



COMPARATIVE ANALYSIS OF THE DEFLECTION EFFICIENCY OF NEGATIVELY CHARGED PARTICLES BY BENT CRYSTALS IN AXIAL AND PLANAR ORIENTATIONS

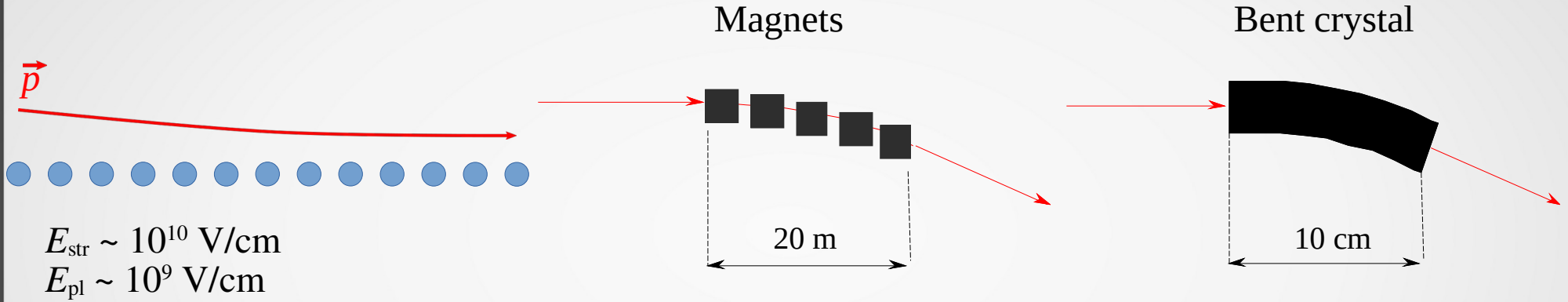
Igor KYRYLLIN

NSC Kharkiv Institute of Physics and Technology

FRENCH-UKRAINIAN WORKSHOP 2026

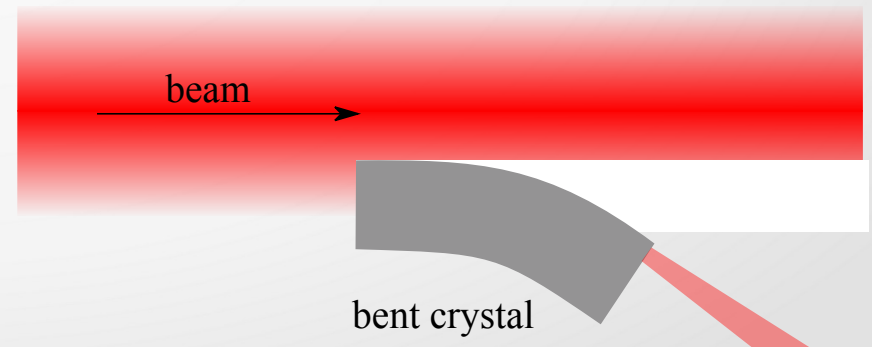
October 12, 2026

Bent crystals and magnetic deflection systems



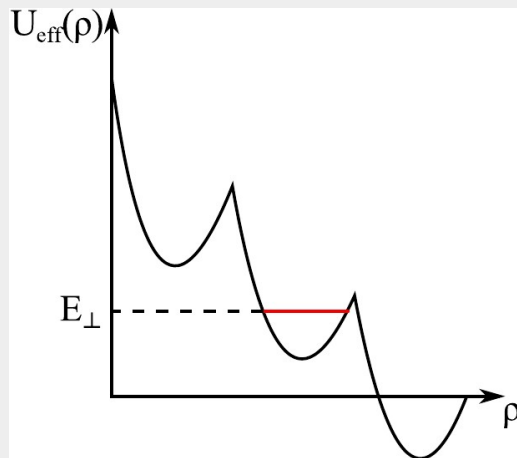
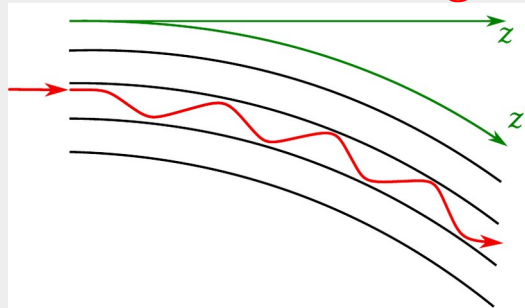
Advantages of bent crystals in comparison with magnetic deflection systems:

- small size
- do not need electricity consumption
- do not need cooling



Mechanisms of deflection

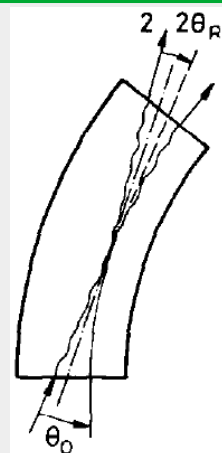
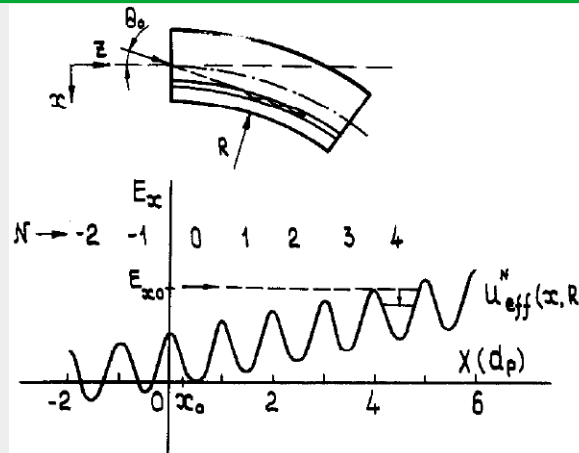
Planar channeling



Tsyganov E. N. Fermilab TM-682, TM-684. 1976.

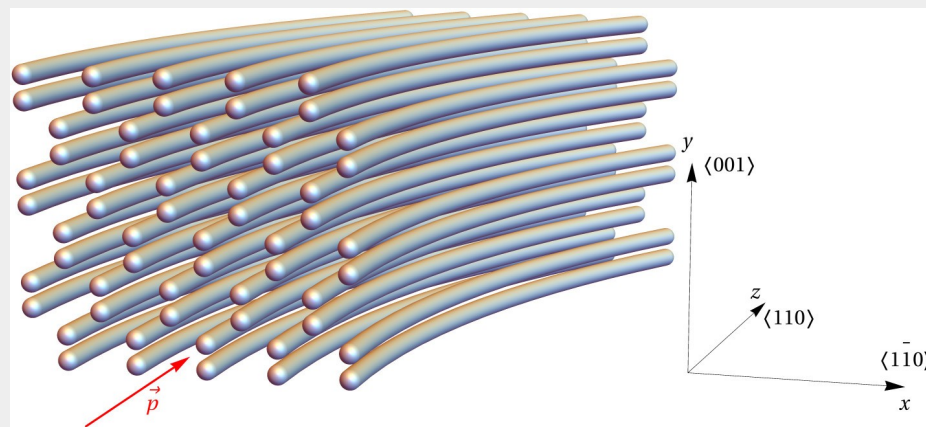
Volume reflection

Taratin A. M., Vorobiev S. A.
Phys. Lett. A. 1986. Vol. 115,
No. 8. P. 398–400.

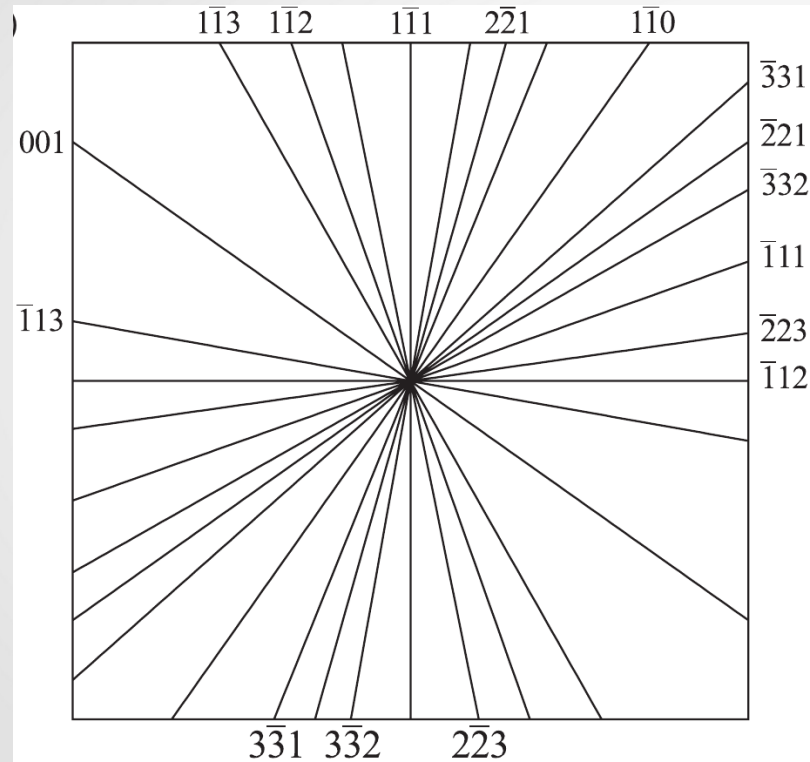


Stochastic deflection

Grinenko A. A., Shul'ga N. F. J.
Exp. Theor. Phys. Lett. 1991.
Vol. 54. P. 524–528.



Stereogram of the most strong atomic planes around the $\langle 110 \rangle$ crystal axis of the silicon crystal



$$R_{(klm)} = R_{(1\bar{1}1)} / \cos(\beta)$$

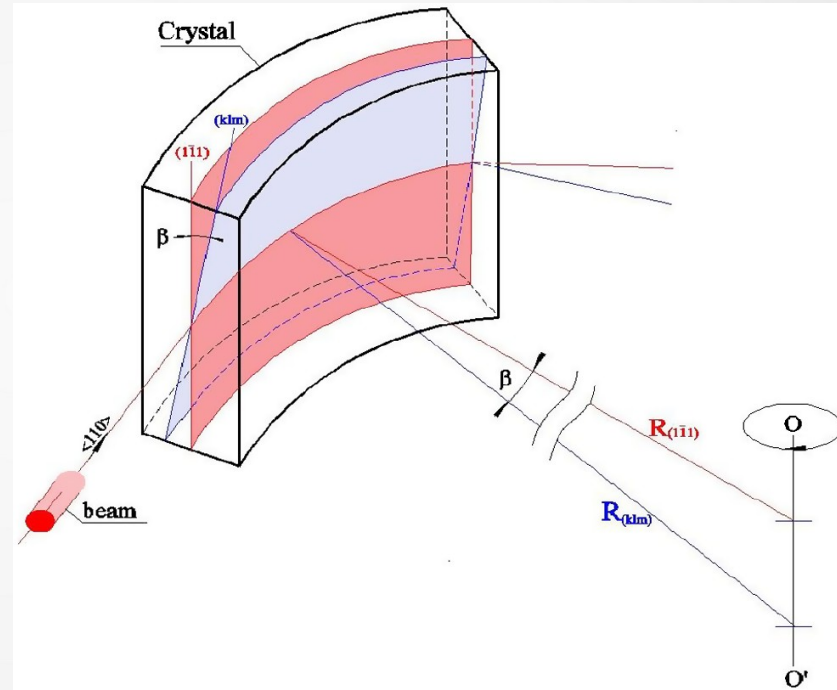


FIG. 9. Beam deflection by the (klm) plane.

Beam profiles for different angles of the goniometer corresponding to the deflection by different crystal planes

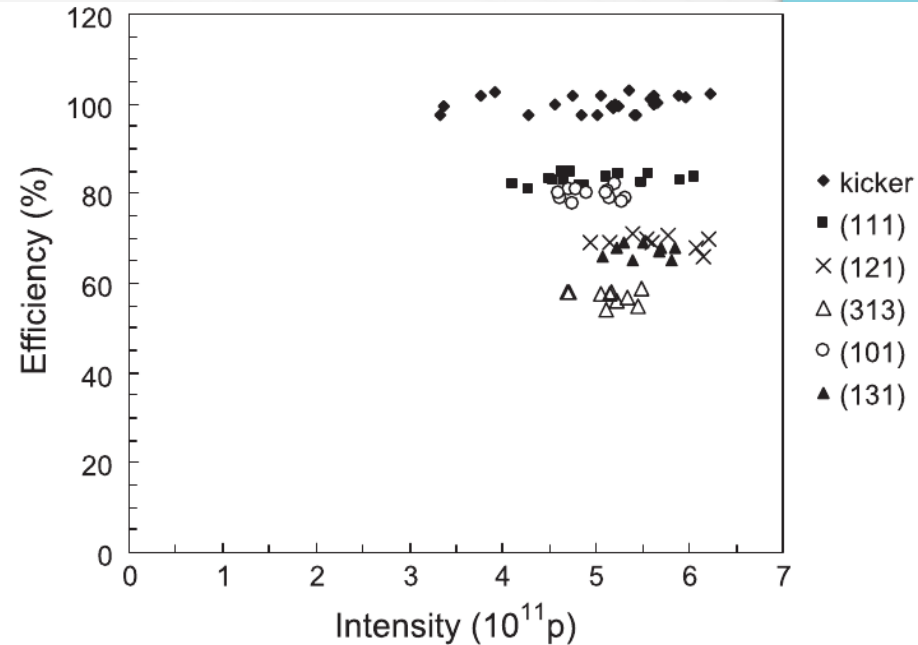
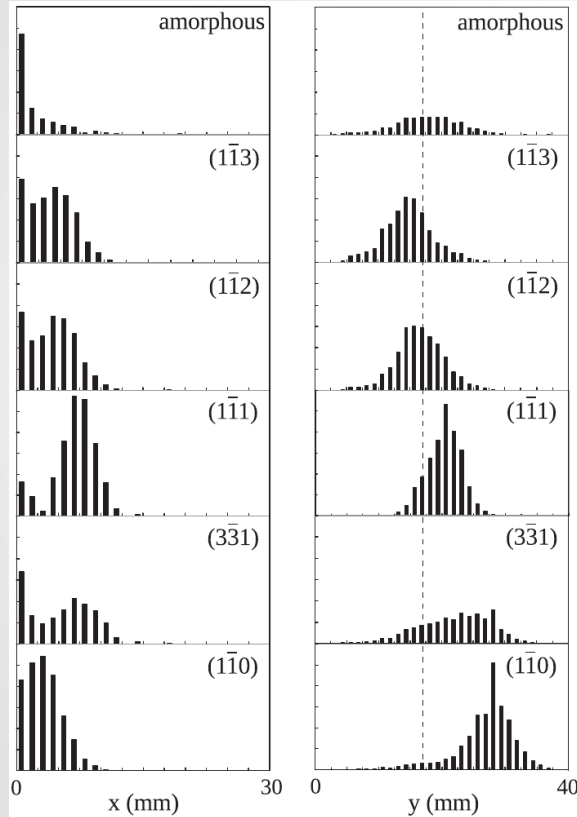
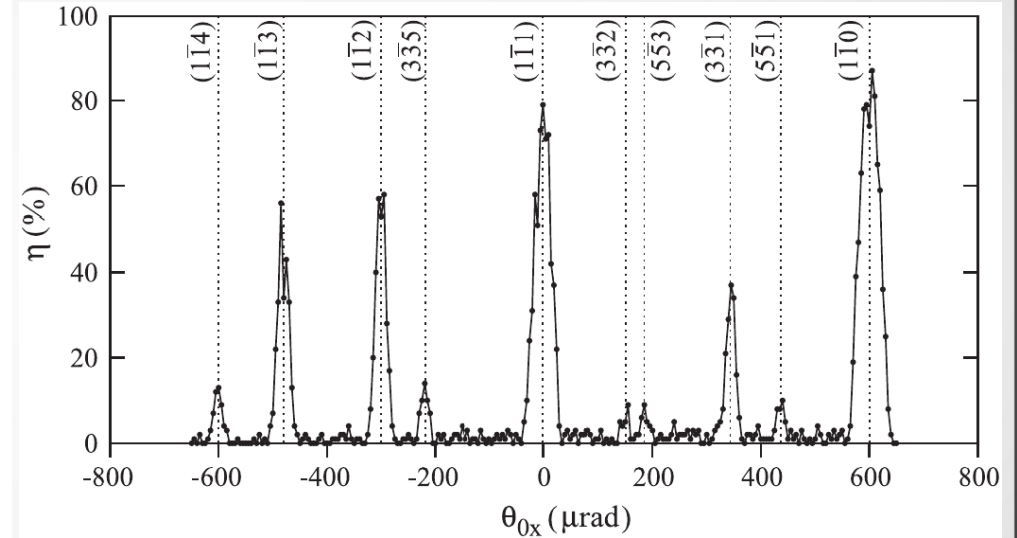
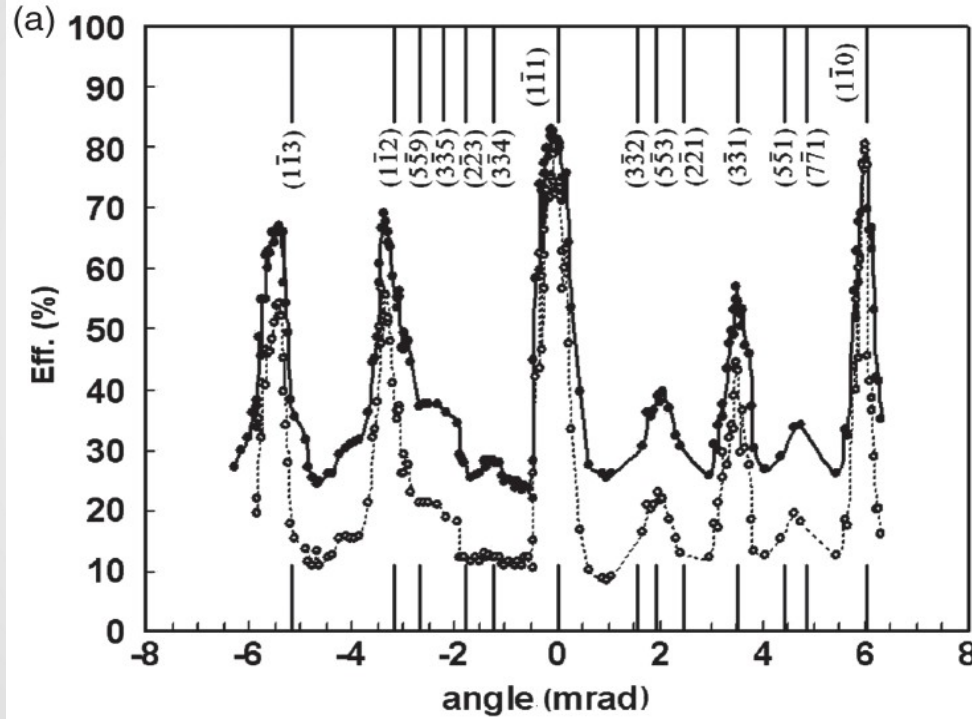


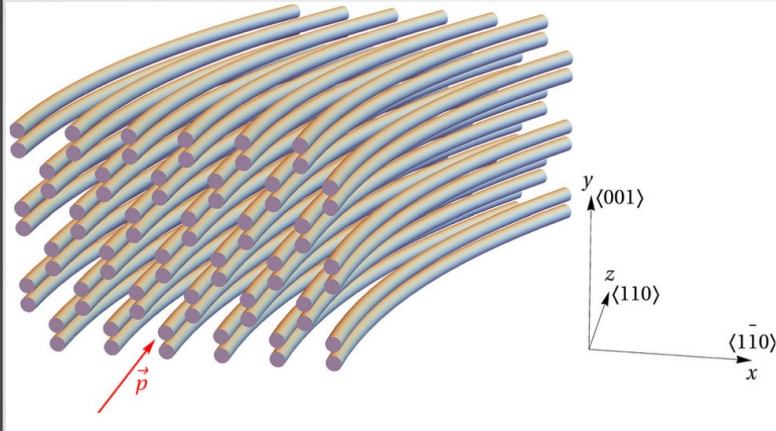
FIG. 5. The comparison of the efficiency of the beam deflection by different crystallographic planes with the extraction by the beam-kicker magnet efficiency.

The efficiency of the accelerated beam extraction as a function of the horizontal angle of the goniometer



Simulation

Influence of incoherent scattering on stochastic deflection of high-energy negative particle beams in bent crystals



$$\overline{\Psi_{inc}^2} = \xi L$$

$$\alpha_{st} = \frac{L_{st}}{R} = \frac{\psi_m^2}{l/R + \xi R}$$

$$R_{opt} = \sqrt{l/\xi}$$

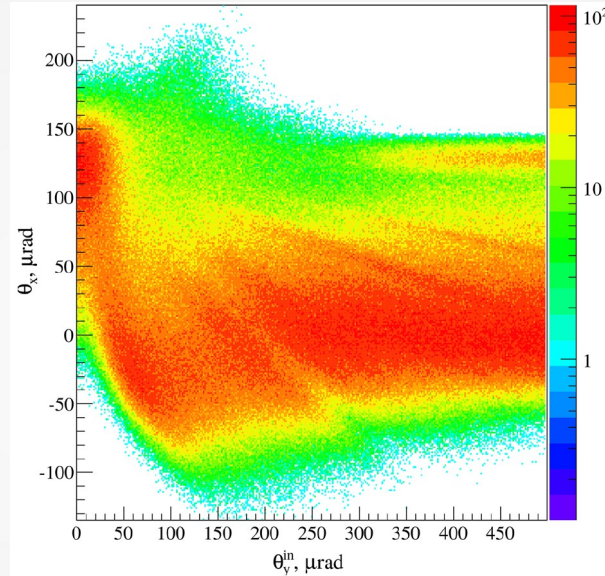


Fig. 2 The dependence of deflection angle of 150 GeV/c π^- -mesons in the direction of crystal bend from the angle between particle initial momentum and the bending plane

The length of the crystal was 1.52 mm and the radius of curvature $R = 11.7$ m

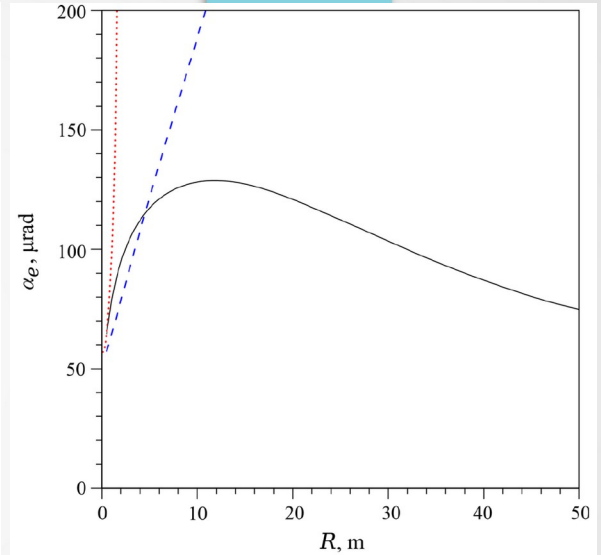


Fig. 5 Dependence of the deflection angle α_e from the radius of curvature R , for 150 GeV/c π^- -mesons. Ideal crystal (dotted line) with $\xi = 0$; above-barrier particles motion with $\xi = 0$ (dashed line); real crystal (solid line)

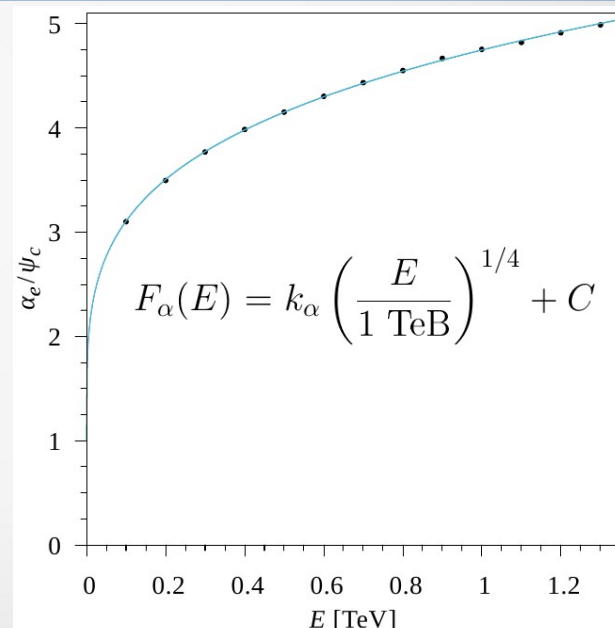
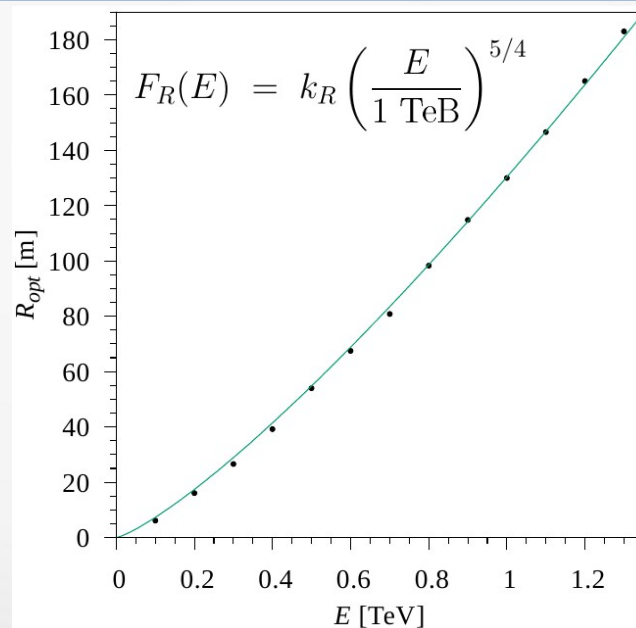
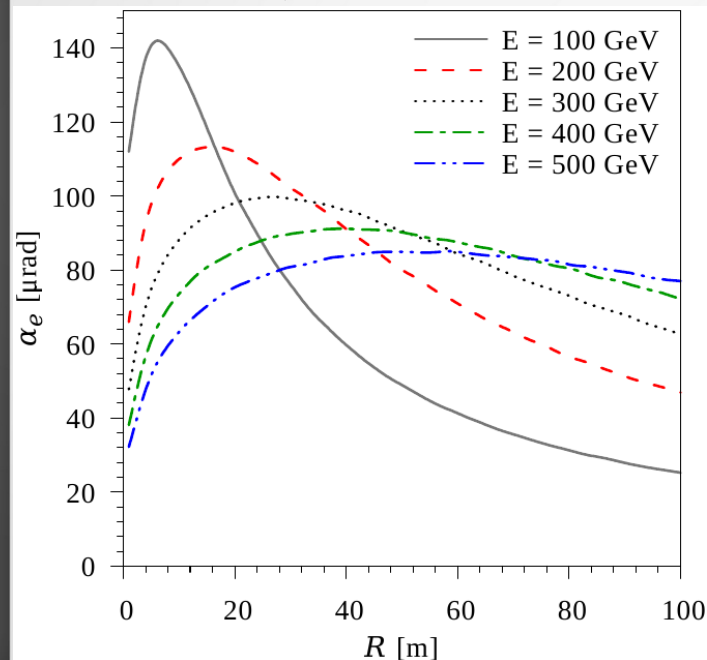
Optimal radius of curvature (stochastic deflection)

$$\overline{\psi_{inc}^2} = \zeta L / E^2 \rightarrow \alpha_{st} = \frac{\psi_m^2}{l/R + \zeta R / E^2} \rightarrow R_{opt} = E \sqrt{l/\zeta}$$

$$l \approx \frac{1}{4\pi da} \sqrt{\frac{E}{U_0}} \rightarrow R_{opt} \propto E^{5/4}$$

$$\psi_m \approx 1,5\psi_c \propto E^{-1/2} \rightarrow \max(\alpha_{st}) = \frac{\psi_m^2}{l/R_{opt} + \zeta R_{opt}/E^2} \propto E^{-1/4}$$

π^- , Si <110>



Optimal radius of curvature (planar channeling)

$$U_p(x) = -\frac{U_0}{d_p^2} \left\{ (2x + d_p)^2 H[-x(d_p + x)] + (2x - d_p)^2 H[x(d_p - x)] \right\},$$

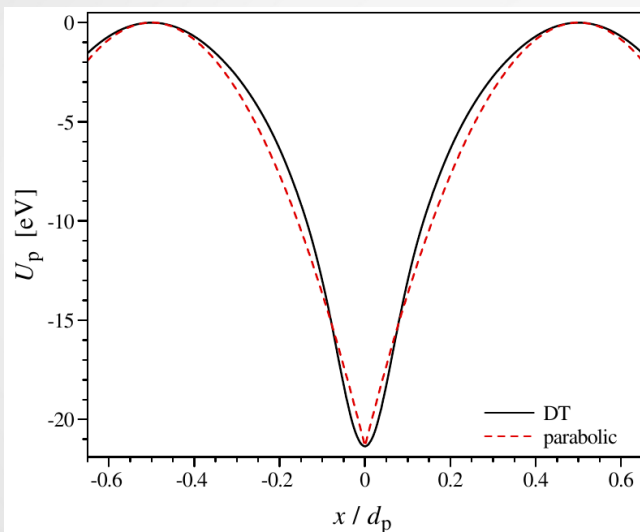
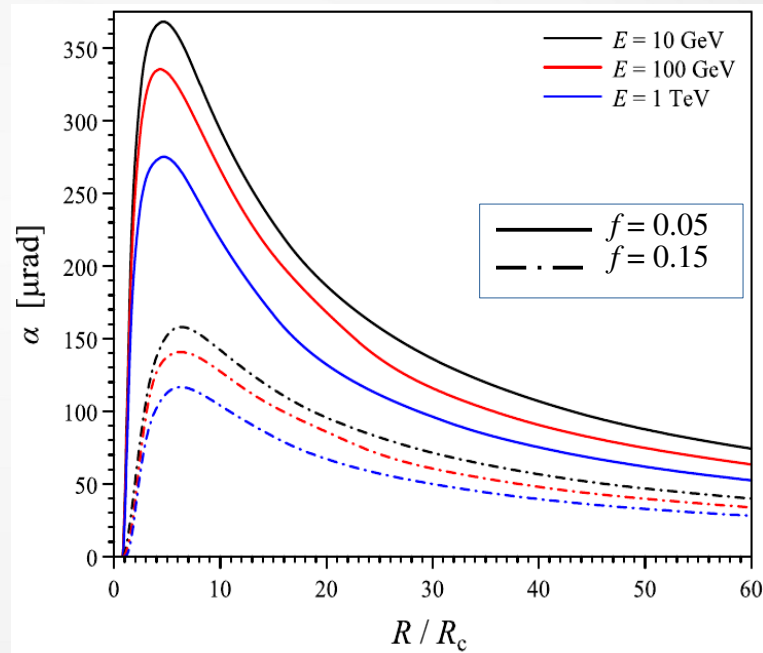


FIG. 1. The potential energy of a particle with a charge, that equals to the charge of an electron, in the field of (110) atomic planes of Si. Black solid line corresponds to the Doyle-Turner approximation (DT), red dashed line corresponds to the parabolic approximation (3).

$$\alpha = \frac{l}{R} = \frac{\theta_c^2}{2\xi^2 R \left[\text{erf}^{-1} \left(\frac{f}{1 - \sqrt{R_c/R}} \right) \right]^2}$$



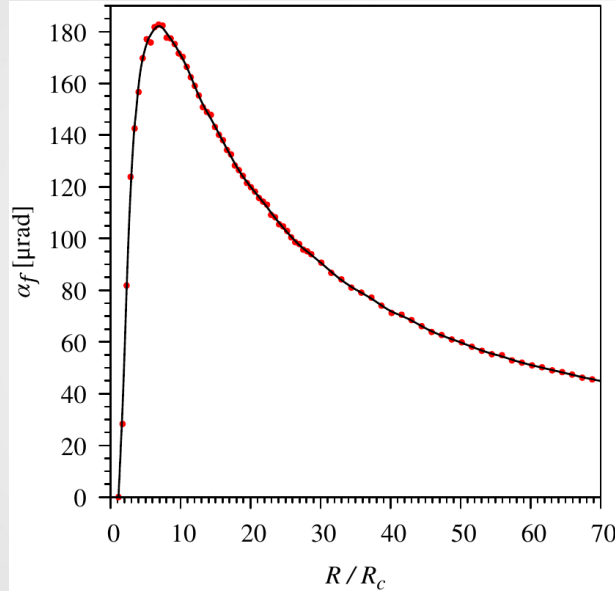
$$U_p(x) = -\frac{2\pi\hbar^2}{m_e d d_s d_p} \sum_{k=1}^4 \alpha_k \theta_3 \left(\pi \frac{x}{d_p}, e^{-\frac{\beta_k + B}{4d_p^2}} \right)$$

Energy dependence of the efficiency of high-energy negatively charged particle beam deflection by planar channeling in a bent crystal

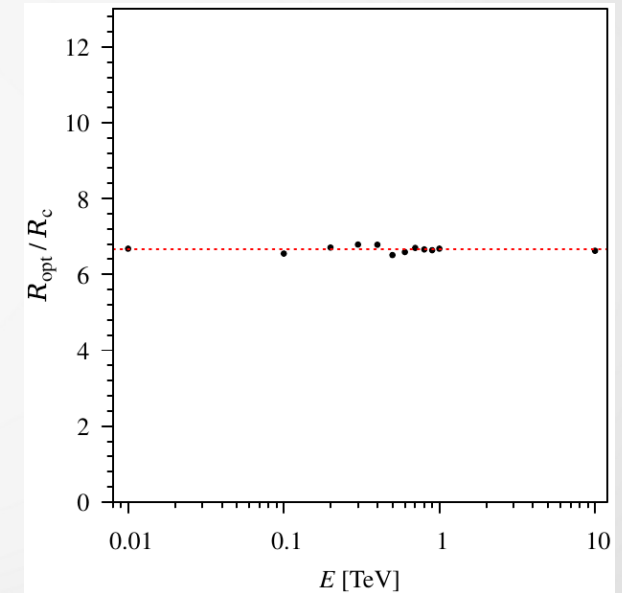
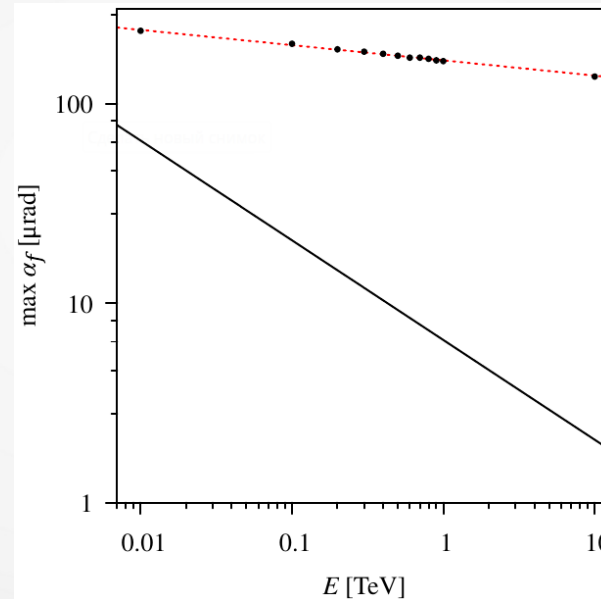
$$U_{\text{eff}}(x) = U_p(x) + Ex/R$$

$$\left. \frac{dU_{\text{eff}}(x)}{dx} \right|_{x=x_0} = 0$$

$$R_c = \frac{E}{|U'_p(x_0)|}$$



The dependence of the maximum deflection angle on which the given fraction $f = 0.1$ of beam particles can be deflected by a bent crystal on the radius of curvature of the crystal



Dependence of the maximum deflection angle of antiprotons with a kinetic energy of 14 GeV on the bending radius of the crystal

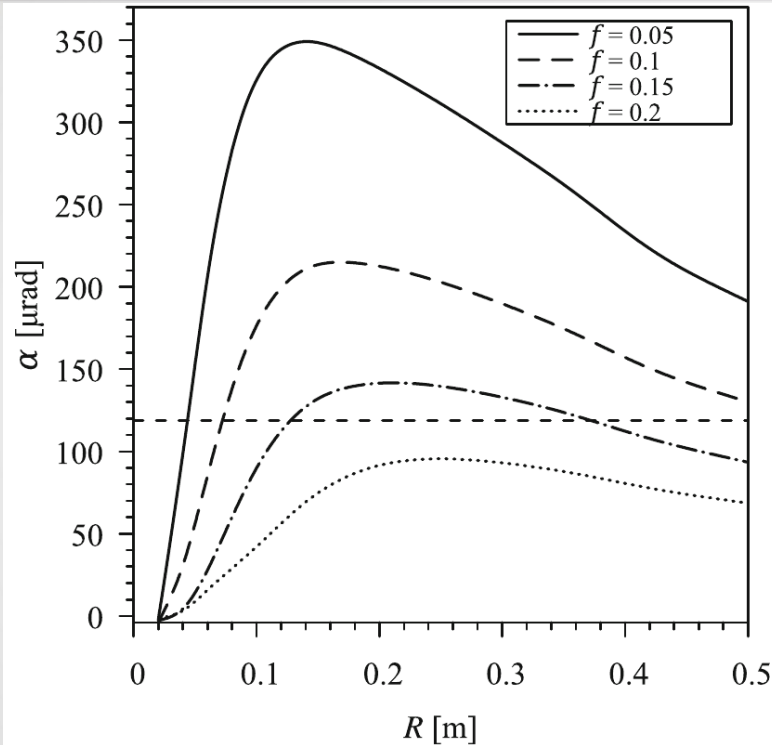


Fig. 3 Dependence of the maximum deflection angle of of antiprotons with a kinetic energy of 14 GeV on the bending radius of the crystal in case of planar channeling

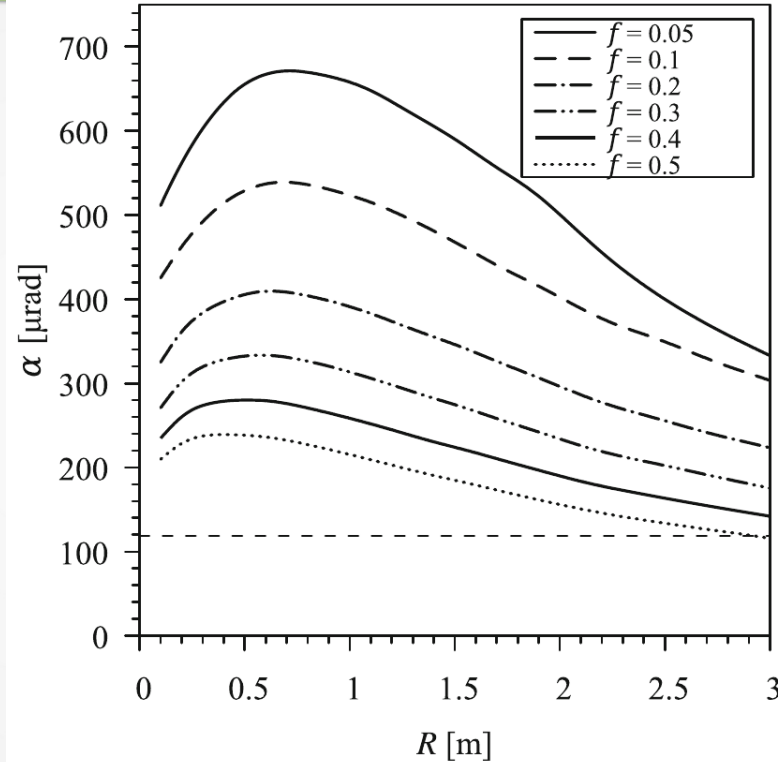


Fig. 4 Dependence of the maximum deflection angle of of antiprotons with a kinetic energy of 14 GeV on the bending radius of the crystal in case of stochastic deflection

$$\psi_c = \sqrt{4Z|qe|/pvd}$$

$$118.8 \mu\text{rad}$$

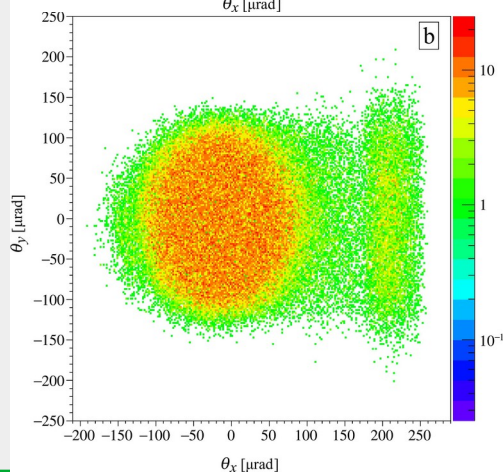
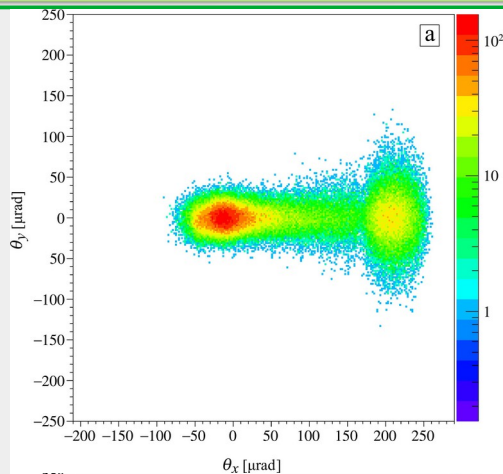
Influence of the initial angular divergence of the beam

Si (110),
planar
channeling,
 $E_{\text{kin}}=14 \text{ GeV}$

$$\Delta\psi_{\text{in}}=0$$

$$\Delta\psi_{\text{in}}=2\psi_{\text{c}}=$$
$$=237.6 \text{ } \mu\text{rad}$$

$R = 15.9 \text{ cm}$
 $\alpha = 212 \text{ } \mu\text{rad}$
 $L = 33.7 \text{ } \mu\text{m}$

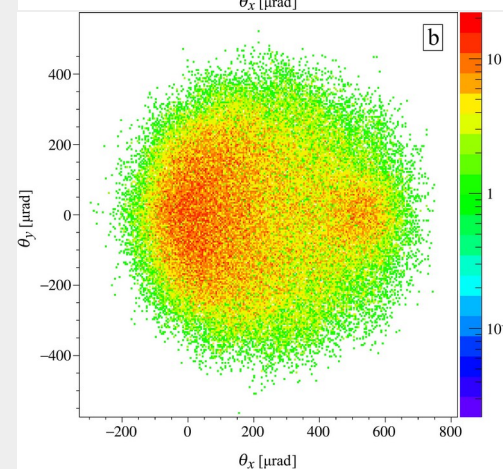
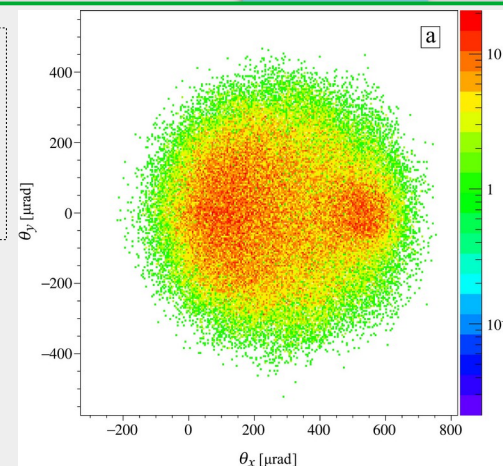


Si <110>,
stochastic
deflection,
 $E_{\text{kin}}=14 \text{ GeV}$

$$\Delta\psi_{\text{in}}=0$$

$$\Delta\psi_{\text{in}}=2\psi_{\text{c}}=$$
$$=237.6 \text{ } \mu\text{rad}$$

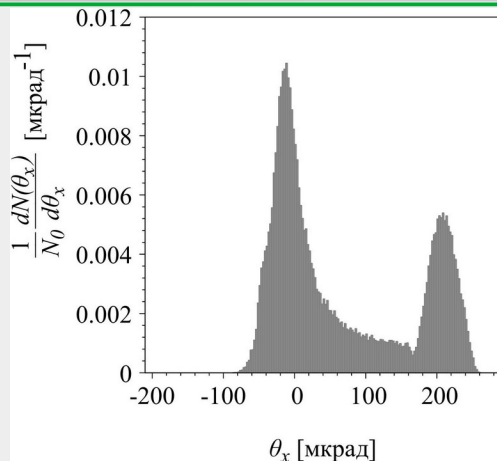
$R = 65 \text{ cm}$
 $\alpha = 540 \text{ } \mu\text{rad}$
 $L = 351 \text{ } \mu\text{m}$



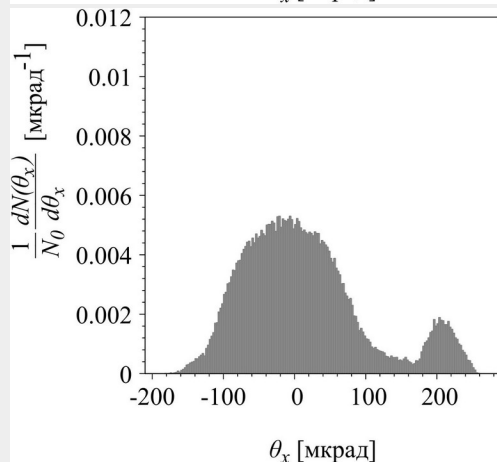
Influence of the initial angular divergence of the beam

Si (110),
planar
channeling,
 $E_{\text{kin}}=14 \text{ GeV}$

$$\Delta\psi_{\text{in}}=0$$



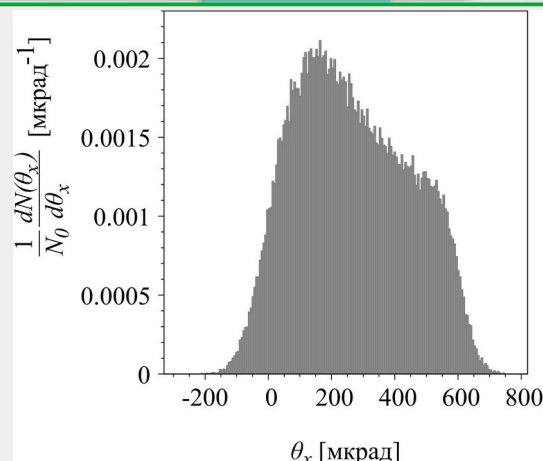
$$\Delta\psi_{\text{in}}=2\psi_{\text{c}}=\\=237.6 \text{ }\mu\text{rad}$$



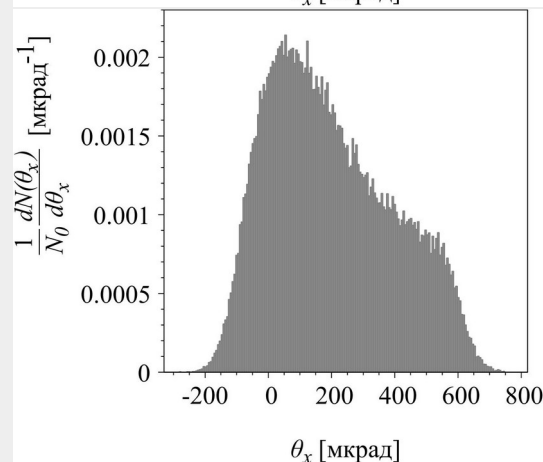
$R = 15.9 \text{ cm}$
 $\alpha = 212 \text{ }\mu\text{rad}$
 $L = 33.7 \text{ }\mu\text{m}$

Si <110>,
stochastic
deflection,
 $E_{\text{kin}}=14 \text{ GeV}$

$$\Delta\psi_{\text{in}}=0$$



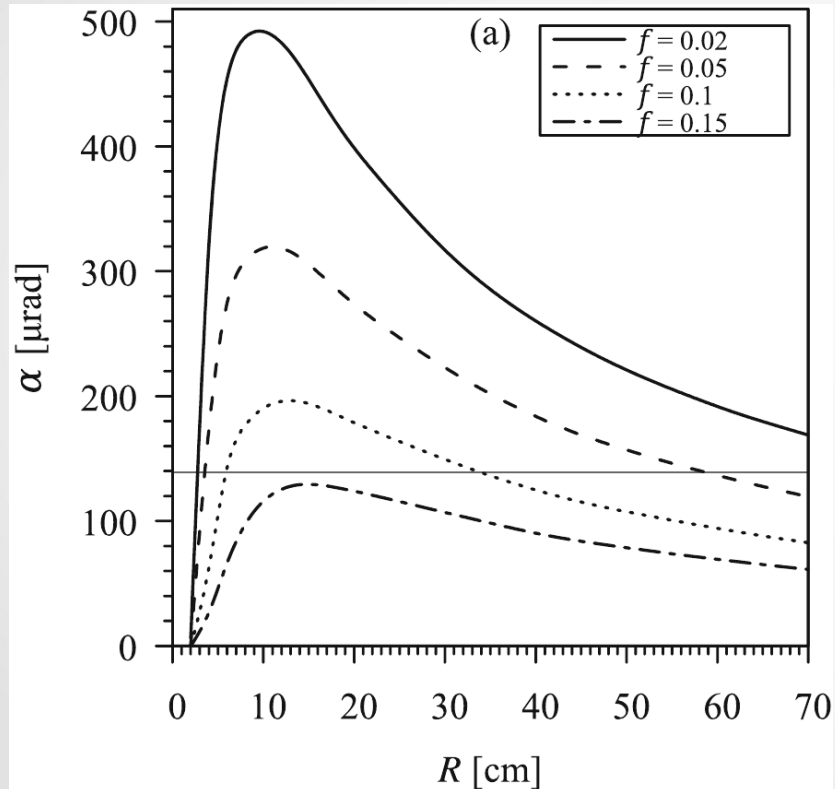
$$\Delta\psi_{\text{in}}=2\psi_{\text{c}}=\\=237.6 \text{ }\mu\text{rad}$$



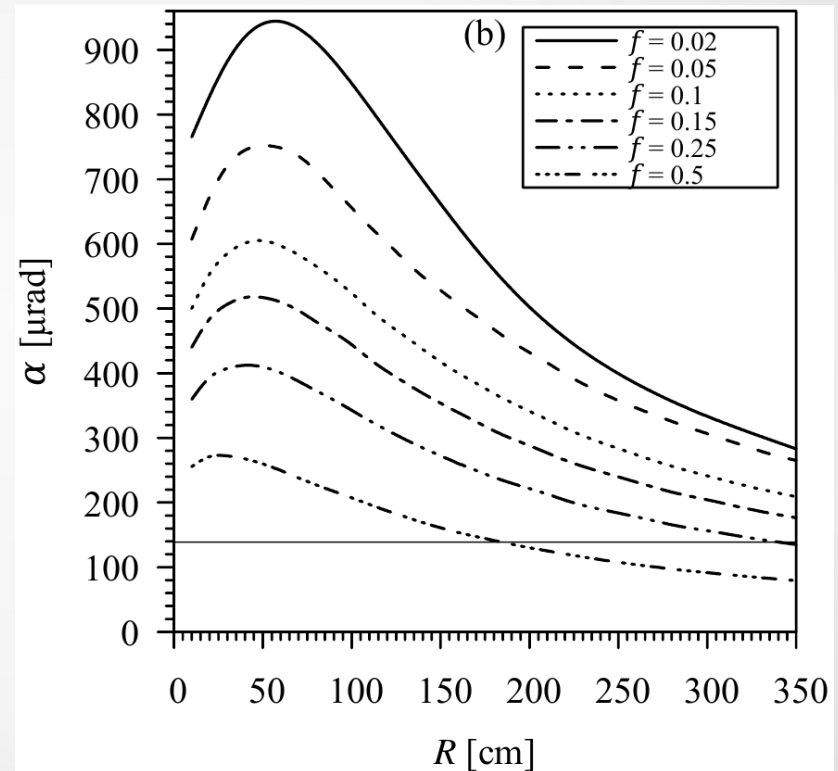
$R = 65 \text{ cm}$
 $\alpha = 540 \text{ }\mu\text{rad}$
 $L = 351 \text{ }\mu\text{m}$

Dependence of the maximum deflection angle of antiprotons with a kinetic energy of 10 GeV on the bending radius of the crystal

Si (110), planar channeling

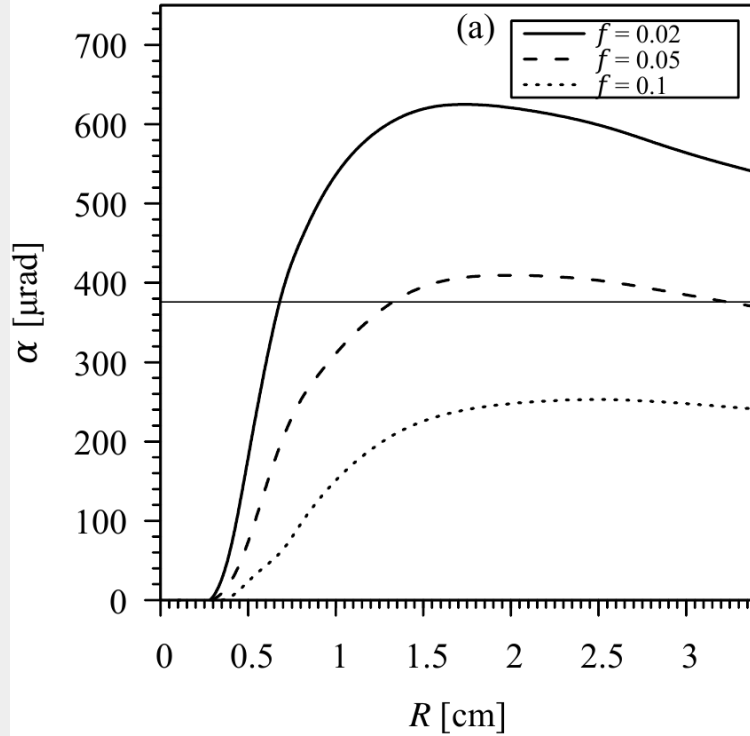


Si <110>, stochastic deflection



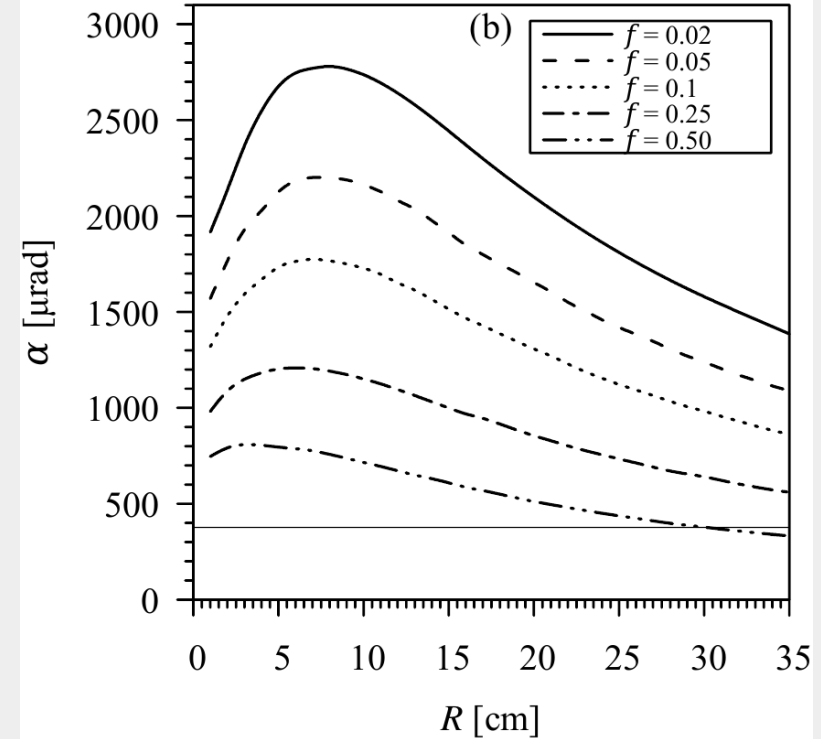
Dependence of the maximum deflection angle of antiprotons with a kinetic energy of 1 GeV on the bending radius of the crystal

Si (110), planar channeling

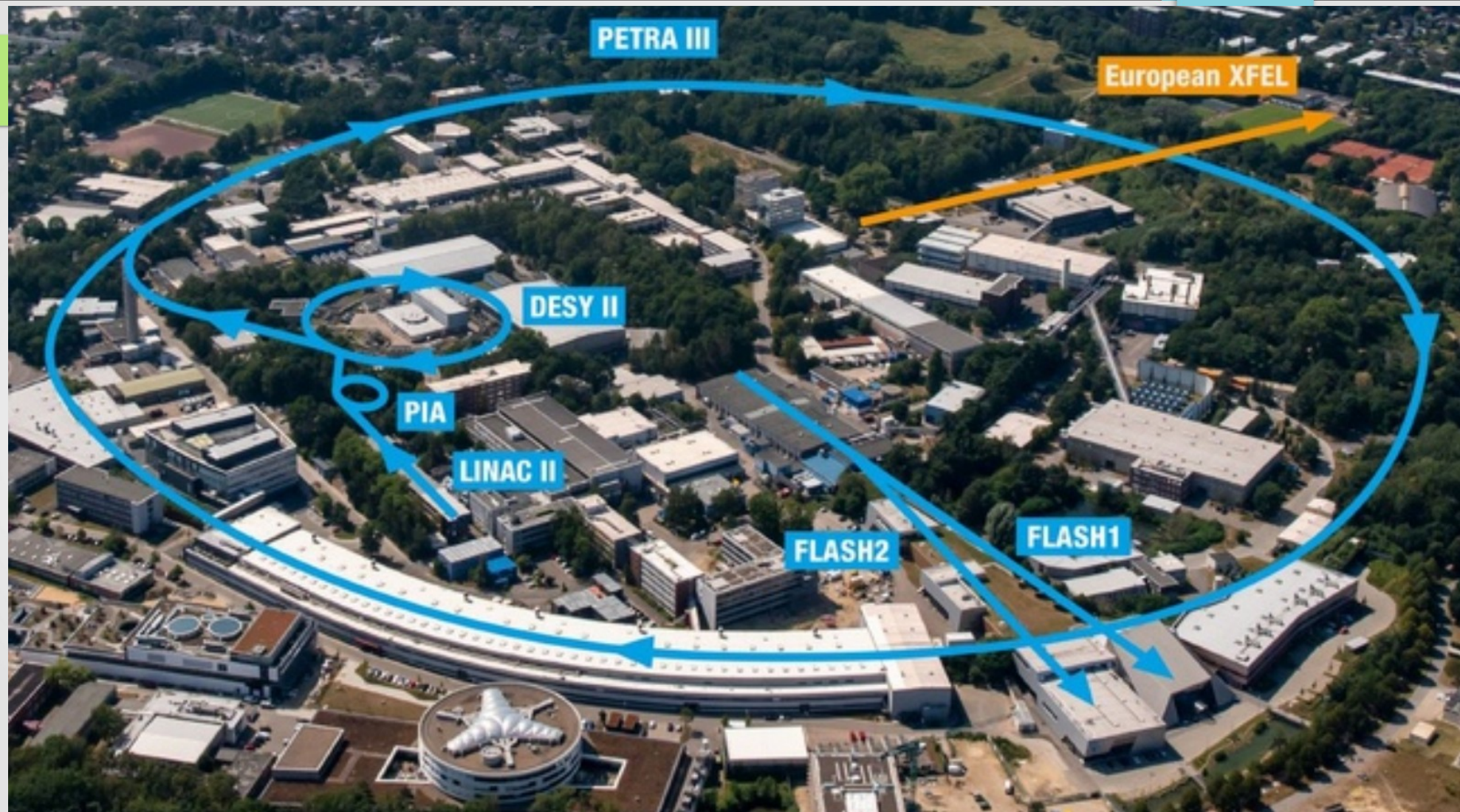


$$\alpha_p = \frac{l_p}{R} = \frac{\theta_c^2}{2\xi_p^2 R \left(\text{erf}^{-1} \left(\frac{f}{1 - \sqrt{\frac{R_c}{R}}} \right) \right)^2}$$

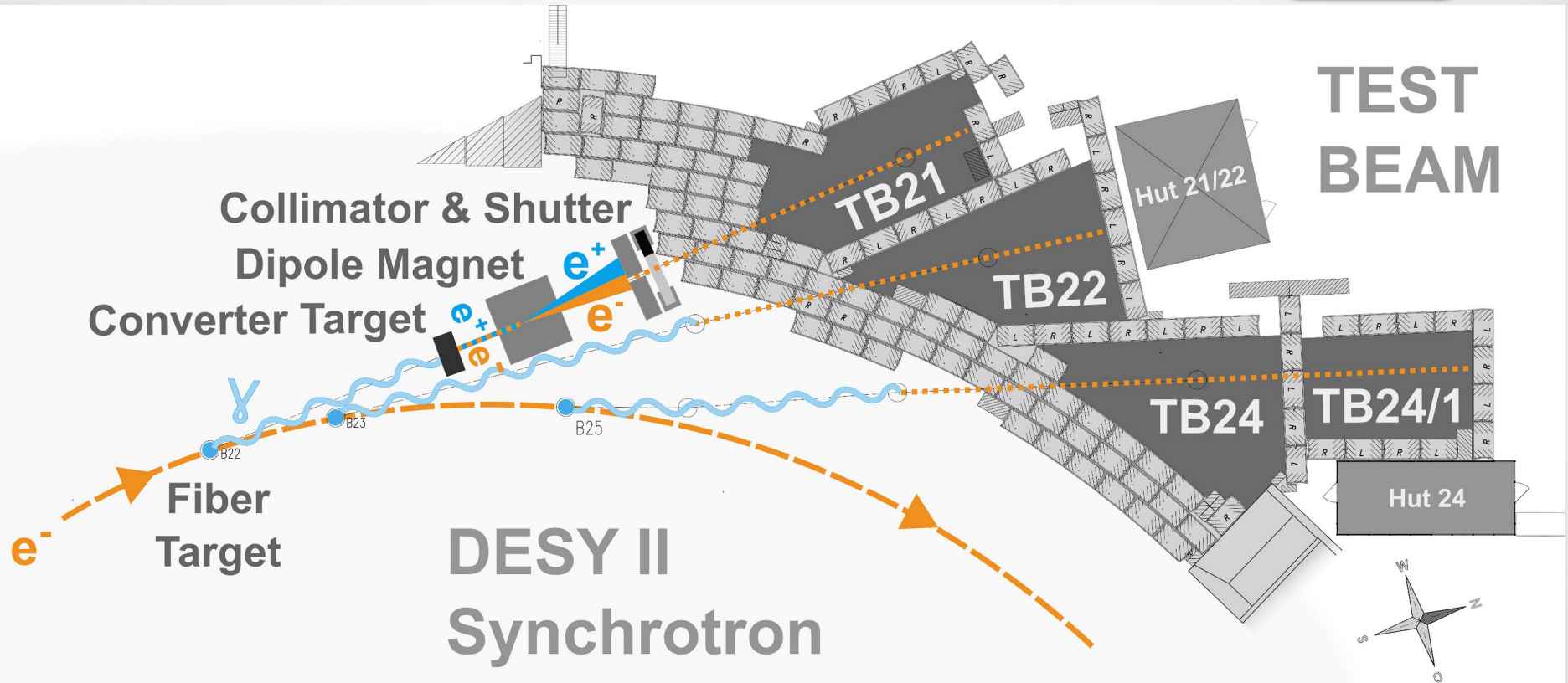
Si <110>, stochastic deflection



$$\alpha_{st} = \frac{L_{st}}{R} = \frac{\psi_m^2}{l/R + \xi R}$$

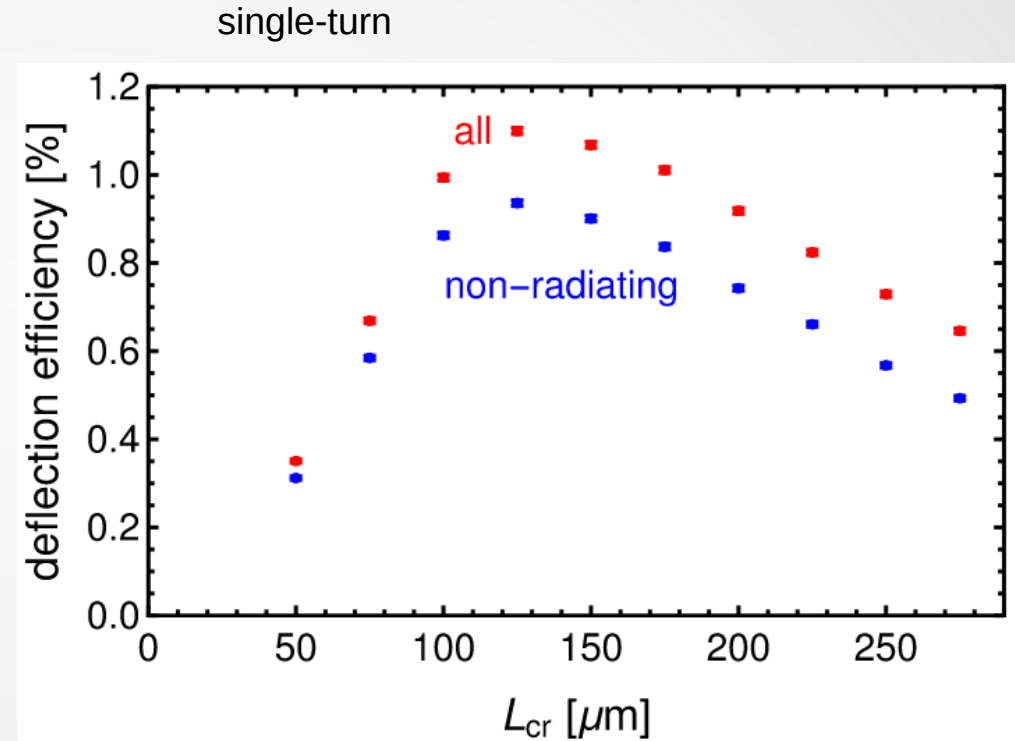
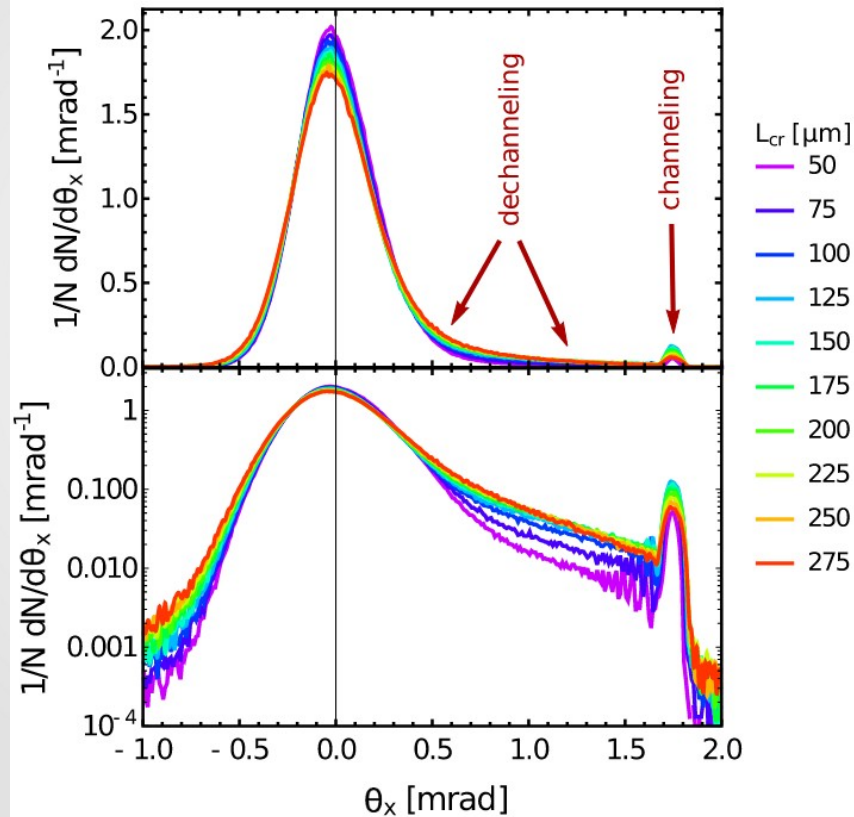


Generation of the DESY Test Beams



A. Sytov, G.Kube, L. Bandiera et al. First design of a crystal-based extraction of 6 GeV electrons for the DESY II Booster Synchrotron

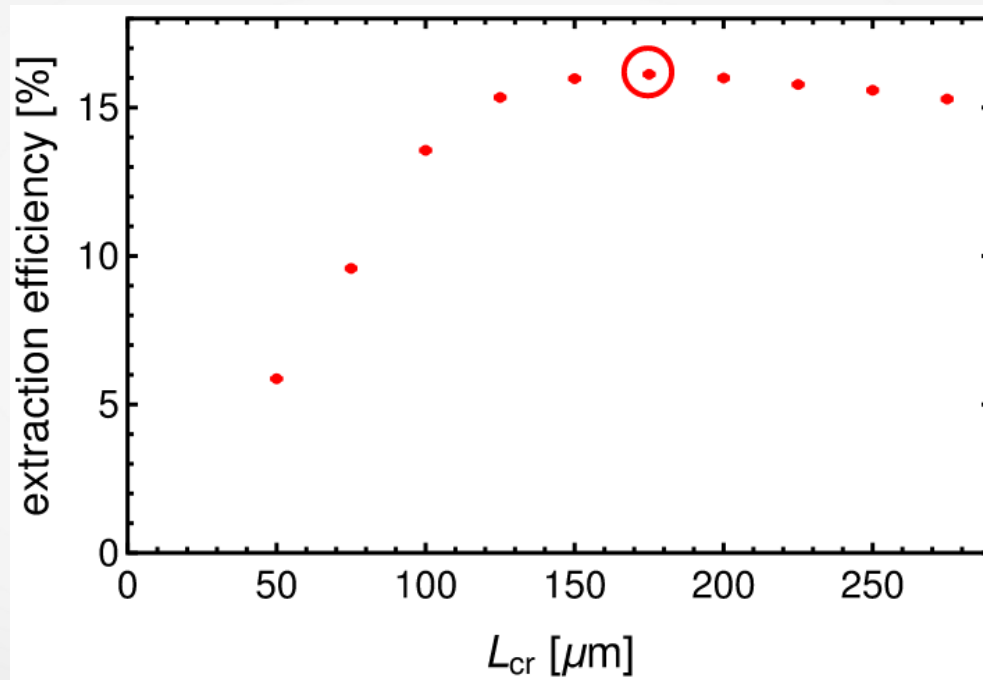
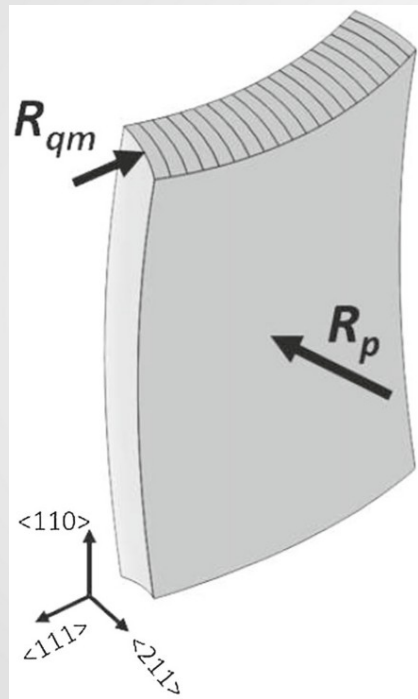
Eur. Phys. J. C (2022) 82:197



A. Sytov, G.Kube, L. Bandiera et al. First design of a crystal-based extraction of 6 GeV electrons for the DESY II Booster Synchrotron

Eur. Phys. J. C (2022) 82:197

multi-turn



$$L = 175 \mu m,$$

$$\alpha = 1.75 \text{ mrad}$$

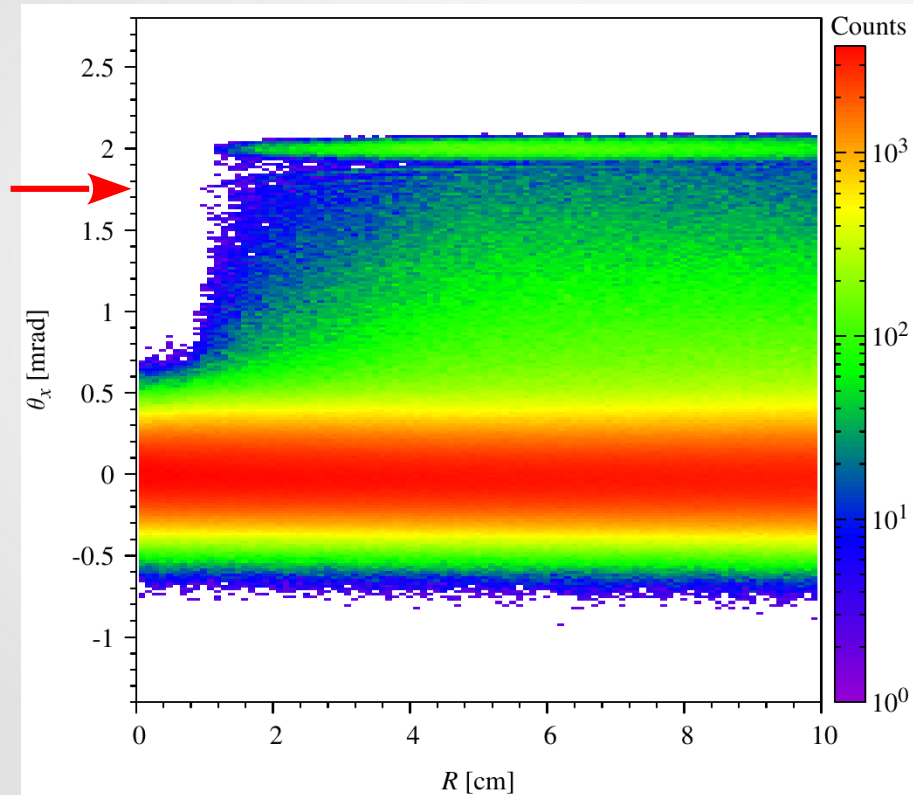


$$R = L / \alpha = 10 \text{ cm}$$

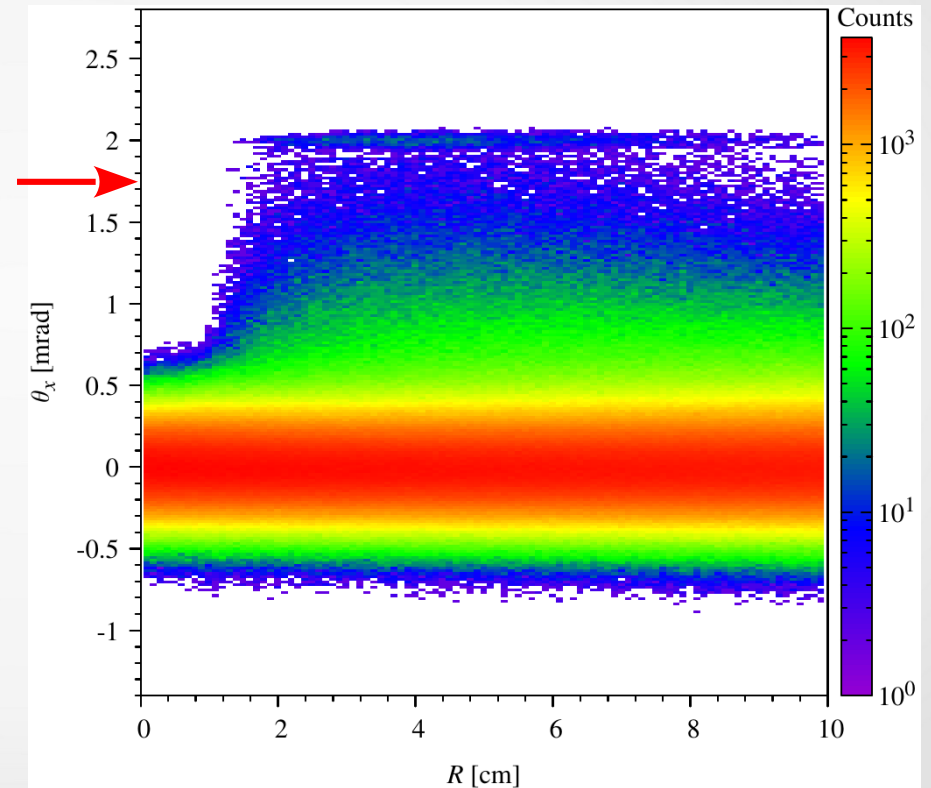
"The maximum number of turns in the accelerator is set to 100, since it is small enough to consider the beam energy as a constant, as well as high enough for charged particles to cross the crystal several times."

- Dependence of deflection angle on R ($\alpha = 2$ mrad)

(111) plane

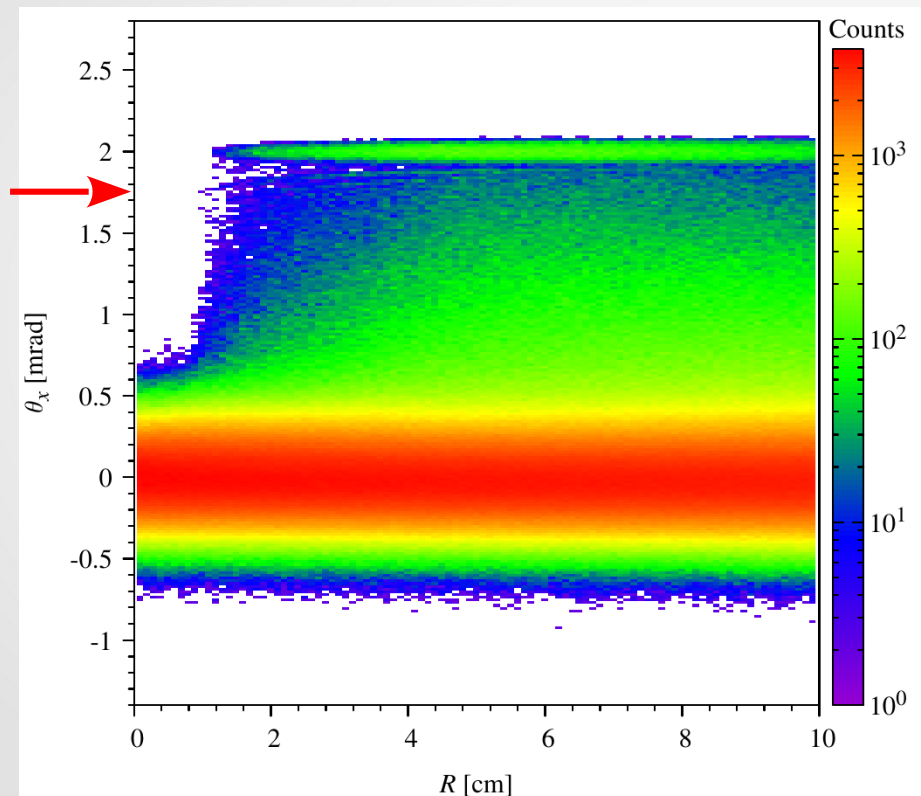


(110) plane



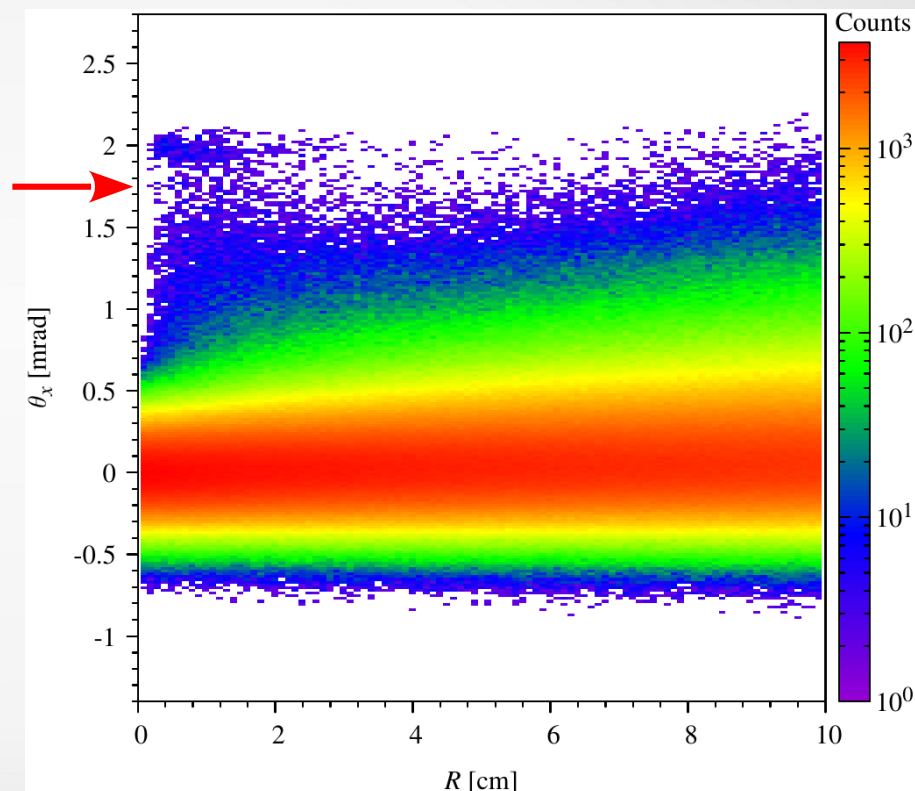
- Dependence of deflection angle on R ($\alpha = 2$ mrad)

(111) plane



$$\theta_c \approx 97.3 \mu\text{rad}$$

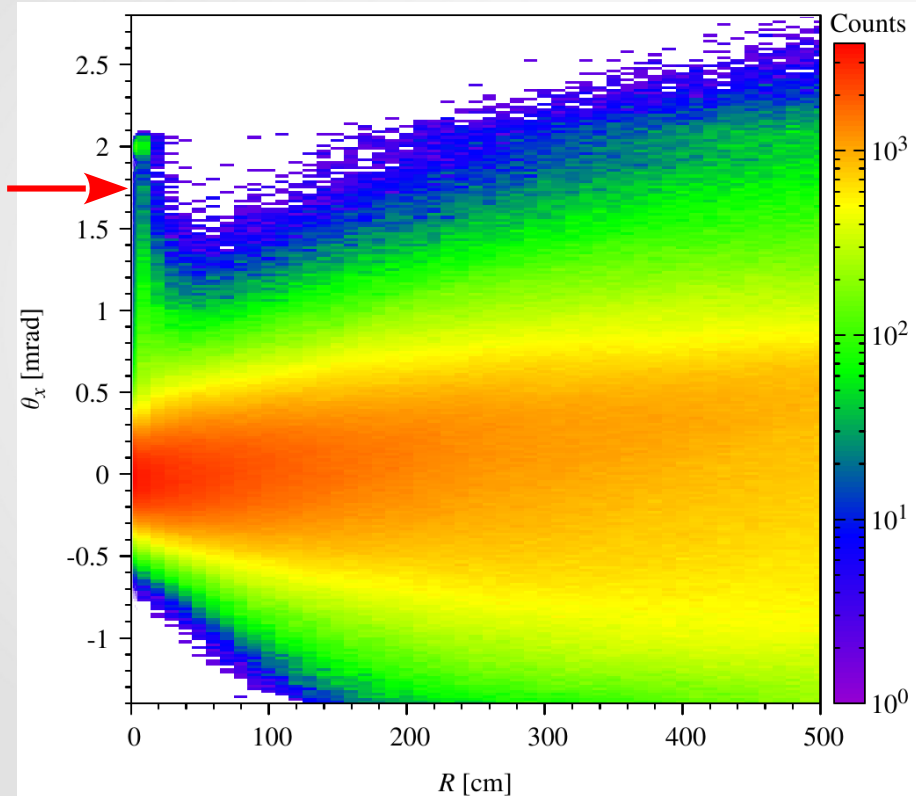
$\langle 110 \rangle$ axis



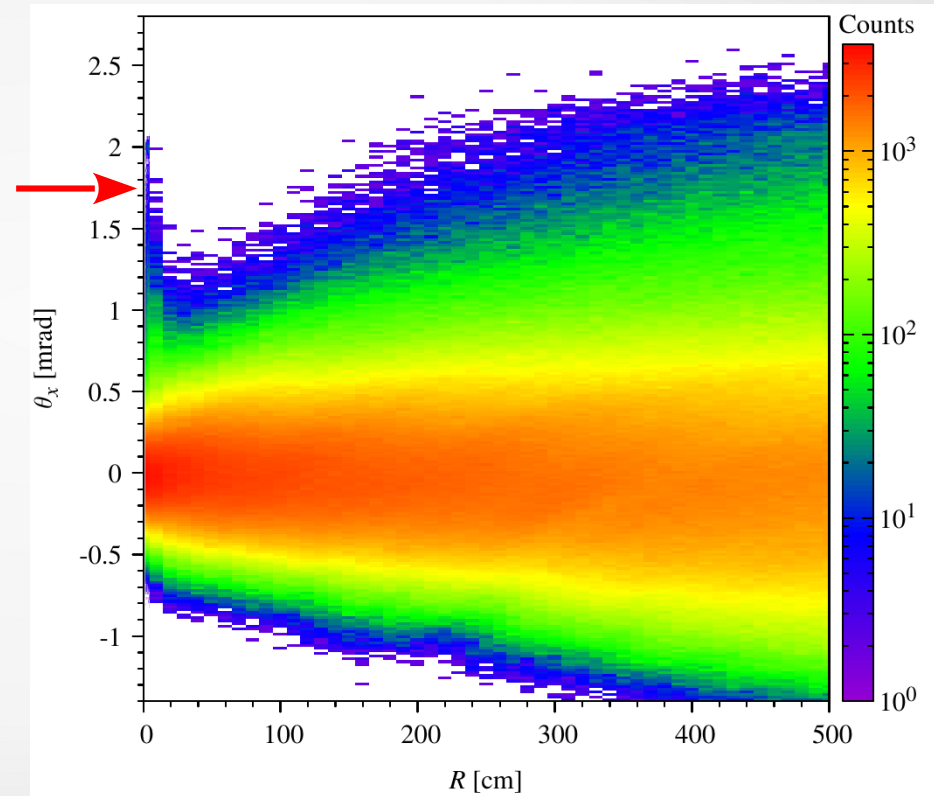
$$\psi_c \approx 187.1 \mu\text{rad}$$

- Dependence of deflection angle on R ($\alpha = 2$ mrad)

(111) plane

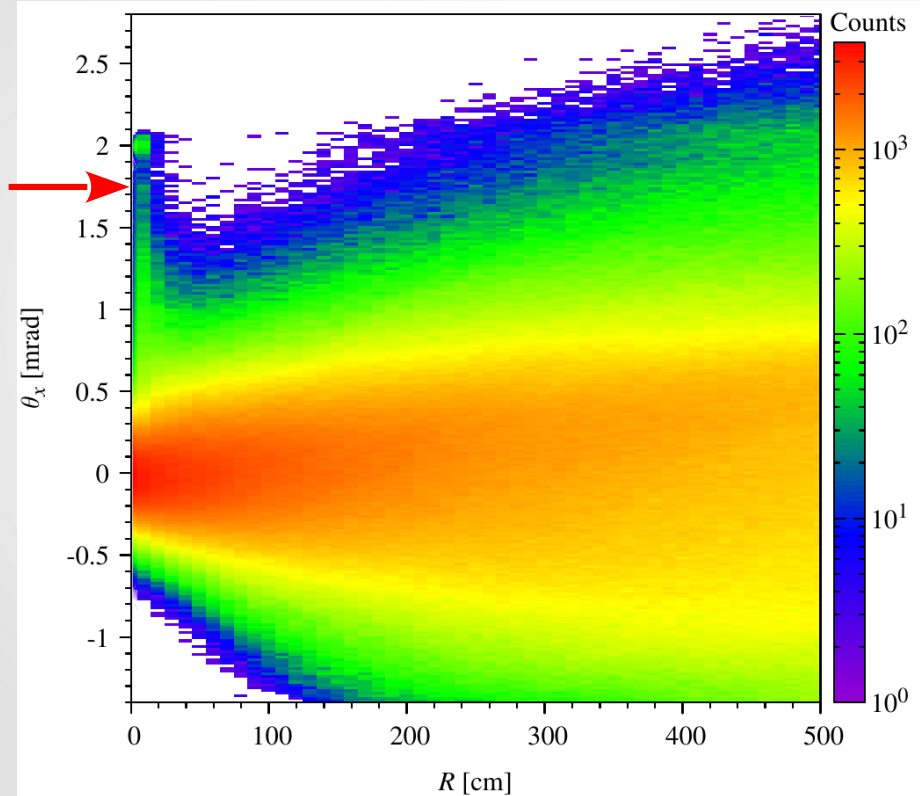


(110) plane



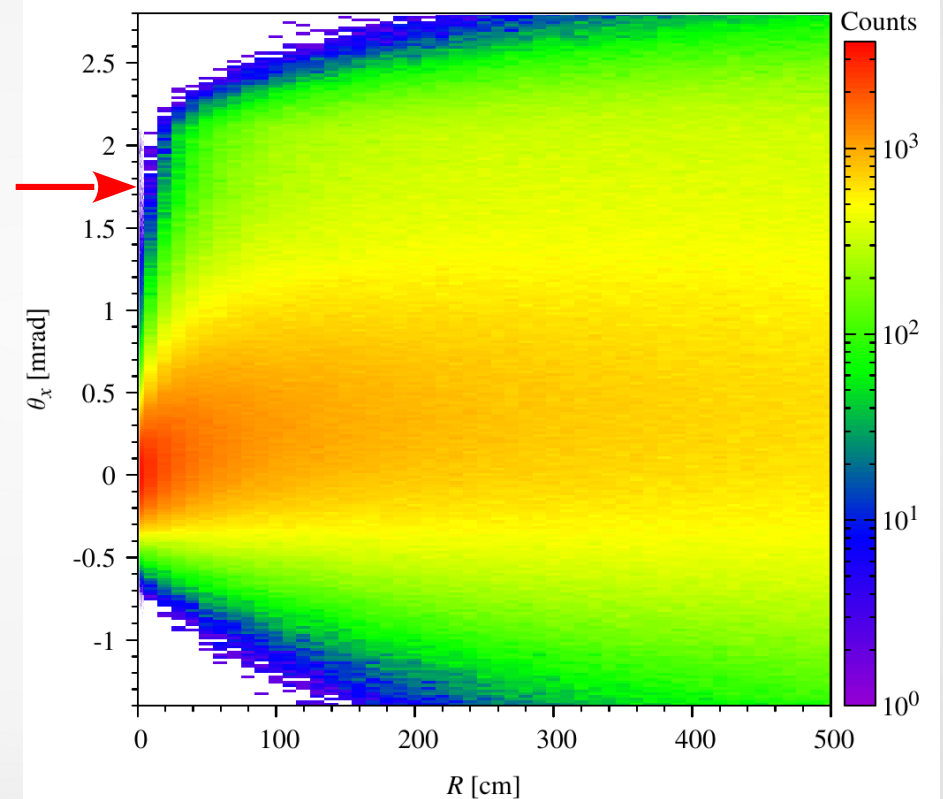
- Dependence of deflection angle on R ($\alpha = 2$ mrad)

(111) plane



$$\theta_c \approx 97.3 \mu\text{rad}$$

<110> axis

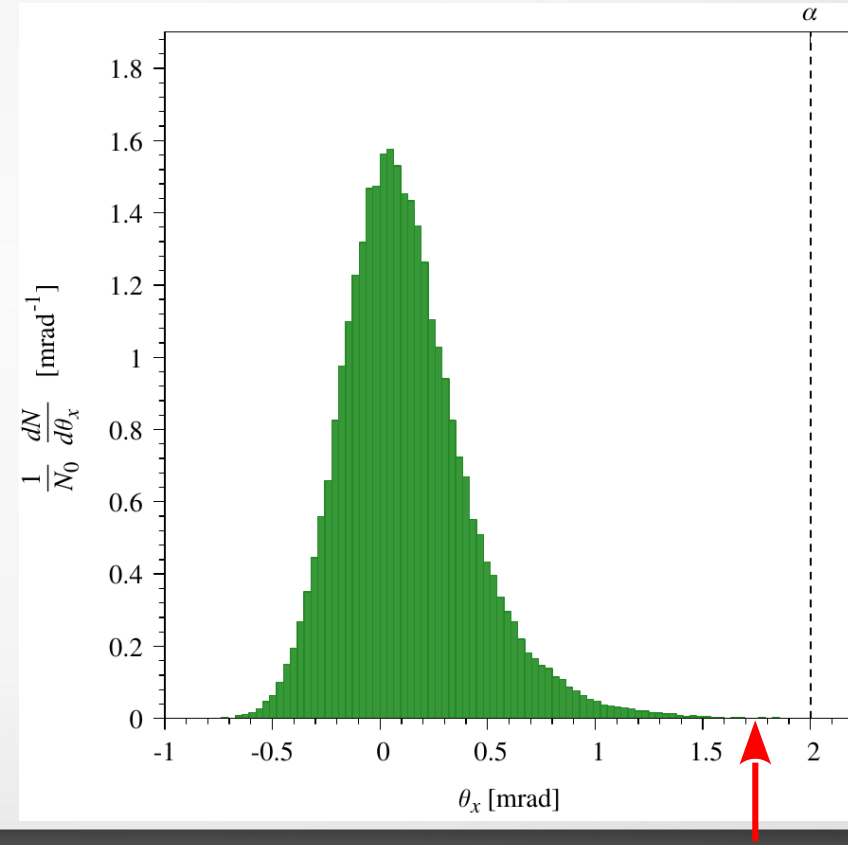
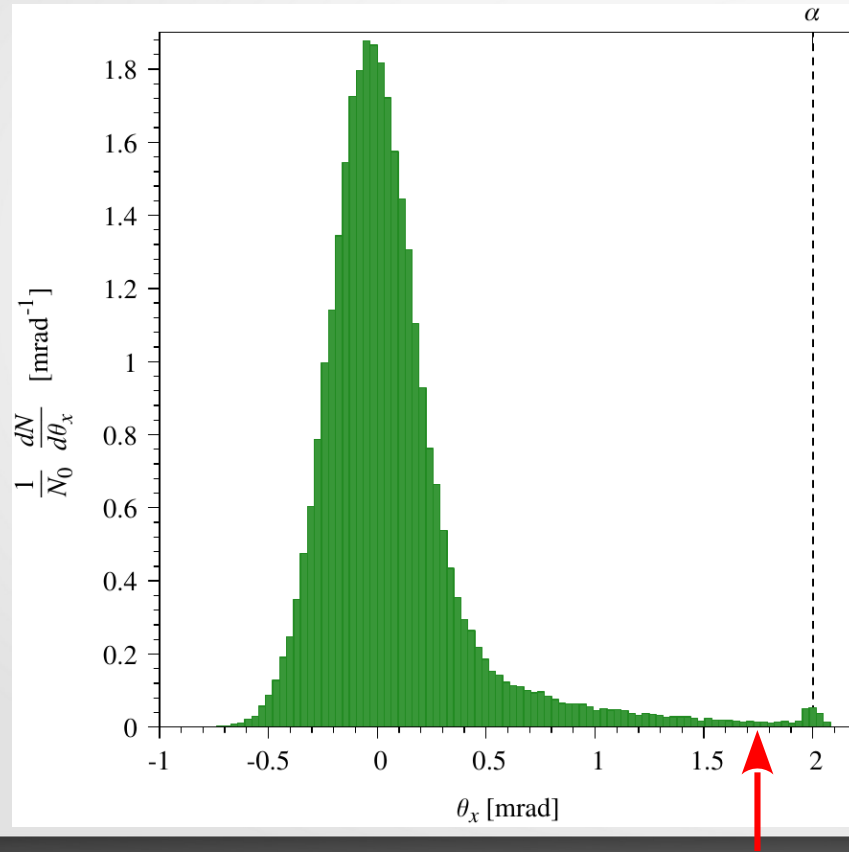


$$\psi_c \approx 187.1 \mu\text{rad}$$

- $R = 8 \text{ cm}$ ($\alpha = 2 \text{ mrad}$, $L = 160 \text{ }\mu\text{m}$)

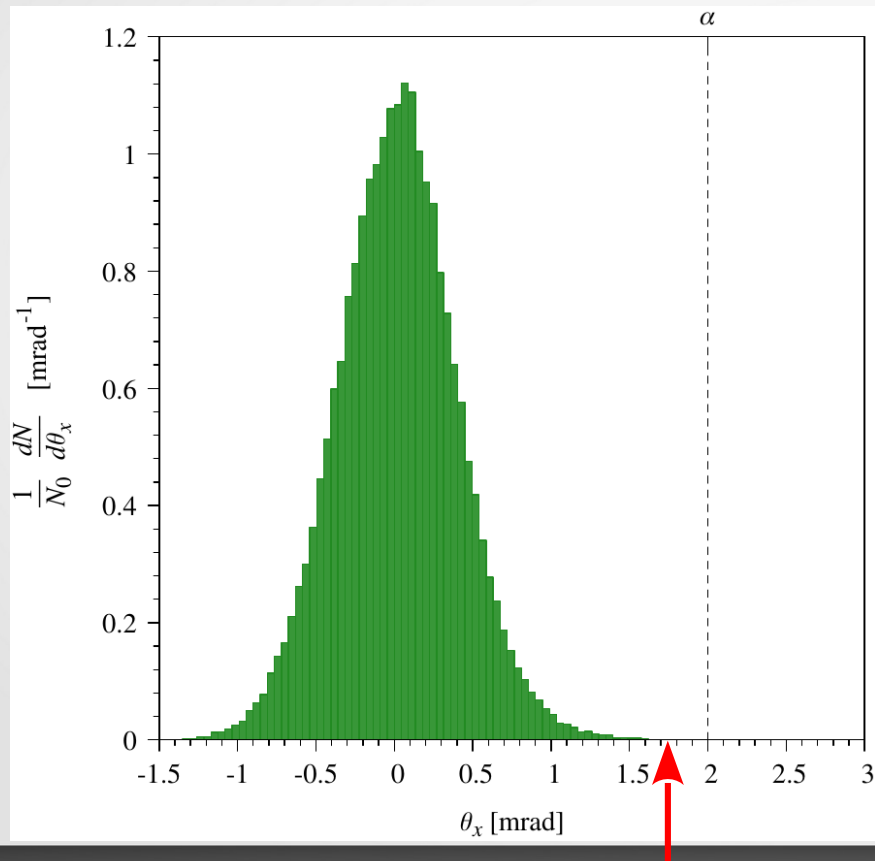
(111) plane

<110> axis

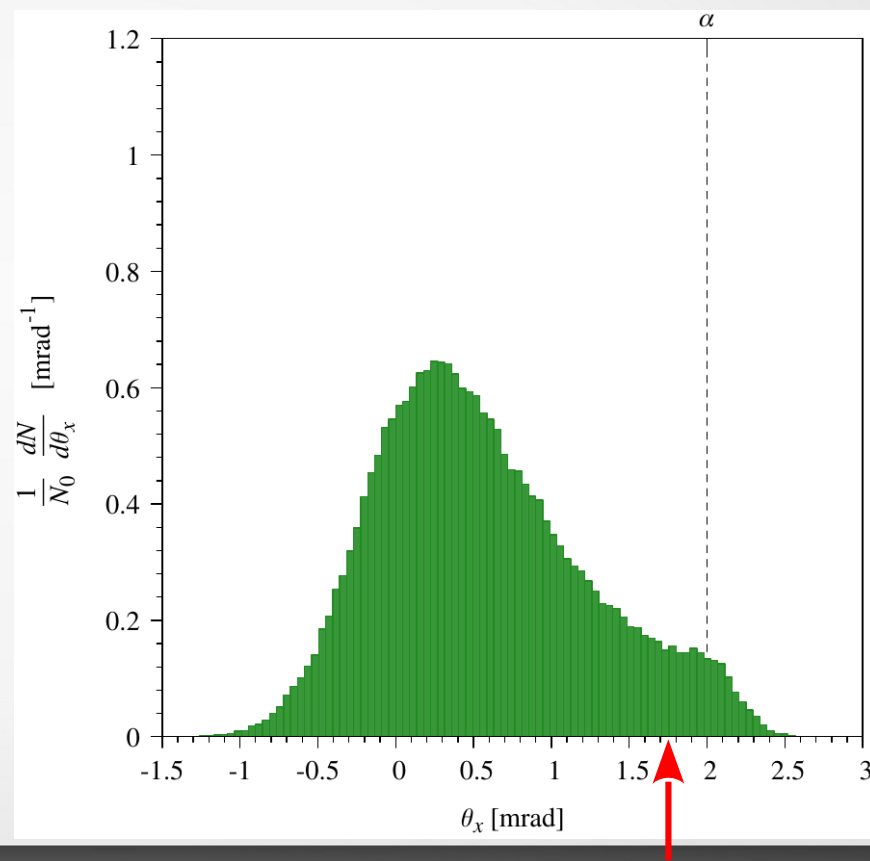


- $R = 1 \text{ m}$ ($\alpha = 2 \text{ mrad}$, $L = 2 \text{ mm}$)

(111) plane

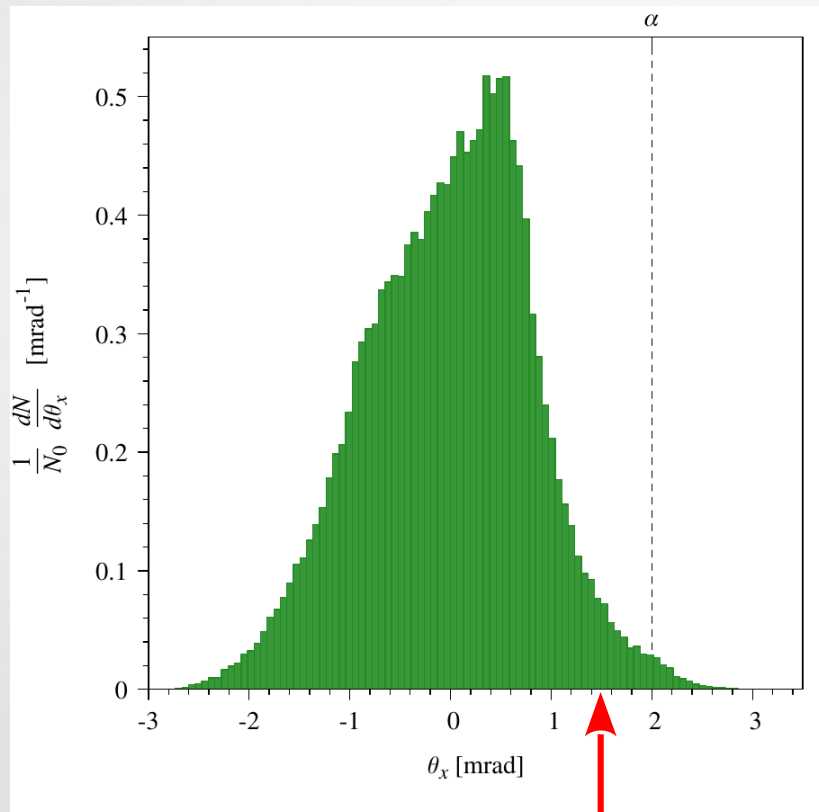


<110> axis

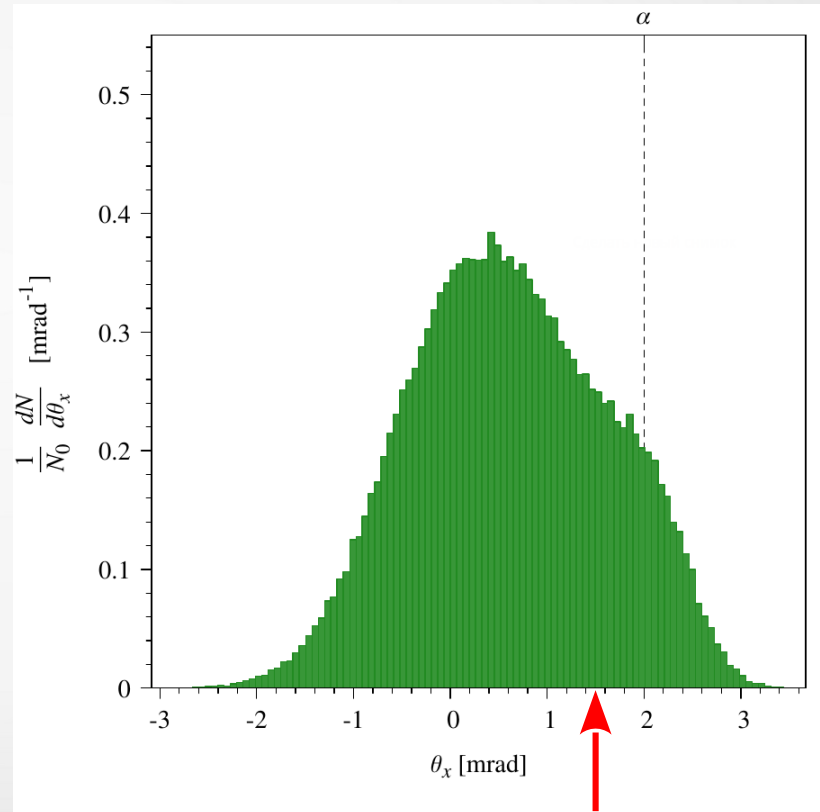


- $R = 5 \text{ m}$ ($\alpha = 2 \text{ mrad}$, 1 cm)

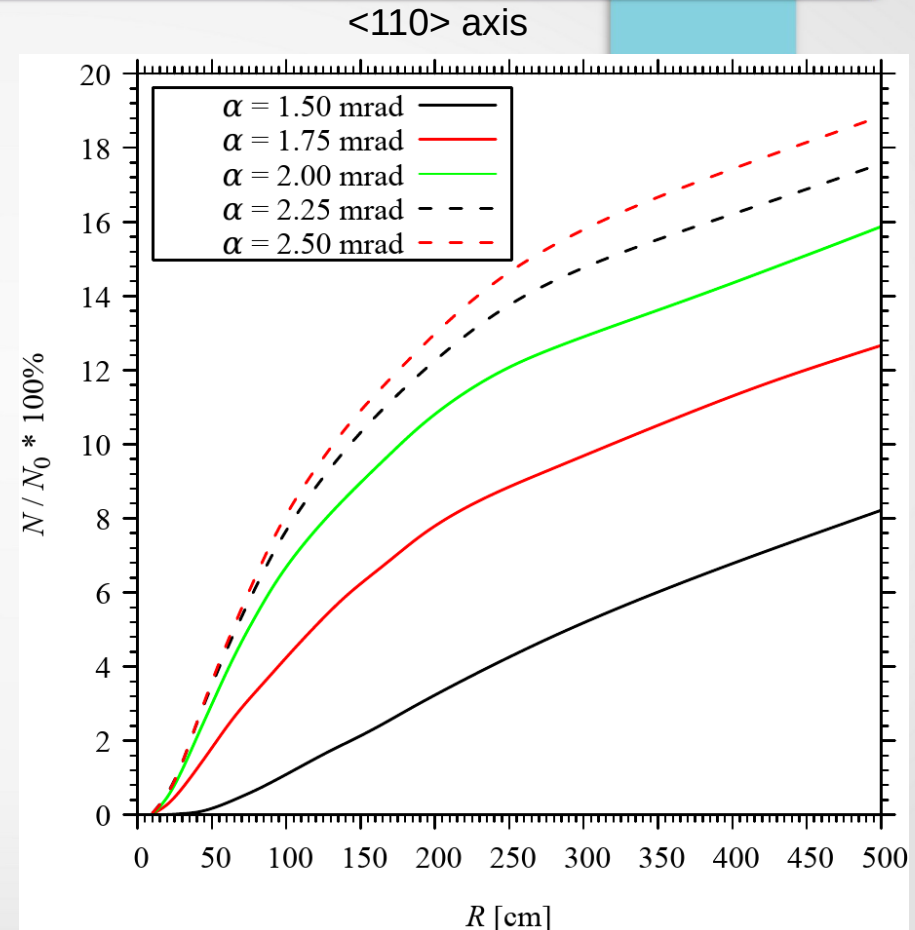
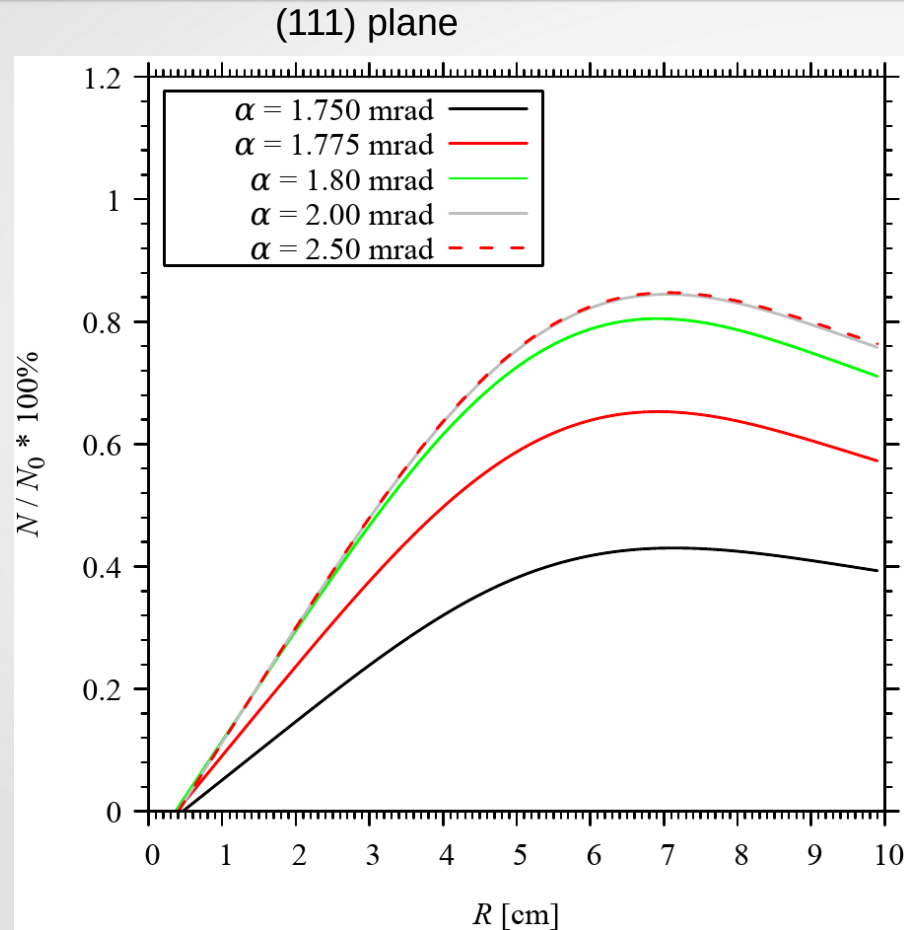
(111) plane



<110> axis



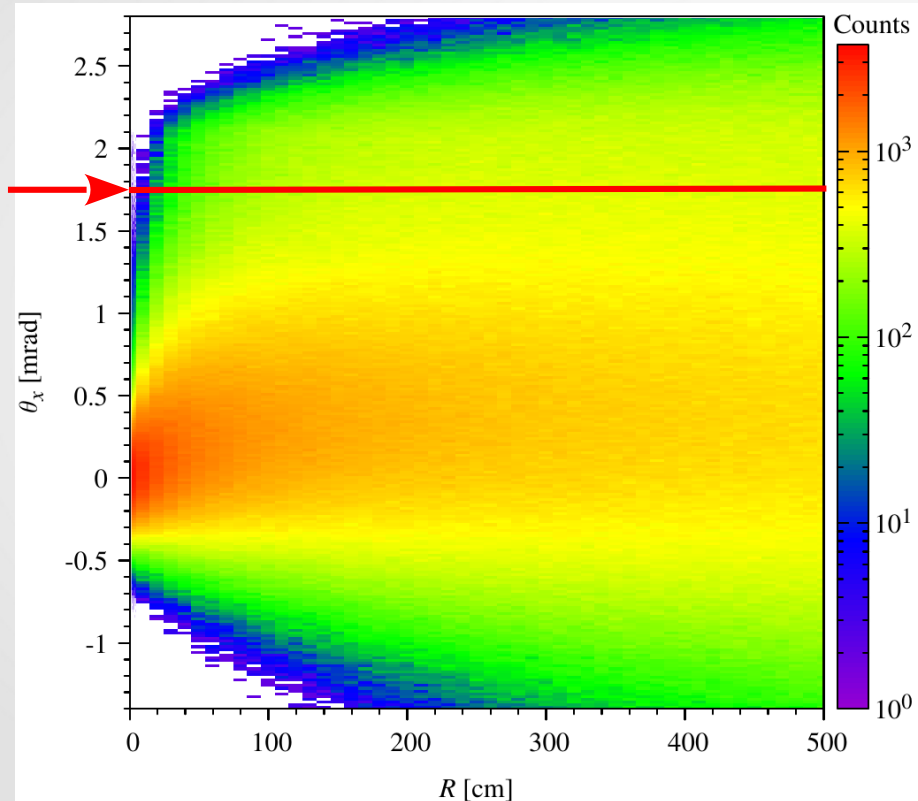
- Deflection efficiency. $\theta_x > 1.75$ mrad



- Deflection efficiency. $\theta_x > 1.75$ mrad

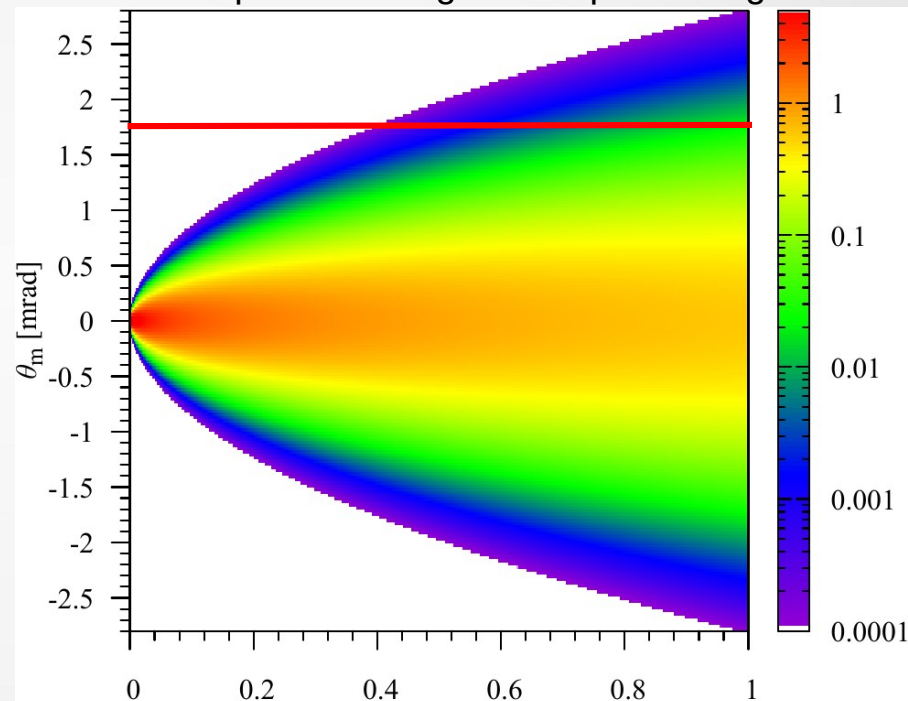
<110> axis

$L = \alpha R$. For $\alpha = 2$ mrad and $R = 5$ m $L = 1$ cm



L [cm]

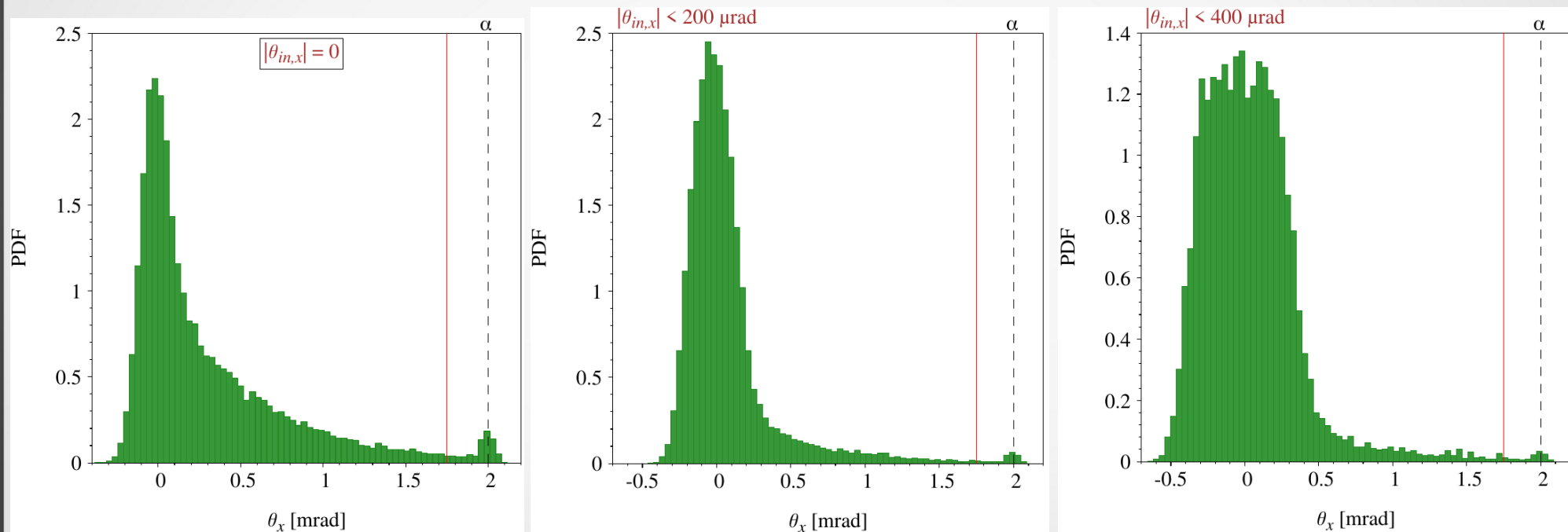
Multiple scattering in amotphous target



$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \sqrt{x/X_0} \left[1 + 0.038 \ln(x/X_0) \right]$$

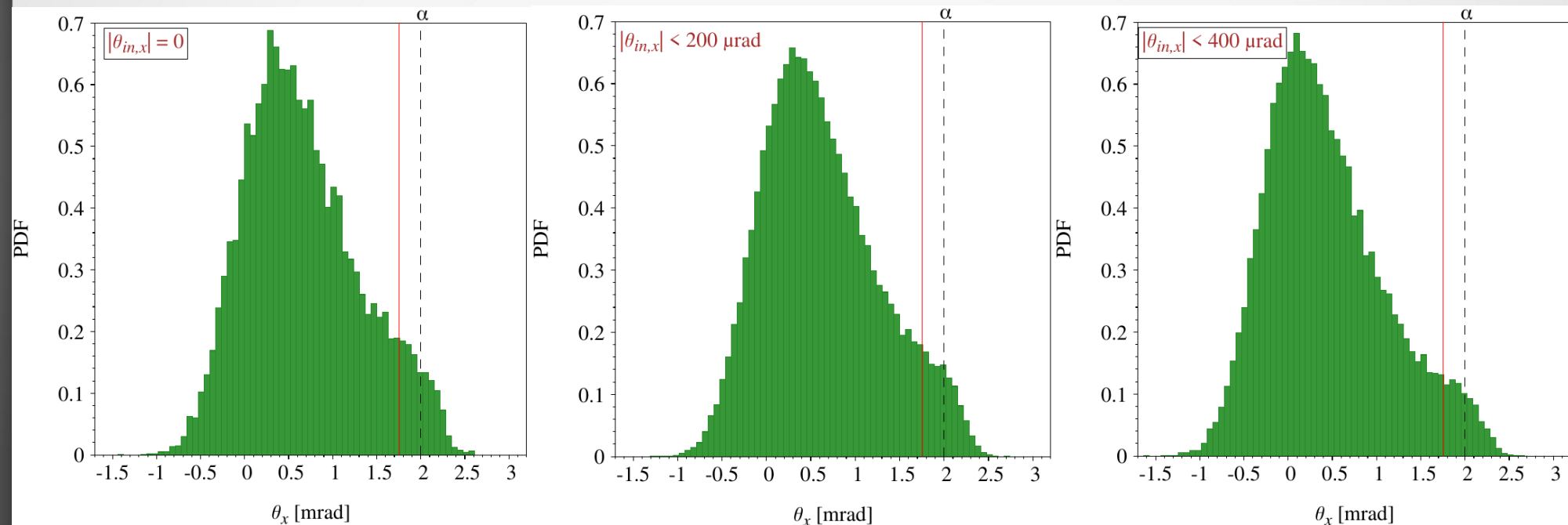
- Initial angular divergence of the beam $\theta_{in,x}$

To study the change in the number of deflected particles from the beam divergence, we assume that the initial angles of particle entry relative to the (111) plane have a uniform distribution. $L = 160 \text{ } \mu\text{m}$, $R = 8 \text{ cm}$.

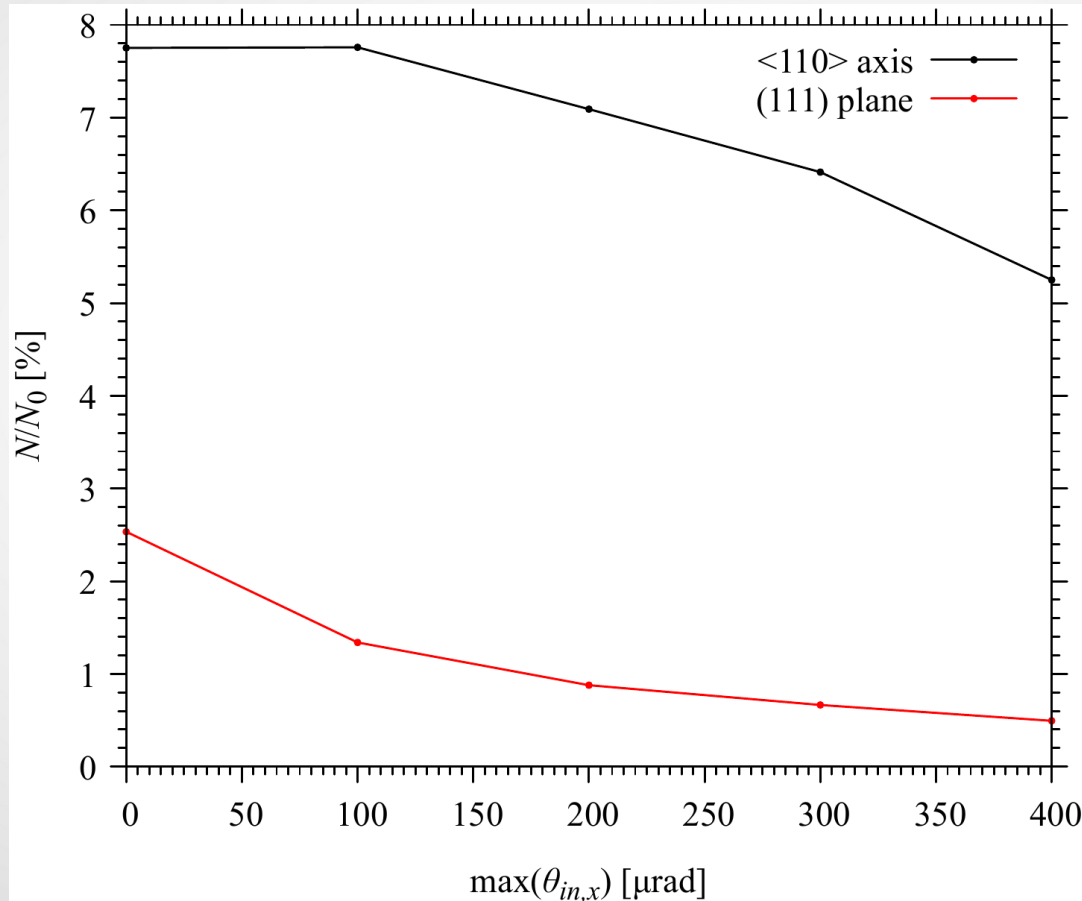


- Initial angular divergence of the beam $\theta_{in,x}$

To study the change in the number of deflected particles from the beam divergence, we assume that the initial angles of particle entry relative to the $\langle 110 \rangle$ axis have a uniform distribution. $L = 2$ mm, $R = 1$ m.



- Initial angular divergence of the beam $\theta_{in,x}$



For axial orientation of the crystal:
 $L = 2 \text{ mm}$, $R = 1 \text{ m}$

For planar orientation of the crystal:
 $L = 160 \mu\text{m}$, $R = 8 \text{ cm}$