Phase-space emulation of quantum dynamics

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Content overview

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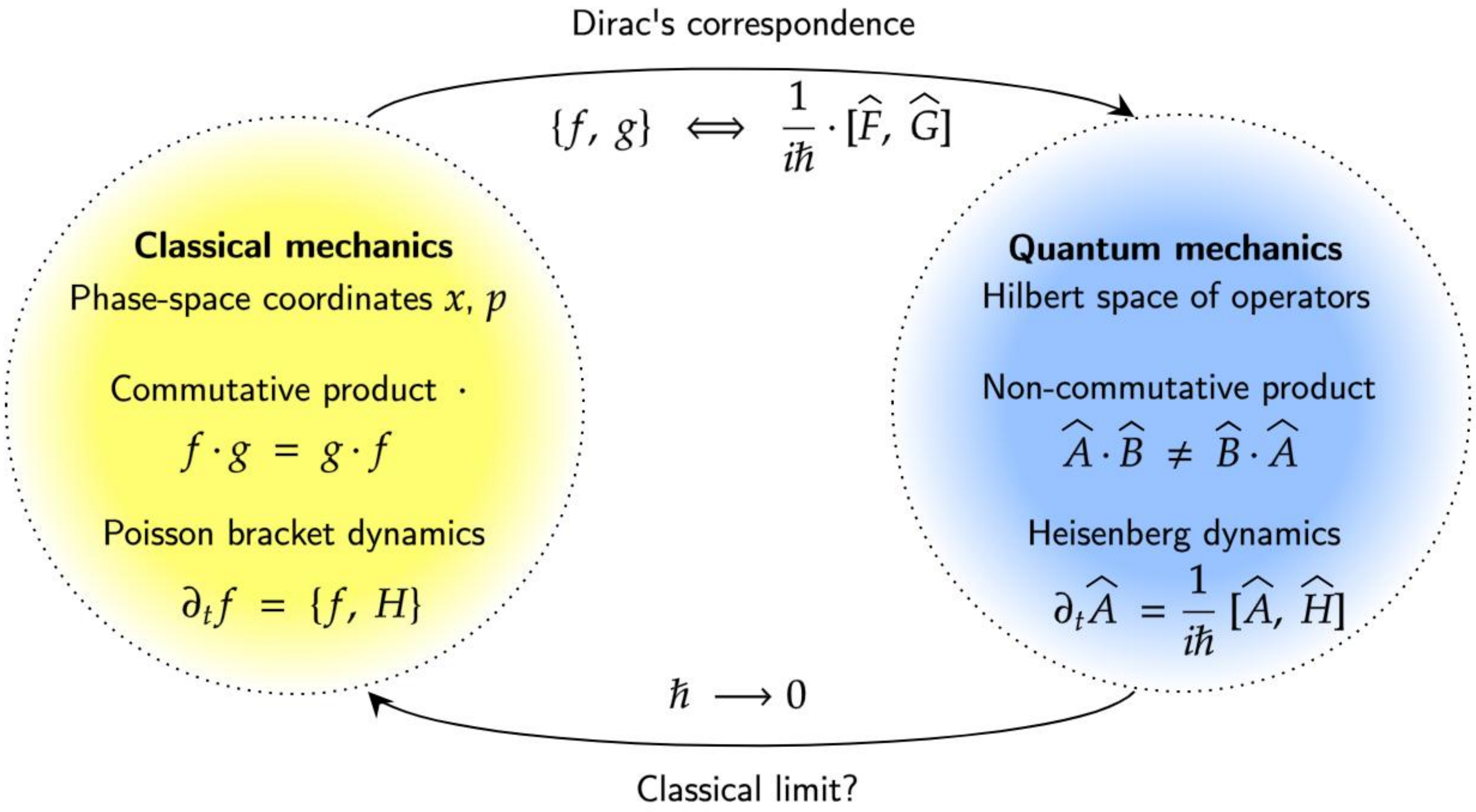
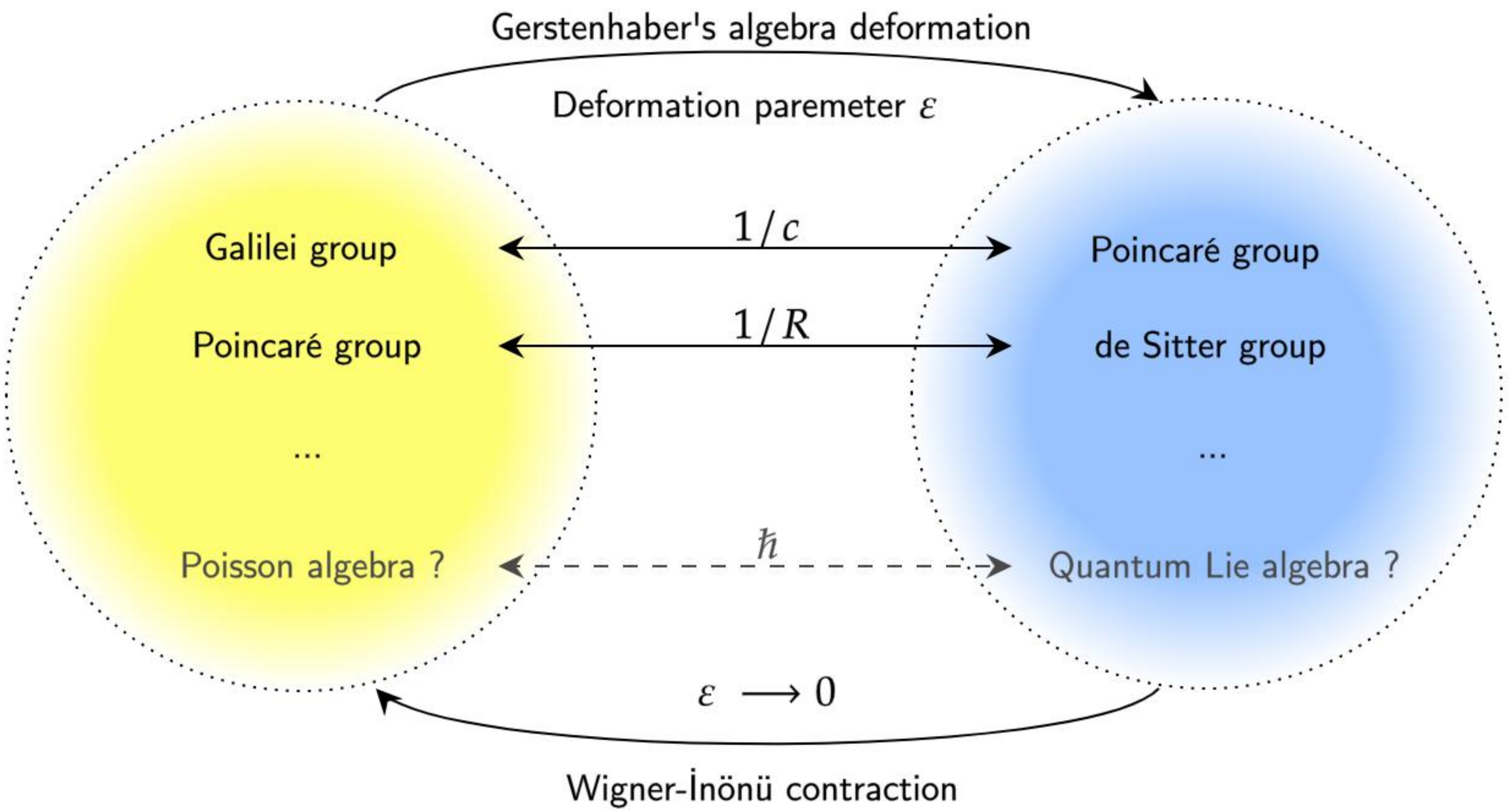
01 Phase-space quantum mechanics

How do we link quantum mechanics to classical mechanics?



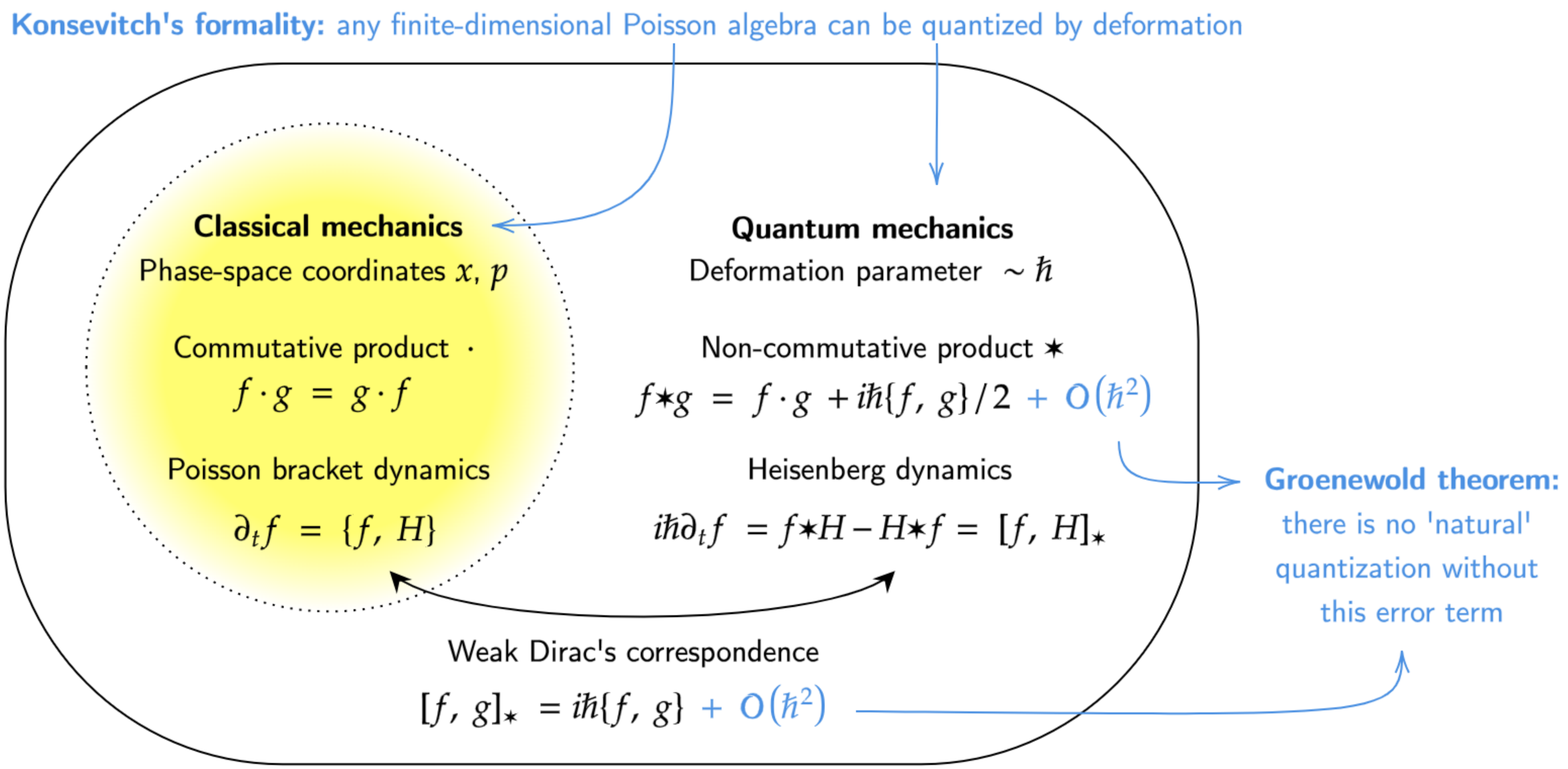
Motivation for deformation quantization

Deformation is a poweful framework to recast known theories!



Deformation quantization

Formal power series, everwhere, all at once



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02 Emulating quantum systems

Phase-space description of a quantum system

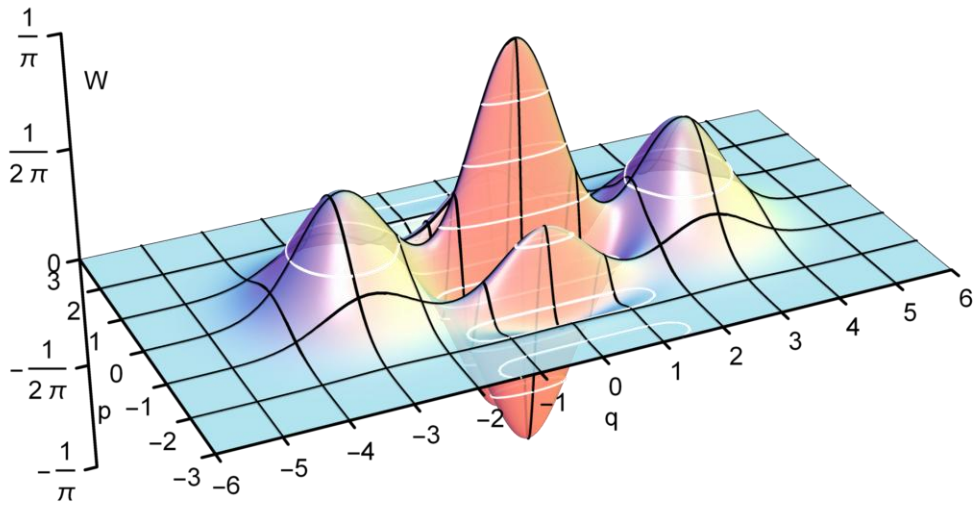
Interpretation of the Wigner transform

The *Weyl symbol* $A_W(x,p)$ of an operator $\widehat{A}$ is a phase-space observable defined by integral transform:

$$A_W(x,\ p) \ = \ 2\int \langle x+y \mid \widehat{A} \mid x \ - \ y\rangle \exp[-2ipy/\hbar] \ dy$$

The *Wigner function* of a quantum state is the Weyl symbol of its density operator $\widehat{\rho}$:

$$W(x,\ p) \ = \rho_W(x,\ p)$$



The von Neumann equation for $\widehat{\rho}$ shows that W evolves as $\partial_t W \ = \{W,\ H\} \ + \ O(\hbar^2)$

Phase-space evolution of a quantum system

Quantum dynamics, now seen as a deformation of classical dynamics

Given an observable $\widehat{O}$, its time-evolution is given by $\langle \widehat{O_t} \rangle = \iint O_W(x_t, p_t) \cdot W(x_0, p_0)\; dx_0\, dp_0$

The *Truncated Wigner approximation* (TWA) approximates it in the Heisenberg picture as follows:

$$\langle \widehat{O_t} \rangle \simeq \iint O_W\left(x_t^{MF}, p_t^{MF}\right) W(x_0, p_0)\; dx_0\, dp_0$$

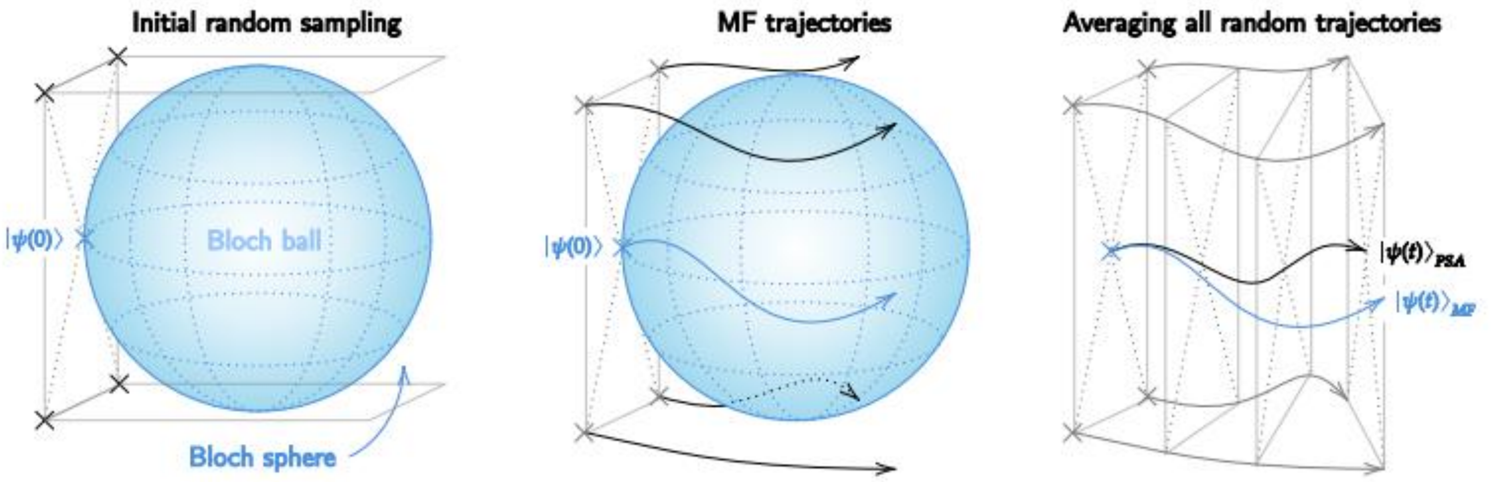
1. Generate many initial random conditions, depending on the initial state
2. MF-evolve each random cost to get the required expectation values
3. Average over all trajectories to get your final estimate

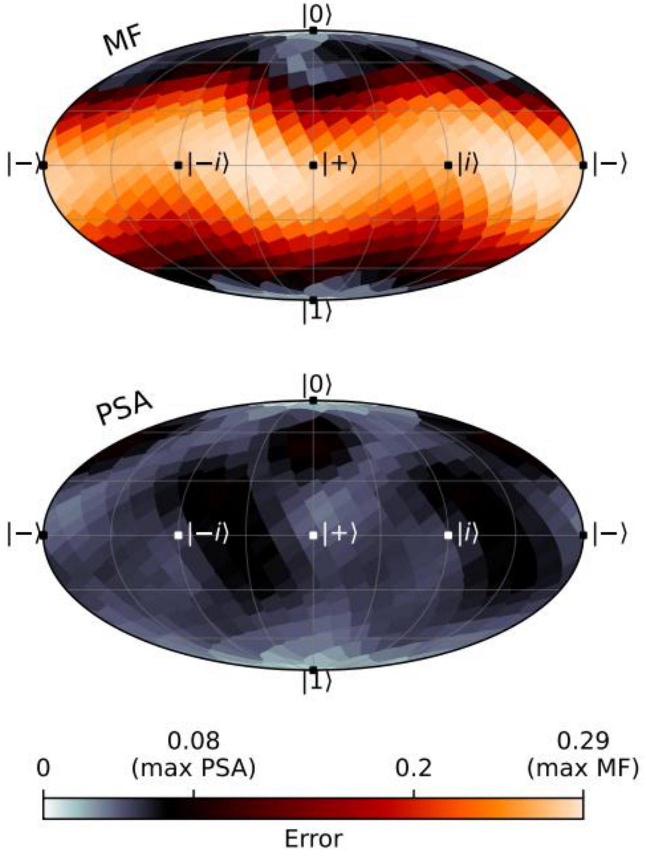
Example of MF evolution for a 2-qubit system under an Ising Hamiltonian $\widehat{\sigma}_0^x \widehat{\sigma}_1^x$:

$$\begin{cases} \partial_t \langle \widehat{\sigma}_i^x \rangle & \propto 0 \\ \partial_t \langle \widehat{\sigma}_i^y \rangle & \propto \langle \widehat{\sigma}_i^z \widehat{\sigma}_{1-i}^y \rangle \\ \partial_t \langle \widehat{\sigma}_i^z \rangle & \propto \langle \widehat{\sigma}_i^y \widehat{\sigma}_{1-i}^x \rangle \end{cases} \Longrightarrow \begin{cases} \partial_t \langle \widehat{\sigma}_i^x \rangle & \propto 0 \\ \partial_t \langle \widehat{\sigma}_i^y \rangle & \propto \langle \widehat{\sigma}_i^z \rangle \cdot \langle \widehat{\sigma}_{1-i}^y \rangle \\ \partial_t \langle \widehat{\sigma}_i^z \rangle & \propto \langle \widehat{\sigma}_i^y \rangle \cdot \langle \widehat{\sigma}_{1-i}^x \rangle \end{cases}$$

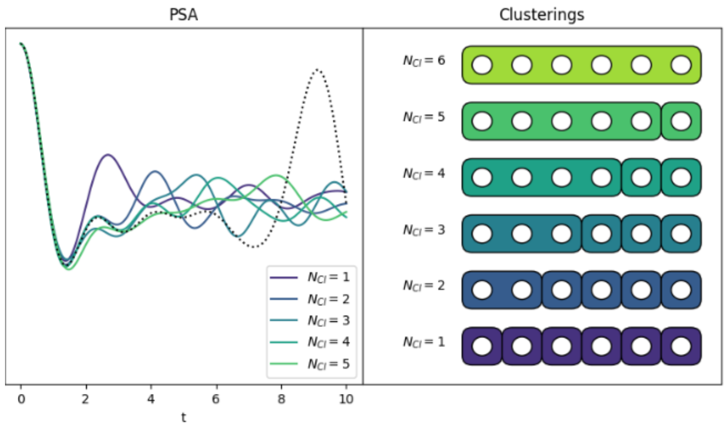
Phase-space simulation of qubit systems

An impressingly rich toy model to probe quantum dynamics and computational methods





Aspects under study: 1-qubit observables, 2-qubit correlations, entropies, scaling cost, perturbations…



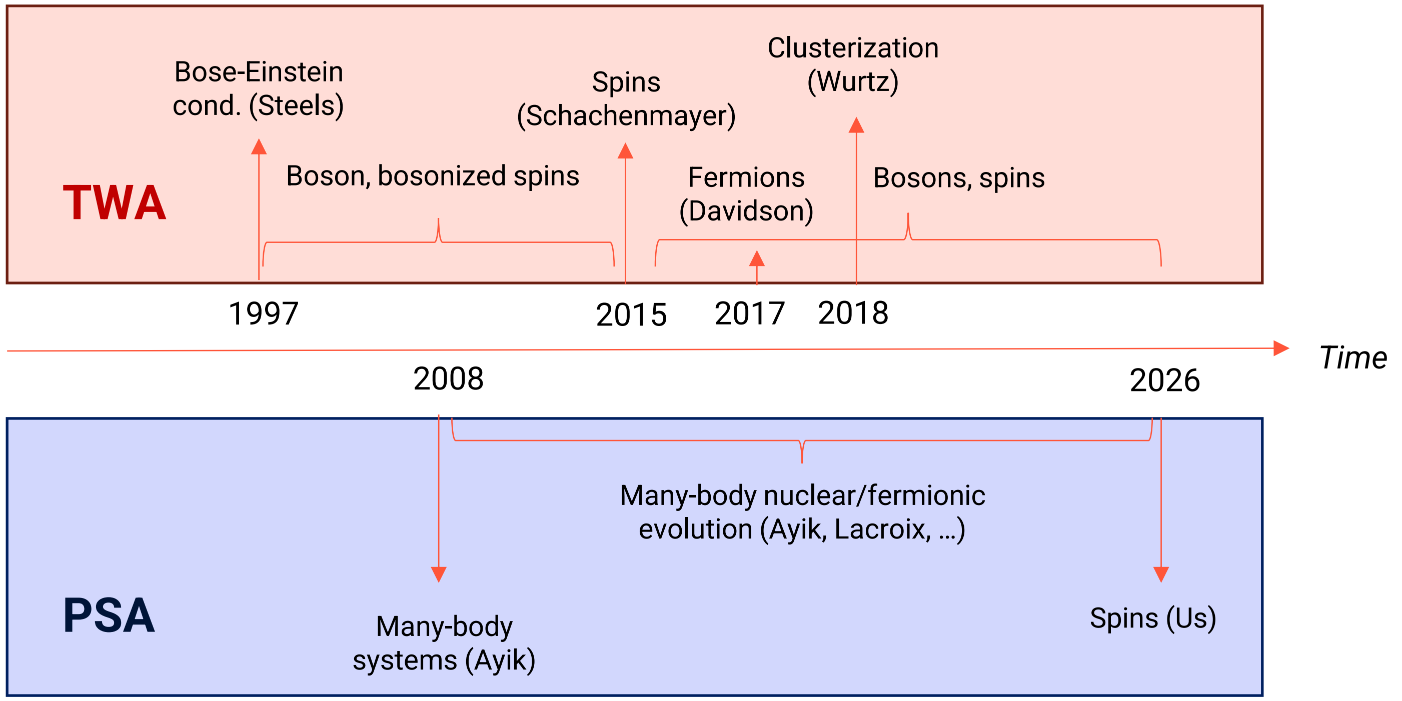
Advantages: scalability, parallelization, geometry & connectivity

Thank you!

Feel free to ask questions :)

Connection between two lines of work

An unexpected twist



Weyl quantization

We first introduce the Weyl quantization map from classical phase-space to the quantum Hilbert space.

Definition:

The Weyl quantization of the polynomial $f(x, p) = (ax + bp)^n$ is $\hat{f} = f(\hat{x}, \hat{p}) = (a\hat{x} + b\hat{p})^n$.

Example: $f(x, p) = 6x^2p^2 \longmapsto \hat{f} = \hat{x}^2\hat{p}^2 + \hat{p}^2\hat{x}^2 + \hat{x}\hat{p}\hat{x}\hat{p} + \hat{x}\hat{p}^2\hat{x} + \hat{p}\hat{x}\hat{p}\hat{x} + \hat{p}\hat{x}^2\hat{p}.$

Theorem:

If f is a polynomial of degree at most 2 and g and arbitrary polynomial, then $i\hbar\widehat{\{f, g\}} = [\hat{f}, \hat{g}].$

Counter-example: $\widehat{L^2 - 3\hbar^2/2} = (\hat{L})^2$

Wigner transform

We now define the inverse of our quantization map.

Theorem: the Weyl quantization admits an inverse defined by the Weyl symbol f_W of an operator $\hat{f}$:

$$f_W(x, p) = 2\int \langle x+y \mid \hat{f} \mid x - y\rangle \exp[-2ipy/\hbar]\ dy$$

The *Wigner function* of a quantum state is the Weyl symbol (or *Wigner transform*) of its density operator:

$$W(x, p) = \rho_W(x, p) = 2\int \langle x+y \mid \hat{\rho} \mid x - y\rangle \exp[-2ipy/\hbar]\ dy$$

The von Neumann equation for $\hat{\rho}$ shows that W evolves as $\partial_t W = \{W, H\} + O(\hbar^2)$