

(gravitational) Ultra-relativistic freeze-out during reheating

Mathieu Gross

Particle physics in the early universe

15/09/2026

Based on : Phys. Rev. D **112**, 103538 and arXiv:2606.25033 [hep-ph]

In collaboration with: Stephen E. Henrich, Fotis Koutroulis Yann Mambrini, Keith A. Olive



Outline

1. Ultra-relativistic freeze-out in radiation domination
2. Ultra-relativistic freeze-out during reheating (UFO)
3. Gravitational ultra-relativistic freeze-out during reheating (GUFO)

Relativistic freeze-out in radiation domination

We use the Boltzmann equation

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$$\langle\sigma v\rangle \sim \frac{T^n}{\Lambda^{n+2}}$$

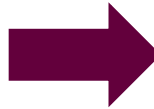
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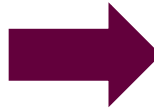
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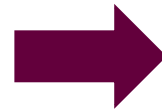
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$$\frac{\Omega_\chi h^2}{0.12} \simeq g \left(\frac{106.75}{g_{FO}} \right) \left(\frac{m_\chi}{170 \text{ eV}} \right)$$



$$17 \text{ eV} \lesssim m_\chi \lesssim 170 \text{ eV} \quad (\text{real scalar})$$

$$11 \text{ eV} \lesssim m_\chi \lesssim 110 \text{ eV} \quad (\text{Majorana fermion})$$

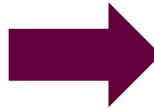
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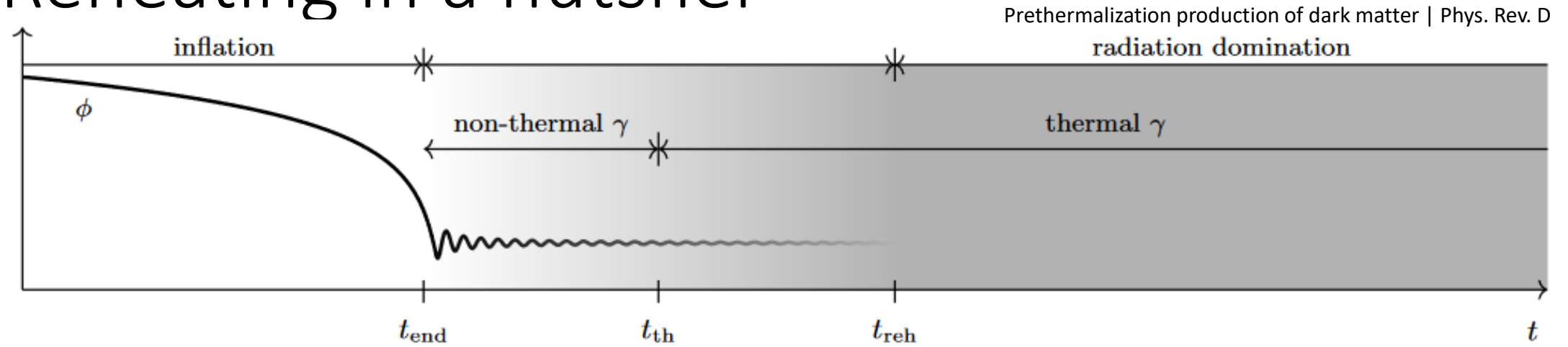
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No dependence on the coupling due to

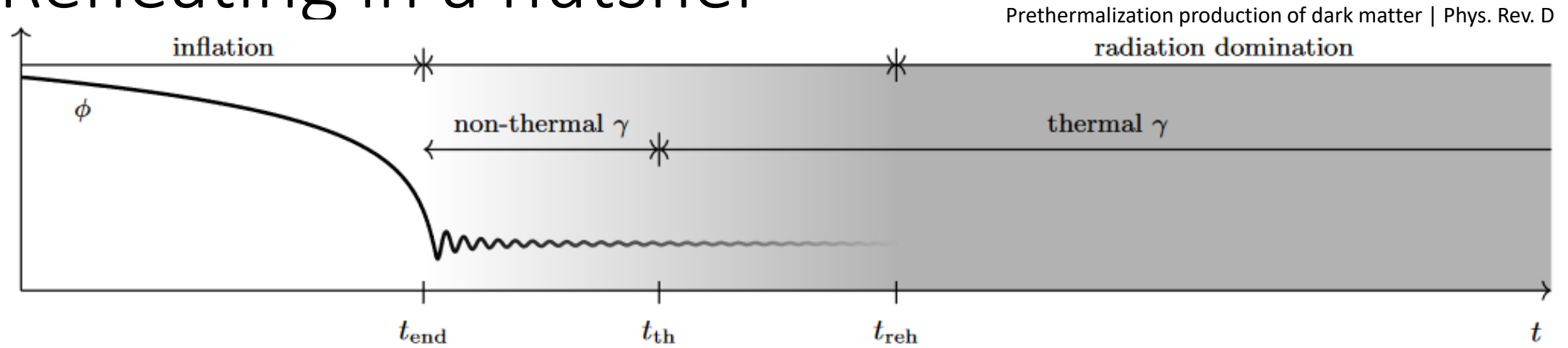
$$T \propto a^{-1}$$

and ruled out by structure formation

Reheating in a nutshell



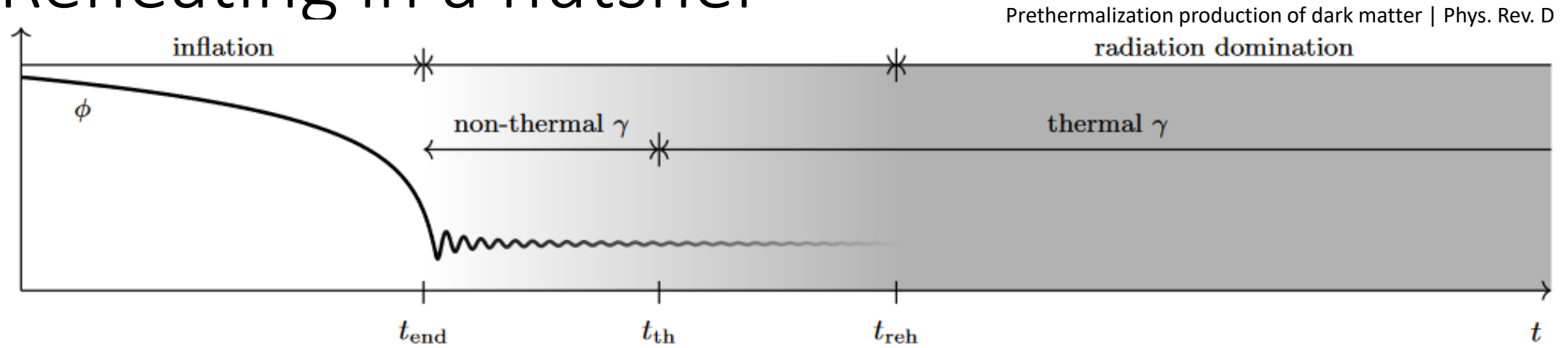
Reheating in a nutshell



Once inflation ends:

$$V(\phi) \simeq \lambda M_P^4 \left(\frac{\phi}{M_P} \right)^k \quad \longrightarrow \quad w_\phi = \frac{k-2}{k+2}$$

Reheating in a nutshell



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Reheating is achieved by coupling the inflaton:

$$\begin{aligned} \dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi &= -(1 + w_\phi)\Gamma_\phi\rho_\phi \\ \dot{\rho}_R + 4H\rho_R &= (1 + w_\phi)\Gamma_\phi\rho_\phi \end{aligned}$$

During reheating:

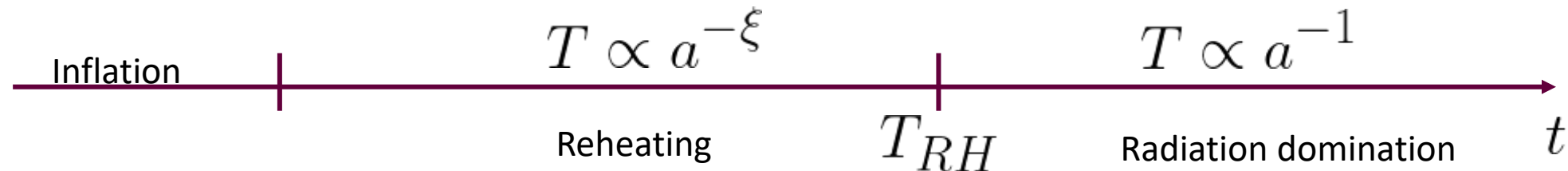
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Reheating in a nutshell

channel	generic	k=2	k=4	k=6
$\phi \rightarrow f \bar{f}$	$T \propto a^{-\frac{3k-3}{2k+4}}$	$T \propto a^{-3/8}$	$T \propto a^{-3/4}$	$T \propto a^{-15/16}$
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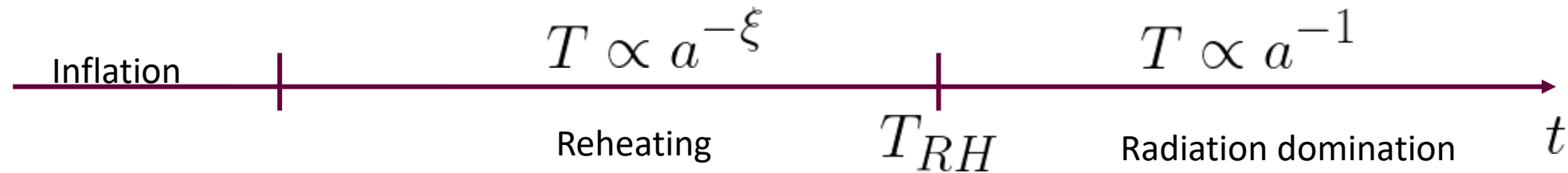
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$$10^{15} \text{ GeV} \gtrsim T_{RH} \gtrsim 1 \text{ MeV}$$

Relativistic freeze-out in radiation domination

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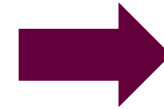
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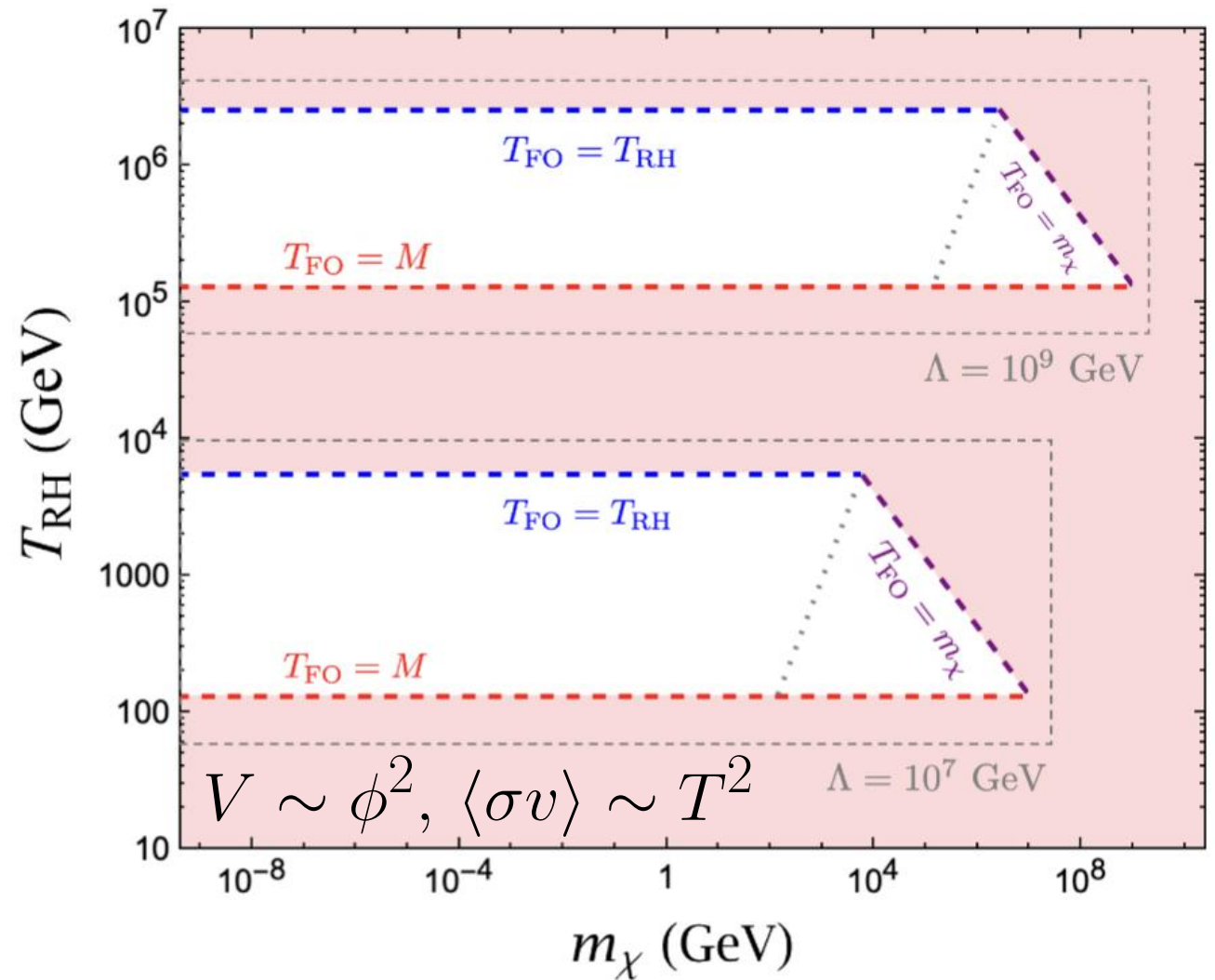

$$T_{FO} = \Lambda \left(\frac{\Lambda}{M_P g \zeta(3)} 2\pi^2 \sqrt{\frac{\alpha}{3}} \right)^{\frac{1}{n+1}}$$

Reheating can save the day since it inject entropy

For the following results we will assume:

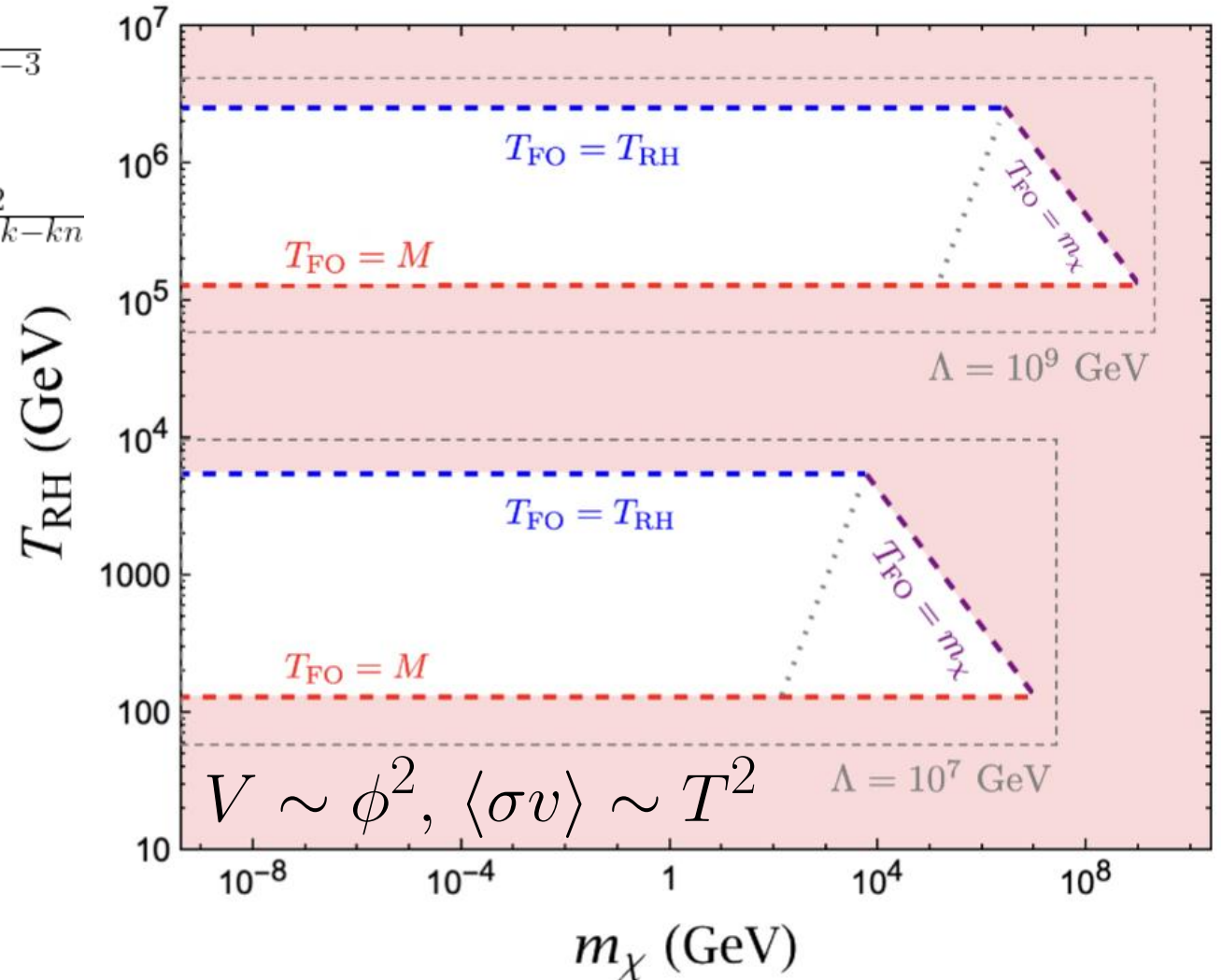

$$\mathcal{L}_{int} \sim y\phi f \bar{f} \quad T \propto a^{-\frac{3k-3}{2k+4}}$$

Is relativistic freeze-out during reheating allowed?



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$$T_{FO} = \left(\frac{3k}{k + 2g_\chi \zeta(3)} \frac{\pi^2}{\sqrt{\frac{\alpha}{3}} \frac{\Lambda^{n+2}}{M_P}} \right)^{\frac{k-1}{k+kn-n-3}} \times T_{RH}^{\frac{2}{n+3-k-kn}}$$

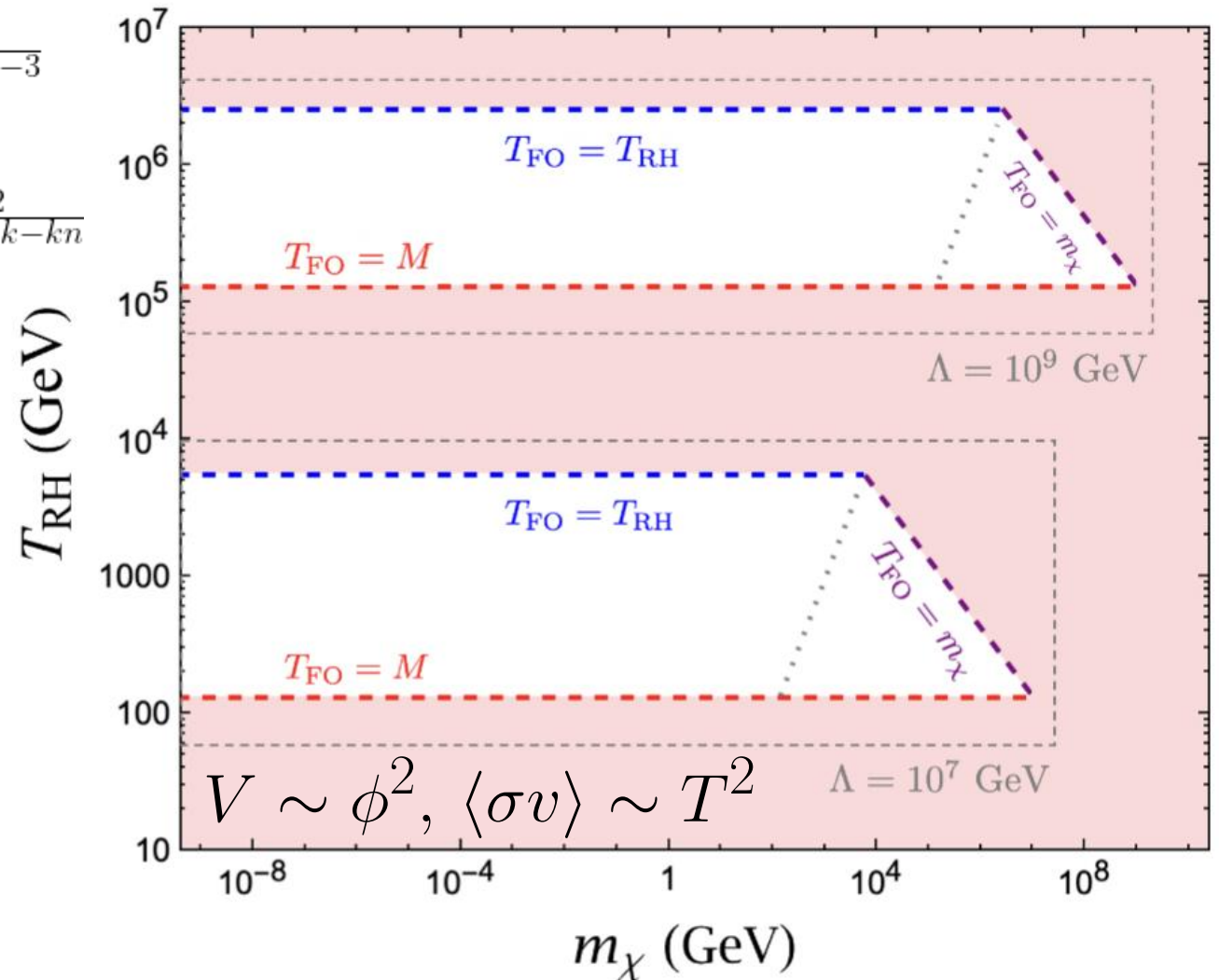


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UFO has 3 requirement:

$T_{FO} > T_{RH}$ Happens during reheating



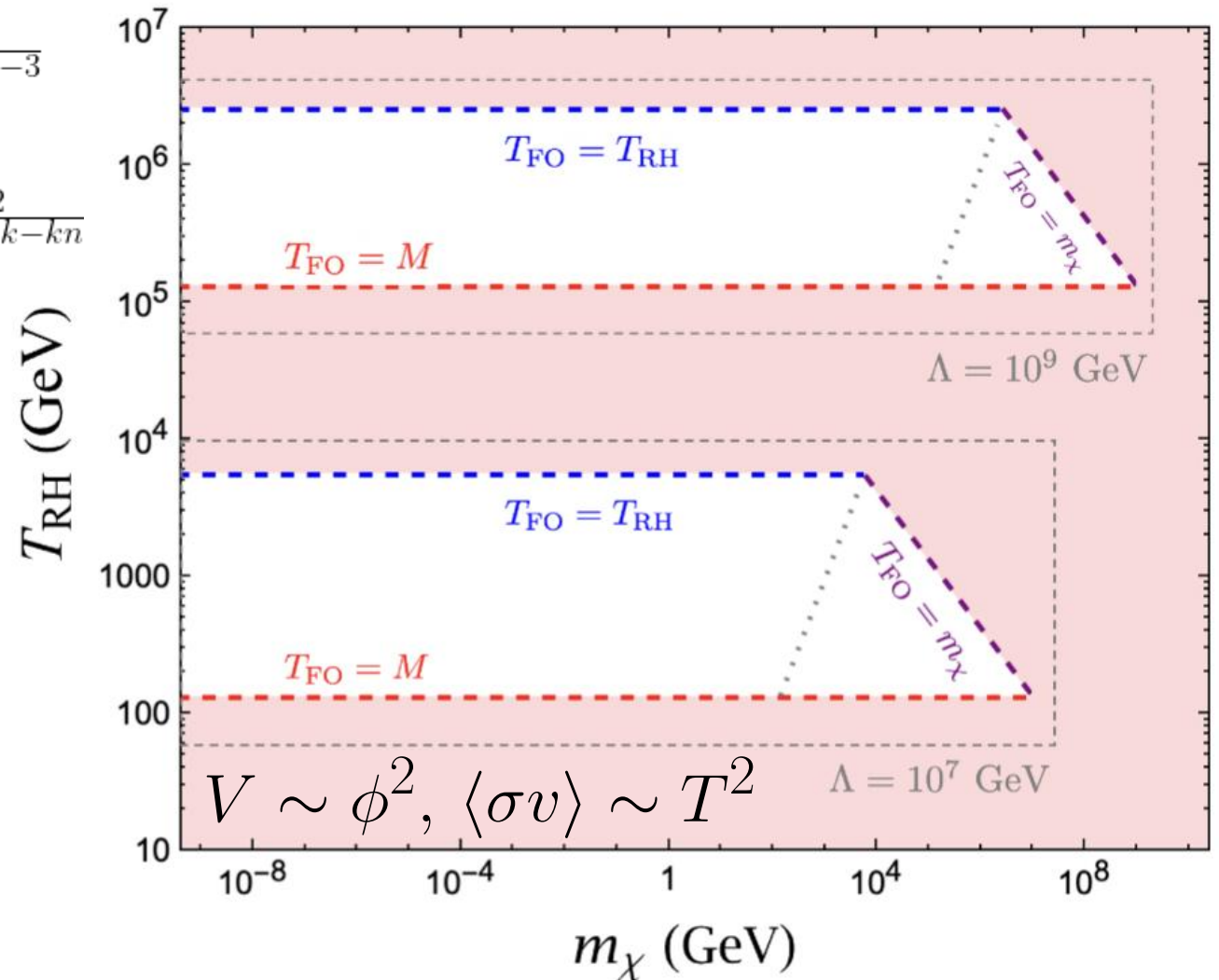
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$T_{FO} > m_\chi$ Relativistic decoupling

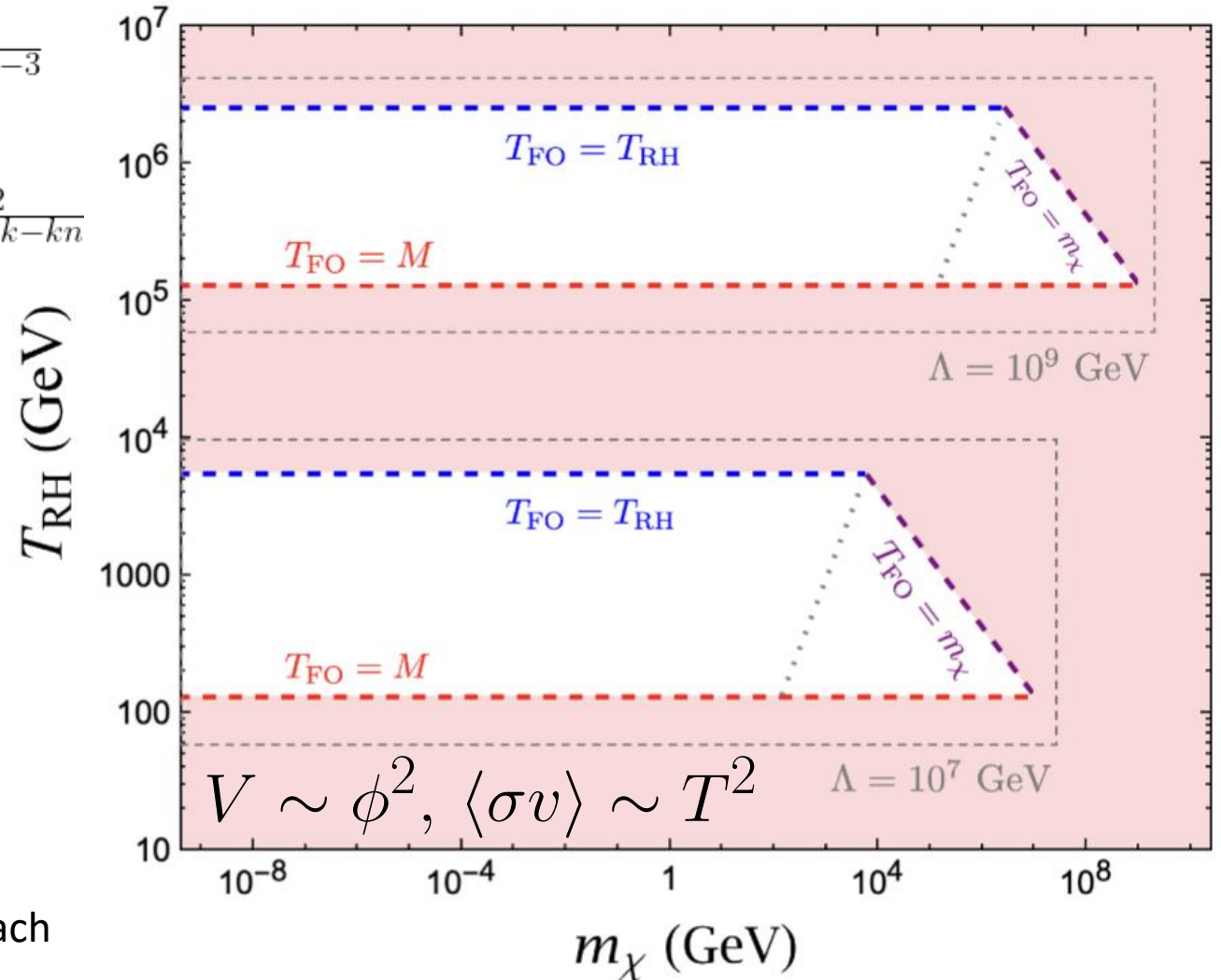


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UFO has 3 requirement:

- $T_{FO} > T_{RH}$ Happens during reheating
- $T_{FO} > m_\chi$ Relativistic decoupling
- $T_{FO} < \Lambda$ Validity of the effective approach



Relativistic freeze-out during reheating

Using the scalings $\Gamma \sim T^{n+3}$ $H \sim T^{\frac{2k}{k-1}}$  $n > n_c = \frac{3-k}{k-1}$

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After decoupling

$\Rightarrow n_{eq} > n_\chi \Rightarrow \frac{d(n_\chi a^3)}{da} \equiv \frac{dY_\chi}{da} \propto a^{\frac{3n+26-8k-3kn}{2k+4}} \Rightarrow$ Freeze-in period after decoupling

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$$Y_\chi(a) = Y_{FO} + \frac{g_\chi^2 \zeta(3)^2}{\pi^4} \sqrt{\frac{3}{\alpha}} \frac{T_{RH}^{n+4} M_P}{\Lambda^{n+2}} a_{RH}^{\frac{(3k-3)(n+6)-6k}{2k+4}} \left(\frac{2k+4}{3n-3nk-6k+30} \right) \times \left[a^{\frac{(3-3k)(n+6)+12k+12}{2k+4}} - a_{FO}^{\frac{(3-3k)(n+6)+12k+12}{2k+4}} \right]$$

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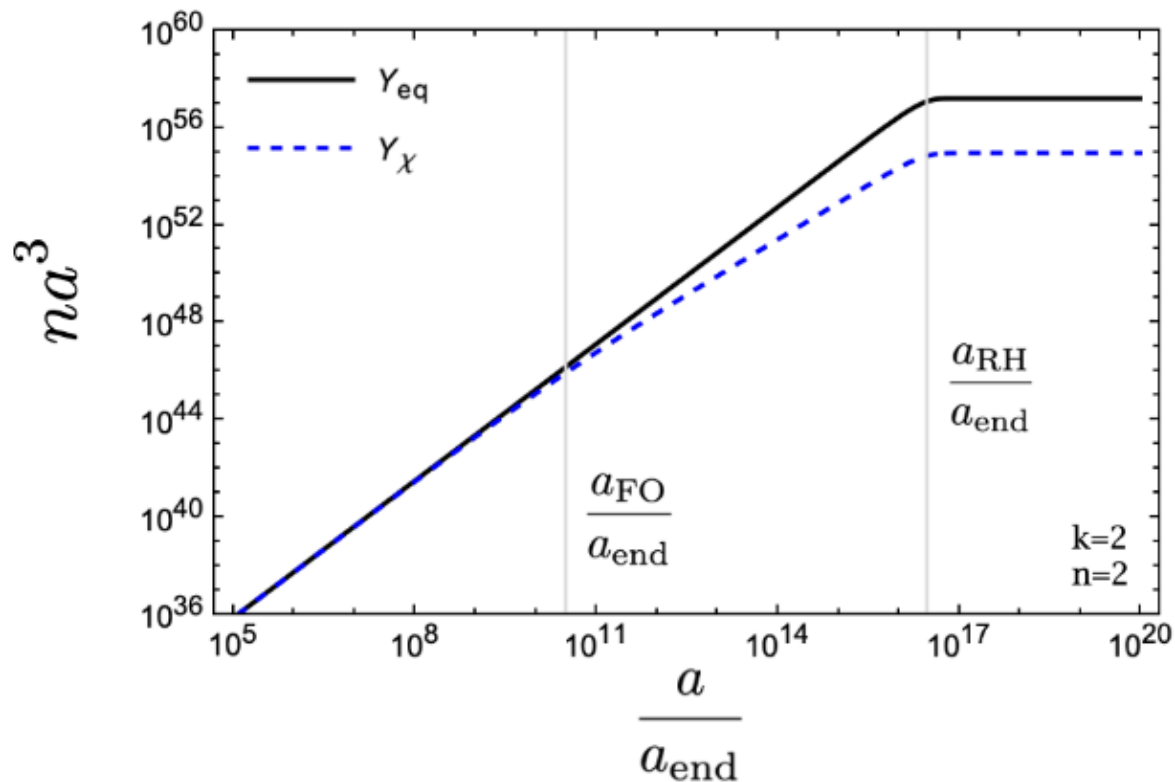
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$\Rightarrow$ $Y_\chi(a_{RH}) = Y_{FO} + Y_{UV} + Y_{IR}$

Relativistic freeze-out during reheating $m_\chi < T_{RH}$

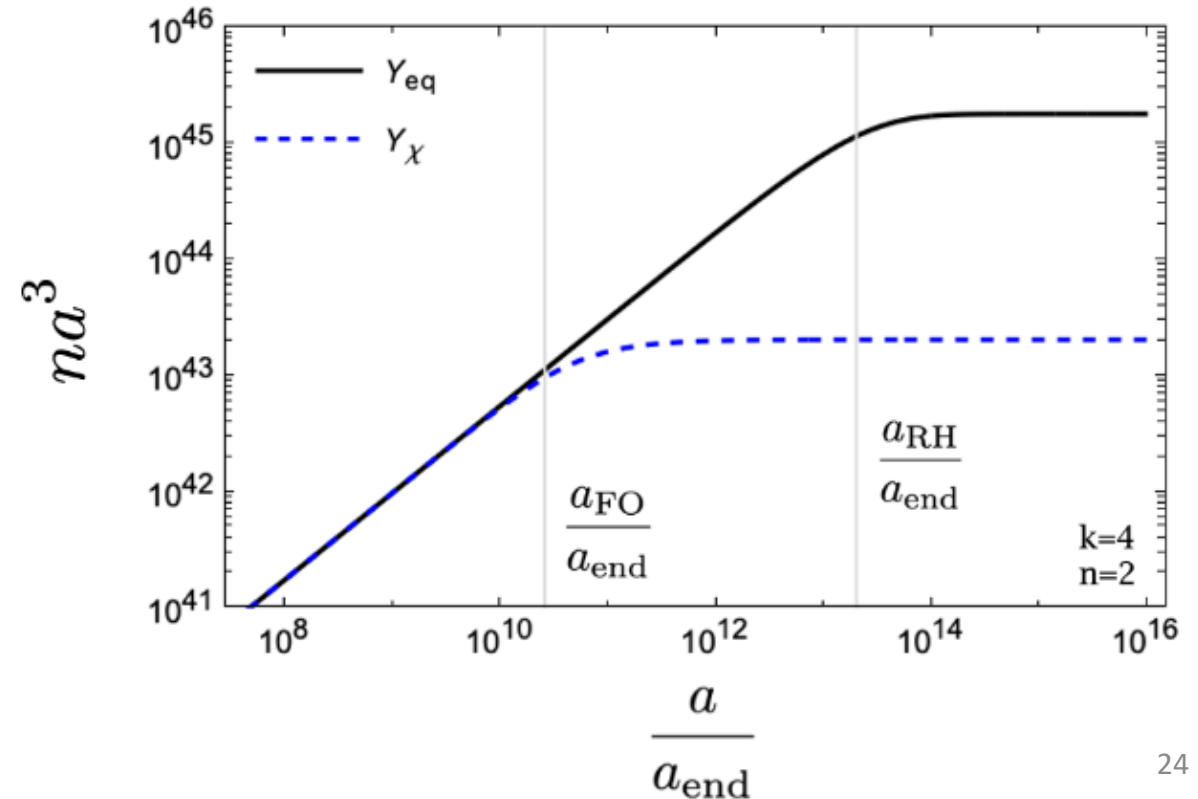
$$Y_\chi(a_{RH}) \simeq Y_{IR}$$

IR dominated UFO



$$Y_\chi(a_{RH}) \approx 3Y_{FO}$$

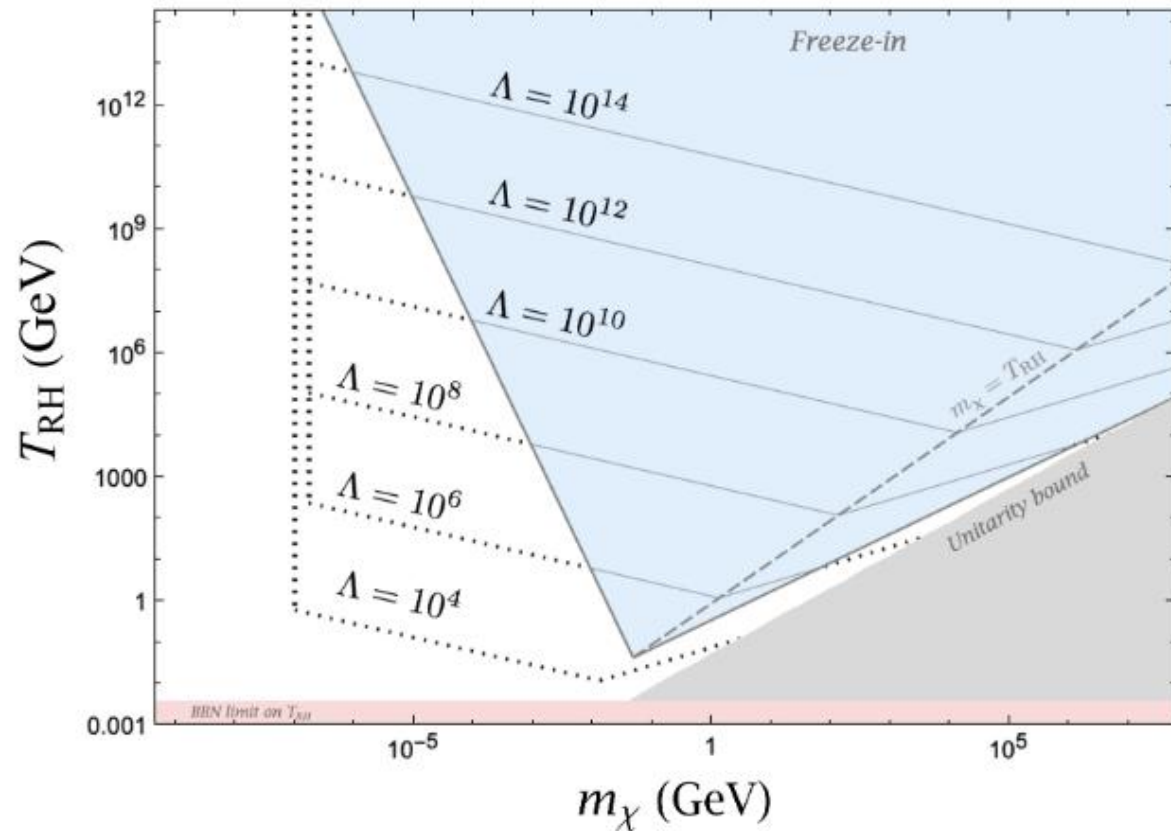
UV dominated UFO



Producing the right relic abundance

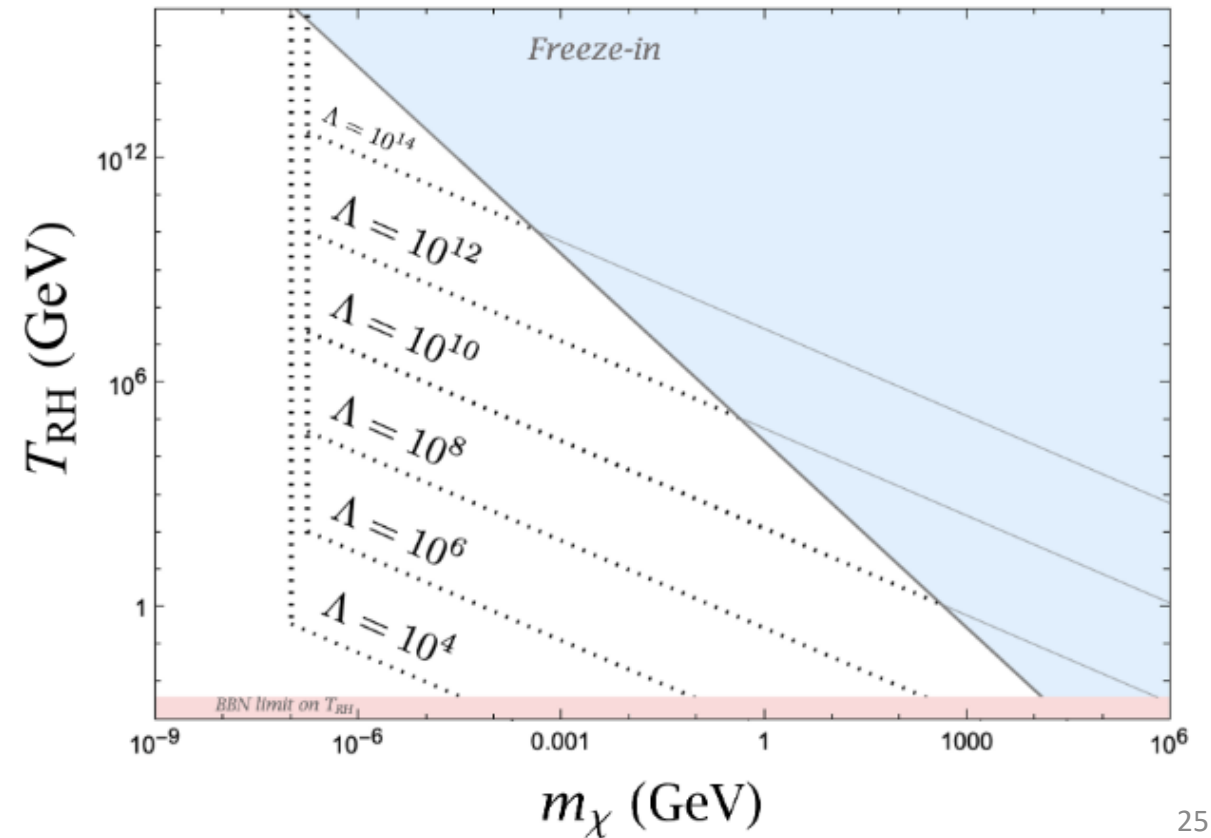
IR dominated UFO

$$V \sim \phi^2, \langle \sigma v \rangle \sim T^2$$



UV dominated UFO

$$V \sim \phi^4, \langle \sigma v \rangle \sim T^2$$



Generalizing to an arbitrary profile

Going back to the generic scaling: $T \propto a^{-\xi}$

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Is decoupling possible?

$$n > \frac{3k - 3\xi(k + 2)}{\xi(k + 2)}$$

	$k = 2$	$k = 4$	$k = 6$
$\phi \rightarrow f\bar{f}$	1	$-\frac{1}{3}$	$-\frac{3}{5}$
$\phi \rightarrow b\bar{b}$	1	5	9
$\phi\phi \rightarrow b\bar{b}$	$\times$	$-\frac{1}{3}$	1
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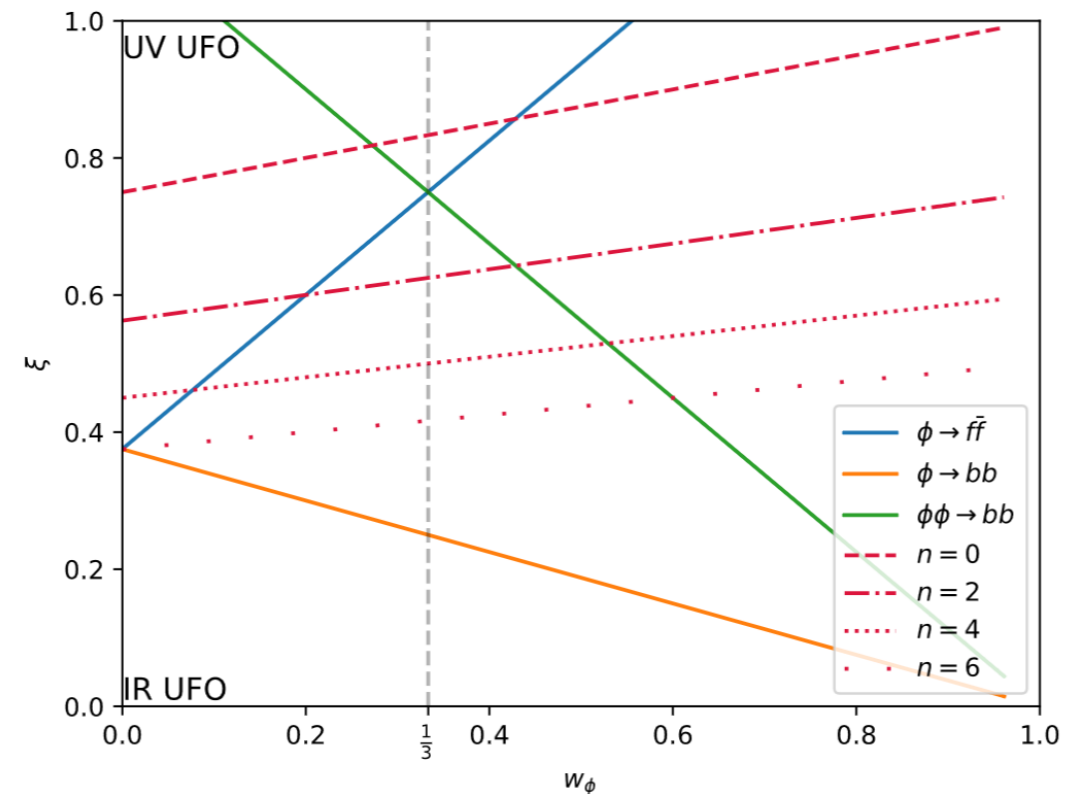
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Will the production be UV or IR?



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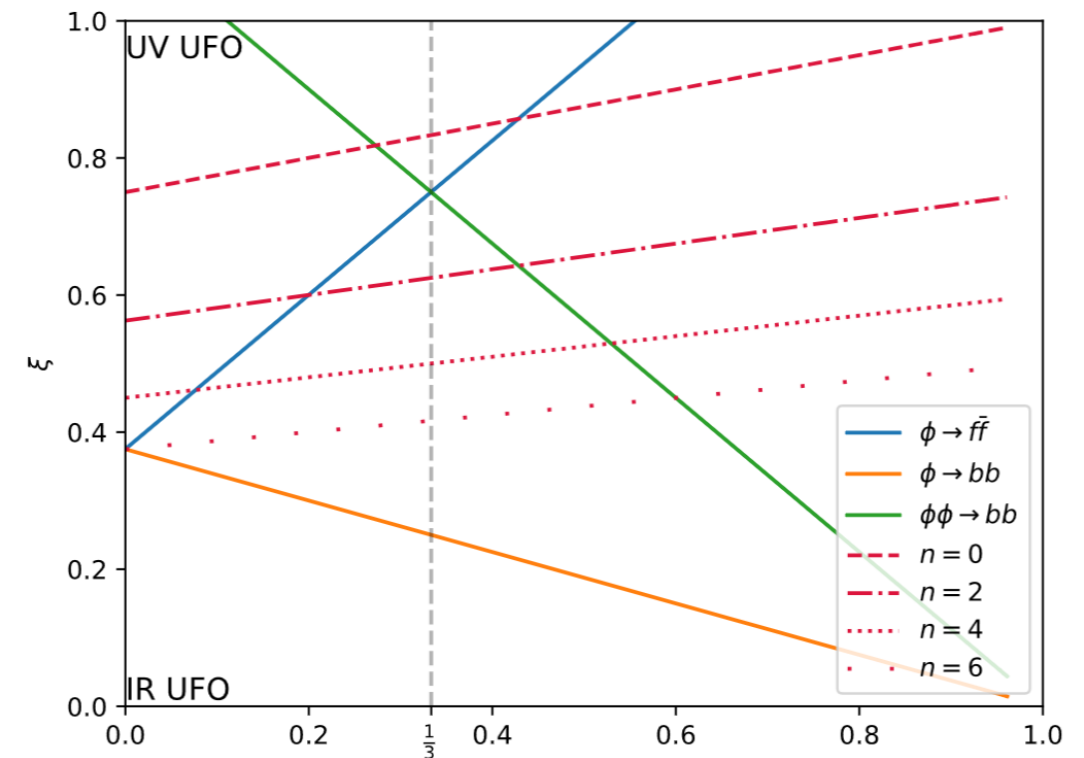
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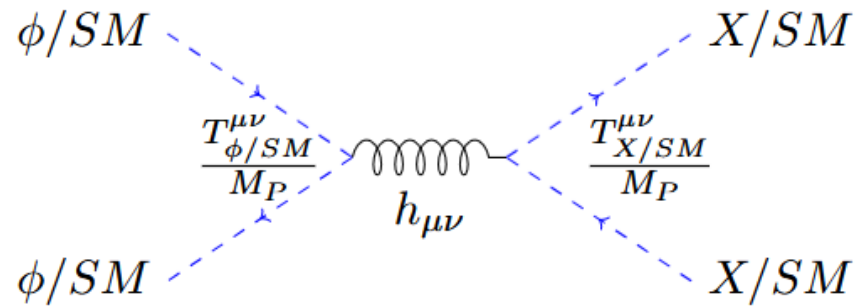
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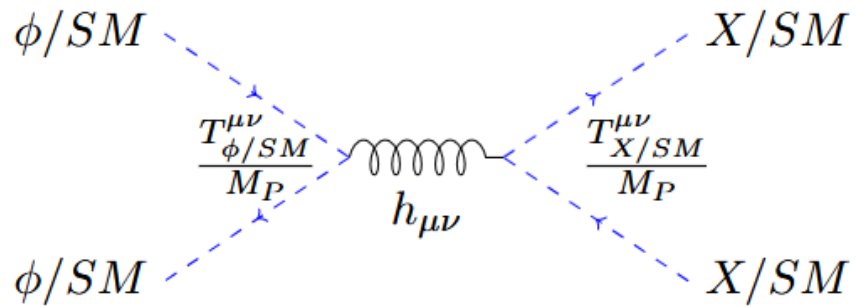
Fermion decay is always the most efficient channel for UFO to happen except if $\xi = 1$

Minimal production of radiation



S. Cléry et al. Phys. Rev. D **105**, 075005

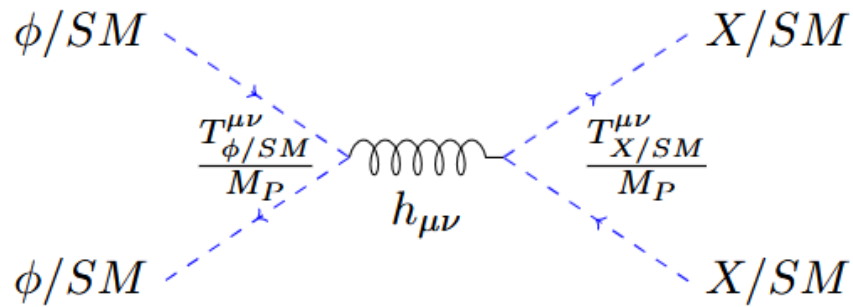
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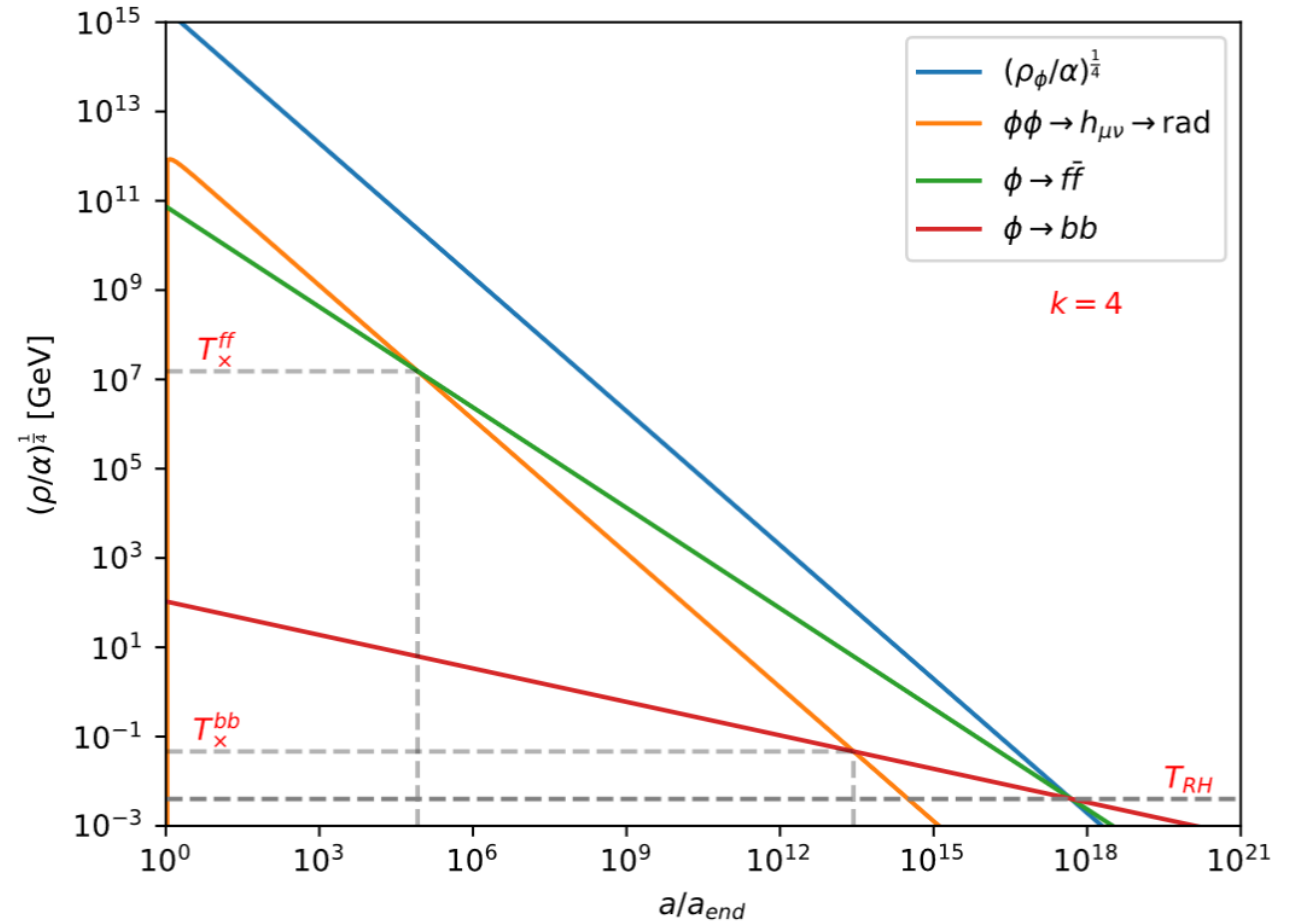
$$\Rightarrow \frac{d\rho_R^h}{dt} + 4H\rho_R^h = N \frac{\rho_\phi^2 \omega}{16\pi^4 M_P^4} \Sigma_k^h$$

Minimal production of radiation

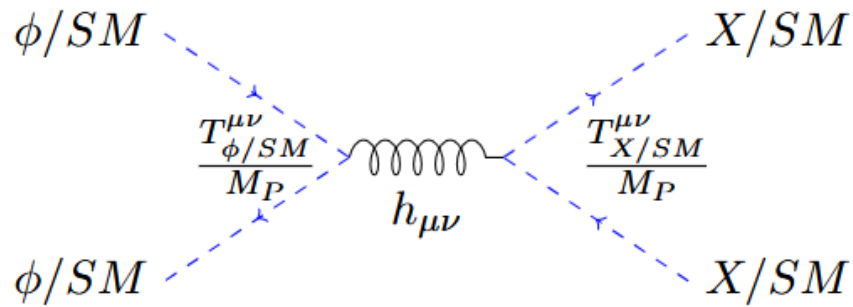


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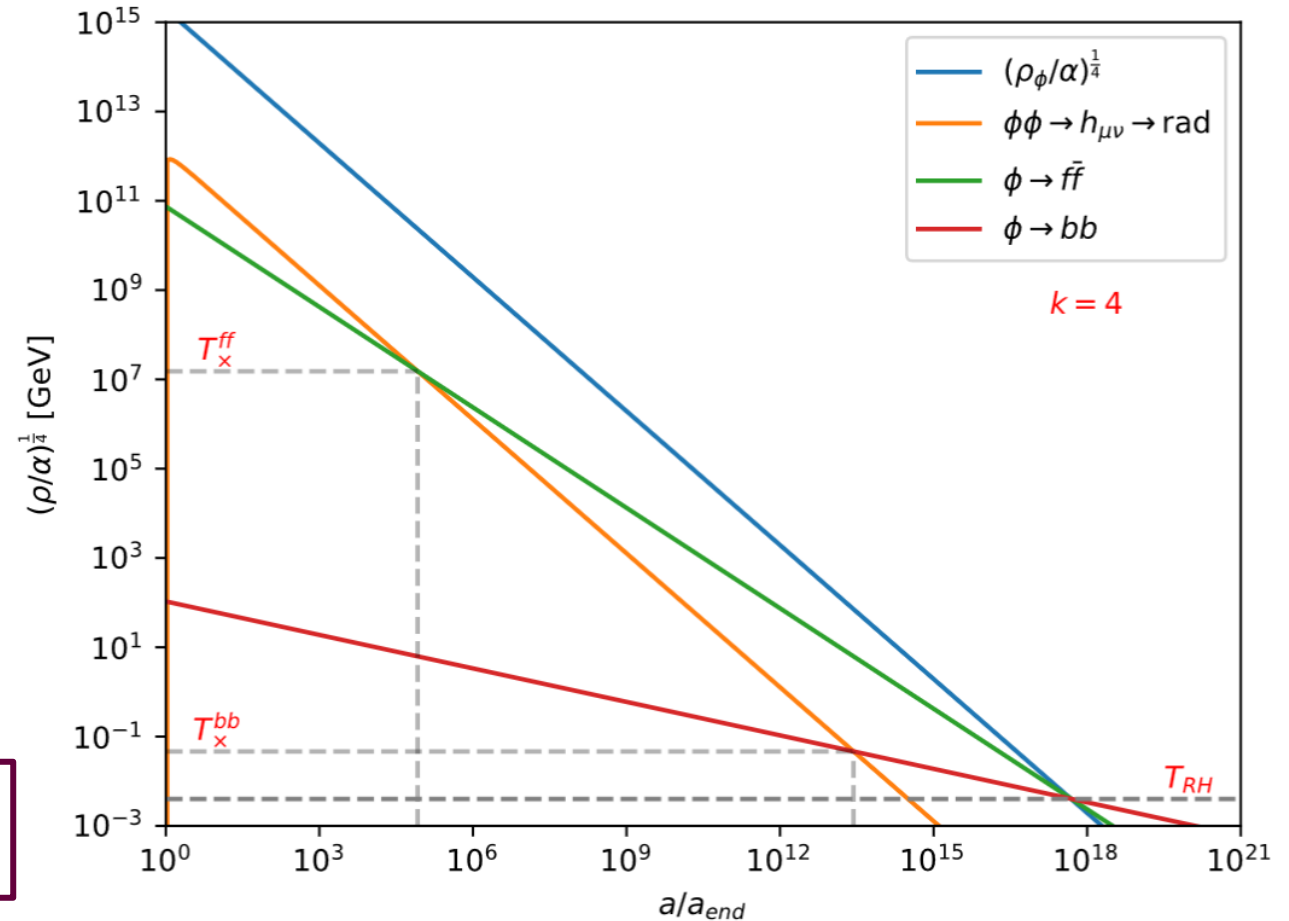
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As UFO decoupling is unrelated to the DM mass, decoupling can happen in this initial phase (GUFO)



GUFO

We can compute the crossing temperature: $T_{\times} = \mathcal{T} T_{RH}^{\frac{\xi(k-4)+3k(1-\xi)}{3k(1-\xi)}}$

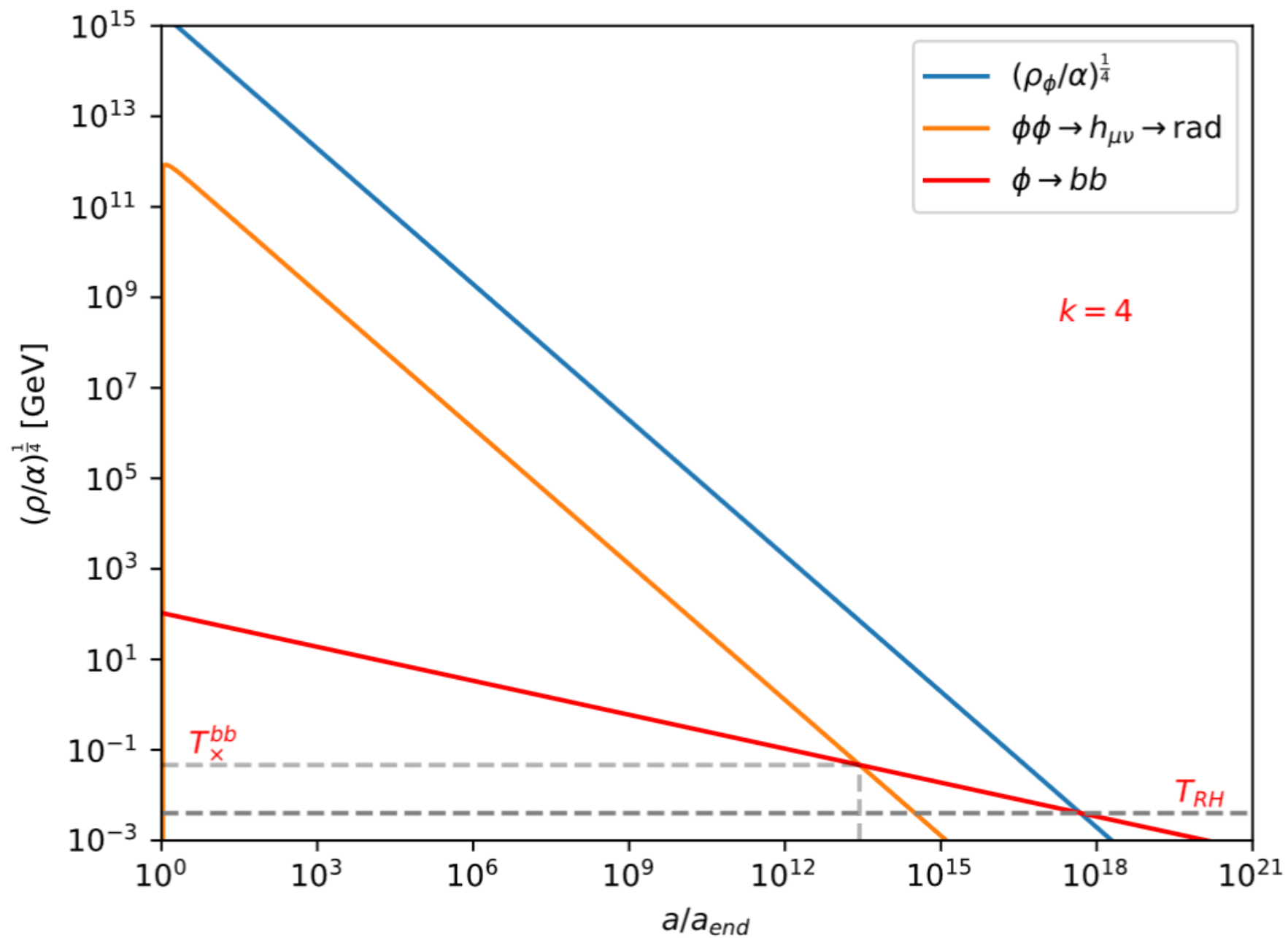
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$$\text{If } T_{FO} > T_{\times} \Rightarrow T_{FO,h} = \left(\frac{\sqrt{3}\alpha\pi^2(1+w_{\phi})\mathcal{T}^{\frac{3k(1-\xi)}{\xi(k+2)}}\Lambda^{n+2}}{2g_{\chi}\zeta(3)M_P} \right)^{\frac{k+2}{(k+2)(n+3)-3k}}$$

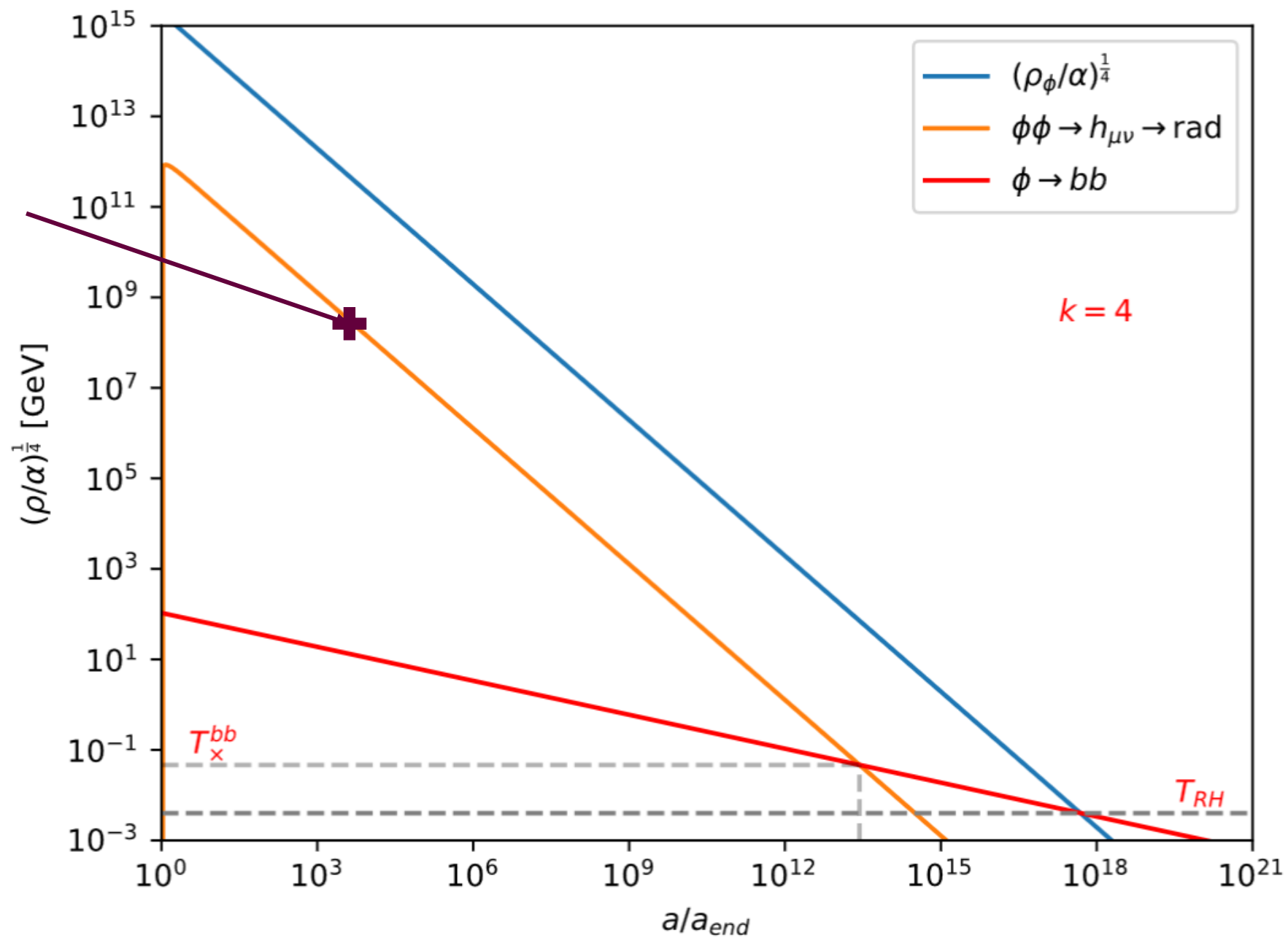
The decoupling temperature do not depend on T_{RH}

GUFO



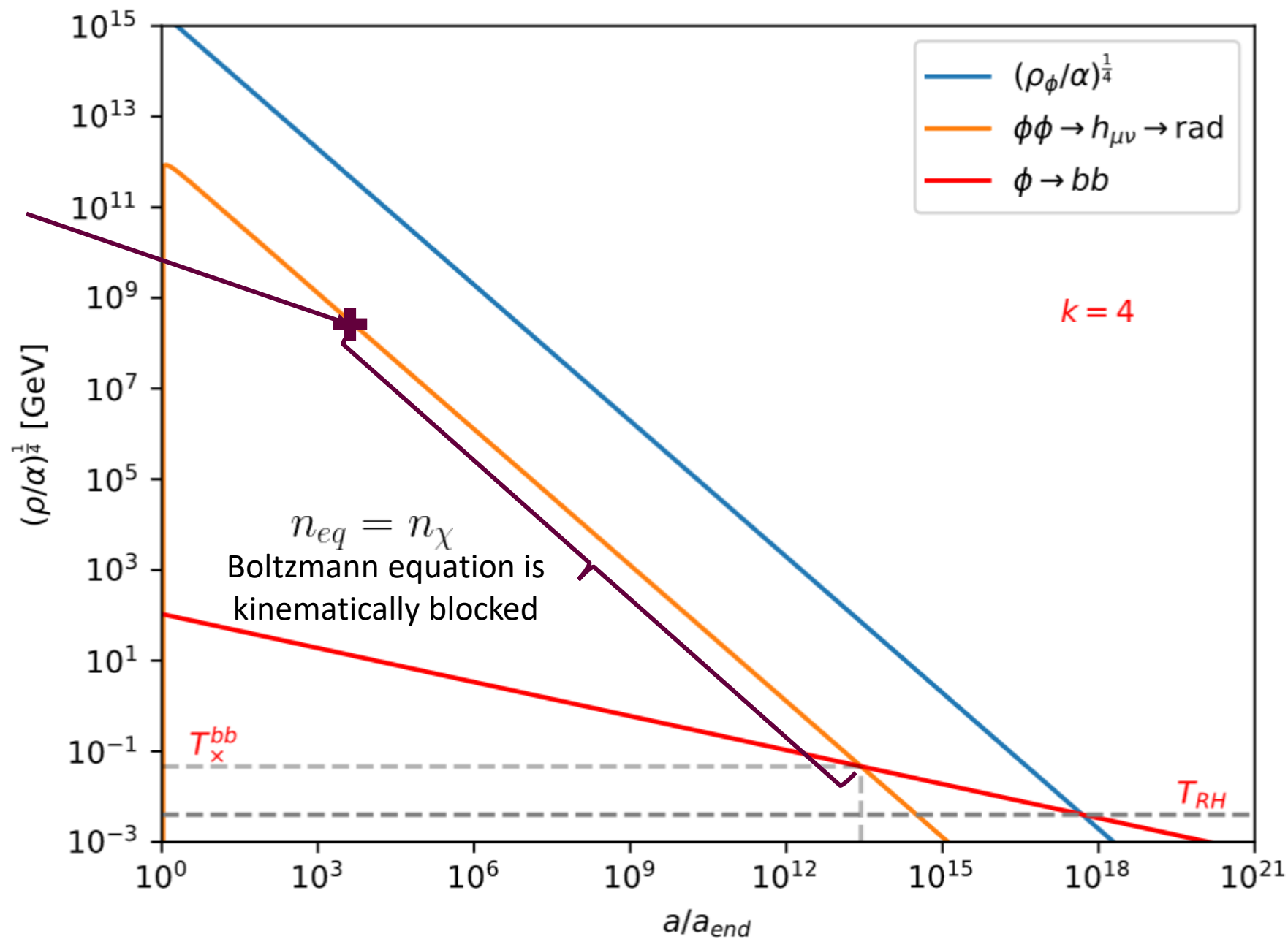
GUFO

decoupling



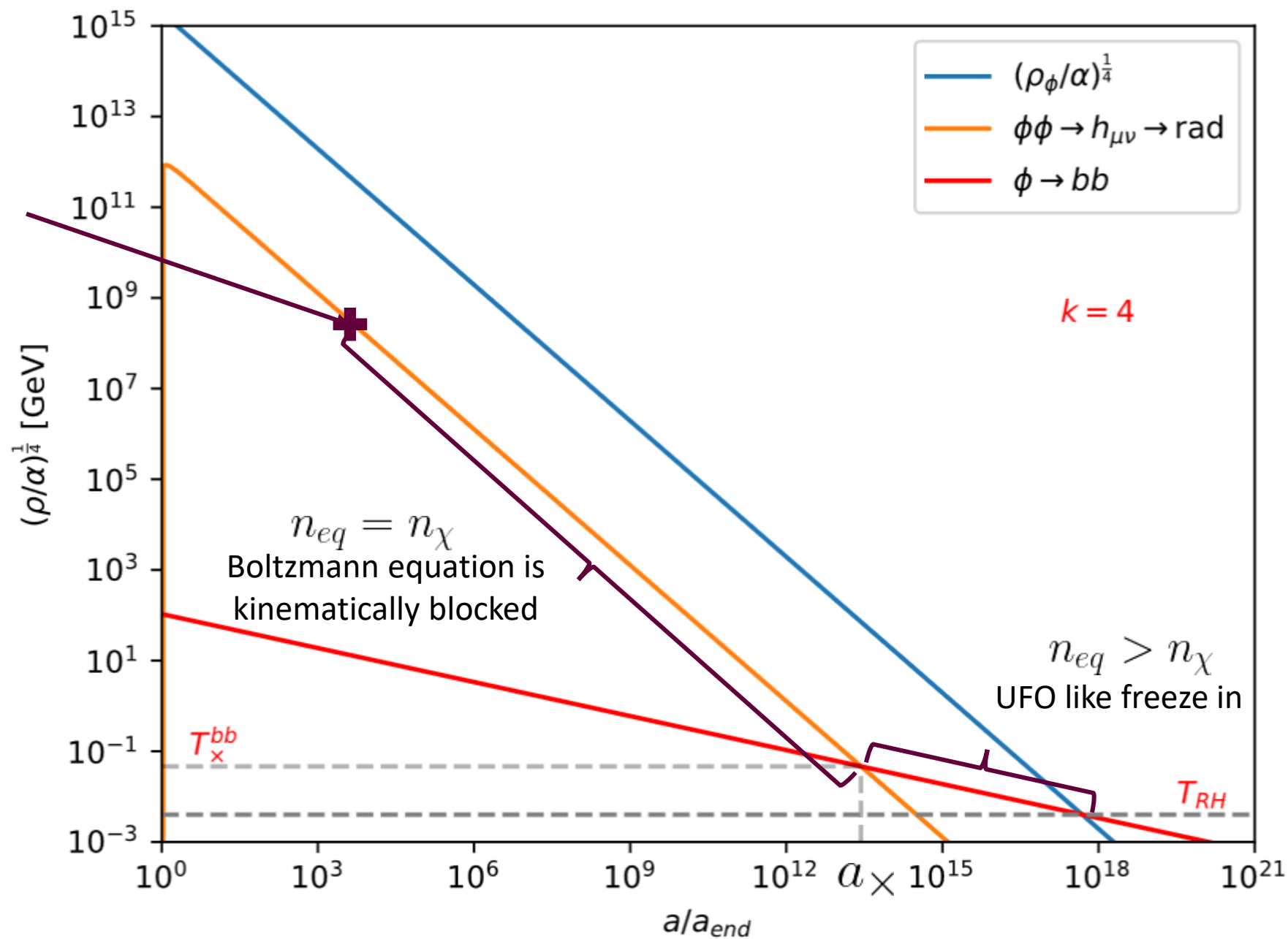
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GUFO

The comoving number density is then given by:

$$Y_{\chi}(a_{RH}) = Y_{FO} + \int_{a_{\times}}^{a_{RH}} \frac{n_{eq}^2 \langle \sigma v \rangle a^2}{H} da$$

GUFO

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$$\rightarrow Y_{\chi}(a_{RH}) = Y_{FO} \times \begin{cases} 1 + \frac{3k}{\xi(n+6)(k+2) - 6(k+1)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} & \text{UV,} \\ 1 + \frac{3k}{6(k+1) - \xi(n+6)(k+2)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} \left(\frac{T_{RH}}{T_{\times}} \right)^{6+n - \frac{6(1+k)}{\xi(k+2)}} & \text{IR, } m_{\chi} < T_{RH} \\ 1 + \frac{3k}{6(k+1) - \xi(n+6)(k+2)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} \left(\frac{m_{\chi}}{T_{\times}} \right)^{6+n - \frac{6(1+k)}{\xi(k+2)}} & \text{IR, } T_{RH} < m_{\chi} < T_{\times}, \end{cases}$$

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The comoving number density is then given by:

$$Y_{\chi}(a_{RH}) = Y_{FO} + \int_{a_{\times}}^{a_{RH}} \frac{n_{eq}^2 \langle \sigma v \rangle a^2}{H} da$$

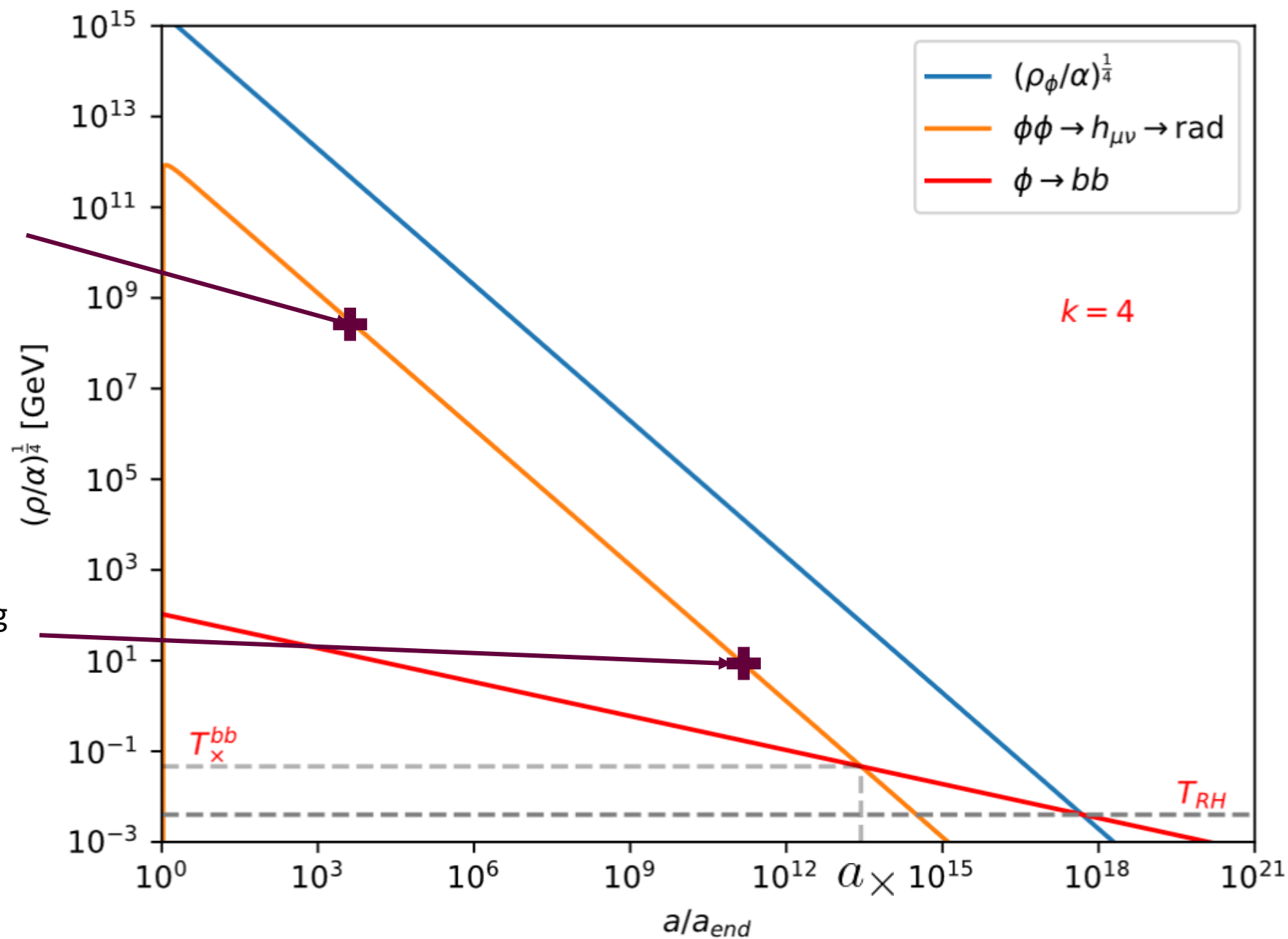
$$\rightarrow Y_{\chi}(a_{RH}) = Y_{FO} \times \begin{cases} 1 + \frac{3k}{\xi(n+6)(k+2) - 6(k+1)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} & \text{UV,} \\ 1 + \frac{3k}{6(k+1) - \xi(n+6)(k+2)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} \left(\frac{T_{RH}}{T_{\times}} \right)^{6+n - \frac{6(1+k)}{\xi(k+2)}} & \text{IR, } m_{\chi} < T_{RH} \\ 1 + \frac{3k}{6(k+1) - \xi(n+6)(k+2)} \left(\frac{T_{\times}}{T_{FO,h}} \right)^{\frac{6}{k+2} + n} \left(\frac{m_{\chi}}{T_{\times}} \right)^{6+n - \frac{6(1+k)}{\xi(k+2)}} & \text{IR, } T_{RH} < m_{\chi} < T_{\times}, \end{cases}$$

UV or IR GUFO depends on the time between decoupling and crossing

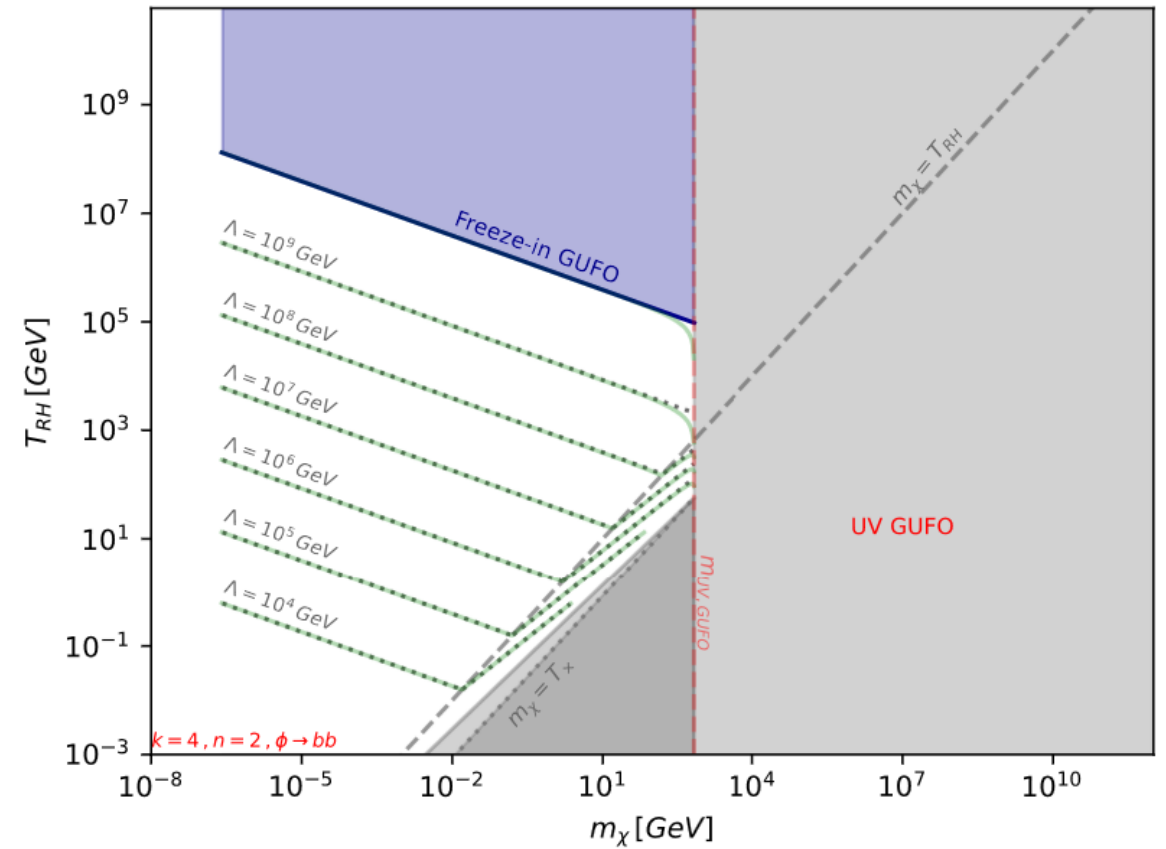
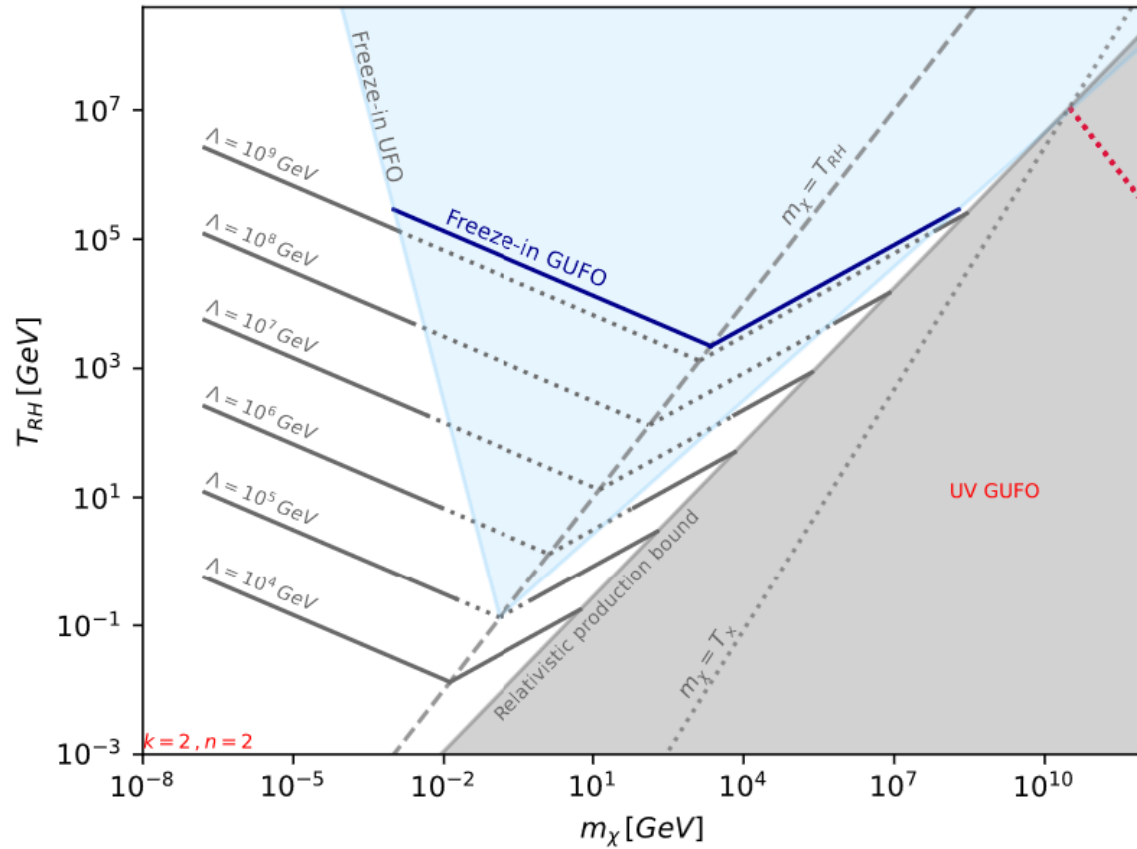
GUFO

Decoupling leading
to UV pheno

Decoupling leading
to IR pheno



Producing the right relic abundance



UFO and light DM constraints

Dark matter need to be compatible with relativistic degrees of freedom: $\Delta N_{eff} < 0.18$

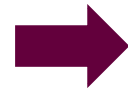
Tsung-Han Yeh *et al* JCAP10(2022)046

UFO and light DM constraints

Dark matter need to be compatible with relativistic degrees of freedom : $\Delta N_{eff} < 0.18$

Tsung-Han Yeh *et al* JCAP10(2022)046

One can compute the DM temperature at BBN:
$$T'_{\text{BBN}} = T_{\text{BBN}} \left(\frac{T_{\text{RH}}}{T_{\text{FO}}} \right)^{\frac{7-n}{3n-3}} \left(\frac{g_{\text{BBN}}}{g_{\text{RH}}} \right)^{\frac{1}{3}} \left(\frac{g_{\text{RH}}}{g_{\text{FO}}} \right)^{\frac{n+2}{6n-6}}$$



$$T_{\text{RH}} \leq 1.3 T_{\text{FO}}$$

Always satisfied!

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Always satisfied!

DM need to be cold at the time of structure formation:
$$m_{\chi} > 5 \text{ keV} \times \left(\frac{T_{\text{RH}}}{T_{\text{FO}}} \right)^{\frac{7-n}{3n-3}}$$

UFO is a viable model to produce dark matter

Conclusion

- Relativistic dark matter production is possible during reheating and is compatible with standard cosmology.
- The independence of the decoupling from the dark matter mass allows the DM production to be modulated by additional radiation production, such as CGPP.
- The UFO/GUFO phenomenology can be seen as a freeze-in with a thermal equilibrium initial condition.

Thank you!

Backup

Gravitationally produced radiation

$$\rho_R^h = N \frac{\sqrt{3} M_P^4 \gamma_k \Sigma_k^h}{16\pi} \left(\frac{\rho_{end}}{M_P^4} \right)^{\frac{2k-1}{k}} \frac{k+2}{8k-14} \times \left[\left(\frac{a_{end}}{a} \right)^4 - \left(\frac{a_{end}}{a} \right)^{\frac{12k-6}{k+2}} \right],$$

It is useful to introduce: $\rho_R^h \sim \rho_{R,e}^h (a_{end}/a)^4$

The reheating temperature must be low enough for GUFO to be possible:

$$T_{RH} < \frac{(\rho_{R,max}^h)^{\frac{3k}{12k-8\xi(k+2)}}}{\alpha^{\frac{1}{4}} \rho_{end}^{\frac{\xi(k+2)}{6k-4\xi(k+2)}}} \left(\frac{a_{max,h}}{a_{end}} \right)^{\frac{3\xi k}{3k-2\xi(k+2)}}$$

The crossing temperature is inevitably a function of the reheating temperature:

$$\frac{T_{\times}}{T_{RH}} = \left(\frac{a_{RH}}{a_{\times}} \right)^{\xi} = \frac{\alpha^{\frac{\xi(k-4)}{12k(1-\xi)}} \rho_{end}^{\frac{\xi(k+2)}{6k(1-\xi)}}}{(\rho_{R,e}^h)^{\frac{\xi}{4(1-\xi)}}} T_{RH}^{\frac{\xi(k-4)}{3k(1-\xi)}} := \mathcal{T} T_{RH}^{\frac{\xi(k-4)}{3k(1-\xi)}}.$$

UV vs IR GUFO

$$\frac{T_{FO,h}}{T_{\times}} \ll \left(\frac{3k}{6(k+1) - \xi(n+6)(k+2)} \right)^{\frac{k+2}{(k+2)(n+6) - 6(k+1)}} \left(\frac{T_{RH}}{T_{\times}} \right)^{\frac{\xi(k+2)(n+6) - 6(k+1)}{\xi(k+2)n + 6\xi}}$$

