

Extracting GPDs from data: status and perspectives

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In collaboration with

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OPTIMIST

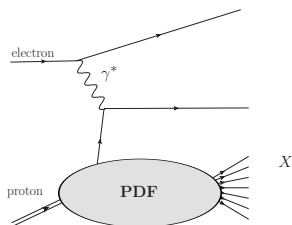


**MY GLASS
IS HALF
FULL**

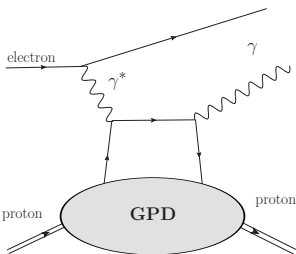
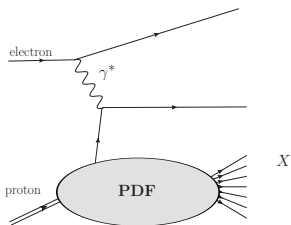
PESSIMIST



**MY GLASS
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EMPTY**

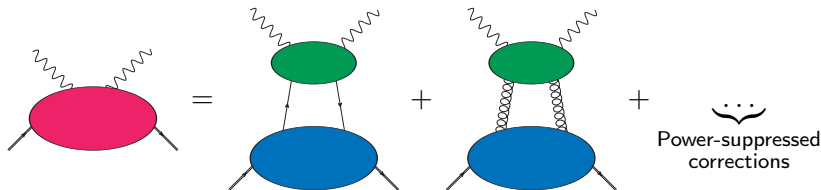


- Deep Inelastic Scattering, only the recoiled electron is characterised.
- The photon virtuality Q^2 is large compared to all other scales.
- The process is inclusive (summed over all possible final states).
- It probes the Parton Distribution Functions (PDF)



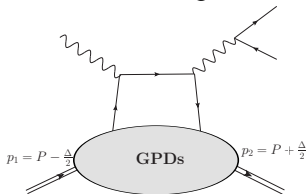
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- It probes the Parton Distribution Functions (PDF)
- Exclusive processes : the final state is **fully** characterised
- Example : Deeply virtual Compton Scattering (DVCS) $ep \rightarrow ep\gamma$
- The photon virtuality Q^2 is large compared to all other scales (including t the total momentum transfer squared).
- It probes Generalised Parton Distributions (GPDs)

- When the photon is strongly virtual : $Q^2 = -q^2 \gg M^2, t$

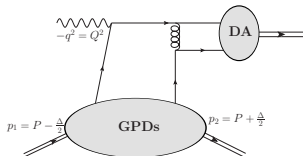


- Decomposition of DVCS between perturbative (green) and non-perturbative (blue) parts.
- Perturbative part \rightarrow description of the interaction between the probe and a parton inside hadron
- Non-perturbative part : description of a parton-hadron amplitude called Generalised Parton Distributions (GPDs)
- GPDs is where the information on the hadron structure lies.

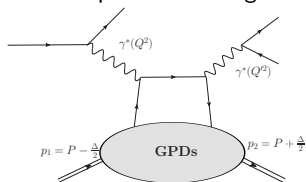
Timelike Compton Scattering



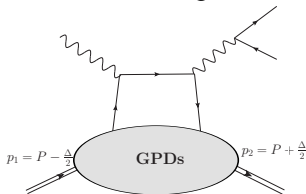
Deeply virtual meson production



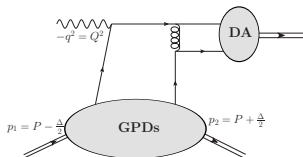
Double Deeply Virtual Compton Scattering



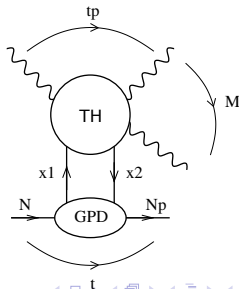
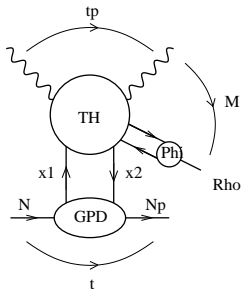
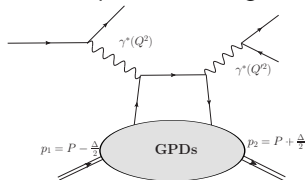
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Deeply virtual meson production



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Generalised Parton Distributions

$$\begin{aligned} & \frac{1}{2} \int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2} | \bar{\psi}^q(-\frac{z}{2}) \gamma^+ \psi^q(\frac{z}{2}) | P - \frac{\Delta}{2} \rangle dz^- |_{z^+=0, z=0} \\ &= \frac{1}{2P^+} \left[H^q(x, \xi, t) \bar{u} \gamma^+ u + E^q(x, \xi, t) \bar{u} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u \right]. \end{aligned}$$

D. Müller et al., Fortschr. Phys. 42 101 (1994)

X. Ji, Phys. Rev. Lett. 78, 610 (1997)

A. Radyushkin, Phys. Lett. B380, 417 (1996)

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- $t = \Delta^2$: the Mandelstam variable
- Caveat! In gauges other than the lightcone one, a Wilson line is necessary to make the GPDs gauge invariant

- Forward limit:

$$H(x, \xi = 0, t = 0) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x)$$

Optical Theorem

- Forward limit:

Optical Theorem

- Polynomiality Property:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t; \mu) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} \xi^{2j} C_{2j}^q(t; \mu) + \text{mod}(m, 2) \xi^{m+1} C_{m+1}^q(t; \mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^1 dx H^q(x, \xi, t; \mu) = F_1^q(t) \quad \int_{-1}^1 dx E^q(x, \xi, t; \mu) = F_2^q(t)$$

Lorentz Covariance

- Forward limit:
- Polynomiality Property:
- Positivity property:

Optical Theorem

Lorentz Covariance

$$\left| H^q(x, \xi, t) - \frac{\xi^2}{1 - \xi^2} E^q(x, \xi, t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1 - \xi^2}}$$

- A. Radyushkin, Phys. Rev. D59, 014030 (1999)
 B. Pire *et al.*, Eur. Phys. J. C8, 103 (1999)
 M. Diehl *et al.*, Nucl. Phys. B596, 33 (2001)
 P.V. Pobilitza, Phys. Rev. D65, 114015 (2002)

Positivity of Hilbert space norm

- Forward limit:
- Polynomiality Property:
- Positivity property:
- Support property:

Optical Theorem

Lorentz Covariance

Positivity of Hilbert space norm

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. B428, 359 (1998)

Relativistic quantum mechanics

- Forward limit:

Optical Theorem

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Continuity at the crossover lines

→ GPDs are continuous albeit non-analytical at $x = \pm\xi$

J. Collins and A. Freund, PRD 59 074009 (1999)

Factorisation theorem

- Forward limit:
Optical Theorem
- Polynomiality Property:
Lorentz Covariance
- Positivity property:
Positivity of Hilbert space norm
- Support property:
Relativistic quantum mechanics
- Continuity at the crossover lines
Factorisation theorem
- Scale evolution property
→ generalization of DGLAP and ERBL evolution equations

D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

Renormalization

- Forward limit: Optical Theorem
- Polynomiality Property: Lorentz Covariance
- Positivity property: Positivity of Hilbert space norm
- Support property: Relativistic quantum mechanics
- Continuity at the crossover lines Factorisation theorem
- Scale evolution property Renormalization

Problem

- There is hardly any model fulfilling all these constraints *a priori*.
- Lattice QCD computations remain very challenging.

Interpretation of GPDs I

2+1D structure of the nucleon



- In the limit $\xi \rightarrow 0$, one recovers a density interpretation:
 - ▶ 1D in momentum space (x)
 - ▶ 2D in coordinate space \vec{b}_\perp (related to t)

M. Burkardt, Phys. Rev. D62, 071503 (2000)

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- Possibility to extract density from experimental data

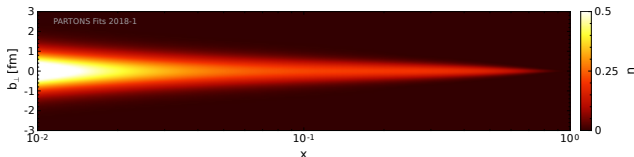


figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

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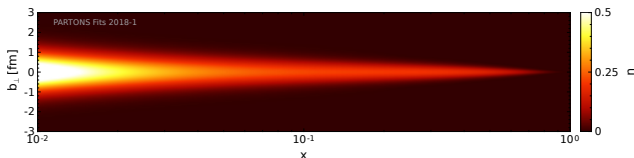


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- Correlation between x and $b_\perp \rightarrow$ going beyond PDFs and FFs.

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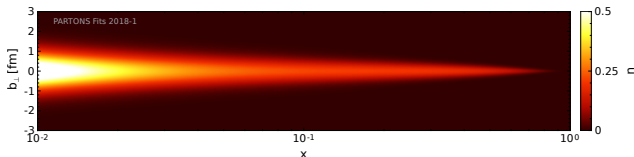
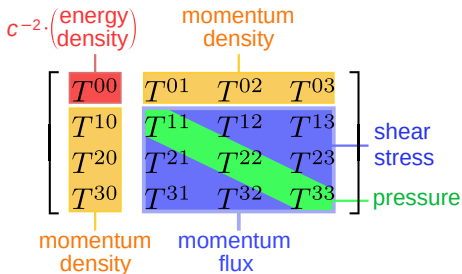


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- Correlation between x and $b_\perp \rightarrow$ going beyond PDFs and FFs.
- Caveat: no experimental data at $\xi = 0$
 \rightarrow extrapolations (and thus model-dependence) are necessary

Interpretation of GPDs II

Connection to the Energy-Momentum Tensor



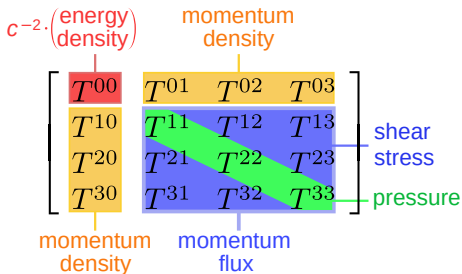
How energy, momentum, pressure are shared between quarks and gluons

Caveat: renormalization scheme and scale dependence

- C. Lorcé *et al.*, PLB 776 (2018) 38-47,
M. Polyakov and P. Schweitzer,
IJMPA 33 (2018) 26, 1830025
C. Lorcé *et al.*, Eur.Phys.J.C 79 (2019) 1, 89

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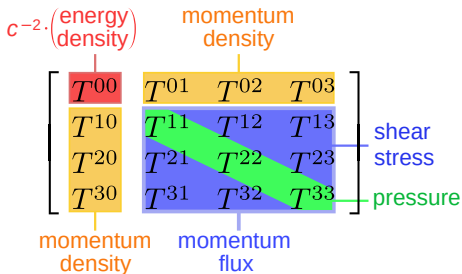
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$$\langle p', s' | T_{q,g}^{\mu\nu} | p, s \rangle = \bar{u} \left[P^{\{\mu\gamma\nu\}} A_{q,g}(t; \mu) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} C_{q,g}(t; \mu) \right. \\ \left. + M g^{\mu\nu} \bar{C}_{q,g}(t; \mu) + \frac{P^{\{\mu i \sigma^\nu\} \Delta}}{2M} B_{q,g}(t; \mu) + \frac{P^{\{\mu i \sigma^\nu\} \Delta}}{2M} D_{q,g}(t; \mu) \right] u$$

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$$\int_{-1}^1 dx x H_q(x, \xi, t; \mu) = A_q(t; \mu) + (2\xi)^2 C_q(t; \mu)$$

$$\int_{-1}^1 dx x E_q(x, \xi, t; \mu) = B_q(t; \mu) - (2\xi)^2 C_q(t; \mu)$$

- Ji sum rule
- Fluid mechanics analogy
 X. Ji, PRL 78, 610-613 (1997)
 M.V. Polyakov PLB 555, 57-62 (2003)

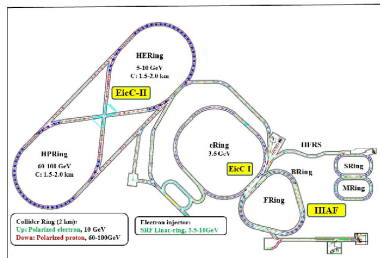
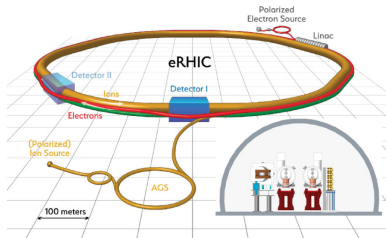
- Current experimental facilities: fixed targets



- Current experimental facilities: fixed targets



- Future facilities: colliders



How are experimental data connected to the 3D structure of the nucleon ?

$$\Im \mathcal{H}(\xi, t, Q^2) = 2 \int_{\xi}^1 \frac{dx}{\xi} \Im \mathcal{C}\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \\ \int_x^1 \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) H\left(\frac{x}{y}, \xi, t, \mu^2\right)$$

DVCS Amplitude extracted
from experimental data

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DVCS Amplitude extracted from experimental data

Targeted GPD function for 3D reconstruction

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Description of probe-parton interaction computed in pQCD

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Description of probe-parton interaction computed in pQCD

Evolution kernel providing a scale resolution tuning

Experimental data

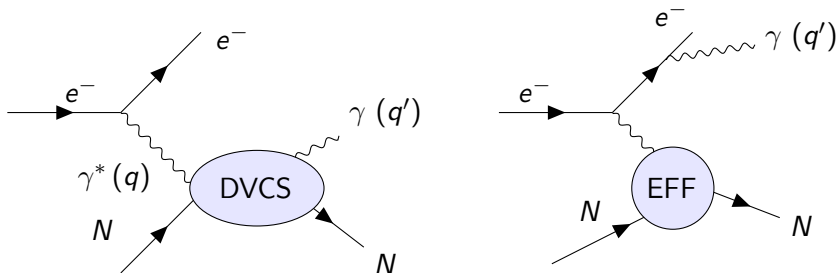
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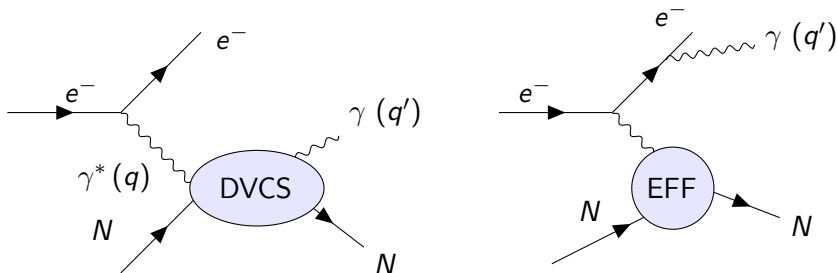
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- We do not measure process amplitudes but cross-sections between an initial and a final state:

$$ep \rightarrow ep\gamma$$

- All processes with the same initial and final state **do** contribute to our measurement



- You need to find a way to disentangle both

DVCS Beam Spin Asymmetry

Exploiting interference



$$\frac{d\sigma}{d\Omega} \propto |BH + DVCS|^2 \propto |BH|^2 + |DVCS|^2 + \underbrace{\text{Interference}}_{\text{our target}}$$

- BH amplitude is well-known \rightarrow access to DVCS amplitude
- BH amplitude is much larger than DVCS one \rightarrow interference term is significantly larger than the pure DVCS one

DVCS Beam Spin Asymmetry

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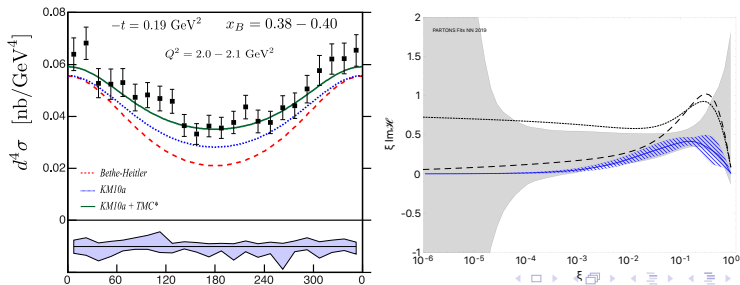


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$$A_{LU} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \propto \Im \mathcal{H}$$

figs. from M. Defurne et al., PRC 92 (2015) 5, 055202, and P. Sznajder et al., EPJC 80 (2020) 2, 171



- Despite mixing with BH and being a rare event, DVCS *is* measurable
- One can build observables directly sensitive to the amplitude by taking advantage of the interference term.
- Good experimental signal for \mathfrak{H} thanks to JLab experiments in the valence region.

$$\mathfrak{H}(\xi, t, Q^2) = 2 \int_{\xi}^1 \frac{dx}{\xi} \mathfrak{S}C\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ \times H\left(\frac{x}{y}, \xi, t, \mu^2\right)$$

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Problem 1: Limited kinematic sensitivity

\mathfrak{H} is generated only when $|x| \geq |\xi|$ in the convolution integral.
How do we recover the inner $-\xi \leq x \leq \xi$ region?

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Problem 1: Limited kinematic sensitivity

\mathfrak{H} is generated only when $|x| \geq |\xi|$ in the convolution integral.
How do we recover the inner $-\xi \leq x \leq \xi$ region?

Problem 2 : Uniqueness of the solution

Are we sure that the solution is unique? In other words, for a linear problem, do we have to handle vanishing eigenvalues?

Solving Problem 1

For experts:

Existence and uniqueness of kinematic completion
from DGLAP to ERBL regions

N. Chouika *et al.*, Eur.Phys.J.C 77 (2017) 12, 906

N. Chouika *et al.*, Phys.Lett.B 780 (2018) 287-293

J.-M. Morgado Chavez *et al.*, Phys.Rev.D 105 (2022) 9, 094012

P. Dall'Olio *et al.*, Phys.Rev.D 109 (2024) 9, 096013

A. Castro *et al.*, Phys.Rev.D 112 (2025) 3, 034009

- The difficult part is the (x, ξ) dependence as it is the one that is convoluted.
- We also have the polynomiality condition:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t; \mu) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} \xi^{2j} C_{2j}^q(t; \mu) + \text{mod}(m, 2) \xi^{m+1} C_{m+1}^q(t; \mu)$$

- In mathematics, this is known as the Ludwig-Helgason consistency condition, indicating that the function H is in the range of the Radon transform.

- The connection between GPDs and DDs is given through:

$$H(x, \xi, t) - \Theta(-\xi \leq x \leq \xi) D\left(\frac{x}{\xi}, t\right) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) F(\beta, \alpha, t)$$

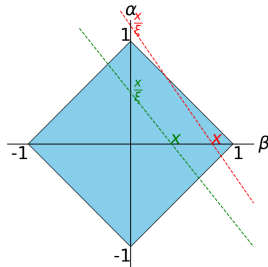
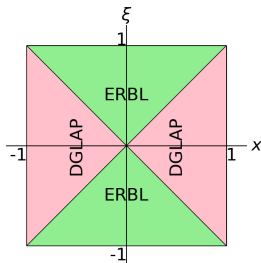
A. Radyushkin, PRD 56 (1997) 5524-5557
 D. Müller et al., Fortschr. Phys. 42 101 (1994)

- The D -term can be reabsorbed as:

$$H(x, \xi, t) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) [F(\beta, \alpha, t) + \xi\delta(\beta)D(\alpha, t)]$$

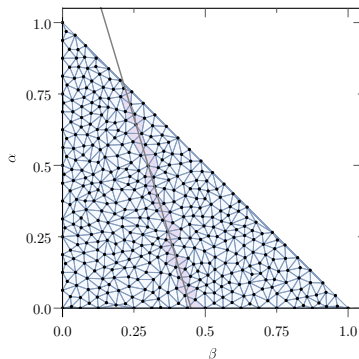
M. Polyakov and C. Weiss, PRD60 114017 (1999)

- The properties of the DD guarantee those of the GPD

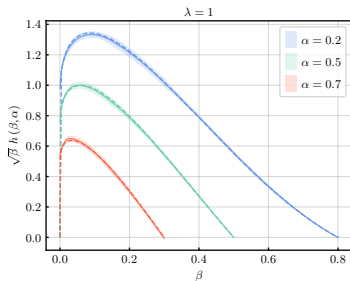
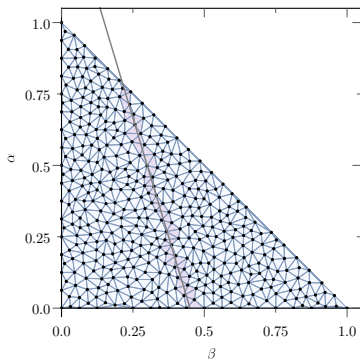


$$\begin{aligned} \mathfrak{H}(\xi, t, Q^2) &= 2 \int_{\xi}^1 \frac{dx}{\xi} \mathfrak{C}\left(\frac{x}{\xi}, \frac{t}{Q^2}, \alpha_s(Q^2); \partial_{\xi}^{(n)}\right) \int_x^{\infty} \frac{dy}{y} K\left(y, \frac{\xi}{x}; Q^2, \mu^2\right) \\ &\quad \times \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta\left(\frac{x}{y} - \beta - \alpha\xi\right) F(\beta, \alpha, t, \mu^2) \end{aligned}$$

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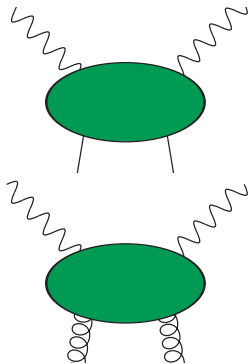
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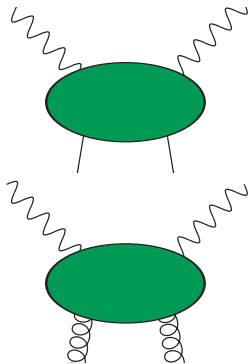
Solving Problem 2

For experts:
Characterising the kernel
of the coefficient function

- H. Dutrieux *et al.*, Phys.Rev.D 103 (2021) 11, 114019
- E. Moffat *et al.*, Phys.Rev.D 108 (2023) 3, 036027
- H. Dutrieux *et al.*, Eur.Phys.J.C 82 (2022) 3, 252
- M. Riberdy *et al.*, Eur.Phys.J.C 84 (2024) 2, 201
- M. Cuic *et al.*, in preparation



- Strongly off-shell (hard) scattering amplitudes between a photon and a parton (quark or gluon)
- They are computed systematically in perturbation theory as an expansion in α_s .
- Currently, the DVCS coefficient function is known at next-to-next-to leading order (NNLO)
- Kinematic higher-twist corrections (i.e. corrections in M^2/Q^2 and t/Q^2) are known but not interpretable in terms of simple Feynman graphs.



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The coefficient function is the only complex function in the convolution integral. We will thus focus on its imaginary part.

- The leading-order and leading-twist coefficient function is given by

$$\mathfrak{S}C\left(\frac{x}{\xi}\right) = -e_q^2 \pi \left[\delta\left(1 + \frac{x}{\xi}\right) - \delta\left(1 - \frac{x}{\xi}\right) \right]$$

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- This is an issue because a function:

$$g(x, \xi) = f(x, \xi) + \frac{x - \xi}{2\xi} f(-\xi, \xi) - \frac{x + \xi}{2\xi} f(\xi, \xi)$$

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Deconvolution

- What happens when we add the other theory constraints that we know?
→ The forward limit of g is not defined and thus g is ruled out
- What happens when we increase the precision on the coefficient functions?

CFF Definition

$$\underbrace{\mathcal{H}(\xi, t, Q^2)}_{\text{Observable}} = \int_{-1}^1 \frac{dx}{\xi} \underbrace{\mathcal{T}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)}_{\text{Perturbative DVCS kernel}} H(x, \xi, t, \mu^2)$$

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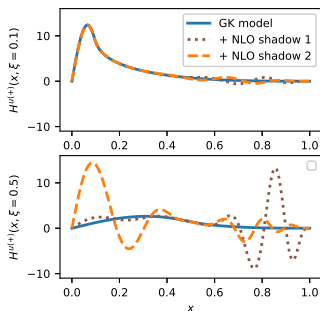
Shadow GPD definition

We define shadow GPD $H^{(n)}$ of order n such that when T is expanded in powers of α_s up to n one has:

$$0 = \int_{-1}^1 \frac{dx}{\xi} T^{(n)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu_0^2}, \alpha_s(\mu_0^2)\right) H^{(n)}(x, \xi, t, \mu_0^2) \quad \text{invisible in DVCS}$$

$$0 = H^{(n)}(x, 0, 0) \quad \text{invisible in DIS}$$

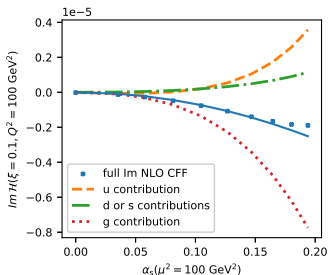
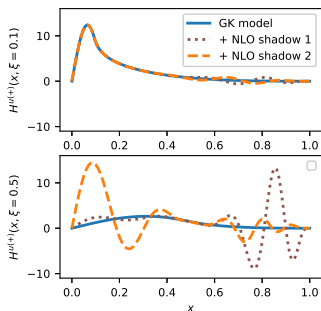
A part of the GPD functional space is invisible to DVCS and DIS combined



• NLO analysis of shadow GPDs:

- ▶ Cancelling the line $x = \xi$ is necessary but **no longer** sufficient
- ▶ Additional conditions brought by NLO corrections reduce the size of the “shadow space”...
- ▶ ... but do not reduce it to 0
→ NLO shadow GPDs

H. Dutrieux *et al.*, PRD 103 114019 (2021)



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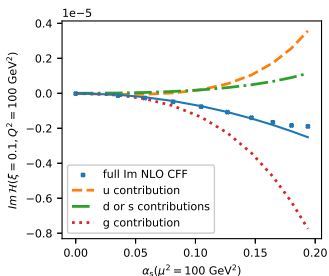
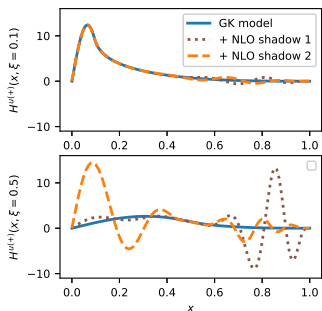
• Evolution

- ▶ it was argued that evolution would solve this issue

A. Freund PLB 472, 412 (2000)
E. Moffat *et al.*, PRD 108 (2023)

- ▶ but in practice it is not the case

H. Dutriex *et al.*, PRD 103 114019 (2021)



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Theoretical uncertainties promoted
to main source of GPDs uncertainties

Problem : infinite dimension of the kernel

- Shadow GPDs are **continuous, eigenfunctions** of an operator.
- We extracted some using a basis of continuous functions, polynomials.
- Can we find a basis such that at any truncation order we are free of shadow GPDs (but of course recover them when $N \rightarrow \infty$) ?

M. Cuic et al., in preparation

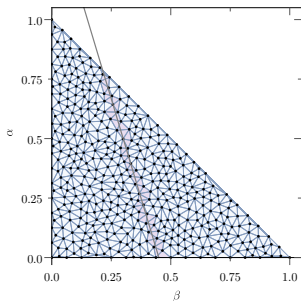
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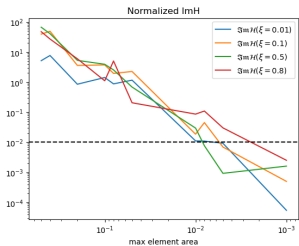
Answer : Finite element method

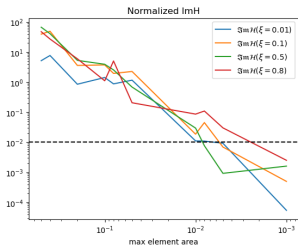
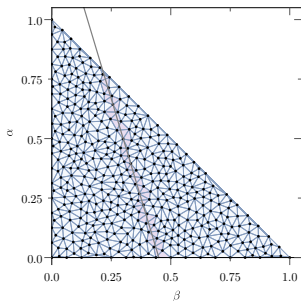
- If the zero modes are continuous function, **let us discretise them**. The discretisation errors shift them out of the kernel of the coefficient function.
- The shadow GPDs become visible and affect the computation of the CFF.

M. Cuic et al., in preparation



- The Double Distribution domain is discretised in terms of elements of maximal size a (discretisation surface).
- Impact of discretised LO shadow GPDs on $\mathfrak{S}\mathcal{H}$ can be assessed
- Impact is clearly visible $\sim 1\% - 10\%$ for an element area of 10^{-2}





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Resolution power

You can define a resolution power a_{\min} once you determine:

- 1 The precision of your data
- 2 The uncertainty target

Statement from 15 years ago

DVCS provides you information only on the diagonal $x = \xi$.

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Modern statement

DVCS provides you with all the knowledge you need on the GPDs with the following caveat:

- The imaginary part of DVCS amplitude on a large kinematic domain is enough to constrain the DD F unambiguously,
- Your resolution power in discretised DD space depends on the precision of your pQCD computation,
- Your ability to determine the D -term depends on your ability to extract the real part of the amplitude.

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Last but not least

Once properly understood, DVCS reveals itself
much richer than previously thought

Thank you for your attention and enjoy your half-full glass

OPTIMIST



**MY GLASS
IS HALF
FULL**

PESSIMIST

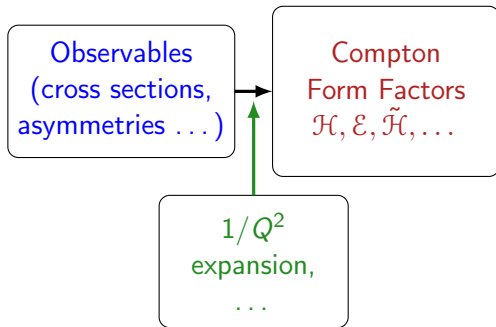


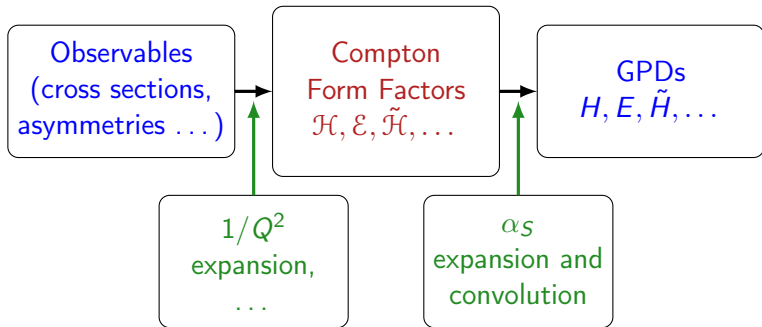
**MY GLASS
IS HALF
EMPTY**

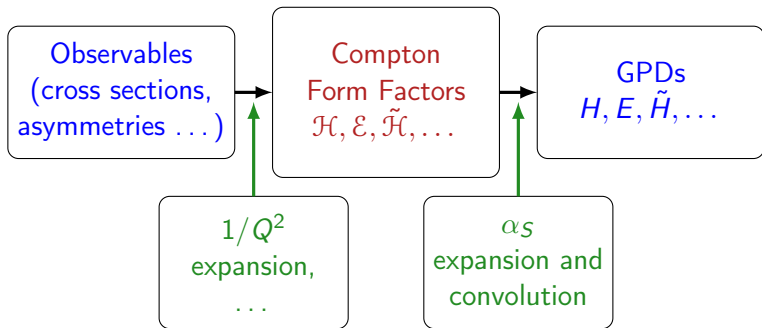
Addendum

- From valence to high energy, how does the picture of the proton change ?
 - ▶ Some collision at very high energy at the LHC are sensitive to GPDs
- Does the 3D map of quarks and gluons strongly differs ?
- What about mesons ? How are quarks and gluons distributed in a pion ?
 - ▶ We may be able to extract observables sensitive to the pion GPDs.
- Can we extract intrinsic gluons, i.e. gluons that do not mix with quarks ?

Observables
(cross sections,
asymmetries ...)







- We were able to define an observable such that \mathfrak{H} can be extracted
- We have then discussed the difficulties of deconvolution and how to bypass them.

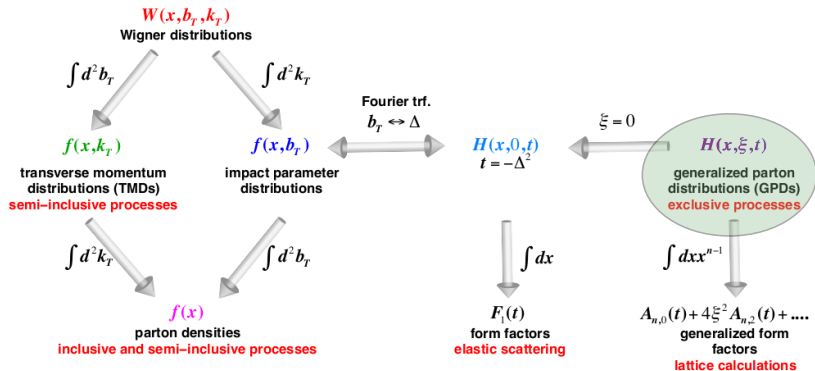
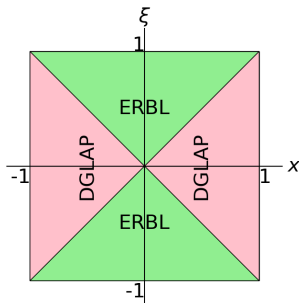
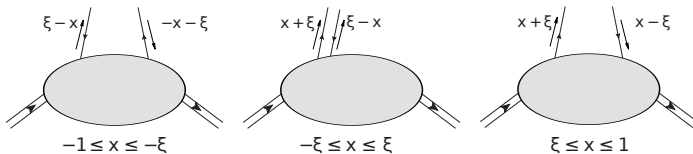


figure from A. Accardi et al., Eur.Phys.J.A 52 (2016) 9, 268

Different values of (x, ξ) yields different light-front interpretations:



- Modifies our understanding of what is probed
- Different type of contributions
- It determines two big regions
- Relevant for evolution equations
- $|\xi| > 1$ region of Generalised Distribution Amplitudes (GDA)

And from them, extract pressure and shear forces following:

$$\varepsilon_a(r) = M \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2} [B_a(t) - 4C_a(t)] \right\},$$

$$p_{r,a}(r) = M \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \left\{ -\bar{C}_a(t) - \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} \left(t^{3/2} C_a(t) \right) \right\},$$

$$p_{t,a}(r) = M \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \left\{ -\bar{C}_a(t) + \frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d}{dt} \left[t \frac{d}{dt} \left(t^{3/2} C_a(t) \right) \right] \right\},$$

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$$s_a(r) = M \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \left\{ -\frac{4}{r^2} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left(t^{5/2} C_a(t) \right) \right\},$$

C. Lorcé et al., Eur.Phys.J.C 79 (2019) 1, 89

Model $H = H_{\text{visible}} + H_{\text{shadow}}$ with two different neural networks fulfilling by construction all the properties but one, the positivity property.

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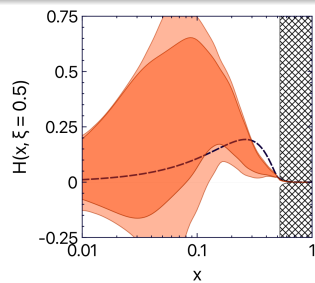
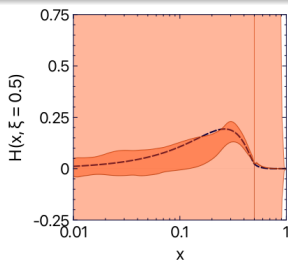
The positivity property

$$\left| H^q(x, \xi, t) - \frac{\xi^2}{1 - \xi^2} E^q(x, \xi, t) \right| \leq \sqrt{\frac{1}{1 - \xi^2} q\left(\frac{x + \xi}{1 + \xi}\right) q\left(\frac{x - \xi}{1 - \xi}\right)}$$

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H. Dutrieux *et al.*, EPJC 82 (2022) 3, 252

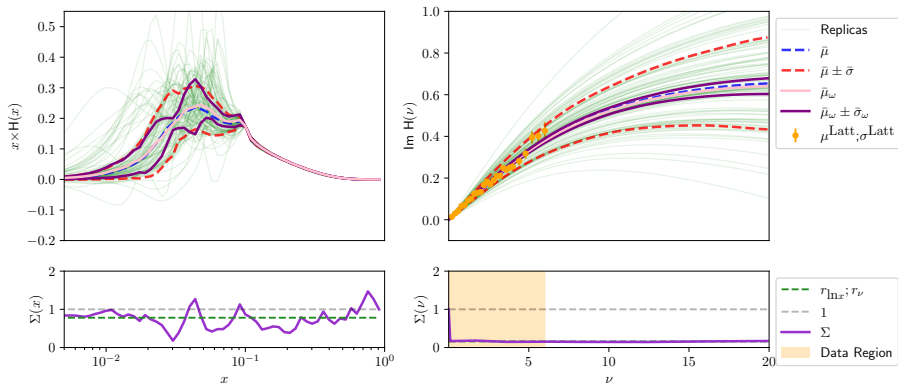


Lattice QCD can now compute matrix elements connected to GPDs:

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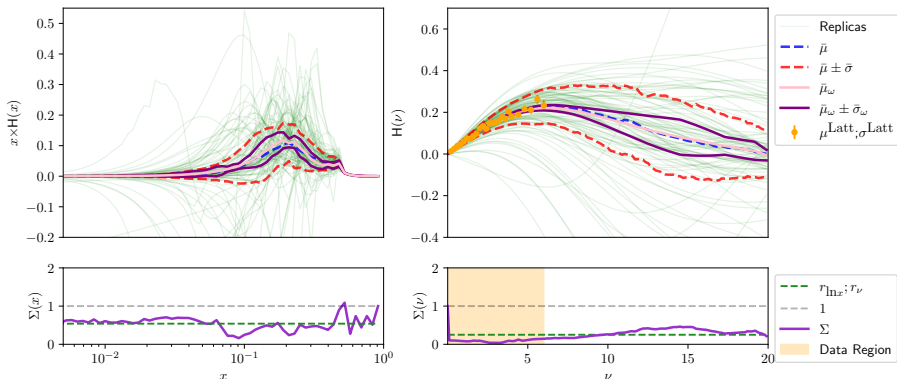
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M. Riberdy et al., Eur.Phys.J.C 84 (2024) 2, 201

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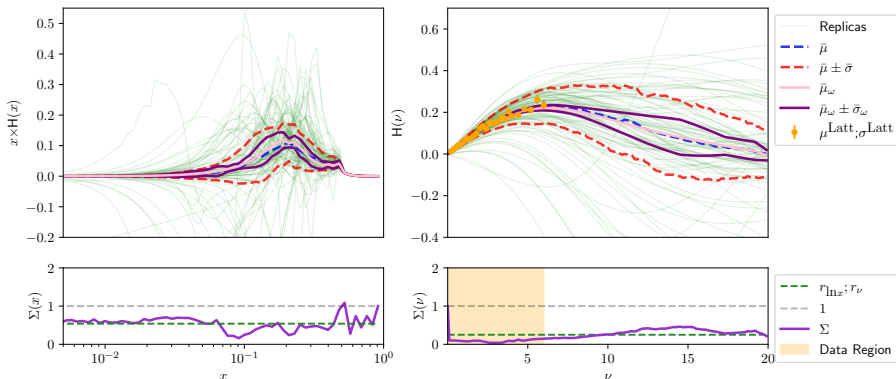
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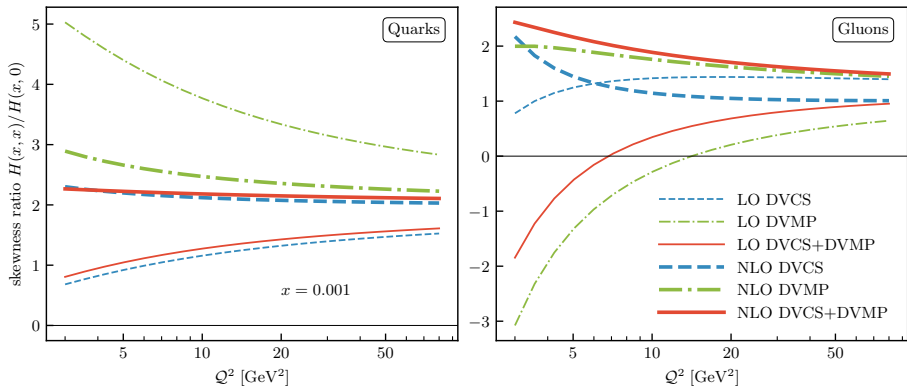
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M. Riberdy et al., Eur.Phys.J.C 84 (2024) 2, 201

We can expect a reduction of the deconvolution uncertainties



M. Cuic *et al.*, JHEP 12 (2023) 192

- Extraction of quark and gluon GPDs from NLO-DVCS and NLO-DVMP
- Inverse problem regularised through moments parametrisation and truncation

- We want our shadow GPDs to fulfill all the good theoretical properties of standard GPDs, especially polynomiality

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which is in one to one correspondence with the polynomiality property

N. Chouika *et al*, EPJC 77 (2017)

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N. Chouika *et al*, EPJC 77 (2017)

- We expanded f_{shadow} on polynomials of order N , so that we have a number of coefficient of order N^2 .

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- For definiteness, we add another constraint such that $f_{\text{shadow}}(1 - \alpha, \alpha) = 0$ (continuity of the DD).

With a number of parameters of order N^2 , we find our first solution for $N = 25$

We impose the following conditions :

- No forward limit $H(x, 0) = 0 \rightarrow N + 2$ equations
- $T \otimes H = (C^{(0)} + \alpha_s C^{(1)} + \alpha_s \ln(Q^2/\mu^2) C^{\text{coll}}) \otimes H = 0$
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Adding Mellin moments (computed on the Lattice) provides other sets of order N equations.

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- We expect CFF computed from evolved NLO shadow GPDs to exhibit an α_s^2 behaviour under evolution (provided that the logs remain small enough).

- Improving the precision computation of the coefficient function (N2LO, NLP)

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H. Dutrieux *et al.*, EPJC 82 (2022) 3, 252

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H. Dutrieux *et al.*, EPJC 82 (2022) 3, 252

- Go to multichannel analysis
 - ▶ Shadow GPDs are process-dependent, *i.e.* some processes can see the shadow GPDs of others
 - ▶ Some exclusive processes are expected *not* to have shadow GPDs at all (but they are harder to measure).
 - ★ Double DVCS is the most obvious one
 - ★ New 2 \rightarrow 3 exclusive processes are also good candidates
 - ▶ View LQCD Ioffe-time ratios as an additional process to be included in a global fit

K. Deja *et al.*, PRD 107 (2023) 9, 094035

R. Boussarie *et al.*, JHEP 02 (2017) 054

O. Grocholski *et al.*, Phys.Rev.D 104 (2021) 11,

J.-W. Qiu and Z. Yu, JHEP 08 (2022) 103

M. Riberdy *et al.*, Eur.Phys.J.C 84 (2024) 2, 201



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- But some bases are more practical than others:
 - ▶ Polynomials are not practical $\alpha^{12}\beta^{13}$ is very small within the 1-ball.
 - ▶ Finite Element Discretisation: Already benchmarked on the inverse Radon Transform; Discretisation \rightarrow matrix product; Intuitive;
 - ▶ Artificial Neural Networks: trendy \rightarrow many modern tools to handle them; Ability to extrapolate; Smoothness;

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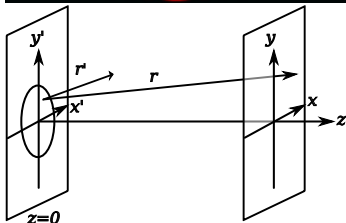
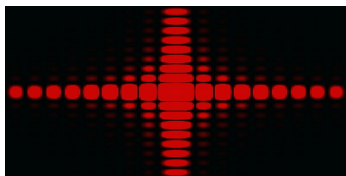
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Status

We have preliminary answers to these points, which makes me more optimistic than I was 4 years ago.

Scattering experiment I

Fraunhofer diffraction



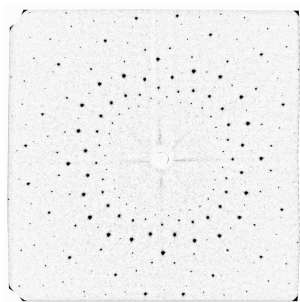
- Far field diffraction $z \gg x', y'$
- Monochromatic wavelength $\lambda \approx 1\mu m$

$$U(x, y, z) \approx \frac{e^{ikz} e^{ik \frac{x^2+y^2}{z}}}{i\lambda z} \underbrace{\iint dx' dy' U(x', y', 0) e^{-ik(\frac{x}{z}x' + \frac{y}{z}y')}}_{\text{Fourier Transform of the aperture}}$$

source : Wikimedia Commons

Scattering experiment II

X ray's scattering



Silicon crystal diffractive pattern

source: UK's national synchrotron

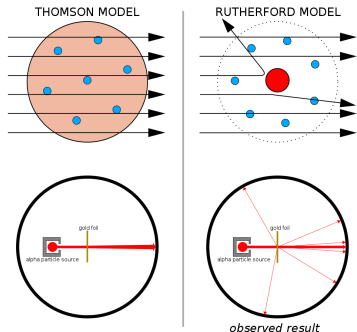
- X-ray wavelength
→ $\lambda \simeq$ typical size $\sim 1\text{nm}$
- Bragg's Law
- Diffraction pattern
→ Fourier transform of electronic density
- Reminder, for a grating one gets

$$I(\theta) \propto \frac{\sin^2(k/2NS \sin \theta)}{\sin^2(k/2S \sin \theta)}$$

- Provide information on the cristal structure

Scattering experiments III

Rutherford experiment



- α particles scattering on a gold foil
- Some of which are scattered at large angles
- Invalidate the Thomson Model (Plum Pudding)
- Allows to develop the Rutherford planetary model

source : Wikimedia Commons

- Scattering without breaking (Fraunhofer diffraction, Bragg's law, etc)
- Fourier transform relation between matter structure and diffraction figure
- Repeats itself for different orders of magnitude
- Can we extend that to hadron structure and probe the spatial organisation of **confined** quarks and gluons ?

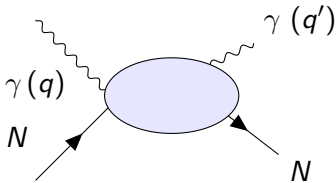
- Scattering without breaking (Fraunhofer diffraction, Bragg's law, etc)
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- Repeats itself for different orders of magnitude
- Can we extend that to hadron structure and probe the spatial organisation of **confined** quarks and gluons ?
- Some order of magnitudes:
 - ▶ typical nucleon radius 1 fm
 - ▶ we thus want a photon wavelength smaller to resolve details within the nucleon
 - ▶ Photon minimal energy : $E = hc/\lambda \approx 1.24\text{GeV}$ Highly energetic gamma ray
 - ▶ NB: the shortest laser wavelength is 0.15 nm.

Deeply virtual Compton Scattering I

Definition and kinematics

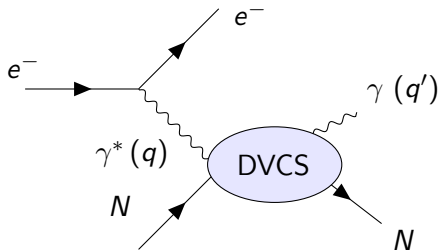


- No beam of 1 – 10GeV photon



Deeply virtual Compton Scattering I

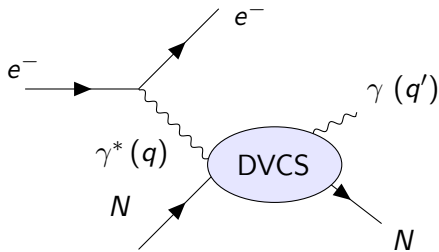
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- We switch to electroproduction with $Q^2 = -q^2$ much larger than the other scales

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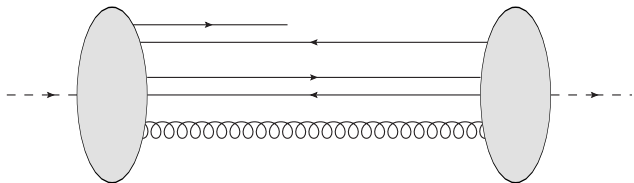
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Kinematics:

$$N_{in} \left(p = P - \frac{\Delta}{2} \right), \quad N_{out} \left(p' = P + \frac{\Delta}{2} \right)$$
$$\Delta^2 = t, \quad P \cdot \Delta = 0, \quad P^2 = M^2 - t/4$$
$$-q^2 = Q^2 \gg M^2, t \quad q'^2 = 0$$

Deeply virtual Compton Scattering II

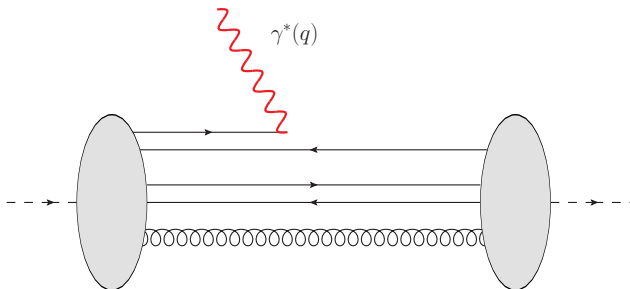
Leading contributions



- we look at a proton flying close to the light-cone
- all constituents (quarks and gluons) are moving collinearly

Deeply virtual Compton Scattering II

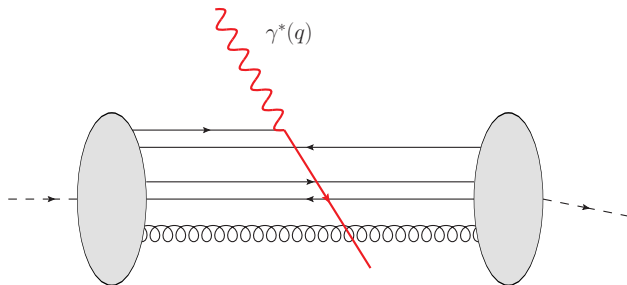
Leading contributions



- we look at a proton flying close to the light-cone
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- a deeply virtual photon deviates a quark, transferring its high virtuality Q^2

Deeply virtual Compton Scattering II

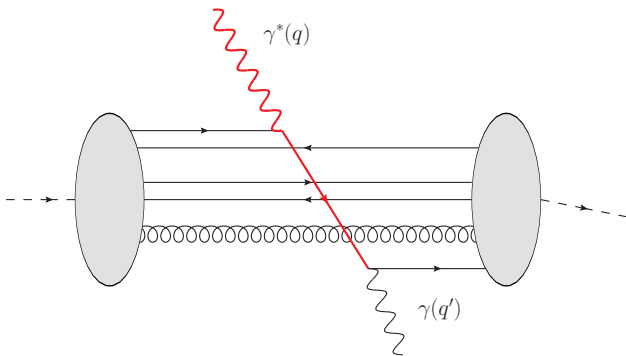
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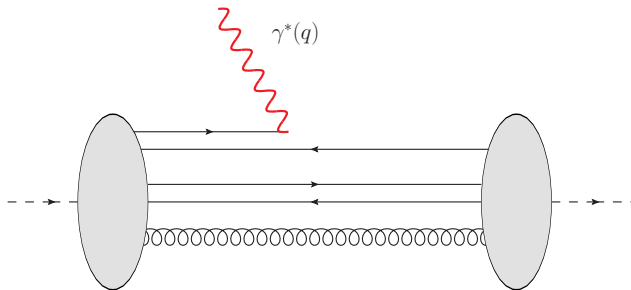
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- we look at a proton flying close to the light-cone
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- a deeply virtual photon deviates a quark, transferring its high virtuality Q^2
- the quark releases the energy before breaking the hadron

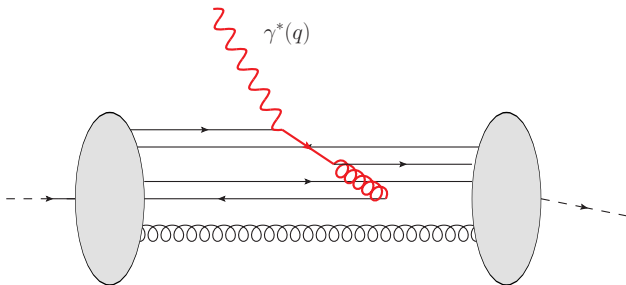
Deeply virtual Compton Scattering II

Subleading contributions



Deeply virtual Compton Scattering II

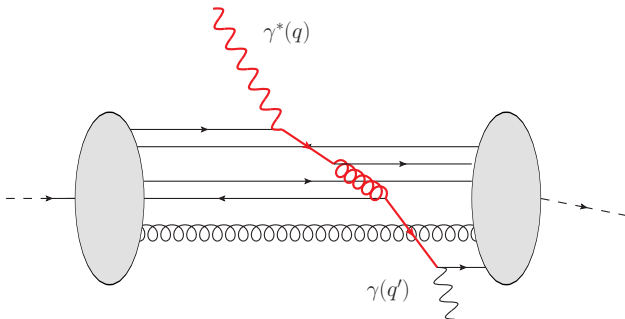
Subleading contributions



- this time, the quark releases energy through a gluon
- the gluon is absorbed by another quark transferring the energy

Deeply virtual Compton Scattering II

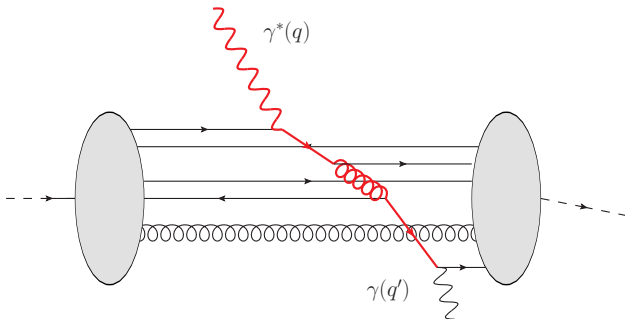
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Virtuality transfers between partons is power suppressed due to additional denominators (caveat : gauge link)

Deeply Virtual Compton Scattering II

Virtuality transfer



The amplitude can be seen as

$$\mathcal{A} \propto \epsilon^\sigma \gamma_\sigma S(k'_2 + q') \gamma_\mu D^{\mu\nu}(k_1 + q - k'_1) \gamma_\nu S(k_1 + q) \gamma_\lambda \epsilon^\lambda \mathcal{O}$$

where, in the lightcone gauge,

$$\begin{aligned} D^{\mu\nu}(k_1 + q - k'_1) &= \frac{i \left(\eta^{\mu\nu} - \frac{n^\mu (k_1 + q - k'_1)^\nu + n^\nu (k_1 + q - k'_1)^\mu}{(k_1 + q - k'_1) \cdot n} \right)}{(k_1 + q - k'_1)^2 + i\epsilon} \\ &= \frac{1}{Q^2} \frac{i \left(\eta^{\mu\nu} - \frac{n^\mu (k_1 + q - k'_1)^\nu + n^\nu (k_1 + q - k'_1)^\mu}{(k_1 + q - k'_1) \cdot n} \right)}{-1 + \frac{2q \cdot (k_1 - k'_1)}{Q^2} + \frac{(k_1 - k'_1)^2}{Q^2} - i\epsilon} \end{aligned}$$

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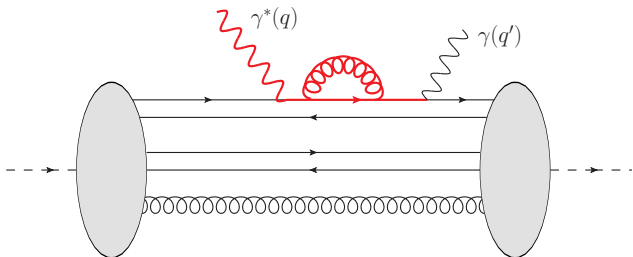
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- The gluon propagator introduces a power suppression
- A complete proof requires the computation of the Dirac trace to ensure that no compensations appear in the numerator

Deeply Virtual Compton Scattering III

Logarithmic corrections



- Loops are not power-suppressed, but provide logarithmic corrections.
- Reason: additional propagators are *integrated* over internal momenta.
- These loops are critical: “scaling violation” in DIS.