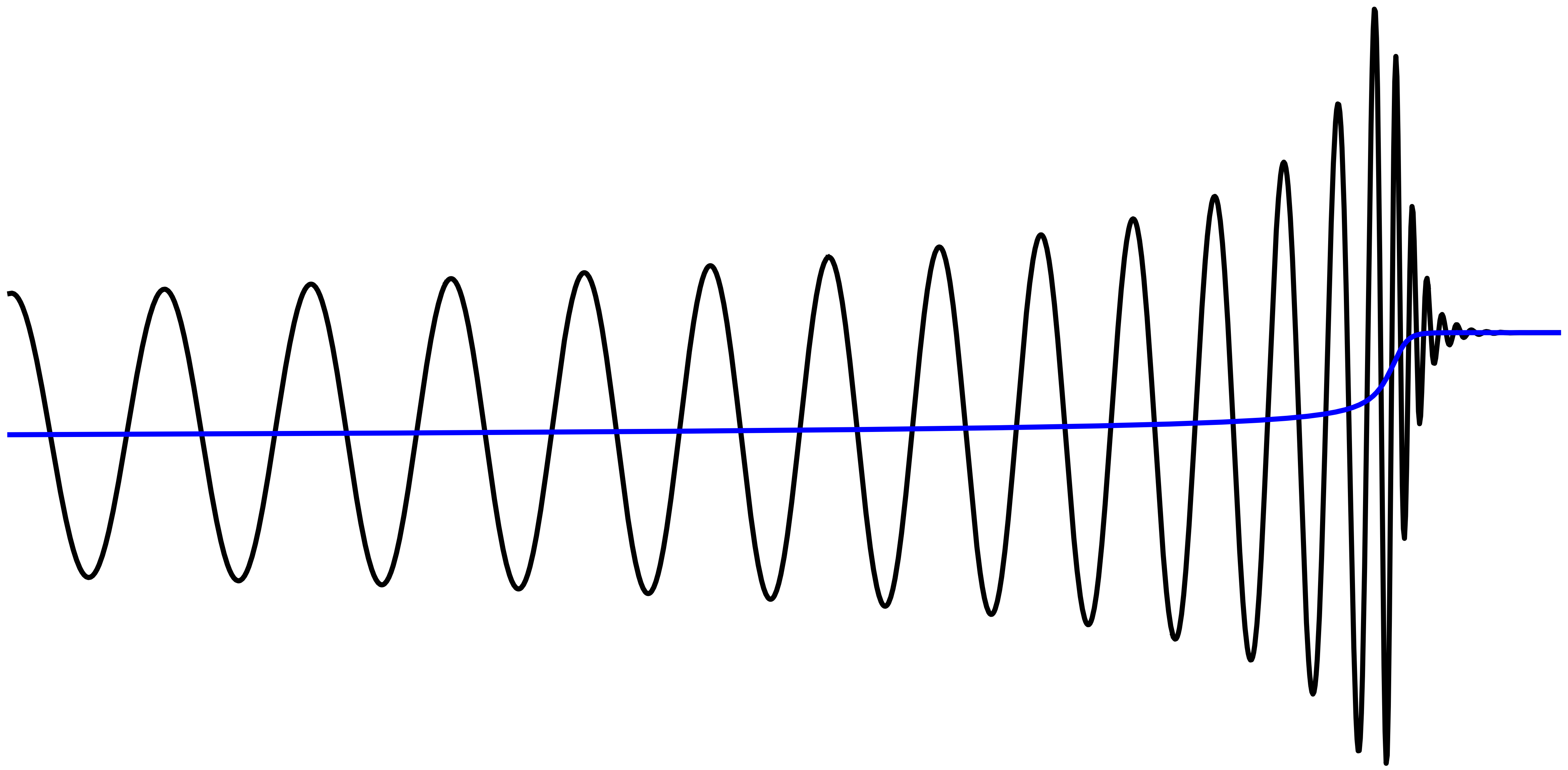
Waveform Models for the Gravitational-wave Memory Effect



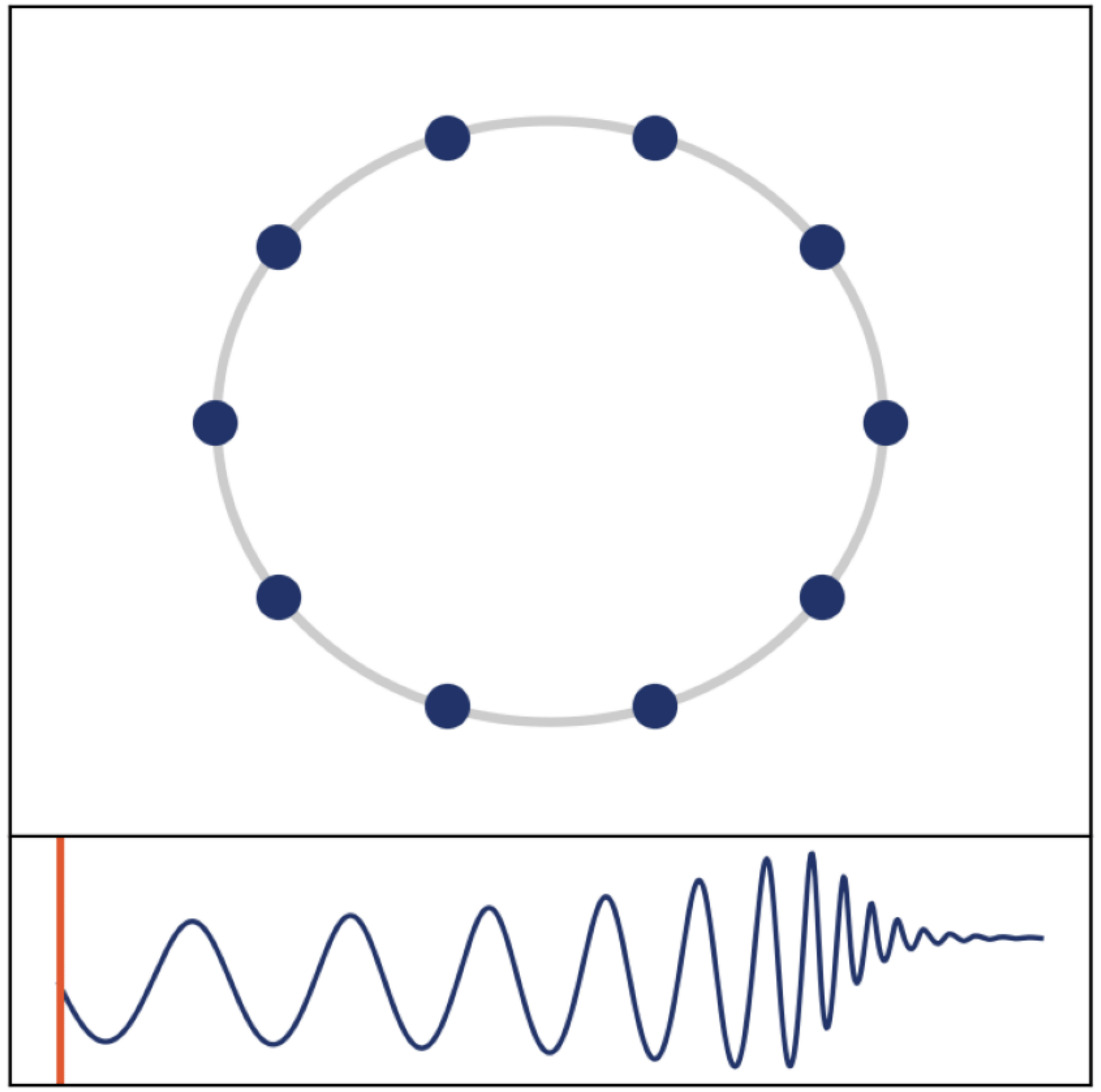
A.E. & David Nichols, arXiv:**2407.19017** & **2504.18635**

Arwa Elhashash - IJCLab

Outline

- The nonlinear GW memory effect from non spinning binary-black-hole mergers.
- Developing waveform modeling for the memory signal:
  - The GW memory signal from Extreme-mass-ratio inspirals
  - A model for the late-time memory signal
  - A time-domain model
  - A frequency-domain model

Waveform models for the memory (h_{20})



KEEFE MITMAN

A lasting gravitational-wave strain after the gravitational waves have passed.

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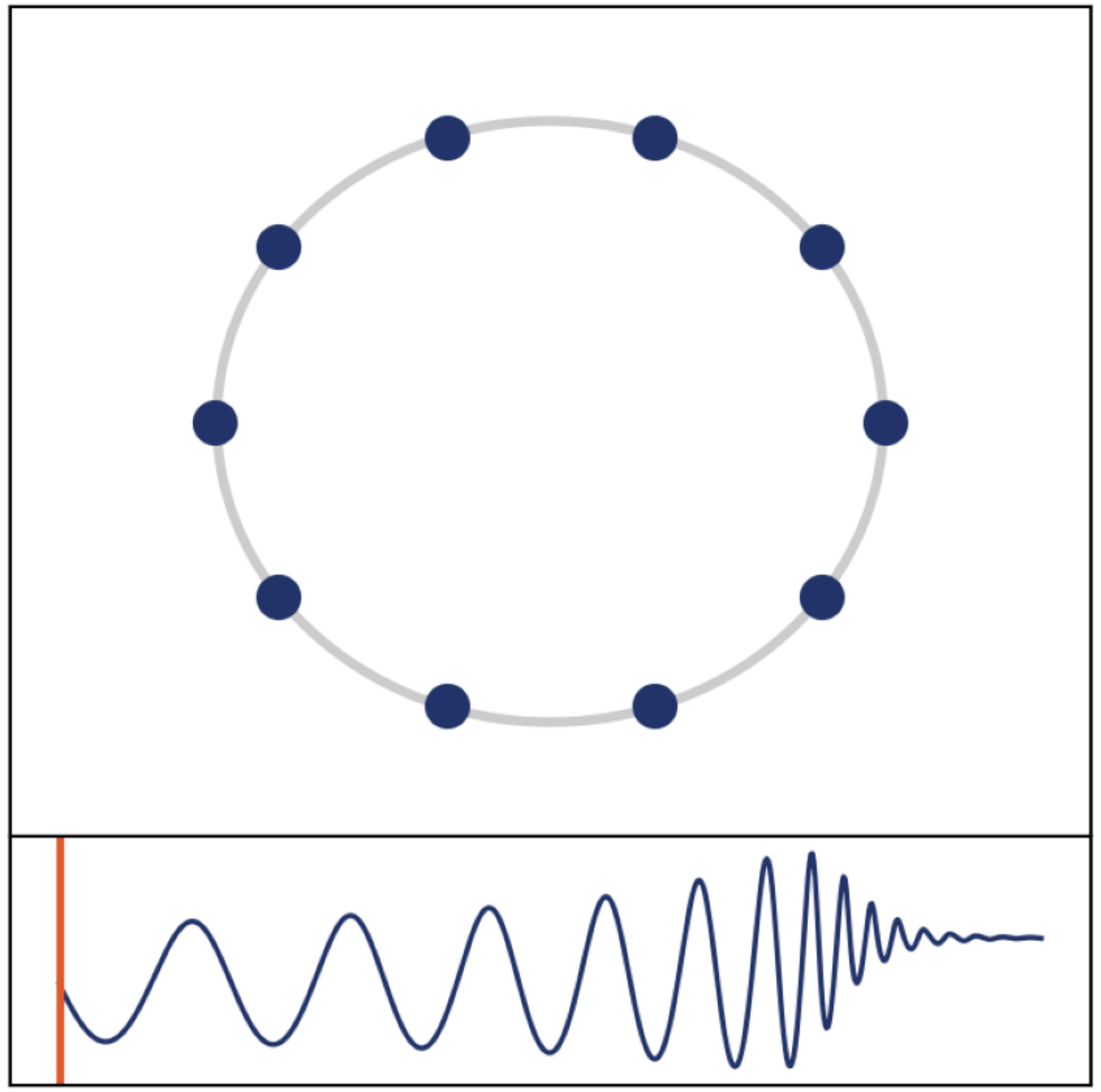

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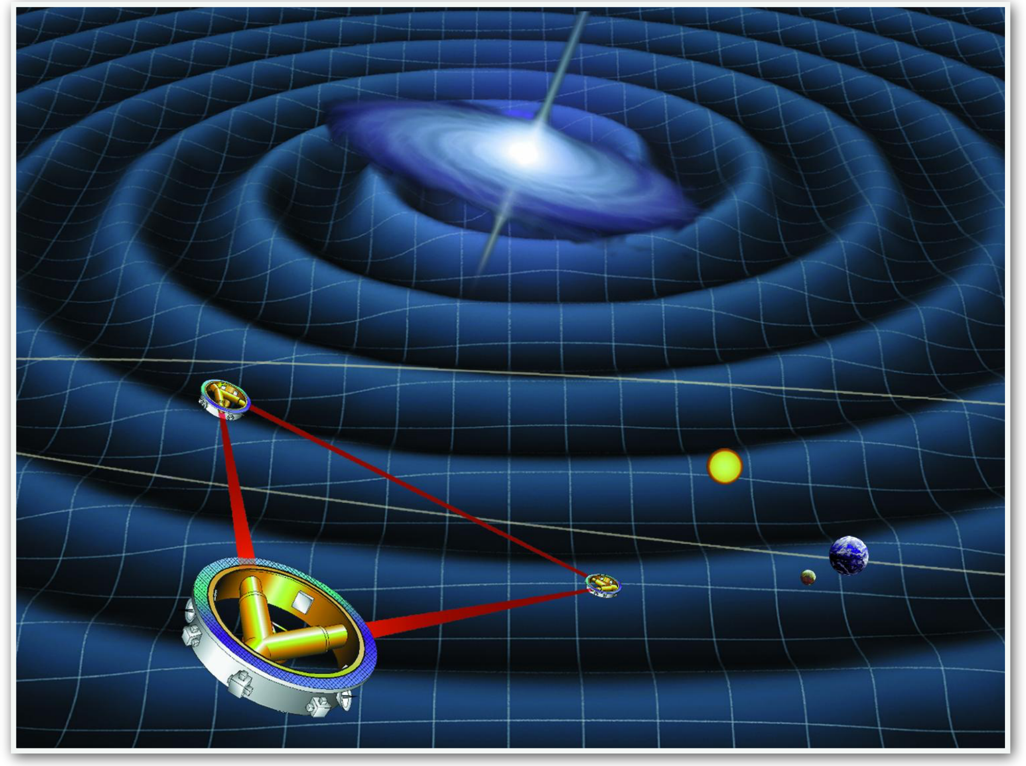
LIGO

In a population of BBH mergers, with the upcoming improvements.



LISA

From individual events.

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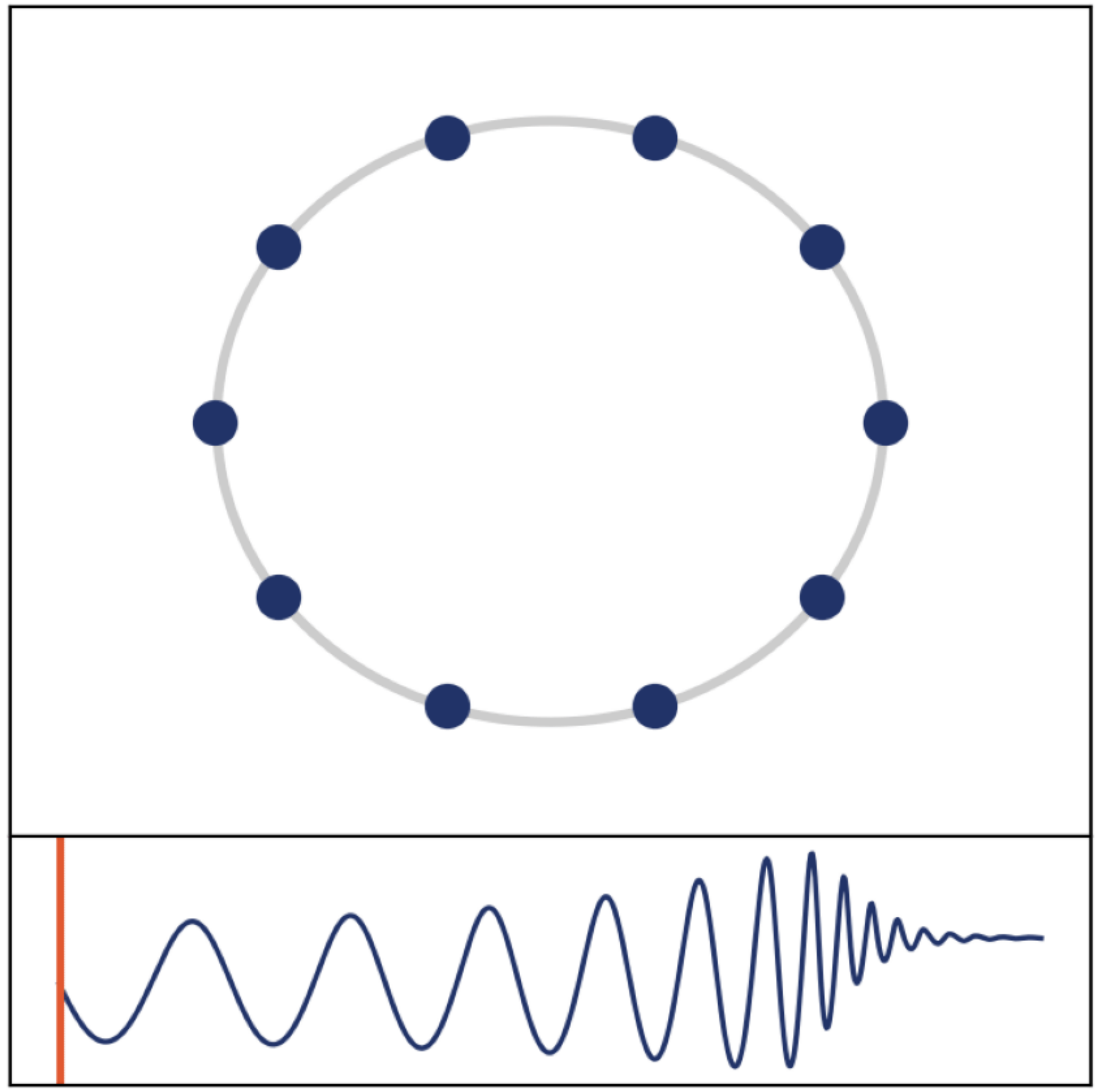

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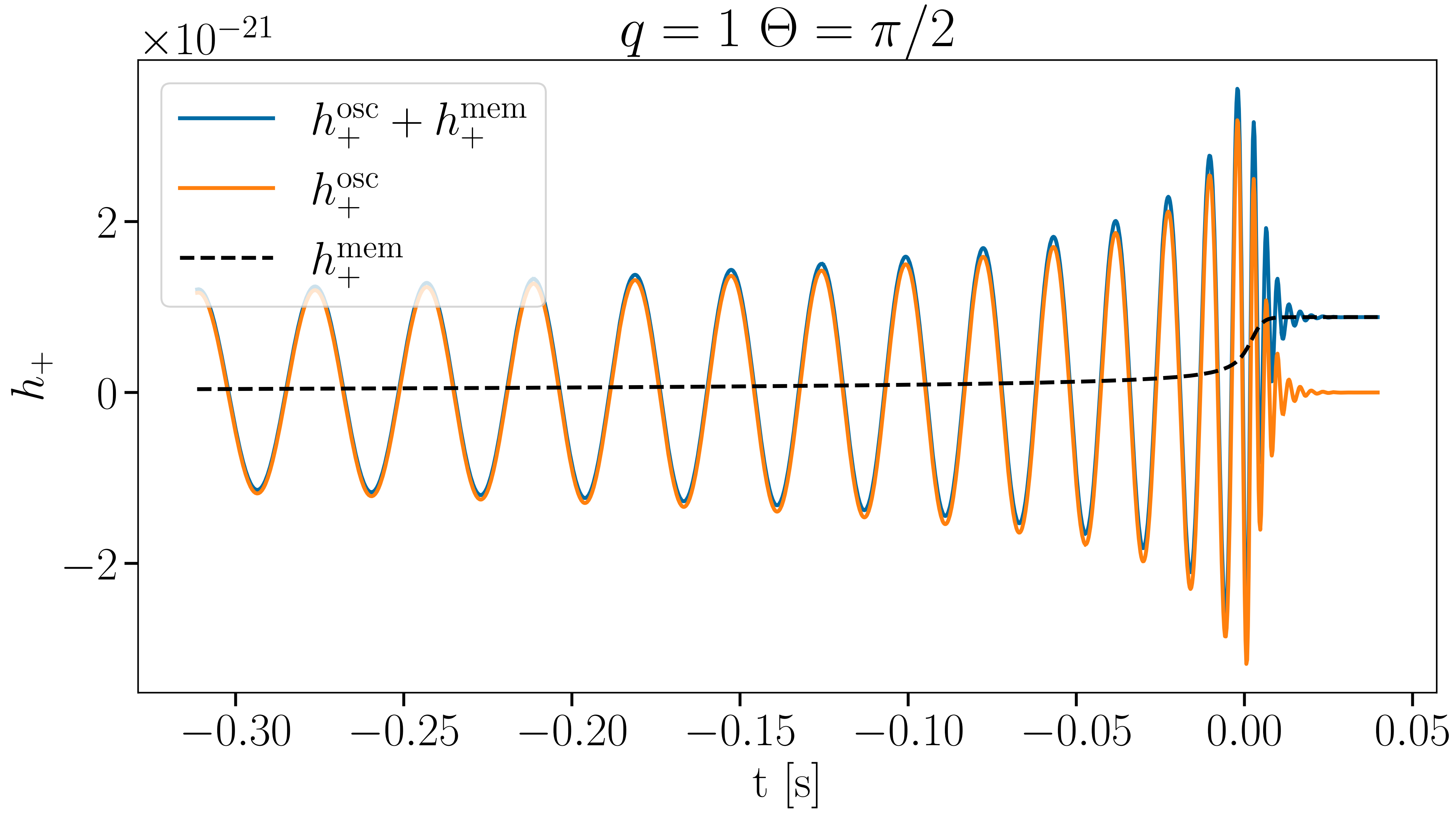

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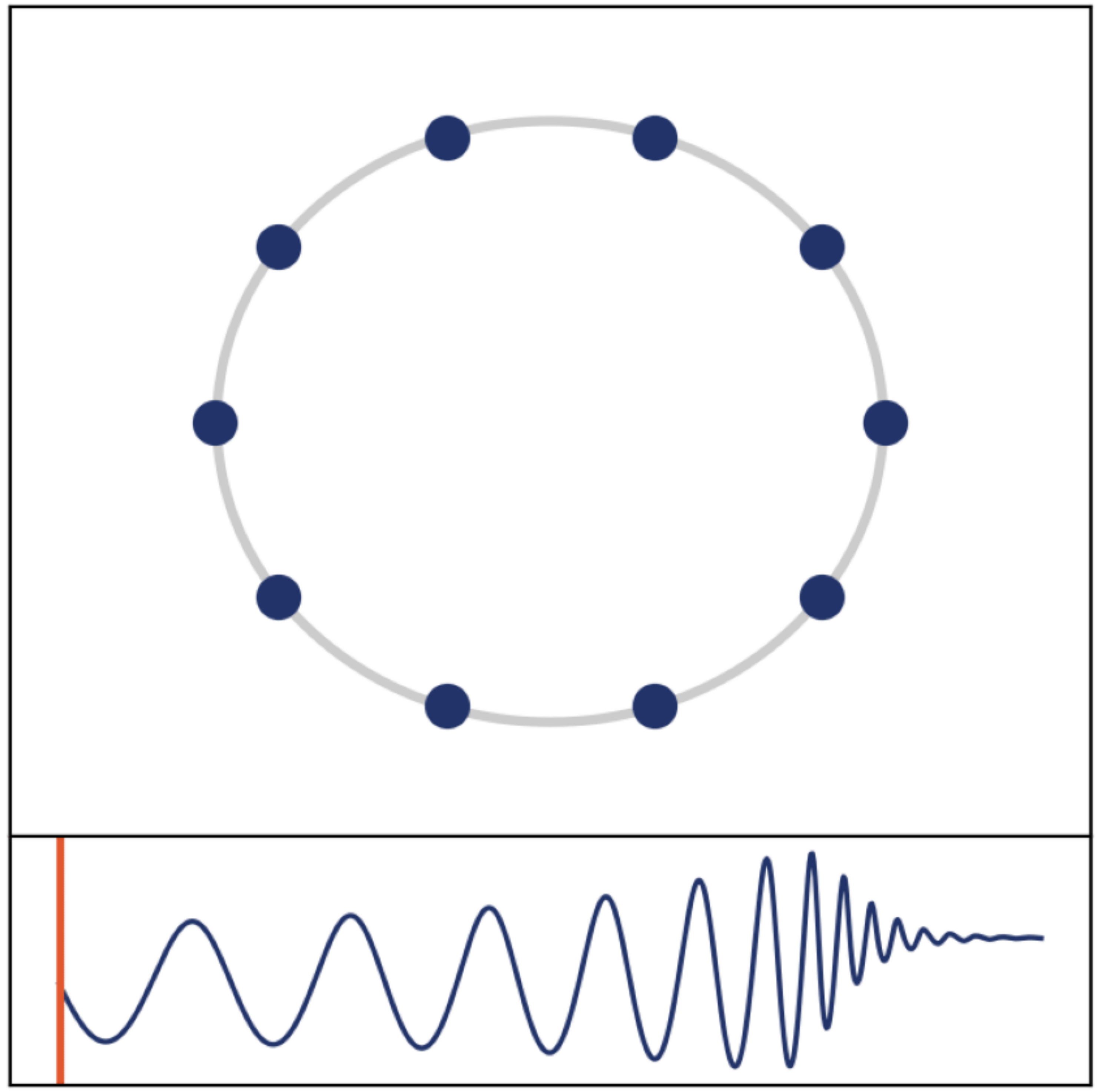

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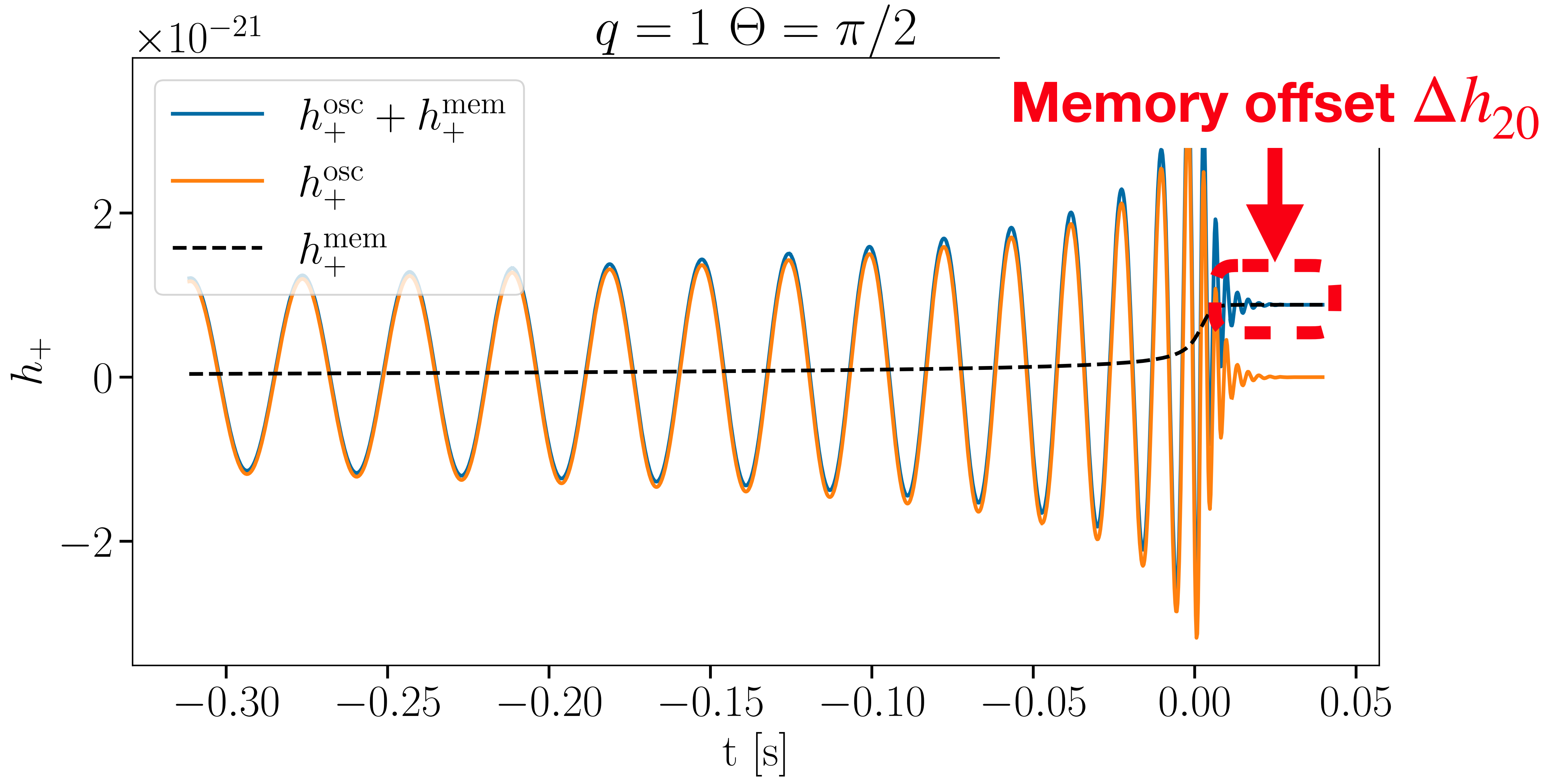

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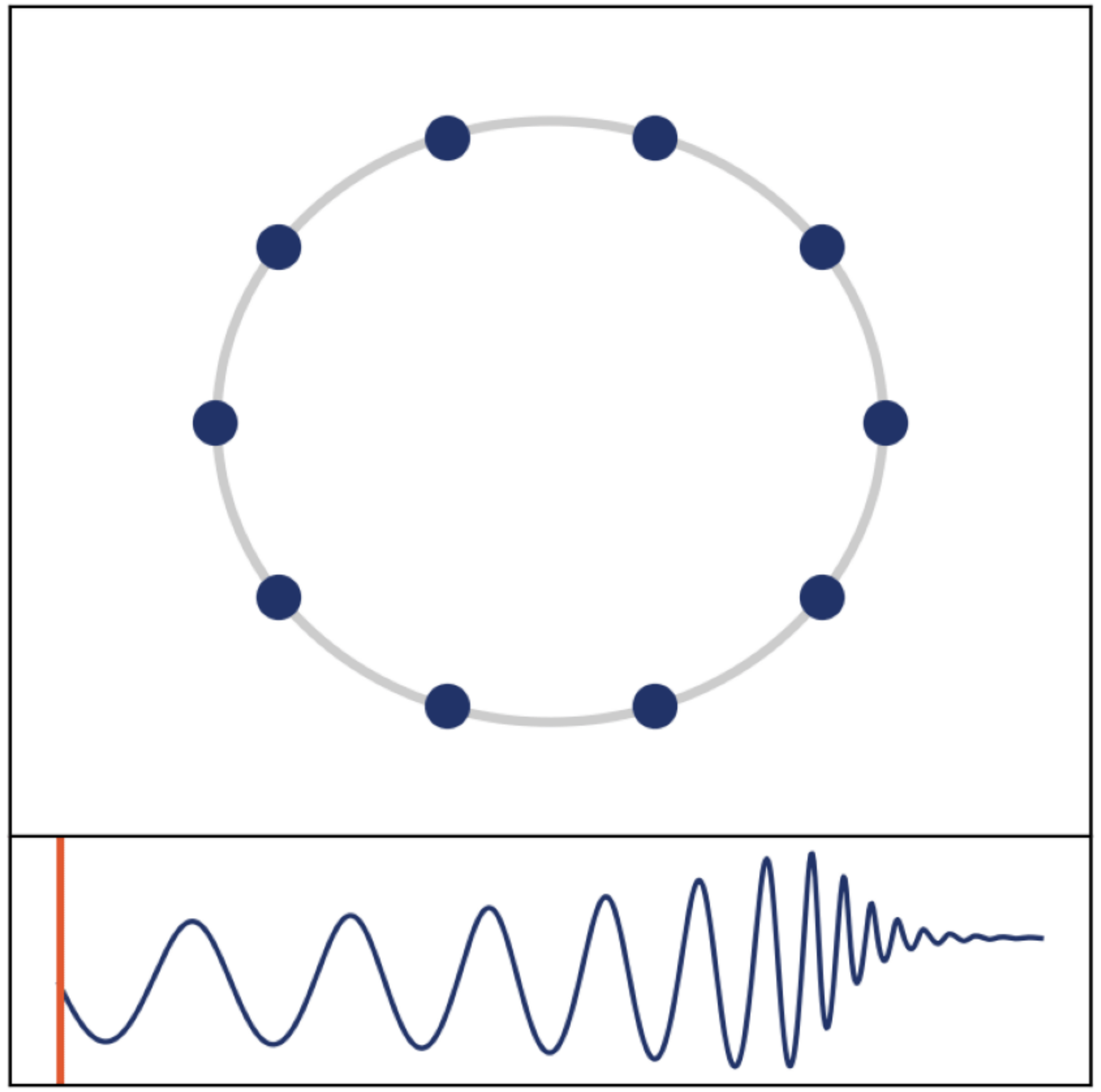

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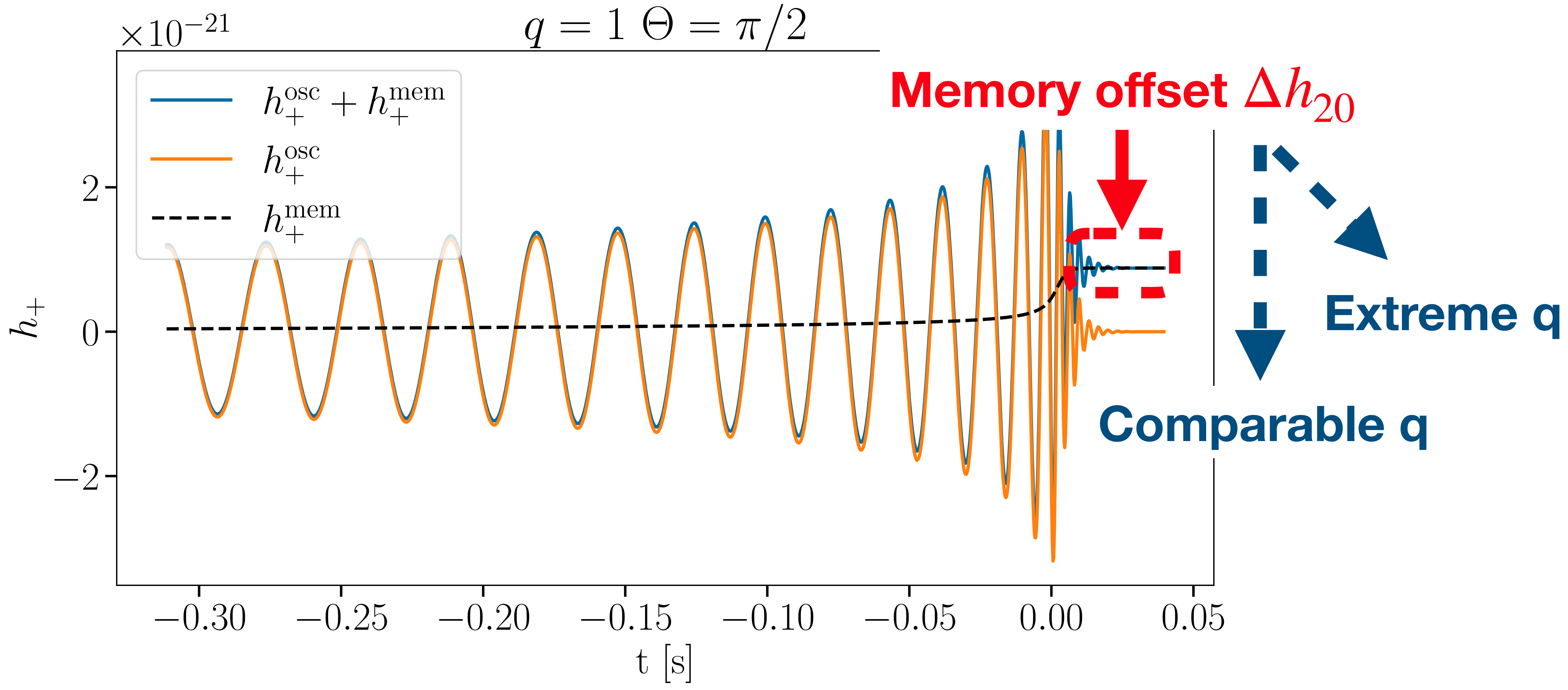

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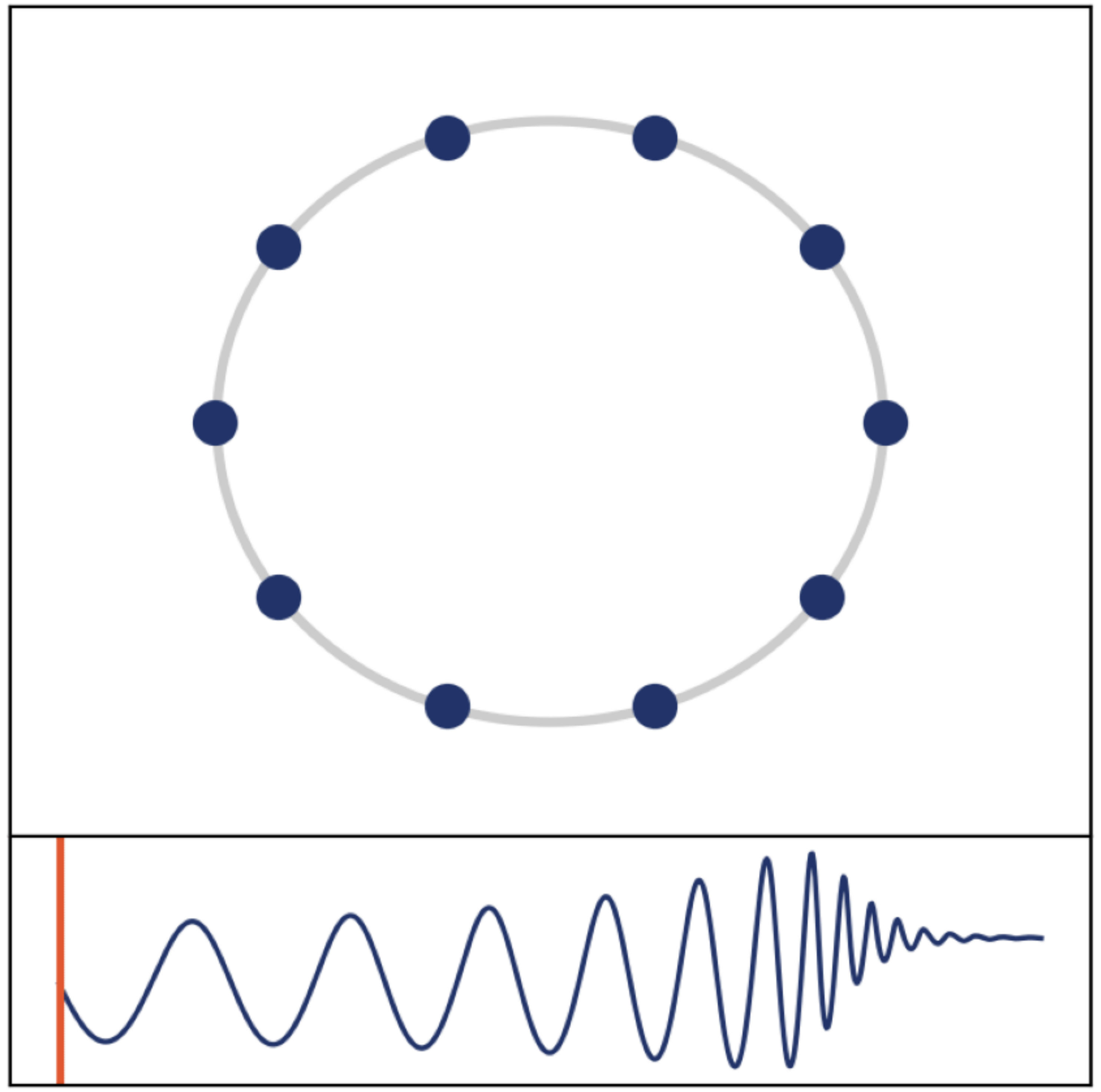

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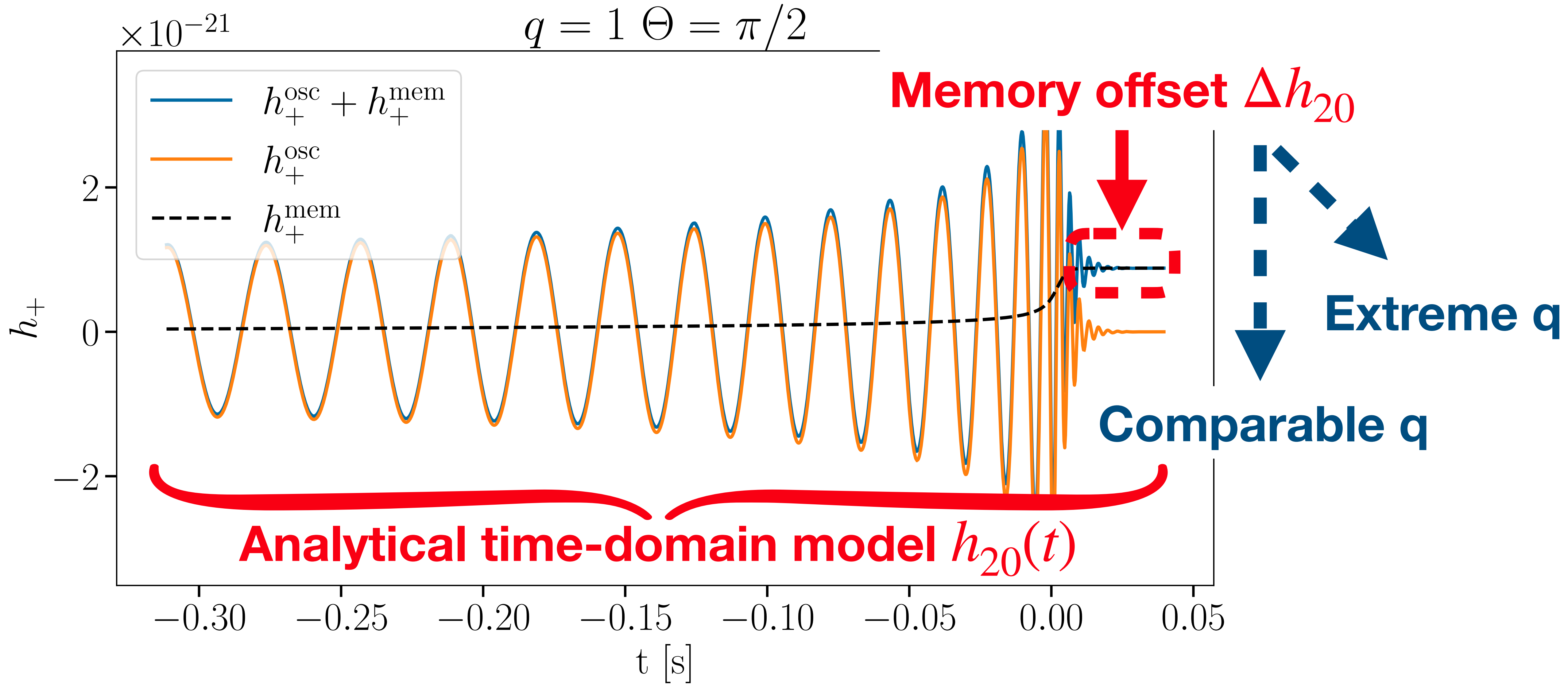

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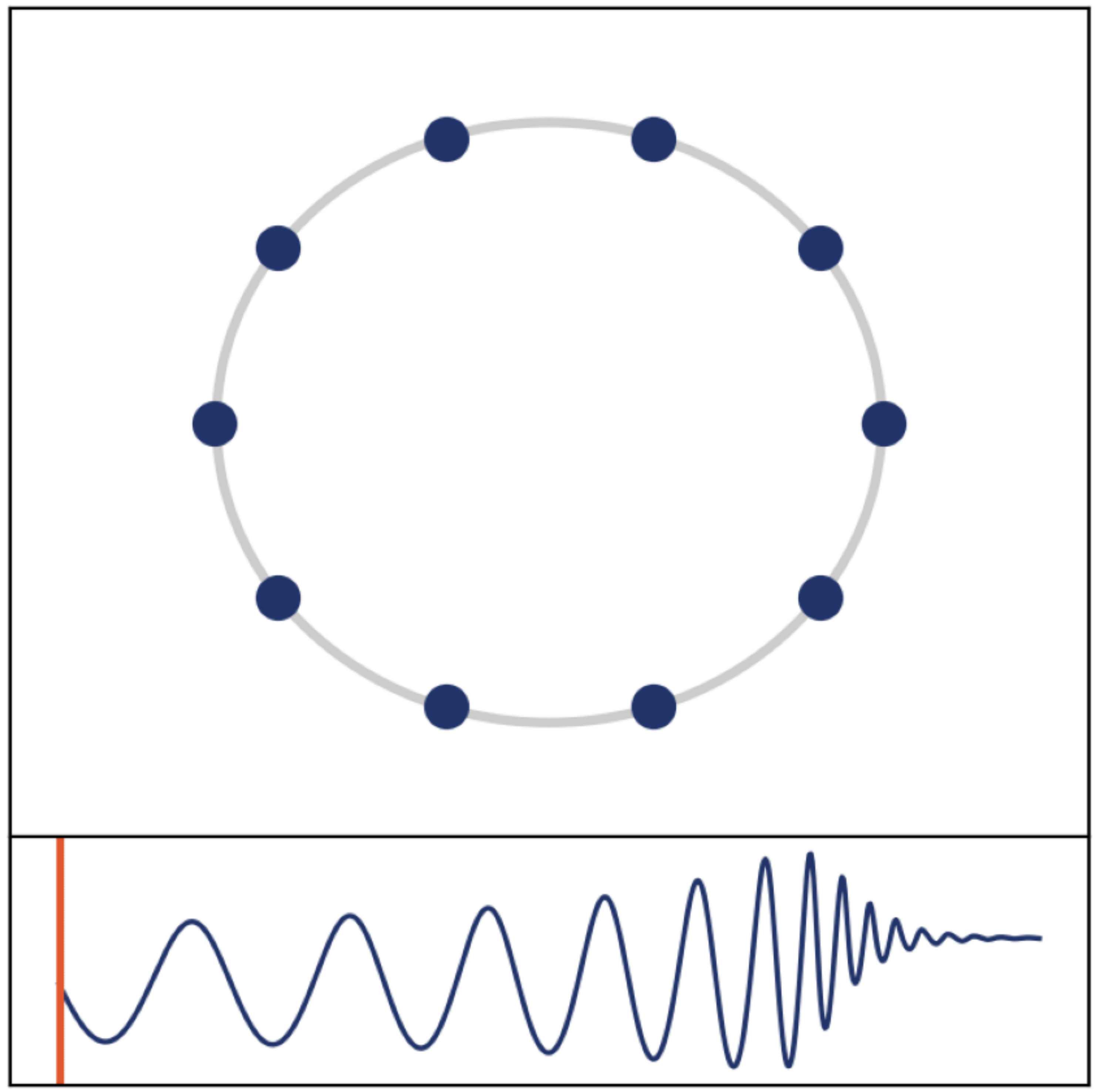

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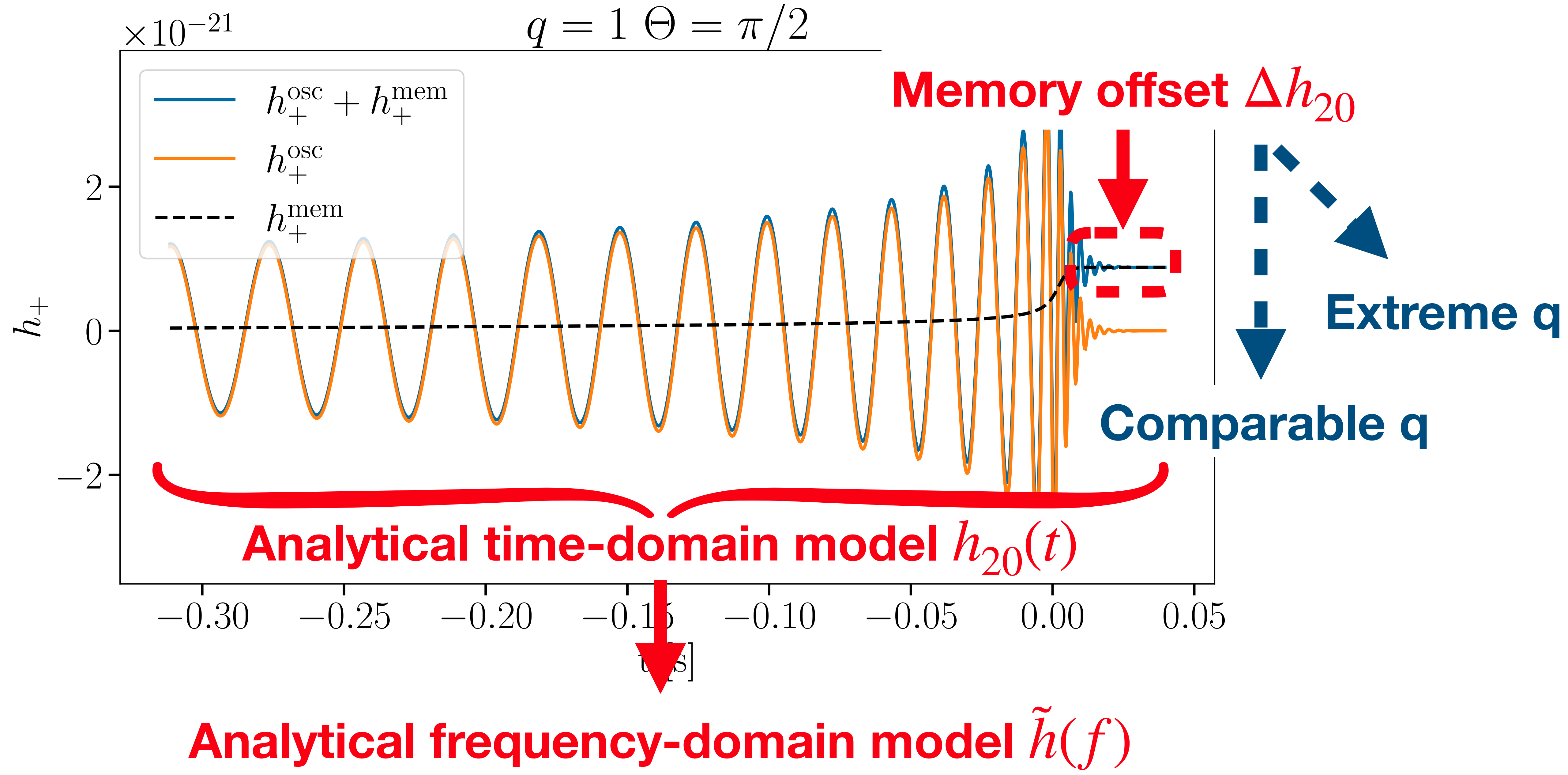

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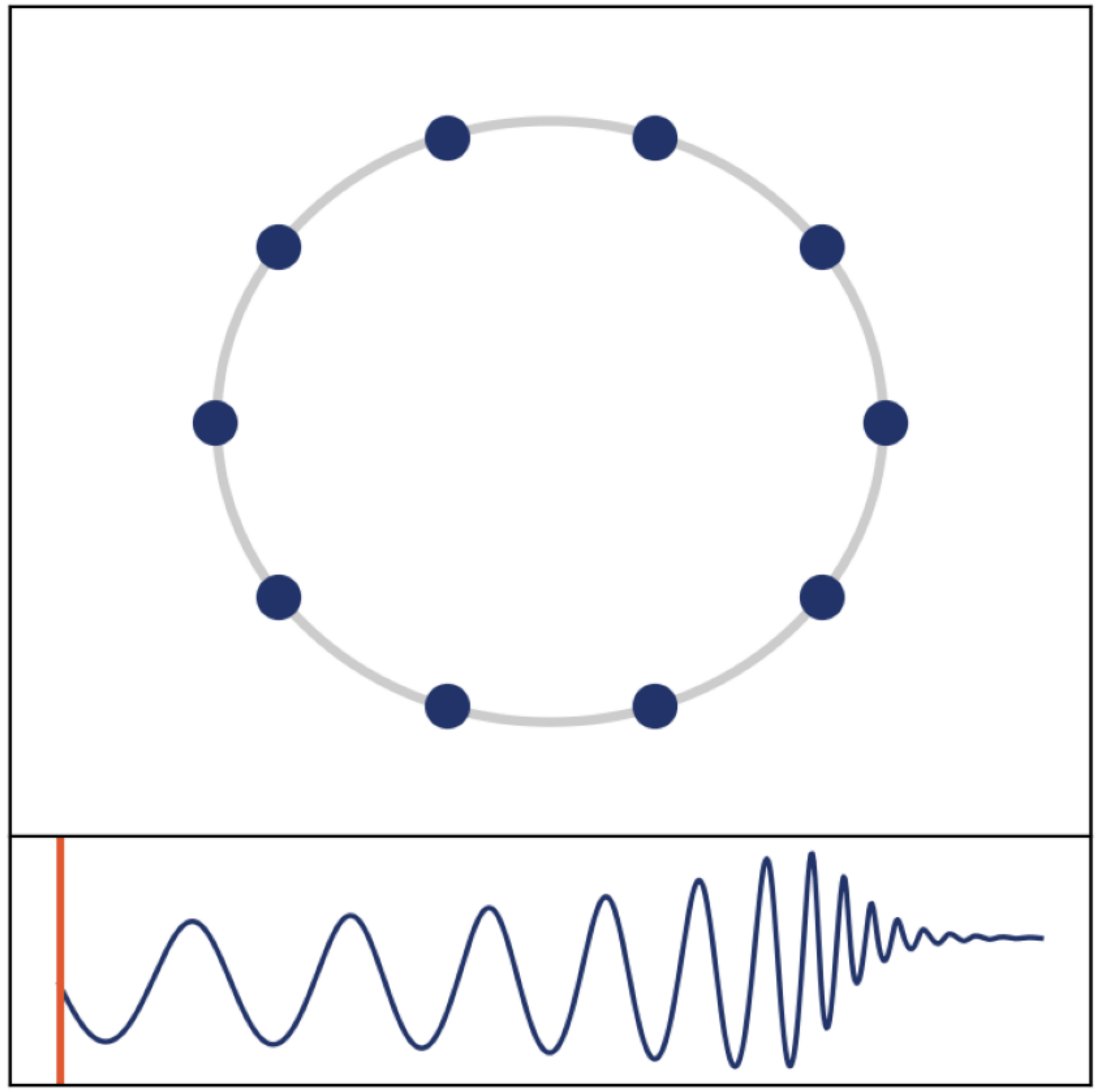

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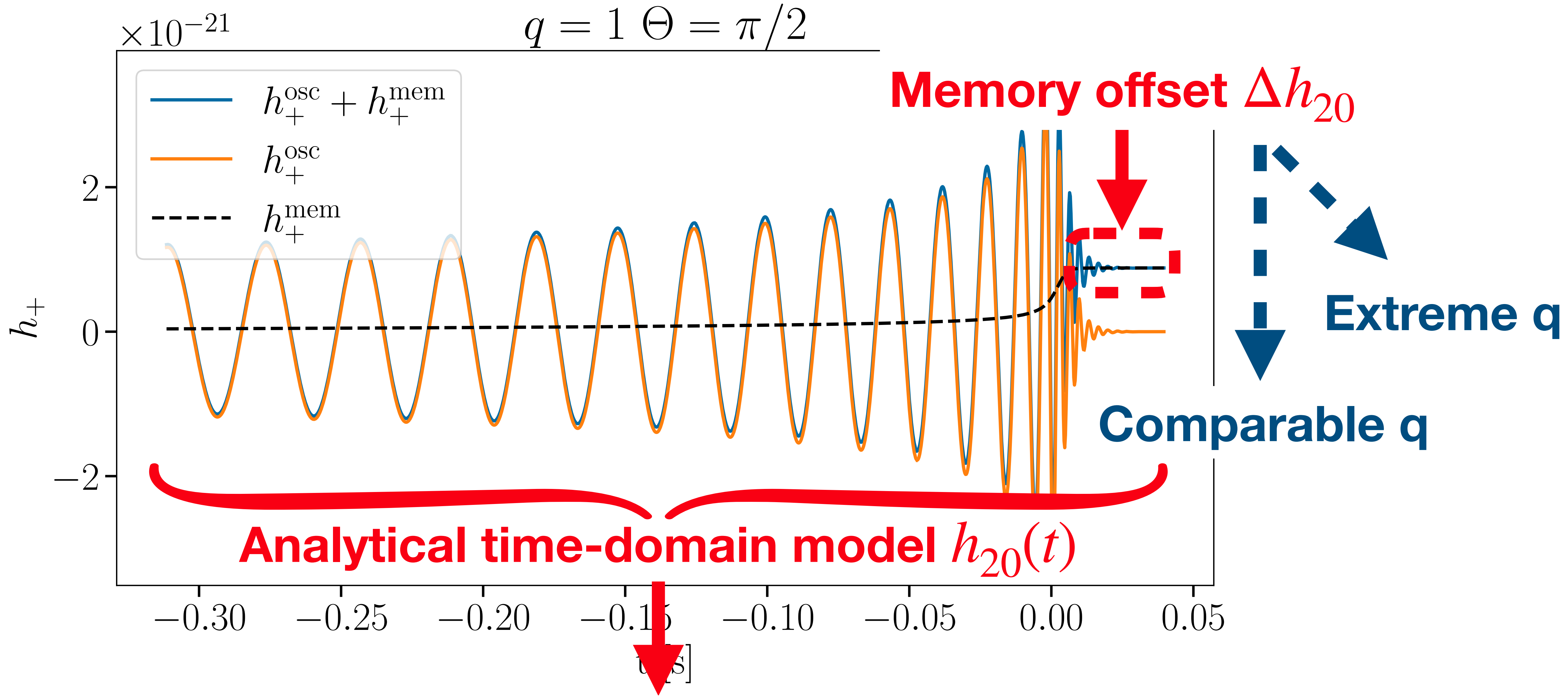

Analytical frequency-domain model $\tilde{h}(f)$

& (Faster) Phenomenological frequency-domain model $\tilde{h}(f)$

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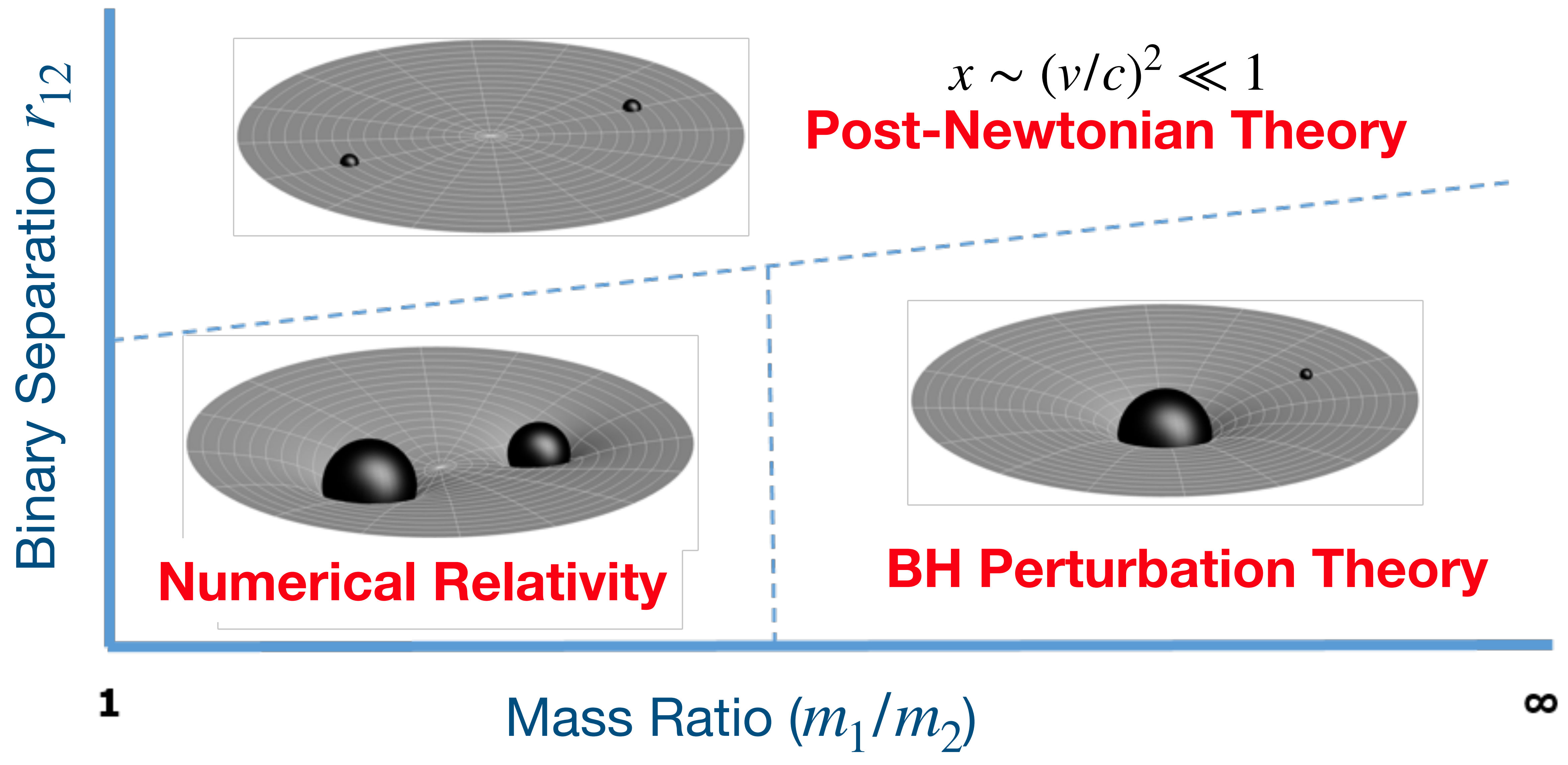

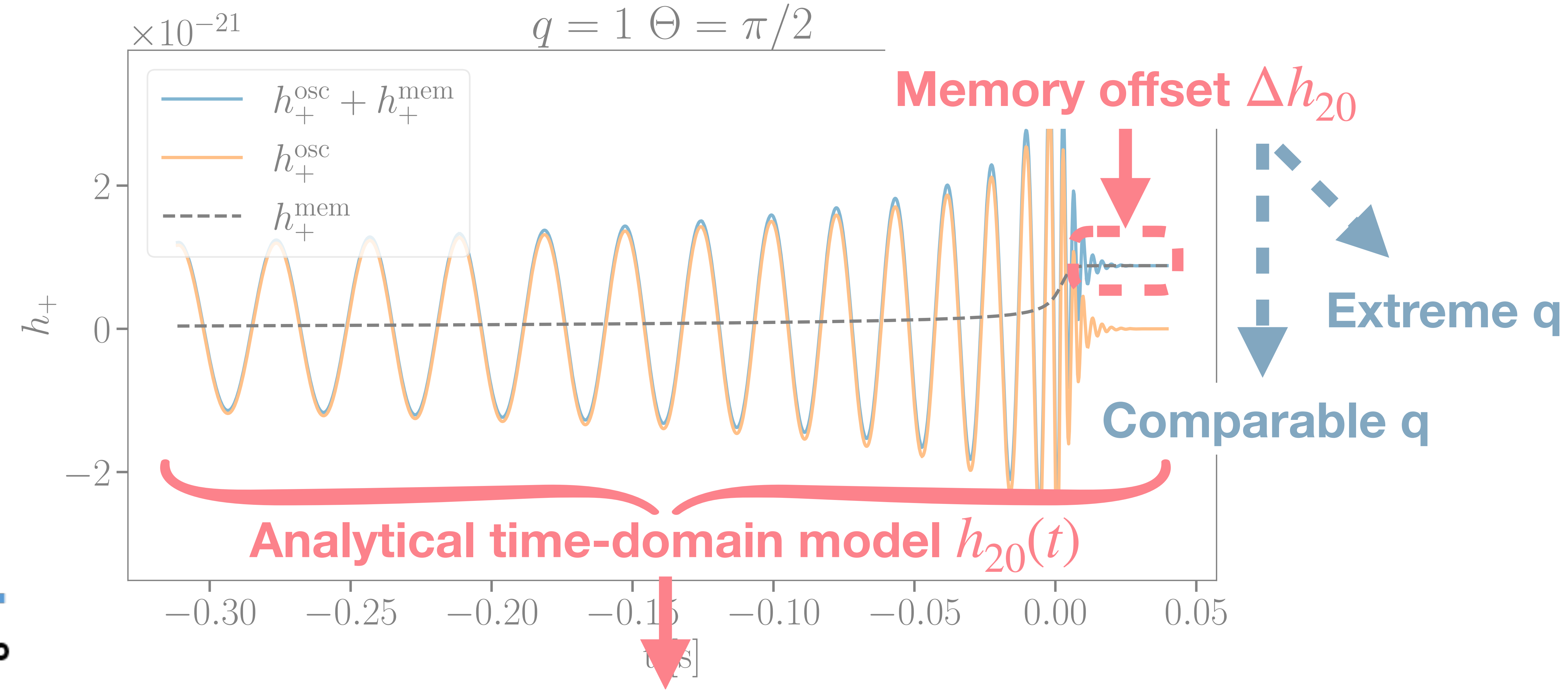

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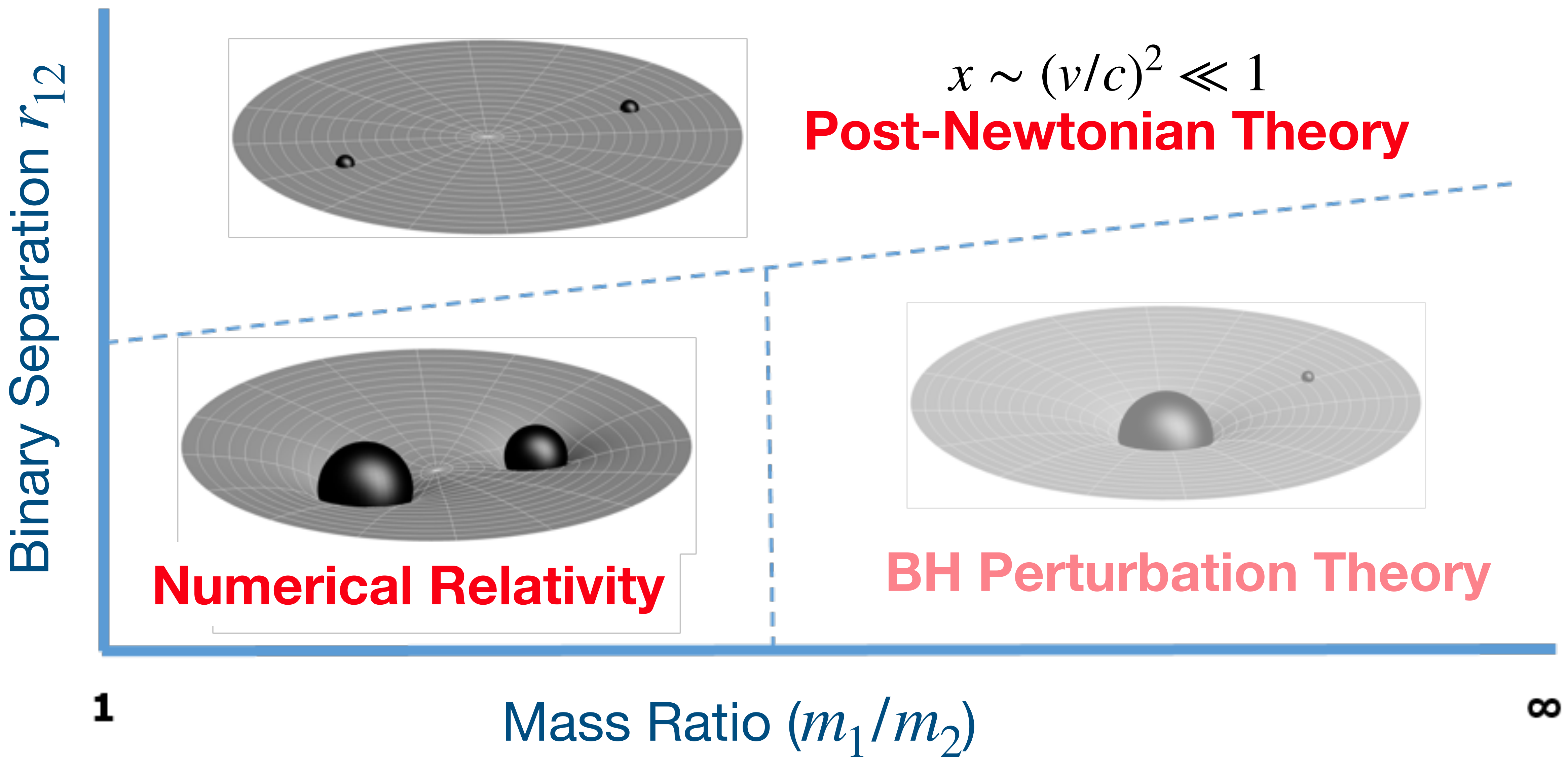

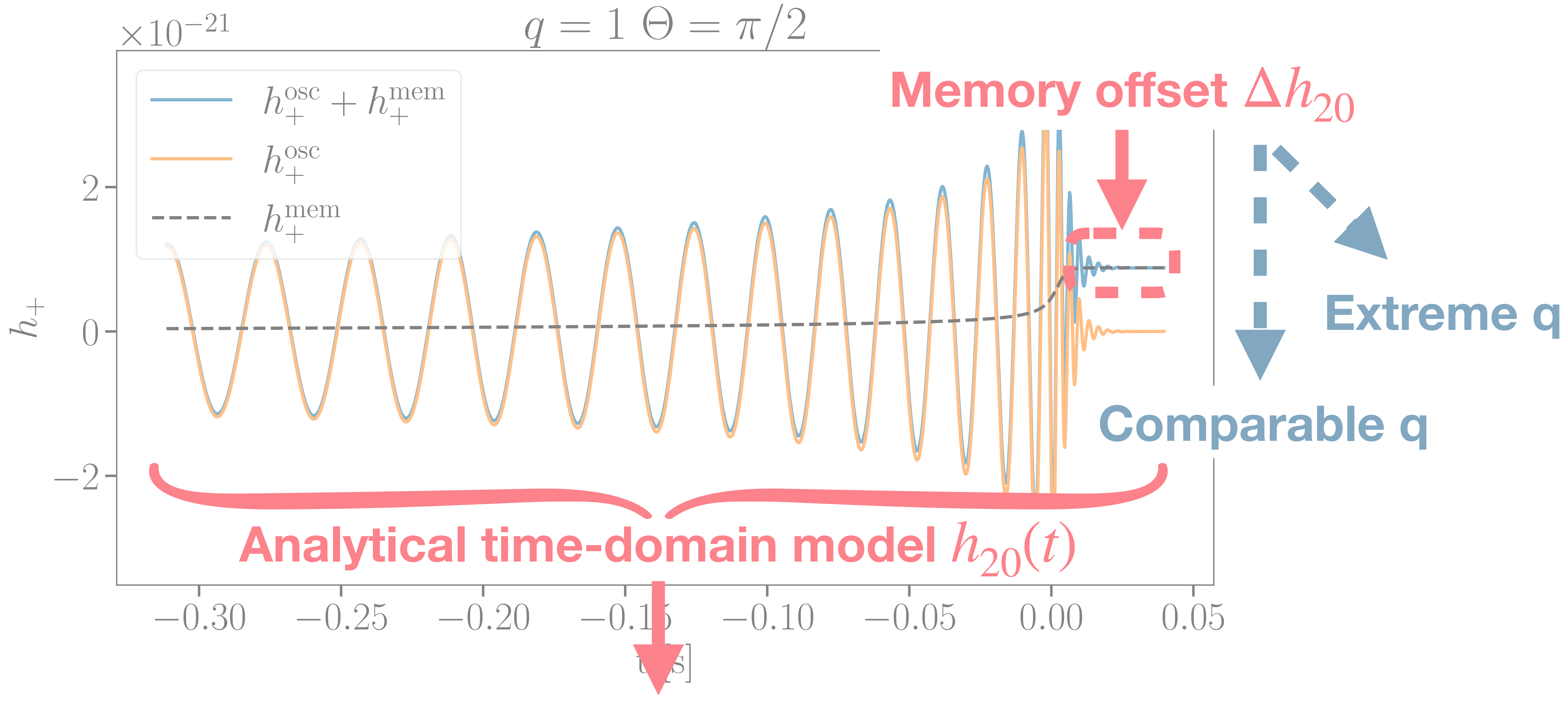

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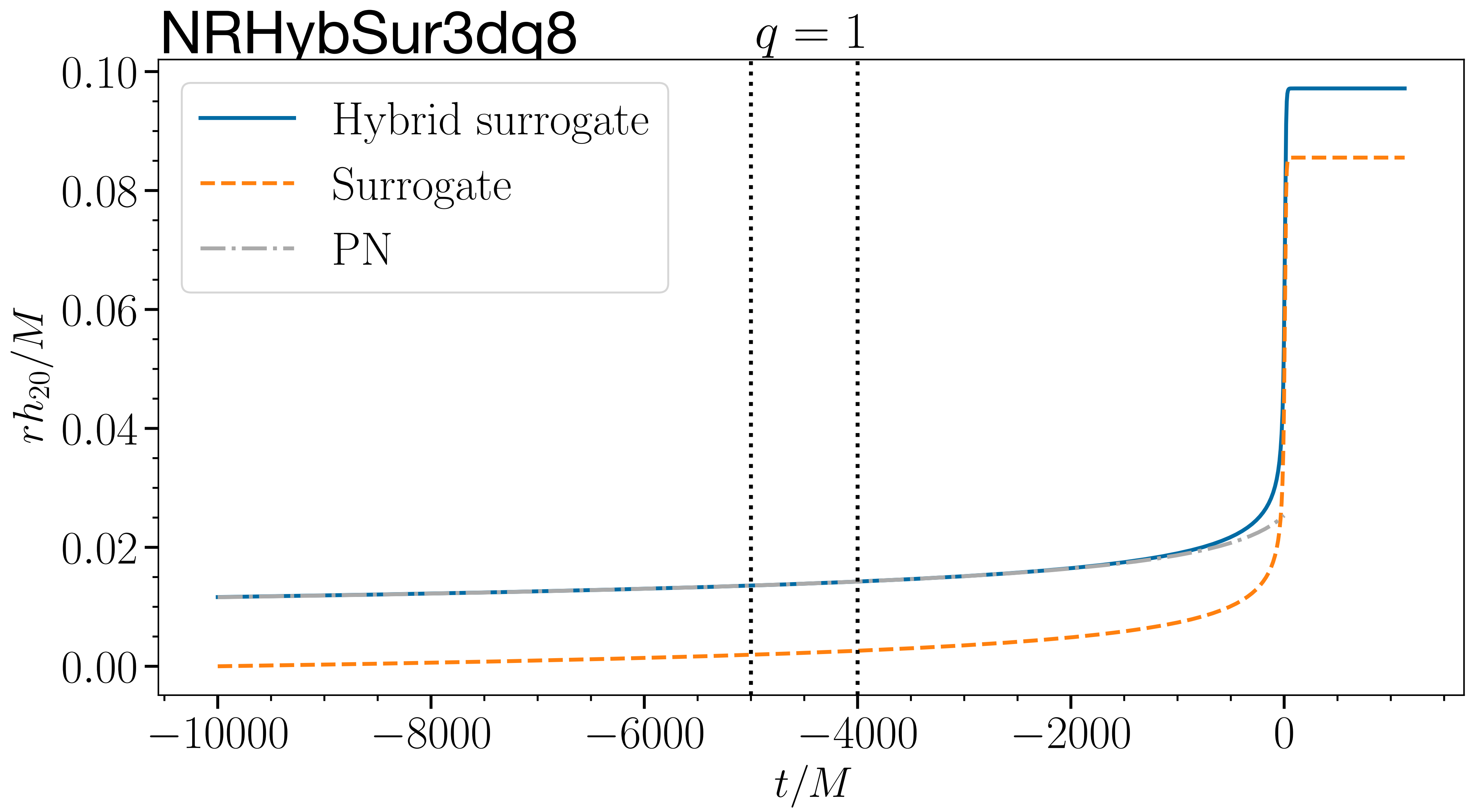

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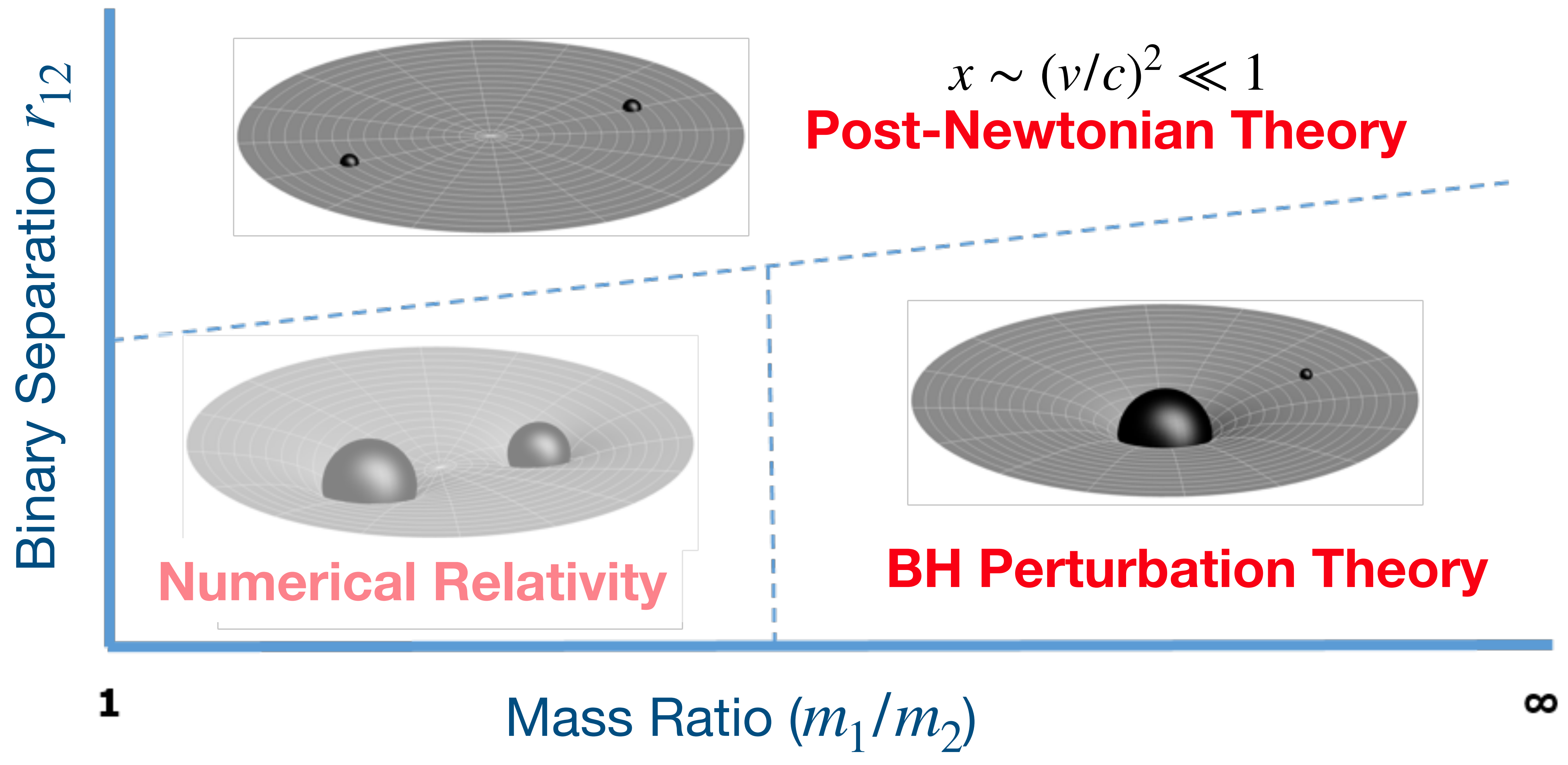

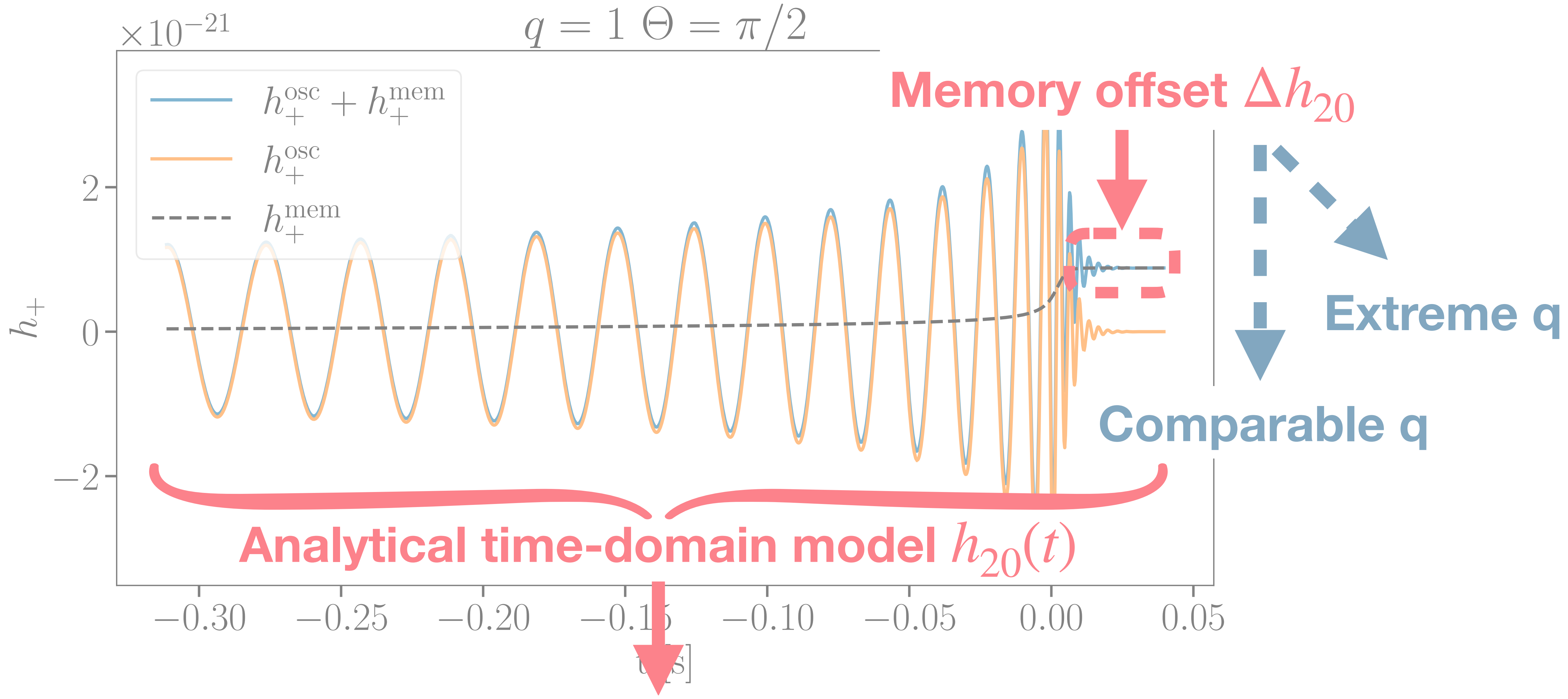

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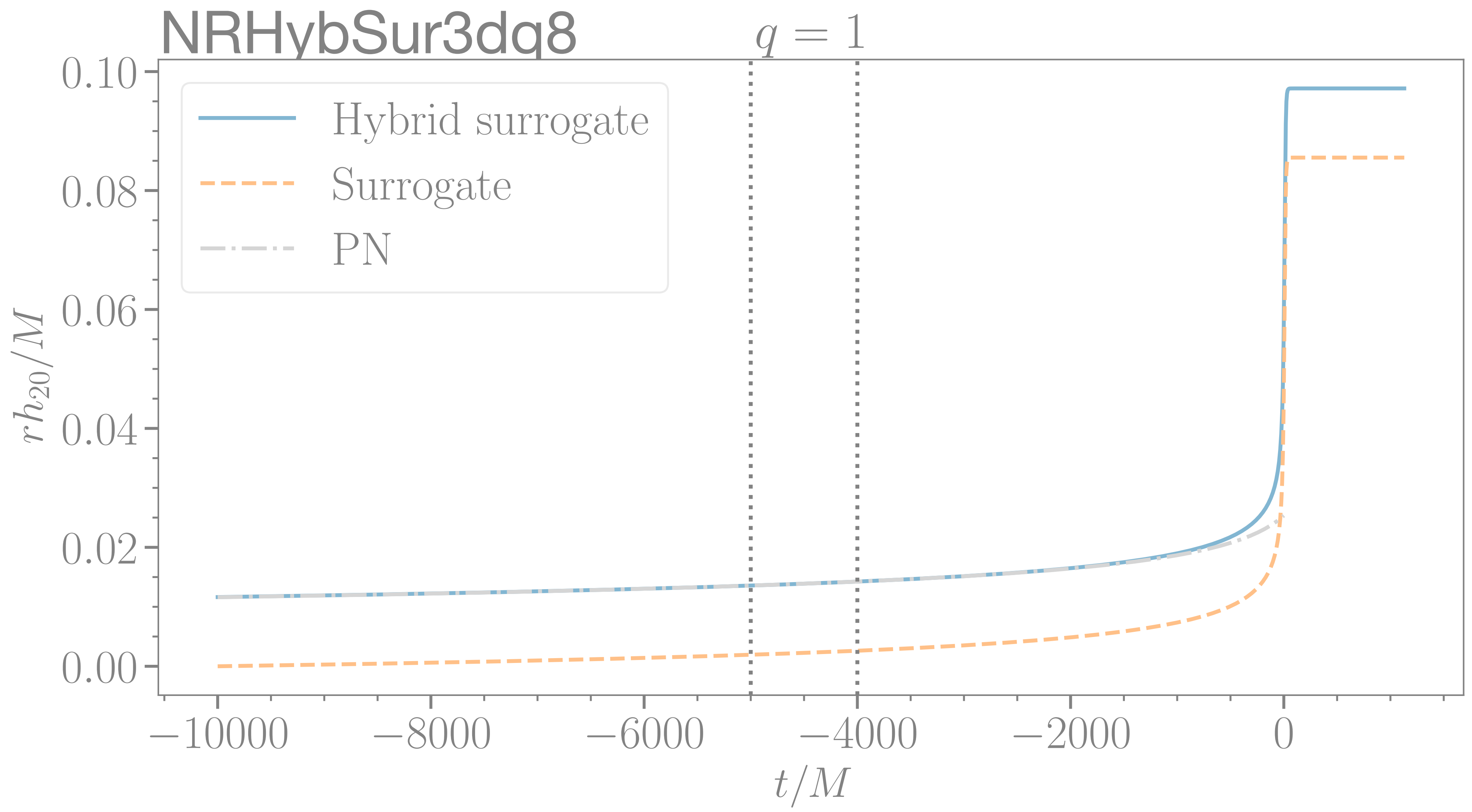
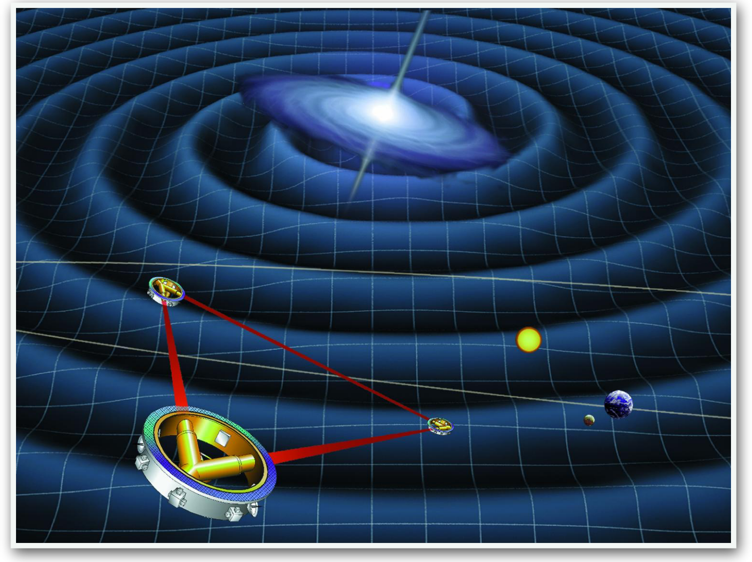
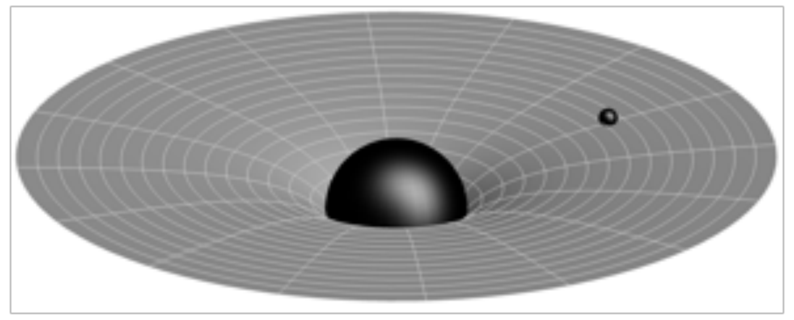




Memory signal from EMRIs



Adiabatic inspiral

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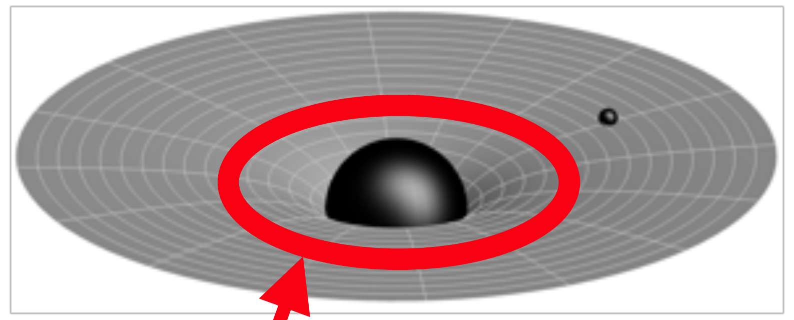
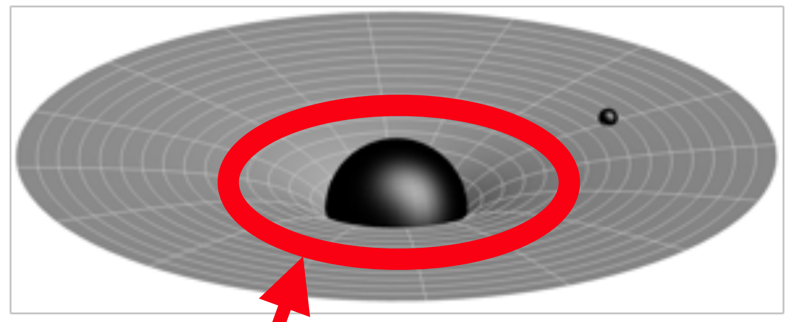


Adiabatic inspiral

Innermost stable circular orbit
(ISCO)

Memory signal from EMRIs

- We used resummed, factorized waveforms h_{lm} $(m \neq 0)$ computed by Fujita for a test particle orbiting a Schwarzschild BH on a quasicircular orbit.
- Computed the 22PN order memory for extreme mass ratio inspirals.



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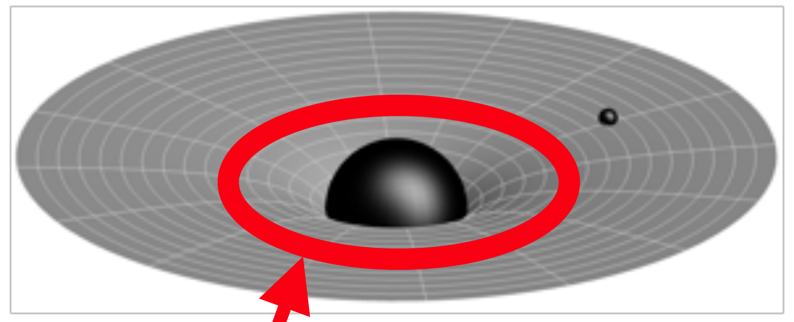

Adiabatic inspiral

$$\eta = m_1 m_2 / M^2 = q/(q+1)^2$$

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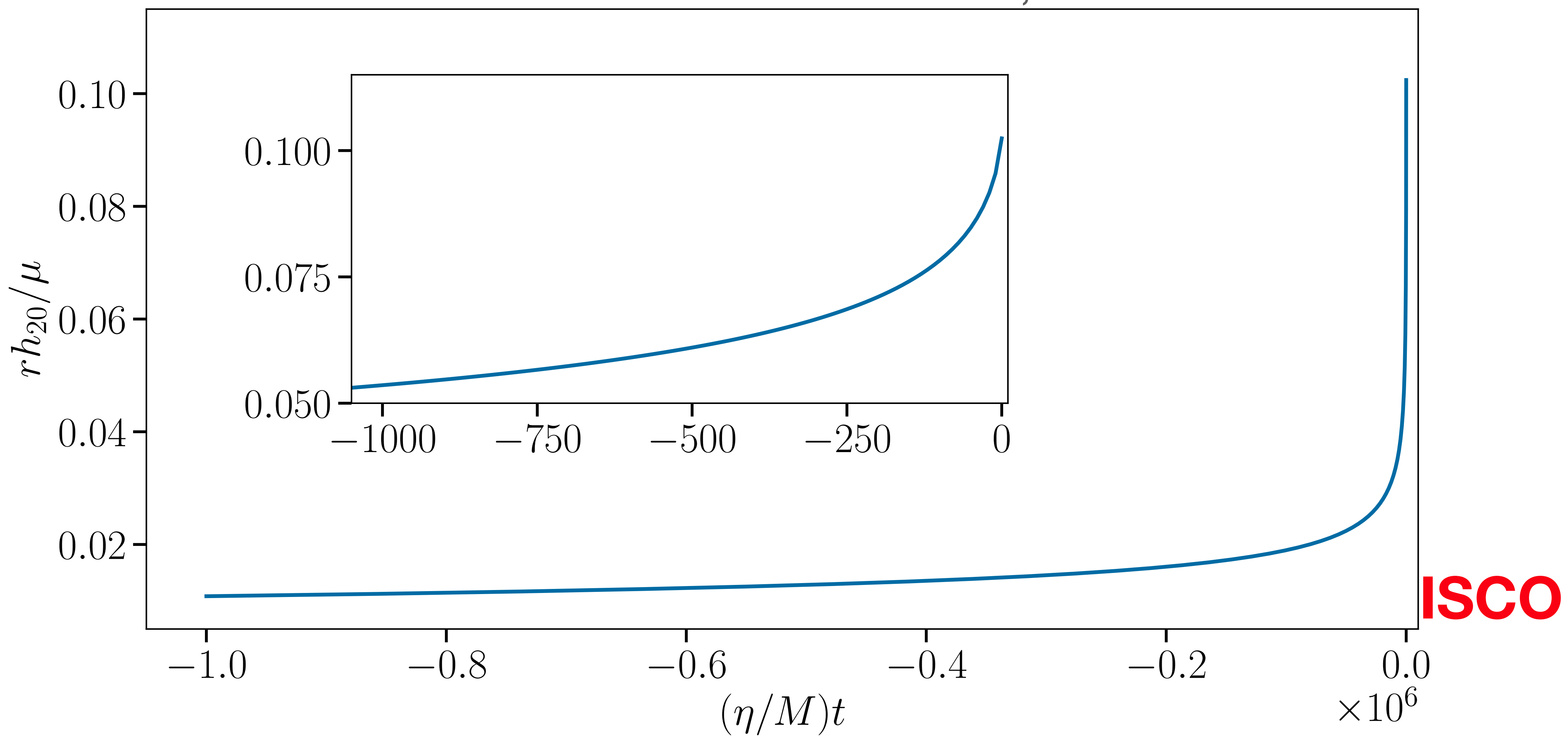
Innermost stable circular orbit (ISCO)

A.E. & David Nichols, arXiv:**2407.19017**



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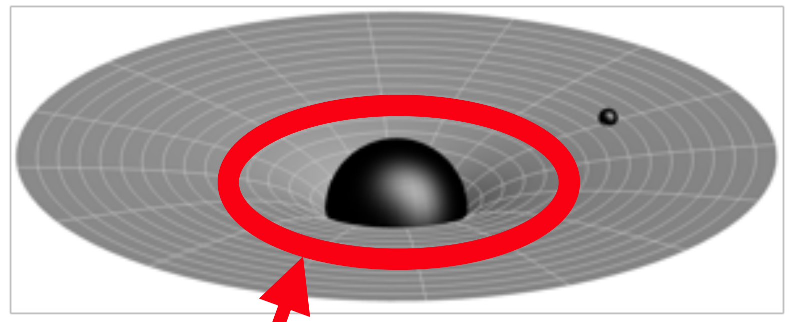

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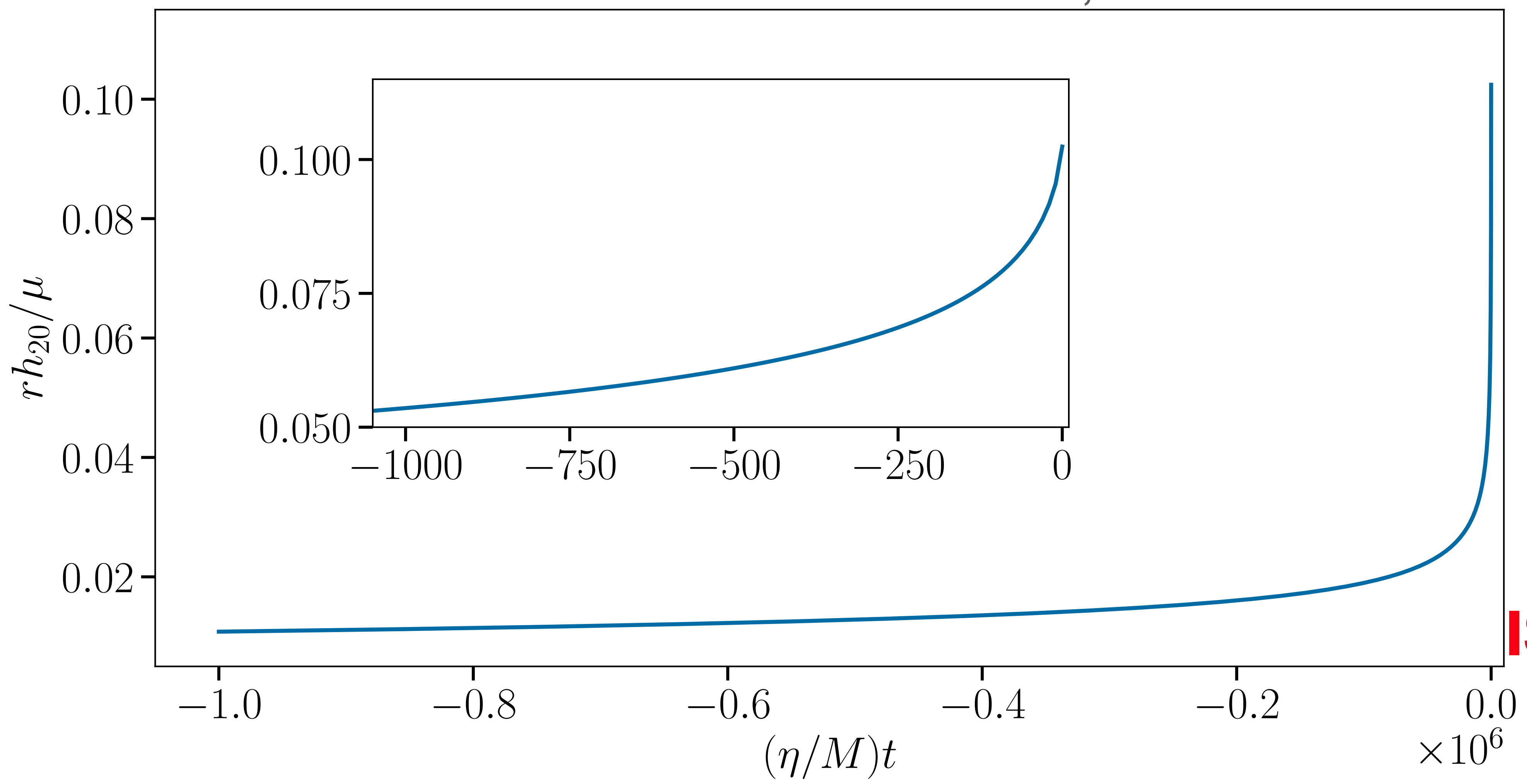

- We fit the memory to a polynomial function and fix the first parameter (c_1) to cover extreme mass ratios ($\eta \ll 1$).

$$\Delta h_{20} = \frac{M}{r} \sum_{i=1}^{6} c_i \eta^i$$

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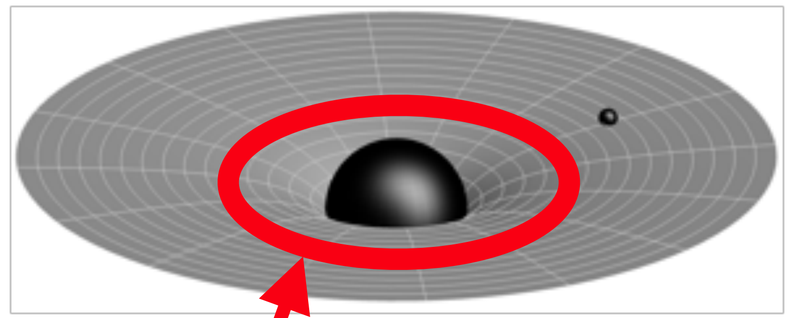

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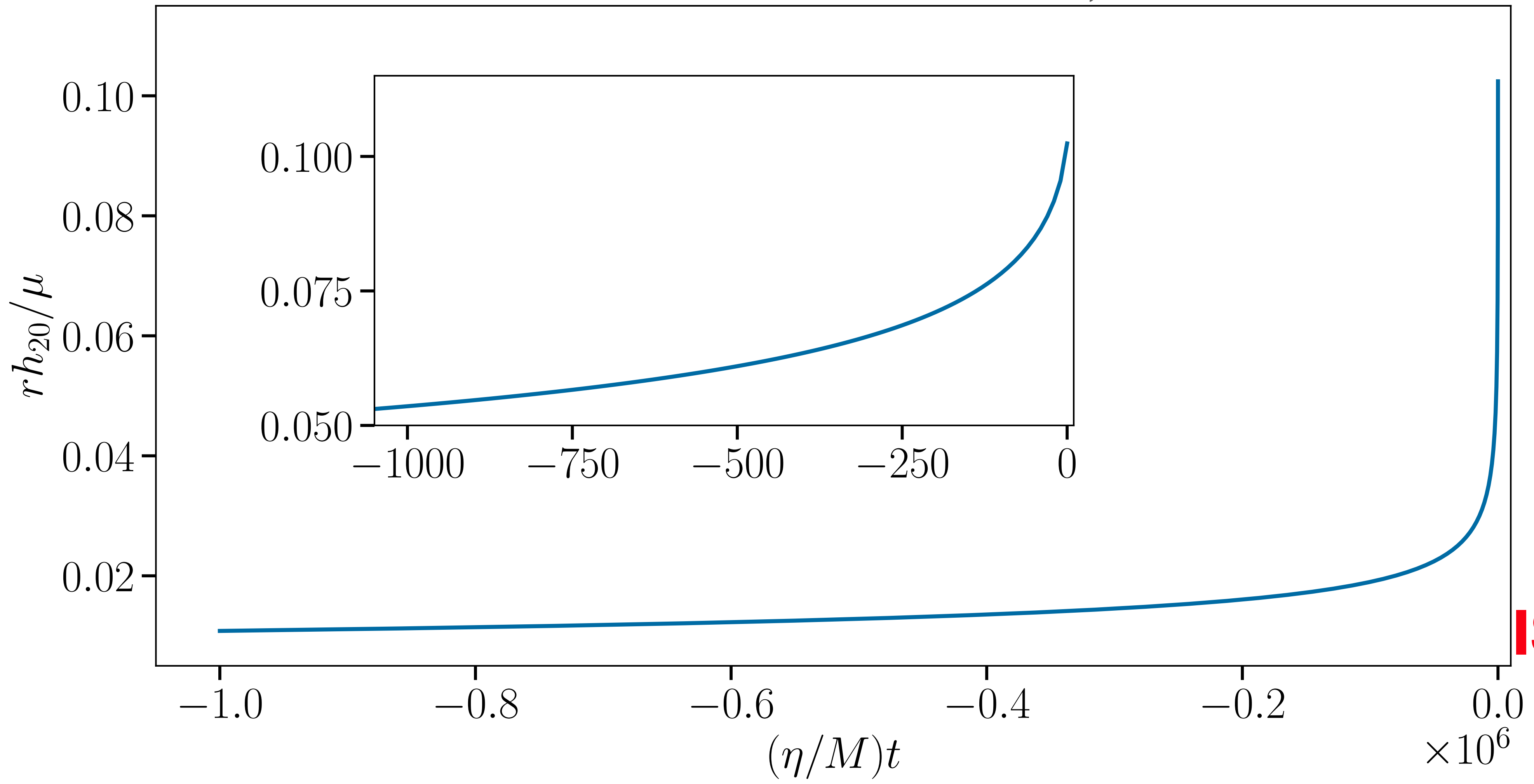

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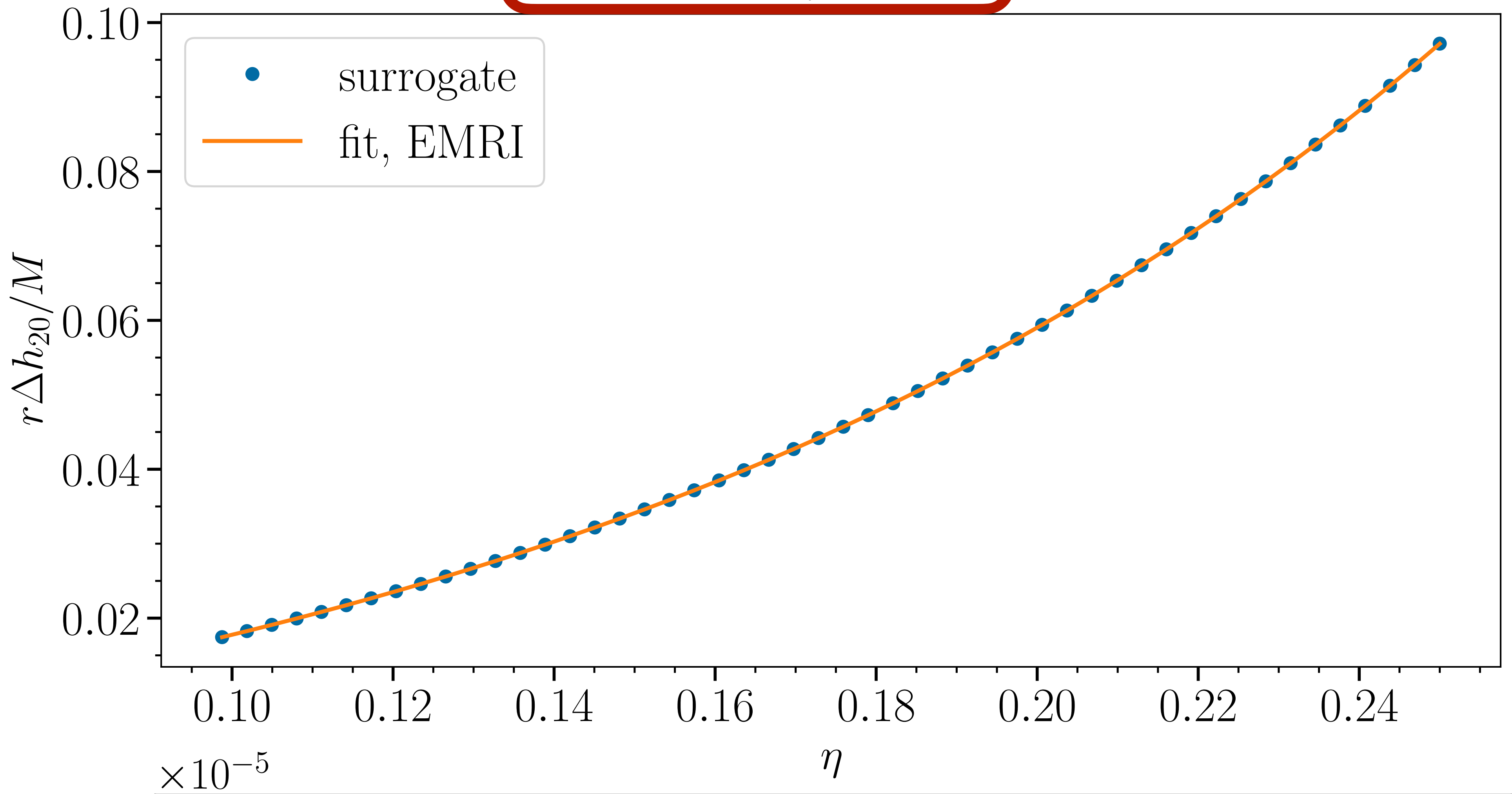
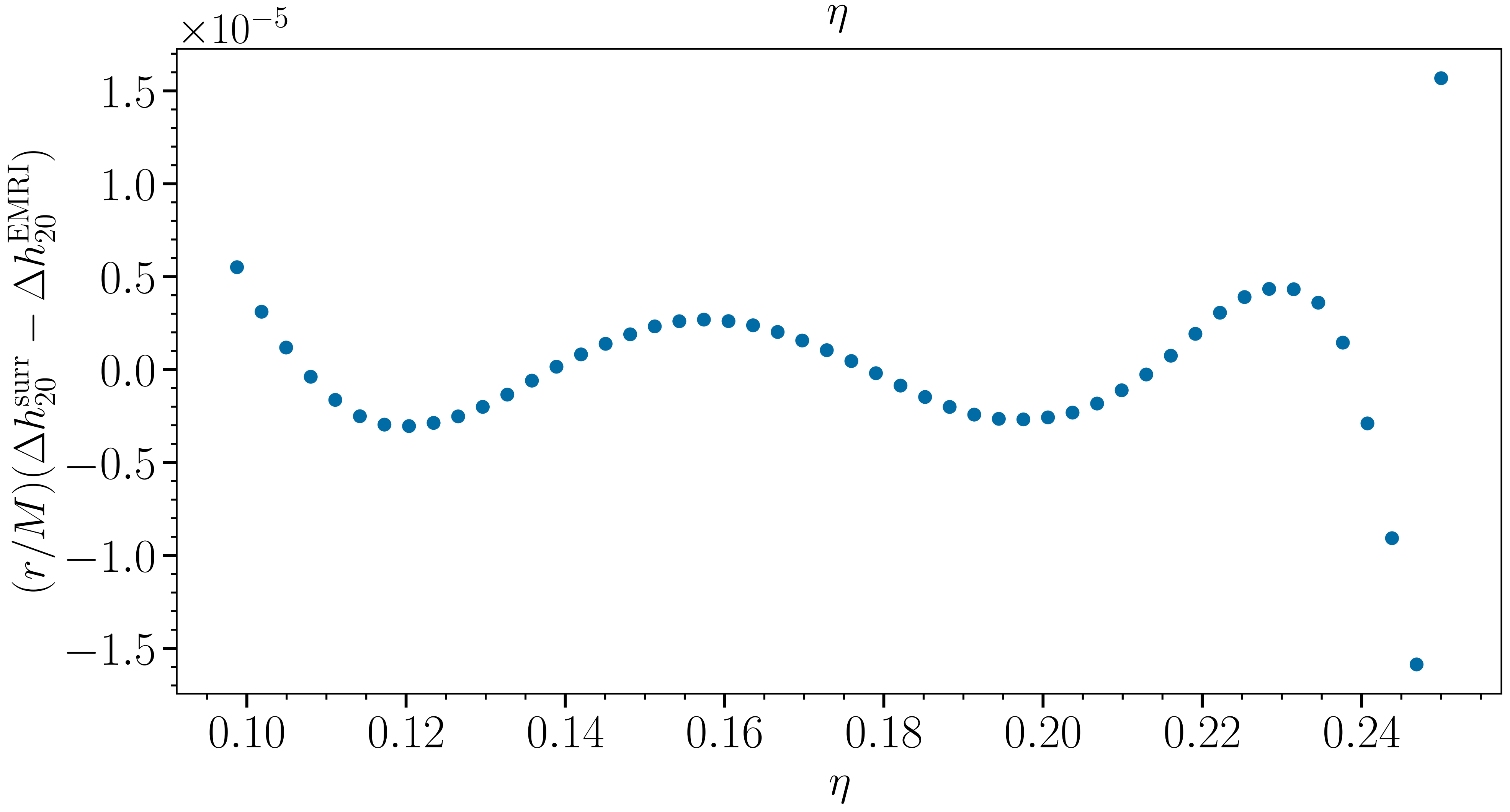
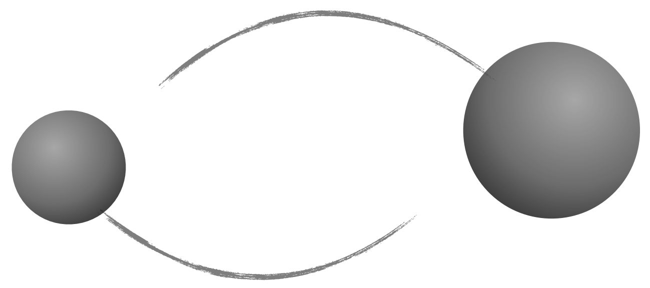


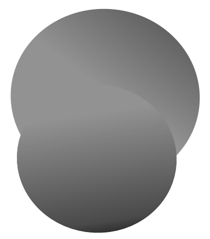


Time-domain model

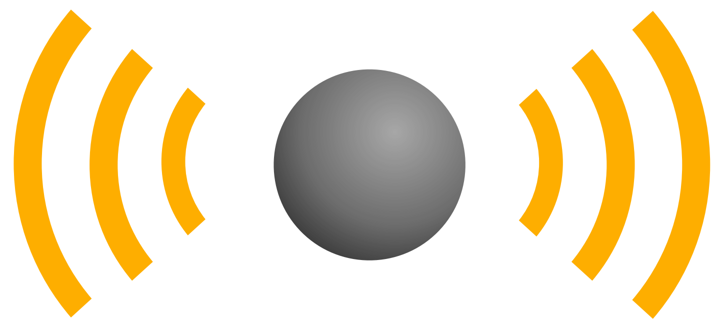
Inspiral



Intermediate

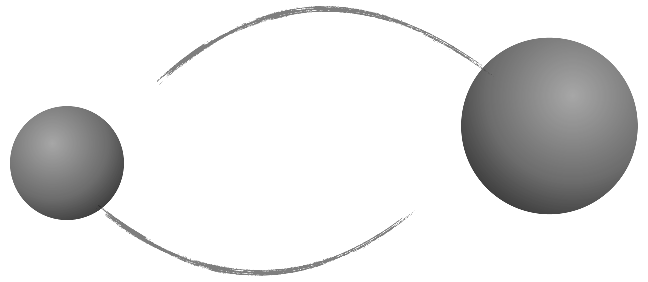


Ringdown



Inspiral

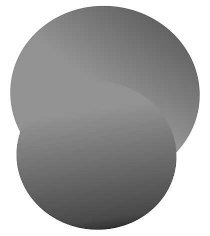
Post-Newtonian Theory



$$h_{20}^{\mathrm{insp}}(t) = h_{20}^{3.5\mathrm{PN}}(x(t))$$

$$x(t) = \frac{1}{4}\left[\frac{\eta}{5M}(t_c - t)\right]^{-1/4}, \; t_c = \sum_{i=0}^{4} t_{c,i}\,\eta^i$$

Intermediate



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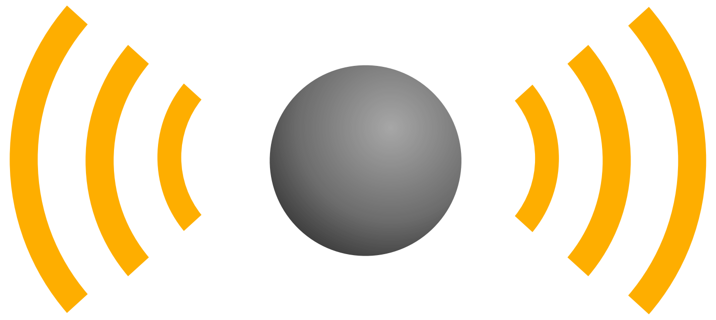

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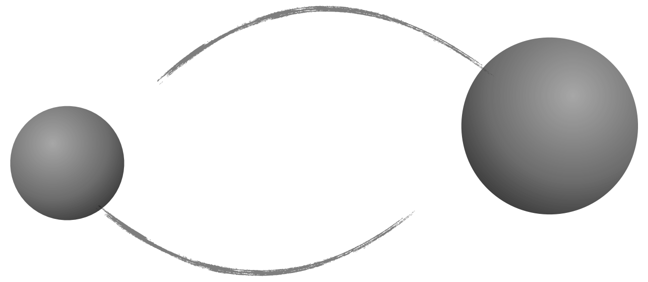

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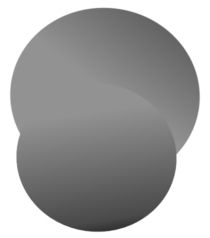
$$C[h_{20}^{\text{surr}}, h_{20}^{\text{insp}}] = \sum_{q=1}^{8} \frac{\int_{t_1}^{t_2} dt\,|\,h_{20}^{surr}(t;q) - h_{20}^{\text{insp}}(t;q)\,|^2}{\int_{t_1}^{t_2} dt\,|\,h_{20}^{\text{surr}}(t;q)\,|^2}$$

Intermediate



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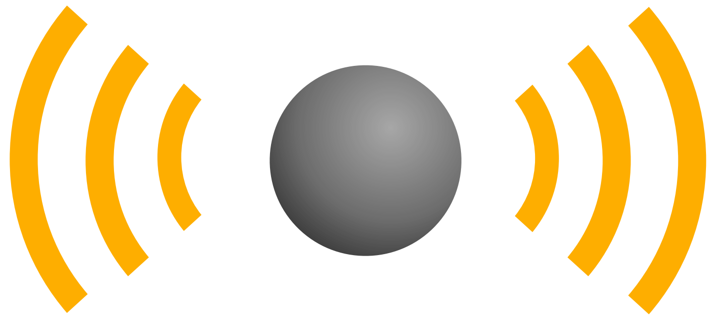

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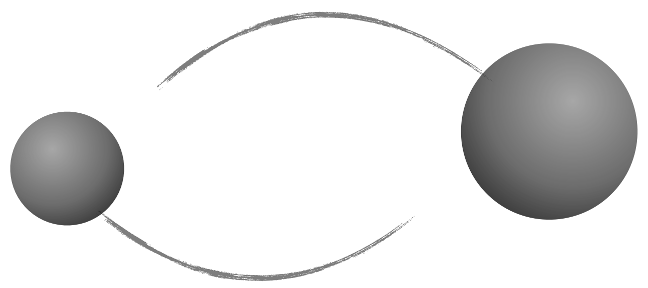

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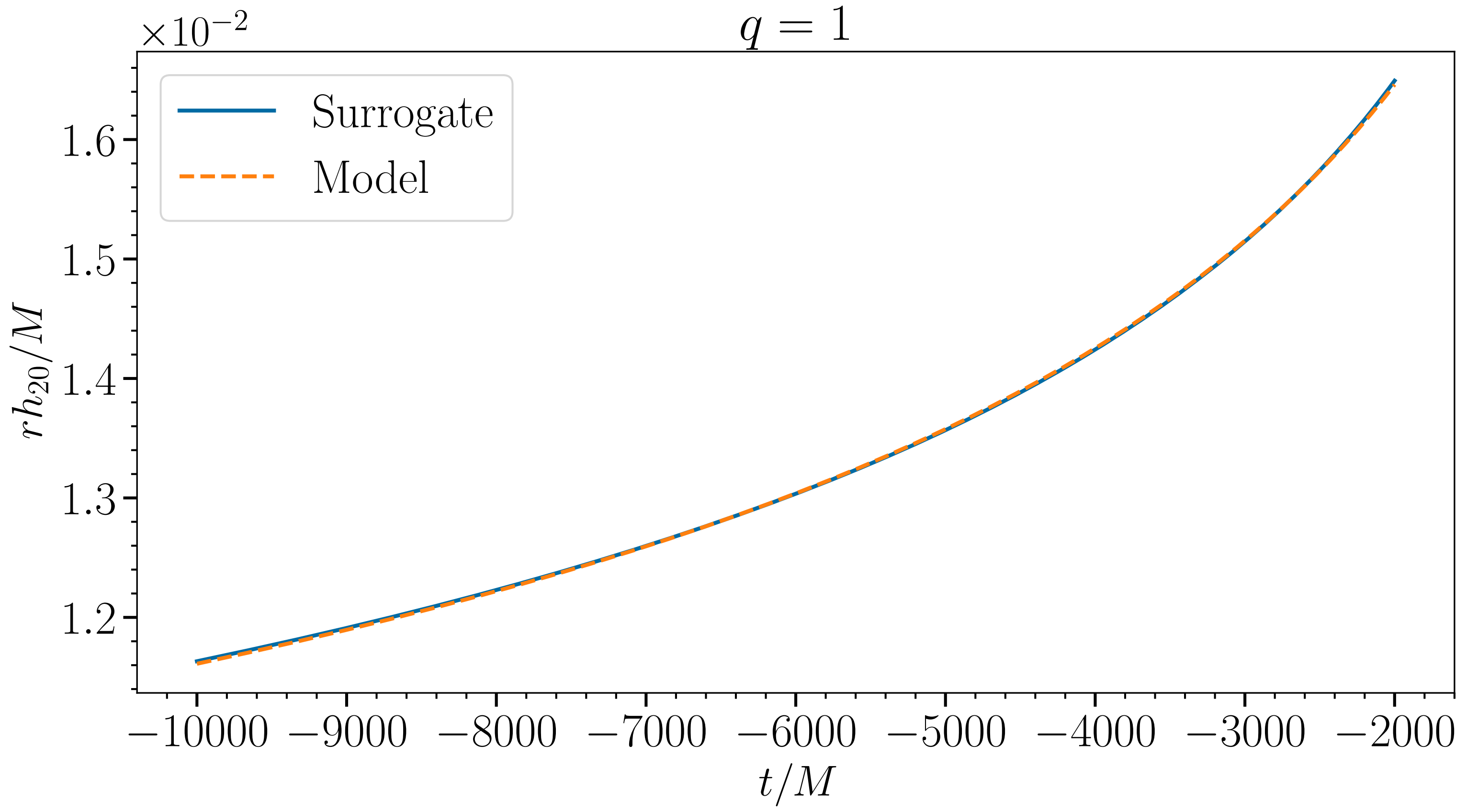

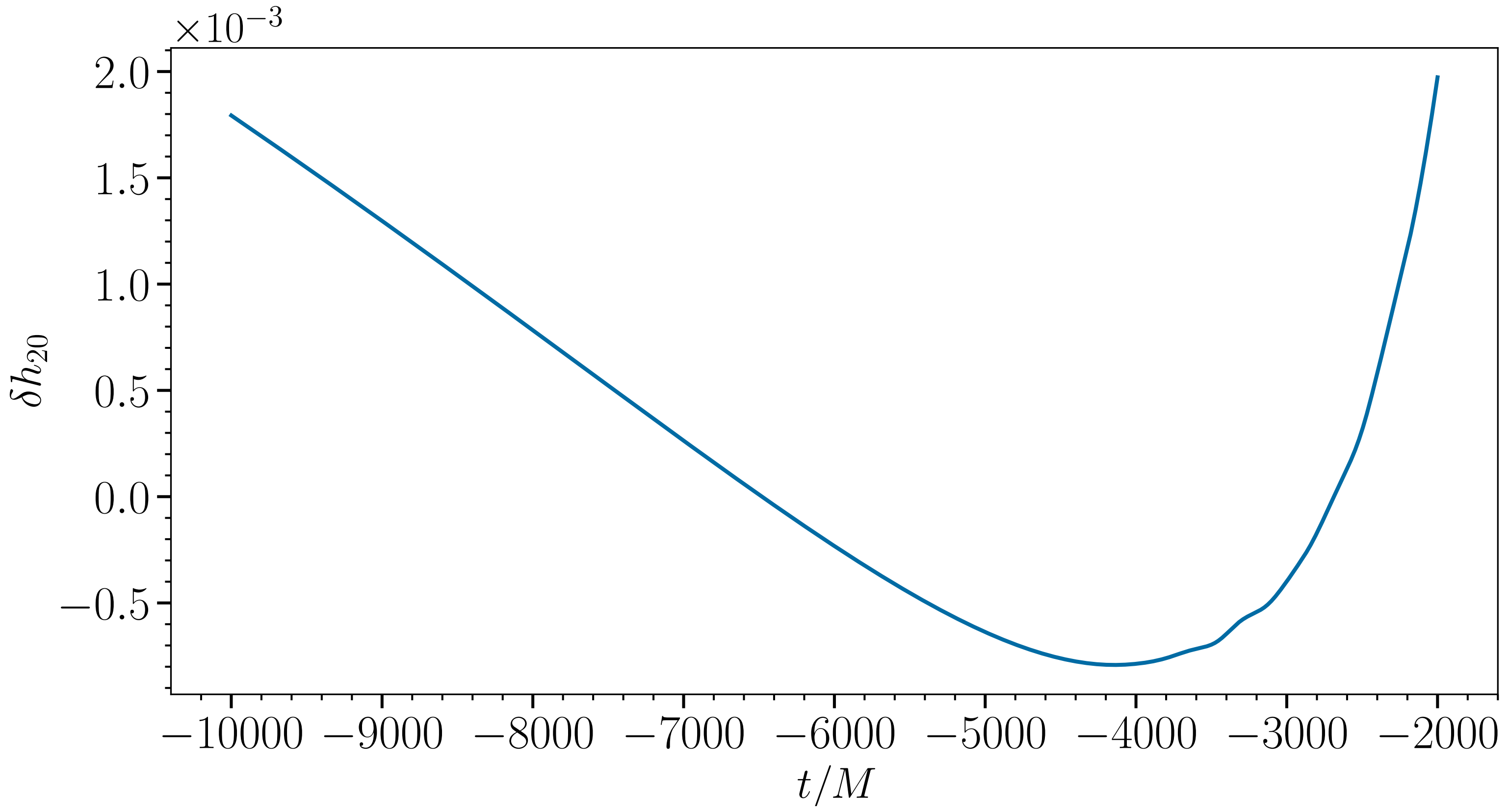

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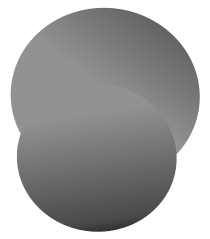

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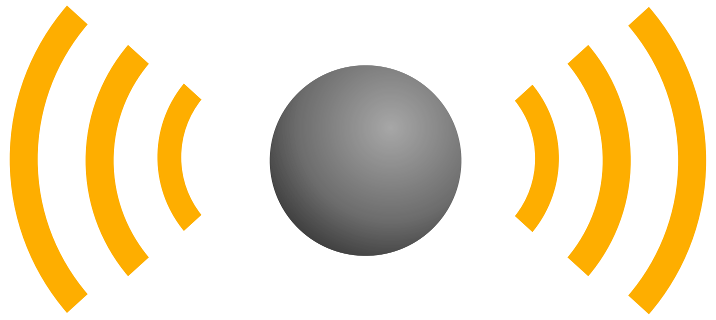

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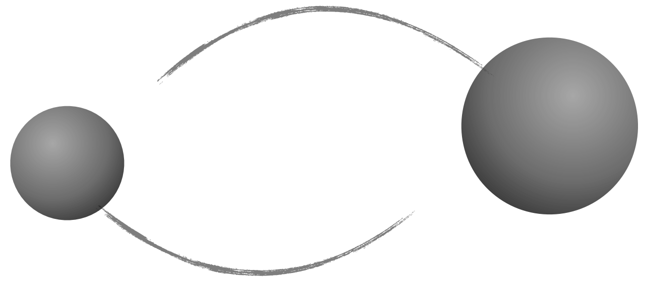

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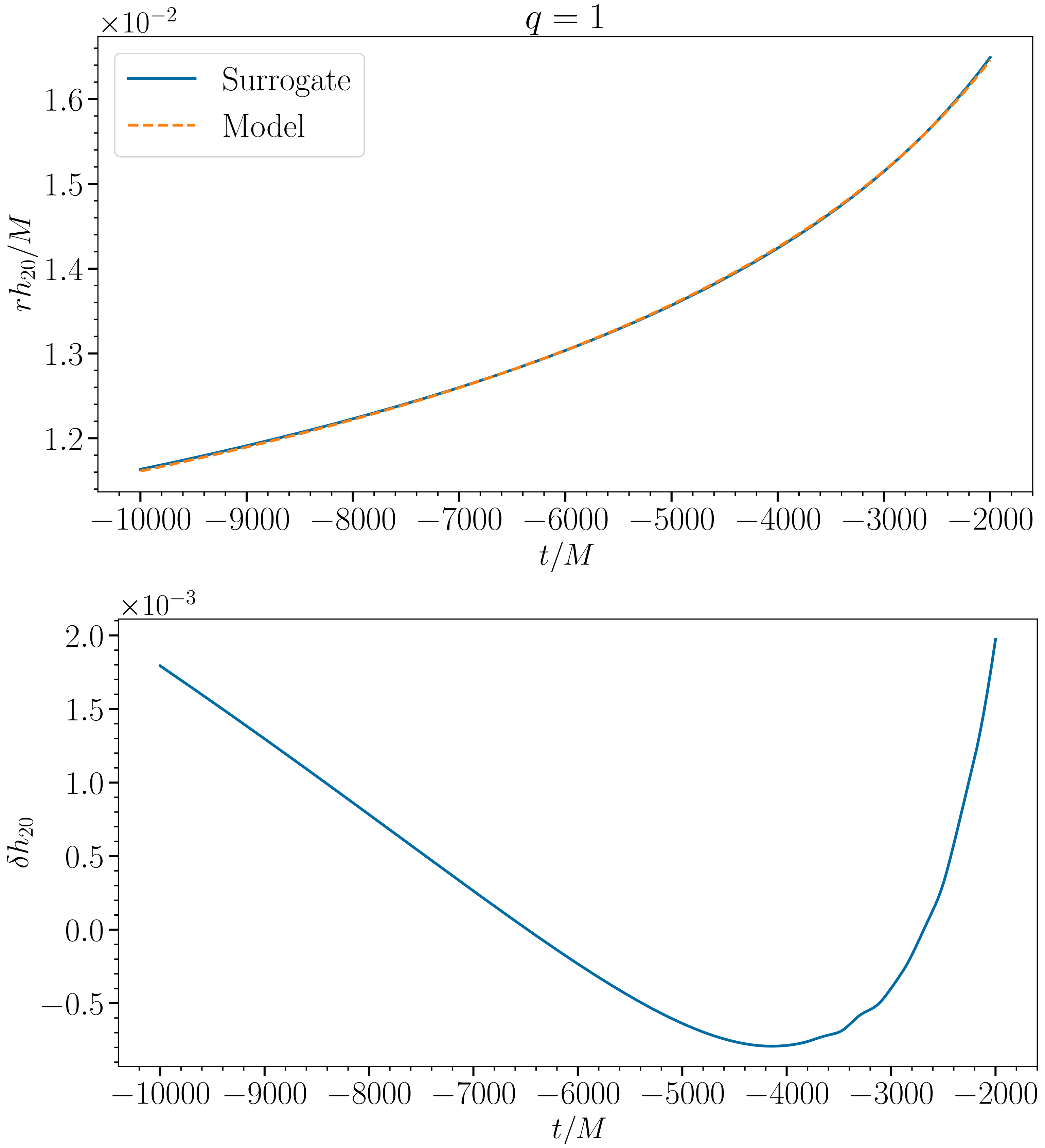

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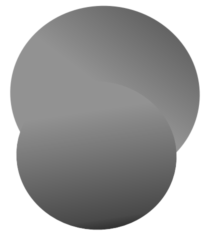

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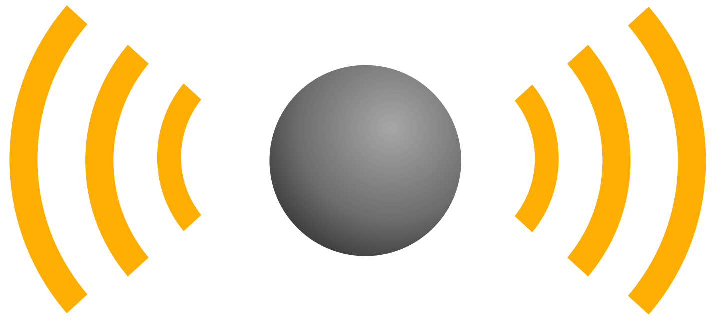
BH Perturbation Theory



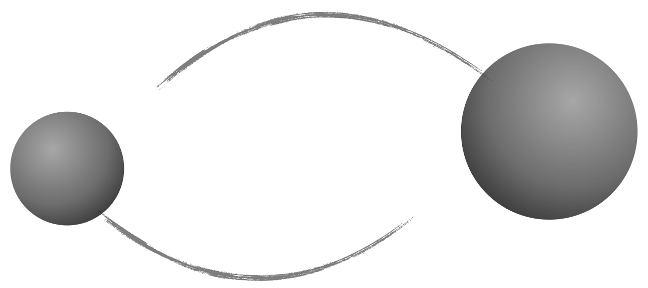
QNMs $[M_f \,\&\, S_f]$ $, N = 7, \omega^+$

$$h_{\text{model}}^{\text{rd}} = \sum_{l,m,n} C_{lmn} e^{-i\omega_{lmn}(t-t_0)/M_f} {}_{-2}S_{lm}(\theta, \phi; a\omega)$$

$$h_{\text{NR}} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}\, {}_{-2}Y_{lm}(\theta, \phi)$$

Inspiral

Post-Newtonian Theory



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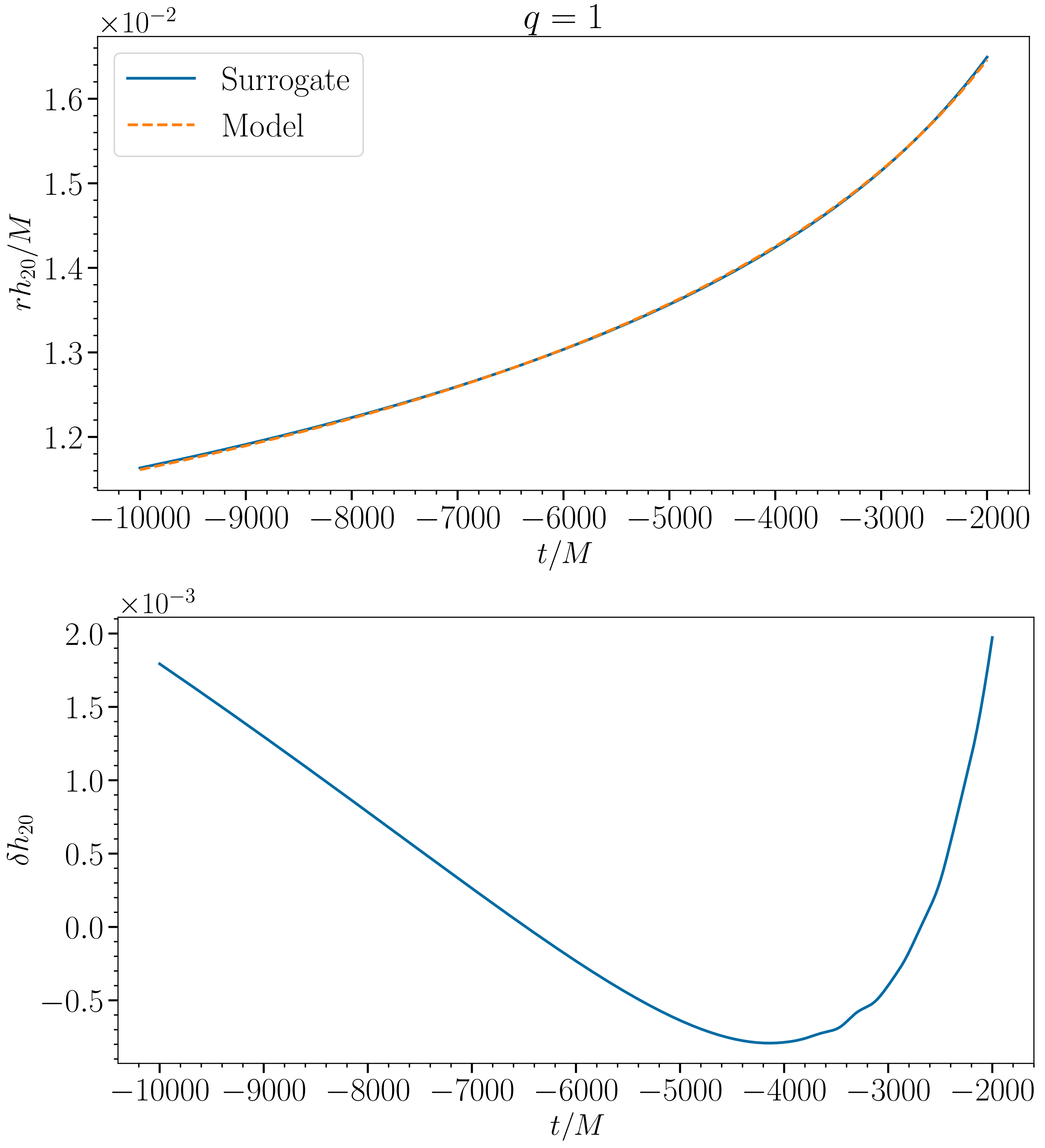

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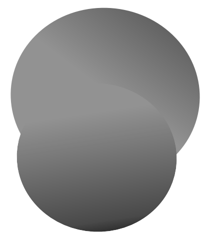

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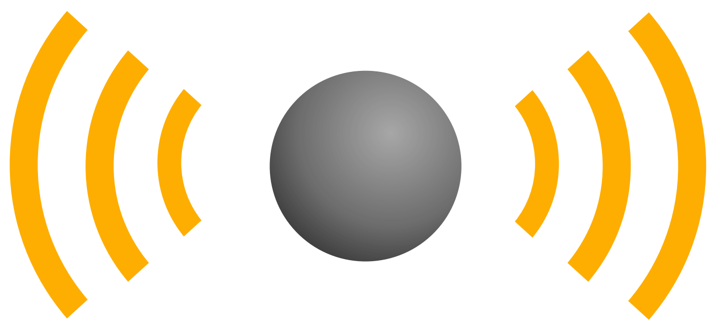

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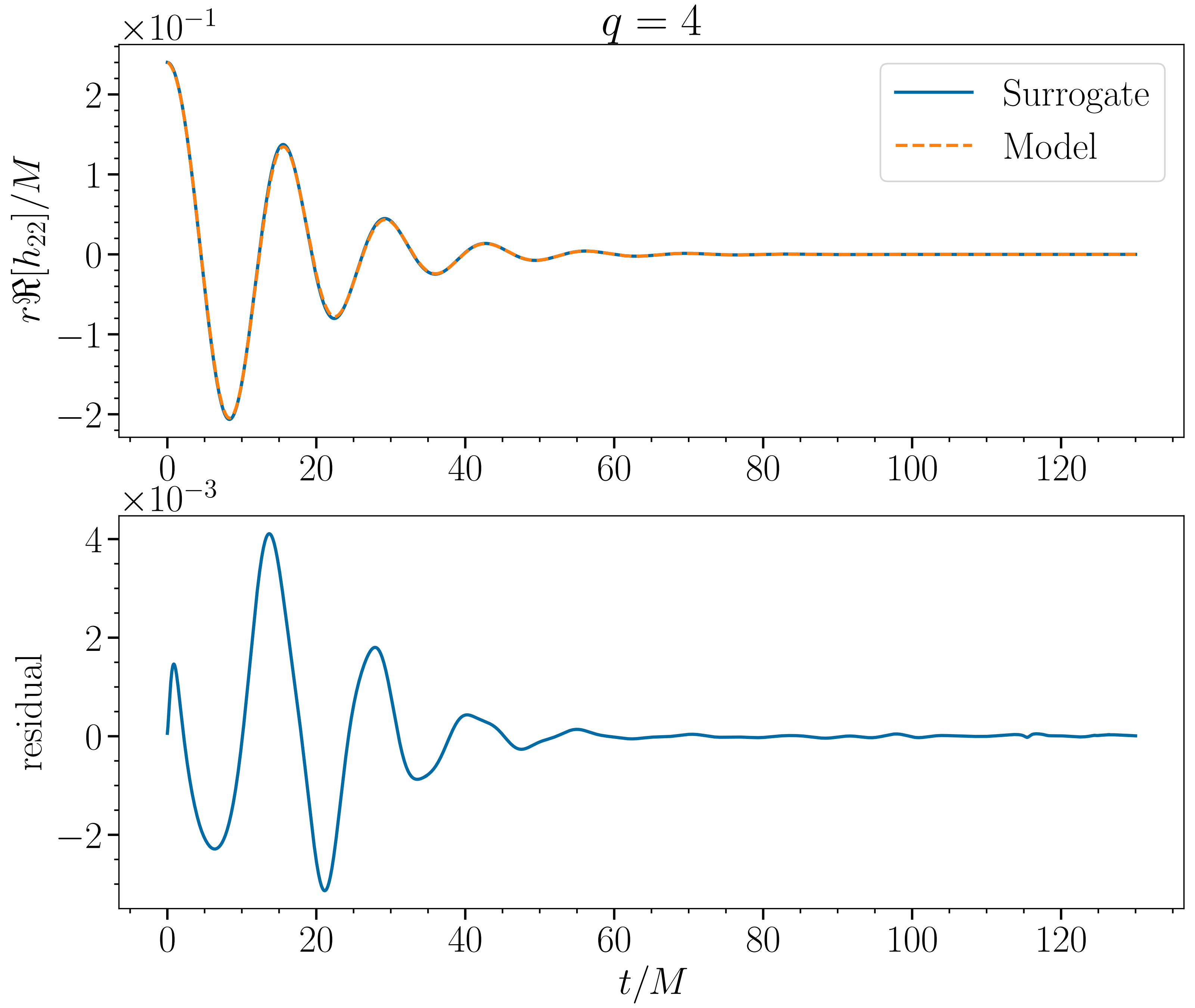
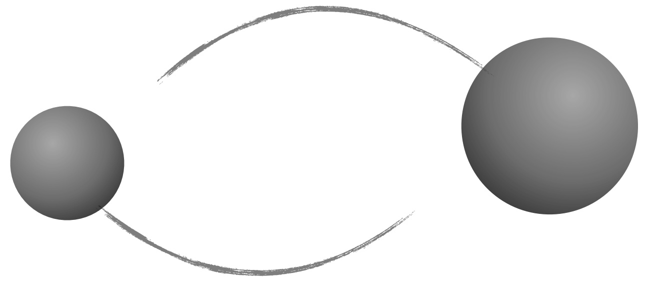
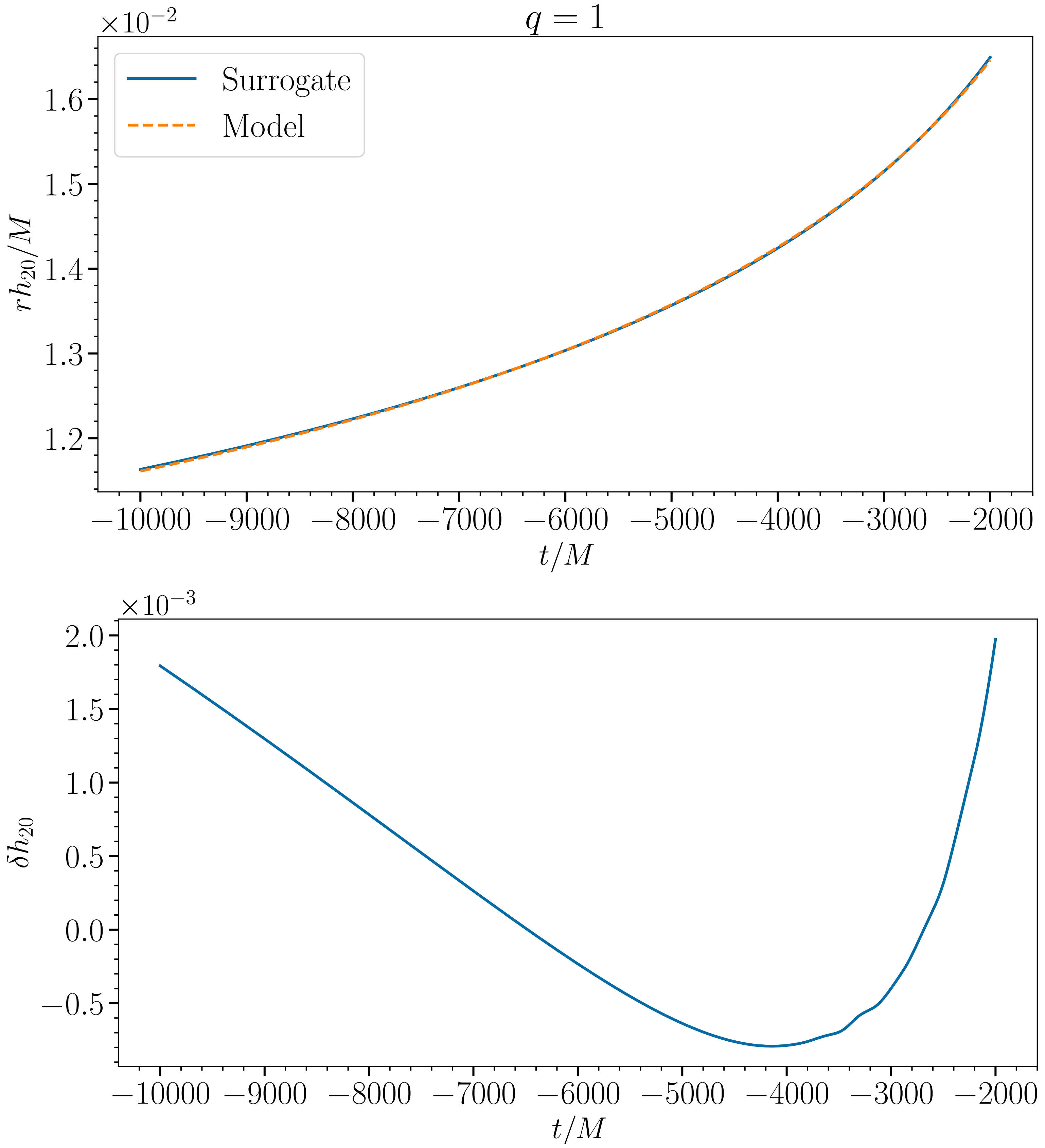


Inspiral

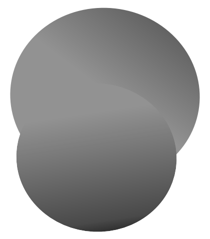
Post-Newtonian Theory



$$h_{20}^{\text{insp}}(t) = h_{20}^{3.5\text{PN}}(x(t))$$

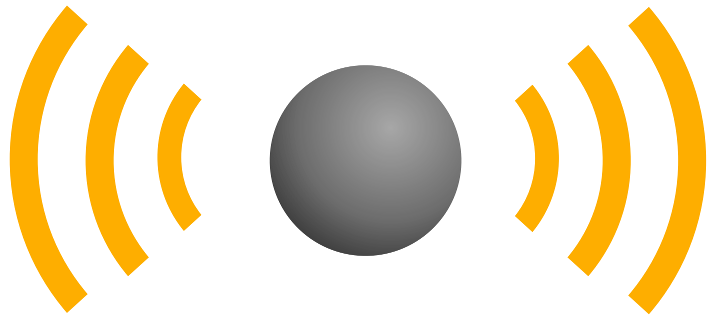


Intermediate

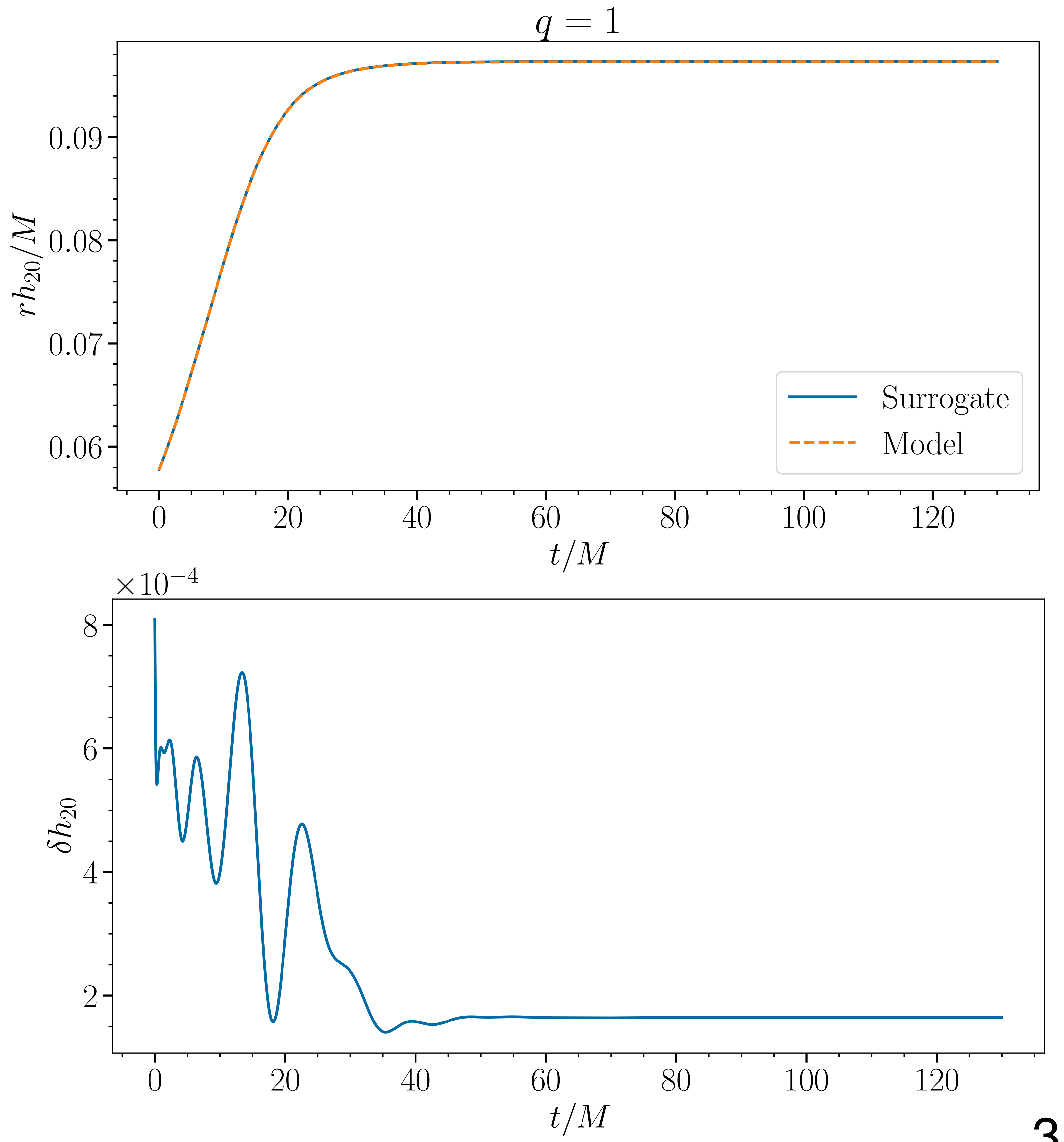


Ringdown

BH Perturbation Theory



QNMs $[M_f \;\&\; S_f]$ $, N = 7, \omega^+$



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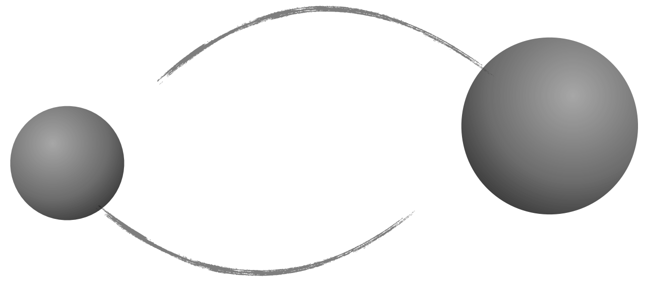

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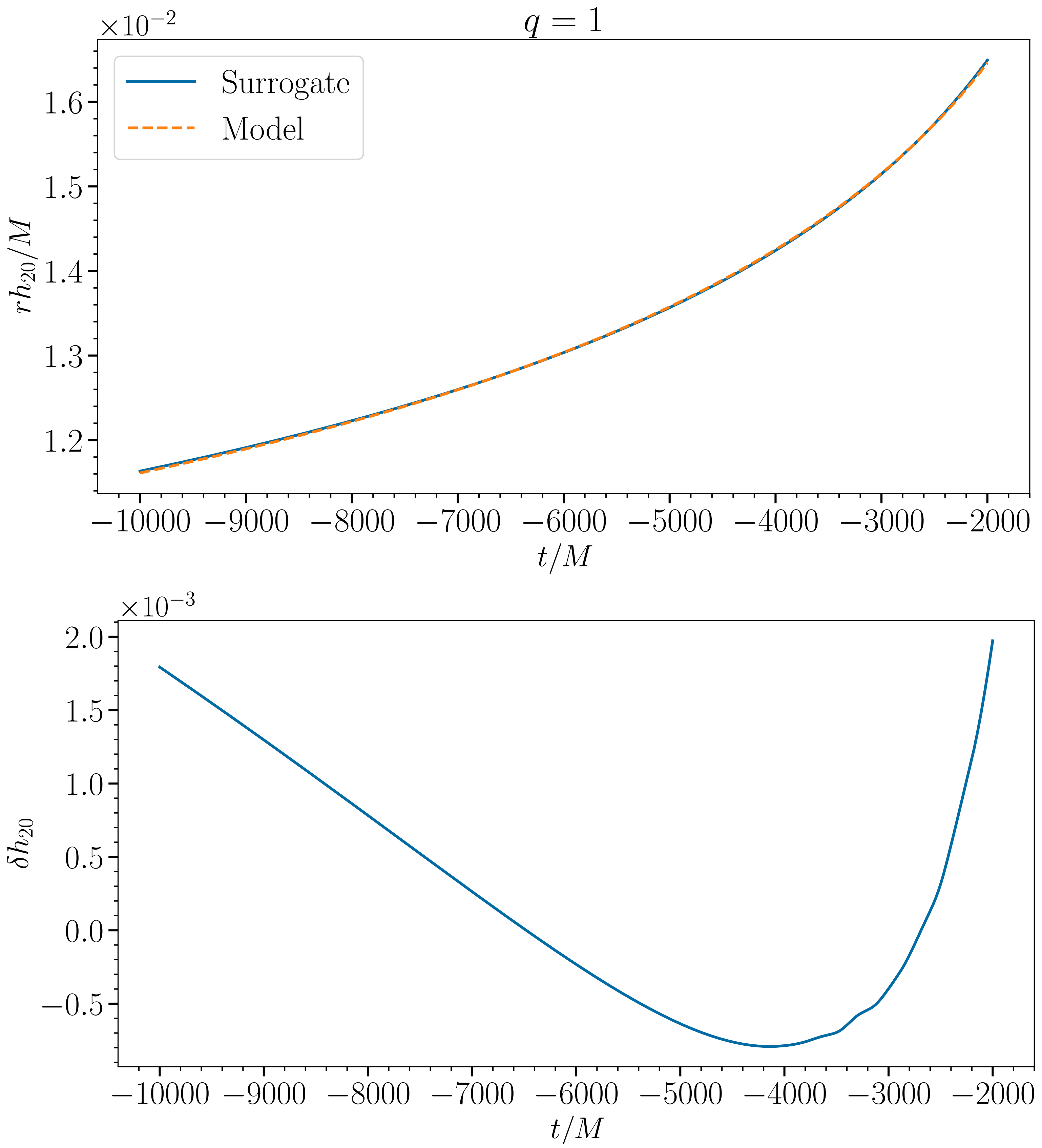
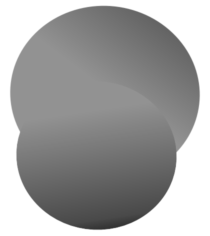


Intermediate

Phenomenological



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Ringdown

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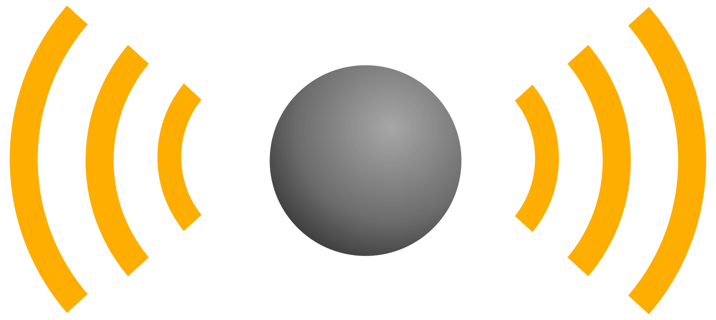

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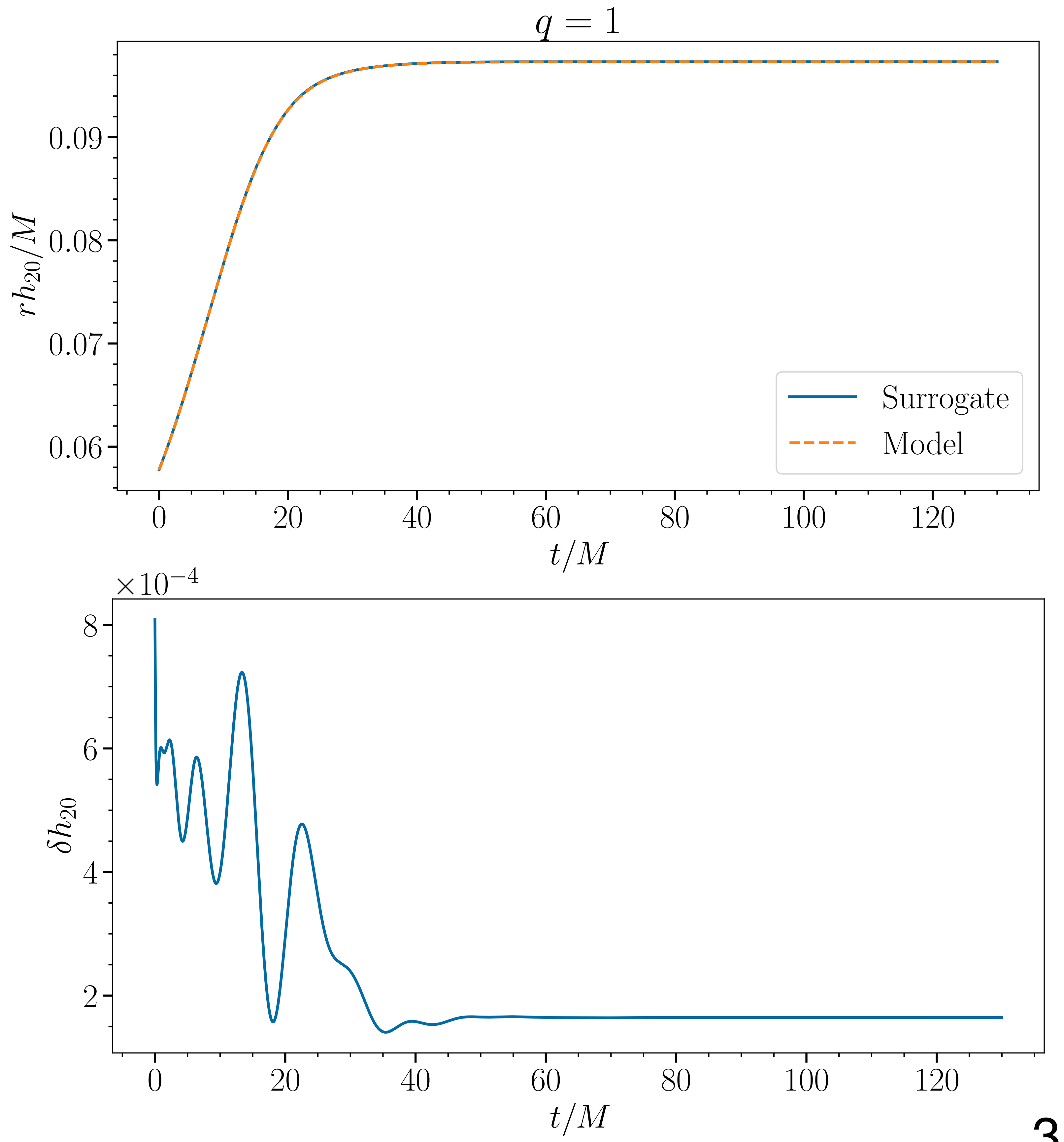

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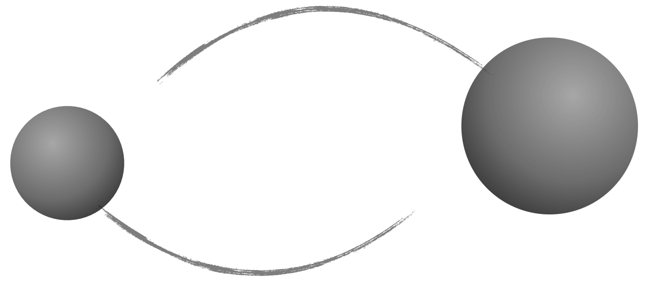

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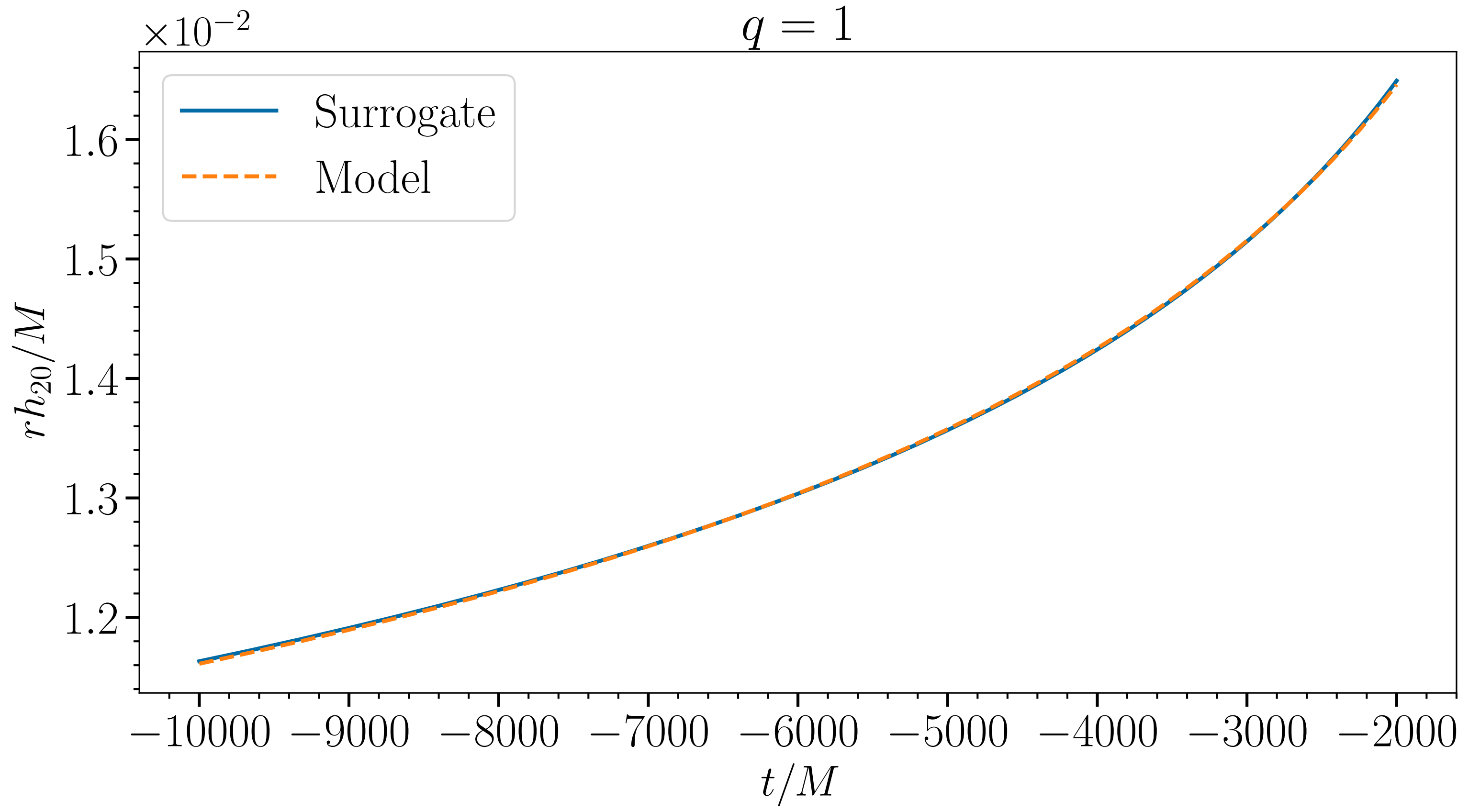

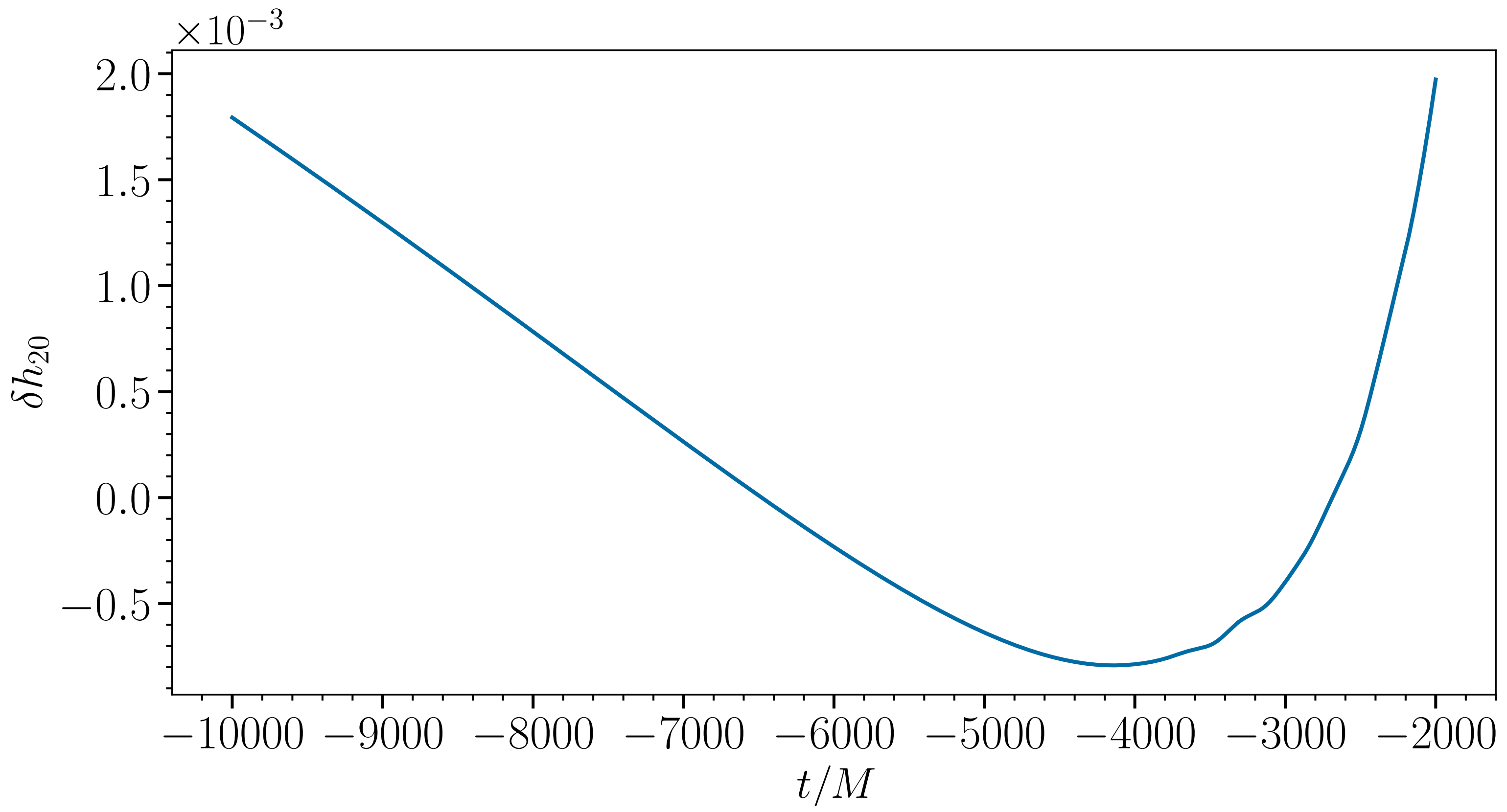

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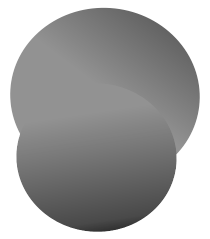

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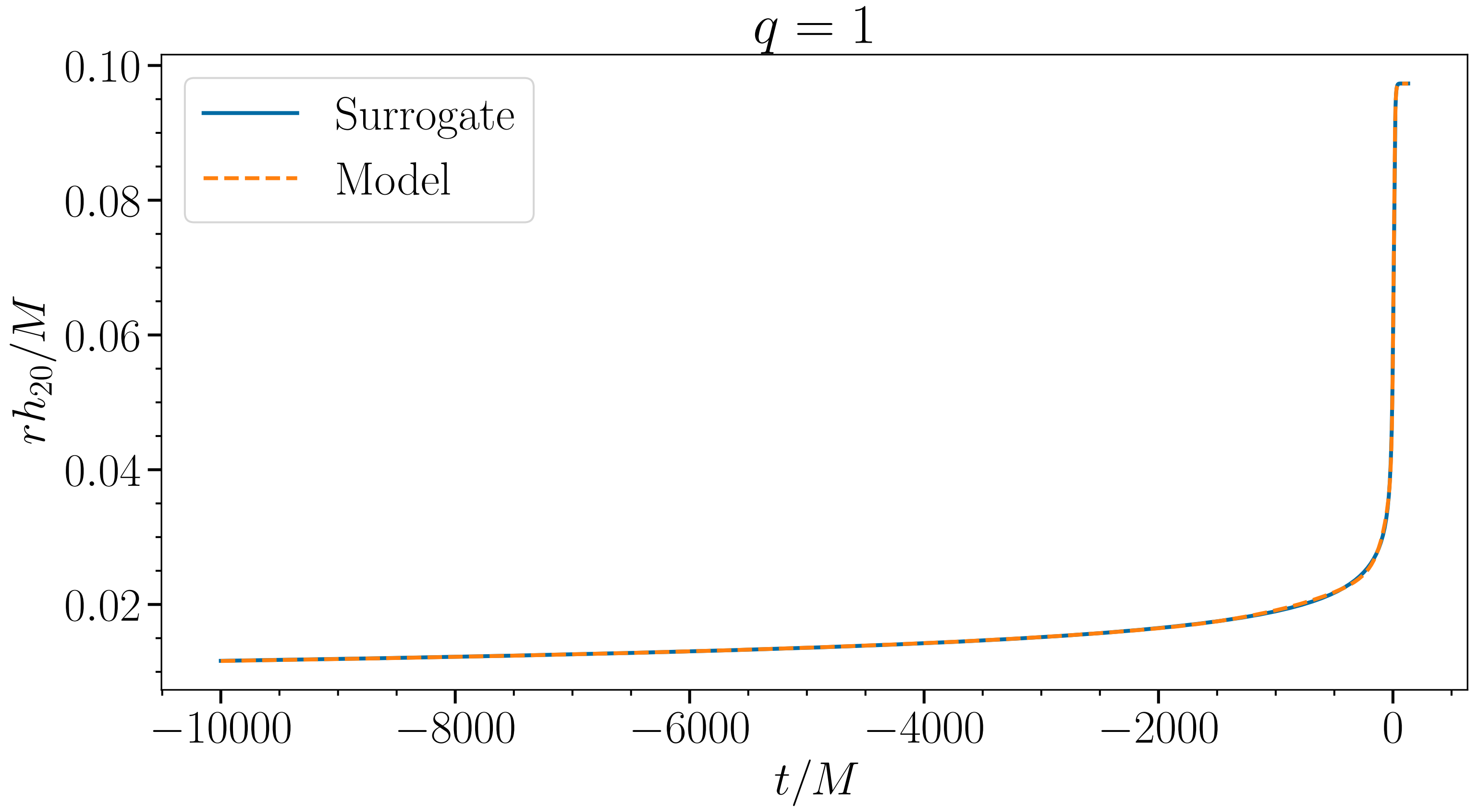
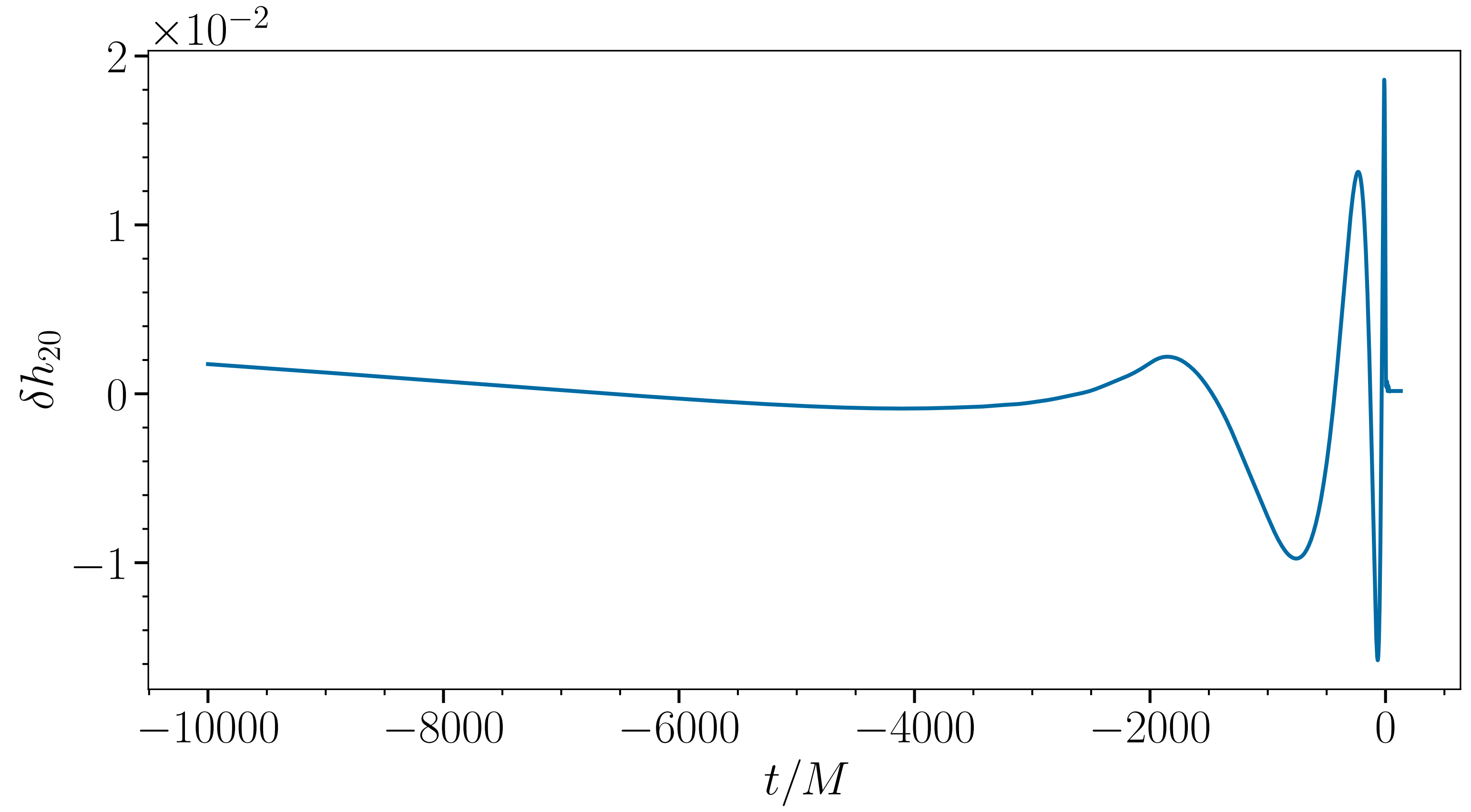
A.E. & David Nichols, arXiv:**2504.18635**





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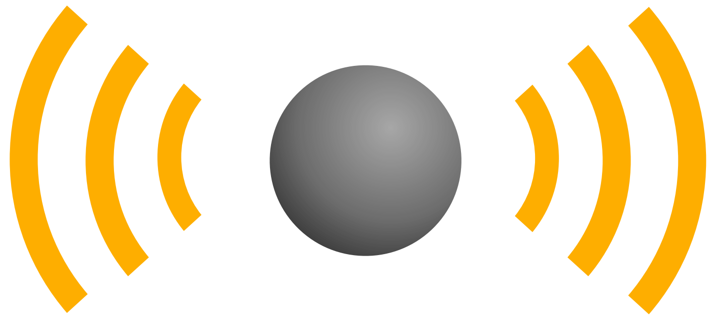

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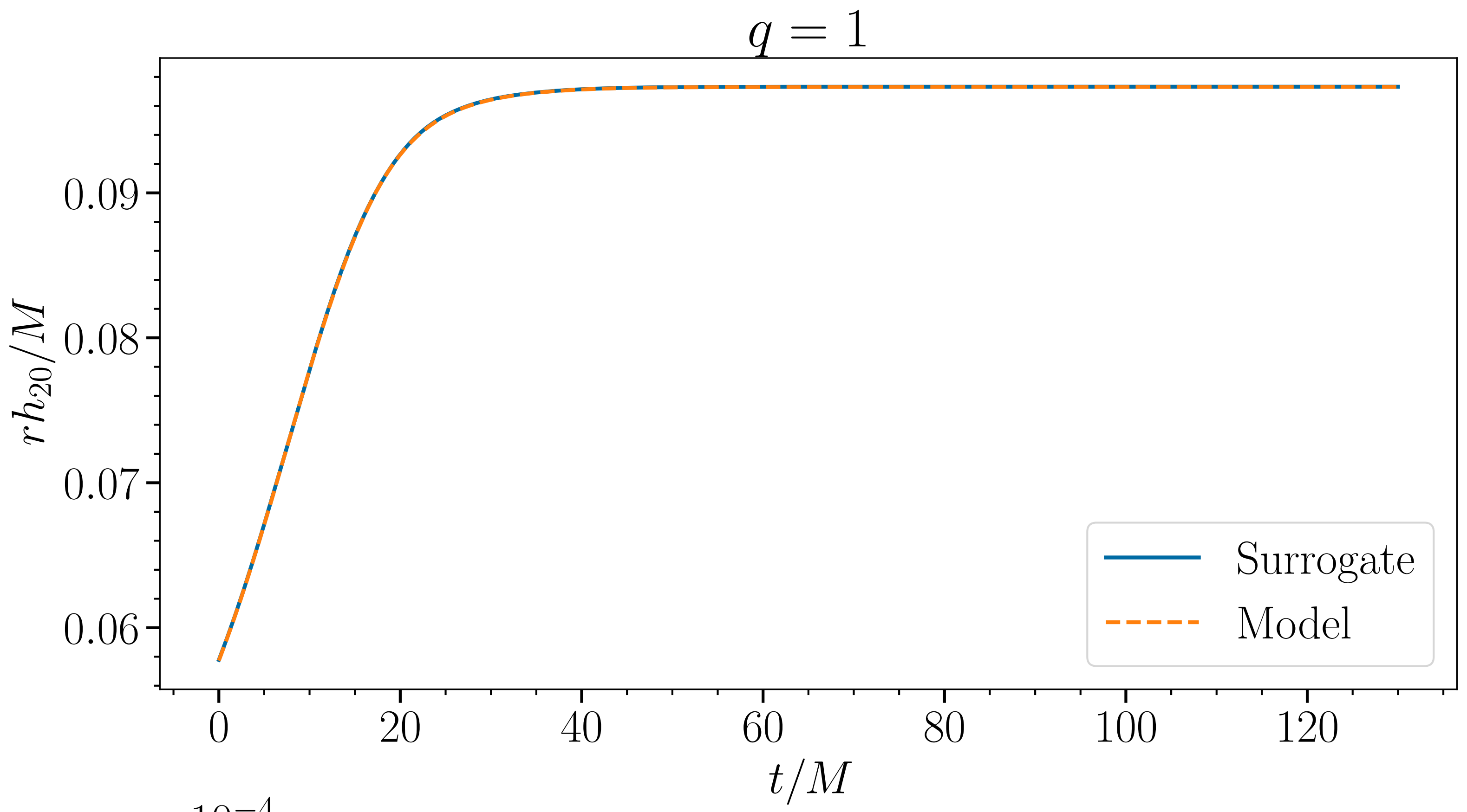

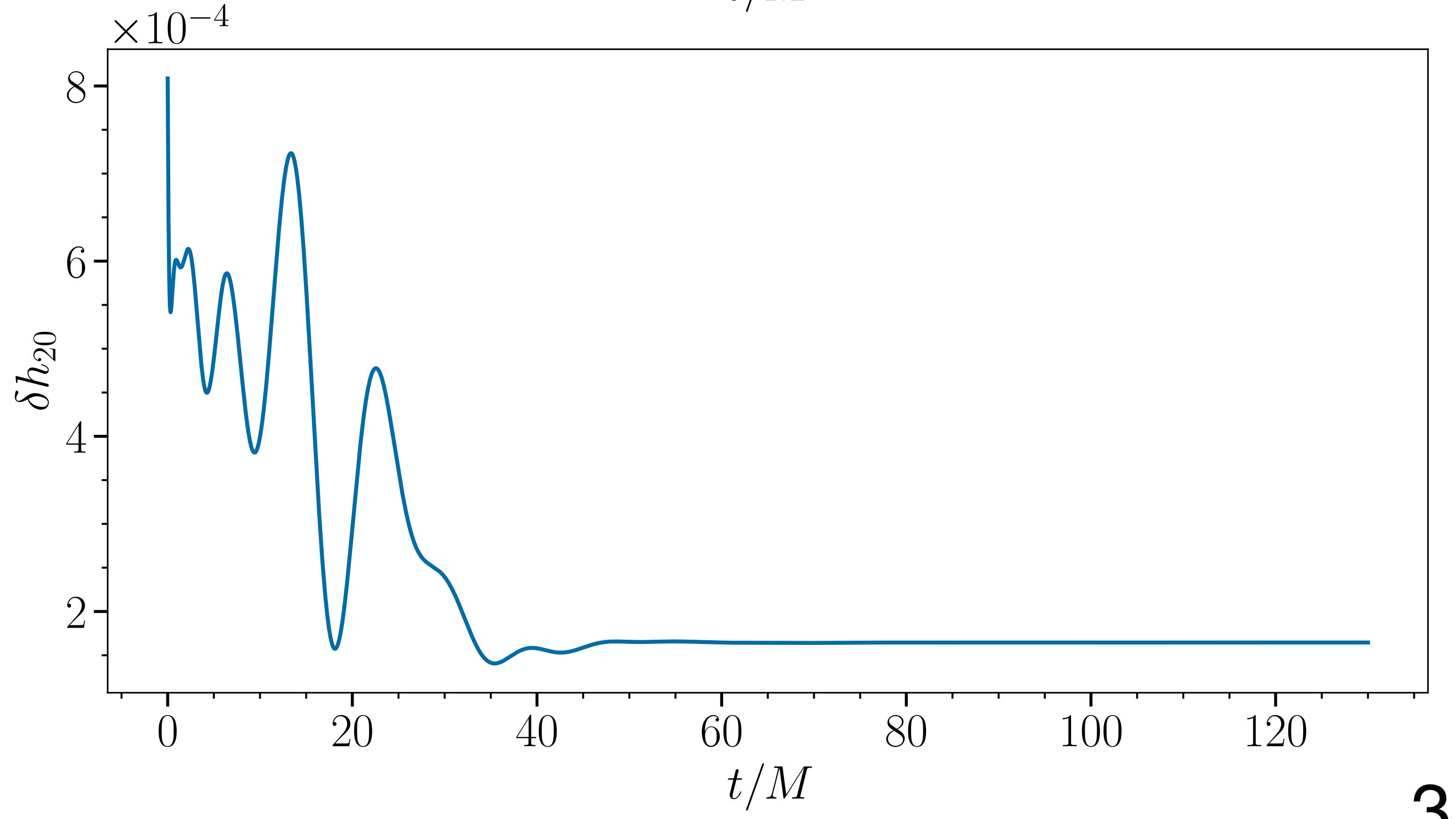

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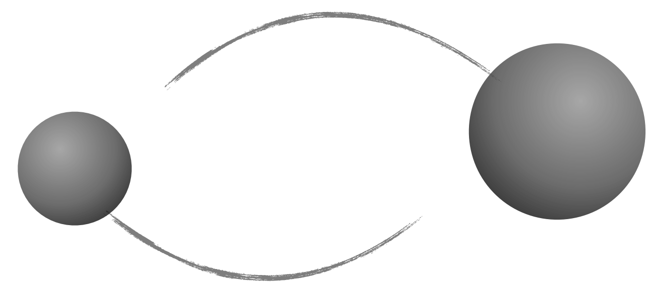

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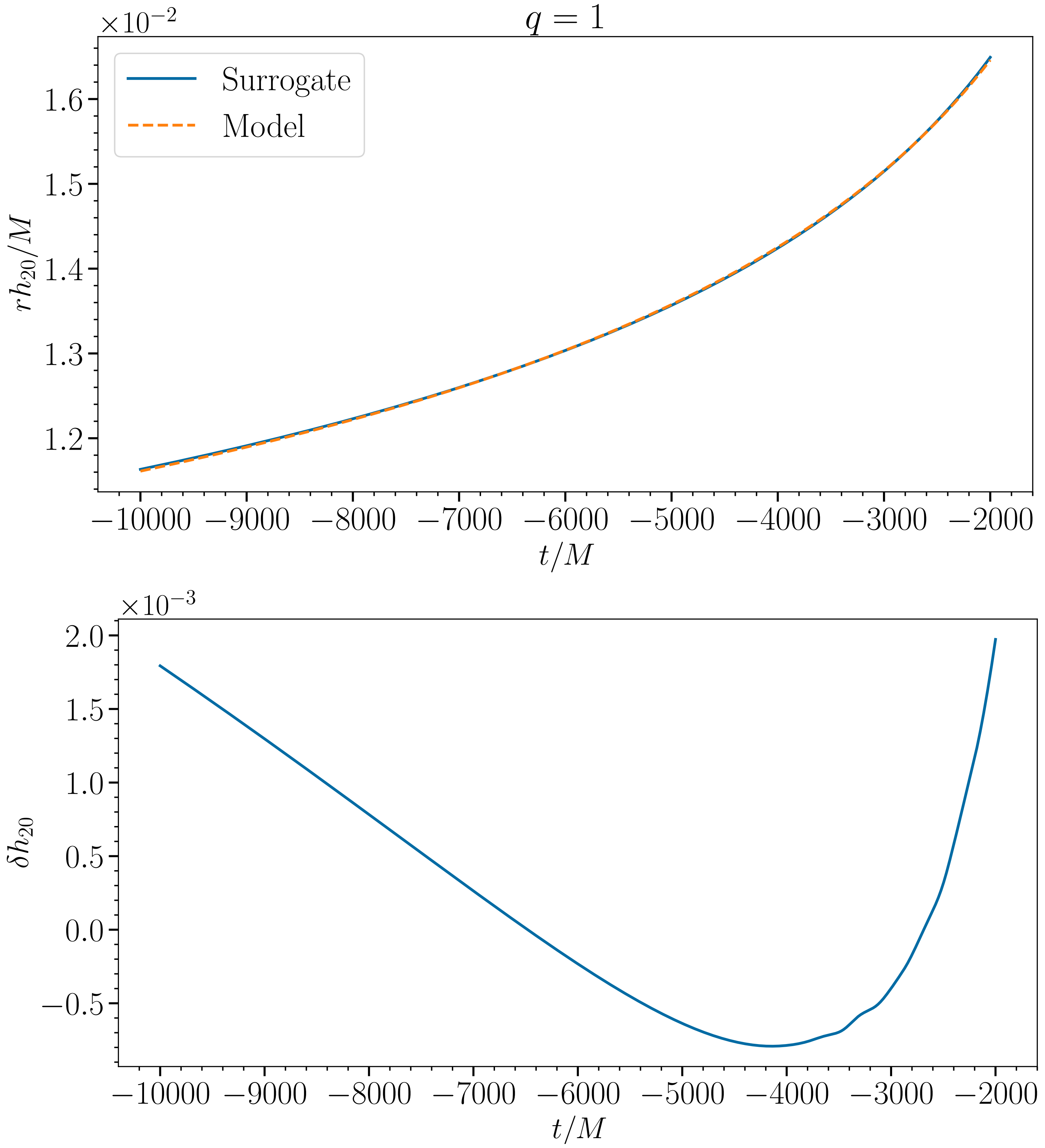
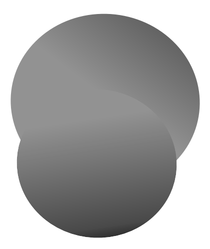


Intermediate

Phenomenological



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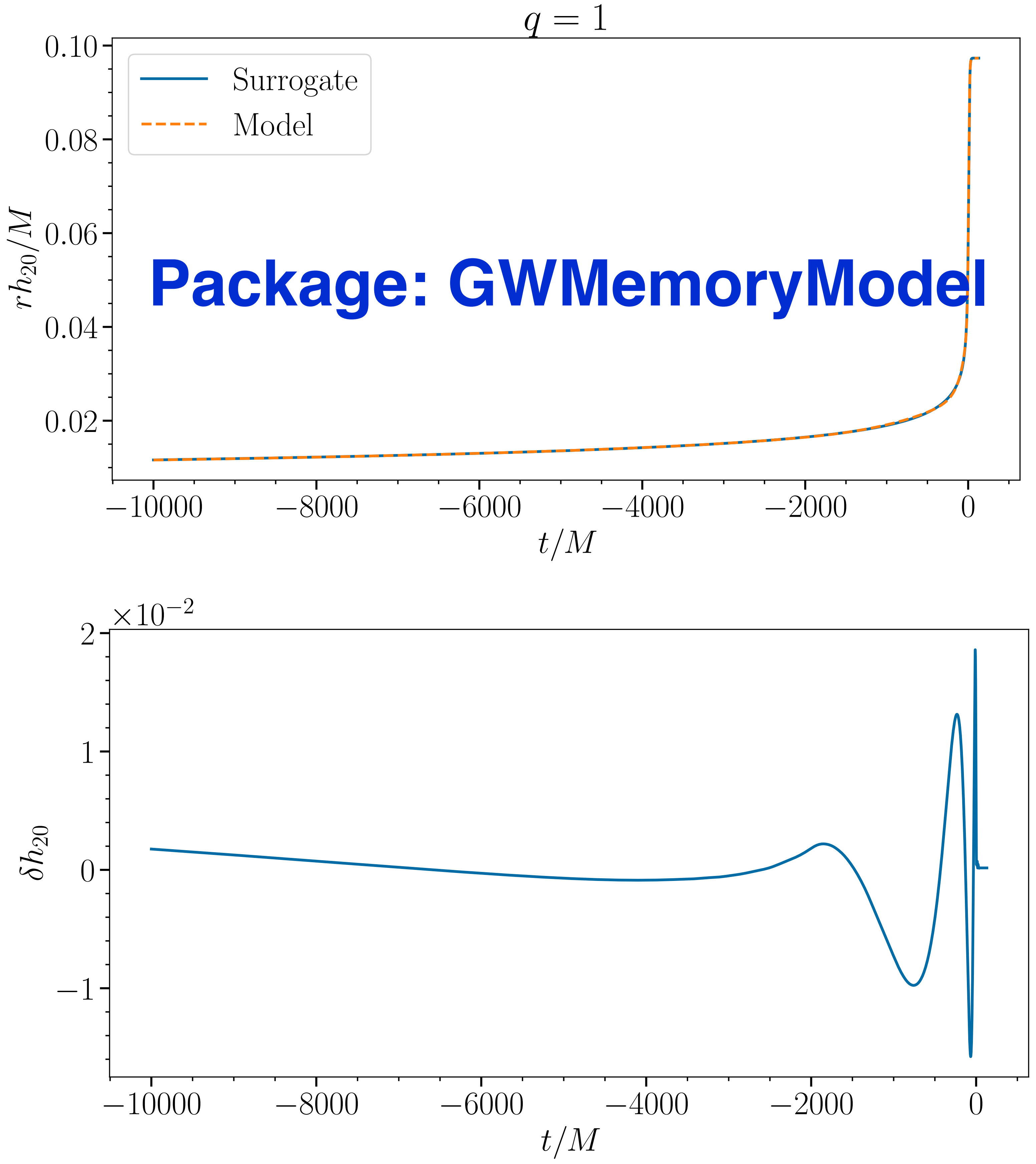

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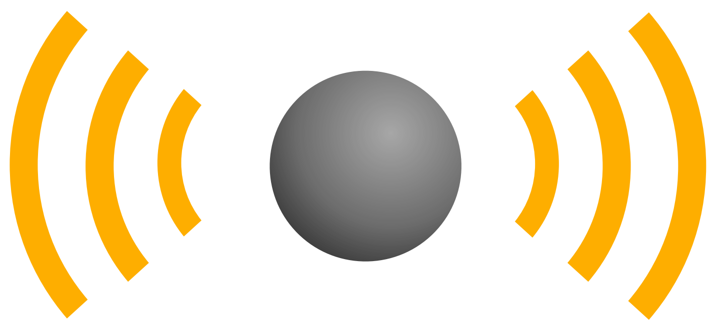

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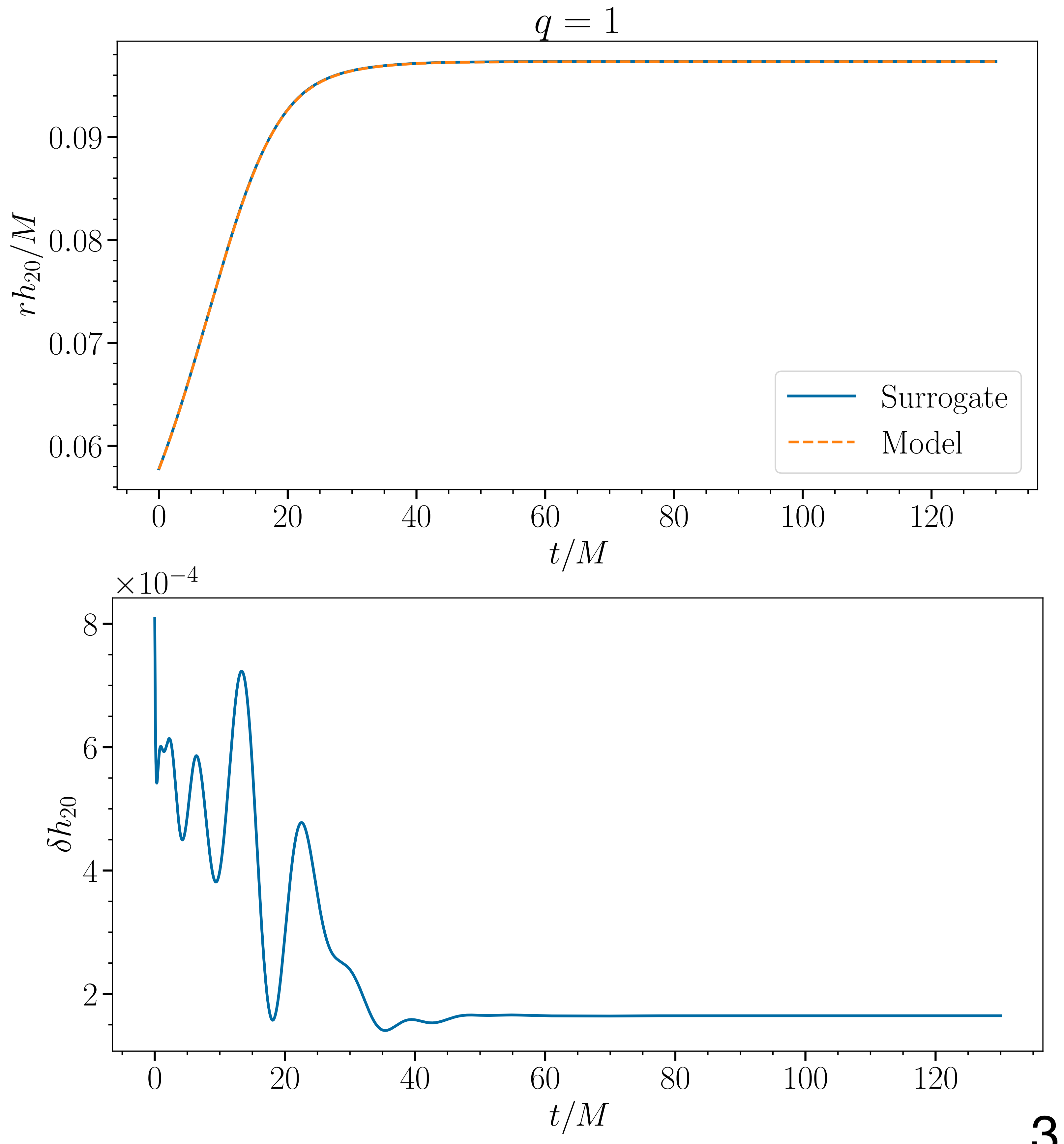

Frequency-domain signal

- Different methods for dealing with the artifacts: SySS [Valencia+ 2024] and LS [Chen+ 2024].
- We introduced a different method, computing the FFT from the time derivative of the memory signal.

$$\tilde{h}_{20}(f) = \frac{\Delta h_{20}}{2}\,\delta(f) + \frac{1}{2\pi i f}\mathscr{F}[\dot{h}_{20}]$$

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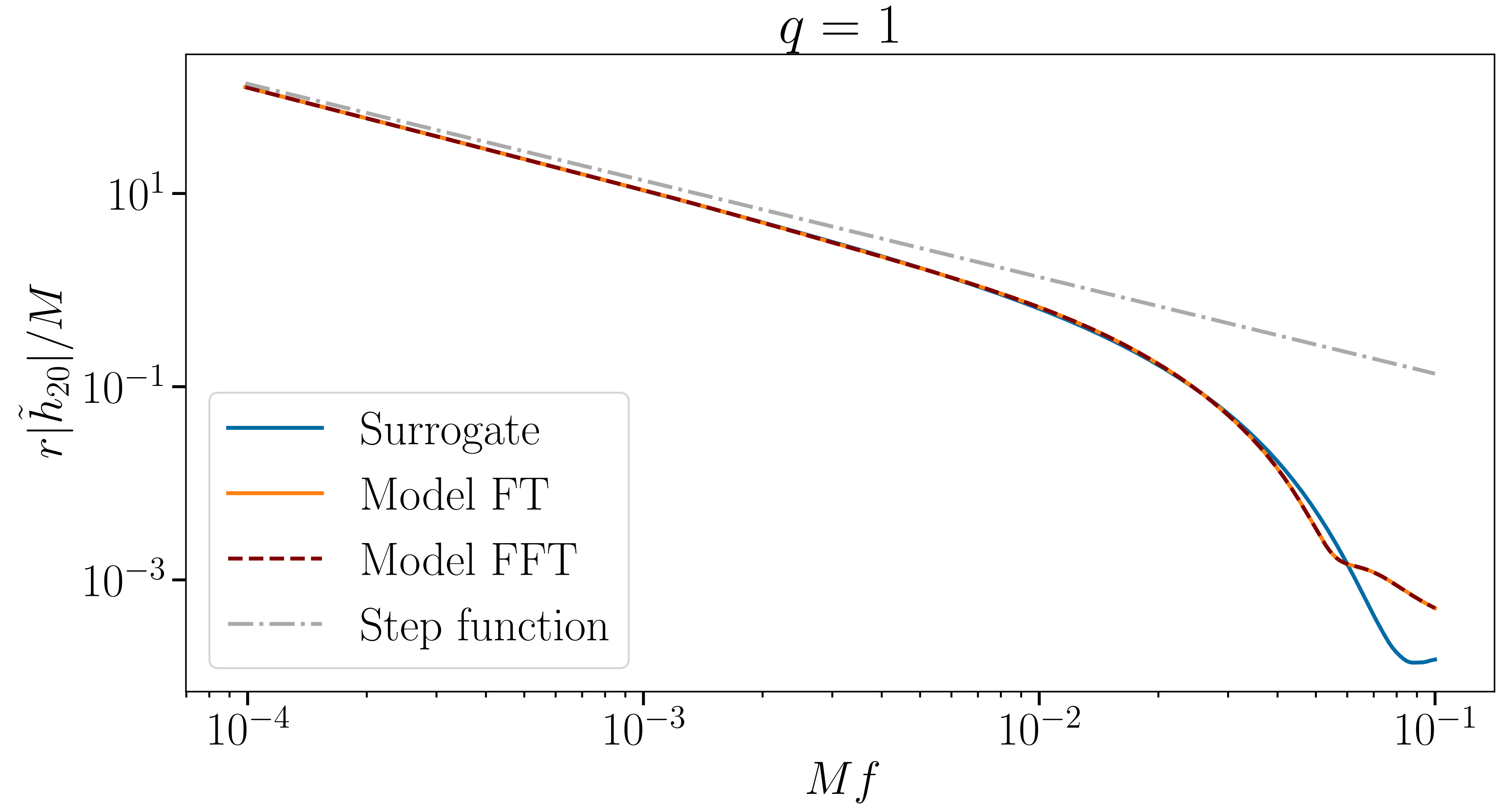
Amplitude



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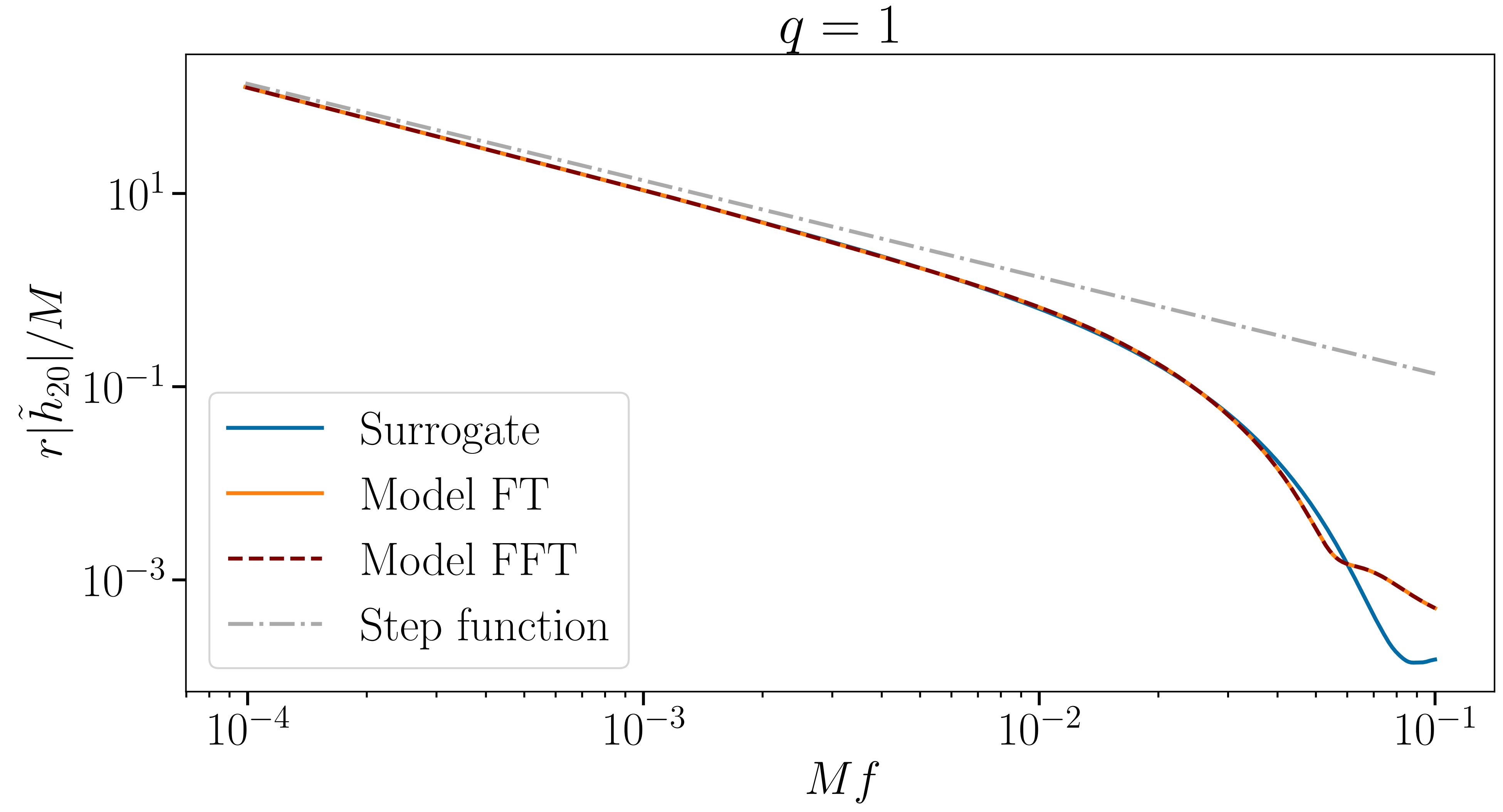
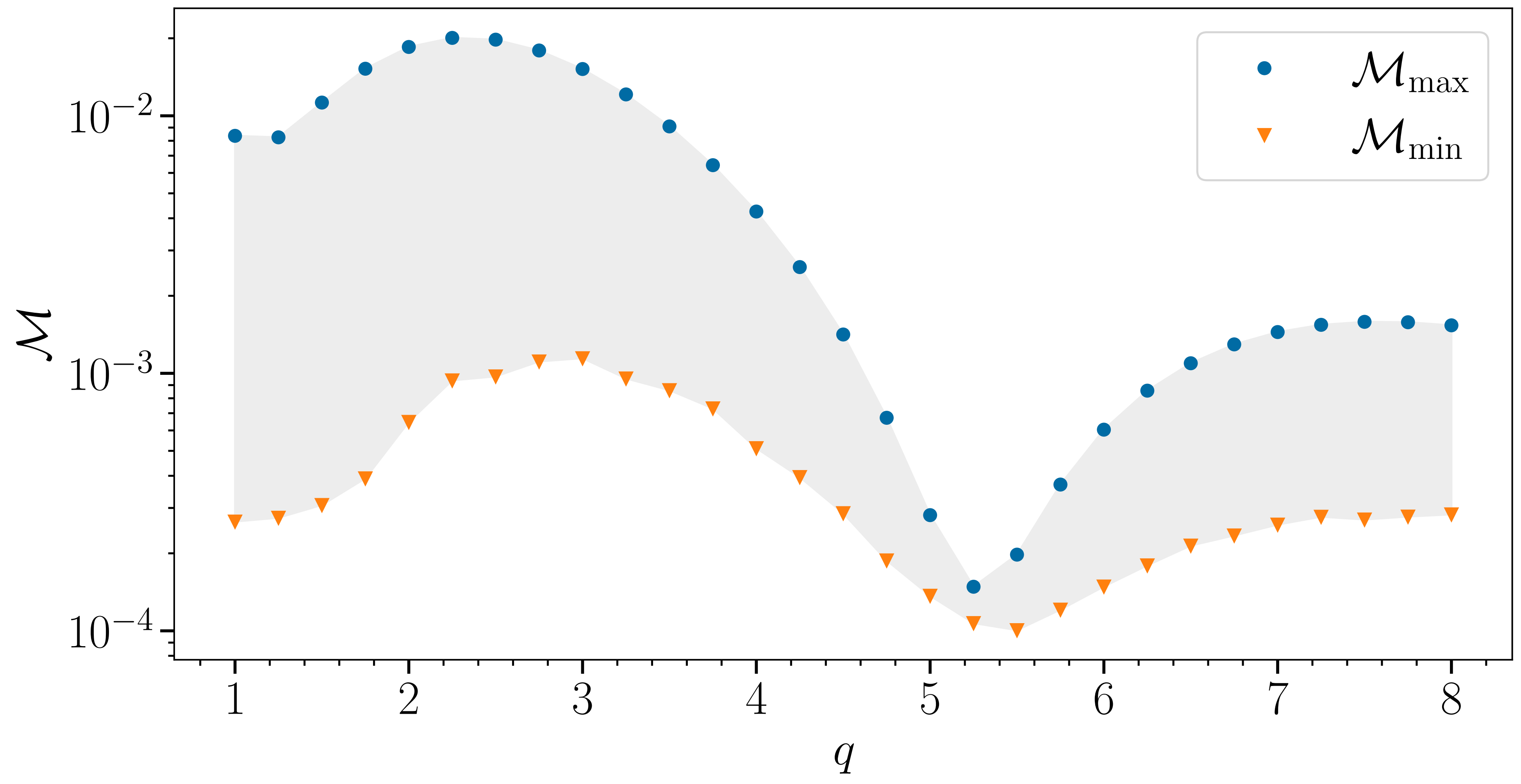


Mismatch

$$\mathscr{M} = 1 - \frac{\langle h_{\rm surr}, h_{\rm model}\rangle}{\sqrt{\langle h_{\rm surr}, h_{\rm surr}\rangle\langle h_{\rm model}, h_{\rm model}\rangle}}$$

$$\langle h_1, h_2\rangle = 4\mathscr{R}\left[\int_{f_{min}}^{f_{max}} df \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}\right]$$

*Advanced LIGO noise curve $m_1 \in [5,100]M_\odot$



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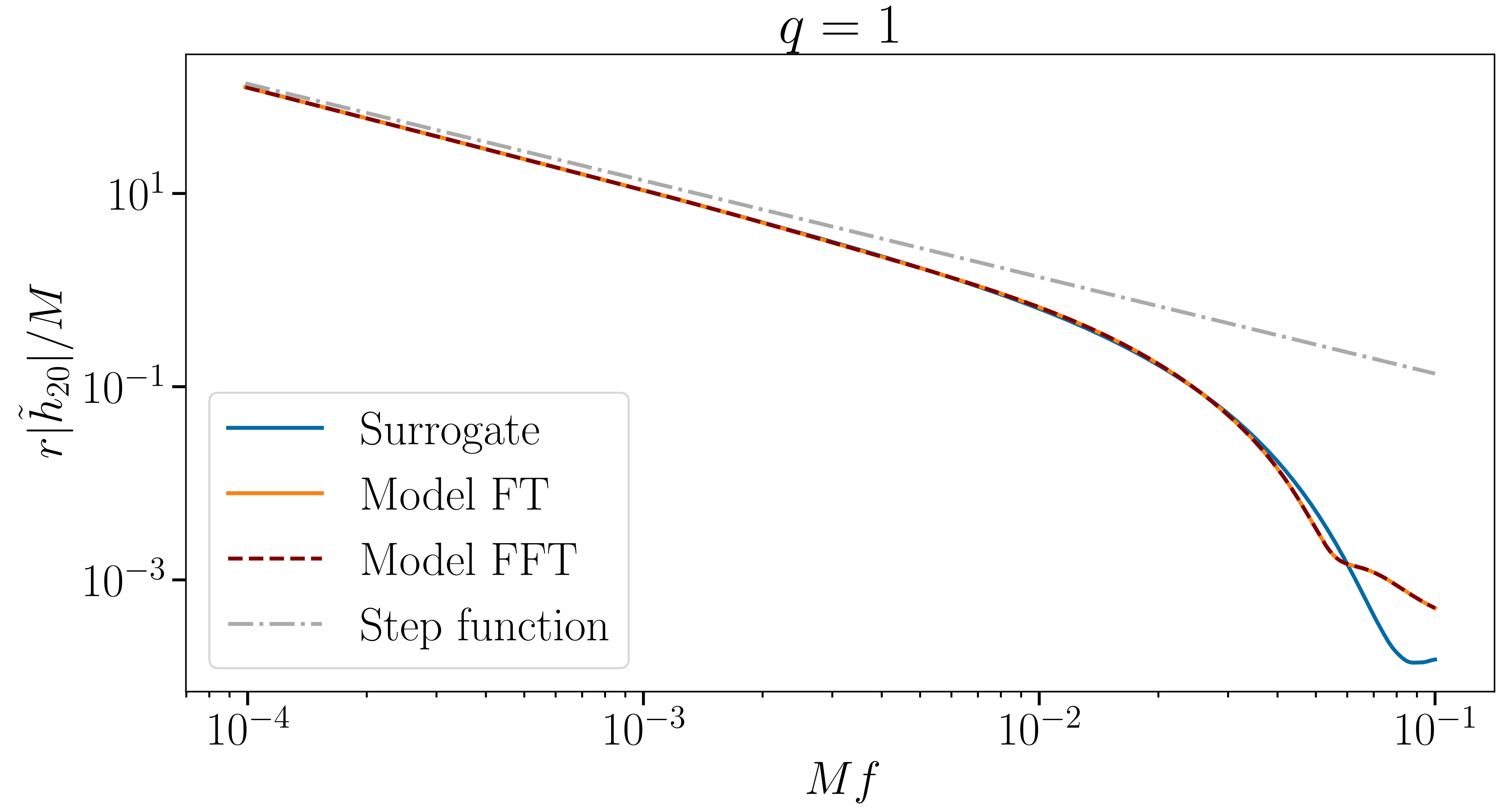
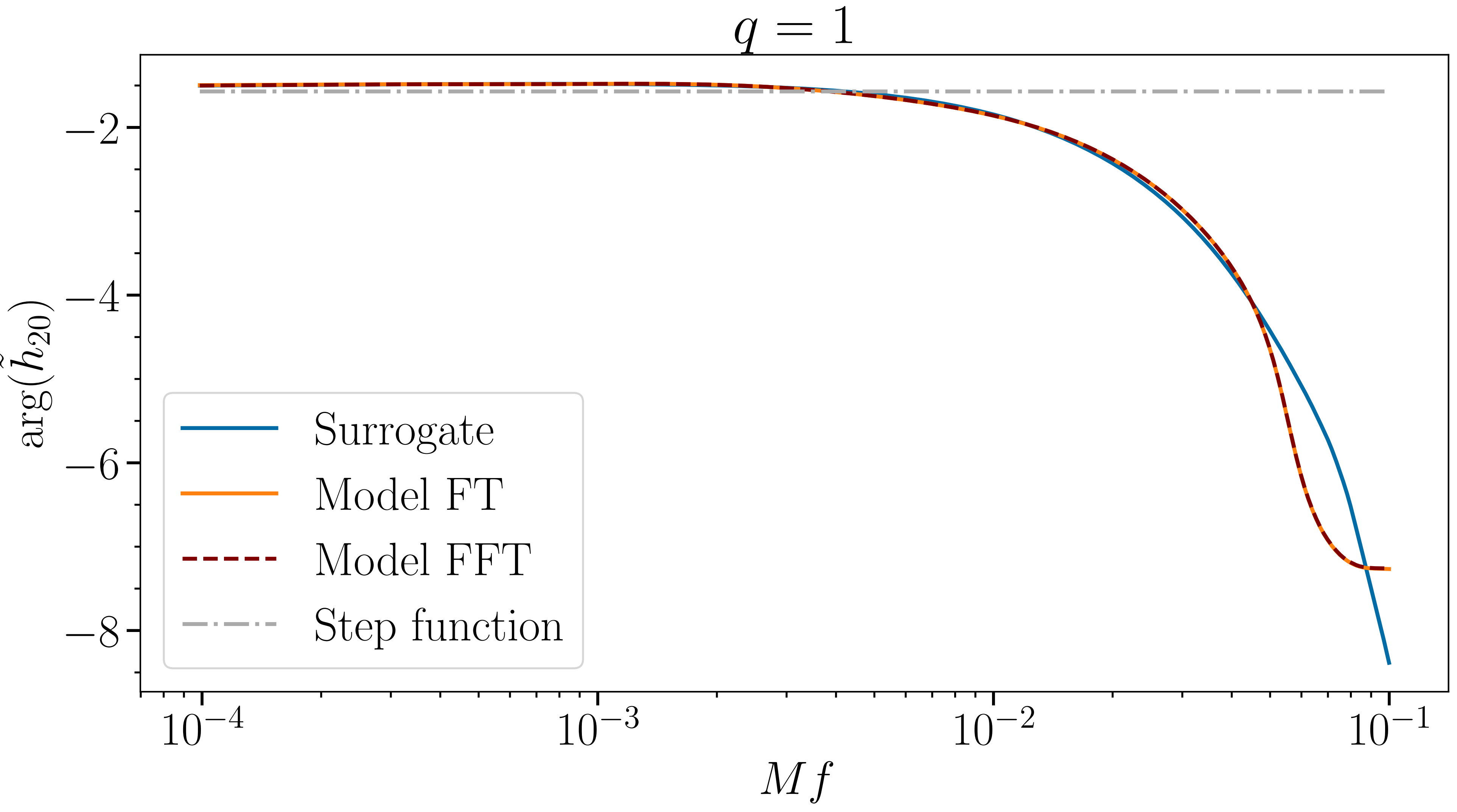


Phase



$-\pi/2$

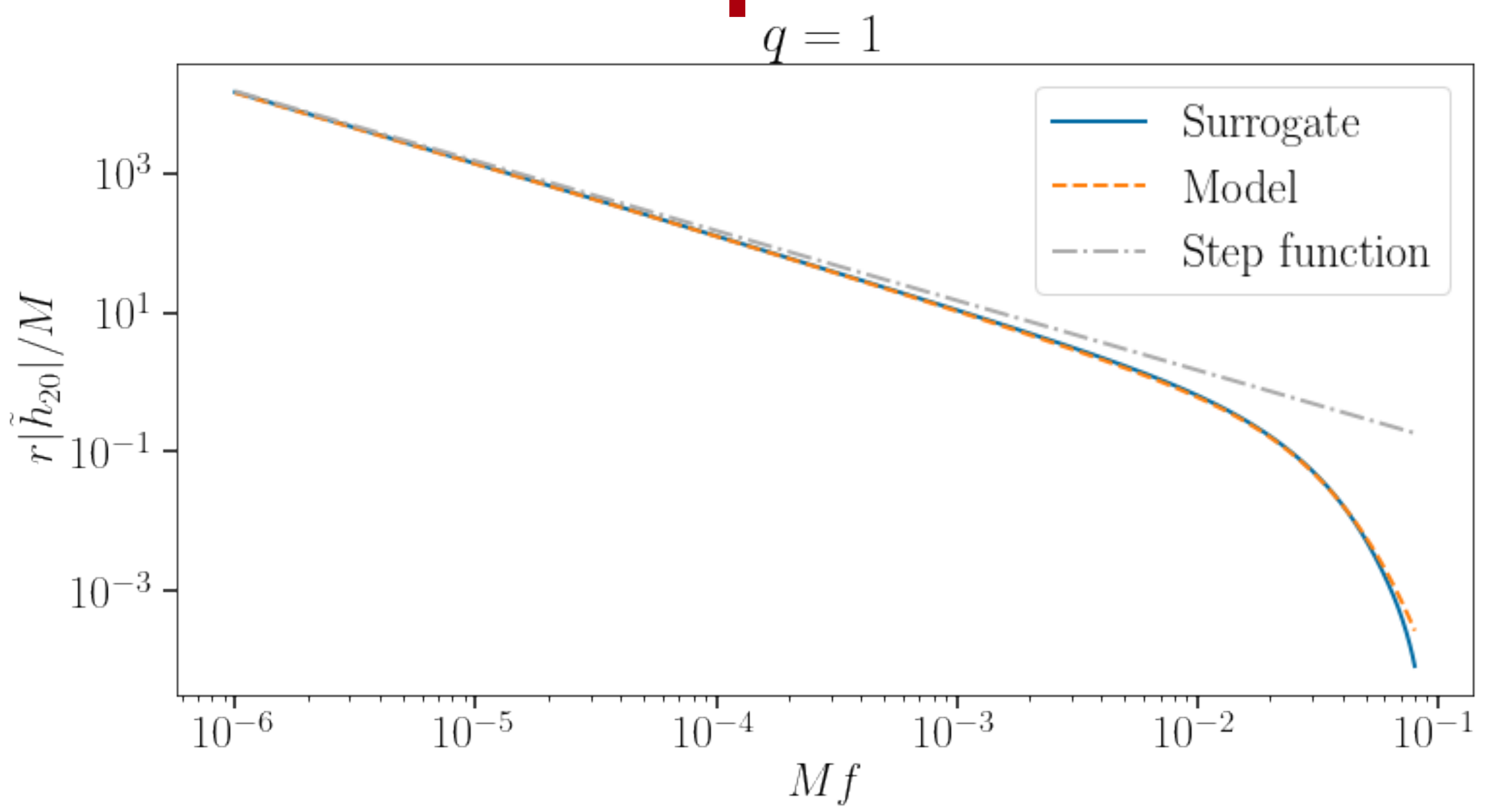
Frequency-domain phenomenological model

- A faster phenomenological model with lower mismatch.

$$\tilde{h}_{20}(f) \equiv |\tilde{h}_{20}(f)|\, e^{i\Phi}$$

$$|\tilde{h}_{20}| \sim A_1 \text{csch}\left(B_1 f\right) + A_2 \text{csch}\left(B_2 f\right) - A_3 f^C \text{csch}\left(B_3 f\right)$$

Amplitude



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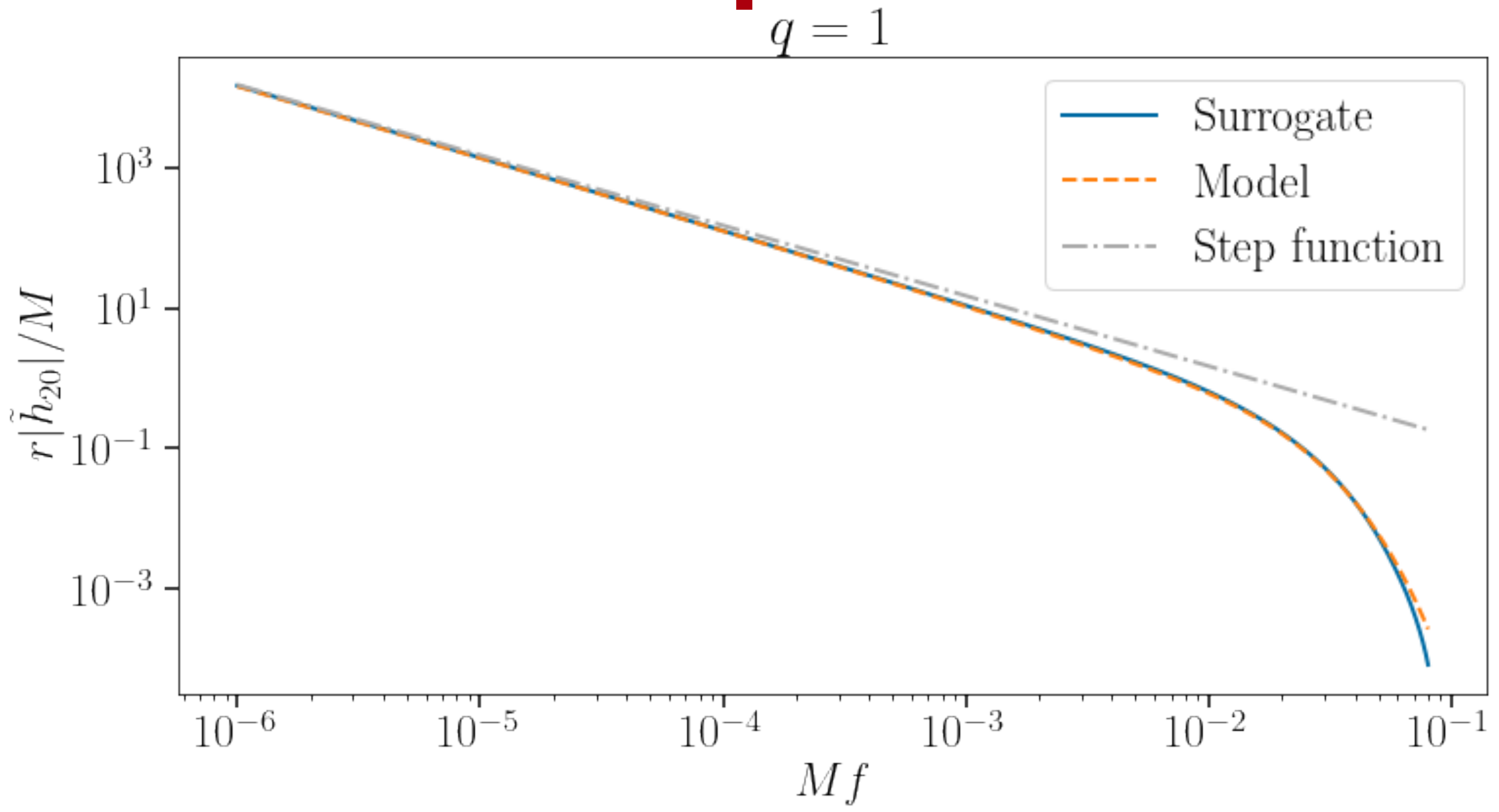

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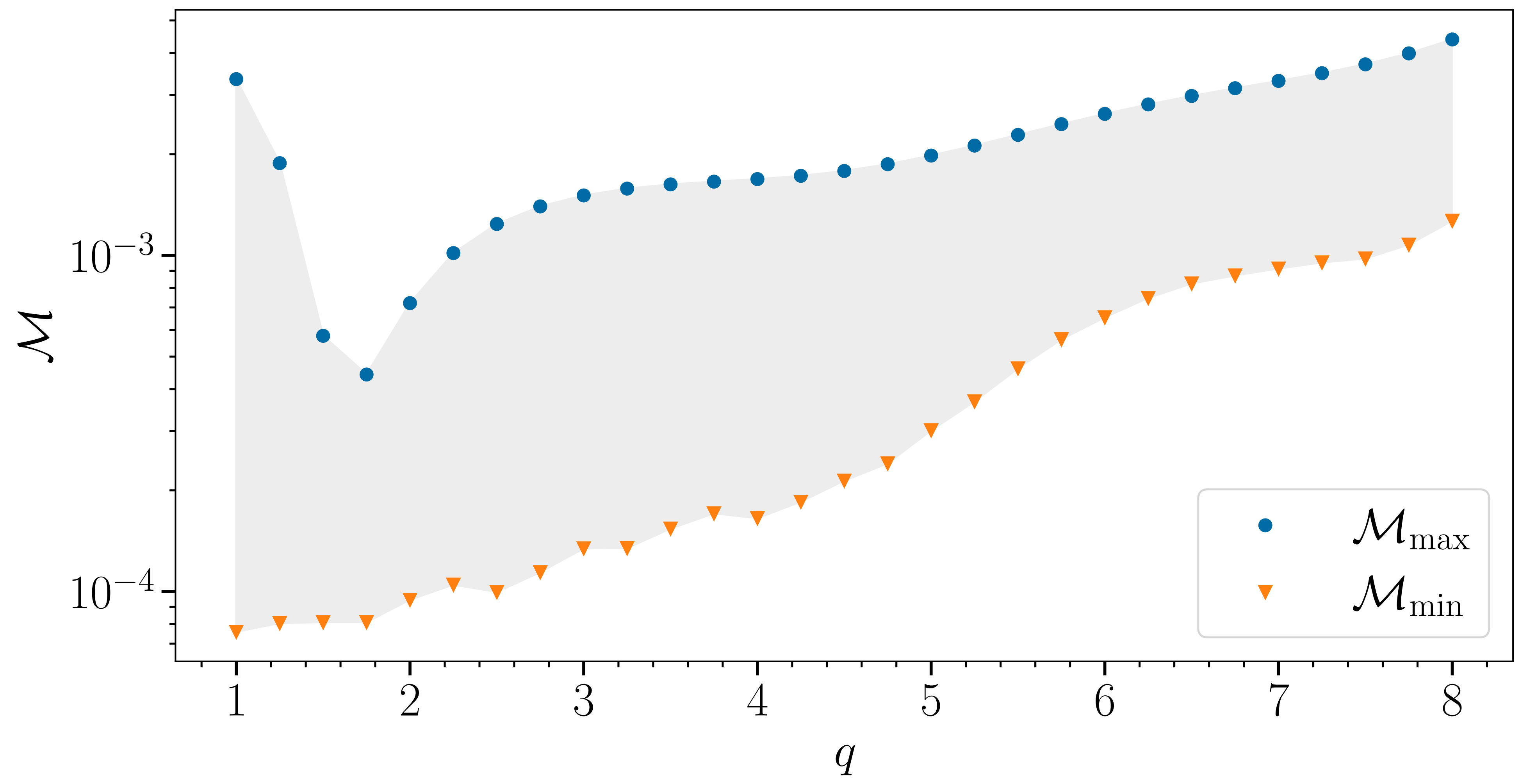

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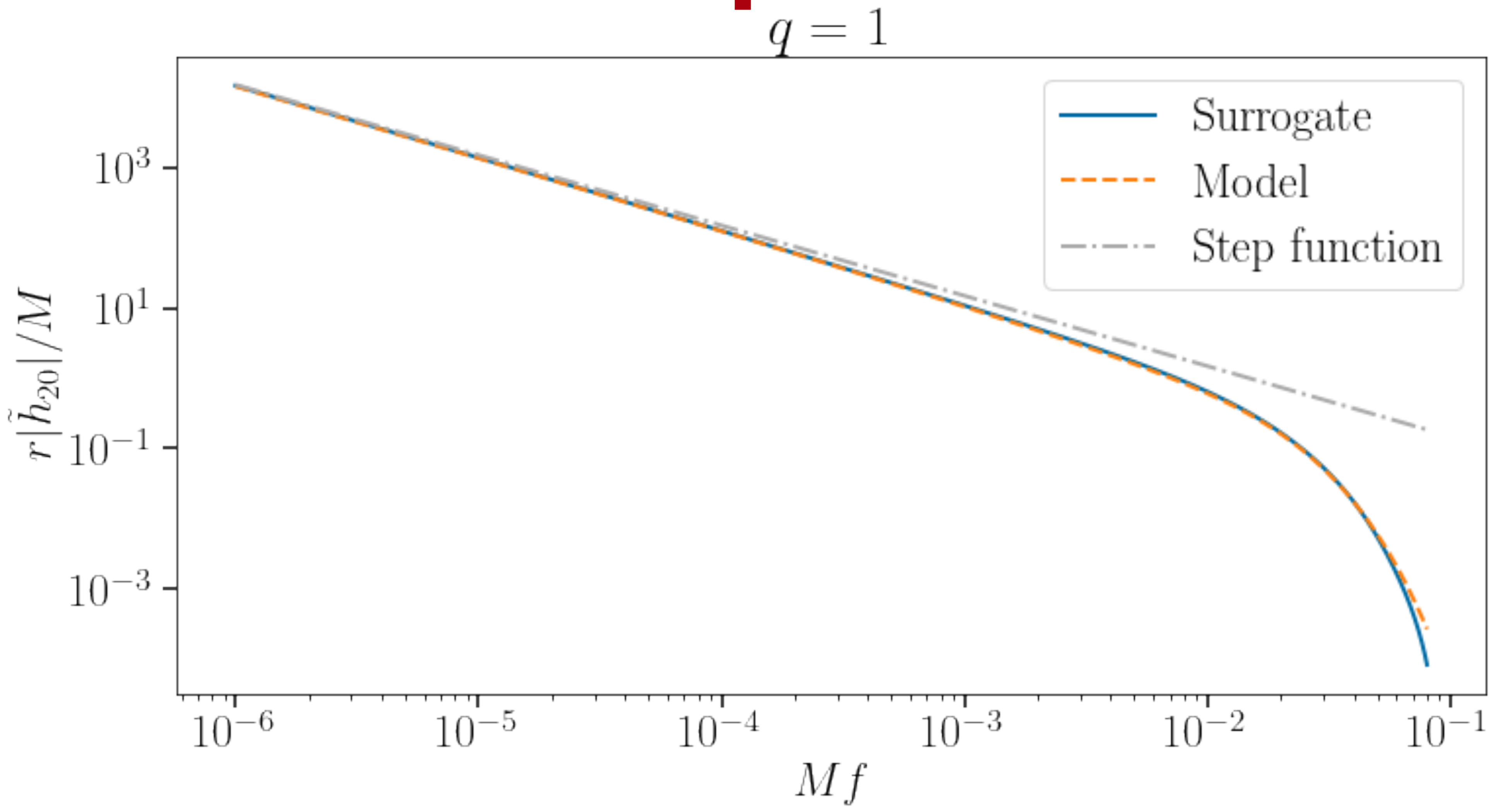
$$\Phi = -\pi/2 + 2\pi t_f f - a_1 f e^{-b_1/f} - a_2 f e^{-f/b_2} + a_3 f^c$$

Amplitude



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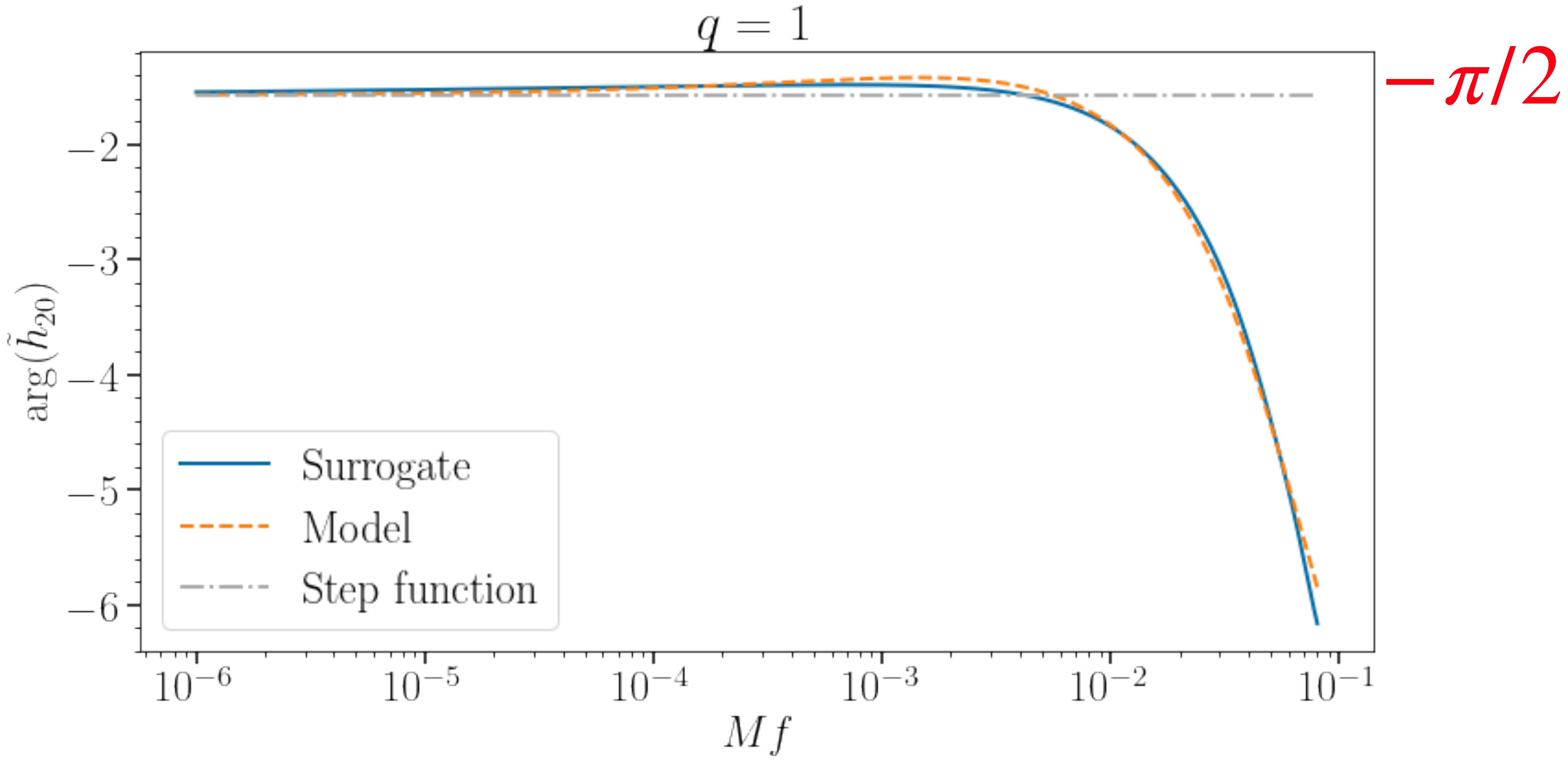
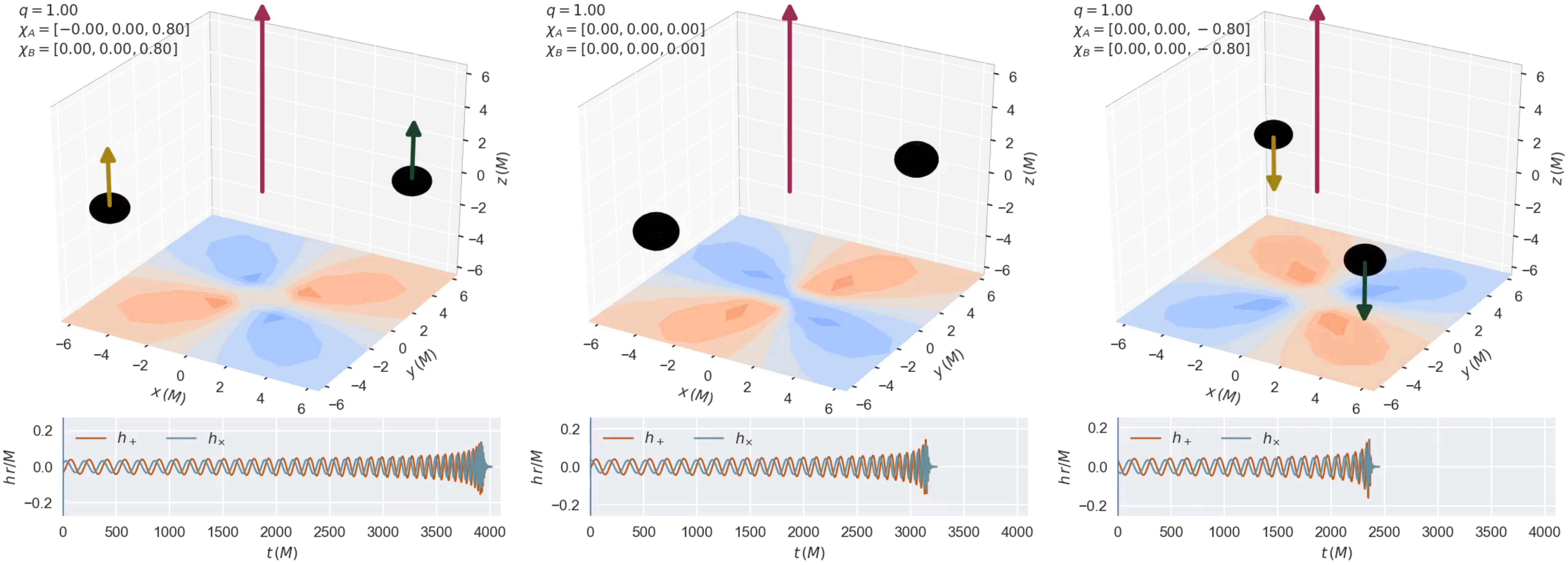


Spin?



Credit: Vijay Varma

Ongoing work

- ✦ Generalize the memory offset model to spinning primaries, and eccentric systems, in order to have a wider coverage of the parameter space of binaries. (w/ Laura Bernard)
- ✦ Generalizing the time- and frequency-domain memory models to include BH's spin, which is necessary for performing the hypothesis test with the mergers observed to date, which have shown evidence for rotation.
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Thanks