

Nonrelativistic conformal collider physics of multi-neutron point production

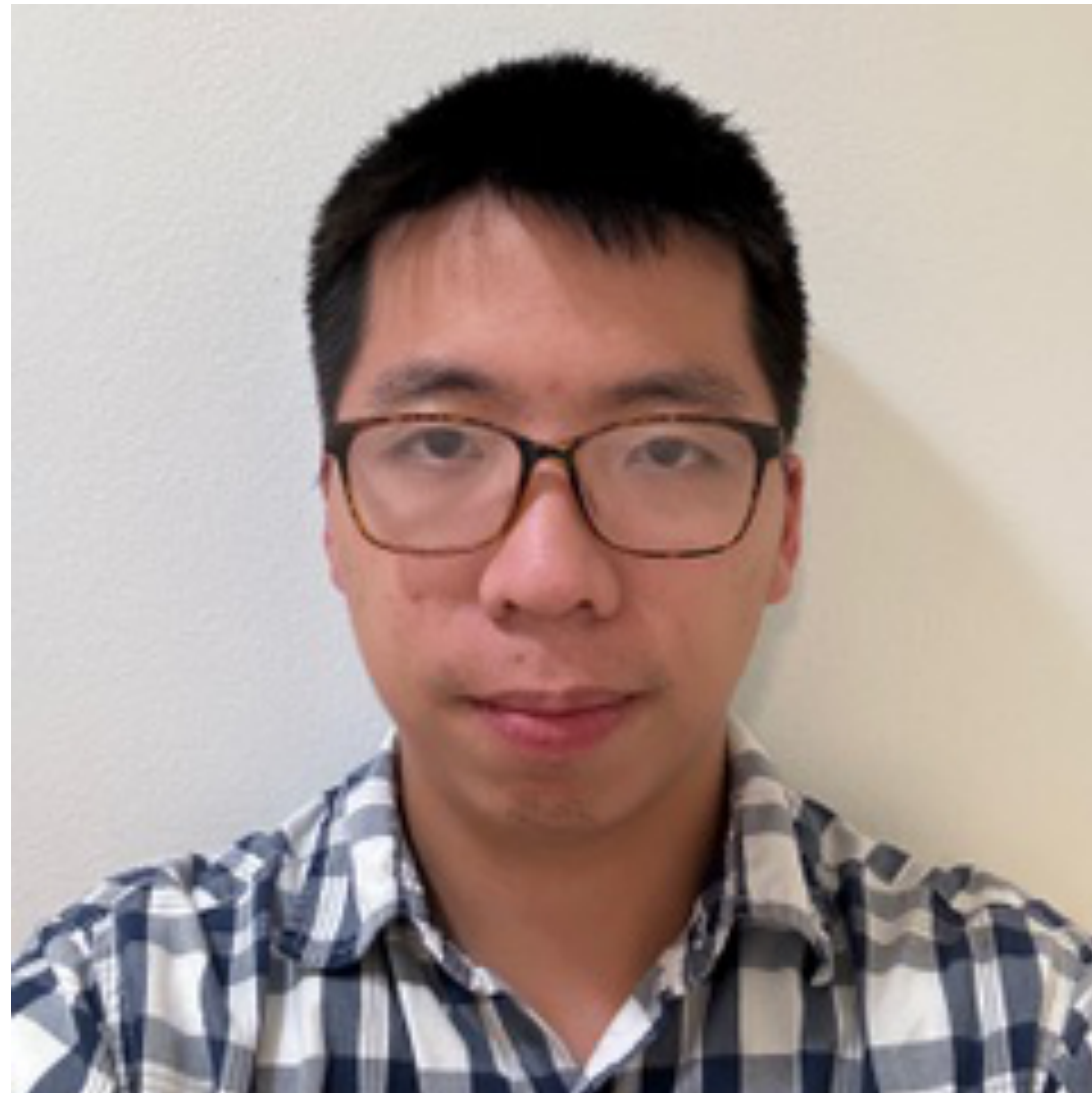
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IJCLab Seminar
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FOR THEORETICAL PHYSICS**

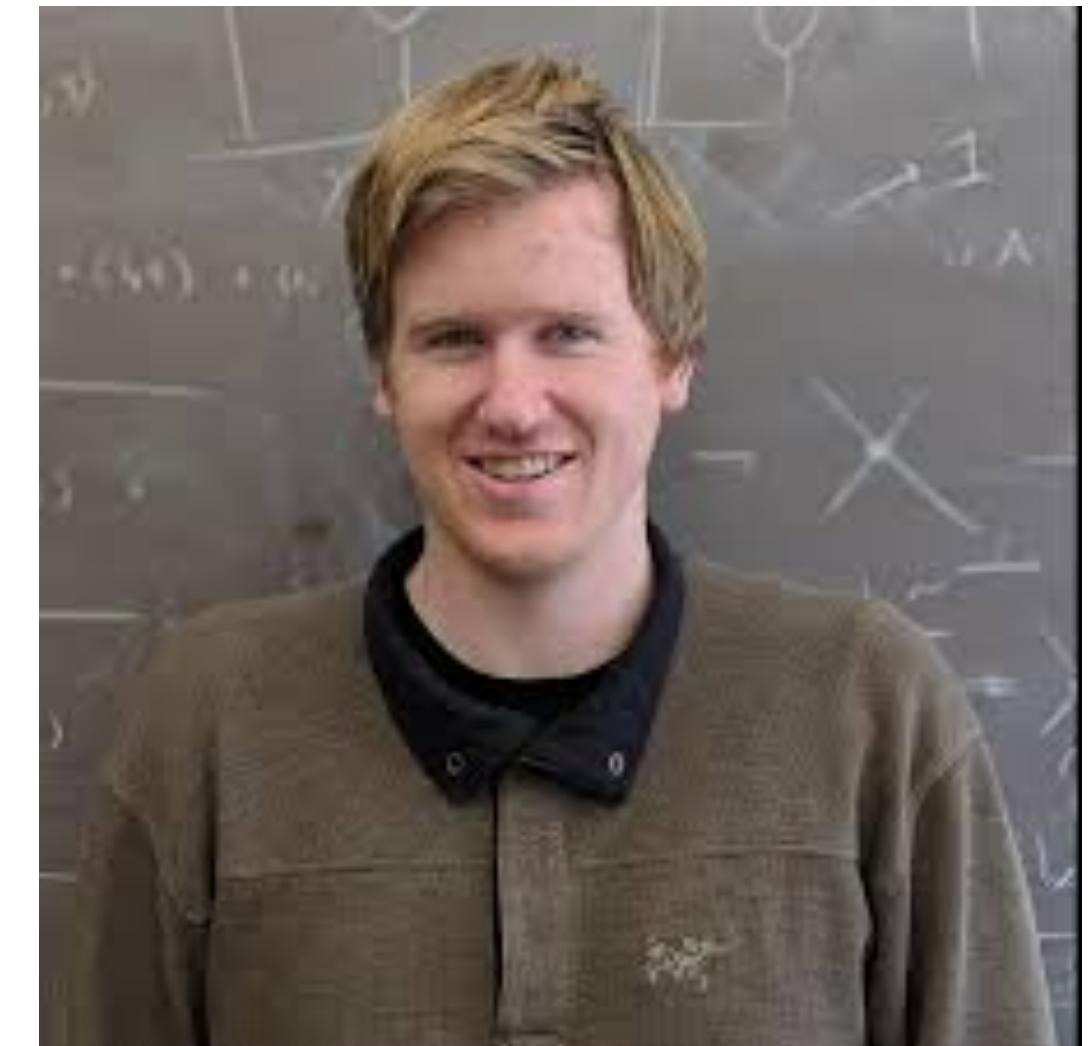
- arXiv:2608.18202 with



Cyuan-Han Chang
(U of Chicago)



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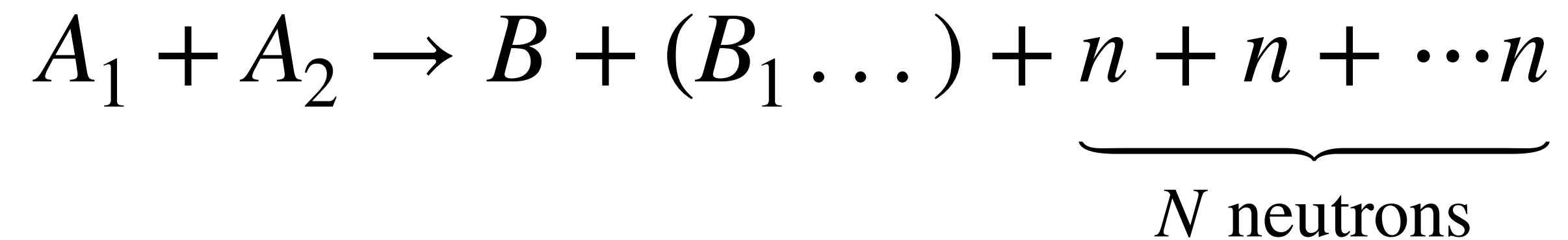
Ian Moul
(Yale University)

Plan

- Motivation
- The problem of point production
- Schrödinger symmetry and nonrelativistic conformal field theory
- Detector operators
- Spectrum of 3 point-produced particles

Motivation

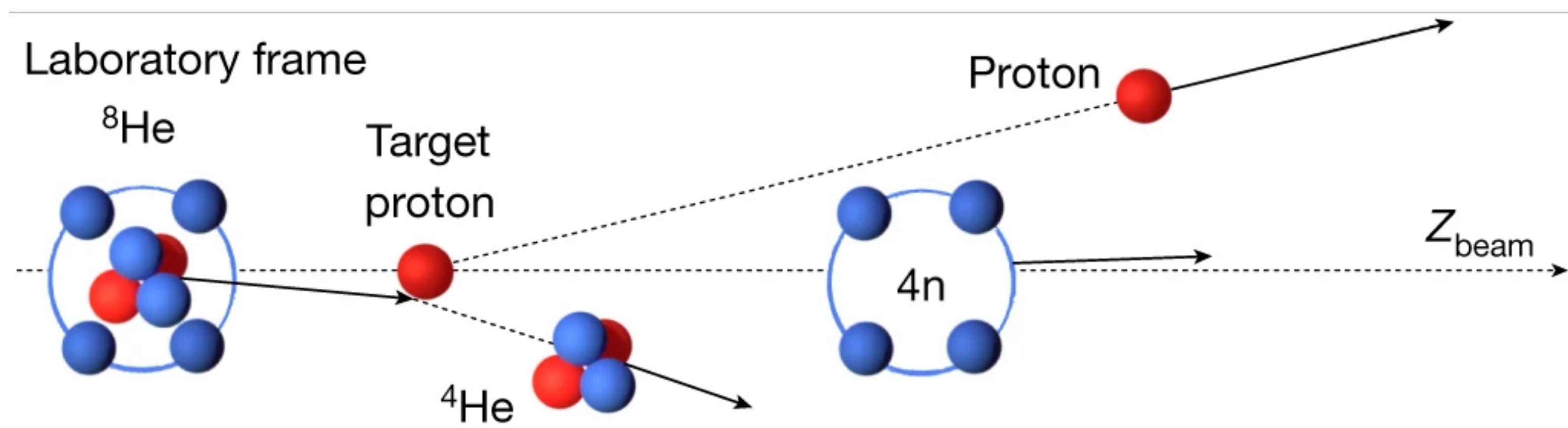
- Many nuclear experiments study reactions



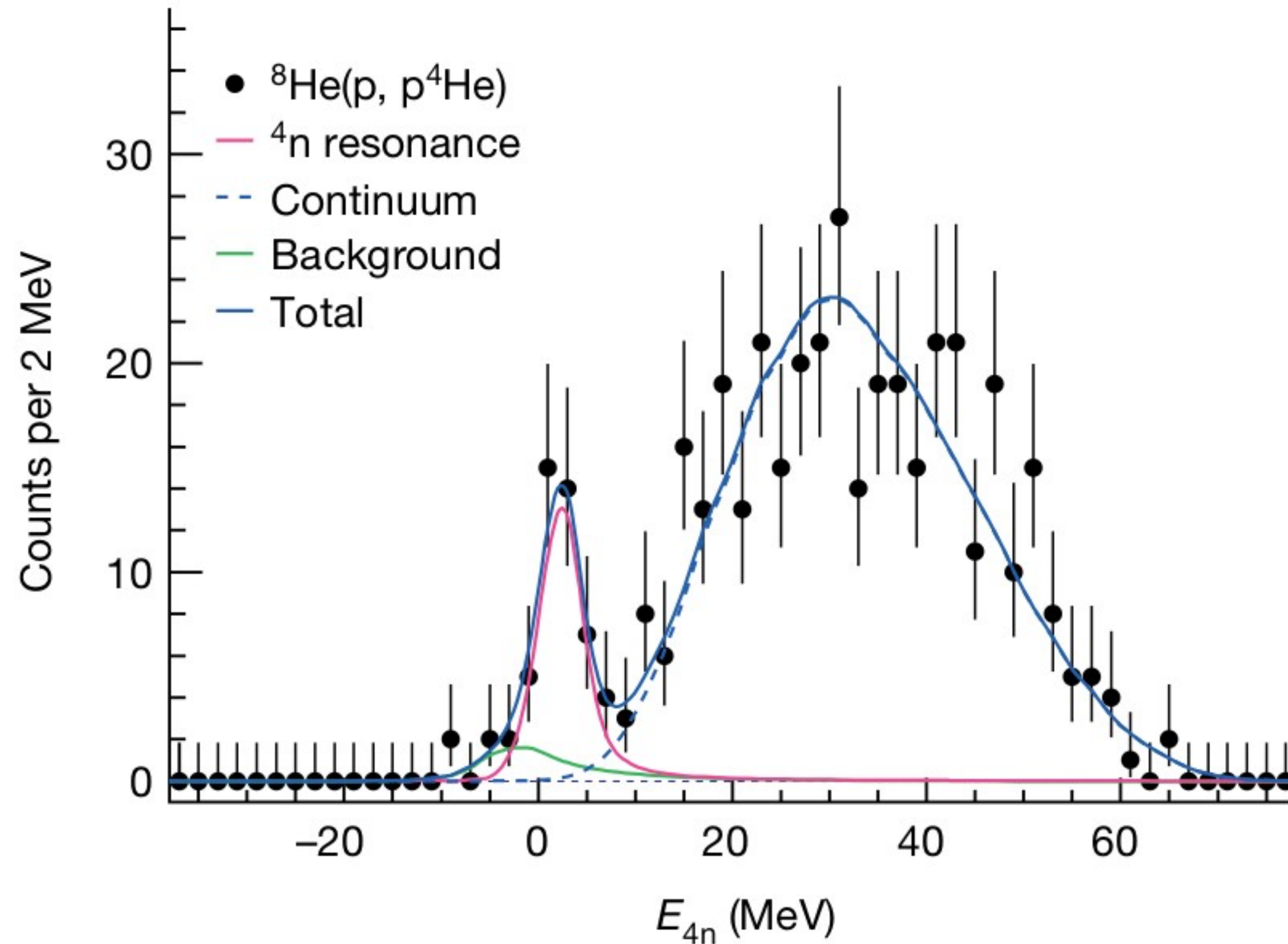
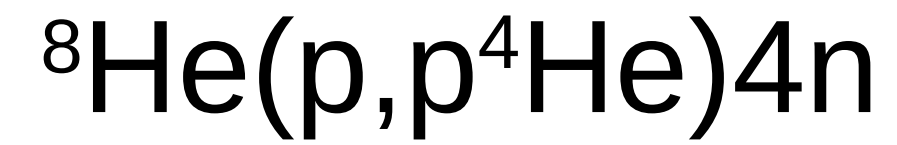
- Neutron-neutron interaction is accidentally fine-tuned

$$|a_{nn}| = 19 \text{ fm} \gg 2 \text{ fm: approximately unitary fermions}$$

- Goal: understanding the effect of neutron-neutron final state interactions



Tetraneutron resonance?



(Duer et al Nature 2022)

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STATE OF NEUTRONS*

Ya. B. ZEL'DOVICH

Submitted to JETP editor October 22, 1959

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The limits of stability (relative to nucleon emission) of light nuclei are considered. The existence (in the sense of stability against decay with emission of a nucleon) of the following nuclei is predicted: He^8 , Be^{12} , O^{13} , $\text{B}^{15,17,19}$, C^{16-20} , N^{18-21} , Mg^{20} . The problem of the possibility of existence of heavy nuclei composed of neutrons only is considered. The problem is reduced to that of a Fermi gas with a resonance interaction between the particles. The energy of such a gas is proportional to $\omega^{2/3}$, where ω is its density. The accuracy of the calculations is not sufficient to determine the sign of the energy and answer the question as to the existence of neutron nuclei.

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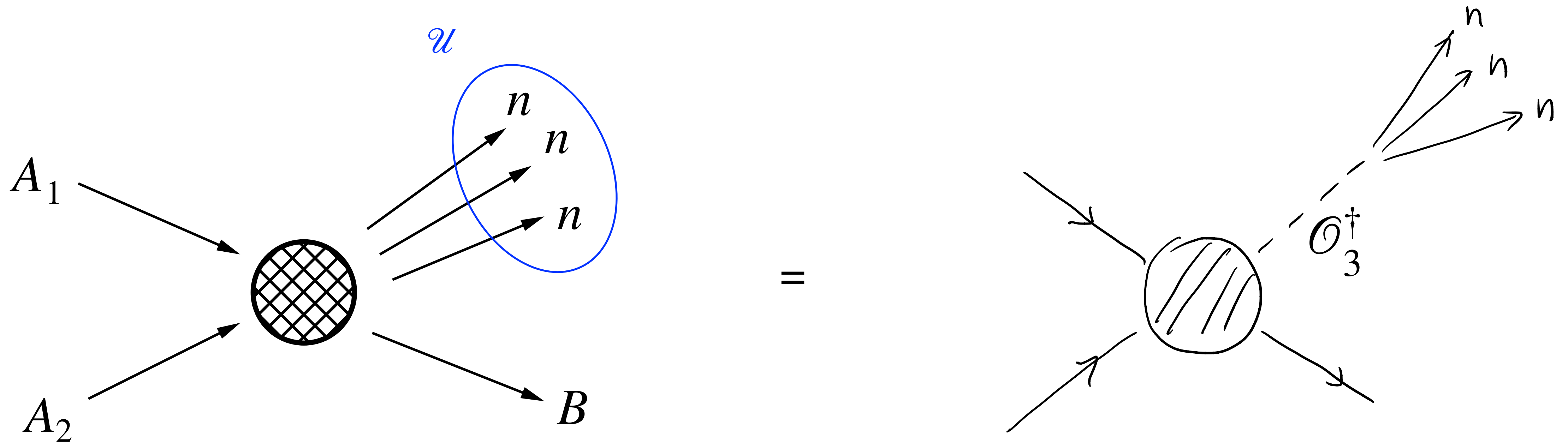
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Point production of neutrons

- Other examples: $\pi^- + {}^4\text{He} \rightarrow p + 3n$
 $\pi^- + {}^4\text{He} \rightarrow \pi^+ + 4n$
- Regime of interest: small neutron relative energy
wavelengths of the neutrons $\gg$ size of reaction region (~ 2 fm)

Factorization



$$\mathcal{A}(A_1 + A_2 \rightarrow B + nnn) = \mathcal{A}(A_1 + A_2 \rightarrow B + \mathcal{O}_3) \mathcal{A}(\mathcal{O}_3 \rightarrow 3n)$$

$$E_{3n} \ll E_{\text{primary}}$$

The problem point production

- Take a local operator $\mathcal{O}_N^\dagger$

act on the vacuum $\int dt d\vec{x} e^{iEt} \mathcal{O}_N^\dagger(t, \vec{x}) |0\rangle$

- measure final particles. Questions:
 - total rate
 - spectrum and correlations between final particles

Conformal collider physics

- A lot of recent progress in formalizing collider physics, jet observables etc
- Key is the energy flow operator

$$\mathcal{E}(\mathbf{n}) = \lim_{r \rightarrow \infty} r^2 \int_{-\infty}^{\infty} dt n^i T^0_i(t, r\mathbf{n})$$

Sterman 1975, Sterman Oderda Korchemsky 1997, Hofman-Maldacena 2008

- Measurements on asymptotic out-state as correlation function
- Simplest in conformal field theory

Schrödinger symmetry and nonrelativistic conformal field theory

Symmetries of NR field theory

$$S = \int dt d^d x \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla \psi|^2}{2m} \right) - \frac{1}{2} \int dt d^d x d^d y V(\vec{x} - \vec{y}) \psi^\dagger(\vec{x}) \psi^\dagger(\vec{y}) \psi(\vec{y}) \psi(\vec{x})$$

- Translation: H, P_i $\psi(t, \vec{x}) \rightarrow \psi(t + a, \vec{x} + \vec{b})$
rotation: M_{ij} $\psi(t, x^i) \rightarrow \psi(t, O^i_j x^j)$
- U(1) symmetry: $\psi \rightarrow e^{i\alpha} \psi$
- Galilean boosts: $\psi(t, \vec{x}) \rightarrow \exp\left(im\vec{v} \cdot \vec{x} - \frac{i}{2}mv^2t\right) \psi(t, \vec{x} - \vec{v}t)$

Galilean algebra

- An algebra formed by H, P_i, J_{ij}, G_i, M

$$[J_{ij}, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i) \quad [J_{ij}, G_k] = i(\delta_{ik}G_j - \delta_{jk}G_i)$$

$$[J_{ij}, J_{kl}] = i(\delta_{ik}J_{jl} - \delta_{jk}J_{il} + \delta_{il}J_{kj} - \delta_{jl}J_{ik})$$

$$[G_i, P_j] = iM\delta_{ij} = imN\delta_{ij}$$

$$[H, G_i] = -iP_i$$

- All other commutators are zero

Additional symmetries of free field

$$S = \int dt d^d x \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla \psi|^2}{2m} \right)$$

- Scale D : $\psi(t, \vec{x}) \rightarrow \lambda^{d/2} \psi(\lambda^2 t, \lambda \vec{x})$
- “Special conformal transformation”

$$C: \quad \psi(t, \vec{x}) \rightarrow \frac{1}{(1 + \lambda t)^{d/2}} \exp \left[\frac{imx^2}{2(1 + \lambda t)} \right] \psi \left(\frac{t}{1 + \lambda t}, \frac{\vec{x}}{1 + \lambda t} \right)$$

The Schrödinger algebra

- $[D, P_i] = iP_i$ $[D, G_i] = -iG_i$ $[D, \mathcal{O}] = i\Delta_{\mathcal{O}}\mathcal{O}$

- $[D, H] = 2iH$
 $[D, C] = -2iC$
 $[C, H] = iD$

SO(2,1) algebra

Current algebra and Schrödinger algebra

- mass density $\rho(\vec{x}) = m \psi^\dagger(\vec{x})\psi(\vec{x})$
- mass current (momentum density) $\vec{\pi} = -\frac{i}{2}(\psi^\dagger \vec{\nabla} \psi - \vec{\nabla} \psi^\dagger \psi)$
- $[\rho(\vec{x}), \vec{\pi}(\vec{y})] = -i\rho(\vec{y}) \vec{\nabla} \delta(\vec{x} - \vec{y})$
 $[\pi_i(\vec{x}), \pi_j(\vec{y})] = -i(\pi_j(\vec{x})\partial_i + \pi_i(\vec{y})\partial_j)\delta(\vec{x} - \vec{y})$ diffeomorphisms

Current algebra and Schrödinger algebra

- Schrödinger algebra can be obtained from

$$M = \int d^d x \rho(\vec{x}) \quad P_i = \int d^d x \pi_i(\vec{x}) \quad J_{ij} = \int d\vec{x} [x_i \pi_j(\vec{x}) - x_j \pi_i(\vec{x})]$$

$$G^i = \int d^d x x^i \rho(\vec{x}) \quad D = \int d^d x x^i \pi_i(\vec{x}) \quad C = \int d\vec{x} \frac{\vec{x}^2}{2} \rho(\vec{x})$$

and $[H, \rho(\vec{x})] = i\partial_i \pi^i(\vec{x})$ continuity eq.

$$[D, H] = 2iH$$

Interacting theories with Schrödinger symmetry

- Theories with $V(\vec{x} - \vec{y}) = \frac{\alpha}{|\vec{x} - \vec{y}|^2}$
- (2+1)d anyons: multivalued wavefunction, boundary condition

$$\psi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n) = \prod_{ij} \frac{(z_i - z_j)^\nu}{|z_i - z_j|^\nu} \underbrace{\phi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n)}_{\text{single valued}}$$

$$H = -\frac{1}{2m} \sum_a \nabla_{x_a}^2$$

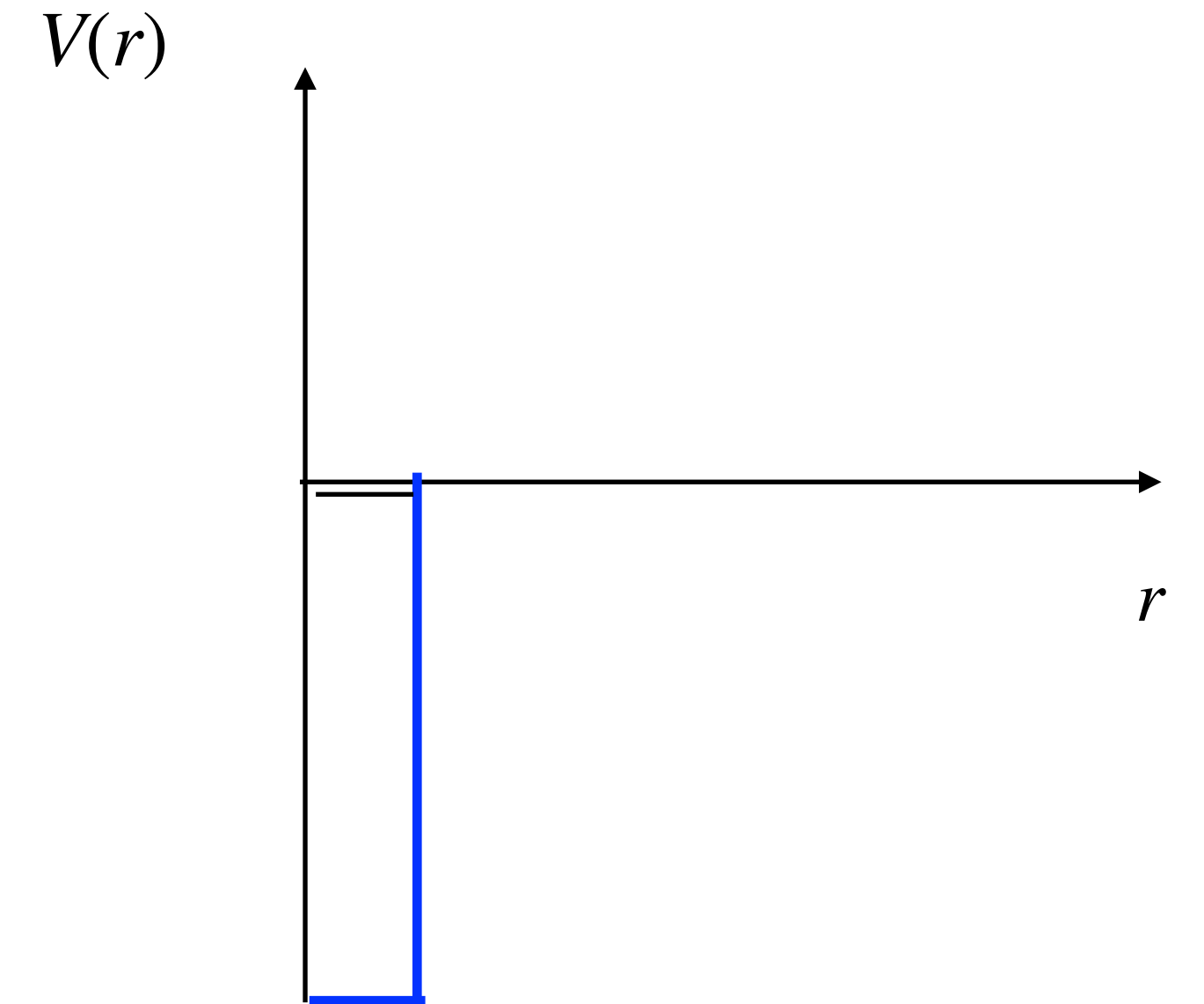
(3+1)D: “Unitary interaction”

- $$V(r) = \begin{cases} -V_0 & r < r_0 \\ 0 & r > r_0 \end{cases}$$

- $V_0 = \frac{\pi^2}{4mr_0^2}$: one bound state at zero energy

- $r_0 \rightarrow 0, V_0 \rightarrow \infty$ limit: scattering length $a \rightarrow \infty$

“unitary limit”



Unitary fermion: boundary condition

- Wave function $\psi(\underbrace{\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m}_{\text{up-spin}}, \underbrace{\vec{y}_1, \vec{y}_2, \dots, \vec{y}_n}_{\text{down-spin}})$
- antisymmetric with respect to exchanging two x 's or two y 's
- When one spin-up and one spin-down particle approach each other:

$$\psi(\dots \vec{x}_i \dots, \dots \vec{y}_j \dots) = \frac{1}{|\vec{x}_i - \vec{y}_j|} C\left(\frac{\vec{x}_i + \vec{y}_j}{2}, \dots\right) + O(\vec{x}_i - \vec{y}_j)$$

- $H = -\frac{1}{2m} \left(\sum \nabla_{x_i}^2 + \sum \nabla_{y_j}^2 \right) \quad \vec{x}_i \neq \vec{y}_j$

Examples of local operators

- Charge-1 operator: $\psi(\mathbf{x}) \quad \Delta_\psi = 3/2$
- Charge-2 operator: naively $\psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{x})$ but

$$\langle 0 | \psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{x}) | \Psi \rangle = \Psi(\mathbf{x}, \mathbf{x}) = \infty$$

- Correct definition of charge-2 operator $O_2(\mathbf{x}) = \lim_{\mathbf{y} \rightarrow \mathbf{x}} |\mathbf{x} - \mathbf{y}| \psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{y})$

$$\text{now } \langle 0 | O_2(\mathbf{x}) | \Psi \rangle = \lim_{\mathbf{y} \rightarrow \mathbf{x}} |\mathbf{x} - \mathbf{y}| \Psi(\mathbf{x}, \mathbf{y}) \quad \Delta[O_2] = 2\Delta[\psi] - 1 = 2$$

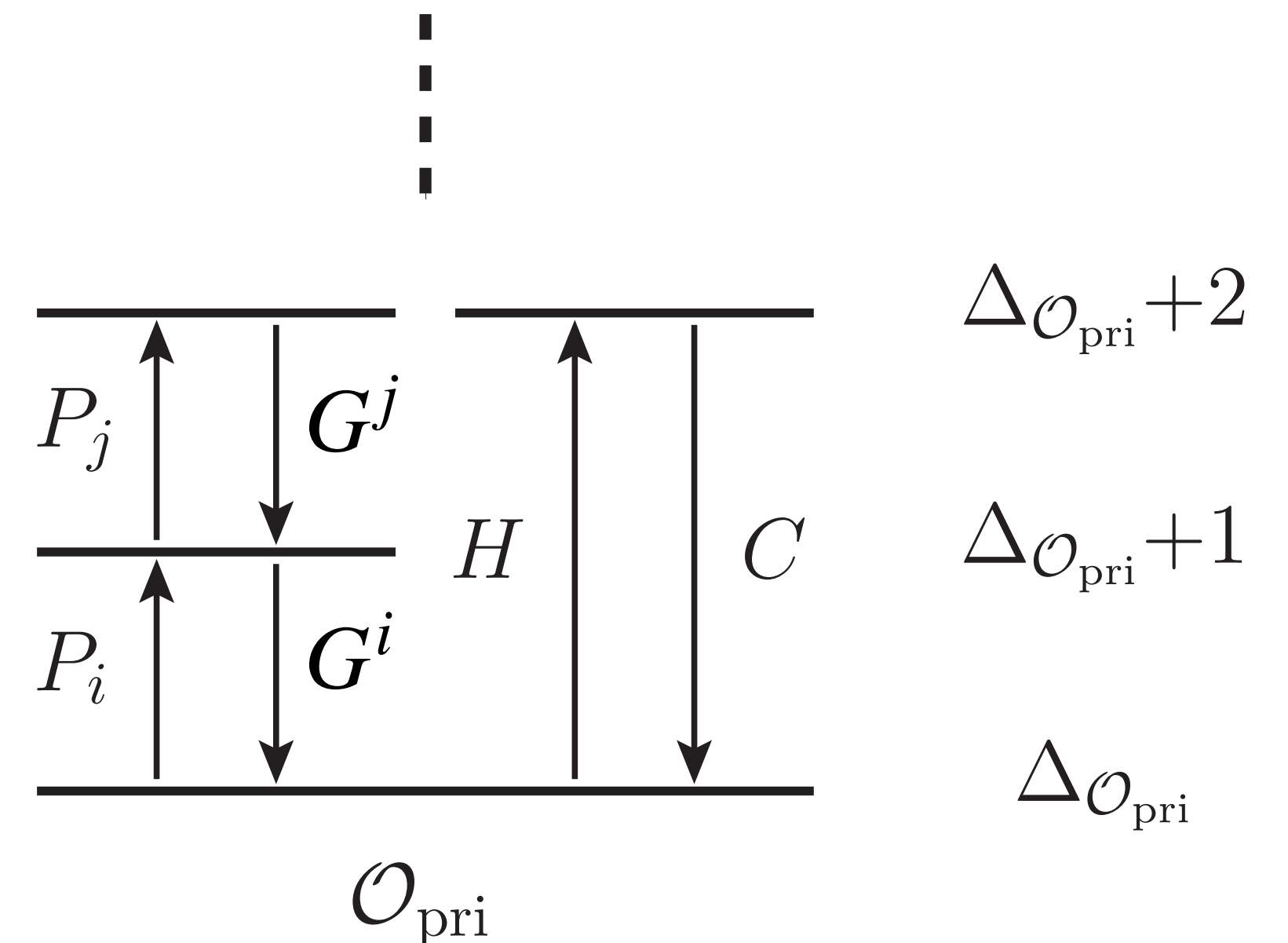
Local operators

- Local operators are classified by mass and dimension:

$$[M, \mathcal{O}^\dagger(\vec{0})] = m_{\mathcal{O}} \mathcal{O}^\dagger(\vec{0}) \quad [D, \mathcal{O}(\vec{0})] = i\Delta_{\mathcal{O}} \mathcal{O}(\vec{0})$$

- Massive operators ($m \neq 0$) form ladders
- Lowest rungs of the ladders: primary operators

$$[G^i, \mathcal{O}(\vec{0})] = [C, \mathcal{O}(\vec{0})] = 0$$



Operator-state correspondence

- Dimension of (lowest dimensional) operator of charge N $O_N(\vec{x})$ is the (ground state) energy of N particles in a harmonic trap in unit of trap frequency

$$O_N(\vec{x})$$
$$[D, O_N(\vec{0})] = \Delta O_N(\vec{0})$$

$$|O_N\rangle$$
$$(H + \omega^2 C) |O_N\rangle = \Delta\omega |O_N\rangle$$

$$C = \frac{1}{2} \int d\vec{x} x^2 \psi^\dagger \psi$$

Lowest operators

-

$$\psi(\vec{x}) \quad \Delta[\psi] = 3/2 \quad [D, \psi(\vec{0})] = \frac{3}{2}i\psi(\vec{0})$$

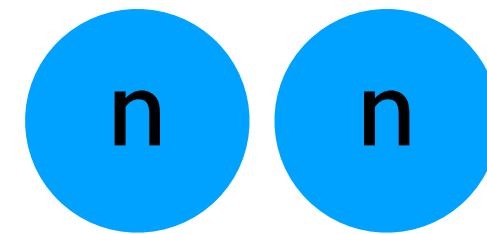
$$O_2(\vec{x}) = \lim_{\vec{y} \rightarrow \vec{x}} |\vec{x} - \vec{y}| \psi_{\uparrow}(\vec{x}) \psi_{\downarrow}(\vec{y}) \quad \Delta[O_2] = 2$$

$$O_3(\vec{x}) \quad \Delta[O_3^{\ell=1}] \approx 4.27272, 6.85825, 8.21643, \dots$$

$$\Delta[O_3^{\ell=0}] \approx 4.66622, 7.62735, 9.61446, \dots$$

V. Efimov 1970

Neutrons: approximate unitary fermions



- $a \approx -19$ fm, $r_0 \approx 2.8$ fm

approximate NRCFT within energy range $\underbrace{\frac{1}{ma_0^2}}_{0.1 \text{ MeV}} \ll E \ll \underbrace{\frac{1}{mr_0^2}}_{5 \text{ MeV}}$

Total rate

Hammer, DTS 2021

- $|O_N, E\rangle = \int dt d\vec{x} e^{iEt} O_N^\dagger(t, \vec{x}) |0\rangle \equiv O_N^\dagger(E) |0\rangle$
- $\langle O_N, E | O_N, E \rangle = \sum_n |\langle n | O_N, E \rangle|^2 \sim E^{\Delta[O_N]-5/2}$
- “Wigner threshold law”
- Momentum distribution of final particles?

Detector operators

- The problem of determining spectrum of final particles can be formalized by defining the *detector operator*

$$\mathcal{M}(\mathbf{v}) = \lim_{t \rightarrow \infty} t^3 \rho(t, \mathbf{v}t)$$

which counts the number of final-state particles moving with velocity $\mathbf{v}$, e.g.,

$$m \frac{dN}{d\mathbf{v}} = \langle O_N, E | \mathcal{M}(\mathbf{v}) | O_N, E \rangle$$

- Formalizing the notion of a detector

Interpretation

- Particles detected by a detector receding with velocity $\mathbf{v}$ to infinity
- In relativistic conformal collider physics, detector operators are functions of $\hat{\mathbf{n}}$: $\mathcal{E}(\mathbf{n}) = \lim_{r \rightarrow \infty} r^2 \int_{-\infty}^{\infty} dt n^i T^0_i(t, r\mathbf{n})$
- In nonrelativistic conformal collider physics, they are functions of $\mathbf{v} = v\mathbf{n}$
 - final particles have arbitrary speed

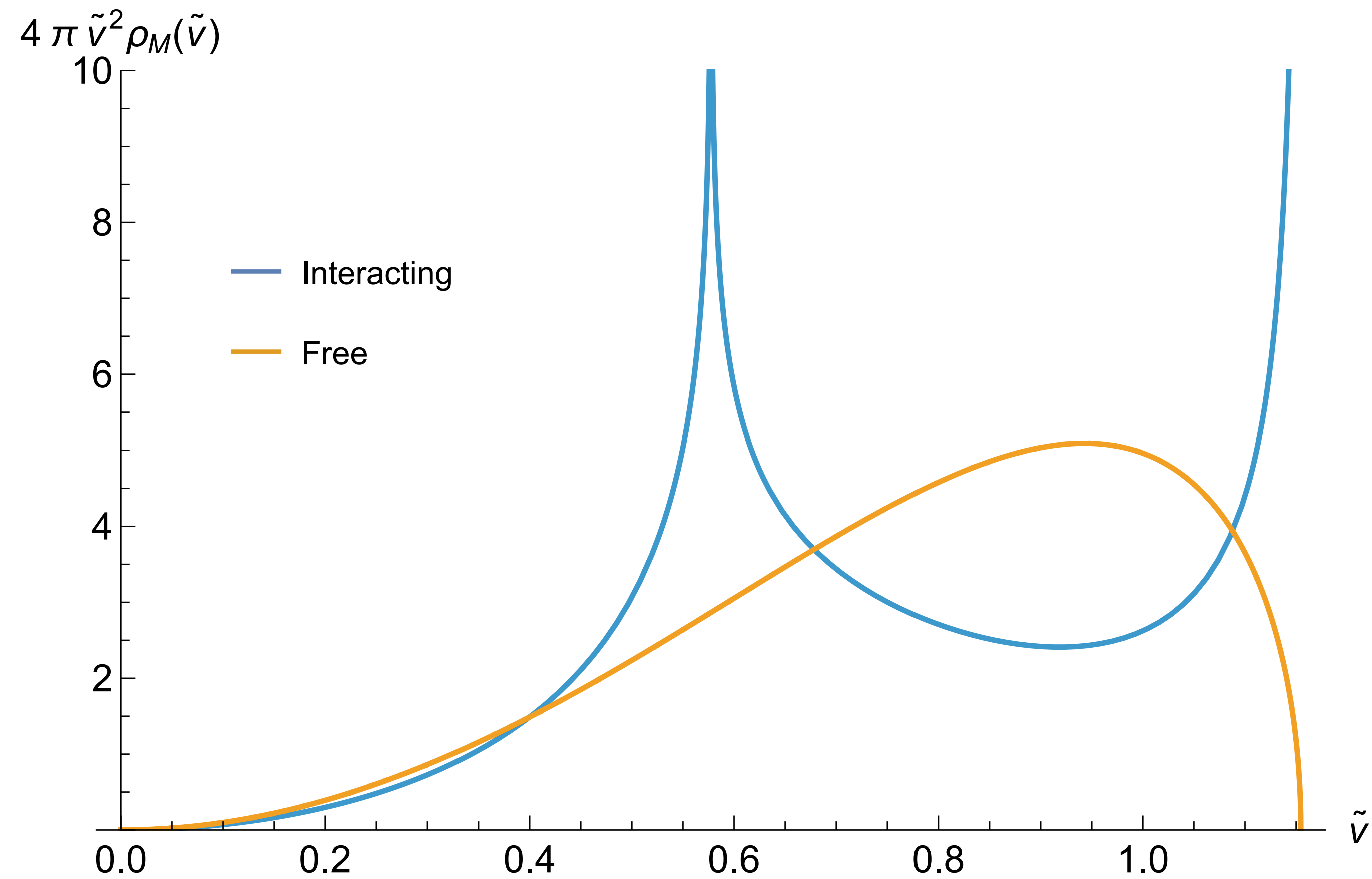
Sum rules (“Ward identities”)

- Total mass of final particles: $Nm = \int d\mathbf{v} \mathcal{M}(\mathbf{v})$
- Total energy: $E = \int d\mathbf{v} \mathcal{M}(\mathbf{v}) \frac{v^2}{2}$

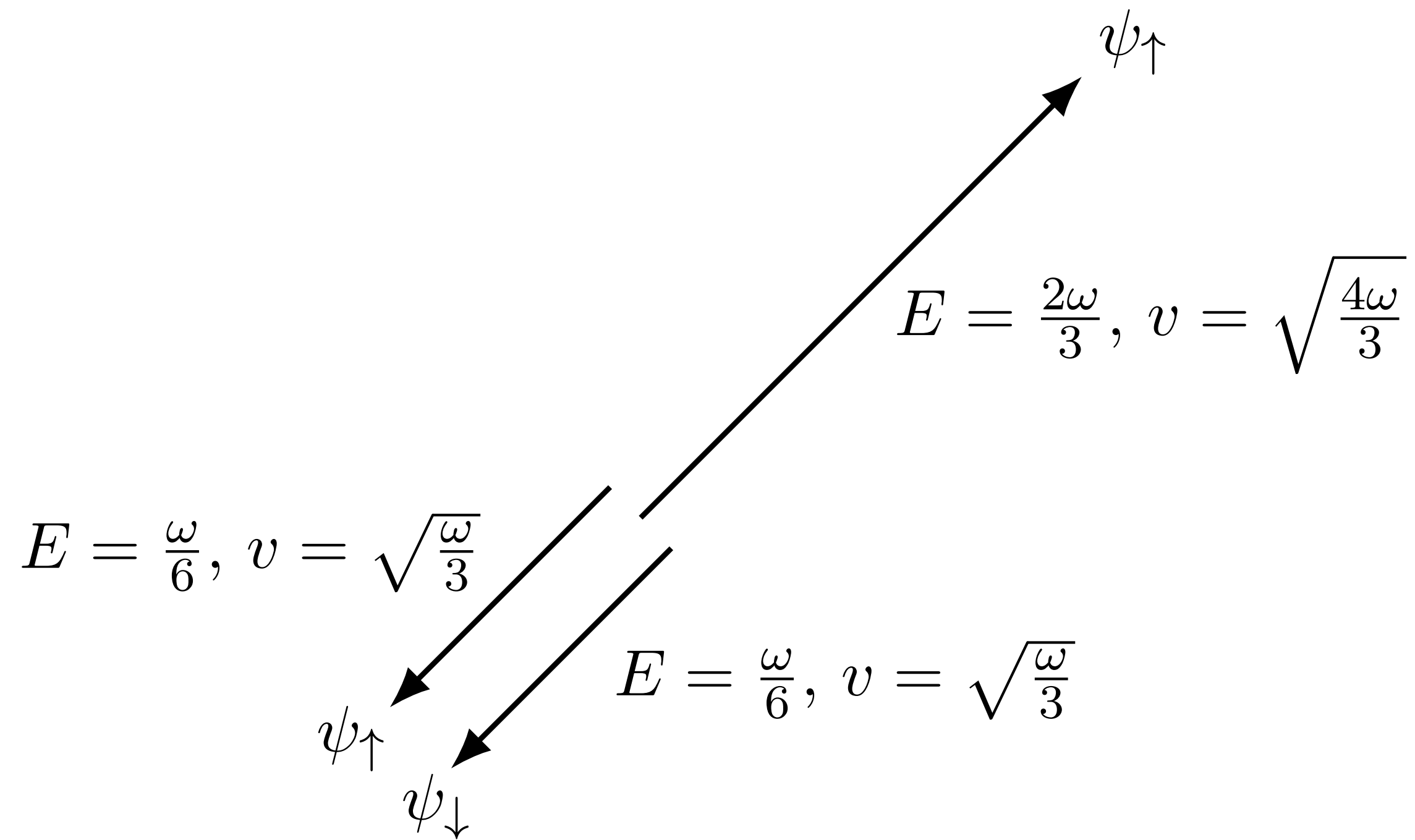
Computing spectrum of final particles

- Start from $m \frac{dN}{d\mathbf{v}} = \langle O_N, E | \mathcal{M}(\mathbf{v}) | O_N, E \rangle$
- Insert complete set of N -body states: can be done exactly for $N = 3$
- For free particles: result can be reproduced by phase space integral

Spectrum for 3 particles



Origin of singularities



$$\rho(v) \sim \frac{1}{\sqrt{\frac{4\omega}{3} - v^2}}, \quad v \rightarrow \sqrt{\frac{4\omega}{3}}.$$

$$\rho(v) \sim \log\left(\frac{\omega}{3} - v^2\right), \quad v \rightarrow \sqrt{\frac{\omega}{3}}.$$

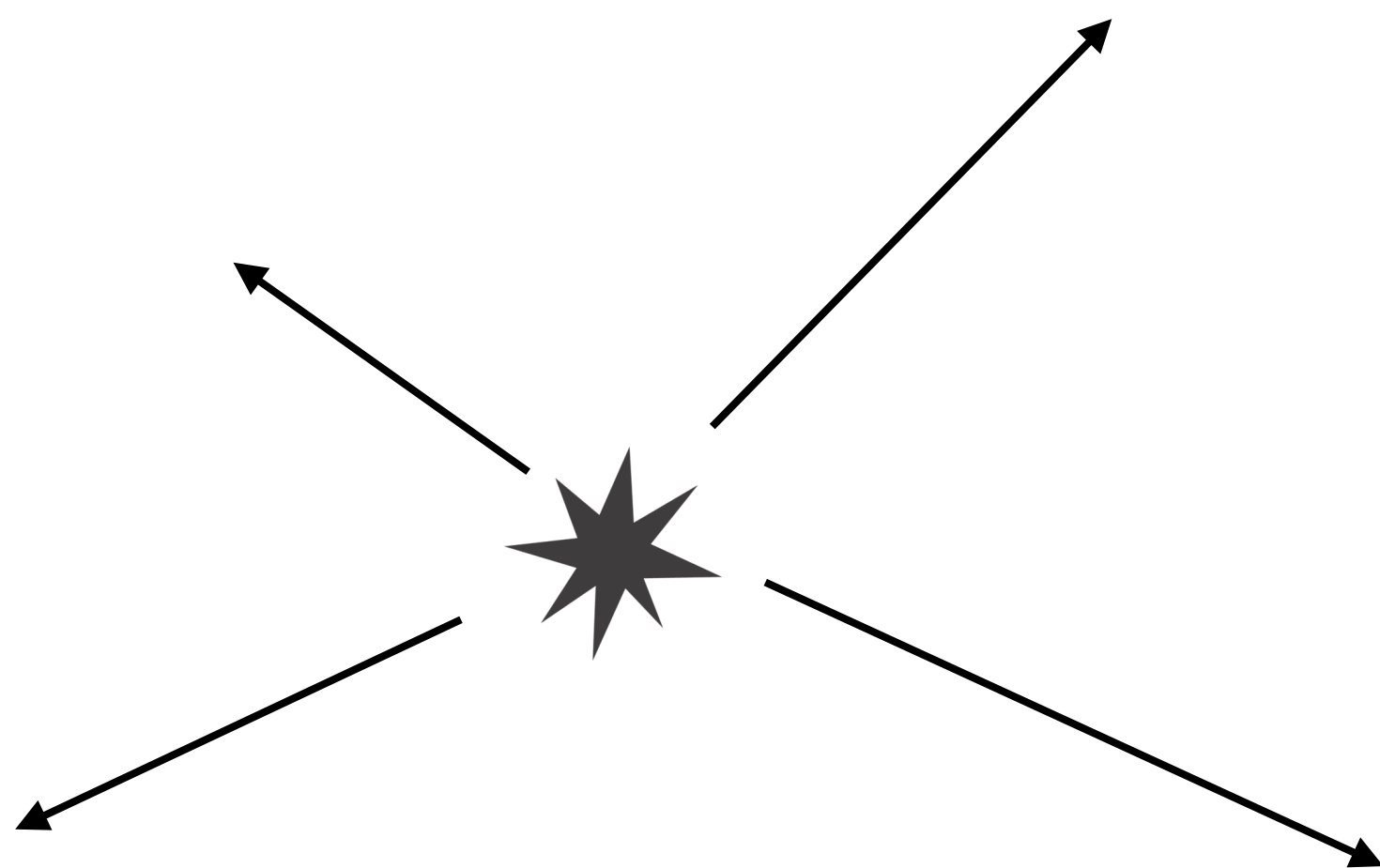
Oscillator frame

- $t = \tan \tau \quad \vec{x}_a = \frac{\vec{y}_a}{\cos \tau} \quad \begin{array}{l} 0 < t < \infty \\ 0 < \tau < \pi/2 \end{array}$

$$\psi(t, \vec{x}_a) = \cos^{3N/2} \tau \exp\left[\frac{i}{2} m \tan \tau \vec{y}_a^2\right] \tilde{\psi}(\tau, \vec{y}_a)$$

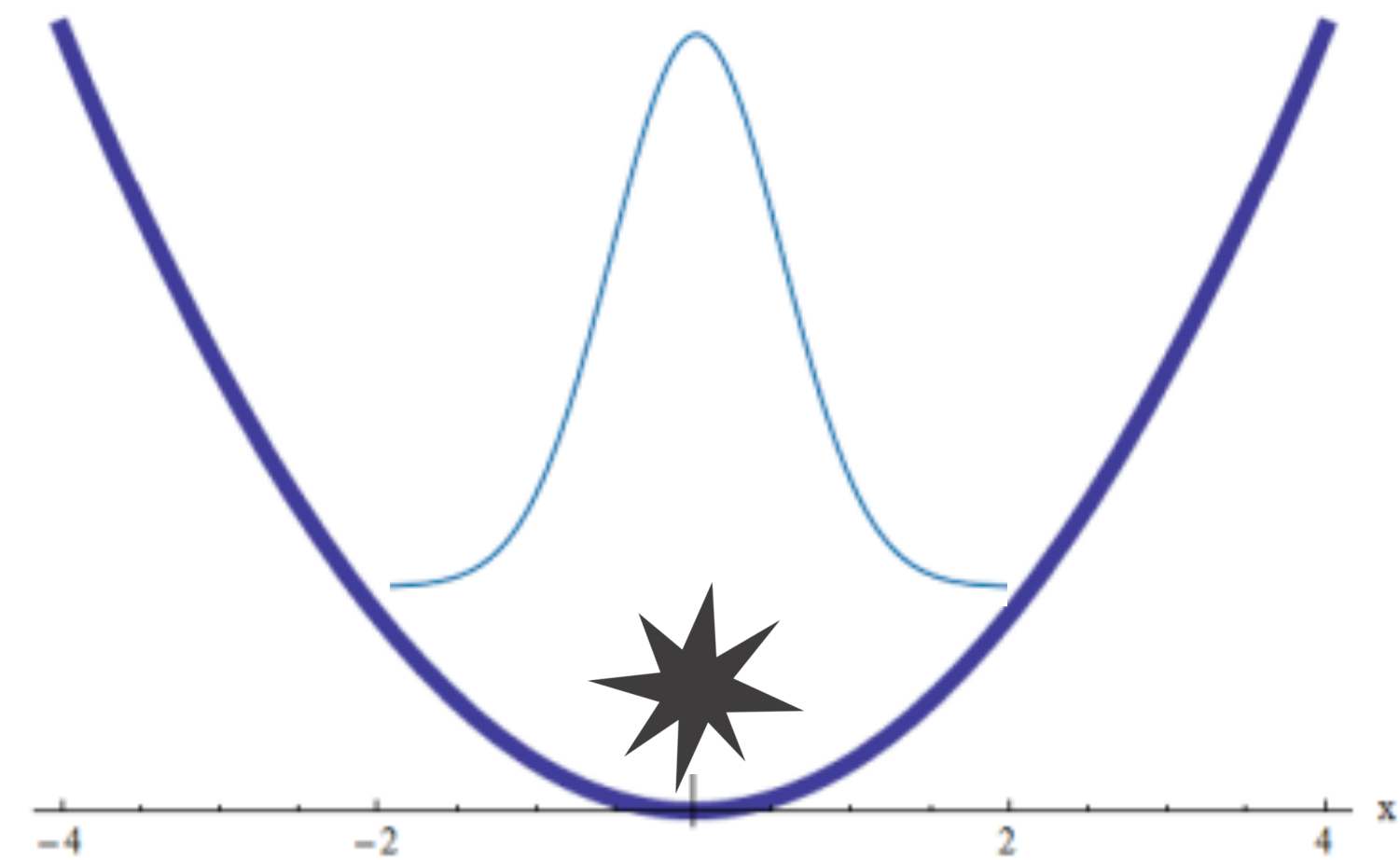
- if $\psi(t, \vec{x})$ satisfies Schrödinger equation in free space, then $\tilde{\psi}(\tau, \vec{y})$ satisfies Schr. equation in harmonic potential $V = \frac{1}{2} m |\vec{y}|^2$

$$i \frac{\partial \tilde{\psi}}{\partial \tau} = - \frac{1}{2m} \sum_a \frac{\partial^2 \tilde{\psi}}{\partial \vec{y}_a^2} + V(\vec{y}_a) \tilde{\psi}$$



$$e^{-iHt}$$

$$t \rightarrow +\infty$$

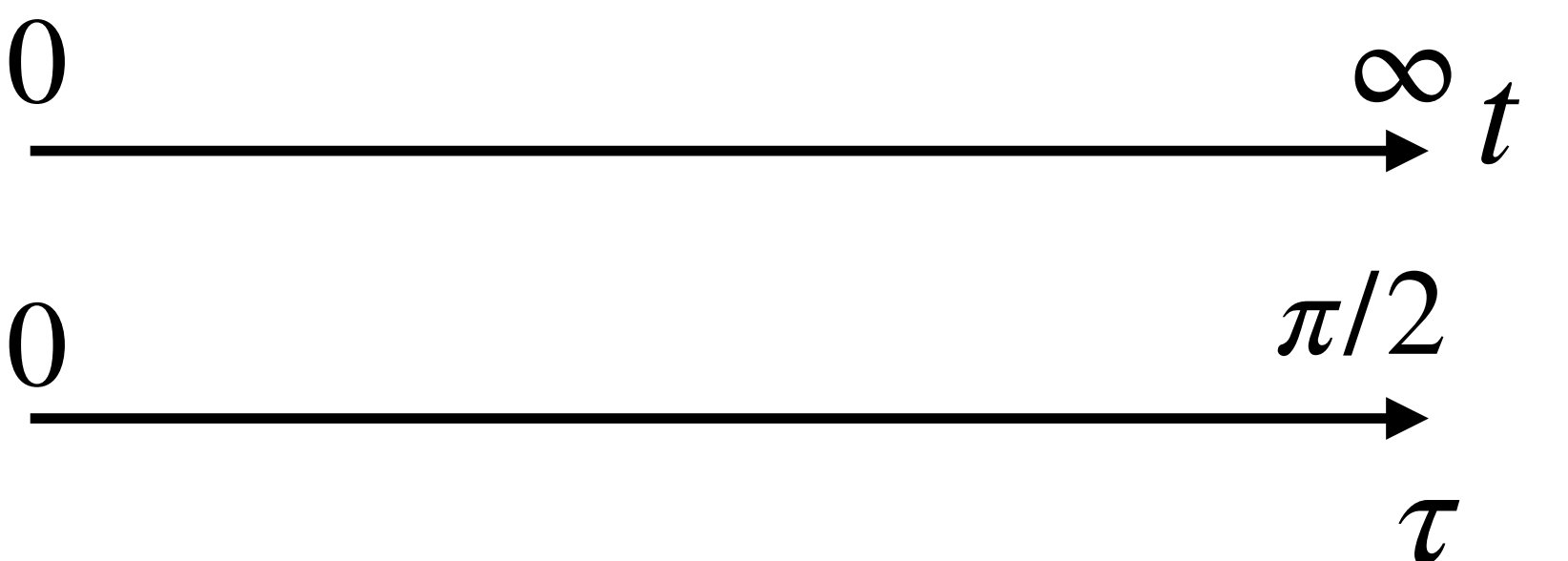


$$e^{-iH_{\text{osc}}\tau}$$

$$\tau \rightarrow +\frac{\pi}{2}$$

“Baby celestial holography”

- Finding the spectrum and correlations of final-state particles is equivalent to finding the properties of a quantum wavefunction

$$\bullet \quad t = \tan \tau \quad \vec{x}_a = \frac{\vec{y}_a}{\cos \tau}$$


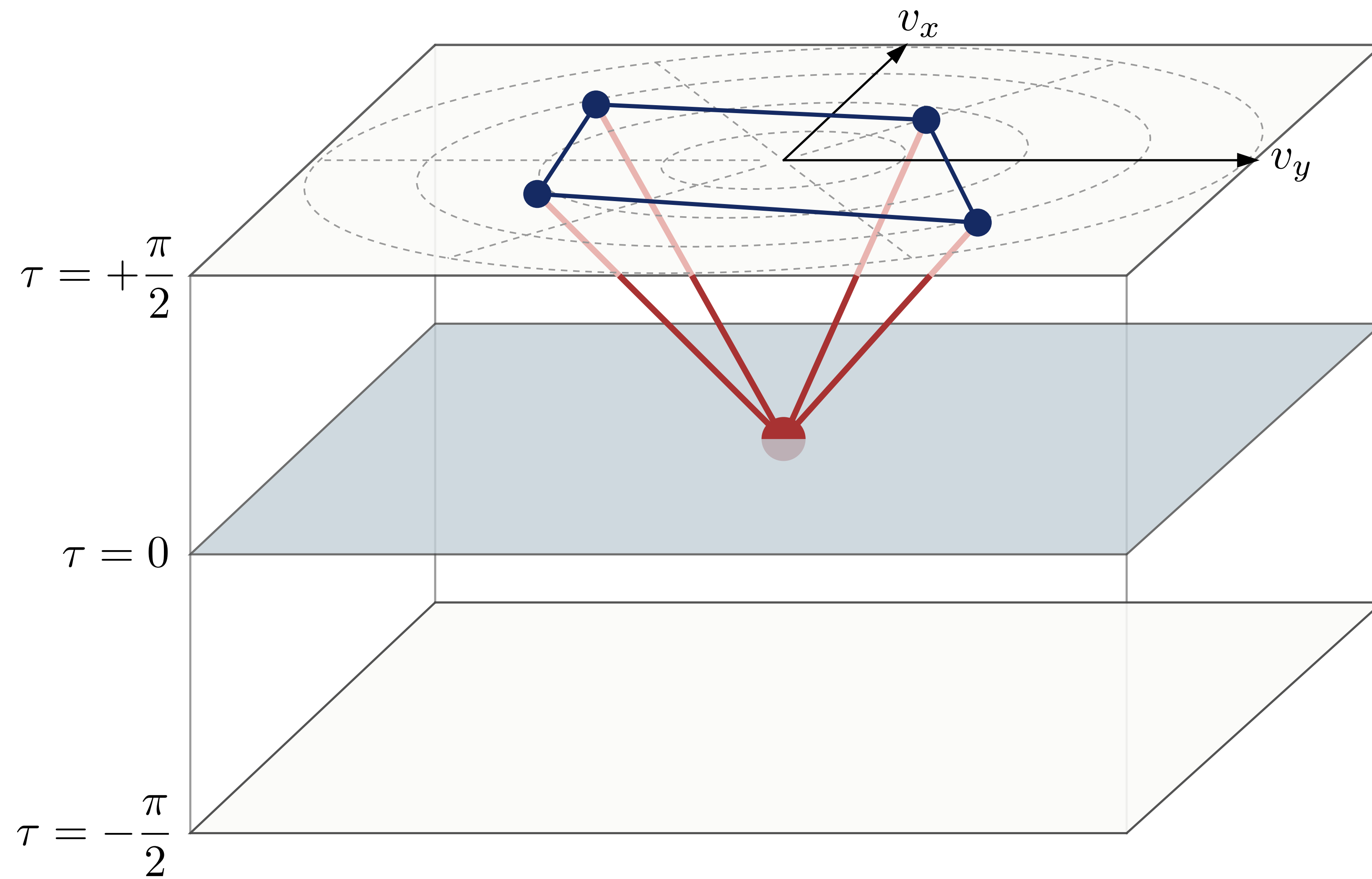
- the detector operators “live” at $t \rightarrow \infty \leftrightarrow \tau = \frac{\pi}{2}$

$$\bullet \quad \mathcal{M}(\mathbf{v}) = m \tilde{\psi}^\dagger \tilde{\psi} \left(\frac{\pi}{2}, \mathbf{v} \right)$$

distribution over momentum

$\rightarrow$ distribution over coordinate

- Express out-state as a state living on a boundary: “celestial holography”



Production of $N \gg 1$ unitarity fermions

- $O_N^\dagger |0\rangle$ is a many-body state: can be described using superfluid effective field theory (superfluid hydrodynamics)

- $\mathcal{L}(\theta) = c \left(\partial_t \theta - \frac{1}{2m} (\vec{\nabla} \theta)^2 \right)^{5/2}$

- $N \gg 1$: path integral is saturated by a complex saddle point

$$\theta_s(\tau, \vec{x}) = \frac{i}{2} \gamma \log \left(\frac{\tau_1 - \tau}{\tau - \tau_2} \right) - \frac{i}{4} m \left[\frac{(\vec{x} - \vec{x}_2)^2}{(\tau - \tau_2)} - \frac{(\vec{x} - \vec{x}_1)^2}{(\tau_1 - \tau)} \right].$$

- $\langle O_N | \mathcal{M}(\mathbf{v}) | O_N \rangle$ computed from this saddle point

Result for $N \gg 1$

- The distribution of particles over v is

$$\mathcal{M}(v) \sim \begin{cases} (v_{\max}^2 - v^2)^{3/2} & v < v_{\max} \\ 0 & v > v_{\max} \end{cases}$$

- $v_{\max} = \sqrt{\frac{16E}{3Nm}}$
- This is also the distribution of particles in harmonic trap in Thomas-Fermi approximation

Compare: production of $N \gg 1$ free bosons

- $\Gamma(\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N) \sim \delta(\mathbf{p}_1 + \mathbf{p}_2 + \dots + \mathbf{p}_N) \delta(p_1^2 + p_2^2 + \dots + p_N^2 - 2mE)$
- $N \gg 1$: microcanonical ensemble $\approx$ canonical ensemble
- Final-state particle Maxwell-distributed:

$$\mathcal{N}(v) \sim v^2 e^{-mv^2/2T_{\text{eff}}}$$

Summary + outlook

- The problem of neutron production in nuclear physics has a lot of common with conformal collider physics in high-energy physics
- Open problem: point production at finite scattering length
 - Universal answer interpolating between free and unitarity fermions
 - can it be studied using ultracold atoms?
- Point production of $N = 4$ neutrons
- How to characterize a finite-size initial state [Lazaukas Hiyama Carbonell PRL 2023](#)
 - inclusion of operators with higher dimensions?

Other detectors

- “Passive” detectors: $[H, \mathcal{M}(\mathbf{v})] = [P^i, \mathcal{M}(\mathbf{v})] = 0$
“Active” detectors: do not commute with energy and momentum
- Example: $\tilde{\mathbf{j}}(\mathbf{v}) = \lim_{t \rightarrow \infty} t^4 [\mathbf{j}(t, \mathbf{v}t) - \rho(t, \mathbf{v}t)\mathbf{v}]$
- $\int d\mathbf{v} \mathbf{v} \times \tilde{\mathbf{j}}(\mathbf{v}) = \text{total angular momentum}$
- can one realize this detector experimentally?
- More “quantum” characteristics of the harmonic-trap wavefunction at $\tau = \pi/2$?