Stabilizing dark matter with quantum scale symmetry

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Mainly based on
Phys.Rev.D 112 (2025) 11, 115024 (arXiv: 2505.02803)
Phys.Rev.D 114 (2026) 1, 016020 (arXiv:2604.03033)
arXiv:2609.06750

in collaboration with

A.Chikkaballi, W.Kotlarski, K.Kowalska, R.R.Lino dos Santos,
G.Giacometti, D.Rizzo, D.Zappalà

National Science Centre Poland

Particle Physics in the Early Universe

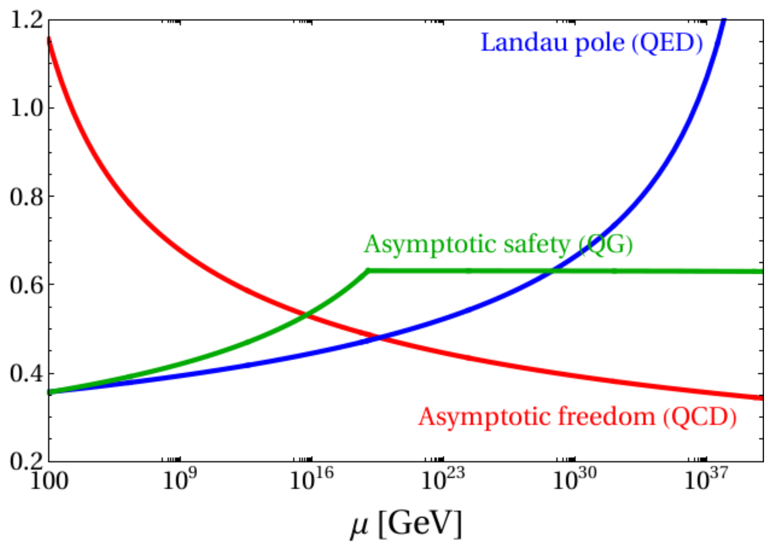
Paris, 15.09.2026

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Quantum scale symmetry (RG fixed points)

Hints from the UV for IR model building

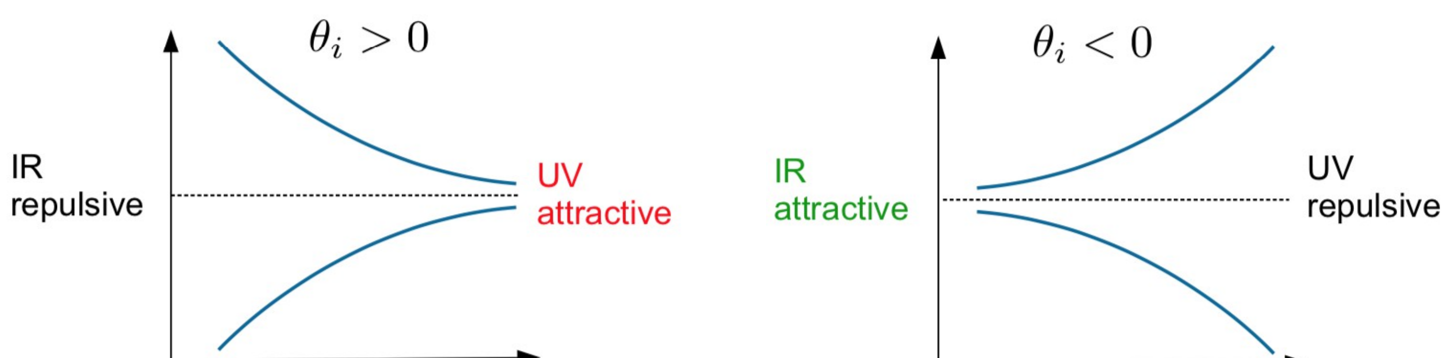
See talk by K. Kowalska



UV interactive FP
(Weinberg '79)

to describe dynamics @FP : $M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}} \longrightarrow \{-\theta_i\}$

stability matrix — critical exponents



Relevant couplings are
<u>free parameters</u> of the theory

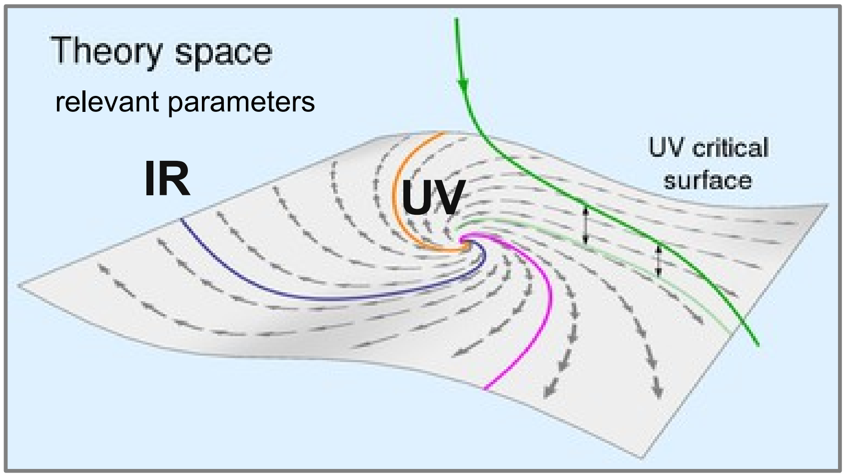
Irrelevant couplings provide
<u>predictions</u>

Quantum scale symmetry (RG fixed points)

Hints from the UV for IR model building

See talk by K. Kowalska

from Wikipedia (created by Andreas Nink)



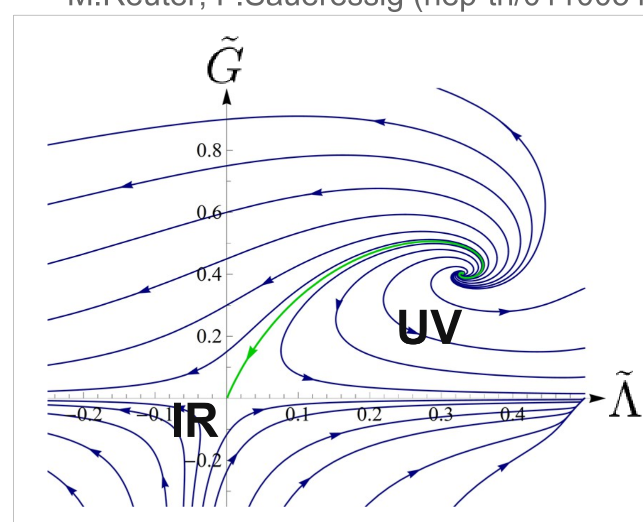
nonperturbative renormalizability

(critical surface has finite dimensions)

asymptotically safe quantum gravity

(FRG) $$\Gamma_k = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,[-R(g)+2\Lambda]$$

M.Reuter, F.Saueressig (hep-th/0110054)



$\tilde{G} = Gk^2$

$\tilde{\Lambda} = \Lambda/k^2$

Quantum scale symmetry (RG fixed points)

See talk by K. Kowalska

UV fixed points of matter RGEs

gauge – Yukawa - quartic

$$\beta_g^{(\text{matter})} - g f_g^{(\text{gravity})} = 0$$
$$\beta_y^{(\text{matter})} - y f_y^{(\text{gravity})} = 0$$
$$\beta_\lambda^{(\text{matter})} - \lambda f_\lambda^{(\text{gravity})} = 0$$

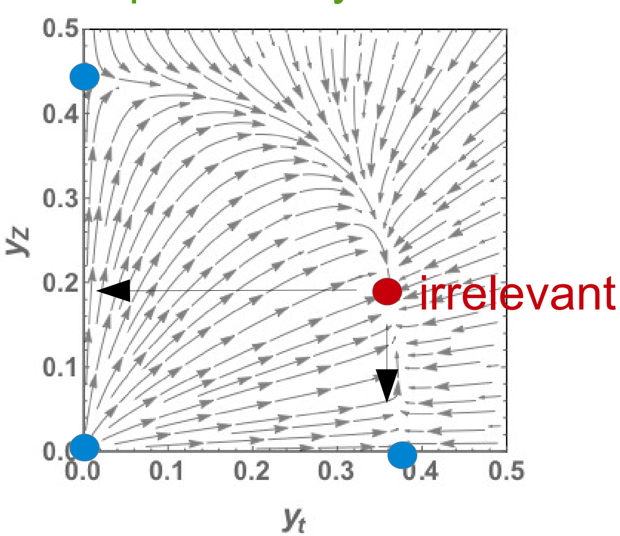
universal corrections ...

… due to gravity (or something else)

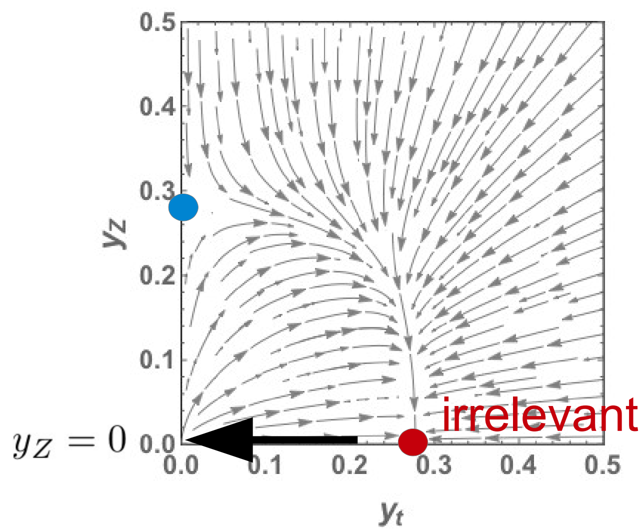
e.g. A.Eichhorn, A.Held (1707.01107)
A.Eichhorn, F.Versteegen (1709.07252)

$$f_g = \frac{\tilde{G}}{4\pi}\frac{1-4\tilde{\Lambda}}{(1-2\tilde{\Lambda})^2} \qquad f_y(\tilde{\Lambda}) = -\tilde{G}\,\frac{96+\tilde{\Lambda}(-235+\tilde{\Lambda}(103+56\tilde{\Lambda}))}{12\pi(3+2\tilde{\Lambda}(-5+4\tilde{\Lambda}))^2}$$
$$f_\lambda = -\tilde{G}\,\frac{165-8\tilde{\Lambda}(61+\tilde{\Lambda}(-49+4\tilde{\Lambda}))}{6\pi(3+2\tilde{\Lambda}(-5+4\tilde{\Lambda}))^2}$$

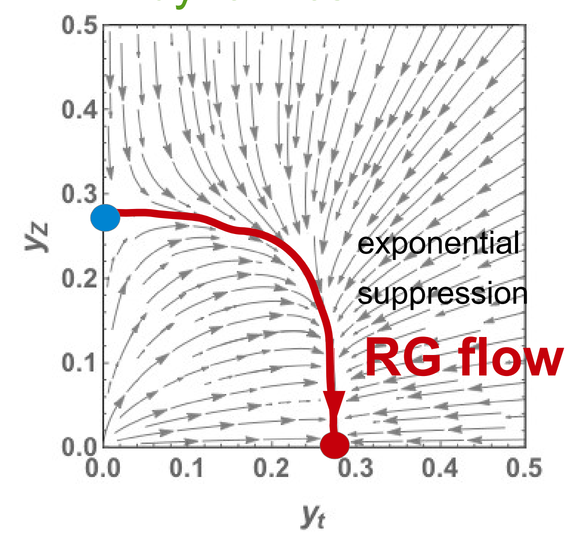
+ predictivity



+ symmetry



+ dynamics



Quantum scale symmetry (RG fixed points)

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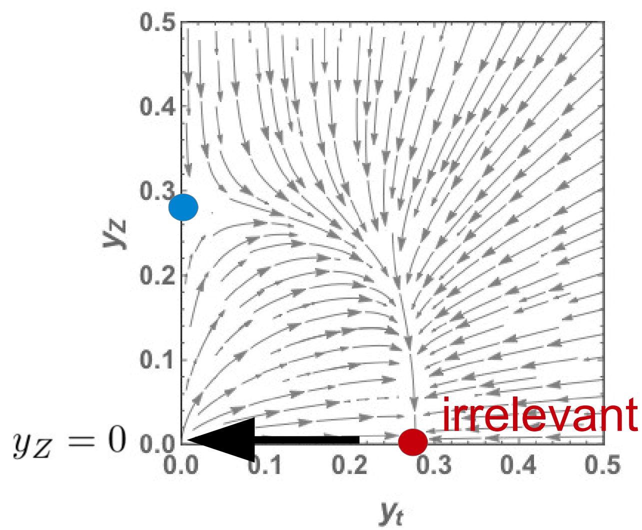
+ symmetry

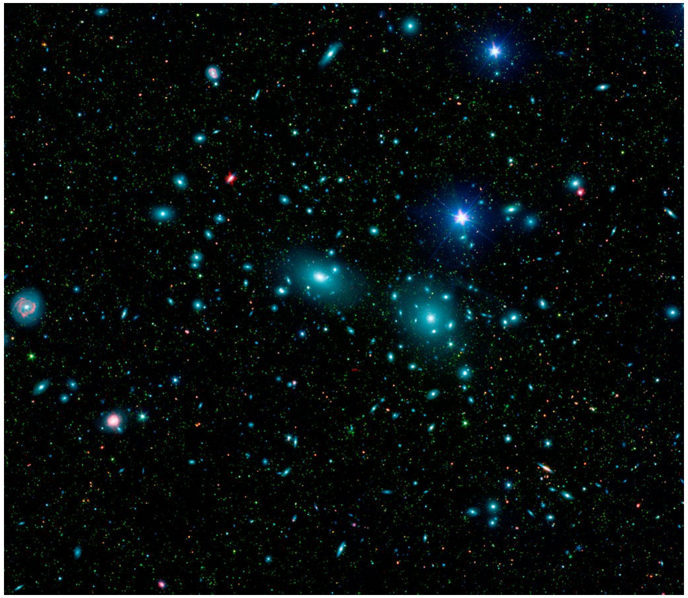


This talk !

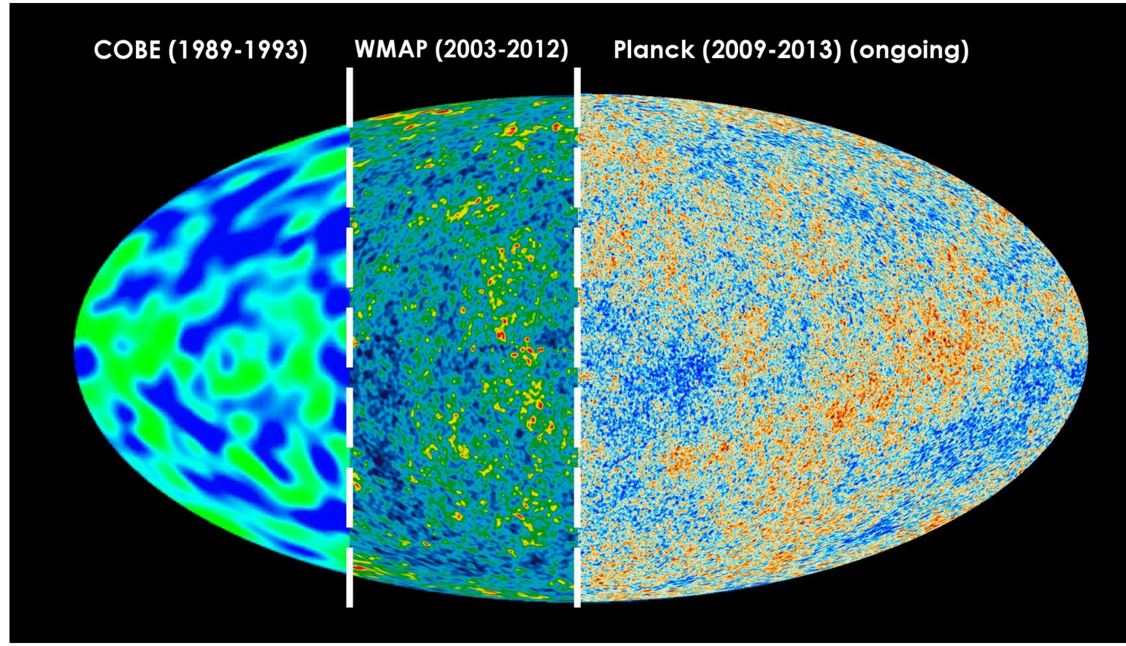
+ symmetry

See talks by J. Smirnov, A. Kamada, K. Sakurai

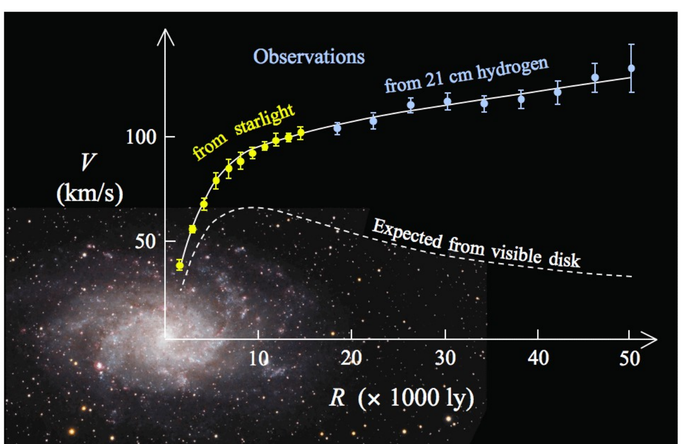
Dark matter



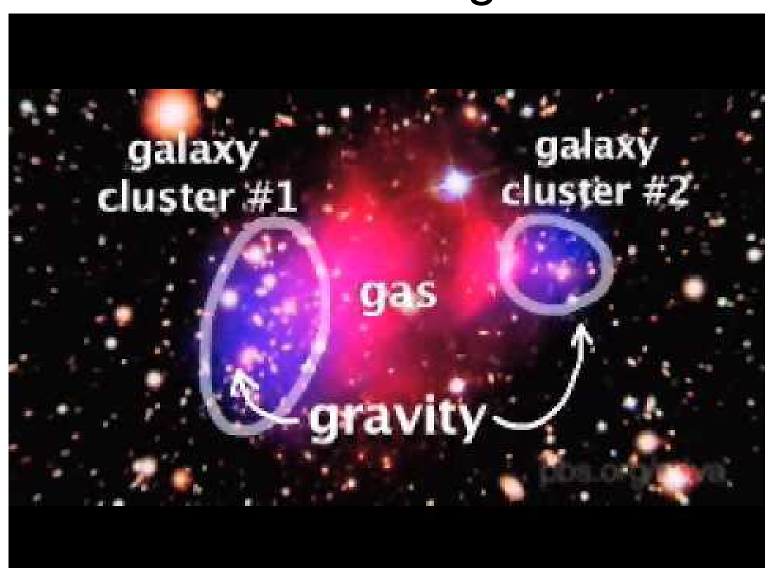
Coma (Zwicky, 1933)

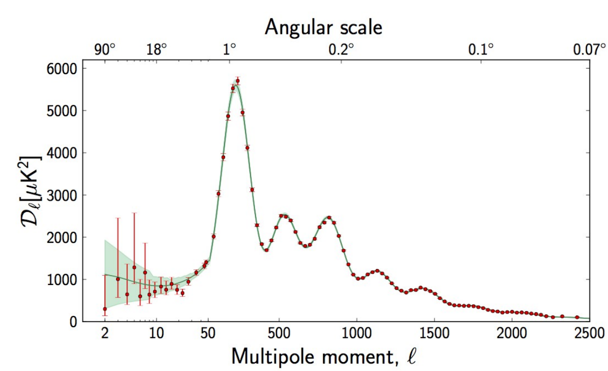


Galaxy / cluster rotation curves



Gravitational lensing





CMB

+ symmetry

See talks by J. Smirnov, A. Kamada, K. Sakurai

Dark matter - the little we know:

- **Invisible** — No (or micro) $U(1)_{em}$ charge
- **Non baryonic** — No $SU(3)_c$ charge
- **Weakly interacting** (or less) — $SU(2)_L$ charge alright
- **Stable** (on cosmological timescales) — **Question is... How?** (kinematics? symmetry?)

A GUT model for DM

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803

Consider SU(6)

x generation

minimal anomaly free setup: $\underbrace{15^{(F)},\ \bar{6}_1^{(F)}}_{\text{SM}},\ \underbrace{\bar{6}_2^{(F)}}_{\text{dark sector ?}}$

+ certain number of scalars:

$$\mathbf{15}^{(S)},\ \mathbf{6_1}^{(S)},\ \mathbf{6_2}^{(S)},\ \mathbf{21}^{(S)},\ \mathbf{35}^{(S)}$$

Yukawa couplings:

$$\begin{aligned}\mathcal{L} \supset\ & y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(S)} + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(S)} \\ & + \tilde{y}_{11}\,\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{12}\,\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{22}\,\bar{\mathbf{6}}_\mathbf{2}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\mathbf{15}^{(S)} \\ & + \hat{y}_{11}\,\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{1}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{12}\,\bar{\mathbf{6}}_\mathbf{1}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{22}\,\bar{\mathbf{6}}_\mathbf{2}^{(F)}\bar{\mathbf{6}}_\mathbf{2}^{(F)}\mathbf{21}^{(S)} \\ & + y_u\,\mathbf{15}^{(F)}\mathbf{15}^{(F)}\mathbf{15}^{(S)} + \text{H.c.}\end{aligned}$$

SU(6) → SU(5) x U(1)$_X$ → SU(3) x SU(2) x U(1) x U(1)$_X$

scalar sector:

$$\bar{\mathbf{6}}_\mathbf{1}^{(S)} \supset \bar{\mathbf{5}}_{-1}^{(S)} \supset \underline{(\mathbf{1},\bar{\mathbf{2}},-\tfrac{1}{2};-1)} + (\bar{\mathbf{3}},\mathbf{1},\tfrac{1}{3};-1)$$

$$\mathbf{15}^{(S)} \supset \mathbf{5}_{-4}^{(S)} \supset \underline{(\mathbf{1},\mathbf{2},\tfrac{1}{2};-4)} + (\mathbf{3},\mathbf{1},-\tfrac{1}{3};-4)$$

Extended 2HDM

$$H_d = \left(\mathbf{1},\bar{\mathbf{2}},-\frac{1}{2};-1\right), \qquad H_u = \left(\mathbf{1},\mathbf{2},\frac{1}{2};-4\right)$$

$$\mathbf{21}^{(S)} \supset \mathbf{1}_{-10}^{(S)} = (\mathbf{1},\mathbf{1},0;-10)$$

$$\bar{\mathbf{6}}_\mathbf{2}^{(S)} \supset \mathbf{1}_{5}^{(S)} = (\mathbf{1},\mathbf{1},0;5)$$

vevs break U(1)$_X$ give mass to Z′

$$s_6 = (\mathbf{1},\mathbf{1},0;5)\ , \qquad s_{21} = (\mathbf{1},\mathbf{1},0;-10)$$

cf. also

E. Ma, Phys. Rev. D 103, 051704 (2021)

A GUT model for DM

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803

Consider SU(6)

x generation

minimal anomaly free setup: $\underbrace{15^{(F)},\ \bar{6}_1^{(F)}}_{\text{SM}},\ \underbrace{\bar{6}_2^{(F)}}_{\text{dark sector ?}}$

Low scale fermions:

	Q	u	$d_{1,2}$	d'	$L_{1,2}$	L'	e	$\nu_{1,2}$
$SU(3)_c$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\bar{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0
$U(1)_X$	2	2	-1	-4	-1	-4	2	5

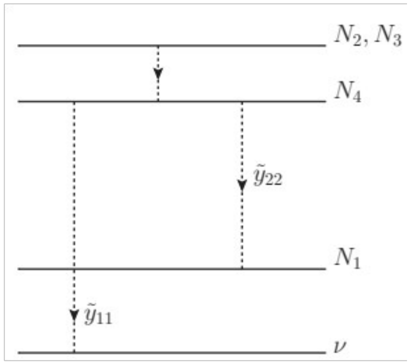
SU(6) → SU(5) x U(1)$_X$ → SU(3) x SU(2) x U(1) x U(1)$_X$

$$\begin{aligned}\mathcal{L}_{\rm IR} \supset\ & 2y_u\, u H_u^{c\dagger} Q + y_d\, d_1 H_d Q + y_e\, e H_d L_1 + y_\nu\, L' H_d^{c\dagger} \nu_1 \\ & + y_D\, d_2 d' s_6 + y_L\, L' L_2 s_6 + y_{\nu_1}\, \nu_1\nu_1 s_{21} + y_{\nu_2}\, \nu_2\nu_2 s_{21} + y'_d\, d_2 H_d Q + y'_e\, e H_d L_2 + y'_\nu\, L' H_d^{c\dagger}\nu_2 \\ & + y'_D\, d_1 d' s_6 + y'_L\, L' L_1 s_6 + 2\tilde{y}_{11}\, \nu_1 H_u^{c\dagger} L_1 + 2\tilde{y}_{22}\, \nu_2 H_u^{c\dagger} L_2 \\ & + \tilde{y}_{12}\left(\nu_1 H_u^{c\dagger} L_2 + \nu_2 H_u^{c\dagger} L_1\right) + \hat{y}_{12}\, \nu_1\nu_2 s_{21} + \text{H.c.}\end{aligned}$$

5 neutral Majorana fermions:

$$M_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & y'_L v_{s_6} & 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u \\ 0 & 0 & y_L v_{s_6} & \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u \\ y'_L v_{s_6} & y_L v_{s_6} & 0 & y_\nu v_d & y'_\nu v_d \\ 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u & y_\nu v_d & y_{\nu_1} v_{s_{21}} & \hat{y}_{12} v_{s_{21}} \\ \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u & y'_\nu v_d & \hat{y}_{12} v_{s_{21}} & y_{\nu_2} v_{s_{21}} \end{pmatrix}$$

DM not stable !!



Stabilizing DM with q.s.s.

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803

$$\mathcal{L} \supset y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)}$$

same gauge quantum numbers

$$16\pi^2\beta^{(1)}(y_{11}) = y_{11}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$
$$16\pi^2\beta^{(1)}(y_{12}) = y_{12}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$
$$16\pi^2\beta^{(1)}(y_{21}) = y_{21}\left(\frac{17}{2}y_{11}^2 + y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$
$$16\pi^2\beta^{(1)}(y_{22}) = y_{22}\left(y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

stability matrix

$$\{-\theta_i\}$$

critical exponents

$\theta_i > 0$ relevant

$\theta_i < 0$ irrelevant

$$\boxed{y_{22}^* = y_{11}^* = y_{12}^* = y_{21}^*}$$

symmetric FP

$$\theta_{11} = \theta_{12} = \theta_{21} = \theta_{22}$$

the same scaling behavior

Stabilizing DM with q.s.s.

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803

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$$16\pi^2\beta^{(1)}(y_{21}) = y_{21}\left(\frac{17}{2}y_{11}^2 + y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{22}) = y_{22}\left(y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

stability matrix

$$\downarrow$$

$$\{-\theta_i\}$$

critical exponents

$\theta_i > 0$ relevant

$\theta_i < 0$ irrelevant

$$\boxed{y_{22}^* \neq 0,\ y_{11}^* = y_{12}^* = y_{21}^* = 0}$$

asymmetric FP

$\theta_{11} > 0$ **relevant**

$\theta_{12}, \theta_{21} < 0$ **irrelevant**

different scaling behavior from FP selection

Stabilizing DM with q.s.s.

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803

most predictive FP allowing for top quark

	y_u^*	y_{22}^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	y_{11}^*	$\tilde{y}_{11}^*$	$\tilde{y}_{12}^*$	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$	y_{12}^*	y_{21}^*
	0.25	0.35	0.38	0.32	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$16\pi^2\times$	θ_u	θ_{22}	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	θ_{11}	$\tilde{\theta}_{11}$	$\tilde{\theta}_{12}$	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$	θ_{12}	θ_{21}
	-5.1	-1.1	-4.7	-3.7	0.63	-0.27	-0.27	-0.27	-3.7	0	0

irrelevant
= predictions

relevant
= free parameter

irrelevant
= forbidden!

$$y_{11} \to y_\nu,\; y_{12} \to y_L',\; y_{21} \to y_\nu'$$

secluded dark sector

$$M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & y_L' v_{s_6} & 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u \\ 0 & 0 & y_L v_{s_6} & \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u \\ y_L' v_{s_6} & y_L v_{s_6} & 0 & y_\nu v_d & y_\nu' v_d \\ 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u & y_\nu v_d & y_{\nu_1} v_{s_{21}} & \hat{y}_{12} v_{s_{21}} \\ \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u & y_\nu' v_d & \hat{y}_{12} v_{s_{21}} & y_{\nu_2} v_{s_{21}} \end{pmatrix} \longrightarrow M_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_L v_{s_6} & 0 & 0 \\ 0 & y_L v_{s_6} & 0 & y_\nu v_d & 0 \\ 0 & 0 & y_\nu v_d & y_{\nu_1} v_{s_{21}} & 0 \\ 0 & 0 & 0 & 0 & y_{\nu_2} v_{s_{21}} \end{pmatrix}$$

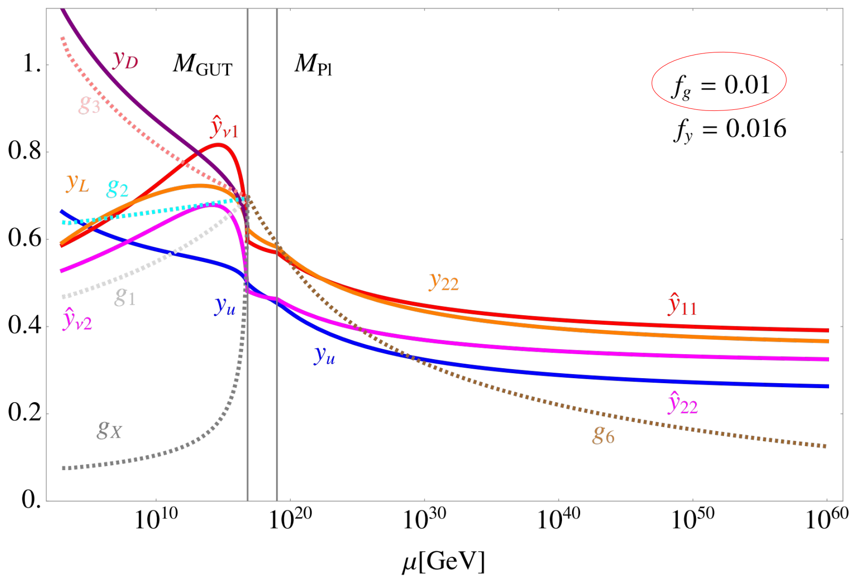
two-component DM

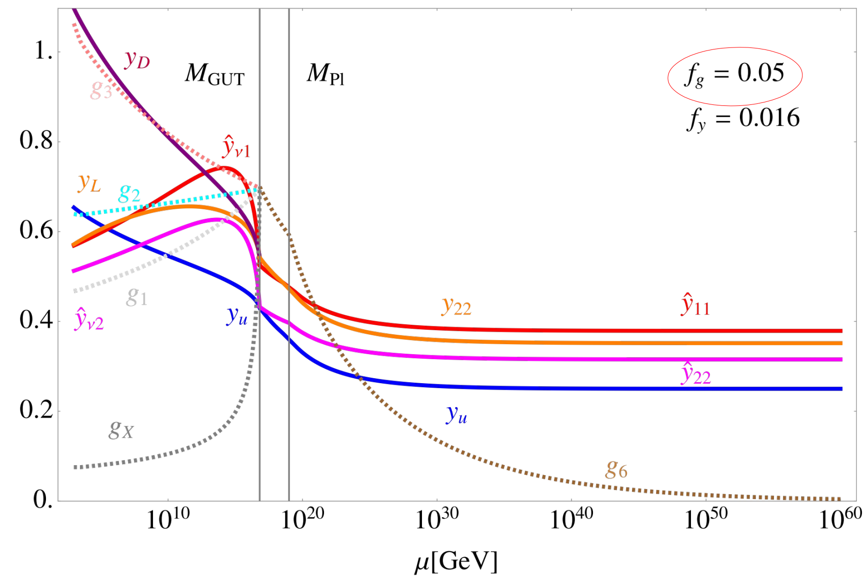
DM is stable (actually, metastable)

cf. E. Ma, '21

Theory uncertainties

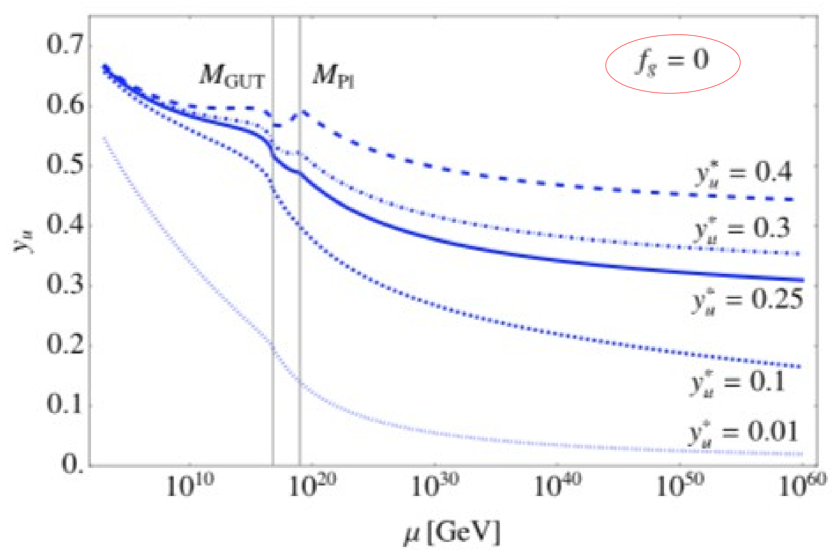
A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS, 2505.02803





gauge – Yukawa system

$$\beta_g^{(\rm matter)} - g f_g^{(\rm gravity)} = 0$$
$$\beta_y^{(\rm matter)} - y f_y^{(\rm gravity)} = 0$$



$f_y \approx 10^{-3} - 10^{-2}$

✓ top mass

to very large extent predictions independent of UV

Compare W. Kotlarski, K. Kowalska, D. Rizzo, EMS, '23

Quantum gravity with proper time

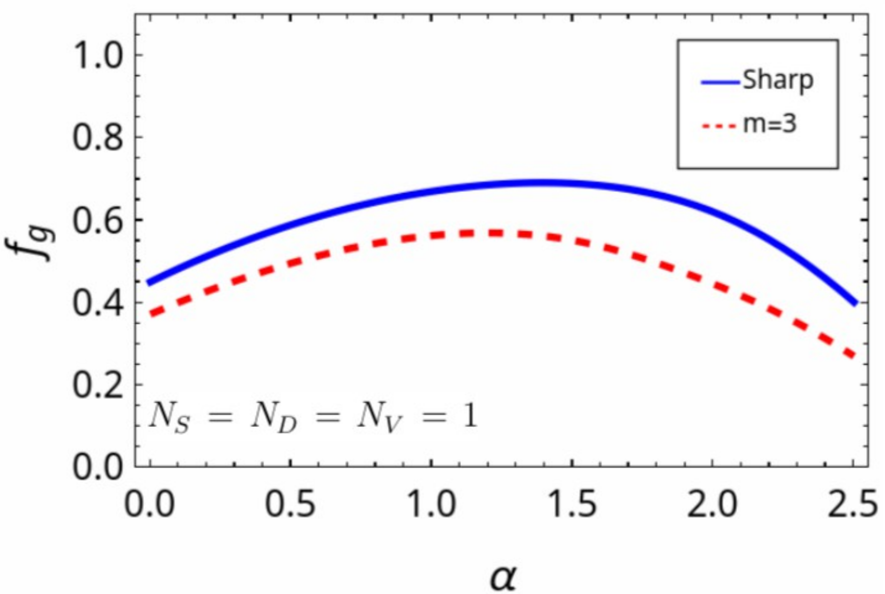
proper time flow eq.

$$k\partial_k \Gamma_k[\phi] = -\frac{1}{2}\int_0^\infty \frac{ds}{s} k\partial_k f_k(s) \mathrm{Tr}\, e^{-s\Gamma_k^{(2)}[\phi]}$$

A.Bonanno, G.Oglialoro, D.Zappala', 2504.07877

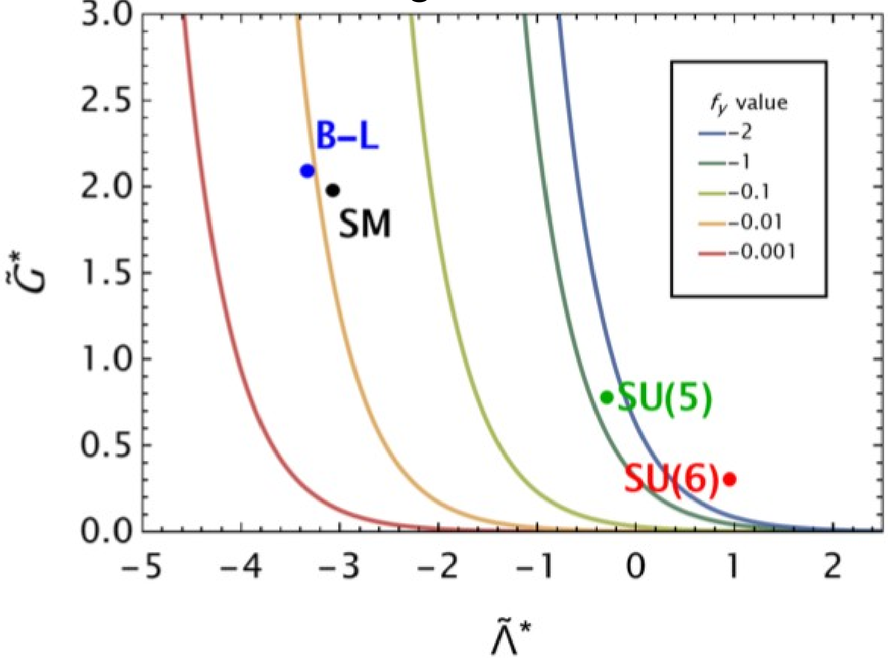
G.Giacometti, K.Kowalska, D.Rizzo, EMS, D.Zappala', 2604.03033

Gauge – large! **OK in GUTs**



$$f_g = \frac{3m\tilde{G}}{2\pi(m-1)}(1+\alpha)$$

Yukawa – negative! not OK in GUTs



$$f_y = -\frac{2\tilde{G}}{\pi}\frac{m}{m-1}\left[3\left(1-2\frac{\tilde{\Lambda}}{m}\right)^{1-m} + 2\alpha\left(1-2\alpha\frac{\tilde{\Lambda}}{m}\right)^{1-m}\right]$$

Yukawa needs higher order – may flip sign!

cf. G.P.de Brito, M.Reichert, M.Schiffer, 2510.09572

One-component DM cases

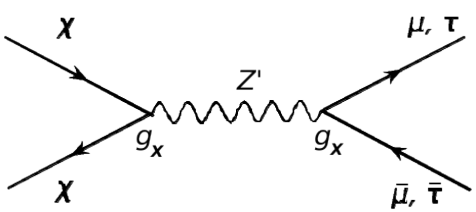
Yukawa and gauge couplings predicted (fixed) by q.s.s.

The only free (relevant) parameters are in the scalar potential (at tanβ=1)

$$v_{s_6} = \langle s_6 \rangle \qquad v_{s_{21}} = \langle s_{21} \rangle$$

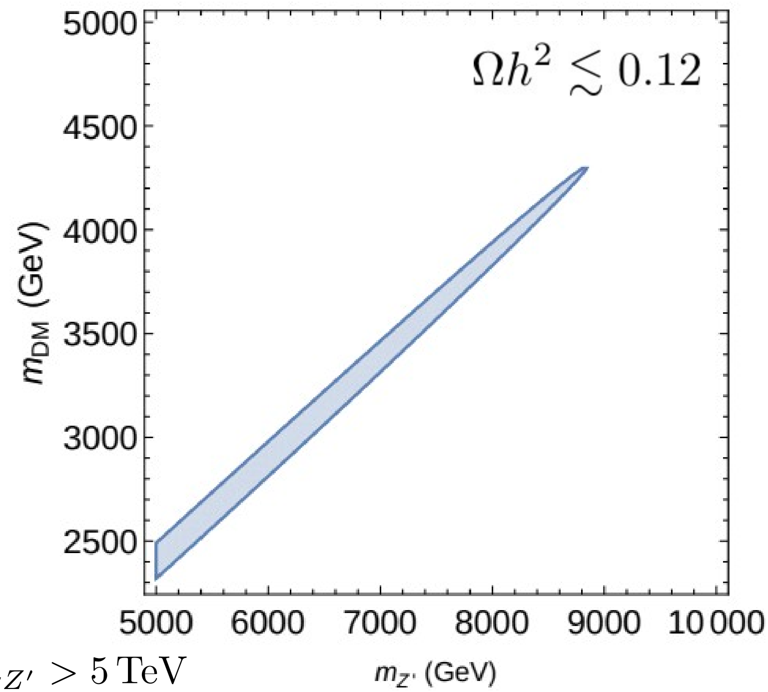
$$H_d = \left(\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2}; -1\right), \qquad H_u = \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}; -4\right)$$
$$s_6 = (\mathbf{1}, \mathbf{1}, 0; 5), \qquad s_{21} = (\mathbf{1}, \mathbf{1}, 0; -10)$$

singlet DM $(v_{s_6} > v_{s_{21}})$



$$m_{\rm DM} = \frac{1}{\sqrt{2}} y_{\nu_1} v_{s_{21}} \qquad m_{Z'} = 5\, g_X \sqrt{v_{s_6}^2 + 4 v_{s_{21}}^2}$$

A. Chikkaballi, K. Kowalska, R.R. Lino dos Santos, EMS '25



$m_{Z'} > 5\,\mathrm{TeV}$

CMS bound
JHEP 07 (2021) 208

One-component DM cases

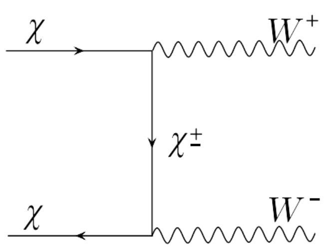
Yukawa and gauge couplings predicted (fixed) by q.s.s.

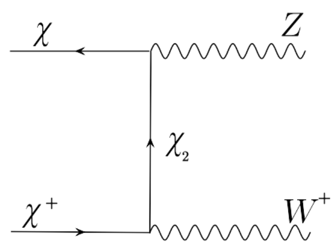
The only free (relevant) parameters are in the scalar potential (at tanβ=1)

$$v_{s_6} = \langle s_6 \rangle \qquad v_{s_{21}} = \langle s_{21} \rangle$$

$$H_d = \left(\mathbf{1}, \bar{\mathbf{2}}, -\frac{1}{2}; -1\right), \qquad H_u = \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}; -4\right)$$
$$s_6 = (\mathbf{1}, \mathbf{1}, 0; 5), \qquad s_{21} = (\mathbf{1}, \mathbf{1}, 0; -10)$$

doublet DM $(v_{s_6} \ll v_{s_{21}})$



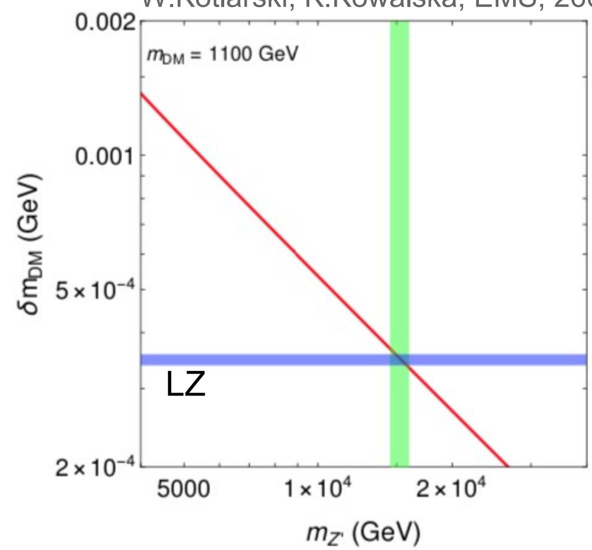


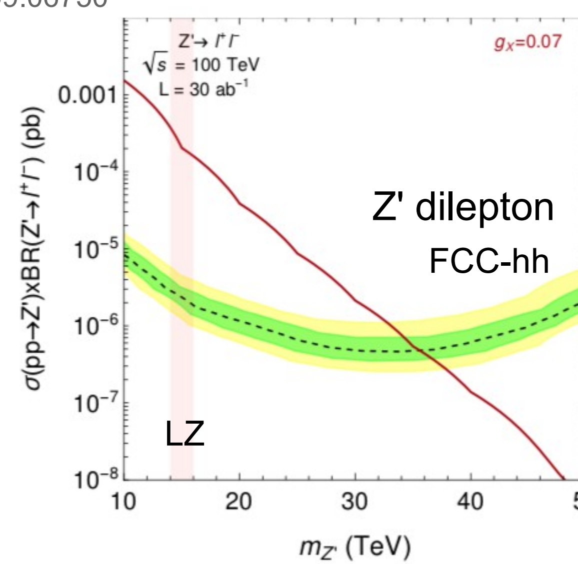
1 TeV "Higgsino-like"

$$\delta m_{\mathrm{DM}} = m_{N_2} - m_{N_1} \simeq \frac{1}{\sqrt{2}} \frac{y_\nu^2 v_d^2}{y_{\nu_1} v_{s_{21}}}$$

LZ inelastic event (2609.02823) $\delta m \approx 340 - 360\,\mathrm{keV}$

W.Kotlarski, K.Kowalska, EMS, 2609.06750





To take home

- **Predictive UV completions** to matter interactions with q.s.s

- q.s.s. can **forbid** the appearance of **unwanted couplings**

- **Control DM decays** and **stabilize it**

  example in SU(6) GUT

  (no discrete and/or global symmetries required)

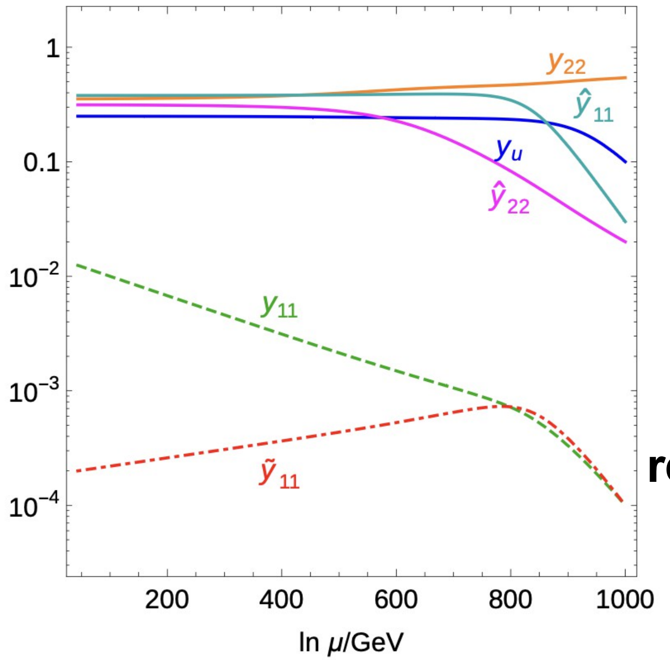
- **Predictivity** implies **phenomenology** (LZ event / FCC ...)

Backup slides

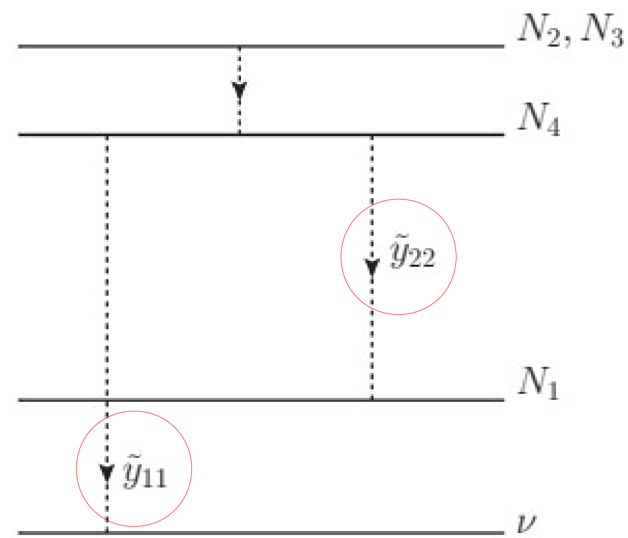
Dynamics

Turn on feeble couplings

dynamical suppression at work in SU(6) DM



C: UV relevant FP



"forbidden'' couplings now appear

dynamical suppression

- Neutrino mass via (light) see-saw ($\tilde{y}_{11}$)
- Controllable decays in dark sector ($\tilde{y}_{22}$) $\to$ DM can be 1-component

Full SU(6) system ...

$$\beta^{(1)}(g_6) = -\frac{38}{3} g_6^3$$

$$\beta^{(1)}(y_{11}) = \left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + y_{22}^2 - 10\tilde{y}_{11}^2 + \frac{5}{2}\tilde{y}_{12}^2 + 7\hat{y}_{11}^2 + \frac{7}{4}\hat{y}_{12}^2 + 12y_u^2 - \frac{91}{4}g_6^2\right) y_{11} + \frac{15}{2}y_{12}y_{21}y_{22} - 5\tilde{y}_{11}\tilde{y}_{12}y_{21} - 5\tilde{y}_{12}\tilde{y}_{22}y_{21} + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}y_{21} + \frac{7}{2}\hat{y}_{12}\hat{y}_{22}y_{21}$$

$$\beta^{(1)}(y_{12}) = \left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + y_{21}^2 + \frac{17}{2}y_{22}^2 - 10\tilde{y}_{11}^2 + \frac{5}{2}\tilde{y}_{12}^2 + 7\hat{y}_{11}^2 + \frac{7}{4}\hat{y}_{12}^2 + 12y_u^2 - \frac{91}{4}g_6^2\right) y_{12} + \frac{15}{2}y_{11}y_{21}y_{22} - 5\tilde{y}_{11}\tilde{y}_{12}y_{22} - 5\tilde{y}_{12}\tilde{y}_{22}y_{22} + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}y_{22} + \frac{7}{2}\hat{y}_{12}\hat{y}_{22}y_{22}$$

$$\beta^{(1)}(y_{21}) = \left(\frac{17}{2}y_{11}^2 + y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \frac{5}{2}\tilde{y}_{12}^2 - 10\tilde{y}_{22}^2 + \frac{7}{4}\hat{y}_{12}^2 + 7\hat{y}_{22}^2 + 12y_u^2 - \frac{91}{4}g_6^2\right) y_{21} + \frac{15}{2}y_{11}y_{12}y_{22} + 5\tilde{y}_{11}\tilde{y}_{12}y_{11} + 5\tilde{y}_{12}\tilde{y}_{22}y_{11} + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}y_{11} + \frac{7}{2}\hat{y}_{12}\hat{y}_{22}y_{11}$$

$$\beta^{(1)}(y_{22}) = \left(y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \frac{5}{2}\tilde{y}_{12}^2 - 10\tilde{y}_{22}^2 + \frac{7}{4}\hat{y}_{12}^2 + 7\hat{y}_{22}^2 + 12y_u^2 - \frac{91}{4}g_6^2\right) y_{22} + \frac{15}{2}y_{11}y_{12}y_{21} + 5\tilde{y}_{11}\tilde{y}_{12}y_{12} + 5\tilde{y}_{12}\tilde{y}_{22}y_{12} + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}y_{12} + \frac{7}{2}\hat{y}_{12}\hat{y}_{22}y_{12}$$

$$\beta^{(1)}(\tilde{y}_{11}) = \left(5y_{11}^2 + 5y_{12}^2 - 24\tilde{y}_{11}^2 + 12\tilde{y}_{12}^2 - 4\tilde{y}_{22}^2 + 14\hat{y}_{11}^2 + \frac{7}{2}\hat{y}_{12}^2 + 12y_u^2 - \frac{35}{2}g_6^2\right)\tilde{y}_{11} + 5\tilde{y}_{12}^2\tilde{y}_{22}$$

$$\beta^{(1)}(\tilde{y}_{12}) = \left(\frac{5}{2}y_{11}^2 + \frac{5}{2}y_{12}^2 + \frac{5}{2}y_{21}^2 + \frac{5}{2}y_{22}^2 - 24\tilde{y}_{11}^2 + 7\tilde{y}_{12}^2 - 24\tilde{y}_{22}^2 - 20\tilde{y}_{11}\tilde{y}_{22} + 7\hat{y}_{11}^2 + \frac{7}{2}\hat{y}_{12}^2 + 7\hat{y}_{22}^2 + 12y_u^2 - \frac{35}{2}g_6^2\right)\tilde{y}_{12} + 5\tilde{y}_{11}y_{11}y_{21} + 5\tilde{y}_{11}y_{12}y_{22} + 7\tilde{y}_{11}\hat{y}_{11}\hat{y}_{12} + 7\tilde{y}_{11}\hat{y}_{12}\hat{y}_{22} + 5\tilde{y}_{22}y_{11}y_{21} + 5\tilde{y}_{22}y_{12}y_{22} + 7\tilde{y}_{22}\hat{y}_{11}\hat{y}_{12} + 7\tilde{y}_{22}\hat{y}_{12}\hat{y}_{22}$$

$$\beta^{(1)}(\tilde{y}_{22}) = \left(5y_{21}^2 + 5y_{22}^2 - 4\tilde{y}_{11}^2 + 12\tilde{y}_{12}^2 - 24\tilde{y}_{22}^2 + \frac{7}{2}\hat{y}_{12}^2 + 14\hat{y}_{22}^2 + 12y_u^2 - \frac{35}{2}g_6^2\right)\tilde{y}_{22} + 5\tilde{y}_{11}\tilde{y}_{12}^2$$

$$\beta^{(1)}(\hat{y}_{11}) = \left(5y_{11}^2 + 5y_{12}^2 - 20\tilde{y}_{11}^2 + 5\tilde{y}_{12}^2 + 16\hat{y}_{11}^2 + 8\hat{y}_{12}^2 + 2\hat{y}_{22}^2 - \frac{35}{2}g_6^2\right)\hat{y}_{11} + \frac{5}{2}\hat{y}_{12}y_{11}y_{21} + \frac{5}{2}\hat{y}_{12}y_{12}y_{22} + \frac{7}{2}\hat{y}_{12}^2\hat{y}_{22}$$

$$\beta^{(1)}(\hat{y}_{12}) = \left(14\hat{y}_{11}\hat{y}_{22} + \frac{5}{2}y_{11}^2 + \frac{5}{2}y_{12}^2 + \frac{5}{2}y_{21}^2 + \frac{5}{2}y_{22}^2 - 10\tilde{y}_{11}^2 + 5\tilde{y}_{12}^2 - 10\tilde{y}_{22}^2 + 16\hat{y}_{11}^2 + \frac{9}{2}\hat{y}_{12}^2 + 16\hat{y}_{22}^2 - \frac{35}{2}g_6^2\right)\hat{y}_{12} - 10\tilde{y}_{11}\tilde{y}_{12}\hat{y}_{11} - 10\tilde{y}_{11}\tilde{y}_{12}\hat{y}_{22} - 10\tilde{y}_{12}\tilde{y}_{22}\hat{y}_{11} - 10\tilde{y}_{12}\tilde{y}_{22}\hat{y}_{22} + 5\hat{y}_{11}y_{11}y_{21} + 5\hat{y}_{11}y_{12}y_{22} + 5\hat{y}_{22}y_{11}y_{21} + 5\hat{y}_{22}y_{12}y_{22}$$

$$\beta^{(1)}(\hat{y}_{22}) = \left(5y_{21}^2 + 5y_{22}^2 + 5\tilde{y}_{12}^2 - 20\tilde{y}_{22}^2 + 2\hat{y}_{11}^2 + 8\hat{y}_{12}^2 + 16\hat{y}_{22}^2 - \frac{35}{2}g_6^2\right)\hat{y}_{22} + \frac{7}{2}\hat{y}_{11}\hat{y}_{12}^2 + \frac{5}{2}\hat{y}_{12}y_{11}y_{21} + \frac{5}{2}\hat{y}_{12}y_{12}y_{22}$$

$$\beta^{(1)}(y_u) = \left(2y_{11}^2 + 2y_{12}^2 + 2y_{21}^2 + 2y_{22}^2 - 4\tilde{y}_{11}^2 + 2\tilde{y}_{12}^2 - 4\tilde{y}_{22}^2 + 36y_u^2 - 28g_6^2\right) y_u$$

… find the FPs

Scalar potential (GUT scale)

$$
\begin{aligned}
V_{\mathrm{SU(6)}} = & -\frac{1}{2}M_{35}^2\mathrm{Tr}(\mathbf{35}^2) + \frac{1}{4}A_{35}[\mathrm{Tr}(\mathbf{35}^2)]^2 + \frac{1}{2}B_{35}\mathrm{Tr}(\mathbf{35}^4) \\
& -M_{21}^2\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + A_{21}[\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21})]^2 + B_{21}\mathrm{Tr}[(\mathbf{21}^\dagger\mathbf{21})^2] \\
& -M_{15}^2\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}) + A_{15}[\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15})]^2 + B_{15}\mathrm{Tr}[(\mathbf{15}^\dagger\mathbf{15})^2] \\
& -M_{61}^2(\mathbf{6_1}^\dagger\mathbf{6_1}) - M_{62}^2(\mathbf{6_2}^\dagger\mathbf{6_2}) - M_{12}^2(\mathbf{6_1}^\dagger\mathbf{6_2} + \mathrm{H.c.}) + B_{61}(\mathbf{6_1}^\dagger\mathbf{6_1})^2 + B_{62}(\mathbf{6_2}^\dagger\mathbf{6_2})^2 \\
& +C_3(\mathbf{6_1}^\dagger\mathbf{6_1})(\mathbf{6_2}^\dagger\mathbf{6_2}) + C_4(\mathbf{6_1}^\dagger\mathbf{6_2})(\mathbf{6_2}^\dagger\mathbf{6_1}) \\
& +\big(C_5(\mathbf{6_1}^\dagger\mathbf{6_2})^2 + C_6(\mathbf{6_1}^\dagger\mathbf{6_1})(\mathbf{6_1}^\dagger\mathbf{6_2}) + C_7(\mathbf{6_2}^\dagger\mathbf{6_2})(\mathbf{6_1}^\dagger\mathbf{6_2}) + \mathrm{H.c.}\big) \\
& +\alpha_1(\mathbf{6_1}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{35}^2) + \beta_1\mathbf{6_1}^\dagger(\mathbf{35}^2)\mathbf{6_1} + \alpha_2(\mathbf{6_2}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{35}^2) + \beta_2\mathbf{6_2}^\dagger(\mathbf{35}^2)\mathbf{6_2} \\
& +\alpha_{12}(\mathbf{6_1}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{35}^2) + \beta_{12}\mathbf{6_1}^\dagger(\mathbf{35}^2)\mathbf{6_2} + \alpha_{21}(\mathbf{6_2}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{35}^2) + \beta_{21}\mathbf{6_2}^\dagger(\mathbf{35}^2)\mathbf{6_1} \\
& +\gamma_1(\mathbf{6_1}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_1\mathbf{6_1}^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6_1} + \gamma_2(\mathbf{6_2}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_2\mathbf{6_2}^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6_2} \\
& +\gamma_{12}(\mathbf{6_1}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_{12}\mathbf{6_1}^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6_2} + \gamma_{21}(\mathbf{6_2}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + \delta_{21}\mathbf{6_2}^\dagger(\mathbf{21}^\dagger\mathbf{21})\mathbf{6_1} \\
& +\rho_1(\mathbf{6_1}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}) + \sigma_1\mathbf{6_1}^\dagger(\mathbf{15}^\dagger\mathbf{15})\mathbf{6_1} + \rho_2(\mathbf{6_2}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}) + \sigma_2\mathbf{6_2}^\dagger(\mathbf{15}^\dagger\mathbf{15})\mathbf{6_2} \\
& +\rho_{12}(\mathbf{6_1}^\dagger\mathbf{6_2})\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}) + \sigma_{12}\mathbf{6_1}^\dagger(\mathbf{15}^\dagger\mathbf{15})\mathbf{6_2} + \rho_{21}(\mathbf{6_2}^\dagger\mathbf{6_1})\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}) + \sigma_{21}\mathbf{6_2}^\dagger(\mathbf{15}^\dagger\mathbf{15})\mathbf{6_1} \\
& +a_1\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15})\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) + a_2\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15})\mathrm{Tr}(\mathbf{35}^2) + a_3\mathrm{Tr}(\mathbf{35}^2)\mathrm{Tr}(\mathbf{21}^\dagger\mathbf{21}) \\
& +b_1\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}\,\mathbf{21}^\dagger\mathbf{21}) + b_2\mathrm{Tr}(\mathbf{15}^\dagger\mathbf{15}\,\mathbf{35}^2) + b_3\mathrm{Tr}(\mathbf{35}^2\,\mathbf{21}^\dagger\mathbf{21}) \\
& +\big[\kappa_1(\mathbf{6_1}\mathbf{15}^\dagger\mathbf{6_1}) + \kappa_2(\mathbf{6_2}\mathbf{15}^\dagger\mathbf{6_2}) + \kappa_3(\mathbf{6_1}\mathbf{15}^\dagger\mathbf{6_2}) \\
& +\kappa_4(\mathbf{6_1}\mathbf{21}^\dagger\mathbf{6_1}) + \kappa_5(\mathbf{6_2}\,\mathbf{21}^\dagger\mathbf{6_2}) + \kappa_6(\mathbf{6_1}\mathbf{21}^\dagger\mathbf{6_2}) \\
& +\lambda_1(\mathbf{6_1}^\dagger\mathbf{35}\,\mathbf{6_1}) + \lambda_2(\mathbf{6_2}^\dagger\mathbf{35}\,\mathbf{6_2}) + \lambda_3(\mathbf{6_1}^\dagger\mathbf{35}\,\mathbf{6_2}) + \lambda_4(\mathbf{6_2}^\dagger\mathbf{35}\,\mathbf{6_1}) \\
& +\eta_1(\mathbf{15}^\dagger\,\mathbf{35}\,\mathbf{15}) + \eta_2(\mathbf{15}\,\mathbf{15}\,\mathbf{15}) + \eta_3(\mathbf{21}^\dagger\mathbf{35}\,\mathbf{21}) + \eta_4(\mathbf{21}^\dagger\mathbf{35}\,\mathbf{15}) + \mathrm{H.c.}\big] \\
& +\xi_1\mathrm{Tr}(\mathbf{21}\,\mathbf{21}^\dagger\mathbf{21}\,\mathbf{15}^\dagger) + \xi_2\mathrm{Tr}(\mathbf{15}\,\mathbf{15}^\dagger\mathbf{15}\,\mathbf{21}^\dagger) + \xi_3\mathrm{Tr}(\mathbf{15}\,\mathbf{15}\,\mathbf{15}\,\mathbf{35}) \\
& +\big(\xi_4(\mathbf{6_1}^\dagger\mathbf{15}\,\mathbf{21}^\dagger\mathbf{6_1}) + \xi_5(\mathbf{6_2}^\dagger\mathbf{15}\,\mathbf{21}^\dagger\mathbf{6_2}) + \xi_6(\mathbf{6_1}^\dagger\mathbf{15}\,\mathbf{21}^\dagger\mathbf{6_2}) + \mathrm{H.c.}\big) \\
& +\xi_7(\mathbf{6_1}\mathbf{15}^\dagger\mathbf{35}\,\mathbf{6_1}) + \xi_8(\mathbf{6_2}\mathbf{15}^\dagger\mathbf{35}\,\mathbf{6_2}) + \xi_9(\mathbf{6_1}\mathbf{15}^\dagger\mathbf{35}\,\mathbf{6_2}) + \xi_{10}(\mathbf{6_2}\mathbf{15}^\dagger\mathbf{35}\,\mathbf{6_1}) \\
& +\xi_{11}(\mathbf{6_1}\mathbf{21}^\dagger\mathbf{35}\,\mathbf{6_1}) + \xi_{12}(\mathbf{6_2}\mathbf{21}^\dagger\mathbf{35}\,\mathbf{6_2}) + \xi_{13}(\mathbf{6_1}\mathbf{21}^\dagger\mathbf{35}\,\mathbf{6_2}) + \xi_{14}(\mathbf{6_2}\mathbf{21}^\dagger\mathbf{35}\,\mathbf{6_1})\,. \qquad \text{(A.1)}
\end{aligned}
$$

Scalar potential (DM scale)

$$
\begin{aligned}
V_{\text{low}} = &\, \mu_d^2\, H_d H_d^\dagger + \mu_u^2\, H_u^\dagger H_u + \mu_6^2 |s_6|^2 + \mu_{21}^2 |s_{21}|^2 \\
&+ z_d \left(H_d H_d^\dagger\right)^2 + z_u \left(H_u^\dagger H_u\right)^2 + z_6 |s_6|^4 + z_{21} |s_{21}|^4 \\
&+ z_{d6} H_d H_d^\dagger |s_6|^2 + z_{d21} H_d H_d^\dagger |s_{21}|^2 + z_{u6} H_u^\dagger H_u |s_6|^2 + z_{u21} H_u^\dagger H_u |s_{21}|^2 \\
&+ z_{du} \left(H_d H_d^\dagger\right) \left(H_u^\dagger H_u\right) + z_{dudu} H_d H_u \left(H_d H_u\right)^\dagger + z_{621} |s_6|^2 |s_{21}|^2 \\
&+ \left(z_{ud621} H_d H_u s_6^\dagger s_{21}^\dagger + w_{du6} H_d H_u s_6 + w_{2166}\, s_{21} s_6^2 + \text{H.c.}\right),
\end{aligned}
$$

Example scalar masses:

$$
\begin{aligned}
M_{a_1}^2 &\simeq -z_{du621} v_{s_{21}} v_{s_6} - \sqrt{2} w_{du6} v_{s_6}\,, \\
M_{a_2}^2 &\simeq -\sqrt{2} w_{2166} \frac{4 v_{s_{21}}^2 + v_{s_6}^2}{2 v_{s_{21}}}\,.
\end{aligned}
\qquad
M_{h^\pm}^2 = z_{dudu} \frac{v_u^2 + v_d^2}{2} - \left(v_{s_{21}} z_{du621} + \sqrt{2} w_{du6}\right) \frac{v_{s_6} \left(v_u^2 + v_d^2\right)}{2 v_d v_u}
$$

More SU(6) fixed points

FP1										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{22}^*	y_{12}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.25	0.37	0.38	0.15	0.17	0.33	0.33	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{22}	θ_{12}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.1	-4.7	-3.6	0	0.62	-3.6	-1.1	0	0	-0.87	-3.8

FP2										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{22}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.26	0.37	0.41	0.17	0.15	0.19	0.29	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{12}	θ_{22}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.1	-3.6	-5.2	0.64	0	0	-1.0	0	0.61	0.18	-3.7

FP3										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{22}^*	y_{21}^*	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$
0.28	0.41	0.41	0.17	0.17	0.0	0.0	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{11}	θ_{12}	θ_{22}	θ_{21}	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$
-5.7	-3.7	-5.1	0	0.66	0.69	0.69	0.69	0.69	0	-3.7

FP4										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	y_{21}^*	y_{22}^*	y_{11}^*	y_{12}^*	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$
0.26	0.41	0.37	0.15	0.17	0.29	0.19	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	θ_{21}	θ_{22}	θ_{11}	θ_{12}	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$
-5.1	-5.2	-3.6	0	0.64	0	-1.0	0	0.61	0	-3.5

FP5										
y_u^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	$\tilde{y}_{11}^*$	$\tilde{y}_{22}^*$	$\tilde{y}_{12}^*$	$\hat{y}_{12}^*$	y_{11}^*	y_{12}^*	y_{21}^*	y_{22}^*
0.26	0.37	0.41	0.17	0.15	0.19	0.29	0.0	0.0	0.0	0.0
θ_u	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	$\tilde{\theta}_{11}$	$\tilde{\theta}_{22}$	$\tilde{\theta}_{12}$	$\hat{\theta}_{12}$	θ_{11}	θ_{12}	θ_{21}	θ_{22}
-5.1	0.86	0.86	0.35	0	-6.4	1.5	1.5	0.48	1.5	0.48