Gravitational Waves from Strongly Supercooled Cosmological Phase Transitions

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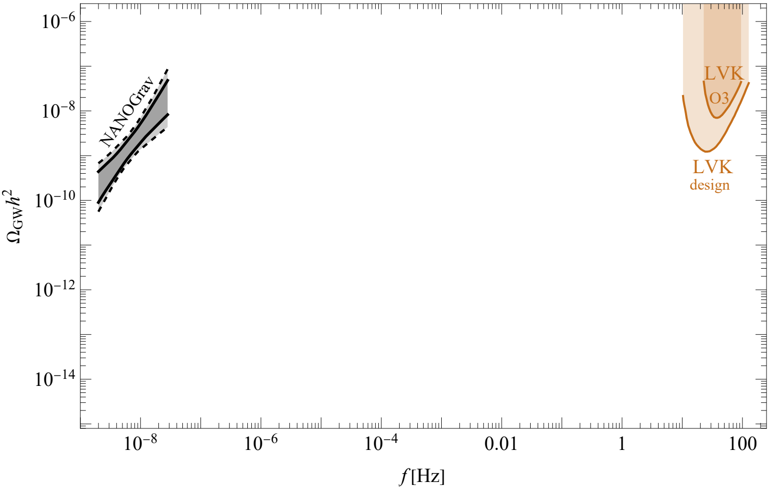

10^{-6}
10^{-8}
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$\Omega_{\rm GW}h^2$
NANOGrav
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O3
LVK
design
10^{-8}
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0.01
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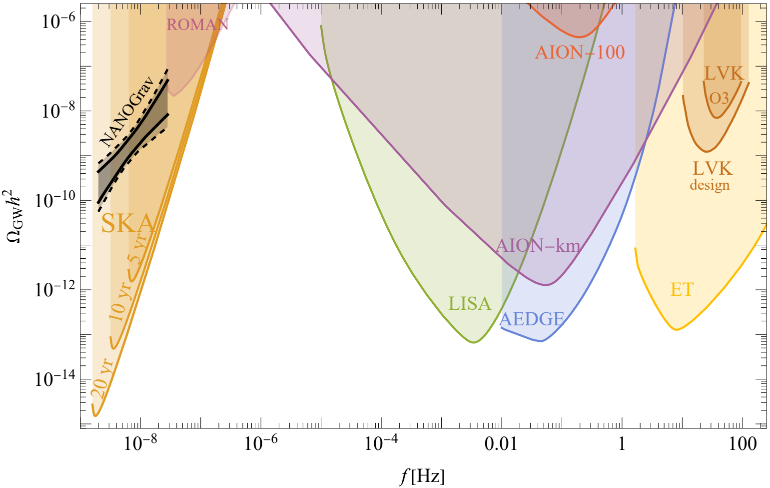
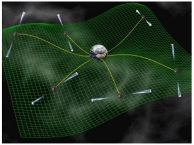
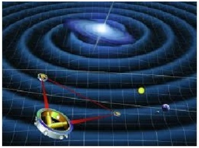
f [Hz]





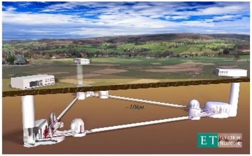
Pulsar Timing

[David Champion/NASA/JPL]



LISA

wiki/Laser_Interferometer_Space_Antenna



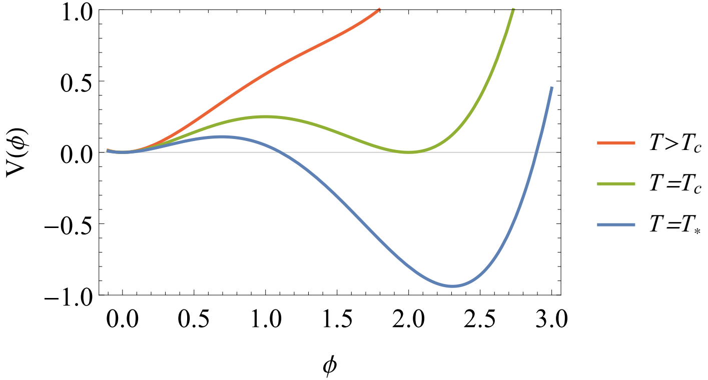
Einstein Telescope

www.et-gw.eu

Symmetry breaking

- Simple high temperature expansion

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi}T\phi^3 + \lambda\phi^4\,, \quad T_0^2 > 0$$



First Order Phase Transition: bubble nucleation

- Temperature corrections to the potential

$$V(\phi, T) = \frac{g_{m^2}}{24}\left(T^2 - T_0^2\right)\phi^2 - \frac{g_m}{12\pi} T\phi^3 + \lambda\phi^4$$

- EOM $\rightarrow$ bubble profile

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial\phi} = 0,$$

$$\phi(r \to \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

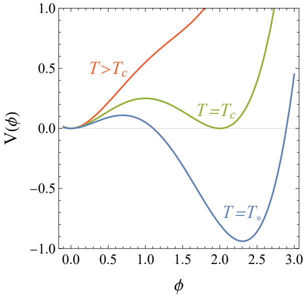
- $\mathcal{O}(3)$ symmetric action

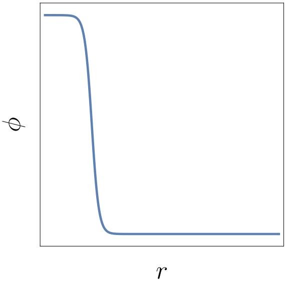
$$S_3(T) = 4\pi \int drr^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- nucleation temperature

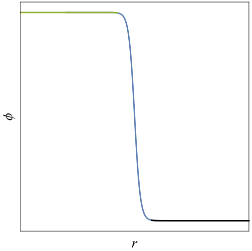
$$\frac{\Gamma}{H^4} \approx \left(\frac{T}{H}\right)^4 \exp\left(-\frac{S_3(T)}{T}\right) \approx 1$$

Linde '81 '83

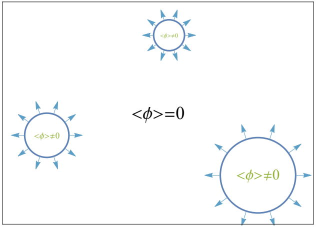




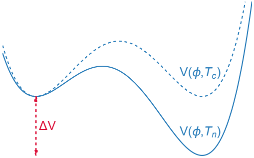
Phase transition parameters



$\Longrightarrow$



- Strength of the transition



$$\alpha \approx \left. \frac{\Delta V - \frac{T}{4}\frac{\partial \Delta V}{\partial T}}{\rho_R} \right|_{T=T_*}, \qquad \Delta V = V_f - V_t$$

- Characteristic scale

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta\,(t-t_0)} \Longrightarrow \frac{\beta}{H} = T\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\Bigg|_{T=T_*} = \frac{(8\pi)^{\frac{1}{3}}}{H R_*}$$

- Bubble wall velocity: v_w

Gravitational waves from a very strong PT

- Strength of the transition

$$\alpha \approx \frac{\Delta V}{\rho_R} \gg 1$$

- Reheating temperature

$$T_{\rm reh} \approx \left(\frac{30}{\pi^2}\frac{1}{g_*}\Delta V\right)^{\frac{1}{4}}$$

- Bubble wall velocity

$$v_w \approx 1$$

- Characteristic scale

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta\,(t-t_0)} \Longrightarrow \frac{\beta}{H} = T\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T=T_p}$$

- Amplitude of the spectrum

$$\Omega_{\rm p} \propto \left(\frac{\alpha}{\alpha+1}\right)^2\left(\frac{\beta}{H}\right)^{-2}$$

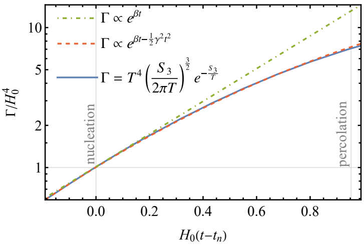
- Peak frequency

$$f_p \propto T_{\rm reh}\frac{\beta}{H}$$

- The probability that the given point is in the false vacuum at time t:

$$P(t) = e^{-I(t)}\,,$$

$$I(t) = \frac{4\pi}{3}\int_{-\infty}^{t} \mathrm{d}t'\,\Gamma(t')a(t')^3 r(t,t')^3$$



- radius of a bubble growing from time t' to t

$$r(t,t') = \int_{t'}^{t} \mathrm{d}t''\,\frac{v_w}{a(t'')}\,,$$

- percollation is typically defined as

$$P(t_p) = e^{-1}$$

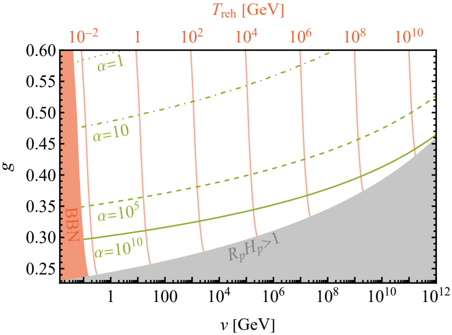
- Simple test for the volume of false vacuum $\mathcal{V}_{\rm false} \propto a(t)^3 P(t)$

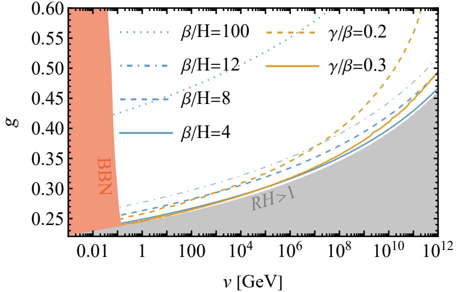
$$\frac{1}{\mathcal{V}_{\rm false}}\frac{d\mathcal{V}_{\rm false}}{dt} = 3H(t) - \frac{dI(t)}{dt} = H(T)\left(3 + T\,\frac{dI(T)}{dT}\right) < 0\,.$$

Model Realisation

- generic classically scale invariant potential

$$V(\phi, T) = \frac{1}{2} g^2 T^2 \phi^2 + \frac{3g^4}{4\pi^2} \phi^4 \left(\ln \frac{\phi^2}{v^2} - \frac{1}{2} \right)$$



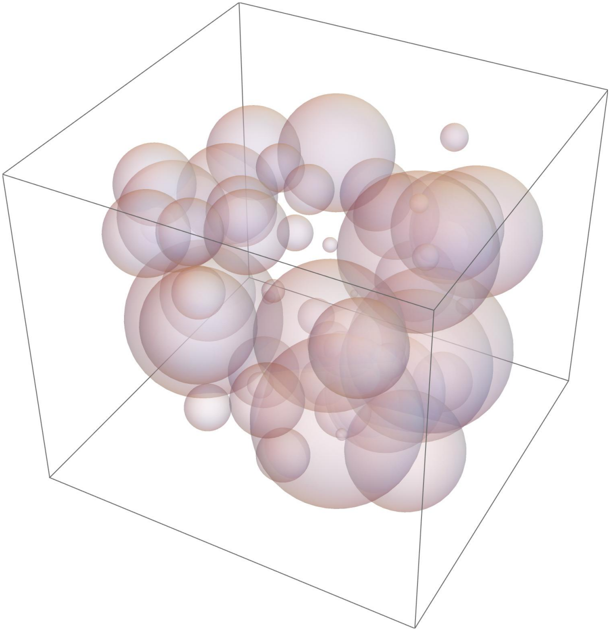


- no percolation coincides with horizon size bubbles

$$\left. \frac{dV_{\rm false}}{dt} \right|_{P=\frac{1}{e}} < 0 \leftrightarrow HR < 1$$

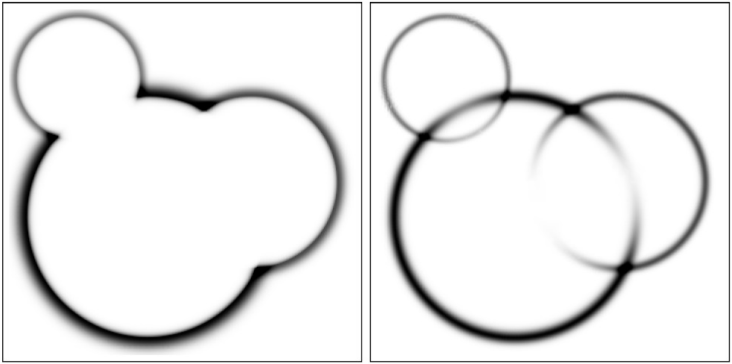
ML, P. Toczek and V. Vaskonen, arXiv:2412.10366

Strong transitions: computation of the GW spectrum

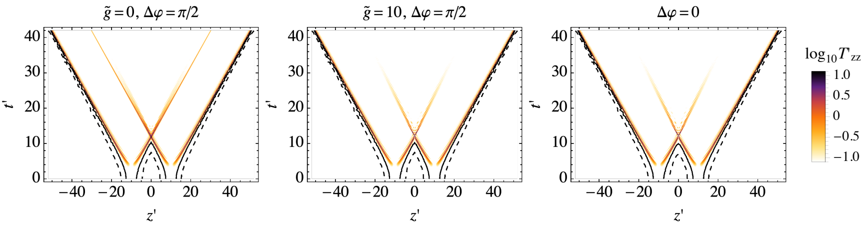


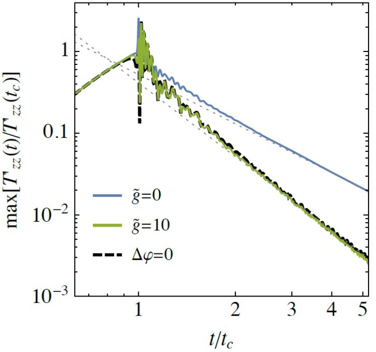
Strong transitions: computation of the GW spectrum

- Envelope: Collided walls disappear
  Kosowsky, Turner and Watkins,PRD 45 (1992) 4514, PRL 69 (1992) 2026,
  Huber and Konstandin 0806.1828
- Bulk Flow model: $E \propto R^{-2}$
  Jinno and Takimoto 1707.03111, Konstandin 1712.06869



Abelian Higgs Model: Energy Scaling





- scaled gauge coupling:

$$\tilde{g} = \frac{gv^2}{\sqrt{\Delta V}}$$

- Global Symmetry breaking:

$$T_{zz} \propto R^{-2}$$

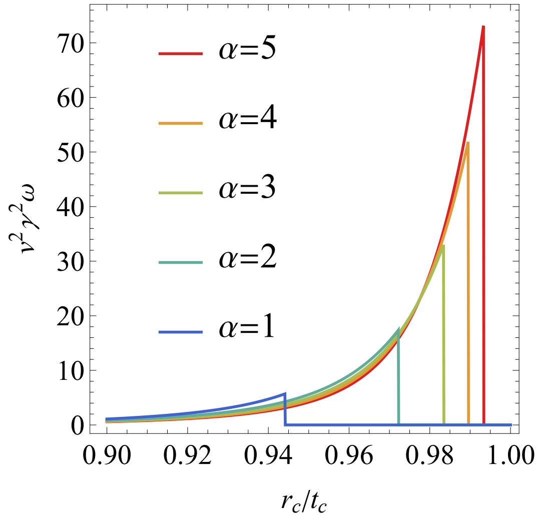
- Gauge Symmetry breaking:

$$T_{zz} \propto R^{-3}$$

ML, Ville Vaskonen arXiv: 2007.04967

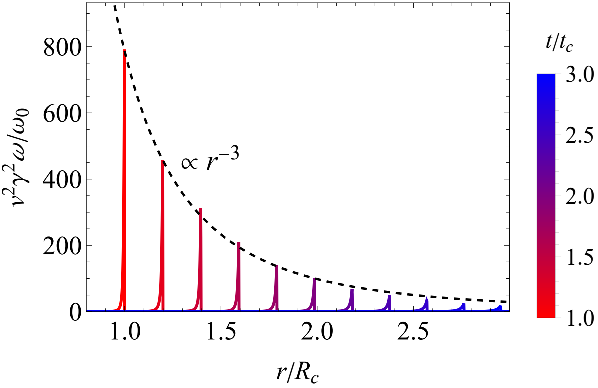
Fluid Shells in strong transitions

- Plasma profiles for $v_w \gtrsim v_J$



Fluid Shell Evolution

- Plasma profile evolution with $\alpha = 20$ and $\gamma_w = 50$



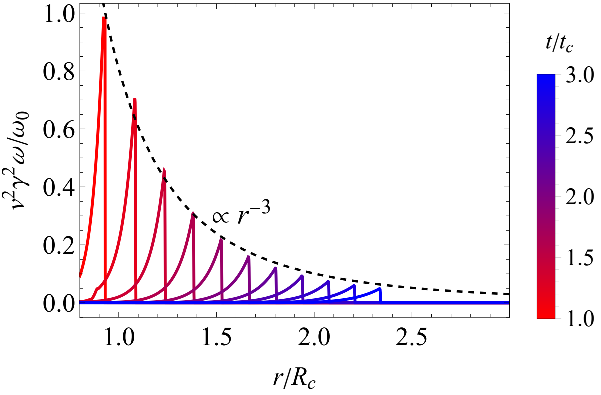
- Fluid shells with $\alpha \gg 1$:

$$T_{zz} \propto R^{-3}$$

ML, Ville Vaskonen arXiv:2208.11697

Fluid Shell Evolution

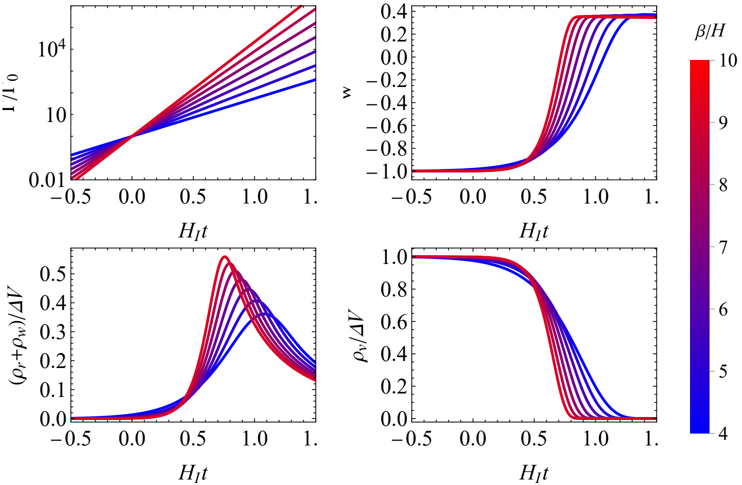
- Plasma profile evolution with $\alpha = 0.5$ and $\gamma_w = 3$



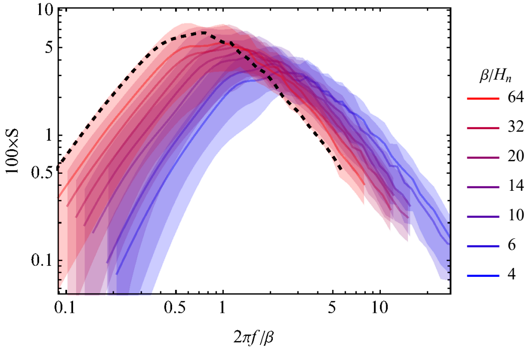
- Friedmann equations

$$H^2 = \frac{1}{3M_P^2}\left(\rho_v + \rho_r + \rho_w\right)\ ,$$

$$\frac{\mathrm{d}(\rho_r + \rho_w)}{\mathrm{d}t} + 4H(\rho_r + \rho_w) = -\frac{\mathrm{d}\rho_v}{\mathrm{d}t}\ .$$



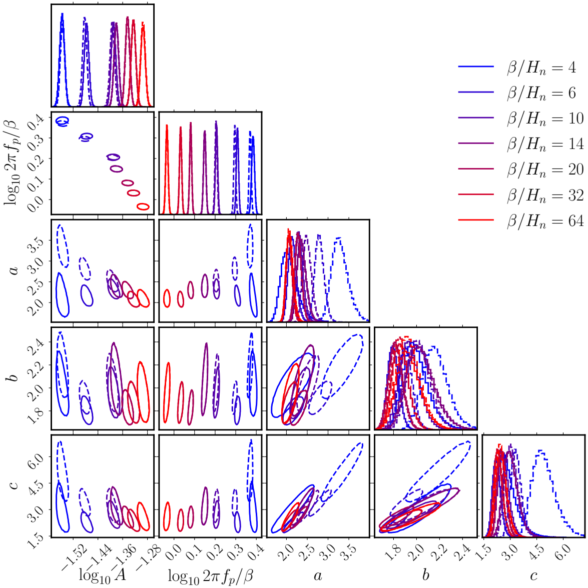
Results including expansion: Spectral shape



- spectral shape

$$\bar{S}(f) = \begin{cases} \dfrac{A(a+b)^c}{\left[b\left(\frac{f}{f_p}\right)^{-\frac{a}{c}} + a\left(\frac{f}{f_p}\right)^{\frac{b}{c}}\right]^c} & \text{for} \quad f > f_H \\ \left(\dfrac{f}{f_H}\right)^3 \dfrac{A(a+b)^c}{\left[b\left(\frac{f_H}{f_p}\right)^{-\frac{a}{c}} + a\left(\frac{f_H}{f_p}\right)^{\frac{b}{c}}\right]^c}, & \text{for} \quad f < f_H \end{cases}$$

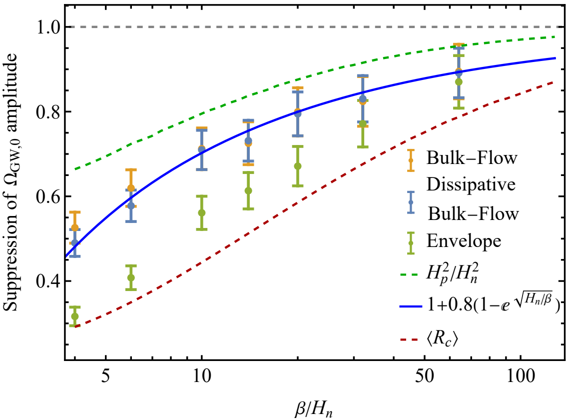
ML and V. Vaskonen, arXiv:2511.15687



- For the dissipative bulk flow model: a=b=c=2.

ML and V. Vaskonen, arXiv:2511.15687

Results including expansion

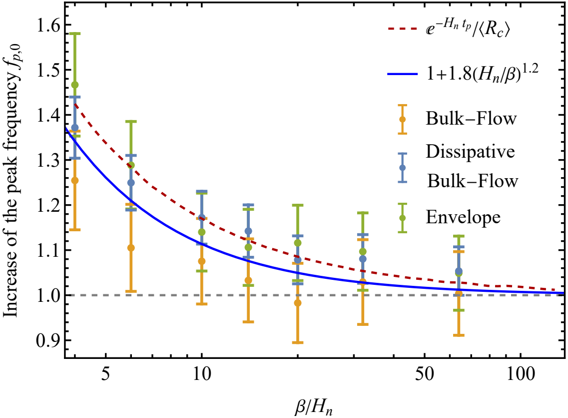


- Impact on the amplitude

$$A \approx 0.06 \left[1 + 0.8\left(1 - e^{\sqrt{H_n/\beta}}\right)\right]$$

ML and V. Vaskonen, arXiv:2511.15687

Results including expansion

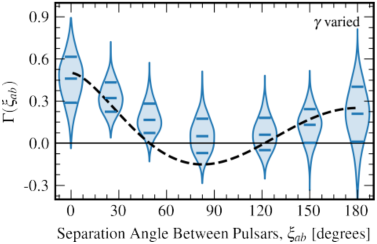


- Impact on the peak frequency

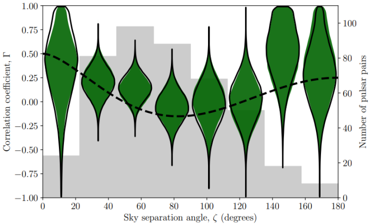
$$\frac{2\pi f_p}{a_p H_p} \approx 0.7 \frac{\beta}{H_n} \left[1 + 1.8 \left(\frac{\beta}{H_n} \right)^{-1.2} \right] .$$

ML and V. Vaskonen, arXiv:2511.15687

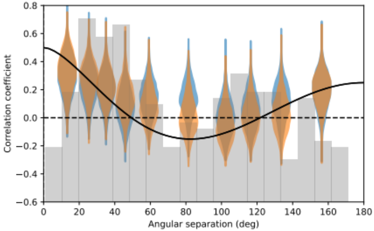
PTA evidence for GWs: Hellings - Downs curve



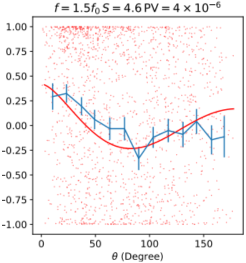
NANOGrav: 2306.16213



PPTA: 2306.16215



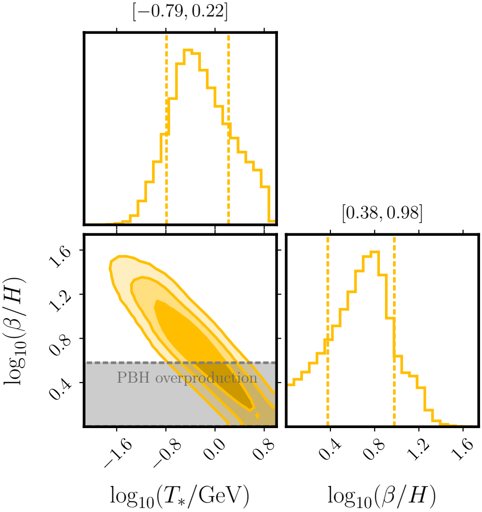
EPTA: 2306.16214



CPTA: 2306.16216

Fit to NANOGrav

- Constraint from PBH overproduction $\beta/H \gtrsim 3.9$
  ML, Piotr Toczek, Ville Vaskonen arXiv: 2305.04924



John Ellis et al. arXiv: 2308.08546

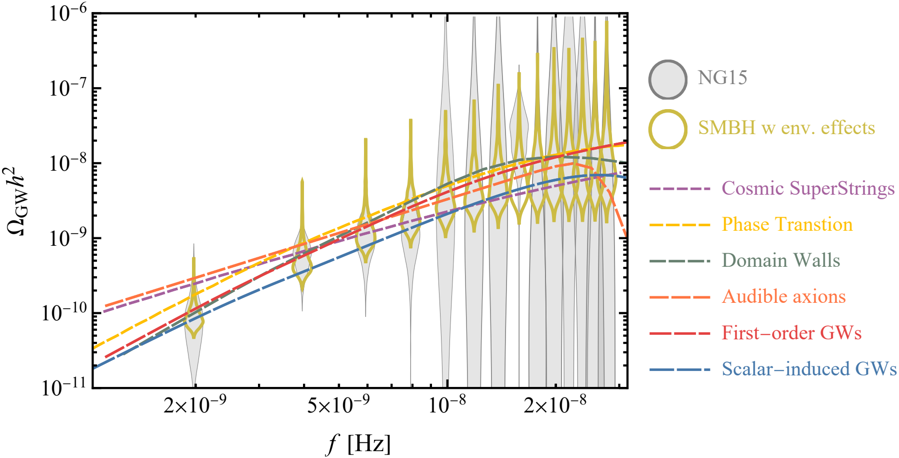
Results from Multi-Model Analysis (MMA)

Scenario	Best-fit parameters	ΔBIC
GW-driven SMBH binaries	$p_{\mathrm{BH}} = 0.25$	6.0
GW + environment-driven SMBH binaries	$p_{\mathrm{BH}} = 1$, $\alpha = 3.8$ $f_{\mathrm{ref}} = 12$ nHz	(BIC = 53.9)
Cosmic (super)strings	$G\mu = 2 \times 10^{-12}$, $p = 6.3 \times 10^{-3}$	-1.2
Phase transition	$T_* = 0.4$ GeV, $\beta/H = 5.5$	-4.9
Domain walls	$T_{\mathrm{ann}} = 0.79$ GeV, $\alpha_* = 0.026$	-5.7
Scalar-induced GWs	$k_* = 10^{7.6}$/Mpc $A = 0.08$, $\Delta = 0.28$	-2.1
First-order GWs	$\log_{10} r = -16$, $n_{\mathrm{t}} = 2.9$ $T_{\mathrm{rh}} = 0.35\,\mathrm{GeV}$	-2.0
"Audible" axions	$m_a = 3.1 \times 10^{-11}$ eV, $f_a = 0.87\, M_{\mathrm{P}}$	-4.2

For each model, we tabulate their best-fit values, and the Bayesian information criterion $BIC \equiv -2\Delta\ell + k \ln 14$ relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the ΔBIC for the best-fit combined SMBH+cosmological scenario.

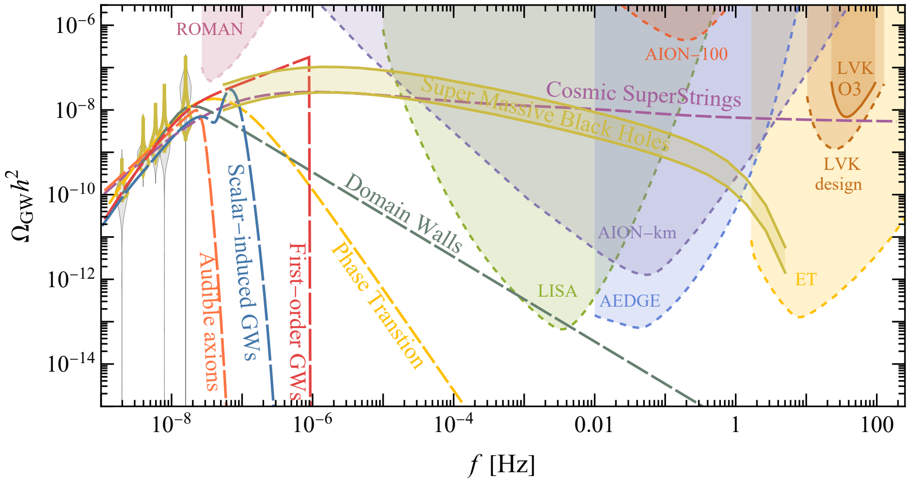
John Ellis et al. arXiv: 2308.08546

Best Fits to NANOGrav including SMBH



John Ellis et al. Phys.Rev.D 109 (2024) 2, 023522 arXiv: 2308.08546

Best Fits to NANOGrav including SMBH



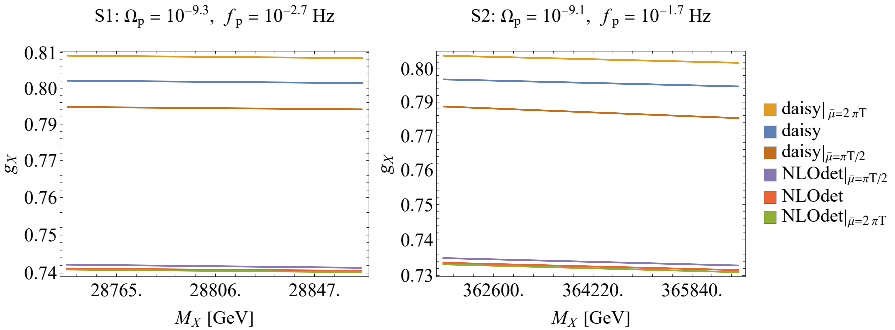
John Ellis et al. Phys.Rev.D 109 (2024) 2, 023522 arXiv: 2308.08546

Conclusions

- Very strong transitions ($\alpha \gg 1$) produce the same GW spectrum regardless of whether the main source is collisions of bubbles or motion of the plasma.
- GW spectra from supercooled transitions including expansion
  - spectral shape is intact, except for a causality-driven tail on super-horizon scales.
  - The peak amplitude and frequency exhibit a weaker dependence on the transition rate β than the usual scaling of $\Omega_p \propto \beta^{-2}$ and $f_p \propto \beta$.
- A very strong first order phase transition is currently one of the leading Early Universe explanations of the PTA signal

Thank you for your attention!

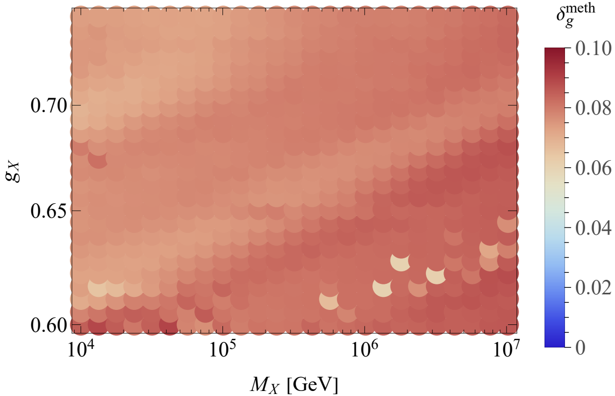
Theoretical uncertainty for strong transitions



- Theoretical uncertainty of about 10% on the coupling

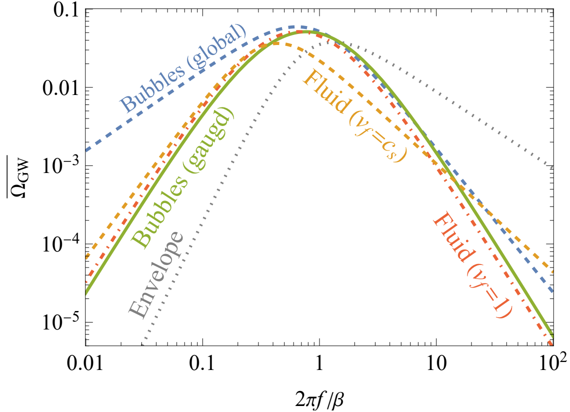
M. Kierkla, ML, P. Schicho, D. Schmitt, B. Świeżewska arXiv:2607.18233

Theoretical uncertainty for strong transitions



- Relative methodological difference in the reconstructed gauge coupling g_X between the daisy and NLOdet methods

M. Kierkla, ML, P. Schicho, D. Schmitt, B. Świeżewska arXiv:2607.18233



- Resulting spectrum:

$$\overline{\Omega_{GW}} = \frac{A\,(a+b)^c}{\left[b\left(\frac{f}{f_p}\right)^{-\frac{a}{c}} + a\left(\frac{f}{f_p}\right)^{\frac{b}{c}}\right]^c}$$

	Bubbles		Fluid
	Global($T \propto R^{-2}$)	Gauged ($T \propto R^{-3}$)	$v_{\rm shell} = 1$
$100\,A$	5.93 ± 0.05	5.13 ± 0.05	5.14 ± 0.04
a	1.03 ± 0.04	2.41 ± 0.10	2.36 ± 0.09
b	1.84 ± 0.17	2.42 ± 0.11	2.36 ± 0.09
c	1.91 ± 0.29	1.45 ± 0.34	3.69 ± 0.48
$2\pi f_p/\beta$	1.33 ± 0.19	0.64 ± 0.09	0.66 ± 0.04

ML, Ville Vaskonen arXiv: 2208.11697
ML, Ville Vaskonen, arXiv: 2007.04967

- redshift of the frequency and amplitude

$$h^2\,\Omega_{\mathrm{GW},0} = \left(\frac{a_*}{a_0}\right)^4 h^2 \Omega_{\mathrm{GW},*} \left(\frac{H_*}{H_0}\right)^2 \simeq 1.64 \times 10^{-5} \left(\frac{100}{g_*}\right)^{1/3} \Omega_{\mathrm{GW},*}$$

$$f = f_* \frac{a_*}{a_0} = \frac{f_*}{H_*}\frac{a_*}{a_0} H_* \simeq \frac{f_*}{H_*} 1.65 \times 10^{-5}\mathrm{Hz} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \left(\frac{T_*}{100\mathrm{GeV}}\right)$$

- Spectrum today

$$h^2\,\Omega_{\mathrm{GW},0} = 1.64 \times 10^{-5} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{\beta}{H}\right)^2 \frac{A(a+b)^c}{\left(b\left[\frac{f}{f_p}\right]^{-\frac{a}{c}} + a\left[\frac{f}{f_p}\right]^{\frac{b}{c}}\right)^c}$$

$$f_{p,0} = \frac{f_p}{\beta}\frac{\beta}{H} 1.65 \times 10^{-5}\mathrm{Hz} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \left(\frac{T_*}{100\mathrm{GeV}}\right)$$