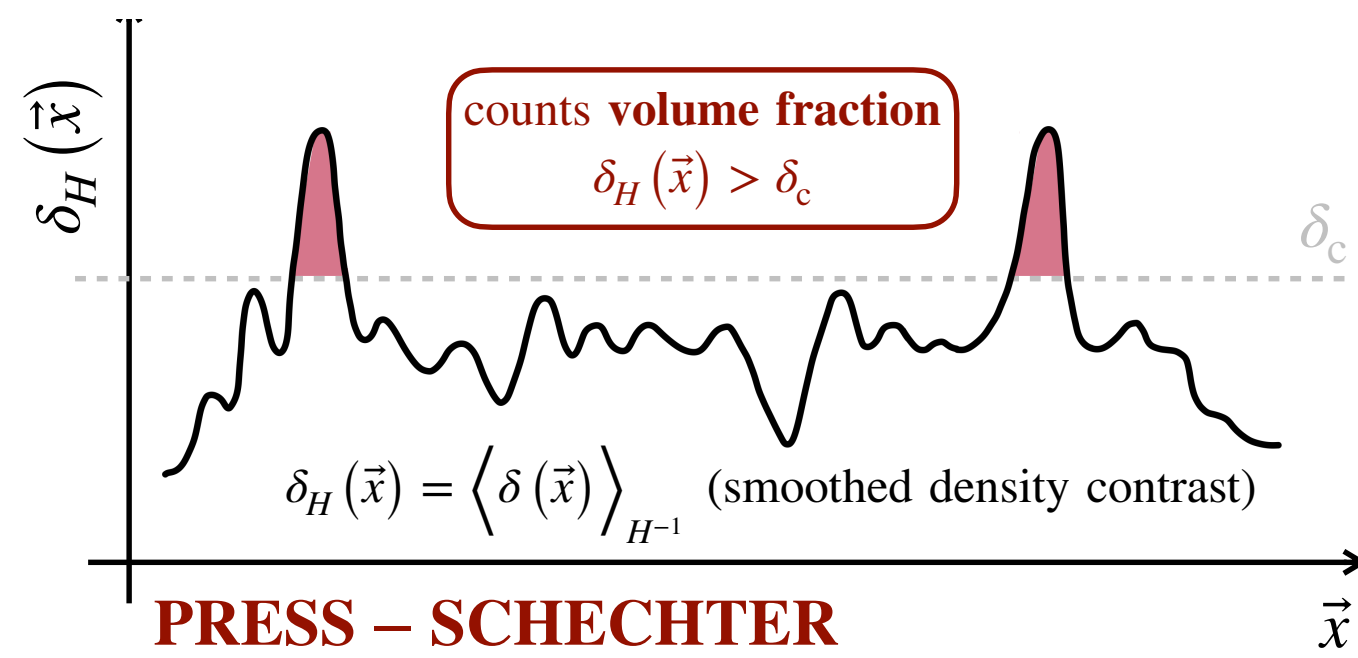


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Press–Schechter: volume fraction with $\delta > \delta_c$

Peak theory: local maxima with $\delta > \delta_c$ (extra factor $\propto (\delta/\sigma)^3$)

Both feed the same collapse law – **critical (Choptuik) scaling** fixes the mass:

$$M(\delta) = \mathcal{K} M_H (\delta - \delta_c)^{\gamma_M} \quad (\mathcal{K} \simeq 4, \gamma_M \simeq 0.36)$$

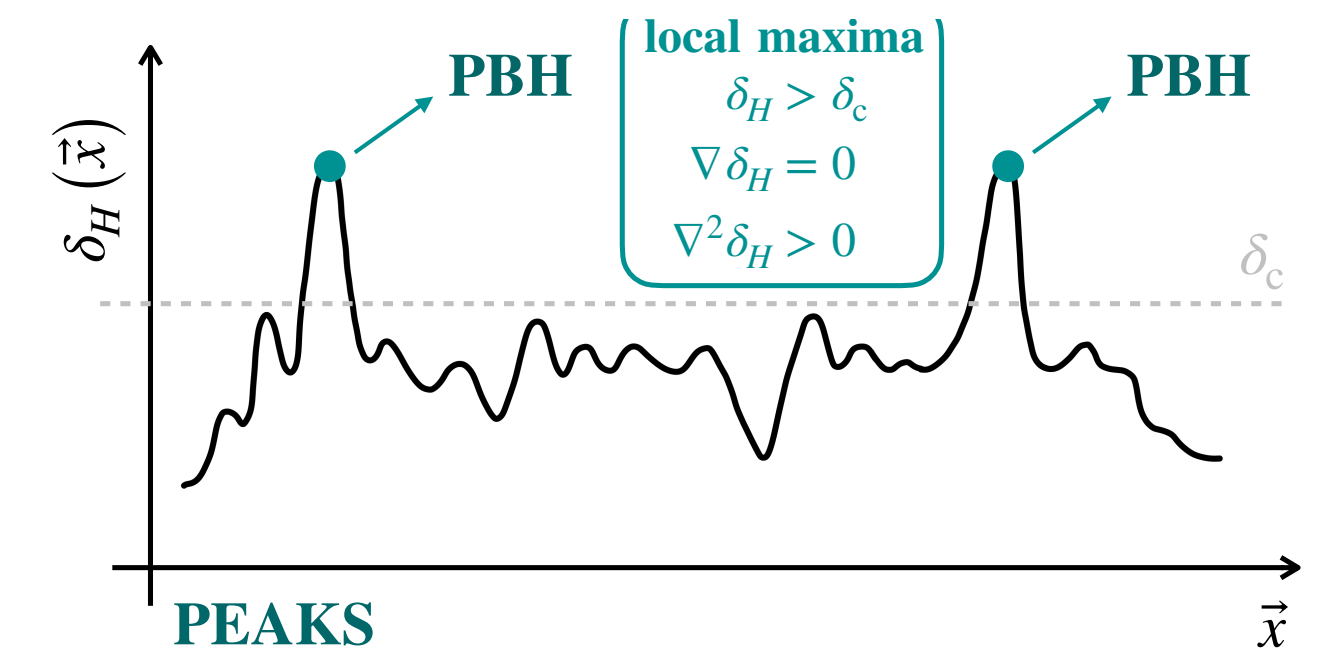
$$\implies M \rightarrow 0 \text{ as } \delta \rightarrow \delta_c^+.$$

Mass fraction (per $\ln M$); the δ_D leaves a Jacobian – count powers of M :

$$\begin{aligned} \frac{d\beta}{d \ln M} &= \int_{\delta_c}^{\infty} d\delta \frac{M}{M_H} P(\delta) \delta_D \left[\ln \frac{M}{M(\delta)} \right] \\ &= \underbrace{\frac{M}{M_H}}_{\propto M} \underbrace{\left| \frac{d\delta}{d \ln M} \right|}_{\propto M^{1/\gamma_M}} \underbrace{P(\delta)}_{\rightarrow P(\delta_c)} \end{aligned}$$

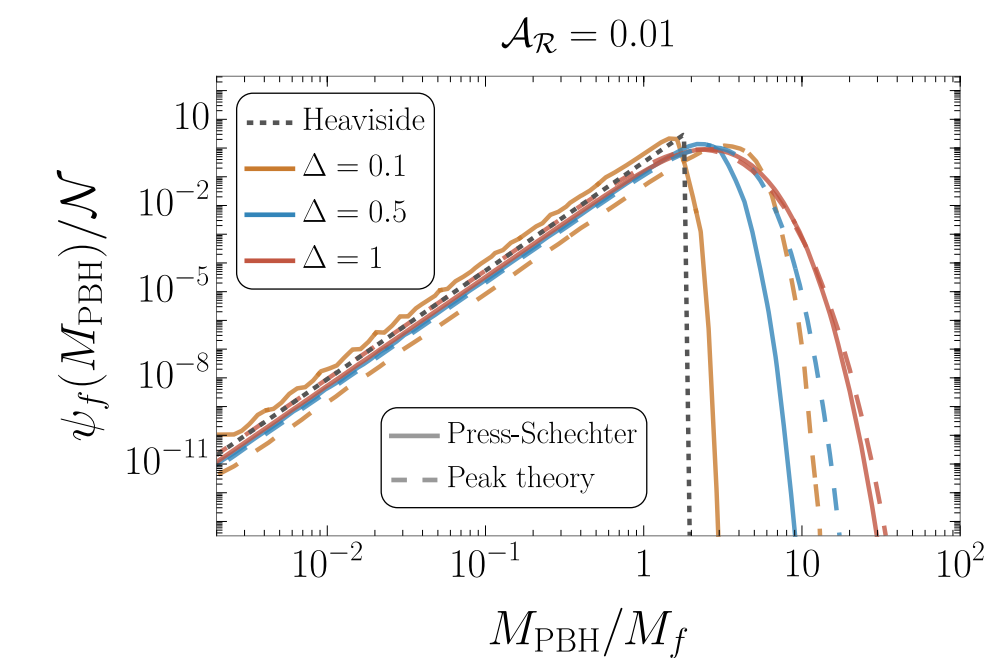
IR ($M \rightarrow 0$, so $\delta \rightarrow \delta_c$): $P \rightarrow P(\delta_c)$ const $\Rightarrow$ only kinematic powers survive:

$$d\beta/d \ln M \propto M^{1+1/\gamma_M} = M^{3.78}$$



same Gaussian core, different prefactor:

$$P_{\text{PS}} \propto e^{-\delta^2/2\sigma^2}, \quad P_{\text{pk}} \propto \left(\frac{\delta}{\sigma}\right)^3 e^{-\delta^2/2\sigma^2}$$



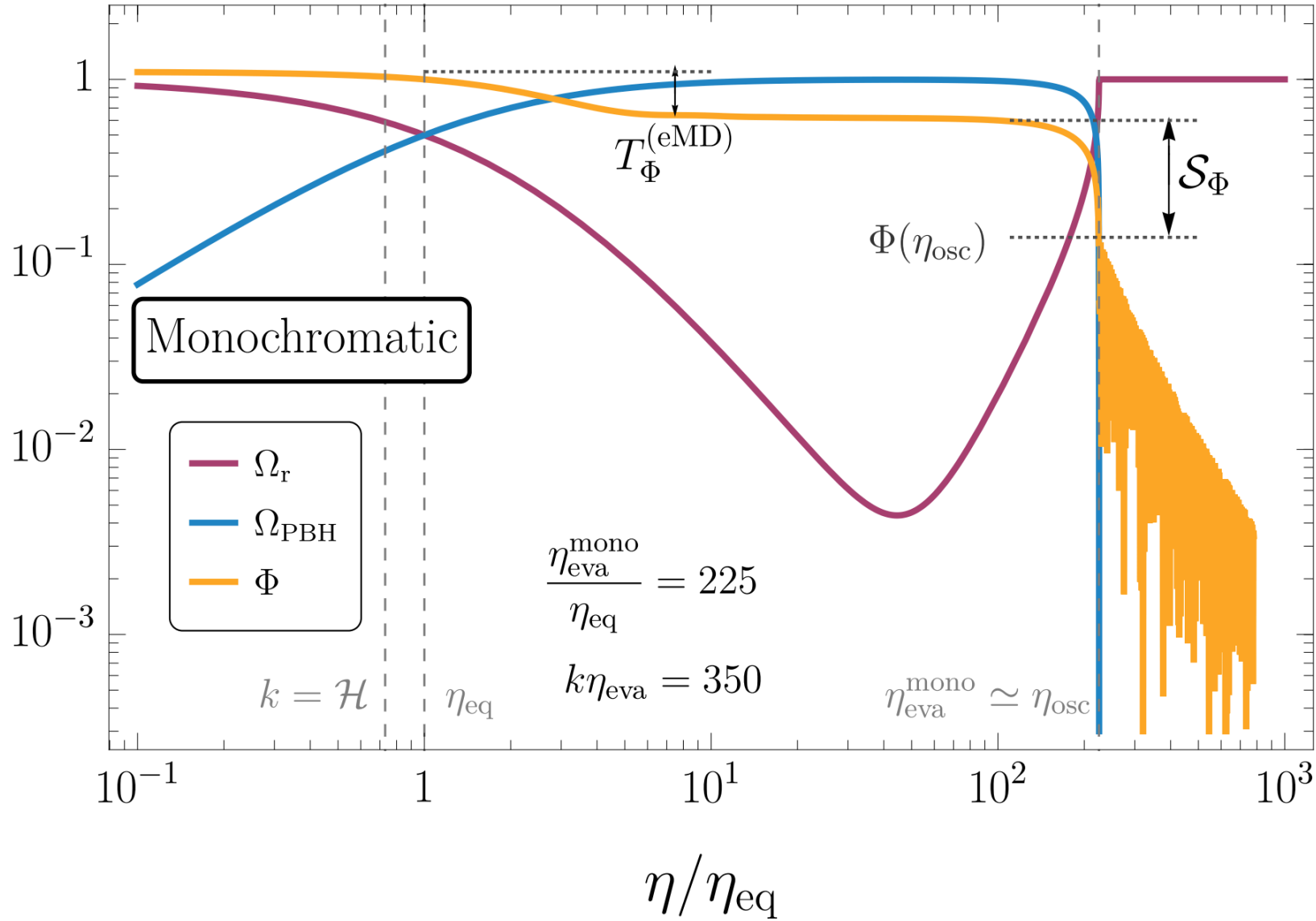
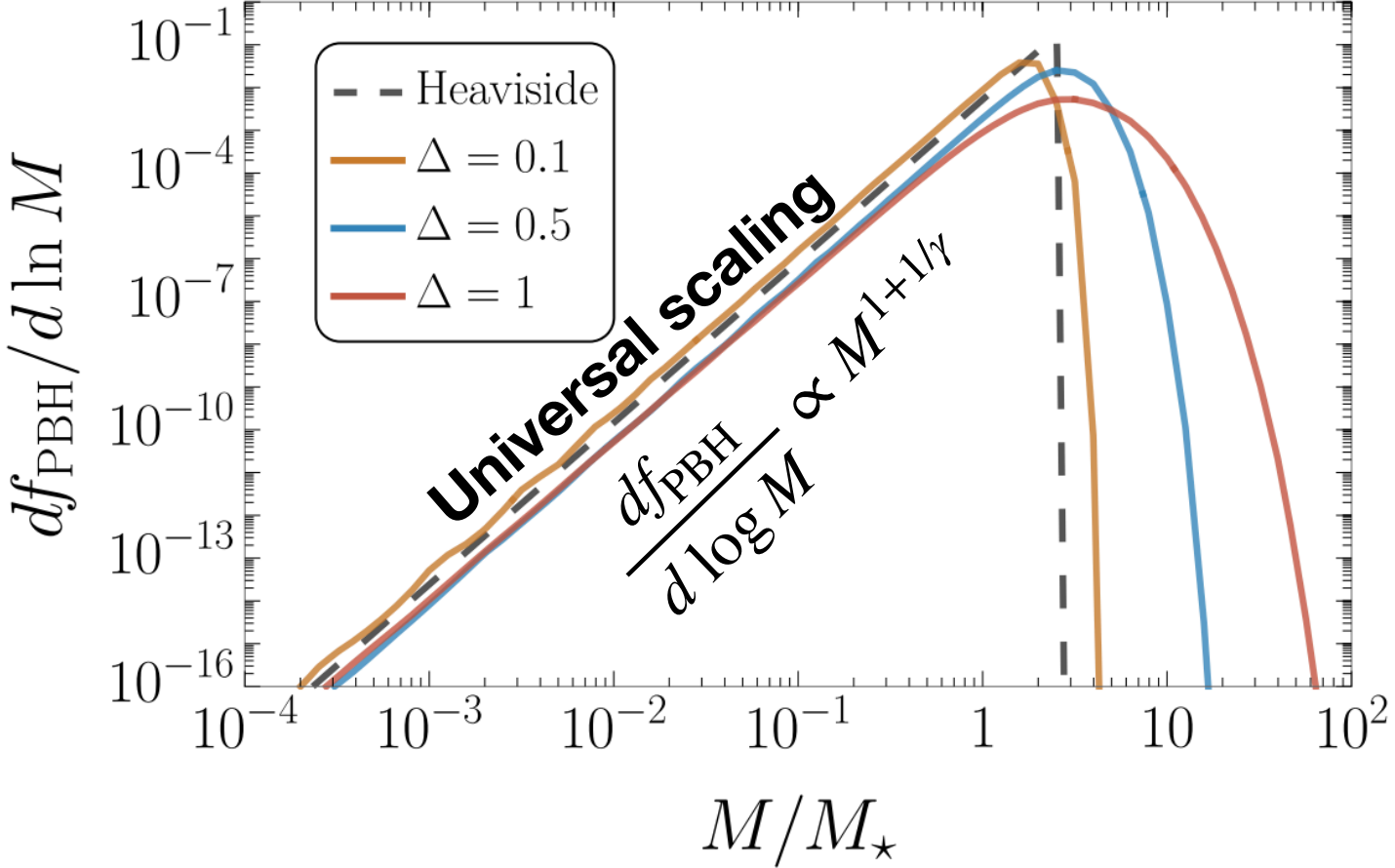
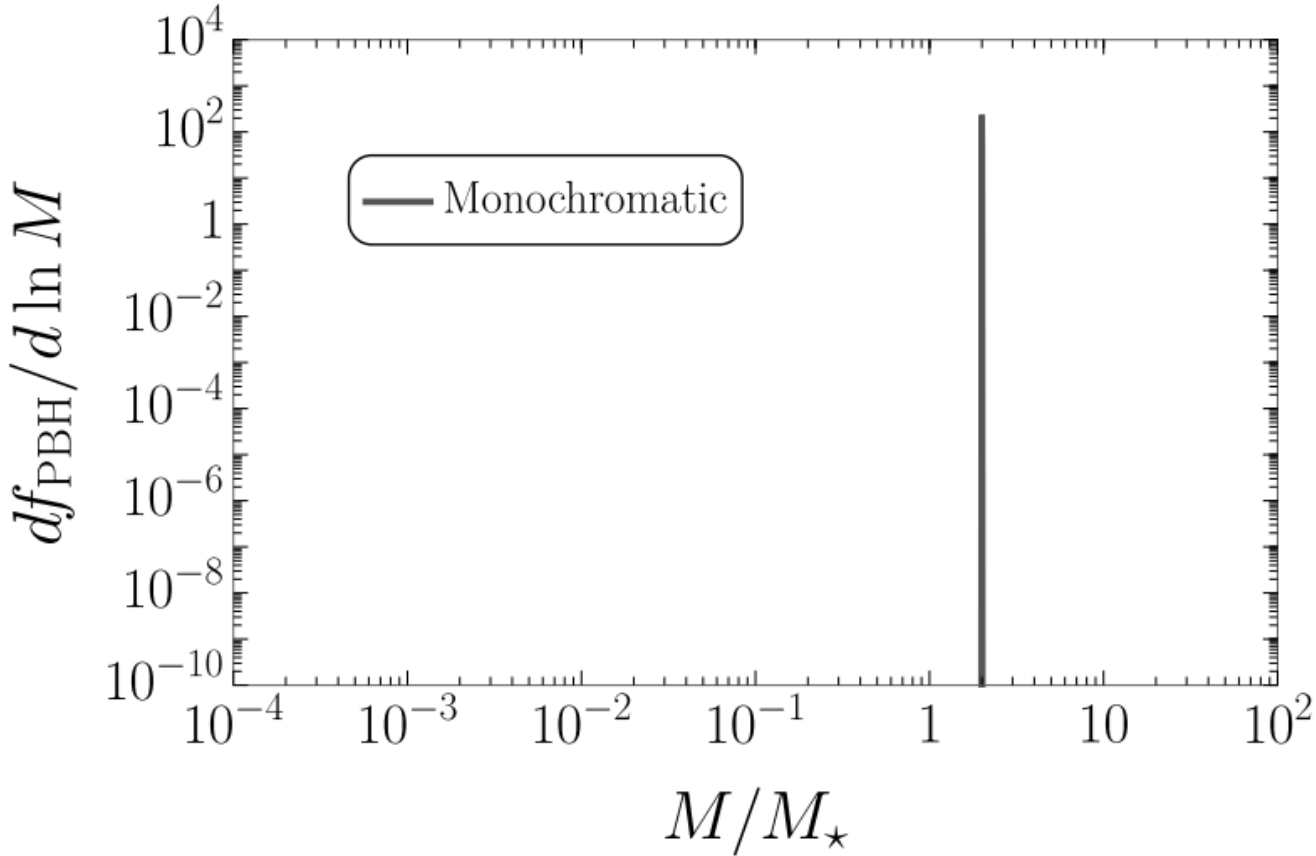
every width Δ , PS (solid) & peak (dashed) $\Rightarrow$ same $M^{3.78}$ IR line; UV cutoff is model-dependent.

Universal: the IR slope is *pure Choptuik kinematics*; $P(\delta_c)$ sets only the amplitude, not the exponent.

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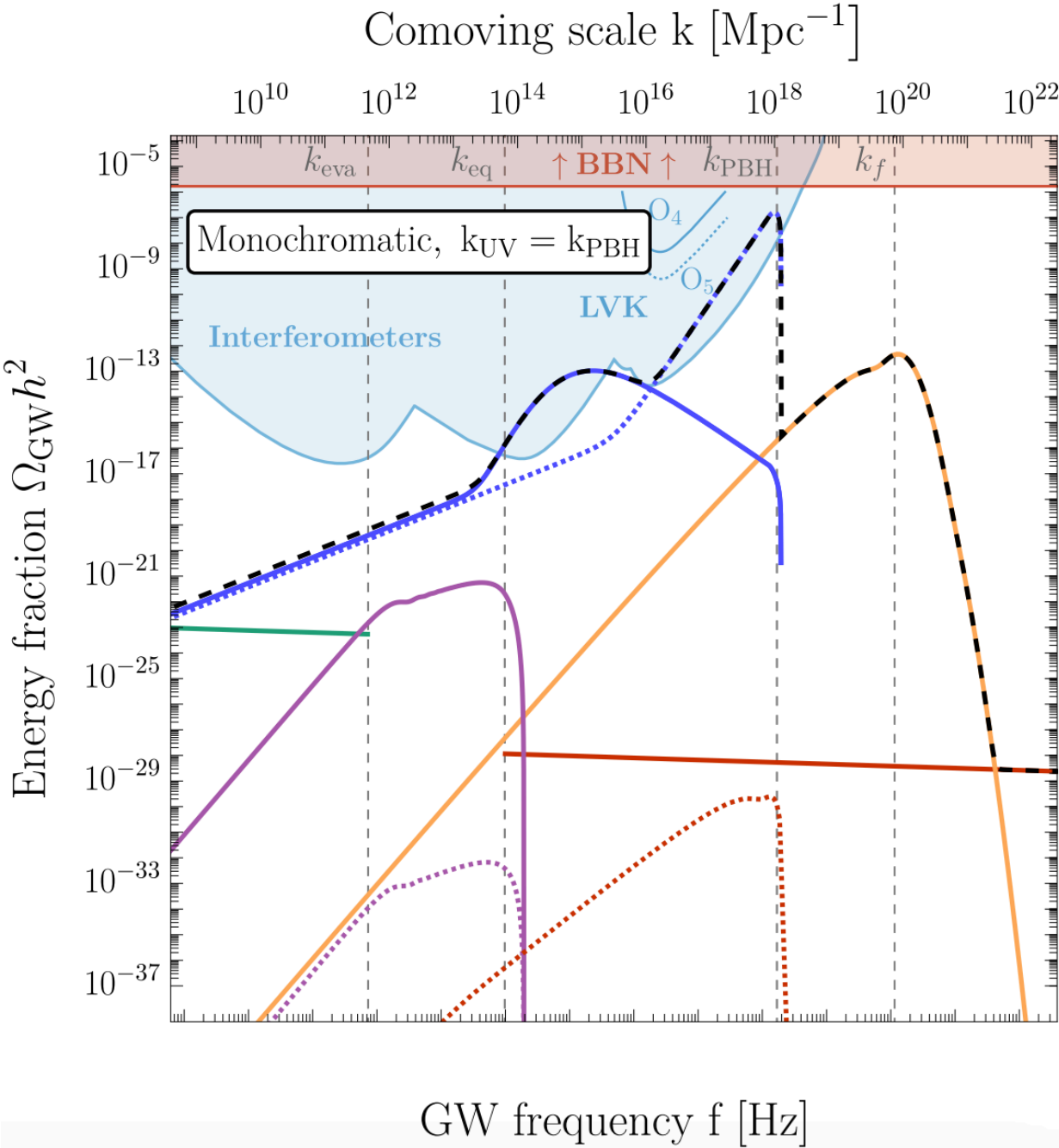
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Tensor perturbation equation

$$\square h_{ij} = \partial_i\Phi\partial_j\Phi$$

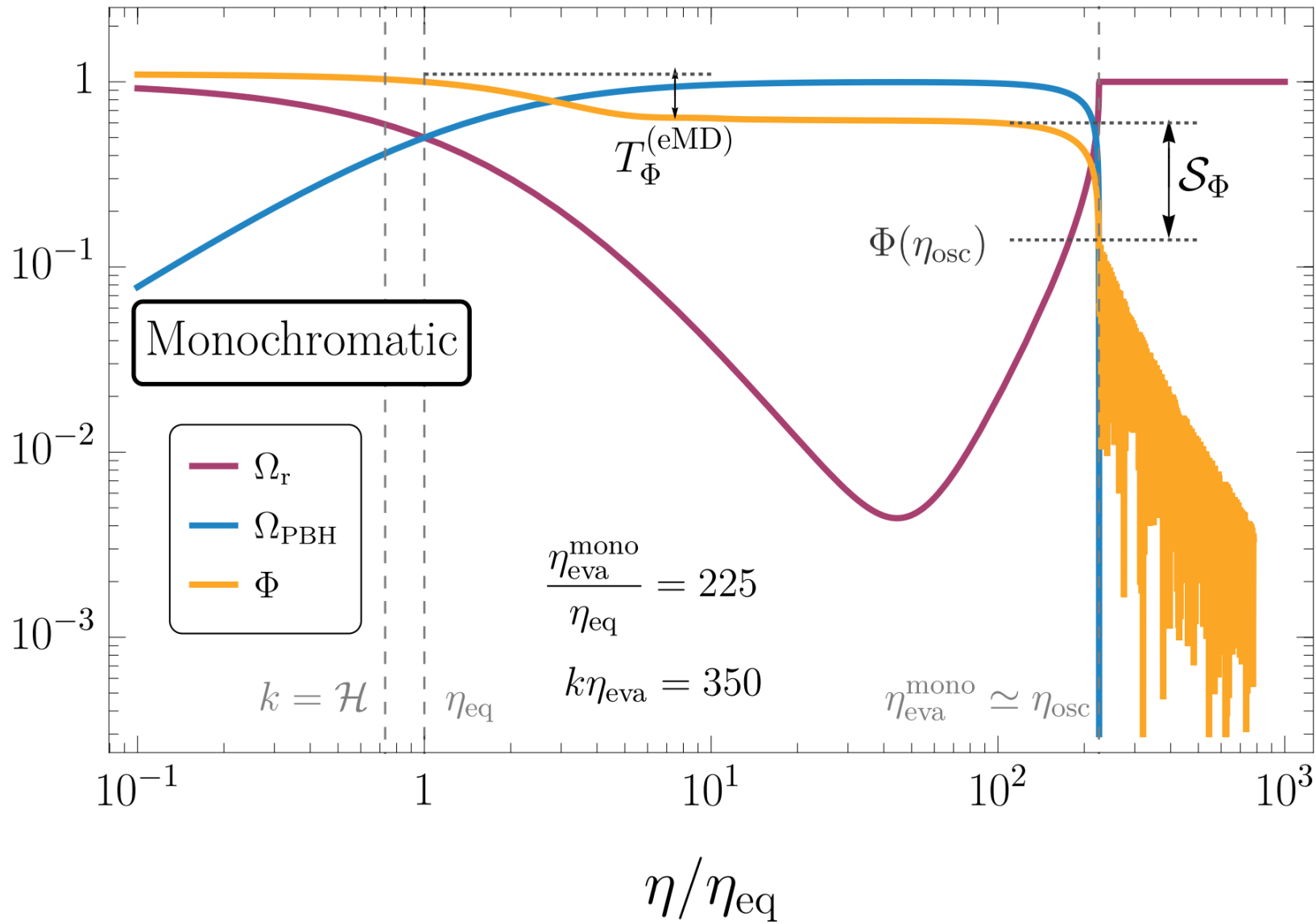
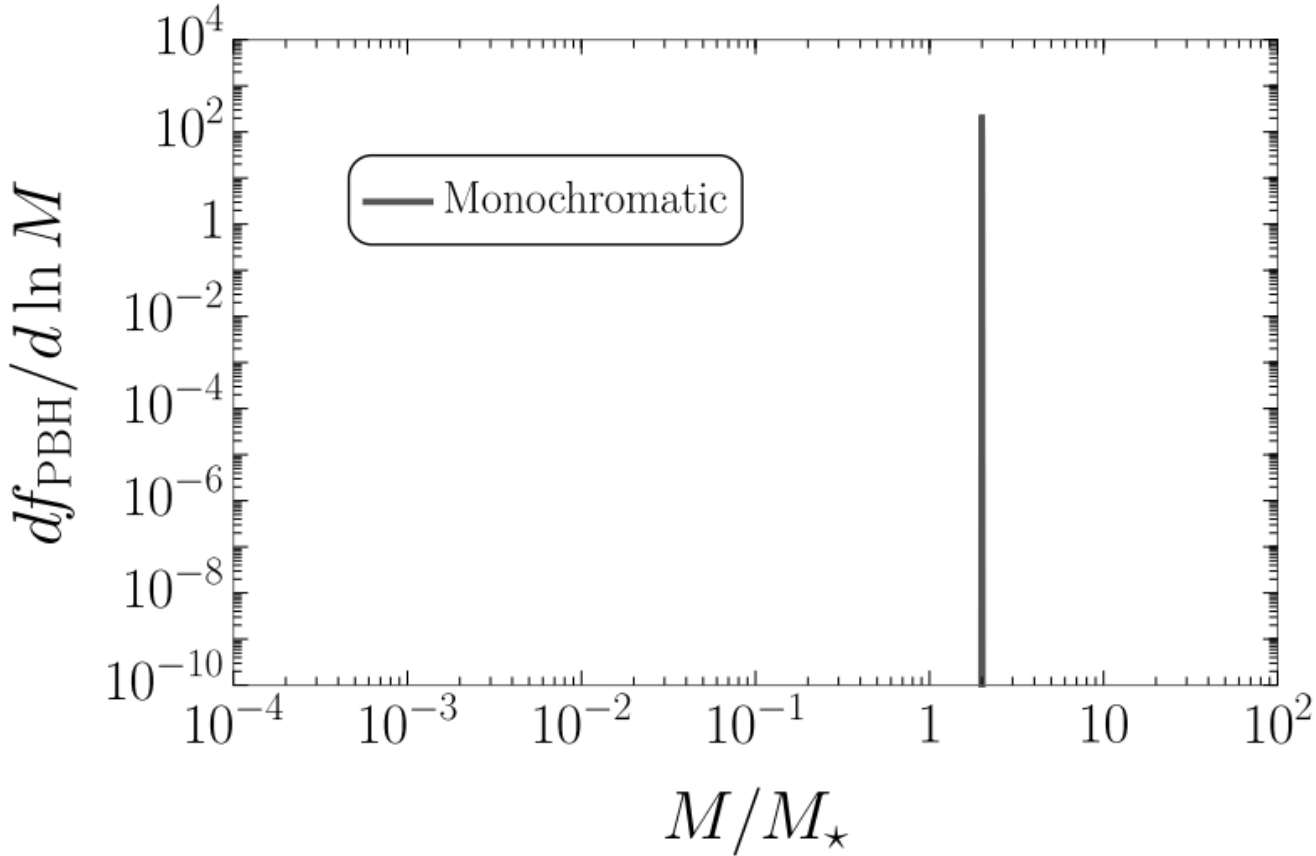
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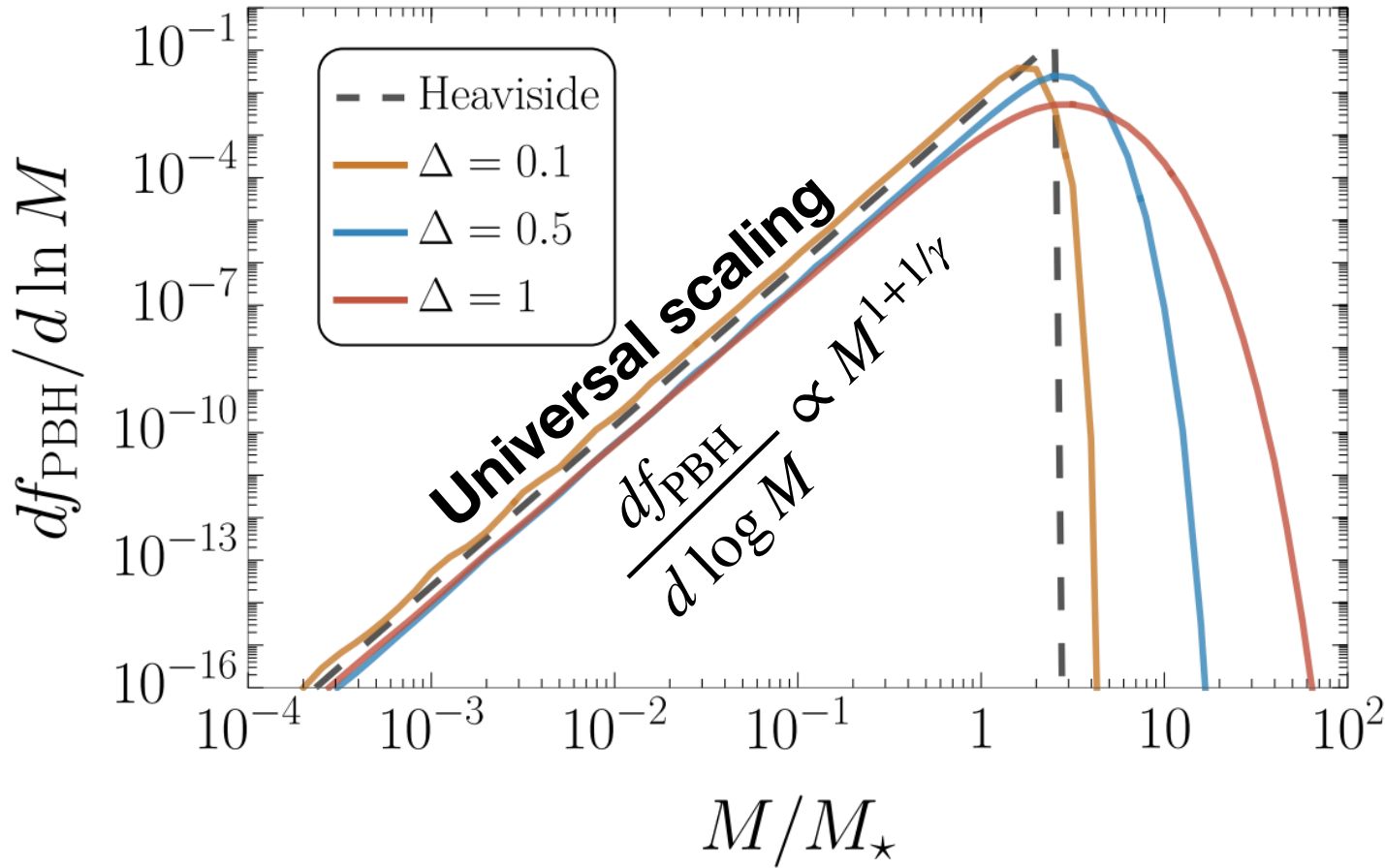
$$\rho'_{\text{PBH}} + 3\mathcal{H}\rho_{\text{PBH}} = -\Gamma\rho_{\text{PBH}}$$

$$\rho'_r + 4\mathcal{H}\rho_r = \Gamma\rho_{\text{PBH}}$$

$$k^2\Phi \simeq \frac{3}{2}\mathcal{H}^2\left(\frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}}\delta_{\text{PBH}} + \frac{\rho_{\text{rad}}}{\rho_{\text{tot}}}\delta_{\text{r}}\right)$$



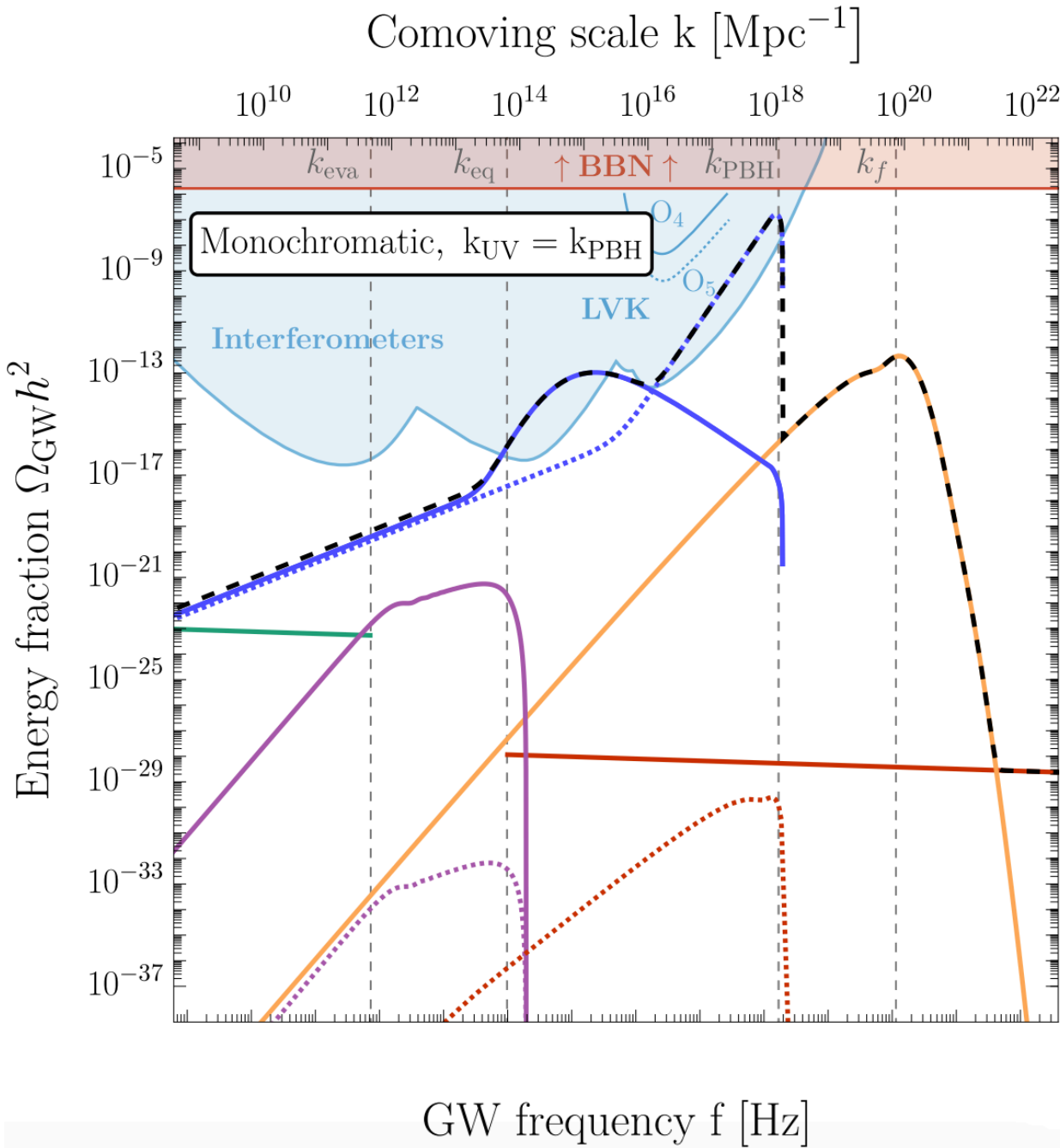
$$\int d\ln M \frac{df_{\text{PBH}}}{d\ln M} \left(\text{all the equations leading to } \Phi \text{ and } h_{ij} \right)$$



Tensor perturbation equation

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$$\begin{aligned}\rho'_{\text{PBH}} + 3\mathcal{H}\rho_{\text{PBH}} &= -\Gamma\rho_{\text{PBH}} \\ \rho'_r + 4\mathcal{H}\rho_r &= \Gamma\rho_{\text{PBH}}\end{aligned}$$

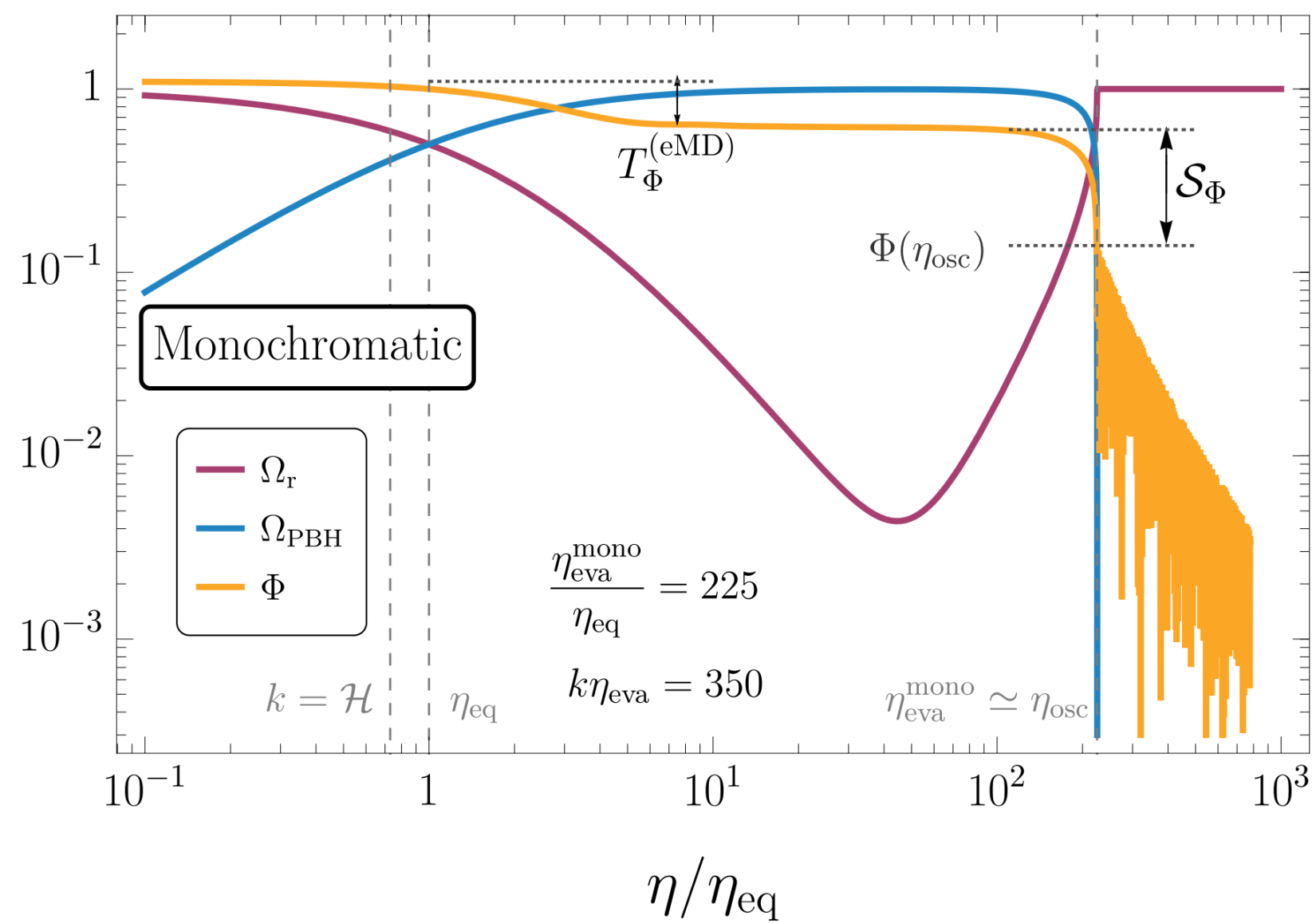
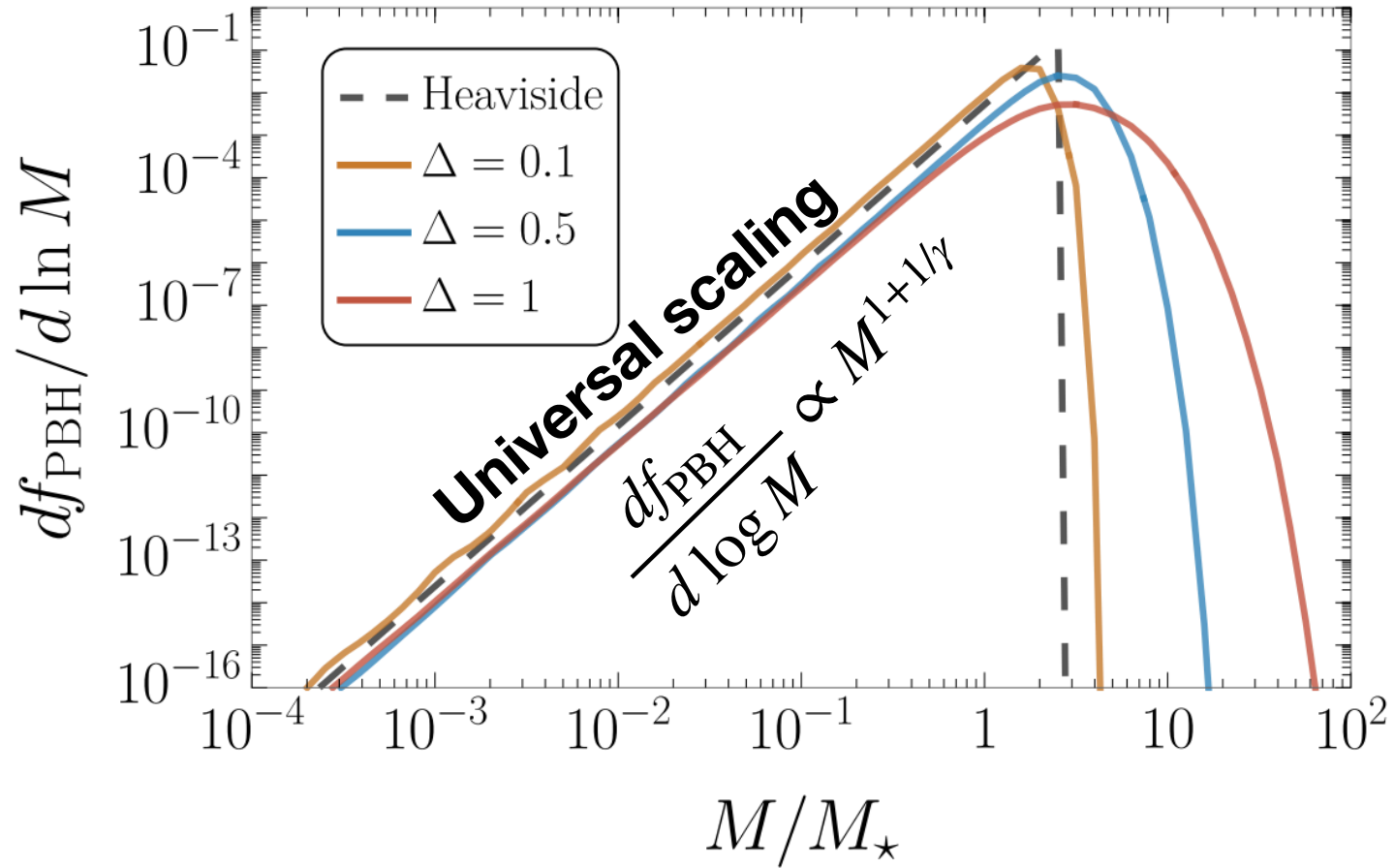
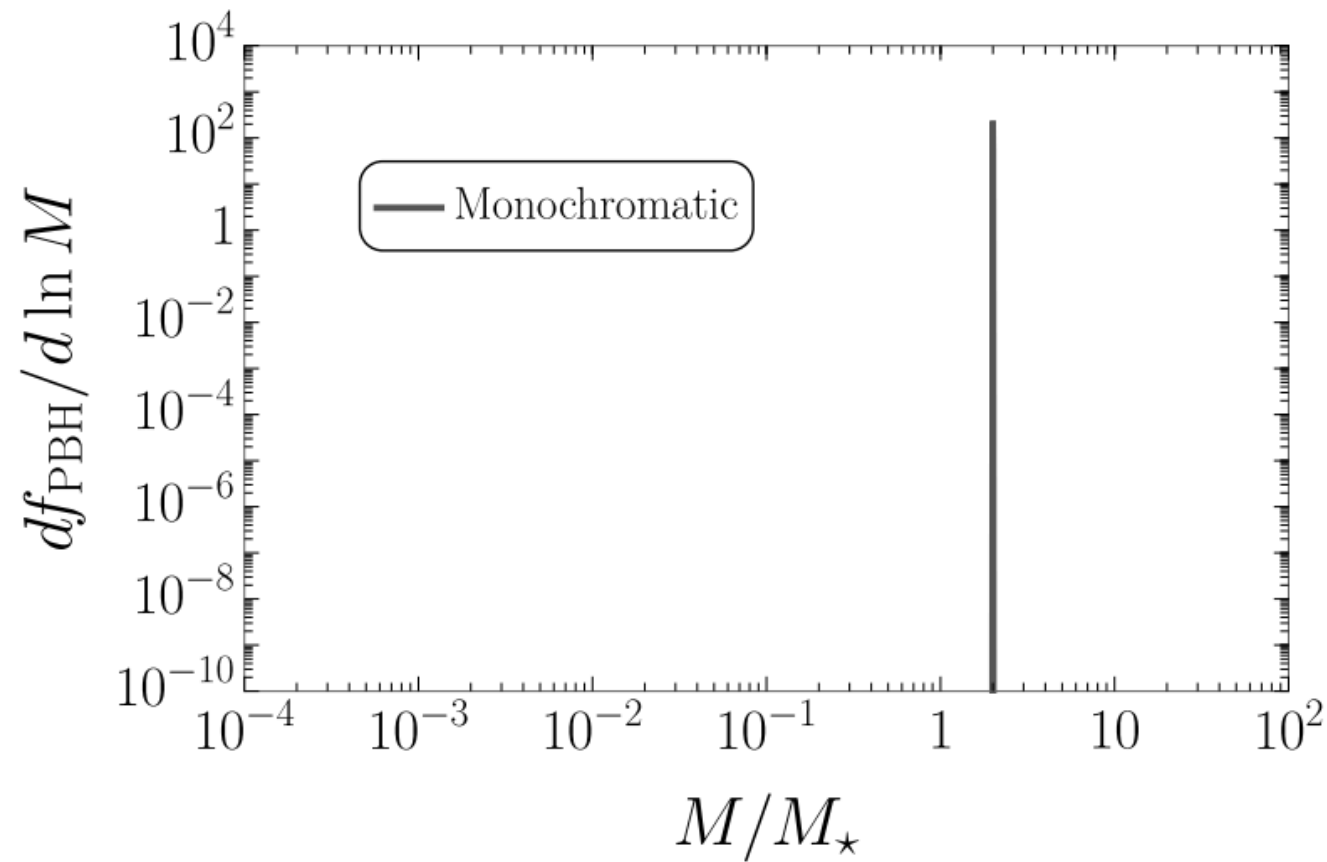
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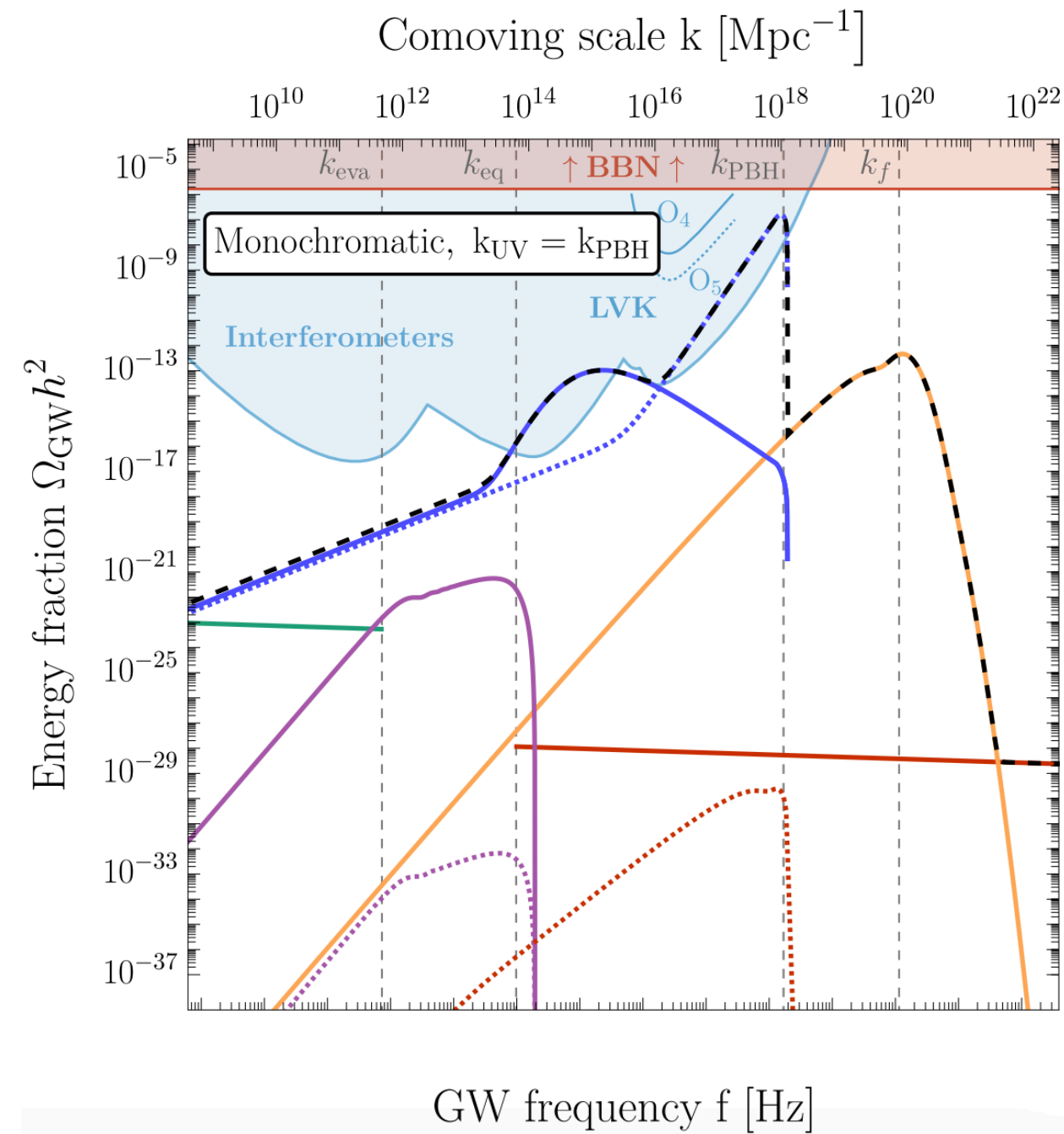
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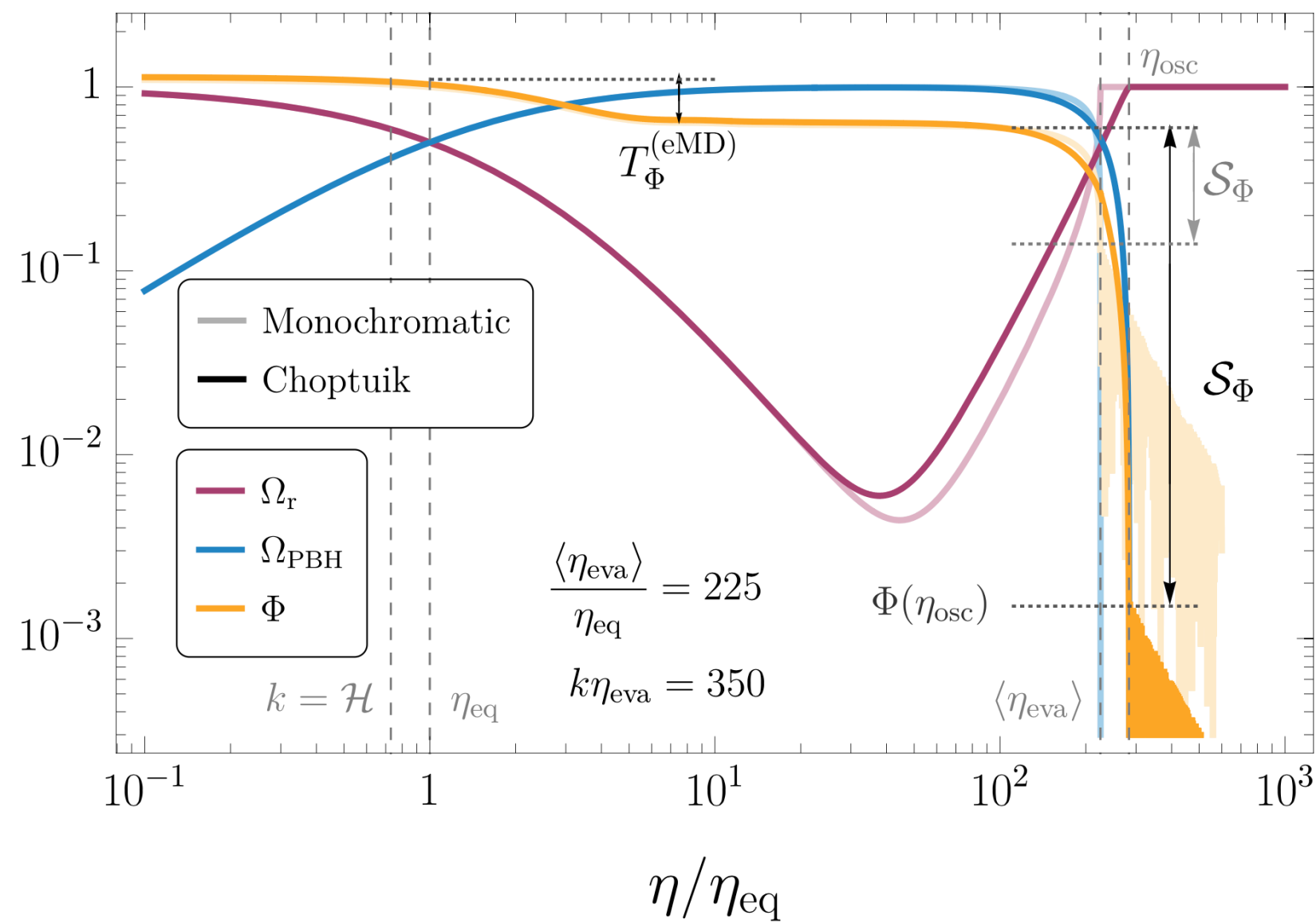
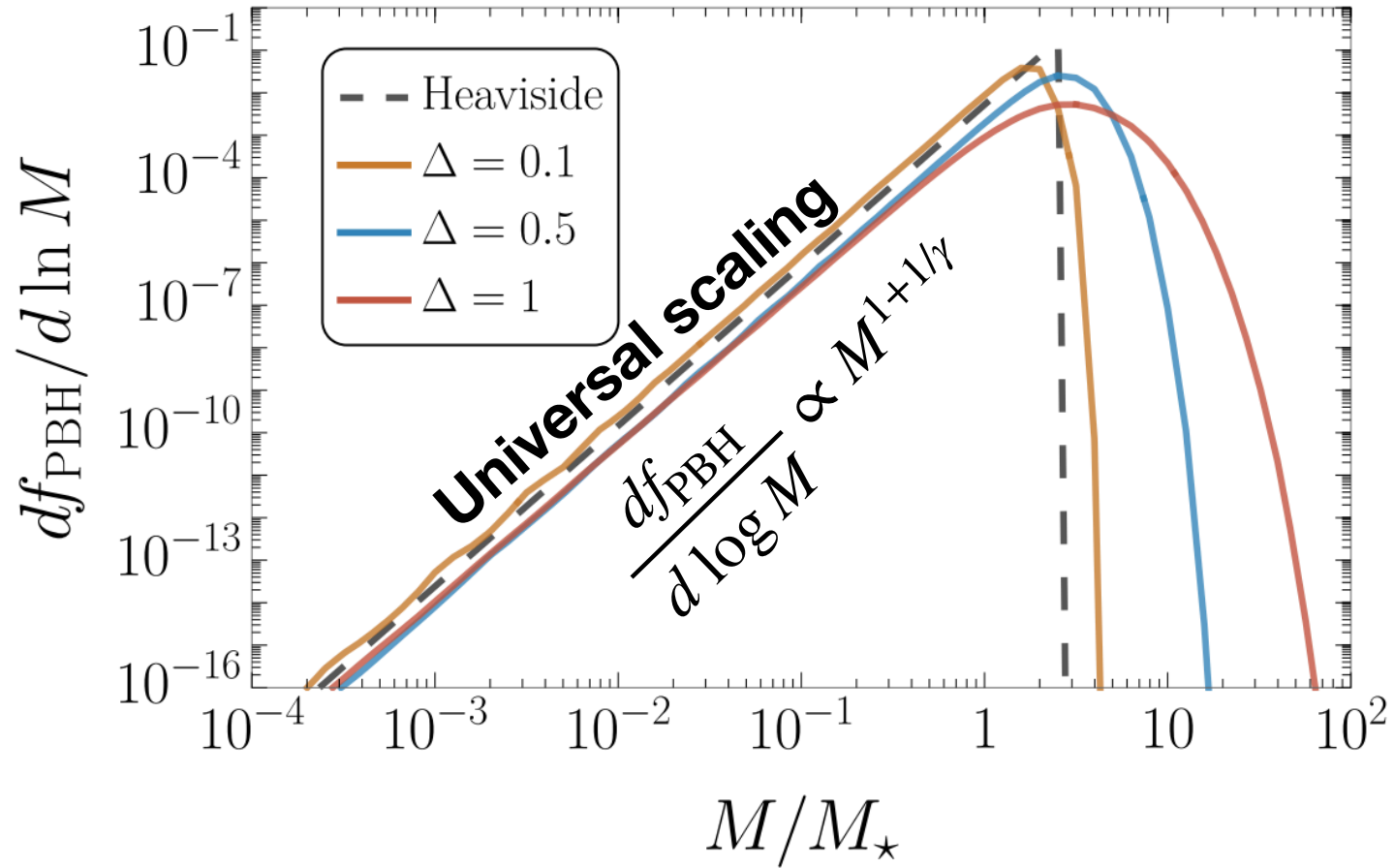
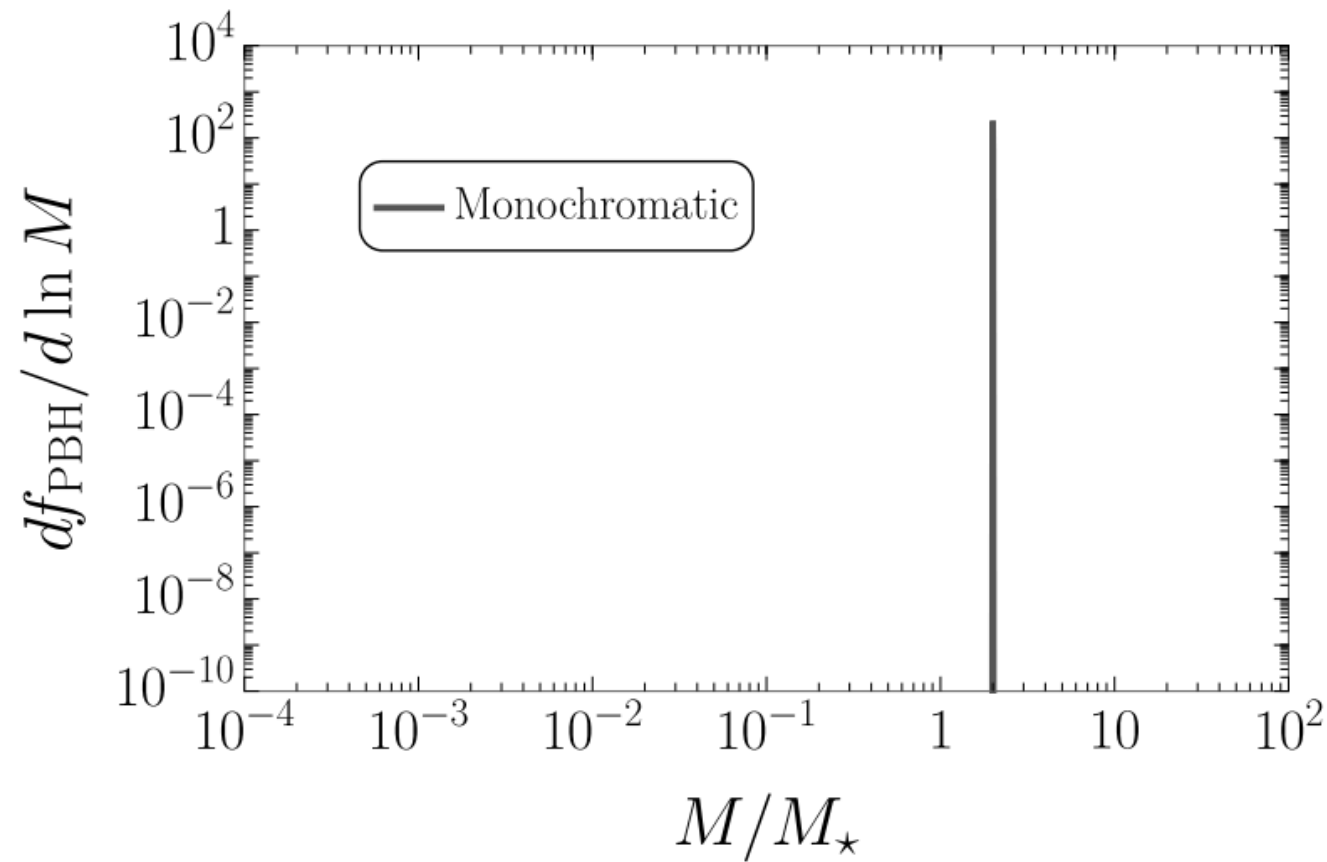
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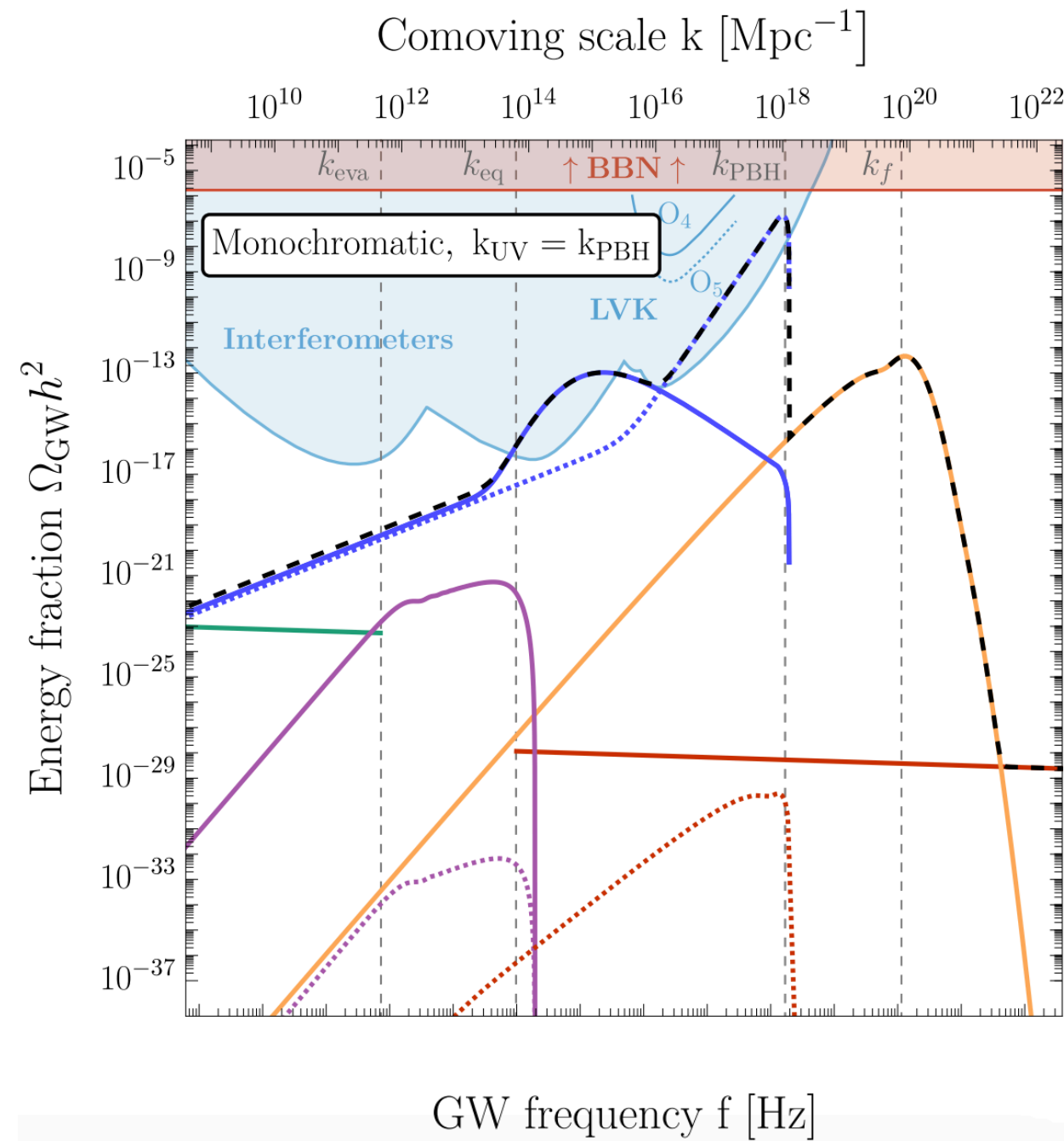


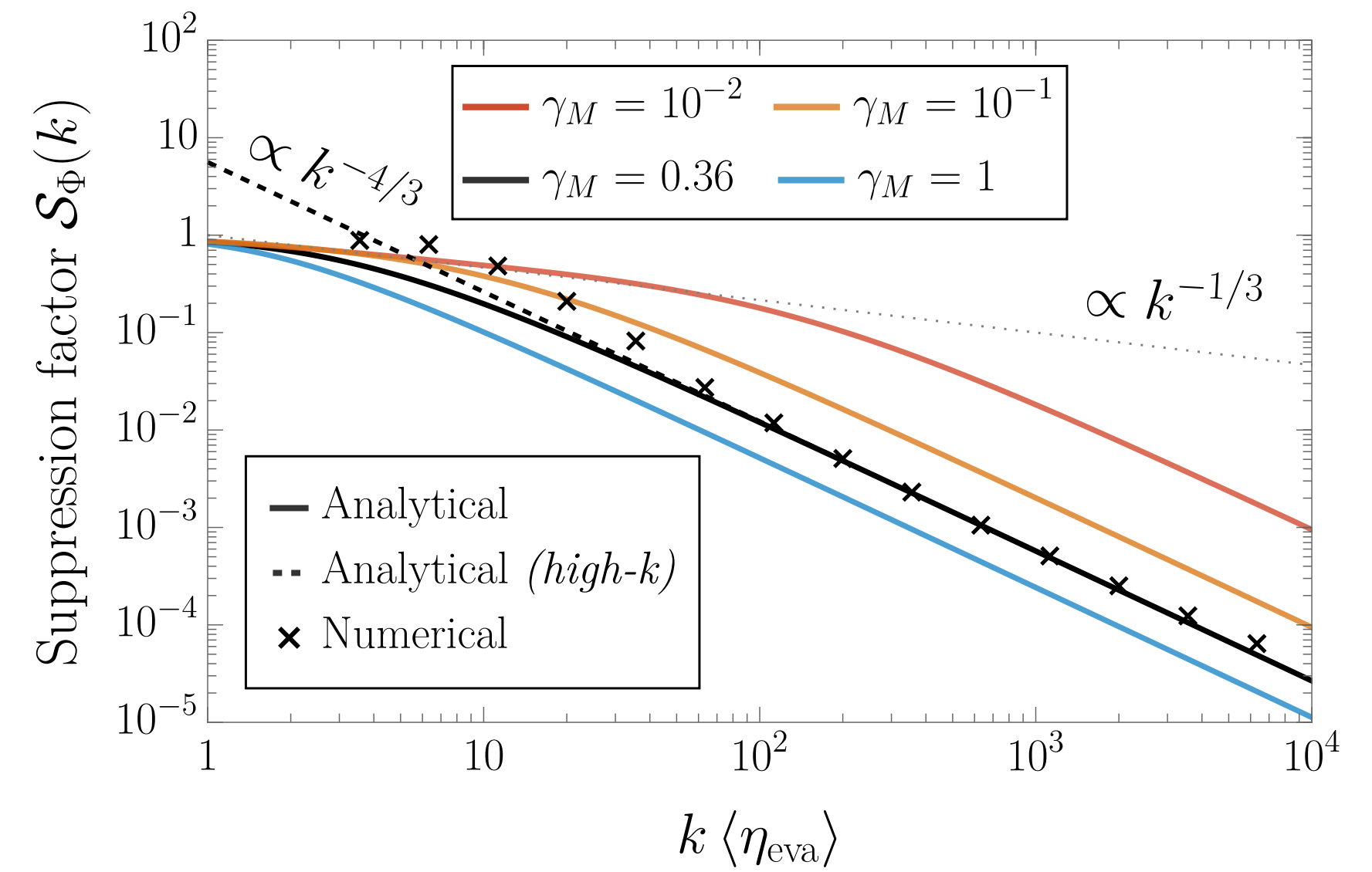
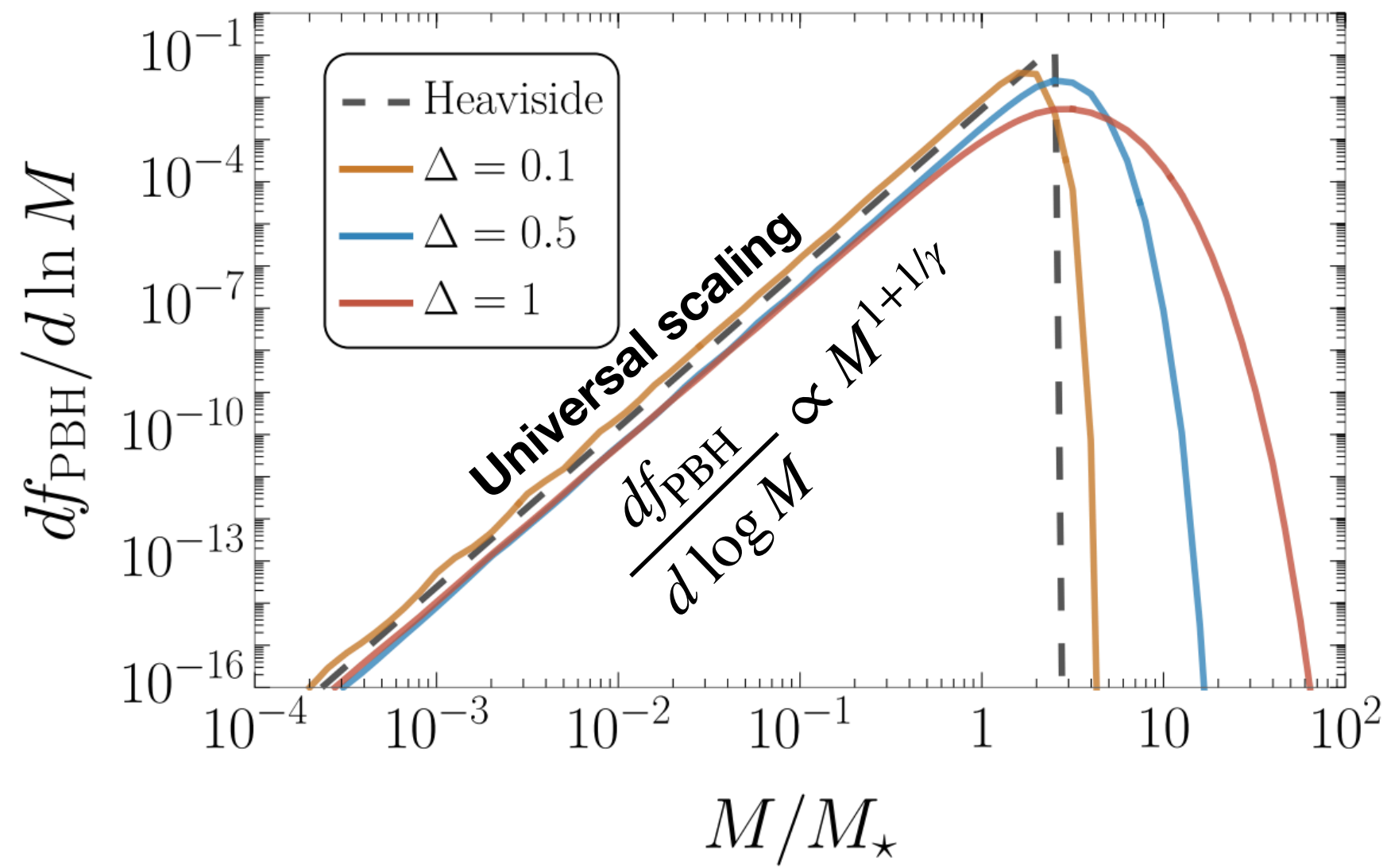
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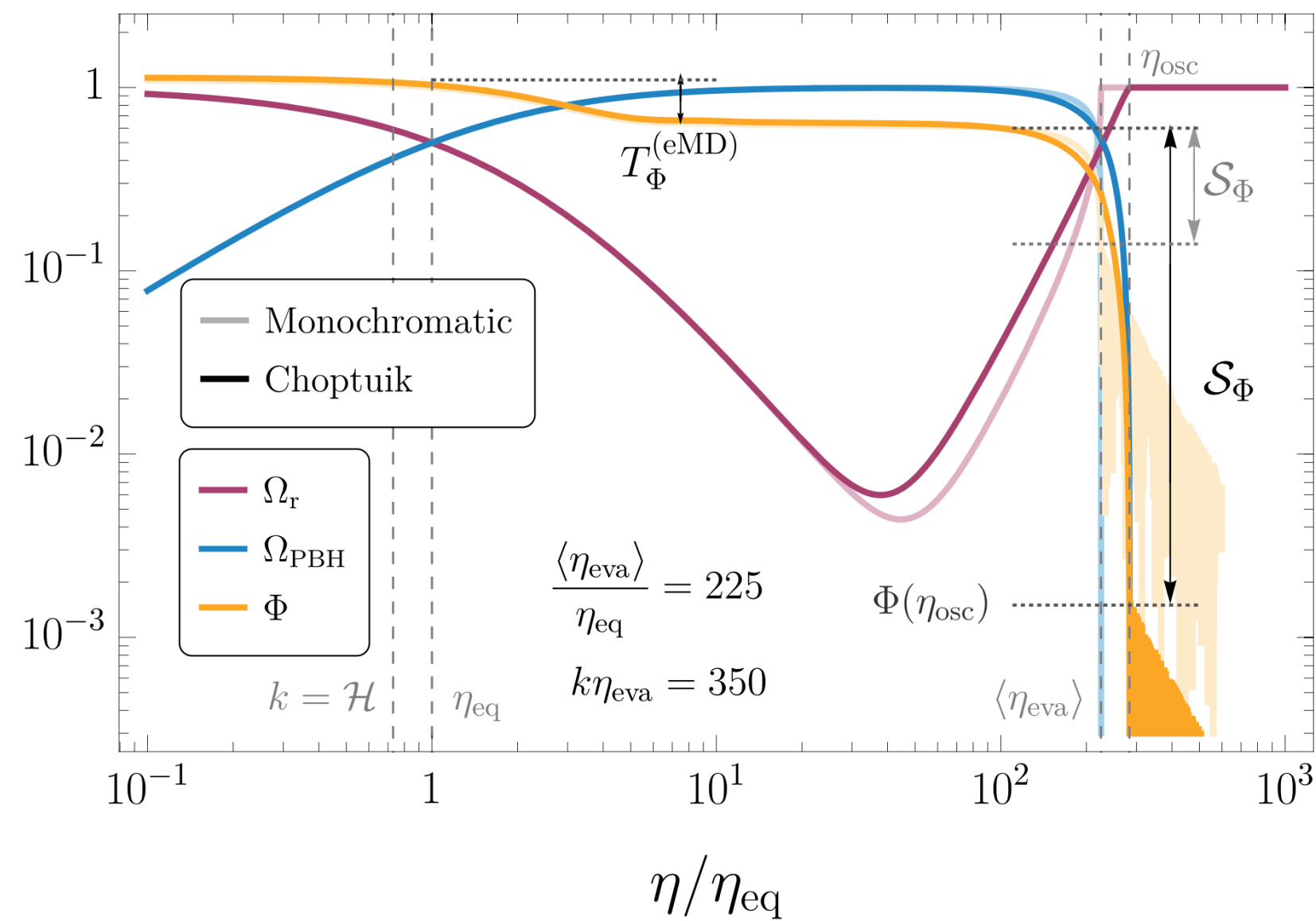




$$\Omega_{\text{GW}}^{(\text{RD}_2)}[X] = \frac{1}{864} \left(\frac{k}{k_{\text{eva}}} \right)^8 \int_0^\infty dv \int_{|1-v|}^{1+v} du \left[4v^2 - (1 + v^2 - u^2)^2 \right]^2$$

$$\times \overline{\mathcal{I}_{\text{osc}}^2}(u, v) \mathcal{S}_\Phi^2(uk) \mathcal{S}_\Phi^2(vk) T_X^{(\text{eMD})^2}(uk) T_X^{(\text{eMD})^2}(vk) \mathcal{P}_X(uk) \mathcal{P}_X(vk),$$

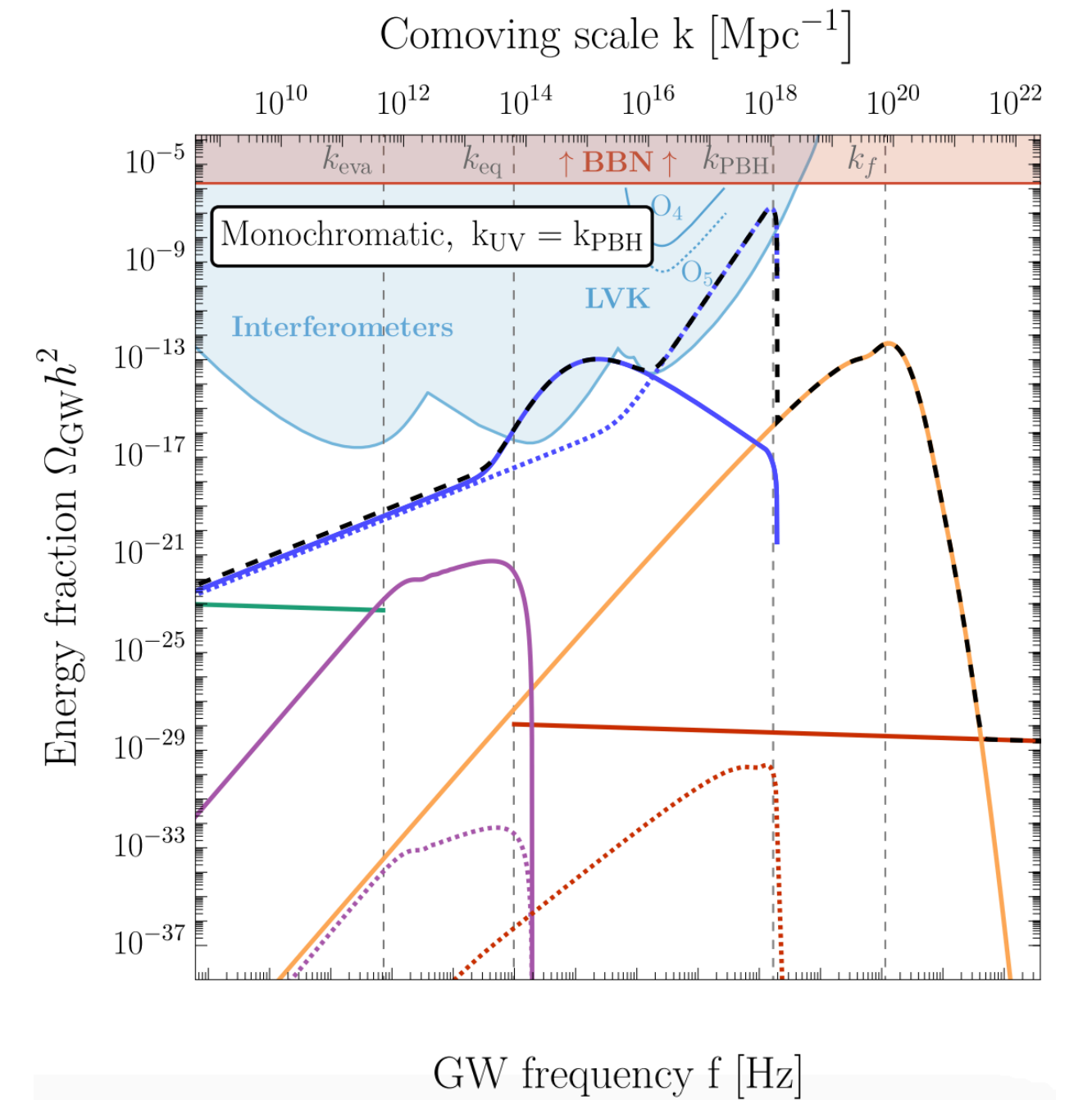
$$X = \{\Phi, S\}$$

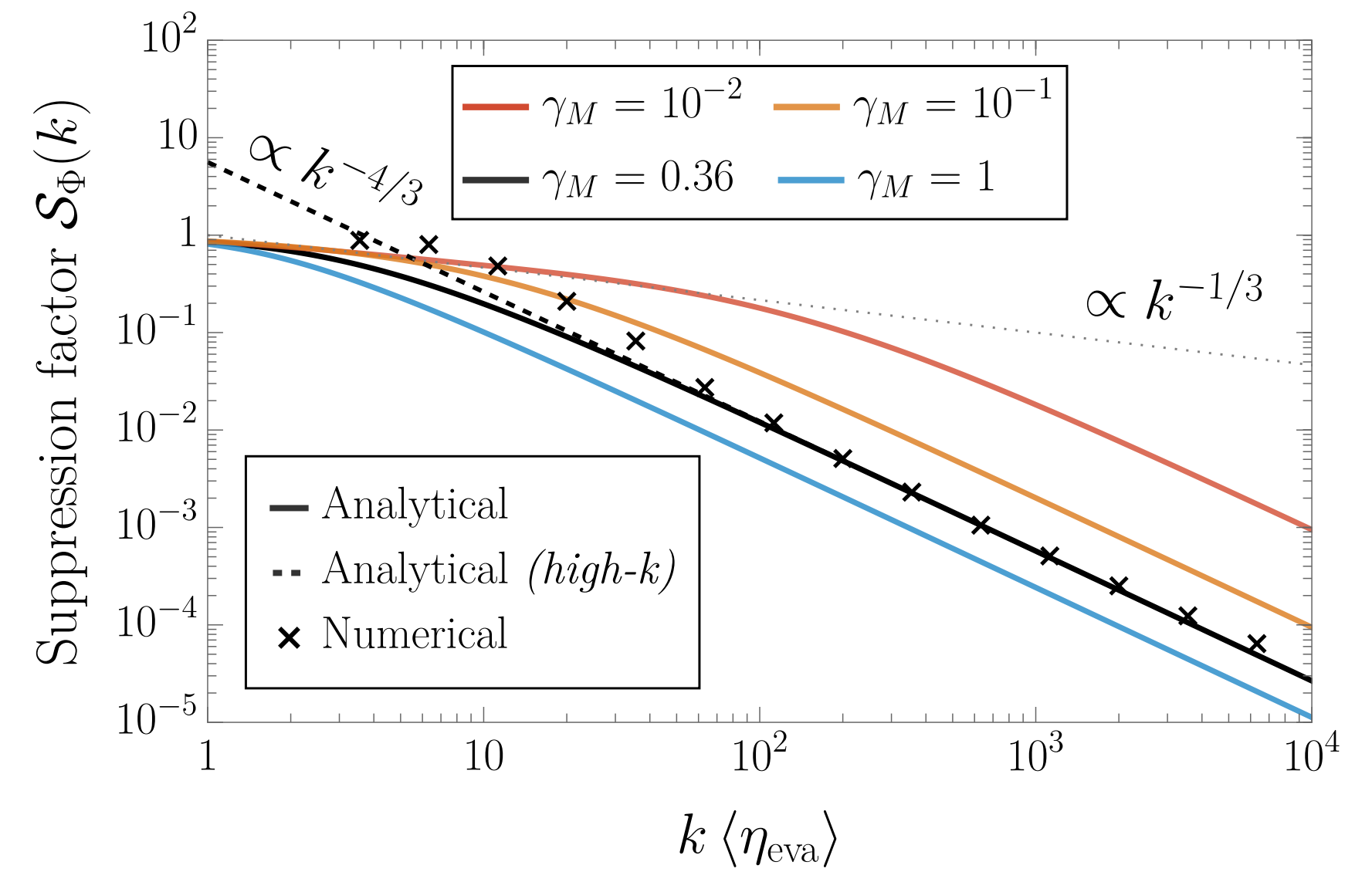
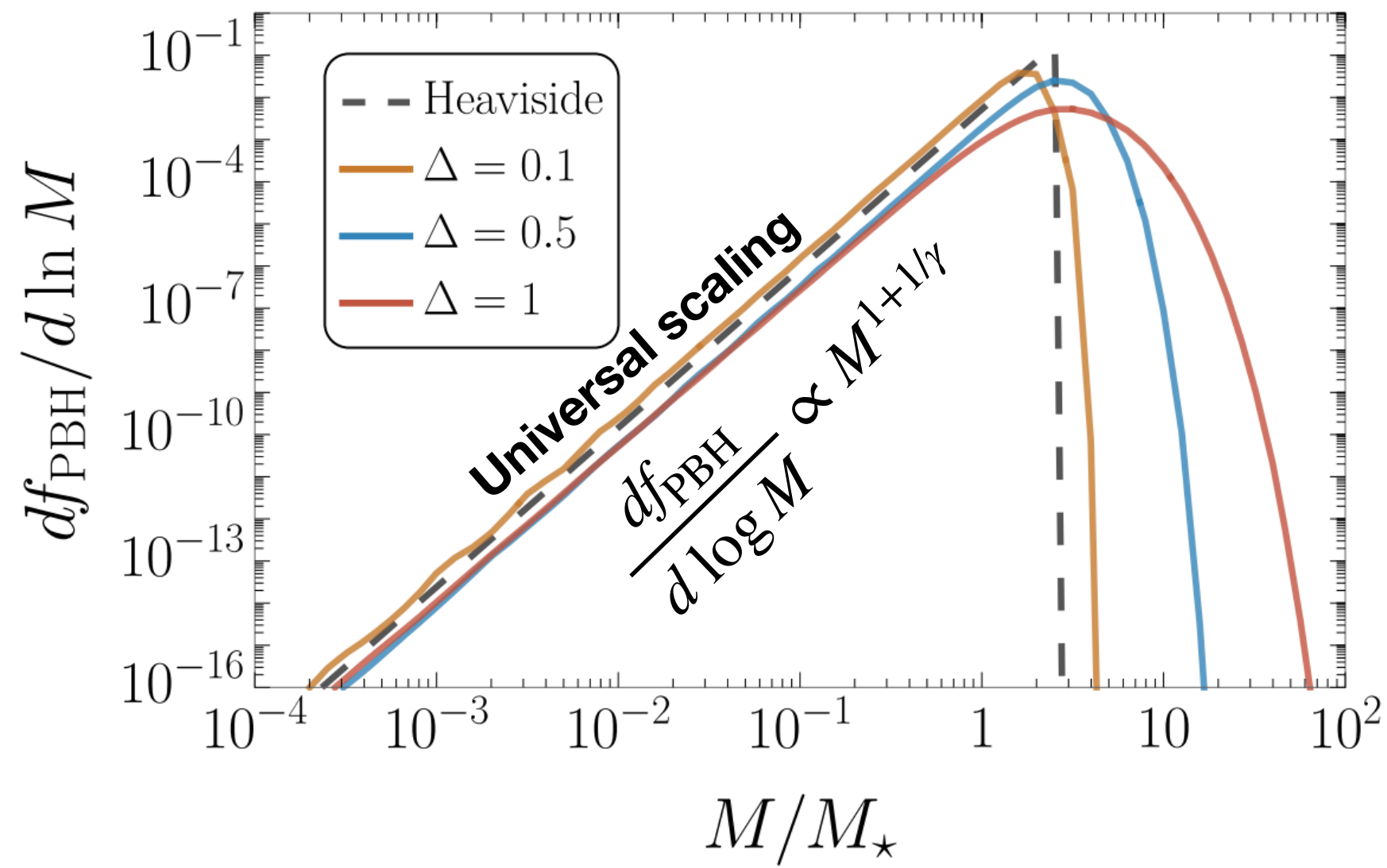


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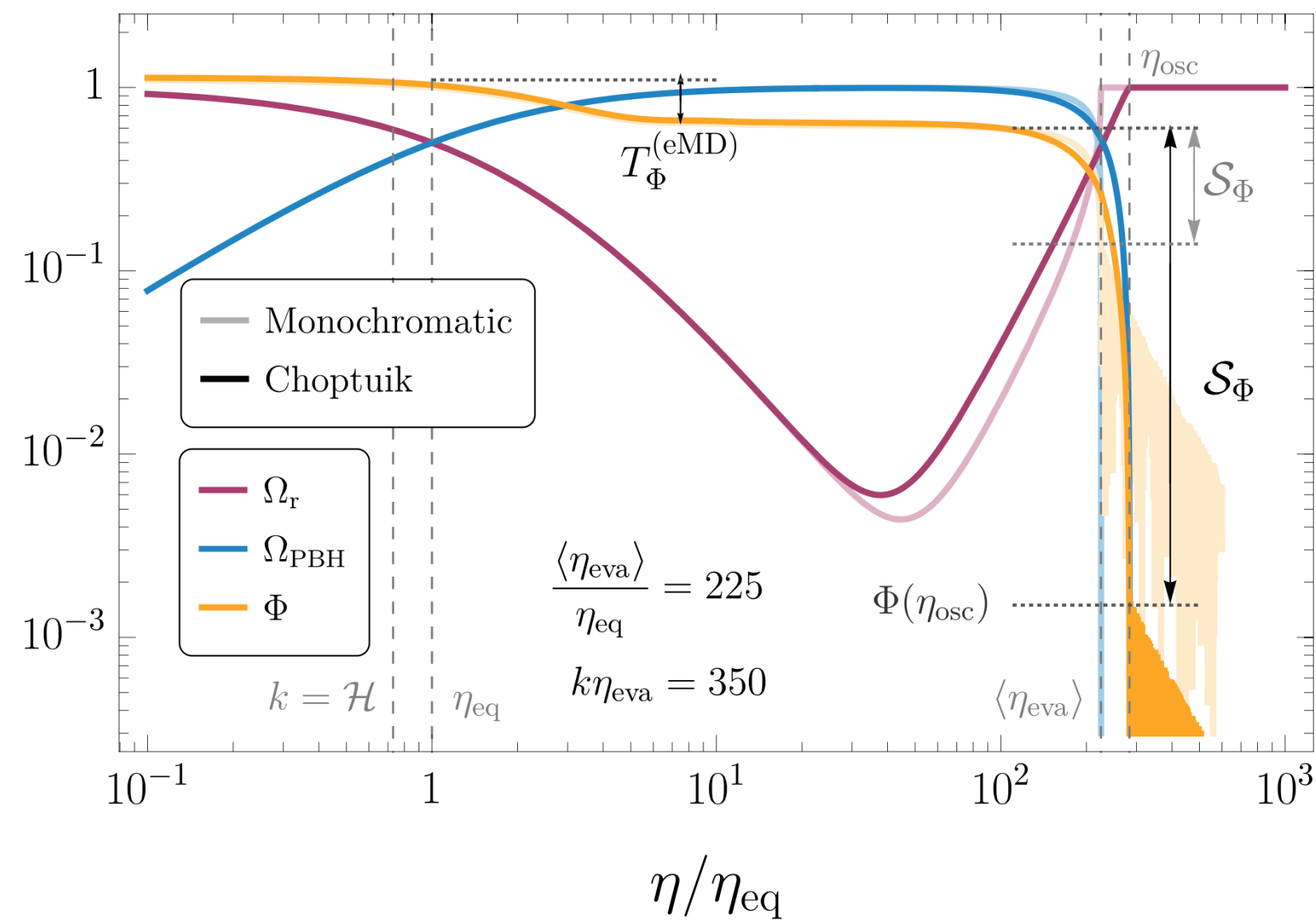




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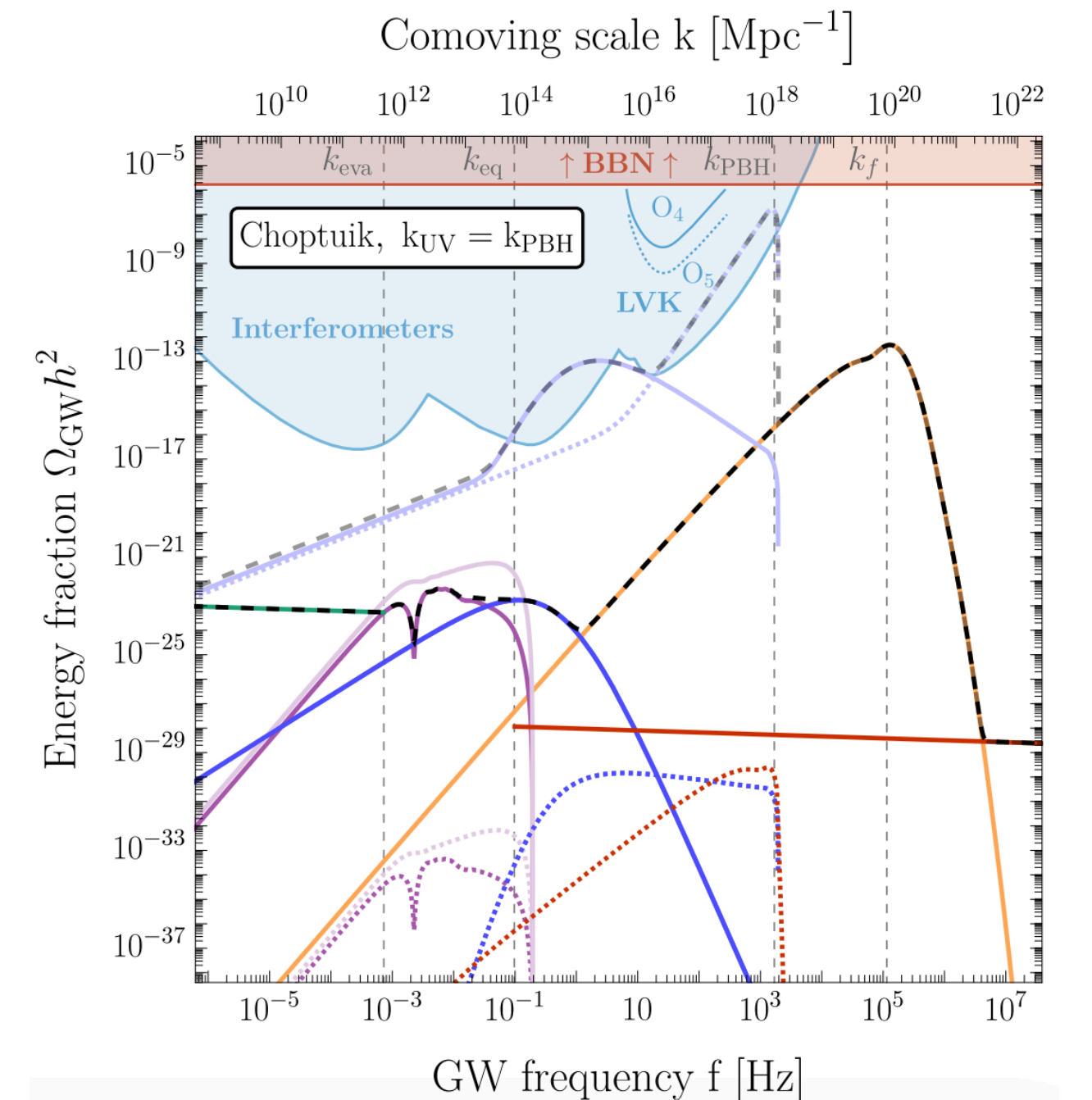
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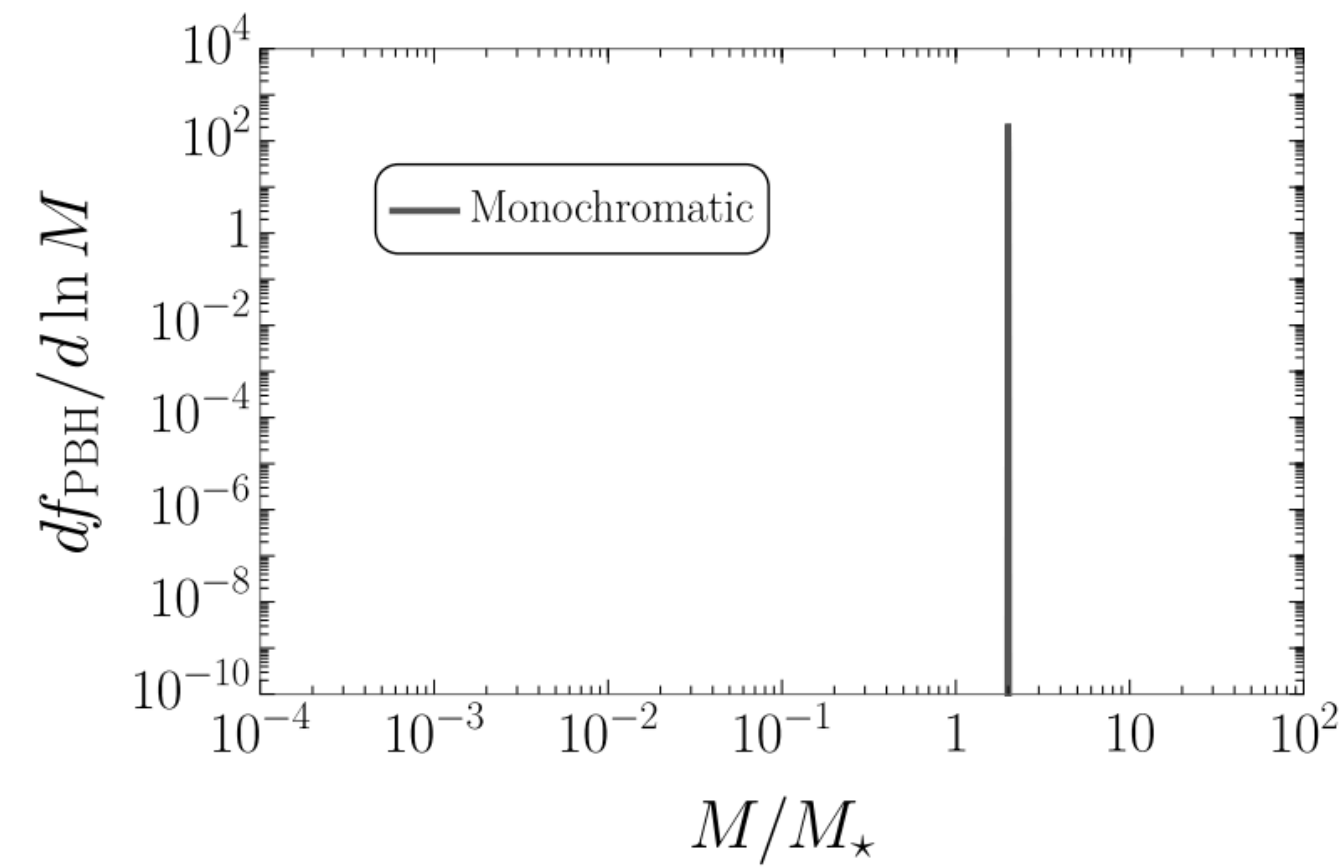


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*Inomata, Kawasaki, Mukaida,
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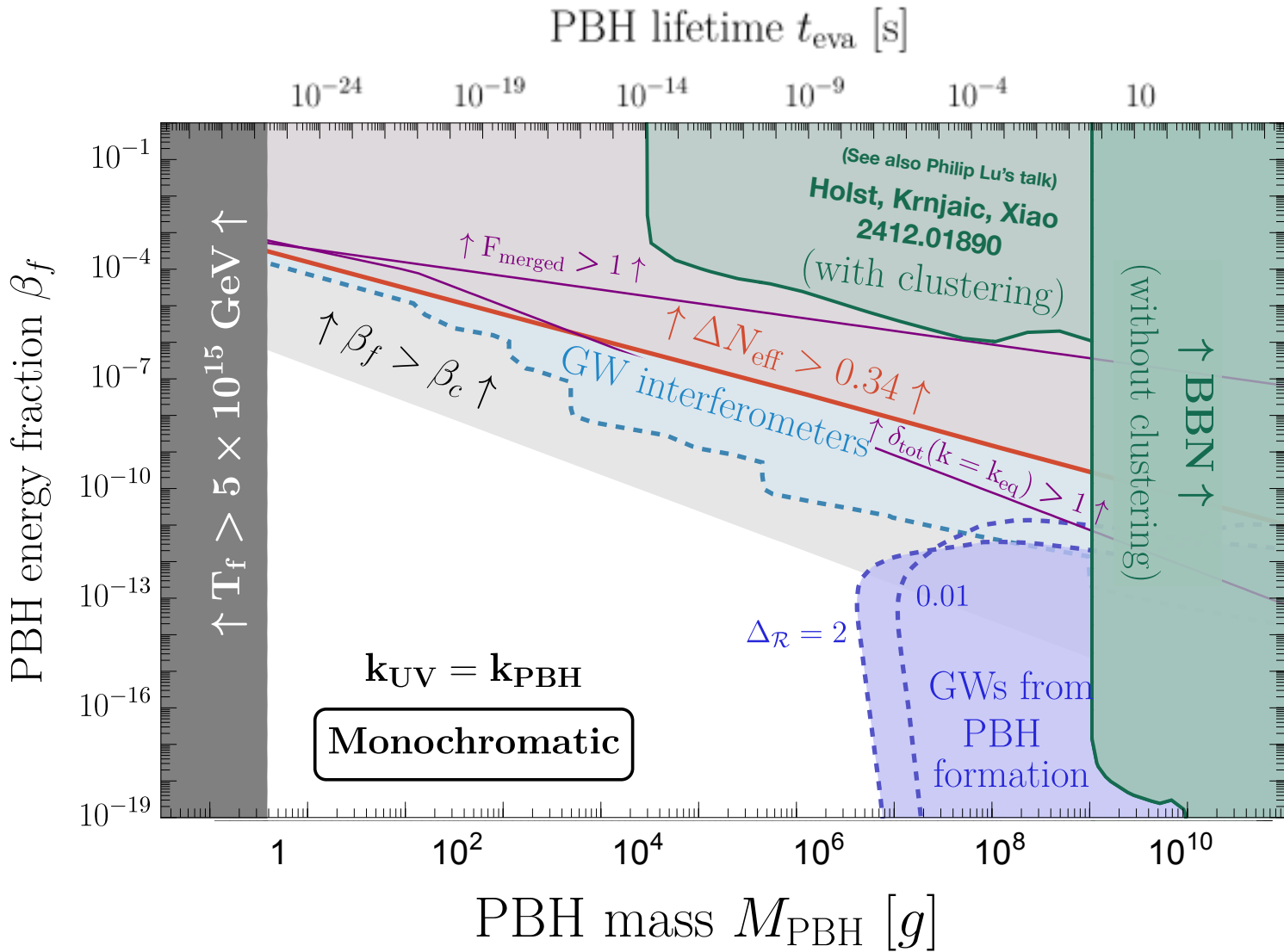
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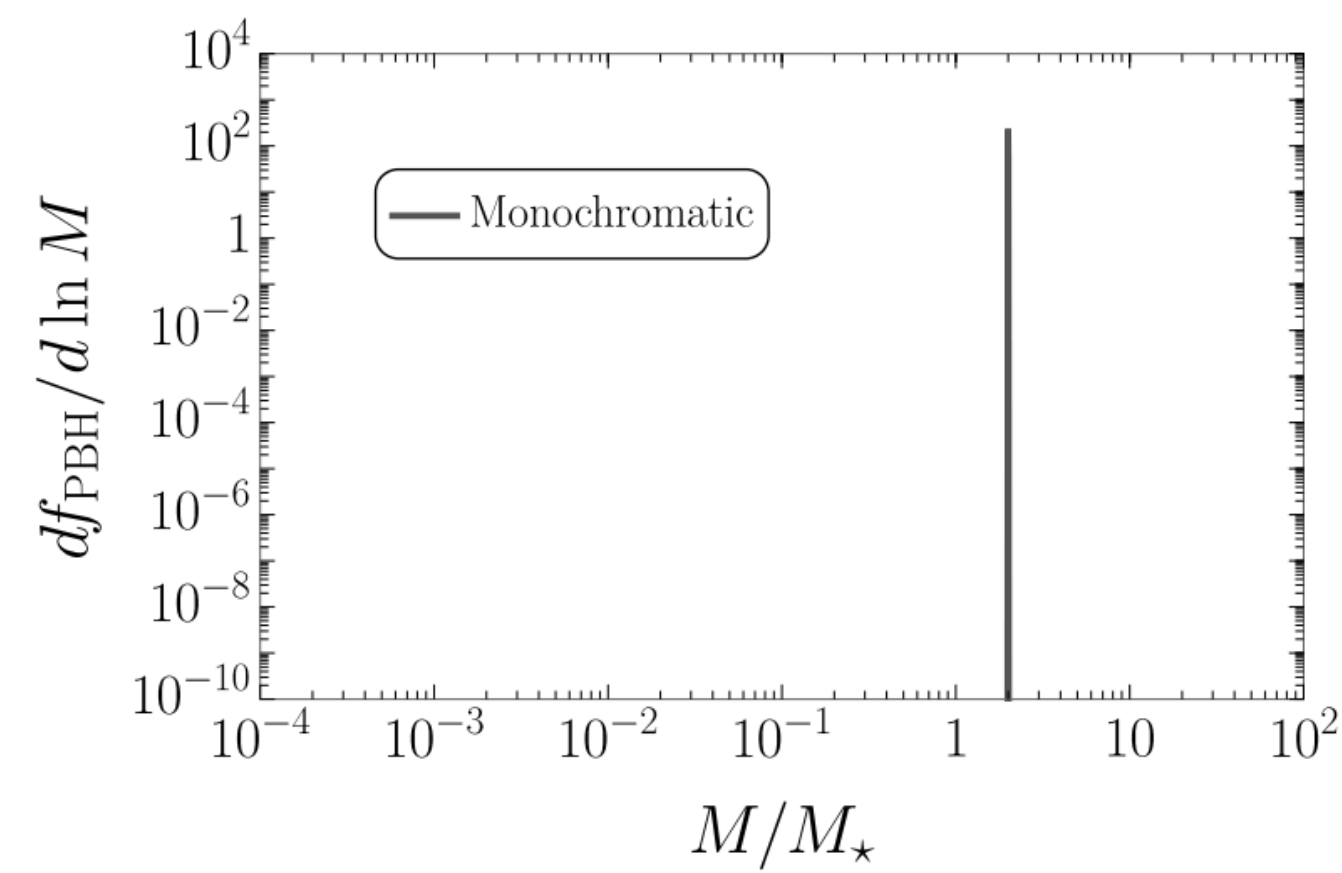
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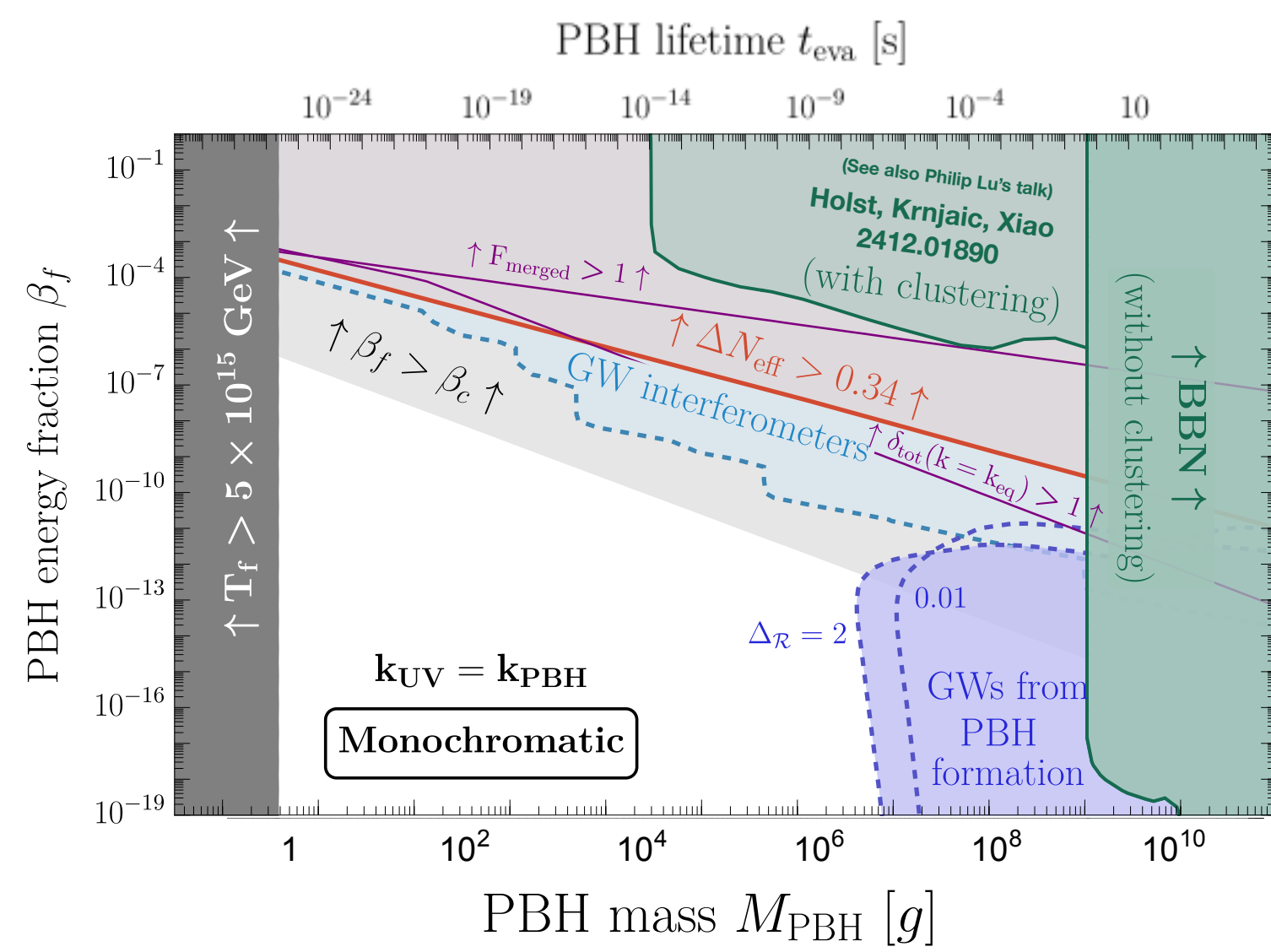
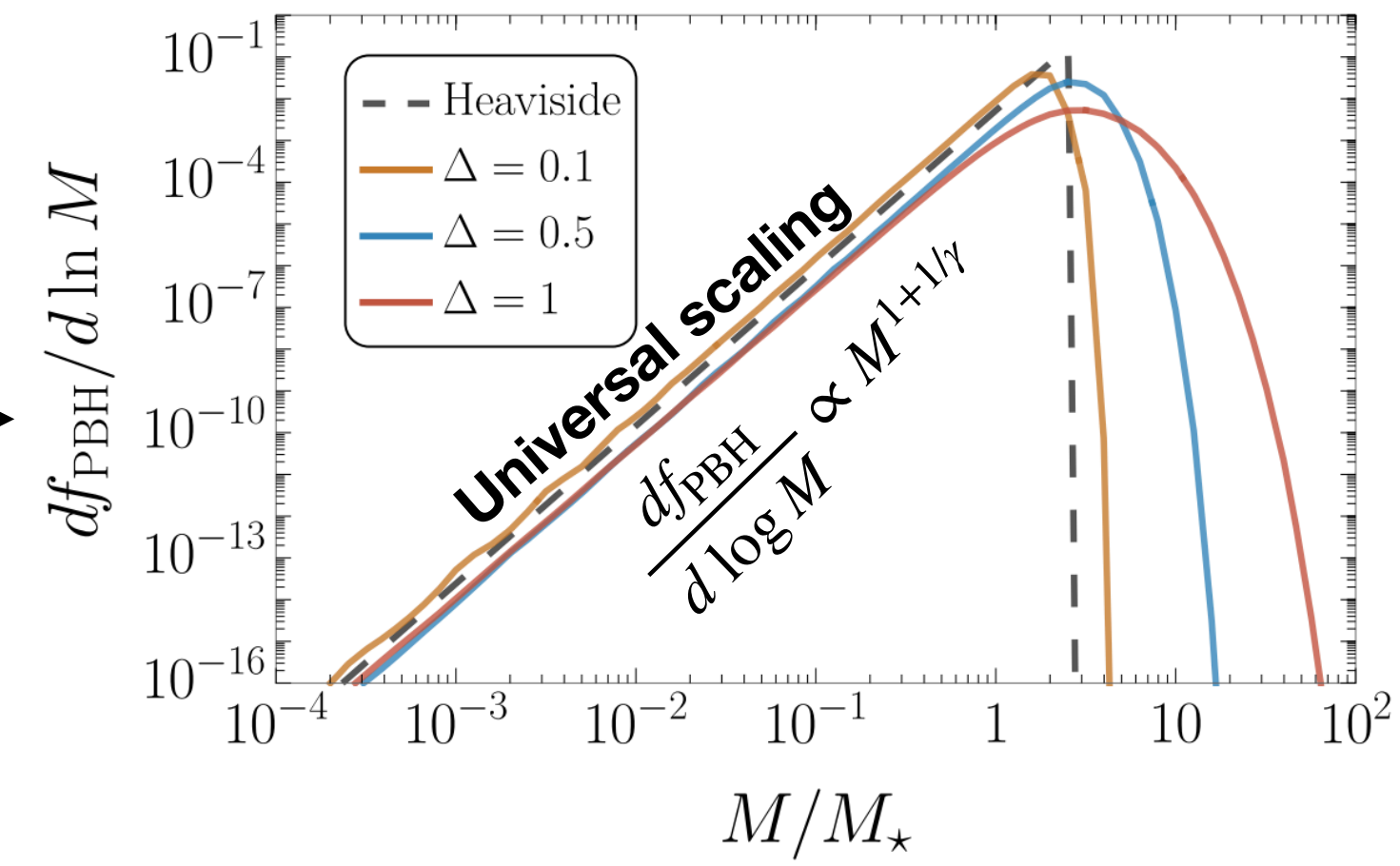
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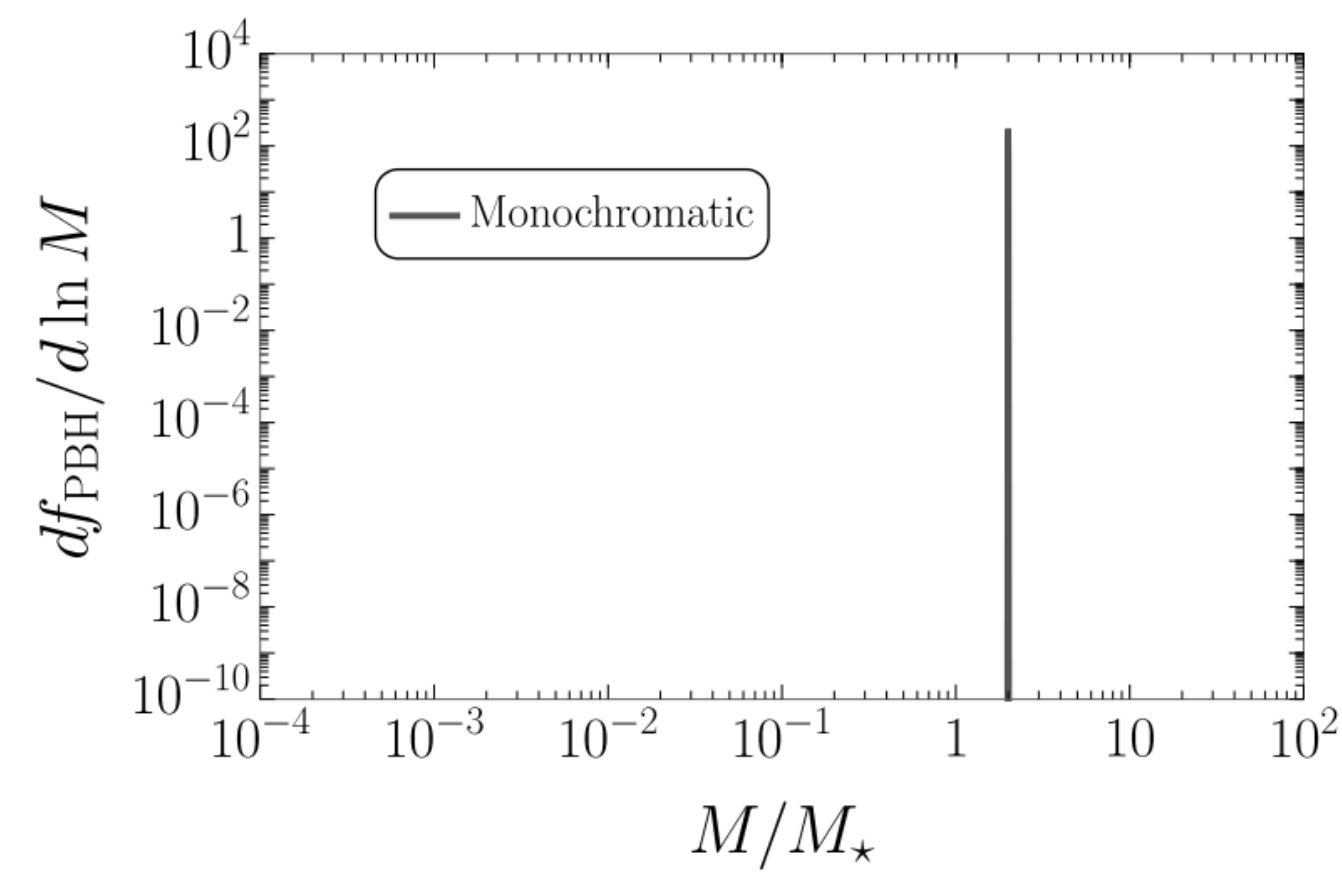
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$$\gamma \simeq 0.36$$

$$M_{\text{PBH}} = M_{\text{H}}(\delta - \delta_c)^\gamma$$

Choptuik's law
(Choptuik 93')





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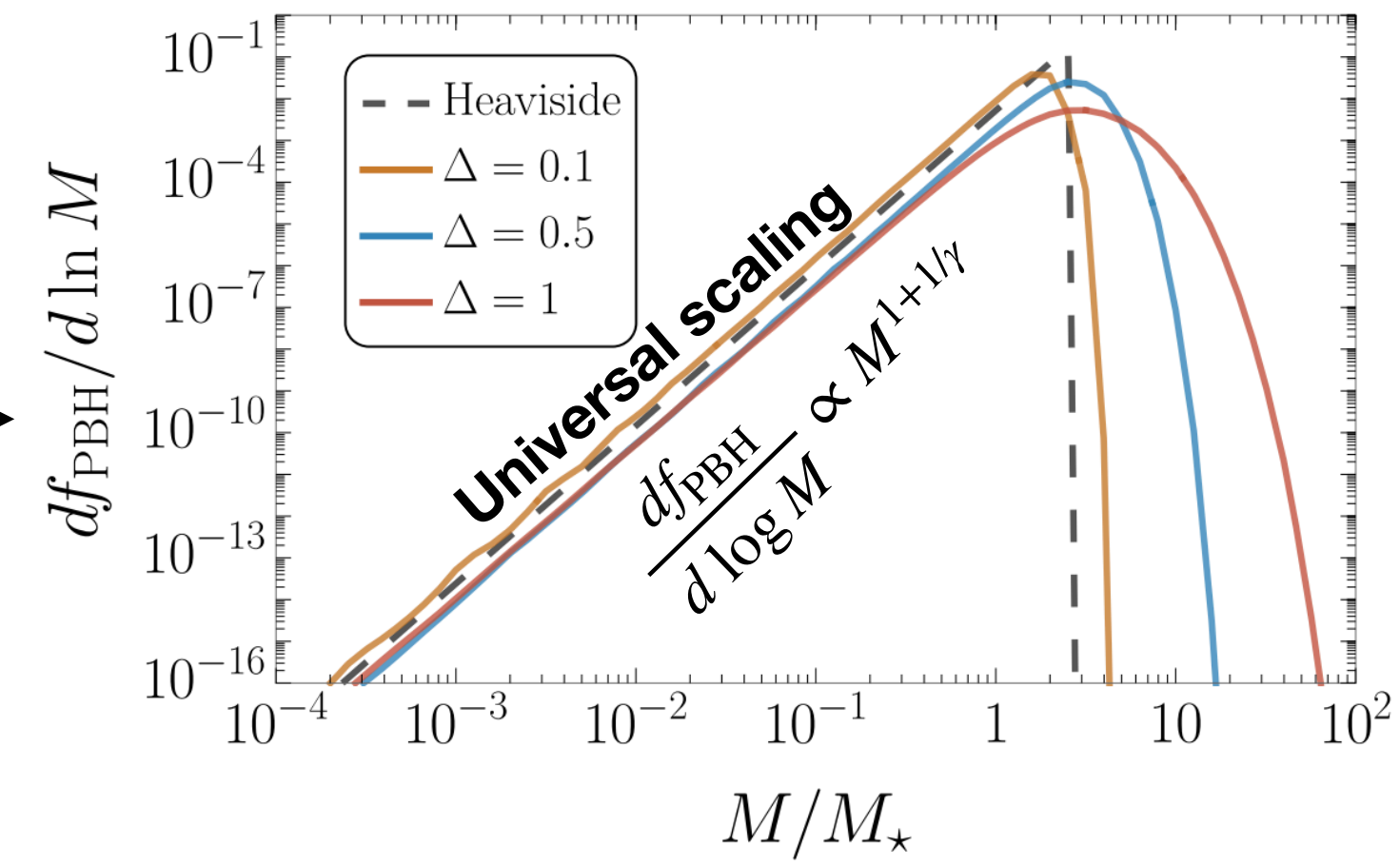
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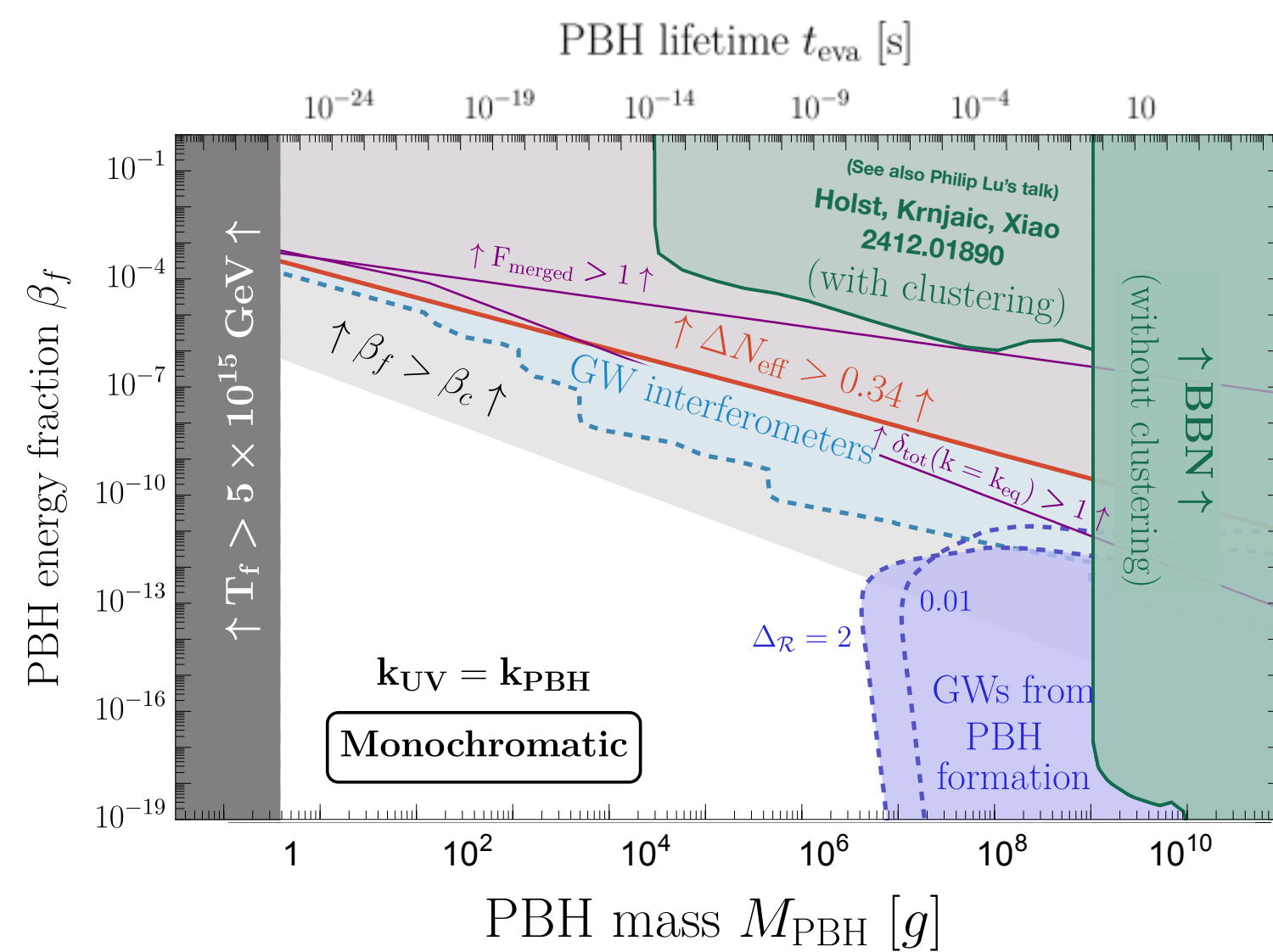


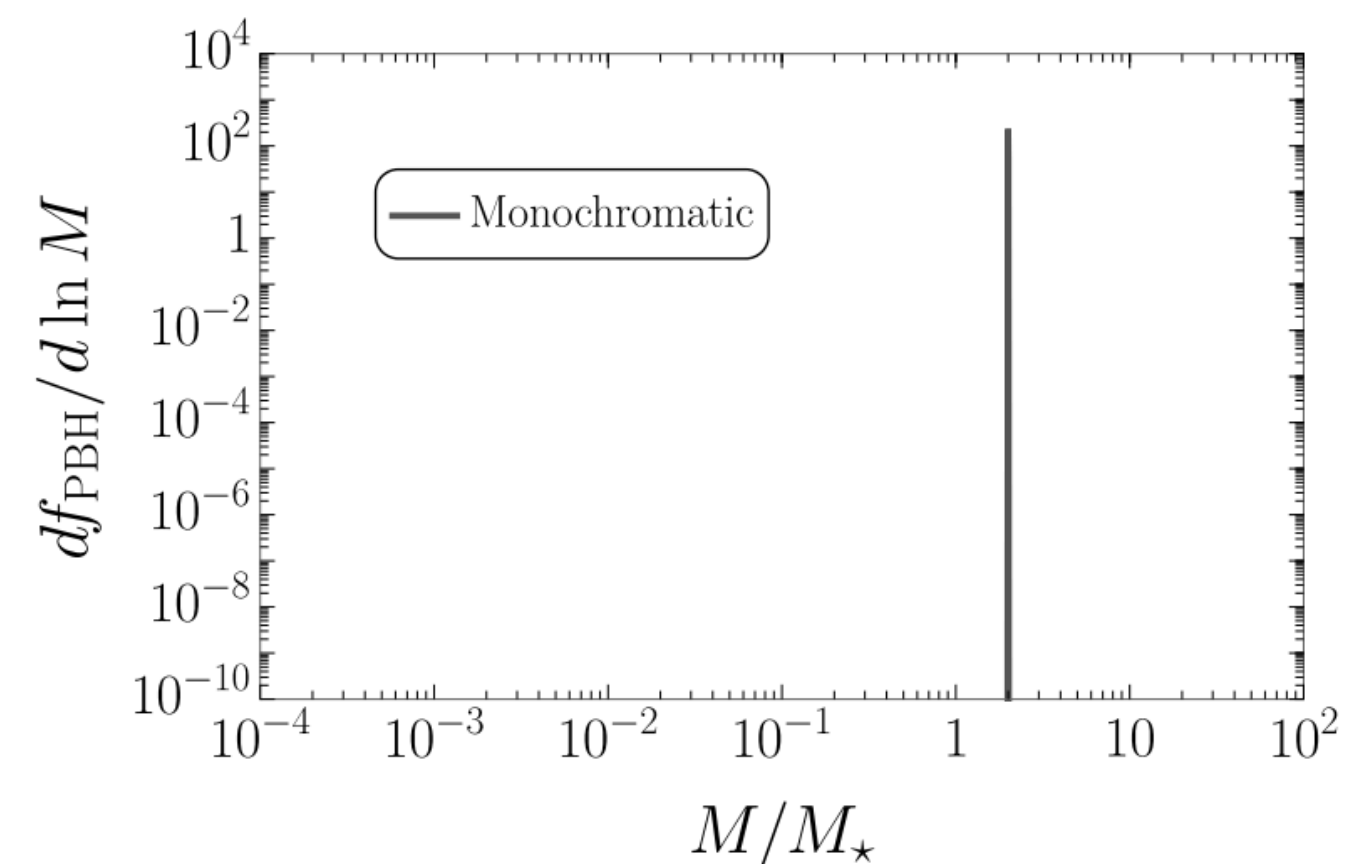
Opening the Window of Ultra-Light PBHs by Exorcising the Poltergeist

(2605.21477)

(To appear in PRL)

Yann Gouttenoire, Nicholas Leister, Pedro Schwaller
PRISMA+ Cluster of Excellence & MITP, Johannes Gutenberg University, 55099 Mainz, Germany





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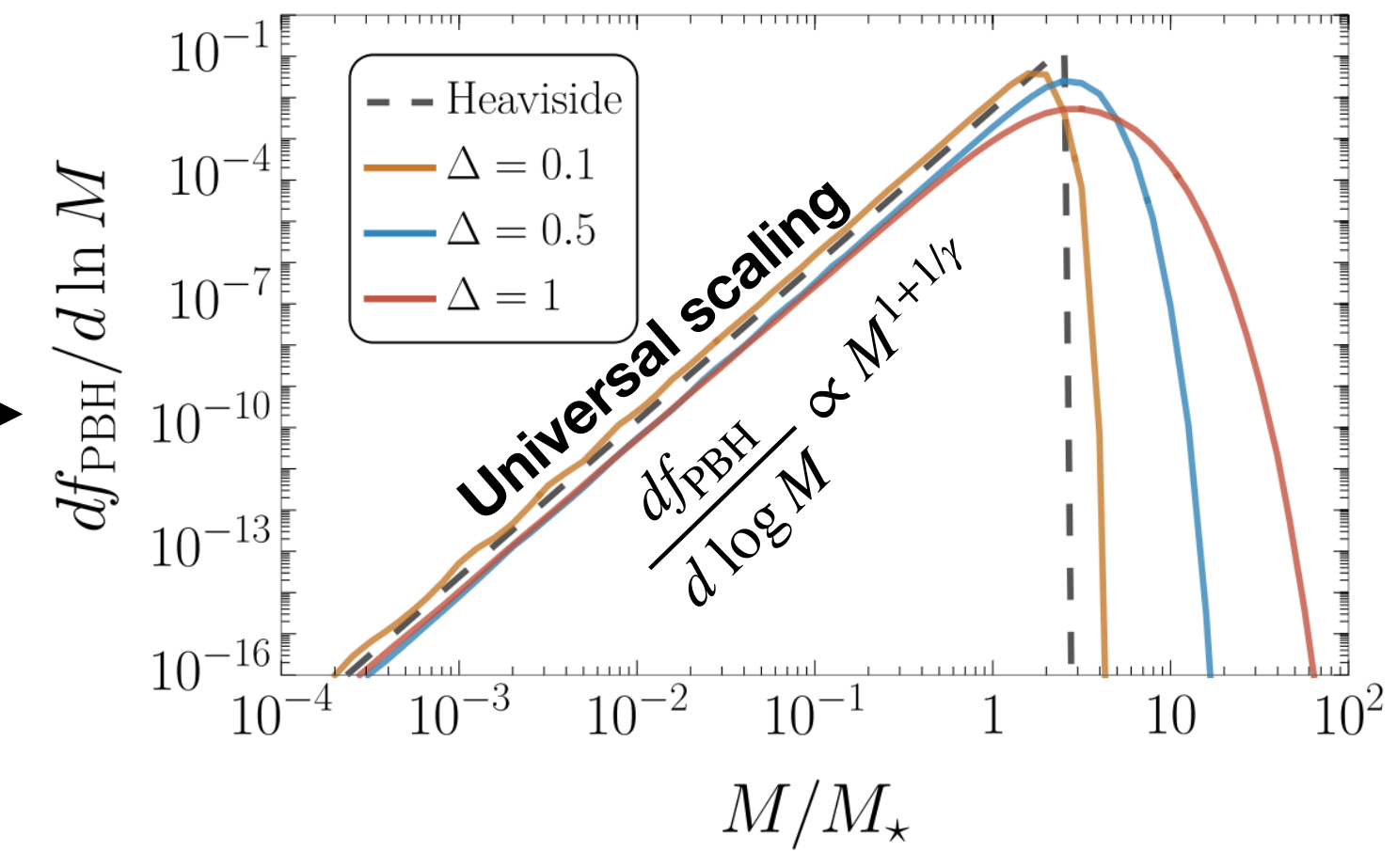
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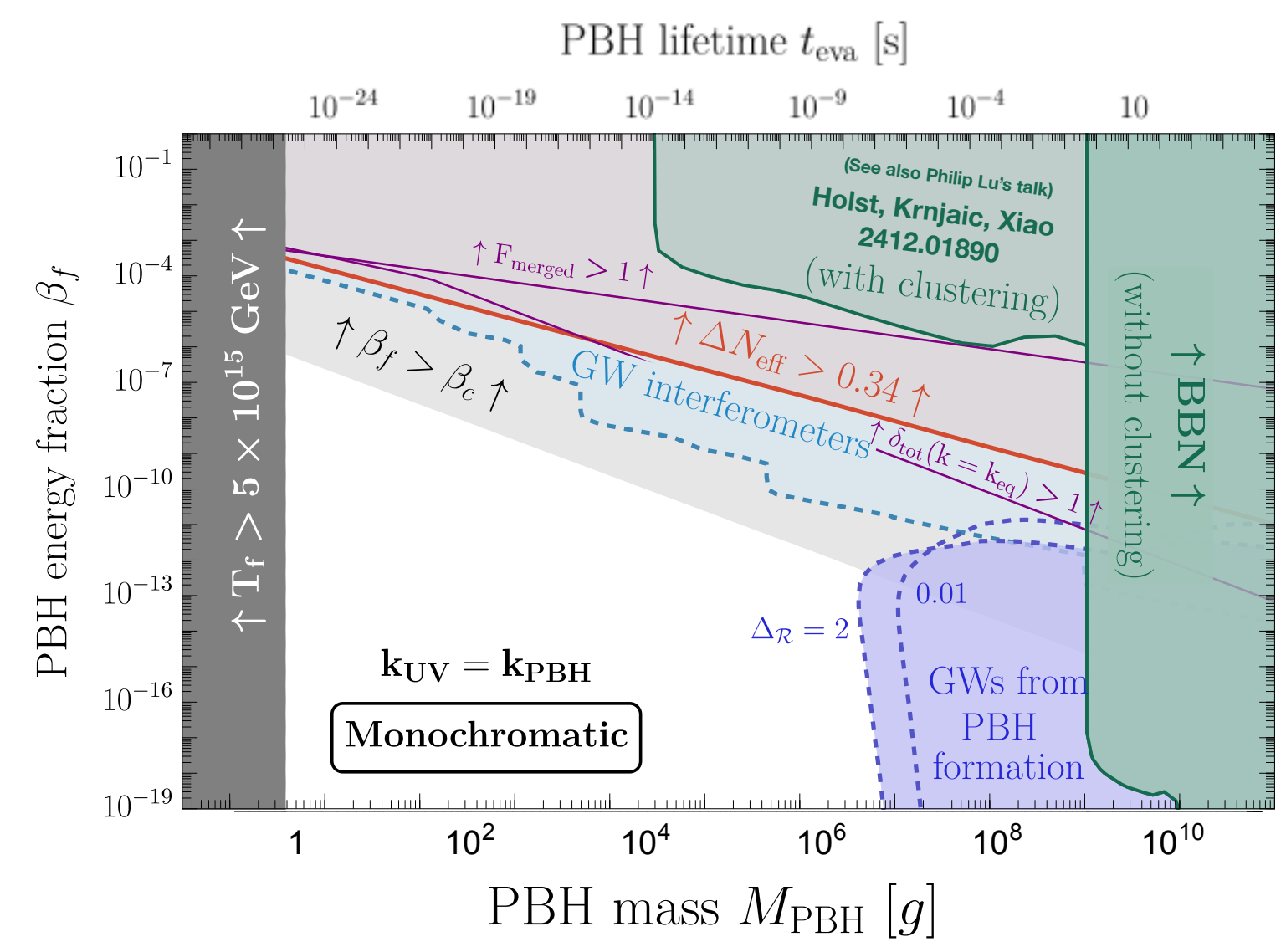


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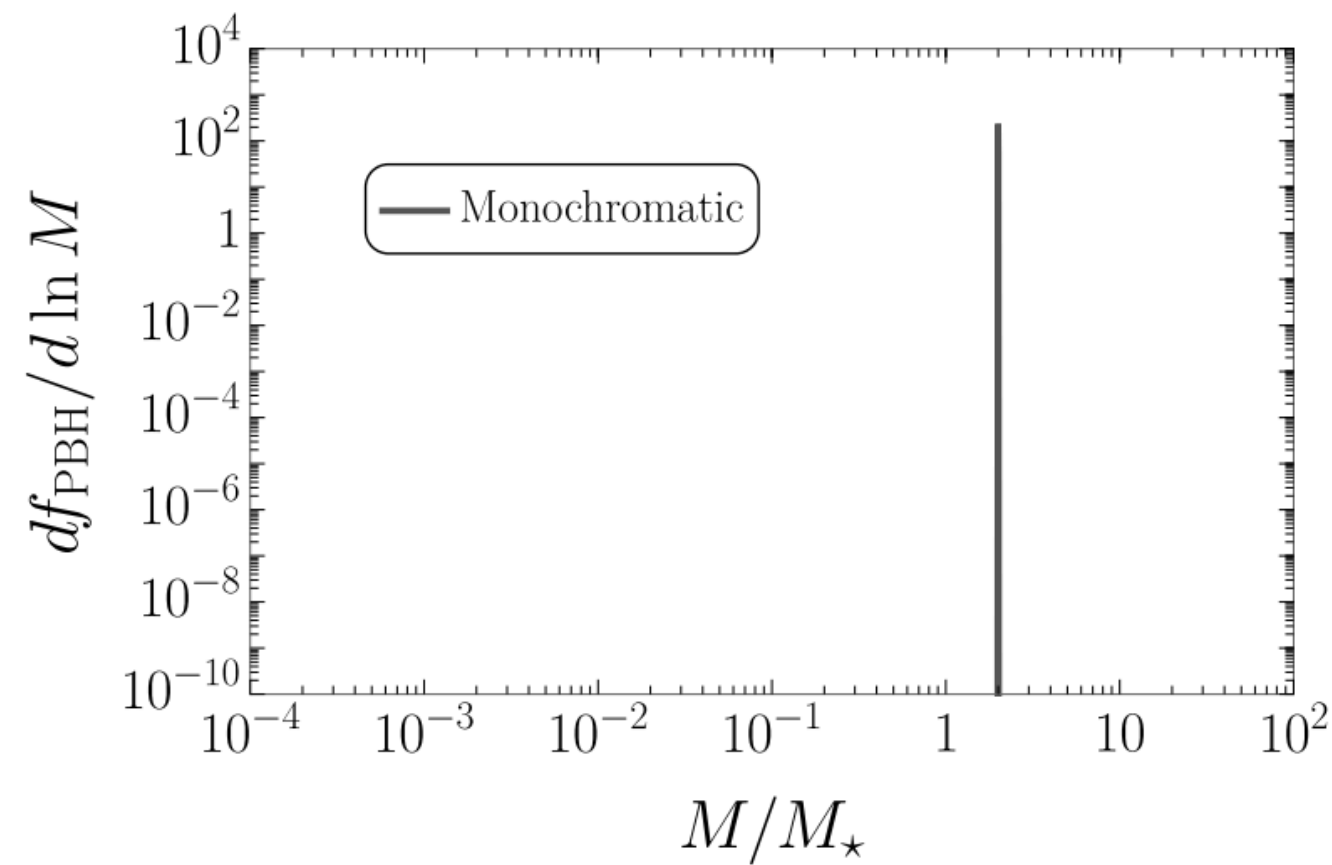
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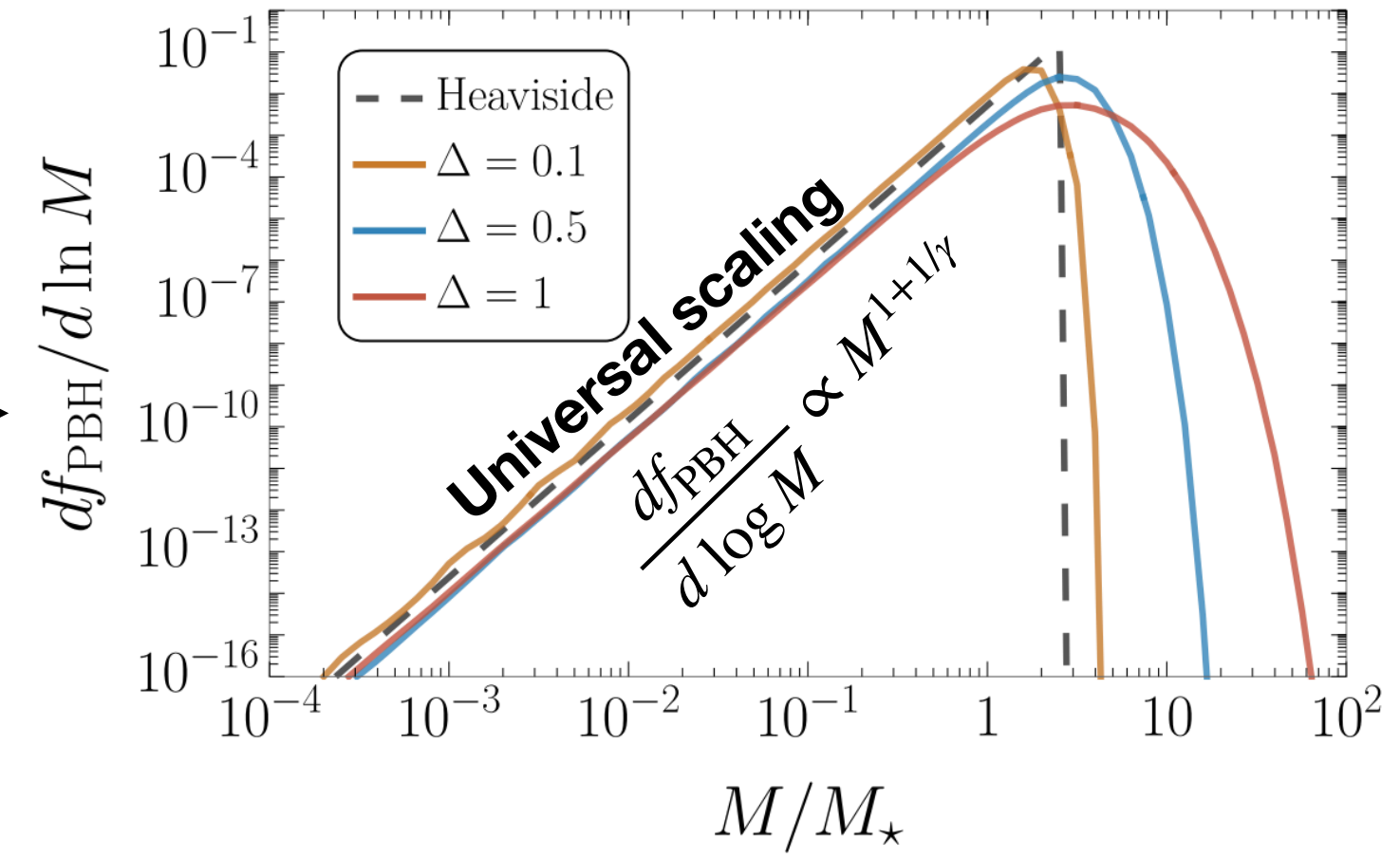
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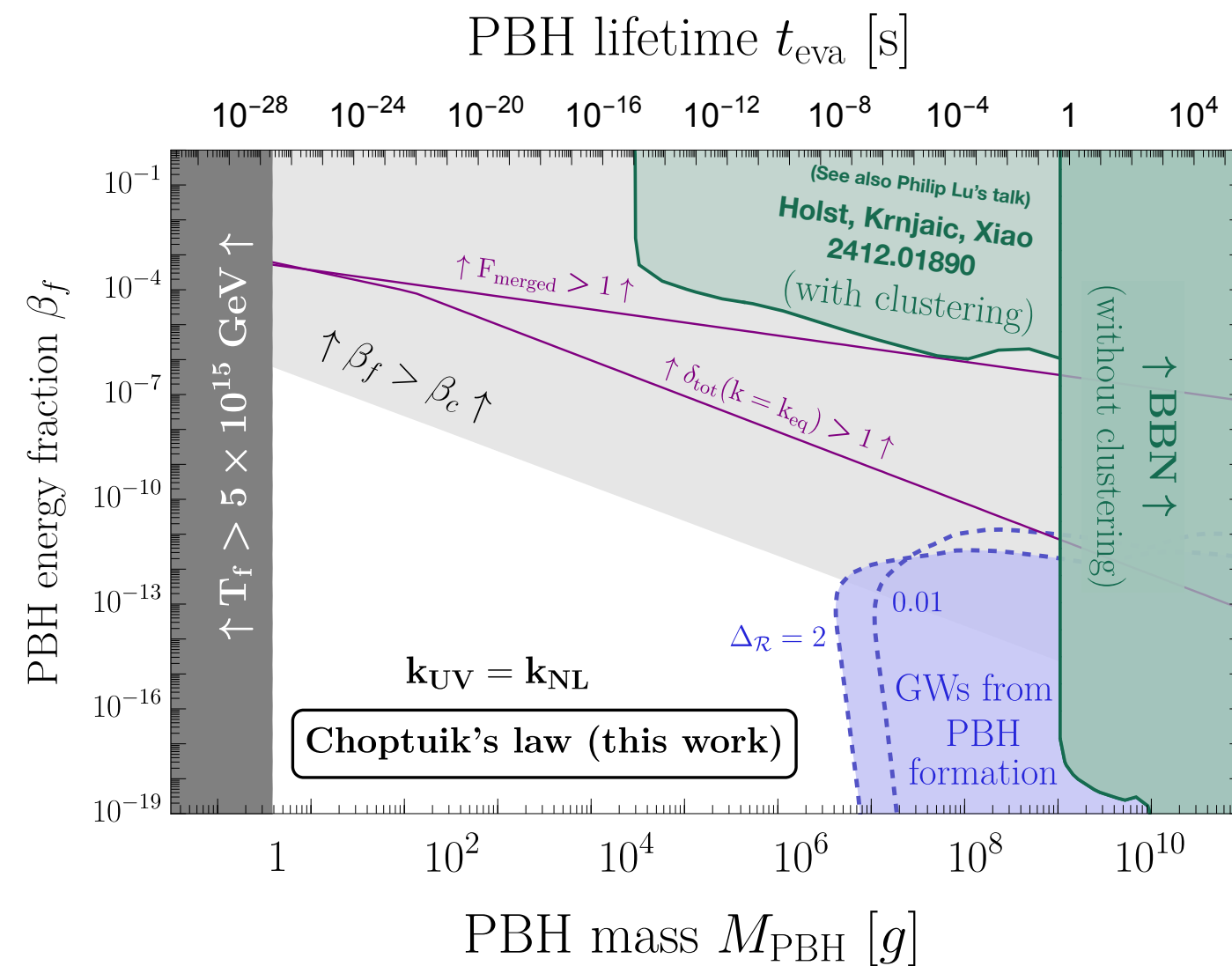
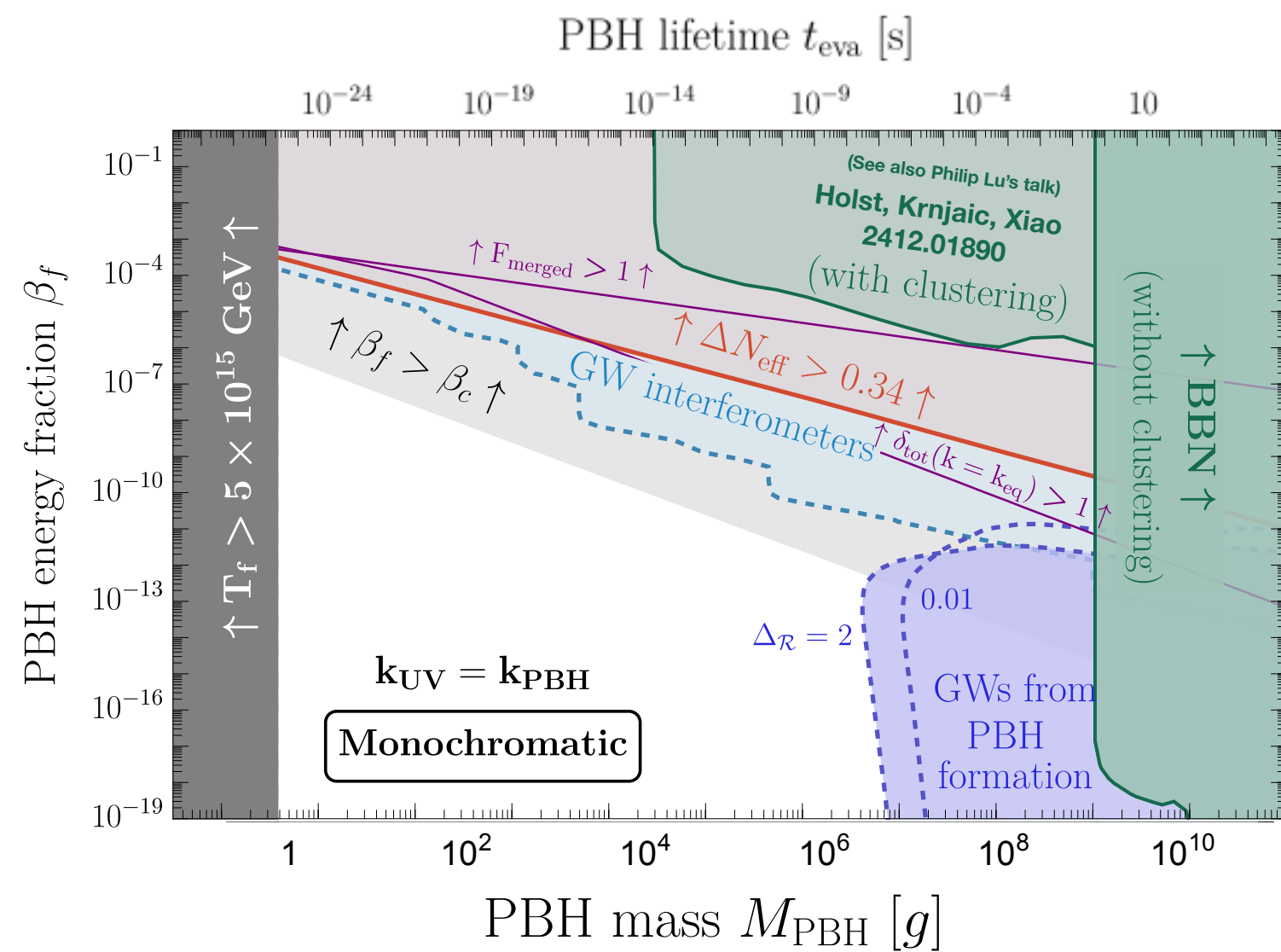
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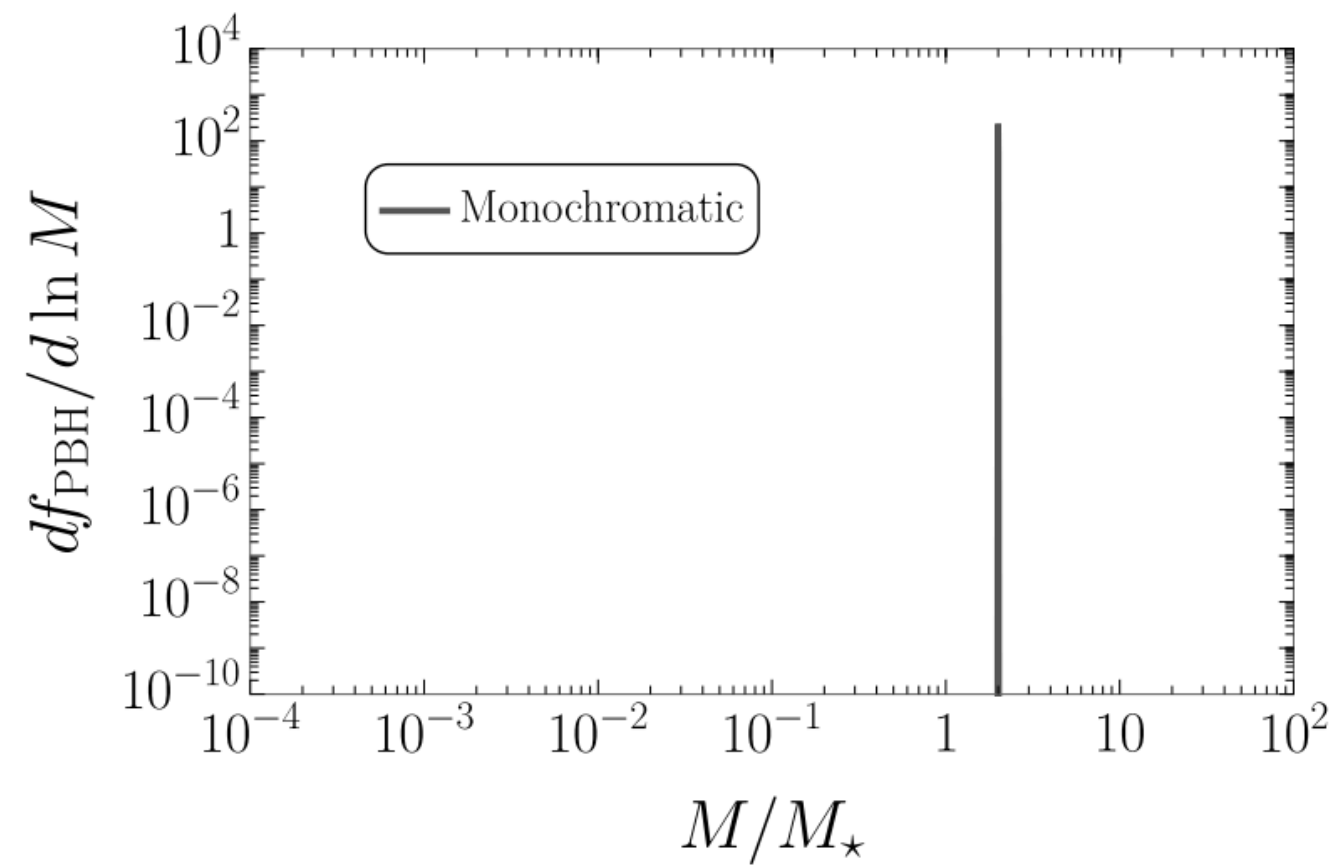
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$$k_{\text{cut}} = k_{\text{NL}} \quad (\text{conservative})$$





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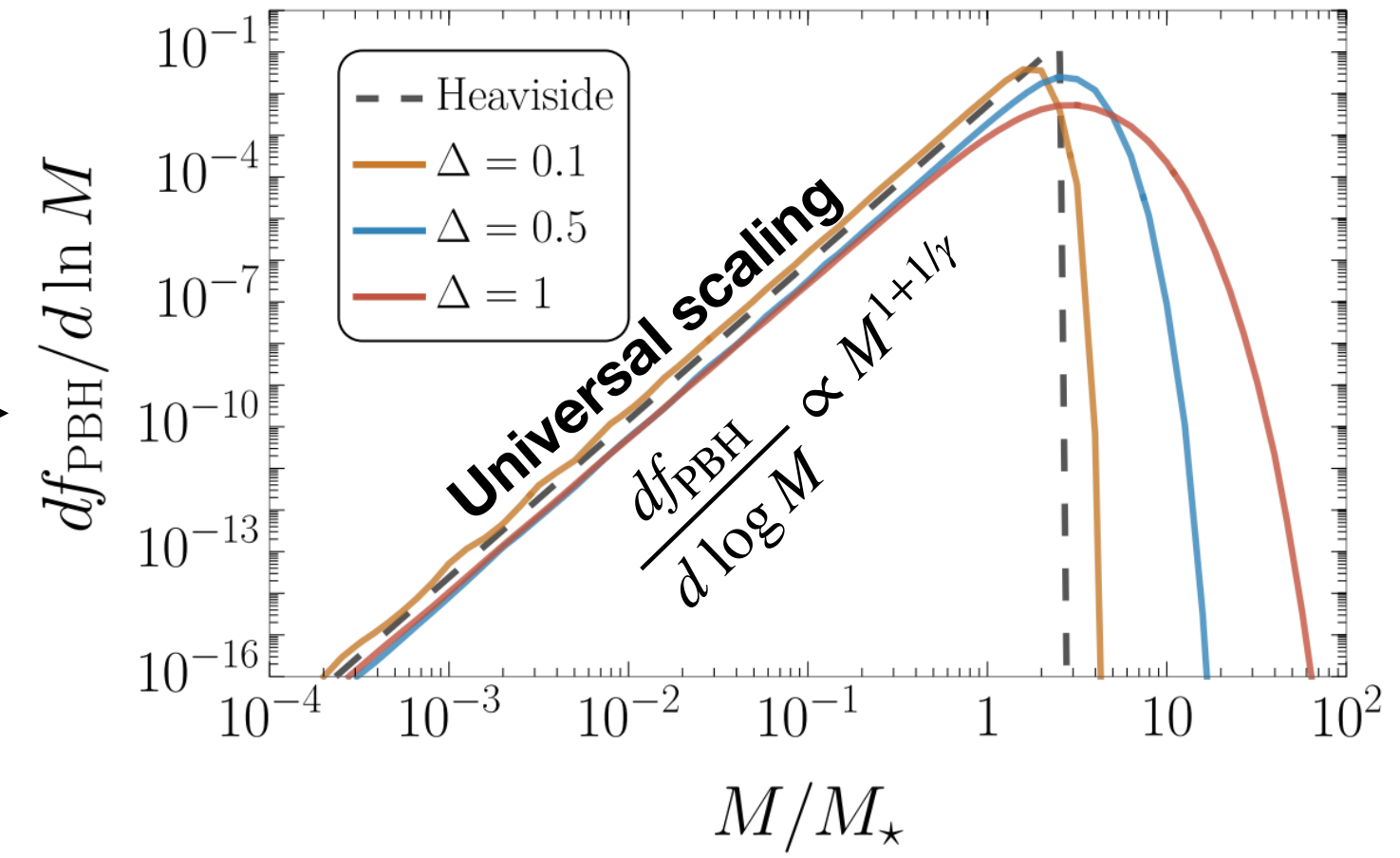
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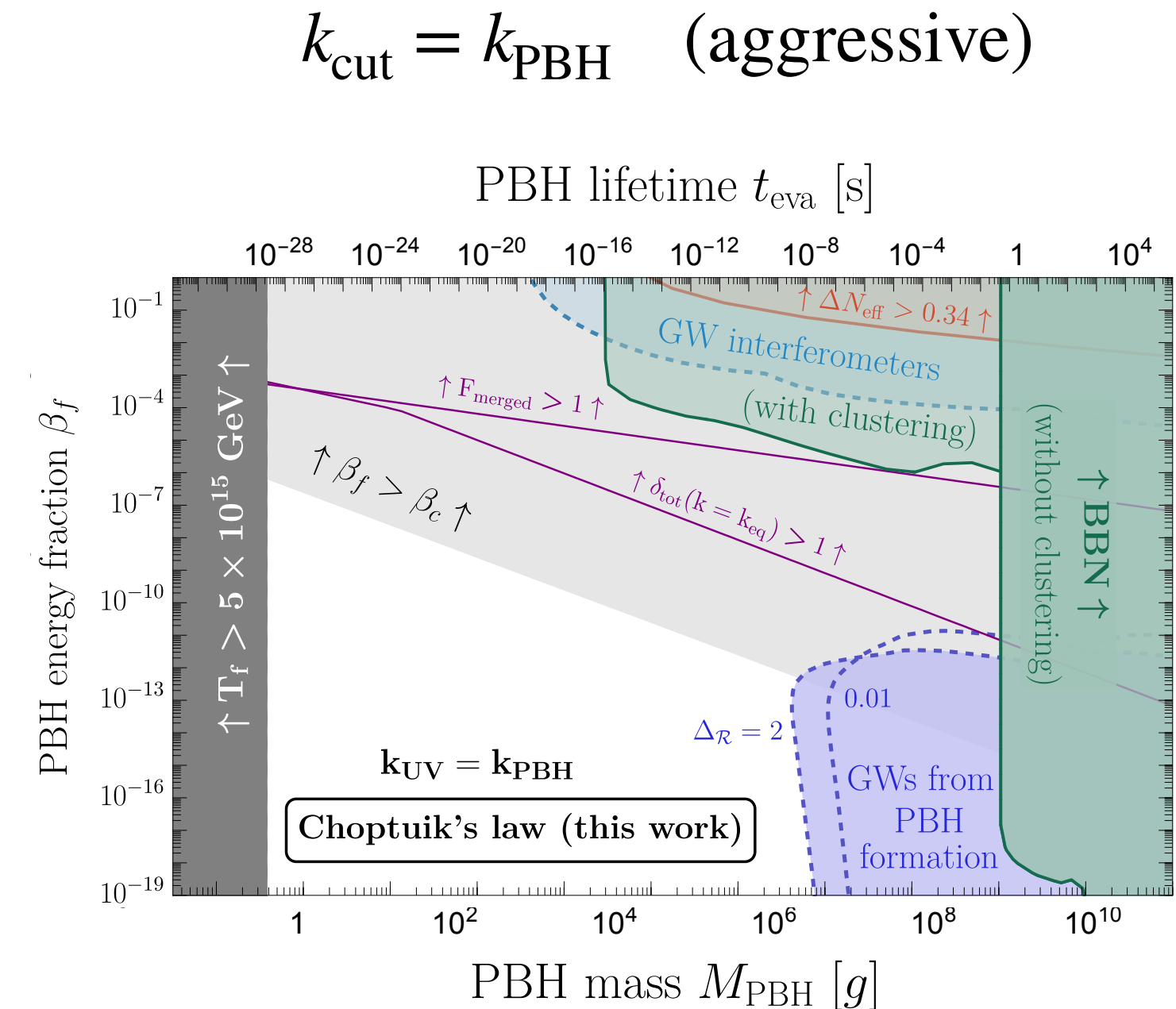
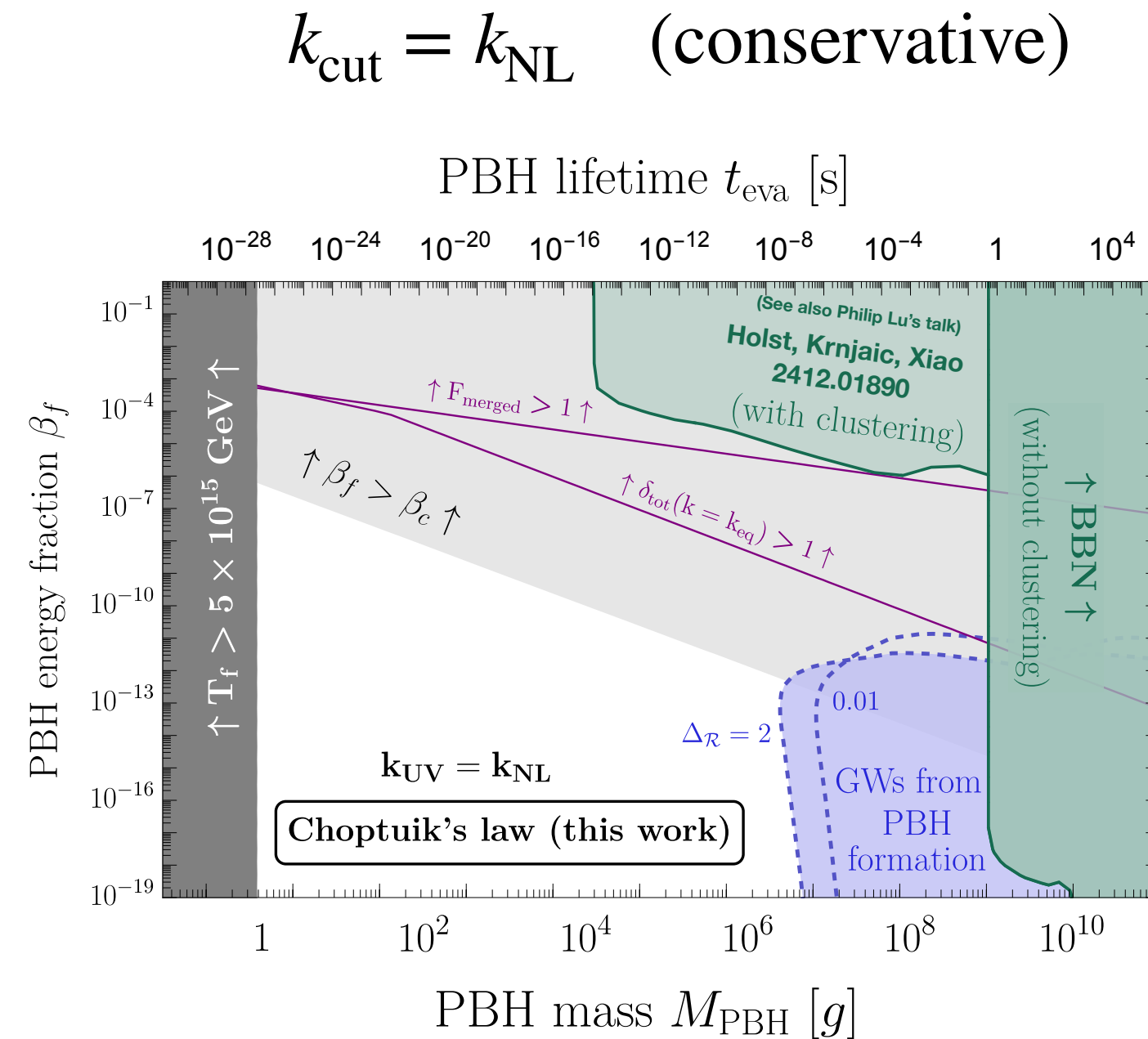
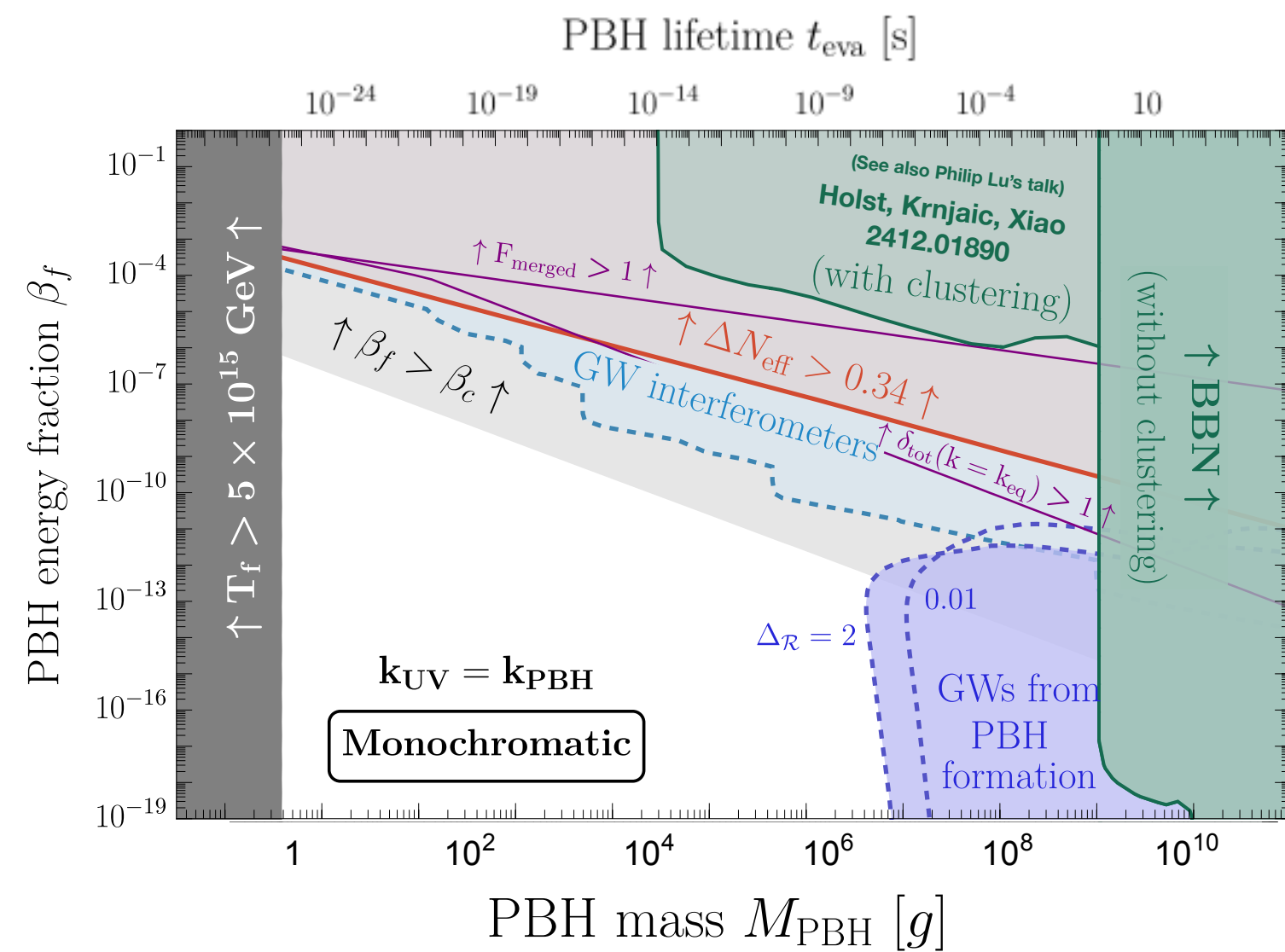


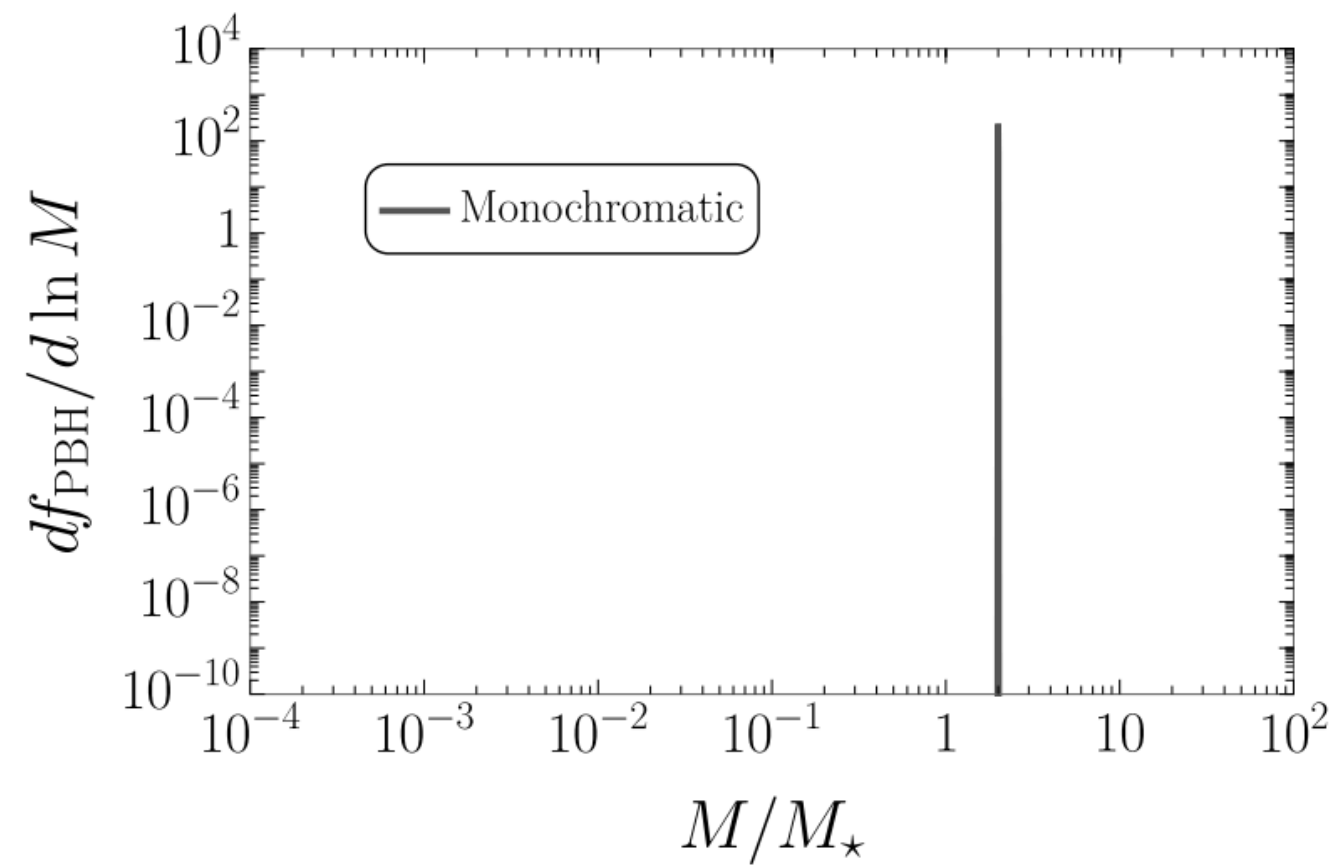
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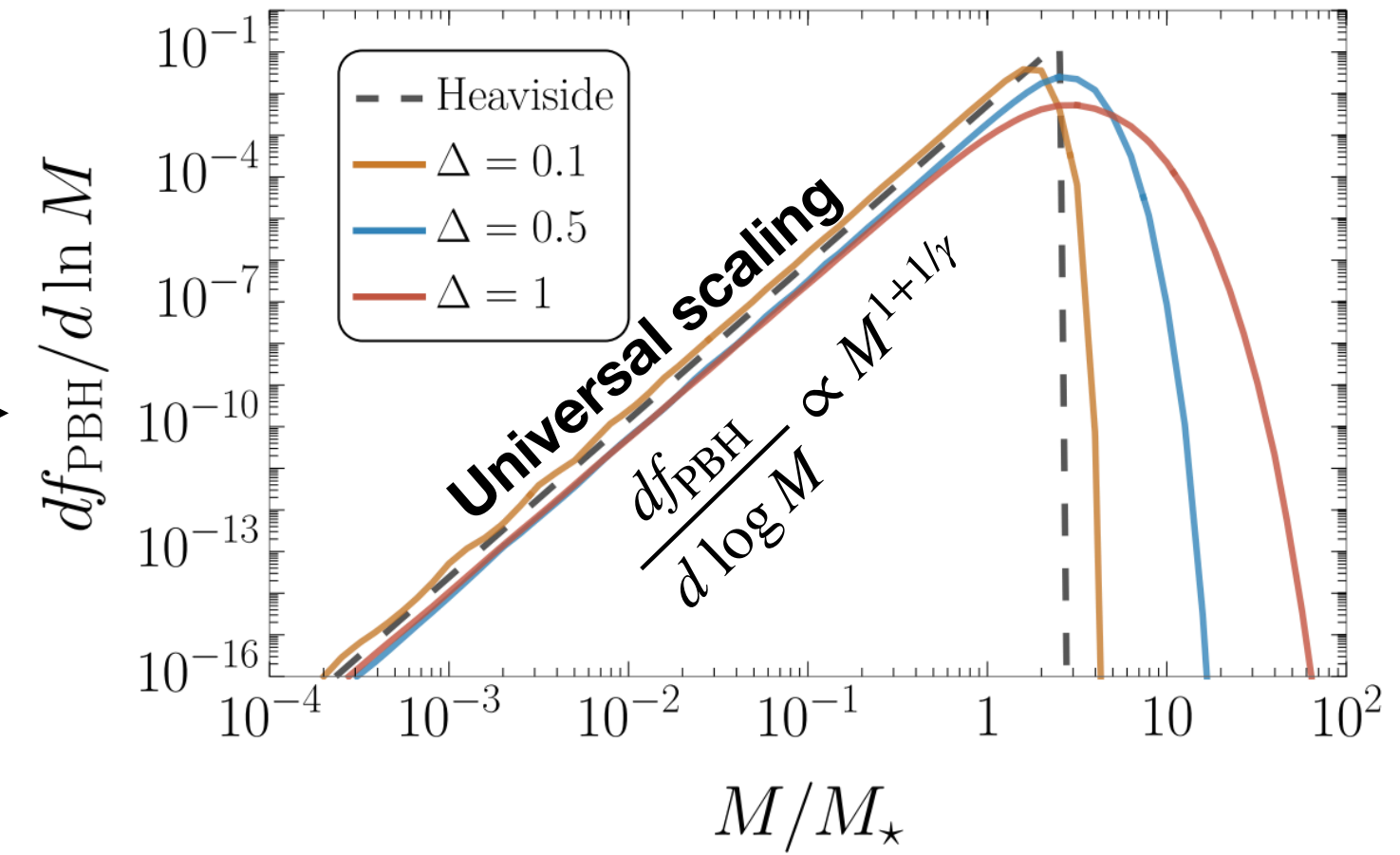
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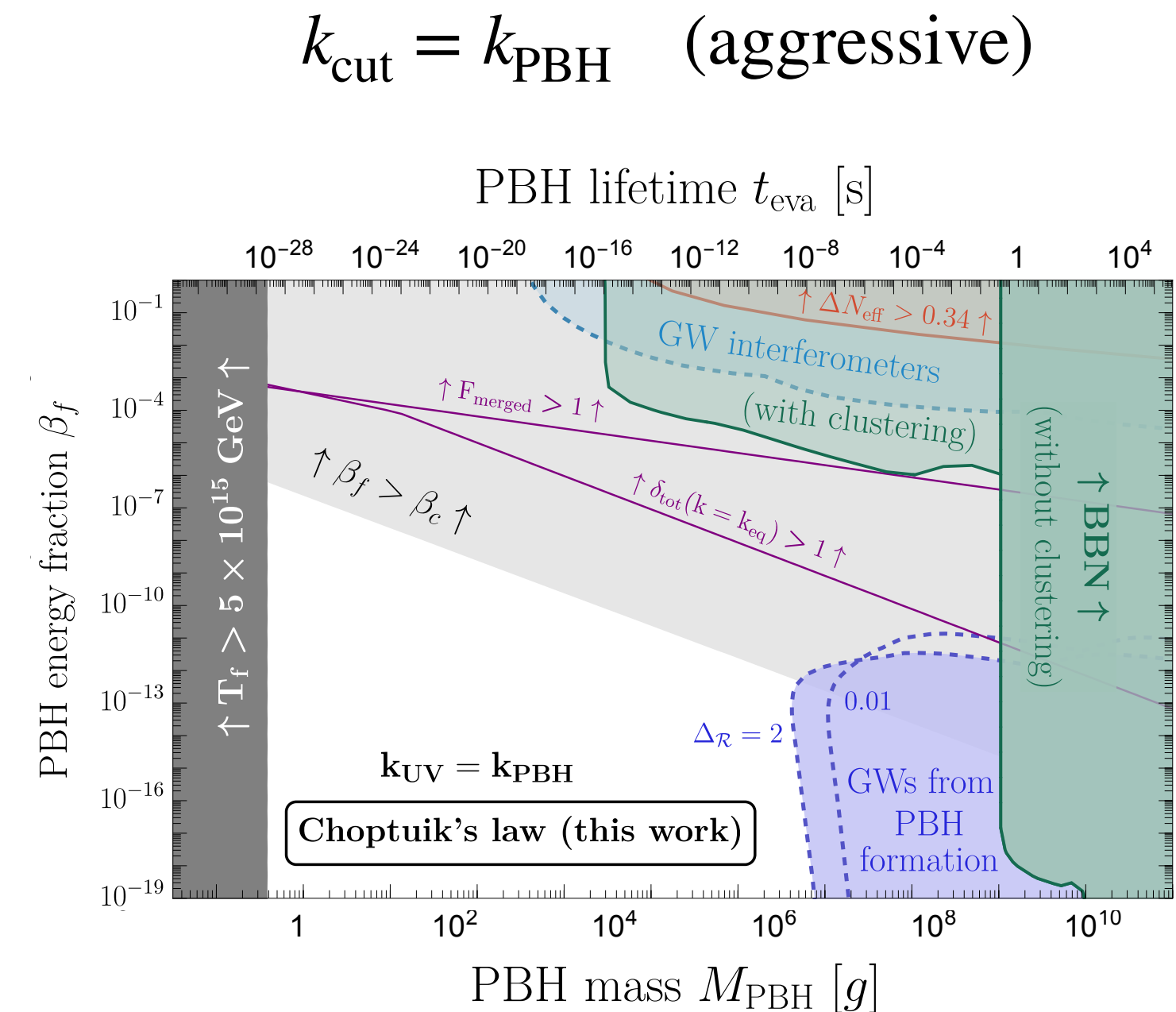
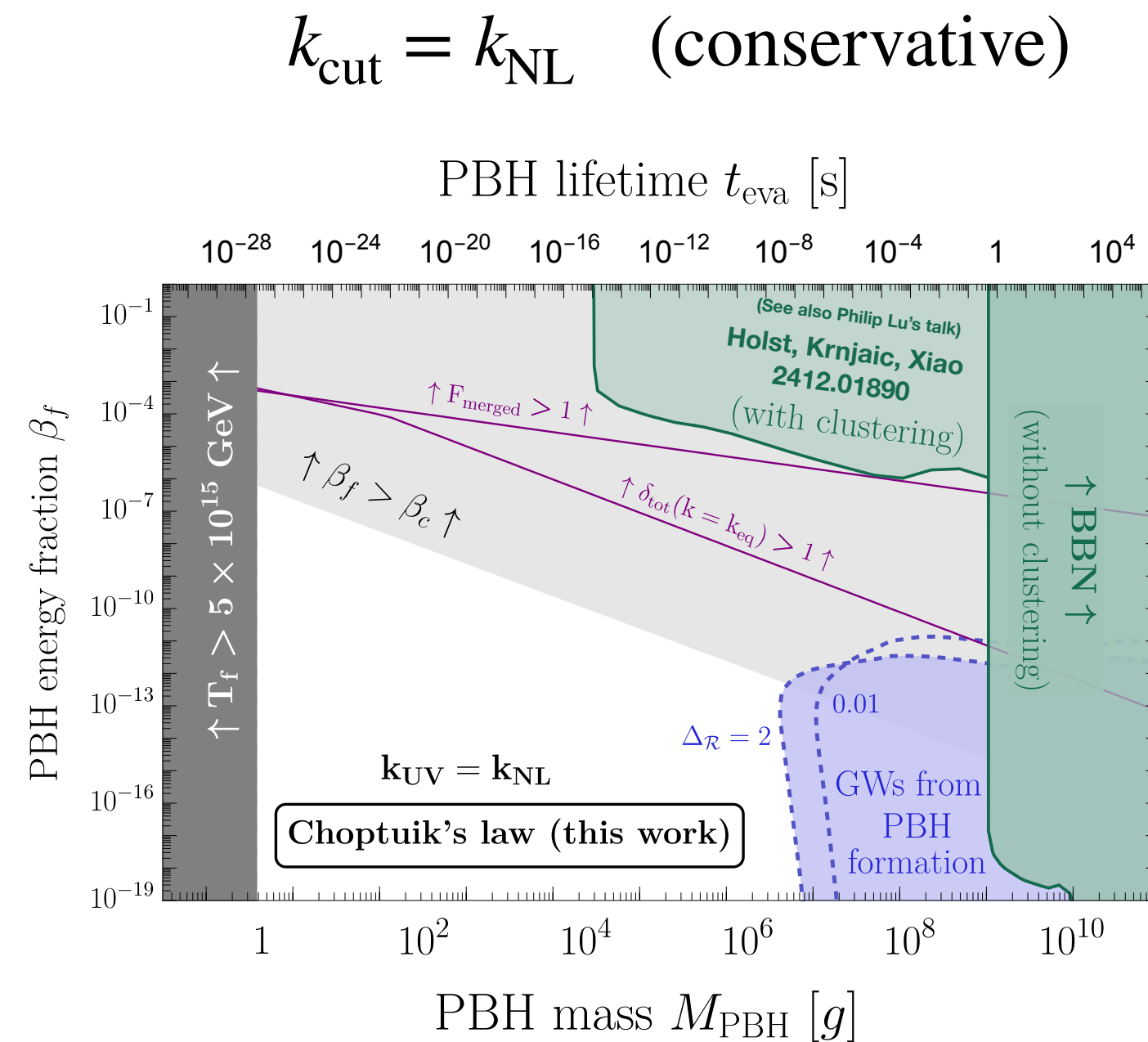
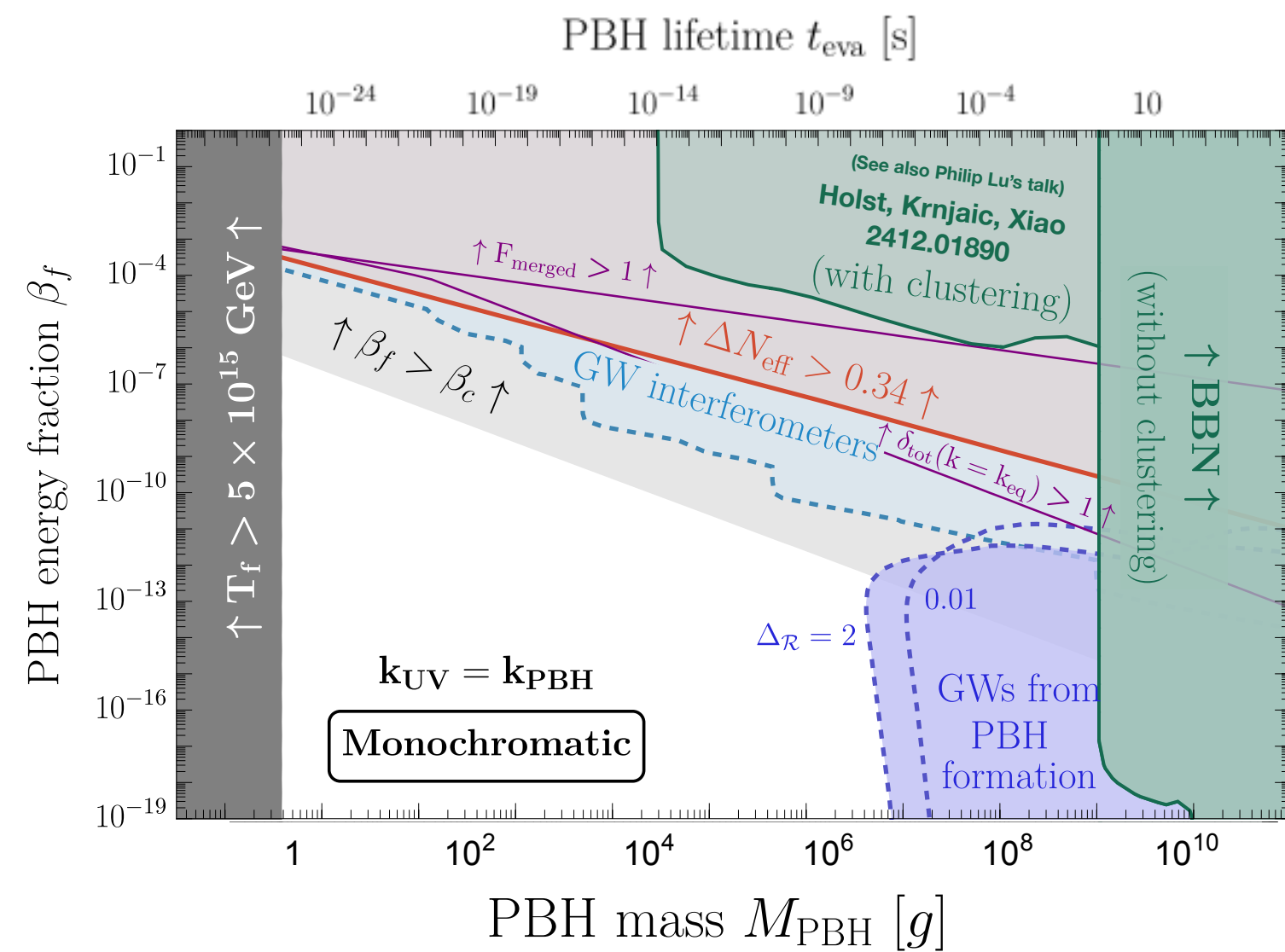


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Gravitational Waves from Black Hole Reheating

The Scalar-Induced Component

(2605.21474)

Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

Gravitational Waves from Black Hole Reheating

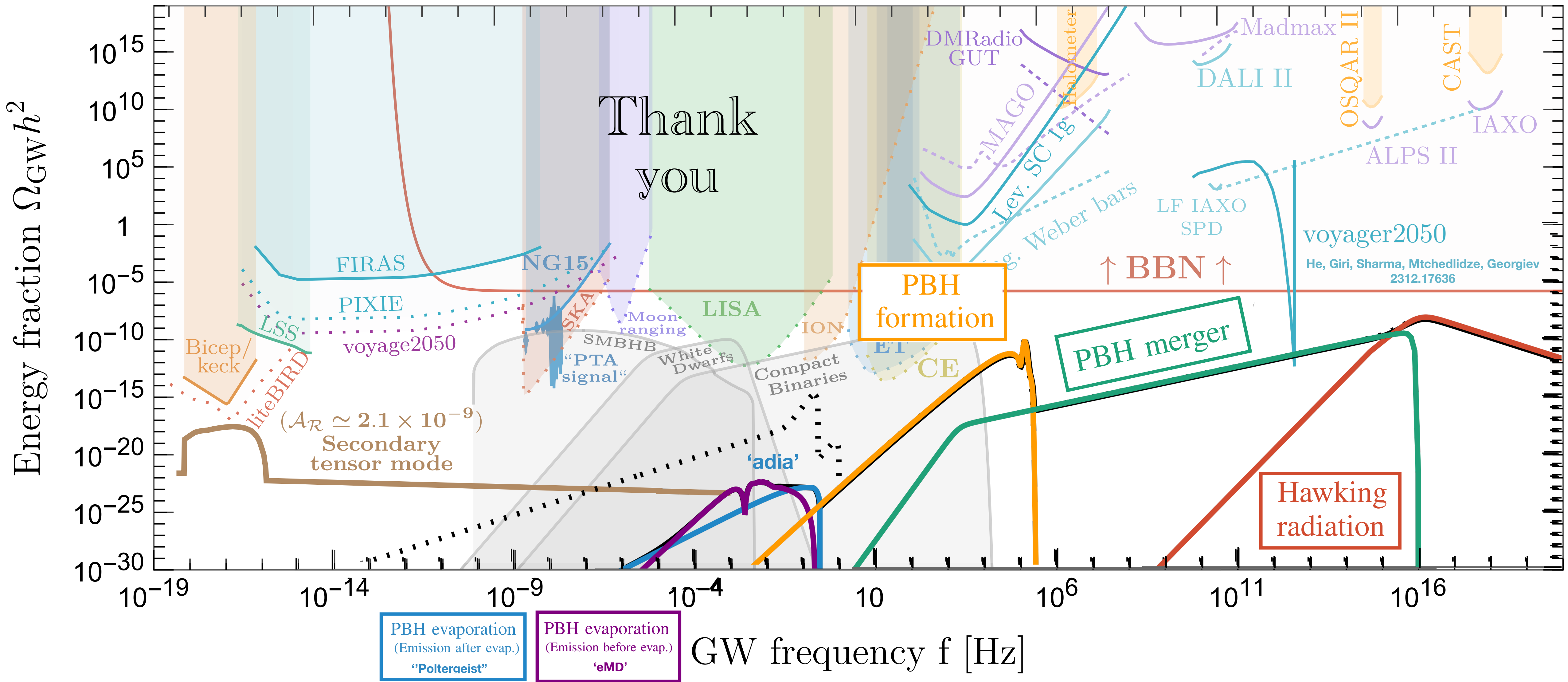
The High-Frequency Component

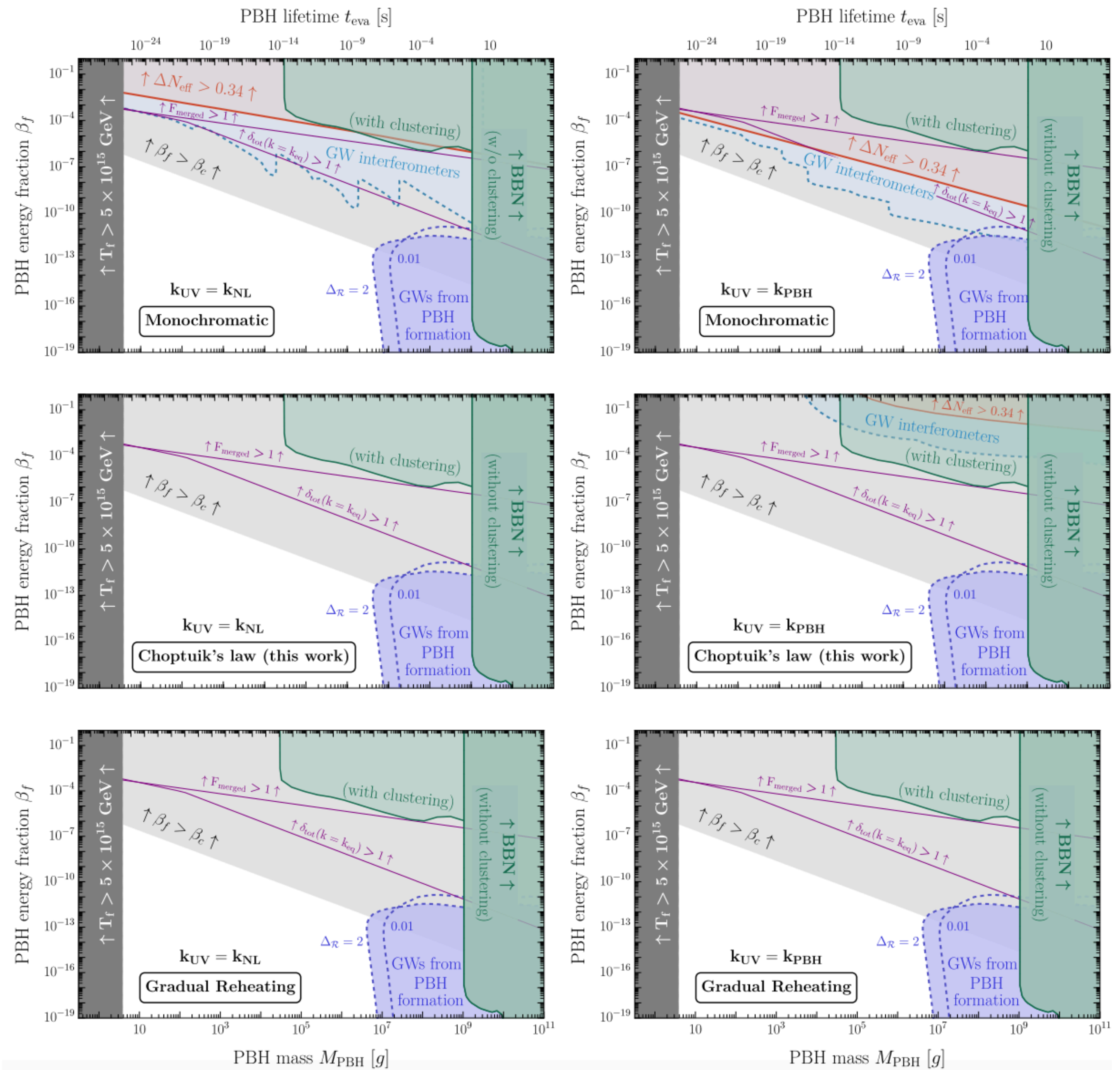
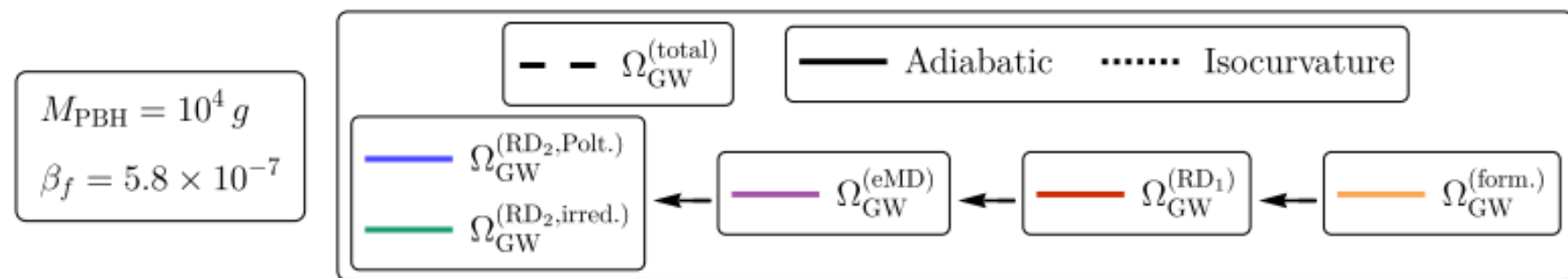
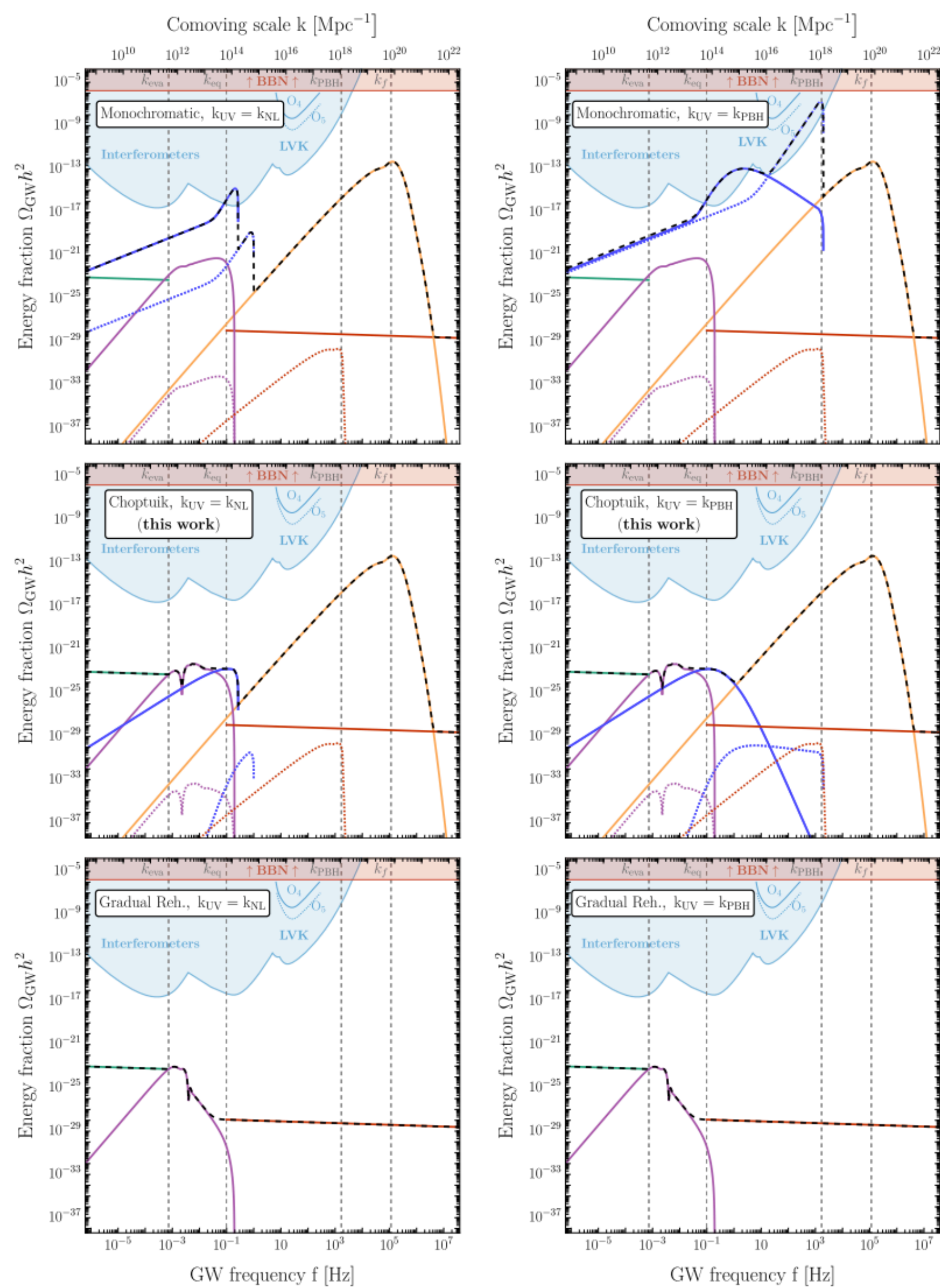
(260X.XXXXX)

Yann Gouttenoire, Nicholas Leister, Pedro Schwaller

$$M_{\text{PBH}} = 10^4 \text{ g}$$

$$\beta = 0.6 \times 10^{-6}$$





The loophole in the BBN bound

Hawking lifetime $\propto M^3$:

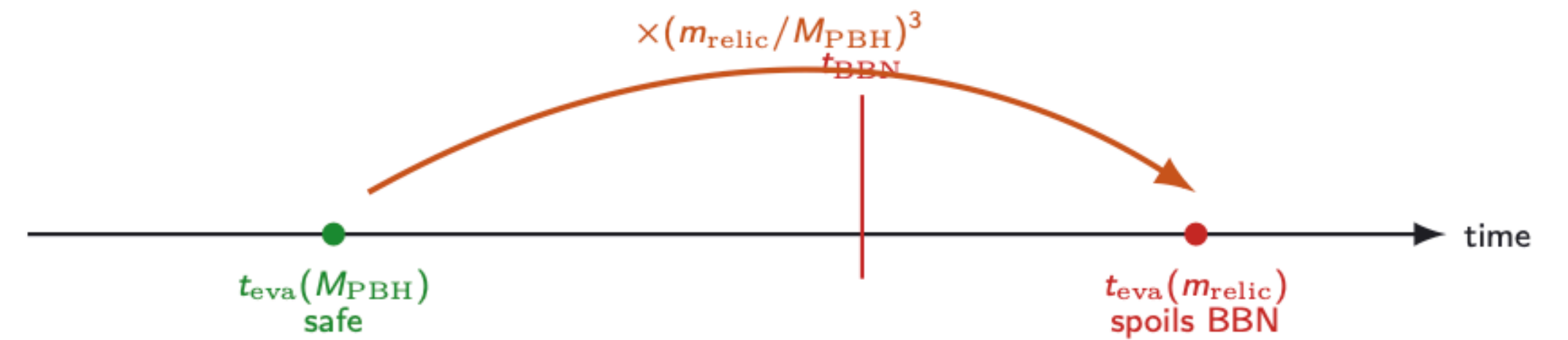
$$t_{\text{eva}} = \frac{M_{\text{PBH}}^3}{3AM_{\text{pl}}^4} \simeq 0.4 \text{ s} \left(\frac{M_{\text{PBH}}}{10^9 \text{ g}} \right)^3$$

$$t_{\text{eva}}(M_{\text{PBH}}) < t_{\text{BBN}} \simeq 10^{-2} \text{ s} \quad \checkmark$$

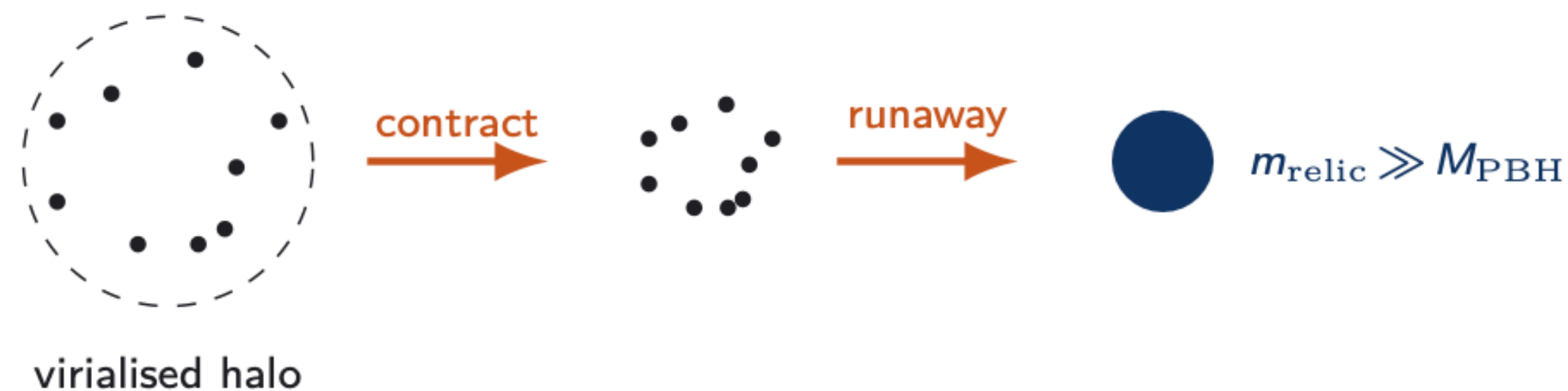
$\Downarrow$ clustering $\Rightarrow m_{\text{relic}} \gg M_{\text{PBH}}$

$$t_{\text{eva}}(m_{\text{relic}}) = t_{\text{eva}}(M_{\text{PBH}}) \underbrace{\left(\frac{m_{\text{relic}}}{M_{\text{PBH}}} \right)^3}_{\text{steep cube}}$$

$$\Rightarrow \boxed{t_{\text{eva}}(m_{\text{relic}}) > t_{\text{BBN}}}$$



Mechanism: clustering → runaway → relic



1. cluster (Poisson shot noise)

$$\delta_i \simeq \sqrt{\frac{\mu_i}{N}} \xrightarrow{\delta \propto a} \delta_{\text{vir}} \simeq 1.69$$

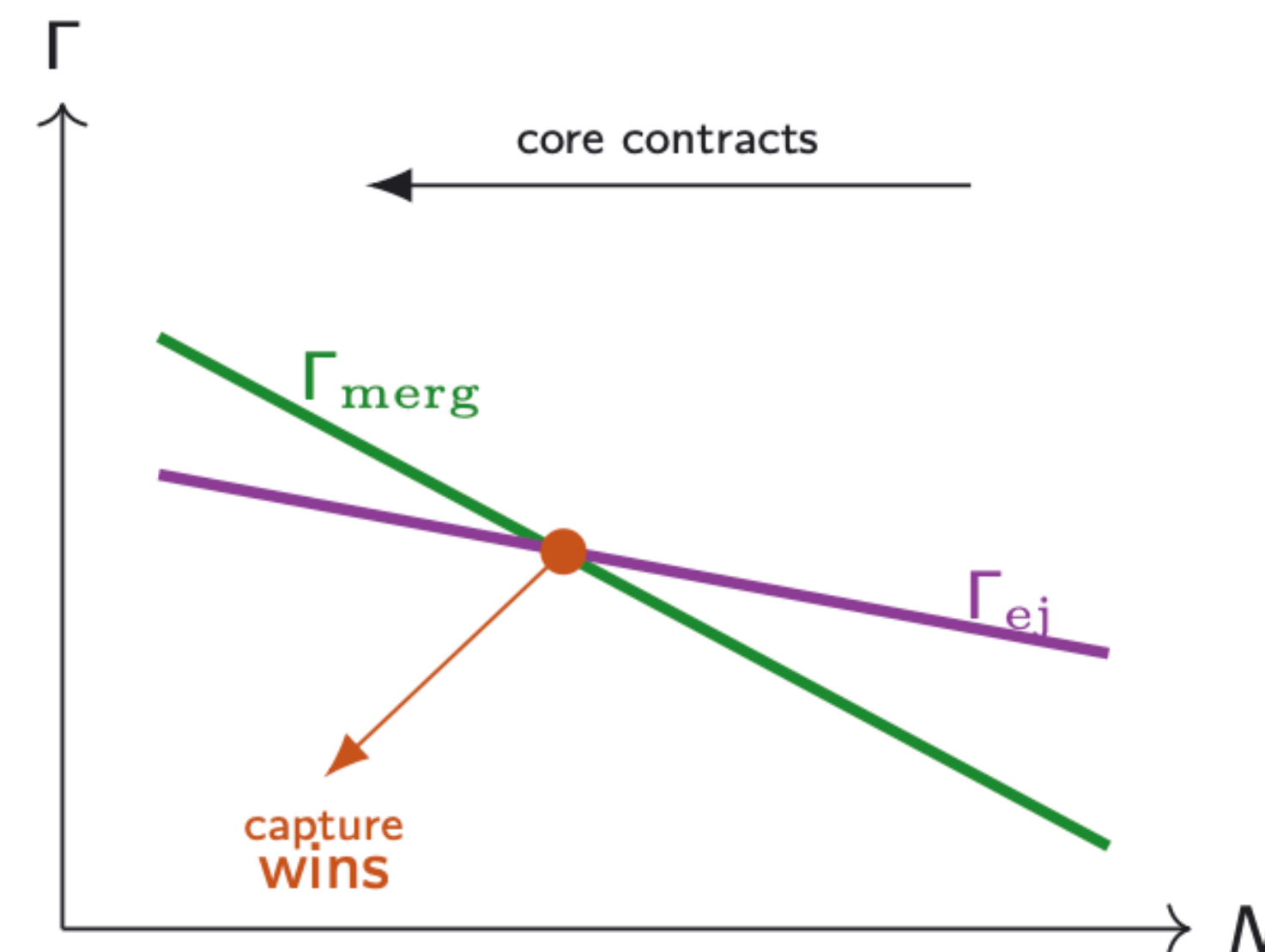
2. runaway (capture vs. ejection)

$$\Gamma_{\text{merg}} \propto N^{-45/14}, \quad \Gamma_{\text{ej}} \propto N^{-5/2}$$

$$\frac{\Gamma_{\text{merg}}}{\Gamma_{\text{ej}}} \propto N^{-5/7} \uparrow \text{ as } N \downarrow \Rightarrow \text{runaway}$$

$$\Rightarrow \text{collapse in } \tau_{\text{col}} \simeq 39 \tau_{\text{rlx}}$$

needs rich, dense clusters & $\tau_{\text{col}} < t_{\text{eva}}$; else ejection wins, $m_{\text{relic}} \rightarrow M_{\text{PBH}}$.



The sharpened bound

runaway completes before evaporation

$$\tau_{\text{col}}(N_{\text{max}}) = t_{\text{eva}}$$

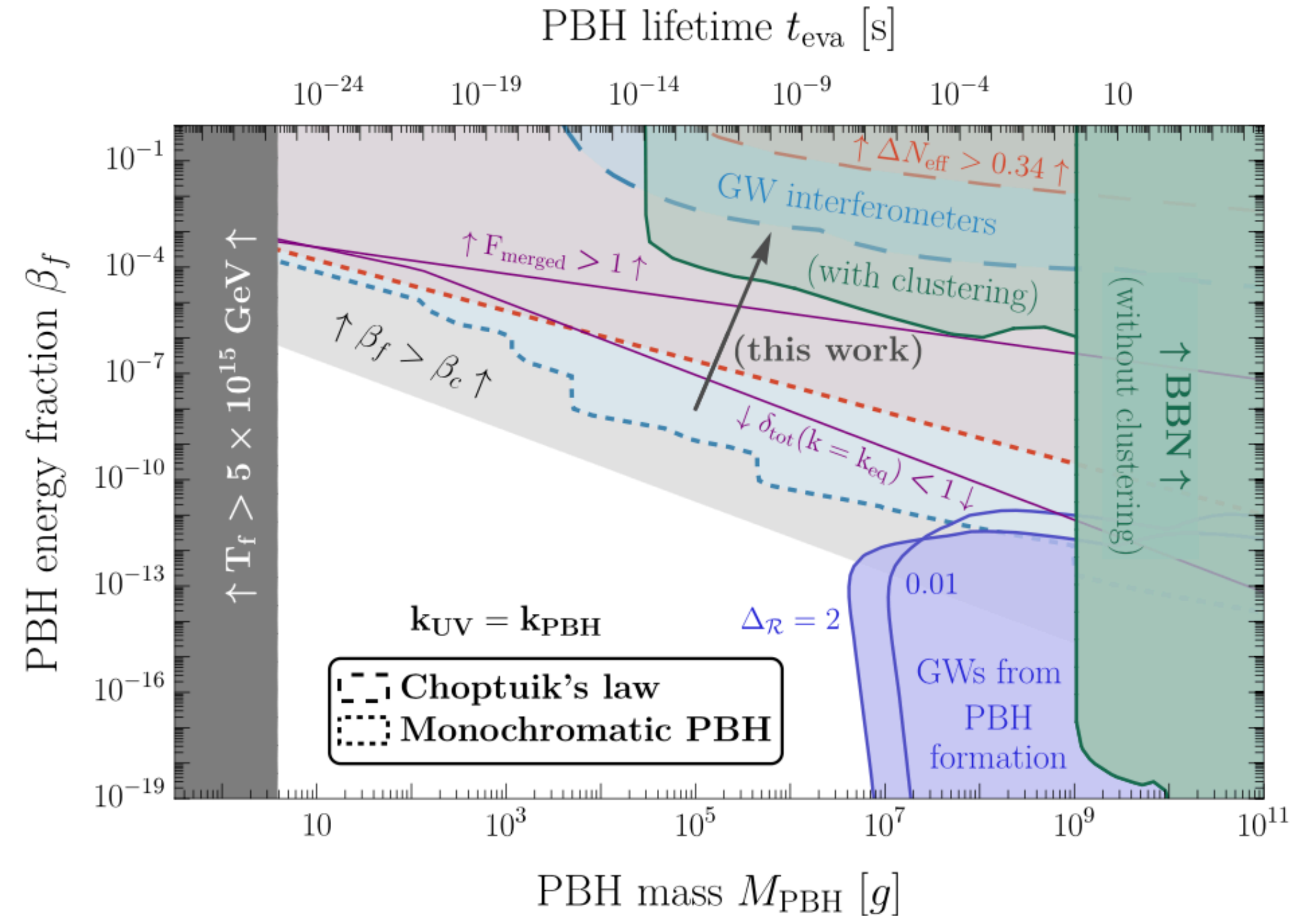
⇓ heaviest relic

$$m_{\text{relic}}^{\text{max}} \simeq 10^{15} \text{g} \left(\frac{M_{\text{PBH}}}{10^9 \text{g}} \right)^{\frac{53}{15}} \left(\frac{t_{\text{eq}}}{10^{-20} \text{s}} \right)^{-\frac{19}{15}}$$

⇓ cube amplifies

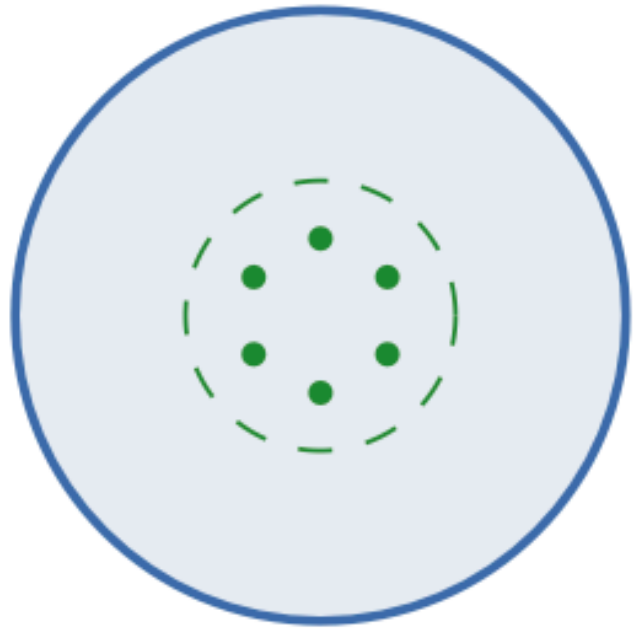
$$t_{\text{eva}}(m_{\text{relic}}^{\text{max}}) = t_{\text{eva}}(M_{\text{PBH}}) \left(\frac{m_{\text{relic}}^{\text{max}}}{M_{\text{PBH}}} \right)^3 < t_{\text{BBN}}$$

new exclusion in $(M_{\text{PBH}}, \beta_f)$, far beyond naive $t_{\text{eva}}(M_{\text{PBH}}) < t_{\text{BBN}}$; isolated Newtonian clusters, $\Delta m_{\text{GW}} \simeq 0.05M$.

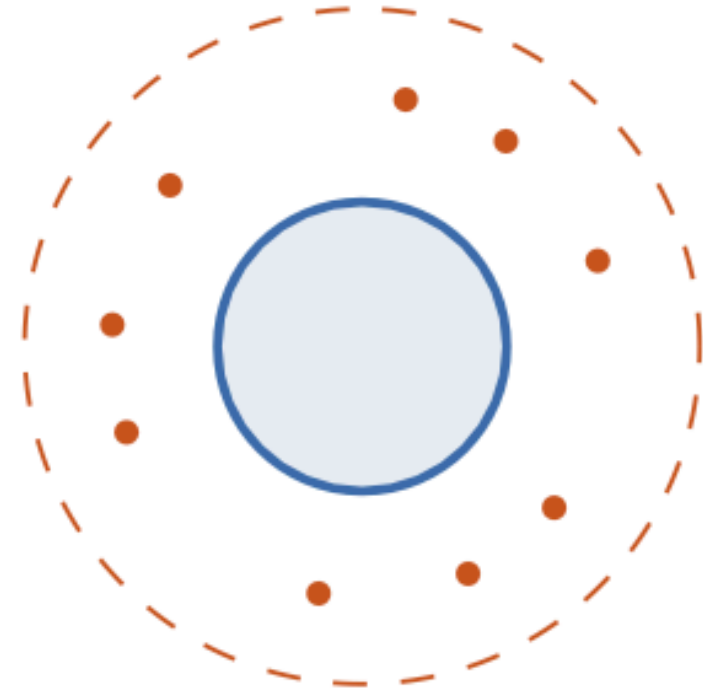


Backup — what is μ_i ?

horizon @ t_{eq} ($1/k_{\text{eq}}$)



small cluster $\subset$ horizon
grown $\Rightarrow \mu_i \approx 1$

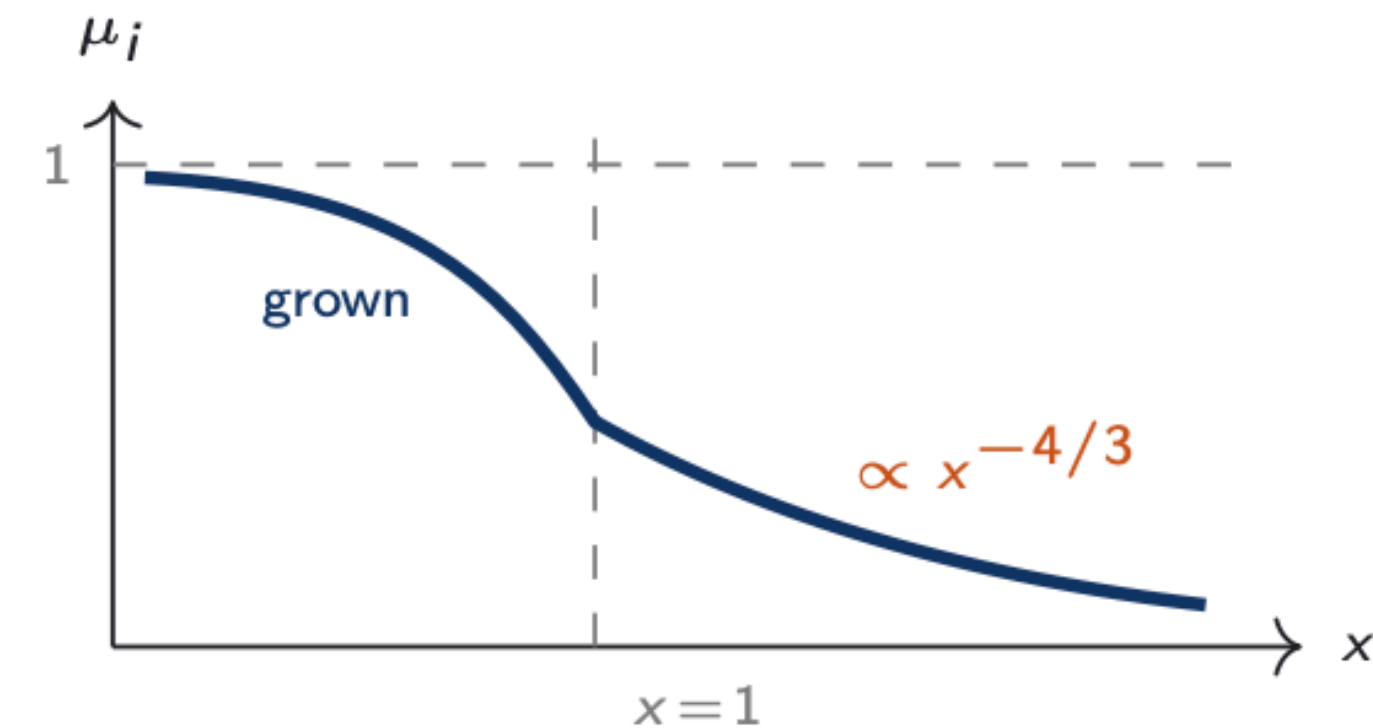


large cluster $\supset$ horizon
super-horizon $\Rightarrow \mu_i \ll 1$

$\mu_i \in (0, 1]$: fraction of the Poisson contrast that has **grown by equality**

$$\mu_i = \begin{cases} 1 - \frac{4}{7}x, & x < 1 \text{ (sub-horizon)} \\ \frac{3}{7}x^{-4/3}, & x > 1 \text{ (super-horizon)} \end{cases}$$

$$x \equiv \frac{M_{\text{cluster}}}{M_{H,\text{eq}}} = \left(\frac{k_{\text{eq}}}{k} \right)^3 \Rightarrow x=1 \iff k=k_{\text{eq}}$$



SIGW language: $\sqrt{\mu_i}$ is the isocurvature *density* transfer; the LF potential transfer is $T_S = -\frac{3}{2} \Omega_{\text{PBH}} \left(\frac{k_{\text{eq}}}{k} \right)^2 \sqrt{\mu_i}$.

The Poltergeist GW signal — mono. PBH reheating: master formula & kernel

1 Master formula — scalar source Φ at 2nd order $\Rightarrow$ induced tensor h

$$h''_{\lambda} + 2\mathcal{H} h'_{\lambda} + k^2 h_{\lambda} = \mathcal{S}_{\lambda}, \quad \mathcal{S}_{ab} = 4 \left[\partial_a \Phi \partial_b \Phi + \frac{2}{3(1+\omega)} \partial_a \left(\Phi + \frac{\Phi'}{\mathcal{H}} \right) \partial_b \left(\Phi + \frac{\Phi'}{\mathcal{H}} \right) \right] \xrightarrow{\text{Green}} h_{\lambda} = \int_0^x \frac{d\bar{x}}{k^2} G_h(x, \bar{x}) \mathcal{S}_{\lambda}$$

$$\Omega_{\text{GW}}(k, \eta) = \frac{k^2}{12 \mathcal{H}^2} \overline{\mathcal{P}}_h, \quad \overline{\mathcal{P}}_h = 8 \int_0^{\infty} dv \int_{|1-v|}^{1+v} du \underbrace{\left[\frac{4v^2 - (1+v^2 - u^2)^2}{4uv} \right]^2}_{\mathcal{K}(u, v)} \overline{I_{\Phi}^2(x, u, v)} \mathcal{P}_{\Phi}(uk) \mathcal{P}_{\Phi}(vk)$$

$$I_{\Phi} = \int_0^x d\bar{x} G_h f_{\Phi}; \quad x \equiv k\eta = k/\mathcal{H}, \quad u = q/k, \quad v = |\mathbf{k} - \mathbf{q}|/k; \quad \text{seed } \mathcal{P}_{\Phi} = \left(\frac{2}{3}\right)^2 \mathcal{A}_{\mathcal{R}}(k/k_{\text{CMB}})^{n_s-1}.$$

2 Sub-horizon limit $k/\mathcal{H} = k\eta \equiv x \gg 1$ ($k \gg k_{\text{eva}}$) $\Rightarrow$ Poltergeist kernel

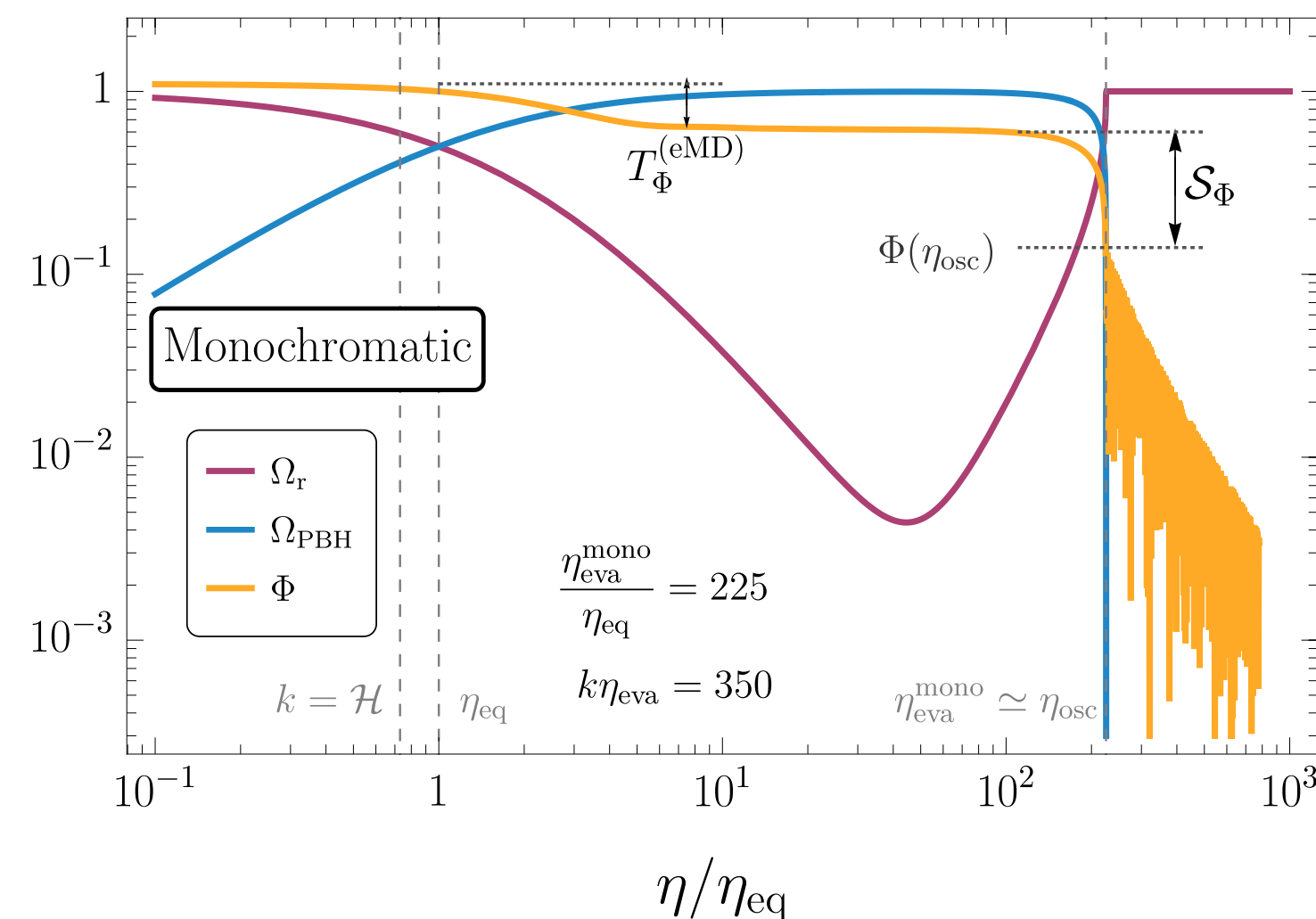
mono.: all PBHs evaporate together $\Rightarrow \Gamma \rightarrow \infty$; the frozen plateau crashes by $\mathcal{S}_{\Phi}$, the radiation fluid *rings*, and the velocity term $\Phi' \Phi'$ dominates the source $\mathcal{S}_{\lambda}$.

$$\Phi_q(\bar{\eta}) \propto \mathcal{S}_{\Phi}(q) T_{\Phi}^{(\text{eMD})}(q) \sin\left[c_s q(\bar{\eta} + \frac{\eta_{\text{eva}}}{2})\right], \quad c_s = \frac{1}{\sqrt{3}}; \quad G_h \propto \sin[k(\eta - \bar{\eta})]; \quad I_{\Phi}^{(\text{RD}_2)} \simeq \frac{c_s^2 uv}{32\bar{x}} x_{\text{eva}}^4 \mathcal{S}_{\Phi}(uk) \mathcal{S}_{\Phi}(vk) T_{\Phi}^{(\text{eMD})}(uk) T_{\Phi}^{(\text{eMD})}(vk) \mathcal{I}_{\text{osc}}$$

$$\Omega_{\text{GW}}^{(\text{RD}_2)}[\Phi] = \frac{1}{864} \left(\frac{k}{k_{\text{eva}}} \right)^8 \int_0^{\infty} dv \int_{|1-v|}^{1+v} du \left[4v^2 - (1+v^2 - u^2)^2 \right]^2 \overline{\mathcal{I}_{\text{osc}}}^2 \mathcal{S}_{\Phi}^2 \mathcal{S}_{\Phi}^2 (T_{\Phi}^{(\text{eMD})})^2 (T_{\Phi}^{(\text{eMD})})^2 \mathcal{P}_{\Phi} \mathcal{P}_{\Phi}$$

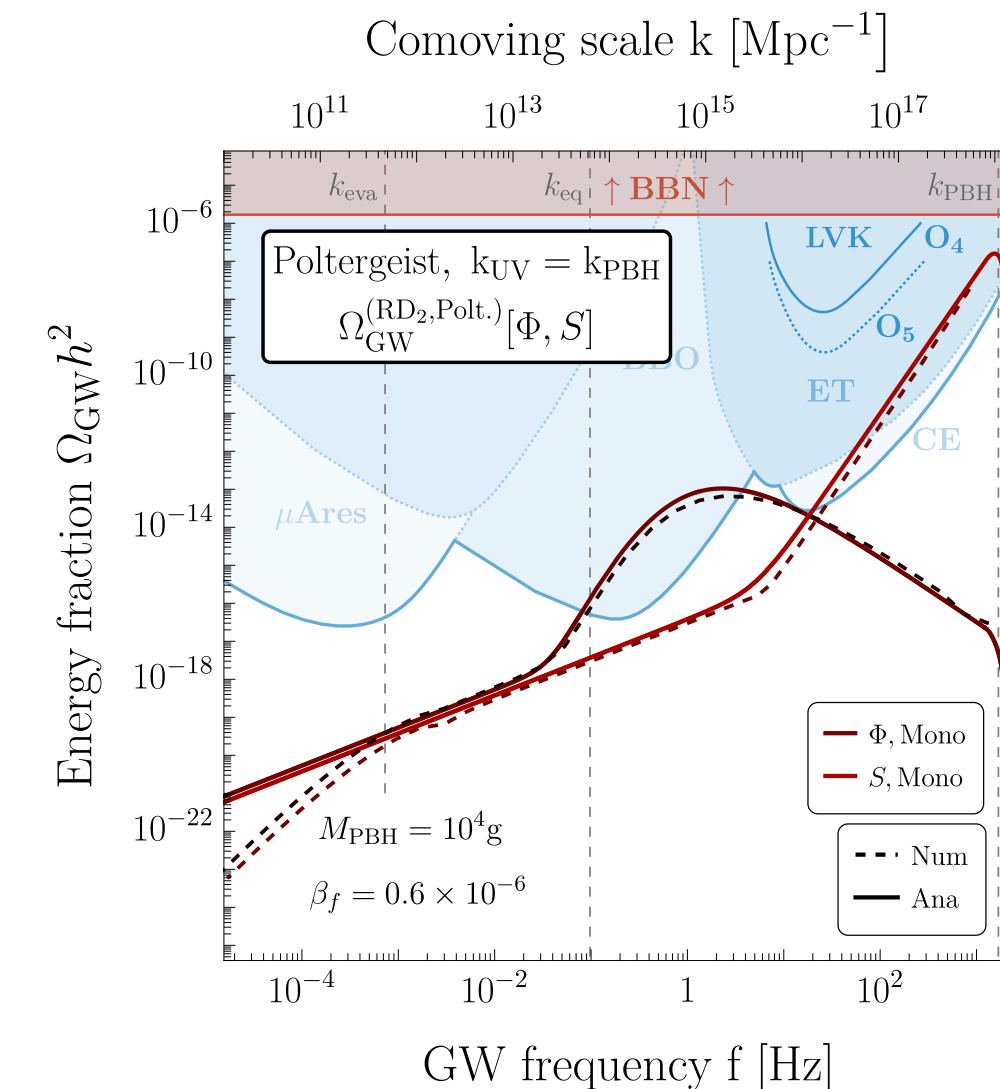
$$\text{resonance (acoustic} \leftrightarrow \text{tensor phase-lock) at } u + v = c_s^{-1} = \sqrt{3}: \quad \overline{\mathcal{I}_{\text{osc}}}^2 \xrightarrow{k \gg k_{\text{eva}}} \frac{\pi c_s}{32} \frac{k_{\text{eva}}}{k} \delta(c_s^{-1} - u - v).$$

The Poltergeist signal — monochromatic limit



Φ frozen on a plateau through eMD, then **crashes by** $\mathcal{S}_\Phi$ at the sharp, simultaneous evaporation ($\eta_{\text{osc}} \simeq \eta_{\text{eva}}$).

Resulting resonant SIGW vs next-gen reach ($M_{\text{PBH}} = 10^4 \text{ g}$,
 $\beta_f = 0.6 \times 10^{-6}$).



Mechanism: monochromatic $\Rightarrow$ all PBHs evaporate *at once* $\Rightarrow$ near-instant matter $\rightarrow$ radiation transition $\Rightarrow$ the frozen sub-horizon Φ resonantly drives the restored radiation fluid into GWs.

$$\Omega_{\text{GW}}^{(\text{RD}_2)} \propto (k/k_{\text{eva}})^8$$

$$\mathcal{S}_\Phi \rightarrow (\sqrt{12}/k\eta_{\text{eva}})^{1/3} \propto k^{-1/3}$$

Mild $k^{-1/3}$ suppression $\Rightarrow$ the $(k/k_{\text{eva}})^8$ enhancement survives $\Rightarrow$ the **loud**, standard Poltergeist (peaks near k_{eq} ; UV-cut $k_{\text{NL}}/k_{\text{PBH}}$). Realistic extended (Choptuik) mass function: $\mathcal{S}_\phi \propto k^{-4/3} \Rightarrow$ orders-of-magnitude weaker.

Inomata, Kawasaki, Mukaida, Terada & Yanagida (2020).

Gravitational waves from the early matter era (eMD)

review · LF 4,5

Why Ω_{GW} keeps growing during the matter era:

1 Frozen potential $\Rightarrow$ frozen source:

$$\Phi' \simeq 0 \Rightarrow \mathcal{S}_\lambda \simeq \text{const}$$

$\downarrow$

2 Constant source $\Rightarrow$ strain stops redshifting:

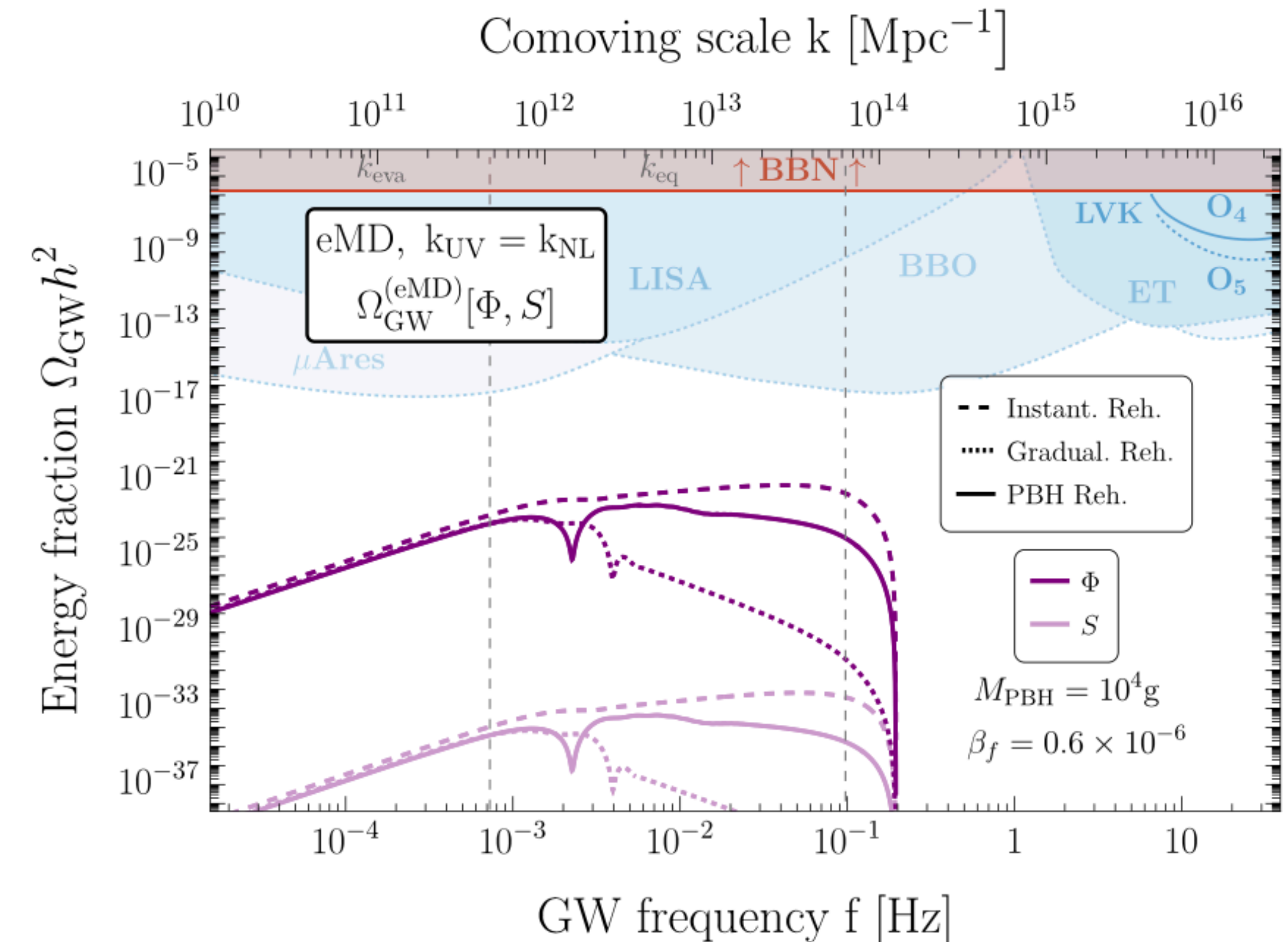
$$h''_\lambda + 2\mathcal{H}h'_\lambda + k^2 h_\lambda = \mathcal{S}_\lambda \Rightarrow h_\lambda \rightarrow \frac{\mathcal{S}_\lambda}{k^2} = \text{const}$$

$\downarrow$

3 Insert into $\Omega_{\text{GW}} = \frac{k^2}{12\mathcal{H}^2} \mathcal{P}_h$:

$$\mathcal{P}_h = \text{const}, \quad \mathcal{H}^2 \propto a^{-1} \quad (\text{MD: } \rho_m \propto a^{-3})$$

$$\Omega_{\text{GW}} \propto \frac{1}{\mathcal{H}^2} \propto a$$



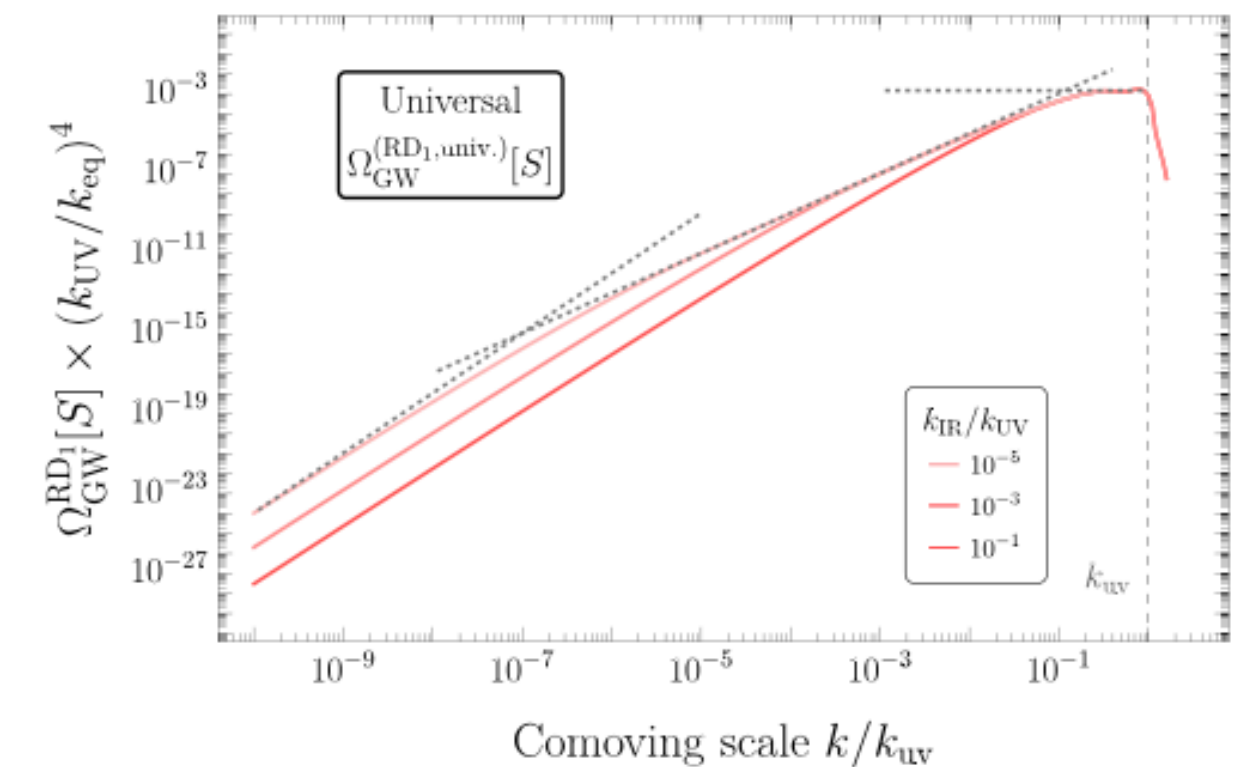
Grows through eMD, cut at k_{eq} — far below BBN & every detector.

► **Robust floor:** only mildly reheating-dependent — the SIGW that **survives** the suppression that kills the **Poltergeist**.

$$\Omega_{\text{GW}} = \frac{k^2}{12 \mathcal{H}^2} \overline{\mathcal{P}}_h, \quad \overline{\mathcal{P}}_h = 8 \int_0^\infty dv \int_{|1-v|}^{1+v} du \underbrace{\mathcal{K}(u,v)}_{\text{fixed geometry}} \sum_{X \in \{\Phi, S\}} \underbrace{\overline{I}_X^2}_{\text{kernel}} \underbrace{\mathcal{P}_X(uk) \mathcal{P}_X(vk)}_{\text{seed}}$$

kernel $I_X = \int_0^x d\bar{x} G_h f_X$ — fixed by the background ω & the seed's ICs; redshift to today via $\mathcal{D}(T_{\text{eva}}, T_0)$.

	2 RD ₁ irr. [Φ]	6 RD ₂ irr. [Φ]	3 RD ₁ univ. [S]
seed $\mathcal{P}_X$ <i>what fluctuates</i>	$A_\zeta (k/k_{\text{CMB}})^{n_s-1}$ (inflation)		$\frac{2}{3\pi} (k/k_{\text{PBH}})^3$
kernel $\overline{I}_X^2$ <i>sourcing efficiency</i>	adiabatic, radiation era		isoc.: $\overline{I}_S^2 \propto \kappa^{-4}$
window <i>which modes form</i>	$k > k_{\text{eq}}$	$k < k_{\text{eva}}$	$k_{\text{eq}} < k < k_{\text{PBH}}$
redshift <i>dilution to today</i>	$D^{-4/3} \mathcal{D}$	$\mathcal{D}$ (not dil.)	$\Omega_{r,0} D^{-4/3}$
spectrum Ω_{GW}^0 <i>what you observe</i>	$Q A_\zeta^2 \left(\frac{k}{k_{\text{CMB}}}\right)^{2(n_s-1)}$	same env.	$\mathcal{A}_S^2 \frac{k^2 k_{\text{eq}}^4}{k_{\text{PBH}}^6} \mathcal{G}(k)$



universal [S] signal

k^3 rise, plateau $\propto (k_{\text{eq}}/k_{\text{PBH}})^4$

► **2 & 6** — one inflationary floor, $\Omega_{\text{GW}} \sim A_\zeta^2 \sim 10^{-18}$: diluted in RD₁, survives in RD₂.

[Ananda, Clarkson & Wands '06 · Baumann, Steinhardt & Takahashi '07 · Kohri & Terada '18]

► **3** — PBH Poisson noise, extra $(k_{\text{eq}}/k_{\text{PBH}})^4$ suppression; *no* RD₂ twin (source evaporated).

[Domènech, Passaglia & Renaux-Petel '21 · Lozanov, Sasaki & Takhistov '23 · Domènech & Tränkle '25]

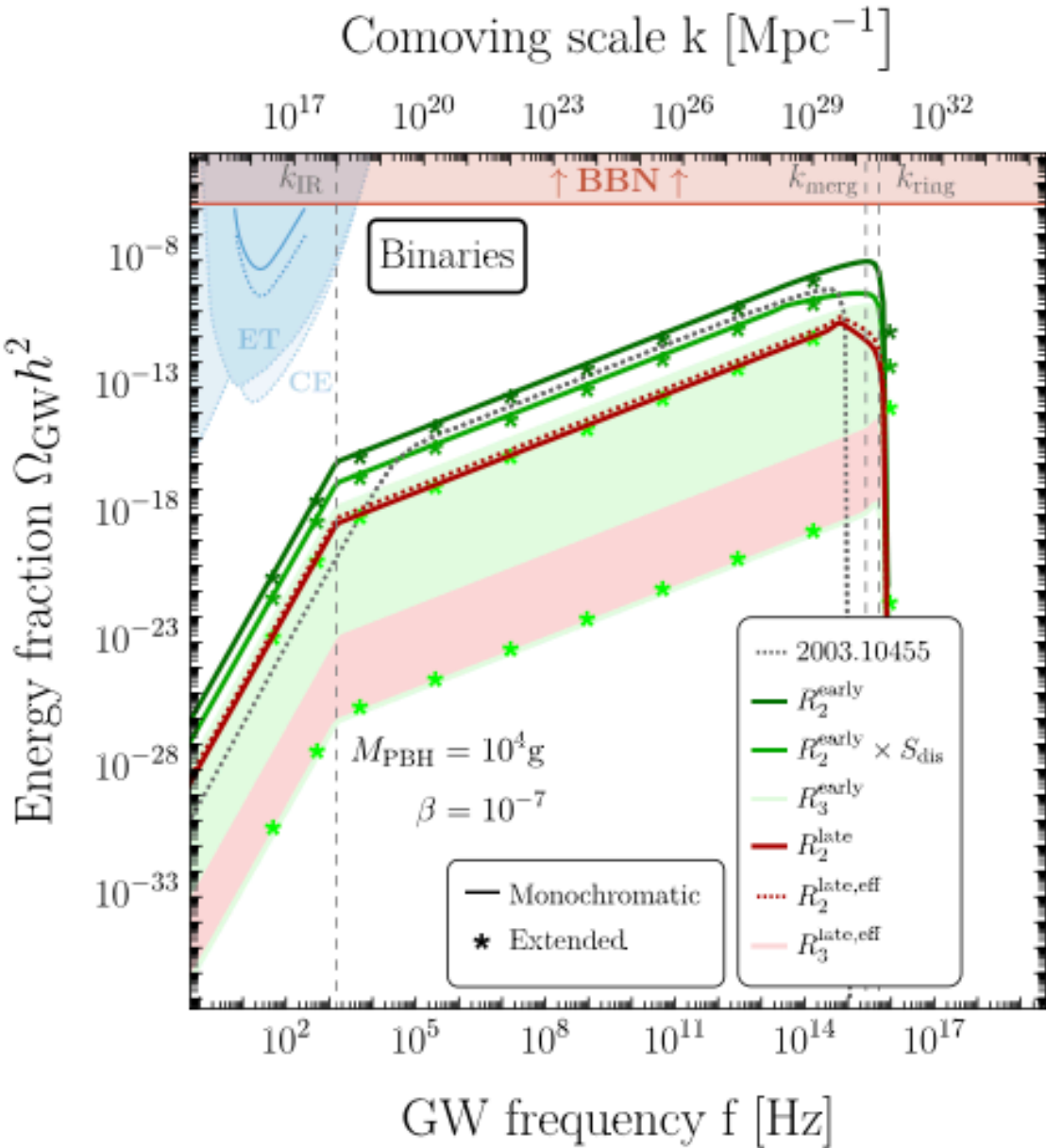
GWs from PBH mergers

PBHs ($M_{\text{PBH}} \lesssim 10^9 \text{ g}$) dominate before BBN $\rightarrow$ **cluster, pair, merge**

Four channels = 2 routes $\times$ 2 bodies

	2-body	3-body
early eMD	decouple + torque $\rightarrow$ eccentric	triple ejection $\rightarrow$ hard
late clusters	GW capture in fly-by	capture + hardening

3-body wins for heavy M_{PBH} , $\gamma \rightarrow 1$



merger background: disrupted (green) vs. undisturbed (dark)

- IR tail $\propto f^3$

peak $0.75 f_{\text{ring}} \sim 10^{15} \text{ Hz}$

disruption $\times 10^{-2}$
- mass function immaterial

above LVK/ET/CE

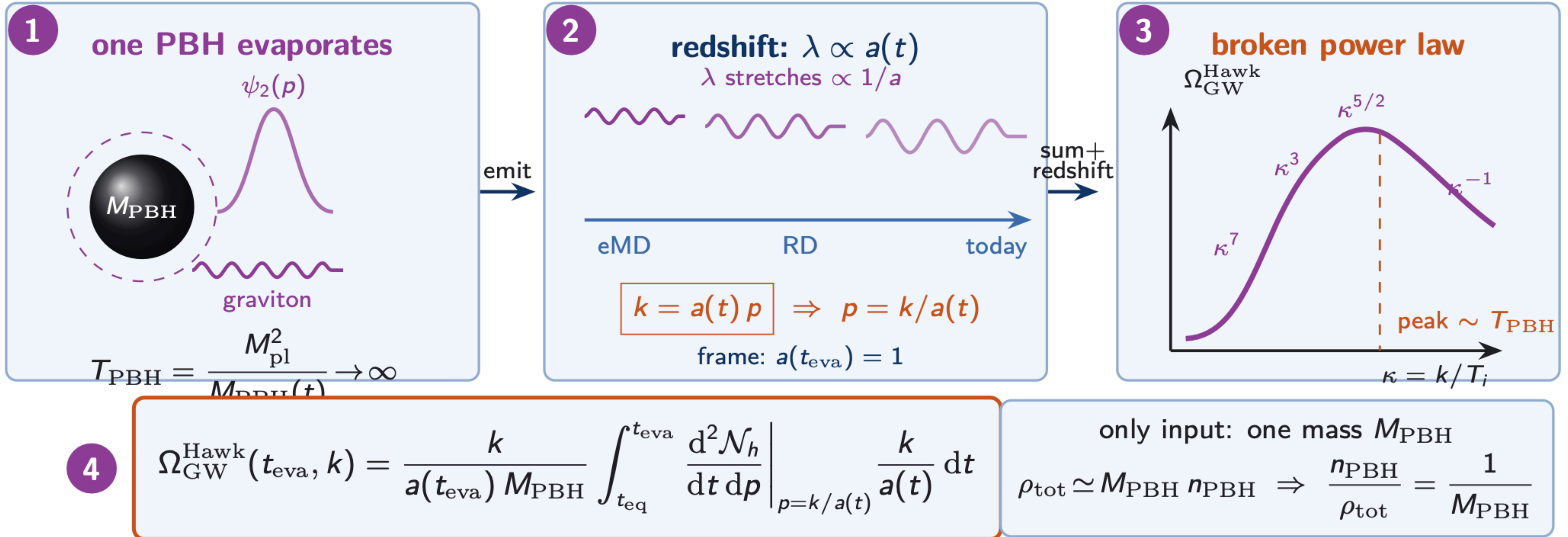
ΔN_{eff} & BBN ok

	2-Body	3-Body ($\gamma = 1$)
EARLY	$\frac{5.6 \times 10^{-7} \beta_f^{16/37}}{G^{188/37} M_{\text{PBH}}^{262/37} t^{34/37}} S_{\text{dis}}(t)$	$\frac{2.4 \times 10^{-6} \beta_f^{274/259}}{G^{186/37} M_{\text{PBH}}^{1790/259} t^{6/7}}$
LATE	$12 \frac{G^{11/63} M_{\text{PBH}}^{11/63}}{t^{263/63} \beta_f^{88/63}} \left(\frac{r_{\text{cl}}}{R_{\text{vir}}} \right)^{52/35}$	$\frac{3 \times 10^{-25} t^{332/63}}{G^{1340/63} M_{\text{PBH}}^{2096/63} \beta_f^{440/63}} \left(\frac{r_{\text{cl}}}{R_{\text{vir}}} \right)^{104/35}$

β_f formation abundance · $S_{\text{dis}}(t)$ disruption survival · $r_{\text{cl}}/R_{\text{vir}}$ cluster compactness · rate per PBH

$$\frac{d^2 \mathcal{N}_h}{dt dp} = \frac{27\pi G_N^2 M_{\text{PBH}}(t)^2}{2\pi^2 (e^{p/T_{\text{PBH}}} - 1)} \psi_2(p) p^2$$

Hawking emission $\times$ spin-2 greybody: $\psi_2 \sim p^4$ (IR), $\psi_2 \rightarrow \text{const}$ (UV)



Hawking gravitons: peak $\sim T_{\text{PBH}}$ redshifted to today (highest- f relic); robust to spin ($a_\star \sim 0.02$) and mass function (mono $\approx$ Choptuik). [Hawking '74; Page '76; MacGibbon–Carr '91; Hooper '20; Cheek '22]