THE SIMPLEST B DECAY(S), PRECISELY !

Cluster of Excellence PRISMA+

Mainz Institute for Theoretical Physics

HEP Seminar (IJCLab)

May 07, 2026   Orsay, France

Max Ferré (they/them)

JGU Mainz

Based on arXiv:2212.14430 [hep-ph] and arXiv:2601.14361 [hep-ph]

In collab. w/ C. Cornella (INFN Padova), M. König and M. Neubert (JGU Mainz)

Motivations

$$\Gamma(B^- \to \ell^- \bar{\nu}_\ell) = G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

- Why leptonic B decays ?
  - Direct determination of the CKM element $|V_{ub}|$
  - Chirality-suppressed in the SM → powerful probe of (pseudo) scalar new physics
  - Testing flavor universality in charged current. Currently $\ell = \tau, \mu$ are measured with large uncertainties but Belle II aims to reduce them to $5 - 6\,\%$ [Belle II Physics Book 1808.10567] , $\mathcal{O}(1\%)$ will be an FCC-ee target.

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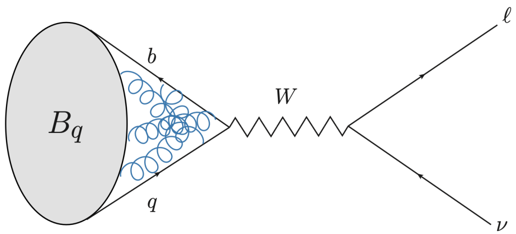

- Why QED corrections are needed?
  - Pure hadronic effects are simple and well-understood:

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p^\mu$$

  and f_{B_q} is known with $\mathcal{O}(1\%)$ precision : $f_{B_q} = 190.0 \pm 1.3$ MeV [FLAG 2411.04268]
  - Not true for QED corrections : potentially few % and currently only partially accounted for.
    → Could compete with QCD uncertainties !
  - The major challenge is to include and estimate the impact of « structure-dependent » corrections.
    → Necessary step before tackling more complicated 3-body decays !

QED corrections: Real and Virtual

Premise: We have in mind the case where the experiment vetoes real radiation with $E_{\text{rad}} > E_{\text{cut}}$, with $E_{\text{cut}} \ll \Lambda_{\text{QCD}}$, e.g. $E_{\text{cut}} \sim 50\,\text{MeV}$. The observable is $\Gamma_{B\to\ell\nu(\gamma)}(E_{\text{cut}})$.

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- Electromagnetic corrections are sensitive to the lepton mass and the experimental energy veto, yielding large (double) logarithmic corrections

$$\alpha \ln\left(\frac{m_\ell}{m_B}\right) \quad , \quad \alpha \ln\left(\frac{E_{\text{cut}}}{m_B}\right) \quad , \quad \alpha \ln\left(\frac{m_\ell}{m_B}\right) \ln\left(\frac{E_{\text{cut}}}{m_B}\right)$$

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- Virtual corrections are unrestricted by such cuts, they live at higher scales and can resolve the partonic substructure of the B meson.

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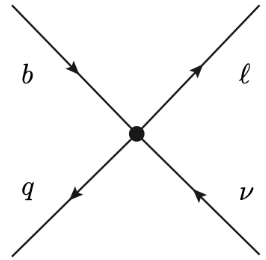
- At energies harder than m_B, the weak effective Lagrangian captures all the *hard* corrections through the Wilson coefficients of the LEFT-operators,

$$\mathscr{L}_{\rm LEFT} \supset L_\ell^{V,LL}\, \mathcal{O}_\ell^{V,LL} \quad \longrightarrow$$



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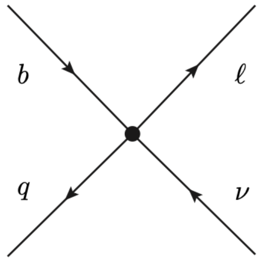

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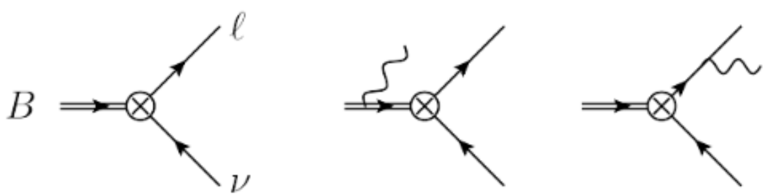
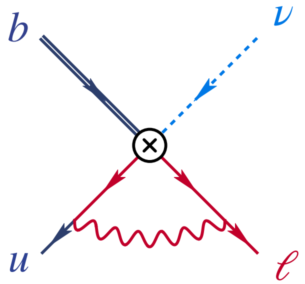
- Radiations softer than Λ_{QCD} sees the meson as point-like(*).



$\longrightarrow$ **Meson-effective-theory**

Structure dependent effects: Intermediate Scales

- Energetic virtual photons with $E_\gamma \sim m_B$ and $\vec{p}_\gamma || \vec{p}_\ell$ can recoil against the *soft* spectator.



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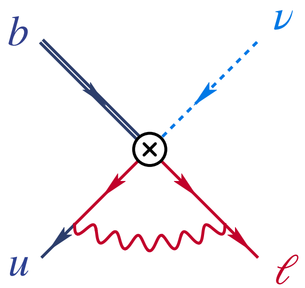

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- The light partons are the displaced on along the light cone, the hadronic *soft* currents become non-local → Light-cone distributions ($\phi\_(\omega), \phi_{3g}(\omega, \omega_g)$)

- When QED is switched on, these distributions are no longer process-universal → QED is sensitive to the direction of the charged external states [Beneke et al (2108.05589)], [Beneke et al (2204.09091)]

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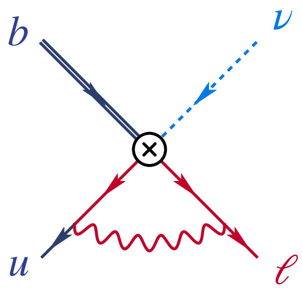

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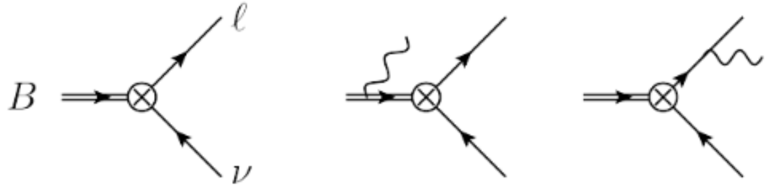
- Complicated to estimate , QED factorization theorems available for only few channels :
  - $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron, 1708.09152,1908.07011]
  - $B \to \pi K, B \to D\pi$ [Beneke, Böer et al 2008.10615,2107.03819]
  - $B \to \ell\nu$ [Cornella et al 2212.14430], [Rowe, Zwicky, 2209.06925] (via interpolating currents)

Structure dependent effects: Mesonic excitations

- Radiations softer than Λ_{QCD} sees the meson as point-like(*).



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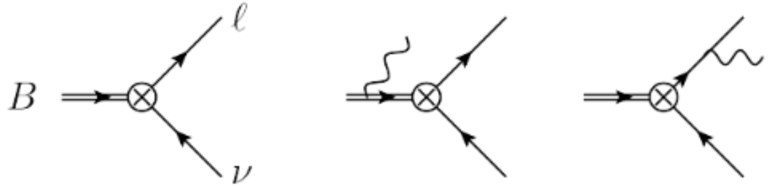

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- (*) For looser veto $E_{\text{cut}} \sim 50 - 150\,\text{MeV}$, contributions from the $B \to B^*\gamma$, $B \to B^*\pi^0$ excitations become sizeable even below the hadronic scale.

$$\delta_{B^*} = \frac{m_{B^*}^2 - m_B^2}{2m_B} \sim 45\,\text{MeV} \ll \Lambda_{\text{QCD}}$$

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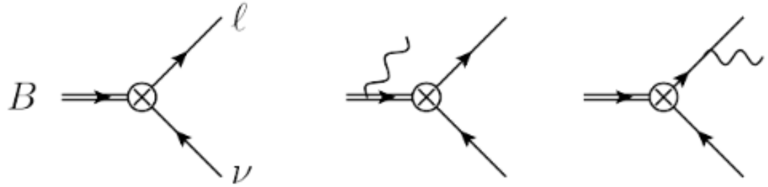

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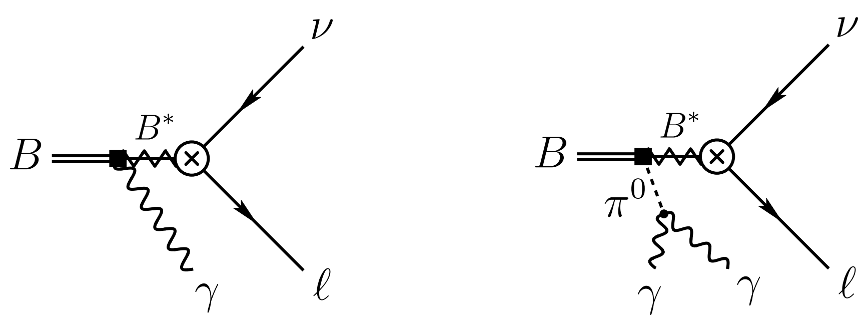
- The B^* vector meson can appear as an intermediate off-shell propagator, allowing for additional decay topologies :



A multi-scale process

- Focusing on $\ell = \mu$ case, including QED effects introduce new scales (both static and dynamic) to which $B^- \to \mu^- \bar{\nu}_\mu$ is sensitive :

m_Z, m_W

m_B, m_b

$\mu_{hc} = \sqrt{\Lambda_{QCD} m_B}$

$m_\mu \sim \Lambda_{QCD}$

$\mu_{us} = E_{\text{cut}}$

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Static scales dictated by the energy cut on the additional final radiation

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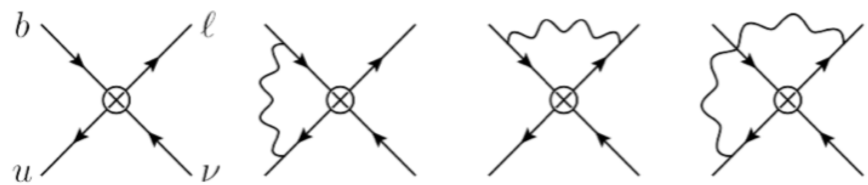

B-meson described as a superposition of Fock-states: $|\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \cdots$

[Beneke et al(2019),JHEP10232]

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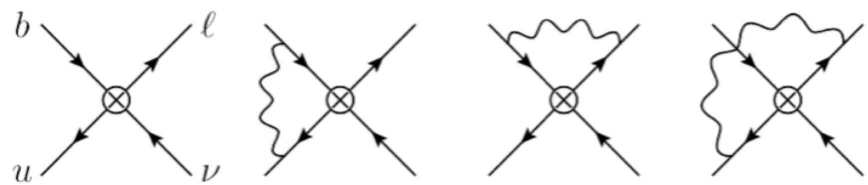

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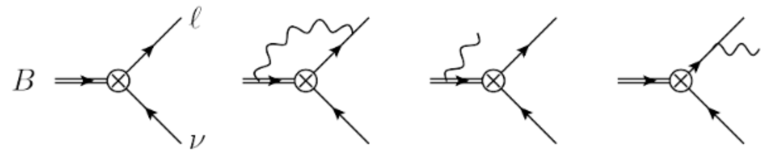
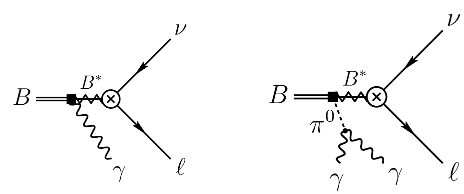
B-meson described as a point-like pseudo-scalar meson…

[e.g. Isidori,Nabeebaccus, Zwicky 2020; Zwicky 2021;Dai Leibovich 2021]



…(*)including mesonic excitations !

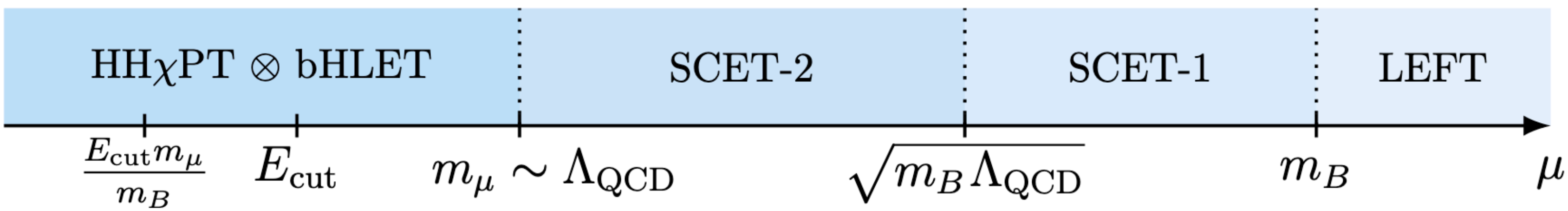
[Becirevic, Haas, Kou (2009) Phys.Lett.B681]



The plan : running down with the scale !

In this talk we are going to embark on an EFT journey :

Goal : Derive a $\mathrm{QED} \times \mathrm{QCD}$ factorization theorem for the decay rate $\Gamma(E_{\mathrm{cut}})$



$$\Gamma \sim W_{usc} \otimes W_{us} \otimes \left| K_i \otimes S_i \otimes J_i \otimes H_i \otimes K_{EW} \right|^2$$

- Partonic picture : Factorization formula for virtual corrections
- Mesonic picture : Factorization formula for the decay rate
- Numerical estimates

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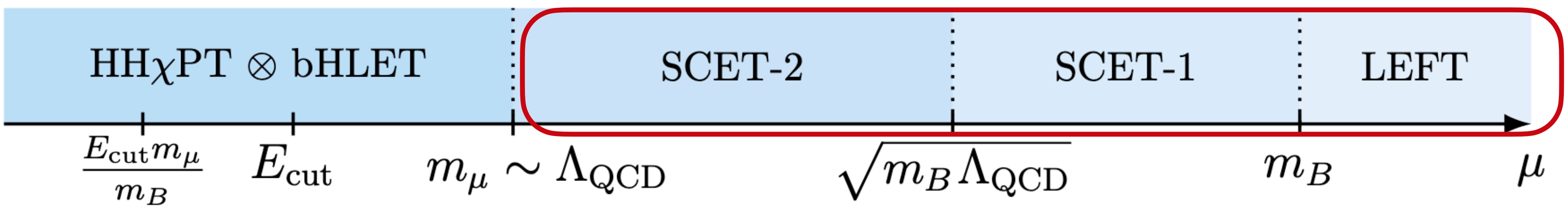

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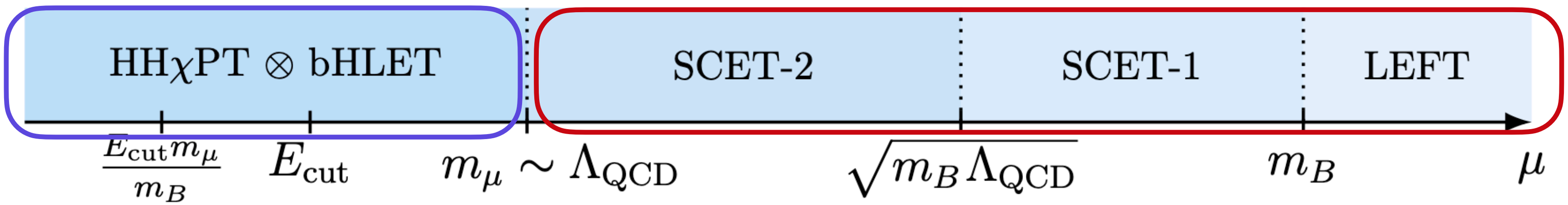

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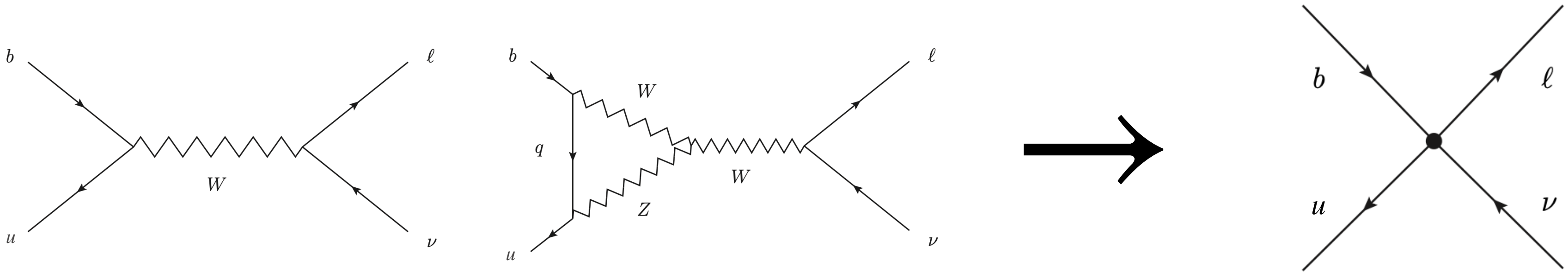
Partonic picture

SMEFT $\rightarrow$ LEFT $\quad \mu \sim m_Z, m_W$

- Below $\mu \sim m_Z, m_W$, heavy SM particles ($Z^0, W^{\pm}, H, t$) are integrated out leading to effective four fermion and dipole operators $\rightarrow$ **LEFT**



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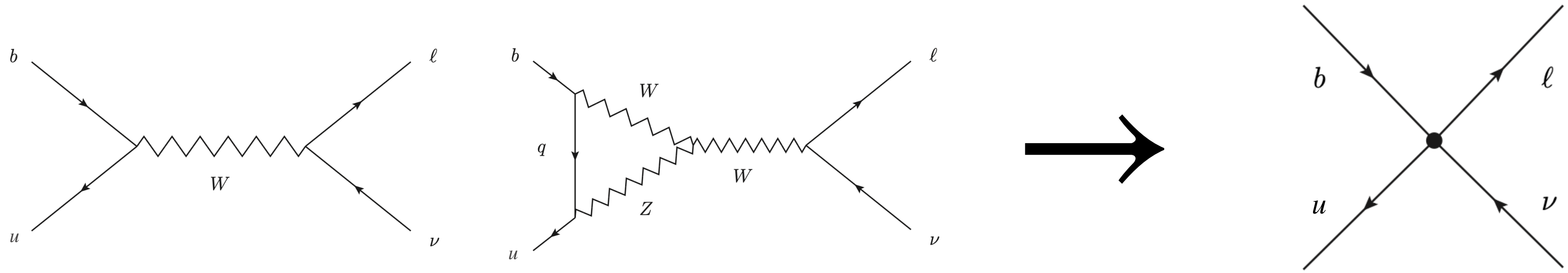

- In the SM, our process is described by a single operator $\quad \mathcal{O}_\ell^{V,LL} = (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$

$$\mathscr{L}_{\text{LEFT}} \supset -\frac{4\,G_F^{(\mu)}}{\sqrt{2}} V_{ub}\, K_{\text{EW}}(\mu)\, \mathcal{O}_\ell^{V,LL}$$

[Marciano and Sirling (1993),PRL713629]

[Dekens et al(2019),JHEP1910]

[Bigi et al(2023),JHEP11163]

At $\mathcal{O}(\alpha)$:
$$K_{\text{EW}}(\mu) = 1 + \frac{Q_\ell\, \alpha}{4\pi}\left[\left(3Q_u \ln\frac{\mu^2}{m_Z^2} + \frac{11}{6}\right) + (Q_b + Q_u)\left(1 + \frac{\kappa}{4}\right)\right]$$

κ : Scheme dependent quantity → cancels against the LEFT matrix elements (see additional slides)

RG improvement

$$K_{\text{EW}}(\mu) = 1 + \frac{Q_\ell\, \alpha}{4\pi}\left[\left(3Q_u \ln\frac{\mu^2}{m_Z^2} + \frac{11}{6}\right) + (Q_b + Q_u)\left(1 + \frac{\kappa}{4}\right)\right]$$

- Evaluated around $\mu \sim 1.5$ GeV, (single) logarithmic corrections can become sizeable → one can resum $\mathcal{O}\left(\alpha \ln(\mu^2/m_Z^2)\right)$ terms using RG evolution equation :

$$\frac{dK_{\text{EW}}(\mu)}{d\ln\mu} = \gamma_{\text{EW}}\, K_{\text{EW}}(\mu)\,; \quad \text{with} \quad \gamma_{\text{EW}} = Q_\ell\, Q_u \frac{3\alpha}{2\pi} = \gamma_{\text{EW}}^{(0)} \frac{\alpha}{4\pi}$$

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- Solving this equation leads to the RG-improved expression

$$K_{\rm EW}(\mu) = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\gamma_0^{\rm EW}/2\beta_0^{\rm QED}}\left[1 + 3Q_\ell Q_u\frac{\alpha}{4\pi}\ln\frac{\mu^2}{m_B^2}\right]K_{\rm EW}(m_Z) \quad \text{with} \quad \beta_0^{\rm QED} = -\frac{4}{3}\sum_f N_c^f Q_f^2$$

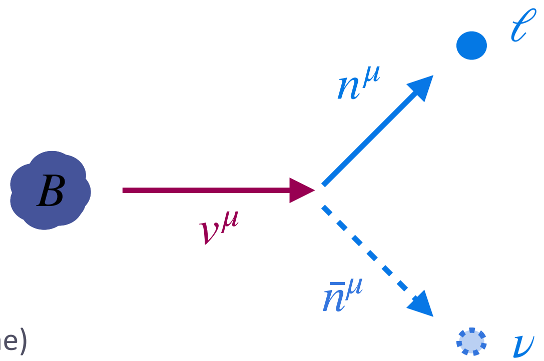
→We include the running of α only from m_Z to m_B : $\sim \mathcal{O}(4\%)$ relative effect

Kinematics and power counting

$$p_B^\mu = m_B \, v^\mu$$

$$v^\mu = (1,0,0,0)$$

(In the B meson rest frame)



$$n^2 = \bar{n}^2 = 0$$

$$n \cdot \bar{n} = 2$$

Power counting: $\lambda_\ell = \frac{m_\ell}{m_B} \sim \lambda = \frac{\Lambda_{QCD}}{m_B}$

Relevant scalings : $p^\mu = (\bar{n} \cdot p, n \cdot p, p_\perp)$

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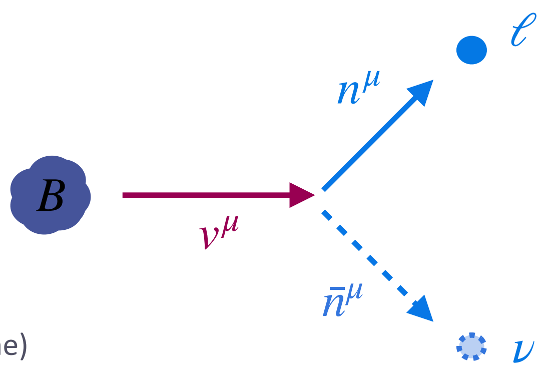

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- The final leptons sit on two opposite lightcones directions:

$$p_\ell^\mu = \frac{m_B}{2}(1+\lambda_\ell^2, 0, 0, 1-\lambda_\ell^2) \simeq \frac{m_B}{2}(1,0,0,1) = \frac{m_B}{2}\, n^\mu$$

$$p_\nu^\mu = \frac{m_B}{2}(1-\lambda_\ell^2, 0, 0, -1+\lambda_\ell^2) \simeq \frac{m_B}{2}(1,0,0,-1) = \frac{m_B}{2}\bar{n}^\mu$$

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$p_\ell \sim (1, \lambda_\ell^2, \lambda_\ell)$ *« collinear »,*

$p_\ell^2 \sim m_\ell^2 \sim \mathcal{O}(\lambda_\ell^2)$

→ given by the lepton virtuality

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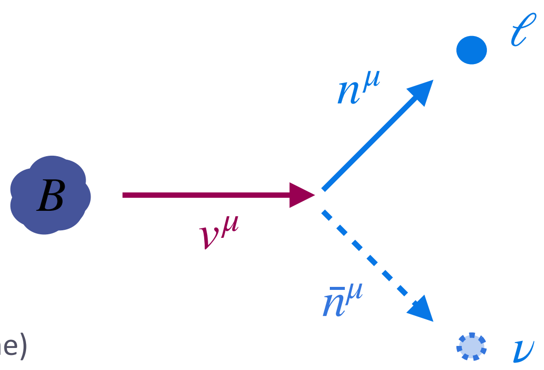

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- The final leptons sit on two opposite lightcones directions:

$$p_\ell^\mu = \frac{m_B}{2}(1+\lambda_\ell^2, 0,0, 1-\lambda_\ell^2) \simeq \frac{m_B}{2}(1,0,0,1) = \frac{m_B}{2}\, n^\mu$$

$$p_\nu^\mu = \frac{m_B}{2}(1-\lambda_\ell^2, 0,0, -1+\lambda_\ell^2) \simeq \frac{m_B}{2}(1,0,0,-1) = \frac{m_B}{2}\bar{n}^\mu$$

- Initial quarks bound in the meson with residual *soft* momenta:

$$p_b = m_b v^\mu + k^\mu \quad , \quad p_q^\mu = k^\mu$$

Power counting: $\lambda_\ell = \dfrac{m_\ell}{m_B} \sim \lambda = \dfrac{\Lambda_{QCD}}{m_B}$

Relevant scalings : $p^\mu = (\bar{n}\cdot p, n\cdot p, p_\perp)$

$p_\ell \sim \left(1, \lambda_\ell^2, \lambda_\ell\right)$ *« collinear »*,

$p_\ell^2 \sim m_\ell^2 \sim \mathcal{O}(\lambda_\ell^2)$

→ given by the lepton virtuality

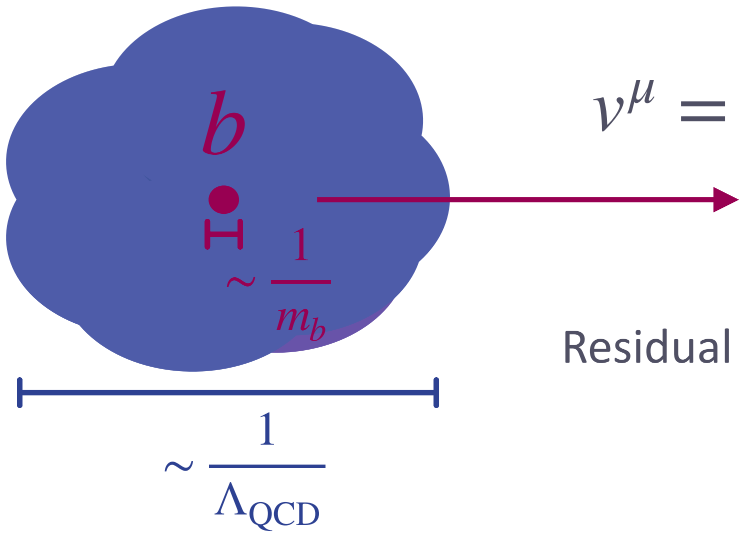
$p_s \sim \left(\lambda, \lambda, \lambda\right)$ *« soft »*,

$p_s^2 \sim \Lambda_{QCD}^2 \sim \mathcal{O}(\lambda^2)$

→ given by the spectator quark virtuality

Heavy quark description

- The B meson is a QCD bound state characterised by two scales : $m_b \gg \Lambda_{\rm QCD}$



$$v^\mu = v_b^\mu - \frac{k^\mu}{m_b}$$

Residual velocity of $\mathcal{O}(\Lambda_{\rm QCD}/m_b) = \mathcal{O}(\lambda)$

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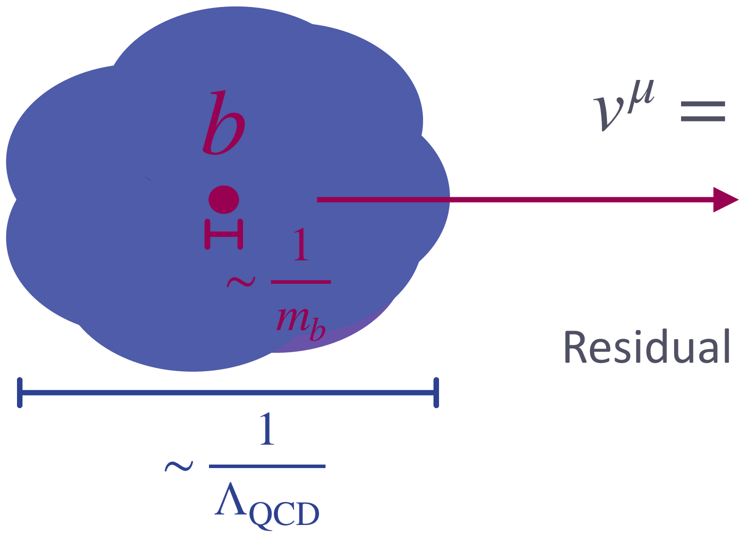

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Residual velocity of $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b) = \mathcal{O}(\lambda)$

- Effectively described by an *on-shell b-quark* + *soft fluctuations* $\rightarrow v_b^\mu$ (almost) conserved

- *Soft fluctuations* of the b quark defines the HQET field :

$$b(x) \rightarrow e^{-im_b(v\cdot x)}\left(1 + \mathcal{O}(\sqrt{\lambda})\right) b_v(x)$$

A glimpse into SCET$_{(I)}$

- Light fields with large momenta but small invariant mass → SCET framework

m_B, m_b "Hard"

$\mu_{hc} = \sqrt{\Lambda_{QCD} m_B}$ " Hard Collinear"

Λ_{QCD} "Soft"

→ In practice, we split each field into modes and integrate out the *hard* ones !

$$u(x) \to u_h(x) + u_{hc}(x) + u_s(x)$$

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Hard-collinear/*Soft* interaction

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Hard-collinear SCET field — *Hard-collinear*/*Soft* interaction

- Key difference vs HQET and LEFT : non-locality along $\bar{n}^\mu$ $\quad \frac{1}{i\bar{n}\cdot\partial} f(x) = \int_{-\infty}^{0} ds\, f(x + s\bar{n})$
- To describe interaction between the *soft* quark and the *collinear* lepton we need an intermediate *hard-collinear* scale → subleading power interactions !

LEFT $\to$ HQET $\otimes$ SCET$_I$ $\quad \mu \sim m_B, m_b$

- The b quark is described by a *soft* HQET field :

$$b(x) \to e^{-im_b(v\cdot x)}\left(1+\mathcal{O}(\sqrt{\lambda})\right) b_v(x)$$

- QED corrections $\to$ *hard-collinear* momentum exchange between partons & lepton.

$\to$ need a SCET$_I$ subleading power description for the different modes of the spectator and the lepton :

$$u(x) \to \left(1+\frac{1}{i\bar{n}\cdot\partial}(i\not{D}^{\perp})\frac{\not{\bar{n}}}{2}\right)\chi_{hc}^{(u)}(x) + \left(1+\frac{1}{i\bar{n}\cdot\partial}eQ_u\not{A}_{hc}^{\perp}\frac{\not{\bar{n}}}{2}\right)u_s(x)$$

$$\ell(x) \to \left(1+\frac{1}{i\bar{n}\cdot\partial}(i\not{D}^{\perp}+m_\ell)\frac{\not{\bar{n}}}{2}\right)\chi_{hc}^{(\ell)}(x) + \left(1+\frac{1}{i\bar{n}\cdot\partial}eQ_\ell\not{A}_{hc}^{\perp}\frac{\not{\bar{n}}}{2}\right)\ell_s(x)$$

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$\to$ given by the lepton virtuality

$p_{hc} \sim \left(1, \lambda, \sqrt{\lambda}\right)$ *« hard-collinear »,*

$p_{hc}^2 \sim \Lambda_{QCD} m_B \sim \mathcal{O}(\lambda)$

$\to$ soft and collinear quark X-talk

$p_s \sim \left(\lambda, \lambda, \lambda\right)$ *« soft »,*

$p_s^2 \sim \Lambda_{QCD}^2 \sim \mathcal{O}(\lambda^2)$

$\to$ given by the spectator quark virtuality

SCET$_I$ operators

$$\mathcal{O}_\ell^{V,LL} = H_i^A O_i^A + \int_0^1 dy \left[H_i^B(y) O_i^B(y) + H_i^C(y) O_i^C(y) \right]$$

We build our SCET$_I$ basis with the following power counting :

$$b_v, u_s \sim \mathcal{O}(\lambda^{3/2}) \qquad \chi_{hc} \sim \mathcal{O}(\lambda^{1/2}) \qquad \mathcal{A}_{hc}^{\perp} \sim \mathcal{O}(\lambda^{1/2})$$

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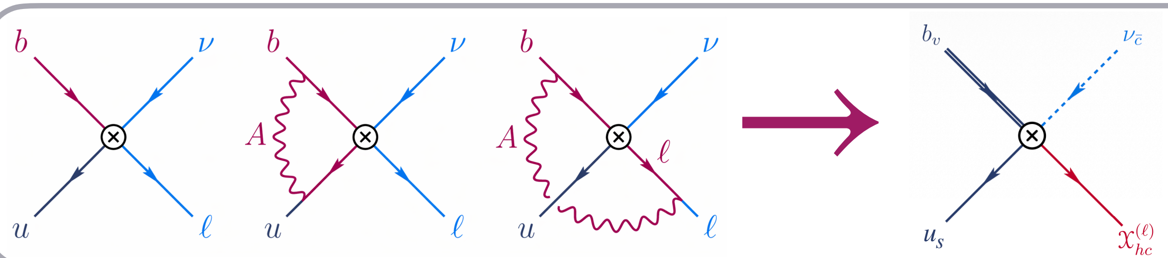

$[O_i^A]$ $(\bar{u}_s \ldots b_v)(\bar{\chi}_{hc}^{(\ell)} \ldots \nu_{\bar{c}})$

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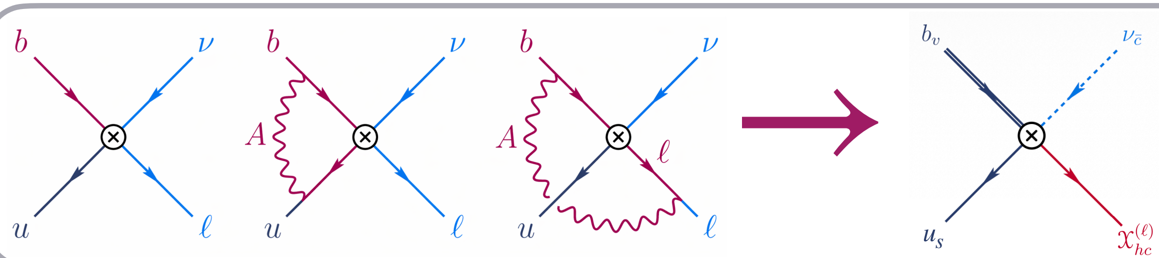

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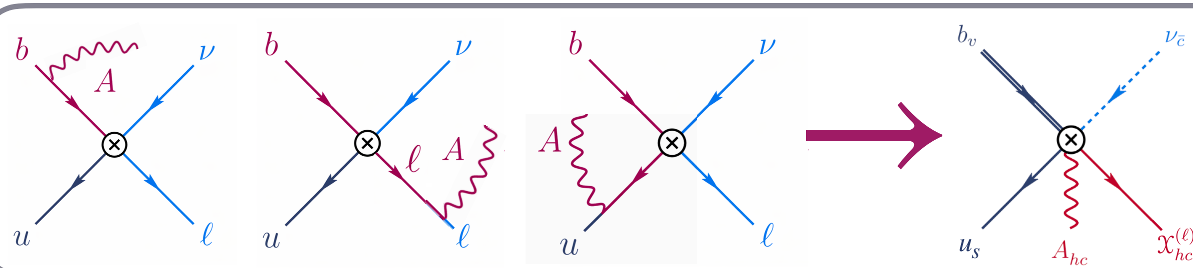

$[O_i^B]$ $(\overline{u}_s \ldots b_v)(\overline{\chi}_{hc}^{(\ell)} \ldots \nu_{\bar{c}})\mathcal{A}_{hc[y]}^{\perp}$

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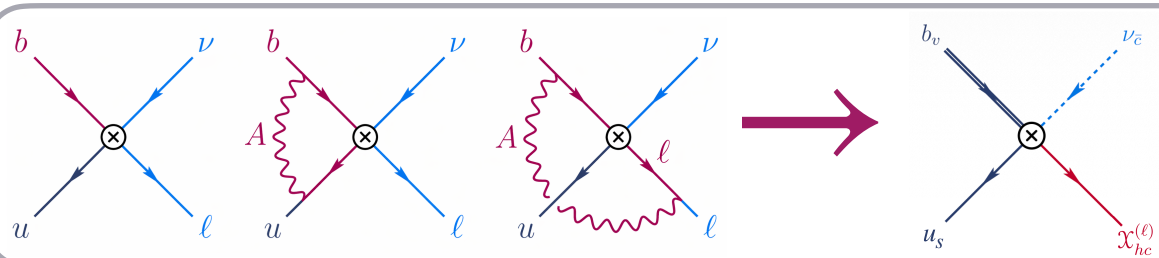

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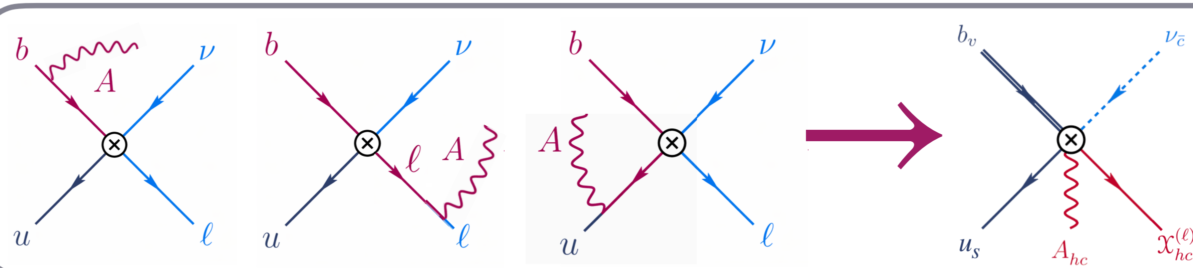

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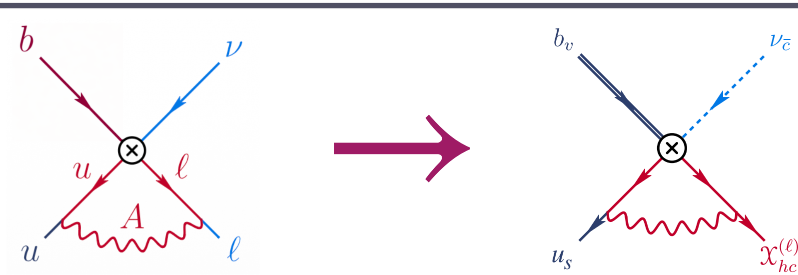

$[O_i^C]$ $(\bar{\chi}_{hc[y]}^{(u)} \dots b_v)(\bar{\chi}_{hc}^{(\ell)} \dots \nu_{\bar{c}})$

$\text{SCET}_I \to \text{SCET}_{II}$ $\quad \mu \sim \mu_{hc} \sim \sqrt{\Lambda_{QCD} m_B}$

- At $\mu \sim \mu_{hc}$, we lower the virtuality removing *hard-collinear* modes $\to$ pure SCET_{II} construction where collinear and *soft* carry the same virtuality :

$$p_c \sim \left(1, \lambda_\ell^2, \lambda_\ell\right), \qquad p_s \sim \left(\lambda, \lambda, \lambda\right), \qquad p_c^2 \sim p_s^2 \sim \mathcal{O}(\lambda^2/\lambda_\ell^2)$$

- Integrating out *hard-collinear* propagators introduces non-localities even in *soft* product :

$$\frac{1}{n\cdot\partial} q_s, \; \left(\frac{1}{n\cdot\partial}\mathcal{G}_s^{\perp}\right)\left(\frac{1}{n\cdot\partial} q_s\right), \cdots$$

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- These operators can overcome the chiral suppression and generate power enhanced structure-dependent contributions : e.g. in $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron 2017, 2019])

- Not true for $B^- \to \mu^- \bar{\nu}_\mu$: for $O_\ell^{V,LL}$ such enhanced contributions are evanescent. But they can appear in presence of generic NP, e.g. for $L_\ell^{V,LR} \neq 0$!

SCET$_{II}$ basis

$$O_i^X(\{y\}) = \sum_{j,X'} J_{O_i^X \to Q_j^{X'}}(\{y\},\{x\},\{\omega\}) \otimes_{x,\omega} Q_j^{X'}(\{x\},\{\omega\})$$

- Starting from our SCET$_I$ operators, we can construct our SCET$_{II}$ basis, at $\mathcal{O}(\alpha)$ we get:

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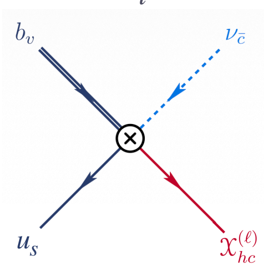

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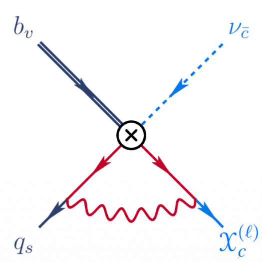

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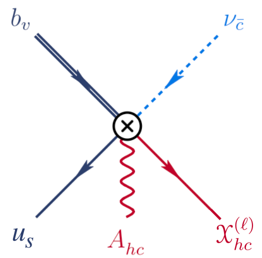

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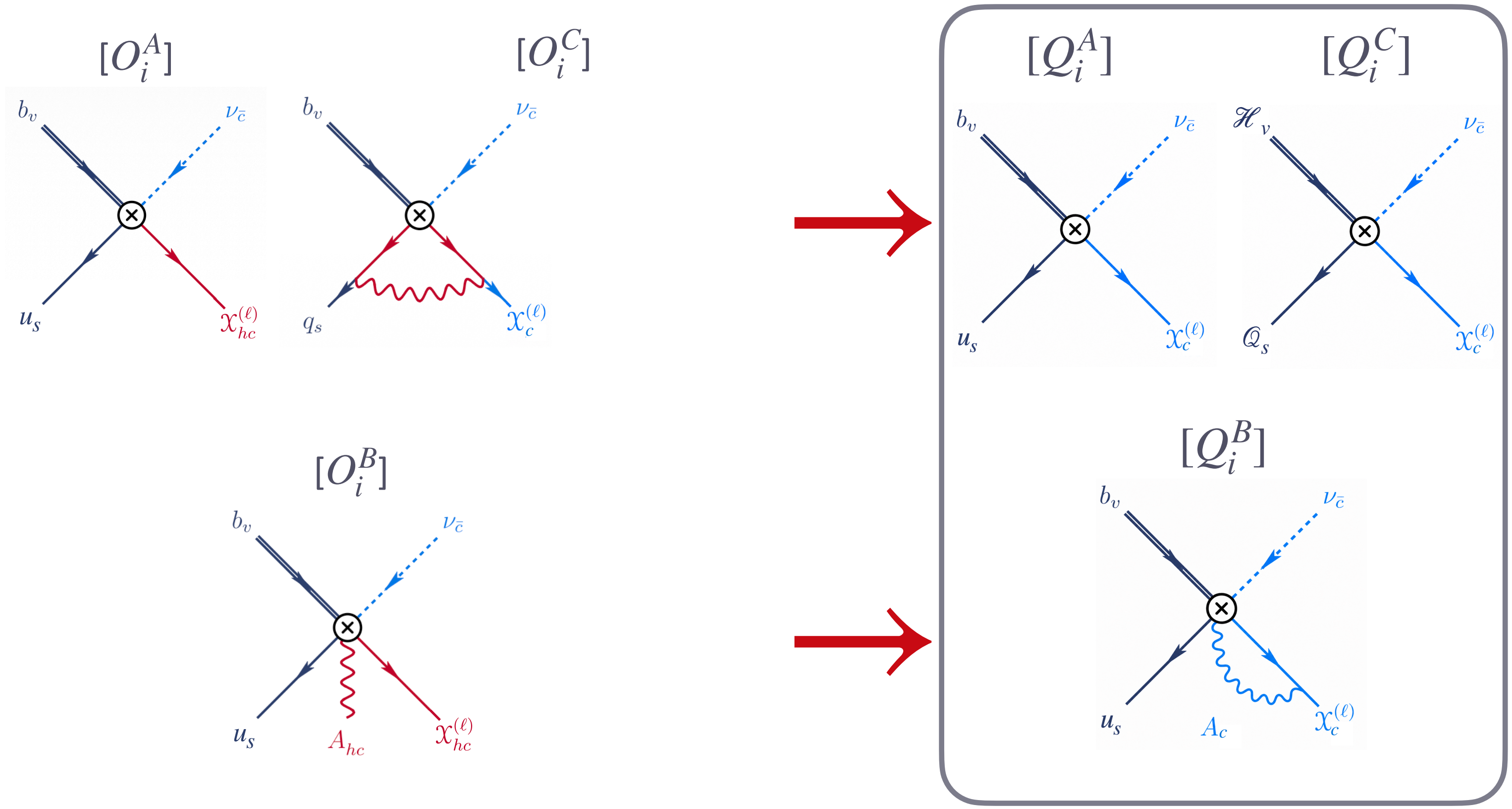

$|B\rangle = \quad |\bar{u}b\rangle \quad \oplus$

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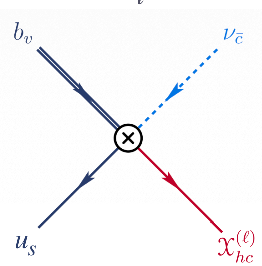

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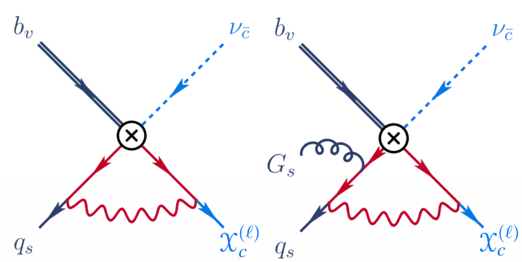

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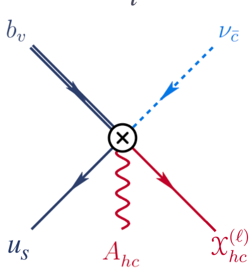
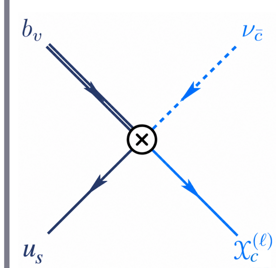


$[Q_i^A]$



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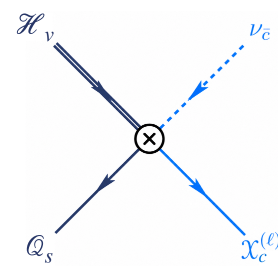
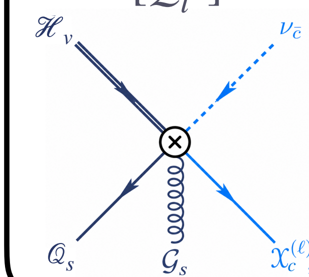


$[Q_i^E]$



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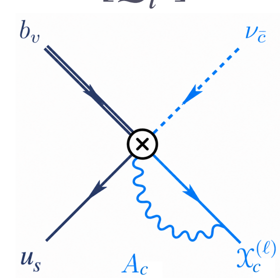

$$|B\rangle = \quad |\bar{u}b\rangle \quad \oplus \quad |\bar{u}bg\rangle \oplus \cdots$$

Factorization formula (virtual corrections)

▸ Taking SCET$_{II}$ matrix elements, we obtain a factorisation theorem for the virtual corrections :

$$\mathscr{A}_{\text{virt}} = -\frac{4\,G_F^{(\mu)}}{\sqrt{2}} V_{ub}\, K_{\text{EW}}(\mu)\, \langle \ell \nu \,|\, \mathscr{L}_{\text{SCET II} \otimes \text{HQET}} \,|\, B \rangle$$

$$= -\frac{4\,G_F^{(\mu)}}{\sqrt{2}} V_{ub}\, K_{\text{EW}}(\mu) \sum_{i,X} H_i^X(\{y\}) \otimes_y \sum_{j,X'} J_{O_i^X \to Q_j^{X'}}(\{y\},\{x\},\{\omega\}) \otimes_{x,\omega} \langle Q_j^{X'}(\{x\},\{\omega\}) \rangle$$

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- SCET$_{II}$ operators factorizes into non perturbative *soft hadronic contributions* and purely leptonic perturbative *collinear functions* :

$$\langle Q_i^{X'}(\{x\},\{\omega\})\rangle = \big\langle 0\,|\,j_{\text{had}}\,|\,B\big\rangle \langle \ell\nu\,|\,j_{\text{lep}}\,|\,0\rangle$$

$$= \underbrace{S_i^{X'}(\{\omega\}) \left\langle 0 \left| Y_n^{(\ell)\dagger} \frac{\varphi_v}{\sqrt{2m_B}} \right| B \right\rangle}_{R^{(\ell,B)}} K_i^{X'}(\{x\})\, \frac{m_\ell}{m_B}\, \langle \ell\nu\,|\,\bar{h}_n P_L \nu_{\bar{c}}\,|\,0\rangle$$

- They define matching coefficients to the low energy theory hadronic and leptonic currents.

Endpoint divergences : Refactorization

- The matrix element of C-type SCET I operators involve an endpoint-divergent integral→ the theory fails to properly separate *hard-collinear modes* with low energy (*hard-collinear* fraction $y \sim \mathcal{O}(\lambda)$) from *soft modes*.

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$$H_1^C(y)\, J_{O_1^C \to Q_1^C}(y, \omega, \Lambda) = H_1^C(y)\, J^{bare}_{O_1^C \to Q_1^C}(y, \omega)$$

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$0 < \eta = \Lambda/m_B \leq 1$: arbitrary scale

$$H_1^C(y)\, J_{O_1^C \to Q_1^C}(y,\omega,\Lambda) = H_1^C(y)\, J^{bare}_{O_1^C \to Q_1^C}(y,\omega) - \theta(\eta - y)[\![H_1^C(y)]\!][\![J_{O_1^C \to Q_1^C}(y,\omega)]\!]$$

Singular for $y \to 0$ $\quad \sim y^{-\epsilon} \quad \sim y^{-1-\epsilon}$

Endpoint divergences : Refactorization

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$0 < \eta = \Lambda/m_B \leq 1$: arbitrary scale $\bar{\omega} = y\,(\bar{n}\cdot p_\ell)/(\bar{n}\cdot v) \sim \mathcal{O}(\lambda)$: new soft scale

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New hadronic parameter !

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Generalized decay constant

$$\ln \frac{\omega_-(\mu)}{\nu} = \int_0^\infty d\omega\, \phi_-^B(\omega,\mu)\, \ln \frac{\omega}{\nu}$$

- $F_-(\Lambda,\mu)$ is an unknown non-perturbative parameter following evolution equations for Λ and μ :

$$\frac{d}{d\ln\Lambda}\frac{F_-(\Lambda,\mu)}{F_{\rm QCD}(\mu)} = -Q_\ell Q_u \frac{\alpha}{2\pi}\tilde{U}_C(\mu,\Lambda)\Bigg[\ln\frac{\mu^2}{\Lambda\omega_-(\mu)} + 1$$
$$-2\int_0^\infty d\omega \int_0^\infty d\omega_g \left(\frac{1}{\omega_g}\ln\frac{\omega+\omega_g}{\omega} - \frac{1}{\omega+\omega_g}\right)\phi_{3g}^B(\omega,\omega_g,\mu) + \mathcal{O}(\alpha_s)\Bigg]$$

$$\frac{d}{d\ln\mu}F_-(\Lambda,\mu) = -\,\gamma_{F_-}(\Lambda,\mu)\,F_-(\Lambda,\mu)$$
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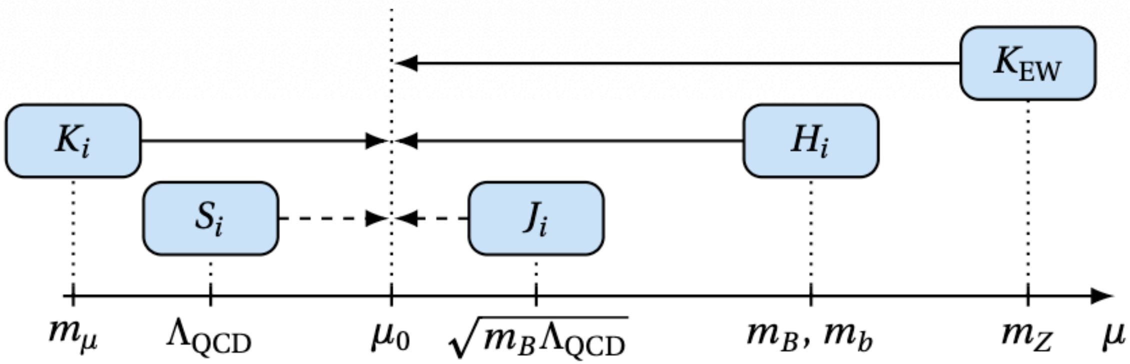
- *Before* refactorization, $F_-(\mu)$ mixes with matrix elements of non-local operators under RGE.
- *After* refactorization, $F_-(\Lambda,\mu)$ renormalizes multiplicatively yet non trivially due to the presence of the subtraction scale Λ.

→Non-linear dependence on $\ln\mu$ in γ : at every step along the μ trajectory, have to evolve the cutoff from Λ to μ. Seems to be a generic feature of SCET NLP problems.

Virtual amplitude, resummed

- We can then RG evolve all component function in the virtual corrections to a common scale $\mu_0 = 1.5\,\mathrm{GeV}$:



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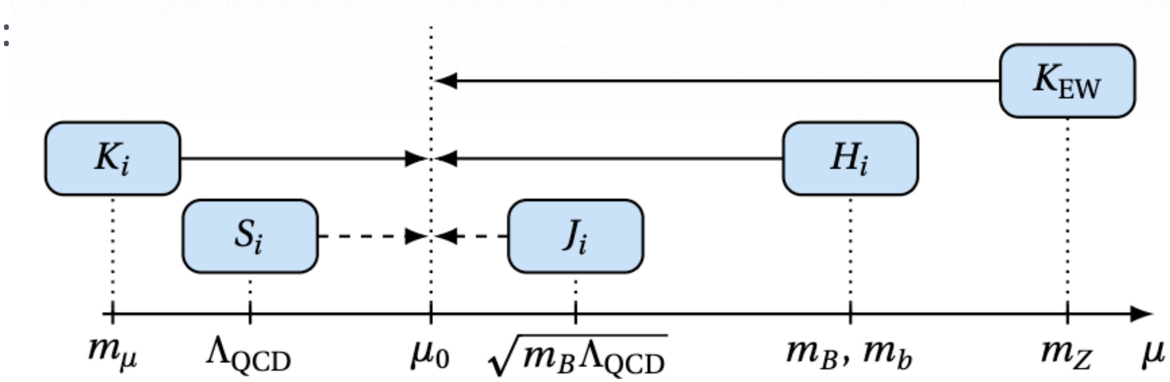

- Virtual corrections are then all contained in the resummed expression :

$$i\mathcal{A}_{\text{virt.}}(B^- \to \ell^- \bar{\nu}_\ell)_{\mu_0} = i\sqrt{2}\, G_F^{(\mu)}\, V_{ub}\, m_\ell\, f_{B^-}\, \bar{u}(v_\ell)\, P_L\, v(p_\nu) \sum_{i,X} T_i^X(\mu_0)$$

$$\sum_{i,X} T_i^X(\mu_0) = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\frac{9}{40}} \left(\frac{\mu_0^2}{m_B\, m_\ell}\right)^{\gamma_{\text{soft}}/2} \mathcal{R}_{\text{virt}}$$

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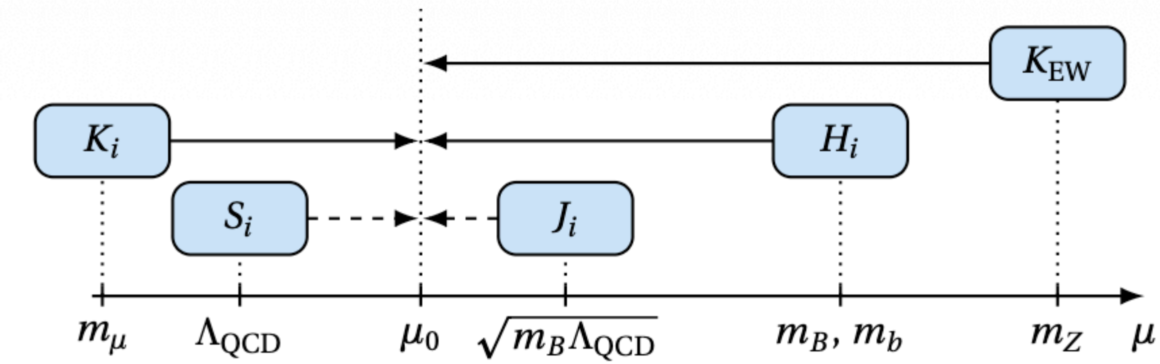

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$$\gamma_{\mathrm{soft}} = Q_\ell^2\,\frac{\alpha}{2\pi}\left(\ln\frac{m_B^2}{m_\ell^2} - 2\right)$$

will cancel scale dependence of real emissions

$$\frac{d}{d\ln\mu_0}\ln\mathscr{R}_{\mathrm{virt}} = \mathscr{O}\left(\alpha\,\alpha_s\,\ln\frac{\mu_0^2}{m_B^2}, \alpha^2\,\ln^2\frac{\mu_0^2}{m_B^2}\right)$$

(Approx.) scale independent

Structure dependent virtual corrections

- All virtual structure dependent are captured in $\mathscr{R}_{\text{virt}}$:

$$\mathscr{R}_{\text{virt}} = \exp\left[-Q_\ell Q_u \frac{\alpha}{\pi}\int_{m_B}^{\mu_0} \frac{d\bar{\omega}}{\bar{\omega}} \tilde{U}_C(\mu',\bar{\omega})\right] \frac{F_-(m_B,\mu_0)}{F_{\text{QCD}}(\mu_0)}$$

$\Lambda = m_B$

(But result is Λ independent)

Two particles contribution

$$\times \left\{ 1 + \frac{\alpha}{4\pi}\left[\frac{3}{2}Q_\ell^2 \ln\frac{\mu^2}{m_\ell^2} - \frac{3}{2}Q_b^2 \ln\frac{\mu^2}{m_B^2} - (2+z)Q_\ell Q_b \ln\frac{m_B^2}{m_\ell^2} + \left(\frac{\pi^2}{12}-\frac{5}{2}\right)Q_\ell^2 \right.\right.$$

$$\left. + \left(-\frac{1}{2} + \frac{z^2 \ln z}{z-1} + z - 2\text{Li}_2(1-z) - \frac{\pi^2}{12}\right) Q_\ell Q_b - (2+3\ln z)Q_b^2 \right]$$

$$+ Q_\ell Q_u \frac{\alpha}{\pi}\int_{m_B}^{\mu} \frac{d\mu'}{\mu'} \frac{\tilde{U}_C(\mu',m_B)}{1-\delta(\mu')}$$

$$-Q_\ell Q_u \frac{\alpha}{2\pi}\left[\frac{1}{1-\delta(\mu_0)} \ln\frac{\mu_0^2}{m_B \omega_-(\mu_0)} - h_1(\delta(\mu_0))\right] \tilde{U}_C(\mu_0, m_B)$$

$$\ln\frac{\omega_-(\mu)}{\nu} = \int_0^\infty d\omega\, \phi_-^B(\omega,\mu)\, \ln\frac{\omega}{\nu}$$

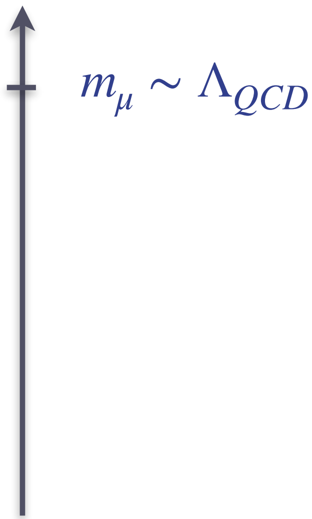
Three particles contribution

$$-Q_\ell Q_u \frac{\alpha}{\pi}\frac{\tilde{U}_C(\mu_0,m_B)}{1-\delta(\mu_0)} \int_0^\infty d\omega \int_0^\infty d\omega_g \frac{\phi_{3g}^B(\omega,\omega_g,\mu)}{\omega_g}\left[\frac{1}{\omega_g}\ln\frac{\omega+\omega_g}{\omega} - \frac{1}{\omega+\omega_g}\right].$$

Mesonic picture

Low-energy theory : modes and scales $\mu < \Lambda_{QCD}$

- After integrating out the physics above $\mu < \Lambda_{QCD}$, few scales remain



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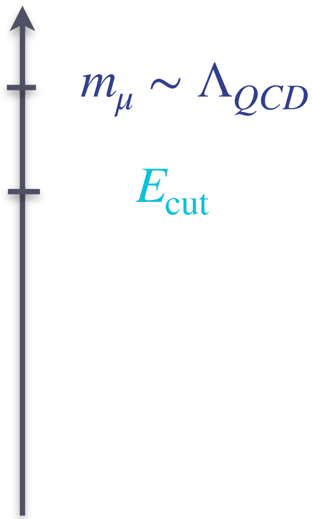

- The experimental radiation veto E_{cut}

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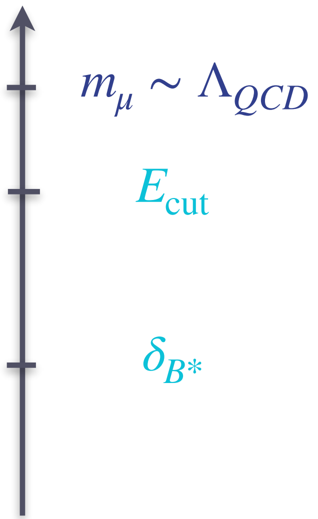

- The experimental radiation veto E_{cut}
- The mass splitting between the B meson

and its first excited state B^*: $\delta_{B*} = \dfrac{m_{B*}^2 - m_B^2}{2m_B} \lesssim E_{\text{cut}} \ll \Lambda_{\text{QCD}}$

$\delta_{B*} \sim \mathcal{O}(\lambda^2)$ $\quad\quad$ $\delta_{B_1'}, \delta_{B_2^*} \sim \mathcal{O}(\lambda)$

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E_{cut}

$m_{\pi,K\eta}$

δ_{B*}

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- Masses of light pseudoscalar mesons :

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$m_\mu \sim \Lambda_{QCD}$

$\mu_{us} \sim E_{\rm cut}$

$m_{\pi,K\eta}$

δ_{B*}

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$m^2_{\pi,K,\eta} \sim m_q \Lambda_{\rm QCD}, \;\; m_{q=u,d,s} \ll \Lambda_{\rm QCD}$

- Relevant modes (region analysis) for the real emissions feel the radiation veto in the B frame (where the measurement is done)

$p^\mu_{\rm us} \sim E_{\rm cut}$ *"ultrasoft modes"*

$p^\mu_{\rm usc} = (p^+_{\rm usc}, p^-_{\rm usc}, p^\perp_{\rm usc}) \sim \left(\dfrac{m_\ell^2}{m_B^2}, 1, \dfrac{m_\ell}{m_B}\right) E_{\rm cut}$ *"ultrasoft-collinear modes"*

= uniform $p \sim E_{\rm cut} m_\ell / m_B$ in the lepton rest frame, boosted to the B one !

The leptonic sector: bHLET

- Since $\Lambda_{\mathrm{QCD}} \sim m_\mu \gg E_{\mathrm{cut}}$, the muon becomes infinitely heavy. We describe it as a boosted heavy lepton field (bHL) with $p_\ell^\mu = m_\ell v_\ell^\mu$:

$$\bar{\chi}_c^{(\ell)}(x) = e^{i m_\ell (v_\ell \cdot x)} \bar{h}_{v_\ell}(x) \frac{\not{\bar{n}} \not{n}}{4} + \cdots$$

[Fleming et al (hep-ph/0703207); Beneke et al (2305.06401)]

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- At leading power, the (V-A) weak leptonic current matches to

$$J^\mu_{\text{lep}}(x) = \bar{\ell}\, \gamma^\mu P_L \nu_\ell \;\rightarrow\; e^{i m_\ell (v_\ell \cdot x)}\, C^{(\ell)\dagger}_{v_\ell}(x) \left[\bar{h}^{(0)}_{v_\ell} \gamma^\mu_\perp P_L \nu_{\bar{c}} + \frac{m_\ell}{\bar{n} \cdot p_\ell} \bar{n}^\mu \bar{h}^{(0)}_{v_\ell} P_L \nu_{\bar{c}} \right]$$

- The lepton is decoupled (superscript $.^{(0)}$) from eikonal radiation via an *ultrasoft-collinear* Wilson line

$$C^{(\ell)}_{v_\ell}(x) = \exp\left\{ i e Q_\ell \int_0^\infty ds\, v_\ell \cdot A_{usc}(x + s v_\ell) \right\}$$

The hadronic sector : HMET

- Below $\mu \sim \Lambda_{QCD}$, quarks hadronize $\to$ point-like meson regime !

- B and B^* mesons nearly mass-degenerate $\to$ collect into superfield $H \sim 2$ of $SU(2)_v$

[Wise 1992]

$$\Phi_B \to \frac{e^{-i\overline{m_B}(v\cdot x)}}{\sqrt{2\overline{m_B}}} H, \quad \text{with} \quad H = \frac{1+\not{v}}{2}(\not{\phi}_v - \varphi_v \gamma_5) \quad \text{with a common rephrasing} \quad \overline{m_B} = m_B + \Lambda_c.$$

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with a common rephrasing $\overline{m_B} = m_B + \Lambda_c$.

- A single subleading dim-5 operator responsible for a $B^{(*)}B^*\gamma$ coupling

$$c_{\text{dip}}^{\text{HMET}}\frac{Q_u e}{8\Lambda_c}\text{Tr}[\sigma_{\mu\nu}\overline{H}^{(0)}H^{(0)}]F_{us}^{\mu\nu} \to \frac{e\, c_{BB^*\gamma}}{2\Lambda_c}v^\mu\rho_v^{\dagger\,\nu\,(0)}\,\varphi_v^{(0)}\,\tilde{F}^{us}_{\mu\nu}+\dots$$

- Mass splitting via insertion of $SU(2)_v$ spurion :

$$\delta_{B^*} = C_{\text{mag}}(\mu)\frac{2\lambda_2(\mu)}{m_b}$$

$$-\frac{\lambda_2(\mu)}{8m_b}\text{Tr}\left[\overline{H}^{(0)}(C_{\text{mag}}\sigma^{\mu\nu})H^{(0)}\sigma_{\mu\nu}\right]$$

~ 3 of $SU(2)_v$, can be read off from $\mathscr{L}_{\text{HQET}}$ at $\mathcal{O}(1/m_b)$

The hadronic sector : HMET

- Neglecting (virtual) QED corrections, the weak hadronic currents in HMET read

$$J^{\mu}_{\rm had}(x) \to e^{-im_B\, v\cdot x}\, \overline{Y}^{(B)}_{v}(x)\, \overline{C}^{(B)}_{\bar{n}}(x_+)\, Y^{(\ell)\dagger}_{n}(x)\, \frac{i\,\mathscr{F}^{\rm HMET}(\mu)}{2\sqrt{2}}\, {\rm Tr}\Big[\Gamma^{\mu}_{\rm weak}\, H^{(0)}\Big]$$

- Currents are introduced as sources of $SU(2)_v$ breaking via the spurion :

$$\Gamma^{\mu}_{\rm weak} = \mathscr{C}^{\rm HQET}_{1}(\mu)\,\gamma^{\mu}P_L + \mathscr{C}^{\rm HQET}_{2}(\mu)\,v^{\mu}P_R \sim \bar{2}_v$$

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- At leading power, our HMET decay constant simply matches to the HQET one

$$\mathscr{F}^{\text{HMET}}(\mu) = \sqrt{m_B}\, f_B \left[1 - \frac{C_F\,\alpha_s}{4\pi}\left(3\ln\frac{m_b}{\mu} - 2\right) + \mathscr{O}(\alpha_s^2)\right] = F_{\text{QCD}}(\mu)$$

Low-energy Lagrangian : bHLET ⊗ HMET

- The leptonic and hadronic Lagrangians take the form

$$\mathscr{L}_\ell = \bar{h}_{v_\ell}(iv_\ell \cdot \partial)\, h_{v_\ell} + \dots$$

$$\mathscr{L}_H = \varphi_v^{\dagger(0)}(iv\cdot\partial)\varphi_v^{(0)} - \delta m_B\, \varphi_v^{\dagger(0)}\varphi_v^{(0)} - \rho_{v,\mu}^{\dagger(0)}(iv\cdot\partial)\rho_v^{\mu(0)} + \delta m_{B^*}\, \rho_{v,\mu}^{\dagger(0)}\rho_v^{\mu(0)}$$
$$+ \frac{ec_{BB^*\gamma}}{2\Lambda_c}\left(v^\mu \rho_v^{\dagger\nu(0)}\varphi_v^{(0)}\tilde{F}^{us}_{\mu\nu} + \mathrm{h.c.}\right) - \frac{iec_{B^*B^*\gamma}}{2\Lambda_c}\,\rho_v^{\dagger\mu(0)}\rho_v^{\nu(0)}F^{us}_{\mu\nu} + \dots\,,$$

- Combining leptonic and hadronic currents, we obtain the effective weak Lagrangian

$$\mathscr{L}_{H\ell} = i\sqrt{2}\, G_F^{(\mu)}\, V_{ub}\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar{n}}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger}$$
$$\times \left\{ \frac{f_B\, m_\ell}{\sqrt{2\, m_B}}\, y_B\, \varphi_v^{(0)}\, \bar{h}_{v_\ell}^{(0)}\, P_L\, \nu_{\bar{c}} - \frac{f_{B^*}\, m_{B^*}}{\sqrt{2\, m_{B^*}}} \left[y_{B^*}^{\perp}\, \bar{h}_{v_\ell}^{(0)}\, \not{\rho}_v^{\perp(0)} P_L\, \nu_{\bar{c}} + y_{B^*}^{\parallel}\, \frac{m_\ell}{m_{B^*}}\, \frac{\bar{n}\cdot\rho_v^{(0)}}{\bar{n}\cdot v}\, \bar{h}_{v_\ell}^{(0)} P_L\, \nu_{\bar{c}} \right] \right\}$$

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- $y_i = 1 + \mathcal{O}(\alpha)$ → QED corrections determined by matching to the partonic picture

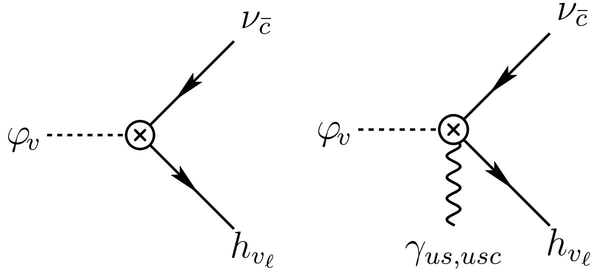
Only for y_B we need the $\mathcal{O}(\alpha)$ piece:

$$y_B(\mu) = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\frac{9}{40}}\left(\frac{\mu^2}{m_B\, m_\ell}\right)^{\frac{\gamma_{\text{soft}}}{2}}\mathcal{R}_{\text{virt}}$$

Decay channel topologies

Decay channel topologies

Direct



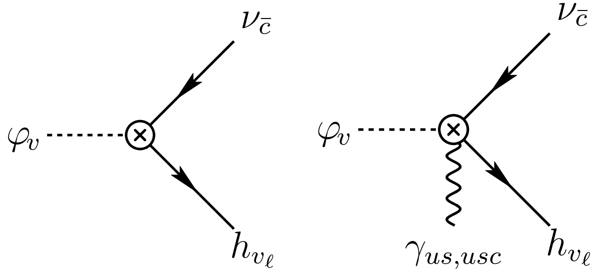
- chirally suppressed $B\ell\nu$ interaction

$$\frac{f_B\, m_\ell}{\sqrt{2\, m_B}}\, y_B\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar{n}}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger} \varphi_v^{(0)}\, \bar{h}_{v_\ell}^{(0)}\, P_L\, \nu_{\bar{c}}$$

- photon emission is unsuppressed

Decay channel topologies

Direct



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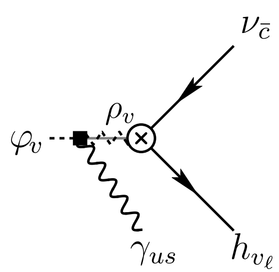

- $BB^*\gamma$ transition is power-suppressed

$$\frac{e\, c_{BB^*\gamma}}{2\Lambda_c} v^\mu \rho_v^{\dagger\,\nu\,(0)}\, \varphi_v^{(0)}\, \tilde F_{\mu\nu}^{us}$$

- subsequent $B^* \to \ell\nu$ decay is unsuppressed

$$-\frac{f_{B^*}\, m_{B^*}}{\sqrt{2\, m_{B^*}}}\, y_{B^*}^{\perp}\, \overline{Y}_v^{(B)}\, \overline{C}_{\bar n}^{(B)}\, Y_n^{(\ell)\dagger}\, C_{v_\ell}^{(\ell)\dagger}\, \bar h_{v_\ell}^{(0)}\, \not{\phi}_v^{\perp(0)} P_L\, \nu_{\bar c}$$

Decay channel topologies

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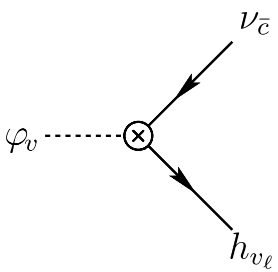

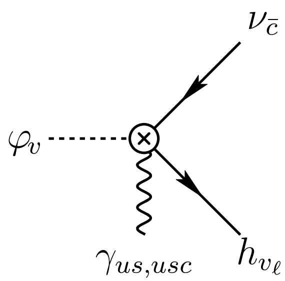

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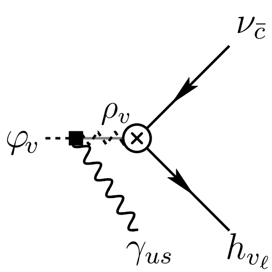
Indirect



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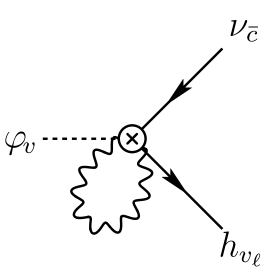
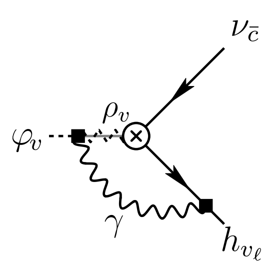
- No extra virtual corrections below Λ_{QCD}

  $\rightarrow$All contained in y_B



scaleless



power-suppressed

Including the pions : $\mathrm{HH}\chi\mathrm{PT}$

- To include the light mesons we implement the chiral $SU(3)_V$: [Wise 1992]
  - H is promoted to a triplet H_a $(a = 1,2,3)$ describing the mesons $(B^{(*)-}, \bar{B}^{(*)0}, \bar{B}_s^{(*)0})$.
  - It couples to the pseudo scalar octet

$$\Sigma = \xi^2 = \exp\left[\frac{i\sqrt{2}\,\pi^a\,\lambda^a}{f_\pi}\right], \quad \text{with} \quad \frac{\pi^a\,\lambda^a}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\,\eta_8 \end{pmatrix},$$

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- After the field redefinition $H \to H\xi^\dagger$, interactions with pions are encoded in

$$\mathcal{L}_{\mathrm{HH}\chi\mathrm{PT}} \supset \frac{1}{4}\mathrm{Tr}\left[\overline{H}H\, v\cdot L\right] - \frac{g}{4}\mathrm{Tr}\left[\overline{H}H \not{L}\gamma_5\right] + \cdots\,, \qquad L^\mu = \Sigma D^\mu \Sigma^\dagger$$

$\to$ they do not couple directly to weak interaction, the only effect of pion loops is to correct $\mathcal{F}^{\mathrm{HMET}}$, $g_{BB^*\gamma}$ and $g = g_{BB^*\pi}$ (our input parameters)!

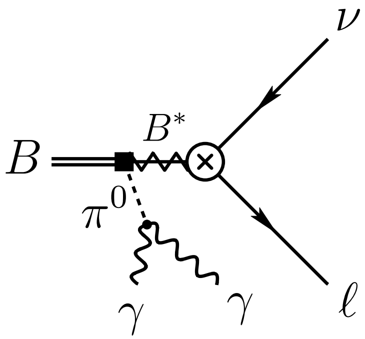
- Conclusion : No relevant $O(\alpha)$ virtual corrections in the hadronic picture.

$\to$ All are scaleless, power suppressed, or already accounted for via our input choice.

Indirect contribution from pions

- The $\mathrm{HH}\chi\mathrm{PT}$ Lagrangian generates an indirect contribution via an on-shell π^0 :

$$\mathscr{L}^{(BB^*\pi^0)}_{\mathrm{HH}\chi\mathrm{PT}} = -\frac{g_{BB^*\pi}}{\sqrt{2}f_\pi}\left(\rho_v^{\dagger\mu(0)}\varphi_v^{(0)}\partial_\mu\pi^0 \; + \; \mathrm{h.c.}\right) \longrightarrow$$



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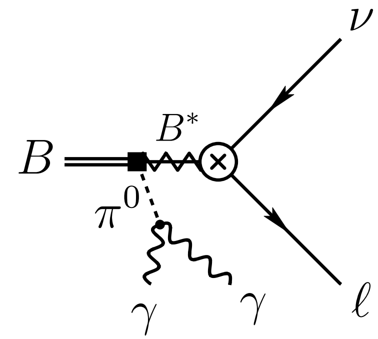

$\rightarrow$ Looks like $B \rightarrow \ell\nu_\ell(\gamma)$ if the Ecal only measures the total energy deposition!

- Looks $\mathcal{O}(\alpha^2)$, but the pion is very narrow and almost exclusively decays to $\gamma\gamma$

$$\Gamma_\pi \approx \Gamma(\pi^0 \rightarrow \gamma\gamma) \approx \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2} \approx 7.8\,\mathrm{eV}$$

Effectively : $\dfrac{1}{(s_{\gamma\gamma} - m_\pi^2)^2 + (m_\pi\Gamma_\pi)^2} \rightarrow \dfrac{\pi}{m_\pi\Gamma_\pi}\delta(s_{\gamma\gamma} - m_\pi^2) = \mathcal{O}(\alpha^{-2})$

- If this background is not remove $\rightarrow$ dominant contribution for $E_{\mathrm{cut}} > m_\pi$!

Resummed direct contribution

- The direct contribution factorizes into : $\Gamma_{\text{dir}}(E_{\text{cut}}) = \Gamma_{\text{tree}} \, | \underbrace{y_B(\mu_0)}_{\text{Non-radiative}} |^2 \underbrace{R(E_{\text{cut}}, \mu_0)}_{\text{Radiative}}$

  with $y_B(\mu) = K_{EW}(\mu) \otimes H_i(\mu) \otimes J_i(\mu) \otimes K_i(\mu) \otimes S_i(\mu)$

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  Non-radiative　　Radiative

- The radiative function can be expressed as squared matrix elements of Wilson lines :

$$W_s(\omega_{us}) = \left[\sum_{n_{us}=0}^{\infty} \prod_{i=1}^{n_{us}} \int d\Pi_i(q_i) \right] \left| \langle n_{us} \gamma_{us}(q_i) \,|\, Y_v^{(B)} Y_n^{\dagger(\ell)} \,|\, 0 \rangle \right|^2 \delta(w_{us} - q_0^{us}) \quad , \quad q_0^{us} = \sum_i q_{0i}$$

$$W_{usc}(\omega_{usc}) = \left[\sum_{n_{usc}=0}^{\infty} \prod_{j=1}^{n_{usc}} \int d\Pi_j(q_j) \right] \left| \langle n_{usc} \gamma_{usc}(q_j) \,|\, C_{\bar{n}}^{(B)} C_{v_\ell}^{\dagger(\ell)} \,|\, 0 \rangle \right|^2 \delta(w_{usc} - q_0^{usc}) \quad , \quad q_0^{usc} = \sum_j q_{0j}$$

  convoluted with the measurement function implementing the radiation veto

$$R(E_{\text{cut}}, \mu) = \int_0^{\infty} d\omega_s \int_0^{\infty} d\omega_{sc} \, \theta\Big(E_{\text{cut}} - \omega_{us} - \omega_{usc}\Big) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$

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  Non-radiative | Radiative

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- Renormalizing in Laplace space yields to the resummed expression :

$$R(E_{\text{cut}}, \mu) = \left(\frac{\mu^2}{m_B\, m_\ell}\right)^{-\gamma_{\text{soft}}} \left(\frac{2E_{\text{cut}}}{m_B}\right)^{2\gamma_{\text{soft}}} \mathscr{W} \quad \text{with} \quad \mathscr{W} = \frac{e^{-2\gamma_E \gamma_{\text{soft}}}}{\Gamma(1 + 2\gamma_{\text{soft}})} \left[1 + \frac{Q_\ell^2\, \alpha}{2\pi}\left(2 - \frac{\pi^2}{3}\right)\right]$$

Numerical estimates

Hadronic input parameters

- Decay constants f_π, f_B, f_{B^*} well determined from the lattice [FLAG 2024, HPQCD 2015, ETMC 2017]
- For LCDA moments, we assume exponential models, yielding [Grozin, Neubert; Braun et al 2017]

$$\int_0^\infty d\omega\, \phi_-^B(\omega,\mu_0)\, \ln\frac{\omega}{\omega_0} = \frac{\lambda_E^2-\lambda_H^2}{18\,\omega_0^2} - \gamma_E$$

$$\int_0^\infty d\omega \int_0^\infty d\omega_g\, \phi_{3g}^B(\omega,\omega_g,\mu_0)\left(\frac{1}{\omega_g}\ln\frac{\omega+\omega_g}{\omega} - \frac{1}{\omega+\omega_g}\right) = \frac{\lambda_E^2-\lambda_H^2}{36\,\omega_0^2}$$

with the inputs $\lambda_E, \lambda_H, \omega_0 = \lambda_B$. [Nishikawa, Tanaka 2014; Khodjamirian et al 2020]

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with the inputs $\lambda_E, \lambda_H, \omega_0 = \lambda_B$. [Nishikawa, Tanaka 2014; Khodjamirian et al 2020]

- The subtracted decay constant $F\_(\Lambda,\mu)$ is a new, unknown QED corrected quantity:

$$F\_(\Lambda,\mu) = F_{\rm QCD}(\mu)\left[1 + \frac{\alpha}{\pi} f^{(1)}(\Lambda,\mu) + \mathcal{O}(\alpha^2)\right]$$

$$f^{(1)}(\mu_0,\mu_0) = 0 \pm 1$$

→ Accounted for in the error budget

- We take $g_{BB^*\gamma}$ and $g_{BB^*\pi}$ from LCSR and lattice, respectively. [Pullin, Zwicky 2021, RBC/UKQCD 2016]

Numerical estimates

$$\frac{\Gamma(E_{\text{cut}})}{\Gamma_{\text{tree}}} = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\frac{9}{20}} \left(\frac{2E_{\text{cut}}}{m_B}\right)^{2\gamma_{\text{soft}}} \mathscr{W}\, \mathscr{R}^2_{\text{virt}}$$

$$+\frac{m_B^2}{m_\ell^2}\frac{f_{B^*}^2 m_{B^*}}{f_B^2 m_B}\left[\frac{\alpha}{6\pi}(g_{BB^*\gamma}E_{\text{cut}})^2 I\left(0,\frac{\delta_{B^*}}{E_{\text{cut}}}\right)+\frac{g_{BB^*\pi}^2}{24\pi^2}\left(\frac{E_{\text{cut}}}{f_\pi}\right)^2 I\left(\frac{m_\pi}{E_{\text{cut}}},\frac{\delta_{B^*}}{E_{\text{cut}}}\right)\right]$$

Numerical estimates

Electroweak logs ≈ 1.014

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Numerical estimates

Radiation-veto logs $\approx 0.94 - 0.96$ for $E_{\text{cut}} \in [25{,}100]\,\text{MeV}$

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Numerical estimates

Radiation-veto logs $\approx 0.94 - 0.96$ for $E_{\text{cut}} \in [25,100]\,\text{MeV}$

Electroweak logs ≈ 1.014

Boost logs $\approx 1.69 \cdot 10^{-3}$

$$\frac{\Gamma(E_{\text{cut}})}{\Gamma_{\text{tree}}} = \left(\frac{\alpha(m_Z)}{\alpha(m_B)}\right)^{\frac{9}{20}} \left(\frac{2E_{\text{cut}}}{m_B}\right)^{2\gamma_{\text{soft}}} \mathscr{W}\, \mathscr{R}^2_{\text{virt}}$$

$$+\frac{m_B^2}{m_\ell^2}\frac{f_{B*}^2 m_{B*}}{f_B^2 m_B}\left[\frac{\alpha}{6\pi}(g_{BB*\gamma}E_{\text{cut}})^2 I\left(0,\frac{\delta_{B*}}{E_{\text{cut}}}\right) + \frac{g_{BB*\pi}^2}{24\pi^2}\left(\frac{E_{\text{cut}}}{f_\pi}\right)^2 I\left(\frac{m_\pi}{E_{\text{cut}}},\frac{\delta_{B*}}{E_{\text{cut}}}\right)\right]$$

Numerical estimates

Radiation-veto logs $\approx 0.94 - 0.96$ for $E_{\text{cut}} \in [25,100]\,\text{MeV}$

Electroweak logs ≈ 1.014

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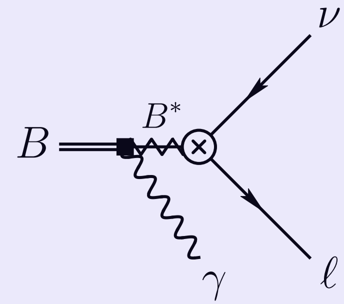
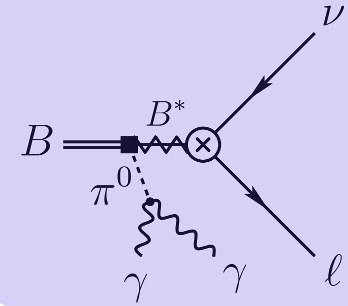
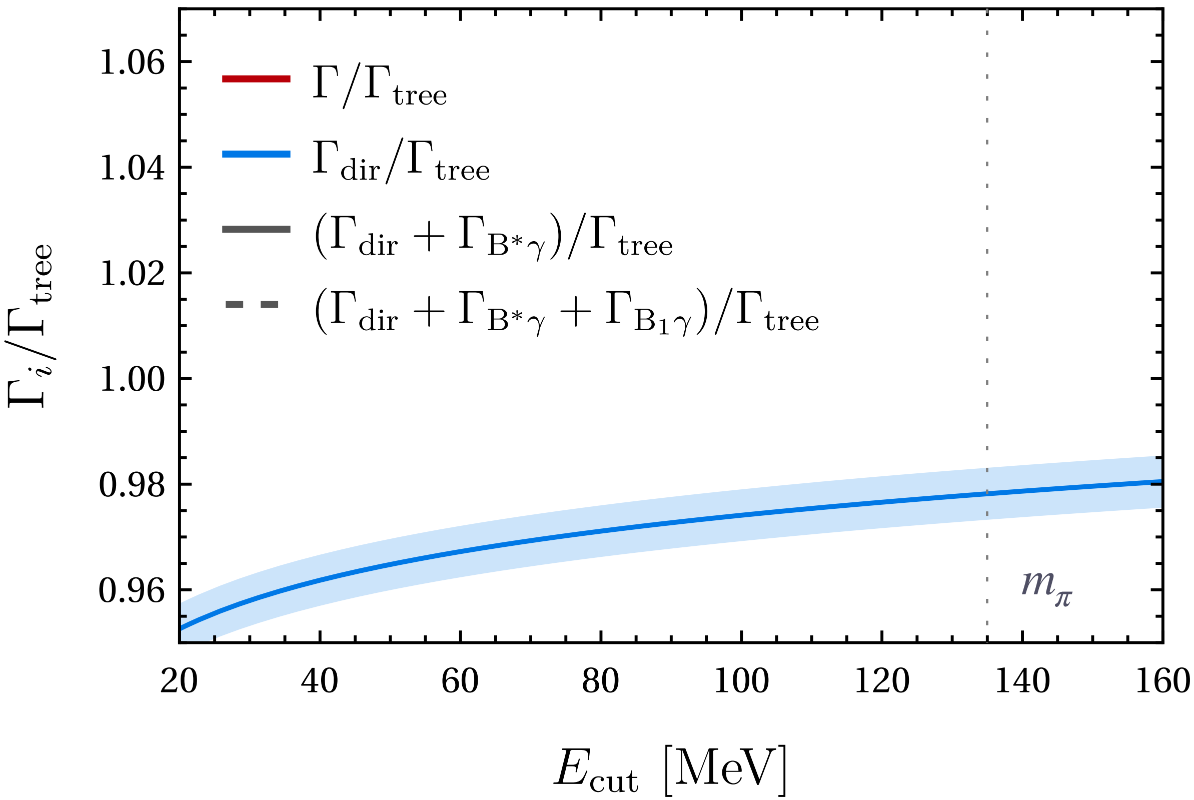
Indirect contributions
(structure-dependent real corrections)



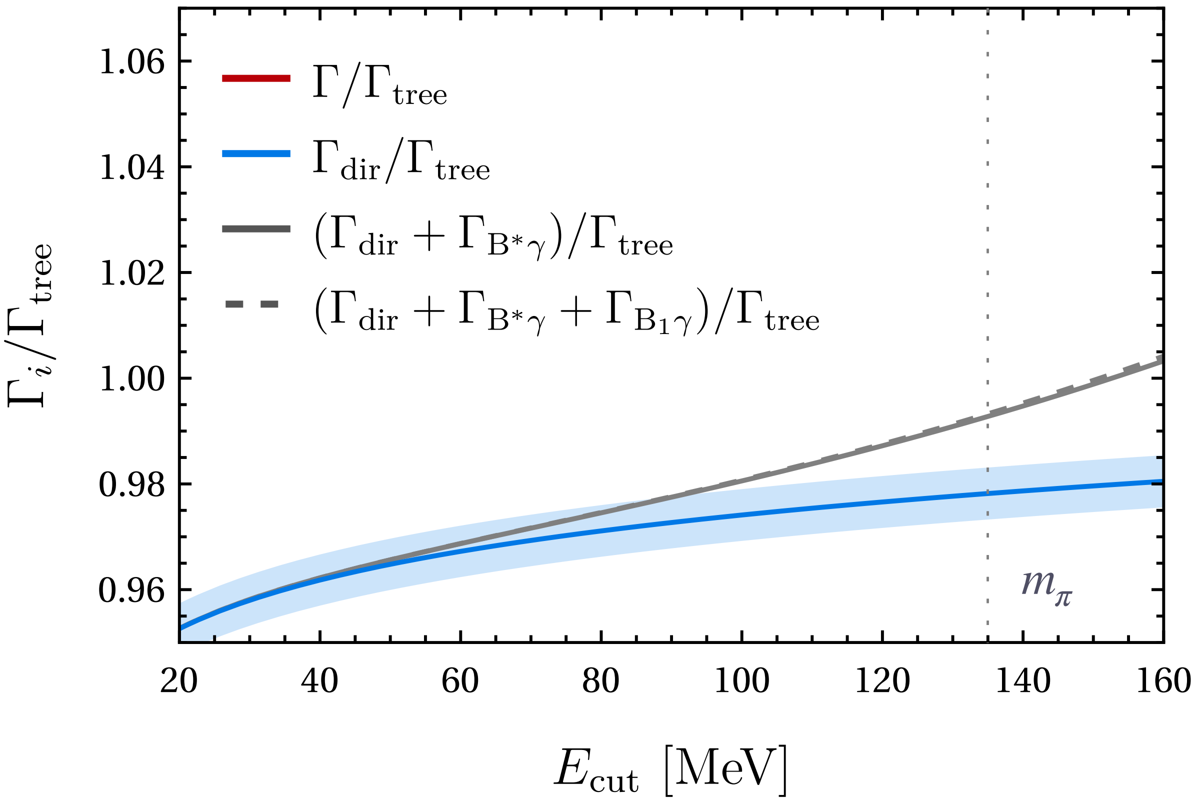


Numerical estimates



Direct contribution only

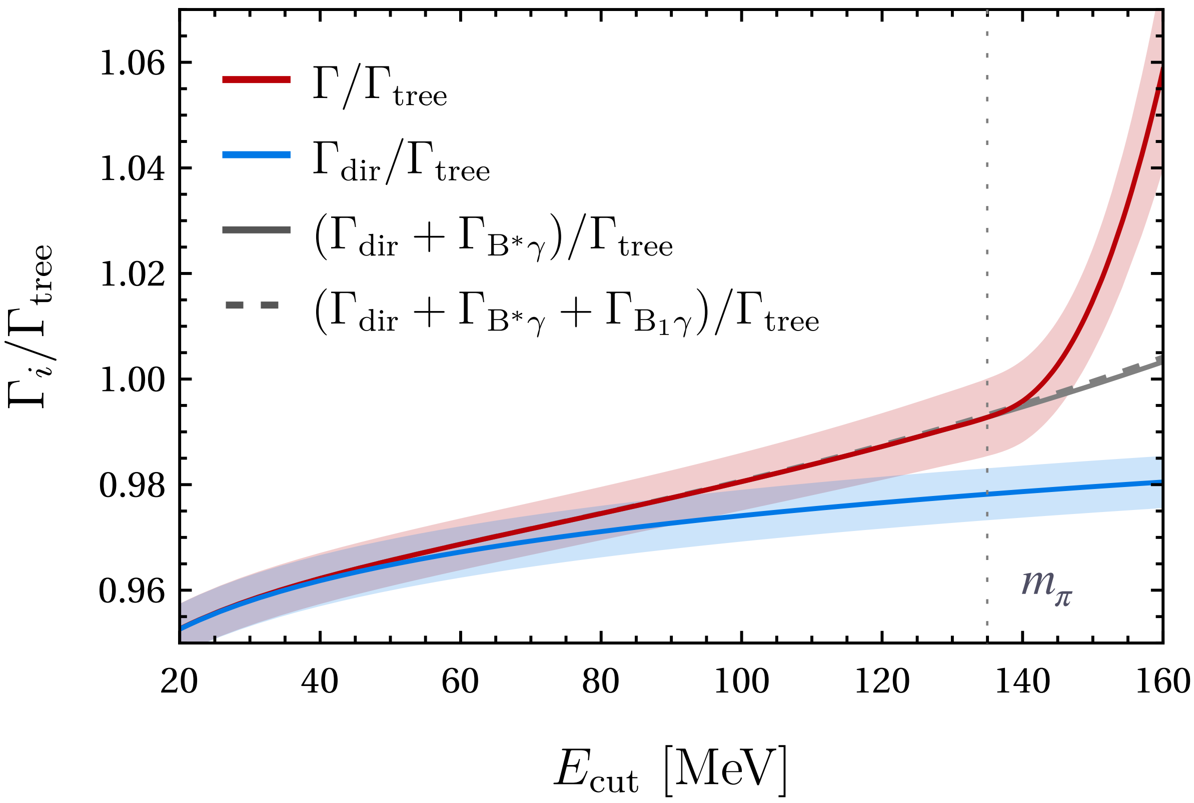
Numerical estimates



$B^*\gamma$ contribution: becomes sizeable around 50 MeV, then rises with E_{cut}

Direct contribution only

Numerical estimates



Pion contribution: dominates quickly for $E_{\rm cut} > m_\pi$

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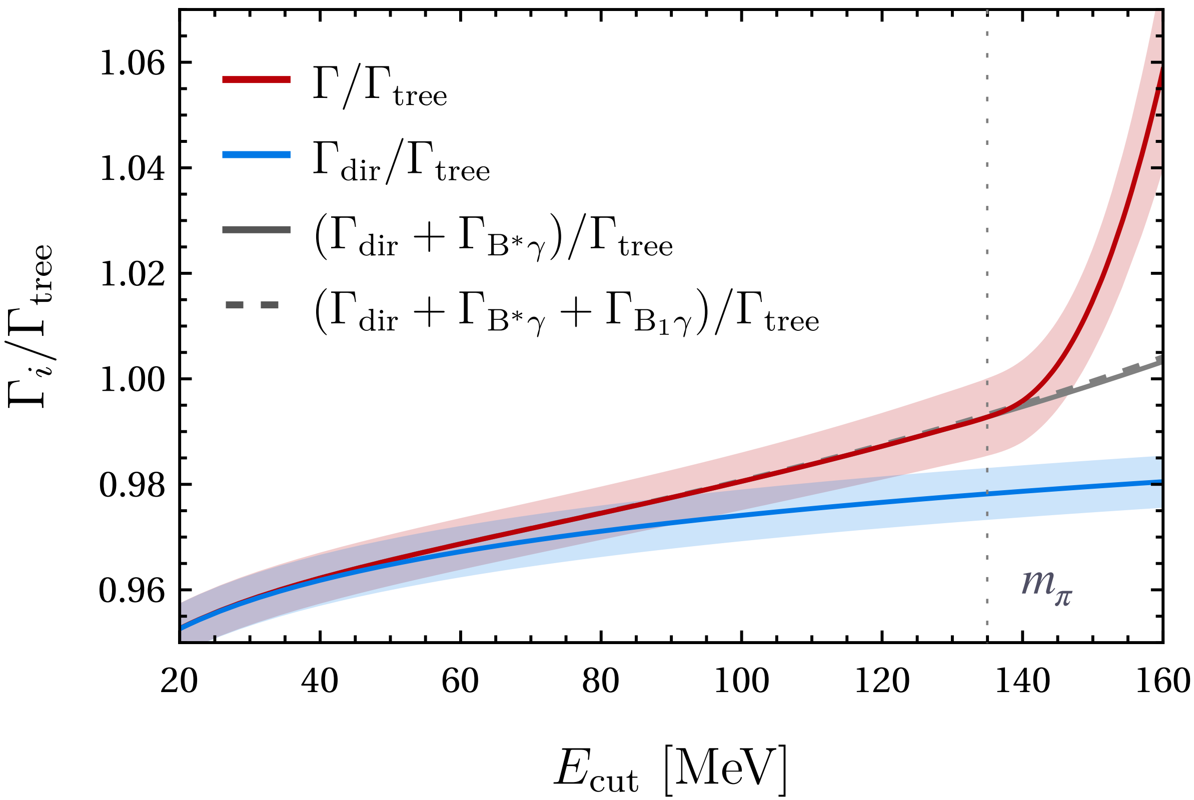

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Direct contribution only

- QED corrections induce a $-3.9(5)\,\% - 0.1(7)\,\%$ shift in the rate for $E_{\rm cut} \in [20\,{\rm MeV}, m_\pi]$
- The structure-dependent component is always positive: $1.6(5) - 3.0(8)\,\%$
- For $E_{\rm cut} < 100\,{\rm MeV}$, the uncertainty on the QED shift is dominated by $\mathscr{R}_{\rm vir}\,(f^{(1)})$, Above, comparable contribution from $g_{BB^*\gamma}$

Conclusions

- The exclusive leptonic decay $B^- \to \mu^- \bar{\nu}_\mu$, is an important channel for test of physics in SM and beyond. Since future experimental accuracy calls for percent precision in the theory prediction, we analysed QED effects beyond point-like approximation.

  $\rightarrow$ **State of the art prediction for the rate including radiation up to $E_{\text{cut}} \ll \Lambda_{\text{QCD}}$**

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- These EFT methods can be adapted to tackle other leptonic channels (τ is work in progress). They need to be extended for semi-leptonic channels ($B \to \pi \ell \nu, B \to D \ell \nu$,...), crucial for the extraction of $|V_{ub}|$ and $|V_{cb}|$.

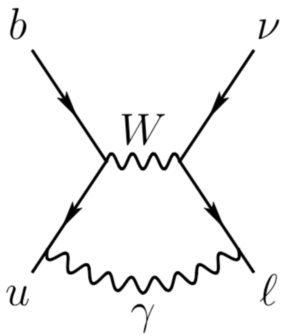
Thanks for listening !

Backup-slides

Reduction scheme

$$K_{\rm EW}(m_Z) = 1 + \frac{Q_\ell\, \alpha}{4\pi}\left[\frac{11}{6} + (Q_b + Q_u)\left(1 + \frac{\kappa}{4}\right)\right]$$

- Box diagrams in the matching generate additional Dirac structures



$$\Gamma = \gamma_\mu \gamma_\nu \gamma_\rho\, P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho\, P_L$$

$$d = 4: \quad \Gamma = 16\, \gamma_\mu P_L \otimes \gamma^\mu P_L$$

$$d \neq 4: \quad \Gamma = \left(16 + \kappa\, \epsilon\right)\, \gamma_\mu P_L \otimes \gamma^\mu P_L$$

- The dependence cancels between the matching coefficient K_{EW} and the matrix element allowing to convert between schemes :

$$A_{\rm LEFT} = -\frac{4G_F^{(\mu)}}{\sqrt{2}} V_{ub} K_{EW}^{[\kappa]}\left(1 - Q_\ell(Q_b + Q_u)\frac{\alpha}{16\pi}\kappa\right)\gamma_\mu \otimes \gamma^\mu + \cdots$$

$$\rightarrow K_{EW}^{[\kappa']} = K_{EW}^{[\kappa]}\left(1 - Q_\ell(Q_b + Q_u)\frac{\alpha}{16\pi}(\kappa - \kappa')\right)$$

- Working with EFTs that decompose Lorentz structures, like SCET, in which

$$\gamma^\mu = n \cdot \gamma \frac{\bar{n}^\mu}{2} + \bar{n} \cdot \gamma \frac{n^\mu}{2} + \gamma_\perp^\mu$$

$\kappa = 0$ is particularly a convenient choice → simplifies construction of evanescent operators and avoids spurious power-enhanced contributions at intermediate steps.

Power enhanced NP contributions

- Including NP physics contributions, LEFT operators describing our process are given by

$$L_\ell^X \equiv [L^X_{\nu edu}]^*_{\ell\ell 31}$$

$$\mathscr{L}_{\text{LEFT}} \ni [L^{V,LL}_{\nu e}]_{1212}\big(\bar\nu_\mu \gamma_\mu P_L \nu_e\big)\big(\bar e\, \gamma^\mu P_L\, \mu\big)$$

$$+L_\ell^{V,LL}\big(\bar u\,\gamma_\mu P_L b\big)\big(\bar\ell\,\gamma^\mu P_L\,\nu_\ell\big) + L_\ell^{V,LR}\big(\bar u\,\gamma_\mu P_R\, b\big)\big(\bar\ell\,\gamma^\mu P_L\,\nu_\ell\big)$$

$$+L_\ell^{S,RL}\big(\bar u\, P_R\, b\big)\big(\bar\ell\, P_L\,\nu_\ell\big) + L_\ell^{S,RR}\big(\bar u P_L b\big)\big(\bar\ell\, P_L\,\nu_\ell\big) + L_\ell^{T,RR}\big(\bar u\,\sigma_{\mu\nu} P_L\, b\big)\big(\bar\ell\,\sigma^{\mu\nu} P_L\,\nu_\ell\big) \ ,$$

- At tree level, this leads to the following expression for the rate :

$$\Gamma^{\text{LEFT}}_{\text{tree}} = \frac{m_\ell^2\, m_B f_{B^-}^2}{64\pi}\left(1-\frac{m_\ell^2}{m_B^2}\right)^2 \left| L_\ell^{V,LL} - L_\ell^{V,LR} + \frac{m_B^2}{m_\ell\,(m_b+m_u)}\left(L_\ell^{S,RL} - L_\ell^{S,RR}\right)\right|^2$$

- Power-enhancement $\sim m_B/m_\ell$ vs NP suppression $\sim m_W^2/\Lambda_{\text{NP}}^2$, the NP contribution competes with the SM one for scales $\Lambda_{\text{NP}} \lesssim 5\ \text{TeV}$!

SCET$_{II}$ basis

$$\mathscr{H}_v = Y_n^{(\ell)\dagger}\overline{Y}_n^{(u)\dagger} b_v \quad , \quad \mathscr{G}_s^\mu = \overline{Y}_n^\dagger (iD_s^\mu \overline{Y}_n)$$

$$\mathscr{Q}_s = u_s \overline{Y}_n^{(u)\dagger} \qquad \omega = n\cdot p_q \,,\, \omega_g = n\cdot p_g$$

- SCET$_{II}$ operators contributing at $\mathcal{O}(\alpha)$:

$$b_v, u_s \sim \mathcal{O}(\lambda^{3/2}) \qquad \mathscr{G}_s^\perp, \omega, \omega_g \sim \mathcal{O}(\lambda) \qquad \chi_c, \mathscr{A}_c^\perp, m_\ell \sim \mathcal{O}(\lambda_\ell)$$

« Local operators »

[A]

$$Q_1^A(\Lambda) = \frac{m_\ell\, \bar n\cdot v}{(\bar n\cdot \mathscr{P}_c)}\left(\bar u_s\left[1-\theta_T\left(-\frac{i\bar n\cdot \overleftarrow{D}_s}{\bar n\cdot v}-\Lambda\right)\right]\frac{\not{\bar n}}{\bar n\cdot v}P_L b_v\, Y_n^{(\ell)\dagger}\right)\left(\bar\chi_c^{(\ell)} P_L \nu_{\bar c}\right)$$

$$Q_2^A = \frac{m_\ell\,(\bar n\cdot v)}{(\bar n\cdot \mathscr{P}_c)}\left(\bar u_s \frac{\not n}{n\cdot v}P_L b_v\, Y_n^{(\ell)\dagger}\right)\left(\bar\chi_c^{(\ell)} P_L \nu_{\bar c}\right)$$

[B]

$$Q_1^B(x) = \frac{\bar n\cdot v}{(\bar n\cdot \mathscr{P}_c)}\left(\bar u_s \frac{\not{\bar n}}{\bar n\cdot v}P_L b_v\, Y_n^{(\ell)\dagger}\right)\left(\bar\chi_c^{(\ell)} \mathscr{A}\!\!\!/_{c[x]}^\perp P_L \nu_{\bar c}\right)$$

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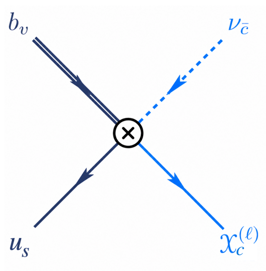
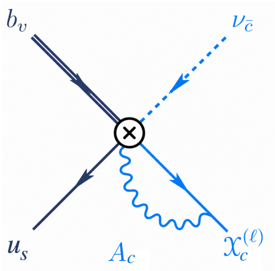




«Non-local operators »

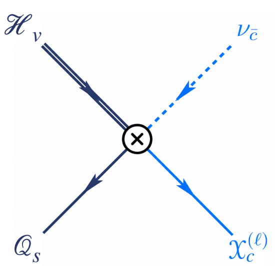
[C]

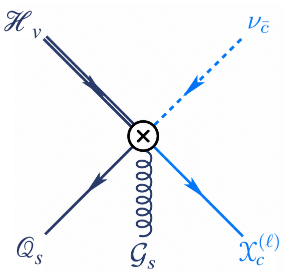
$$Q_1^C(\omega) = \frac{m_\ell\, \bar n\cdot v}{(\bar n\cdot \mathscr{P}_c)}\left(\overline{\mathscr{Q}}_{s[\omega]} \frac{\not{\bar n}}{\bar n\cdot v}P_L \mathscr{H}_v\right)\left(\bar\chi_c^{(\ell)} P_L \nu_{\bar c}\right)$$

$$Q_2^C(\omega) = \frac{m_\ell\, \bar n\cdot v}{(\bar n\cdot \mathscr{P}_c)}\left(\overline{\mathscr{Q}}_{s[\omega]} \frac{\not n}{n\cdot v}P_L \mathscr{H}_v\right)\left(\bar\chi_c^{(\ell)} P_L \nu_{\bar c}\right)$$

[E]

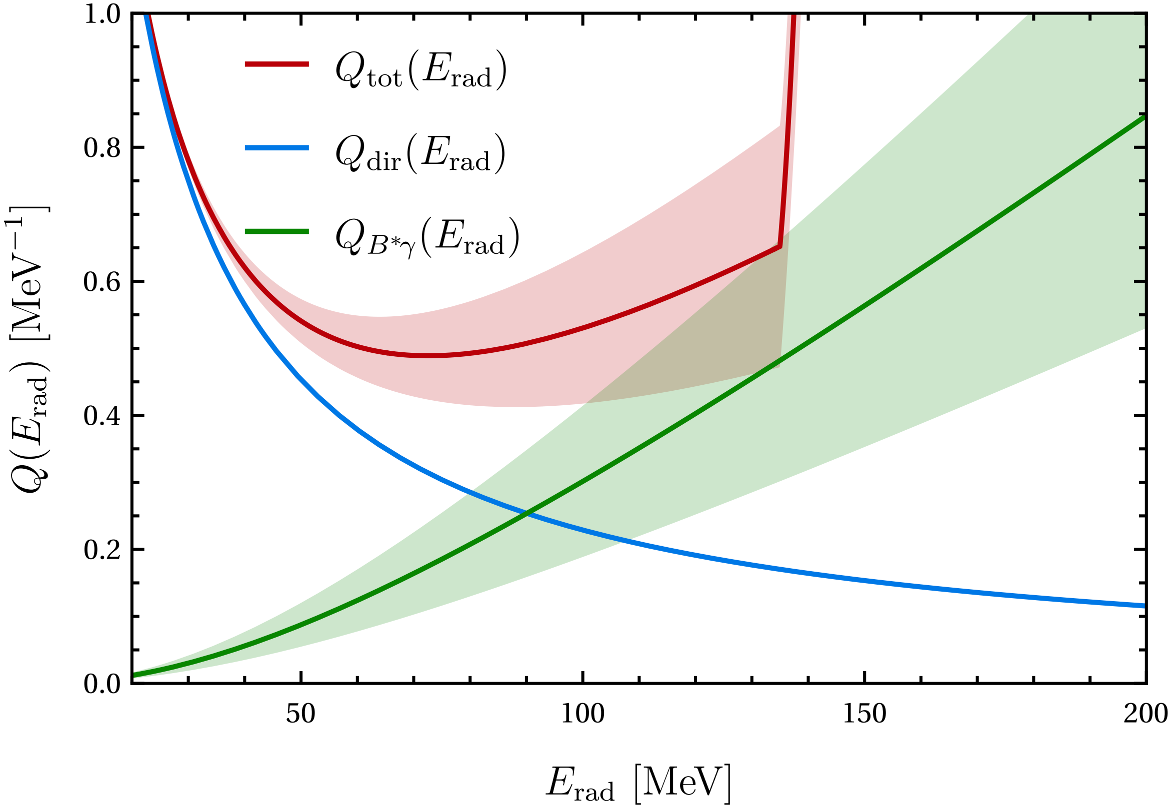
$$Q_2^E(\omega,\omega_g) = \frac{m_\ell\, \bar n\cdot v}{\omega(\bar n\cdot \mathscr{P}_c)}\left(\overline{\mathscr{Q}}_{s[\omega]} \mathscr{G}\!\!\!/_{s[\omega_g]}^{\perp\mu} \frac{\not n}{n\cdot v}P_R \mathscr{H}_v\right)\left(\bar\chi_c^{(\ell)} P_L \nu_{\bar c}\right)$$





Radiation-energy spectrum

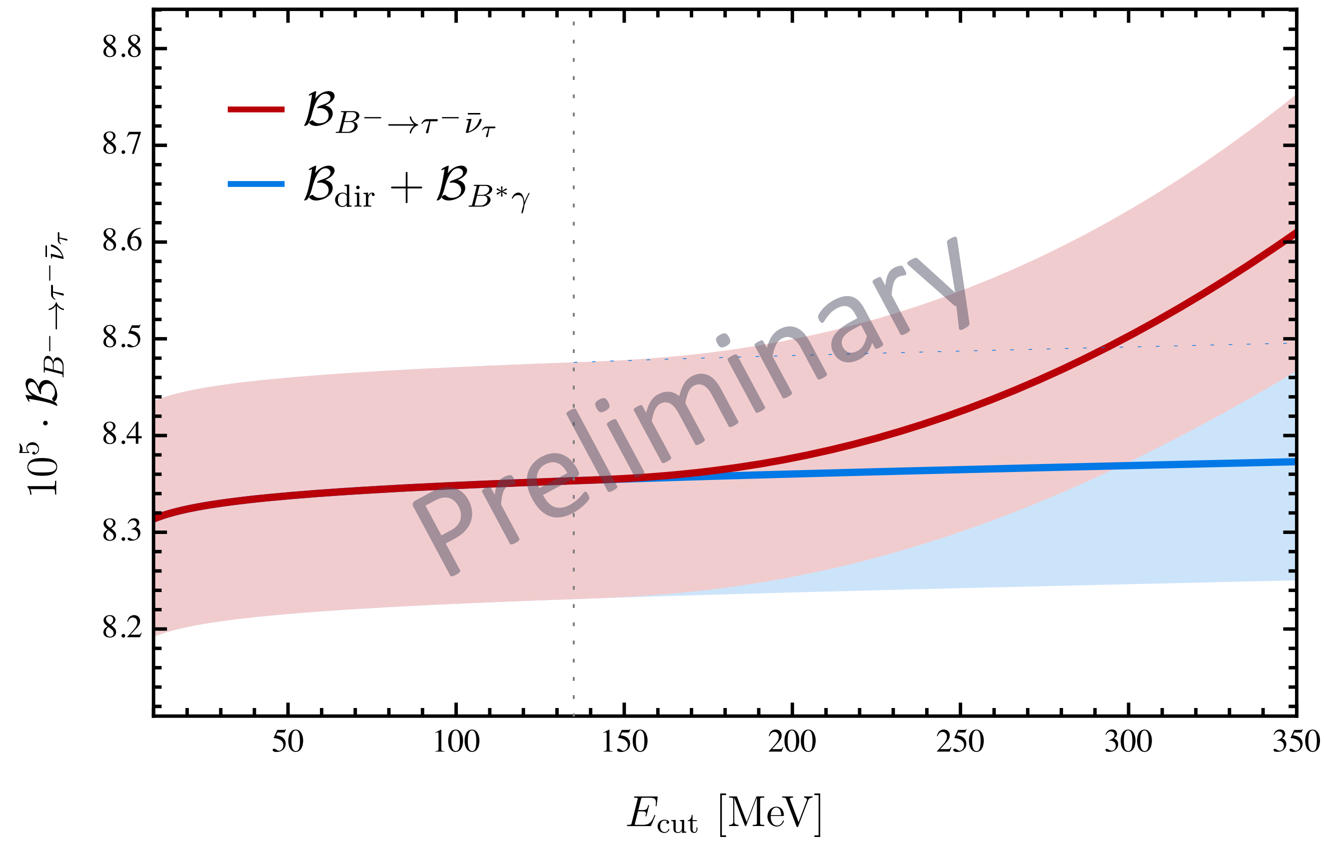
$$Q(E_{\rm rad}) = \frac{4\pi}{\alpha}\frac{1}{\Gamma_{\rm tree}}\frac{d\Gamma(E_{\rm rad})}{dE_{\rm rad}}\,,$$



$$Q_{\rm dir}(E_{\rm rad}) \sim E_{\rm rad}^{-1+2\gamma_{\rm soft}}$$

$$Q_{B^*\gamma}(E_{\rm rad}) \sim \frac{E_{\rm rad}^3}{(E_{\rm rad}+\delta_{B^*})^2}$$

Numerical estimates for the τ channel



Lepton flavor universality (LFU) ratio

$$R_{\tau\mu} = \frac{\Gamma_{B^-\to\tau^-\bar{\nu}_\tau}}{\Gamma_{B^-\to\mu^-\bar{\nu}_\mu}} \cdot \left(\frac{m_\mu}{m_\tau} \cdot \frac{m_B^2 - m_\mu^2}{m_B^2 - m_\tau^2} \right)^2 ,$$