Lecture III:

The Theory of Neutrino Flavor Oscillations

"Anyway, in as far as the neutrino masses are negligible compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effect such as neutrino oscillations."

Froggatt & Nielsen

Neutrino Flavor Oscillations

How they are induced by mass and mixing

(1) if neutrinos have mass (last lecture and this one)

flavor eigenstates

interaction states

$$|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle \neq |\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$$

mass eigenstate

propagating states

CC Interactions are not diagonal in the mass basis (as for quarks!)

$$\frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha,j} (U_{\alpha j}\, \overline{e}_\alpha \gamma^\mu P_L \nu_j) + \text{h.c.}$$

mixing matrix element

Neutrino Flavor Oscillations

How they are induced by mass and mixing

If neutrinos have mass (last lecture)

flavor eigenstates

interaction states

$$|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle \neq |\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$$

mass eigenstate

propagating states

a flavor eigenstate is a superposition of mass eigenstates

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j\rangle \qquad \alpha, \beta = e, \mu, \tau$$

unitary mixing matrix elements

Neutrino Flavor Oscillations

How they are induced by mass and mixing

(2) Neutrinos are ultra relativistic

for neutrinos

$$p \gg m \quad \rightarrow E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p}$$

So ν_i and ν_j with masses m_i and m_j in the same beam ($p_i = p_j$) experience an energy difference

$$\Delta E = E_i - E_j = \frac{m_i^2 - m_j^2}{2p} \simeq \frac{m_i^2 - m_j^2}{2E}$$

because neutrinos have very small masses

this is a very small number

Neutrino Flavor Oscillations

How they are induced by mass and mixing

as we will see the oscillations depend on the combination

$$\frac{m_i^2 - m_j^2}{2E} L = \frac{\Delta m_{ij}^2}{2E} L$$

L = baseline (distance of propagation between production and detection)

this combination can be sizable @ large baselines

Neutrino Flavor Oscillations

How they are induced by mass and mixing

as we will see the oscillations depend on the combination

$$\frac{m_i^2 - m_j^2}{2E} L = \frac{\Delta m_{ij}^2}{2E} L$$

L = baseline (distance of propagation between production and detection)

this combination can be sizable @ large baselines

The observation of oscillations will depend on the Energy of the neutrinos (source) and on the baseline (detector position)

If we know Δm_{ij}^2 we can calculate the appropriate baseline for $\neq$ neutrino sources to observe oscillation

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

@ t=0 neutrino flavor eigenstates are produced as a superposition of mass eigenstates

$$|\nu_\alpha(t=0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t=0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

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@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j(t)\rangle$$

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@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t)\rangle$$

The transition amplitude to a different flavor ν_β is

$$\mathscr{A}_{\alpha\beta} = \langle \nu_\beta | \nu_\alpha(t)\rangle$$

Probability of the transition

$$P_{\alpha\beta}(t) = |\mathscr{A}_{\alpha\beta}|^2 = |\langle \nu_\beta | \nu_\alpha(t)\rangle|^2$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

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$$|\nu_\alpha(0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j(0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j(t)\rangle$$

Probability of the transition

$$P_{\alpha\beta}(t) = \left| \sum_i \sum_j U_{\beta i} U^*_{\alpha j} \langle \nu_i(0) | \nu_j(t) \rangle \right|^2$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

Under the plane wave approximation

$$|\nu_j(t)\rangle = e^{-iHt}|\nu_j(0)\rangle = e^{-iE_j t}|\nu_j(0)\rangle$$

E_j **is the energy of the free streaming neutrino**

$$\mathcal{A}_{\alpha\beta}(t) = \sum_i \sum_j U_{\beta i} U^*_{\alpha j} e^{-iE_j t} \langle \nu_i(0) | \nu_j(0) \rangle = \sum_i U_{\beta j} U^*_{\alpha j} e^{-iE_j t}$$

as $\langle \nu_i(0) | \nu_j(0) \rangle = \delta_{ij}$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

Under the plane wave approximation

$$|\nu_j(t)\rangle = e^{-iHt}|\nu_j(0)\rangle = e^{-iE_jt}|\nu_j(0)\rangle$$

E_j **is the energy of the free streaming neutrino**

$$\mathscr{A}_{\alpha\beta}(t) = \sum_i \sum_j U_{\beta i}U^*_{\alpha j}\, e^{-iE_jt}\langle\nu_i(0)|\nu_j(0)\rangle = \sum_i U_{\beta j}U^*_{\alpha j}\, e^{-iE_jt}$$

as $\langle\nu_i(0)|\nu_j(0)\rangle = \delta_{ij}$

survival/oscillation probability

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k}\mathrm{Re}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k}\mathrm{Im}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin(\Delta_{kj})$$

$$\Delta_{kj} = (E_k - E_j)t$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})$$

this term conserves CP (same for $\bar{\nu}$)

$$U \to U^*$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

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this term violates CP (changes sign for $\bar{\nu}$)

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

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this term conserves CP (same for $\bar{\nu}$)

$U \rightarrow U^*$

this term violates CP (changes sign for $\bar{\nu}$)

If $\beta = \alpha$ there is no CP violation as $\mathrm{Im}[\,|U_{\alpha k}|^2 |U_{\alpha j}|^2] = 0$

(no CP violation in disappearance experiments)

CP violation can only manifest if $\beta \neq \alpha$ (appearance experiments)

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin(\Delta_{kj})$$

phase differences

ultra-relativistic

mass squared difference

$$\frac{\Delta_{kj}}{2} \equiv \frac{(E_k - E_j)}{2\hbar}\frac{L}{c} = \frac{\Delta m^2_{kj}c^4}{4E}\frac{L}{\hbar c} = 1.27\left(\frac{\Delta m^2_{kj}}{\mathrm{eV}^2}\right)\left(\frac{\mathrm{GeV}}{E}\right)\left(\frac{L}{\mathrm{km}}\right) \qquad \Delta m^2_{kj} = m^2_k - m^2_j$$

[only we do not use natural units]

this for if useful for calculations

$$\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E}L$$

(back to natural units)

- only depend on mass squared differences (no sensitivity to the absolute mass scale)

neutrino energy (source) & baseline (source-detector)

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})$$

phase differences

ultra-relativistic

$$\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E} L = \pi \frac{L}{L_\odot}$$

$$\Delta m^2_{kj} = m^2_k - m^2_j$$

oscillation length

- We can define

$$L^{kj}_\odot = \frac{4\pi E}{\Delta m^2_{kj}}$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

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ultra-relativistic

phase differences

$$\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E}L = \pi\frac{L}{L_\odot} \qquad \Delta m^2_{kj} = m^2_k - m^2_j$$

- We can define

oscillation length

$$L^{kj}_\odot = \frac{4\pi E}{\Delta m^2_{kj}}$$

- if $L \ll L^{kj}_\odot$ the distance is too short for this phase difference to develop - no oscillation can be observer due it
- if $L \gg L^{kj}_\odot$ the distance is long enough for this phase for the system to oscillate many time $\overline{\sin^2\left(\frac{\Delta_{kj}}{2}\right)} \to \frac{1}{2} \quad \overline{\sin\left(\Delta_{kj}\right)} \to 0$
- only for $L \approx L^{kj}_\odot$ we can see the oscillation pattern due to this phase difference

Two Flavor Oscillations in Vacuum

simplest but still very useful case

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

U **is real**

only one phase difference $\Delta m^2_{21} = m^2_2 - m^2_1$

only oscillation length $L_\odot = \dfrac{4\pi E}{\Delta m^2_{21}}$

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U is real

only one phase difference $\Delta m_{21}^2 = m_2^2 - m_1^2$

only oscillation length $L_\odot = \dfrac{4\pi E}{\Delta m_{21}^2}$

$\Delta m_{21}^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2$

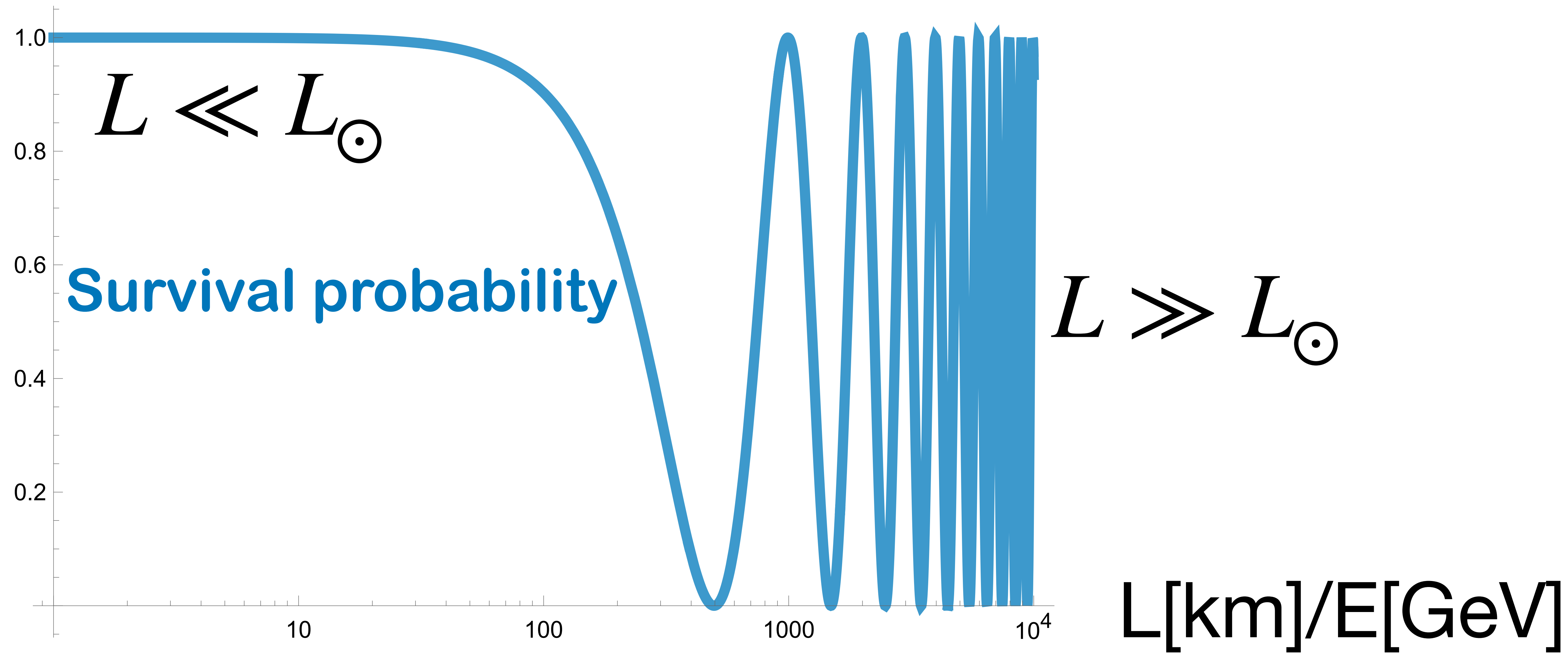
$\sin^2(2\theta) = 1.0$

Oscillation probability

$$P_{\alpha\beta}(t) = \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$

Survival probability

$$P_{\alpha\alpha}(t) = 1 - \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$



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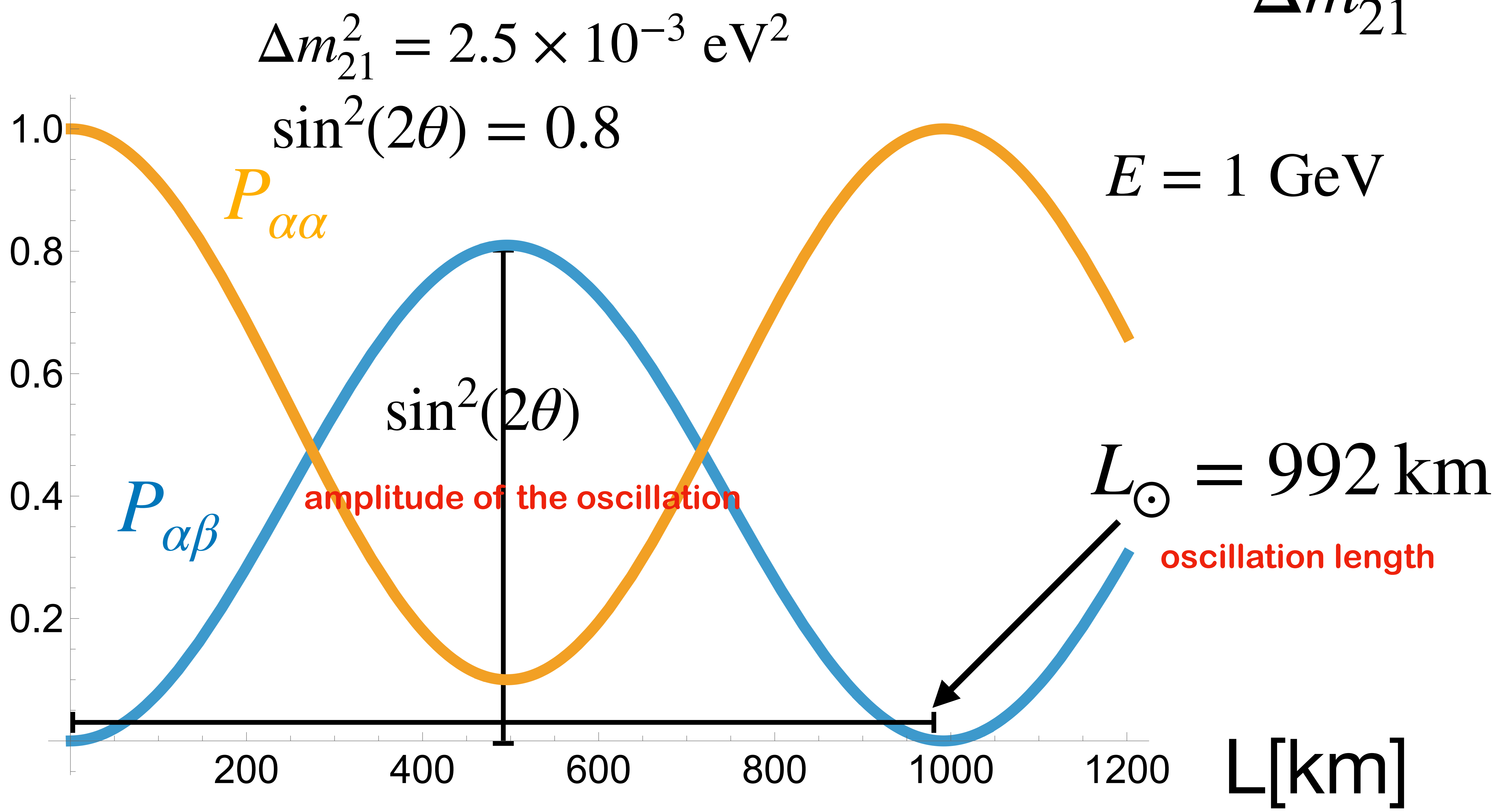

Three Flavor Oscillations in Vacuum

We have 3 flavors in nature

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Unitary Matrix

$$UU^\dagger = I \quad \Longrightarrow \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} = \delta_{\alpha\beta} \quad \sum_{\alpha=e}^{\tau} U_{\alpha j} U^*_{\alpha k} = \delta_{jk}$$

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\imath\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\imath\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\imath\delta} & c_{23}c_{13} \end{pmatrix} V$$

Majorana Phases

3 mixing angles + 1 phase

$$c_{ij} \equiv \cos\theta_{ij} \quad s_{ij} \equiv \sin\theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

$$V = \mathrm{diag}(\alpha_1, 1, \alpha_3)$$

do not enter oscillation
[show that]

Oscillations are the same for DIRAC & MAJORANA neutrinos !

Three Flavor Oscillations in Vacuum

We have 3 flavors in nature

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

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Majorana Phases

3 mixing angles + 1 phase

$$c_{ij} \equiv \cos\theta_{ij} \quad s_{ij} \equiv \sin\theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

$$V = \mathrm{diag}(\alpha_1, 1, \alpha_3)$$

$$\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$$

only two independent

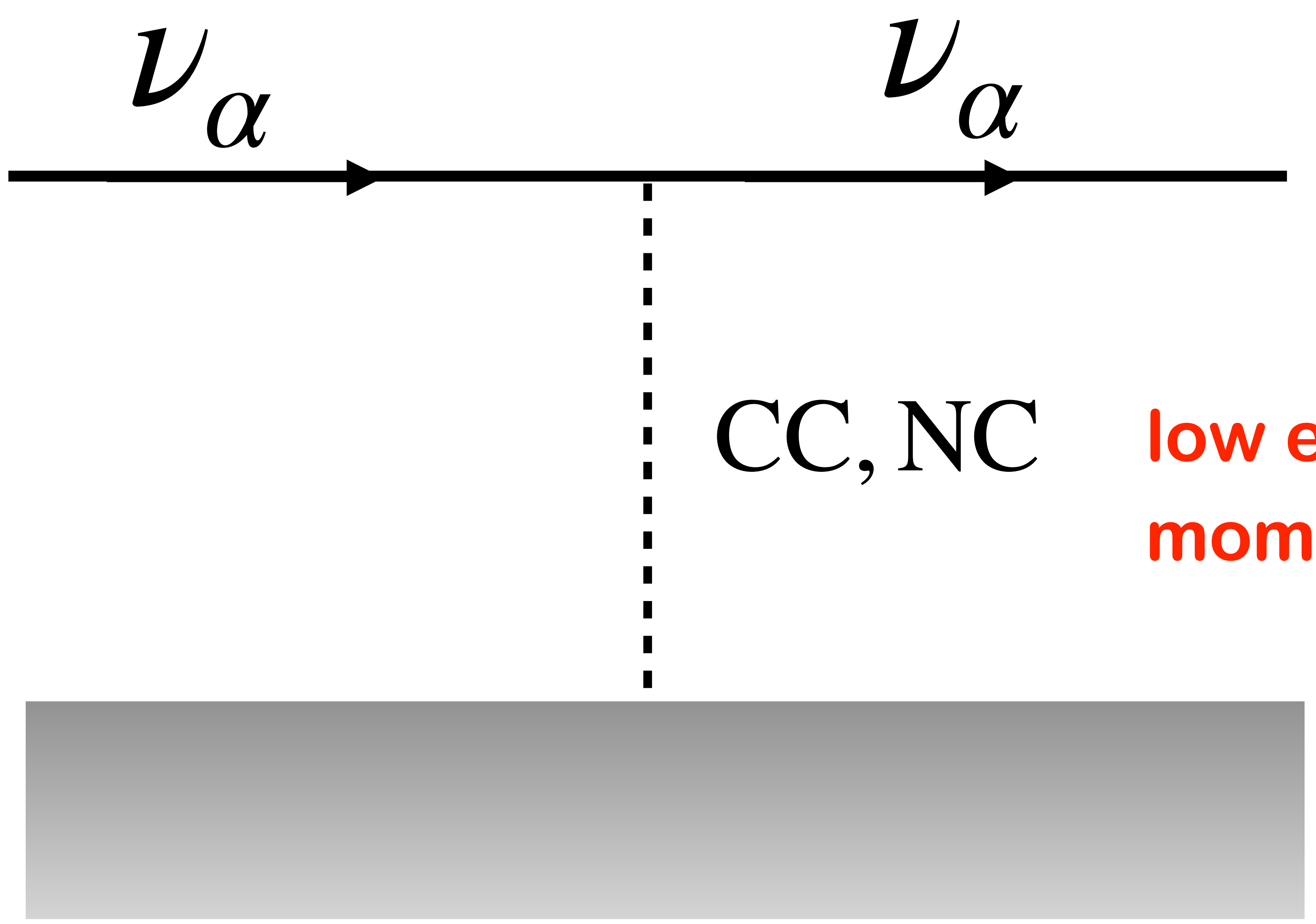
Neutrino Flavor Oscillations in Matter

Basic Concepts

incoherent processes (capture, finite angle scattering) $\sigma \propto G_F^2$

coherent forward scattering effects $\propto G_F$ (a lot larger!)

they lead to a tiny effective potential for neutrinos in matter



low energy limit
momentum transfer $\ll M_W, M_Z$

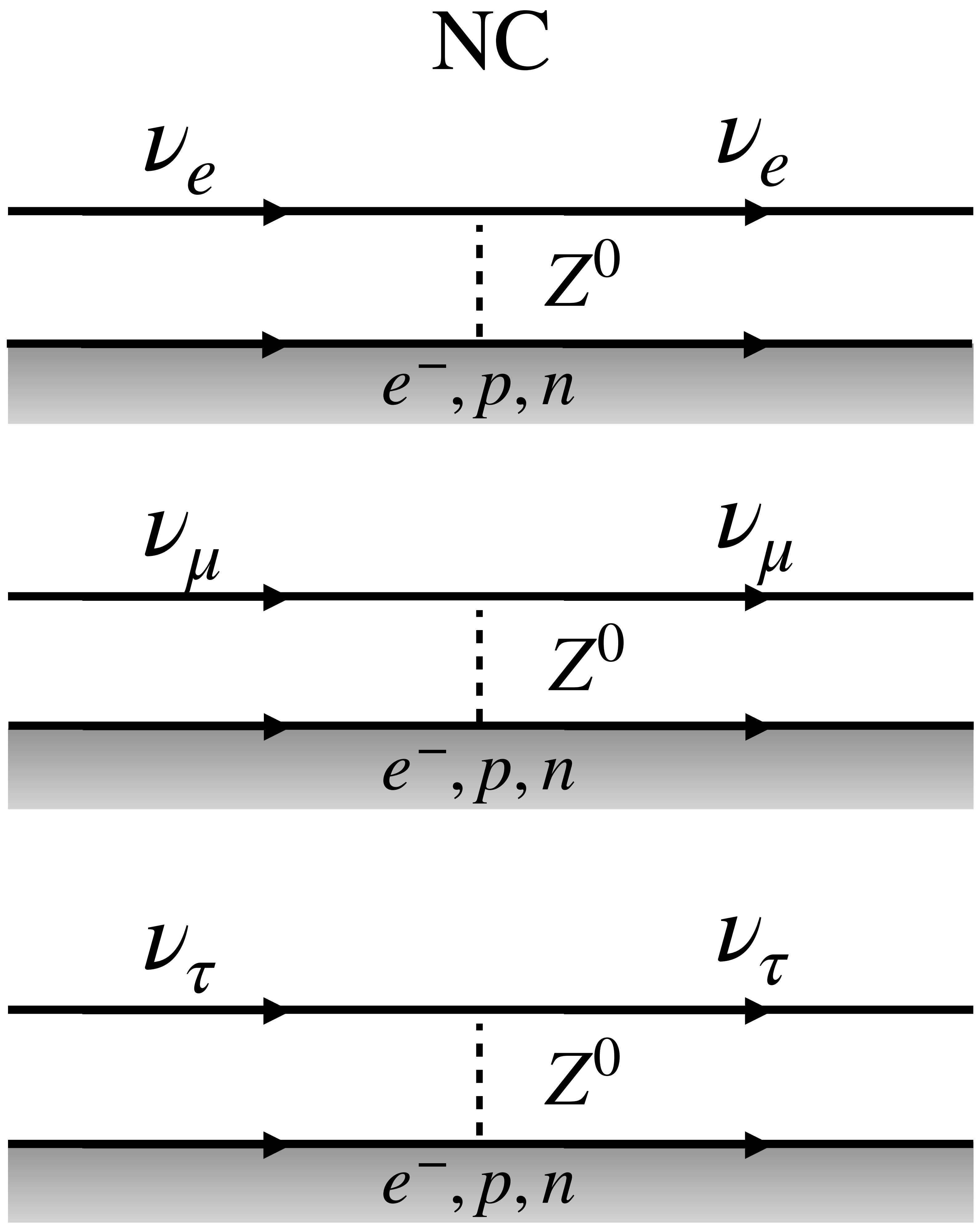
ordinary matter (electrons, nucleons)

[L. Wolfenstein (1978)]

[S. Mikheyev and A. Yu Smirnov (1985)]

Neutrino Flavor Oscillations in Matter

Basic Concepts

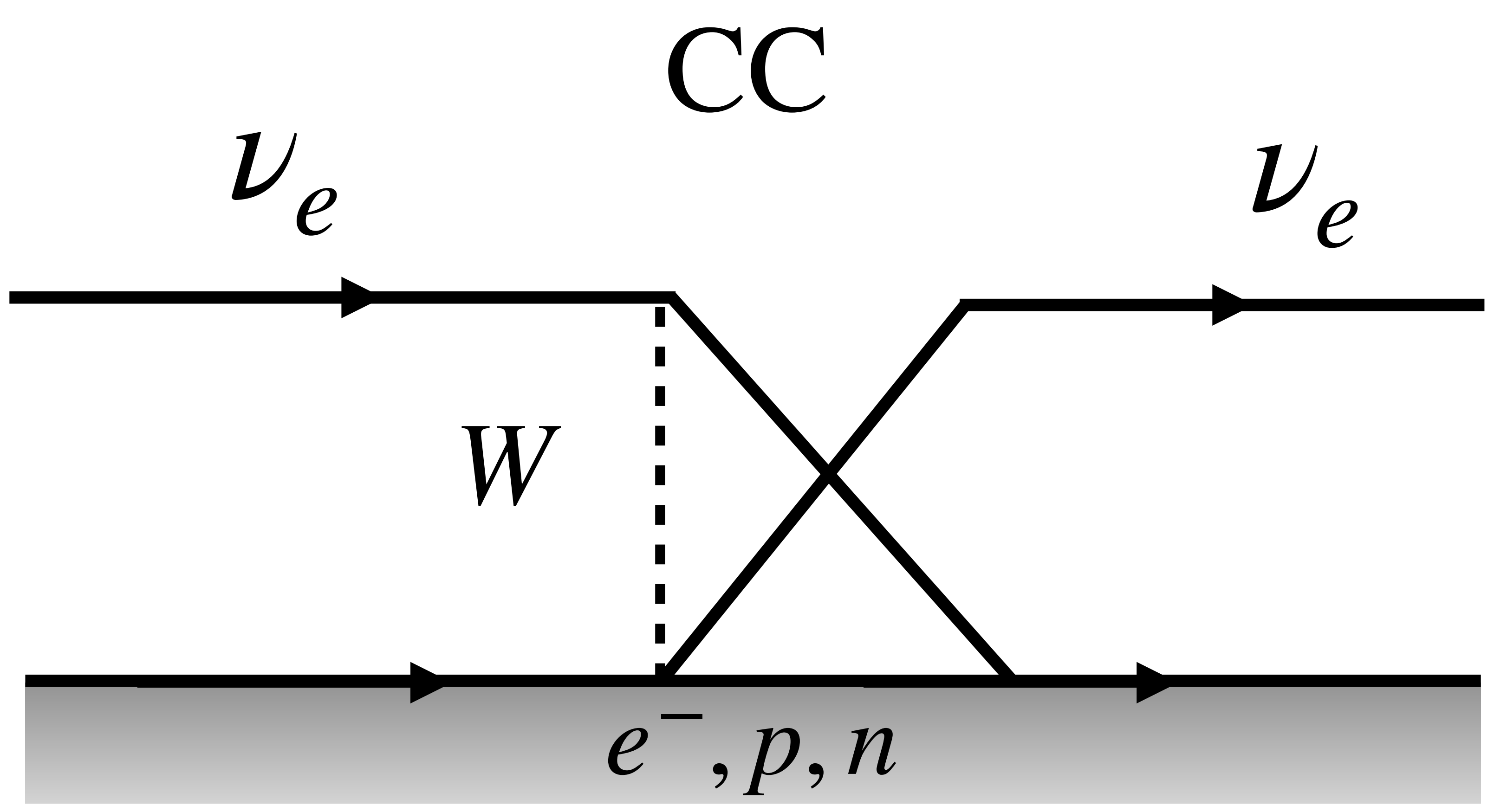


affects all neutrinos in the same way

gives rise to an unobservable common phase

Neutrino Flavor Oscillations in Matter

Basic Concepts



singles out ν_e

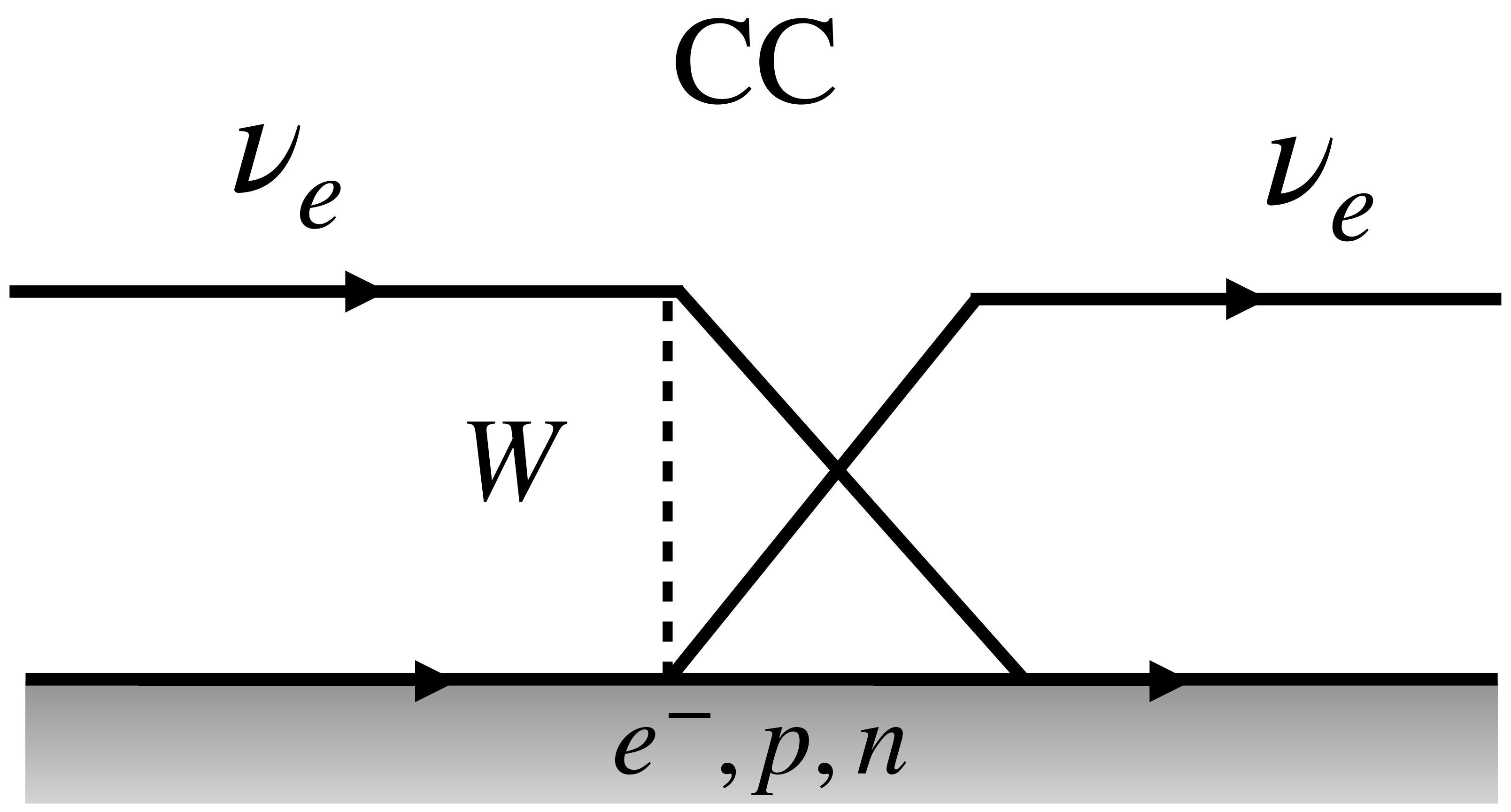
for a homogeneous and isotropic medium (Earth, Sun)

$$V_{cc}(r) = \sqrt{2} G_F n_e(r)$$

electron number density

Neutrino Flavor Oscillations in Matter

Basic Concepts



for a homogeneous and isotropic medium (Earth, Sun)

$$V_{cc}(r) = \sqrt{2} G_F n_e(r)$$

electron number density

singles out ν_e

and $\overline{\nu}_e$: $V_{cc}(r) \rightarrow -V_{cc}(r)$

gives rise to an extra phase difference among the flavors

Neutrino Flavor Oscillations in Matter

order of magnitude of the effect

electron fraction — matter density

$$V_{\rm cc}(r) = \sqrt{2} G_F n_e(r) \simeq 7.6 \left(\frac{n_e}{n_p + n_n}\right)\left(\frac{\rho}{\rm g/cm^3}\right) \times 10^{-14}\,{\rm eV}$$

electron fraction $\sim \frac{1}{2}$

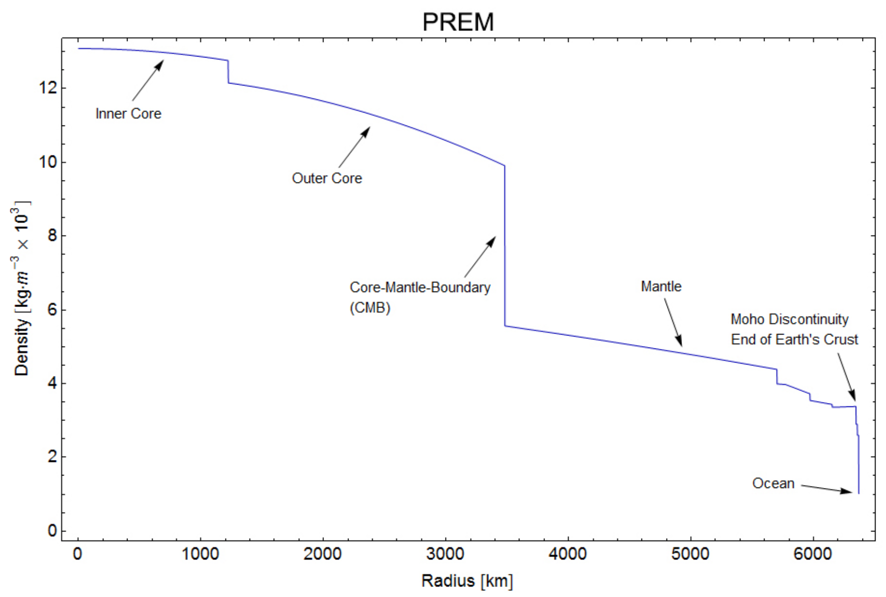
$\rho = 3\,{\rm g/cm^3} \implies V_{\rm cc} \sim 1 \times 10^{-13}\,{\rm eV}$ — **Earth's crust**

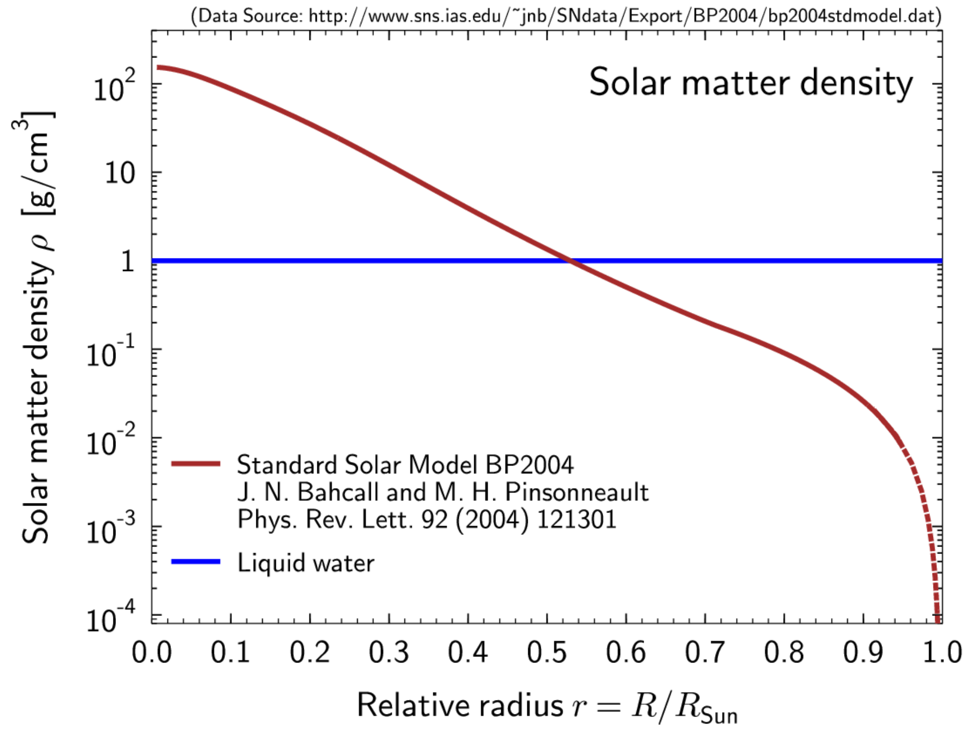
$\rho = 10\,{\rm g/cm^3} \implies V_{\rm cc} \sim 4 \times 10^{-13}\,{\rm eV}$ — **Earth's core**

$\rho = 100\,{\rm g/cm^3} \implies V_{\rm cc} \sim 4 \times 10^{-12}\,{\rm eV}$ — **Sun's core**

$\rho = 10^{14}\,{\rm g/cm^3} \implies V_{\rm cc} \sim 4\,{\rm eV}$ — **Supernova**

[see Alexander Friedland]





Neutrino Flavor Oscillations in Matter

For 3 flavors

We have to solve the Schrödinger evolution equation

$$i\frac{d}{dr}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = H \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix}$$

for the Hamiltonian

matter contribution (dynamics)

$$H(r) = \frac{1}{2E} U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{\rm cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

vacuum contribution

Neutrino Flavor Oscillations in Matter

For 3 flavors

It is useful to write this like this

$$A_{cc}(r)$$

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} 2EV_{cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

Neutrino Flavor Oscillations in Matter

For 3 flavors

It is useful to write this like this

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \overbrace{\begin{pmatrix} \boxed{2EV_{cc}(r)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{A_{cc}(r)} \right]$$

$$\Delta m^2 \sim 2 \times 10\,\mathrm{MeV} \times 4 \times 10^{-12}\,\mathrm{eV} \sim 8 \times 10^{-5}\,\mathrm{eV}^2$$ 10 MeV ν @ Sun's core

$$\Delta m^2/(2EV_{cc}) \sim \mathcal{O}(1)$$

$$\Delta m^2 \sim 2 \times 10\,\mathrm{GeV} \times 10^{-13}\,\mathrm{eV} \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$ 10 GeV ν @ Earth's crust

Neutrino Flavor Oscillations in Matter

For 3 flavors

This Hamiltonian can be diagonal at each point r of the evolution

$$H(r) = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A_{cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$= \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix} \tilde{m}_1{}^2(r) & & \\ & \tilde{m}_2{}^2(r) & \\ & & \tilde{m}_3{}^2(r) \end{pmatrix}\widetilde{U}^\dagger(r)$$

instantaneous mass eigenstates

$$\begin{pmatrix} |\nu_e(r)\rangle \\ |\nu_\mu(r)\rangle \\ |\nu_\tau(r)\rangle \end{pmatrix} = \tilde{U}(r)\begin{pmatrix} |\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle \\ |\widetilde{\nu_3}(r)\rangle \end{pmatrix}$$

Neutrino Flavor Oscillations in Matter

For 3 flavors

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in generally, solve numerally

Neutrino Flavor Oscillations in Matter

For 3 flavors

This Hamiltonian can be diagonal at each point r of the evolution

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A_{cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

in generally, solve numerally

but there are useful analytic solutions

$dA_{cc}(r)/dr = 0$ constant matter density

$dA_{cc}(r)/dr$ is small

i.e. adiabatic transition

we will discuss them for 2 flavors

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+V_{\mathrm{cc}} & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}$$

$$\Delta m^2=\Delta m^2_{21}$$

in constant matter density

mixing in matter

$$\widetilde{U}=\begin{pmatrix}\cos\theta_{\mathrm{m}} & \sin\theta_{\mathrm{m}}\\ -\sin\theta_{\mathrm{m}} & \cos\theta_{\mathrm{m}}\end{pmatrix}$$

$$|\nu_e\rangle=\cos\theta_{\mathrm{m}}\,|\widetilde{\nu_1}\rangle+\sin\theta_{\mathrm{m}}\,|\widetilde{\nu_2}\rangle$$
$$|\nu_x\rangle=-\sin\theta_{\mathrm{m}}\,|\widetilde{\nu_1}\rangle+\cos\theta_{\mathrm{m}}\,|\widetilde{\nu_2}\rangle$$

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+V_{\rm cc} & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}$$

$$\Delta m^2=\Delta m^2_{21}$$

in constant matter density

mixing in matter

$$\widetilde{U}=\begin{pmatrix}\cos\theta_{\rm m} & \sin\theta_{\rm m}\\ -\sin\theta_{\rm m} & \cos\theta_{\rm m}\end{pmatrix}$$

$$|\nu_e\rangle=\cos\theta_{\rm m}|\widetilde{\nu_1}\rangle+\sin\theta_{\rm m}|\widetilde{\nu_2}\rangle$$

$$|\nu_x\rangle=-\sin\theta_{\rm m}|\widetilde{\nu_1}\rangle+\cos\theta_{\rm m}|\widetilde{\nu_2}\rangle$$

mass squared splitting in matter

$$\Delta\widetilde{m}^2=\sqrt{(\Delta m^2\cos 2\theta-2E\,V_{\rm cc})^2+(\Delta m^2\sin 2\theta)^2}$$

$$\sin^2\theta_{\rm m}=\frac{1}{2}\left(1+\frac{(2EV_{\rm cc}-\Delta m^2\cos 2\theta)}{\Delta\widetilde{m}^2}\right)$$

if $2EV_{\rm cc}\gg\Delta m^2$

$\sin^2\theta_m\to 1\quad |\nu_\alpha\rangle\to|\widetilde{\nu}_2\rangle$

if $2EV_{\rm cc}\ll\Delta m^2\cos 2\theta$

we recover vacuum solution

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+V_{\rm cc} & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}$$

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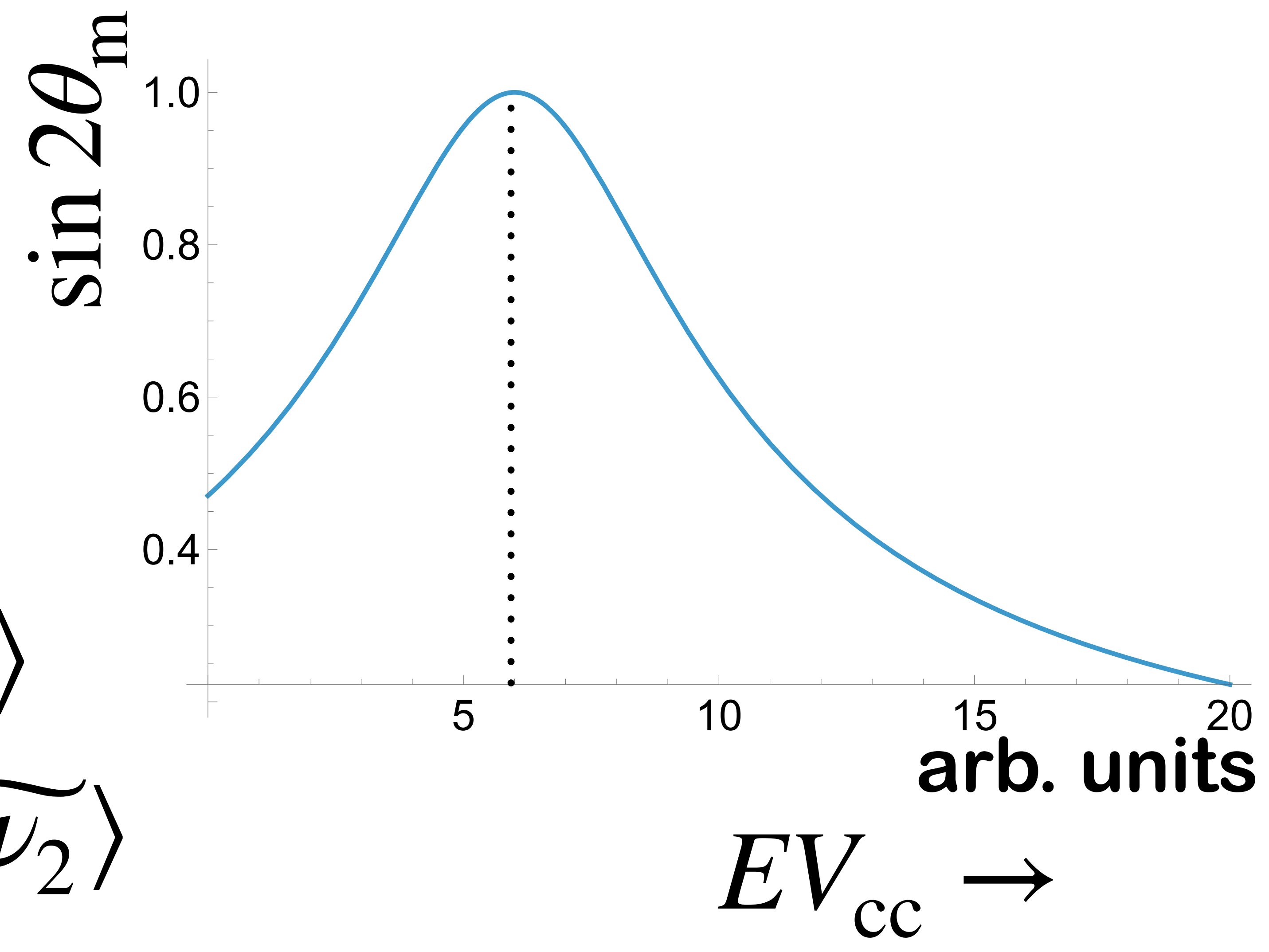

mass squared splitting in matter

$$\Delta\widetilde{m}^2=\sqrt{(\Delta m^2\cos 2\theta-2E\,V_{\rm cc})^2+(\Delta m^2\sin 2\theta)^2}$$

mixing in matter

$$\sin 2\theta_{\rm m}=\frac{\sin 2\theta}{\sqrt{(\cos 2\theta-2EV_{\rm cc}/\Delta m^2)^2+\sin^2 2\theta}}$$

MSW resonance condition

$$\Delta m^2\cos 2\theta=2E\,V_{\rm cc}$$

maximal mixing in matter

$$\theta_m=45^\circ$$

if happens either $\Delta m^2>0$ or $\cos 2\theta<0$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix} |\nu_e\rangle \\ |\nu_x\rangle \end{pmatrix} = H \begin{pmatrix} |\nu_e\rangle \\ |\nu_x\rangle \end{pmatrix}$$

$$H = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix} \tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r) \end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix} |\nu_e(r)\rangle \\ |\nu_x(r)\rangle \end{pmatrix} = \widetilde{U}(r)\begin{pmatrix} |\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle \end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} \qquad H = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix}|\nu_e(r)\rangle \\ |\nu_x(r)\rangle\end{pmatrix} = \widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle\end{pmatrix}$$

$$\Rightarrow \quad i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

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$$\Rightarrow \quad i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix}$$

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Adiabatic Transition in Matter

2 flavors

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} \qquad \textbf{(1)}$$

slowly varying matter density (Sun)

if $\frac{d}{dr}\widetilde{U}(r) \simeq 0$

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} \simeq i\,\widetilde{U}(r)\frac{d}{dr}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} \qquad \textbf{(2)}$$

Adiabatic Transition in Matter

2 flavors

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix} \quad \textbf{(1)}$$

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$\widetilde{U}(r)^\dagger \times$ **(1) using (2)**

$$i\frac{d}{dr}\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix}$$

Equations decouple !

Mass eigenstates evolve independently

They do not mix!

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\widetilde{\nu}_1\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}|\widetilde{\nu}_1\rangle(r_i) \qquad \phi_1/2E$$

$$|\widetilde{\nu}_2\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}|\widetilde{\nu}_2\rangle(r_i) \qquad \phi_2/2E$$

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\widetilde{\nu}_1\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}|\widetilde{\nu}_1\rangle(r_i)$$

$\phi_1/2E$

$$|\widetilde{\nu}_2\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}|\widetilde{\nu}_2\rangle(r_i)$$

$\phi_2/2E$

$$\begin{pmatrix} |\nu_e(r_i)\rangle \\ |\nu_x(r_i)\rangle \end{pmatrix} = \widetilde{U}(r_i)\begin{pmatrix} |\widetilde{\nu_1}(r_i)\rangle \\ |\widetilde{\nu_2}(r_i)\rangle \end{pmatrix} = \begin{pmatrix} \cos\widetilde{\theta}(r_i) & \sin\widetilde{\theta}(r_i) \\ -\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i) \end{pmatrix}\begin{pmatrix} |\widetilde{\nu_1}(r_i)\rangle \\ |\widetilde{\nu_2}(r_i)\rangle \end{pmatrix}$$

Flavor eigenstates @ the production point

$$\begin{pmatrix} |\nu_e(r_f)\rangle \\ |\nu_x(r_f)\rangle \end{pmatrix} = \widetilde{U}(r_f)\begin{pmatrix} |\widetilde{\nu_1}(r_f)\rangle \\ |\widetilde{\nu_2}(r_f)\rangle \end{pmatrix} = \begin{pmatrix} \cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f) \\ -\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f) \end{pmatrix}\begin{pmatrix} |\widetilde{\nu_1}(r_f)\rangle \\ |\widetilde{\nu_2}(r_f)\rangle \end{pmatrix}$$

Flavor eigenstates @ the end of evolution

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\tilde{\nu}_1\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}|\tilde{\nu}_1\rangle(r_i) \qquad |\tilde{\nu}_2\rangle(r_f) = e^{-i\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}|\tilde{\nu}_2\rangle(r_i)$$

$\phi_1/2E$ $\qquad$ $\phi_2/2E$

$$\begin{pmatrix}|\nu_e(r_i)\rangle\\|\nu_x(r_i)\rangle\end{pmatrix} = \widetilde{U}(r_i)\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_i) & \sin\widetilde{\theta}(r_i)\\-\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix}$$

Flavor eigenstates @ the production point

$$\begin{pmatrix}|\nu_e(r_f)\rangle\\|\nu_x(r_f)\rangle\end{pmatrix} = \widetilde{U}(r_f)\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f)\\-\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix}$$

Flavor eigenstates @ the end of evolution

$$= \begin{pmatrix}\cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f)\\-\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f)\end{pmatrix}\begin{pmatrix}e^{-i\phi_1/2E} & 0\\0 & e^{-i\phi_2/2E}\end{pmatrix}\begin{pmatrix}\cos\widetilde{\theta}(r_i) & -\sin\widetilde{\theta}(r_i)\\\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i)\end{pmatrix}\begin{pmatrix}|\nu_e(r_i)\rangle\\|\nu_x(r_i)\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

Survival Propapility of ν_e

$$P_{ee} = |\mathscr{A}_{ee}|^2 = |\langle \nu_e(r_f)|\nu_e(r_i)\rangle|^2 = |\cos\widetilde{\theta}(r_f)\,\cos\widetilde{\theta}(r_i) + e^{-i\frac{(\phi_2-\phi_1)}{2E}}\,\sin\widetilde{\theta}(r_f)\,\sin\widetilde{\theta}(r_i)|^2$$

the interference term average out

$\Delta\phi \gg E$ [S. Parke (1986)]

$$P_{ee} \simeq \cos^2\theta_{12}\,\cos^2\widetilde{\theta}(r_i) + \sin^2\theta_{12}\,\sin^2\widetilde{\theta}(r_i)$$

vacuum value @ the exit of the Sun

Reactor $\bar{\nu}_e$ Disappearance Measurement

Useful way to write the exact survival probability in vacuum

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\;\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

[H. Minakata, H. Nunokawa, S. J. Parke, RZF (2007)]

$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{21}$$

[H. Nunokawa, S. J. Parke, RZF (2005)]

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij}}{4E}L = \pi\,\frac{L}{L^{ij}_{\rm osc}}$$

effective atmospheric

$$\Delta m^2_{\rm atm}$$

for reactor experiments

JUNO's Concept

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

A medium baseline reactor neutrino oscillation experiment

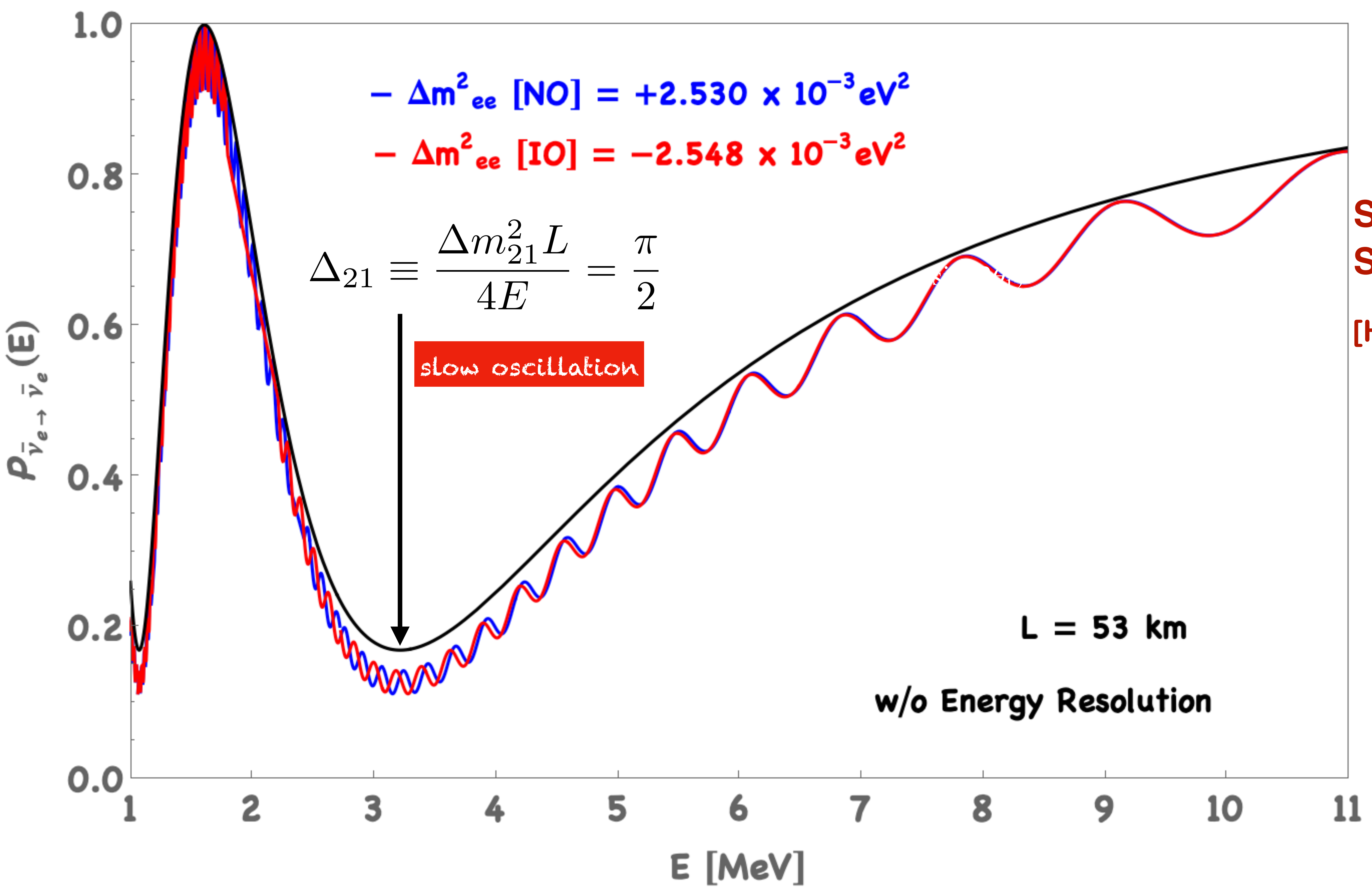
$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\ \cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

JUNO's Concept

A medium baseline reactor neutrino oscillation experiment

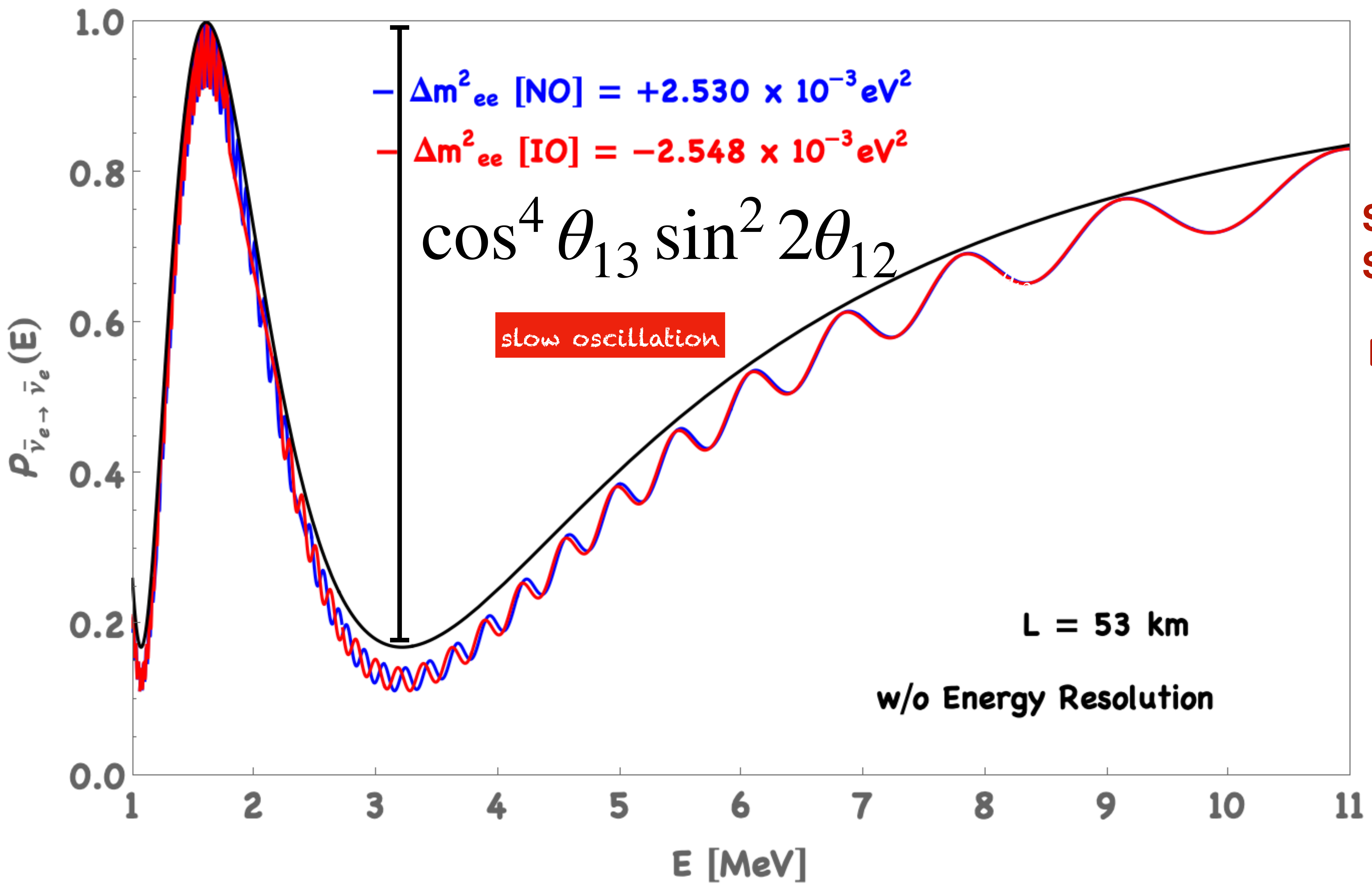


SADO ≡
Several-tens of km Antineutrino DetectOr

[H.Minakata, H. Nunokawa, W.Teves, RZF (2005)]

JUNO's Concept

A medium baseline reactor neutrino oscillation experiment



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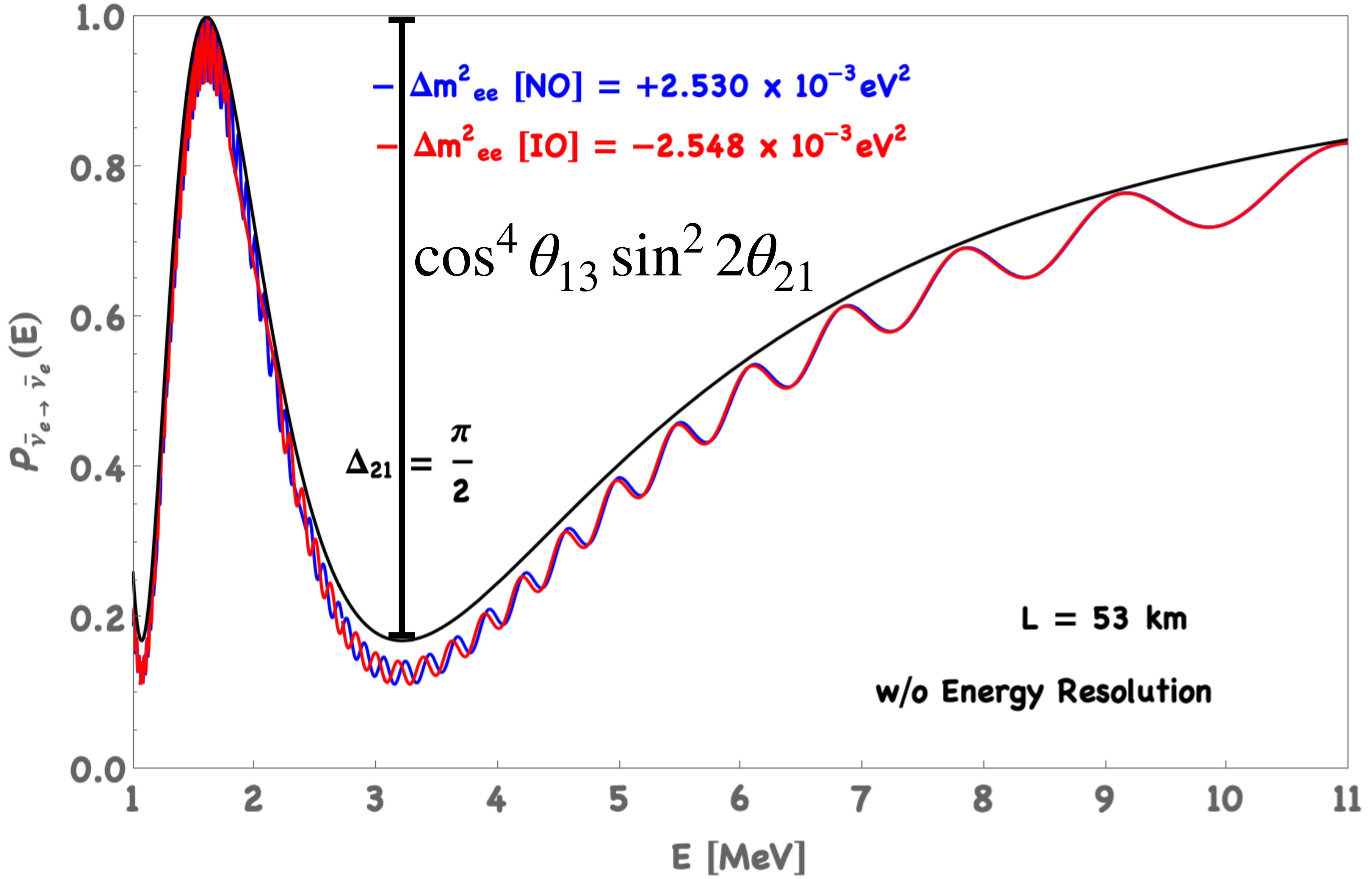
solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

	100 days	6 years
Δm^2_{21}	$\sim 1.0\,\%\,(2.5\%)$	$\sim 0.3\,\%$
$\sin^2\theta_{12}$	$\sim 1.9\,\%\,(3.9\%)$	$\sim 0.5\,\%$



JUNO's Concept

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E} \sim \frac{\pi}{2}$$

A medium baseline reactor neutrino oscillation experiment

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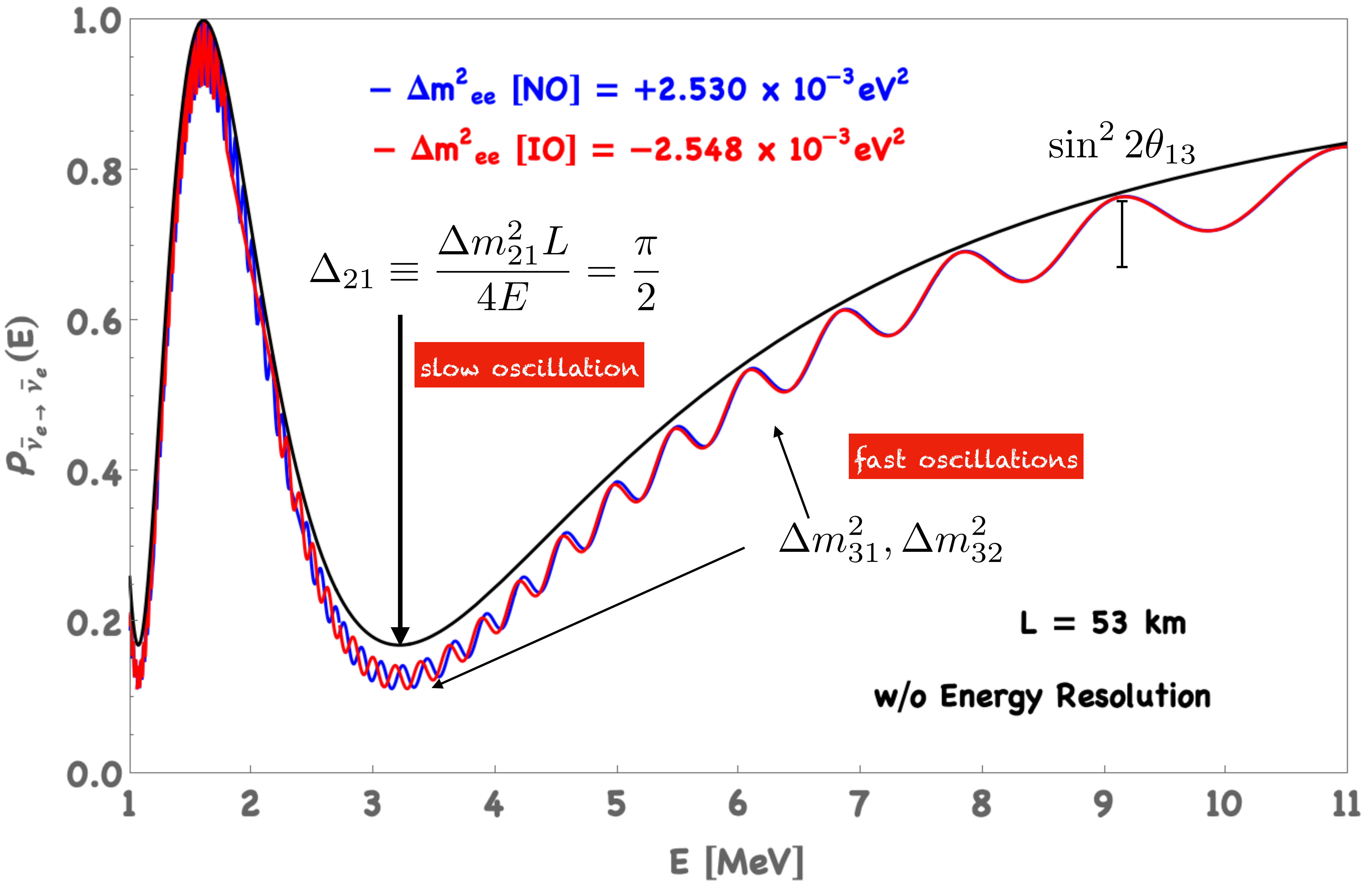
JUNO's Concept

A medium baseline reactor neutrino oscillation experiment

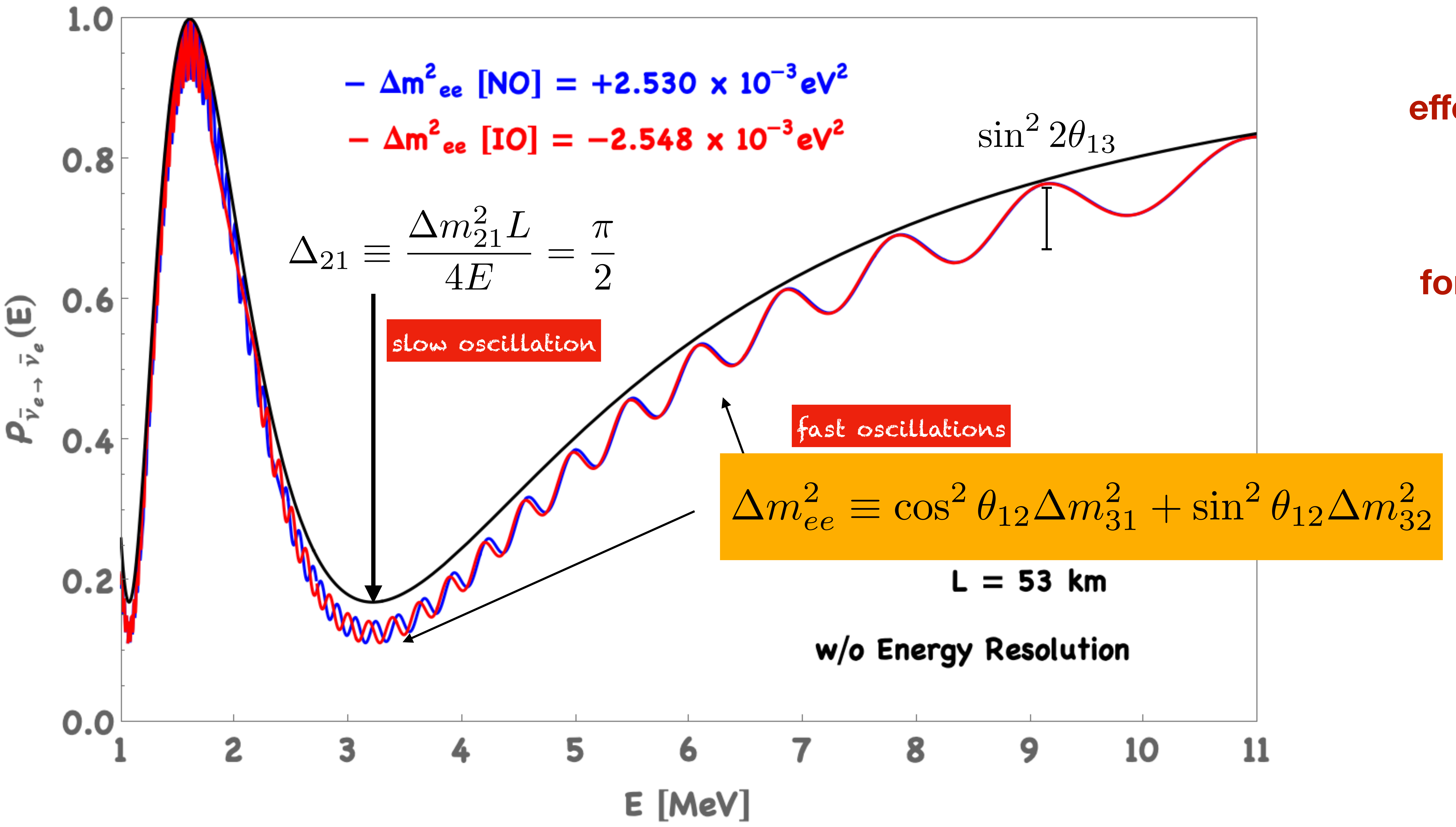


low and high frequency modes present

[S.T.Petcov, M Piai (2002) & S. Choubey+ (2003)]

JUNO's Concept

A medium baseline reactor neutrino oscillation experiment



effective atmospheric

$\Delta m^2_{\rm atm}$

for reactor experiments

JUNO's Concept

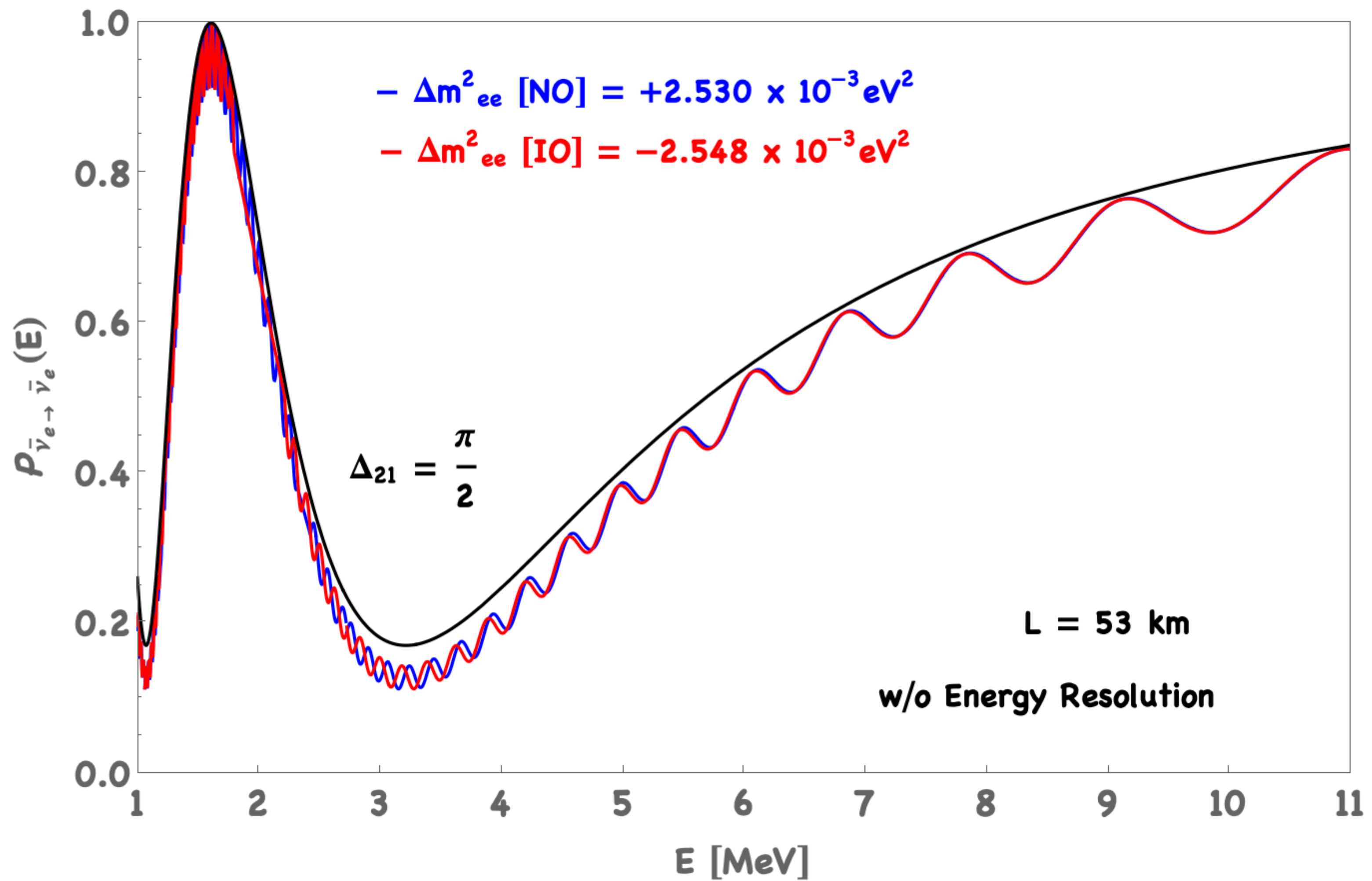
Flagship Measurement - The Neutrino Mass Ordering

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left[1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos(2|\Delta_{ee}| \pm \Phi_\odot) \right] - P_\odot$$

in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21}$$



$$\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2 \theta_{12} + \Delta m^2_{32} \sin^2 \theta_{21}$$

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

JUNO's Concept

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E} \sim \frac{\pi}{2}$$

Flagship Measurement - The Neutrino Mass Ordering

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

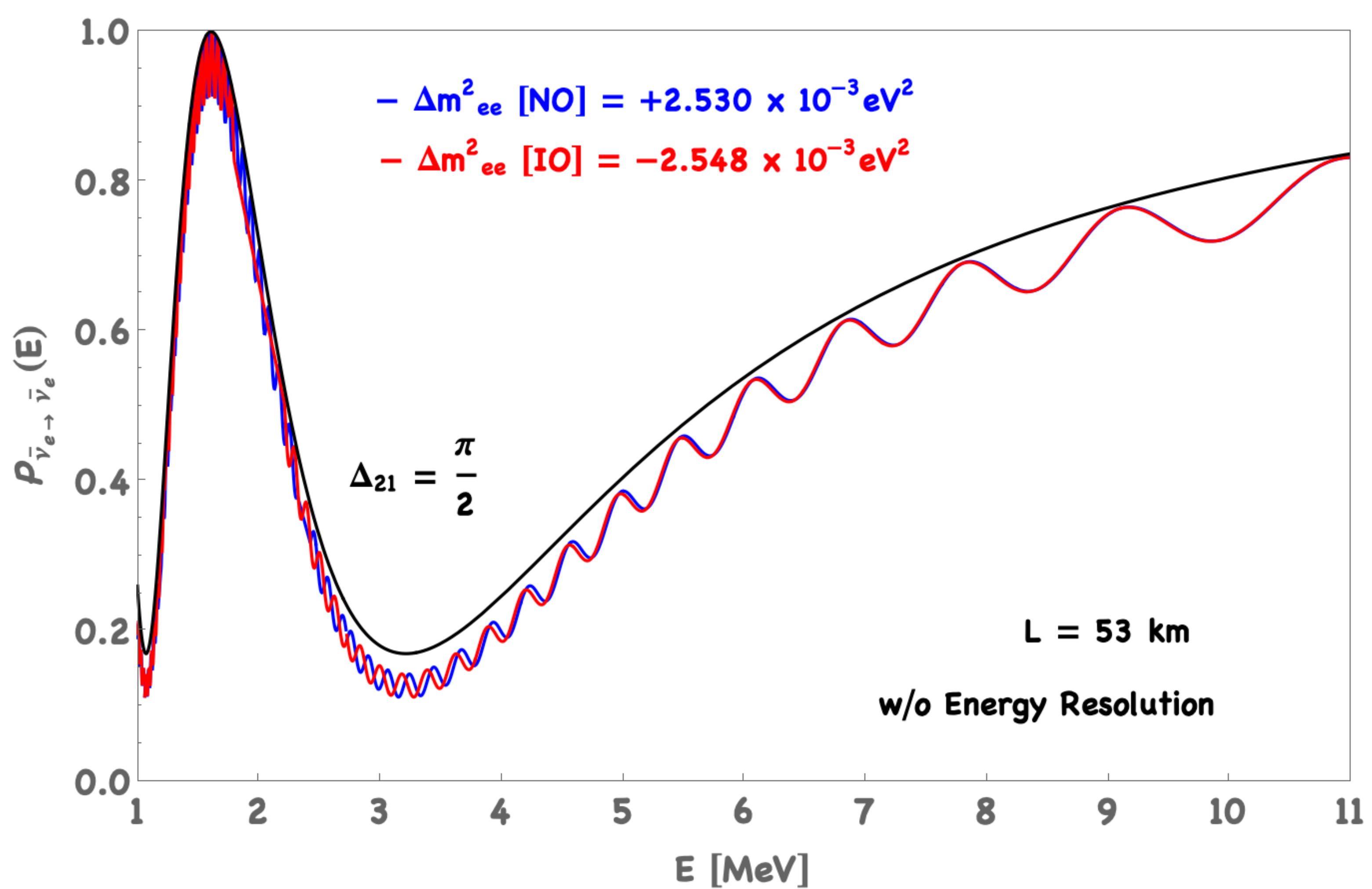
in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

phase

$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$



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JUNO's Concept

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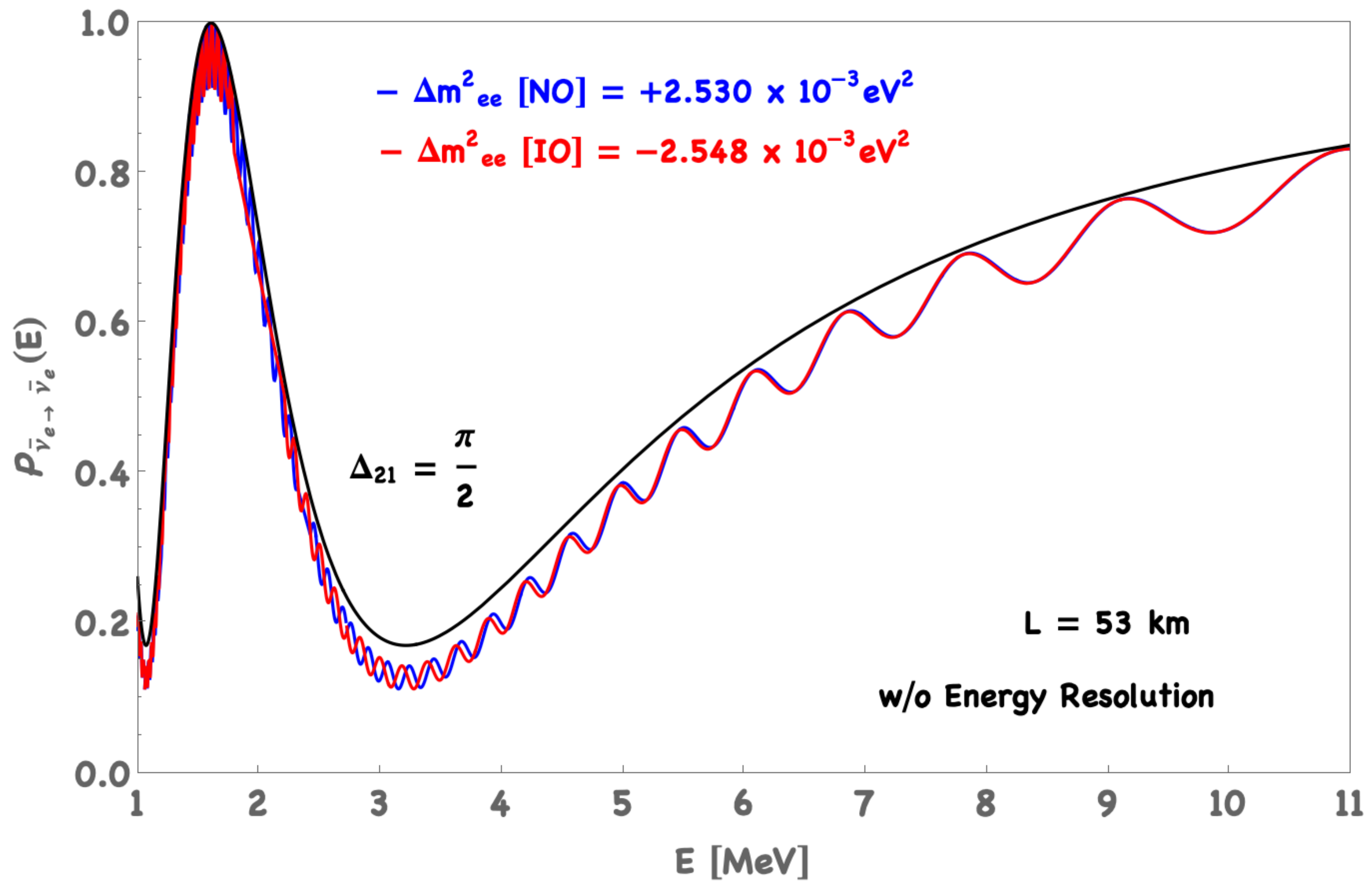
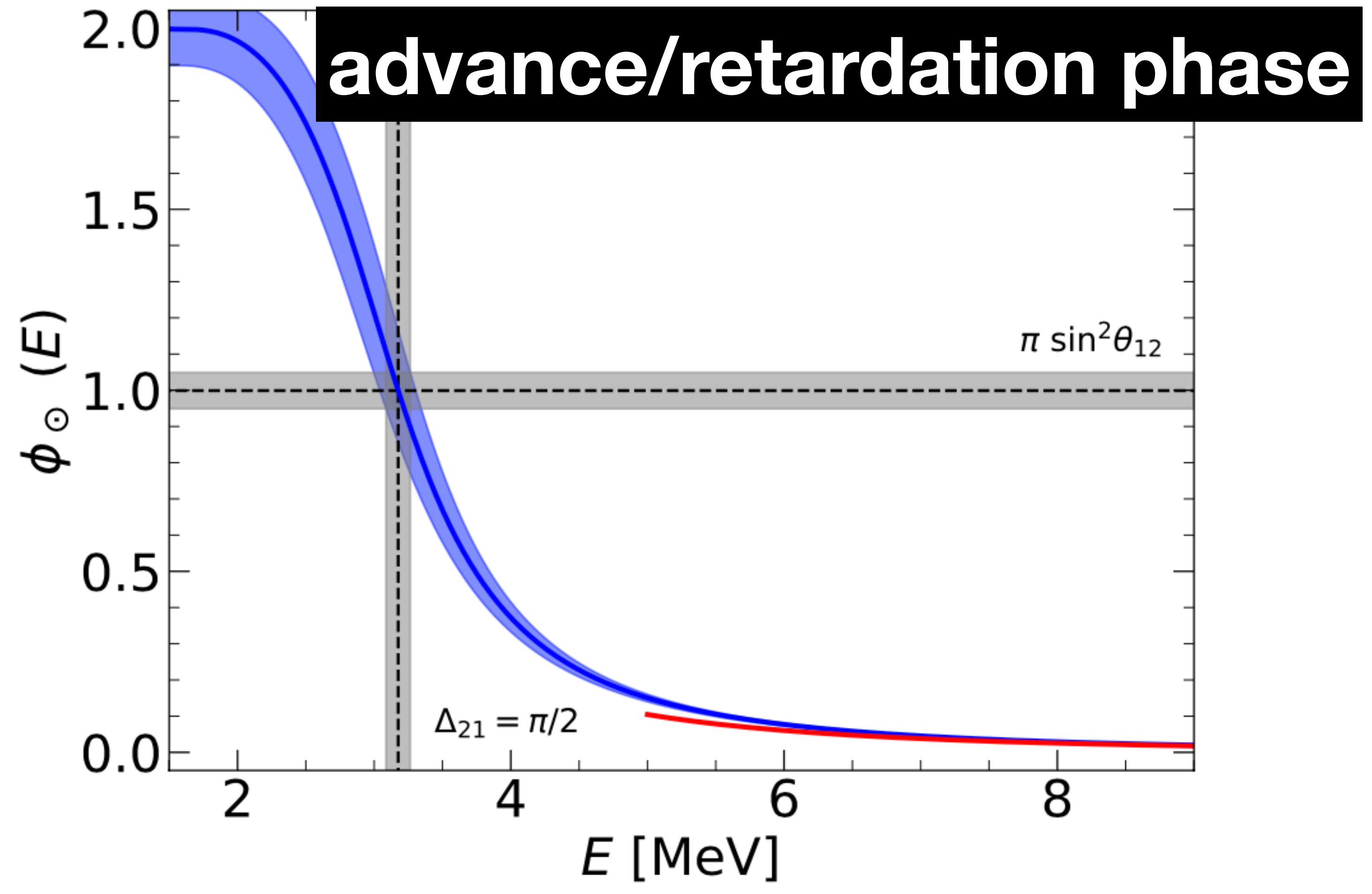


retardation/advancement of the phase result in a change of the "effective fast oscillation scale"

$$|\Delta m^2_{ee}|_{IO} > |\Delta m^2_{ee}|_{NO}$$



JUNO's Concept

Flagship Measurement - The Neutrino Mass Ordering

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

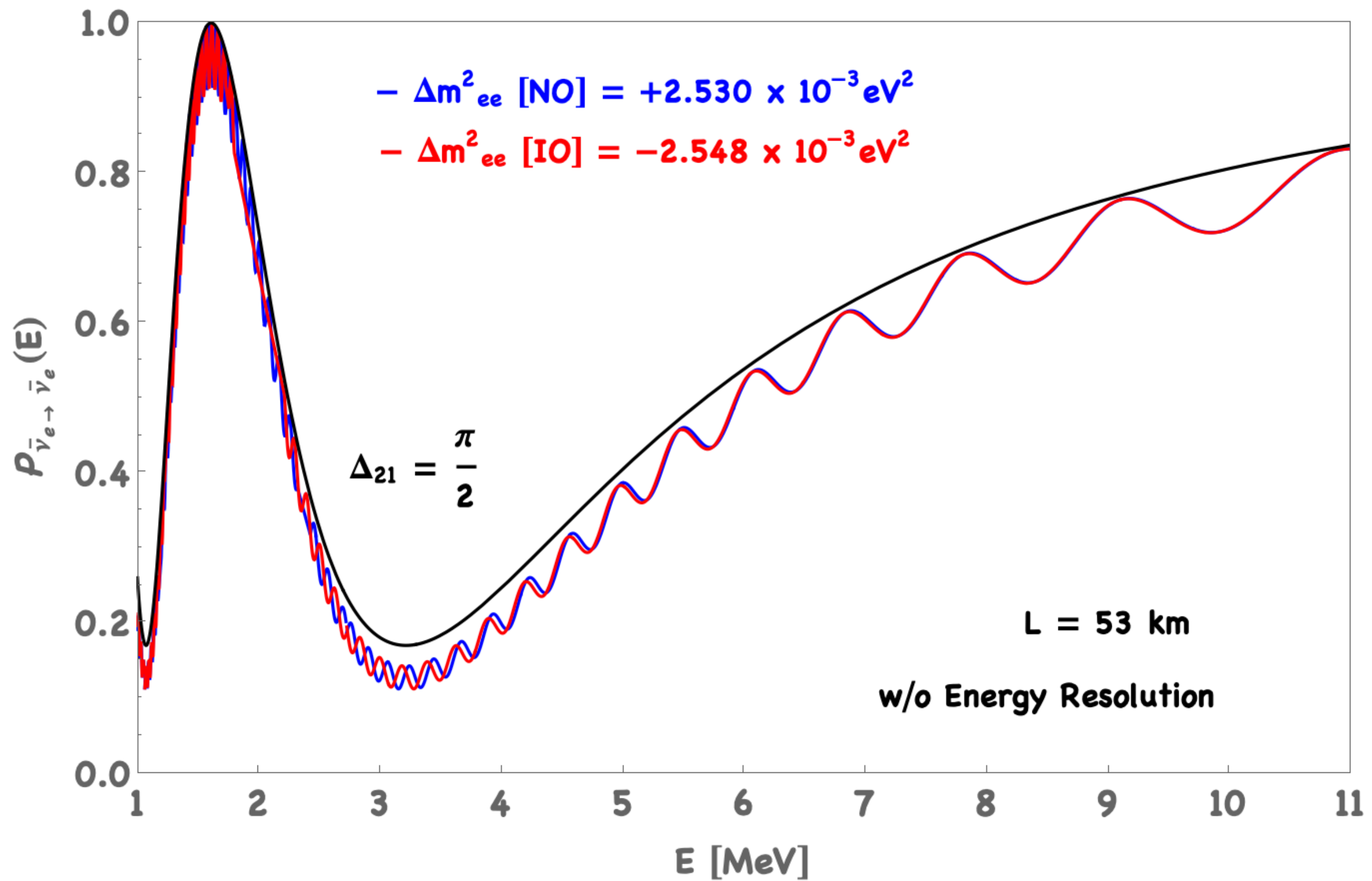
in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

phase

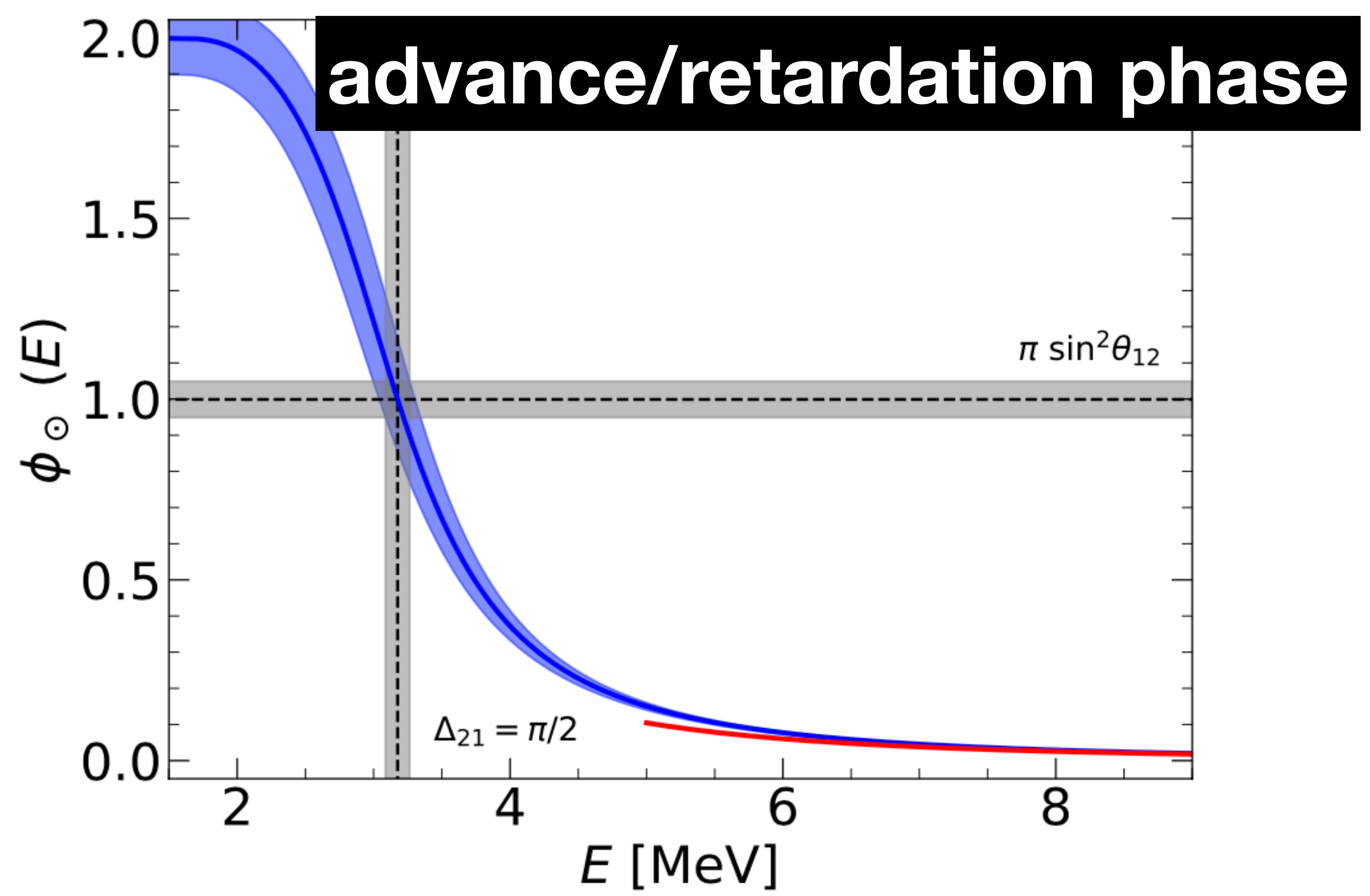
$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$



retardation/advancement of the phase result in a change of the "effective fast oscillation scale"

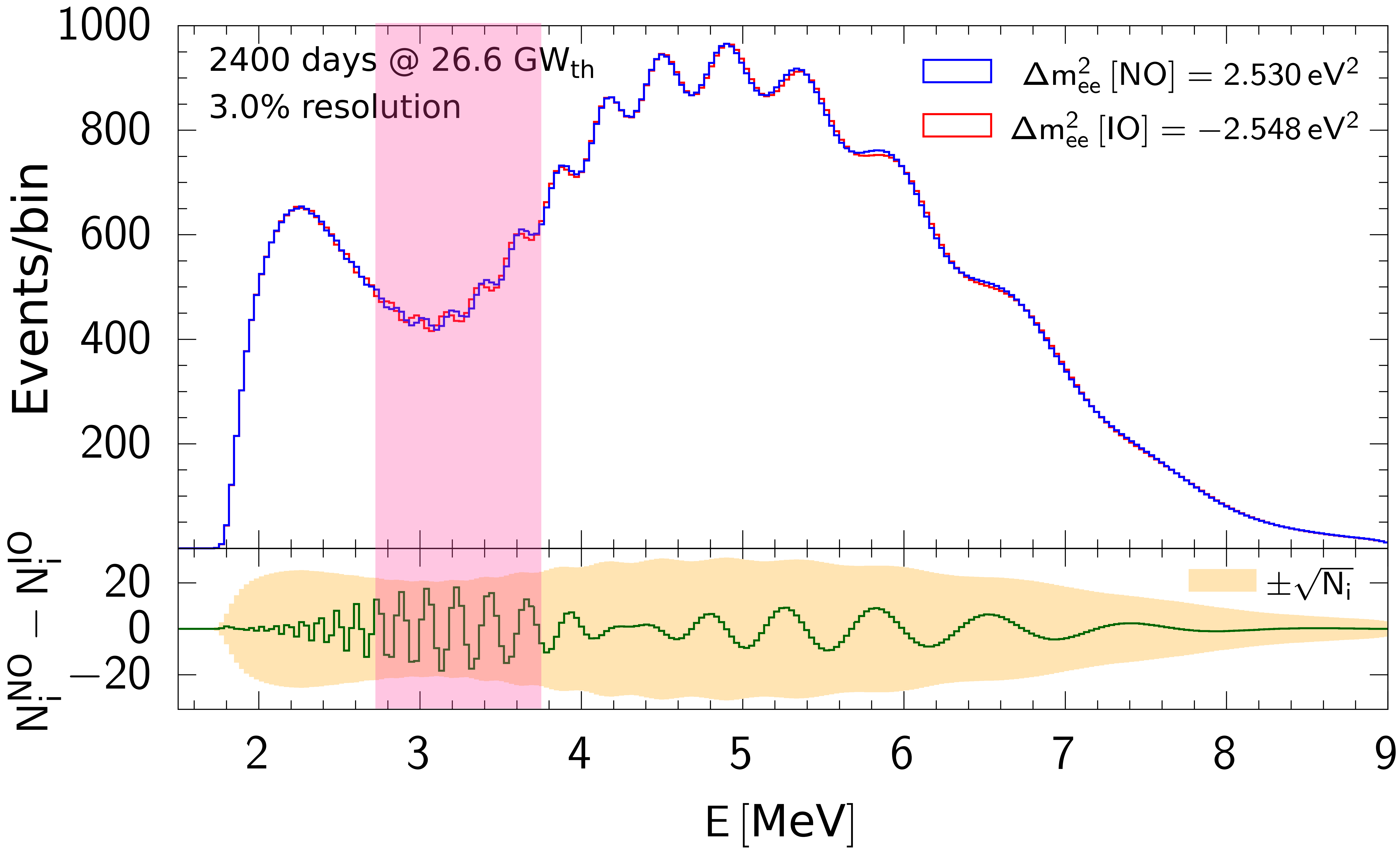
$$|\Delta m^2_{ee}|_{NO} \neq |\Delta m^2_{ee}|_{IO}$$

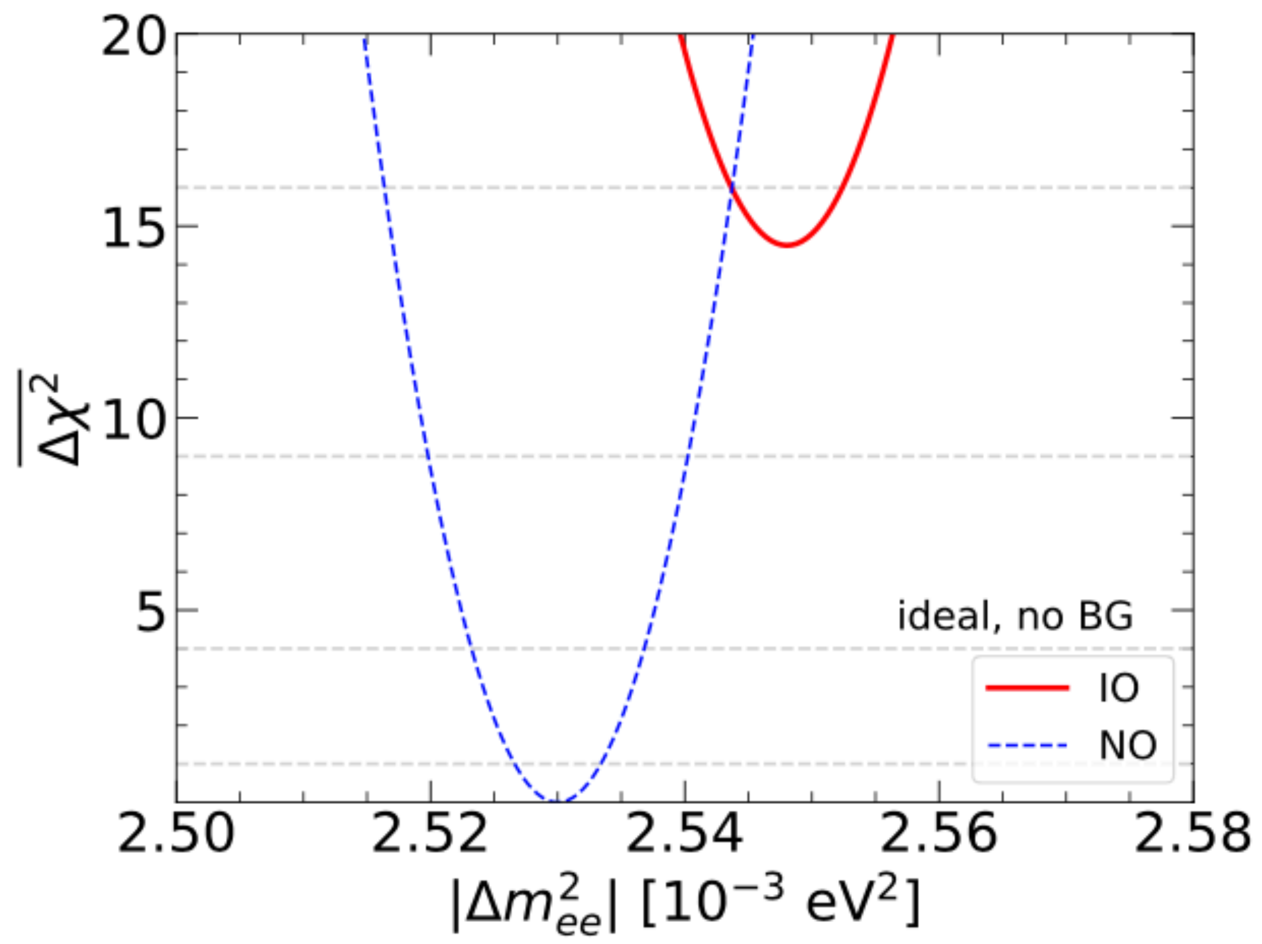
100 days	6 years
$\sim 0.8\,\%$	$\sim 0.2\,\%$



JUNO's Determination of the Ordering

How challenging is this?





single core
no other systematics
no background events

every single effect counts !