Theoretical aspects of Quantum Chromodynamics at the LHC and the EIC

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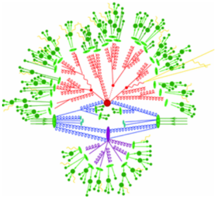
THE ULAM PROGRAMME

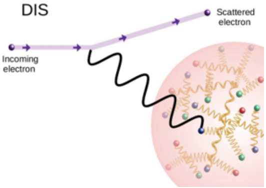
Quantum Chromodynamics

- **Quantum Chromodynamics (QCD)** is the theory that describes the strong interaction between quarks and gluons, which governs the structure of protons, neutrons, and nuclear matter.
- The theory is **asymptotically free** at high energy (short-distance) and exhibits the so-called **color confinement** at low energy (long-distance)

- Hadron-hadron colliders



- Lepton-hadron colliders



- Main guiding principle ⟶ **QCD Factorization**

Higher twist (h.t.) ↙

$$O = F_{\text{n.p.}} \otimes \sigma_{\text{p.}}(Q^2) + \mathcal{O}(\Lambda^2_{\text{QCD}}/Q^2)$$

↗ Non-perturbative methods (e.g. **Lattice QCD**) & extraction from data

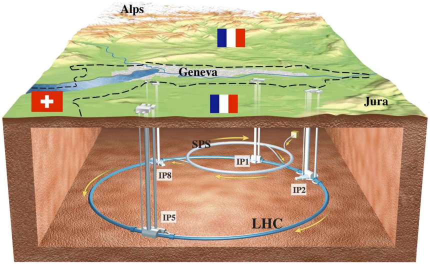
↖ **Perturbative** expansion in the strong coupling $\alpha_s(Q^2) \ll 1$

World's largest particle collider

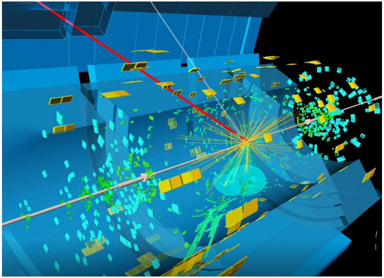
- Hadron-hadron collider
- 27 km ring, $\sqrt{s} = 13.6$ TeV
- High **luminosity** environment
- **Precision test of SM, BSM search**

Four main experiments

- ATLAS (general purpose, mainly *pp* collisions)
- CMS (general purpose, mainly *pp* collisions)
- ALICE (heavy ions)
- LHCb (flavour physics)



CERN accelerator complex



ATLAS detector

Precision physics at the LHC: Collinear factorization

- Collinear factorization in hadron-hadron collision

$$\sigma_{H_1H_2}(p_1,p_2) = \sum_{ij} \int dx_1 \int dx_2 f_i^{H_1}(x_1,Q) f_j^{H_2}(x_1,Q) \hat{\sigma}_{ij}(x_1p_1, x_2p_2, Q) + \text{h.t.}$$

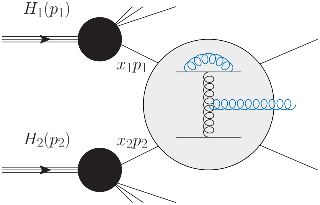
Parton distribution function (PDF) **Partonic cross section**

- Modern Scattering Amplitudes tools

↓

$$\hat{\sigma}_{ij} = \hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}_{ij}^{(1)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2)} + \ldots$$

Up to next-to-next-to-next-to-LO (N^3LO)



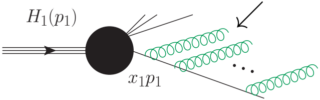
- $f_i(x, Q_0^2)$ is non-perturbative

Renormalization group equation in Q^2

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi]

$f_i(x, Q^2)$

Collinear (initial state) radiation



Resummation of $\alpha_s \ln(Q^2/Q_0^2)$ enhanced terms

Precision physics at the LHC: high-energy factorization

- **Regge limit** of QCD: $s \gg Q^2 \gg \Lambda^2_{\text{QCD}}$

Scale hierarchy $\Longrightarrow$ $\alpha_s \ln(s/Q^2) \sim 1 \longrightarrow$ All order resummation needed!

- **Eikonal** expansion: $\sigma = \sigma_0 + \frac{1}{s}\sigma_1 + ... = \sigma_0 +$ subeikonal terms

$$\sigma_0 = \sum_n \left[A_n\ \alpha_s^n(Q^2) \ln^n(s/Q^2) + B_n\ \alpha_s^{n+1}(Q^2) \ln^n(s/Q^2) + ... \right]$$

Leading logarithmic approximation (LLA) [Balitsky, Fadin, Kuraev, Lipatov]

Next-to-LLA

- Strongly-order-in-rapidity radiation

1) "Initial state" radiation

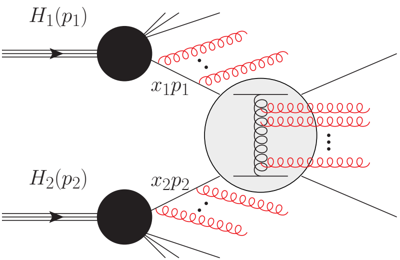
$\downarrow$

Small-x evolution of PDFs

2) Also "final state" radiation

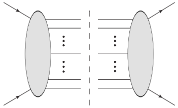
$\downarrow$

Affects partonic cross sections $\hat{\sigma}$

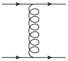
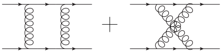
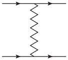
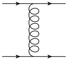
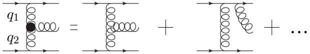
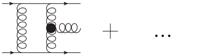
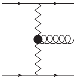
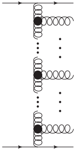
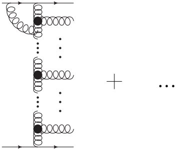
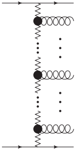


BFKL approach

$$\sigma_{AB\to X} = \frac{\mathrm{Im_s}\mathcal{A}_{\mathrm{AB\to AB}}}{s} = \frac{1}{2s}\sum_n$$

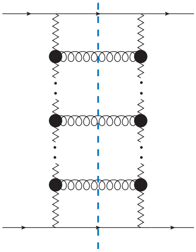


$\mathcal{A}_{AB\to \bar{A}\bar{B}+n}$

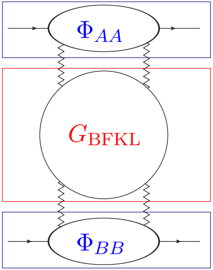
	• Born level	• 1-loop	• All-order
$n = 0:$	 $\sim \frac{s}{t}$ t-channel **gluon** + Eikonal coupling		 $= \left(\frac{s}{-t}\right)^{\omega(t)}$  Gluon Reggeization
$n = 1:$	 $\gamma(q_1, q_2) \longleftarrow$ Lipatov Vertex		
$n:$	 Multi-Regge kinematics $\downarrow$ Strong rapidity ordering		 $\leftarrow \mathcal{A}_{AB\to \bar{A}\bar{B}+n}$

BFKL approach

$$\sigma_{AB\to X} = \frac{1}{2s}\sum_n$$



$=$

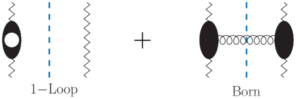


⟵ Impact Factor

Process-dependent, off-shell coefficient function

↖ Green function (universal)

- $G_{\text{BFKL}} \longrightarrow$ Iteration of a perturbative kernel ($\mathcal{K}_{\text{BFKL}}$) $\longrightarrow$



- $\vec{k}_T$ (transverse momentum) **factorization**

$$\sigma_{AB\to X} = \int \frac{d^2\vec{q}_1}{(2\pi)}\frac{d^2\vec{q}_2}{(2\pi)}\frac{\Phi_{AA}(\vec{q}_1)}{(\vec{q}_1^{\,2})^2}\underbrace{G_{\text{BFKL}}(\vec{q}_1,\vec{q}_2;s)}\frac{\Phi_{BB}(\vec{q}_2)}{(\vec{q}_2^{\,2})^2} \sim s^{\omega_P}$$

↙ Controls the whole energy behavior & satisfy the **BFKL equation**

↖ power-like growth

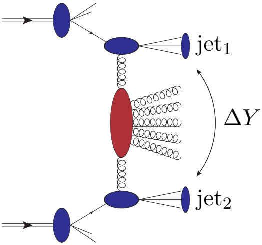
BFKL resummation at colliders

- Proton-proton collision at center-of-mass energy $S = (P_1 + P_2)^2$

$$\ln\left(\frac{S}{Q^2}\right) = \ln\left(\frac{x_1 x_2 S}{Q^2}\right) + \ln\left(\frac{1}{x_2}\right) + \ln\left(\frac{1}{x_1}\right)$$

Forward/backward

(e.g. Mueller-Navelet jets)



$$\sigma = \Phi_1 \otimes G_{\mathrm{BFKL}} \otimes \Phi_2$$

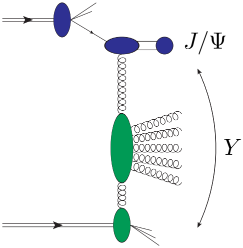
$$\searrow \qquad \swarrow$$

$$\Phi_{\mathrm{i}}(x_i) \otimes_{x_i} f(x_i)$$

$$\alpha_s \ln\left(\frac{x_1 x_2 S}{|p_{1\perp}||p_{2\perp}|}\right) = \alpha_s \Delta Y \sim 1$$

Single forward

(e.g. forward J/Ψ production)



$$\sigma = \Phi \otimes \mathcal{U}_{\mathrm{BFKL}}$$

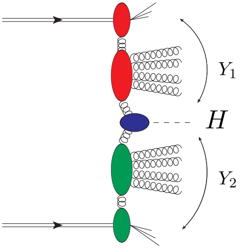
$$\downarrow$$

$$\mathcal{U}_{\mathrm{BFKL}} = \Phi_{\mathrm{prot}} \otimes G_{\mathrm{BFKL}}$$

$$\alpha_s \ln(1/x_2) = \alpha_s Y \sim 1$$

Central

(e.g. central Higgs production)



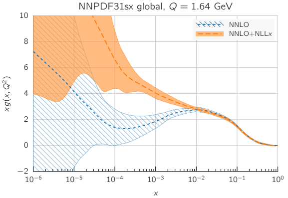
$$\sigma = \mathcal{U}_{\mathrm{BFKL}} \otimes \Xi_{\mathrm{centr.}} \otimes \mathcal{U}_{\mathrm{BFKL}}$$

$$\alpha_s \ln(1/x_2) = \alpha_s Y_2 \sim 1$$

$$\alpha_s \ln(1/x_1) = \alpha_s Y_1 \sim 1$$

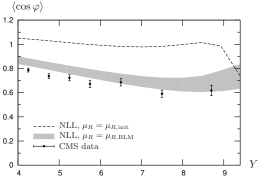
BFKL resummation at colliders: why does it matter?

- Hadron structure at small-x



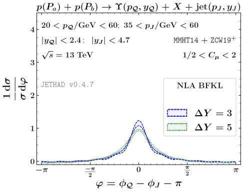
PDF improvement at small-x [Bonvini]

- Parton radiation induced decorrelation phenomena



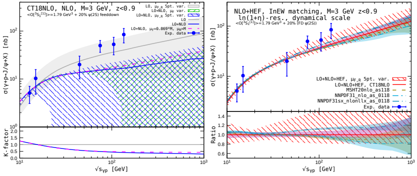
Mueller-Navelet jets

[Ducloué, Szymanowski, Wallon]



$\mathcal{Q}$+jet [Celiberto, **M.F.**]

- Precision physics at LHC



Instabilities of J/Ψ production due to lack of resummation

[Lansberg, Nefedov, Ozelick]

- Future experiments (e.g. Future Circular Collider)

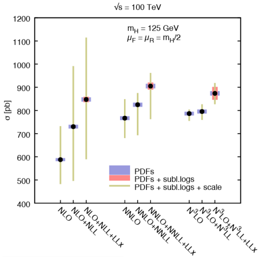
$p\,p \longrightarrow H$ at 100 TeV

[Bonvini, Marzani]

A few percents at LHC

$\downarrow$

10 percent at FCC



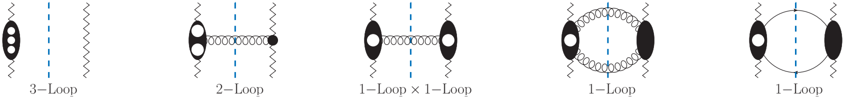
What this requires? $\longrightarrow$ **Precision** (beyond leading logarithms)

Higher orders: BFKL evolution

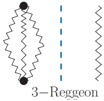
- Leading order (LO) BFKL kernel [Fadin, Kuraev, Lipatov] [Lipatov, Balitsky]

- Next-to-LO kernel [Fadin, Lipatov] [Ciafaloni, Camici] [Fadin, Fiore, Papa] [Fadin, Gorbachev]

- Next-to-next-to-LO kernel (ongoing) [Fadin, **M.F.**, Papa] [Del Duca, Marzucca, Verbeek] [Gardi—Abreu, De Laurentis, Falcioni, Maher, Milloy, Vernazza—Byrne, Del Duca, Dixon, Smillie] [Buccioni, Caola, Chakraborty, Devoto, Gambuti, von Manteuffel, Tancredi]



+ { Reggeization breaking terms $\xrightarrow{e.g.}$
[Del Duca, Glover]
[Caron-Huot—Gardi, Vernazza]
[Fadin, Lipatov]



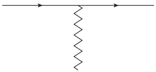
}

Higher orders: impact factors

- Full NLL cross section = Full NLL Green function + NLO impact factors

- Known NLO impact factors

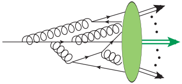
1) Parton-to-Parton transition
Φ_{pp} (both quark and gluon)
[Fadin, Fiore, Kotsky, Papa]
[Ciafaloni—Colferai—Rodrigo]



2) Proton-to-hadron transition

$$\text{PDF} \otimes \ \Phi_{pp} \otimes \text{FF}$$

[Ivanov, Papa]

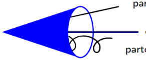


Fragmentation function

3) Proton-to-jet transition

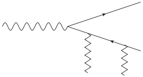
$$\text{PDF} \otimes \ \Phi_{pp} \cdot \mathcal{S}_{jet}$$

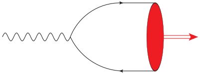
[Bartels, Colferai, Vacca]
[Caporale, Murdaca, Ivanov, Papa, Perri]



Jet algorithm

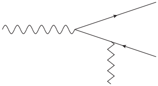
4) Exclusive $\gamma^* \to \rho_L$ meson $\longrightarrow$ $\Phi_{\gamma^* \rho_L} \otimes$ DA
[Ivanov, Kotsky, Papa]





Distribution amplitude

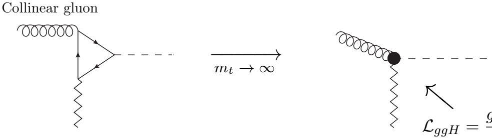
5) DIS impact factor (inclusive $\gamma^* \to q\bar{q}$) $\longrightarrow$ $\Phi_{\gamma^* \gamma^*}$
[Bartels, Colferai, Gieseke, Kyrieleis]



Higher orders: impact factors (Higgs)

- LO Higgs impact factor

  [Del Duca, Schmidt]



$\mathcal{L}_{ggH} = \frac{g_H}{4} F^a_{\mu\nu} F^{\mu\nu,a} H$

- **NLO** Higgs impact factor in the infinite top-mass limit

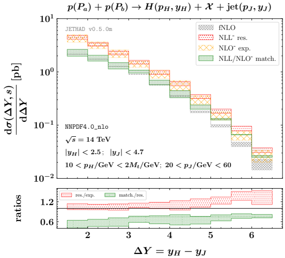
[Nefedov][Hentschinski, Kutak, van Hameren][Celiberto, **M.F.**, Ivanov, Mohammed, Papa]

- Does Reggeization holds in presence of dim-5 operator?

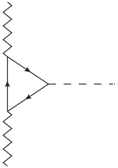
  $\Longrightarrow$ Proof at 1-loop [**M.F.**, Nefedov, Papa] and 2-loop [Del Duca, Falcioni]

- **Finite top-mass** extension (real part) [Celiberto, Delle Rose, **M.F.**, Gatto, Papa]

- **Phenomenology**: Higg-jet (hadron) [Celiberto, **M.F.**, Ivanov, Mohammed, Papa]



- **Frontier**: **NLO Central** impact factor



Quarkonium and NRQCD

- **Quarkonium** ($\mathcal{Q}$) = Heavy quark-antiquark bound state ($J/\Psi, \Upsilon$)

long distance m.e.

$m_Q \gg \Lambda_{\rm QCD}$

Quark are moving slowly within the $\mathcal{Q}$ $\xrightarrow{\text{EFT}}$ **Non-Relativistic QCD** $\xrightarrow{\text{factorization}}$ $\sigma(H) = \sum_k C_k \, \langle \mathcal{O}_k^H \rangle$

$v \ll 1$

$k \equiv {}^{2s+1}L_J^{[c]}$

- Conceptual breakthrough: $\mathcal{Q}$ produced not only from a $Q\bar{Q}$[singlet] in C_k

$$|J/\psi\rangle = O(1)\,|c\bar{c}\left[{}^3S_1^{(1)}\right]\rangle + O(v)\,|c\bar{c}\left[{}^3P_J^{(8)}\right] + g\rangle + O(v^2)\,|c\bar{c}\left[{}^1S_0^{(8)}\right] + g\rangle + O(v^2)\,|c\bar{c}\left[{}^3S_1^{(8)}\right] + gg\rangle + \dots$$

- Interface between Quarkonium and semi-hard physics is **natural** and **inevitable!**

$$\sqrt{s} \gg m_Q \gg \Lambda_{\rm QCD}$$

- An ideal test of BFKL: J/ψ plus jet hadroproduction

[Boussarie, Ducloué, Szymanowski, Wallon]

(NLL in fragmentation approx.) [Celiberto, **M.F.**]

- BFKL resummation cures instabilites of inclusive Quarkonium production

Inclusive hadroproduction and photoproduction [Lansberg, Nefedov, Ozcelik]

Exclusive photoproduction [Flett, Lansberg, Nabeebaccus, Nefedov, Sznajder, Wagner]

Higher orders: impact factors (Quarkonium)

- Precision Quarkonium phenomenology requires complete **NLO impact factors**

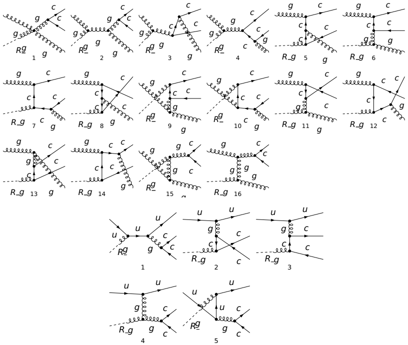
  [**M.F.**, Lansberg, Nefedov, Szymanowski, Wallon]

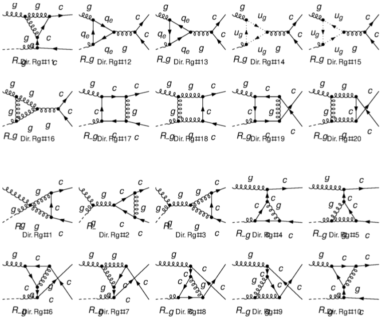
- Complicated automation chain:

  1) Diagrams generation ⟶ `FeynArts`

  2) IBP-reduction ⟶ `FIRE`

  3) General QFT Manipulation ⟶ `FeynCalc` + `FORM`

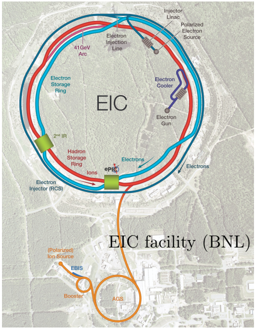




The Electron-Ion Collider (EIC)

Next-generation electron-ion collider

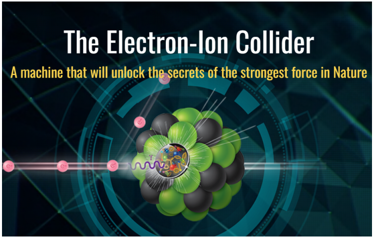
- Electron-proton and electron-ion collisions
- $\sqrt{s} \sim$ 20–140 GeV
- High **luminosity** and polarization
- Imaging the **quark-gluon structure** of matter



EIC facility (BNL)

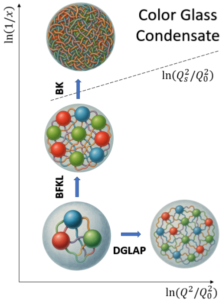
Physics goals

- 3D structure of nucleons (TMDs, GPDs)
- Gluon saturation and small-x physics
- Spin decomposition of the proton
- Nuclear structure and QCD in nuclei



Potential of the Electron-Ion Collider

- **Dense gluonic matter**



Small-x $\rightarrow$ High-density regime

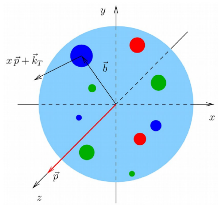
Saturation $\downarrow$

Collective & non-linear dynamics

$\downarrow$

Color Glass Condensate

- **Multi-dimensional tomography**



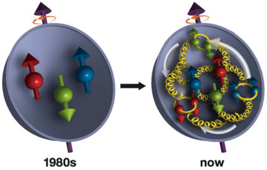
$\mathcal{W}(x,\vec{b},\vec{k}_T)$	
$\mathcal{F}(x,\vec{k}_T)=\int d^2\vec{b}\,\mathcal{W}(x,\vec{b},\vec{k}_T)$	$\mathcal{G}(x,\vec{b})=\int d^2\vec{k}_T\,\mathcal{W}(x,\vec{b},\vec{k}_T)$
$f(x)=\int d^2\vec{k}_T\,\mathcal{F}(x,\vec{k}_T) = \int d^2\vec{b}\,\mathcal{G}(x,\vec{b})$	$F(\vec{b})=\int dx\,\mathcal{G}(x,\vec{b})$
$Q=\int dx\,f(x)=\int d^2\vec{b}\,F(\vec{b})$	

More general distributions

$\updownarrow$

Less inclusive observables

- **Proton spin**



Proton spin decomposition

[Jaffe, Manohar] [Ji]

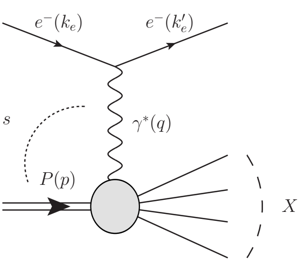
$$\frac{1}{2} = S_q + S_g + L_q + L_g$$

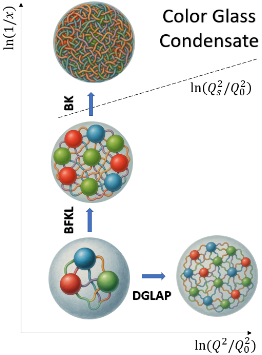
$S_q \rightarrow$ Quark spin

$S_g \rightarrow$ Gluon spin

$L_{q,g} \rightarrow$ Orbit. angular momenta

- **Deep Inelastic Scattering (DIS)**

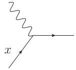




- DIS variables

$$Q = \sqrt{-q^2} \longrightarrow \text{related to the transverse resolution } \Delta x_\perp = \frac{1}{Q}$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{Q^2 + s}$$



(naive parton model)

Saturation physics

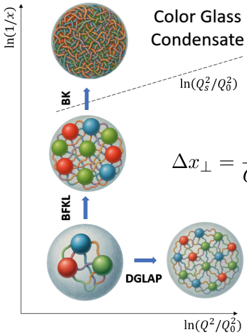
- DIS total cross-section (BFKL)

$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\ ) \otimes_{\vec{k}} \mathcal{F}_g(x, \vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0} = \left(\frac{1}{x}\right)^{\omega_0}$$

Growth of the gluon distribution at small-x



$$\Delta x_\perp = \frac{1}{Q}$$

A too fast growth

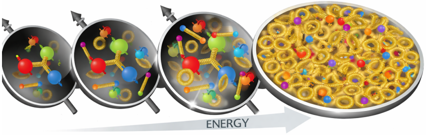
$\sigma_{\text{tot}} \lesssim c \ln^2 s$

Martin-Froissart bound

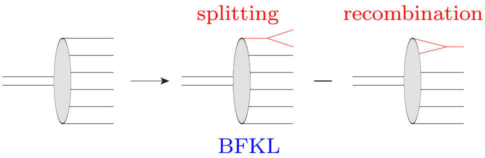
$\uparrow$

Unitarity limit

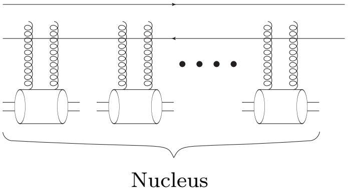
$\longrightarrow$ Evolution is bringing the proton into an infinitely dense gluonic system $\longrightarrow$ must saturate before



- **Saturation effects**
  - *i.* Very dense system $\Longrightarrow$ Non-linear recombination effects
    [Gribov, Levin, Ryskin (1981-1983)] [Mueller and Qiu (1985)]



  - *ii.* In large nuclei $\Longrightarrow$ Multiple re-scattering ($\alpha_s^2 A^{1/3}$ resummation)
    [Glauber (1959)—Gribov (1969)], [Kovchegov (1999)]



- **Dense QCD** system → collective and non-linear dynamics

The Color Glass Condensate

- **High occupation number** $\Longrightarrow$ Classical field description [McLerran, Venugopalan]

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda_{\text{QCD}}^2 \qquad \alpha_s(Q_s^2) \ll 1 \qquad A_{\text{classical}} \sim \frac{1}{g_s(Q_s^2)}$$

$\nearrow$ saturation scale

- Small-x partons evolves on a very short time w.r.t large-x parton ($\tau_{\text{fast}} \gg \tau_{\text{slow}}$)

Small-x partons
Dynamical degrees of freedom
$\downarrow$
Obtained by solving the classical Y.M. equations: $[D_{\mu\nu}, F^{\mu}] = J_{\nu}(\rho)$ $\leftarrow$

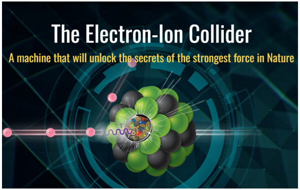
Large-x partons
Static degrees of freedom
$\downarrow$
Act as a static color charge density ρ

Color
Color degrees of freedom

Glass
Separation of time scales and disordered system

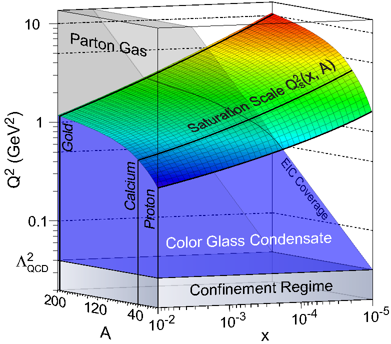
Condensate
System as dense as possible

- Quantum corrections $\longrightarrow$ **Non-linear small-x**



- Saturation scale

$$Q_s^2(A,x) \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\rm QCD}$$



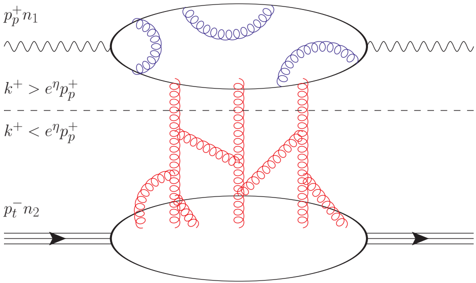
- **Perturbative control** on gluonic saturation

$$\Lambda^2_{\rm QCD} \ll Q^2 \lesssim Q_s^2$$

- At the **EIC** the saturation scale Q_s will be in the perturbative range
- At the **LHC** the saturation can be tested in Ultra Peripheral Collision

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$
- n_1^μ, n_2^μ are light-cone vectors (+/- directions)



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[McLerran, Venugopalan (1994)] [Balitsky (1996-2001)]

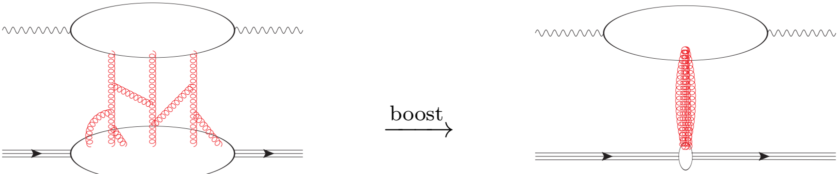
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

Shockwave approach

- Large longitudinal Boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+) B(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$$

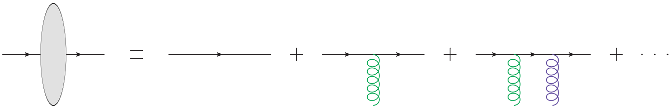
Shockwave approximation

- Independence from x^- $\implies$ conservation of p^+ (**eikonal approximation**)
- Light-cone gauge $A \cdot n_2 = 0 \implies A \cdot b = 0 \implies$ *Simple effective Lagrangian*

Shockwave approach

- Multiple interactions with the target → **path-ordered Wilson lines**

$$V^{\eta}_{\vec{z}_i} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right)\right]$$



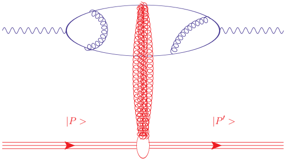
$$V_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) + (ig)^2\int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_i\right)\theta\left(z_{ij}^+\right) + \cdots$$

- Factorization in the Shockwave approximation

$$\mathcal{M}^\eta = N_c\int d^d z_{1\perp} d^d z_{2\perp}\,\Phi^\eta(z_{1\perp}, z_{2\perp})\left\langle P'\left|\left[\frac{1}{N_c}\mathrm{Tr}\left(V^\eta_{\vec{z}_1} V^{\eta\dagger}_{\vec{z}_2}\right) - 1\right](\vec{z}_1, \vec{z}_2)\right|P\right\rangle$$

- **Dipole operator**

$$\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}\left(V^\eta_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j}\right) - 1$$



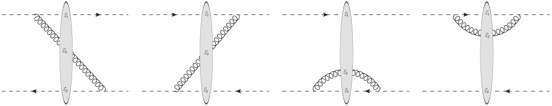
Balitsky-JIMWLK evolution equations

- **Balitsky-JIMWLK evolution equations** for the dipole
  [Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

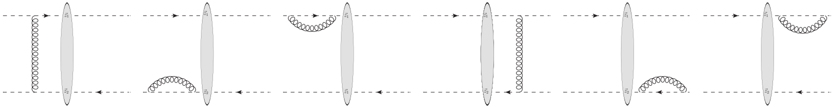
$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 \vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2} \vec{z}_{31}^{\,2}} \right) \Big[\underbrace{\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12}}_{\text{BFKL}} - \mathcal{U}^{\eta}_{13} \mathcal{U}^{\eta}_{32} \Big]$$

$$\frac{\partial \mathcal{U}^{\eta}_{13} \mathcal{U}^{\eta}_{32}}{\partial \eta} = \cdots \qquad \longleftarrow \quad \textit{Balitsky hierarchy}$$

- Double dipole contribution and Dipole contribution



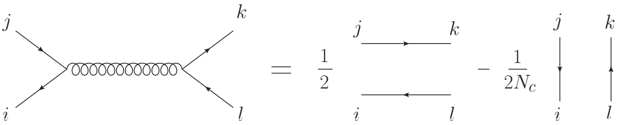
- Dipole contribution



- Gluon with rapidity slightly above the cut-off: $\eta + \Delta\eta$

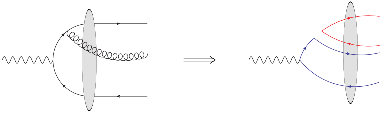
Balitsky-Kovchegov evolution equation

- **Large-N_c limit** ['t Hooft (1974)]



$$t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$

- Double dipole → Dipole × dipole



$$\langle \mathcal{U}^\eta_{13}\mathcal{U}^\eta_{32}\rangle \to \langle \mathcal{U}^\eta_{13}\rangle\,\langle \mathcal{U}^\eta_{32}\rangle$$

- Hierarchy of equations broken → closed non-linear BK equation

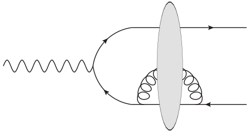
[Balitsky (1995)] [Mueller (1994-1995)] [Kovchegov (1999)]

$$\frac{\partial\langle \mathcal{U}^\eta_{12}\rangle}{\partial\eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3\left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}}\right)\left[\langle \mathcal{U}^\eta_{13}\rangle + \langle \mathcal{U}^\eta_{32}\rangle - \langle \mathcal{U}^\eta_{12}\rangle - \langle \mathcal{U}^\eta_{13}\rangle\langle \mathcal{U}^\eta_{32}\rangle\right]$$

with $\langle \mathcal{U}^\eta_{12}\rangle \equiv \langle P'|\mathcal{U}^\eta_{12}|P\rangle$

Precision frontier in CGC: NLO calculations

- Quantum corrections to the semi-classical approach ⟶ **QCD coupling corrections**



$$\nearrow \quad \Phi^{(0)} + \alpha_s \ln\left(\frac{1}{x_g}\right)\Phi^{(0)} \otimes \mathcal{K}_{\mathrm{BK}} + \alpha_s \Phi^{(1)}_{\mathrm{finite}}$$

↙ Resummed into LO evolution

↓ NLO corrections impact factors

- NLO evolution of the dipole → two-loop corrections in the small rapidity limit

[Balitsky, Chirilli] [Grabovsky] [Kovner, Lublinsky, Mulian]

- NLO IFs: inclusive processes

[Balitsky, Chirilli] [Chirilli, Xiao, Yuan]
[Beuf] [Roy, Venugopalan]
[Caucal, Salazar, Schenke]
Stebel, Venugopalan]
[Beuf, Lappi, Paatelainen, Penttala]
[Bergabo, Jalilian-Marian]
[Taels, Altinoluk, Beuf, Marquet]
[Altinoluk, Beuf, Favrel, **M.F.**]

- NLO IFs: exclusive processes

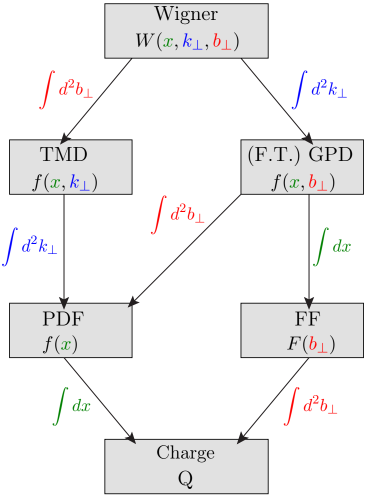
[Boussarie, Grabovsky, Szymanowski, Wallon]
[Lappi, Mäntysaari, Pentalla]
[**M.F.** Grabovsky, Li, Szymanowski, Wallon]

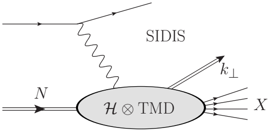
↓

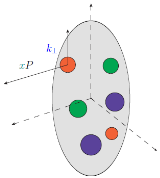
Excellent tools for
hadron tomography

Genealogy of parton distributions: Inclusive processes

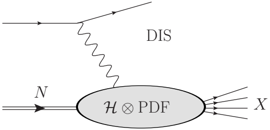
Genealogy of
parton distributions

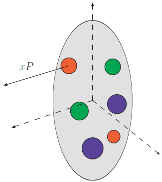






Transverse Momentum distribution

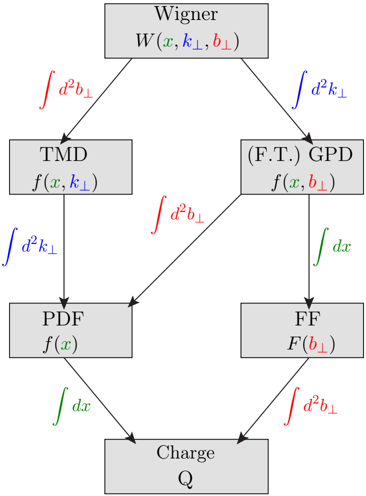


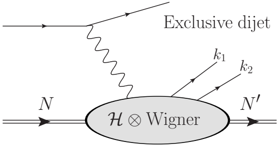


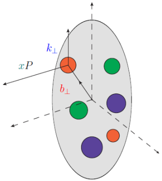
Parton distribution function

Genealogy of parton distributions: Exclusive processes

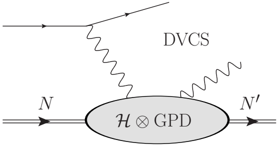
Genealogy of
parton distributions

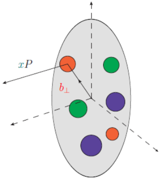




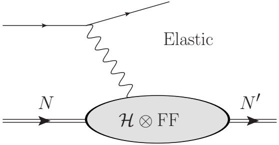


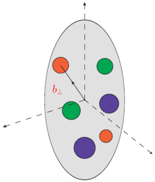
Wigner distribution





Generalized Parton distribution





Form Factor

- Gluon dipole Wigner distribution

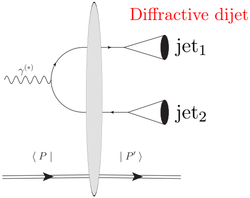
$$xW\left(x,\vec{q}_\perp,\vec{b}_\perp\right)=\frac{2}{P^+(2\pi)^3}\int dz^+d^2\vec{z}_\perp\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}e^{i\vec{q}_\perp\cdot\vec{z}_\perp-ixP^-z^+}$$

$$\times\left\langle P+\vec{\Delta}_\perp/2\right|\mathrm{Tr}\left[U_+\,F_a^{+i}\left(\vec{b}_\perp+z_\perp/2\right)U_-\,F_a^{+i}\left(\vec{b}_\perp-z_\perp/2\right)\right]\left|P-\vec{\Delta}_\perp/2\right\rangle$$

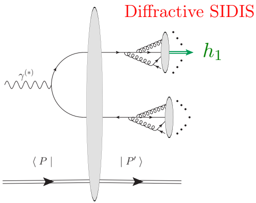
- GTMD distribution (F.T. of Wigner) at small-x [Dominguez, Marquet, Xiao, Yuan]

$$xW\left(x,\vec{q}_\perp,\vec{\Delta}_\perp\right)\approx\frac{2N_c}{\alpha_s}\left(q_\perp^2-\frac{\Delta_\perp^2}{4}\right)\langle P'|\,\tilde{\mathcal{U}}(\vec{q}_\perp,\vec{\Delta}_\perp)\,|P\rangle$$

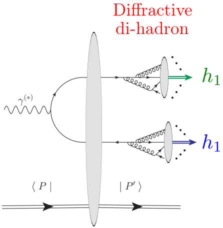
- Ideal processes to probe the Wigner



(LO) [Hatta, Xiao, Yuan]
(NLO) [Boussarie, Grabovsky, Szymanowski, Wallon]



(LO) [Hatta, Xiao, Yuan]
(NLO) [**M.F.**, Grabovsky, Li, Szymanowski, Wallon]



(LO+NLO)
[**M.F.**, Grabovsky, Li, Szymanowski, Wallon]

Higher-twist effects

- Sensitivity to saturation effects: Q^2 (hard scale) $\lesssim$ Q_s^2 (saturation scale)

$$Q_s^2(A,x) \sim \left(\frac{A}{x}\right)^{1/3} \Lambda_{\mathrm{QCD}}^2 \sim \; 2\text{–}3 \text{ GeV}^2 \text{ (at the EIC)}$$

- Formalism for higher-twist effects in the context of saturation physics

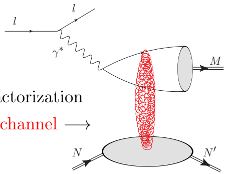
Applications beyond the scopes of saturation ↓ [Boussarie, **M.F.**, Szymanowski, Wallon]

- Helicity amplitudes ratio of Deeply Virtual Meson Production

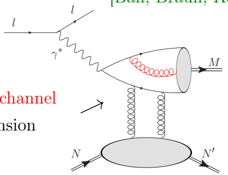
These observables appears beyond twist-2 (leading one) ⟶ Breakdown of Collinear factorization at twist-3 ⟶ New factorization scheme

- Small-x EFT [Balitsky] [McLerran, Venugopalan]
- Higher-twist Collinear factorization [Ball, Braun, Koike, Tanaka]

All-twist factorization in the t-channel ⟶



Systematic s-channel twist expansion ↗



- Most general description of DVMP at high-energy [Boussarie, **M.F.**, Szymanowski, Wallon]

Deeply Virtual meson production at HERA

- Ratio of helicity amplitudes

$$R^{11} = \mathcal{A}^{11}/\mathcal{A}^{00}$$

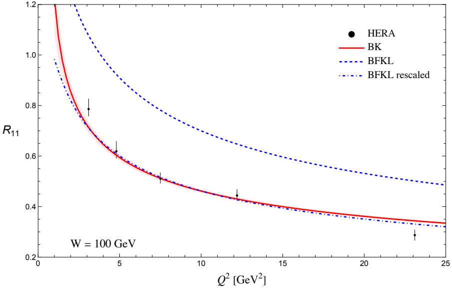
$\mathcal{A}^{11}$ is the $\gamma_T \longrightarrow M_T$ (no-flip) transition

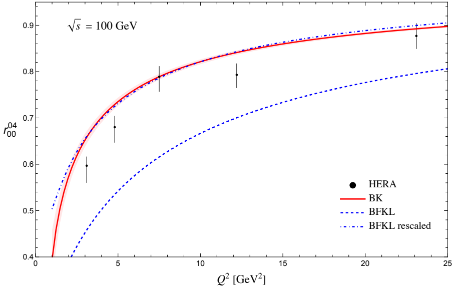
$\mathcal{A}^{00}$ is the $\gamma_L \longrightarrow M_L$ transition

- Spin-density matrix element r_{00}^{04}

$$r_{00}^{04} = \frac{\varepsilon + R_{10}^2}{R_{11}^2 + \varepsilon + R_{10}^2 + R_{-11}^2 + 2\varepsilon R_{01}^2}$$

ε = photon polarization parameter





[Boussarie, Delle Rose, **M.F.**, Papa, Szymanowski, Wallon]

- Low-Q^2 data needs full NLO or twist-4 corrections

[Boussarie, Delle Rose, **M.F.**, Papa, Szymanowski, Wallon (ongoing)]

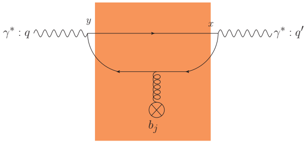
Subeikonal corrections

- Eikonal expansion: $\sigma = \sigma_0 + \frac{\mathbf{1}}{\mathbf{s}}\sigma_{\mathbf{1}} + ... = \sigma_0 +$ **next-to-eikonal** $+ ...$

$$\text{In the CGC} \implies b^{\mu}_{\text{gluons}}(x^+, x^-, \vec{x}) = \delta(x^+)B(\vec{x}, x^- = 0)n_2^{\mu}$$

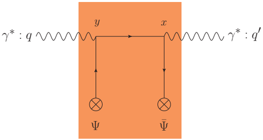
- **Next-to-eikonal (NEik)** corrections are sizeable in the EIC kinematics

- Beyond Shockwave & Transverse gluons



- Beyond static
  ↓
  non-zero p^+ transfer between projectile and target

- Fermionic exchange



- Evolution of the new CGC operators? ⟶ Loop corrections

[Altinoluk, Beuf, Favrel, **M.F.**]

$$F_T\Big|^{\text{NLO, NEik}}_{\Psi,\text{ rap. \& UV}} \sim x\,\frac{\alpha_s}{2\pi}\,q_f(x, \boldsymbol{r}=0)\int_{\alpha}^{1}\frac{dz}{z}\int^{\boldsymbol{r}^2<\Lambda^2} d^{2-2\epsilon}\boldsymbol{r}\,\frac{1}{(\boldsymbol{r}^2)^{1-2\epsilon}}$$

Both UV & rapidity $\implies$ Signal of a double logarithms evolution (e.g. $\alpha_s \ln^2(1/x)$)

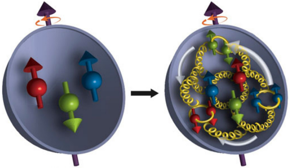
Subeikonal corrections and spin physics

- Eikonal approximation $\longrightarrow$ Spins of projectile and target do not talk!
- Spin of the proton

[Jaffe, Manohar] [Ji]

$$\frac{1}{2} = S_q + S_g + L_q + L_g$$

Spin — Orbital angular momenta (OAM)



$$S_q(Q^2) = \frac{1}{2}\int_{x_{\min}}^{1} dx\ \Delta\Sigma(x,Q^2) \qquad S_g(Q^2) = \int_{x_{\min}}^{1} dx\ \Delta G(x,Q^2)$$

in real experiments $x_{\min} \neq 0$

$$S_q(Q^2) + S_g(Q^2) \neq \frac{1}{2}$$

- Missing contribution to the spin $\longrightarrow$ OAM and/or small-x behavior of $\Delta\Sigma$ and ΔG
- Pioneering works suggests a sizable contribution at small-x [Bartels, Ermolaev, Ryskin]
- **NEik CGC**
  - $\nearrow$ Determination of the asymptotics for $x \to 0$ (double logs evolution) [Kovchegov, JAM]
  - $\searrow$ Access to spin-dependent distributions at finite but small-x ($x \sim x_{\min}$)

Spin-spin correlations at the EIC

Spin correlations

↓

Observables inspired by **Quantum Information**

- $Q\bar{Q}$-pair production as two qubit system

$$S_{\alpha\alpha'\beta\beta'} \propto \bar{v}_{\beta'}\Gamma u_{\beta}\bar{u}_{\alpha'}\Gamma^{\dagger}u_{\alpha'} = \frac{A}{4}\left(\delta_{\alpha\beta}\delta_{\alpha'\beta'} + C_{ab}\,\xi^{\dagger}_{\alpha}\sigma^{a}\xi_{\beta}\eta^{\dagger}_{\alpha'}\sigma^{a}\eta_{\beta'}\right)$$

↑ Spin-density matrix ↖ Spin-spin correlation matrix

- Huge activity at the LHC [Atlas, (2024)] [CMS, (2024)]
  - *i.* Large set of **new observables** to test Standard Model
  - *ii.* Possibility to investigate features of quantum mechanics at the highest accessible energy scales (e.g. spin **Entanglement** between particles)
- Similar program at the **Electron-Ion Collider** [Qi, Guo, Xiao (2025)][Cheng, Han, Trifinopoulos (2025)]
- Spin correlation in the **high-energy of limit QCD** (exclusive $Q\bar{Q}$-pair production) [**M.F.**, Hatta]
- Extension to back-to-back inclusive $Q\bar{Q}$-pair production [**M.F.**, Hatta, Xiao (to appear)]

Merci de votre attention!