Lecture IV:

Evidences of Masses and Mixings - Current Status

"Anyway, in as far as the neutrino masses are negligible compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effect such as neutrino oscillations."

Froggatt & Nielsen

WHAT HAVE WE ALREADY LEARNED FROM OSCILLATION EXPERIMENTS ?

HOW DO WE CLASSIFY OSCILLATION EXPERIMENTS?

Four Basic Types

$$\nu_\alpha \to \nu_\alpha$$
$$\overline{\nu}_\alpha \to \overline{\nu}_\alpha$$

disappearance experiments

we talk about survival probability

$$\nu_\alpha \to \nu_\beta$$
$$\overline{\nu}_\alpha \to \overline{\nu}_\beta$$

appearance experiments

we talk about oscillation probability

HOW DO WE COMPARE DATA WITH THEORY?

WHAT DO EXPERIMENTS MEASURE?

[Experiment]
# of neutrinos

[Type of Source]
ν Flux

[Theory]
survival probability

$$N_\alpha(L) = A \int \Phi_\alpha(E)\,\sigma(E)\,P(\nu_\alpha \rightarrow \nu_\alpha; E, L)\,\epsilon(E)\,dE$$

number of targets x time
[Experiment]

x-section
[Theory]

detector efficiency
[Experiment]

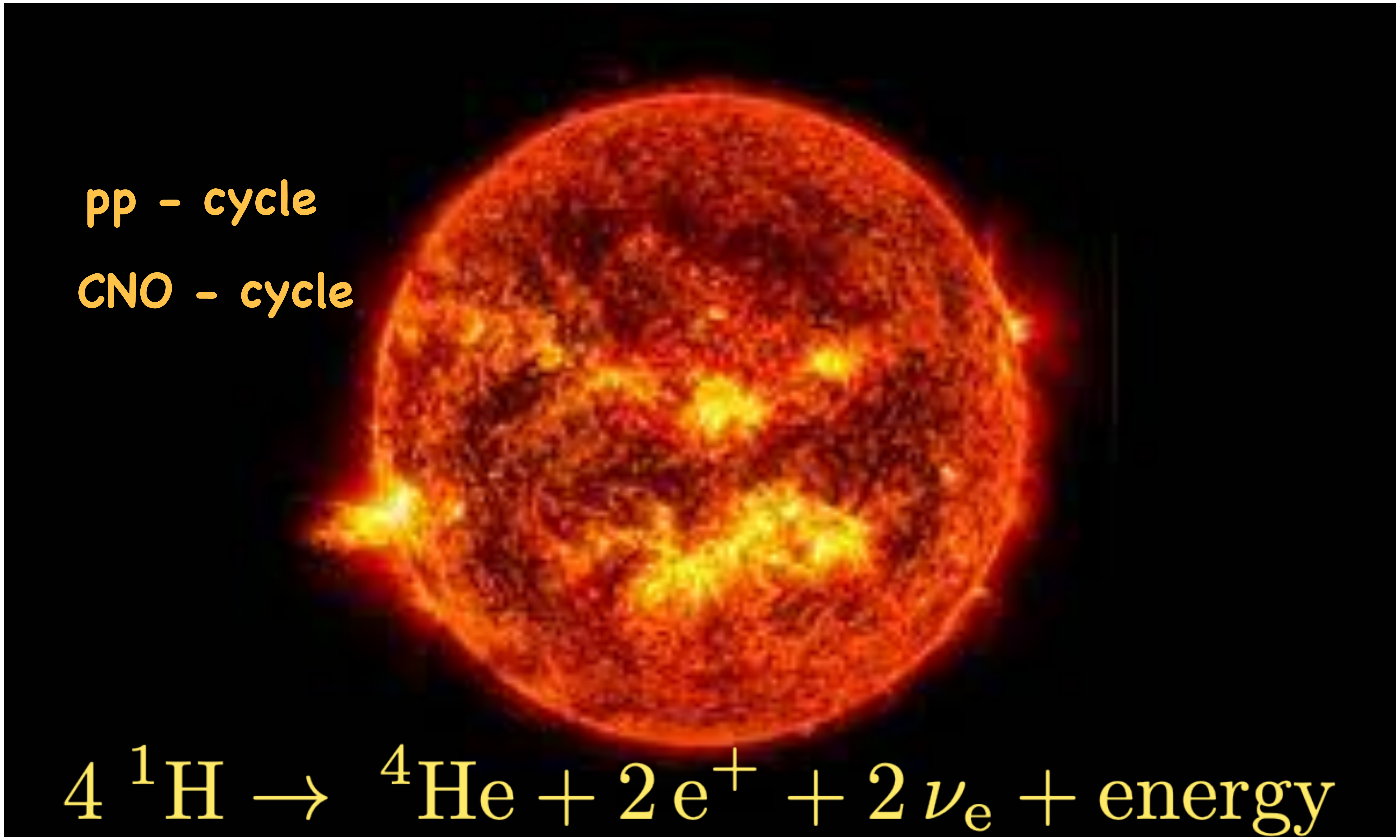
Solar Neutrinos

Solar Neutrinos

Standard Solar Model

[H. A. Bethe, “Energy production in stars” (1939)]

[J. Bahcall et al. (2001)]



[Vinyoles et al. (2017)]

[@NASA]

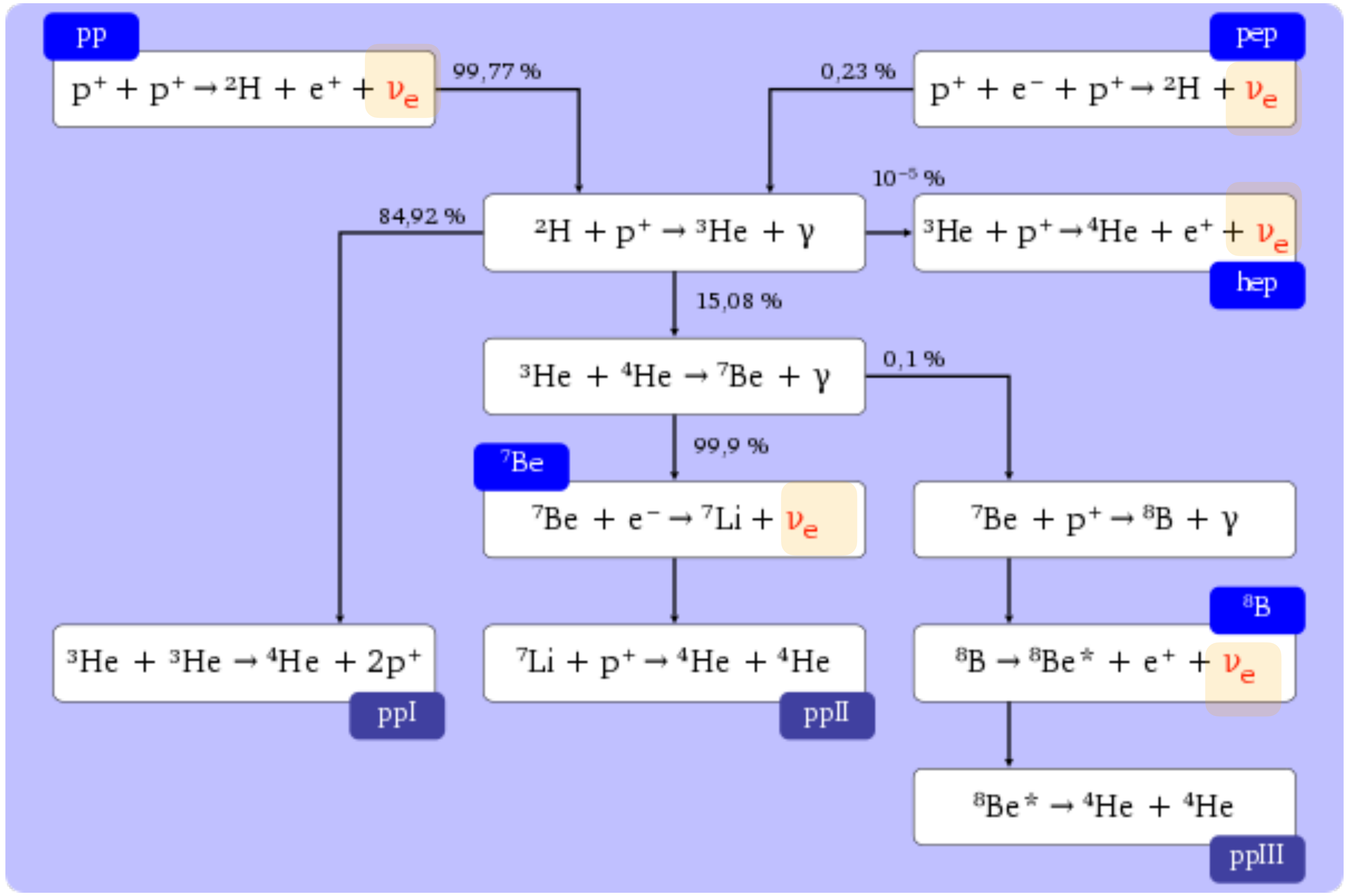
only electron neutrinos are produced in the Sun

Solar Neutrinos

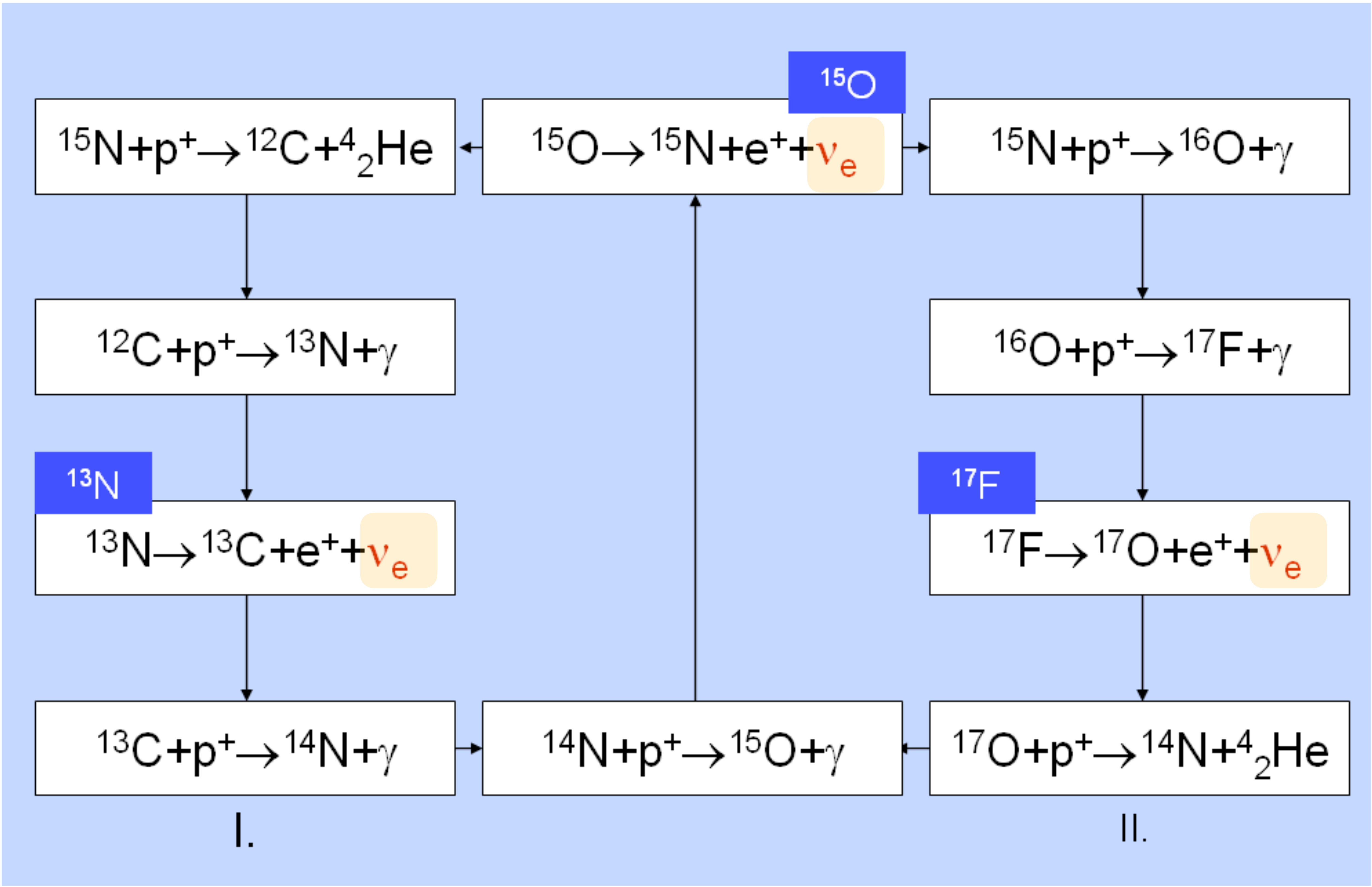
Standard Solar Model

chains of thermonuclear reactions

pp-cycle (99% of energy output of the Sun)



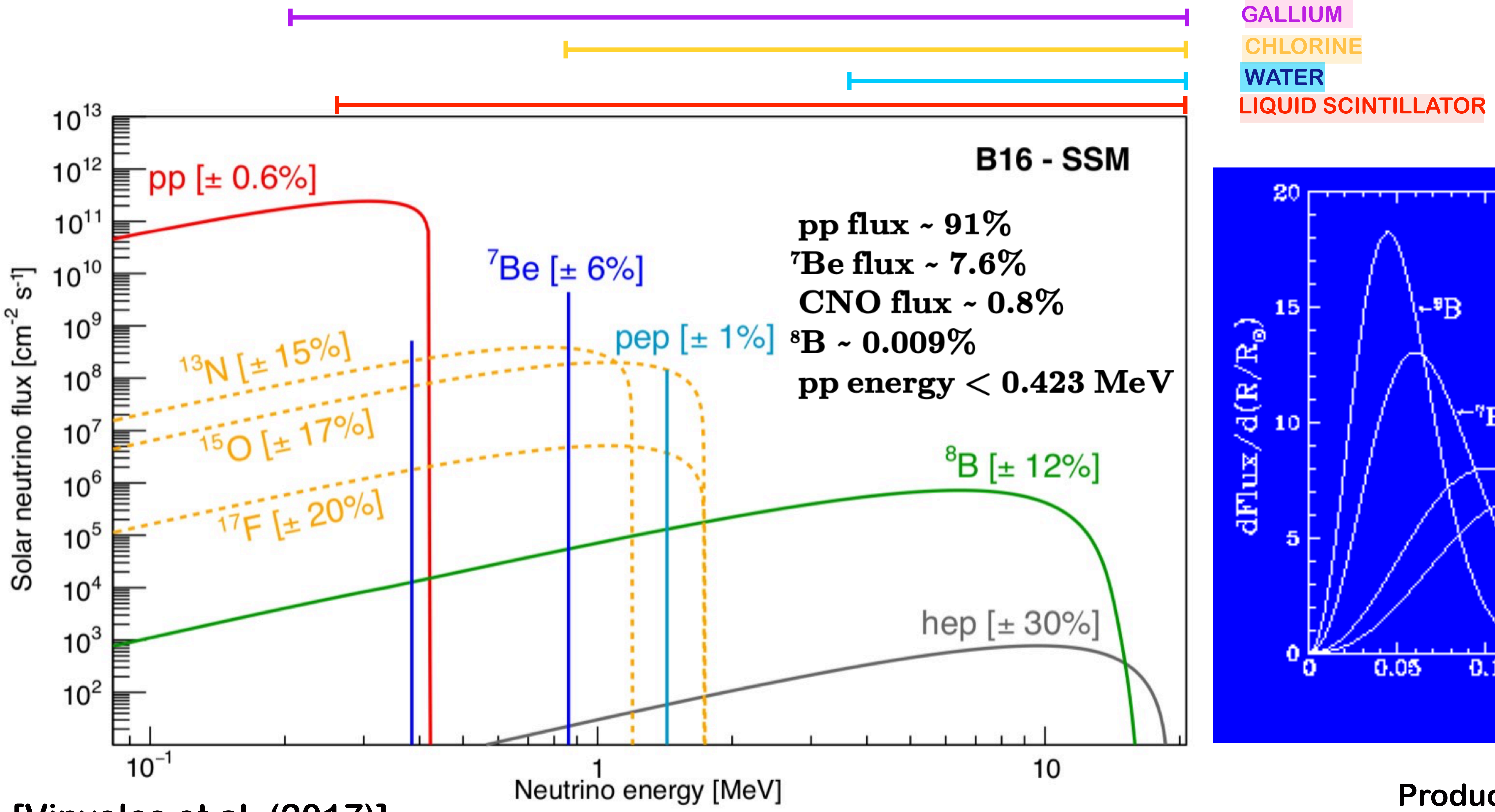
CNO-cycle



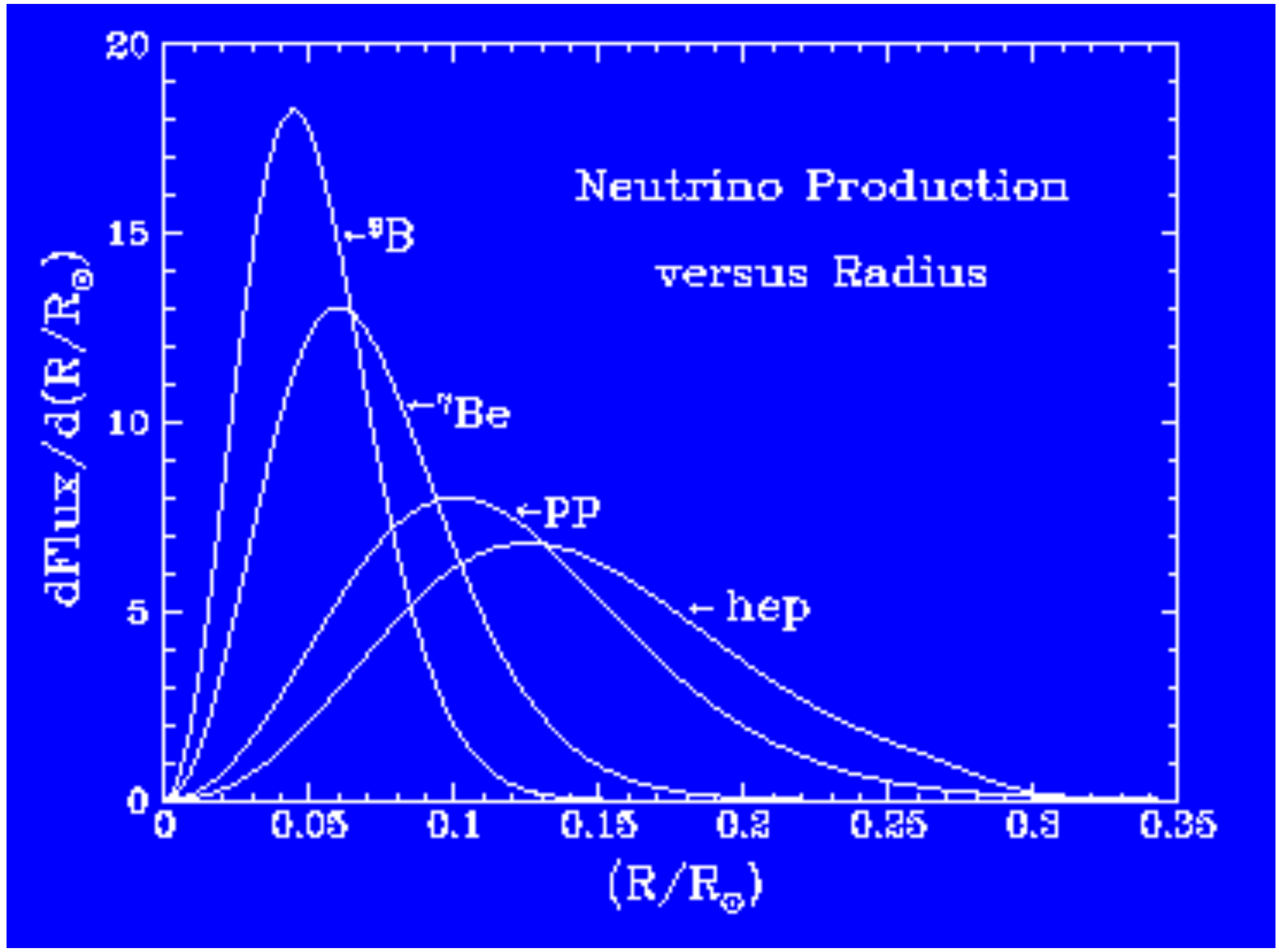
[@Wikipedia]

Solar Neutrino Flux

Standard Solar Model



GALLEX/GNO/SAGE
HOMESTAKE
KAMIOKANDE/SuperK/SNO
BOREXINO



[Vinyoles et al. (2017)]

Energy Distribution

Production Point Distribution

[@J.Bahcall]

Solar Neutrinos

ν_e Survival Probability

Solar neutrinos undergo matter effects inside the Sun and also when they travel through the Earth (ignored here)
In the Sun the problem reduces to an effective 2 flavor problem

[show this]

$$P_{3\nu}(\nu_e \to \nu_e) = s_{13}^4 + c_{13}^4\, P_{2\nu}(\nu_e \to \nu_e; V_{CC} \to c_{13}^2\, V_{CC})$$

$$P_{2\nu}(\nu_e \to \nu_e; V_{CC} \to c_{13}^2 V_{CC}) = |\mathcal{A}_{ee}(R_\odot)|^2$$

Survival Probability at the surface

Solar Neutrinos

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Survival Probability at the surface

finally we have to take into account the energy distribution and the distribution of the production point

$$P_{2\nu}^X = \overline{\cos^2\theta_m}\,\cos^2\theta_{12} + \overline{\sin^2\theta_m}\,\sin^2\theta_{12} \qquad X = \mathrm{pp}, {}^7\mathrm{Be}, {}^8\mathrm{B}, \mathrm{etc.}$$

averaged over production point and energy distribution

Solar Neutrinos

Adiabatic Transition

$\nu_e \equiv \nu_2$

Sun's core

in our current notation

$\Delta m^2_{21} \equiv \delta m^2_\odot$

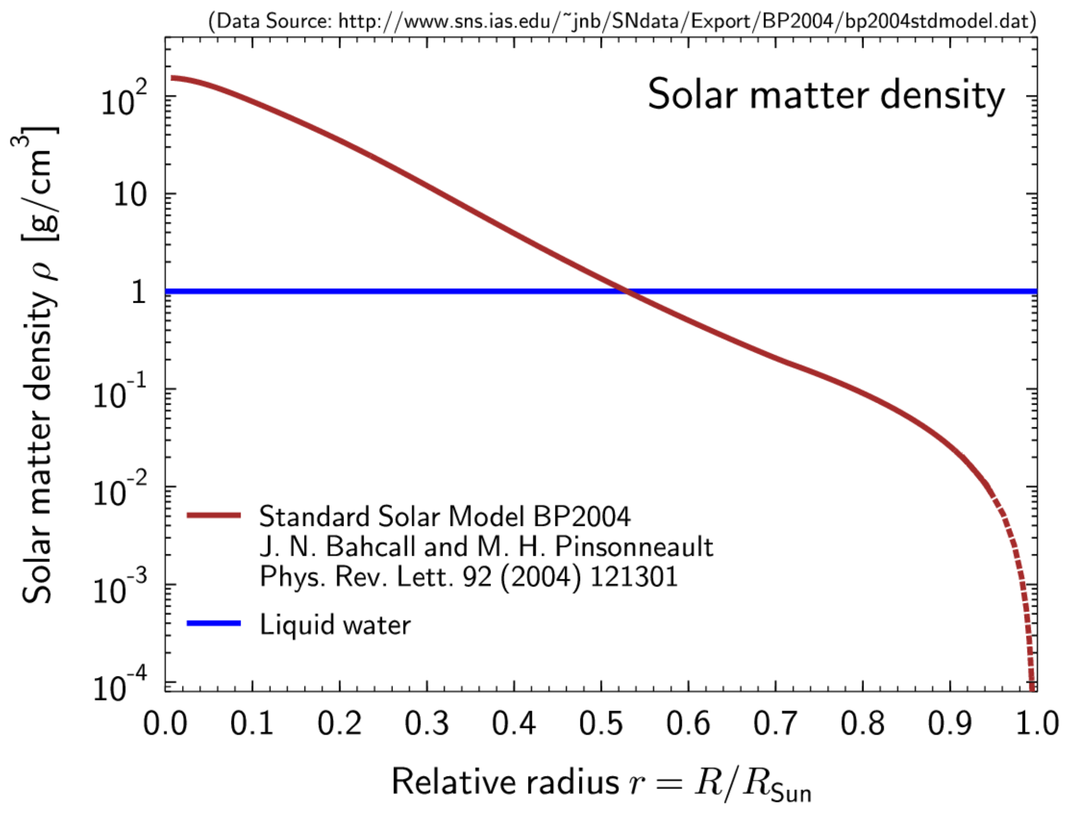
$\sin^2\theta_{12} \equiv \sin^2\theta_\odot$

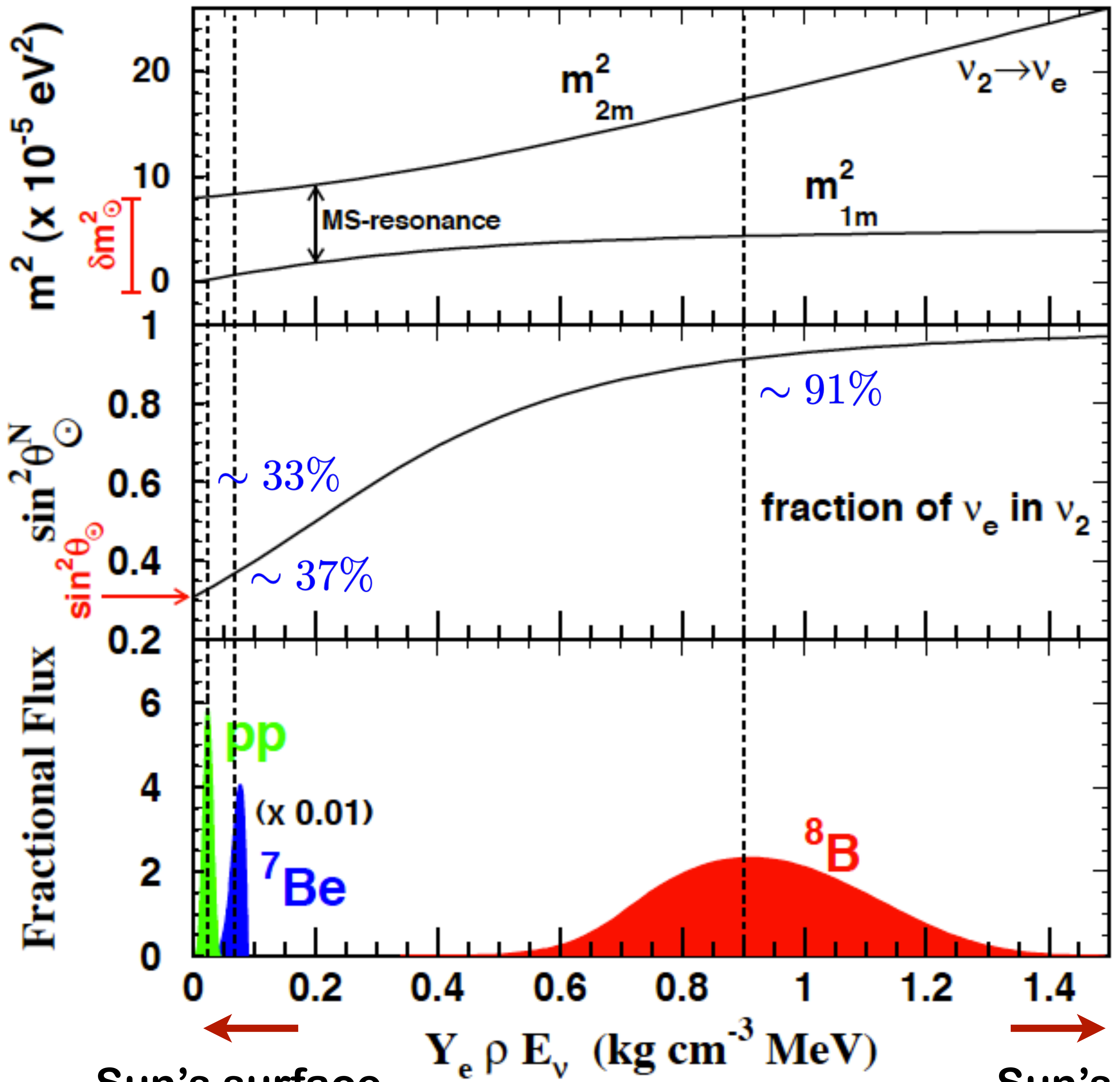
$\sin^2\theta_m \equiv \sin^2\theta^N_\odot$

Solar neutrinos

tell us that

$\Delta m^2_{21} > 0$



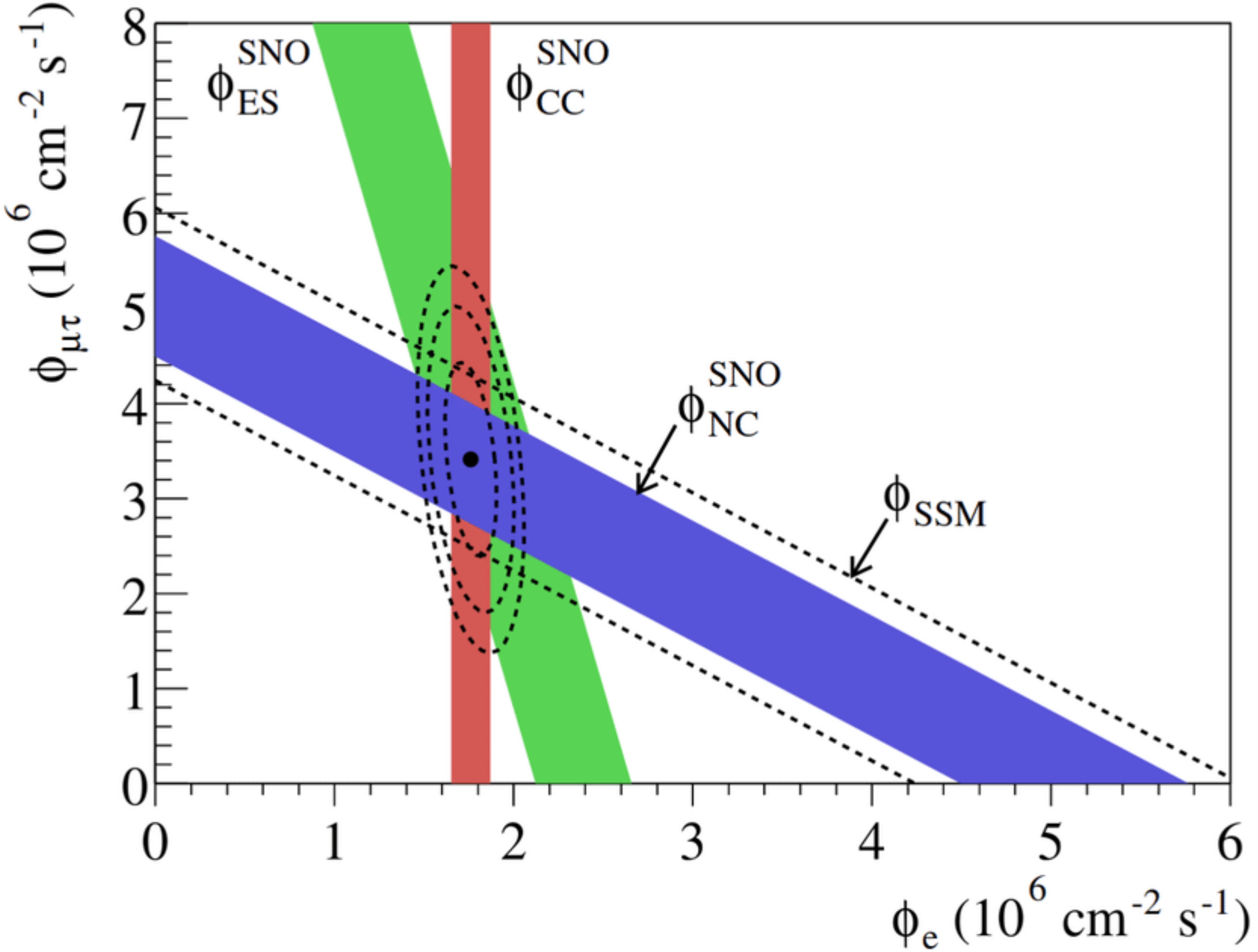


Sun's surface

Sun's core

Solar Neutrinos

SNO Historical Results



[Taken from SNO Collab. (2003)]

$\mathrm{CC}: \nu_e + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{e}^-$

$\mathrm{E}_{\mathrm{threshold}} = 1.4\,\mathrm{MeV}$

$\mathrm{NC}: \nu_\alpha + \mathrm{d} \rightarrow \nu_\alpha + \mathrm{p} + \mathrm{n}$

$\mathrm{E}_{\mathrm{threshold}} = 2.2\,\mathrm{MeV}$

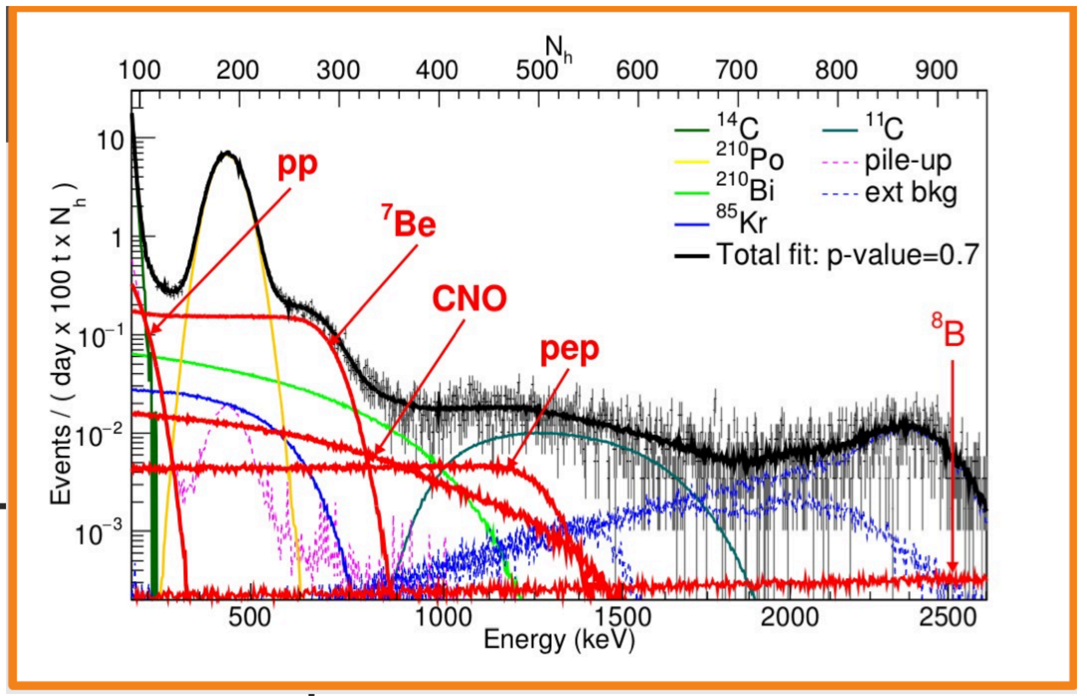
$\mathrm{ES}: \nu_\alpha + \mathrm{e}^- \rightarrow \nu_\alpha + \mathrm{e}^-$

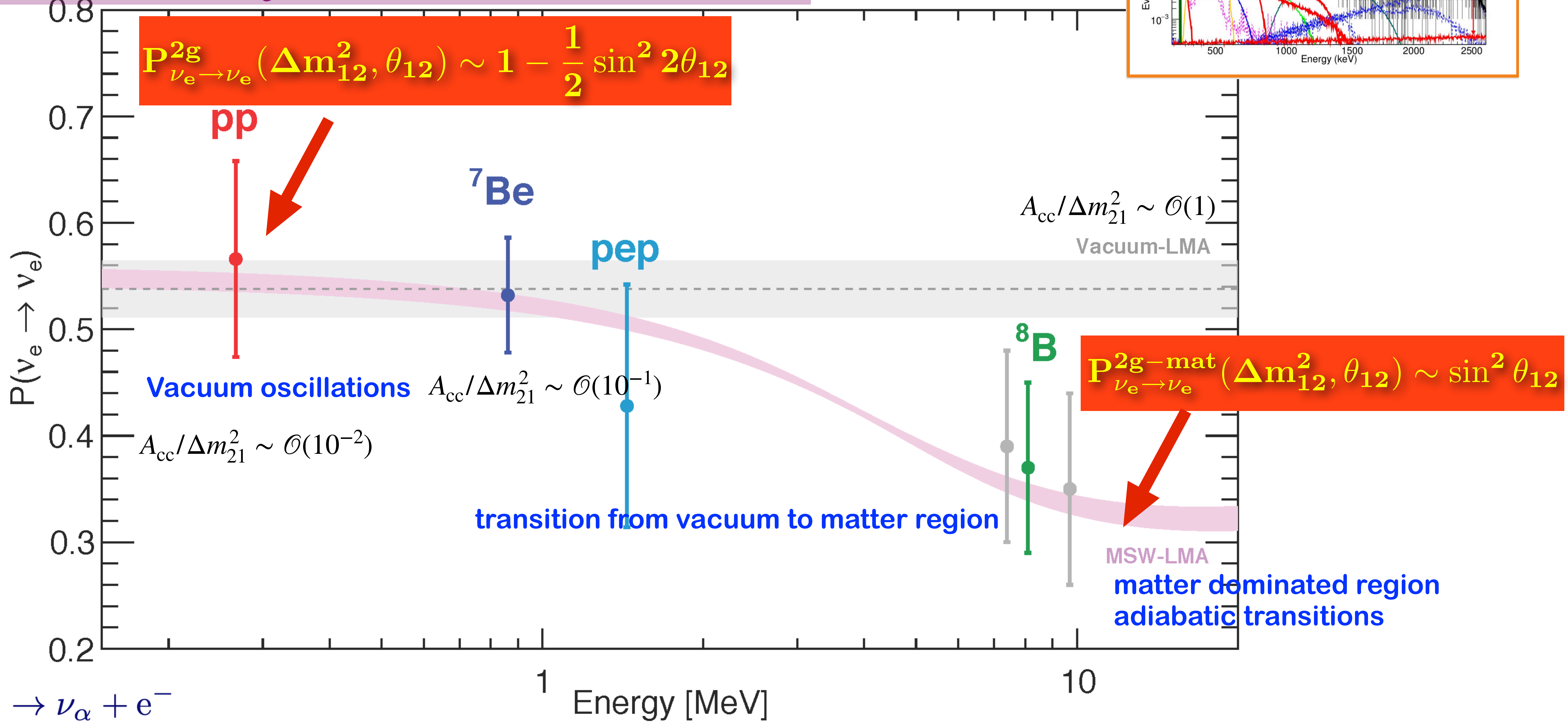
$\mathrm{E}_{\mathrm{threshold}} = 0$

Solar Neutrinos

General Picture of ν_e transitions in the Sun

$$A_{cc} = 2EV_{cc}$$





$$\mathrm{ES}: \nu_\alpha + e^- \to \nu_\alpha + e^-$$

[Taken from Borexino Collab. (2018)]

$$\mathrm{P}^{3g}_{\nu_e\to\nu_e} = \sin^4\theta_{13} + \cos^4\theta_{13}\, \mathrm{P}^{2g-mat}_{\nu_e\to\nu_e}(\Delta m^2_{12}, \theta_{12})$$

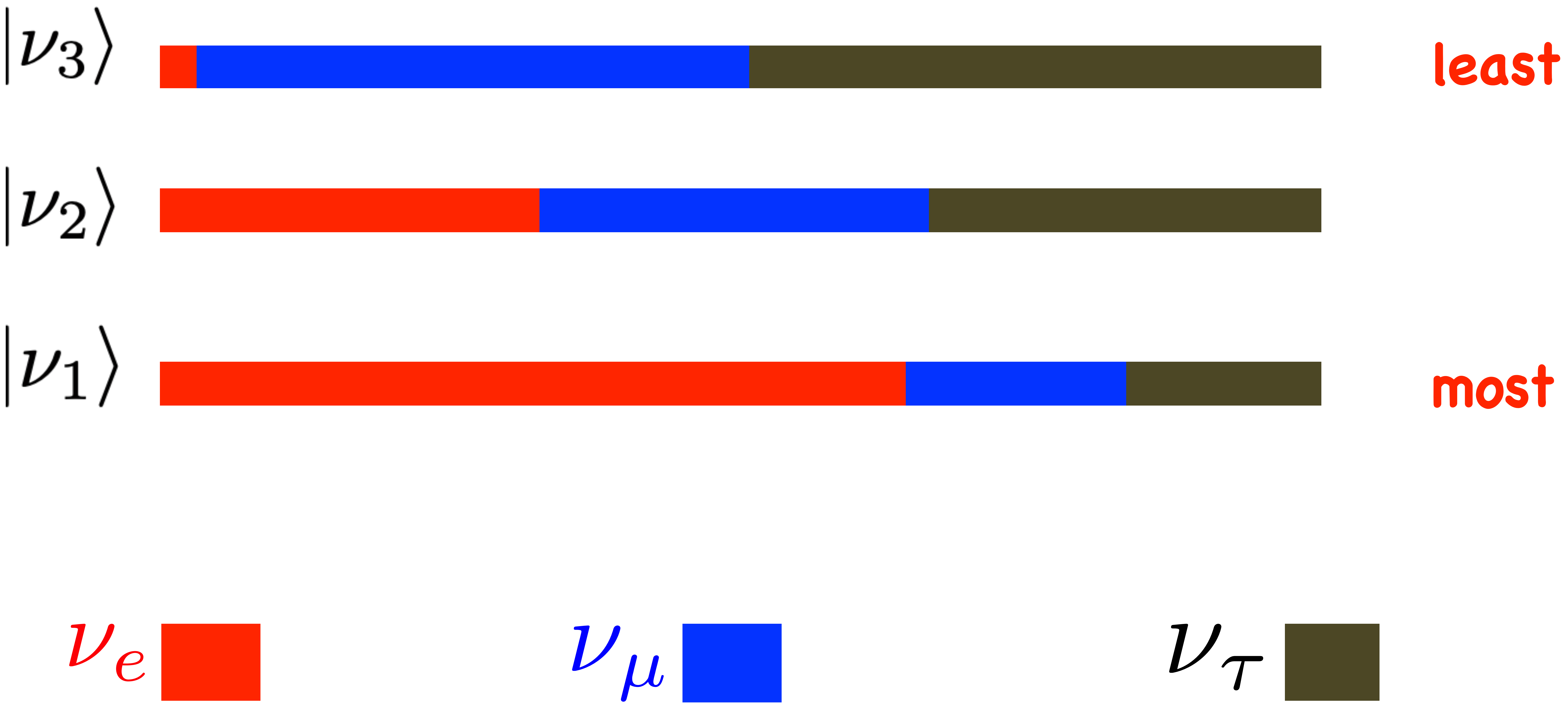
Allows us to Label the Mass Eigentates

Using data results

using $|\nu_e\rangle$ content

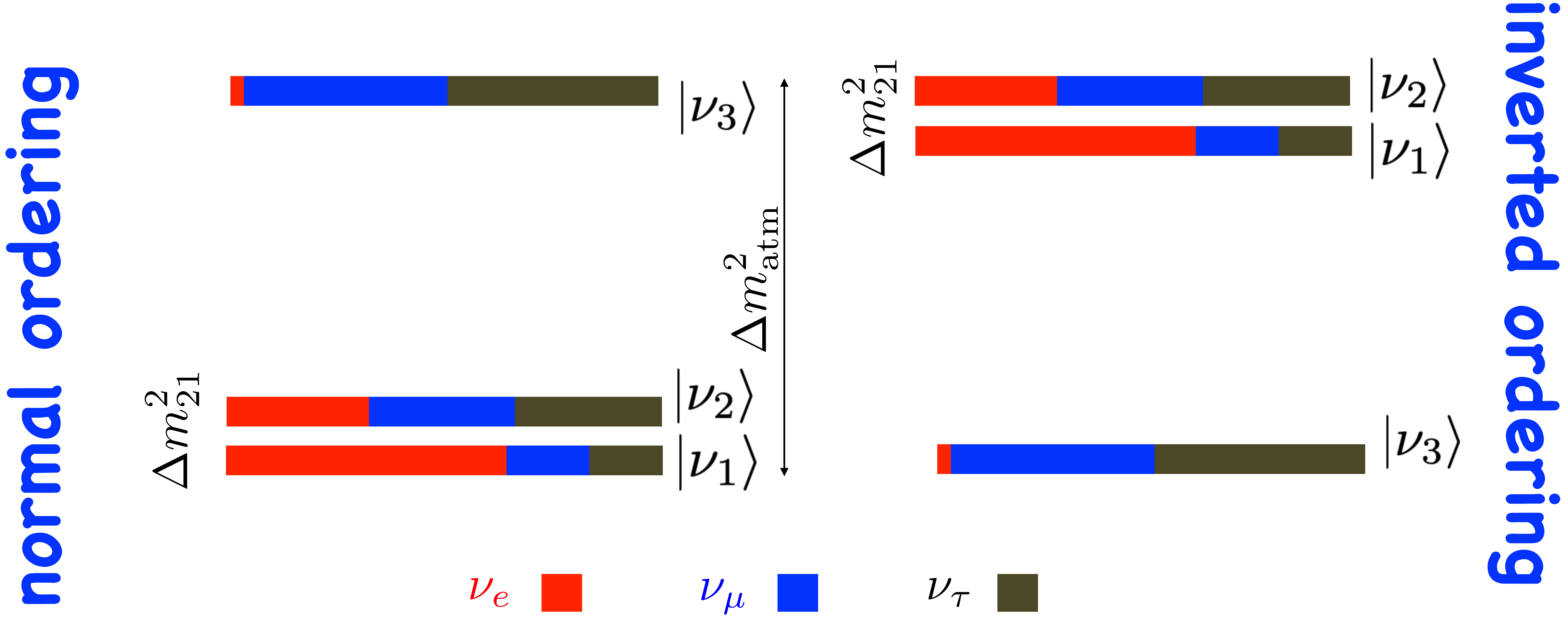
since SNO we know that

$$|U_{e1}| > |U_{e2}| > |U_{e3}| \quad [\sin^2\theta_{12} \simeq 0.3]$$



What is the neutrino mass Ordering?

We currently have two possibilities



Reactor Neutrinos

Reactor Neutrinos

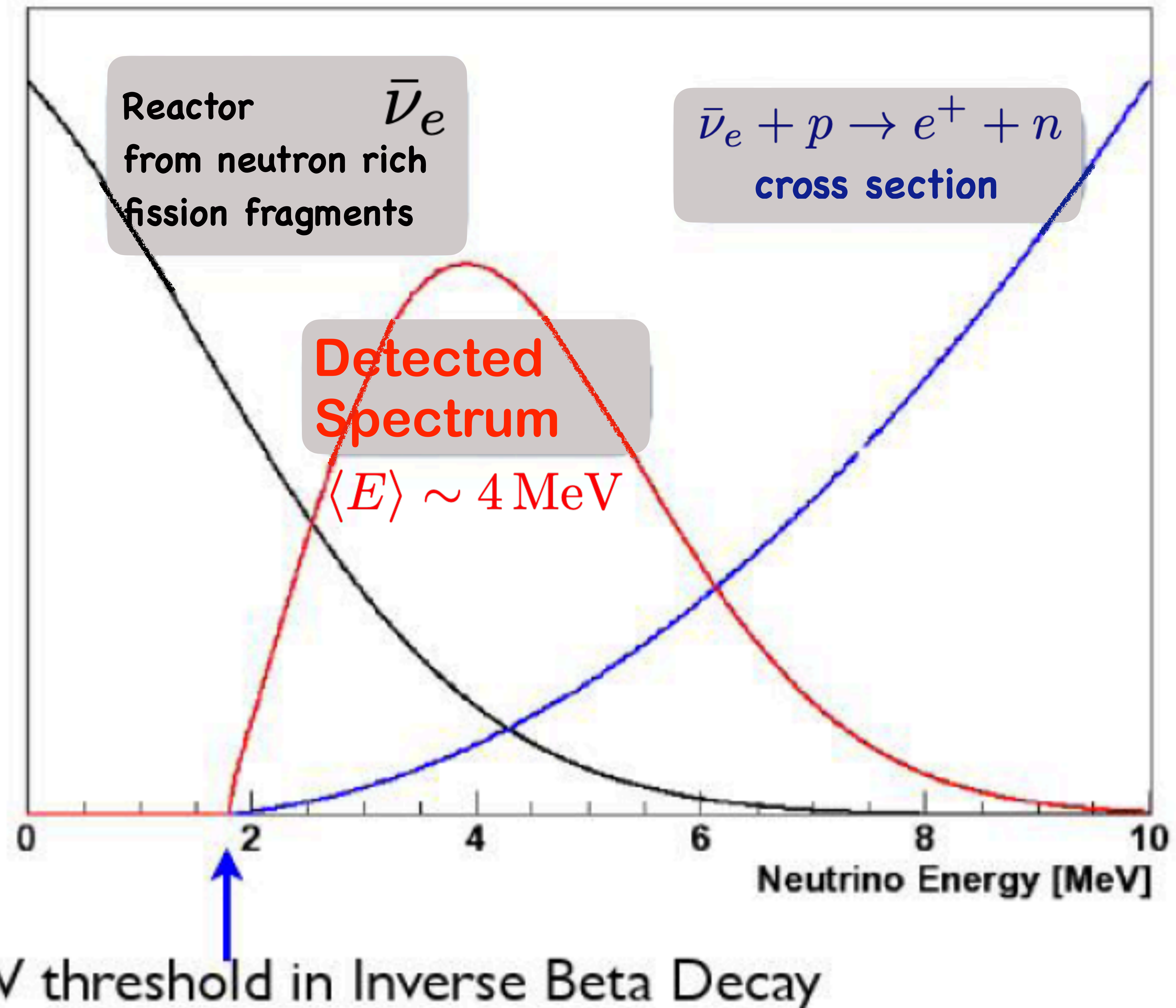
Disappearance of antineutrinos

commercial reactors fuel

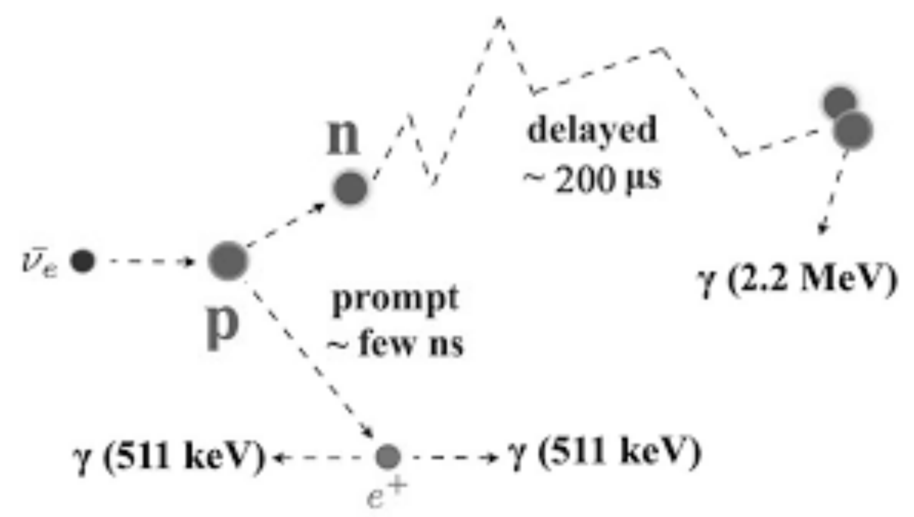
$^{235}\mathrm{U}(\sim 50-60\%),^{239}\mathrm{Pu}(\sim 25-35\%),^{238}\mathrm{U}(\lesssim 8\%),^{241}\mathrm{Pu}(\lesssim 6\%)$

Neutrino Flux from commercial nuclear reactors

$$2\times 10^{20}\quad \bar{\nu}_e/\mathrm{GW_{th}}$$



Liquid Scintillator Detectors



Reactor Neutrino Oscillations

Disappearance of antineutrinos

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

in vacuum

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

solar term

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

effective atmospheric Δm^2_{atm} for reactor experiments

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

$+\mathrm{NO} \mid -\mathrm{IO}$

retardation/advancement of the phase

$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$

Very useful for JUNO

Reactor Neutrino Oscillations

Disappearance of antineutrinos

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in vacuum

since we know

$\Delta m^2_{ee} \sim 3 \times 10^{-3}\,\mathrm{eV}^2$ $\quad\Delta m^2_{21} \sim 7 \times 10^{-5}\,\mathrm{eV}^2$

$L^{ee}_{\odot} \sim 3\ \mathrm{km}$ $\quad L^{12}_{\odot} \sim 140\ \mathrm{km}$

$\langle E \rangle \sim 4\ \mathrm{MeV}$

Reactor Neutrino Oscillations

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averaged out

in vacuum

since we know

$\Delta m^2_{\mathrm{ee}} \sim 3 \times 10^{-3}\,\mathrm{eV}^2$ $\quad$ $\Delta m^2_{21} \sim 7 \times 10^{-5}\,\mathrm{eV}^2$ $\quad$ $\langle E \rangle \sim 4\ \mathrm{MeV}$

$L^{\mathrm{ee}}_\odot \sim 3\ \mathrm{km}$ $\quad$ $L^{12}_\odot \sim 140\ \mathrm{km}$

KamLAND has a baseline about 180 km

SNO+ has a baseline > 240 km

$$L \approx L^{12}_\odot \gg L^{ee}_\odot$$

Reactor Neutrino Oscillations

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averaged out

in vacuum

since we know

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the formula reduces to

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = s^4_{13} + c^4_{13}\, P_{2\nu}(\bar{\nu}_e \to \bar{\nu}_e)$$

$$P_{2\nu}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}$$

Reactor Neutrino Oscillations

Disappearance of antineutrinos

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negligible

in vacuum

negligible

negligible

since we know

$\Delta m^2_{ee} \sim 3\times 10^{-3}\,\mathrm{eV}^2$

$L^{ee}_\odot \sim 3$ km

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$L^{12}_\odot \sim 140$ km

$\langle E\rangle \sim 4$ MeV

Daya Bay/Reno/Double Chooz have a baseline of 1 km

$$L \approx L^{ee}_\odot \ll L^{12}_\odot$$

Reactor Neutrino Oscillations

Disappearance of antineutrinos

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negligible

negligible

in vacuum

negligible

since we know

$\Delta m^2_{ee} \sim 3\times 10^{-3}\,\mathrm{eV}^2$ $\quad$ $\Delta m^2_{21} \sim 7\times 10^{-5}\,\mathrm{eV}^2$ $\quad$ $\langle E\rangle \sim 4$ MeV

$L^{ee}_\odot \sim 3$ km $\quad$ $L^{12}_\odot \sim 140$ km

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

effective atmospheric scale for ν_e disappearance

Daya Bay/Reno/Double Chooz have a baseline of 1 km

$$L \approx L^{ee}_\odot \ll L^{12}_\odot$$

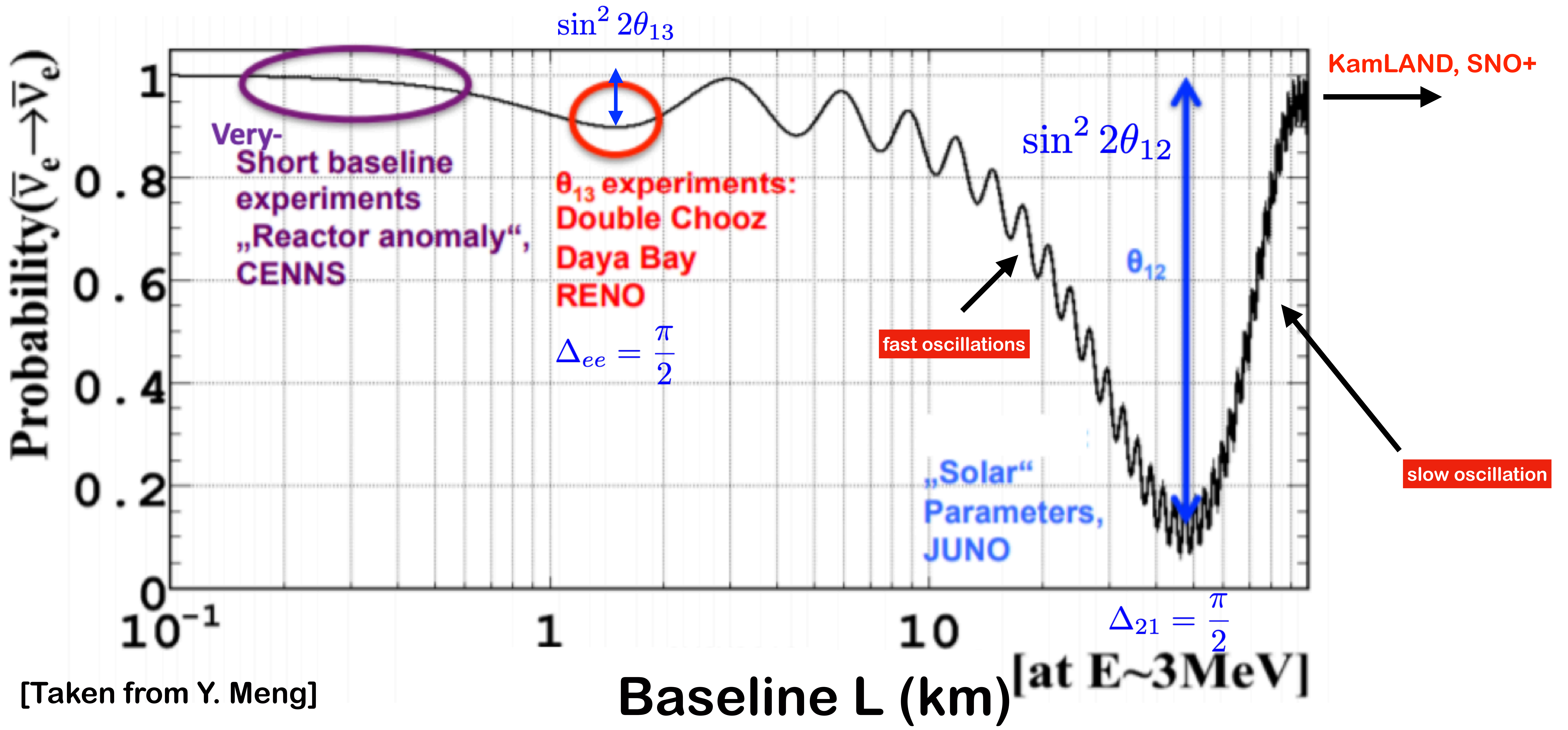
the formula reduces to

$$P_{\bar\nu_e \to \bar\nu_e} = 1 - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$$

+ matter effects are negligible

Reactor Neutrino Oscillations

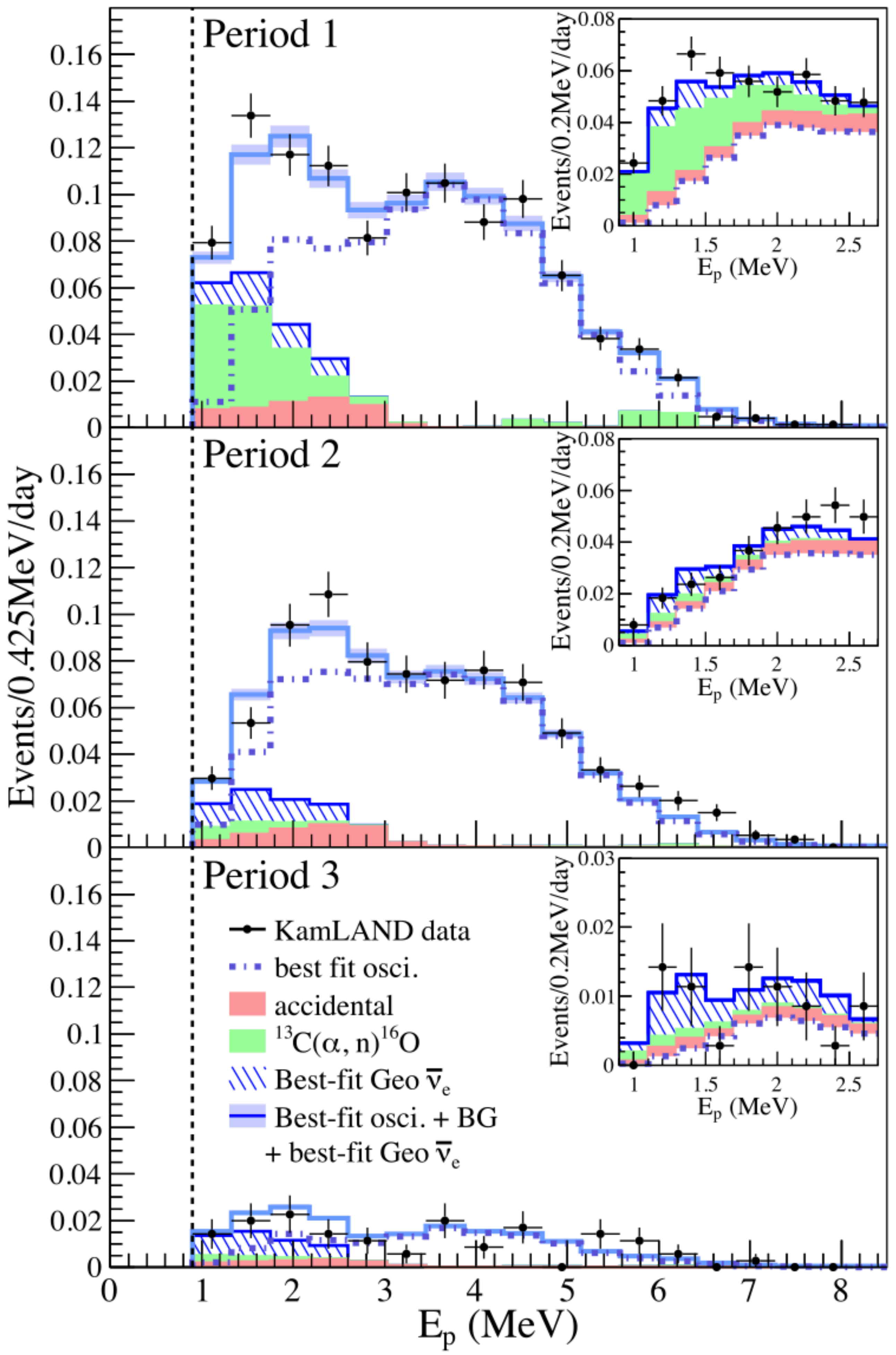
Disappearance of antineutrinos

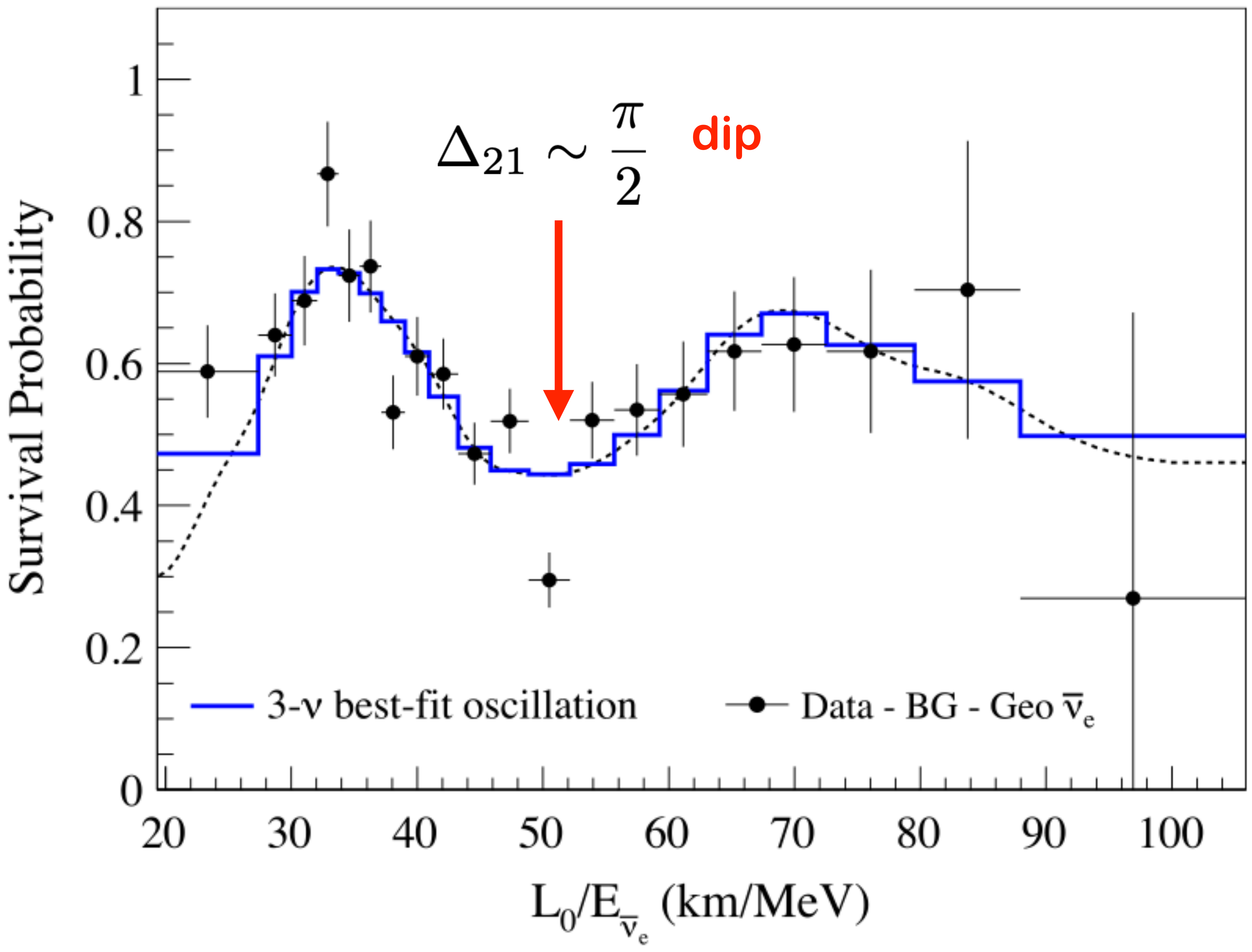


[Taken from Y. Meng]

Reactor Neutrinos

Results from KamLAND





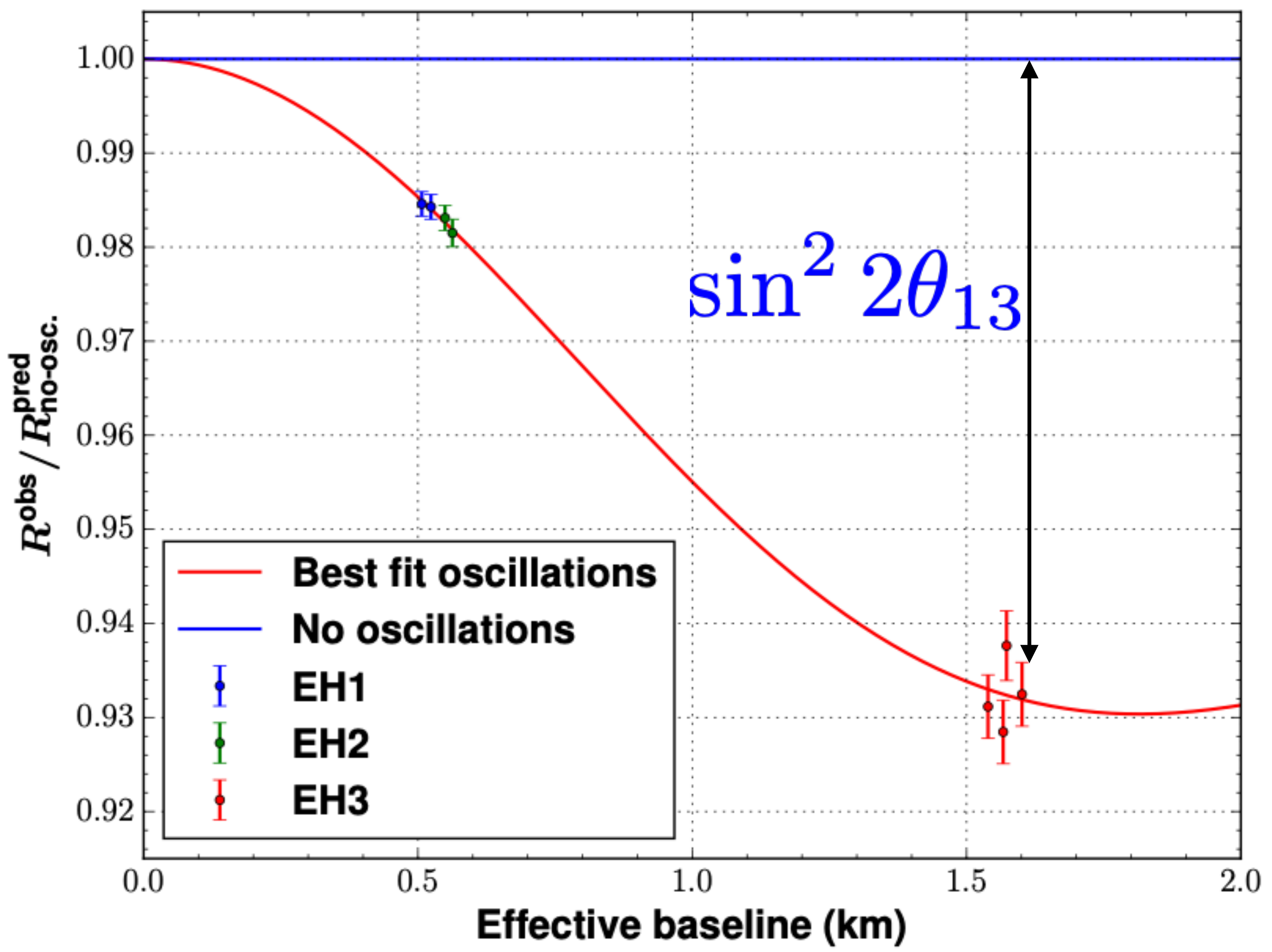
[Taken from KamLAND Collab. (2016)]

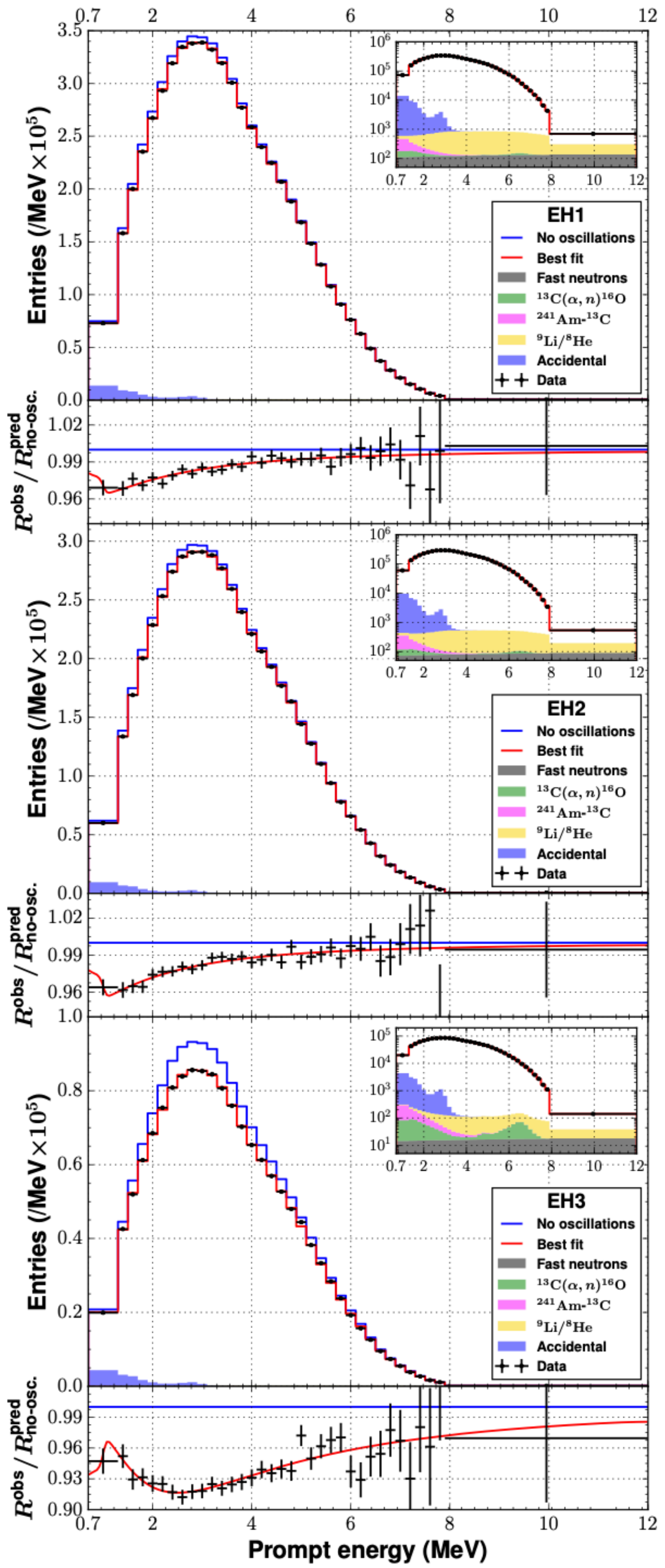
Reactor Neutrinos

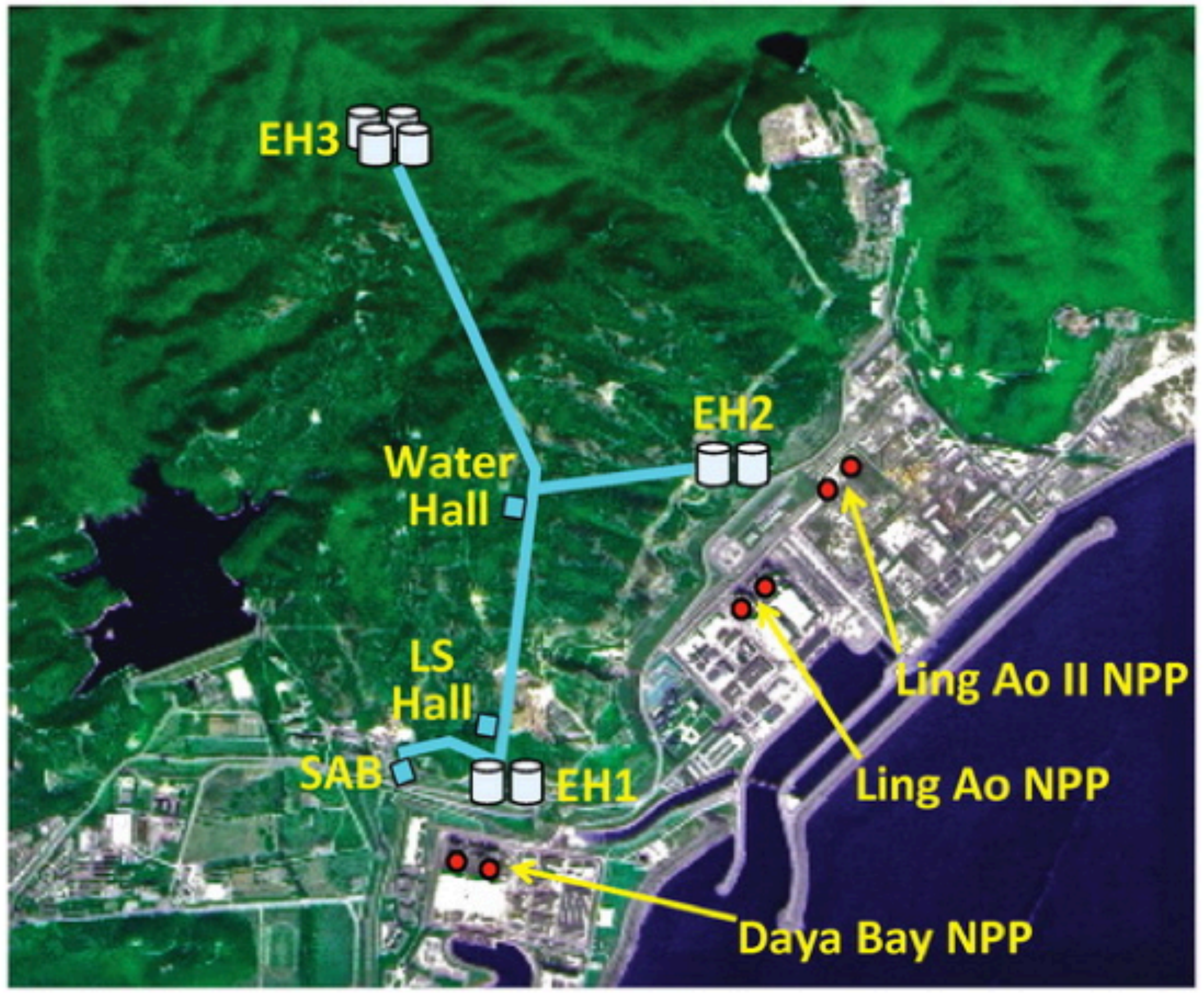
Results from Daya Bay

Dominates the measurement of $\sin^2\theta_{13}$

2.5%







[Taken from Daya Bay Collab. (2017)]

JUNO's Concept

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E} \sim \frac{\pi}{2}$$

A medium baseline reactor neutrino oscillation experiment

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

JUNO's Concept

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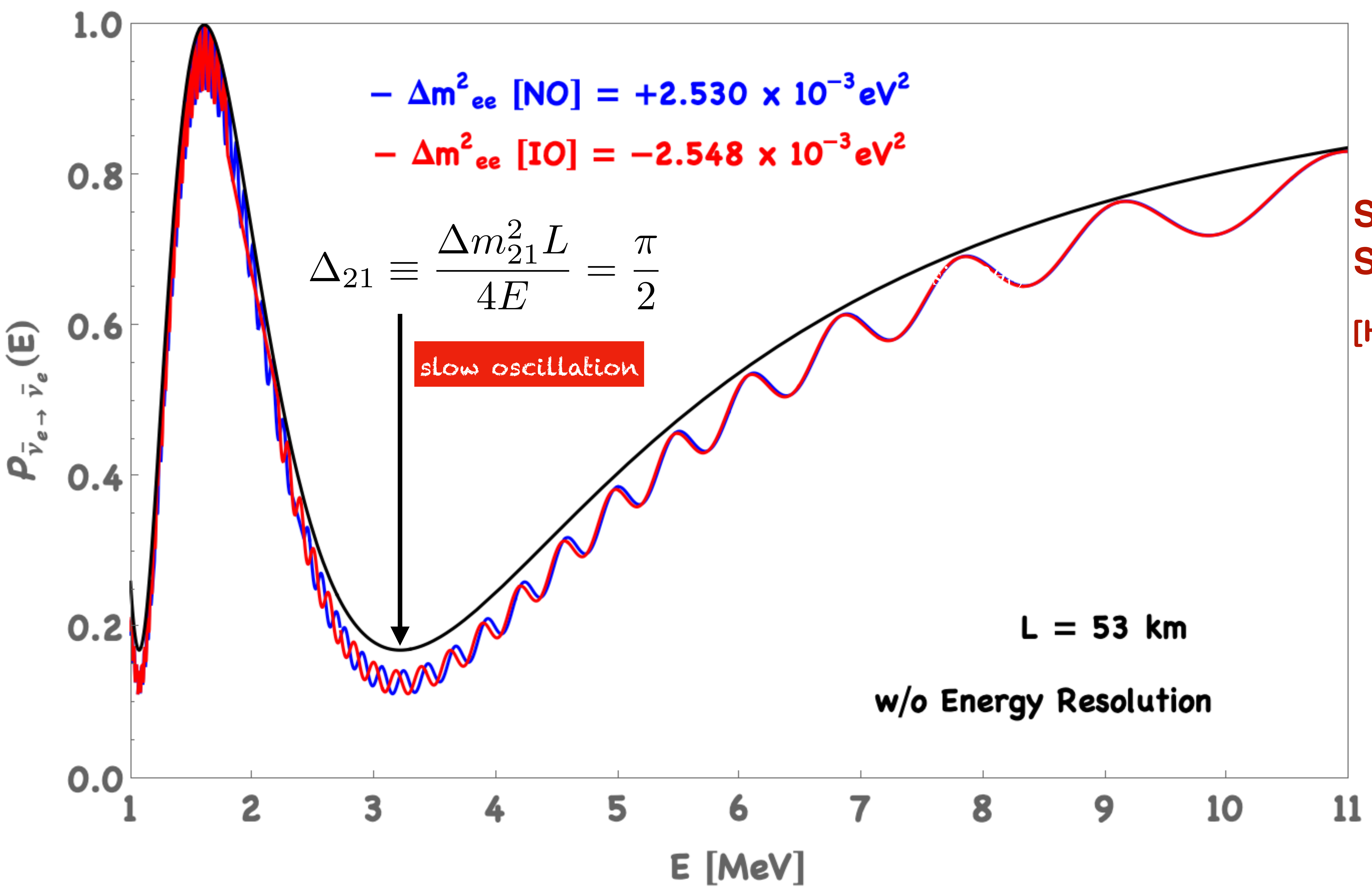
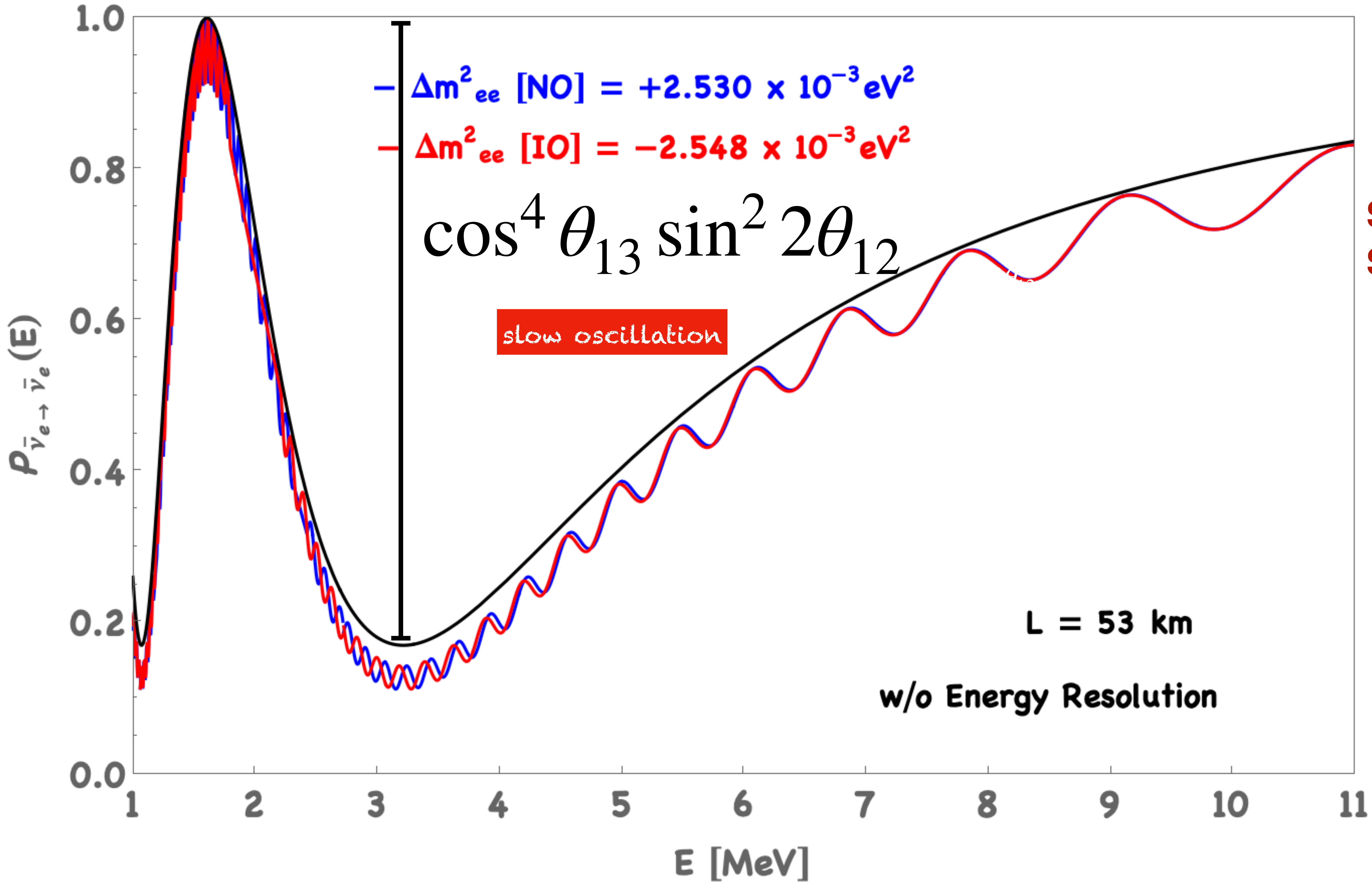


SADO ≡
Several-tens of km Antineutrino DetectOr

[H.Minakata, H. Nunokawa, W.Teves, RZF (2005)]

JUNO's Concept

A medium baseline reactor neutrino oscillation experiment



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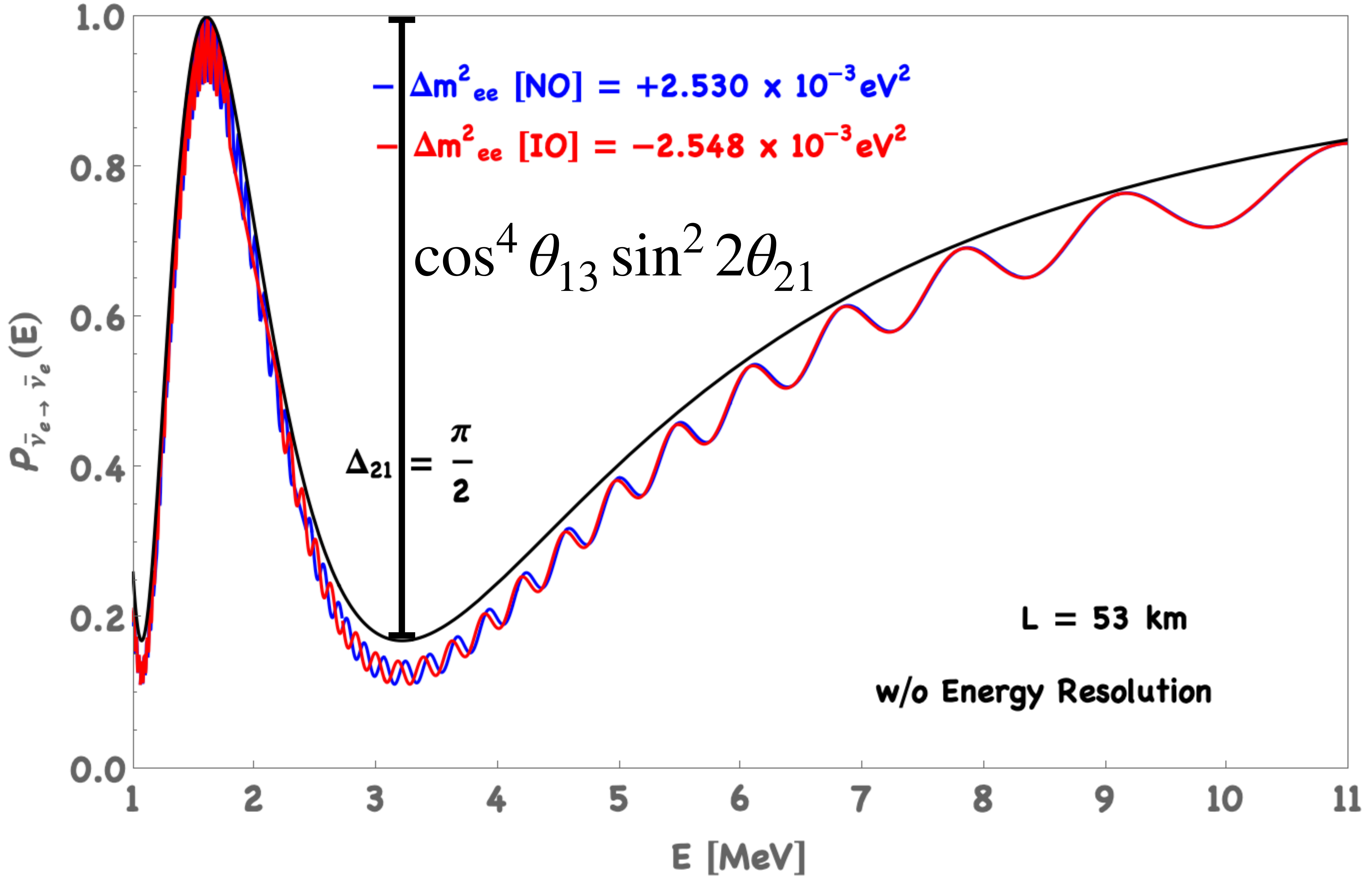
solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

	100 days	6 years
Δm^2_{21}	$\sim 1.0\,\%\,(2.5\%)$	$\sim 0.3\,\%$
$\sin^2\theta_{12}$	$\sim 1.9\,\%\,(3.9\%)$	$\sim 0.5\,\%$



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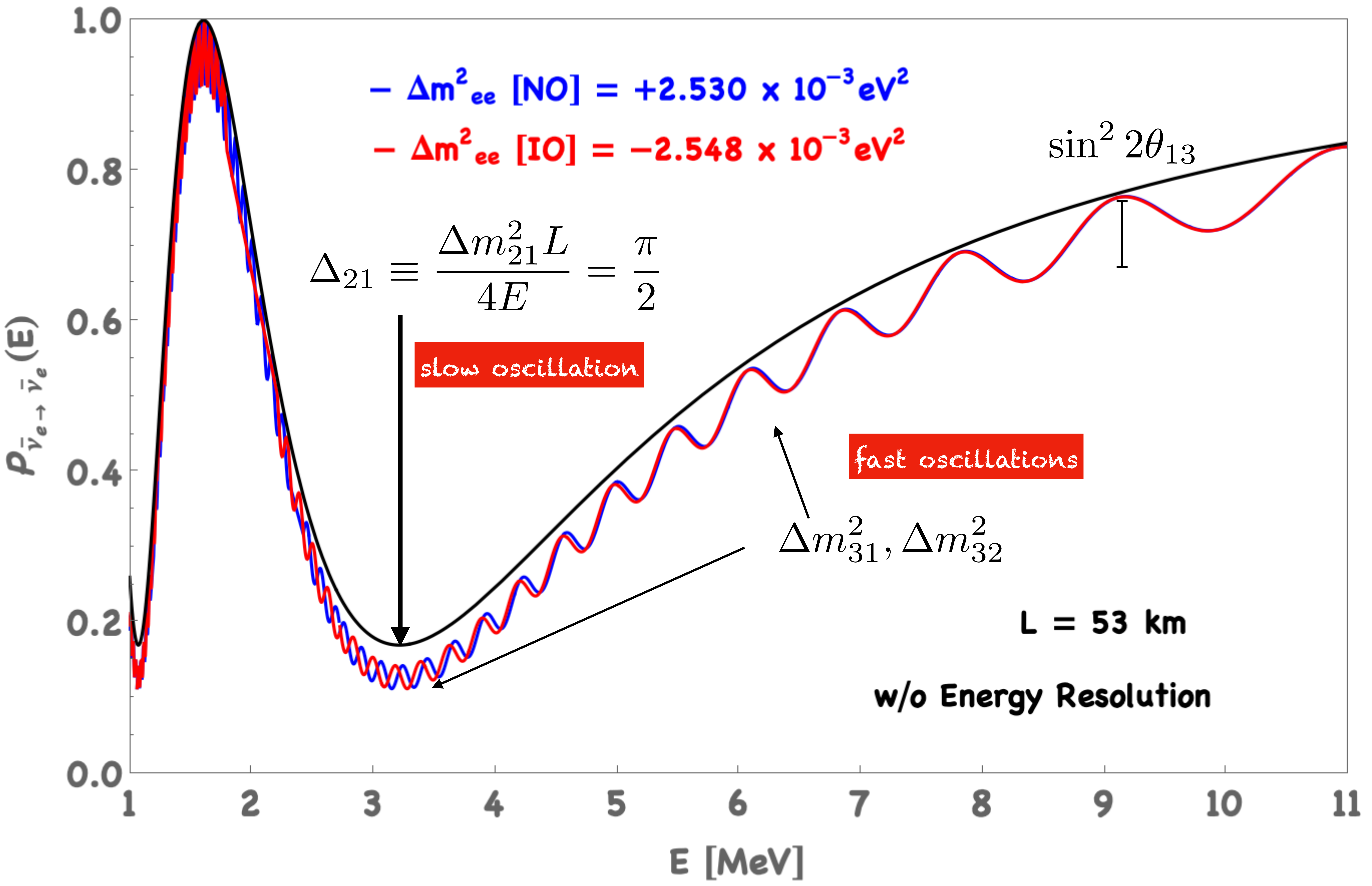
JUNO's Concept

A medium baseline reactor neutrino oscillation experiment

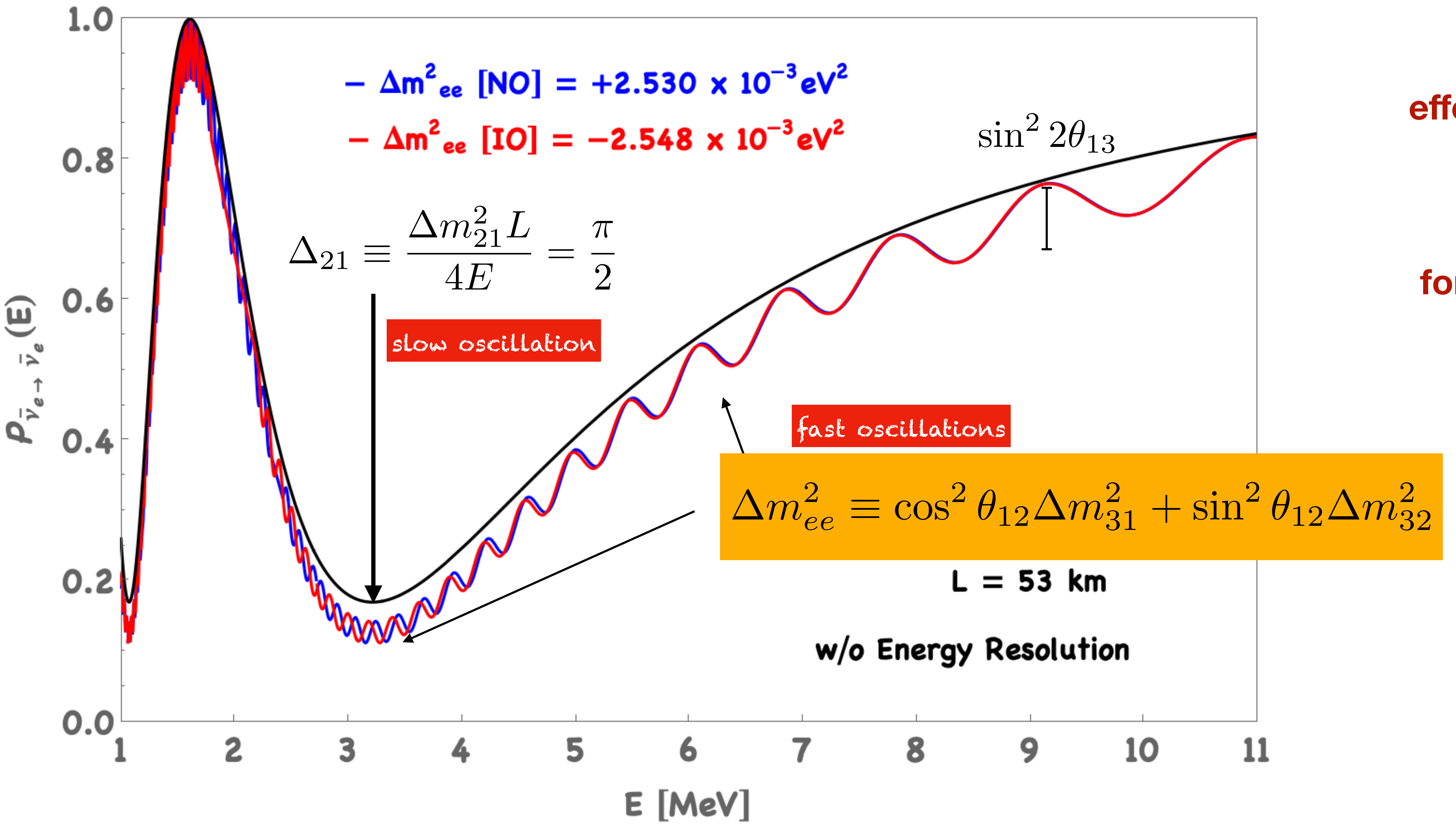


low and high frequency modes present

[S.T.Petcov, M Piai (2002)
&
S. Choubey+ (2003)]

JUNO's Concept

A medium baseline reactor neutrino oscillation experiment



effective atmospheric

Δm^2_{atm}

for reactor experiments

JUNO's Concept

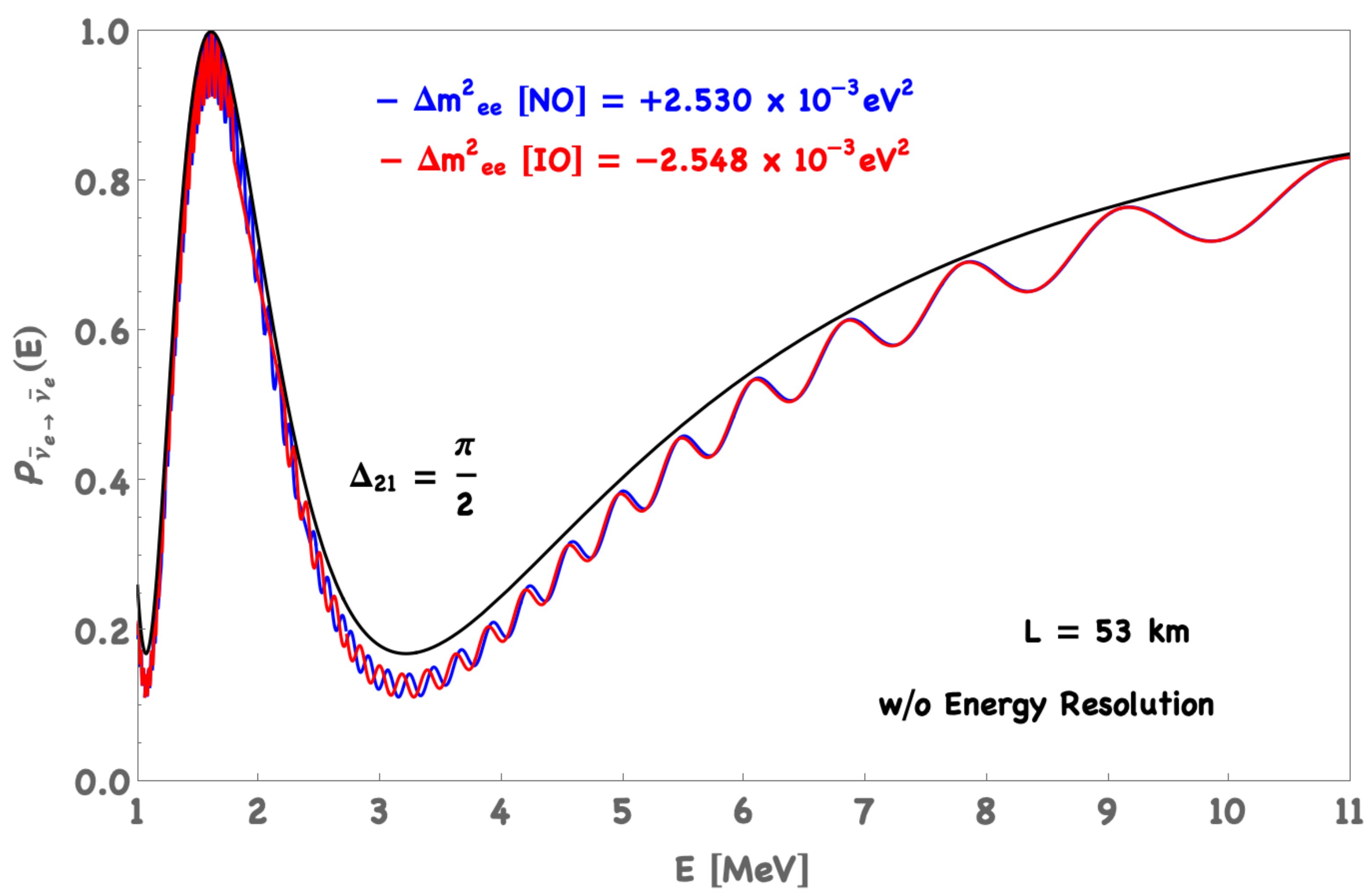
Flagship Measurement - The Neutrino Mass Ordering

$$P_{\overline{\nu}_e \to \overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\ \cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$



$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{21}$$

$$\Delta_{ee} \equiv \frac{\Delta m^2_{ee} L}{4E}$$

JUNO's Concept

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in vacuum

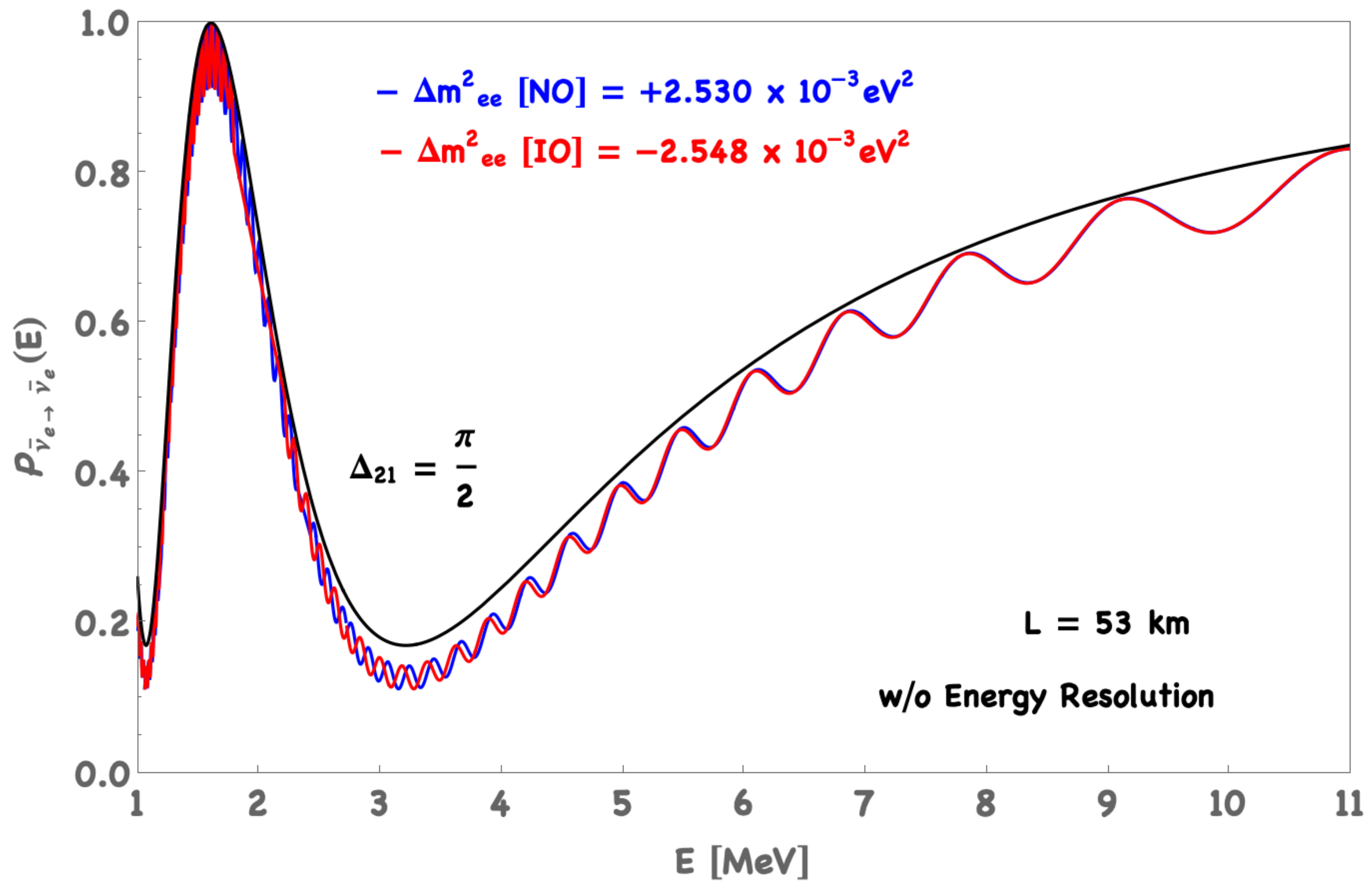
solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

phase

$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$

$$\Delta m^2_{ee} \equiv \Delta m^2_{31}\cos^2\theta_{12} + \Delta m^2_{32}\sin^2\theta_{21}$$



+ matter effects are negligible

JUNO's Concept

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in vacuum

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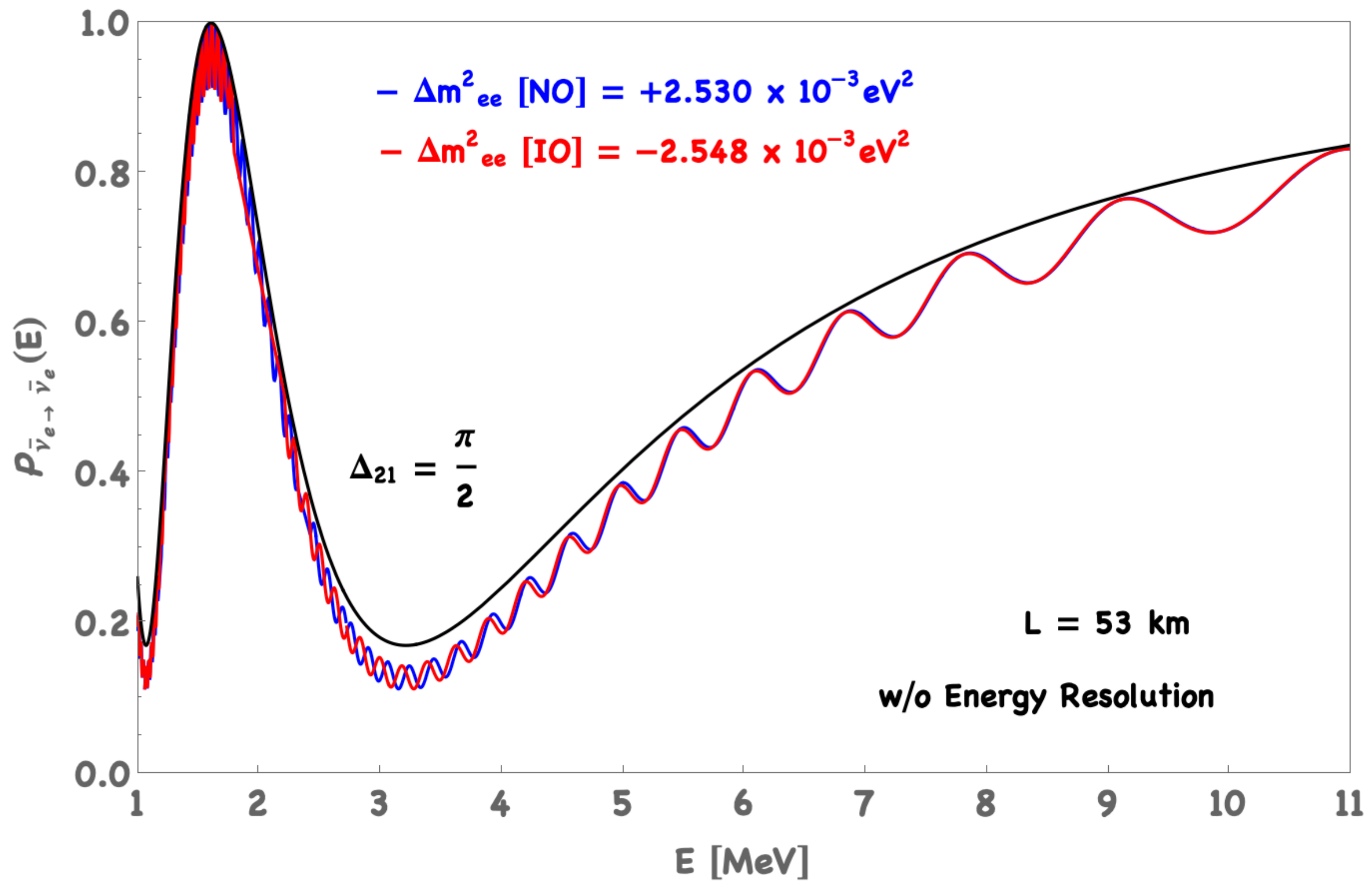
phase

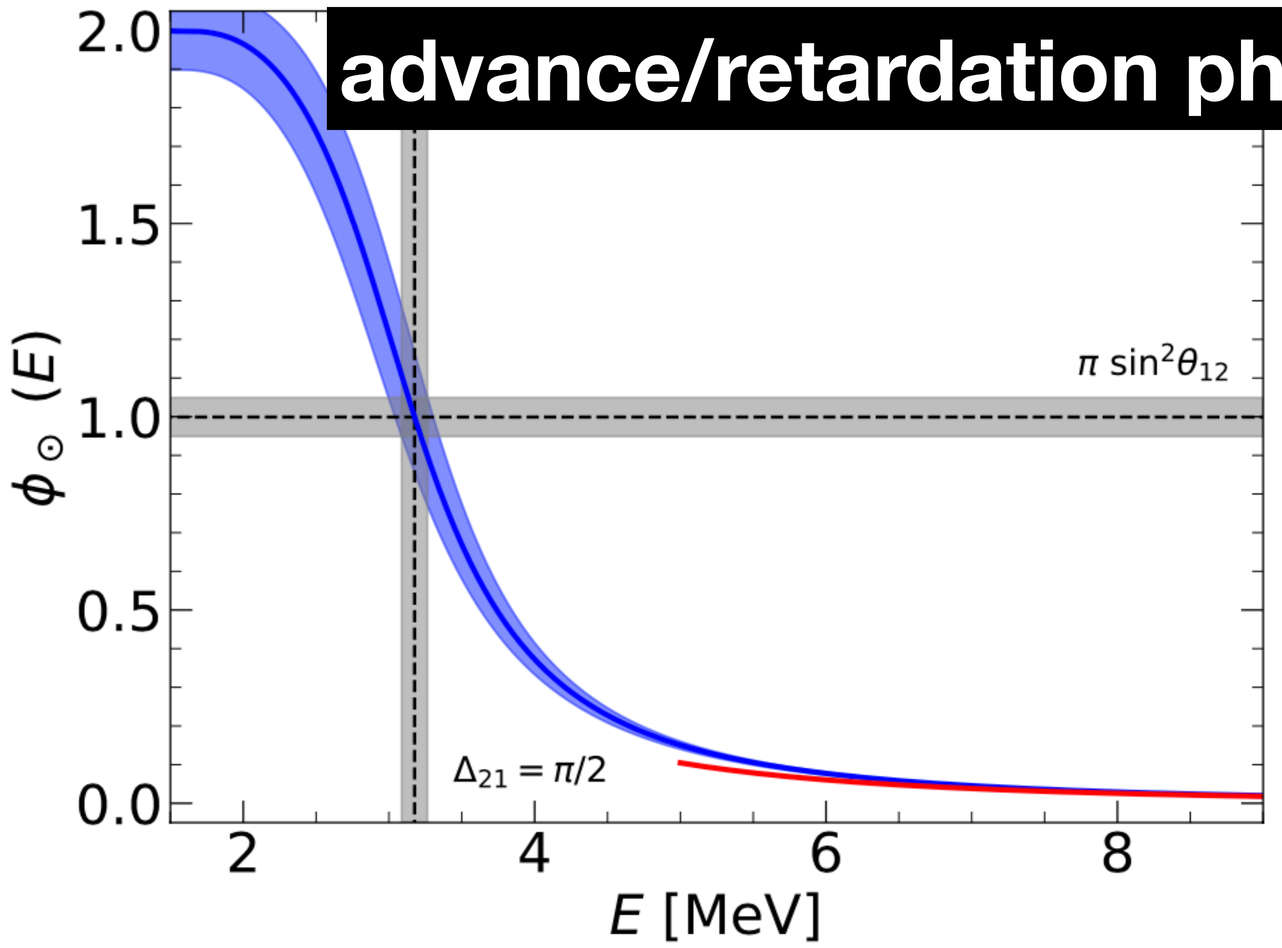
$$\Phi_\odot = \arctan\left(\cos 2\theta_{12} \tan \Delta_{21}\right) - \Delta_{21} \cos 2\theta_{12}$$



retardation/advancement of the phase result in a change of the "effective fast oscillation scale"

$$|\Delta m^2_{\rm ee}|_{\rm IO} > |\Delta m^2_{\rm ee}|_{\rm NO}$$

advance/retardation phase



JUNO's Concept

Flagship Measurement - The Neutrino Mass Ordering

$$P_{\overline{\nu}_e\to\overline{\nu}_e} = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}}\,\cos(2|\Delta_{ee}| \pm \Phi_\odot)\right] - P_\odot$$

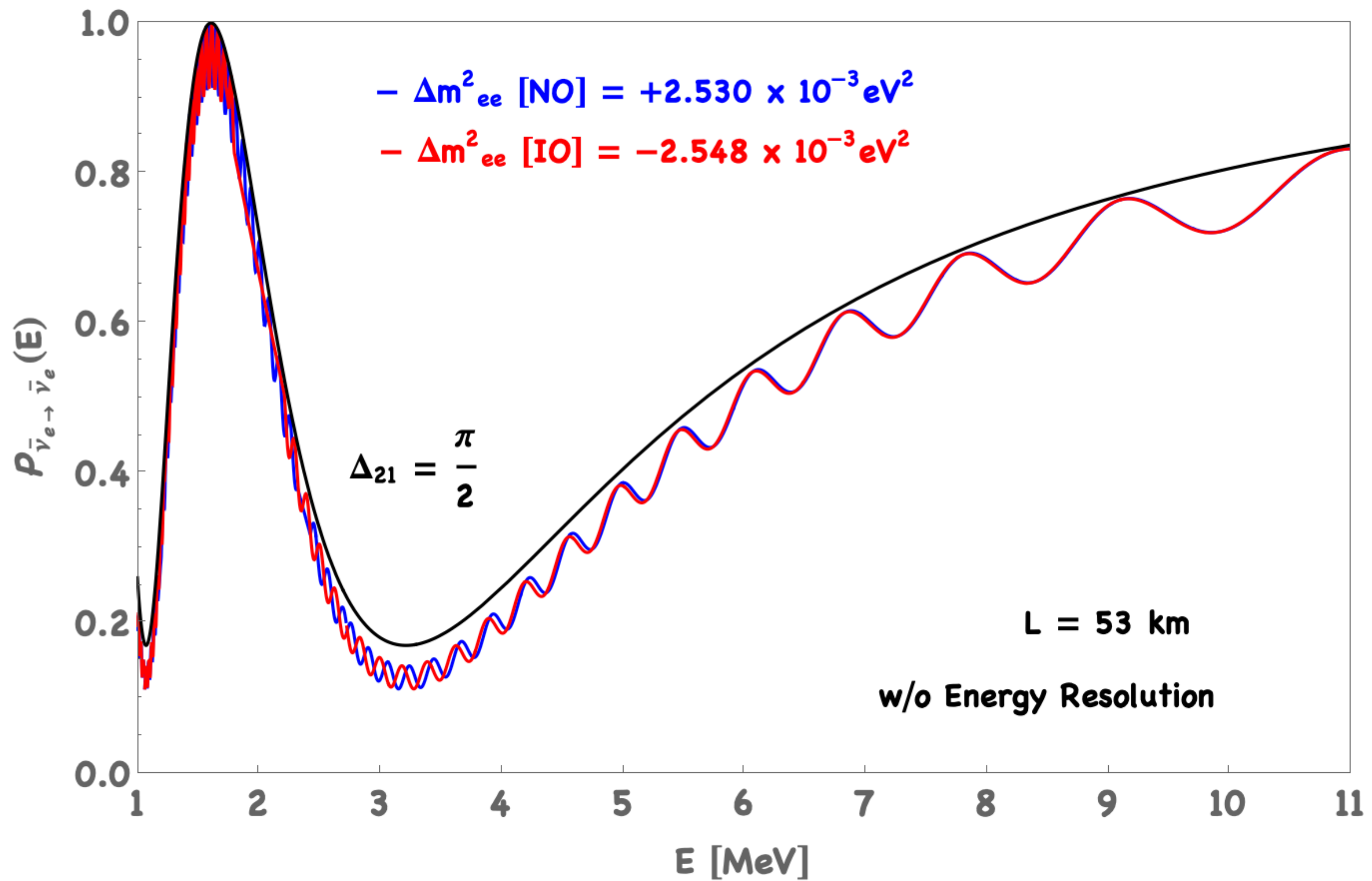
in vacuum

solar term

$$P_\odot = \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\Delta_{21}$$

phase

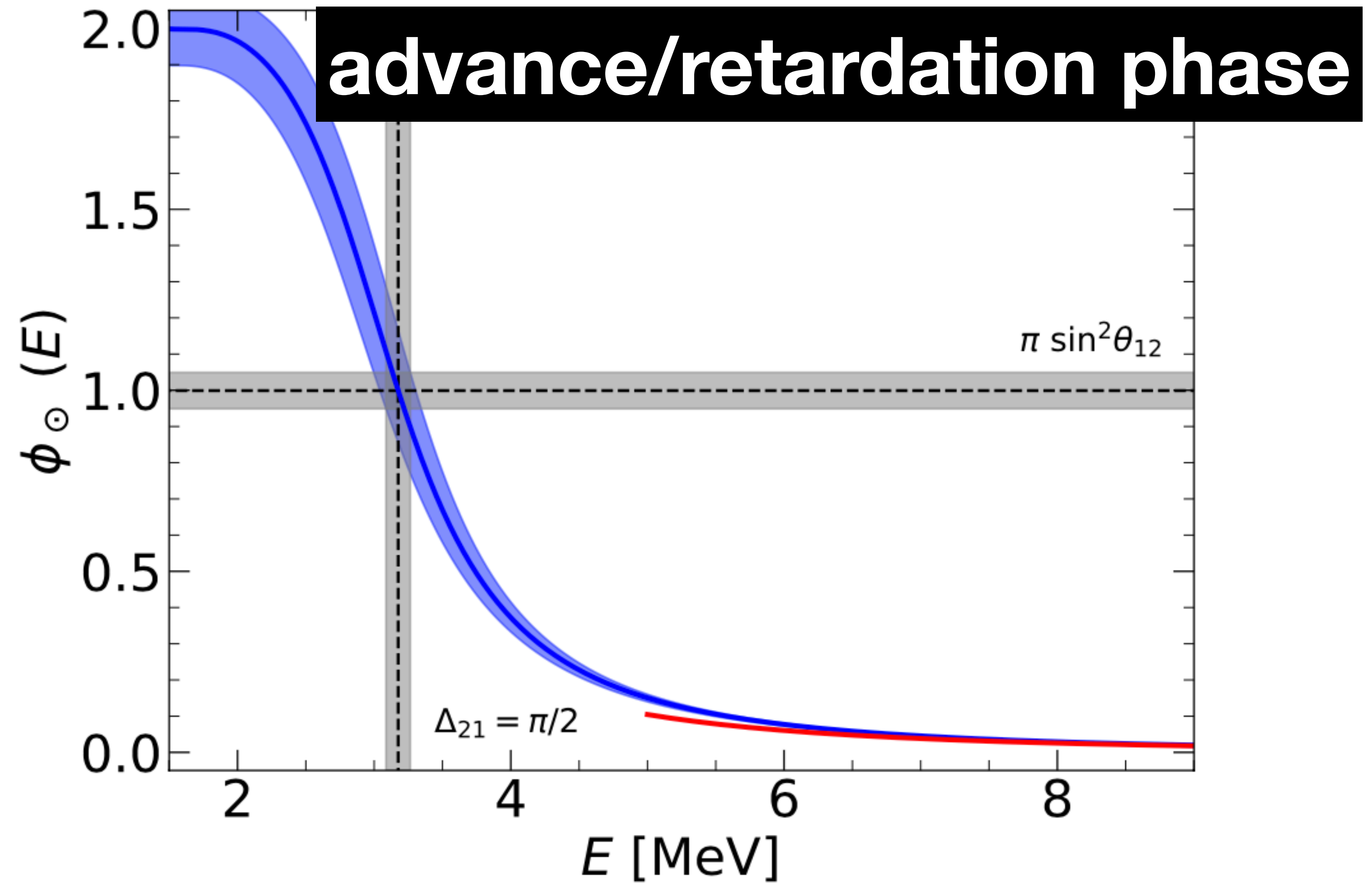
$$\Phi_\odot = \arctan\left(\cos 2\theta_{12}\tan\Delta_{21}\right) - \Delta_{21}\cos 2\theta_{12}$$



retardation/advancement of the phase result in a change of the "effective fast oscillation scale"

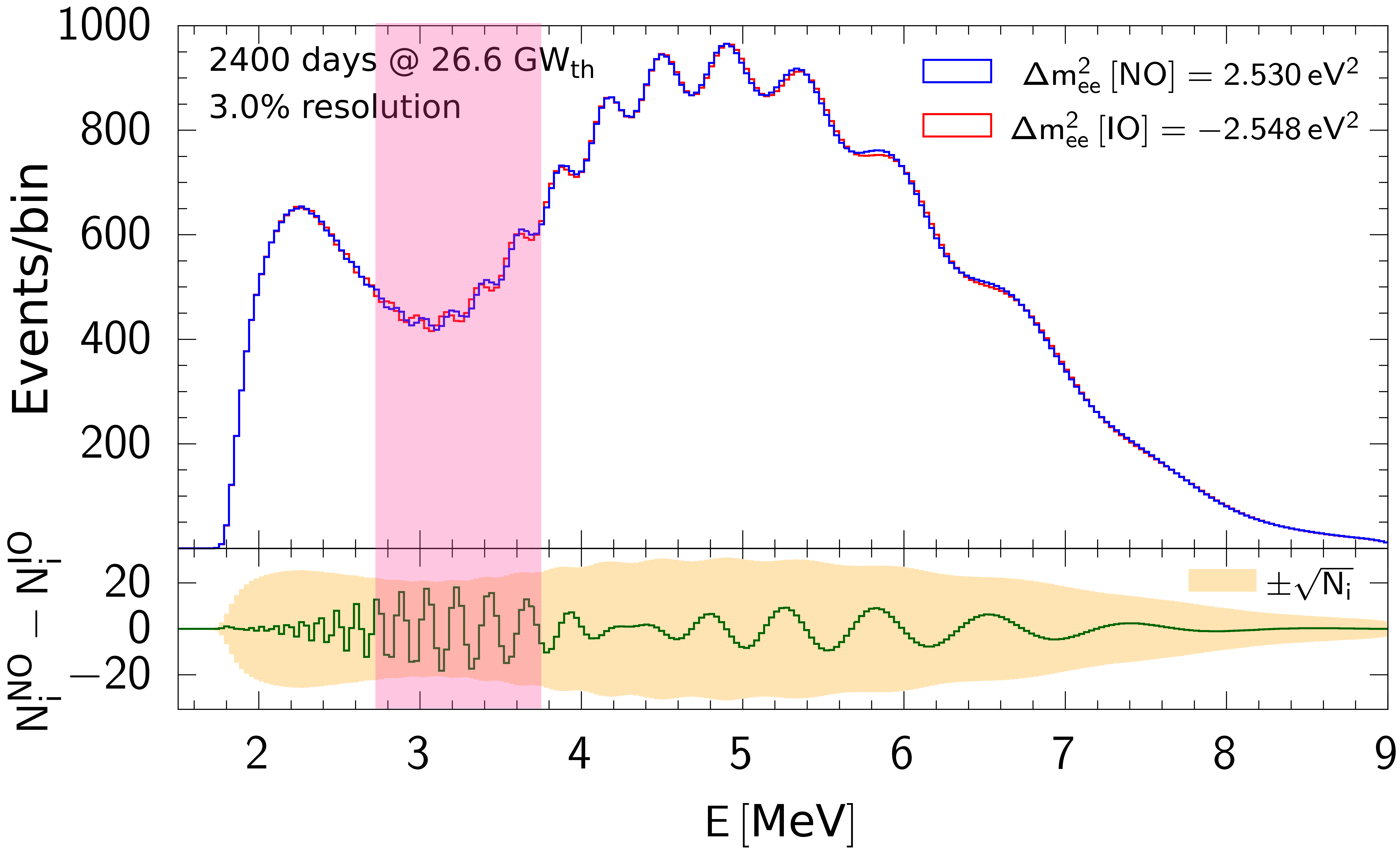
$$|\Delta m^2_{ee}|_{NO} \neq |\Delta m^2_{ee}|_{IO}$$

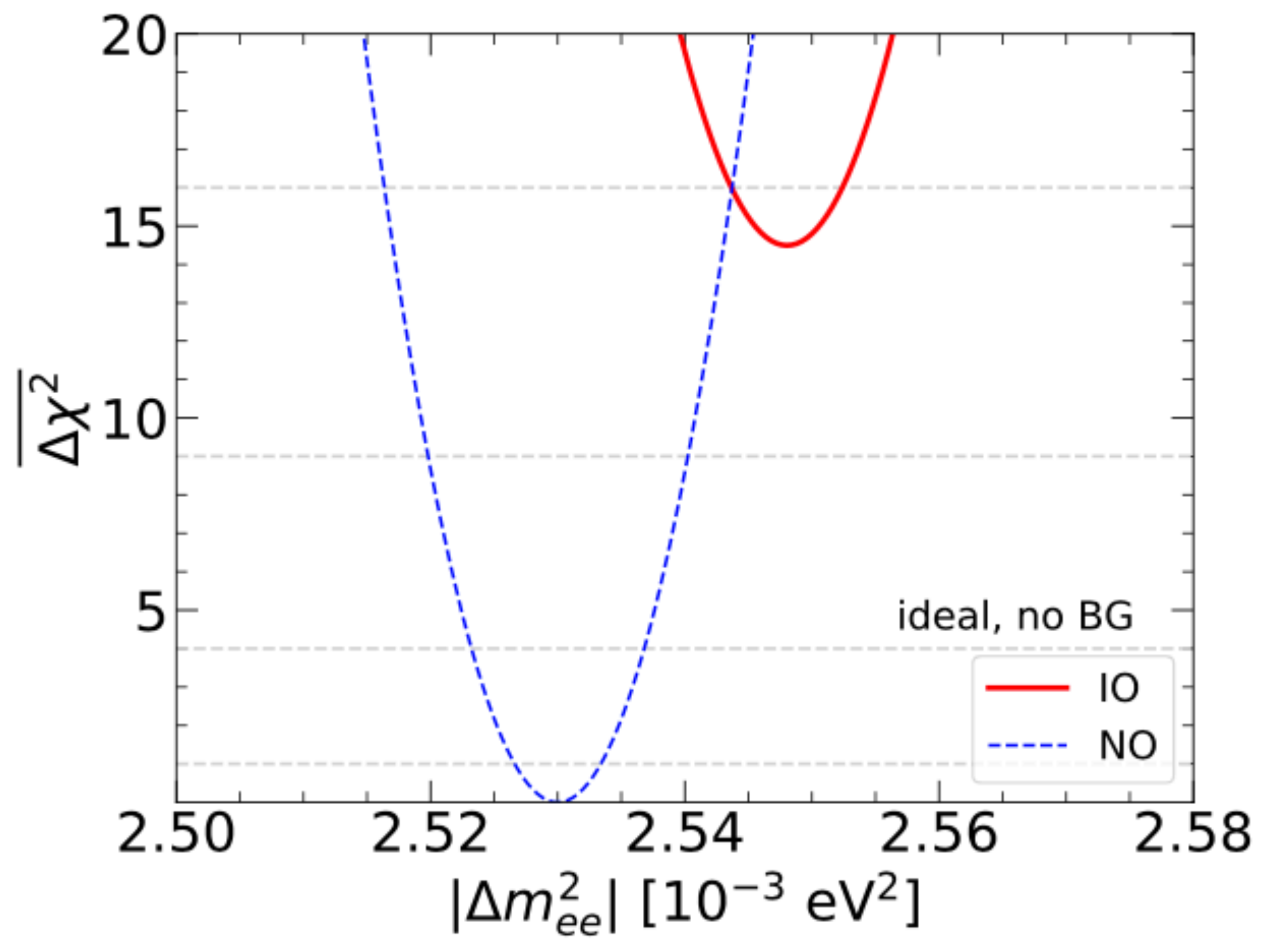
100 days	6 years
$\sim 0.8\,\%$	$\sim 0.2\,\%$



JUNO's Determination of the Ordering

How challenging is this?





single core
no other systematics
no background events

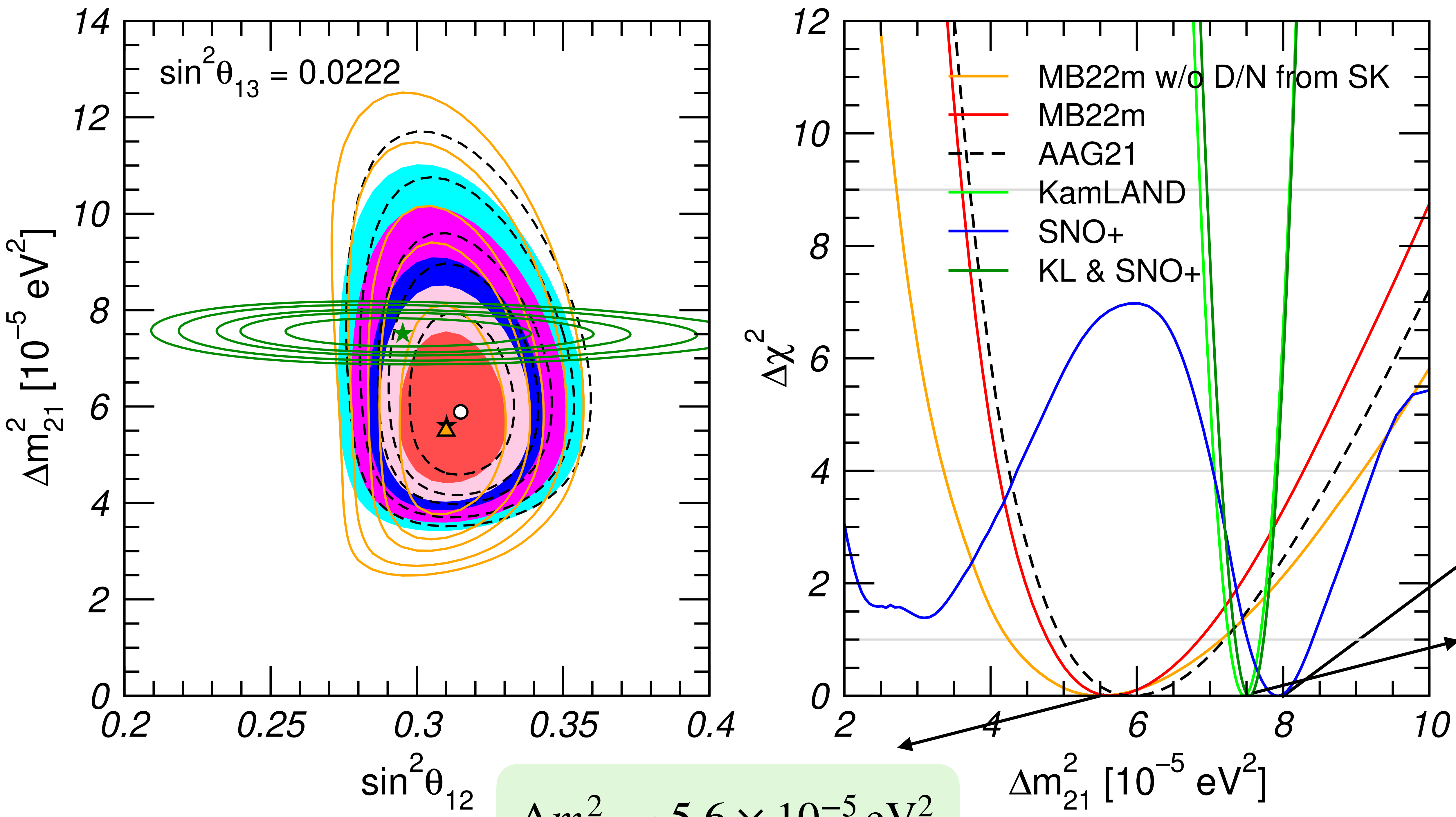
every single effect counts !

Solar Neutrino Parameters

Current status: combined results for the 12-sector

EXPERIMENT	Dominant Dependence	Sub-dominant Dependence
Solar (Cl,Ga,SK,SNO,Borexino)	θ_{12}	$\Delta m^2_{21}, \theta_{13}$
Reactor LBL (KamLAND, SNO+)	Δm^2_{21}	θ_{12}, θ_{13}

NuFIT 6.0 (2024)



tension on Δm^2_{21}

$\Delta m^2_{21} \sim 7.96 \times 10^{-5}\,\mathrm{eV}^2$

$\Delta m^2_{21} \sim 7.5 \times 10^{-5}\,\mathrm{eV}^2$

$\Delta m^2_{21} \sim 5.6 \times 10^{-5}\,\mathrm{eV}^2$

MB22m = high metallicity
AAG21 = low metallicity

Solar Neutrino Parameters

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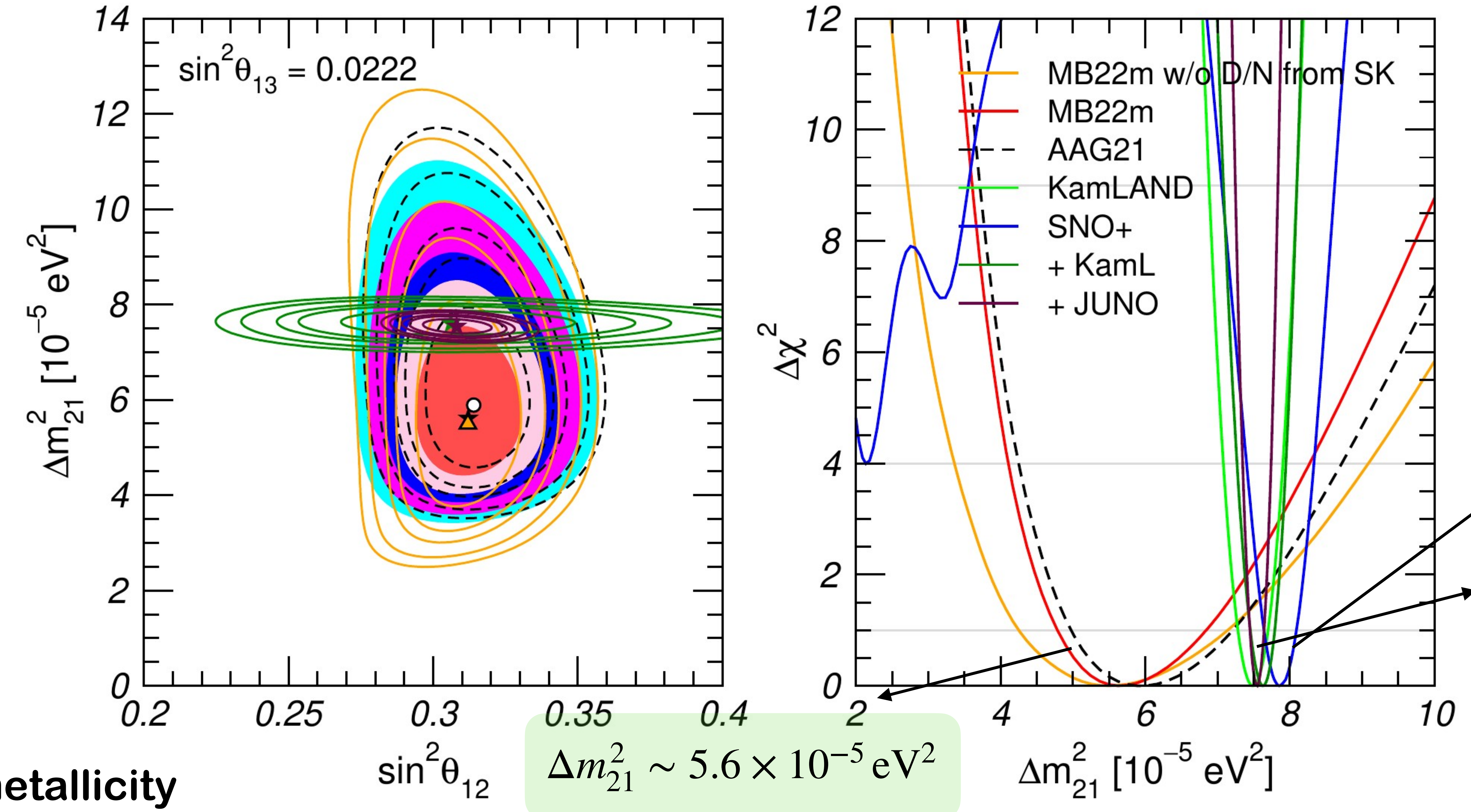
NuFIT 6.1 (2025)



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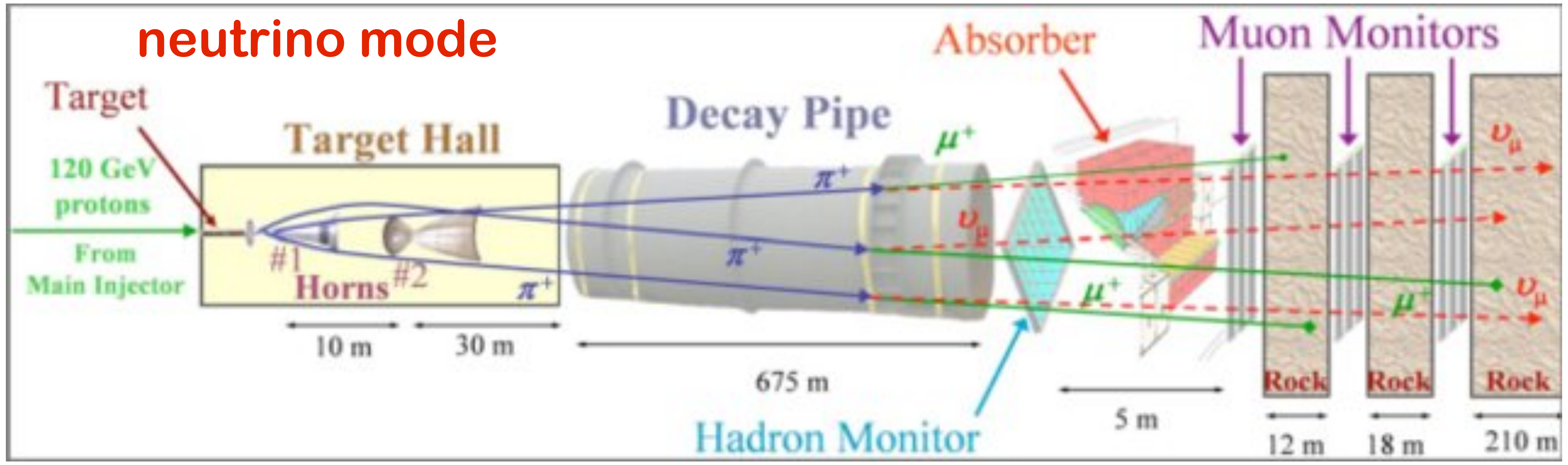
Accelerator Neutrinos

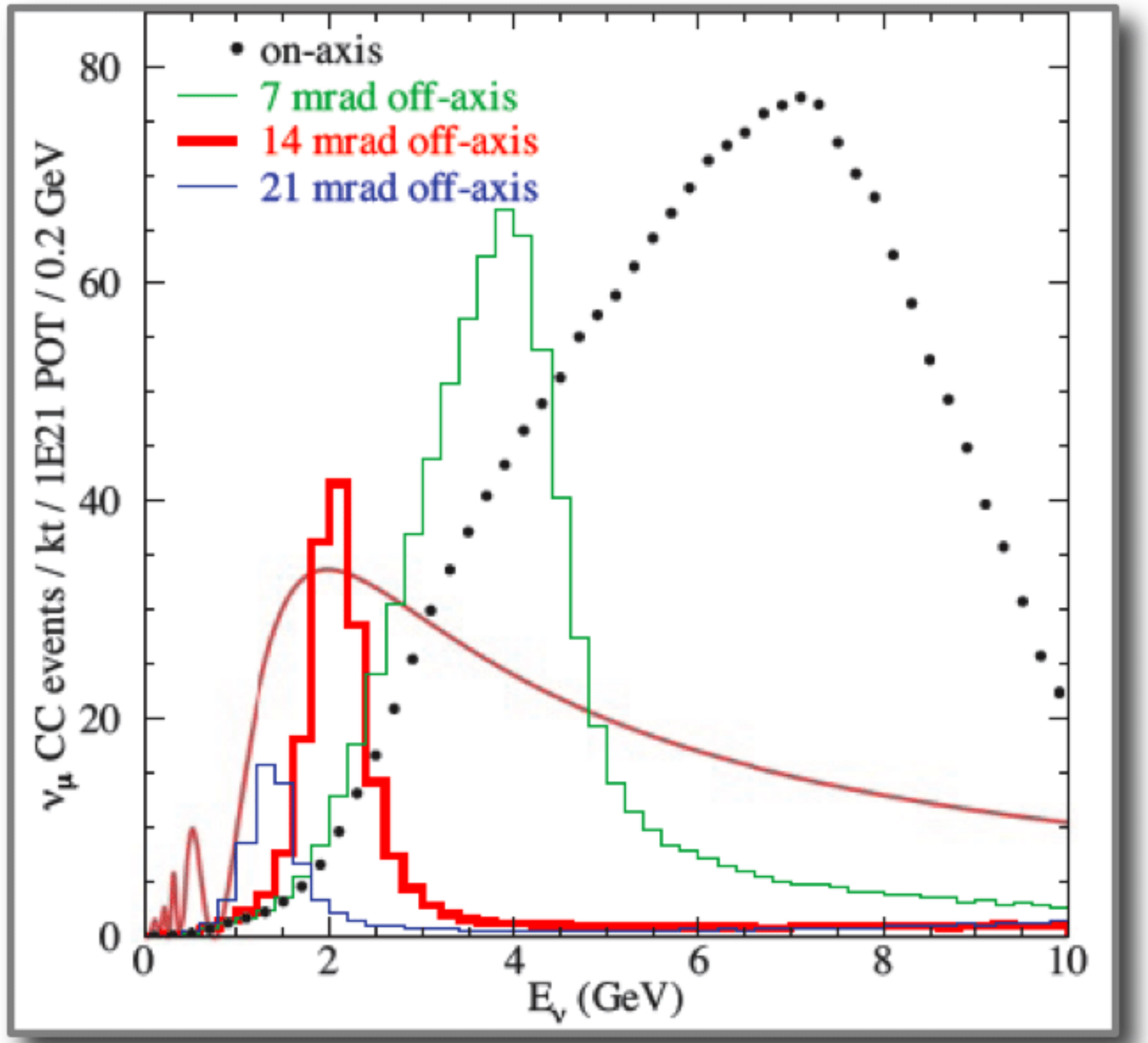
Accelerator Neutrinos

[see Chris Marshall & Matthew Toups]

Disappearance and appearance for neutrinos and antineutrinos



Off-axis : the beam is collimated in energy



NuMI neutrino beam @ FNAL

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$K^+ \to \mu^+ + \nu_\mu$$

are the main processes for the neutrino flux

$\nu_\mu \to \nu_\mu$

$\bar{\nu}_\mu \to \bar{\nu}_\mu$

disappearance modes

$\nu_\mu \to \nu_e$

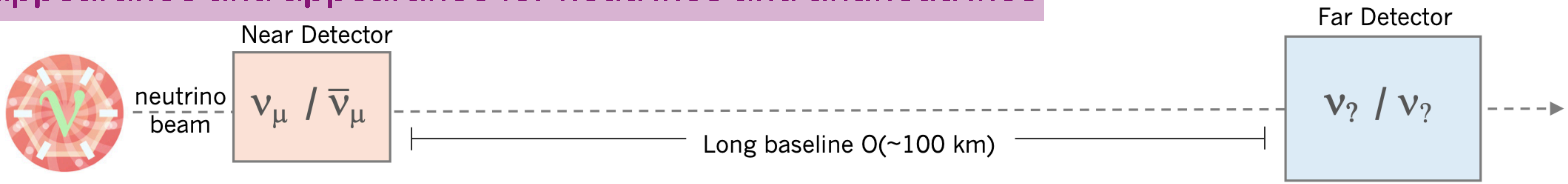
$\bar{\nu}_\mu \to \bar{\nu}_e$

appearance modes

[taken from NOvA Collab.]

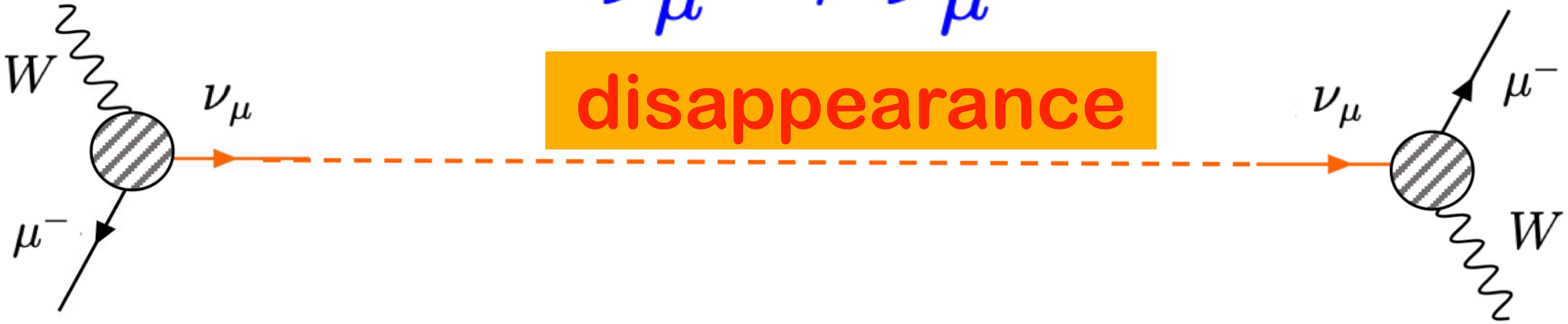
Accelerator Neutrinos

Disappearance and appearance for neutrinos and antineutrinos



$$\nu_\mu \to \nu_\mu$$

disappearance



$$\nu_\mu \to \nu_e$$

appearance

Accelerator Disappearance Probability

Sensitivity to the 32/31-sectors

$$\nu_\mu \to \nu_\mu \quad \& \quad \bar{\nu}_\mu \to \bar{\nu}_\mu$$

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)\sin^2\Delta_{\mu\mu} + 4|U_{\mu 2}|^2|U_{\mu 1}|^2\sin^2\Delta_{21} + \mathcal{O}(\Delta^2_{21})$$

vacuum

Accelerator Disappearance Probability

Sensitivity to the 32/31-sectors

$\nu_\mu \to \nu_\mu$ & $\bar{\nu}_\mu \to \bar{\nu}_\mu$

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)\sin^2\Delta_{\mu\mu} + 4|U_{\mu 2}|^2|U_{\mu 1}|^2 \sin^2\Delta_{21} + \mathcal{O}(\Delta_{21}^2)$$

negligible

vacuum

effective atmospheric $\Delta m^2_{\rm atm}$ for accelerator experiments

$$\Delta m^2_{\mu\mu} = s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32} + s_{13}\Delta m^2_{21}\sin 2\theta_{12}\tan\theta_{23}\cos\delta \approx s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32}$$

MINOS baseline L = 735 km	$\langle E\rangle \sim 3$ GeV	$\Delta_{21} \sim 0.02$
T2K baseline L = 295 km	$\langle E\rangle \sim 0.6$ GeV	$\Delta_{21} \sim 0.05$
NOvA baseline L = 810 km	$\langle E\rangle \sim 2$ GeV	$\Delta_{21} \sim 0.04$

Accelerator Disappearance Probability

Sensitivity to the 32/31-sector

$$\nu_\mu \to \nu_\mu \quad \& \quad \bar{\nu}_\mu \to \bar{\nu}_\mu$$

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)\sin^2\Delta_{\mu\mu} + 4|U_{\mu 2}|^2|U_{\mu 1}|^2\sin^2\Delta_{21} + \mathcal{O}(\Delta^2_{21})$$

vacuum

effective atmospheric $\Delta m^2_{\rm atm}$ for accelerator experiments

$$\Delta m^2_{\mu\mu} = s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32} + s_{13}\Delta m^2_{21}\sin 2\theta_{12}\tan\theta_{23}\cos\delta \approx s^2_{12}\Delta m^2_{31} + c^2_{12}\Delta m^2_{32}$$

so for these experiments to very good approximation we can write

$$P_{\nu_\mu\to\nu_\mu} \approx 1 - 4s^2_{23}c^2_{13}(1-s^2_{23}c^2_{13})\sin^2\Delta_{\mu\mu} = 1 - \sin^2 2\theta_{\rm eff}\sin^2\Delta_{\mu\mu}$$

+ matter effects are negligible

Accelerator Appearance Probability

Sensitivity to the 32/31-sector: exploring matter effects

$$\nu_\mu \to \nu_e \text{ \& } \bar{\nu}_\mu \to \bar{\nu}_e$$

$$P_{\nu_\mu \to \nu_e} \approx P_{\text{atm}} + 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos(\Delta_{32} + \delta) + P_{\text{sol}}$$

in vacuum

sensitive to δ

$$\sqrt{P_{\text{sol}}} \equiv c_{23} c_{13} \sin 2\theta_{12} \sin \Delta_{21}$$

$$\sqrt{P_{\text{atm}}} \equiv s_{23} \sin 2\theta_{13} \sin \Delta_{31}$$

matter effects are important because they increase the difference between neutrino & antineutrinos

Accelerator Appearance Probability

Sensitivity to the 32/31-sector: exploring matter effects

$\nu_\mu \to \nu_e$

atmospheric term

solar term

octant

$$P_{\nu_\mu\to\nu_e} \approx 4s_{23}^2 s_{13}^2 c_{13}^2 \frac{\sin^2(\Delta_{31} - aL)}{(\Delta_{31} - aL)^2}\Delta_{31}^2 + 4s_{12}^2 c_{12}^2 c_{13}^2 c_{23}^2 \frac{\sin^2(aL)}{(aL)^2}\Delta_{21}^2$$

mass ordering

interference term

CP phase

$$+8\frac{J}{\sin\delta}\frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)}\Delta_{31}\frac{\sin(aL)}{(aL)}\Delta_{21}\cos(\Delta_{31} + \delta)$$

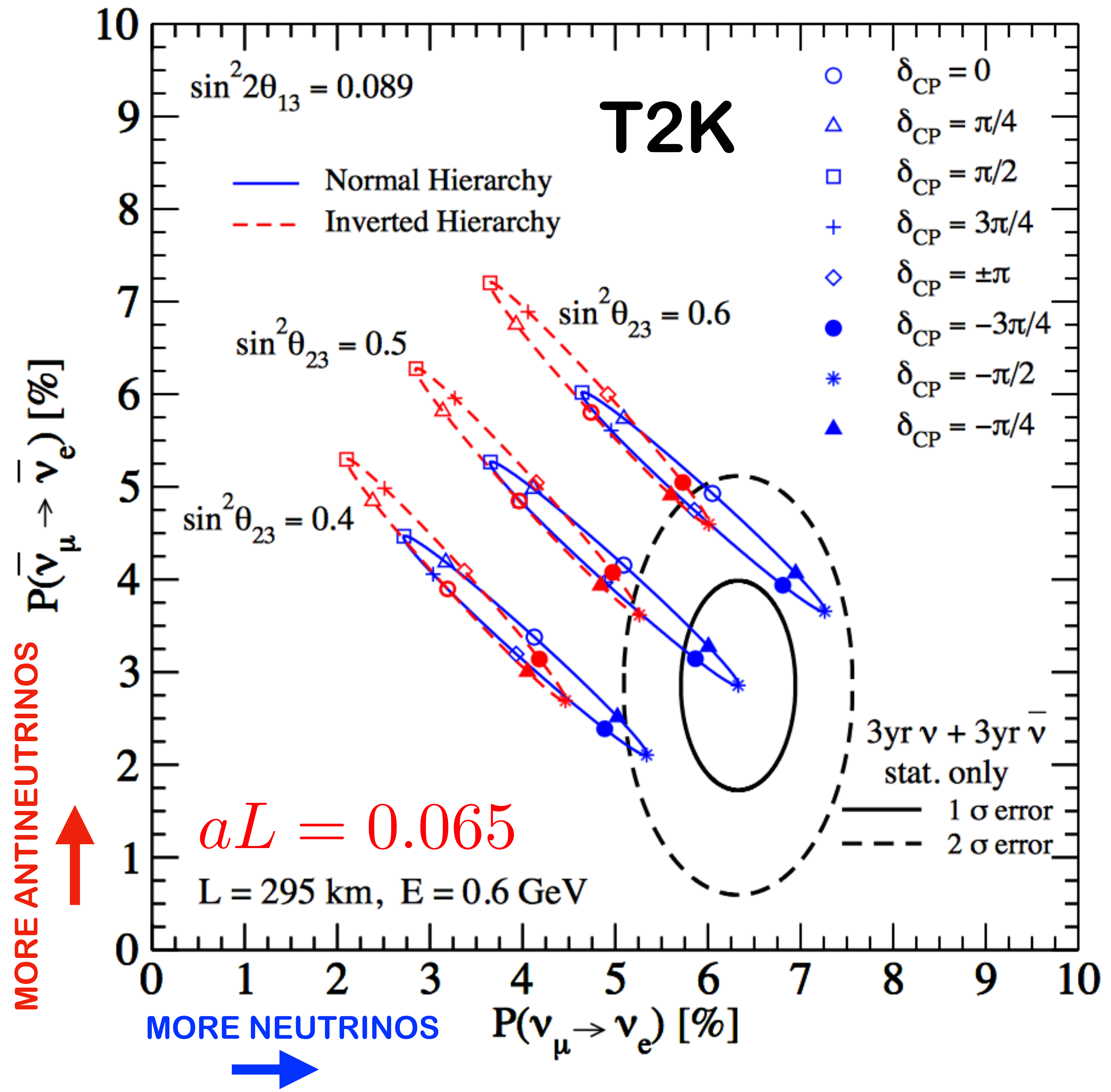
$\bar{\nu}_\mu \to \bar{\nu}_e$

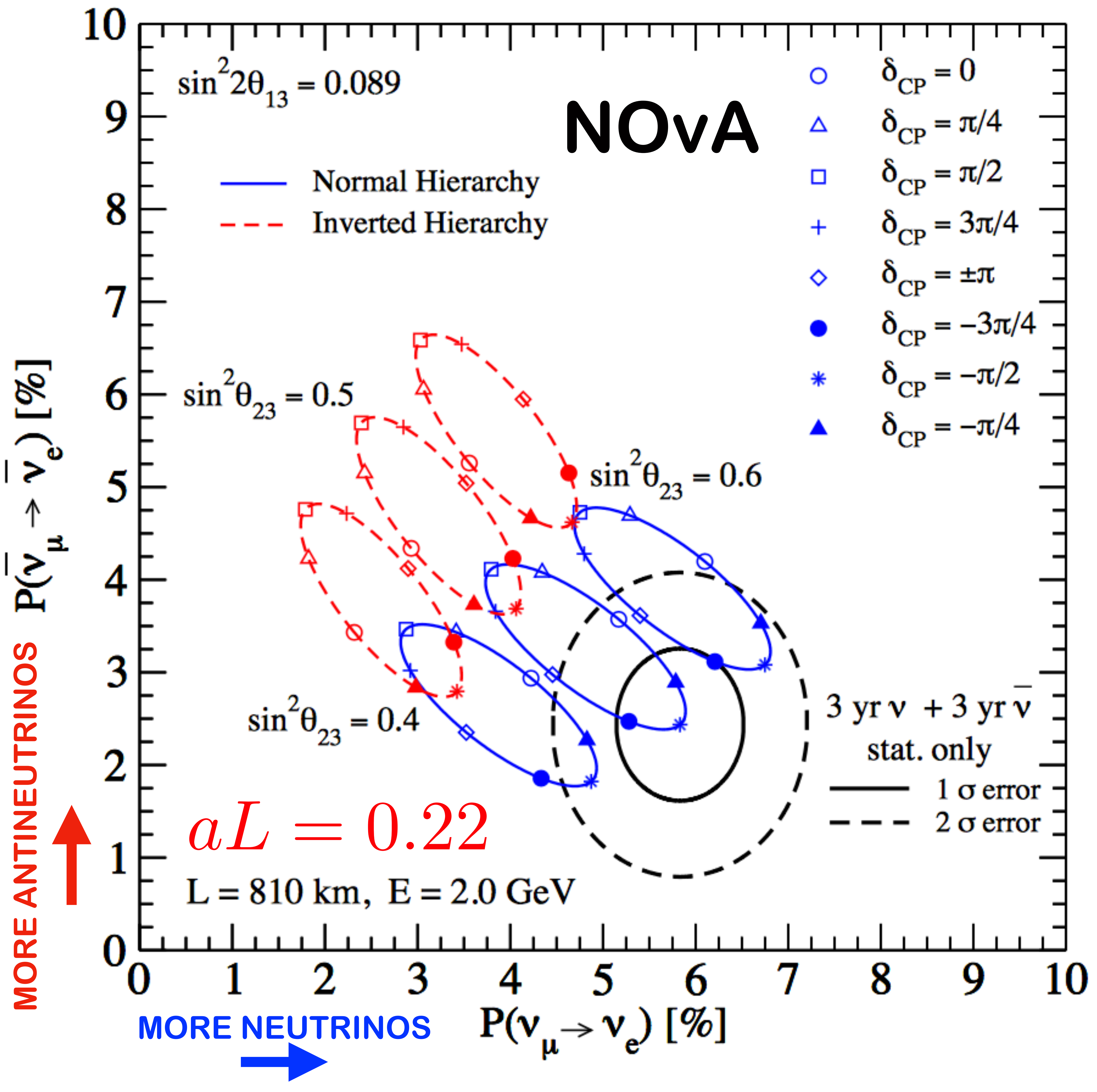
$$P_{\bar{\nu}_\mu\to\bar{\nu}_e} = P_{\nu_\mu\to\nu_e}(\delta \to -\delta,\ aL \to -aL)$$

effect of the matter potential

$$a \equiv \frac{G_F n_e}{\sqrt{2}} \approx \frac{1}{3500\ \text{km}}\left(\frac{\rho}{3\ \text{g/cm}^3}\right)$$

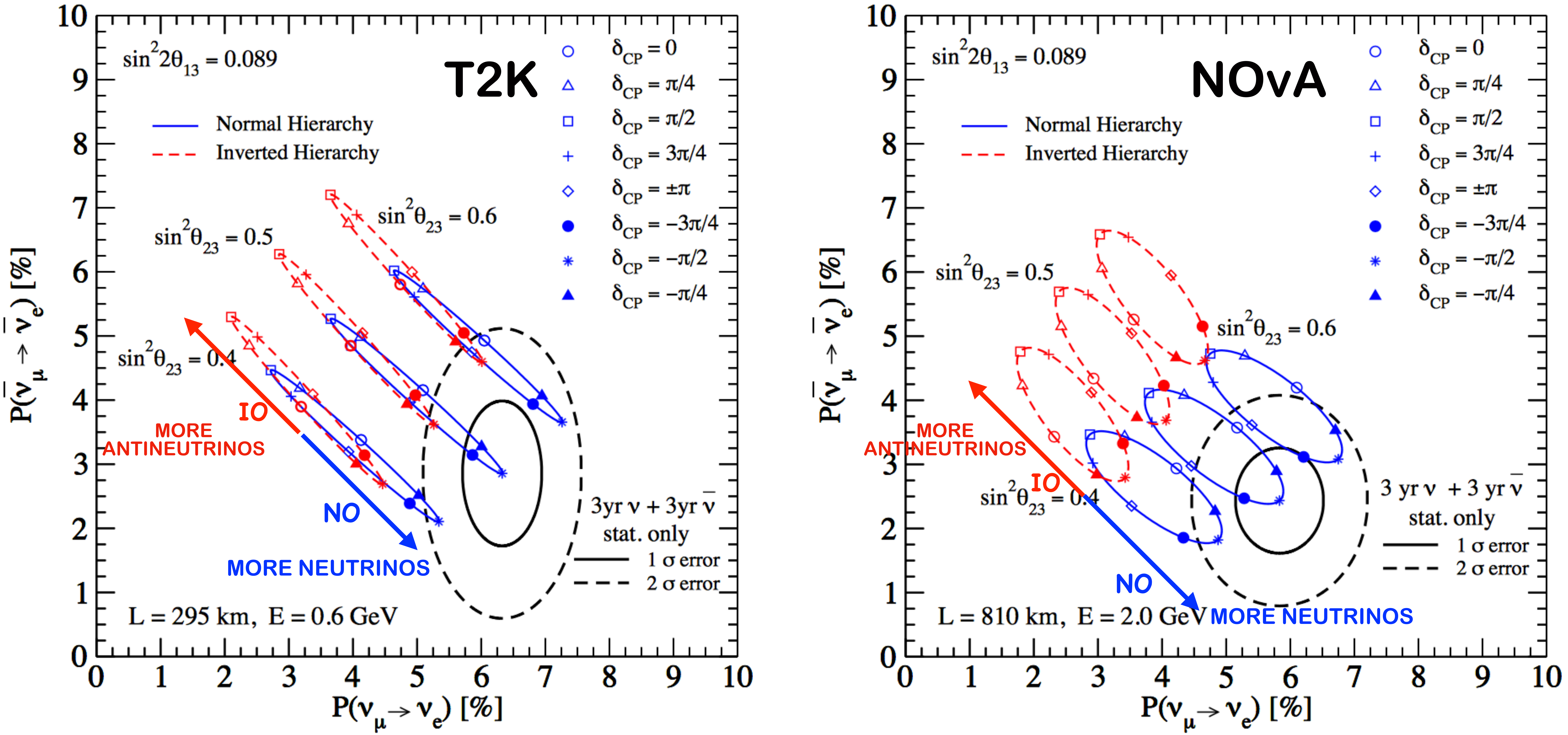
Bi-probability plots





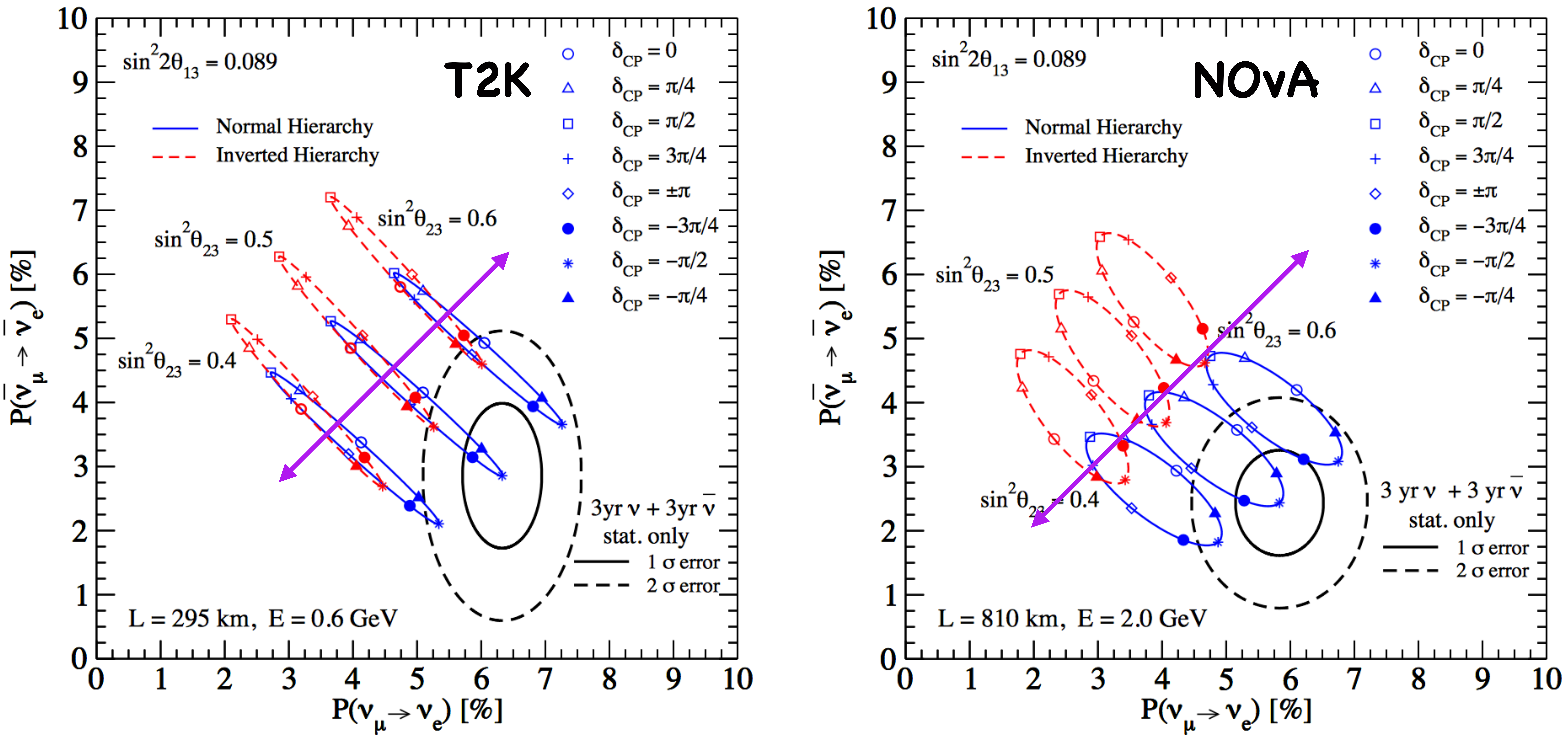
Bi-probability plots

Mass Ordering



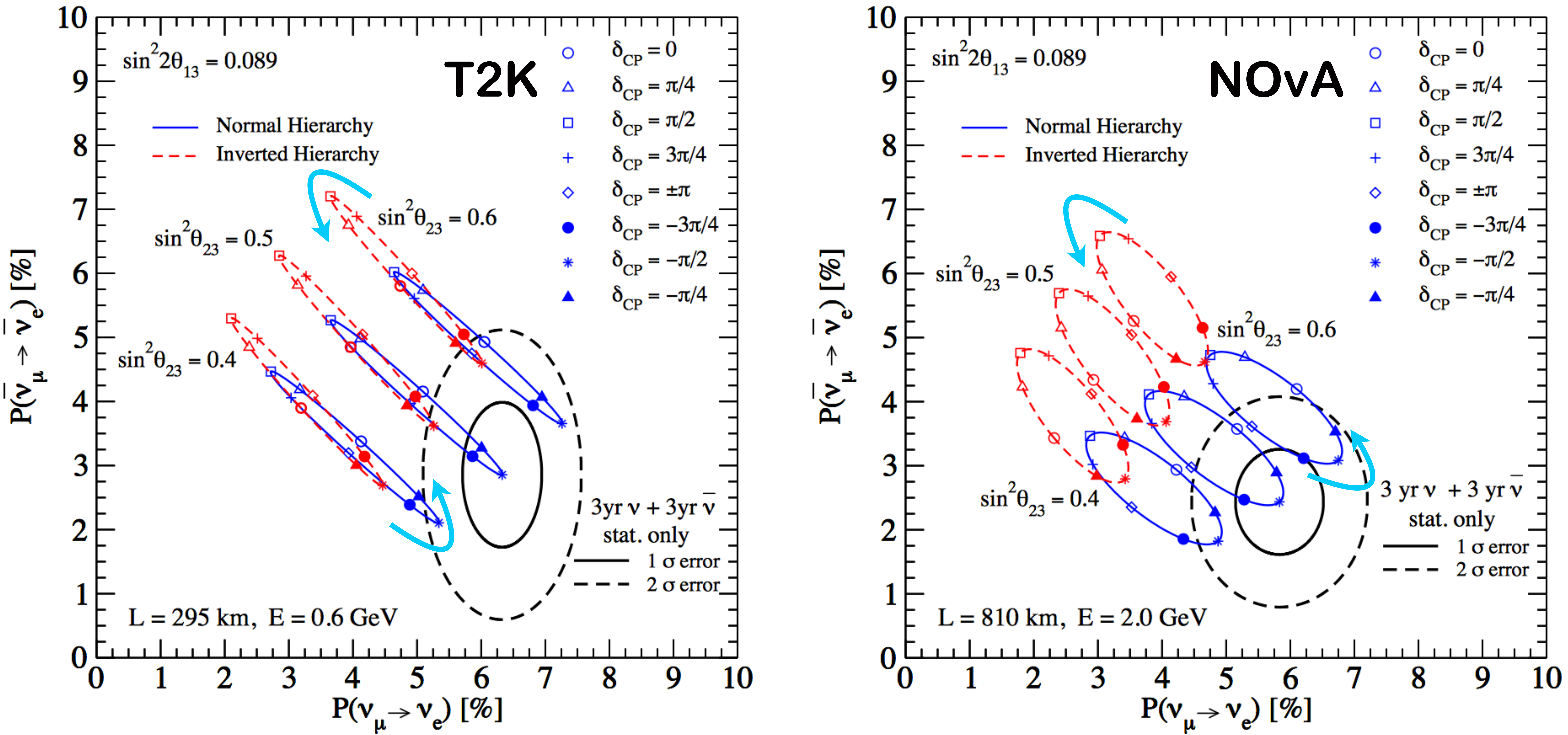
Bi-probability plots

octant θ_{23}



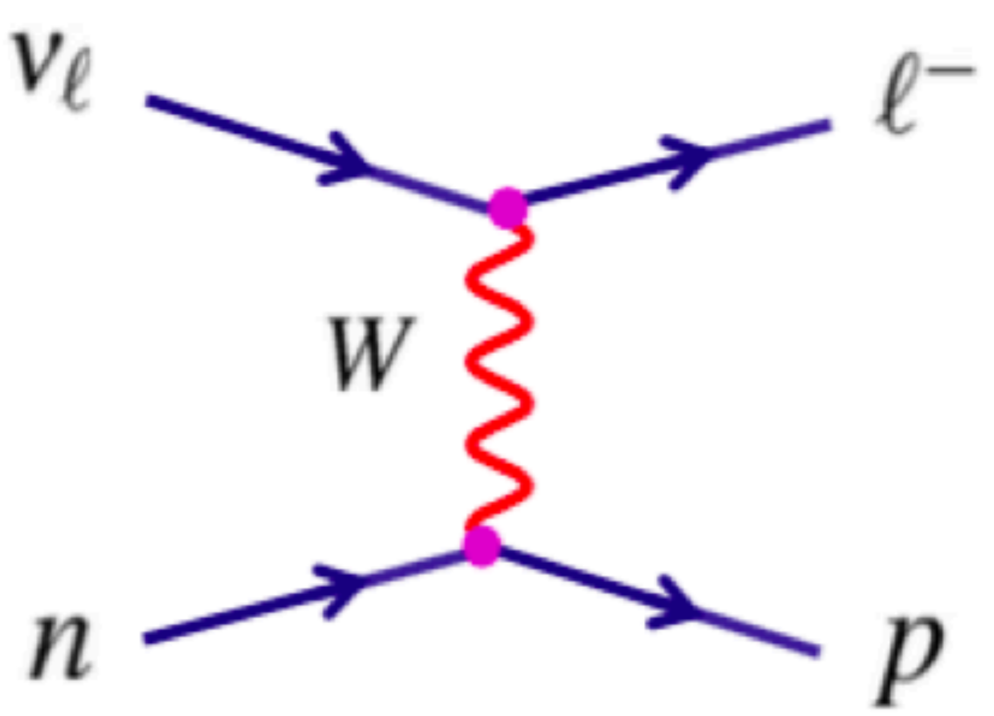
Bi-probability plots

CP Phase

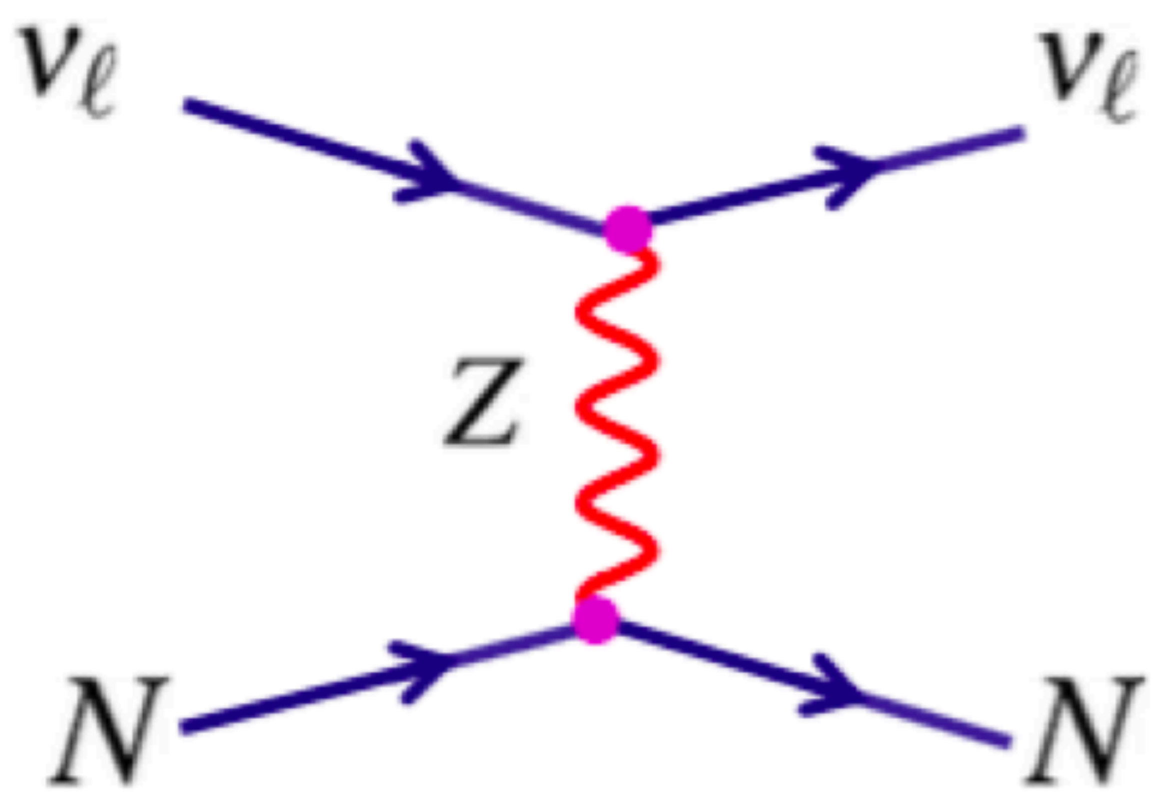


Accelerator (& Atmospheric) Neutrinos

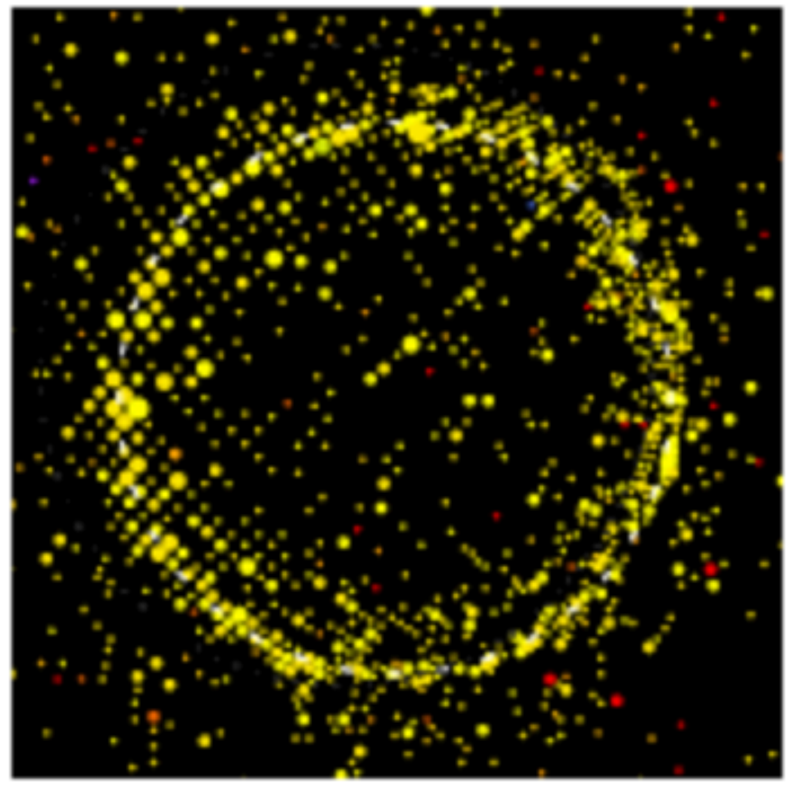
How neutrinos are measured?



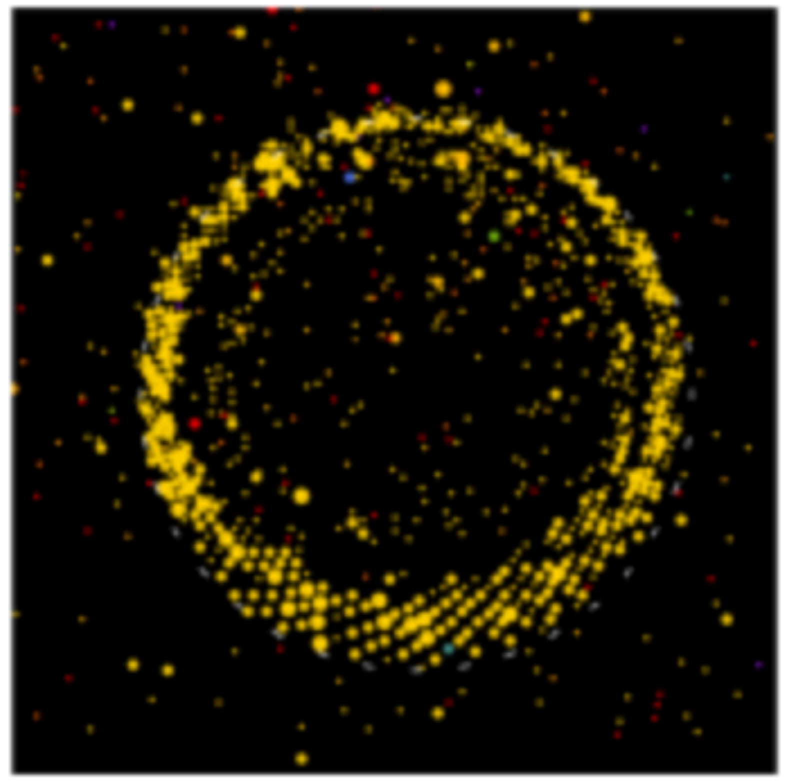
Charged Current (CC) interaction reveals the flavor from the outgoing charged lepton



Neutral Current (NC) interactions mediated by Z boson is indistinguishable for the the 3 flavors



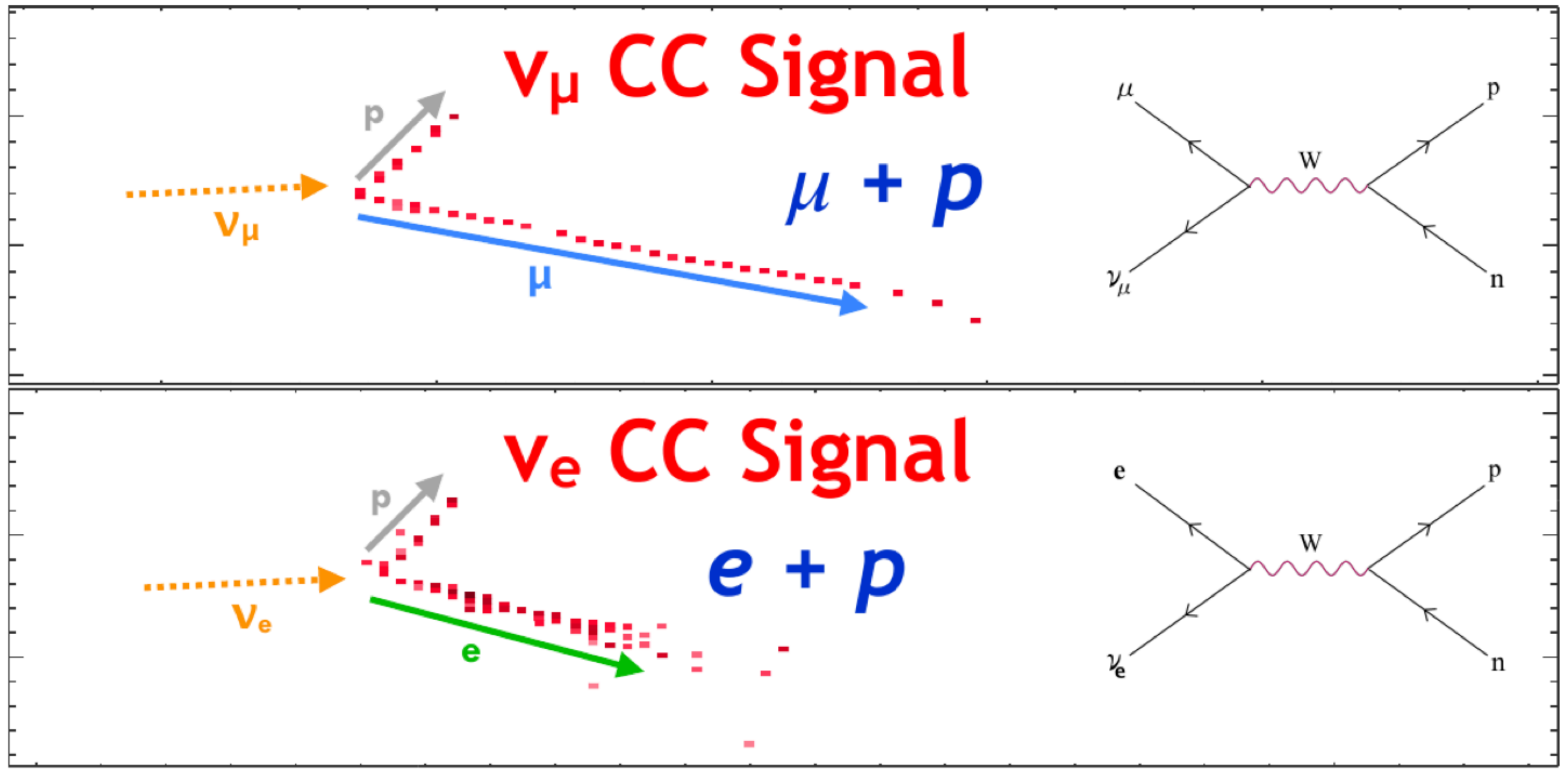
ν_e-like



ν_μ-like

T2K/Super-Kamiokande

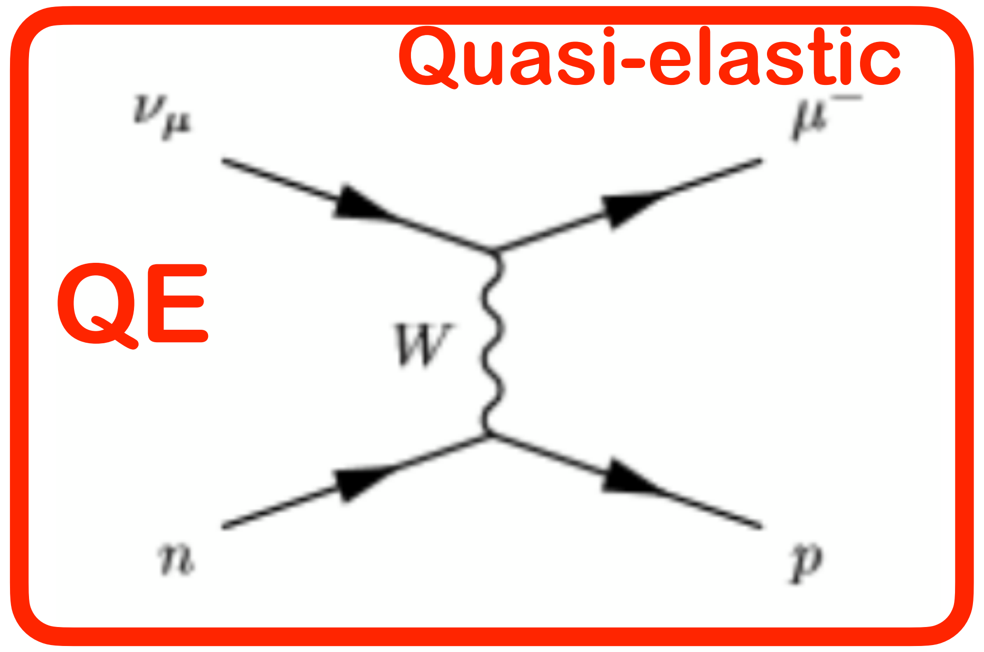
NOvA

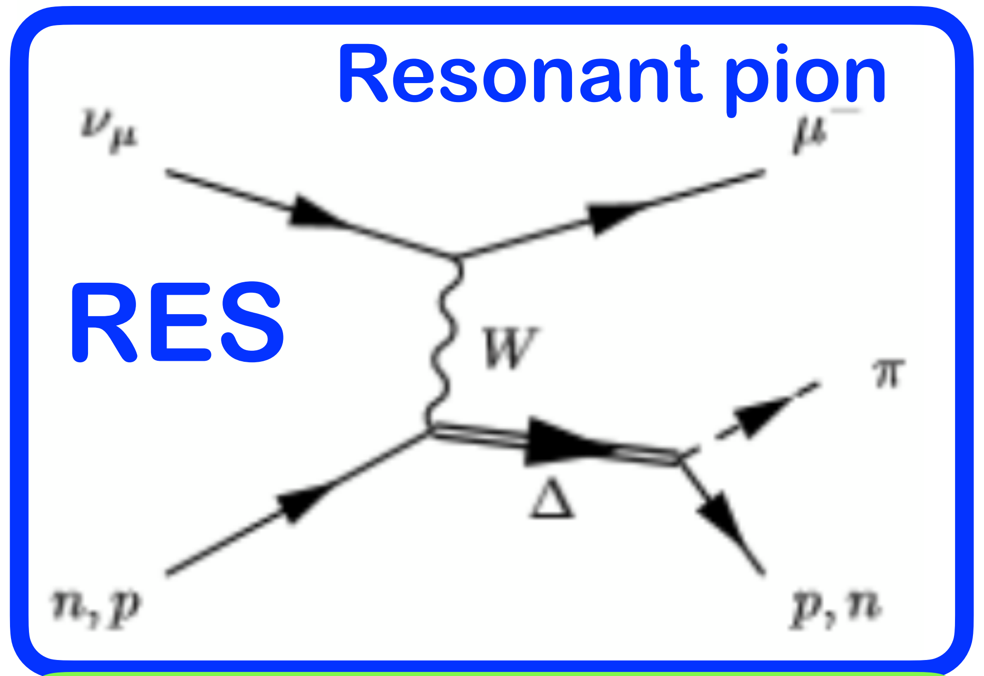


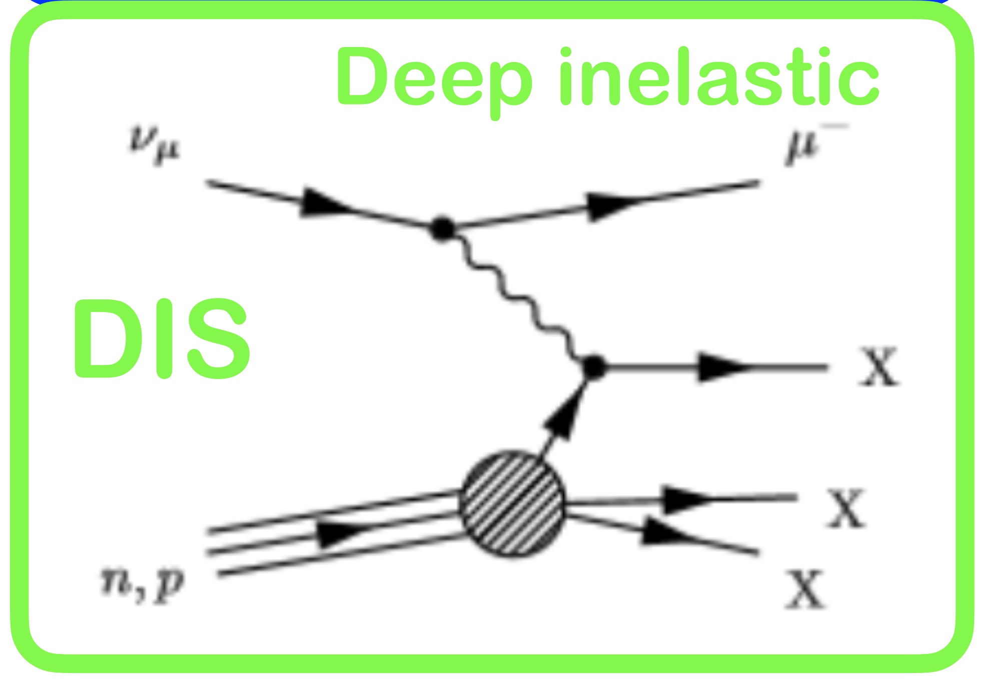
Accelerator (& Atmospheric) Neutrinos

x-sections

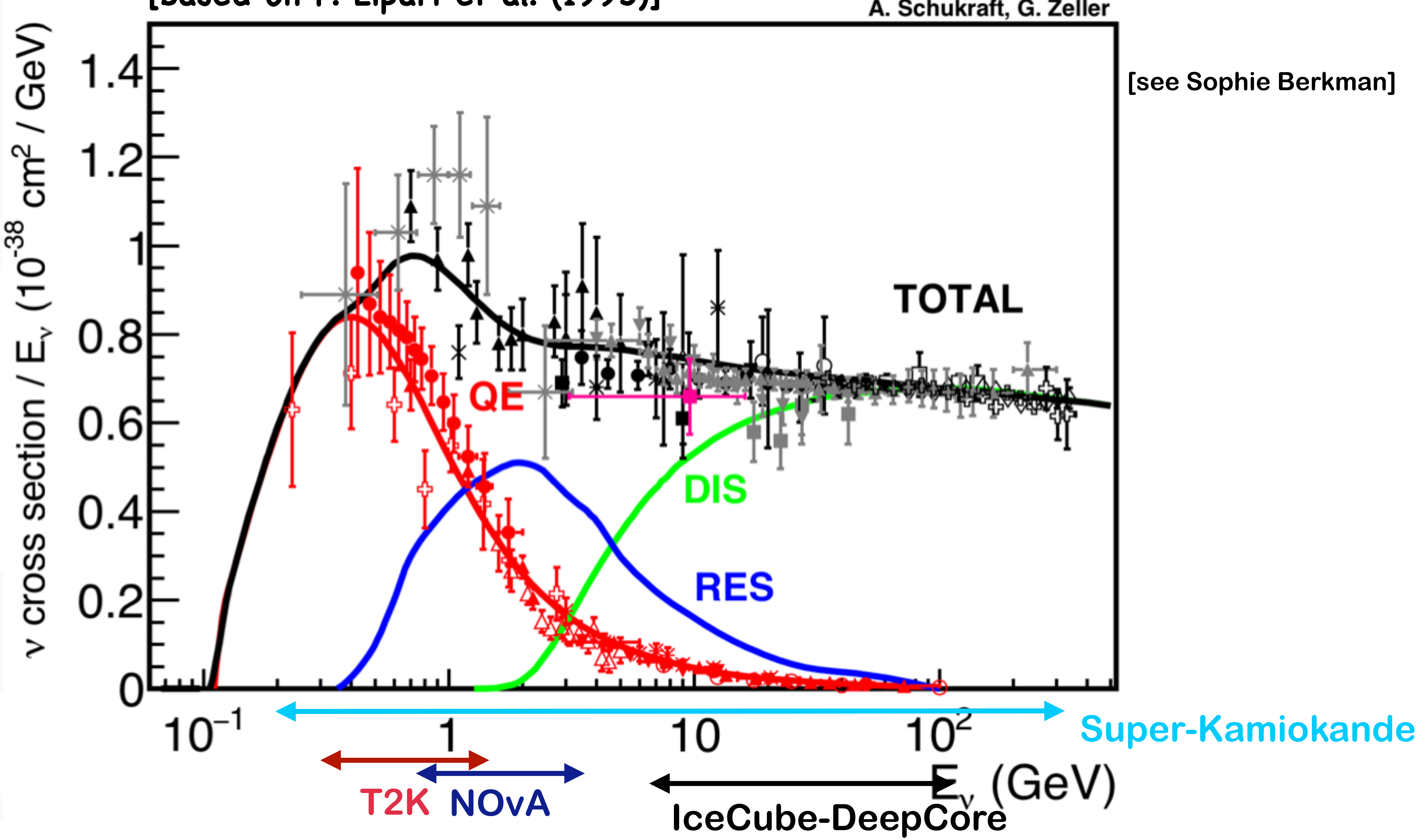
CC-interaction







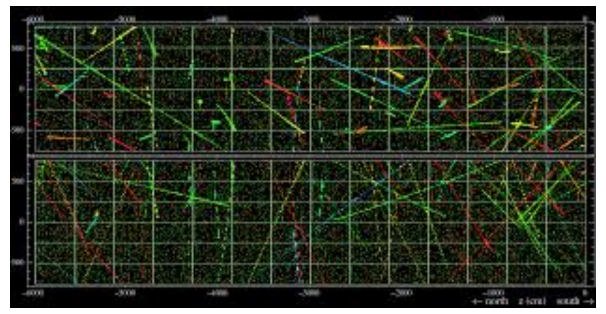
[based on P. Lipari et al. (1995)]



[see Sophie Berkman]

Accelerator Neutrinos

Results from NOvA



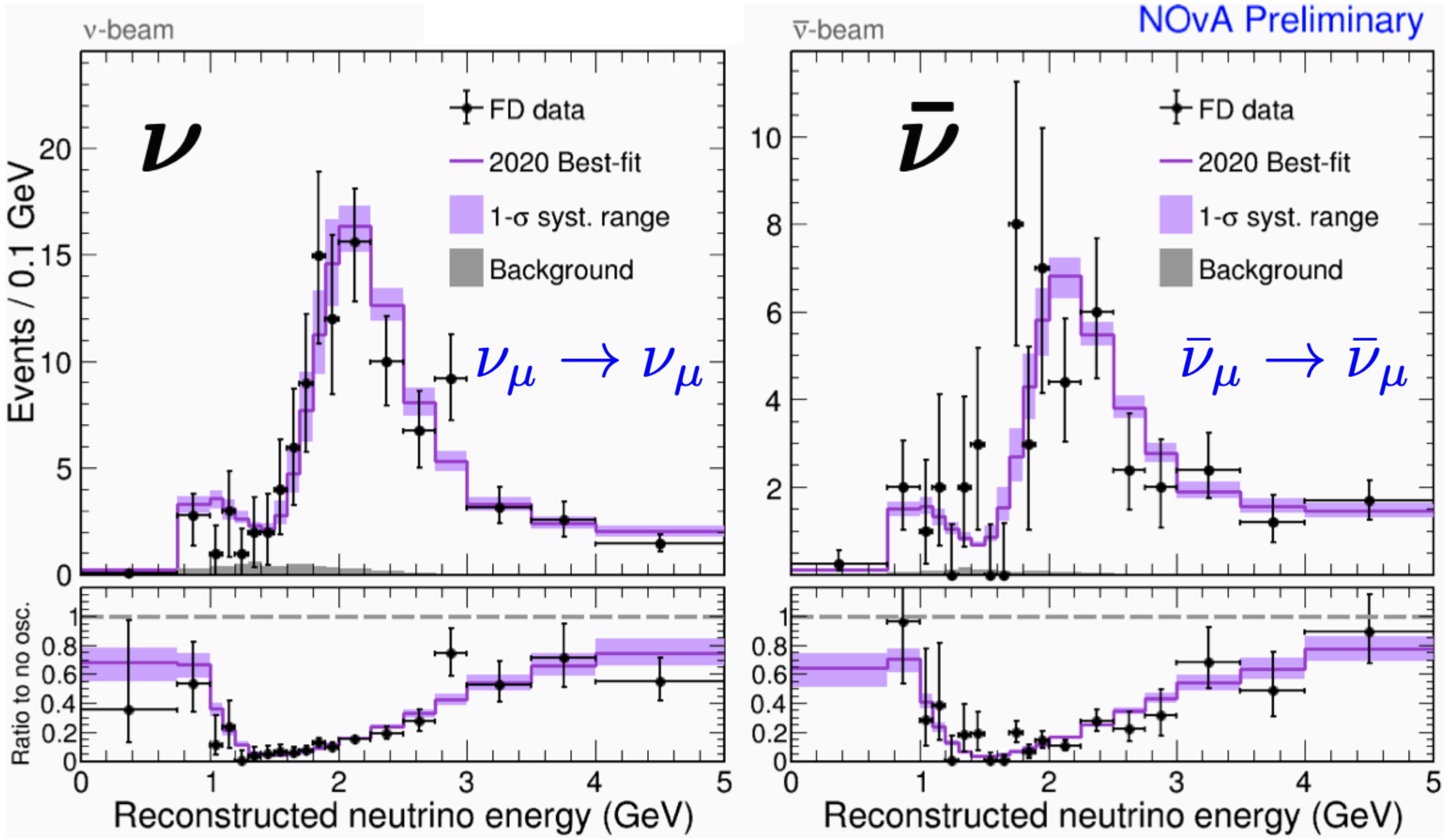
211 ν_μ CC candidates observed			
$\nu_\mu \to \nu_\mu$ 201.1		$\bar{\nu}_\mu \to \bar{\nu}_\mu$ 12.6	
NC	cosmic	other	**total**
2.6	5.0	1.0	**222.3**$^{+16.3}_{-15.7}$

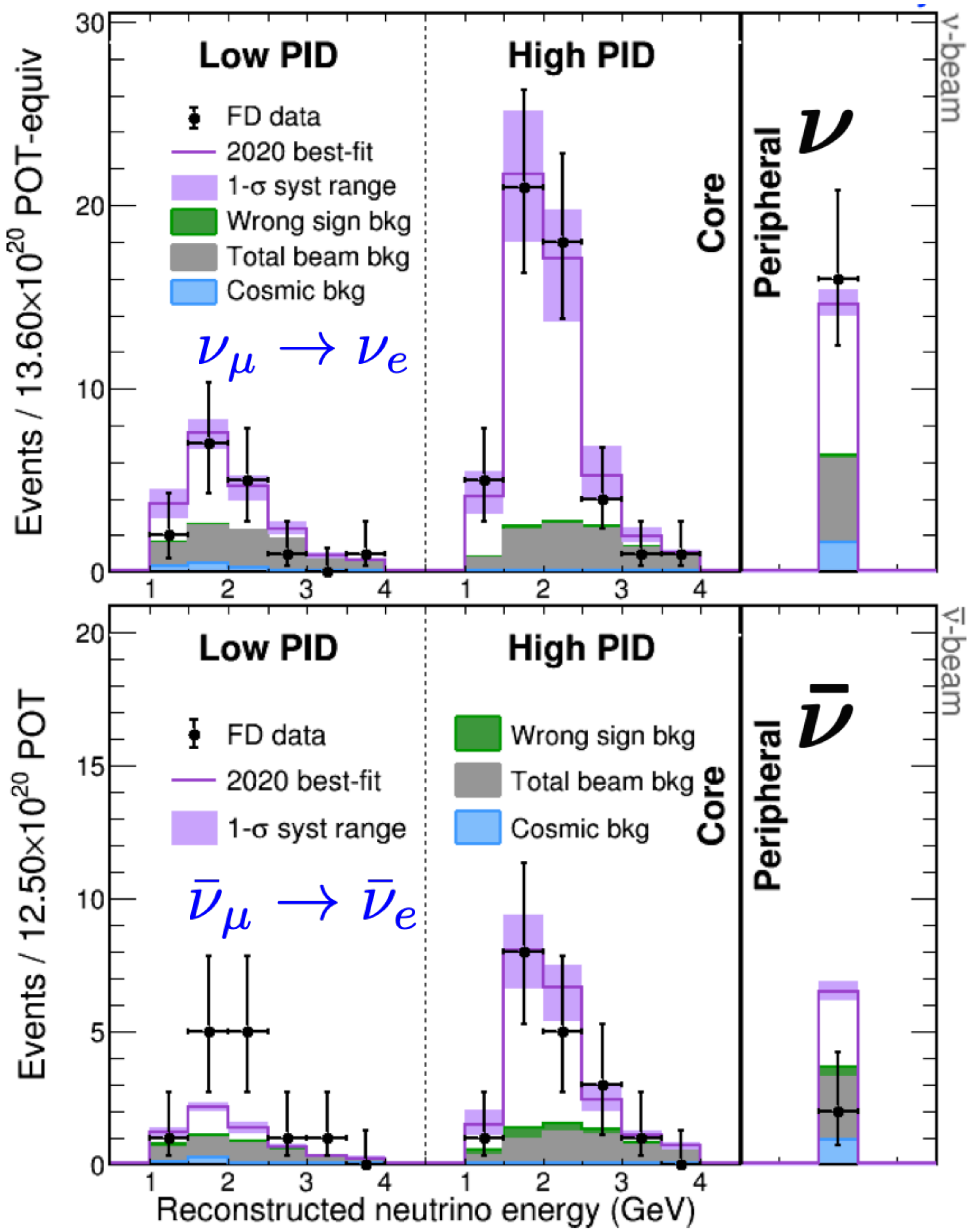
105 $\bar{\nu}_\mu$ CC candidates observed			
$\bar{\nu}_\mu \to \bar{\nu}_\mu$ 77.2		$\nu_\mu \to \nu_\mu$ 26.0	
NC	cosmic	other	**total**
0.8	0.9	0.4	**105.4**$^{+7.8}_{-7.7}$

82 ν_e CC candidates observed					
Signal $\nu_\mu \to \nu_e$			**59.0**		
$\bar{\nu}_e$	beam ν_e	ν_μ	NC	cos.	other
1.0	14.1	1.7	6.3	3.1	0.5
total 85.8$^{+4.1}_{-4.2}$ (background 26.8)					

33 $\bar{\nu}_e$ CC candidates observed					
Signal $\bar{\nu}_\mu \to \bar{\nu}_e$			**19.2**		
ν_e	beam ν_e	ν_μ	NC	cos.	other
2.3	7.3	0.4	2.2	1.6	0.3
total 33.2$^{+1.5}_{-1.7}$ (background 14.0)					

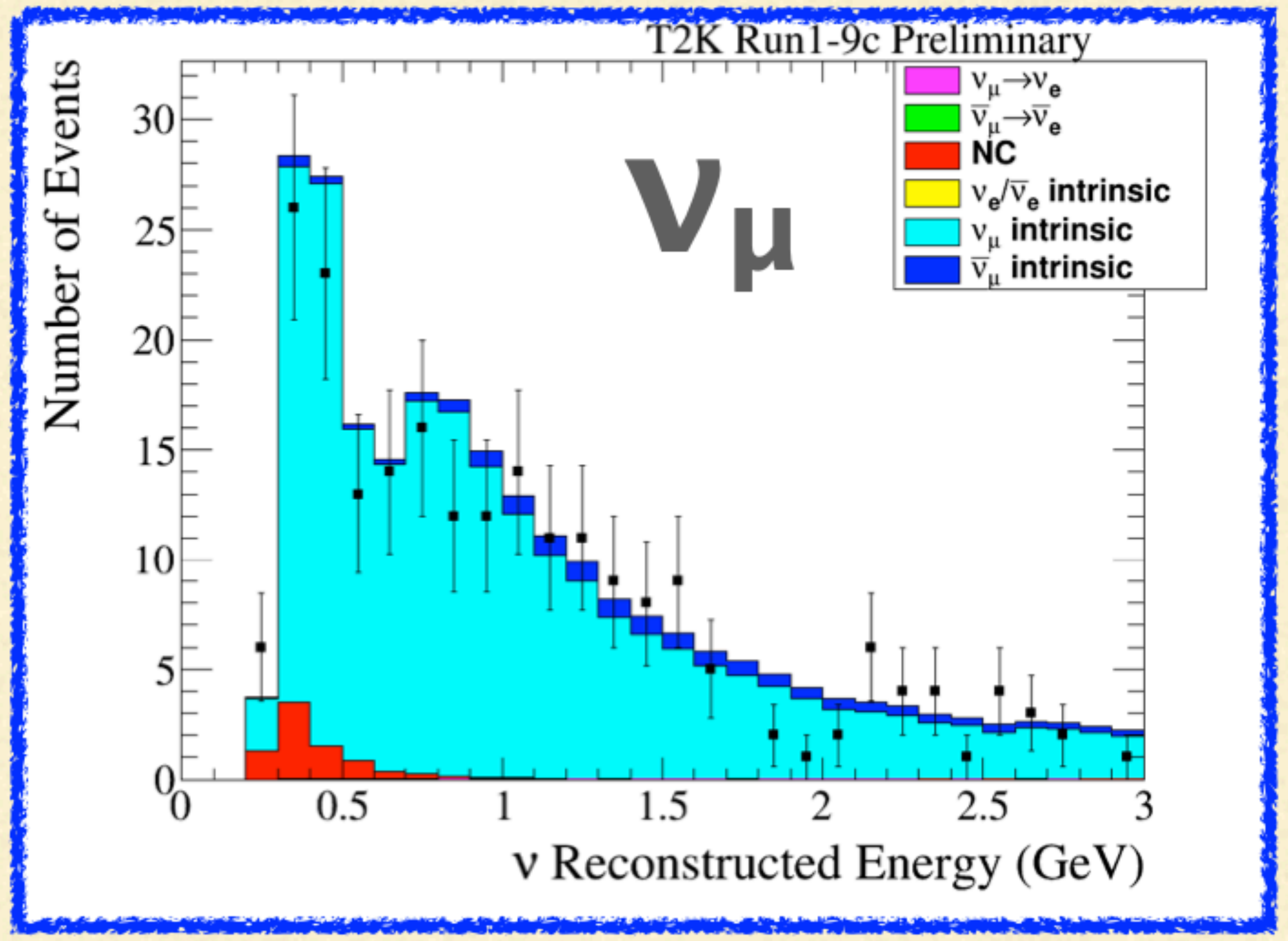
[Taken from NOvA Collab. (2022)]

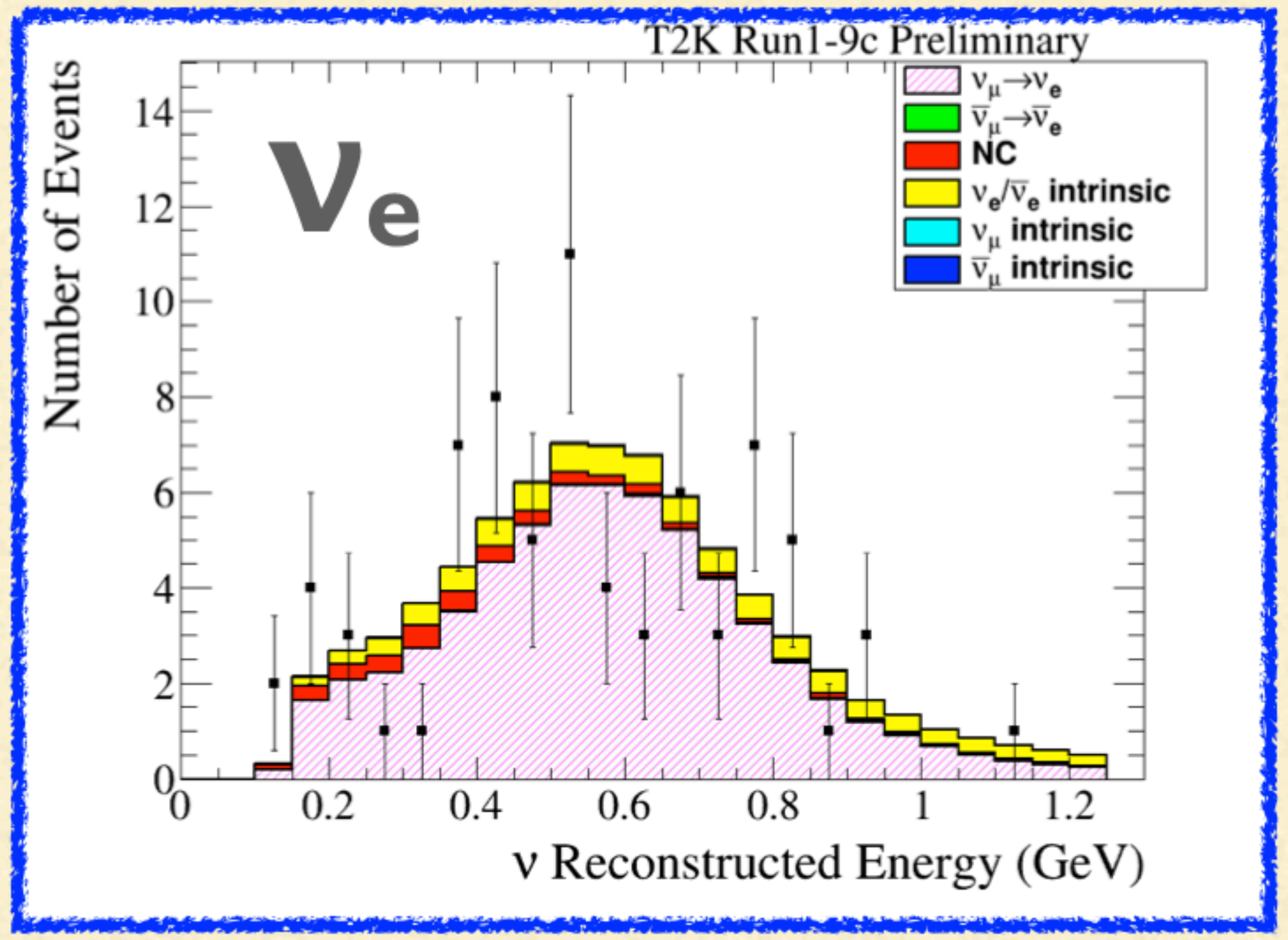


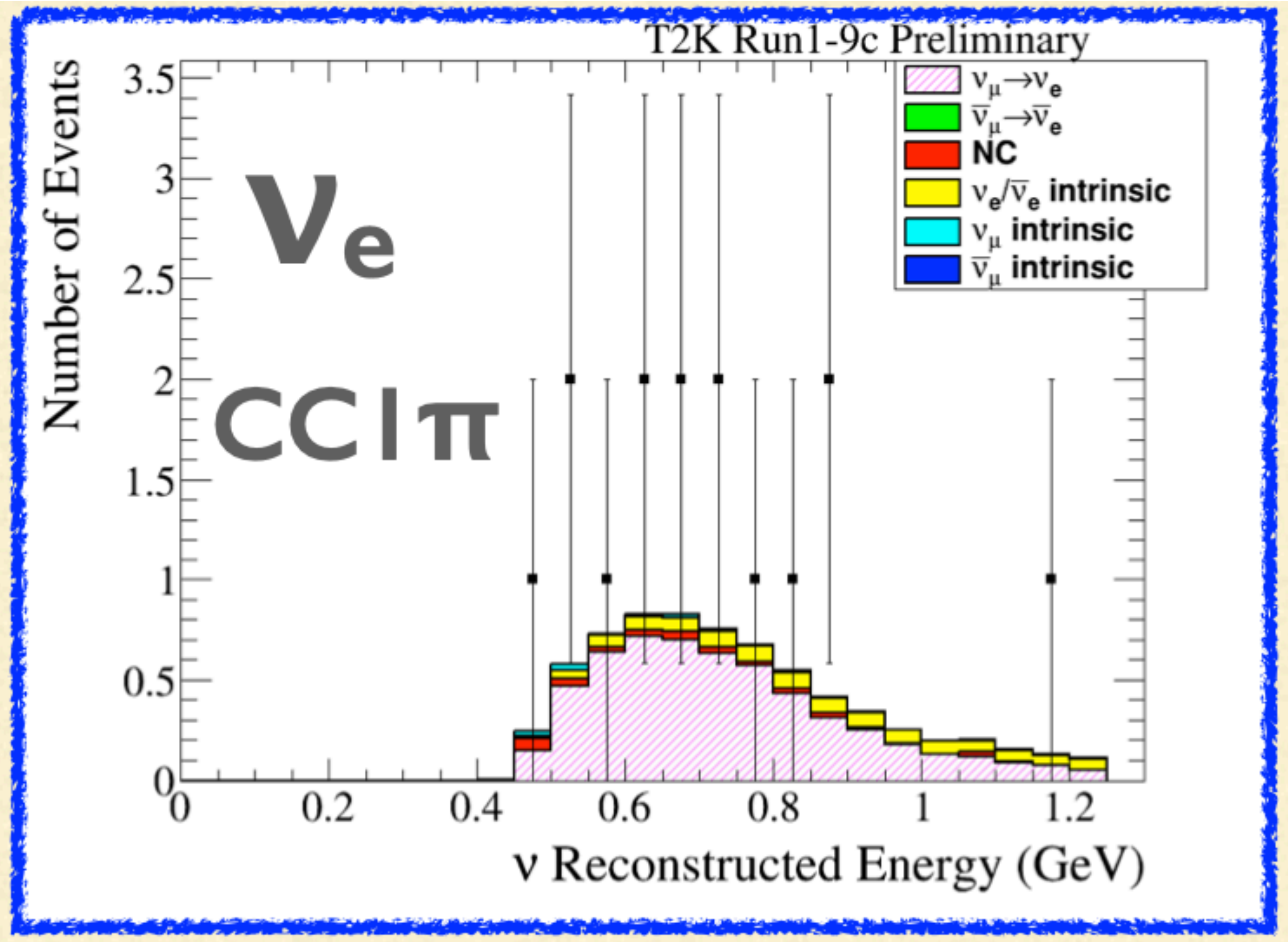


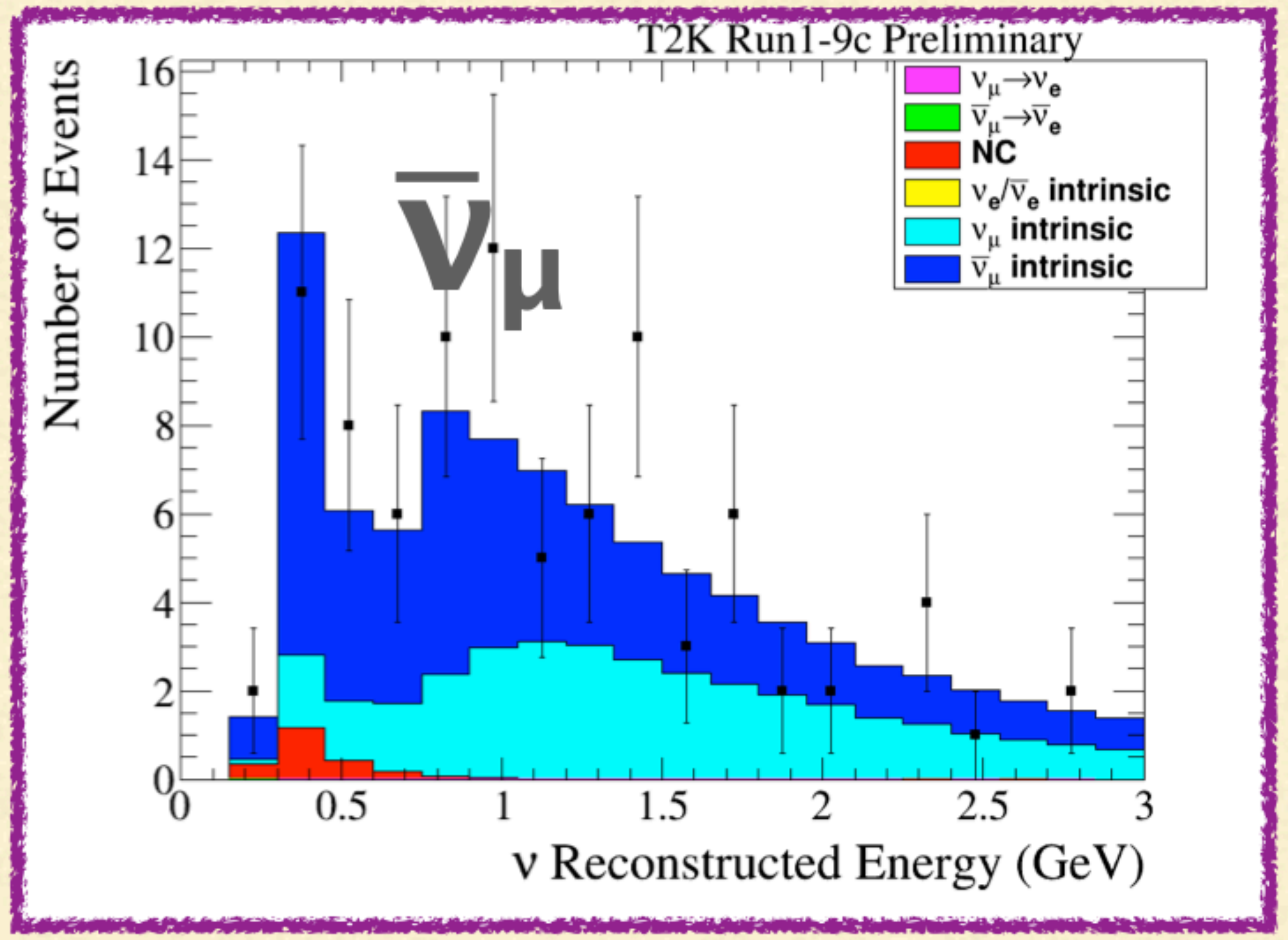
Accelerator Neutrinos

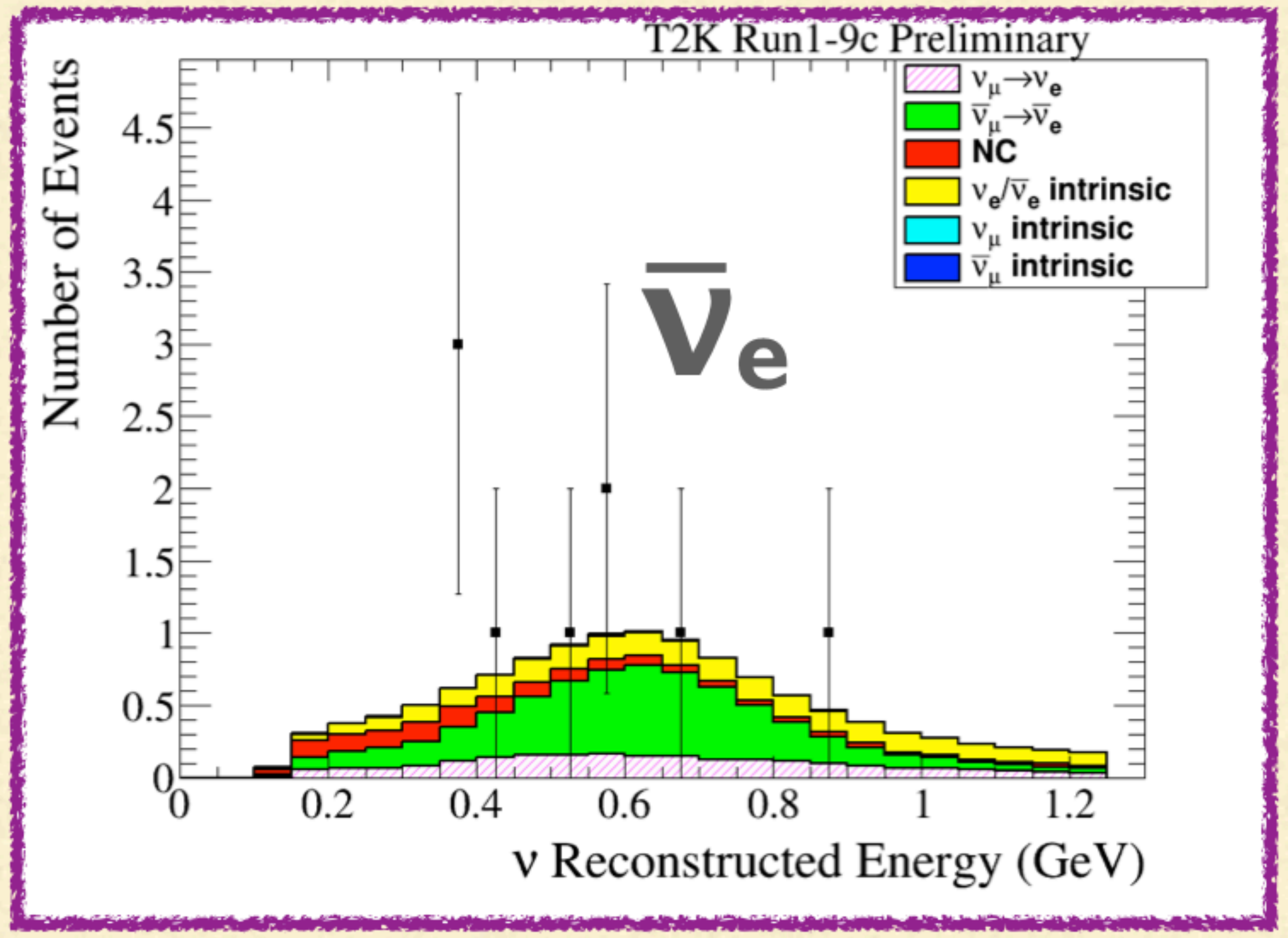
Results from T2K











[Taken from T2K Collab. (2020)]

Energy distribution

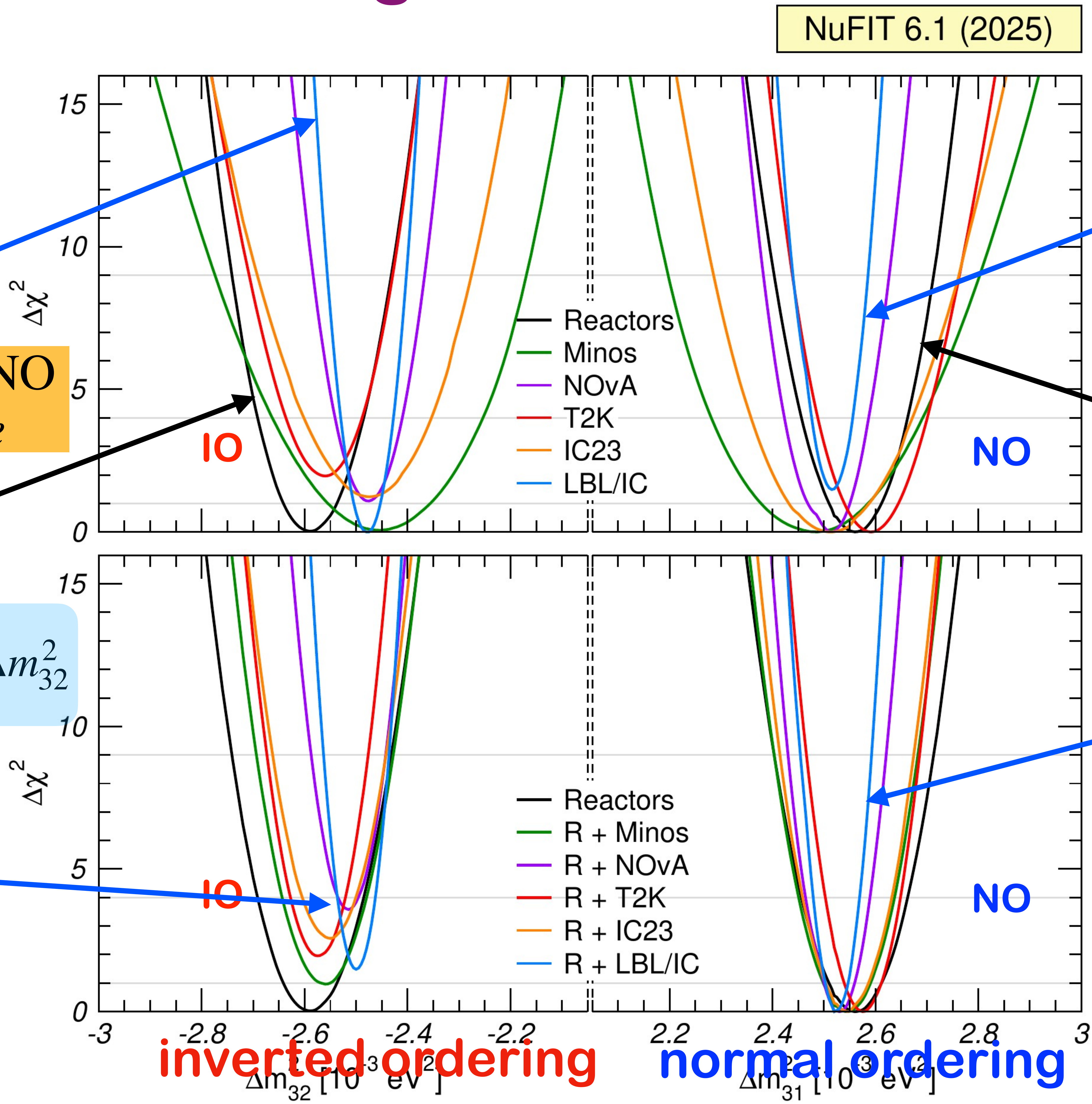
ν-mode

anti-ν-mode

*T2K Preliminary

Disappearance Experiments Synergy

Neutrino Mass Ordering



$\Delta m^2_{31}|^{\rm NO}_\mu > |\Delta m^2_{32}|^{\rm IO}_\mu$

LBL

$\Delta m^2_{\mu\mu} \simeq \sin^2\theta_{12}\,\Delta m^2_{31} + \cos^2\theta_{12}\Delta m^2_{32}$

Reactors Combined

LBL+Reactors

LBL

$|\Delta m^2_{32}|^{\rm IO}_e > \Delta m^2_{31}|^{\rm NO}_e$

Reactors Combined

$\Delta m^2_{\rm ee} \equiv \cos^2\theta_{12}\,\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$

LBL+Reactors

Disappearance Experiments Synergy

Mass Ordering Sum Rule

@ the moment no disappearance experiment has any sensitivity to the ordering

few $\%$

$$(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\overline{\cos\delta})\,\Delta m^2_{21}$$

$$\Delta m^2_{31}|^{NO}_{\mu} = \Delta m^2_{31}|^{NO}_{e}$$

if NO is true

$$\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3} eV^2$$

Precision $\sim 1.2\,\%$

$$|\Delta m^2_{32}|^{IO}_{e} = |\Delta m^2_{32}|^{IO}_{\mu}$$

if IO is true

$$|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3} eV^2$$

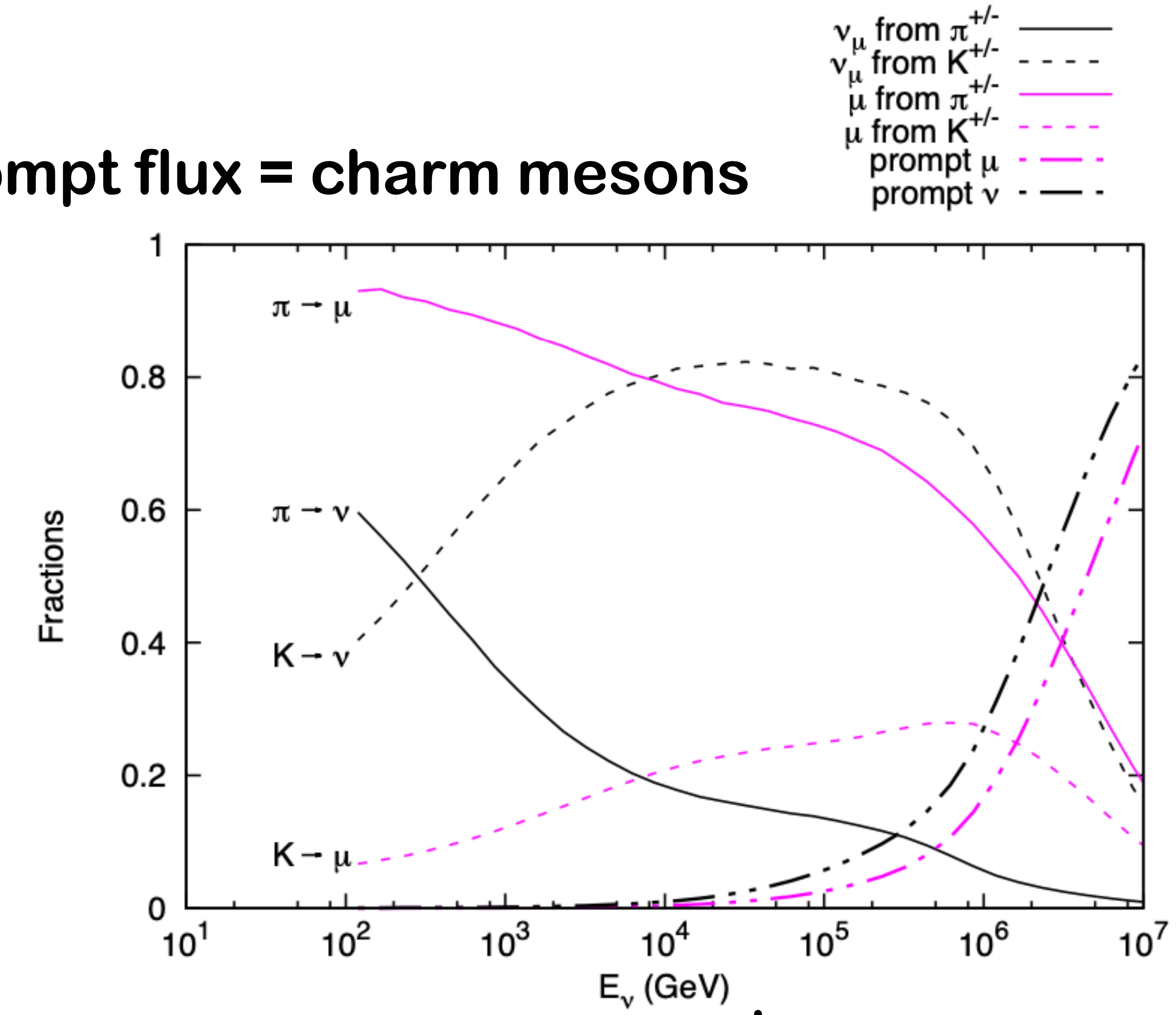
this will change soon with JUNO

Atmospheric Neutrinos

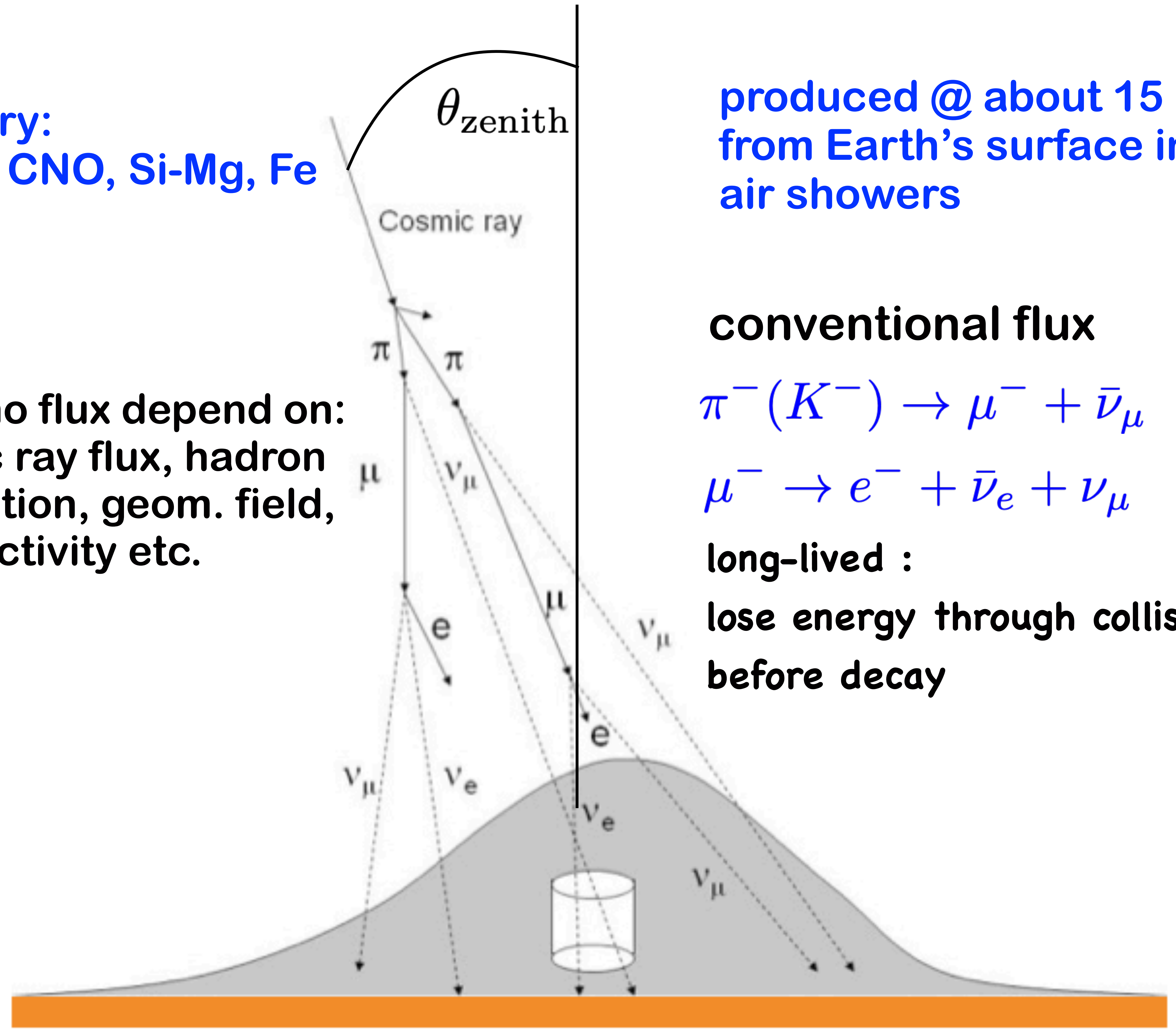
Atmospheric Neutrinos

Sources of the Flux

prompt flux = charm mesons



primary:
p, He, CNO, Si-Mg, Fe



produced @ about 15 km from Earth's surface in air showers

neutrino flux depend on: cosmic ray flux, hadron interaction, geom. field, solar activity etc.

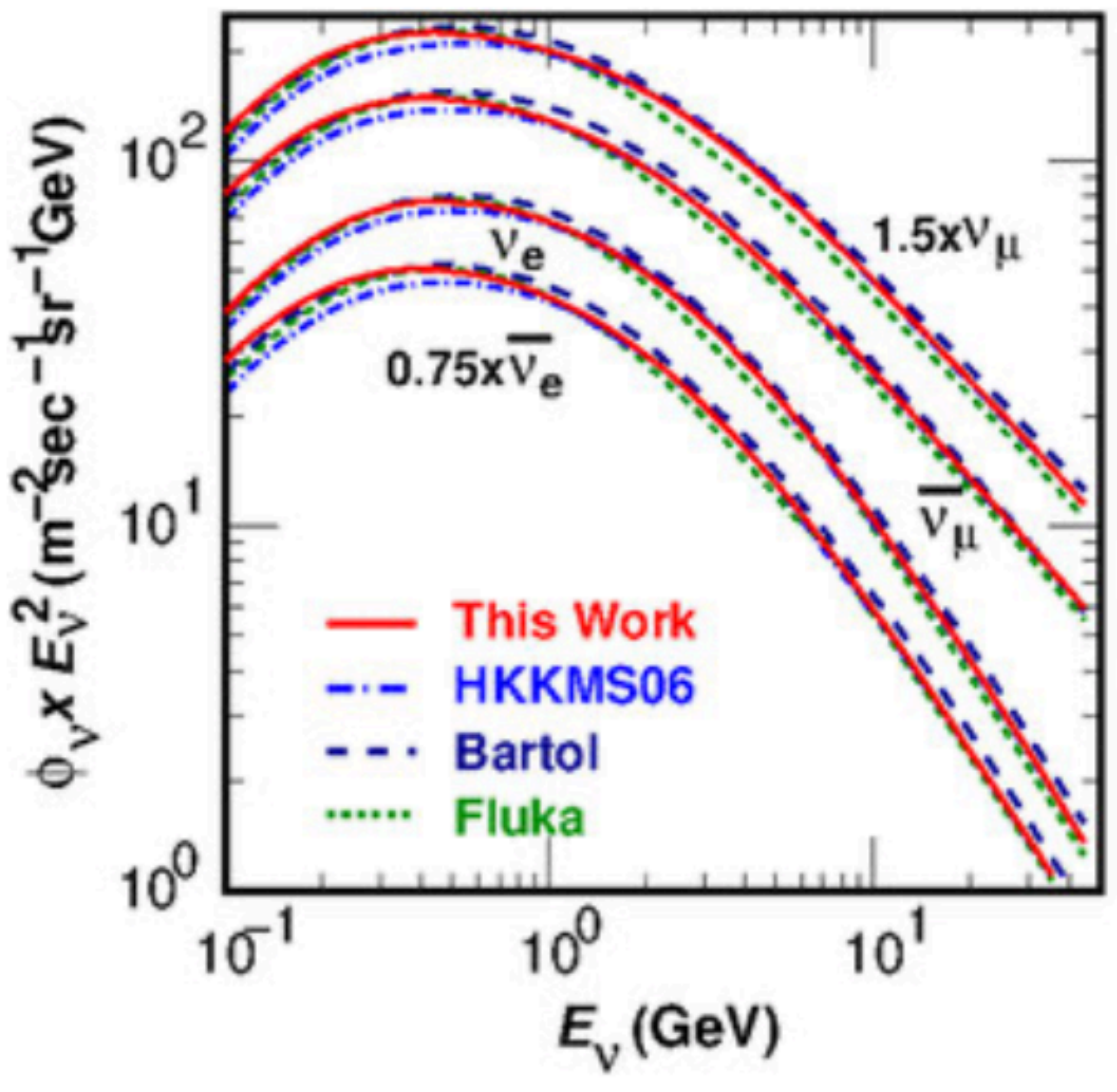
conventional flux

$$\pi^-(K^-) \to \mu^- + \bar{\nu}_\mu$$

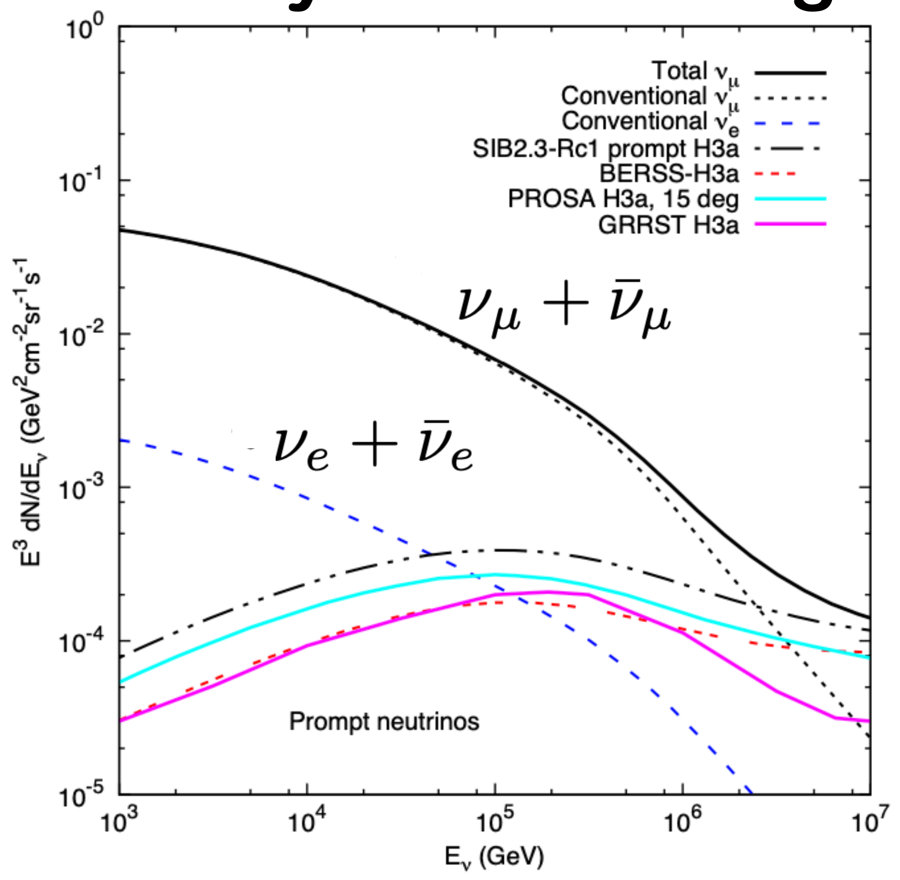
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

long-lived :

lose energy through collisions before decay



prompt: mesons decay before losing energy



unoscillated flux averaged over zenith angle

Atmospheric Neutrinos

Sensitivity to the 32/31-sectors

Experiments count $N_{\nu_e+\bar{\nu}_e}$ & $N_{\nu_\mu+\bar{\nu}_\mu}$ as a function of the zenith angle

have to take into account matter effect for neutrinos traveling through the Earth

$$P_{\nu_e\to\nu_e} \qquad P_{\nu_e\to\nu_\mu} \qquad P_{\nu_e\to\nu_\tau}$$

$$P_{\nu_\mu\to\nu_\mu} \qquad P_{\nu_\mu\to\nu_e} \qquad P_{\nu_\mu\to\nu_\tau}$$

Δm^2_{31} (or Δm^2_{32})

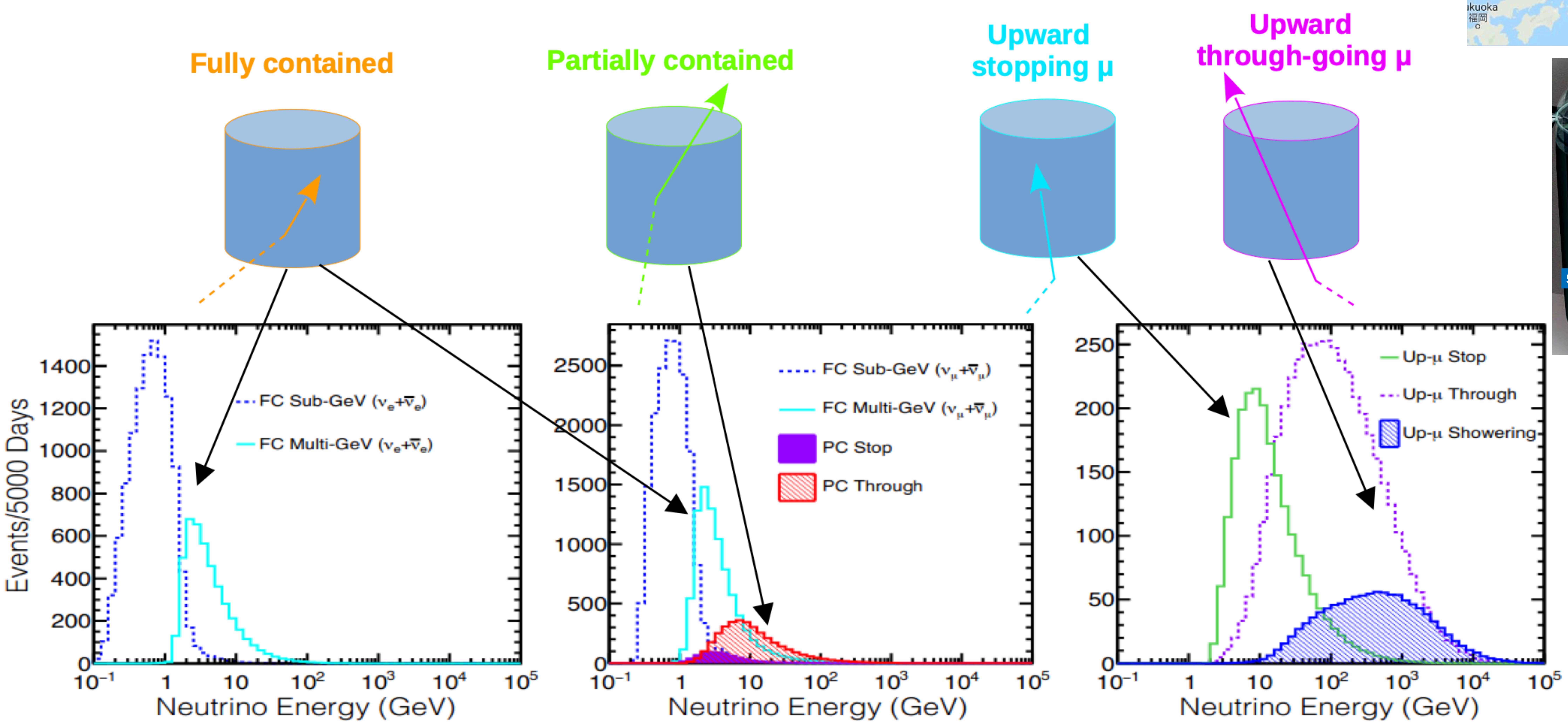
θ_{32}

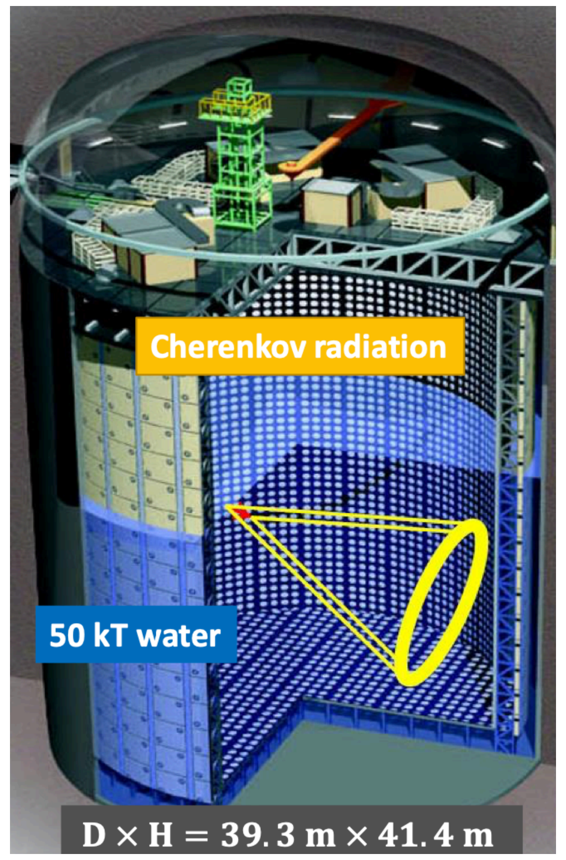
+ for antineutrinos

disappear

Atmospheric Neutrinos

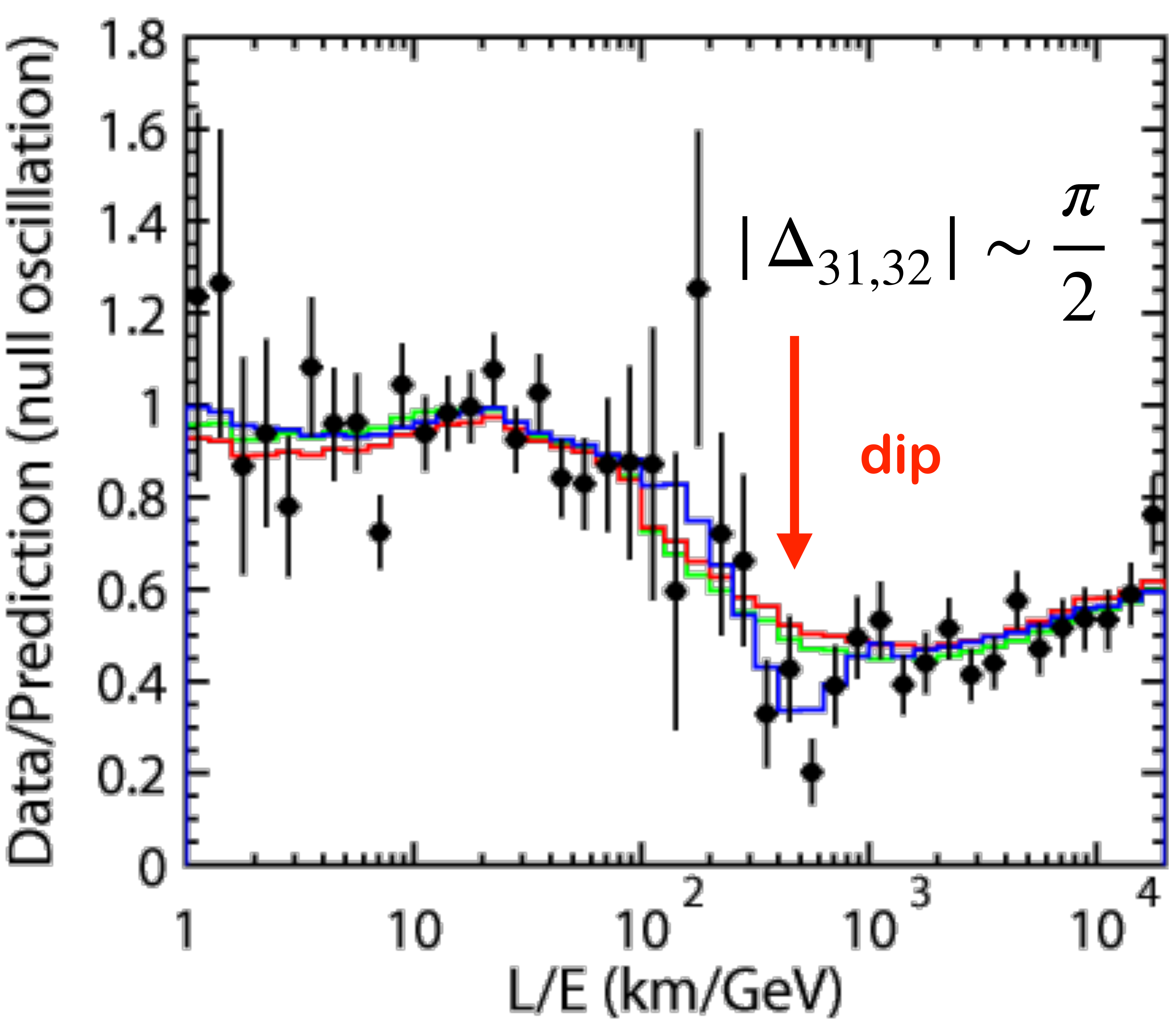
SuperKamikande Event Classification

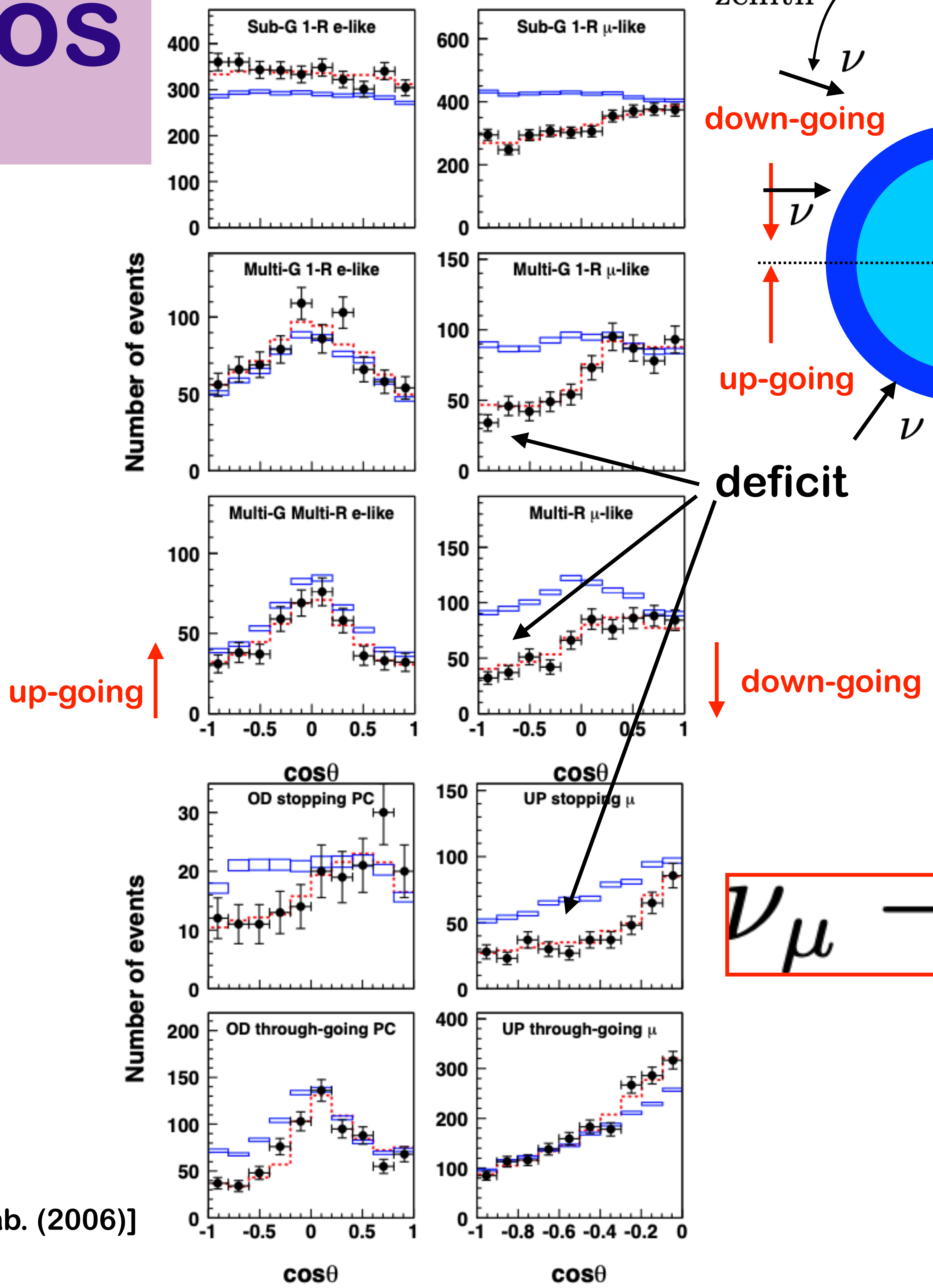


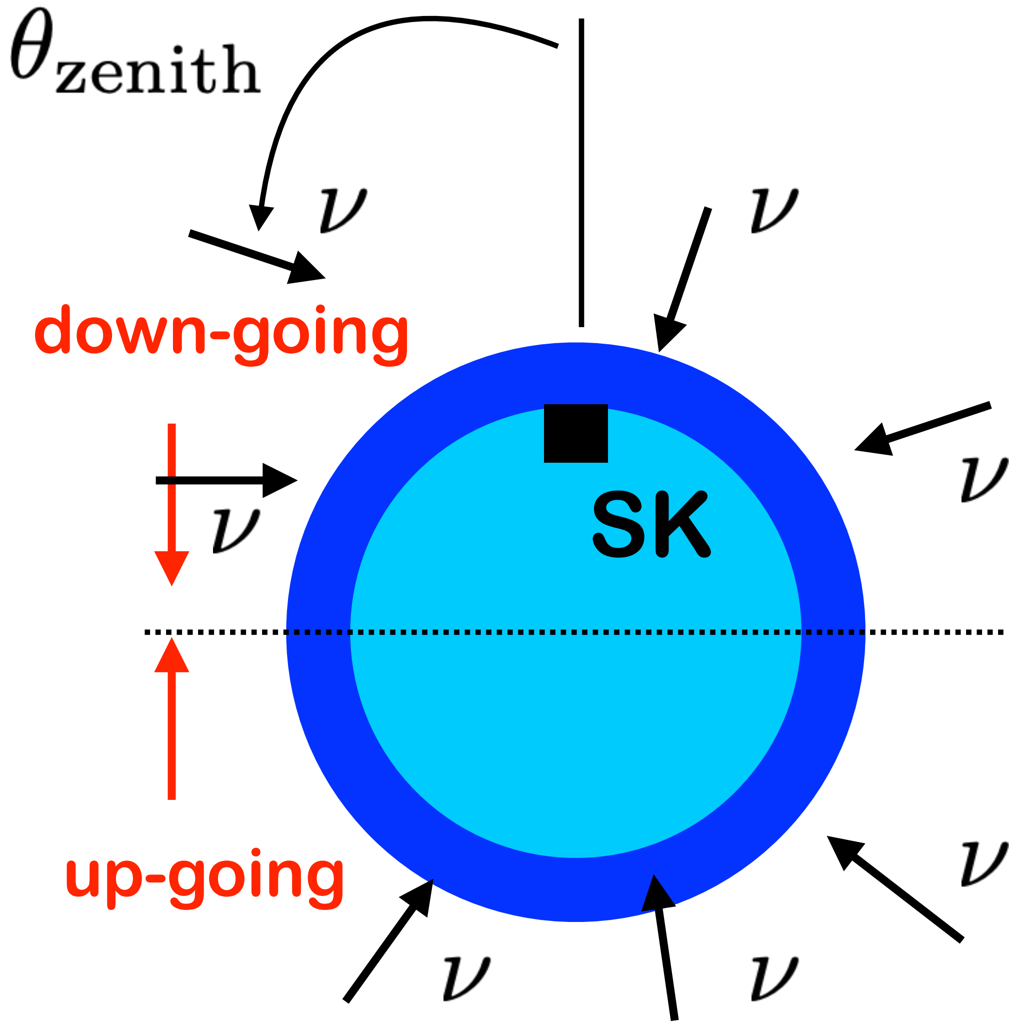


Atmospheric Neutrinos

SuperKamikande Observations





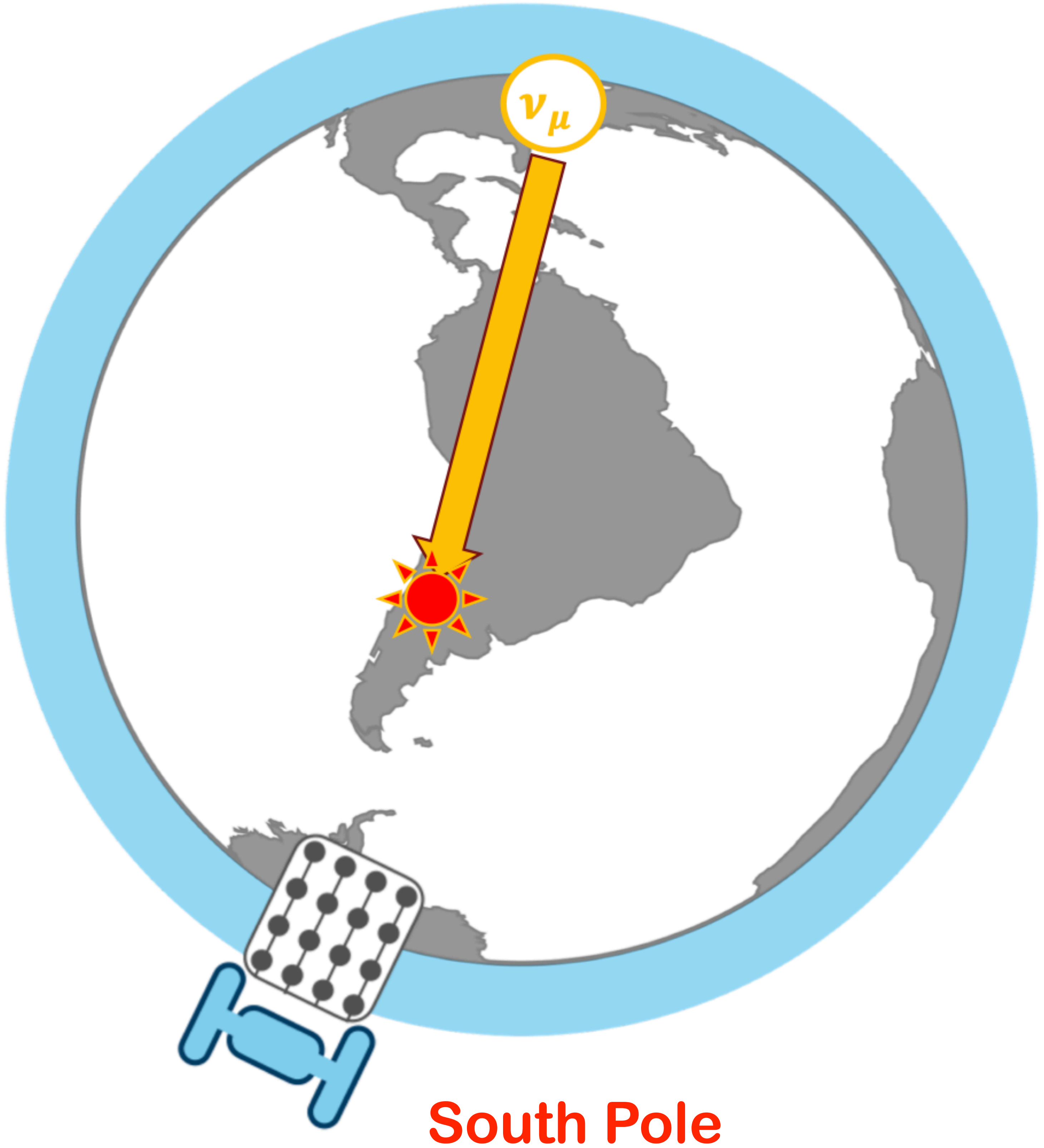


$$\nu_\mu \to \nu_\tau$$

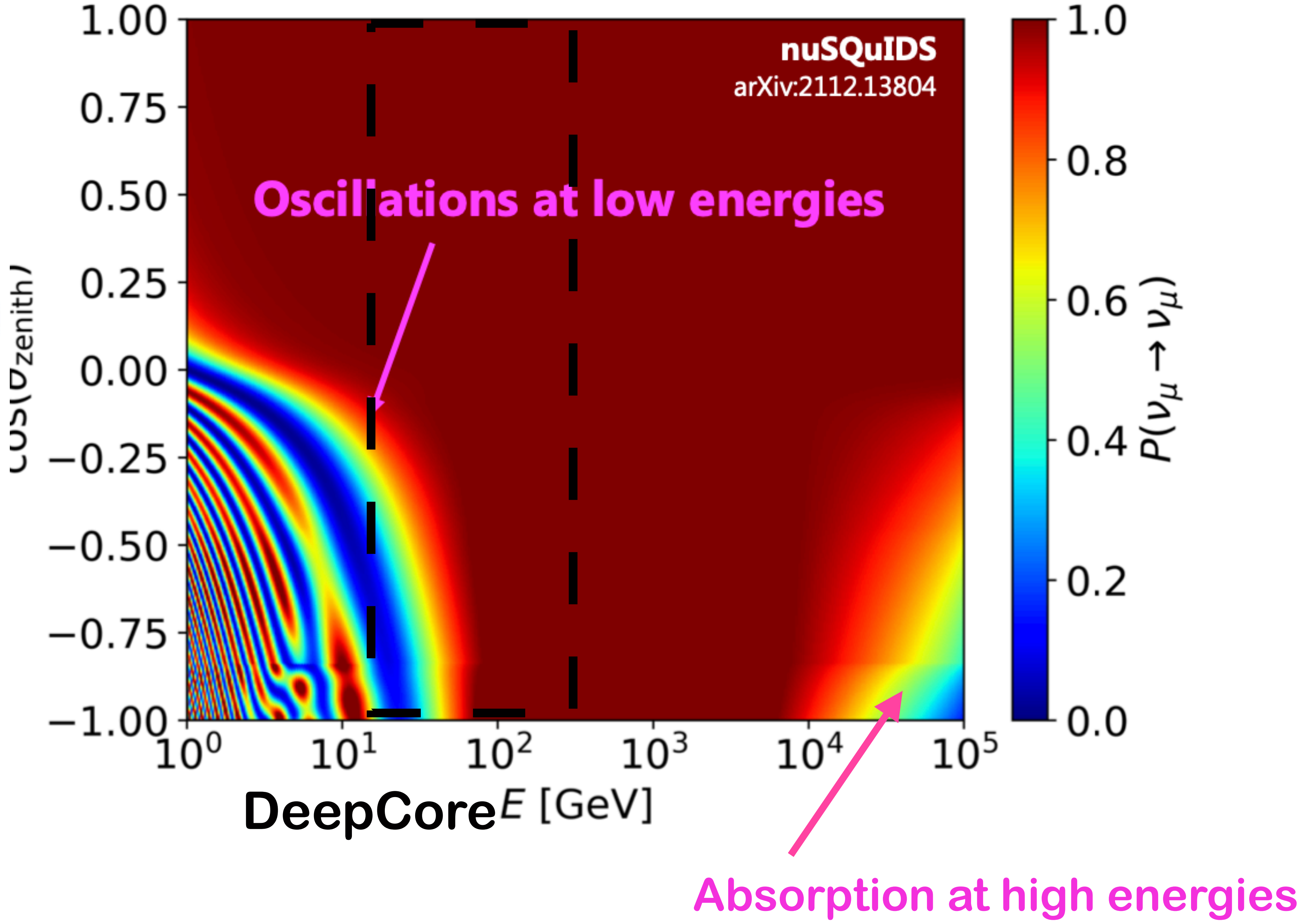
[Super-Kamiokande Collab. (2006)]

Atmospheric Neutrinos

IceCube/DeepCore Observations



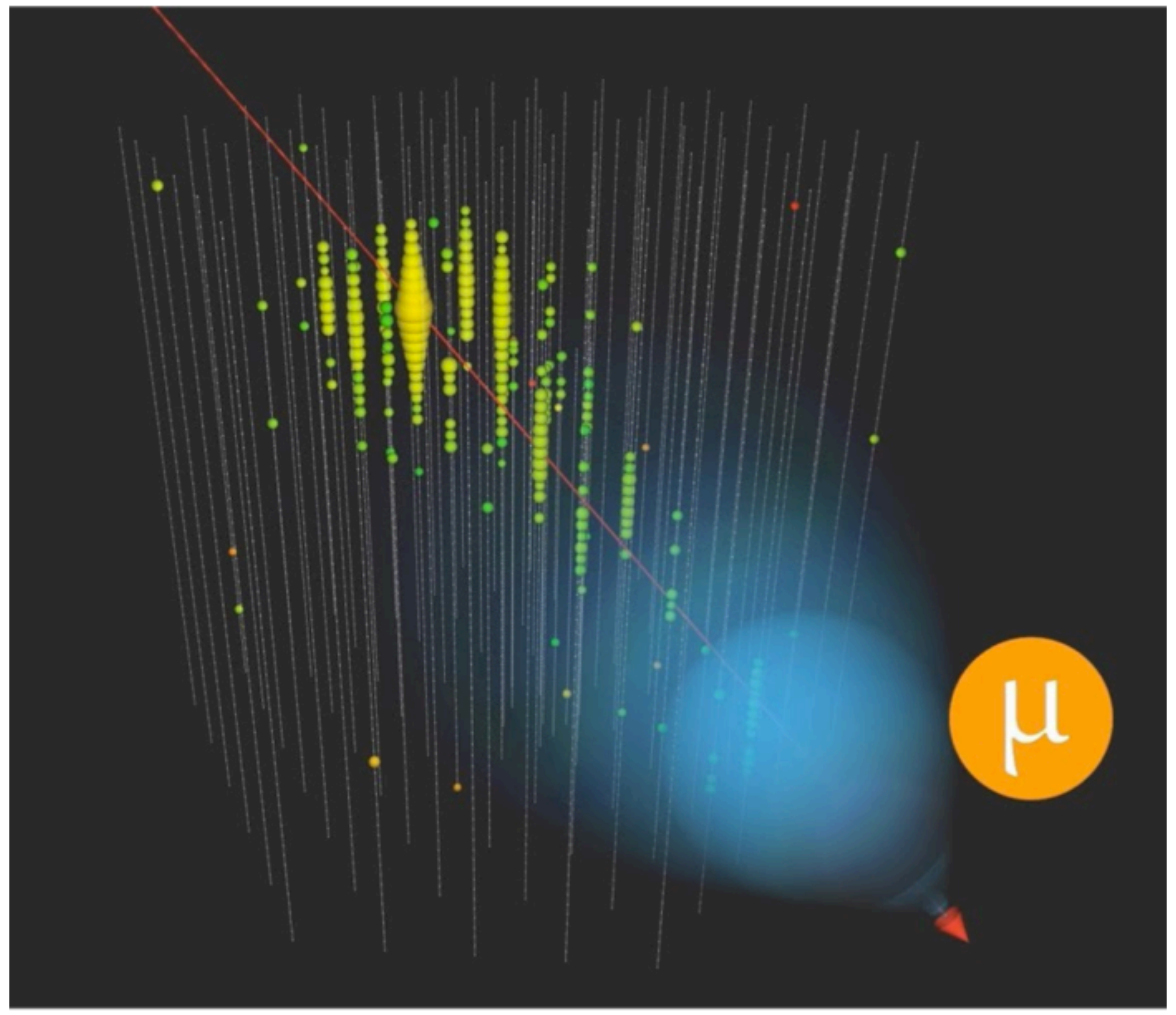
Neutrinos propagate across the Earth



Detection via Cherenkov light emission from secondary particles produced in $\nu - N$ interactions :

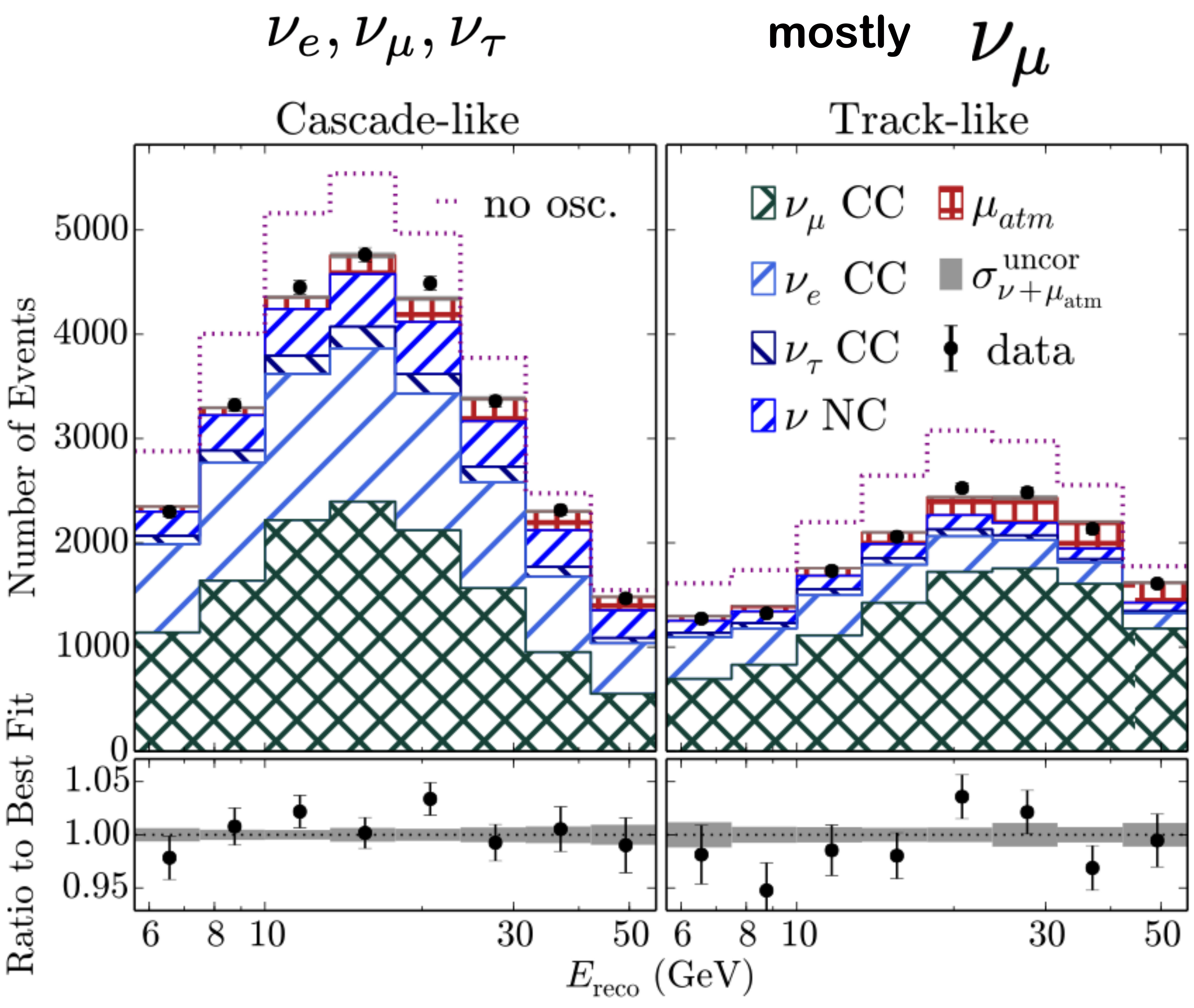
tracks : μ

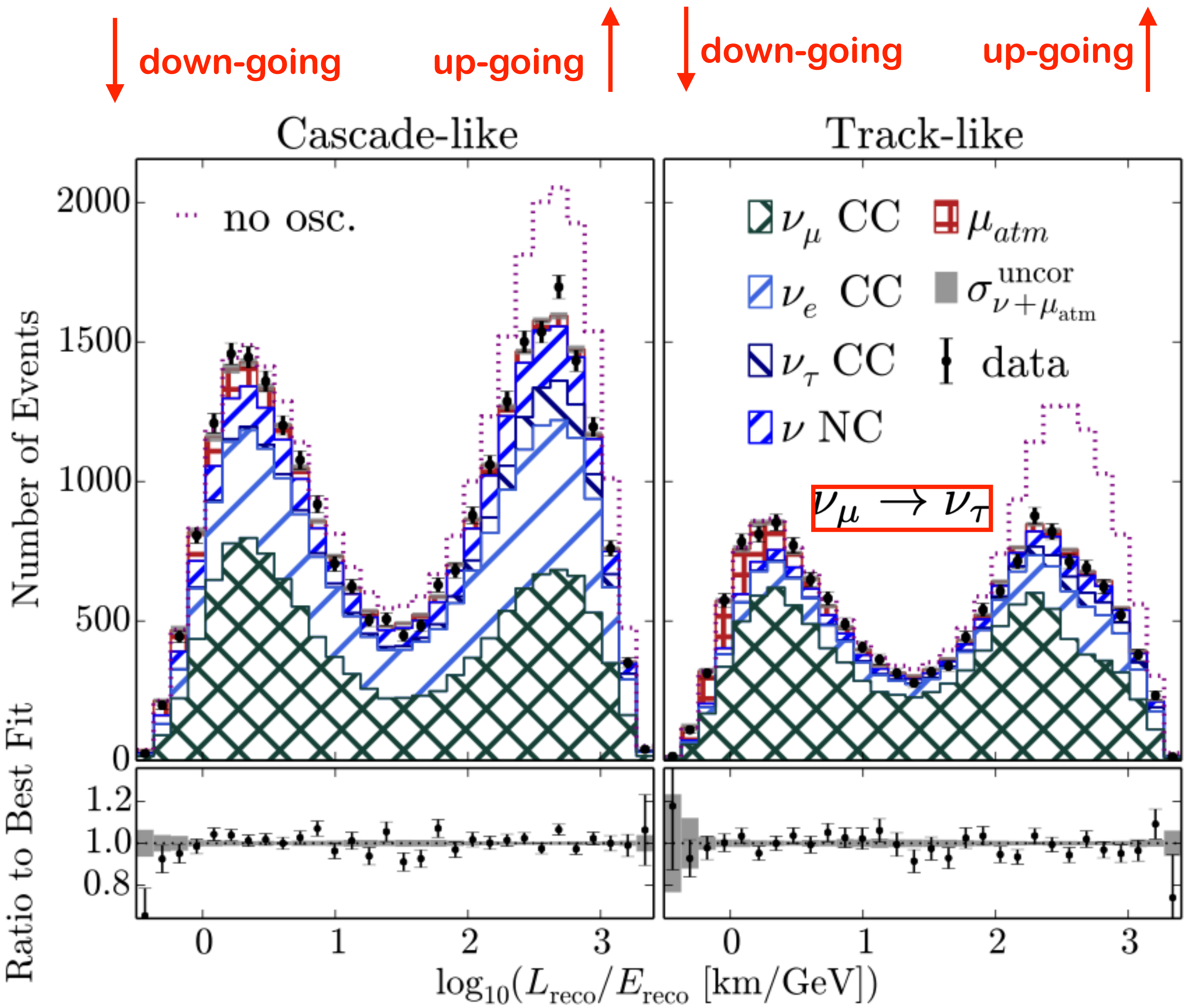
cascades: $e, \tau, \text{hadrons}$



Atmospheric Neutrinos

IceCube/DeepCore Observations





[IceCube/DeepCore Collab. (2017)]

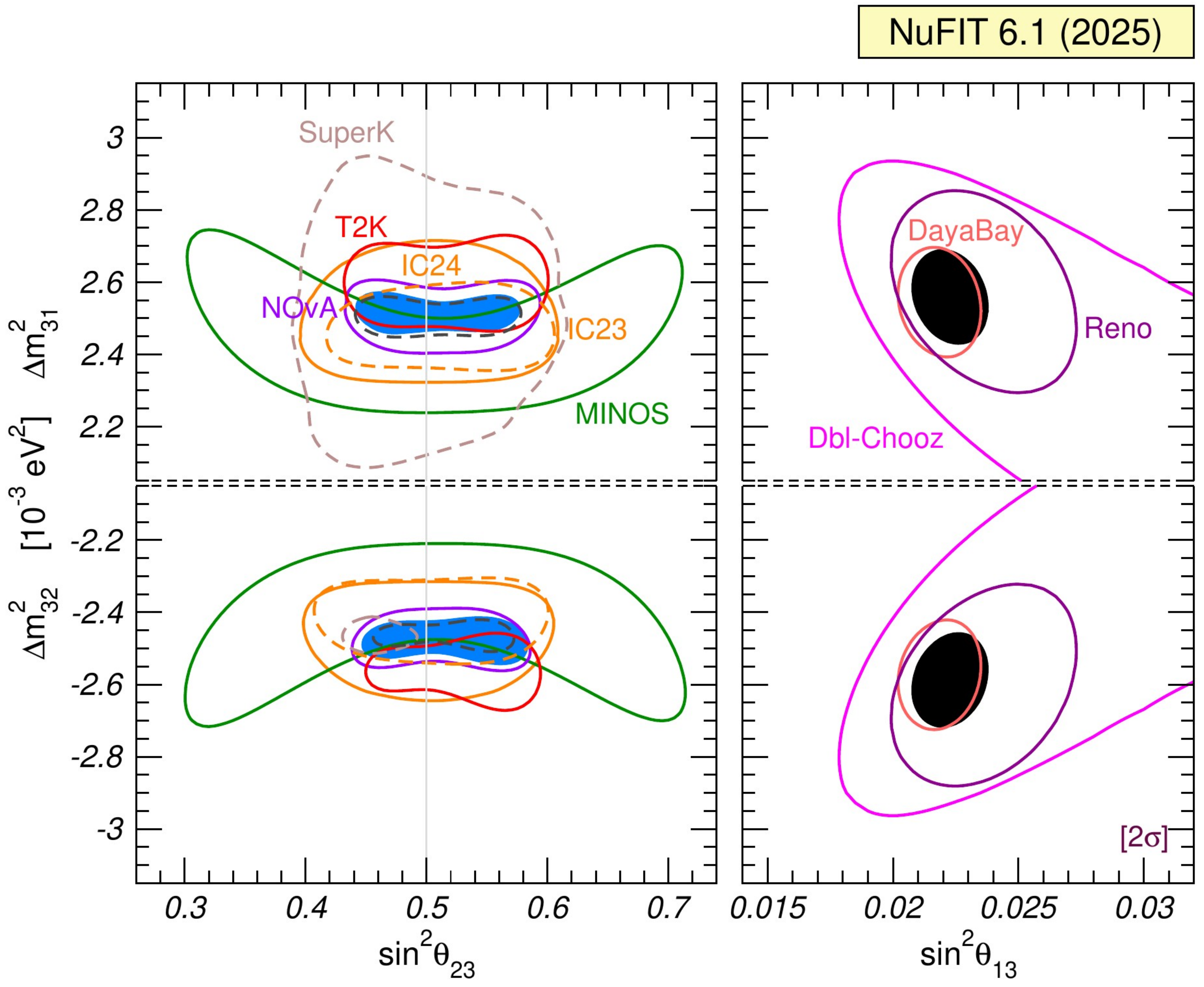
Atmospheric Neutrino Parameters

Current status: combined results for the 32/31-sectors

EXPERIMENT		Dominant Dependence	Sub-dominant Dependence	
Atmospheric		θ_{23}	$\Delta m^2_{\rm atm}, \theta_{13}, \delta$	
Accelerator disappearance		$\Delta m^2_{\rm atm}$ $\Delta m^2_{\mu\mu}$	θ_{23}	
Accelerator appearance	$\nu_\mu \to \nu_e$	θ_{13}	δ, θ_{23}	
Reactor (Daya Bay, RENO, DC)		θ_{13}	$\Delta m^2_{\rm atm}$	Δm^2_{ee}

normal ordering

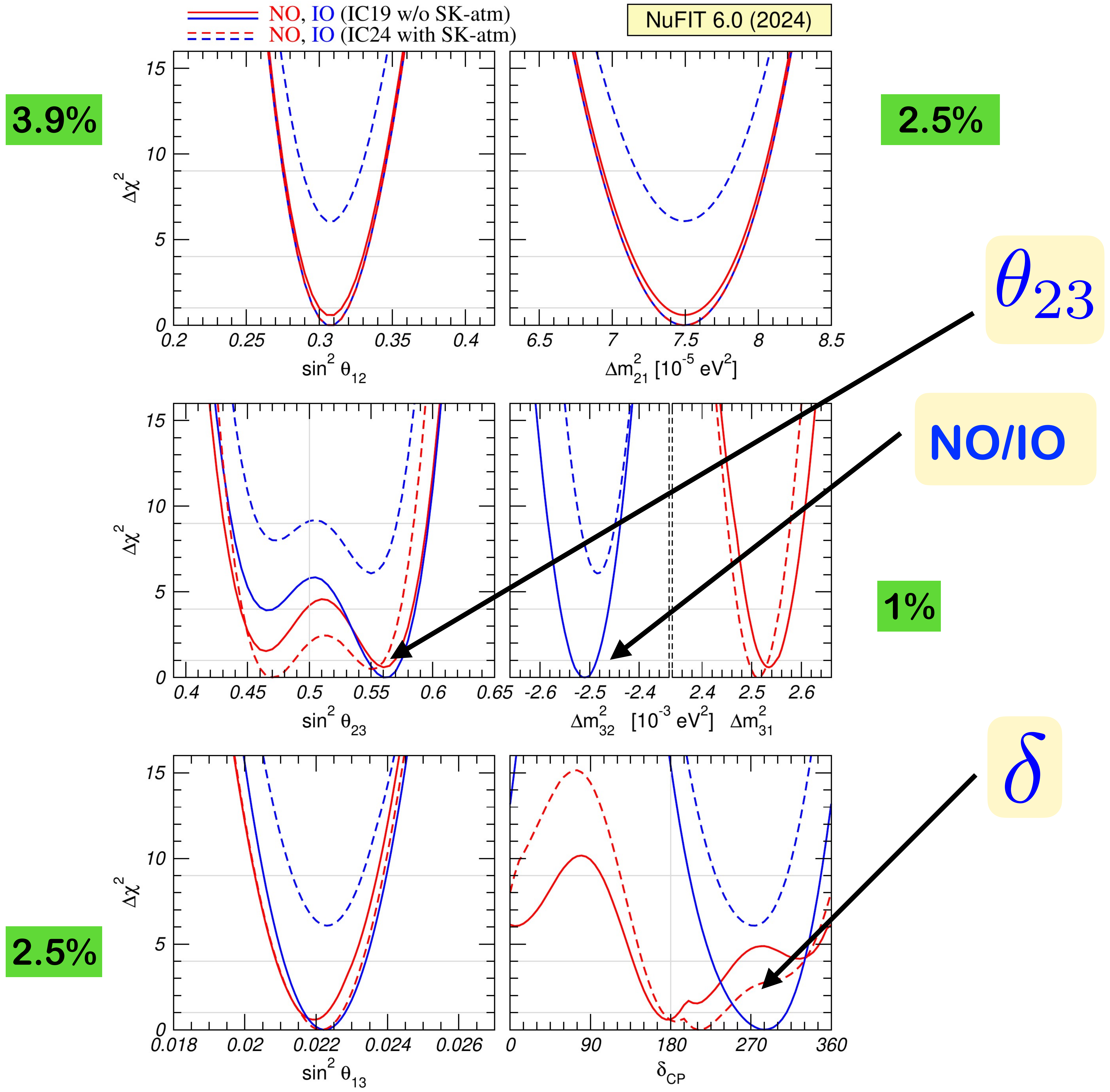
inverted ordering

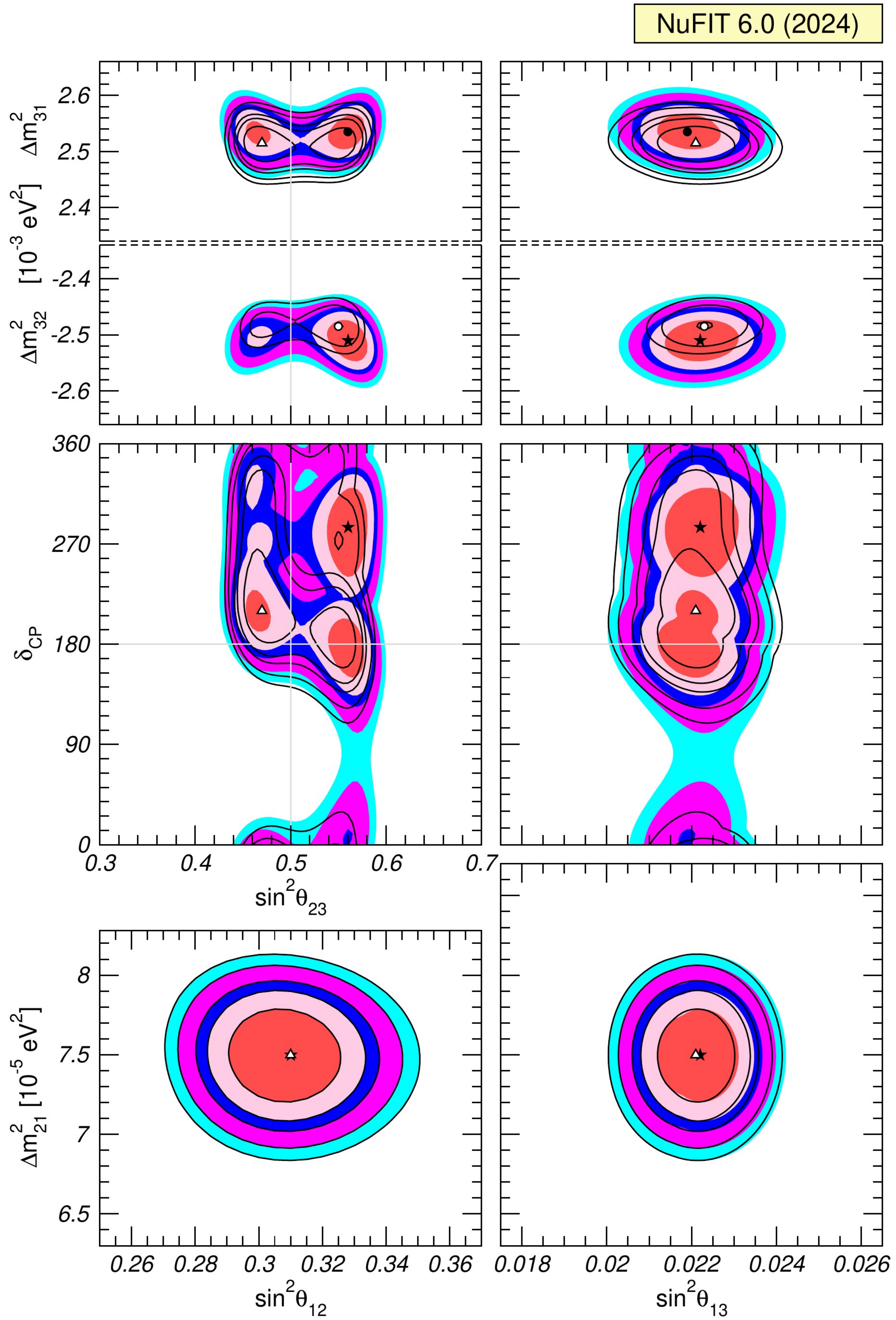


- 23/13-sector now best determined by accelerator experiments + reactors
- consistent $\Delta m^2_{\rm atm}$

$$\Delta m^2_{\rm atm} = \Delta m^2_{31} \quad \text{or} \quad \Delta m^2_{32}$$

Status of the Standard Paradigm

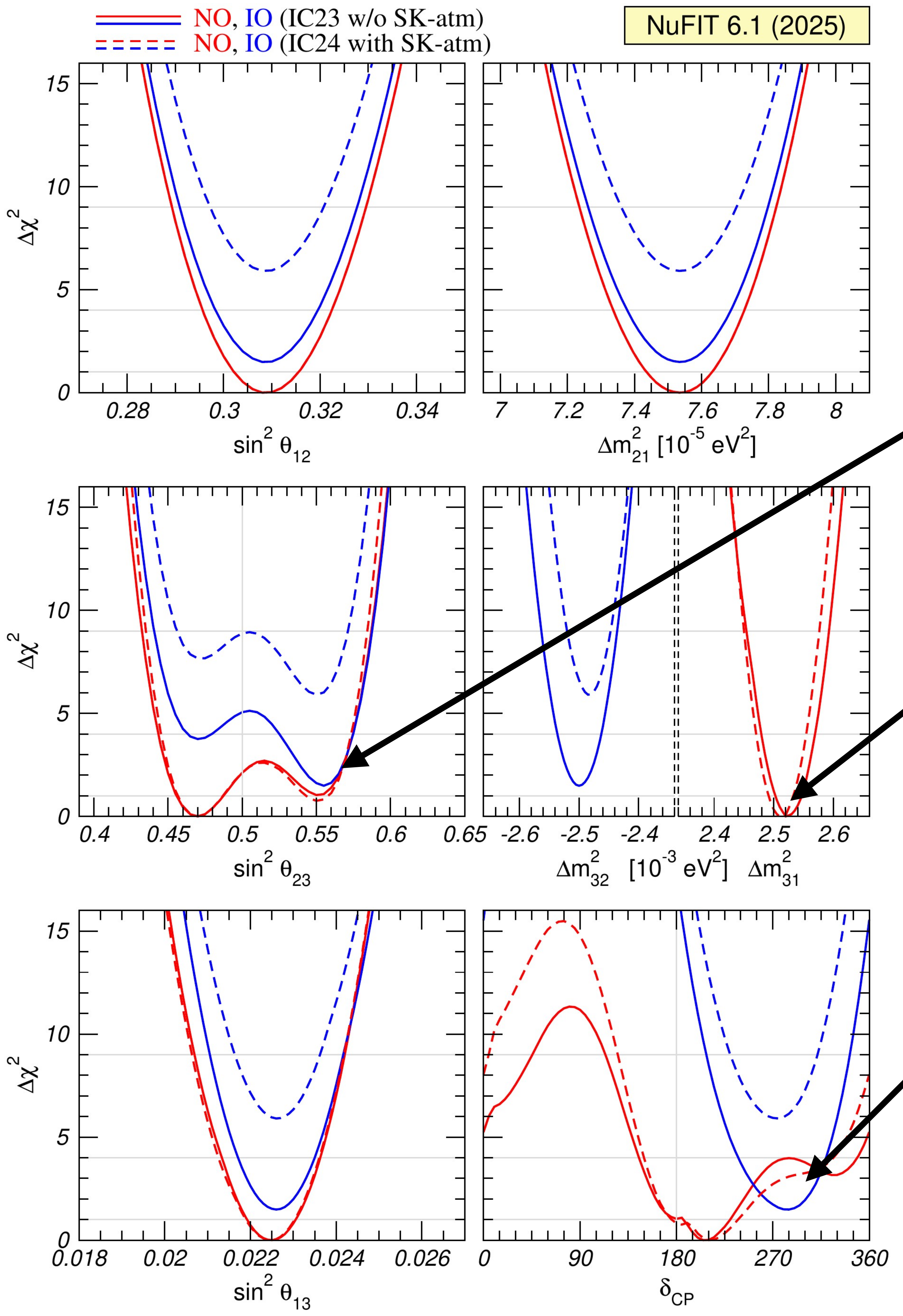




Status of the Standard Paradigm

2.2%

1.3%



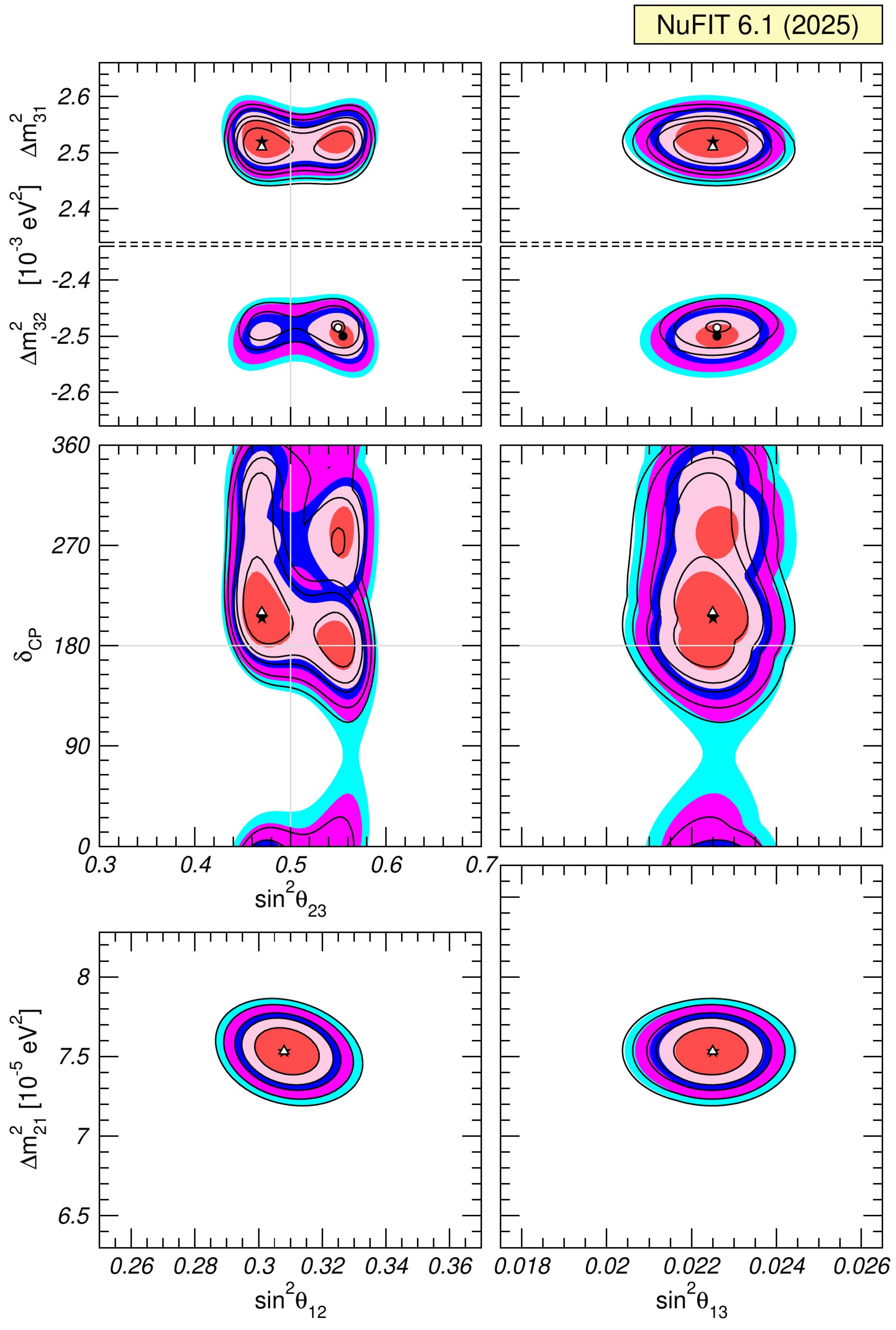
θ_{23}

NO/IO

1%

δ

2.5%

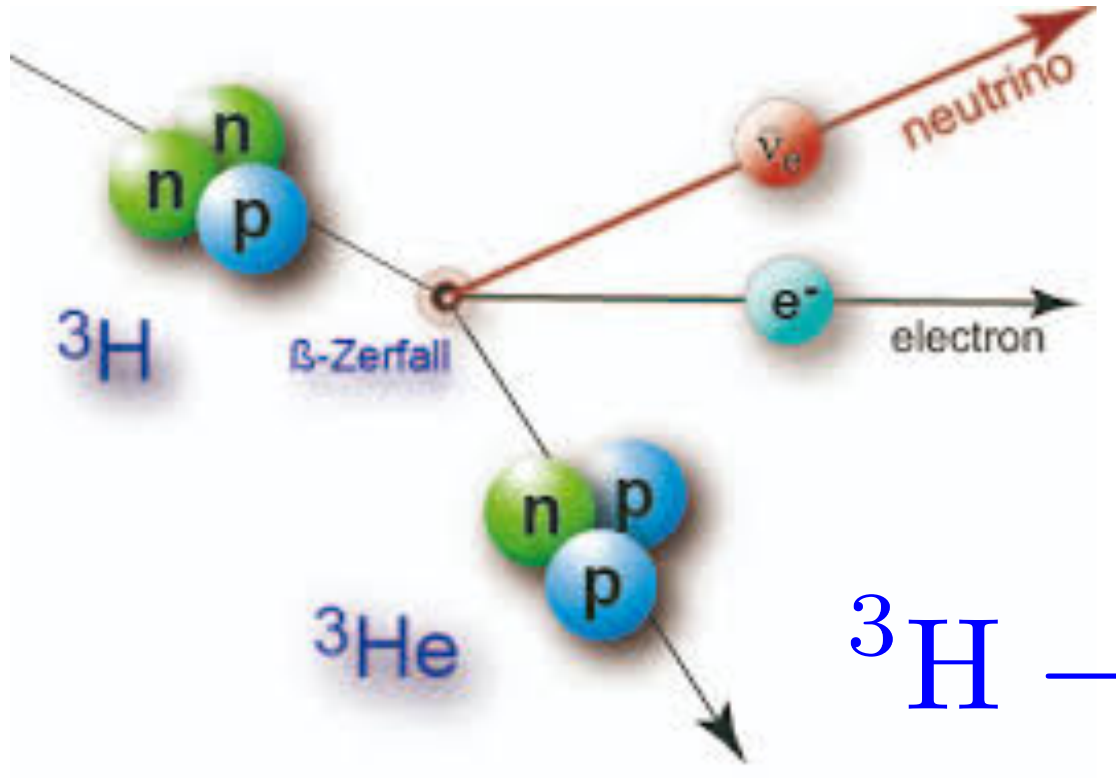


WHAT ELSE DO WE KNOW ABOUT NEUTRINO MASSES?

Effective Neutrino Mass

Absolute Neutrino Mass Scale

Tritium β-decay



$${}^{3}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}$$

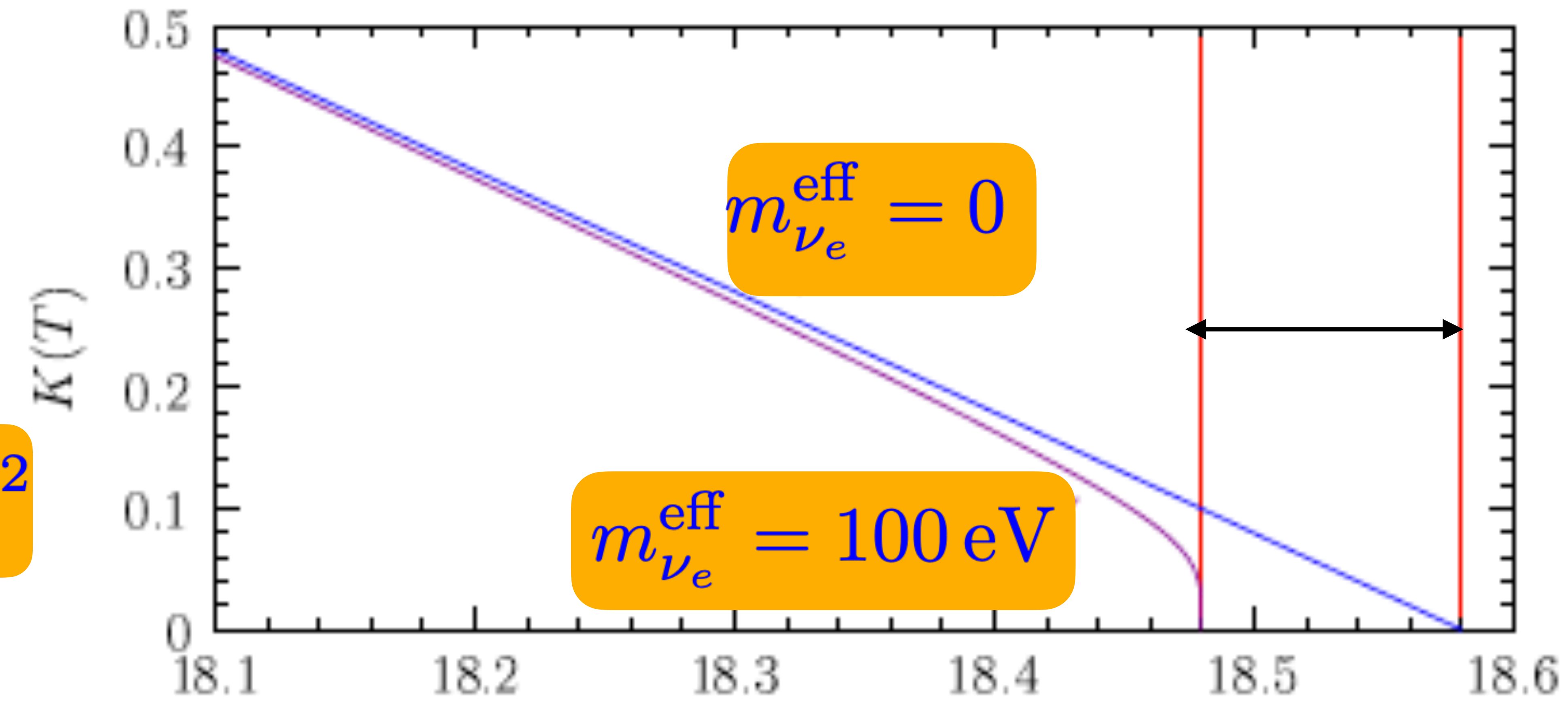
Valid for Dirac or Majorana neutrinos

$$m_{\nu_\alpha}^{\mathrm{eff}} = \sqrt{\sum_i m_i^2 |U_{\alpha i}|^2}$$

pure kinematics
model independent

$$\frac{d\Gamma}{dT} \propto |\mathcal{M}|^2 \, F(E)\, p\, E(Q-T)\sqrt{(Q-T)^2 - (m_{\nu_e}^{\mathrm{eff}})^2}$$

Kurie plot:



T $\quad Q - m_{\nu_e}^{\mathrm{eff}} \quad Q$

$$K(T) = \sqrt{(Q-T)\sqrt{(Q-T)^2 - (m_{\nu_e}^{\mathrm{eff}})^2}}$$

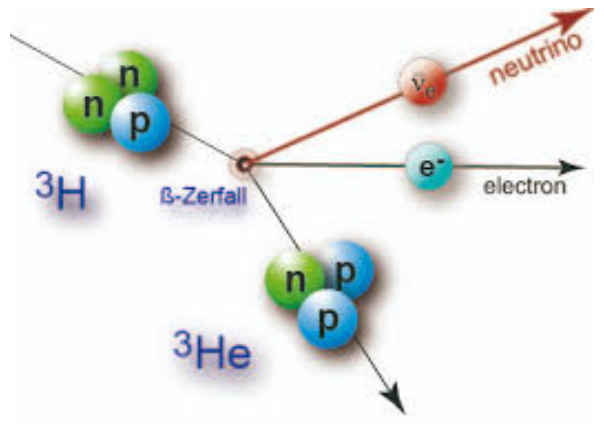
$$Q = M_{\mathrm{H}} - M_{\mathrm{He}} - m_e = 18.58\ \mathrm{keV}$$

[see Elise Novitski]

Effective Neutrino Mass

Absolute Neutrino Mass Scale

Tritium β-decay



Valid for Dirac or Majorana neutrinos

$$m_{\nu_\alpha}^{\text{eff}} = \sqrt{\sum_i m_i^2 |U_{\alpha i}|^2}$$

pure kinematics
model independent

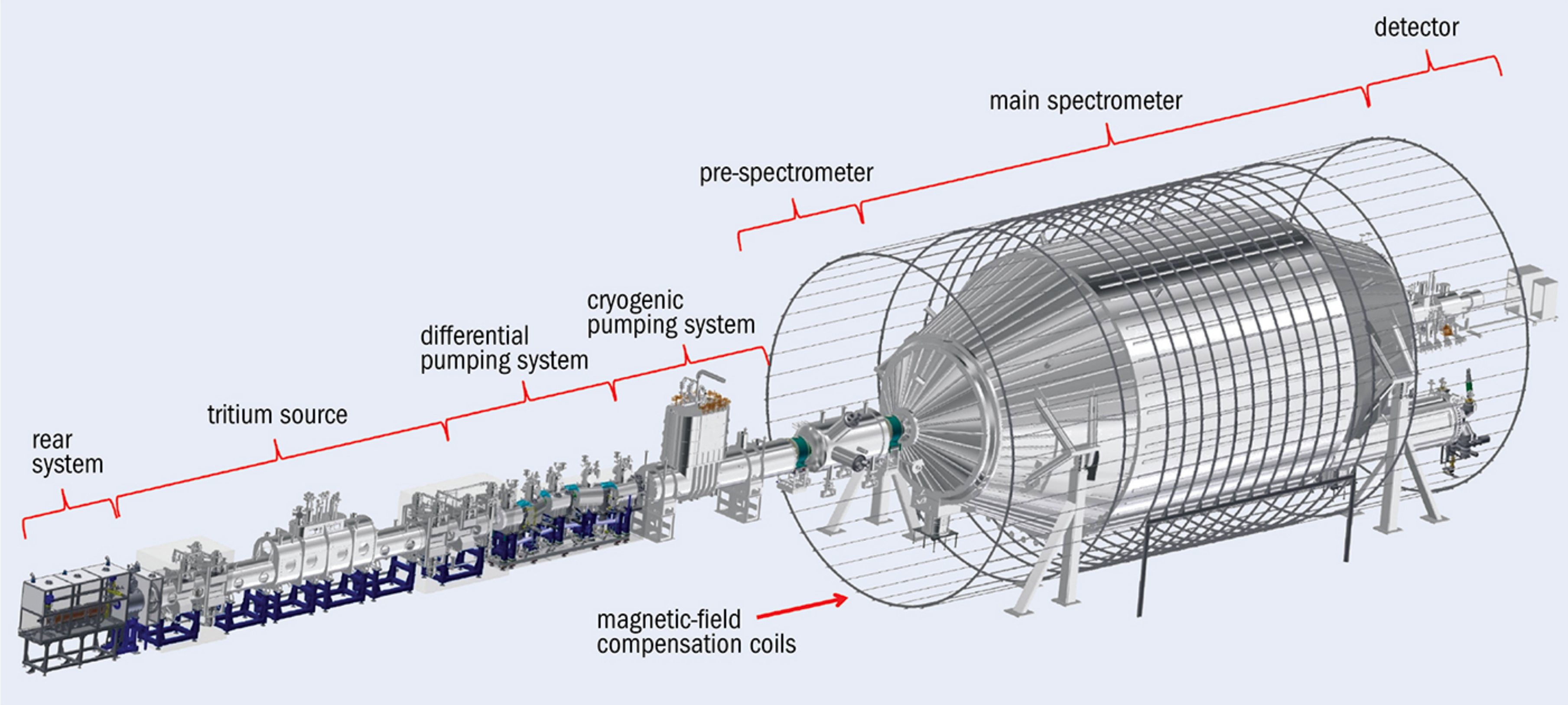
neutrino mass scale

$$(m_{\nu_e}^{\text{eff}})^2 = \begin{cases} m_0^2 + \Delta m_{21}^2 c_{13}^2 s_{12}^2 + \Delta m_{31}^2 s_{13}^2 & \text{normal ordering} \\ m_0^2 + \Delta m_{21}^2 c_{13}^2 s_{12}^2 - \Delta m_{31}^2 c_{13}^2 & \text{inverted ordering} \end{cases}$$

current bound

$m_{\nu_e}^{\text{eff}} < 0.45 \,@\, 90\,\%\,\text{CL}$

[KATRIN Collab., Science 388 (2005) 6743]



$m_{\nu_e}^{\text{eff}} \sim (0.2 - 0.3)\ \text{eV}$

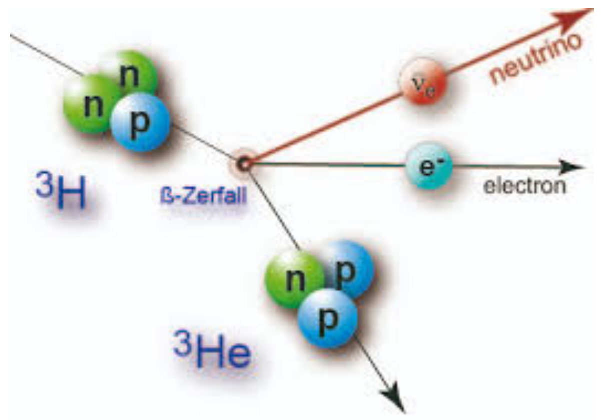
KATRIN future sensitivity

[see Elise Novitski]

Effective Neutrino Mass

Absolute Neutrino Mass Scale

Tritium β-decay



Valid for Dirac or Majorana neutrinos

pure kinematics
model independent

$$m^{\text{eff}}_{\nu_\alpha} = \sqrt{\sum_i m_i^2 \, |U^{\nu 2}_{\alpha i}|^2}$$

neutrino mass scale

$$(m^{\text{eff}}_{\nu_e})^2 = \begin{cases} m_0^2 + \Delta m^2_{21} c^2_{13} s^2_{12} + \Delta m^2_{31} s^2_{13} \\ m_0^2 + \Delta m^2_{21} c^2_{13} s^2_{12} - \Delta m^2_{31} c^2_{13} \end{cases}$$

normal ordering $m^{\text{eff}}_{\nu_e} > 0.0085$ eV (NO)

inverted ordering $m^{\text{eff}}_{\nu_e} > 0.048$ eV (IO)

current bound

$m^{\text{eff}}_{\nu_e} < 0.45$ eV @ 90 % CL

[KATRIN Collab., Science 388 (2005) 6743]

other limits

$m^{\text{eff}}_{\nu_\mu} < 190$ keV

$\pi^- \to \mu^- + \bar{\nu}_\mu$

$m^{\text{eff}}_{\nu_\tau} < 18.2$ MeV

$\tau \to n\pi + \nu_\tau \qquad n \geq 3$

Effective Majorana Neutrino Mass

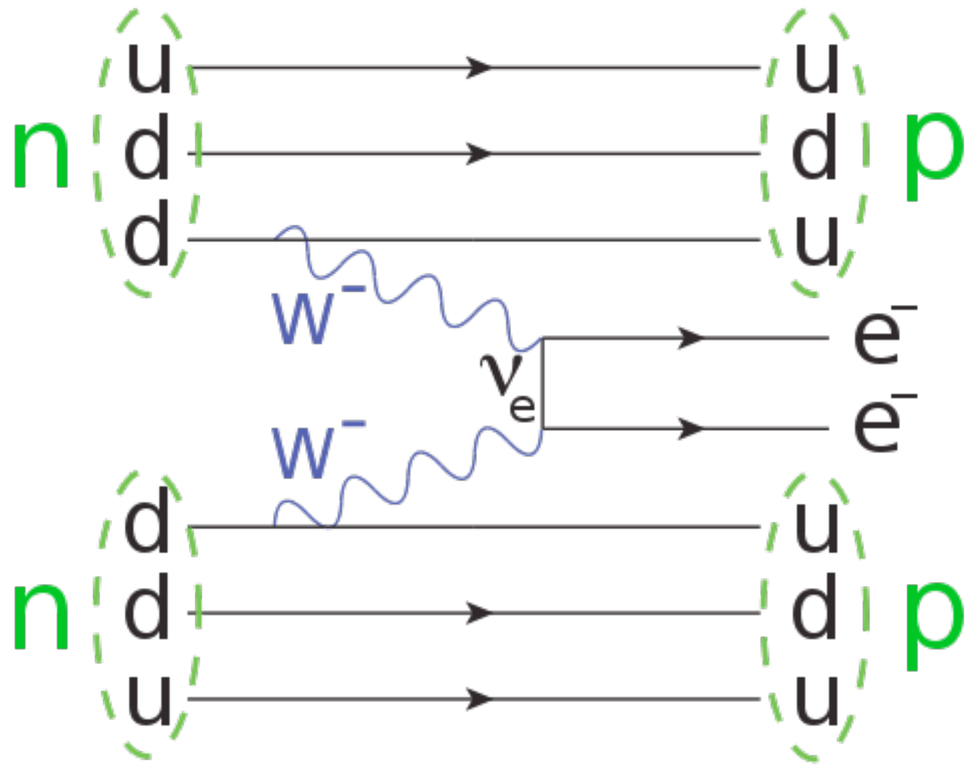
$0\nu\beta\beta$ - decay

Majorana Neutrino | Dirac Neutrino

$$\nu = \bar{\nu} \quad \times \quad \nu \neq \bar{\nu}$$

Neutrinoless Double Beta Decay

$$(A, Z) \rightarrow (A, Z+2) + 2e^-$$



$$|\Delta L| = 2$$

[see Benjamin Jones]

Effective Majorana Neutrino Mass

$0\nu\beta\beta$ - decay

Only occurs for Majorana neutrinos

phase space integral

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\,|M_{0\nu}|^2\,m^2_{\beta\beta}$$

if neutrino mass is the only source of $\not{L}$

half-life of the decay

nuclear matrix element

$$m_{\beta\beta} = \sum_i m_i\,U^2_{ei}$$

Normal Ordering

$$m_1 \equiv m_0$$

$$m_2 \equiv \sqrt{m_0^2 + \Delta m^2_{21}}$$

$$m_3 \equiv \sqrt{m_0^2 + \Delta m^2_{21} + \Delta m^2_{32}}$$

Inverted Ordering

$$m_1 \equiv \sqrt{m_0^2 - \Delta m^2_{21} - \Delta m^2_{32}}$$

$$m_2 \equiv \sqrt{m_0^2 - \Delta m^2_{32}}$$

$$m_3 \equiv m_0$$

$$m_{\beta\beta} = \left| m_1 c^2_{12} c^2_{13} e^{2i\alpha_1} + m_2 s^2_{12} c^2_{13} + m_3 s^2_{13} e^{2i\alpha_3} \right|$$

Effective Majorana Neutrino Mass

$0\nu\beta\beta$ - decay

Only occurs for Majorana neutrinos

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\,|M_{0\nu}|^2\, m^2_{\beta\beta}$$

if neutrino mass is the only source of $\not{L}$

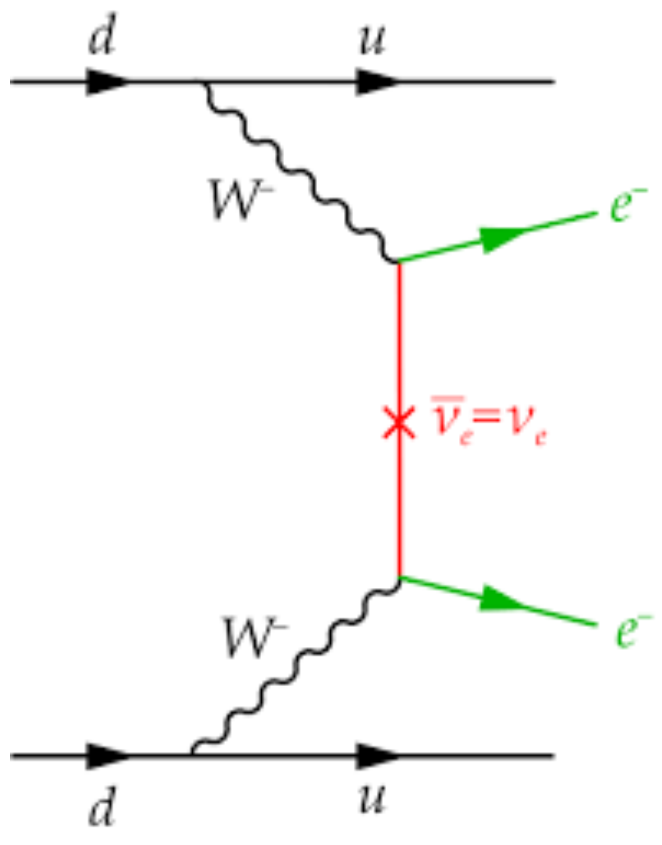
$$m_{\beta\beta} = \sum_i m_i\, U^2_{ei}$$

NO/IO

$$m_{\beta\beta} = f(m_0, \text{order}, \text{Majorana phases})$$

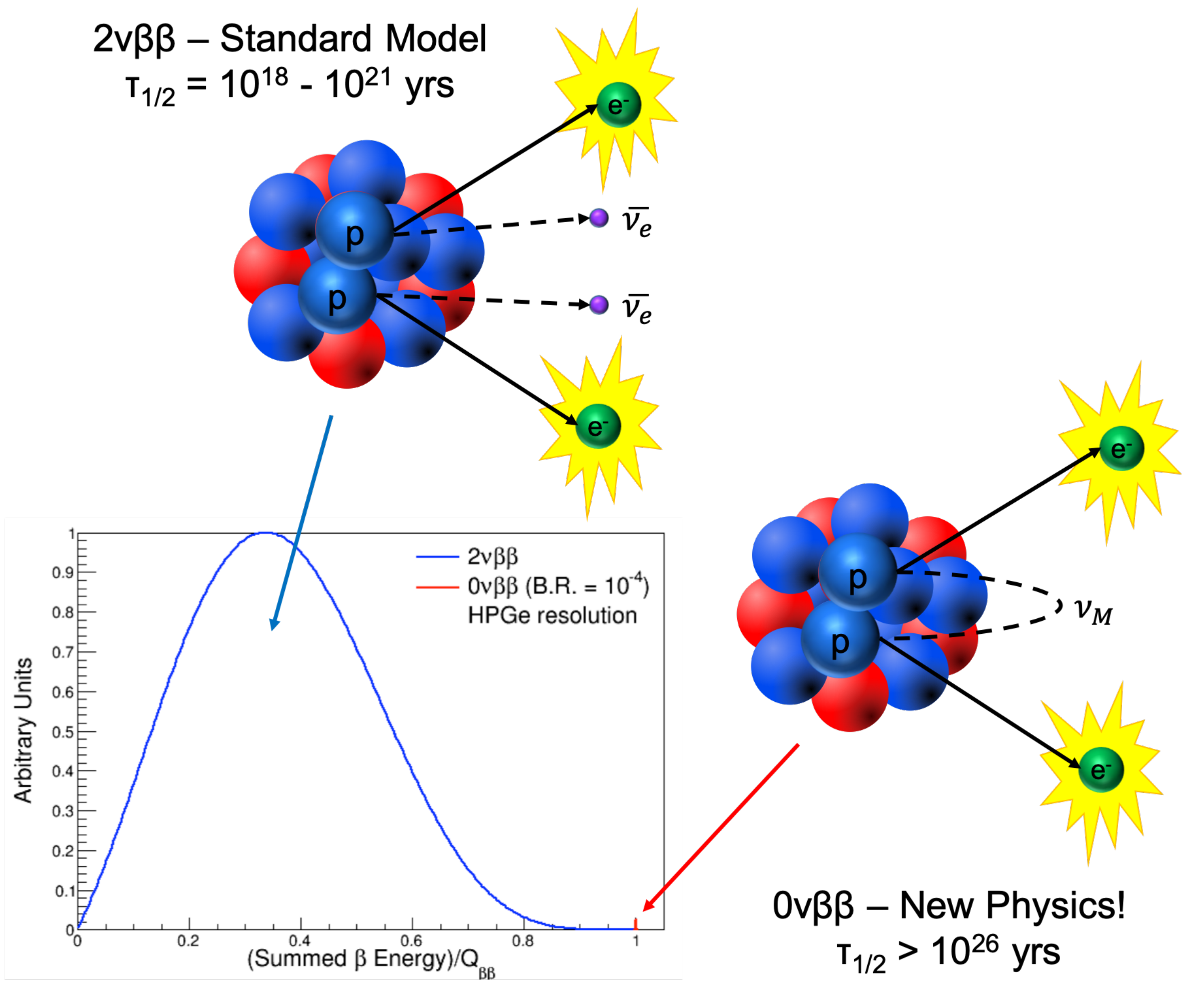
α_1, α_3

neutrino mass scale



$\propto Q^5$

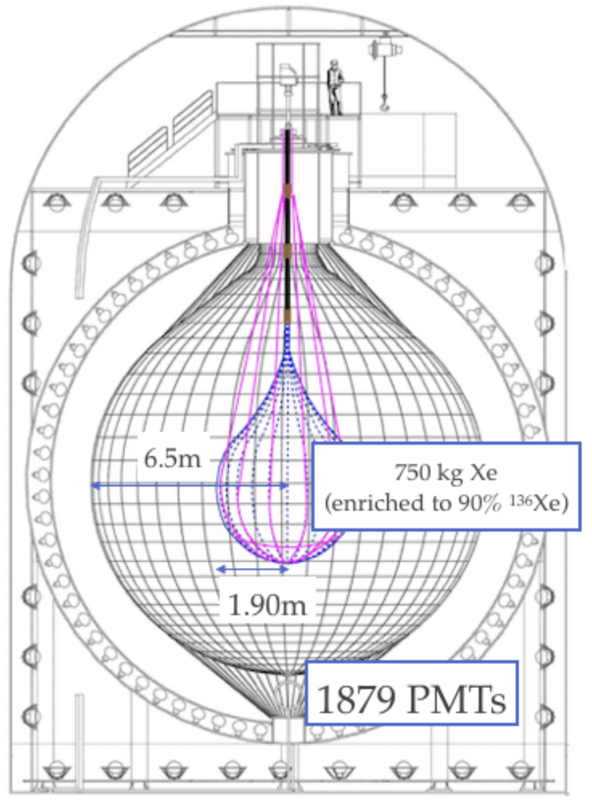
Isotope	Natural abundance (%)	$Q_{\beta\beta}$ (MeV)
^{48}Ca	0.187	4.263
^{76}Ge	7.8	2.039
^{82}Se	8.7	2.998
^{96}Zr	2.8	3.348
^{100}Mo	9.8	3.035
^{116}Cd	7.5	2.813
^{130}Te	34.08	2.527
^{136}Xe	8.9	2.459
^{150}Nd	5.6	3.371



current bound

$$m_{\beta\beta} < (28 - 122)\,\text{meV}$$

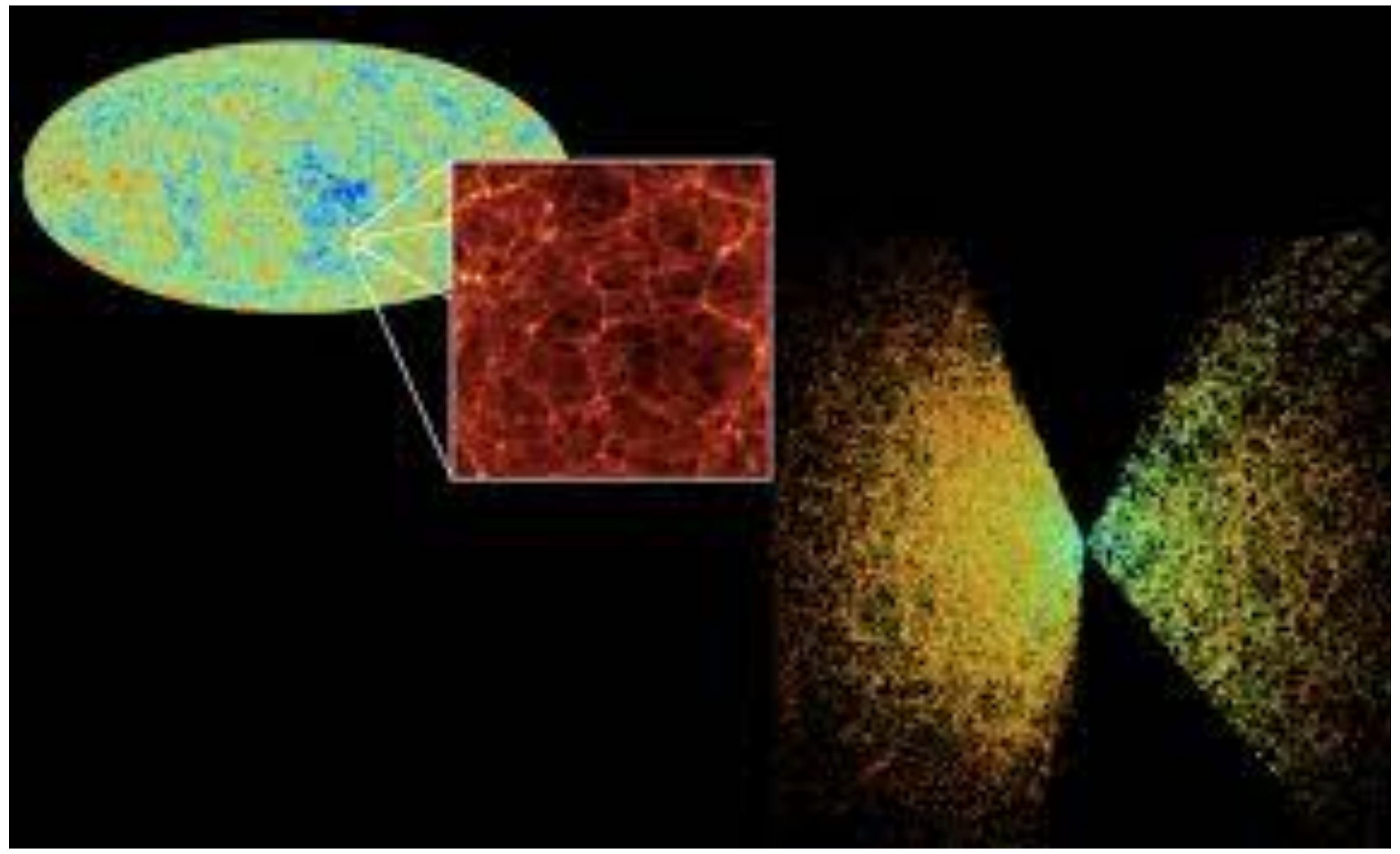
[KamLAND-Zen Collab., arXiv:2406.11438]



$$T^{0\nu}_{1/2} > 3.8 \times 10^{26}\,\text{yr}$$

Sum of Neutrino Masses

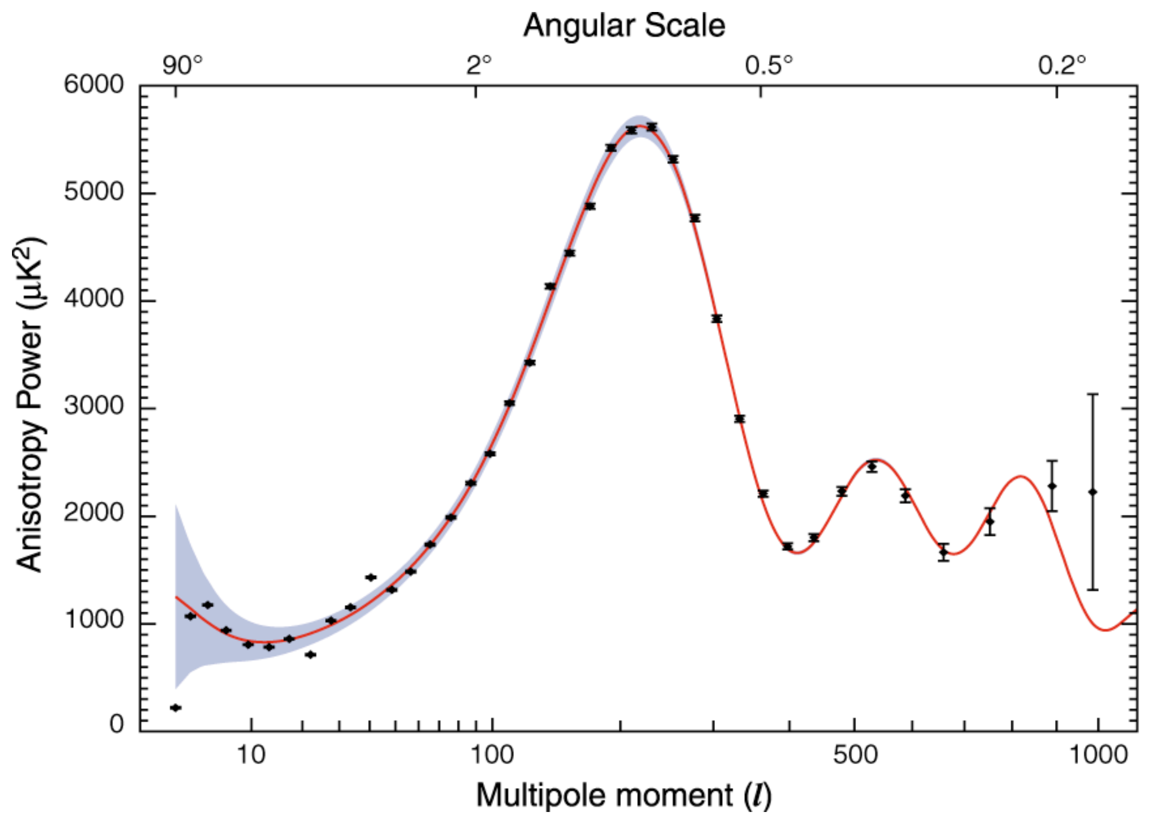
Absolute Neutrino Mass Scale



COSMOLOGY

Valid for Dirac or Majorana neutrinos

$$\sum_i m_i$$



neutrino masses affect CMB and the growth of large scale structures

$$\sum_i m_i = f(m_0, \mathrm{order}, \Delta \mathrm{m}^2_{21}, \Delta \mathrm{m}^2_{\mathrm{atm}})$$

$$\sum_i m_i < 0.26 \text{ eV}$$

current bound

[Planck Collab., Astron.Astrophys. 641 (2020)]

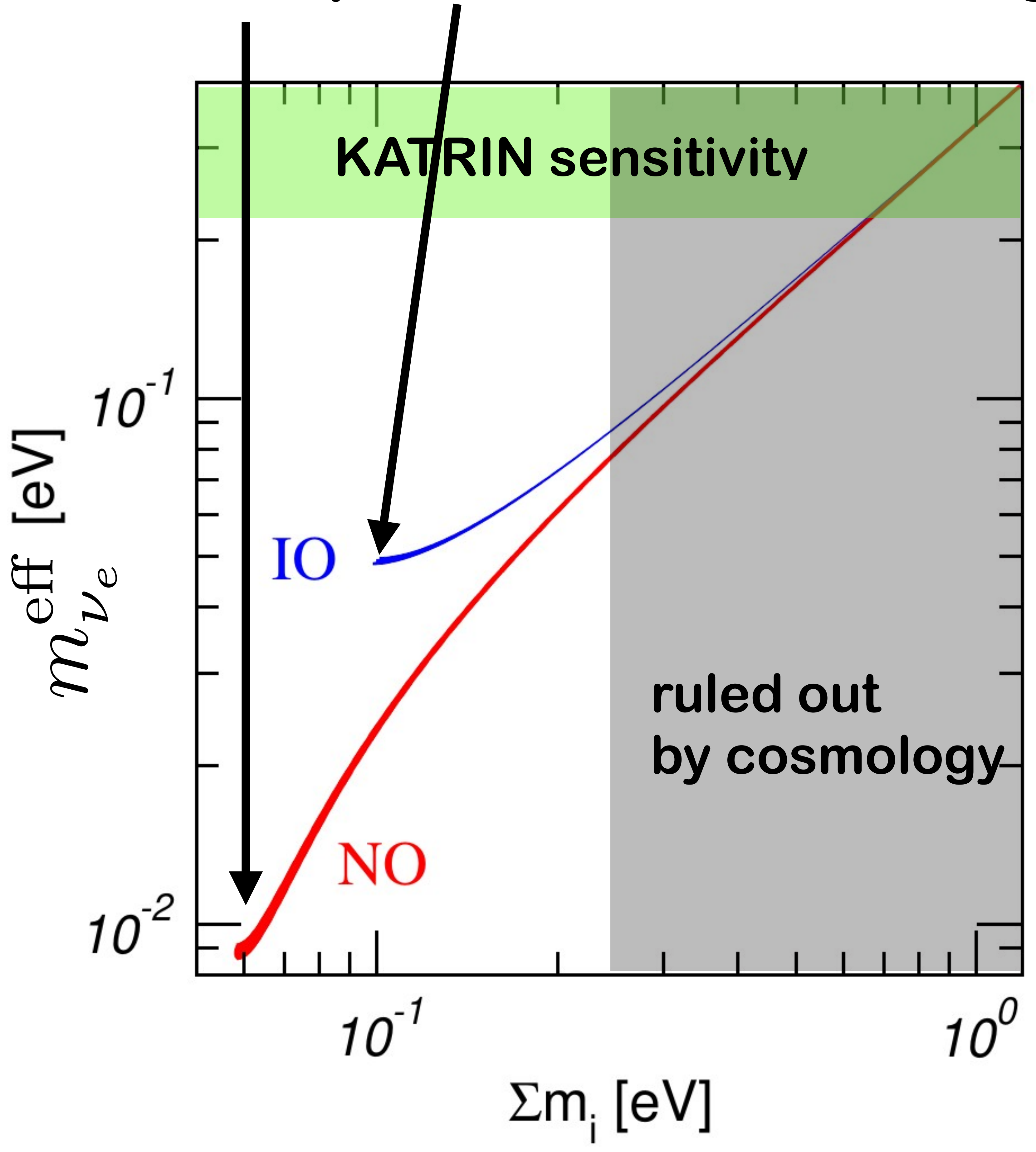
[see Yvonne Wong]

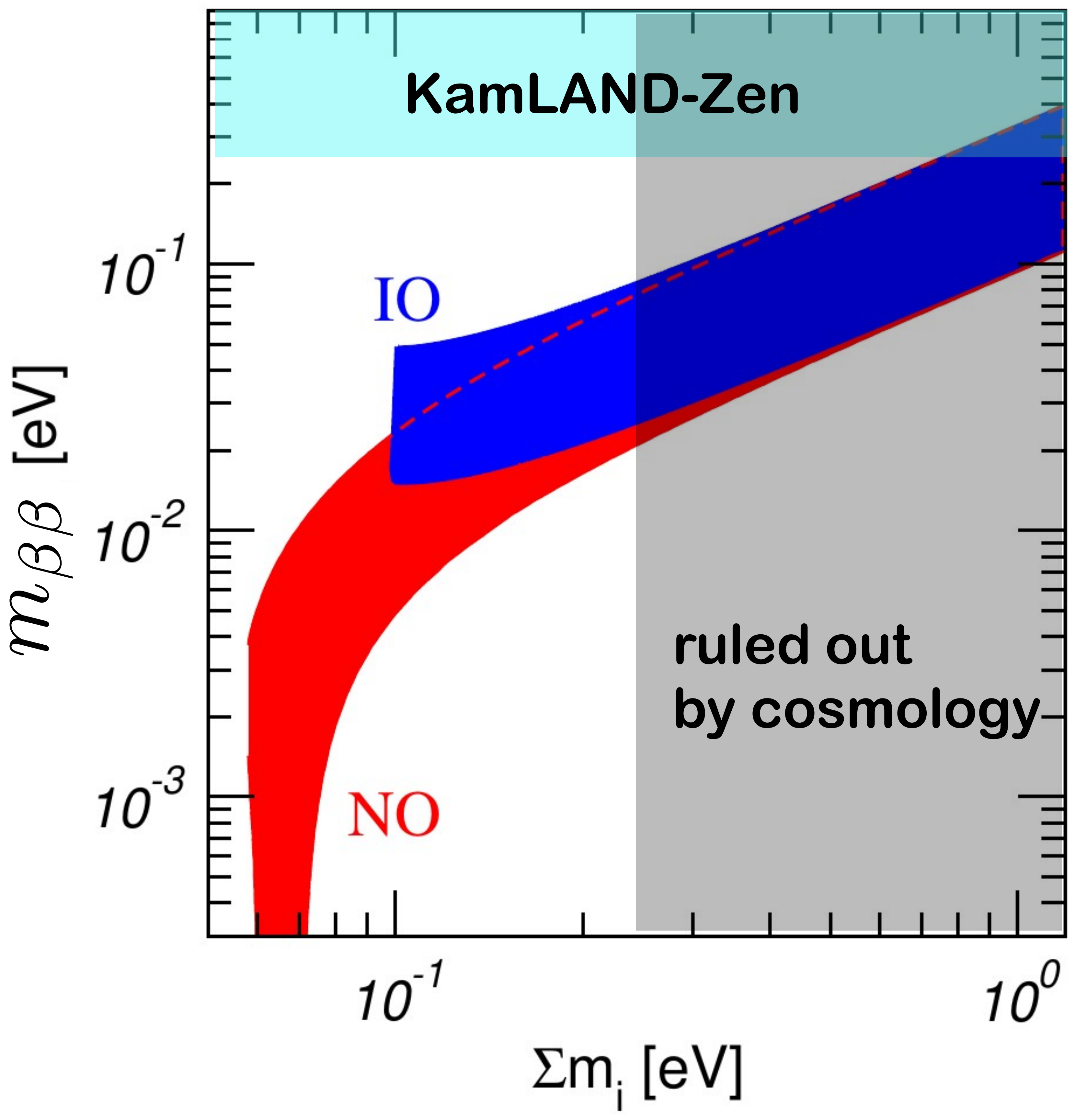
Correlation Among Measurements

Absolute Neutrino Mass Scale

lower bound depends on ordering

NuFIT 4.0 (2018)





SUMMARY OF LECTURE II

What have we learned?

- We have a theoretical framework for neutrino flavor oscillations based on masses & mixings in vacuum and in matter
- This framework explains well the available oscillation data
- We still do not the neutrino mass ordering
- We still do not know if we have CP violation in the leptonic sector
- We still do not know the octant of θ_{23}
- We still do not know the absolute neutrino mass scale
- We still do not know if neutrinos are Dirac or Majorana particles