Neutrino mass and gravitational waves from RG fixed points

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in collaboration with

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Mainly based on:
JHEP 08 (2022) 262 (arXiv: 2204.00866)
JHEP 11 (2023) 224 (arXiv: 2308.06114)

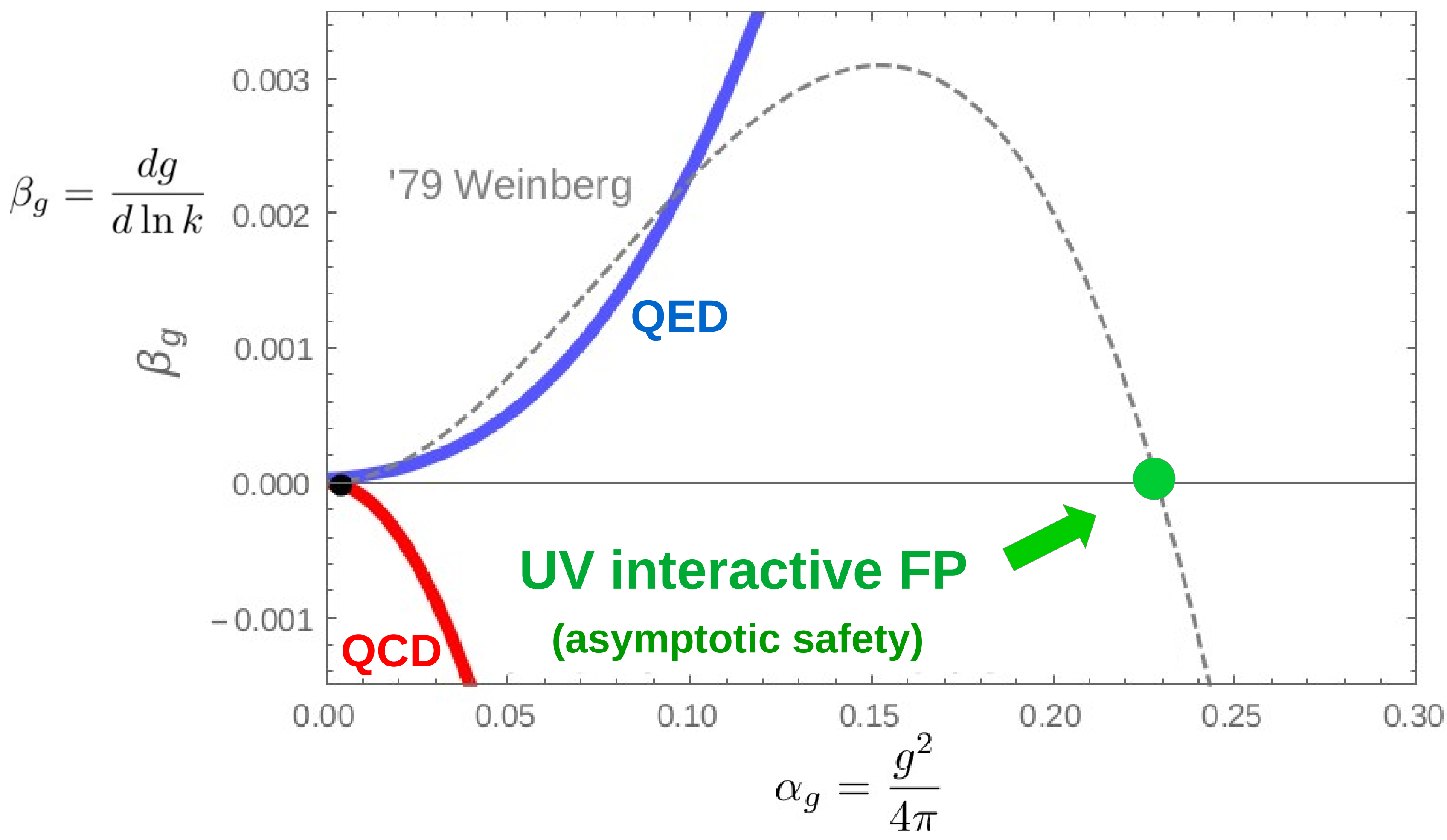
Particle Physics in the Early Universe

Paris, 15.09.2026

National Science Centre Poland

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Asymptotic safety

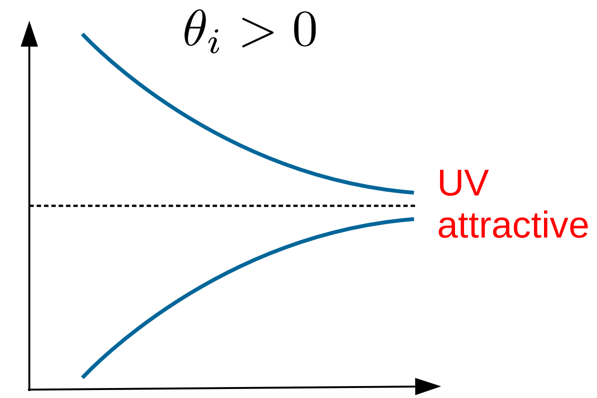


Weinberg '79: improve UV behavior of G_N (non-perturbative renormalizability)

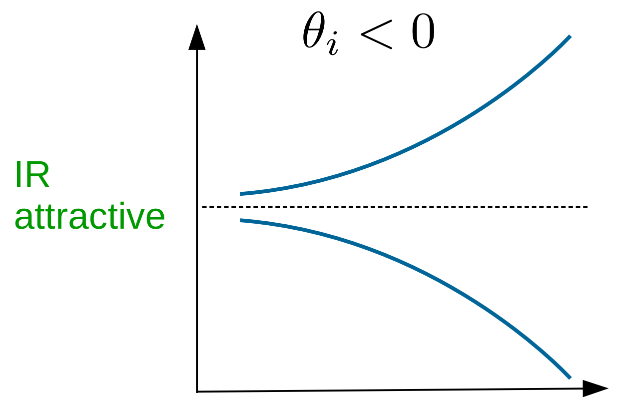
Critical surface and predictivity

$$\boxed{\beta_i(\{\alpha_j^*\}) = 0} \longrightarrow M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}} \longrightarrow \{-\theta_i\}$$

stability matrix — critical exponents



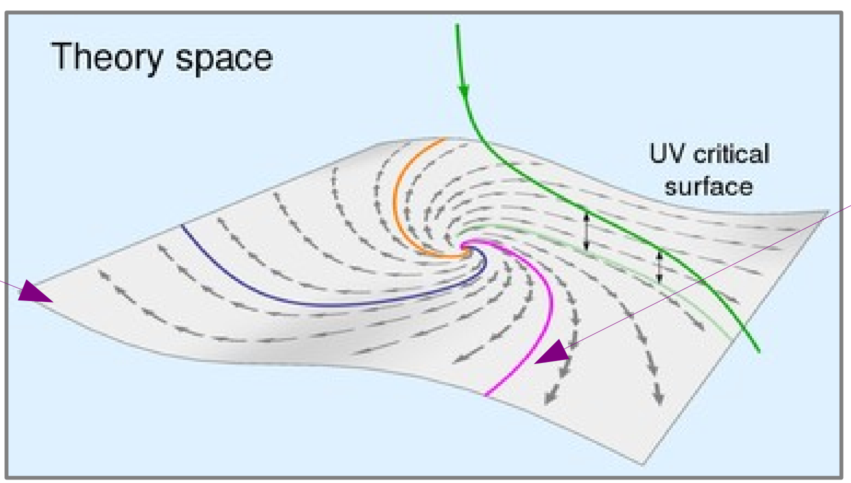
Relevant couplings are
free parameters of the theory



Irrelevant couplings provide
predictions

span the
UV critical surface

- **Non-perturbative renormalizability:** critical surface is finite-dimensional



from Wikipedia

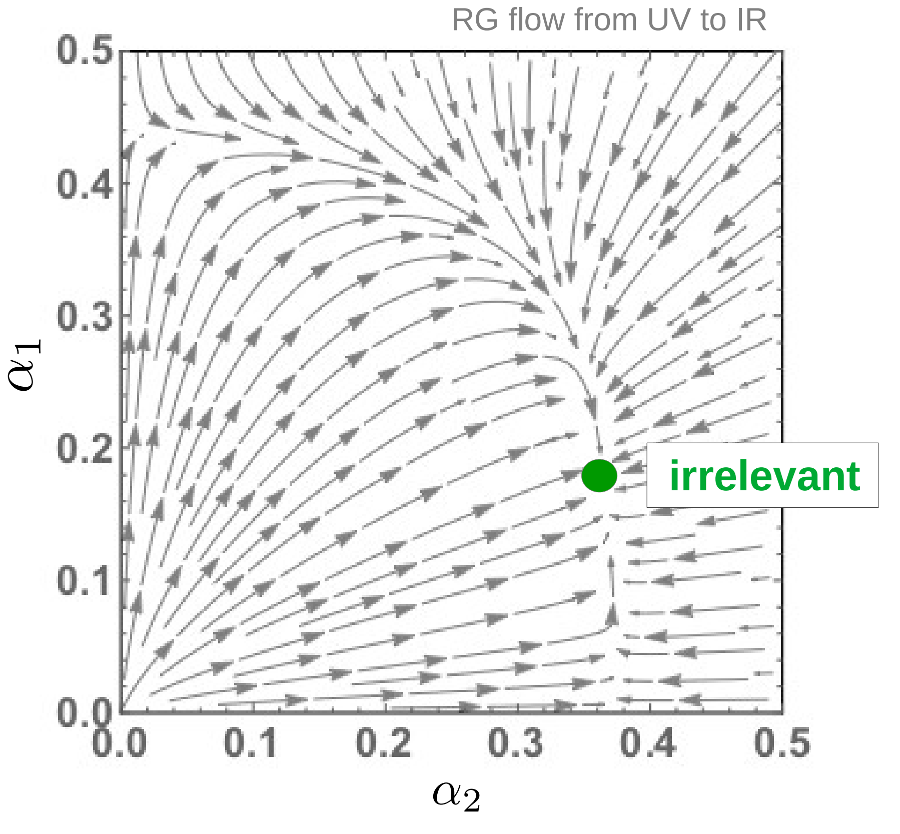
can only deviate from the FP along the critical surface

- **Max predictivity:** max number of irrelevant couplings

Phenomenological implications of AS

$$\beta_i(\{\alpha_j^*\}) = 0$$

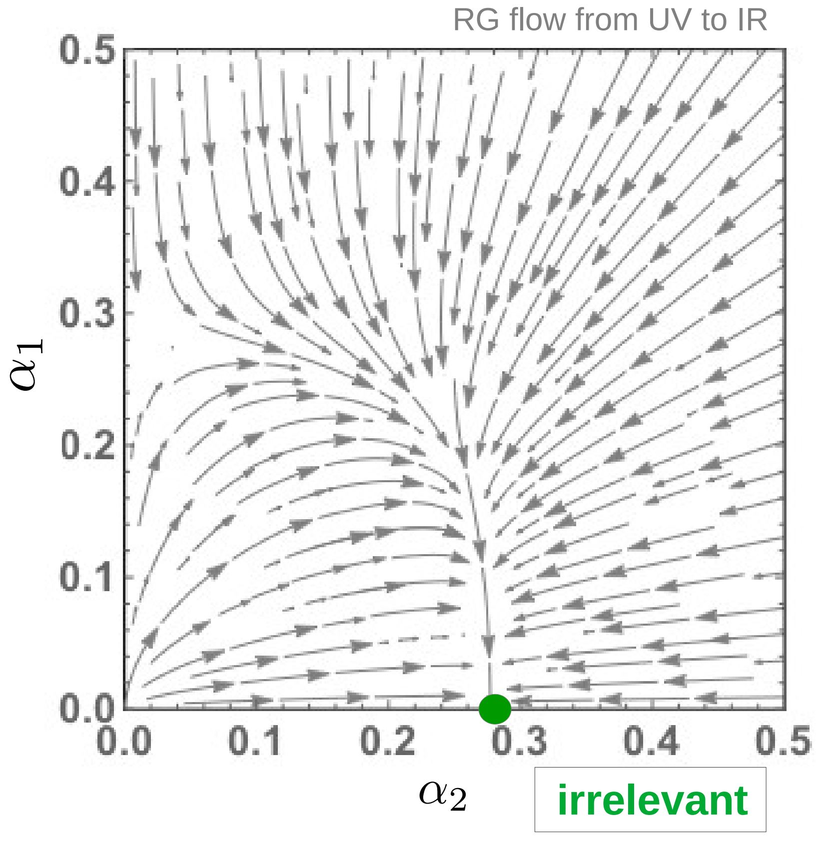
A. Enhanced predictivity



Phenomenological implications of AS

$$\beta_i(\{\alpha_j^*\}) = 0$$

B. Enhanced symmetry

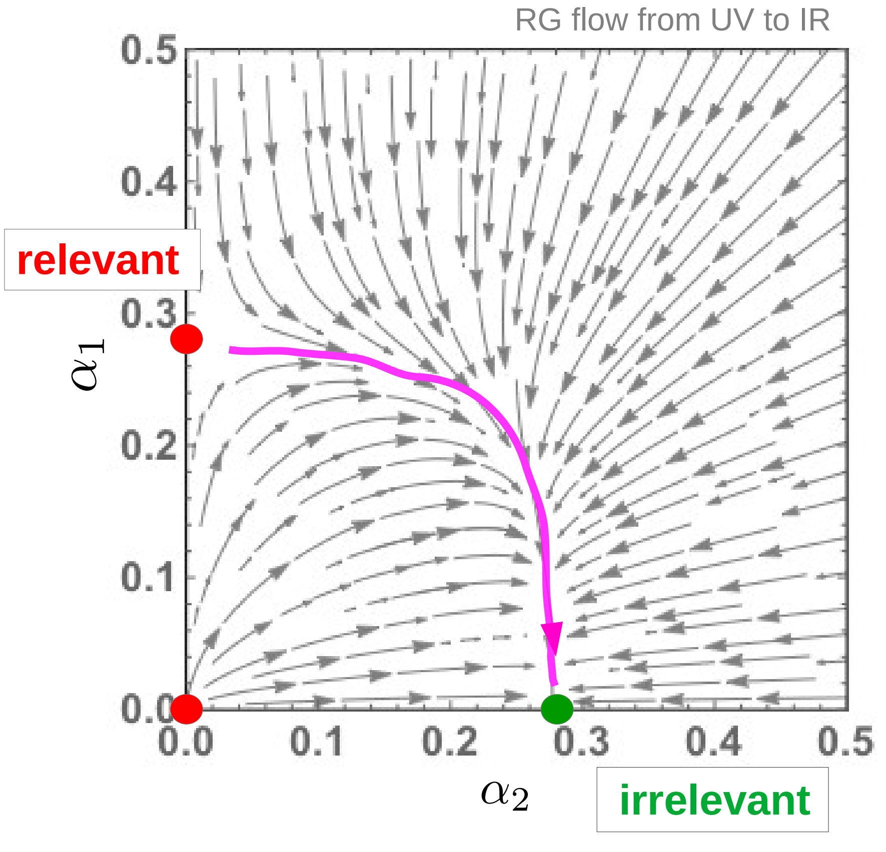


see E. Sessolo's talk this afternoon (dark matter)

Phenomenological implications of AS

$$\beta_i(\{\alpha_j^*\}) = 0$$

C. Dynamical suppression



this talk: Dirac neutrino mass

Asymptotic safety in gravity and matter

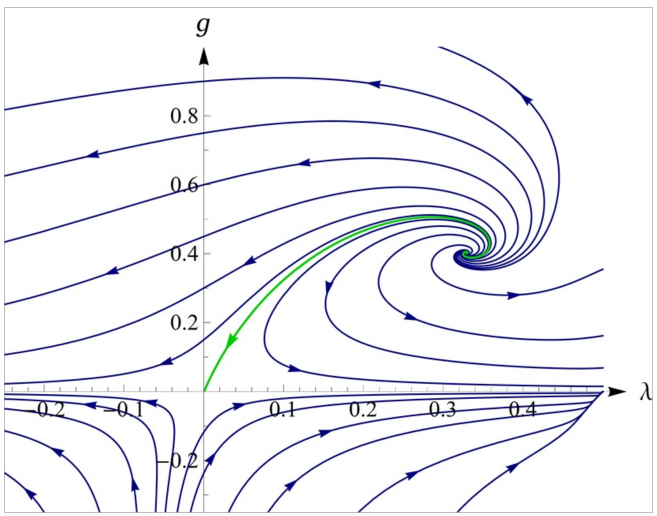
Reuter '96, Reuter, Saueressig '01, Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Pawlowski *et al.* '18 …

Trans-Planckian fixed point for gravity

$$S_{\rm EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,(-R(g)+2\Lambda)$$

$$\beta_g \equiv \frac{dg}{d\ln k} = 0 \qquad \beta_\lambda \equiv \frac{d\lambda}{d\ln k} = 0$$

M. Reuter, F. Saueressig , PRD 65, 065016 (2002)



- **Fixed point persists w/ more interaction terms**
- **Critical surface has finite dimension**

Trans-Planckian corrections to matter RGEs

$k > M_{\rm Pl}$

universal correction

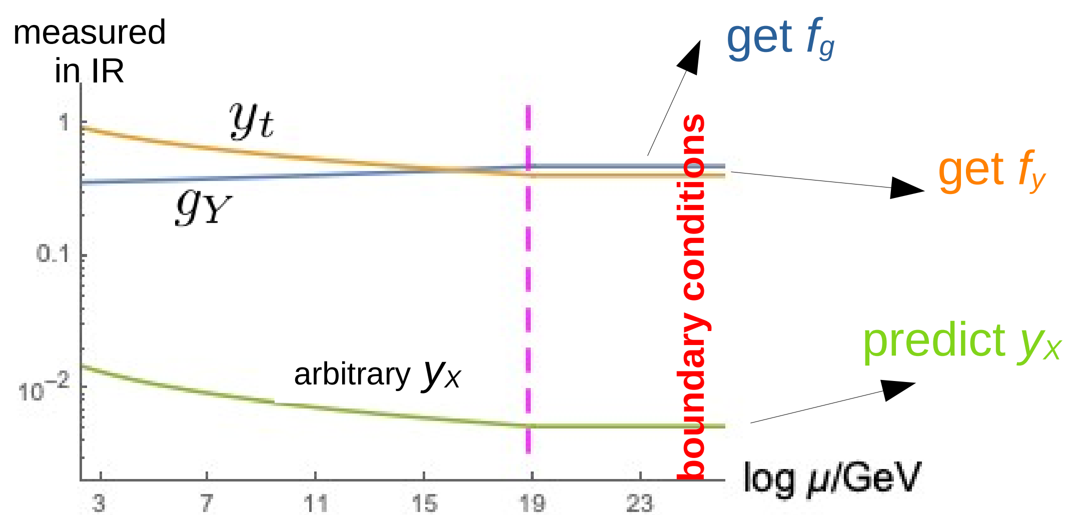
$$\beta_g = \beta_g^{\rm SM+NP} - g\, f_g \quad = 0$$

$$\beta_y = \beta_y^{\rm SM+NP} - y\, f_y \quad = 0$$

$$\beta_\lambda = \beta_\lambda^{\rm SM+NP} - \lambda\, f_\lambda \quad = 0$$

TP fixed points for matter

Parametric approach



see E. Sessolo's talk for proper time derivation of f_g and f_y

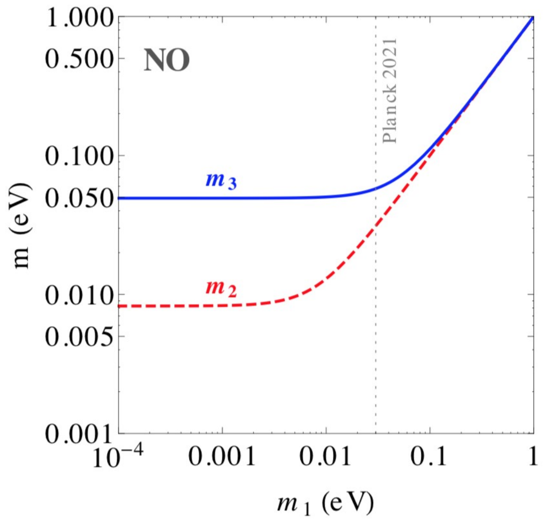
Neutrino masses

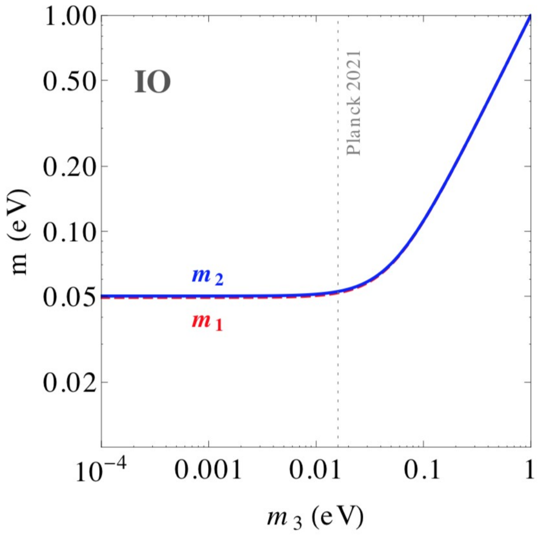
NuFIT5.1 (2021)   2007.14792

$$\begin{aligned} &\Delta m^2_{21} = 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\mathrm{eV}^2, \\ \text{NO:}\quad &\Delta m^2_{31} = 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\mathrm{eV}^2, \\ \text{IO:}\quad &\Delta m^2_{32} = -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\mathrm{eV}^2, \end{aligned}$$

DESI (2025)   2507.12401

$$m_{\nu_{\mathrm{lightest}}} < 0.02\ \mathrm{eV}$$





either Dirac neutrino ...

$$\mathcal{L}_D = -y_\nu^{ij}\,\nu_{R,i}\,(H^c)^\dagger L_j + \mathrm{H.c.}$$

$$m_\nu = \frac{y_\nu v_H}{\sqrt{2}}$$

- 10^{-13} Yukawa coupling
- Lepton number is conserved

… or Majorana neutrino

e.g. Type 1 see-saw

$$\mathcal{L}_M = \mathcal{L}_D - \frac{1}{2} M_N^{ij}\nu_{R,i}\nu_{R,j} + \mathrm{H.c.}$$

$$m_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_N \end{pmatrix} \qquad m_\nu = y_\nu^2 v_h^2/(\sqrt{2} M_N)$$

- *O*(1) Yukawa coupling
- Lepton number is violated

Neutrino – top system

KK, S.Pramanick, E.M.Sessolo, JHEP 08 (2022) 262

SM + RHN:

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y = 0$$ get f_g=0.0096

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t = 0$$ get f_y: [-10^{-4}, 10^{-3}]

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu = 0$$ predict

$\Rightarrow$ $g_Y^*, y_t^* \sim \mathcal{O}(1)$

$\beta_\nu \equiv \frac{dy_\nu}{dt} = 0$ $\rightarrow$ two IRR solutions for neutrino FP:

1. $$y_\nu^{*2} = \frac{32\pi^2}{5}f_y + \frac{3}{10}g_Y^{*2} - \frac{6}{5}y_t^{*2}$$ (interactive)

2. $$y_\nu^* = 0$$ (Gaussian)

Neutrino – top system

KK, S.Pramanick, E.M.Sessolo, JHEP 08 (2022) 262

SM + RHN:

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y$$

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get f_y : [-10^{-4}, 10^{-3}]

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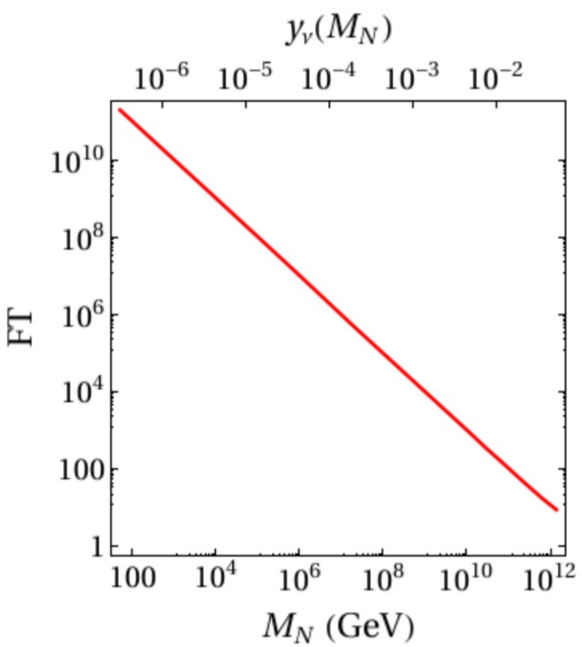
1. $y_\nu^{*2} = \frac{32\pi^2}{5} f_y + \frac{3}{10} g_Y^{*2} - \frac{6}{5} y_t^{*2}$ (interactive)

large fine tuning of f_y to get small Yukawa

large Yukawa coupling → Majorana neutrino

$$m_\nu = y_\nu^2 v_h^2/(\sqrt{2}M_N)$$

AS prediction for the Majorana mass (heavy)



Neutrino – top system

KK, S.Pramanick, E.M.Sessolo, JHEP 08 (2022) 262

SM + RHN:

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu$$

f_y: [-10^{-4}, 10^{-3}]

$\beta_\nu \equiv \dfrac{dy_\nu}{dt} = 0$ → two IRR solutions for neutrino FP:

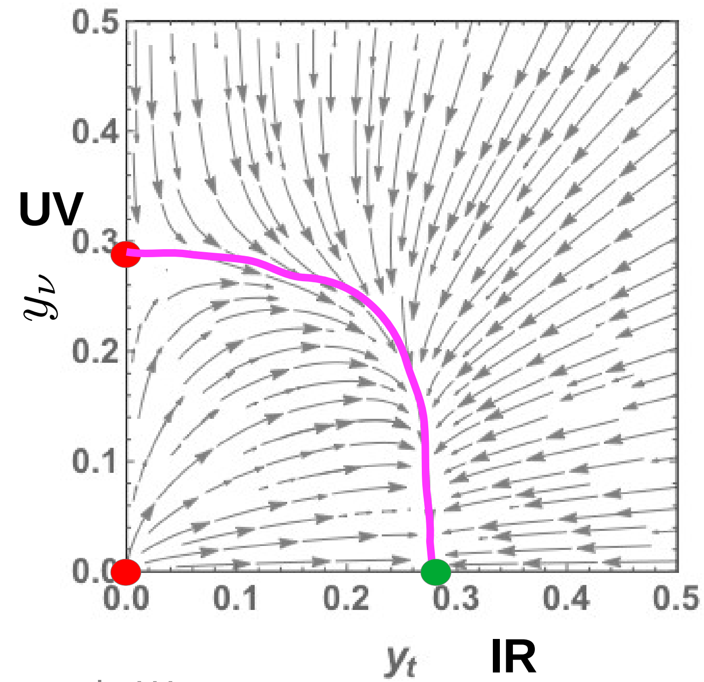
2. $y_\nu^* = 0$ (Gaussian)

Irrelevant if *fy* is small enough!

$$f_y < f_{\nu,tY}^{\text{crit}} \approx 0.0008$$

small Yukawa coupling → Dirac neutrino

Relevant FPs provide a UV completion



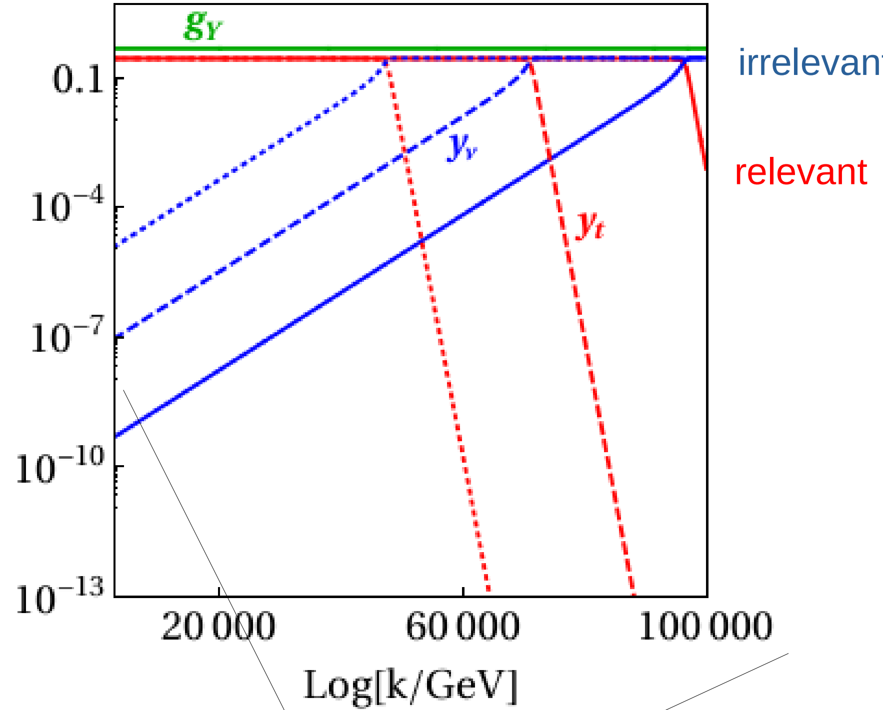
A dynamical mechanism!

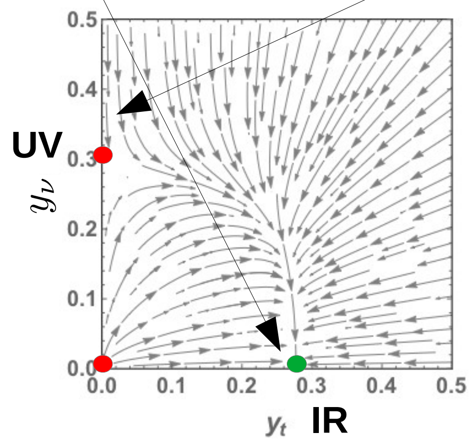
Integrated curve in blue :

$$y_\nu(t;\kappa) \approx \left(\frac{16\pi^2 f_y}{e^{f_y(\kappa - t)} + 5/2}\right)^{1/2}$$

$$\kappa \approx \frac{1}{16\,\pi^2}\ln\left(\frac{M_{\rm UV}}{M_{\rm Pl}}\right)$$

KK, S.Pramanick, E.M.Sessolo, JHEP 08 (2022) 262





No fine tuning

Smallness of the neutrino Yukawa due to the "distance" of the Planck scale from infinity

Emergent effective scale M_{UV} ("transplankian seesaw")

Neutrinos can be Dirac naturally

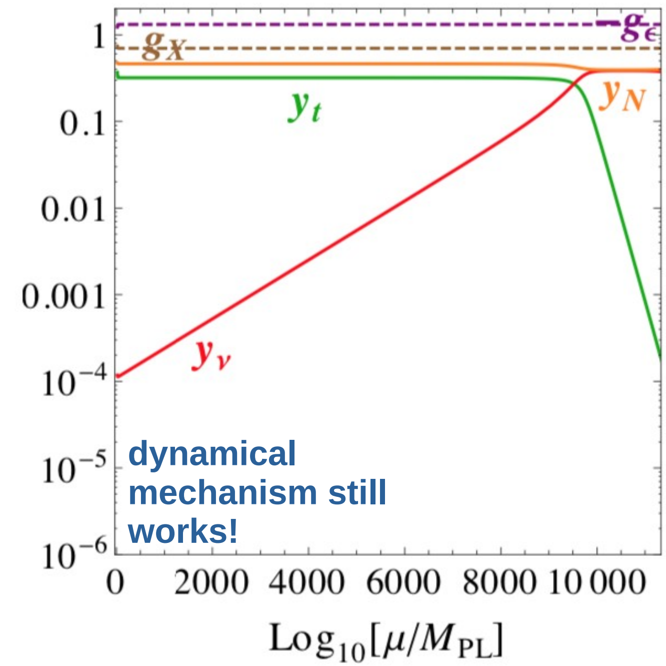
What about Majorana mass << 10^{14} GeV?

Gauged B-L model

SM + gauged U(1)$_{B\text{-}L}$ + QG:

A. Chikkaballi, KK, E.M.Sessolo, JHEP 11 (2023) 224

- Dirac neutrino Yukawa $y_\nu^* = 0$ (irrelevant → feasible dynamical mechanism)
- $g_Y^* = 0$ (relevant → f_g free param.)
- New gauge sector g_X, g_ε (irrelevant → predictions)



- New scalar vev breaking U(1)$_{B\text{-}L}$ (relevant → free param.)
- New Yukawa coupling y_N (irrelevant → predictions) $\mathcal{L}_M = -y_N^{ij} S \nu_{R,i} \nu_{R,j} + \mathrm{H.c.}$

(Majorana mass term)

$y_N^* \neq 0$

Majorana neutrino

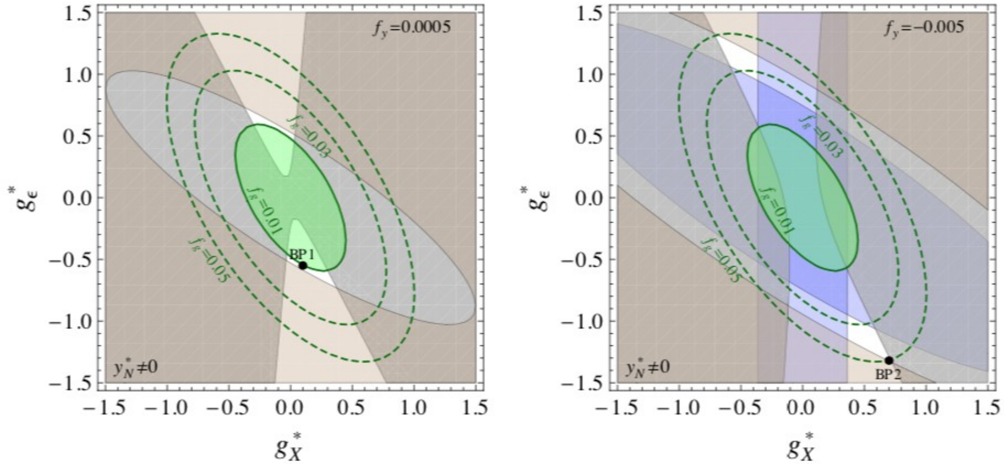
("light")

$y_N^* = 0$

Dirac neutrino

Predictions for B-L model

A. Chikkaballi, KK, E. M. Sessolo,
JHEP 11 (2023) 224, (2308.06114)

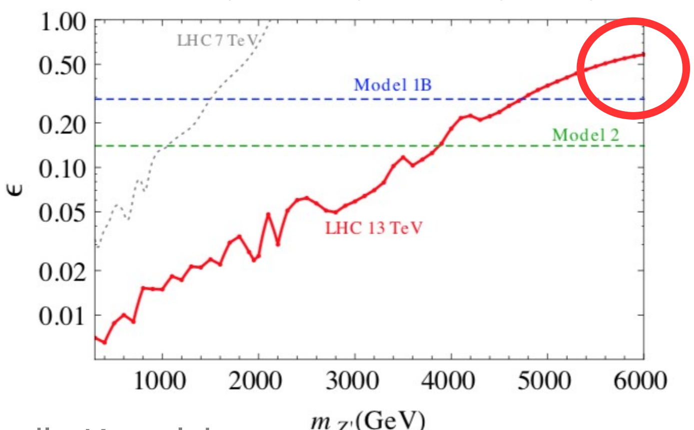


$\longrightarrow$ g_X , g_ϵ

(top mass, irrelevant y_ν)

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	g_X ($10^{5,7,9}$ GeV)	g_ϵ ($10^{5,7,9}$ GeV)	y_N ($10^{5,7,9}$ GeV)	
BP1	0.01	0.0005	0.10	−0.55	0.12	0.29, 0.29, 0.30	−0.26, −0.27, −0.28	0.16, 0.16, 0.16	**Majorana**
BP2	0.05	−0.005	0.70	−1.32	0.47	0.40, 0.41, 0.44	−0.52, −0.56, −0.61	0.42, 0.44, 0.45	**Majorana**
BP3	0.02	−0.0015	0.10	−0.75	0.0	0.12, 0.12, 0.12	−0.33, −0.35, −0.37	0.0	**Dirac**
BP4	0.03	−0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0	**Dirac**

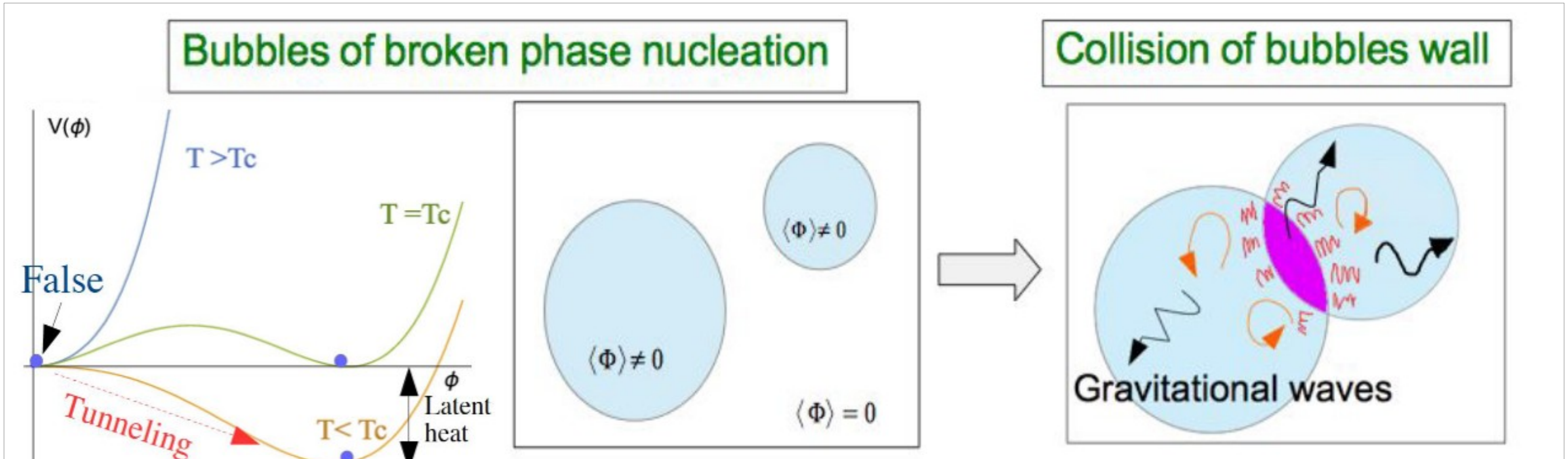
Chikkaballi, Kotlarski, Kowalska, Rizzo, Sessolo '22



large kinetic mixing implies $v_S >> v_H$

Gravitational waves from FOPT

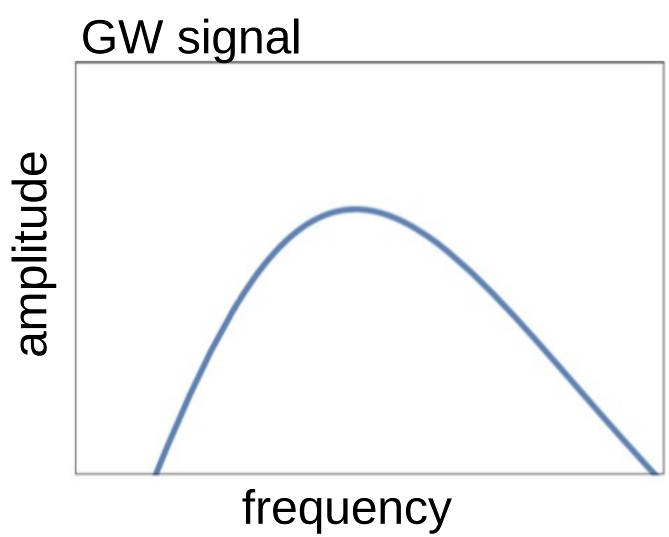
talks by M.Lewicki,
R. Durrer,
B.Świeżewska



K. Hashino,
talk at the YITP, Kyoto, 2021

Latent heat α

Nucleation speed β/H*



← < ϕ vev > →

Linde '81, '83, Turner *et al.* '92-'94; Guth *et al.* '80, '81, Caprini *et al.* '07-'09, Hindmarsh *et al.* '13-'17, Ellis *et al.* '18 … many more

Possible signals in GW

Possible gravitational-wave (GW) signatures from FOPT?

<u>predictions</u> have strong discriminating features... may show up in GW amplitude!

	g_X ($10^{5,7,9}$ GeV)	y_N ($10^{5,7,9}$ GeV)	
BP1	0.29, 0.29, 0.30	0.16, 0.16, 0.16	Majorana
BP2	0.40, 0.41, 0.44	0.42, 0.44, 0.45	Majorana
BP3	0.12, 0.12, 0.12	0.0	Dirac

⇒

NO GW SIGNAL
in the conformal limit
due to large y_N

… nucleation/percolation T is too low
… FOPT stop-condition not satisfied

Possible signals in GW

Possible gravitational-wave (GW) signatures from FOPT?

predictions have strong discriminating features... may show up in GW amplitude!

	$g_X\ (10^{5,7,9}\ \mathrm{GeV})$	$y_N\ (10^{5,7,9}\ \mathrm{GeV})$	
BP1	0.29, 0.29, 0.30	0.16, 0.16, 0.16	Majorana
BP2	0.40, 0.41, 0.44	0.42, 0.44, 0.45	Majorana
BP3	0.12, 0.12, 0.12	0.0	Dirac

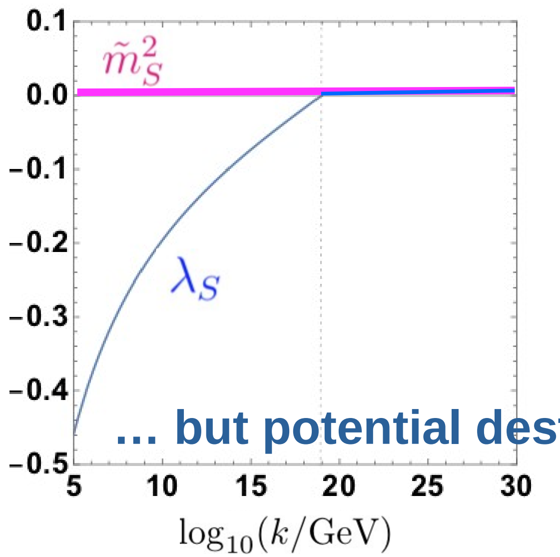
$\Rightarrow$

NO GW SIGNAL
in the conformal limit
due to large y_N

… nucleation/percolation T is too low
… FOPT stop-condition not satisfied

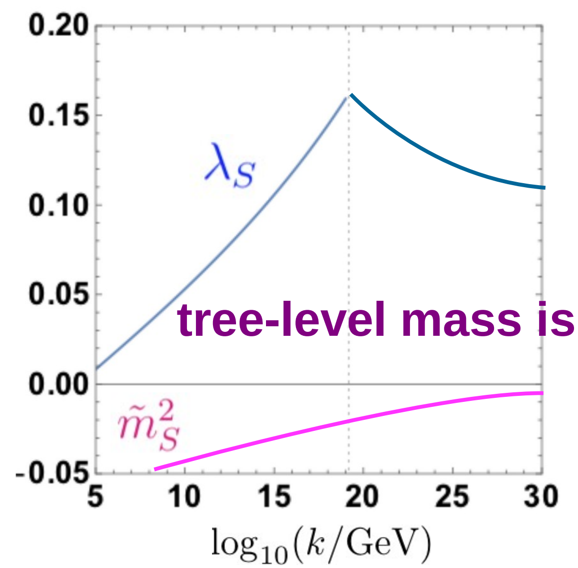
What about the scalar potential?

$\boxed{\tilde{m}_S^{2*} = 0}$ **irrelevant** (conformal limit)



… but potential destabilized !

$\boxed{\tilde{m}_S^{2*} = 0}$ **relevant**

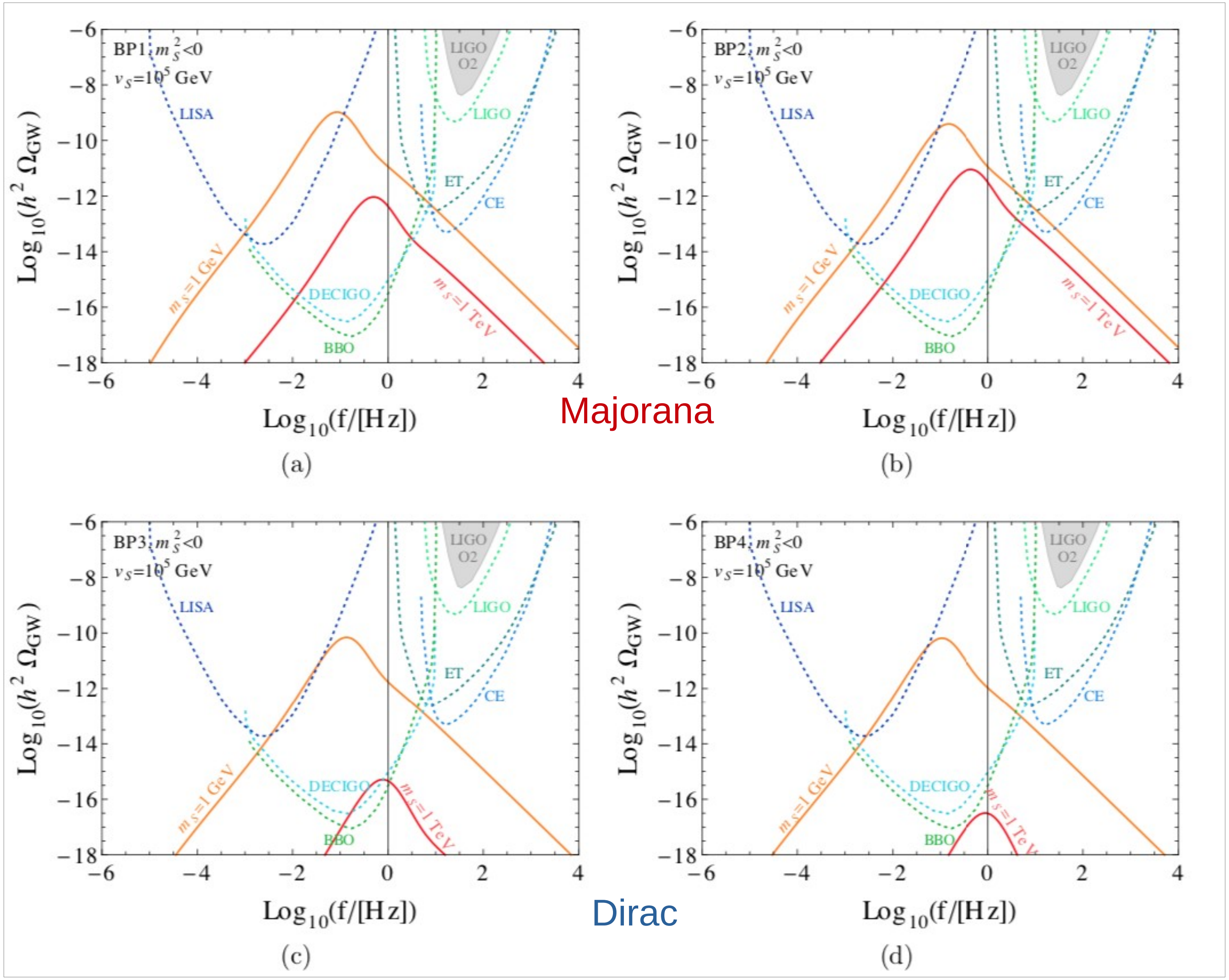


tree-level mass is allowed

Gravitational waves

Signal is now visible...

A. Chikkaballi, KK, E. Sessolo, 2308.06114

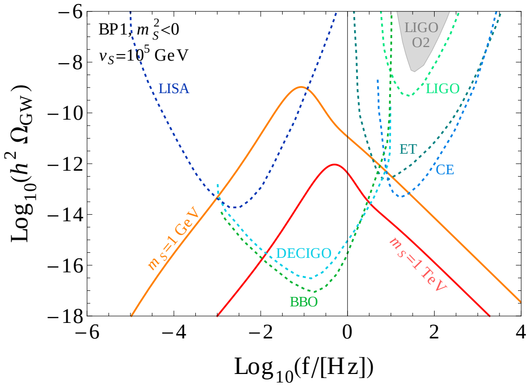


… but determined the relevant parameter

Majorana vs Dirac in GWs

A. Chikkaballi, KK, E. Sessolo, 2308.06114

Majorana neutrino



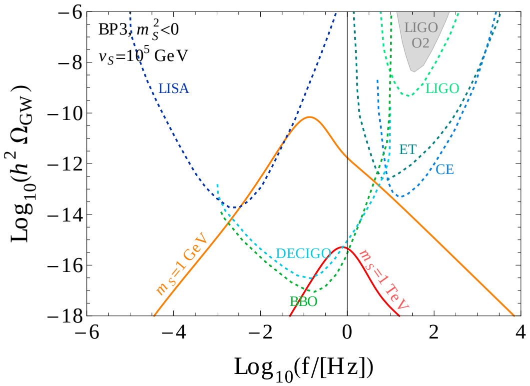
BP1

$$m_S = 1\ \mathrm{TeV} : \alpha = 0.27,\ \beta = 185$$
$$m_S = 1\ \mathrm{GeV} : \alpha = 10^{10},\ \beta = 50,\ T_n = 15\ \mathrm{GeV}$$

BP2

$$m_S = 1\ \mathrm{TeV} : \alpha = 0.88,\ \beta = 187$$
$$m_S = 1\ \mathrm{GeV} : \alpha = 10^{11},\ \beta = 79,\ T_n = 8; \mathrm{GeV}$$

Dirac neutrino



BP3

$$m_S = 1\ \mathrm{TeV} : \alpha = 0.02,\ \beta = 227$$
$$m_S = 1\ \mathrm{GeV} : \alpha = 10^{9},\ \beta = 189,\ T_n = 10; \mathrm{GeV}$$

BP4

$$m_S = 1\ \mathrm{TeV} : \alpha = 0.01,\ \beta = 229$$
$$m_S = 1\ \mathrm{GeV} : \alpha = 10^{8},\ \beta = 201,\ T_n = 11; \mathrm{GeV}$$

Reconctruction of thermodynamical parameters from measured spectra ⇒ **Discrimination Majorana vs Dirac neutrino**

C.Gowling, M.Hindmarsh, D. Hooper, J.Torrado,JCAP 04 (2023) 061
M. Kierkla, M. Lewick, P. Schicho, D. Schmitt, B. Swiezewska, arXiv:2607.18233

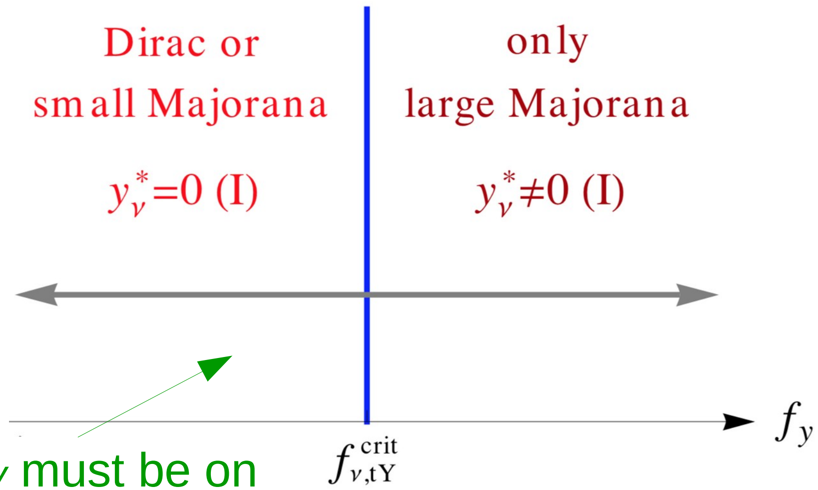
Conclusions

- AS (based on quantum gravity) offers a **predictive UV completion.**
- Naturalness of small couplings enforced via **IR attractive Gaussian FPs** of RG flow.
- AS can be used to make the neutrino (or other) Yukawa coupling **arbitrarily small dynamically.**
- Gravitational waves could provide a way of **discriminating between Dirac and Majorana** neutrinos (in the AS-based setup).

Backup slides

Is neutrino special?

KK, S.Pramanick, E.Sessolo
JHEP 08 (2022) 262



fy must be on this side of *f*crit ...

… and there's an *f*crit for each fermion ...

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2}\left[\alpha_X\, y_X^2 + \alpha_Z y_Z^2 - \alpha_Y\, g_Y^2\right] - f_y\, y_X$$

$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2}\left[\alpha'_X\, y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y\, g_Y^2\right] - f_y\, y_Z$$

$$f_{Z,XY}^{\mathrm{crit}} = \frac{g_Y^{*2}}{16\pi^2}\,\frac{\alpha'_X\alpha_Y - \alpha'_Y\alpha_X}{\alpha_X - \alpha'_X}$$

… but top mass is good only if... $-1\times10^{-4} \lesssim f_y \lesssim 1\times10^{-3}$

Z	$\alpha'_{X=t}$	α'_Y	$f^{\mathrm{crit}}_{Z,tY}$	
u, c	3	$\frac{17}{12}$	-20.0×10^{-4}	TOP BAD
b	$\frac{3}{2}$	$\frac{5}{12}$	1.17×10^{-4}	TOP OKAY
d, s	3	$\frac{5}{12}$	22.3×10^{-4}	TOP GOOD
ν_i	3	$\frac{3}{4}$	8.22×10^{-4}	TOP GOOD
e, μ, τ	3	$\frac{15}{4}$	-119×10^{-4}	TOP BAD

Is neutrino special?

KK, S.Pramanick, E.Sessolo
JHEP 08 (2022) 262

… running CKM makes f_{crit} smaller

No CKM:

$$16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - 3y_t^{*2} + \frac{5}{12}g_Y^{*2}$$

Running CKM:

$$\Rightarrow \quad 16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - \frac{3}{2}\left(1+|V_{tb}|^2\right)y_t^{*2} + \frac{5}{12}g_Y^{*2}$$

FP = 0

R. Alkofer *et al.* (2003.08401)

… but top mass is good only if... $-1\times10^{-4} \lesssim f_y \lesssim 1\times10^{-3}$

Z	$\alpha'_{X=t}$	α'_Y	$f^{\text{crit}}_{Z,tY}$	
u,c	3	$\frac{17}{12}$	-20.0×10^{-4}	TOP BAD
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ν_i	3	$\frac{3}{4}$	8.22×10^{-4}	TOP GOOD
e,μ,τ	3	$\frac{15}{4}$	-119×10^{-4}	TOP BAD

… perhaps the neutrino is special after all

Does it work in the full SM?

KK, S.Pramanick, E.Sessolo, JHEP 08 (2022) 262

PMNS parametrization

$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}$$

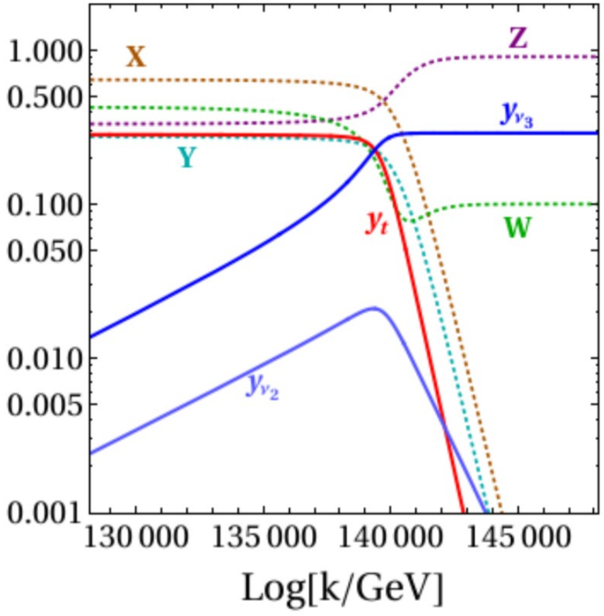
$$\theta_{12} = \arctan\sqrt{\frac{Y}{X}}$$
$$\theta_{13} = \arccos\sqrt{X+Y}$$
$$\theta_{23} = \arcsin\sqrt{\frac{1-W-Z}{X+Y}}$$

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}$$

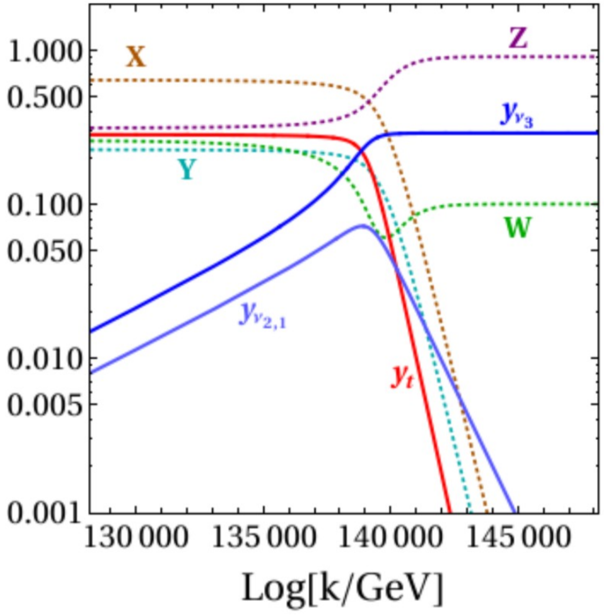
PMNS fit $X \in [0.64 - 0.71]$ $Y \in [0.26 - 0.34]$ $Z \in [0.05 - 0.26]$ $W \in [0.21 - 0.48]$

Normal ordering works! (no solution found with IO)

Hierarchical NO



Degenerate NO



The mechanism is more generic...

In pairs of Yukawa interactions one can use the "large" Y_L to drive down the "small" Y_S...

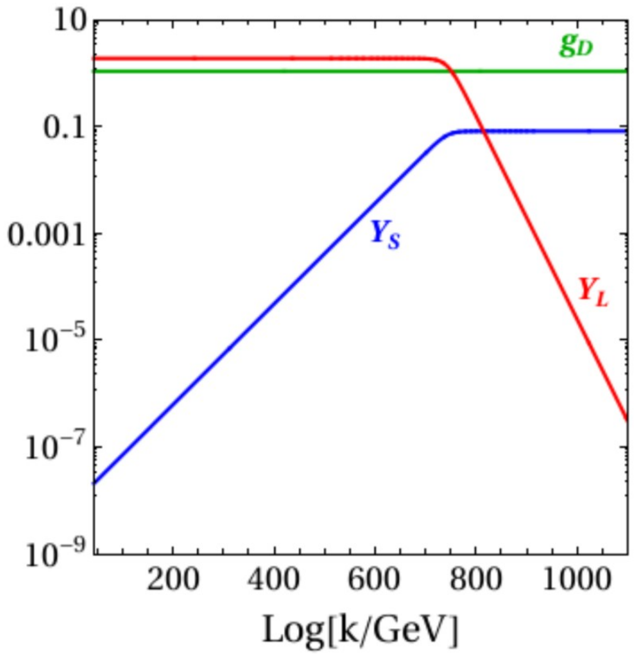
$$\mathcal{L} \supset Y_S\, \chi_R \Phi \chi_L + Y_L\, \psi_R \Phi \psi_L + \mathrm{H.c.}$$

Recall that...

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2}\left[\alpha_X\, y_X^2 + \alpha_Z y_Z^2 - \alpha_Y\, g_Y^2\right] - f_y\, y_X$$
$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2}\left[\alpha'_X\, y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y\, g_Y^2\right] - f_y\, y_Z$$

… thus we want ...

$$f_{Z,XY}^{\mathrm{crit}} = \frac{g_Y^{*2}}{16\pi^2}\,\frac{\alpha'_X\alpha_Y - \alpha'_Y\alpha_X}{\alpha_X - \alpha'_X} > f_y\,(\mathrm{from\ UV})$$



… it happens often (but not always) if $Q_\psi \gg Q_\chi$ (gauge charge)

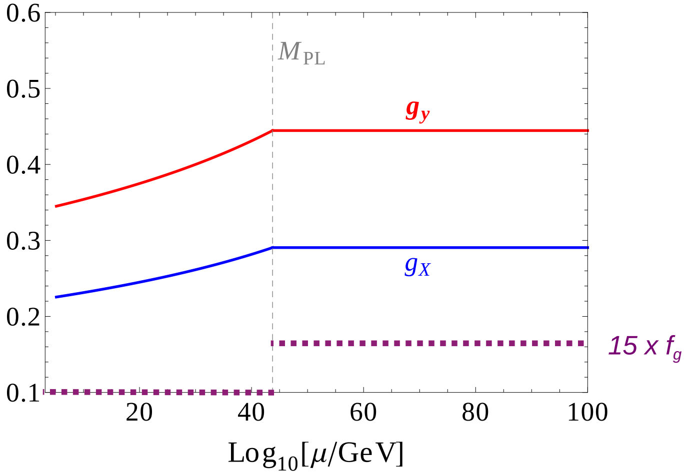
Can use it to justify freeze-in, feebly interacting models, etc...

How robust are low-scale predictions?

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
Eur. Phys. J. C, 83 (2023) 644

assumptions:

- 1-loop matter RGEs
- Planck scale set at 10^{19} GeV
- Gravity parameters *f* are constant
- Gravity decouples instantaneously

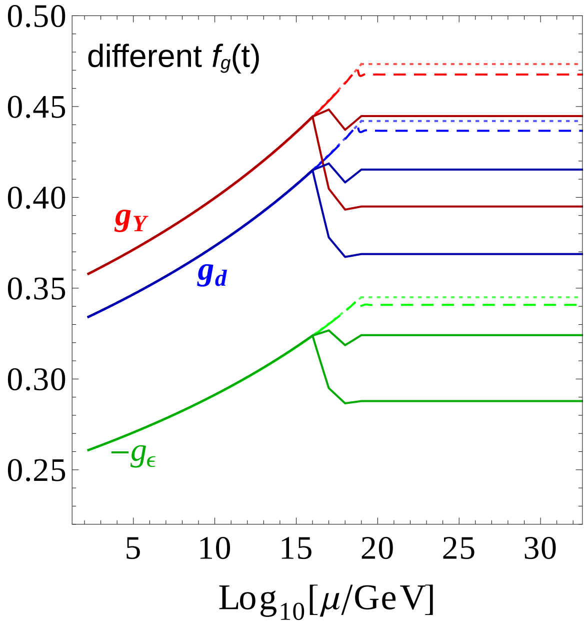


The gauge coupling ratios do not depend on f_g
(due to the universality of QG)

Invariant under the RGE flow

PREDICTIONS VERY STABLE

$$\delta g \lesssim 0.1\%$$



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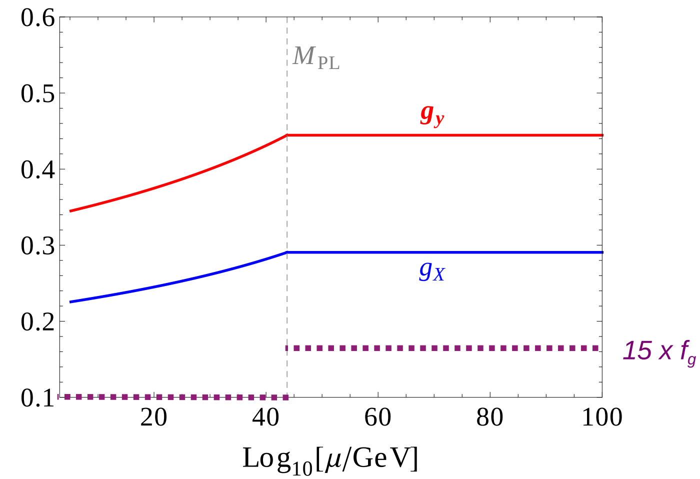

The Yukawa ratios depend on the other FPs

fixed f_g and f_y

shift due to the running f_g, f_y

$$\left(\frac{y^*_{LQ}}{y^*_t}\right)^2 (1\text{ loop}) \approx A + B g_Y^{*2}/y_t^{*2} + C\delta(y^*_t, g^*_Y)$$

$y^*_2 \ll y^*_1$ **PREDICTIONS UNSTABLE**

$y^*_2 \approx y^*_1$ $\boxed{\delta y \lesssim 20\%}$

+ fousing, realistic UV running