



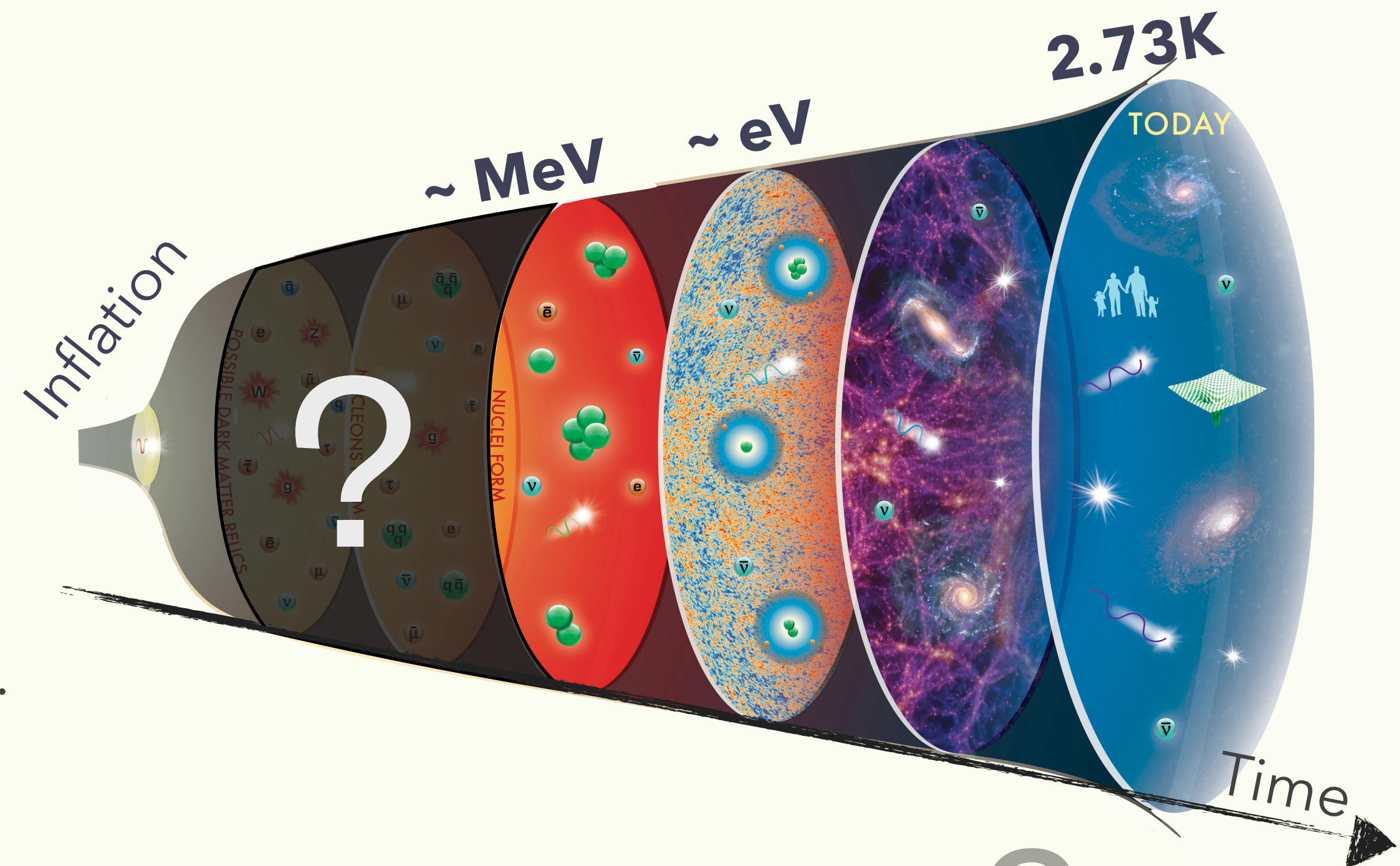
Thermalization Process during Reheating & Gravitational Waves

Kyohei Mukaida @KEK, SOKENDAI

Based on work w/ K. Harigaya, T. Tsuji, M. Yamada

Beginning of the Hot Universe?

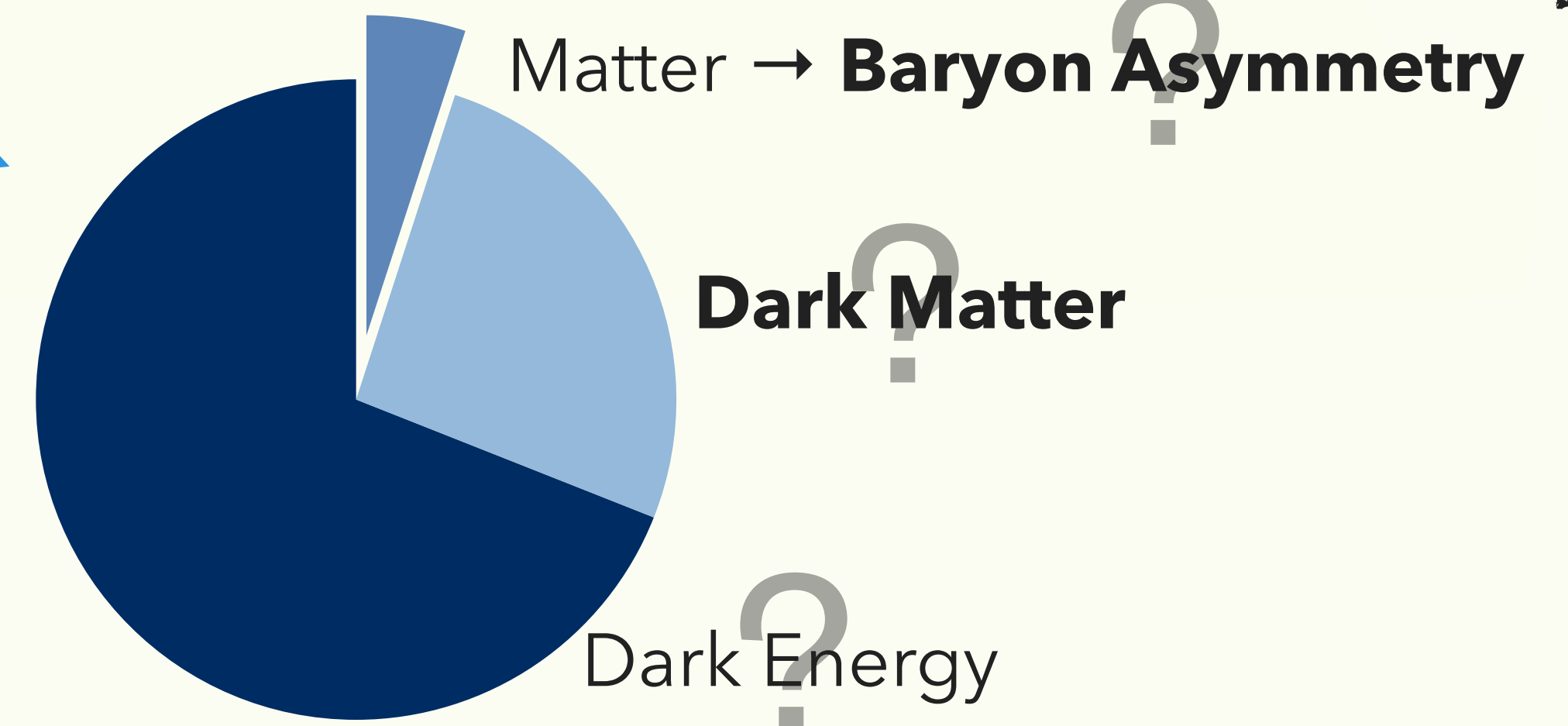
- Cosmic inflation (accelerated expansion)
 - Solves horizon/flatness problems
 - Provides the origin of density fluctuations
- ◉ **Dilutes everything** including baryons, dark matter...



Reheating

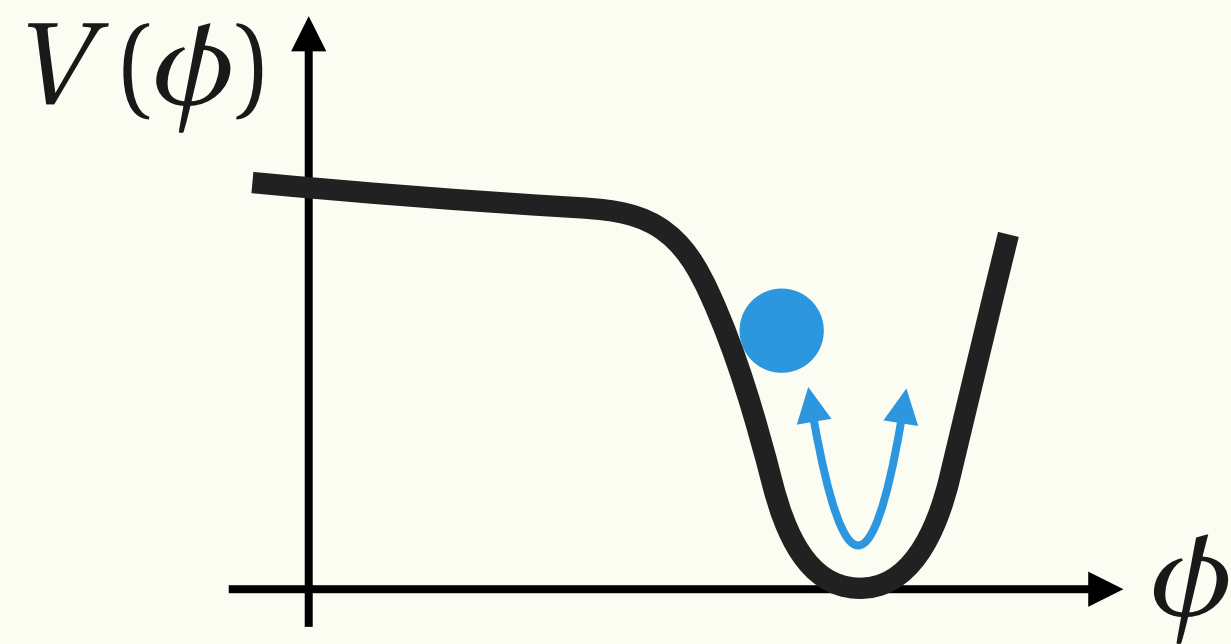
Hot Universe

- Big Bang Nucleosynthesis
- Cosmic Microwave Background

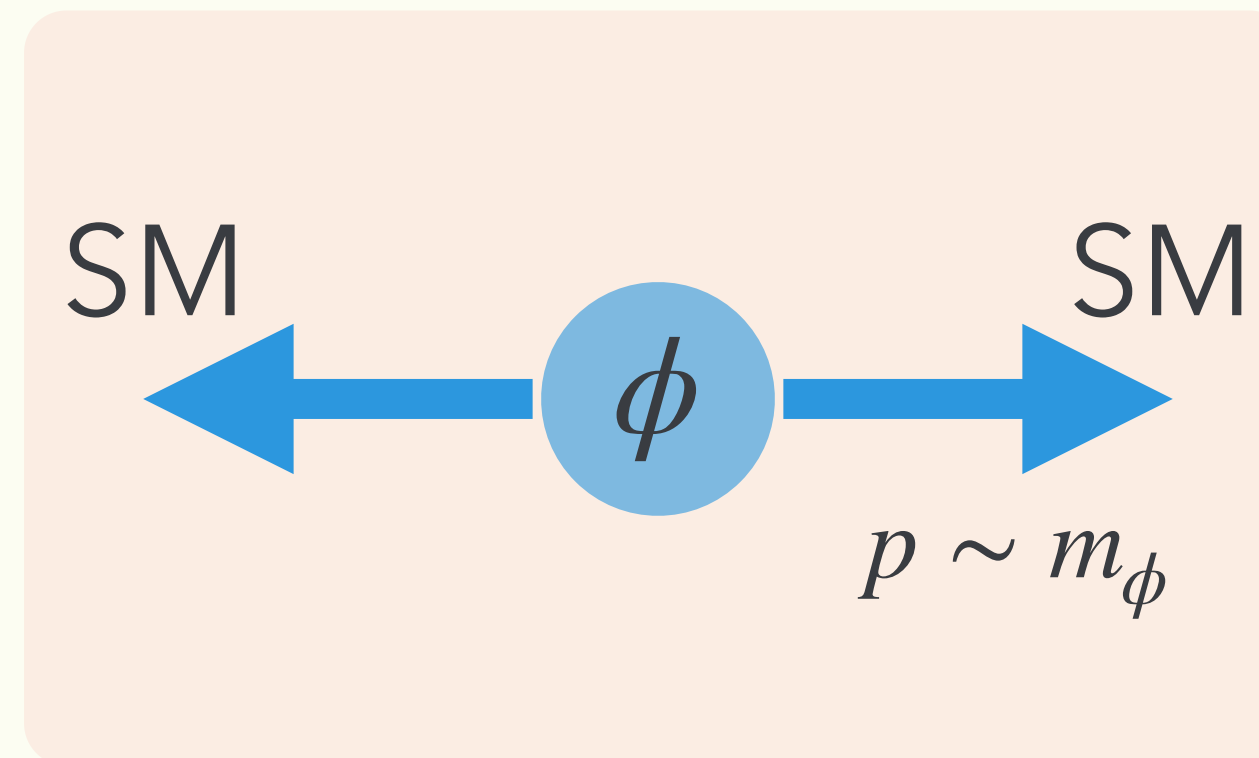


Reheating after Inflation

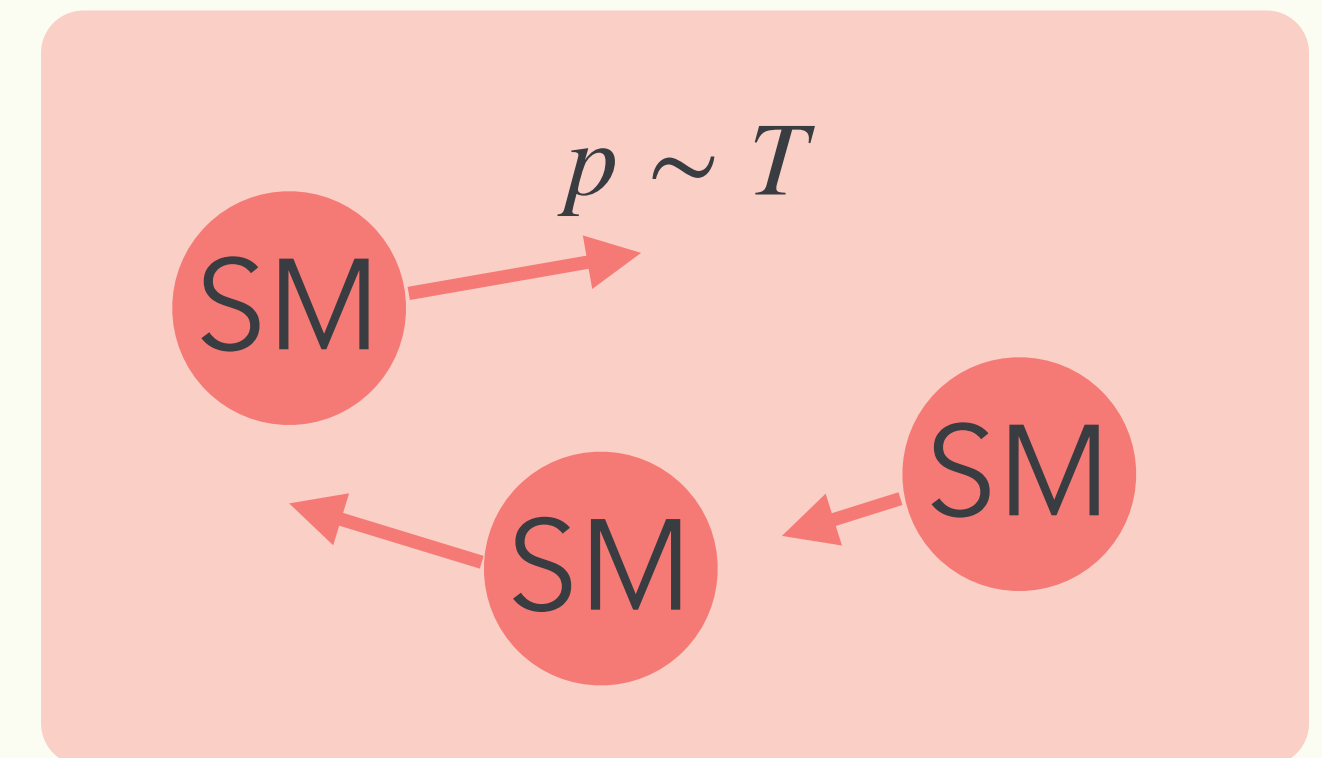
- Textbook picture of reheating after inflation



Inflaton oscillation



Inflaton decay



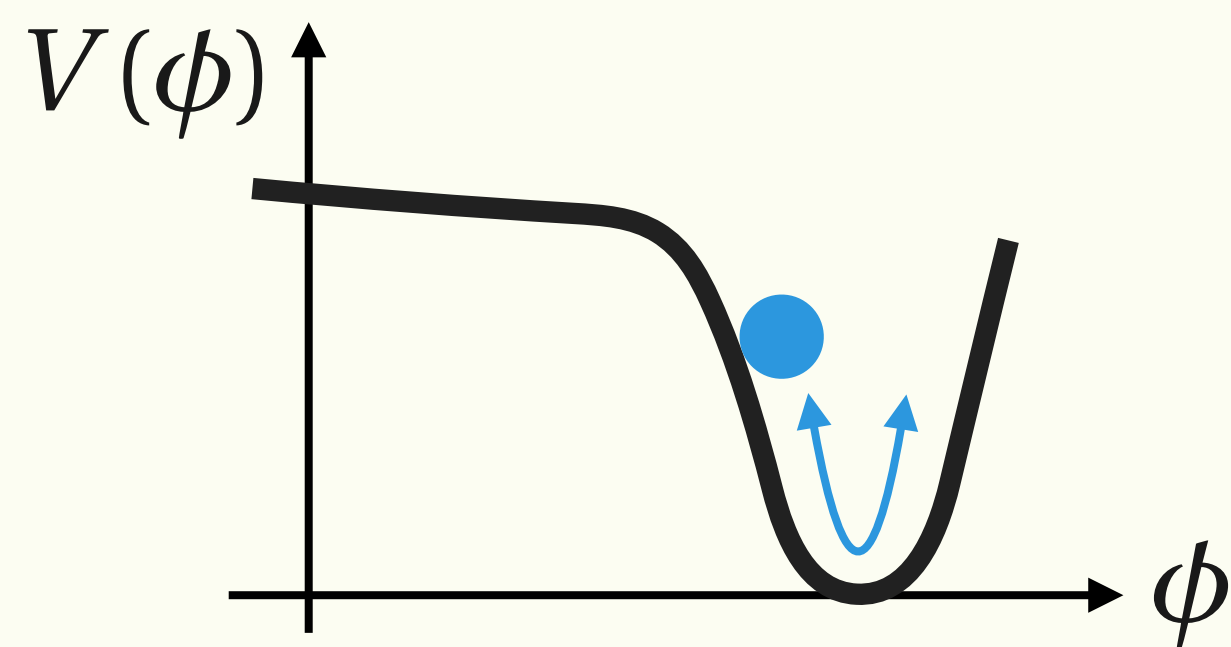
Thermal plasma

$$T_R \sim \sqrt{M_{\text{Pl}} \Gamma_\phi} \sim \sqrt{m_\phi / M_{\text{Pl}}} m_\phi \text{ @ dim 5 Planck-suppressed decay}$$

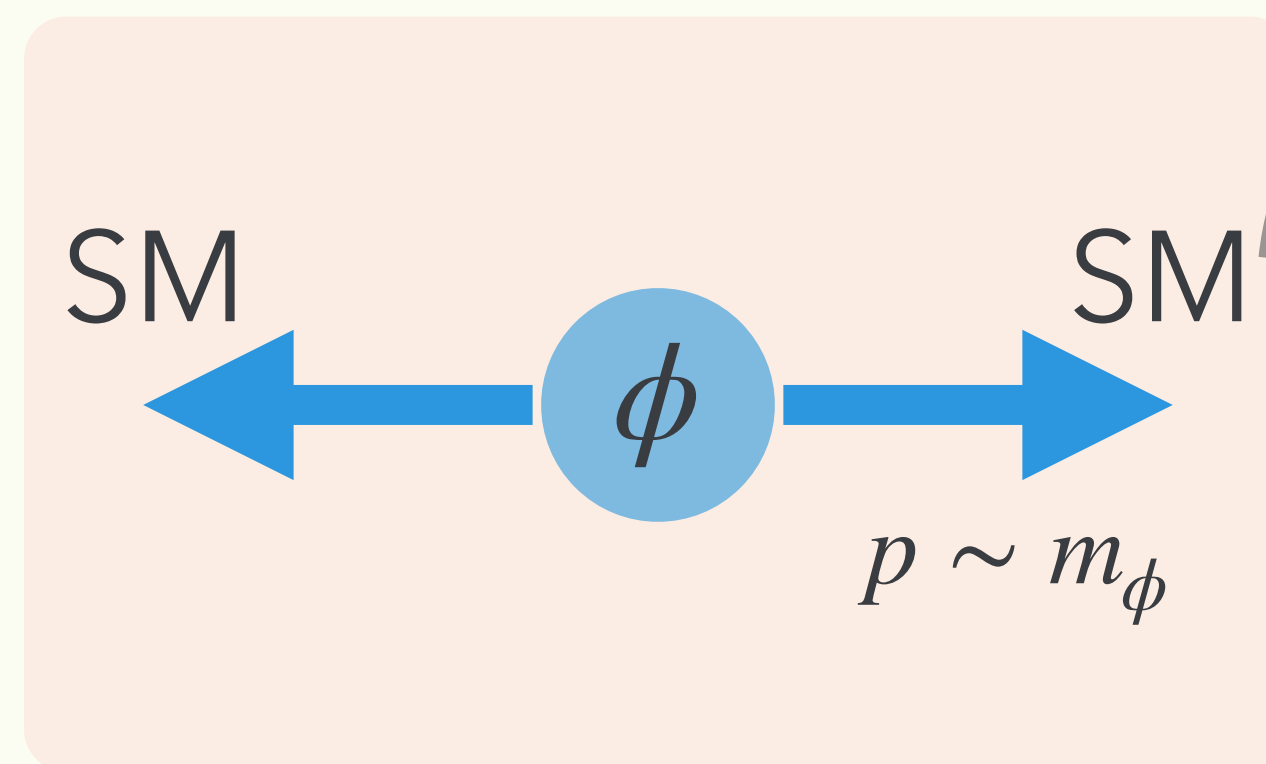
Time

Reheating after Inflation

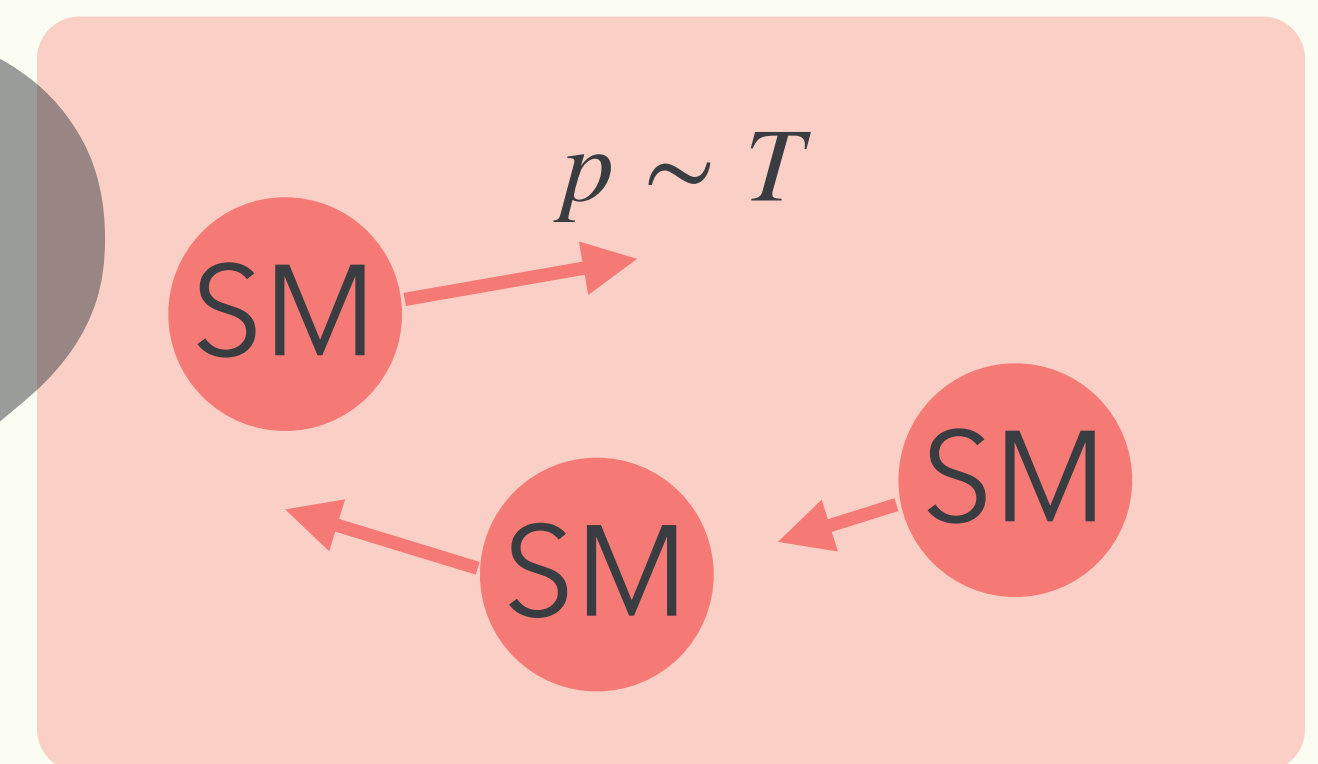
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Inflaton oscillation



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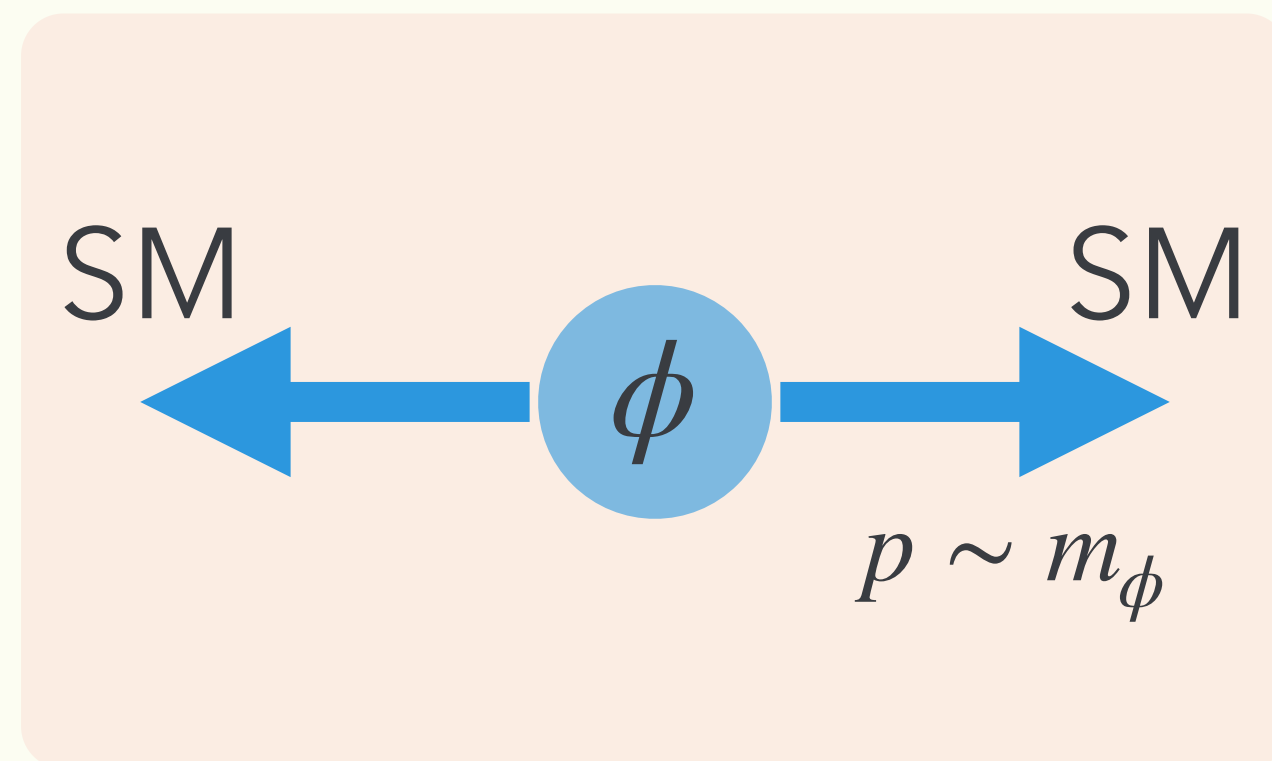
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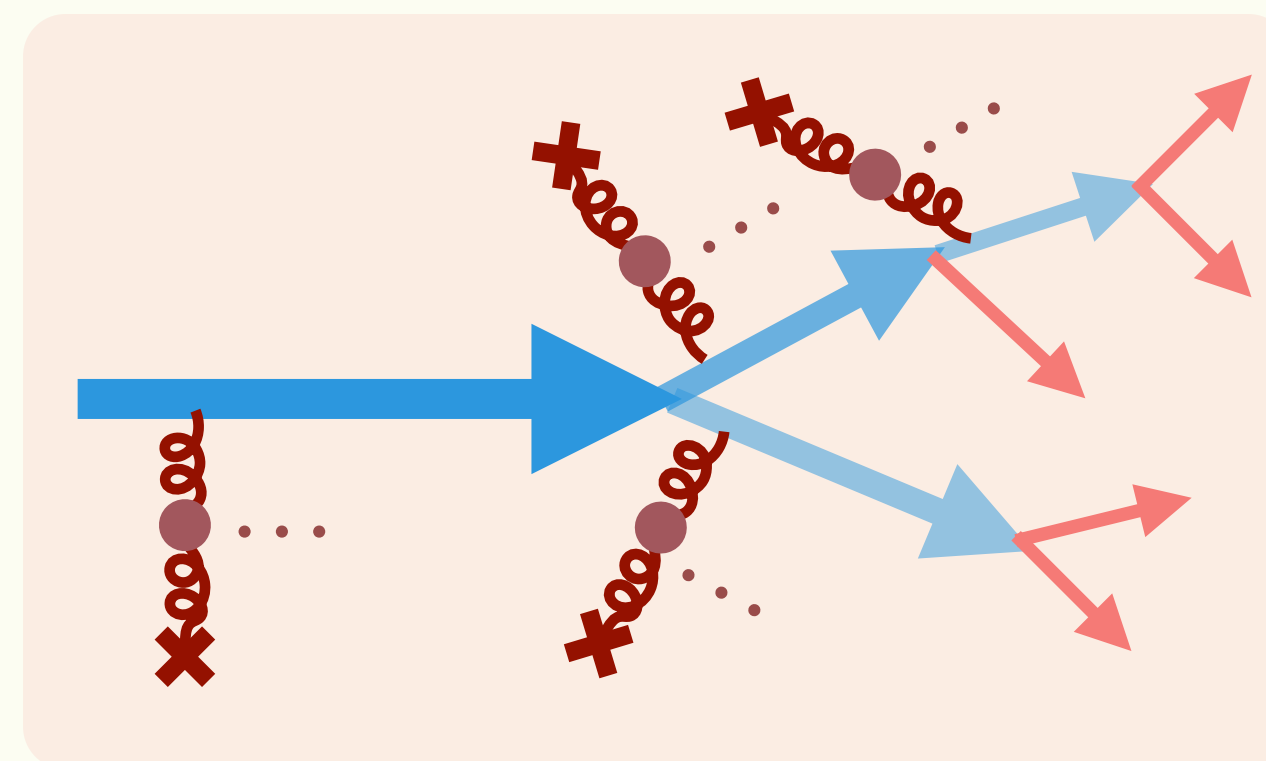
Time

Beyond Instantaneous Thermalization

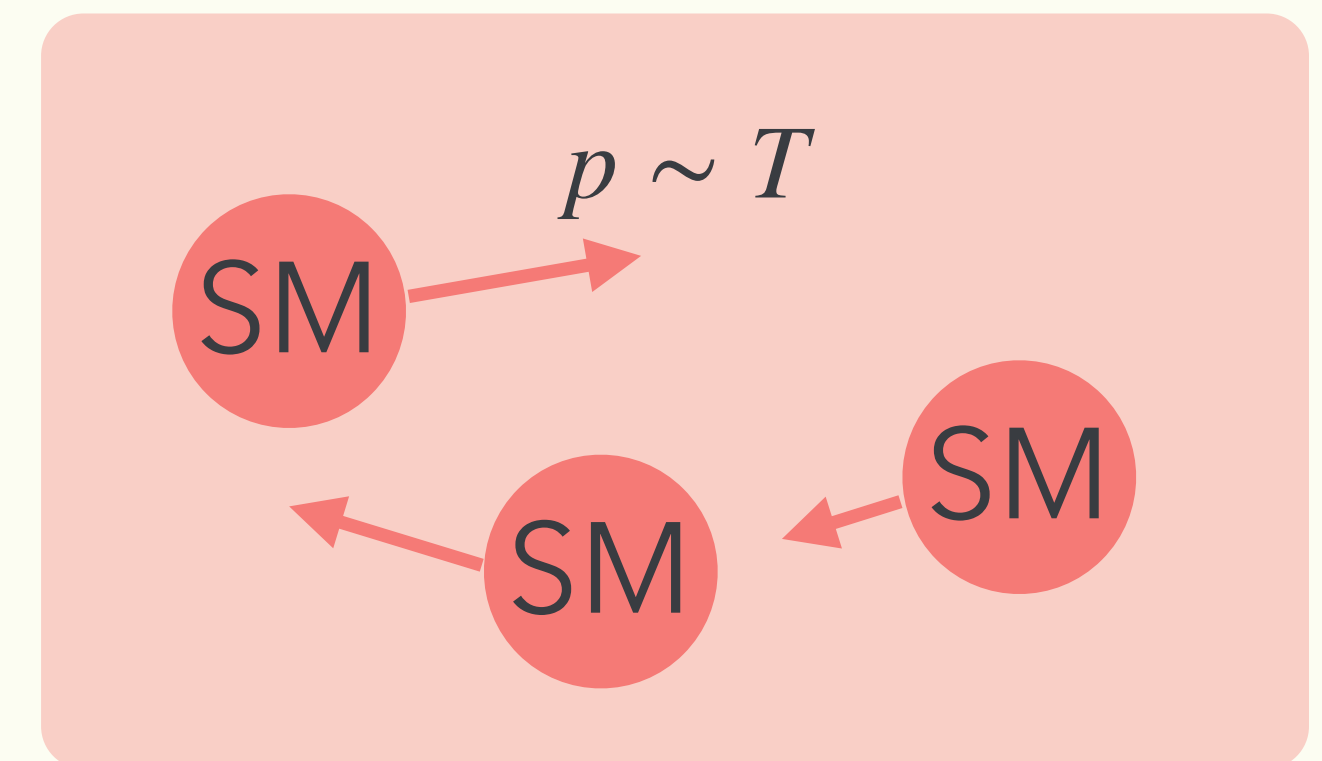
- In-medium cascades are the bottleneck



Inflaton decay



In-medium cascade



Thermal plasma

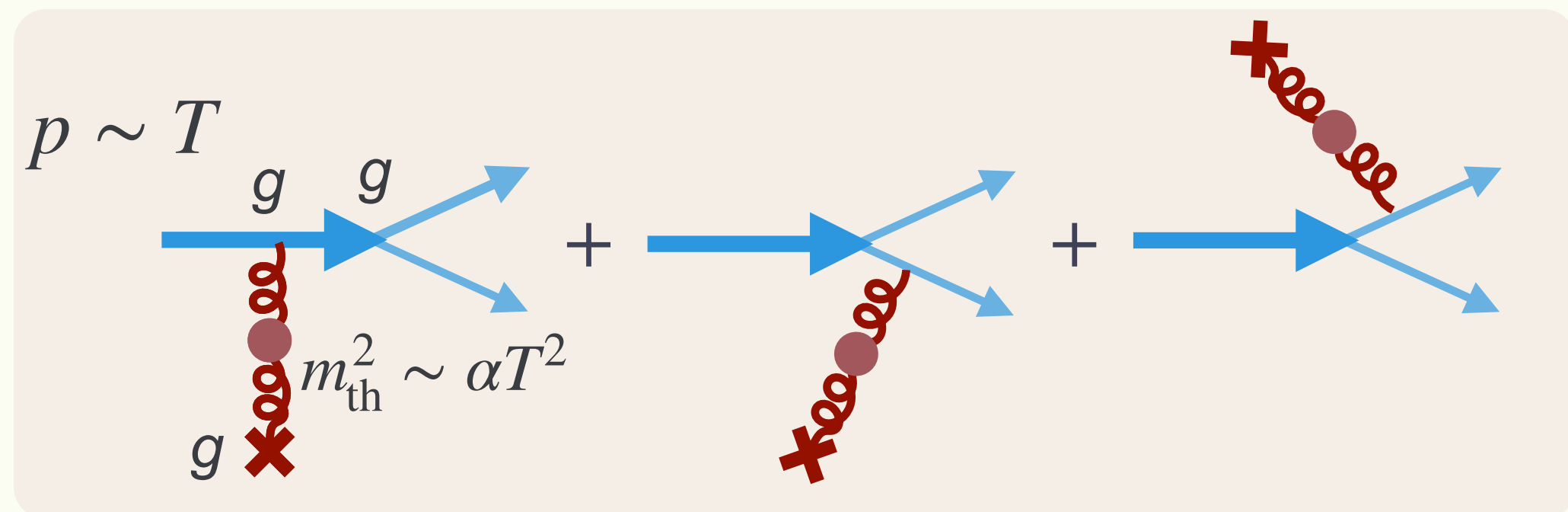
Thermalization during Reheating

via In-medium Cascades

In-medium Cascade of Energetic Particles

- Bethe–Heitler vs. Landau–Pomeranchuk–Migdal (LPM)

- Bethe–Heitler formula

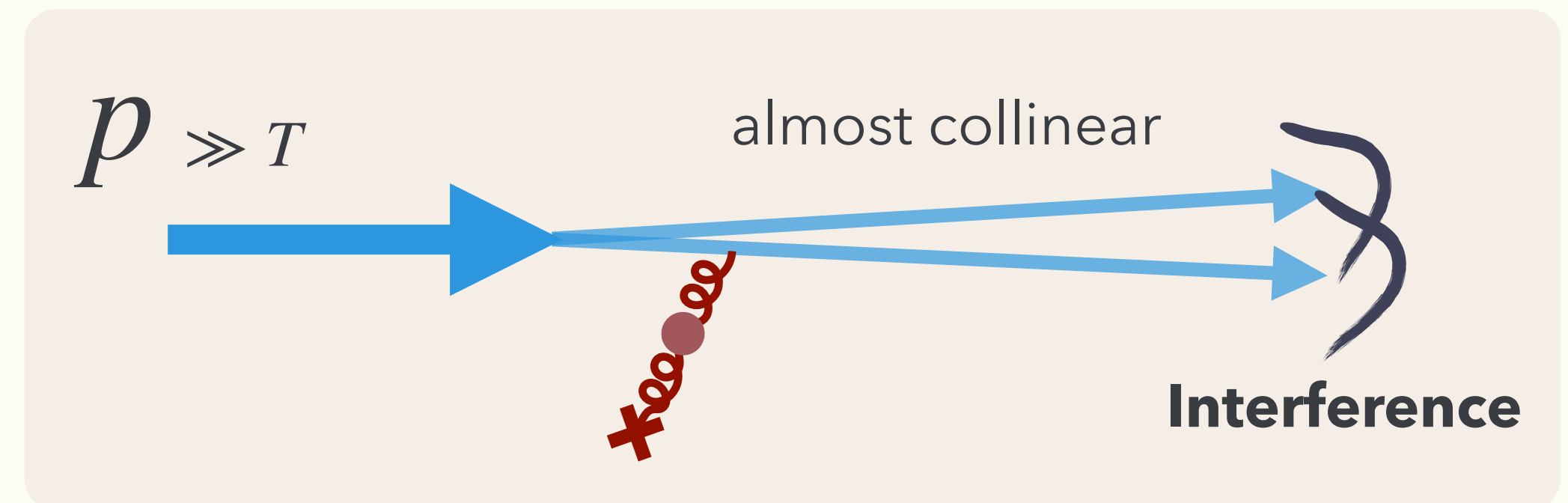


Scale-independent splitting

$$\Gamma_{\text{BH}} \sim \alpha^3 \frac{T^3}{m_{\text{th}}^2} \sim \alpha^2 T$$

- **LPM** suppression

[Landau, Pomeranchuk; Migdal]

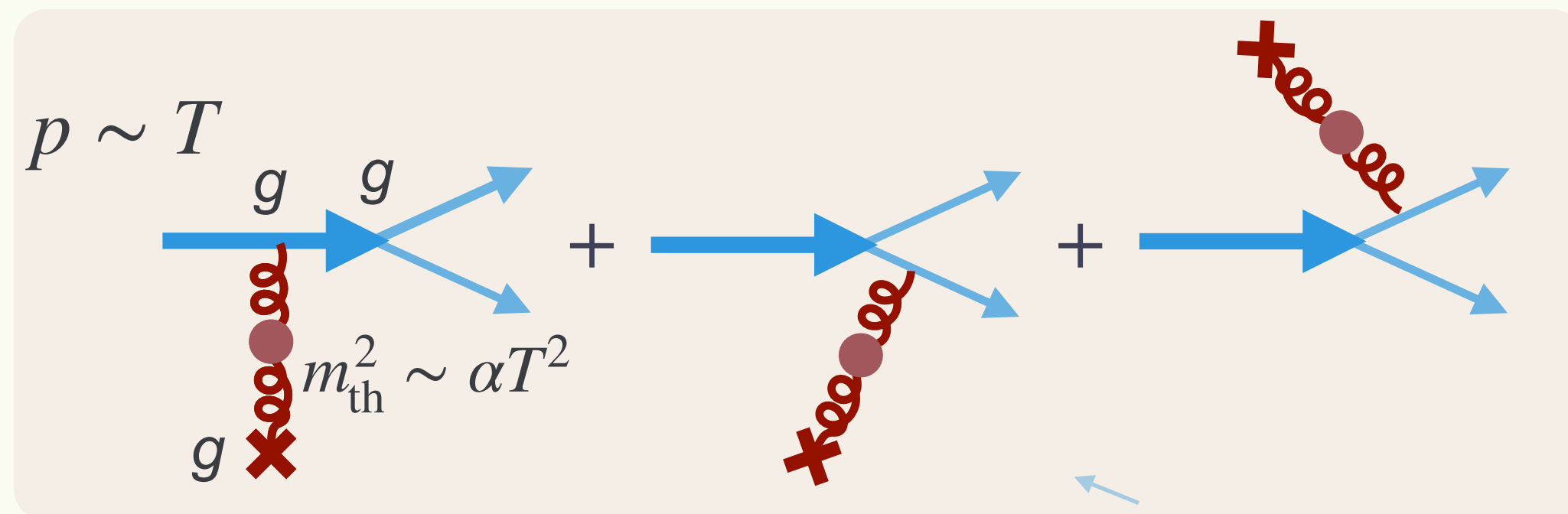


coherent multiple scatterings

In-medium Cascade of Energetic Particles

- Bethe–Heitler vs. Landau–Pomeranchuk–Migdal (LPM)

- Bethe–Heitler formula

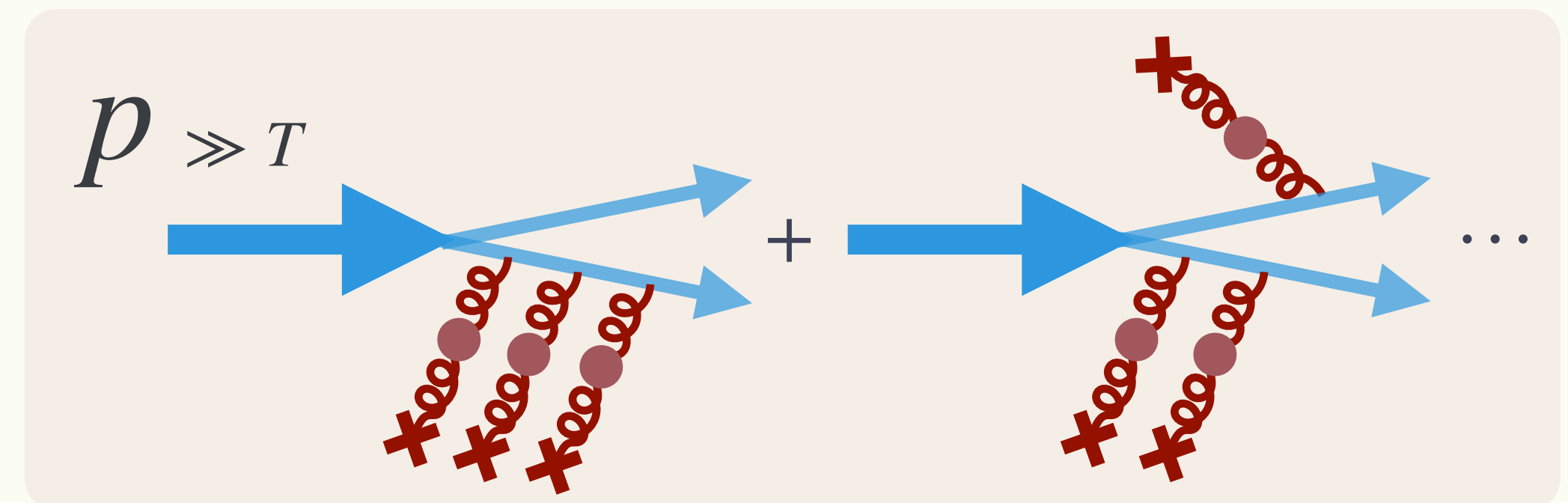


Scale-independent splitting

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[Landau, Pomeranchuk; Migdal]



coherent multiple scatterings

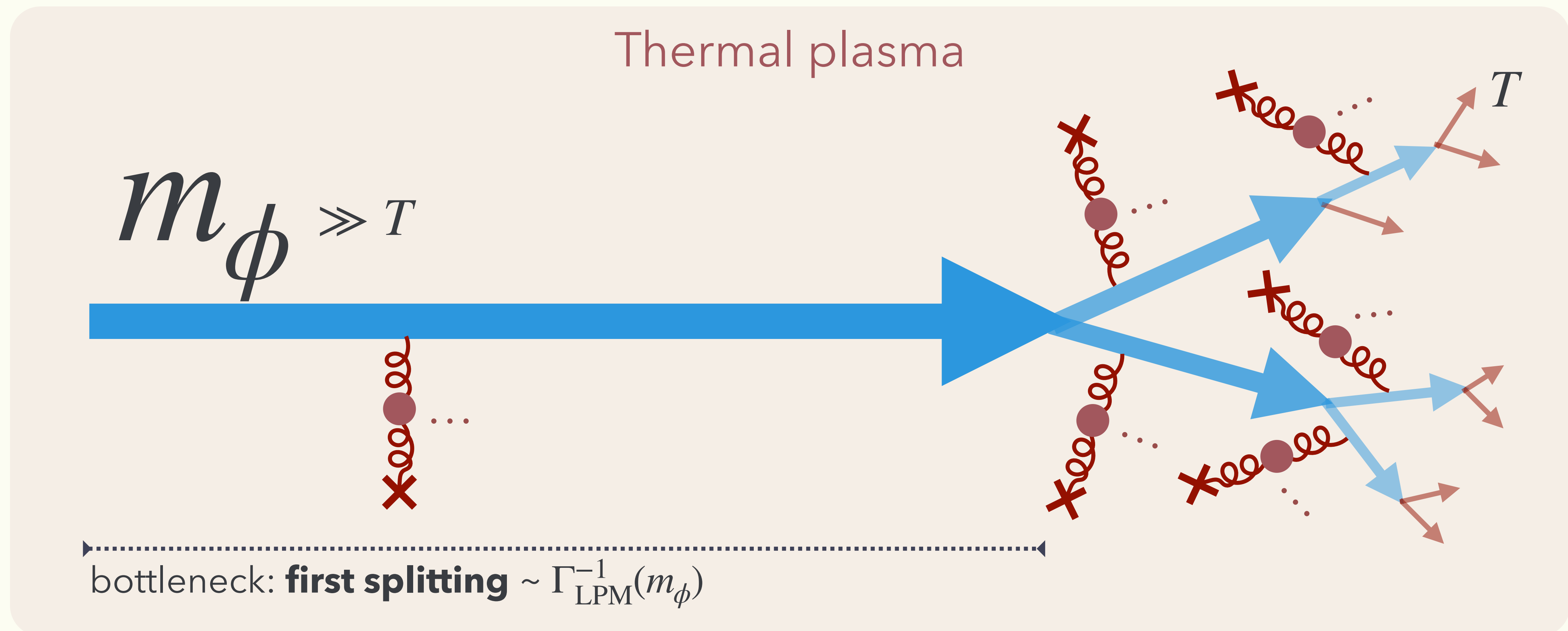
p-dependent splitting: **LPM suppression**

$$\Gamma_{\text{LPM}} \sim \alpha t_{\text{form}}^{-1}(p) \sim \alpha^2 T \times \sqrt{\frac{T}{p}}$$

[Arnold, Moore, Yaffe '01,'02,'03; Arnold, Dogan '08]

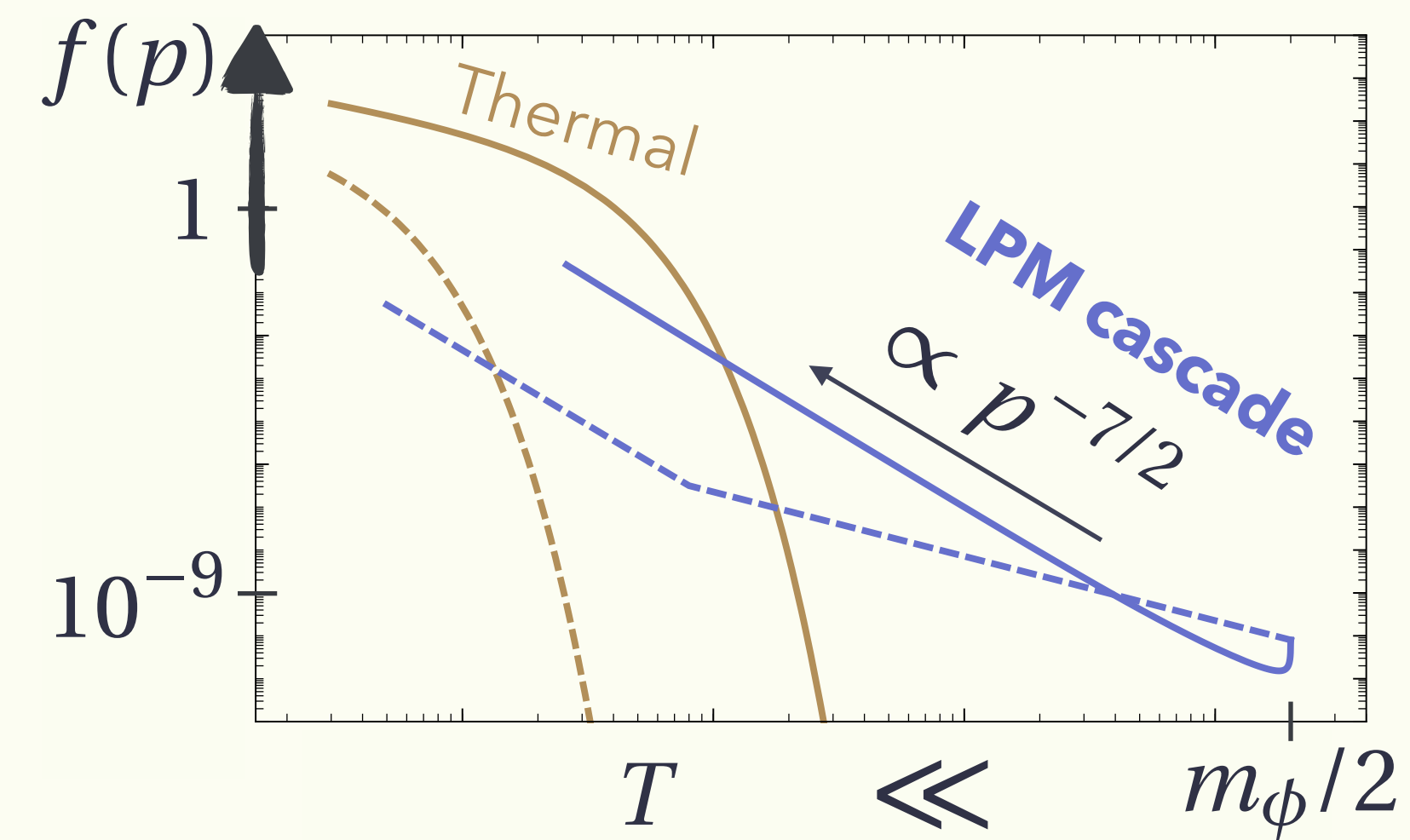
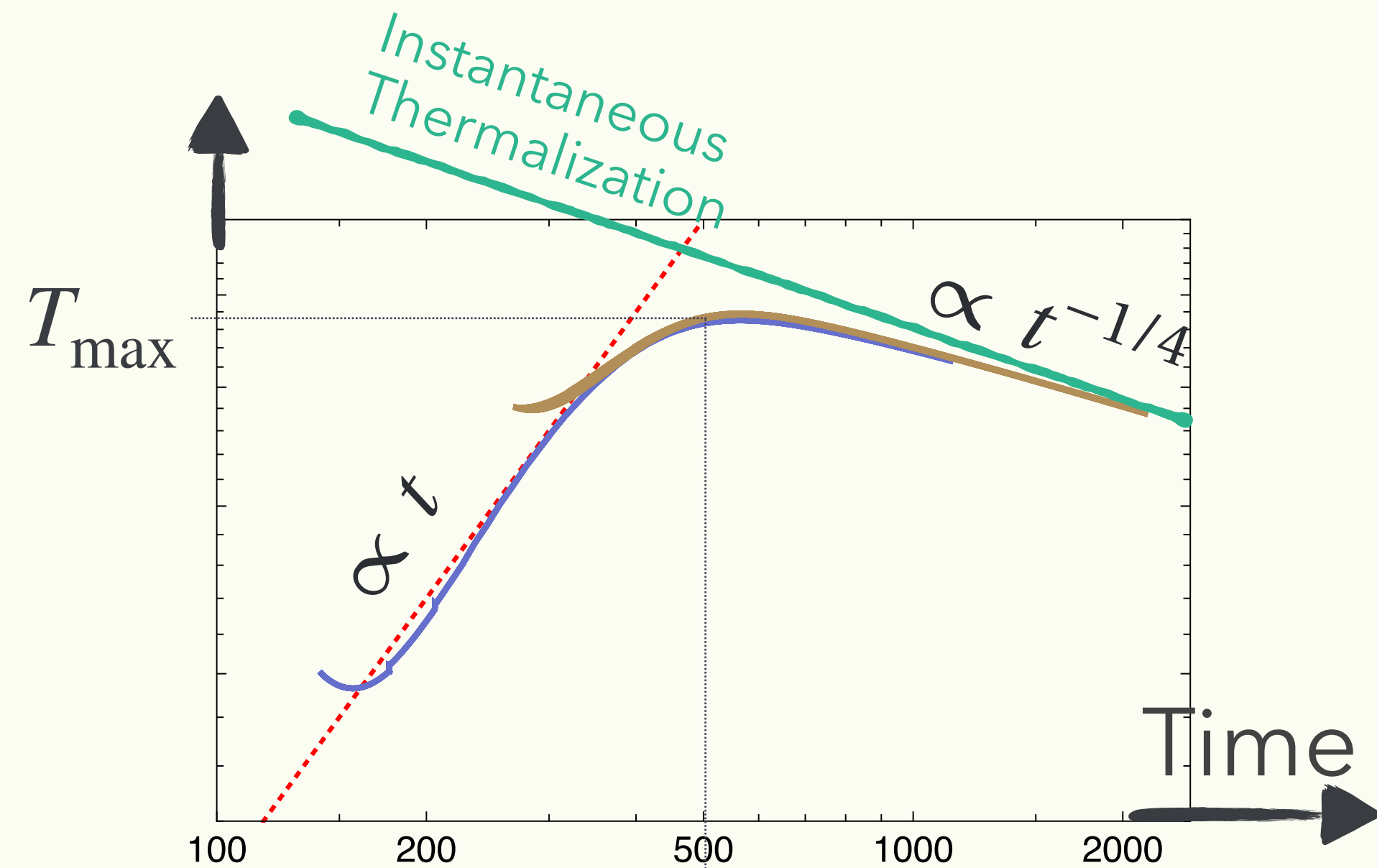
In-medium Cascade of Energetic Particles

- Cascade via LPM-suppressed multiple splittings



Does This Change the Evolution of the Universe?

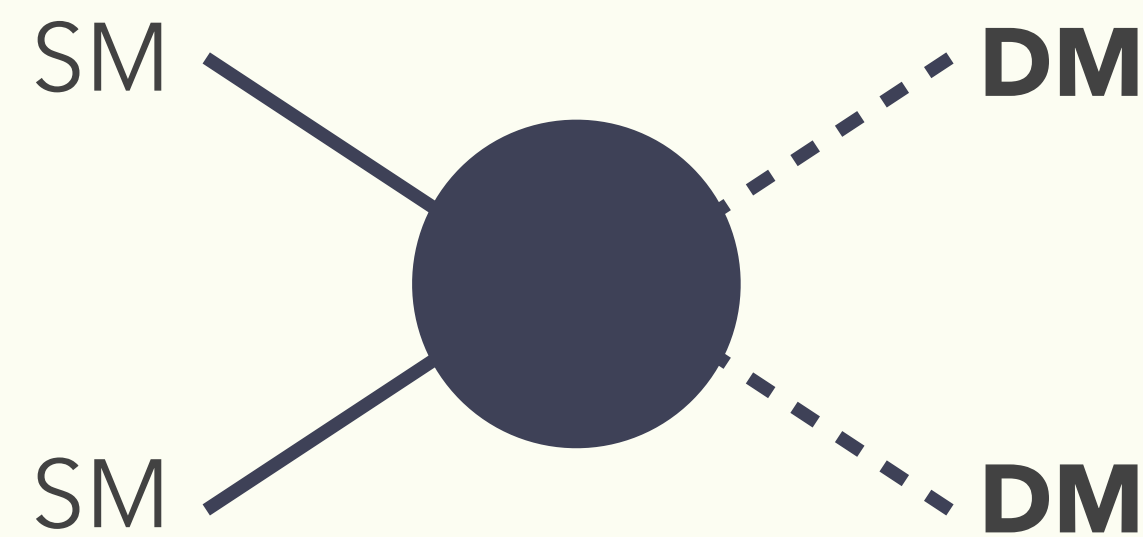
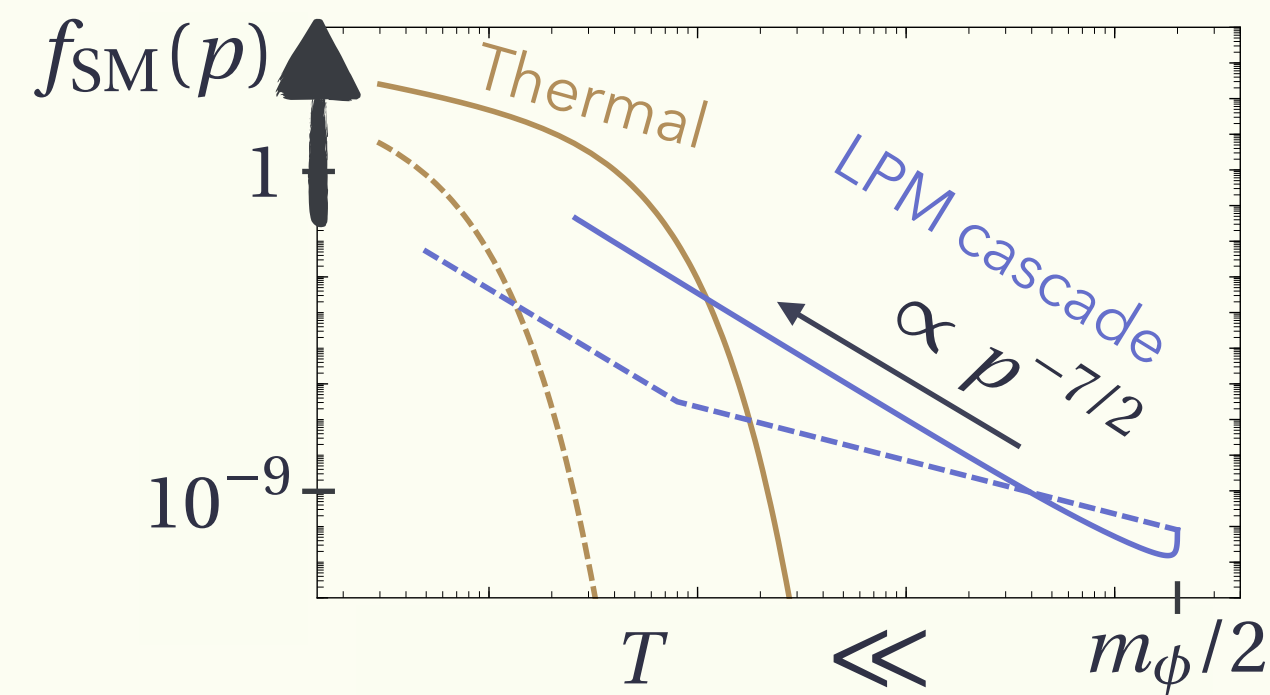
- Suppression of the Maximal Temperature



$$\Gamma_{\text{split}}(m_\phi) \lesssim H \quad \Gamma_{\text{split}}(m_\phi) \gtrsim H \quad \Rightarrow \quad \frac{T_{\text{max}}}{T_{\text{max}}^{(\text{inst})}} \sim 10^{-3} \left(\frac{\alpha}{0.1} \right)^{\frac{4}{5}} \left(\frac{T_R}{10^3 \text{ GeV}} \right)^{\frac{3}{10}} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^{-\frac{9}{20}}$$

Does This Change the Evolution of the Universe?

- Non-thermal Heavy Dark Matter Production



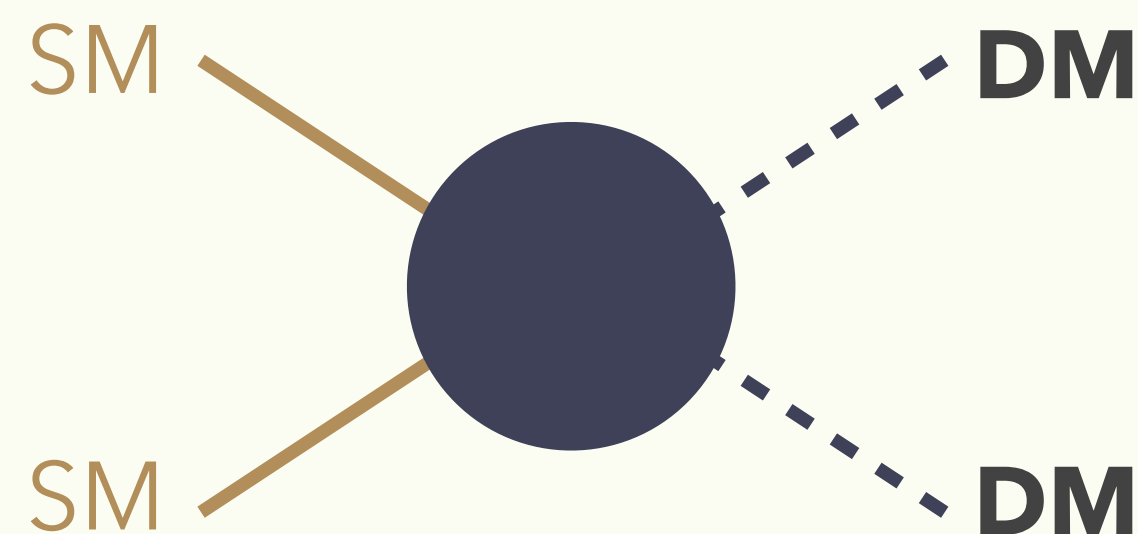
E.g. $W W \rightarrow \text{DM DM}$

$$\langle \sigma v \rangle \simeq \frac{\alpha_{\text{DM}}^2}{m_{\text{DM}}^2} \quad \Rightarrow \quad \frac{\rho_{\text{DM}}}{s} \simeq 0.11 \alpha_{\text{DM}}^2 \frac{T_{\text{R}}^3}{m_{\text{DM}}^2}$$

"Universal" production

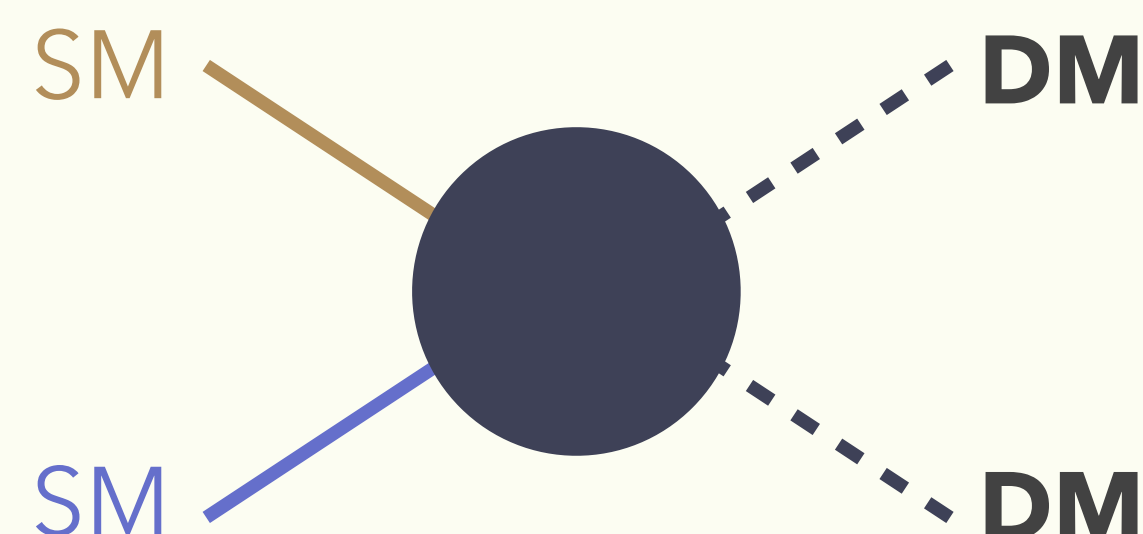
Thermal + Thermal

[Giudice, Kolb, Riotto '01]



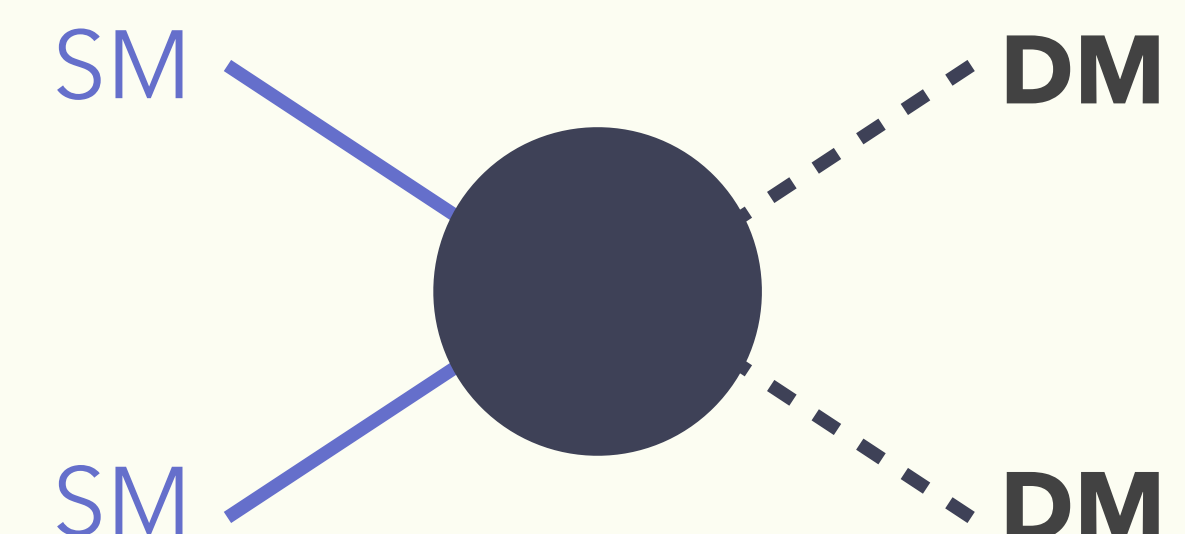
Thermal + LPM

[KM+ 1402.2846; Drees, Najjari '21]



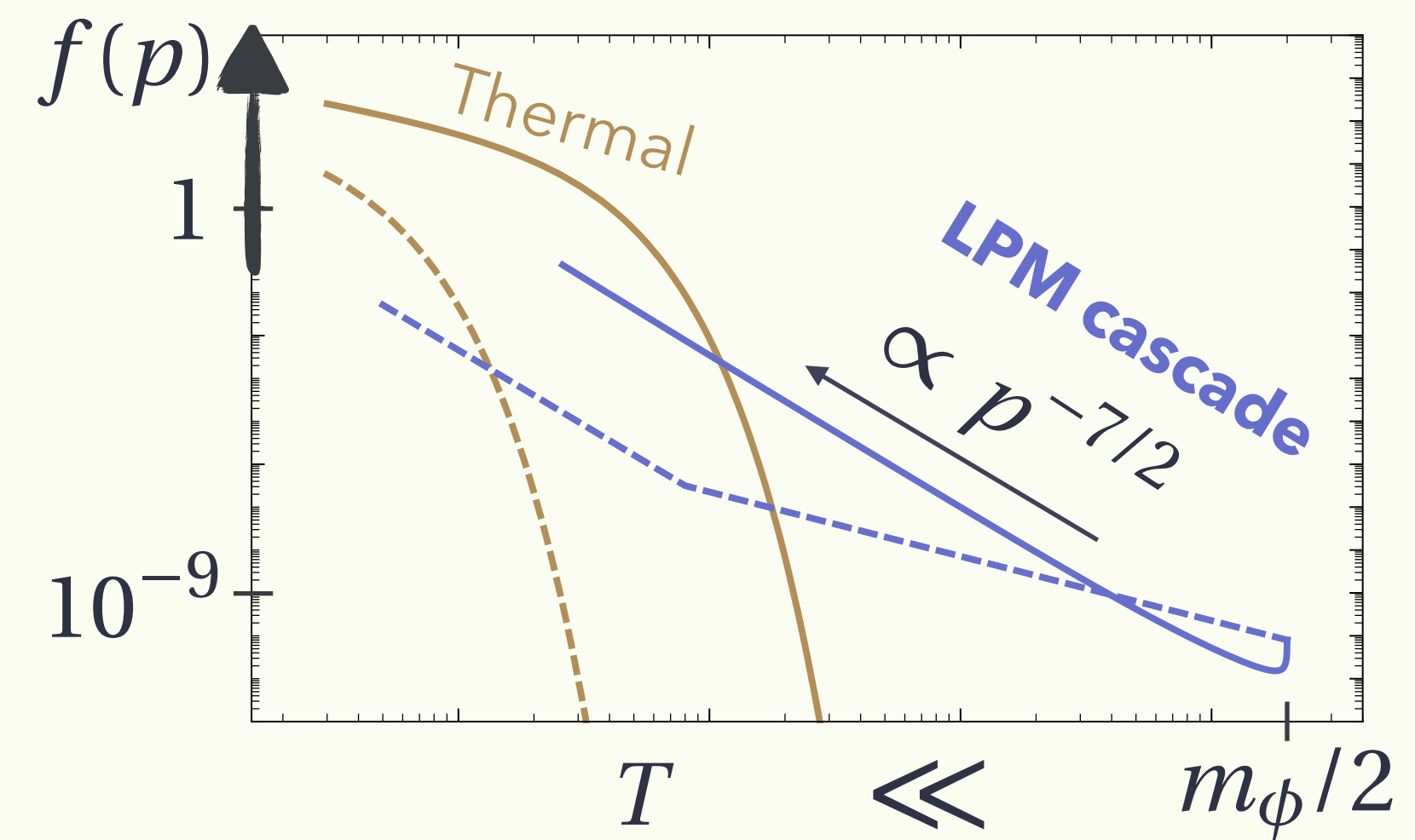
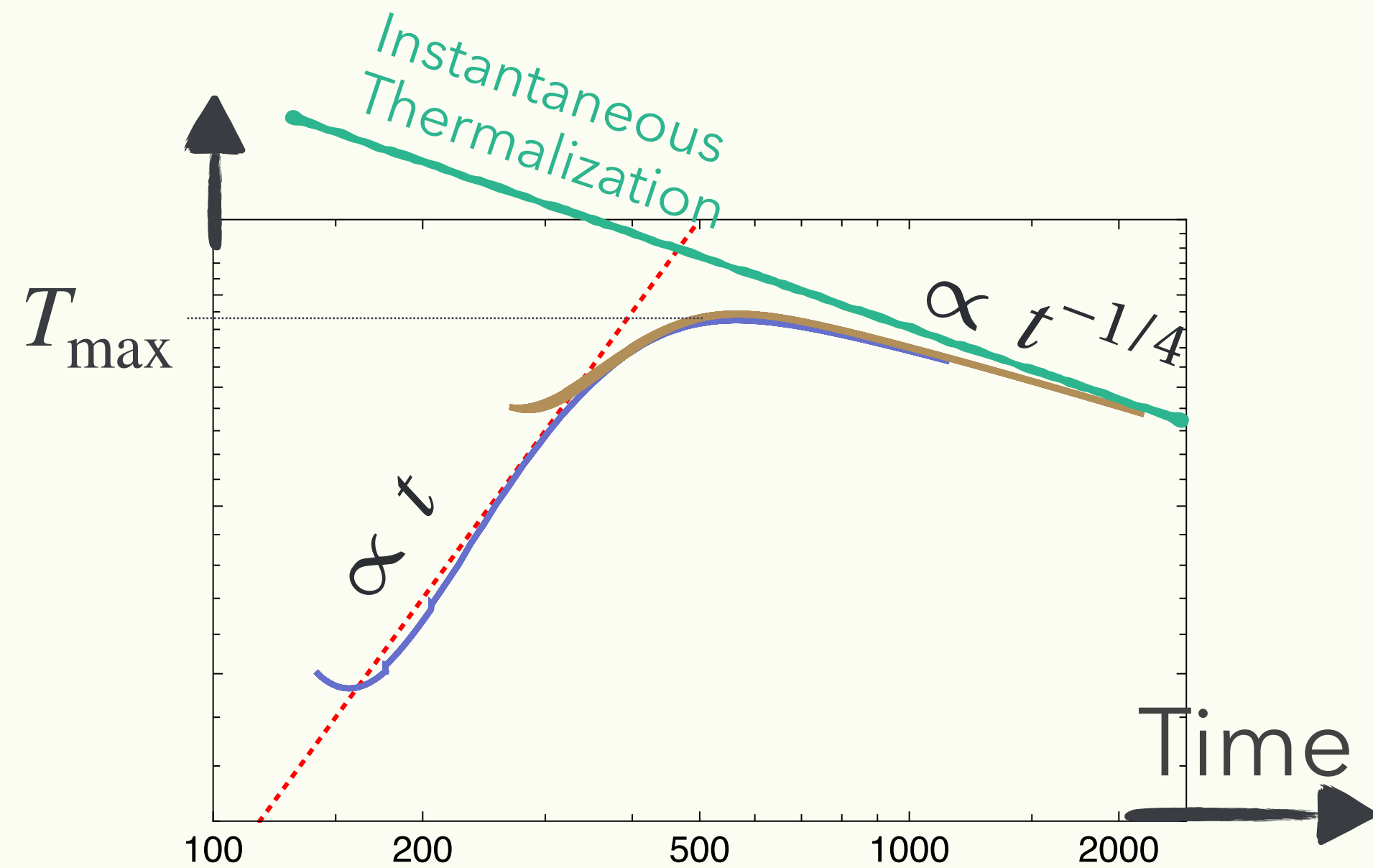
LPM + LPM

[KM+ 1901.11027; Garcia, Amin '18]

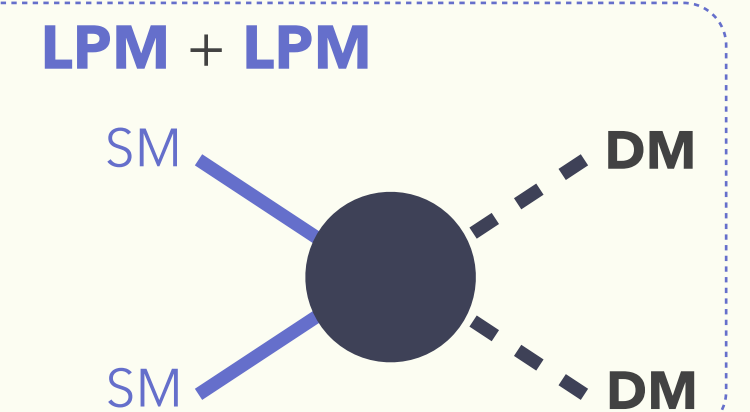
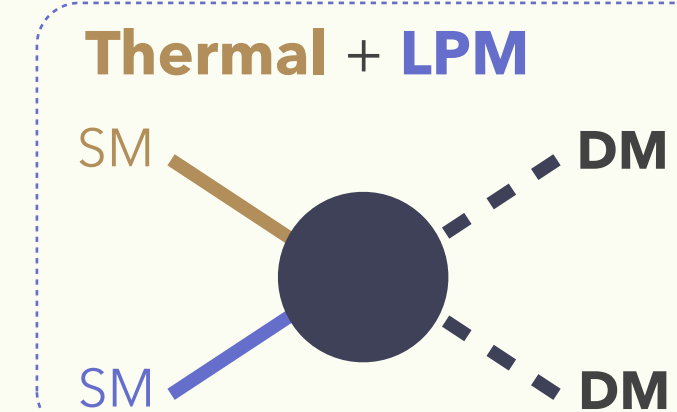
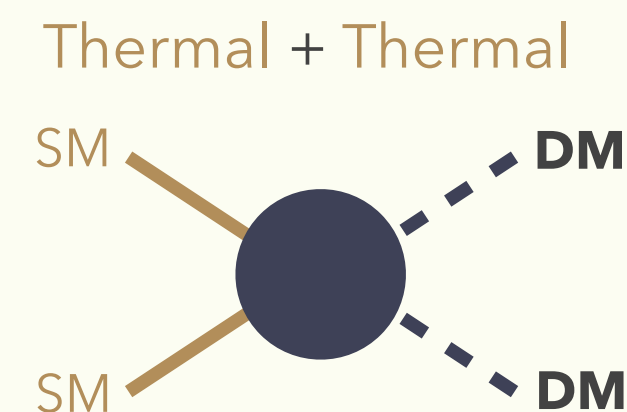


Thermalization & Reheating

- Implications: Suppression of the maximal T & DM production



$$\frac{T_{\max}}{T_{\max}^{(\text{inst})}} \sim 10^{-3} \left(\frac{\alpha}{0.1} \right)^{\frac{4}{5}} \left(\frac{T_R}{10^3 \text{ GeV}} \right)^{\frac{3}{10}} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^{-\frac{9}{20}}$$



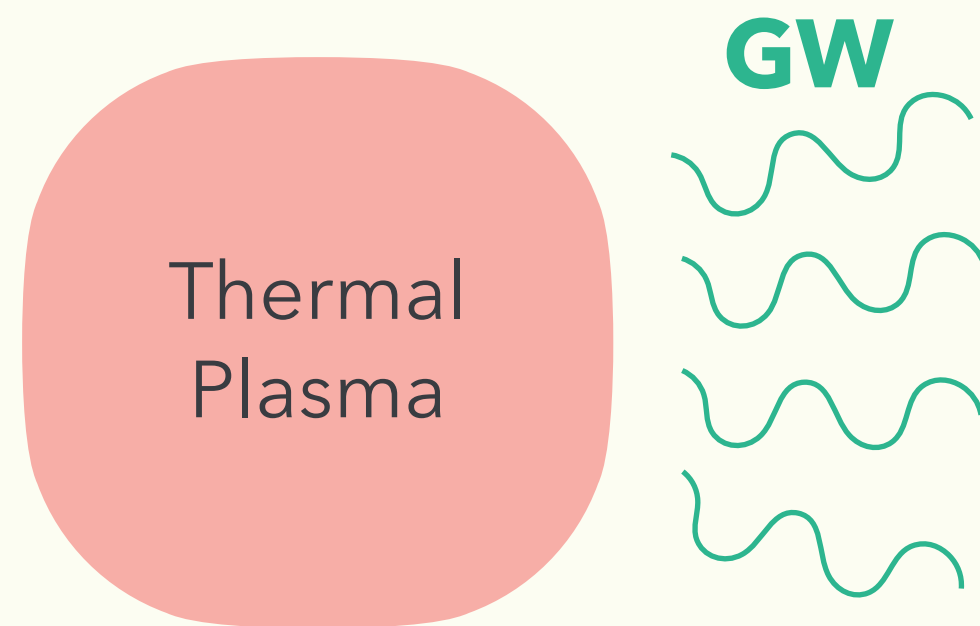
High-Frequency GWs

& In-medium Cascades

High-Frequency GWs as a Probe of Reheating

- Two generic, often dominant, contributions in the literature

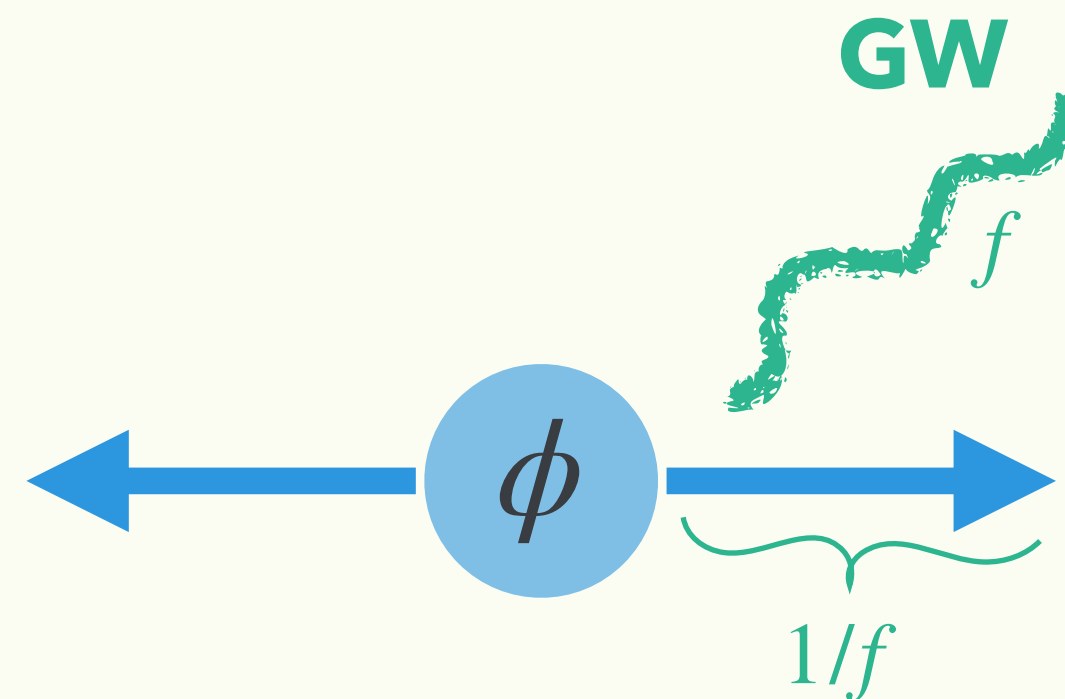
Inevitable Thermal GWs



Cosmic **Gravitational**
Microwave Background

[Ghiglieri+ 1504.02569, 2004.11392; ...;
Bernal, Xu 2410.21385]

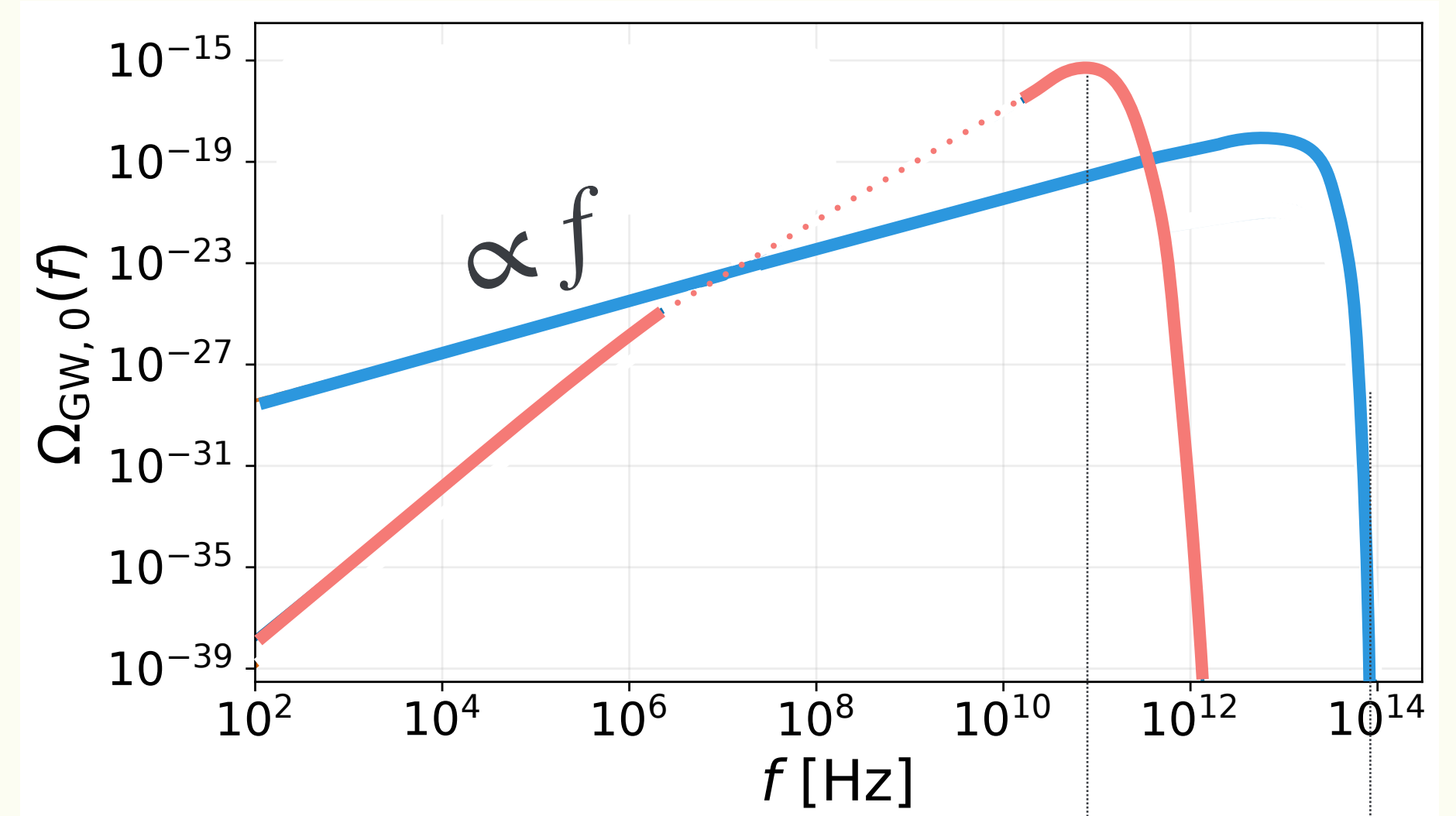
GW Emission @ Reheating



Graviton brems @ vacuum
persistent directional flow $\propto f$

[Nakayama, Tang 1810.04975; ...;
Bernal+, 2311.12694; Cline, Xu 2605.16201]

[See e.g., Xu+ 2505.08868]

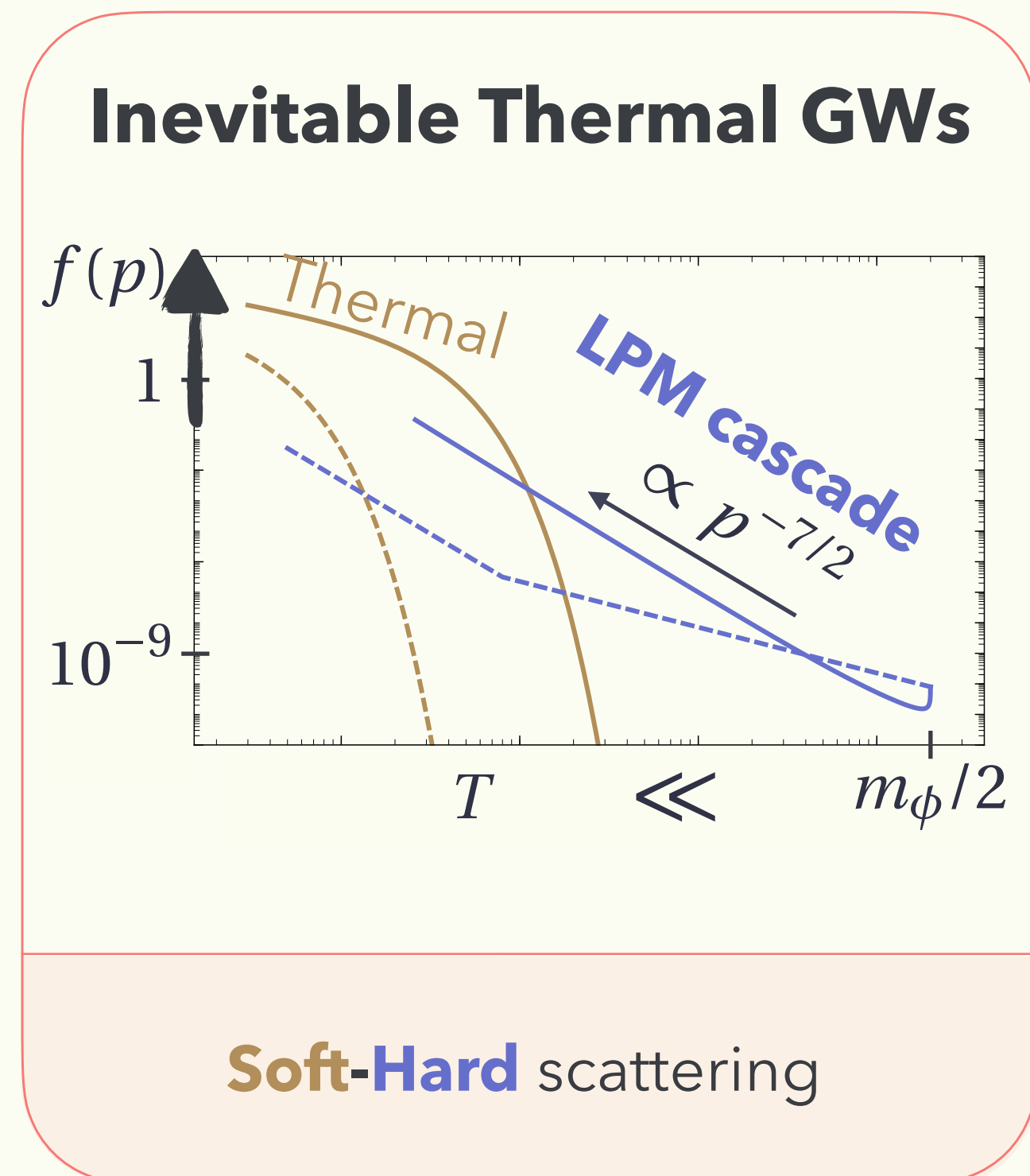


80 GHz

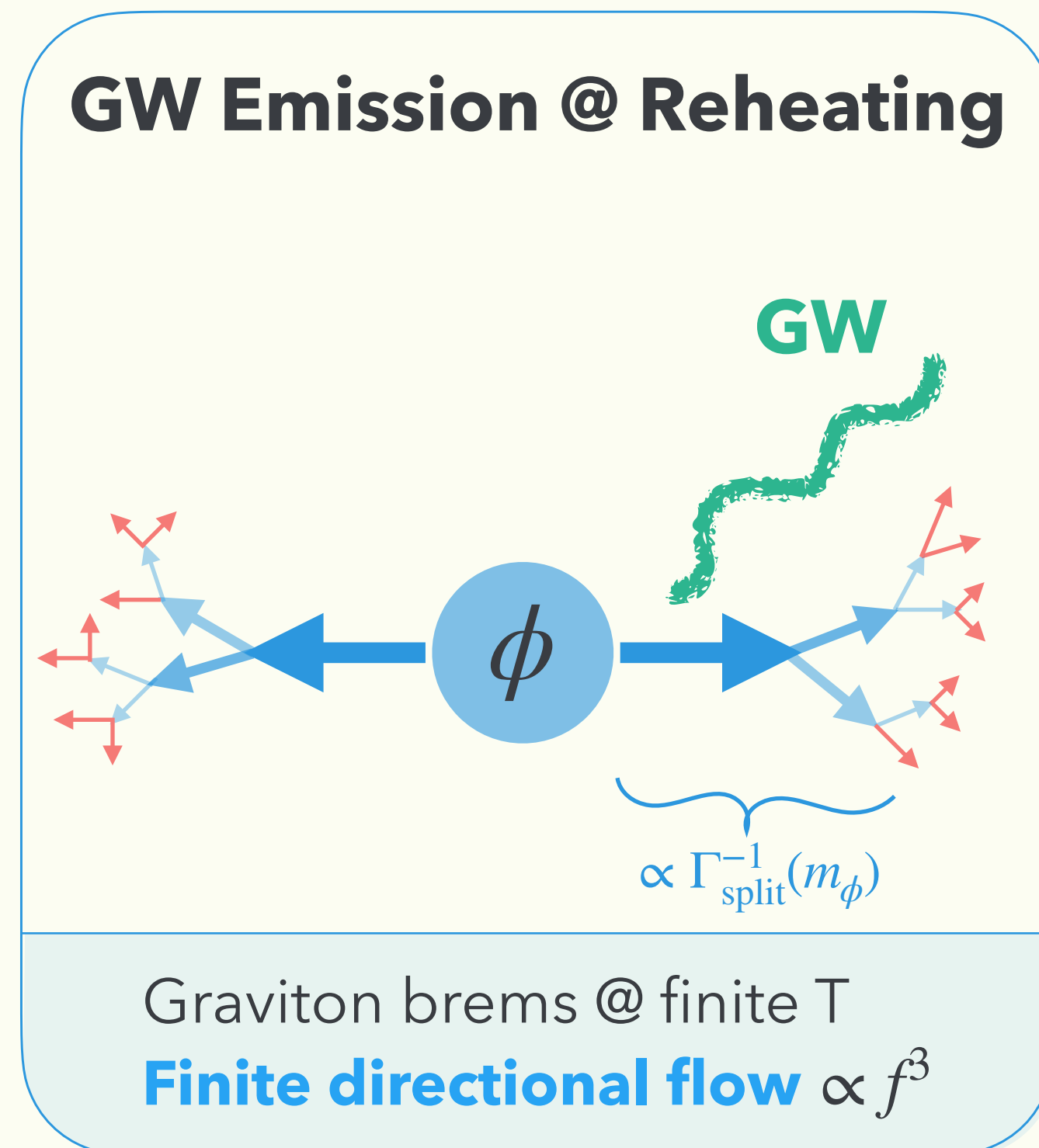
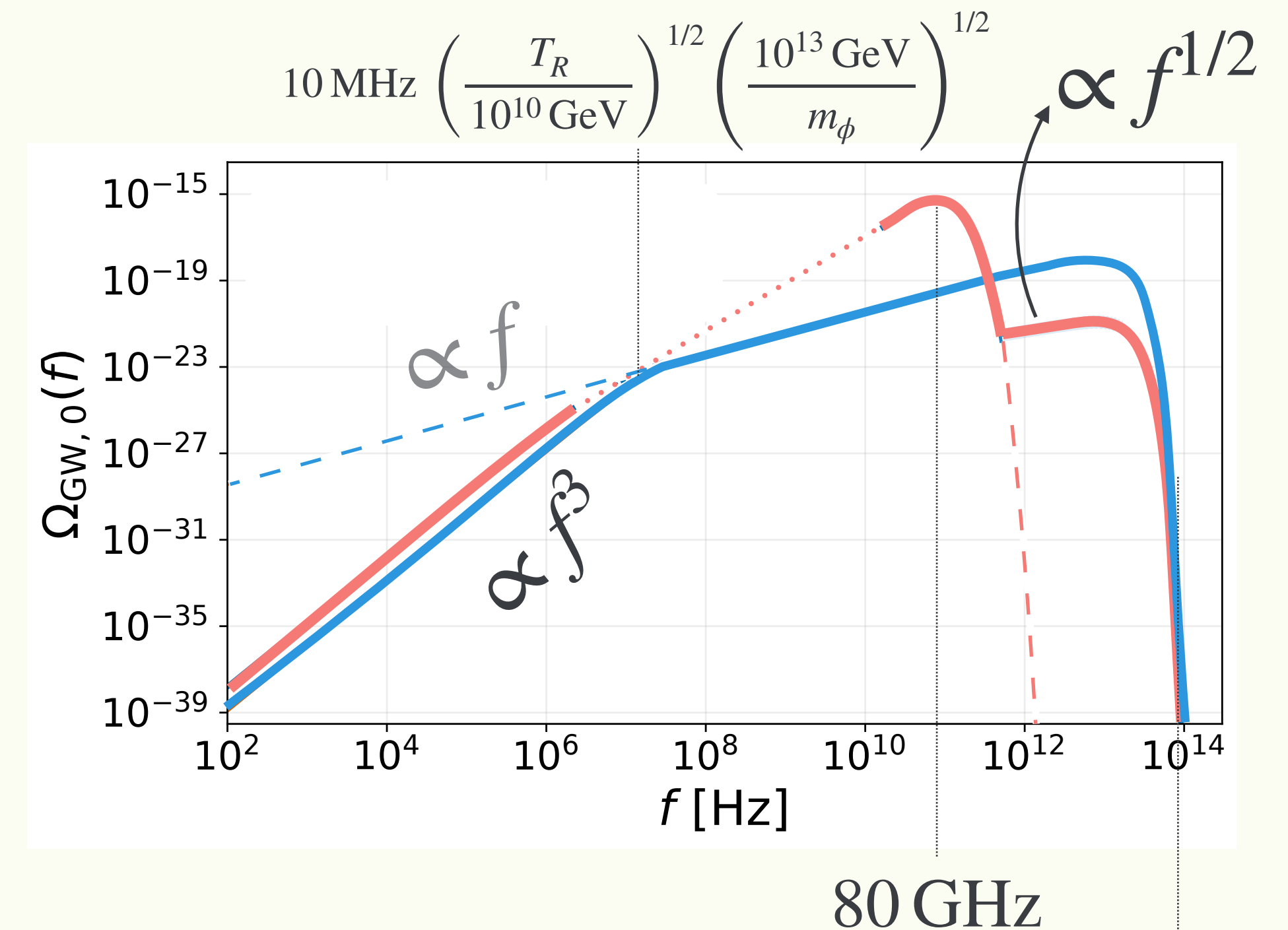
$$10^5 \text{ GHz} \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^{-1} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)$$

High-Frequency GWs as a Probe of Reheating

- In-medium cascades change the spectrum qualitatively



[KM, Tsuji 2609.01480]

[KM, Tsuji 2609.01480;
see also Murayama+ 2608.24994]

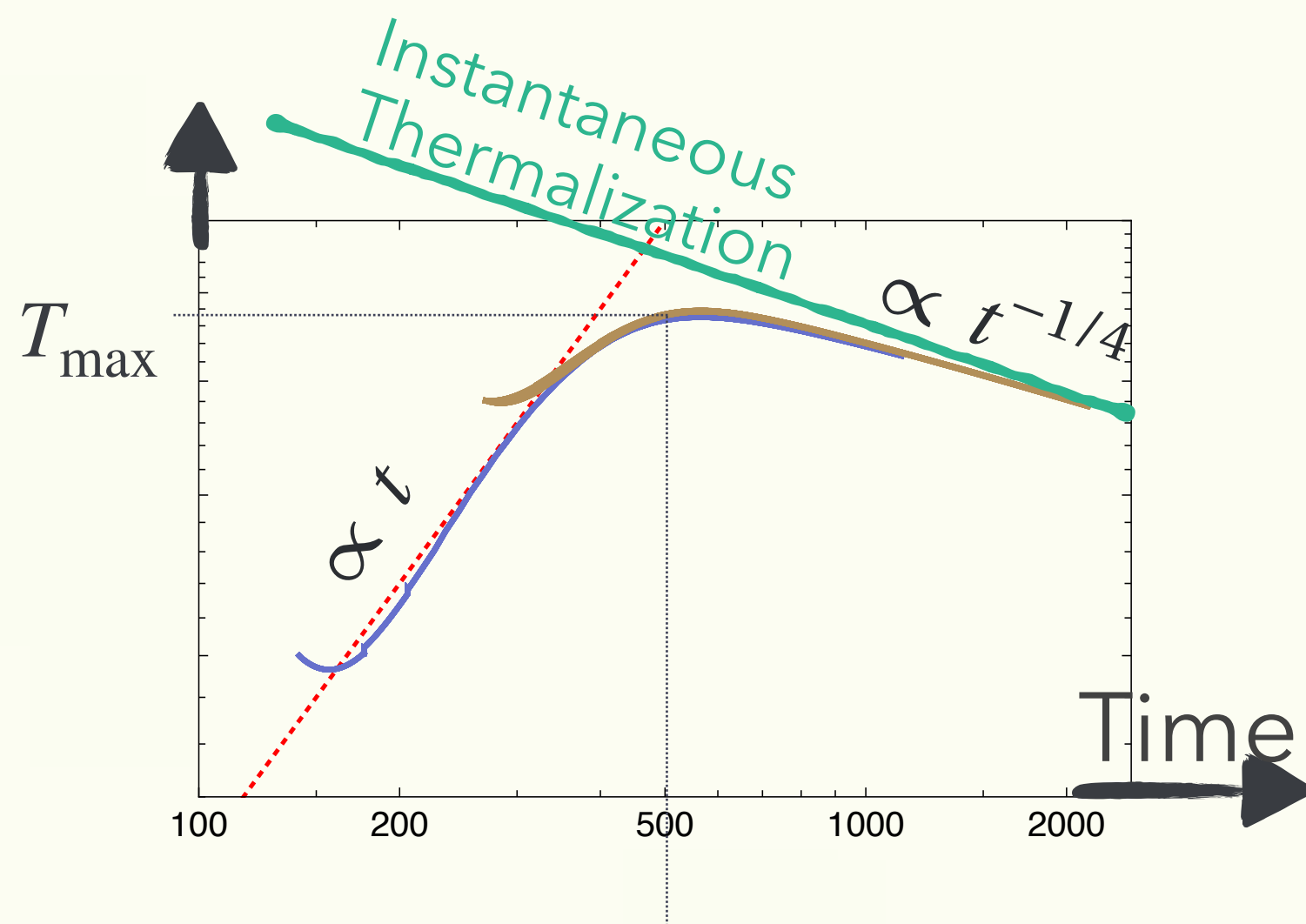
$$10^5 \text{ GHz} \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^{-1} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)$$

Summary

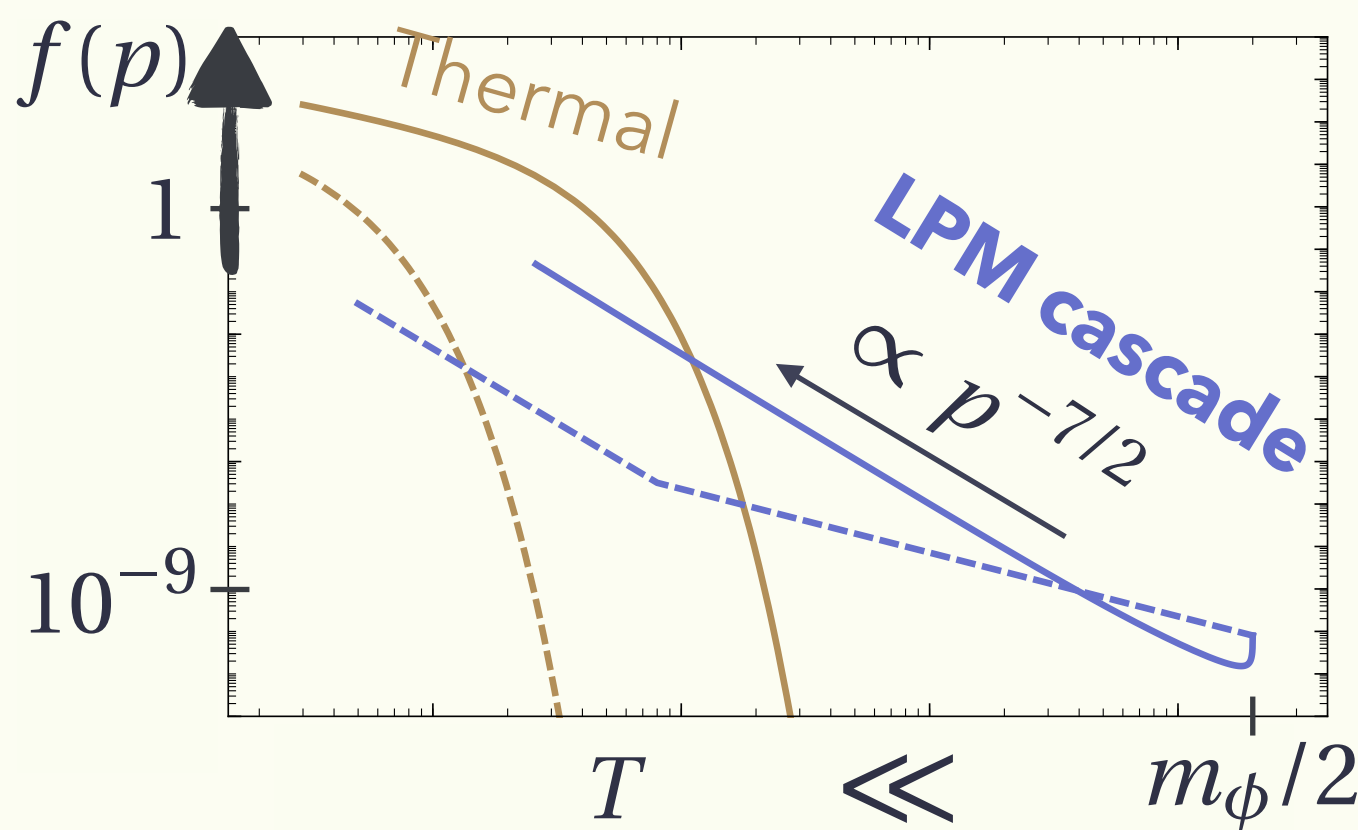
- Thermalization during Reheating & Its Implications

[KM, Tsuji 2609.01480]

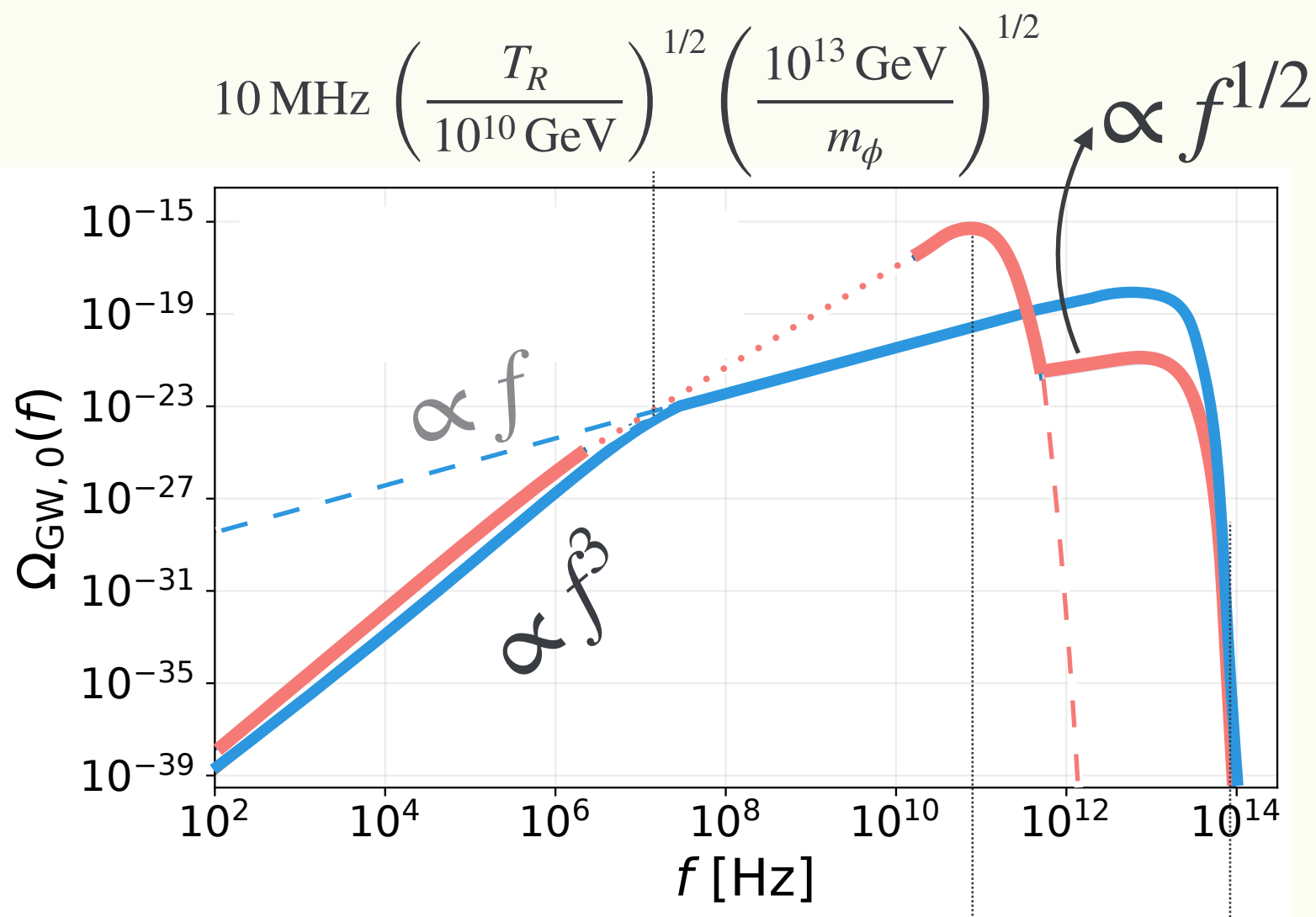
- **Suppression of $T_{\max}$**



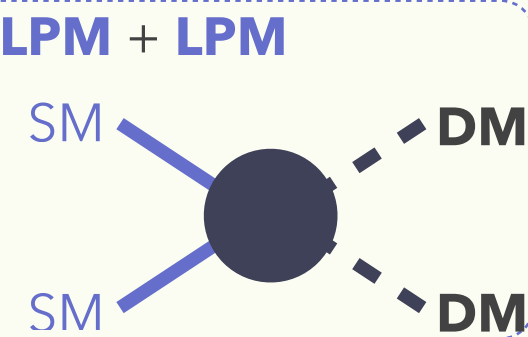
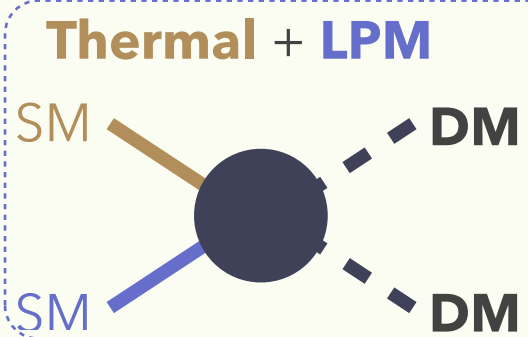
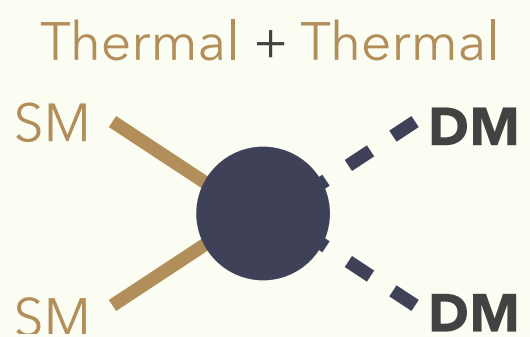
- **DM production during reheating**



- **Cascades leave imprints on HFGWs**



$$\frac{T_{\max}}{T_{\max}^{(\text{inst})}} \sim 10^{-3} \left(\frac{\alpha}{0.1} \right)^{\frac{4}{5}} \left(\frac{T_R}{10^3 \text{ GeV}} \right)^{\frac{3}{10}} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)^{-\frac{9}{20}}$$



80 GHz

$$10^5 \text{ GHz} \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^{-1} \left(\frac{m_\phi}{10^{13} \text{ GeV}} \right)$$

Backup

Effective Kinetic Theory of SM

- LPM-suppressed splitting function for SM

[**KM**, Yamada 2208.11708;
Arnold, Morre, Yaffe '01,'02,'03; Arnold, Dogan '08]

- Kinetic equation

$$\mathcal{L} f_s(\mathbf{x}, \mathbf{p}, t) = \mathcal{S} + \mathcal{C}_{2 \leftrightarrow 2}[f_s] + \mathcal{C}_{"1 \leftrightarrow 2"}[f_s]$$

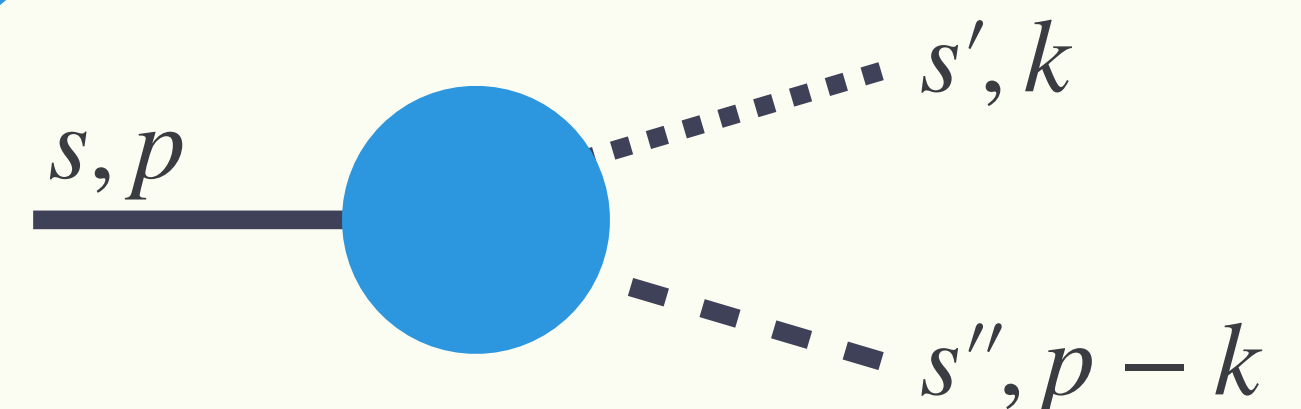
w/ $s = e_f, L_f, u_f, d_f, Q_f, \phi, B, W, g$

Source Elastic scat. **LPM-suppressed**
e.g., inflaton, PBHs, ... splitting

- **Splitting function** for effective "1 to 2" processes

$$\mathcal{C}_{"1 \leftrightarrow 2"}[f_s] \supset -\frac{(2\pi)^3}{p^2 v_s} \sum_{s', s''} \int_0^p dk \gamma_{s \leftrightarrow s' s''}(p; k, p-k) f_s(p) \\ + (\text{inverse})$$

Splitting function



Effective Kinetic Theory of SM (cont'd)

- LPM-suppressed splitting function for SM

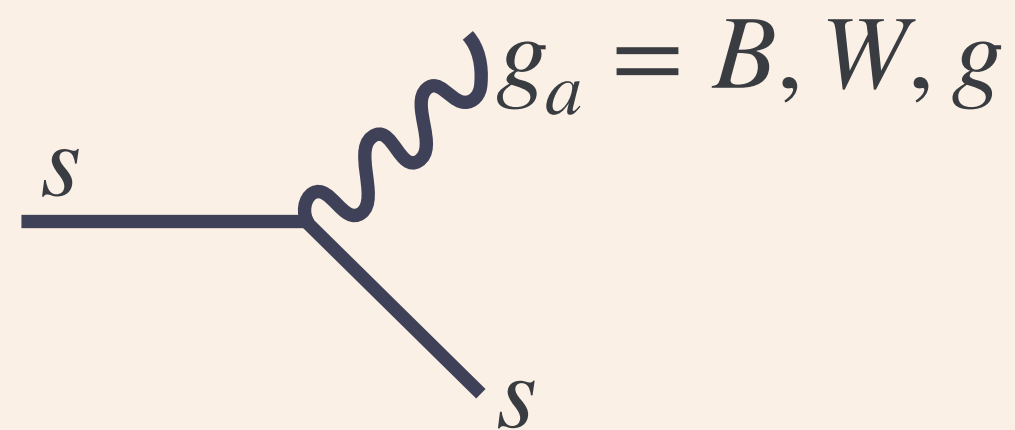
[**KM**, Yamada 2208.11708;
Arnold, Morre, Yaffe '01,'02,'03; Arnold, Dogan '08]

- SM splitting function

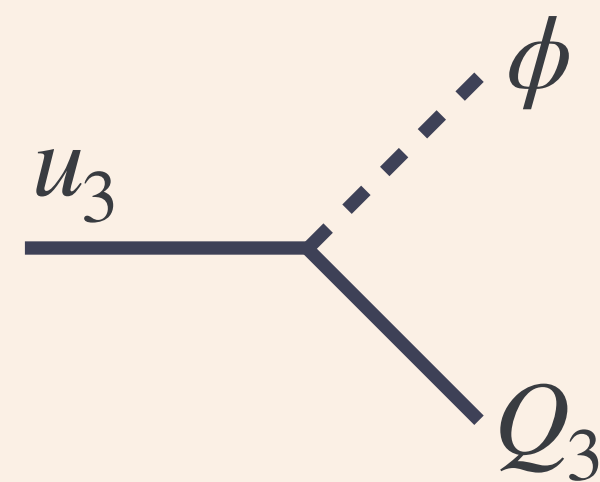
$$\gamma_{s \rightarrow s's''}(p; xp, (1-x)p) = \frac{1}{2} \frac{\alpha_{ss's''}}{(2\pi)^4 \sqrt{2}} \times \frac{P_{s \rightarrow s's''}^{(\text{vac})}(x)}{x(1-x)} \times \mu_{\perp}^2(1, x, 1-x; s, s', s'')$$

vacuum DGLAP splitting function

SM gauge + top Yukawa

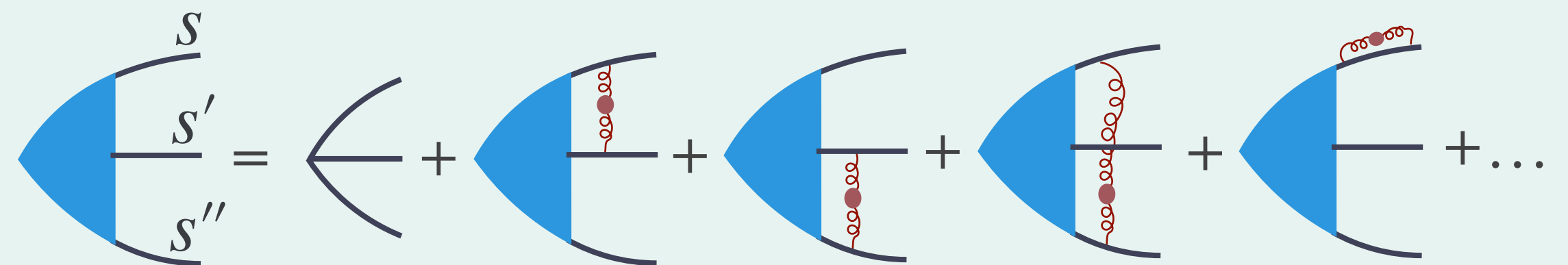


$$\alpha_{sg_a s} = d_{\mathbf{R}_s}^{(a)} C_{\mathbf{R}_s}^{(a)} \alpha_a$$



$$\alpha_{u_3 \phi Q_3} = y_t^2 / (4\pi)$$

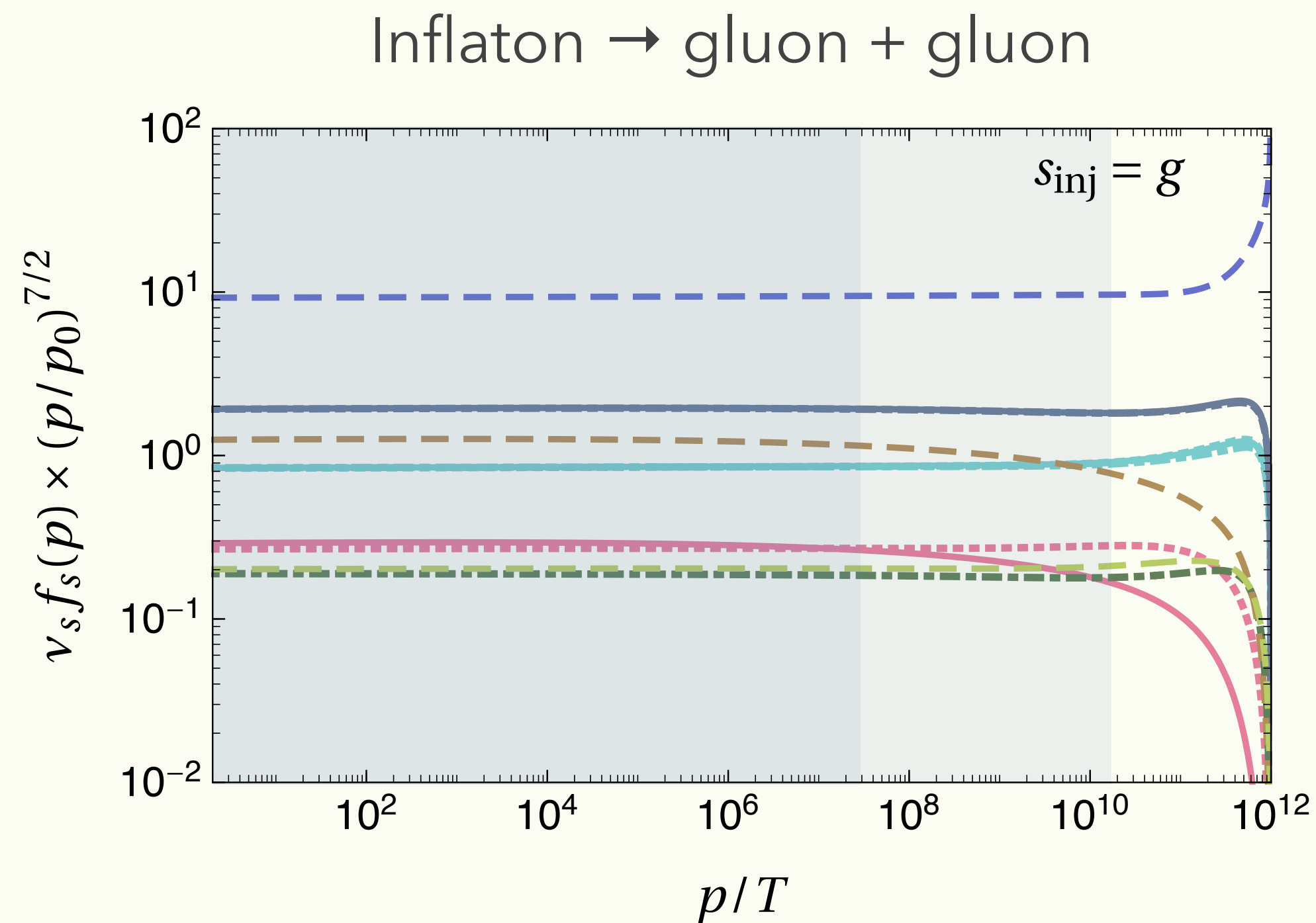
LPM suppression



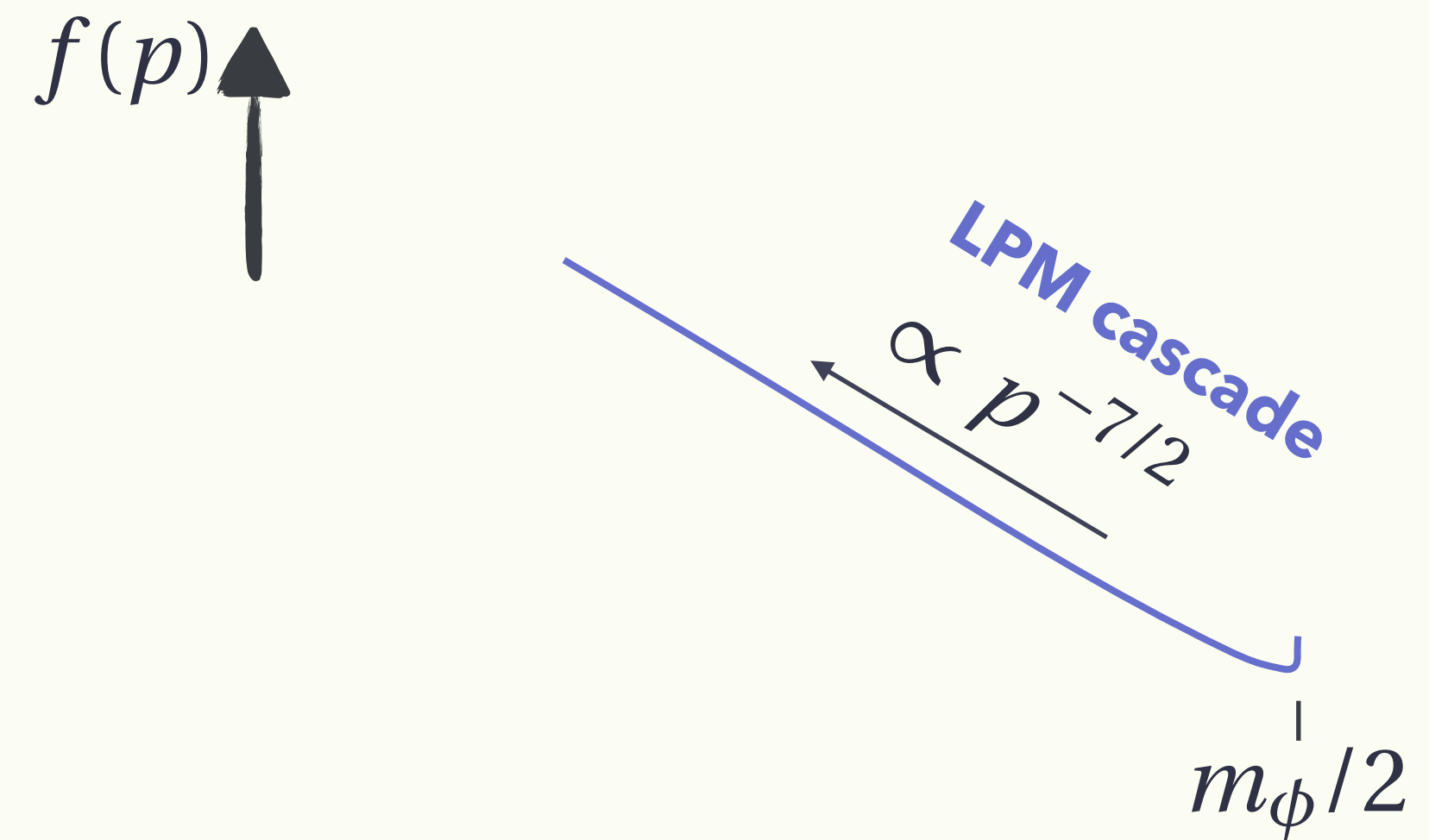
i.e., self-consistent equation for vertex func.

Does This Change the Evolution of the Universe?

- Universality in the Composition of LPM cascade



[**KM**, Yamada, 2208.11708]



Universality in the Spectrum

[KM, Yamada 2208.11708]

